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QCD Sum Rule at Finite Temperature

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To my parents

Abstract

We propose a new type of QCD sum rule which is valid even at finite temperature. This sum rule gives us knowledge about temperature dependence of vacuum condensate in quantum chromo dynamics. We apply this sum rule to charmonium and F-meson channels. Through these channels we can investigate temperature dependence of gluon condensate and nature of these mesons at finite temperature ($\leq$ 130 MeV).

The values of vacuum condensates should be channel independent. We find quantitative agreement of the temperature dependence of gluon condensate and verify this universality in these channels. Based on this universality, the behaviour of masses, widths and thresholds of these mesons is discussed at finite temperature.

Contents

§1. Introduction

It is considered that the fundamental theory of strong interaction is quantum chromo dynamics (QCD). The QCD theory describes interaction between colored quarks which are considered as particles constructing hadrons. There are many evidences which support the hypothsis of quark, and it explains experimental facts very systematically. But they appear only as some combinations which form colorless states and do not come out as a single quark. This is so-called confinement of quarks. It is one of important problems in current physics. Enormous efforts have been devoted to derive this confined structure of quarks from this fundamental QCD theory.

The confinement of quarks means that asymptotic one particle state of single quark can not exist in the real world and perturbation theory based on asymptotic expansion does not work well in QCD. In other words, true QCD vacuum deviates from usual perturbative one. As a result of this fact, vacuum expectation values of various operators do not vanish, and we can investigate the structure of true QCD vacuum by these quantities. Especially the gluon condensate $\langle G_{\mu\nu}G_{\mu\nu}\rangle$ plays an important role in confinement[1] and the chiral condensate $\langle\bar{\psi}\psi\rangle$ has relation with dynamical quark mass.[2),3)] There have been several approaches to non-perturbative QCD, i.e., confined perturbation theory,[4] lattice gauge theory,[5] instanton approach[6)~8)] etc. The QCD sum rule is one of these methods to analyze the non-perturbative aspects of QCD, and determines these vacuum expectation values from experimental data.

On the other hand, the analysis of QCD at finite temperature gives another point of view on this confinement problem. Monte Carlo calculation is the most powerful method in this field.[9] It suggests strongly that hadrons melt into quarks and gluons at very high temperature and/or very high density, and the phase transition from hadron phase to quark-gluon phase does occur. At such a phase transition point, gluon condensate is expected to vanish and quarks will be deconfined. There is possibility that such a high temperature and density state can be obtained in ultra-relativistic heavy ion collisions and is called quark gluon plasma (QGP).[10] Experimental researches for such a new phase have started recently and are a current topic. Moreover if vacuum condensates have temperature dependence as expected from Monte Carlo simulations, hadrons will show change of nature at finite temperature through this dependence. We consider it is probable that the hadron physics under environment of very high temperature and density before the transition also has rich contents. From these points of view, if we find some way to apply QCD sum rule at finite temperature, we will be able to derive knowledge about the phase transition or hadron physics at finite temperature. We discuss this possibility in this paper.

The pioneering work on QCD sum rule was done by M.A. Shifman, A.I. Vainstein and V.I. Zakharov in 1979.[11] Since then it has been applied in various ways and made a great success in the study of non-perturbative nature of QCD and resonance states of hadrons.[12)∼14)] QCD sum rule is a semi-phenomenological approach to derive vacuum expectation values of QCD from experimental data. It is based on the combination of dispersion relation and optical theorem. We consider a dispersion relation for invariant form factor of vacuum expectation value of time ordered current-current correlator :

$$\Pi(Q^2) = \frac{1}{\pi} \int ds \, \frac{\mathrm{Im}\Pi(s)}{s + Q^2} \quad .$$

The $\mathrm{Im}\Pi(s)$ on the right hand side (hadron side) can be related to cross section corresponding to the current through optical theorem and we can input experimental data . We calculate $\Pi(Q^2)$ on the left hand side (QCD side) by the use of QCD with operator product expansion (OPE).[15)] The OPE method decomposes an arbitrary operator to the products of Wilson coefficient and local operator and enables us to treat confined region and asymptotically free region separately. In QCD we have $G^a_{\mu\nu} G^a_{\mu\nu}$ and $\bar{\psi}\psi$ etc. as these local operators. We can determine vacuum expectation values of these operators requiring the agreement of both hand sides of the dispersion relation. Thus QCD sum rule determines universal vacuum expectation values of local operators from experimental data.

The first trial to extend this sum rule to finite temperature was done by A.I. Bochkarev and M.E. Shaposhnikov in

1984.[16] Their extension was based on the idea of the sum rule for spectral density. They compaired theoretical spectral density calculated in QCD and hadronic one integrating them with an appropriate weight function. But their calculation is only perturbative one and both hand sides of dispersion relation do not saturate each other. So constant non-perturbative effect was introduced to adjust the sum rule at zero temperature and fixed it at this value. The parameters in this sum rule were mass, decay constant and threshold. They applied this sum rule to ρ meson channel with increasing temperature and determined temperature dependence of these parameters. They discussed the phase transition by the criterion that it takes place when the threshold becomes lower than squared resonance mass. Thier result on critical temperature was 130 MeV, which is rather low compared with the result of lattice calculation.

We consider their argument contains a problem. If condensates have no temperature dependence, it is inconsistent with the above deconfinement picture that it takes place when gluon condensate vanish. Is the assumption of temperature independence of non-perturbative effects correct?

We propose a new type of QCD sum rule which is consistent with our confinement picture. We extend the sum rule by Shifman et al. to finite temperature to take non-perturbative efects into account. It can be done by replacing the vacuum expectation in time ordered current-current correlator by Gibbs ensemble average. The noted points by this substitution are : 1) Wilson coefficients should be calculated in terms of finite temperature perturbation theory and 2) there is no experimental data input to $\mathrm{Im}\Pi(s)$. There are two kinds of perturbation theory at finite temperature, i.e., thermofield dynamics and Matsubara's method. We adopt Matsubara's method[17)∿19)] to calculate Wilson coefficients, which gives the correct values at discrete points on the imaginary energy axis ($q_0(n) = 2\pi in/\beta$). As for hadron side, we use resonance continuum model to parametrize $\mathrm{Im}\Pi(s)$. To cure the lack of experimental data, the physical parameters (masses, widths) in the resonance continuum model are related to vacuum condensates by some models. Thus we have dispersion relation at finite temperature on $q_0(n)$. If we can perform analytic contination of Wilson coefficients from points $q_0(n)$, sum rule for spectral density proposed by Bochkarev and Shaposhnikov can be applicable. But the analytic continuation is difficult in Matsubara's scheme except C_I (see (2·4)). So we consider the discrete mode sum of this dispersion relation with some weight functions. This is our new proposal of discrete mode sum rule. It allows us to determine temperature dependence of vacuum condensate with appropriate choice of weight functions.

In this paper, we concentrate on the analysis of gluon condensate, which is one of key condensates of true QCD vacuum, and hadron physics at finite temperature. We apply our sum rule to charmonium channel first. Because of heavy mass of charm quark we can extract gluon condensate as a dominant vacuum condensate in this channel. We investigate temperature dependence of gluon condensate. Next we study F-meson channel. F-meson is considered as the bound state of charm and strange quark. Strange quark is also supposed to be heave enough to neglect other condensates, we can investigate gluon condensate in this channel too. Gluon condensate is a vacuum structure of QCD and should be channel independent. Analysis of different channels allows us to study this universality. Based on this universality, quantitative estimation of temperature dependence of masses, widths and thresholds of charmonium and F-meson is obtained. These are our main results.

In §2. we propose "discrete mode sum rule," and consider some usage of this sum rule. Application to charmonium is also mentioned in this section, which shows the prototype of the way how our sum rule works. In §3. we describe the application to F-meson channel. The summary and the discusions are in §4. Appendix A is the explanation about the sum rule at zero temperature and the techniques used in it, which is also useful at finite temperature. Appendix B contains the explanation of sum rule for spectral density by Bochkarev and Shaposhnikov and some discussions. We show the method used to calculate Wilson coefficient of gluon condensate in Appendix C. Appendix D is devoted to semi-relativistic potential model used in F-meson analysis.

The §2. is a work in collaboration with Professor K.Hirose, Professor T.Kanki and Dr. O.Miyamura. The §3. is my contribution.

§2. Proposal of QCD sum rule at finite temperature

QCD sum rule at finite temperature is also based on the combination of dispersion relation and optical theorem as in the zero temperature case. The difference is to consider Gibbs ensemble average instead of vacuum expectation value. The current-current correlator is defined as

$$\Pi_{\mu\nu}(x-y) = \langle\langle\, T\, j^a_\mu(x)\, j^b_\nu(y)\, \rangle\rangle$$

$$= \mathrm{Tr}\, \exp(-\beta(H-\mu N))\, T\, j^a_\mu(x)\, j^b_\nu(y) \,/\, \mathrm{Tr}\, \exp(-\beta(H-\mu N)) \,, \tag{2·1}$$

where H is the hamiltonian, N is number operator, β is temperature inverse and μ is chemical potential. It is decomposed into a tensor part and an invariant form factor. In momentum space we have

$$\Pi_{\mu\nu}(q) = T_{\mu\nu}(q)\, \Pi(Q^2) \,, \tag{2·2}$$

where we set $Q^2 = -q^2$ for later convenience. The dispersion relation for $\Pi(Q^2)$ is the following :

$$\Pi(Q^2) = \frac{1}{\pi}\int_0^\infty ds\, \frac{\mathrm{Im}\Pi(s)}{s + Q^2} \,. \tag{2·3}$$

The OPE is a short distance expansion with respect to x-y. It is independent of Gibbs ensemble average and also valid at finite temperature. We apply OPE to the time ordered product of currents (the inside of << >> in (2·1)) in combination with the usual perturbative expansion (see Appendix A). As OPE decomposes operator into products of a Willson coefficient C_i and a normal ordered local operator O_i, we have

$$C_I \langle\langle I \rangle\rangle + \sum_i C_i \langle\langle O_i \rangle\rangle = \frac{1}{\pi} \int_0^\infty ds \, \frac{\mathrm{Im}\Pi(s)}{s + Q^2} , \qquad (2\cdot 4)$$

where I is the unit operator. The Wilson coefficients can be calculated perturbatively and have relation with short distance structure of the theory and Gibbs ensemble average of local operators reflect non-perturbative effects at large distance.

To study vacuum expectation values of QCD, we need some modification of (2·4). Because QCD has asymptotically free nature, usual perturbative part C_I and continuum part in $\mathrm{Im}\Pi(s)$ saturate each other. The relation (2·4) holds almost trivially and we cannot obtain information about vacuum expectation values by direct comparison of both hand sides. We have to find some method to enhance resonance region in $\mathrm{Im}\Pi(s)$ where the information about condensates is rich. We can notice easily there are two ways of such method in the analogy of sum rule at zero temperature. One is the "moment" sum rule which compares the derivatives with respect to Q^2 at the origin, which was adopted by Shifman et al. The other one is the "continuous weight function" sum rule which compares both hand sides after the integration with respect to Q^2 multiplying an appropriate weight function which enhances resonance region. Both methods require the knowledge of Wilson coefficient on the whole complex energy plane. It is true that thermo field dynamics gives a way to calculate these quantities at finite temperature, but the calculation is so complicated and impractical at higher order.

Then we propose a new type of sum rule at finite temperature :

(Discrete Mode Sum Rule)

$$\sum_{n=-\infty}^{+\infty} g_n \left(C_I(n)\langle\langle I\rangle\rangle + \sum_i C_i(n)\langle\langle O_i\rangle\rangle \right)$$

$$= \frac{1}{\pi}\int_0^\infty ds \sum_{n=-\infty}^{+\infty} g_n \frac{\mathrm{Im}\Pi(s)}{s + q_0(n)^2} , \qquad (2\cdot5)$$

where we choose momentum rest frame (the frame $\vec{q} = 0$ is also adopted in what follows). The g_n and $C_i(n)$ represent their values on $q_0(n)$ and the later quantities can be calculated by Matsubara's method. This type of sum rule reduces the complexity of calculation very much.

It is not always allowed to exchange the order of integration and summation on the hadron side. There is some restriction to weight function g_n. The criterion to exchange them is given by following Hardy's theorem :

If $\sum_{n=-\infty}^{+\infty} \int_0^\infty dx \mid g_n(x) \mid$ or $\int_0^\infty \sum_{n=-\infty}^{+\infty} \mid g_n(x) \mid$

converges, $$\sum_{n=-\infty}^{+\infty} \int_0^\infty dx\, g_n(x) = \int_0^\infty \sum_{n=-\infty}^{+\infty} g_n(x) . \qquad (2.6)$$

In the real applications, we choose weight fuctions which satisfy this theorem.

2.1 Usage of discrete mode sum rule

In this section, we discuss several possibilities of discrete weight functions and the parametrization of $\mathrm{Im}\Pi(s)$ at finite temperature. The desirable features expected to these functions are : a) they enhance resonance region and b) they allow analytic calculation as much as possible. We consider next functions as an example :

$$g(n) = \cos(nx) \ , \qquad (2\cdot 7)$$

where x is a parameter. We can perform the mode sum on the hadron side analytically. We find

$$\sum_{n=-\infty}^{+\infty} \frac{\cos(Q_0(n)\cdot x)}{s + Q_0(n)^2} = \left(\frac{\beta}{2\pi}\right)^2 \frac{\cosh[\sqrt{s}(\beta/2-x)]}{\sinh(\sqrt{s}\beta/2)} \ , \qquad (2\cdot 8)$$

where $0 \leqq x \leqq \beta$. This resultant function damps very rapidly as $\exp(-x\cdot s)$ when s tends to infinity. $\mathrm{Im}\Pi(s)$ is considered to tend to a constant as s becomes large because of asymptotically free nature of QCD. So the integration with respect to s on the right hand side of (2·4) converges and Hardy's theorem guarantees the validity of exchanging summation and integration for this weight function. This damping factor becomes strong when x is large. In other words, x plays the role of cut off parameter on the s axis. In general, oscillating weight functions on the imaginary axis are supposed to give damping factors on s and this is the reason why we consider this weight function. Especially in zero temperature limit, the result reduces to $\exp(-x\cdot s)$ and give Laplace transformation of $\mathrm{Im}\Pi(s)$ to the variable x. Our purpose to enhance the resonance region can be achieved by taking appropriate values of x. If x is sufficiently small, (2·4) with this weight functions is saturated by the asymptotic region. But as x becomes large, the weight of the resonance region also increases.

When we use this type of weight function, we must pay attension to the compatibility of this function and OPE. The OPE is one of short distance expansions and it breaks down at large distance. In momentum space, we can not rely on this expansion near the origin. But naive cosine type weight function has large weight for small values of momentum and some device is needed to suppress such a region. Here we consider subtracted cosine type one as follows :

$$g_n(\beta) = \frac{2\pi}{\beta}\left(\cos\left(\frac{2\pi n}{\beta}x\right) - \cos\left(\frac{2\pi n}{\beta}y\right)\right) . \qquad (2\cdot 9)$$

It is obvious that the part near $n = 0$ is suppressed by this weight function. We will use this function in the applications.

Another possible choice is $g_n(\beta) = n \sin(\frac{2\pi n}{\beta} i)$.

Next we describe the treatment of $\mathrm{Im}\Pi(s)$ at finite temperature. We also adopt resonance continuum model in this case. At zero temperature we can use experimental data to parametrize $\mathrm{Im}\Pi(s)$, which enables us to determine vacuum expectation values. But we have no such data at finite temperature. It is of course possible to find solutions for all parameters (condensates, masses, widths and threshold) satisfying the sum rule. But parameters are so many that there still remain some ambiguities. One way to reduce the number of parameters is to use some models which relate these parameters. For example, next combination is one of possible choices in the channels where gluon condensate is a dominant condensate. First using a potential model with linear confining potential, we have masses and widths as functions of string tension. Secondly we consider flux tube model which relates string tension and gluon condensate. Then we get them as functions of gluon condensate. This procedure eliminates mass and width parameters and makes us possible to determine parameters precisely. This combination is used in both charmonium and F-meson analysis.

We require the agreement of both hand sides of (2·5) to determine remaining parameters. When vacuum expectation values about confinement concerning confinement is concerned, the appropriate values of x is larger than the inverse of the resonance masses. These parameters are fixed by adjusting (2·5) to hold in the wide region of x. We can determine higher condensates at larger x. But it is noted that too large x becomes meaningless and the results may depend on model which we take in parametrizing $\mathrm{Im}\Pi(s)$. Such a simple parametrization as resonance-continuum model cannot include information about higher condensate.

2.2 Analysis of charmonium

We apply our new sum rule at finite temperature to analyze charmonium channel.[20] We consider current-current correlator for vector current (Fig.1a):

$$j_\mu(x) = \bar{c}(x)\, \gamma_\mu\, c(x) \; . \tag{2.10}$$

Charm quark mass is about 1.5 GeV and so heavy that the pair creation from vacuum owing to vacuum fluctuation or absorption of quark-antiquark pair into vacuum do not occur. The quark propagator is not modified from free one. In othe words, the vacuum of such a heavy quark does not much deviate from asymptotic vacuum and there are no condensats due to non-perturbative effect. So the problem completely reduces to a free motion of quark in external gluon field which is under non-perturbative situation. Such argument leads us to perturbative expansion of quark propagator and application of OPE for gluon to take non-perturbative effect into account. Then combination of this perturbation and OPE allows us to calculate the right hand side of (2•4) for charmonium. Using technique of Fock-Schwinger gauge[21] and OPE in momentum space, we find next dispersion relation :

$$C_I(Q^2) + C_G(Q^2)\langle\langle G^a_{\mu\nu} G^a_{\mu\nu}\rangle\rangle = \frac{1}{\pi}\int_0^\infty ds \, \frac{\mathrm{Im}\Pi(s)}{s + Q^2} \tag{2•11}$$

in very good approximation neglecting higher condensates.

Here we make a comment on Lorentz covariance of condensates. At finite temperature, Lorentz covariance breaks down generally unless we introduce extended concept of 4-dimensional Lorentz covariant temperature. The origin of this breakdown consists in the fact that the frame is chosen to make the thermal medium at rest. We also chose this frame, so the local operators which appear in OPE are not necessary Lorentz invariant. This means the gluon condensate does not nees to have the form $\langle\langle G^a_{\mu\nu} G^a_{\mu\nu}\rangle\rangle$. Colour electric condensate $\langle\langle G^a_{0i} G^a_{0i}\rangle\rangle = \langle\langle E^a E^a\rangle\rangle$ and clour magnetic one $\langle\langle G^a_{ij} G^a_{ij}\rangle\rangle = \langle\langle B^a B^a\rangle\rangle$ can condense independently with different Wilson coefficients. This separation have relation with the tensorial form of gluon propagator. It has no Lorentz covariance anymore and keeps only O(3) invariance, which allows various tensorial components of gluon propagator. But in this paper we use finte temperature version of usual Lorentz covariant form of gluon propagator. This is the reason why we have Lorentz invariant $\langle\langle G^a_{\mu\nu} G^a_{\mu\nu}\rangle\rangle$ in (2•11). Of course it may be more complicated in the actual situation at finite temperature. But we expect that $\langle\langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle\rangle$ plays an important role in confinement even at finite temperature. Independent analysis of $\langle\langle E^a E^a \rangle\rangle$ and $\langle\langle B^a B^a \rangle\rangle$ requires more strict calculation.

We show results of calculation of Wilson coefficients. We consider C_I up to two loop order. It is convenient to decompose these coefficients into two part,

$$C_W = C_W + \Delta C_W \; , \tag{2$\cdot$12}$$

where C_W is an any Wilson coefficient and C_W is a contribution from zero temperature and ΔC_W is one from finite temperature.

C_I at one loop level (C_I^0) can be derived easily. It is represented by one loop diagram in Fig.2 and the results are as follows:

$$C_I^0 = \frac{1}{4\pi^2} \int_{4m_c^2}^{+\infty} ds \, \frac{1}{s + Q^2} \, \frac{(s + 2m_c^2)\sqrt{s - 4m_c^2}}{\sqrt{s}^3} \tag{2$\cdot$13a}$$

$$\Delta C_I^0 = \frac{1}{4\pi^2} \int_{4m_c^2}^{+\infty} ds \left(\frac{1}{s + Q^2} - \frac{1}{Q^2} \right) \left(\tanh(\beta\sqrt{s}/4) - 1 \right) \times \frac{(s+2m_c^2)\sqrt{s-4m_c^2}}{\sqrt{s}^3} \; . \tag{2$\cdot$13b}$$

C_I at two loop level have contributions from three diagrams in Fig.3 and we have[11)]

$$C_I^1 = \frac{1}{24\pi^2} \int_{4m_c^2}^{+\infty} ds \, \frac{1}{s + Q^2} \, (3-v^2) \left(2\pi - \frac{2\pi^2-3}{4\pi} \, v(v+3) \right) \tag{2$\cdot$14}$$

where $v = (1 - 4m_c^2/s)^{1/2}$. But ΔC_I^1 requires a two loop calculation and is very complicated. We have estimated this quntity numerically and find its contribution is negligible.

As for C_G term (Fig.4), Shifman et al. obtained C_G as follows :

$$C_G = \frac{\alpha}{48\pi Q_0^2} \left(\frac{3(a+1)(a-1)^2}{a^2} \frac{1}{2\sqrt{a}} \ln \left(\frac{\sqrt{a}+1}{\sqrt{a}-1} \right) - \frac{3a^2-2a+3}{a^2} \right) . \tag{2·15}$$

ΔC_G is given after lengthy calculation and the result for total C_G is as follows :

$$C_G = \frac{\alpha}{36} \frac{1}{Q_0^4} \int_0^\infty dt \quad f_1 \frac{1}{T(r)^5} \frac{\tanh(\pi\hat{m}\sqrt{1+t^2})}{\sqrt{1+t^2}}$$
$$+ f_2 \frac{1}{T(r)^4} \frac{\pi\hat{m}}{\cosh^2(\pi\hat{m}\sqrt{1+t^2})} + f_3 \frac{1}{T(r)^3} \frac{(\pi\hat{m})^2}{\sqrt{1+t^2}} \frac{\tanh(\pi\hat{m}\sqrt{1+t^2})}{\cosh^2(\pi\hat{m}\sqrt{1+t^2})}$$
$$+ f_4 \frac{1}{T(r)^2} \frac{(\pi\hat{m})^3}{\cosh^2(\pi\hat{m}\sqrt{1+t^2})} \left(1 - \frac{3}{2\cosh^2(\pi\hat{m}\sqrt{1+t^2})} \right) \tag{2·16}$$

where

$$f_1 = -1024(3r-10)t^8+512(3r^2+6r+34)t^6-64(15r^3-60r^2-288r-16)t^4$$
$$-32(47r^3+6r^2-236r+288)t^2-32(10r^3+71r^2+148r+96) ,$$

$$f_2 = 12r(r+2)(r+3)(r+4) , \quad f_3 = 2r(r+2)(r+4) , \quad f_4 = 2r(r+4) \tag{2·17}$$

with $T(r) = 4t^2+4+r$, $r = Q_0^2/m^2$ and $\hat{m} = \beta m/2\pi$.

Next we consider the relation between gluon condensate and resonance masses and widths. In heavy quarkonium, non-relativistic potential models have made remarkable success.[22] There are several potentials which reproduce mass spectrum. We use a refined version of such potentials, which is proposed by Buchmüller and Tye,[23]

$$V(r) = -\frac{3}{4}\frac{\alpha(r)}{r} + kr , \tag{2·18}$$

where the k is string tension and $\alpha(r)$ is a running coupling constant motivated from QCD and defined as

$$\alpha(r) = \frac{8}{b_0}\int_0^\infty dt\ \frac{\sin(\Lambda tr)}{t}\left(\frac{1}{\ln(1+t^2)} - \frac{1}{t^2}\right) , \tag{2·19}$$

where $b_0 = 9$ and $\Lambda = 250$ MeV. We have neglected spin-spin interaction term. Variational method with this potential gives the masses of charmonium. The variational criterion is

$$\delta \langle\psi_{trial}|\ -\frac{p^2}{2m} + V(r)\ |\psi_{trial}\rangle = 0 . \tag{2·20}$$

We have used Gaussian trial function with extension parameter, which is used as the variational parameter. Varying k, we have charmonium masses as a function of the string tension. As for the widths, we use next relation :

$$|\psi(0)|^2 = \frac{M}{2\pi}\langle\psi_{sol}|\ \frac{dV(r)}{dr}\ |\psi_{sol}\rangle , \tag{2·21}$$

where ψ_{sol} is the solution of (2•20), because direct value of $\psi(0)$ is not so reliable, though once integrated this relation is rather correct. Leptonic decay width is proportional to this quantity, so it is also a function of the string tension. This is the first step.

Secondly we relate this string tension and gluon condensate. We use the relation motivated from flux-tube model.[24] In this model, we suppose that color-electric flux lines from quark to anti-quark are squeezed into a flux tube. In this picture energy has contributions from the vacuum energy and the field energy.

$$E_{total} = E_{vac.} + E_{field} \qquad (2\cdot22)$$

with

$$E_{vac.} = BV = B\sigma r \ , \qquad (2\cdot23a)$$

$$E_{field} = \frac{E^2}{8\pi} V = \frac{2\pi Q^2}{\sigma} r \ , \qquad (2\cdot23b)$$

where B is the bag constant, σ is the cross section of the flux tube and $E = \frac{4\pi Q}{\sigma}$. The effect of $E_{vac.}$ will be to compress the flux lines as much as possible and E_{field} has opposite effect. These effects are balanced at energy minimum, i.e., $\frac{\partial E}{\partial \sigma} = 0$ and we have $\sigma = Q\sqrt{2\pi/B}$. At this cross section, total energy is thus

$$E = 2Q\sqrt{2\pi B}\; r \; . \tag{2·24}$$

The bag constant can be related with gluon condensate as follows :

$$B = -\varepsilon_{vac} = -\frac{1}{4}\langle 0|T_{\mu\mu}|0\rangle$$

$$= \frac{b\alpha}{96\pi}\langle G^a_{\mu\nu}G^a_{\mu\nu}\rangle \quad , \tag{2·25}$$

with $b = 11N_c - 2N_f = 27$ (for $N_c = N_f = 3$). For quark and anti-quark, we have $Q^2 = \frac{4}{3}\alpha$. Substituting these results for the bag constant and charge, we obtain final relation between the string tension and the gluon condensate.

$$k = \sqrt{3}\cdot\alpha\,\langle\langle G^a_{\mu\nu}G^a_{\mu\nu}\rangle\rangle \; . \tag{2·26}$$

Then this completes the relation between the resonance masses and widths and the gluon condensate, and also our sum rule at the same time. We show the dependence of charmonium mass and its leptonic decay width on these parameters in Fig.5a and Fig.5b.

To check the reliability of our new sum rule, we investigate whether it reproduces the value of gluon condensate obtained by Shifman et al. in the zero temperature limit. We parametrize $\mathrm{Im}\Pi(s)$ using the resonance-continuum model with experimental data :

$$J/\psi \ : \ M_{J/\psi} = 3095 \text{ MeV} \ , \quad \Gamma^{e^+e^-}_{J/\psi} = 4.50 \text{ keV} \ ,$$

$$\psi' \ : \ M_{\psi'} = 3684 \text{ MeV} \ , \quad \Gamma^{e^+e^-}_{\psi'} = 1.95 \text{ MeV} \quad \text{etc.}$$

and

$$S = (4.2 \text{ GeV})^2 \ . \qquad (2\cdot 27)$$

As for QCD side, we take

$$m_c = 1.41 \text{ GeV} \ , \quad \alpha = 0.24 \ , \qquad (2\cdot 28)$$

following Shifman et al. In Fig.6a and 6b, we can see contributions from each component of QCD side and hadron side, respectively. The contribution of gluon condensate becomes dominant as x becomes larger. We define L/R-ratio as the ratio of left and right hand side. It should be unity if sum rule holds well. We show in Fig.7, the result of this quantity versus cut off parameter x in two cases : 1) $\langle\langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle\rangle = 0$, 2) $\langle\langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle\rangle = 0.1884 \text{ GeV}^4$. L/R-ratio begins to deviate at small x in the case 1). We can see that $C_G \langle\langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle\rangle$ term in the left hand side gives negative effect and the value in case 2) keeps L/R-ratio unity in wider range in x and improves the sum rule very much. This shows that our sum rule is compatible with that of Shifman et al., and we have verified it works well at zero temperature.

We suppose the deviation of L/R-ratio from unity in the region where x is larger than 5 GeV^{-1} occurs because higher order condensates become important in such a region. Shifman et al. used "moment" sum rule which is completely independent our sum rule. This compatibility strongly supports the reliability of our sum rule.

We proceed to finite temperature case. Fig.8 shows L/R-ratio for parameter values fixed at zero temperature in several finite temperature cases. Horizontal axis is the same cut off parameter x. When temperature is between 0 and 50 MeV, sum rule holds well. But it begins to break down at 70 MeV and L/R-ratio becomes worse in the region where x is larger than 2 GeV^{-1} as temperature becomes higher than that. This shows that some modification of parameters is needed to keep the sum rule and we can see they have temperature dependence. We plot L/R-ratio for various values of gluon condensate at T = 100 MeV in Fig.9. The sum rule is recovered as gluon condensate becomes smaller. Searching the best fit, we can determine the value of gluon condensate at this temperature, i.e.,

$$\langle\langle G_{\mu\nu} G_{\mu\nu} \rangle\rangle = 0.14\ \mathrm{GeV}^4 \quad (T = 100\ \mathrm{MeV})\ . \qquad (2\cdot 29)$$

In Fig.10, we show the temperature dependence of gluon condensate. It decrease gradually as temperature becomes high. It becomes 80% of its zero temperature value at T = 100 MeV. From this dependence, we can derive temperature dependence of masses and widths. We show the results in Fig.11a and 11b. J/ψ mass shifts toward lower side by about 30 MeV and ψ' mass does about 80 MeV at $T = 100$ MeV, i.e.,

$$M_{J/\psi} = 3065 \text{ MeV} ,$$

$$M_{\psi'} = 3600 \text{ MeV} \qquad (T = 100 \text{ MeV}) . \qquad (2\cdot30)$$

We can also see that the decrease of width is 20% at the same temperature, i.e.,

$$\Gamma^{e^+e^-}_{J/\psi}(T = 100 \text{ MeV}) = 0.8\, \Gamma^{e^+e^-}_{J/\psi}(T = 0 \text{ MeV}) . \qquad (2\cdot31)$$

In next section, we consider F-meson channel.

§3. Application to F-meson channel

We apply our QCD sum rule at finite temperature to F-meson channel. As shown in §2., we have applied the sum rule to charmonium channel and investigated the temperature dependence of gluon condensate. The analysis of F-meson channel also tells us about this dependence. As gluon condensate is a nature of the QCD vacuum, it is supposed to be channel independent. Gluon condensate should have this universality, which we can study by the analysis of this different channel.

In charmonium case, we can parametrize $\mathrm{Im}\Pi(s)$ on the hadron side by its leptonic decay width through vector current. In F-meson channel, we parametrize the imaginary part by the decay constant f_F. A differnt point in these cases is a lack of experimental value of f_F. Then first we determine its zero temperature value by the use of parameters fixed in charmonium analysis. After that applying our sum rule at finite temperature, we investigate temperature dependence of gluon condensate in this channel and discuss about the universality of this dependence.

In this channel, another difficulty arises in potential model. Non-relativistic potential model have works well and reproduces the mass spectrum of charmonium rather precisely. We have applied this model to s-$\bar{s}$ system, but we cannot obtain correct ϕ-meson mass. It shows that non-relativistic approximation breaks down bacause strange quark mass is not sufficiently heavy. We have to take relativistic effect into account. This is also the case for F-meson. In the F-meson, heavy charm quark exists near the center almost at rest and the strange quark forms a relativistic orbit around the charm quark. The general framework to treat such semi relativistic atom like mesons was developed by Morishita et al. The basic idea is to perform non-relativistic approximation only to heavy quark starting from fully relativistic theory. We adopted their method to derive the string tension dependence of mass and the value of the wave function at the origin which have relation with the decay constant.

3.1 Sum rule in F-meson channel

First we explain the QCD side of the dispersion relation in F-meson channel. The main decay mode of F-meson is $F \to \phi \pi$. It suggests that F-meson is composed of charm and strenge quark ($c\bar{s}$ or $\bar{c}s$). Generally the lowest state in any channel should have the same quantum numbers of the current. So the corresponding current is a pseudo-scalar one (Fig.2b):

$$j_5(x) = \frac{1}{2}\left(c(x)\gamma_5\bar{s}(x) + \bar{c}(x)\gamma_5 s(x)\right) . \qquad (3\cdot1)$$

In charmonium case, the assumption that there is no $\langle c\bar{c}\rangle$ condensate in QCD vacuum is completely justified because of its heavy mass. In F-meson case, as strange quark is lighter than charm quark, it may seem that the condensate of the strange quark should be taken into account. But strange quark mass is about 150 MeV and still heavy to condense in the vacuum. So we suppose that the assumption that there is no quark condensate is still valid in F-meson channel, though the approximation becomes slightly worse. Under this assumption the same argument and method in §2. lead us to the dispersion relation :

$$C_I(Q^2) + C_G(Q^2)\langle\langle G^a_{\mu\nu}G^a_{\mu\nu}\rangle\rangle = \frac{1}{\pi}\int_0^\infty ds\,\frac{\mathrm{Im}\Pi(s)}{s+Q^2} \tag{3・2}$$

which has same form as before and Wilson coefficients are calculated by the current (3・1).

If we treat D-meson, which is composed of charm and u,d quarks, such assumption breaks down since u,d quarks condence and have non-vanishing vacuum expectation values.

We summerize the results for Wilson coefficients in the following. In F-meson channel, the tensor structure is simpler than that of charmonium but the mass differnce makes the calculation and results complicated especially in higher order. Using the notation as before, C_I^0 can be calculated from a diagram in Fig.12. The result is

$$C_I^0 = \frac{1}{\pi}\int_{s_0}^{+\infty} ds\,\frac{1}{s+Q^2}\,\frac{3}{8\pi}\,\frac{\bar{s}^2}{s}\,v\;, \tag{3・3}$$

where $\bar{s} = s - (m_1-m_2)^2$, $v^2 = 1 - 4m_1m_2/\bar{s}$. ΔC_I^0 is given by calculating the same diagram by the use of Mastubara's method and we have

$$\Delta C_I^0 = \frac{3}{2} \int_0^{+\infty} \frac{4\pi k^2 dk}{(2\pi)^3} \frac{1}{\exp(\beta\sqrt{k^2+m_1^2}) + 1}$$

$$\times \frac{-4\left(Q^2(2k^2+ m_1(m_1+ m_2)) + m_1(m_1+ m_2)(m_1- m_2)^2\right)}{\sqrt{k^2+ m_1^2} \quad \left((Q^2- m_1^2+ m_2^2)^2 + 4Q^2(k^2+ m_1^2)\right)}$$

$$+ (m_1 \leftrightarrow m_2) . \tag{3.4}$$

on $Q_0(n) = 2\pi n/\beta$. In the equal mass case, this reduces to usual dispersion type but the mass inequality forbids such representation. C_I^1 is expressed by three diagrams in Fig.13 and is rather complicated.[25)]

$$C_I^1 = \frac{1}{\pi} \int_{s_0}^{+\infty} ds \frac{1}{s + Q^2} \frac{3}{8\pi} \frac{\bar{s}^2}{s} v \frac{4}{3} \frac{\alpha}{\pi v}$$

$$\times \Big\{ (1+v^2) \Big(\frac{\pi^2}{6} + \ln(\frac{1+v}{1-v}) \ln(\frac{1+v}{2})$$

$$+ 2l(\frac{1-v}{1+v}) + l(\frac{1+v}{2}) - l(\frac{1-v}{2})$$

$$+ \frac{1}{2} \Big(-4l(v_1) + l(v_1^2) + l(\frac{1+v_1}{2}) - l(\frac{1-v_1}{2})$$

$$-4l(v_2) + l(v_2^2) + l(\frac{1+v_2}{2}) - l(\frac{1-v_2}{2}) \Big) \Big)$$

$$+ \left(\frac{19}{16} - 3v + \frac{1}{8}v^2 + \frac{3}{16}v^4 \right) \ln\left(\frac{1+v}{1-v}\right)$$

$$+ \frac{29}{8}v - \frac{3}{8}v^2 + 6v \ln\left(\frac{1+v}{2}\right) - 4v \ln(v)$$

$$+ M(v,v_1,v_2) \quad \Big\} \tag{3$\cdot$5a}$$

where

$$M(v,v_1,v_2) = \frac{1}{2}(1+v^2) \left(\ln\left(\frac{1+v_1}{1-v_1}\right) \ln\left(\frac{v_1}{v}\right) + \ln\left(\frac{1+v_2}{1-v_2}\right) \ln\left(\frac{v_2}{v}\right) \right)$$

$$+ \frac{1}{2} v \left(\left(\frac{1}{v_1} - \frac{1}{v}\right) \ln\left(\frac{1+v_1}{1-v_1}\right) + \left(\frac{1}{v_2} - \frac{1}{v}\right) \ln\left(\frac{1+v_2}{1-v_2}\right) \right)$$

$$+ 2 v \ln(q/\bar{q}) + \frac{3}{2} v \ln(m_1/m_2)$$

$$+ v \left(\frac{(m_1-m_2)^2}{q^2} v \ln\left(\frac{1+v}{1-v}\right) - \frac{m_1^2 - m_2^2}{q^2} \ln(m_1/m_2) \right) , \tag{3$\cdot$5b}$$

with

$$v_1 = \frac{\bar{s}\ v}{s + m_1^2 - m_2^2} , \qquad v_2 = \frac{\bar{s}\ v}{s - m_1^2 + m_2^2}$$

and

$$l(x) = - \int_0^x dt \frac{\ln(1-t)}{t} .$$

We neglect temperature dependence of C_I^1 (ΔC_I^1) for the present. We have estimated ΔC_I^1 and found it negligible in the charmonium case. We expect the situation is not different in this case. As for C_G , it is given by diagrams in Fig.14. We calculate these diagrams following Yazaki et al. We have to enumerate

$$\frac{2\pi}{\beta} \sum_{-\infty}^{+\infty} \int \frac{d^3k}{(2\pi)^3}$$

$$\mathrm{Tr}\ (\gamma_5 S_F(k)\gamma_\mu S_F(k)\gamma_\rho S_F(k)\gamma_\rho S_F(k)\gamma_5 \hat{S}_F(k')\gamma_\mu \hat{S}_F(k')$$

$$+ \gamma_5 S_F(k)\gamma_\mu S_F(k)\gamma_\rho S_F(k)\gamma_5 \hat{S}_F(k')\gamma_\rho \hat{S}_F(k')\gamma_\mu \hat{S}_F(k')$$

$$+ \gamma_5 S_F(k)\gamma_\mu S_F(k)\gamma_5 \hat{S}_F(k')\gamma_\rho \hat{S}_F(k')\gamma_\rho \hat{S}_F(k')\gamma_\mu \hat{S}_F(k')$$

$$+ \gamma_5 S_F(k)\gamma_\mu S_F(k)\gamma_\rho S_F(k)\gamma_\rho S_F(k)\gamma_\mu S_F(k)\gamma_5 \hat{S}_F(k')$$

$$+ \gamma_5 S_F(k)\gamma_5 \hat{S}_F(k')\gamma_\mu \hat{S}_F(k')\gamma_\nu \hat{S}_F(k')\gamma_\nu \hat{S}_F(k')\gamma_\mu \hat{S}_F(k')\ )\ , \tag{3·6}$$

where S_F and $\hat{S}_F$ are fermion propagator of heavy and light quark respectively, and $k' = k-q$. Analytic calculation is difficult and complicated because of the inequality of masses. We take internal mode sum analytically by the use of mathematical formula and integrate numerically with respect to $|k|$. See Appendix C for further details.

Secondary we mention the semi relativistic potential model used on the hadron side, which enables us to treat strange quark relativistically. This method was developed by J.Morishita, T.Morii and M.Kawaguchi.[26)] Their basic strategy of semi-relativistic method is treating light quark relativistically and heavy quark non-relativistically. Starting from fully relativistic potential model with scalar and vector potential introduced by Fermi and Yang, Foldy transformation is applied for heavy quark. This procedure reduces the number of components by eight and the hamiltonian is expanded in the inverse power series of heavy quark mass. To investigate mass spectrum, we separate the hamiltonian inti two part. One is upto 1st inverse power of heavy quark mass and the other is the remainder. Then we apply many dimensional variational method to the former hamiltonian and treat the latter by perturbation. The trial function is a Gaussian type, which is a product of polynomial and Gaussian weight $\exp(-r^2/(2\lambda^2))$, and the extension parameter λ is taken as the variational parameter. One attention is here in order. The hamiltonian is still partially relativistic and has no energy positivity. They cure this point by the use of virial theorem, which says that the virial should vanish for the stationary solution. Then we vary the extension parameter observing the virial and fix the value when the virial vanish.

We try this method for various values of string tension contained in the scalar potential and obtained mass spectrum as a function of string tension. As for the value of wave function at the origin, the direct value of the trial function is not so reliable that we used a relation which corresponds to (2·21) in charmonium case to determin the value.

In Table 1, we show the value of fitted extension parameter, mass of F-meson, and $4\pi|\Psi(0)|^2$. We can see that as string tension becomes smaller, the extension parameter becomes larger and the wave function becomes broader. Masses gradually decrease as string tension becomes smaller, which shows the same tendency as in the charmonium case. The values of wave function at the origin fluctuate slightly comparing the smooth change of mass, but it also decreases as the string tension becomes smaller. The string tension dependence of mass and $4\pi|\Psi(0)|^2$ are shown in Fig.15a and 15b, respectively.

We introduce the decay constant of F-meson by the following matrix element in the standard way,

$$\langle F|\bar{c}\gamma_\mu\gamma_5 s|0\rangle = -if_F P_\mu . \tag{3·7}$$

The resonance part of $\mathrm{Im}\Pi(s)$ can be expressed by the use of this decay constant as follows :

$$\mathrm{Im}_{res.}\Pi(s) = \frac{\pi}{8} f_F^2 M_F \delta(s-M_F^2) , \tag{3·8}$$

for pseudo scalar current.

Let the light quark be non-relativistic, so that one finds that

$$f_F^2 M_F \propto |\Psi(0)|^2 , \tag{3·9}$$

for ordinary wave function. In what follows, we use this relation for relativistic quarks as well. The quantity $|\Psi(0)|^2$ has been already given as functions of gluon condensate, and we find the decay constant f_F as a function of gluon condensate. As for continuum part, Bochkarev and Shaposhnikov used temperature dependent resonance continuum model requiring the saturation above threshold. But we simply parametrize by the use of asymptotic value of $\mathrm{Im}C_I$ based on the asymptotically free nature of QCD :

$$\mathrm{Im}_{cont.}\Pi(s) = C_I(\infty)\theta(s-S_0) , \tag{3·10}$$

where $C_I(\infty) = \lim_{s\to\infty} \mathrm{Im}C_I(s)$ and S_0 is an effective threshold. This completes the parametrization of our sum rule.

We adopt cosine type one in the choice of weight function. In the actual use of our sum rule, we fix $y = \frac{\beta}{2}$ and we have the following explicit form as our sum rule.

$$\frac{2\pi}{\beta}\sum_{n=1}^{+\infty}\left(\cos\left(\frac{2\pi n}{\beta}\right) - (-)^n\right)\left(C_I(n) + C_G\langle\langle G^a_{\mu\nu}G^a_{\mu\nu}\rangle\rangle\right)$$
$$= \int_0^\infty ds\ \mathrm{Im}\Pi(s)\ \frac{\cosh\left(\left(\frac{\beta}{2}-x\right)\sqrt{s}\right)-1}{\sinh\left(\frac{\beta}{2}\sqrt{s}\right)} , \tag{3·11}$$

where $0 \leqq x \leqq \frac{\beta}{2}$. If we take zero temperature limit, factors $\frac{2\pi}{\beta}(-)^n$ and $1/\sinh(\beta\sqrt{s}/2)$ on left and right hand side respectively tend to zero. This shows that sum rule (3·12) has the same zero temperature limit as one with naive cosine weight function and the meaning of x as cut off parameter does not change.

By the use of some models, we have reduced the number of parameters and finally we have two parameters, i.e., gluon condensate and effective threshold. As x plays a role of dumping parameter on the s axis ($\exp(-x\sqrt{s})$ at T=0), when x is small this factor is not so effective and information contained in $\mathrm{Im}\Pi(s)$ at large s is enhanced in the sum rule. On the other hand, when x is large, this factor becomes effective and the weight on the resonance region increases. So the non-perturbative effect appears in the large x region. As for our parameters, the threshold is related with the small x region and the gluon condensate with large x region. Thus, the effective regions of these parameters are different in x and we can determine both threshold and gluon condensate by this single sum rule naturally.

3·2 Results

Before proceeding to the analysis at finite temperature, we have to fix decay constant f_F at zero temperature. We take

$$M_F = 1.970 \text{ GeV} \quad \text{and} \quad S_0 = (3.5 \text{ GeV})^2 , \tag{3·12}$$

on the hadron side and

$$m_c = 1.41 \text{ GeV}, \quad m_s = 0.15 \text{ GeV},$$

$$\alpha = 0.23 \quad \text{and} \quad \langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle = 0.1884 \text{ GeV}^4 \tag{3•13}$$

on the QCD side. Using these values we check the L/R-ratio for various values of f_F. The contributions from each terms in QCD side and hadron side are shown in Fig.16a and 16b, respectively, in zero temperature limit. We can see on the hadron side that we have contribution from continuum part in the small x region but generally at large x it decreases very rapidly as mentioned above. On QCD side, the contribution from gluon condensate has positive sign contrary to the charmonium case. But it also increses as x becomes large and we can see that non-perturbative effect plays an essential role in saturating the resonance contribution on the hadron side. We plot L/R-ratio for several values of f_F in Fig.17. As the best value for keeping the L/R-ratio unity, we find

$$f_F = 0.80 \text{ GeV} . \tag{3•14}$$

This is one of our result. This value is rather large comparing $f_D = 0.22$ GeV and $f_B = 0.14$ GeV ,which was found by Shuryak[27]) (D and B means D-meson and B-meson, respectively). The charm quark and strange quark are in the same doublet, but charm and u,d or beauty and u,d do not belong to the same one. So suppresion from Cabbibo angle is expected for f_D and f_B. This would be the reason why f_F is larger than f_D and f_B.

Next we proceed to finite temperature case. The L/R-ratio is shown in Fig.18 at $T = 100$ MeV for parameters fixed at $T = 0$ MeV. L/R-ratio deviates largely from unity, which shows some modification of the parameter values is necessary at finite temperature. We can see L/R-ratio for various values of gluon condensate in Fig.19. The L/R-ratio becomes near to unity as gluon condensate decreases and the sum rule is recovered. We search the value of gluon condensate which keep the L/R-ratio unity in widest region of x and it gives the value of gluon condensate at $T = 100$ MeV . We have

$$\langle\langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle\rangle = 0.15 \text{ GeV}^4 \quad (T = 100 \text{ MeV}) \ . \qquad (3\cdot15)$$

Investigating the sum rule at several temperature in this way, we find temperature dependence of the gluon condensate. The result is shown in Fig.20. We can also find temperaure dependence of effective threshold at small values of x. The result is in Fig.21. It also decreases and becomes $(3.0 \text{ GeV})^2$ at $T = 100$ MeV from the value $(3.5 \text{ GeV})^2$ at $T = 0$ MeV . We can also find temperature dependence of mass and decay constant of F-meson from that of gluon condensate. They are in Fig.22a and 22b, respectively. Typically we can see that F-meson mass shifts by about 20 MeV toward lower side at $T = 100$ MeV , i.e.,

$$M_F = 1940 \text{ MeV} \qquad (T = 100 \text{ MeV}) . \qquad (3\cdot16)$$

The decay constant also decrease and at the same temperatue its value becomes 90% of the value at zero temperature.

$$f_F(T = 100 \text{ MeV}) = 0.9\, f_F(T = 0 \text{ MeV}) . \qquad (3\cdot17)$$

Comparing results obtained in charmonium channel and F-meson channel, the region where sum rule holds is small in the latter case. Effects of higher condensates becomes dominant at large x and they increase for small mass. We suppose that the break down of the sum rule at small values of x in F-meson channel is caused by small strange quark mass through this general mechanism.

In F-meson channel, we have measured temperature dependence of effective threshold quantitatively. It is true that it shifts toward lower side as temperature becomes higher but it has a value about $(3.0 \text{ GeV})^2$ at maximum temperature which we have studied and it is still larger than squared resonance mass. We can not observe the phenomenon that effective threshold becomes lower than squared resonance mass within the temperature we have investigated.

The temperature dependences obtained from analyses of different channels have same tendency that they decrease as temperature becomes high. The decrese is slightly larger in charmonium channel but the difference is small. They show almost same behaviour quantitatively in both channel. This fact indicates the universality of temperature dependence of gluon condensate. We consider this universality assures the reliability of the temperature dependence of hadronic parameters, i.e., masses, widths and decay constant.

§4. Summary and discussion

We have considered QCD sum rule at finite temperature based on the success at zero temperature. It gives a method to investigate non-perturbative effect at finite temperature, which is interesting in connection with hadron and quark-gluon phase transition and hadron physics at finite temperature. Finite temperature QCD sum rule was considered by Bochkarev and Shaposhnikov first to study the phase transition. But they neglected the temperature dependence of non-perturbative effects. We have proposed a new sum rule which enables us to take this dependence into account. Our basic strategy is to extend the method by Shifman et al. to finite temperature substituting vacuum expectation value by Gibbs ensemble average. There were two altered points : 1) we use finite temperature perturbation theory in the calculation of Wilson coefficients and 2) the lack of experimental data is supplemented by spectroscopic models depending on vacuum condensates. We adopt Matsubara's method for 1) and parametrize masses and widths (or decay constant) by gluon condensate in terms of the combination of potential models and flux tube model to cure the lack of 2). This latter procedure reduces the number of parameters and we can determine remaining parameters without ambiguity. Essential point here is that Matsubara's method gives correct values of Wilson coefficients only on $q_0(n) = \frac{2\pi i n}{\beta}$, so that we summed up the both hand sides of dispersion relation with a weight function on these points.

This is our proposal of discrete mode sum rule. It has a merit that it does not need analytic continuation which is difficult for Wilson coefficients of higher consensates. It simplifies the calculation very much and information contained in this sum rule is not less than that of continuous version. Some proper choice of weight function on imaginary energy axis leads to damping factor on the real axis. Choosing appropriate strength of this damping factor, we can enhance resonance region where information on non-perturbative effects is rich. The criterion to determine vacuum expectation value is the agreement of both hand sides in as wider region of damping parameter as possible.

To check our sum rule, we compared our results and that of Shifman et al. in charmonium channel at zero temperature. Their sum rule was moment type one and completely different from our discrete mode sum rule, but both predictions about gluon condensate agree each other. This fact supports the reliability of our new sum rule.

We have applied our sum rule to charmonium channel first, and next proceed to F-meson analysis. In these channels, we can extract gluon condensate effectively. We concentrate on temperature dependence of gluon condensate and hadron physics at finite temperature. We have found gluon condensate decrease gradually as temperatere becomes higher. Typically it becomes 80% at $T = 100$ MeV . This behaviour is seen in both channels and the amounts of the decrease show quantitative agreement. Gluon condensate is a nature of true QCD vacuum and its temperature dependence should be channel independent. We have verified this universality through the analyses of these different chanels. We summerize the results on hadron physics at finite temperature.

1) Charmonium masses shift toward lower side. J/ψ and ψ' masses decrease about 30 MeV and 80 MeV at $T = 100$ MeV, respectively.

2) Leptonic decay widths of charmonium also decrease and at $T = 100$ MeV they becomes 80% of their zero temperature values.

3) As there is no experimental data on decay constant of F-meson, we have determined its value by the use of parameter values fixed by charmonium channel. The result is $f_F = 0.8$ GeV at zero temperature.

4) F-meson mass shifts to the lower side by 20 MeV at $T = 100$ MeV.

5) The decay constant of F-meson also shifts to the lower side and it becomes 80% of its zero temperature value at $T = 100$ MeV.

6) Effective threshold decreases from $(3.5\ \mathrm{GeV})^2$ to about $(3.0\ \mathrm{GeV})^2$ at $T = 100$ MeV.

The value of f_F is reasonable in comparison with f_D or f_B determined by Shuryak. We consider that verified universality gives a support on these results. We cannot see the phenomenon that threshold becomes lower than squared resonance mass within the temperature we have investigated. Unfortunately we cannot apply our sum rule at higher temperature than 130 MeV. We used oscillating weight functin on the imaginary energy axis to make damping factor on the real axis. Such behaviour of weight function results in the periodicity of cut off parameter x. The parameter x has a period β and the significant region becomes smaller as temperature becomes higher. This limits the highest temperature of our sum rule. To overcome this defect, we need alternative weight functions. One of possible choices is $(-)^n {}_nC_\lambda$ with appropriate normalization factor, which makes difference on $q_0(n)$ and directly corresponds to moment sum rule by Shifman et al.

We have assumed that gluon propagator is proportional to metric tensor and it is Lorentz covariant. We have condensate $\langle\langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle\rangle$ as a result of this assumption. But at finite temperature, there is no Lorentz covariance, color electric and magmetic field can condense independently. We will be able to know these precise information on finite temperature condensate decomposing gluon propagator by O(3) invariant tensors.

Other interesting applications are analyses of different channels. By these analyses, we will be able to confirm the universality of temperature dependence of gluon condensate and study that of other condensates. Especially, we can investigate temperature dependence of chiral condensate in D-meson channel which is another typical condensate of QCD vacuum. Because D-meson contains light u or d quark, chiral condensate becomes important in this channel. The development from F-meson channel to D-meson channel is rather straightforward with respect to potential model as the semi-relativistic one is also applicable to this channel, though some other technical difficulties are expected. We hope it will be realized in near future.

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All numerical calculations for this work have been carried out on ACOS Computer System at the Computer Center of Osaka University.

Appendix A

The basic object in QCD sum rule is the current-current commutator, which is defined by

$$\Pi_{\mu\nu}(x-y) = \langle 0| \ T \ j^{a}_{\mu}(x) \ j^{b}_{\nu}(y) \ |0\rangle \quad , \qquad (A\cdot1)$$

where j_{μ} is a current and given by

$$j^{V}_{\mu}(x) = \bar{q}(x) \ \gamma_{\mu} \ q(x) \qquad (A\cdot2)$$

in vector current. In momentum space, we have

$$\Pi_{\mu\nu}(q) = i \int d^{4}x \ \exp(iq(x-y)) \ \langle 0| \ T \ j^{V}_{\mu}(x) \ j^{V}_{\nu}(y) \ |0\rangle$$

$$= (\ q_{\mu}q_{\nu} - g_{\mu\nu}q^{2} \ ) \ \Pi(Q^{2}) \qquad (A\cdot3)$$

by the use of gauge invariance. The dispersion relation for $\Pi(Q^{2})$ is the following,

$$\Pi(Q^{2}) = \frac{1}{\pi} \int_{0}^{\infty} ds \ \frac{\mathrm{Im} \ \Pi(s)}{s + Q^{2}} \quad . \qquad (A\cdot4)$$

To derive non-perturbative effects, we calculate $\Pi(Q^{2})$ on the left hand side by QCD with operator product expansion and input experimental data to $\mathrm{Im}\Pi(Q^{2})$. The right hand side is represented by Willson coefficients and vacuum expectation values of local operators through operator product expansion. The vacuum expectation values reflect the non-perturbative effects of QCD. These are determined from experimental data through (A·4) and we can obtain information about universal vacuum structure of QCD.

The force between quarks becomes stronger as they are separated at larger distance, i.e., there works confining force between quarks which is non-perturbative effect of QCD. On the other hand, the interaction between quarks becomes smaller at shorter distance and they behave like free particles, which shows that QCD has asymptotically free nature. This fact means that there exists some scale Λ which characterizes QCD and we can treat QCD perturbatively at smaller distance than Λ but non-perturbative effects dominate at larger distance than Λ. Operator product expansion (OPE) gives a general framework of treating such a situation, i.e., different phenomena with different scales. The basic idea of OPE is to expand operators for small x/Λ. It is a short distance expansion and it contains further information about long distance behaviour as we take higher order terms into account.

In QCD, we can extract gluon condensate $\langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle$, chiral condensate $\langle \bar{\Psi}\Psi \rangle$ and other vacuum condensate as vacuum expectation values of local operators by applying OPE to the time ordered current-current commutator (left hand side of (A·4)). So we can determine these universal vacuum parameters from QCD sum rule with the input of hadron side data.

To know how these condensates can be extracted actually, we need some technical preparation on Fock-Schwinger gauge. Fock-Schwinger gauge condition was introduced by Fock in quantum electro dynamics (QED), and then, independently, by Schwinger. After that it has been rediscovered several times in the context of QCD. It is given by

$$(x - x_f)^\mu A^a_\mu(x) = 0 , \qquad (A\cdot5)$$

where x_f is an arbitary fixed point in the space and plays the role of gauge parameter. It keeps Lorentz invariance but breaks transformational symmetry. The latter should be restored in gauge invariant quantities and x_f disappears. This serves as an additional check of correctness of calculation. Hereafter, we take $x_f = 0$ for simplicity.

This gauge condition has two main virtues, 1) the gauge field is expressed by field strength, 2) it allaws covariant Taylor expansion.

1) Usually field strength $G^a_{\mu\nu}(x)$ is expressed by gauge field $A^a_\mu(x)$:

$$G^a_{\mu\nu}(x) = \frac{\partial}{\partial x^\mu} A^a_\nu(x) - \frac{\partial}{\partial x^\nu} A^a_\mu(x) + f^{abc} A^b_\mu(x) A^c_\nu(x) ,$$

where f^{abc} 's are structure constans of the gauge group. But in this gauge, the gauge field $A^a_\mu(x)$ is expressed by field strength $G^a_{\mu\nu}(x)$ vice versa :

$$A^a_\mu(x) = \int_0^1 d\alpha \ \alpha \ x_\rho \ G^a_{\rho\mu}(\alpha x) \ , \qquad (A\cdot 6)$$

which can be derived easily as follows.

Following identity :

$$A^a_\mu(x) = \frac{\partial}{\partial x^\mu} (x^\rho \ A^a_\rho(x)) - x^\rho \ \frac{\partial A^a_\rho(x)}{\partial x^\mu} \qquad (A\cdot 7)$$

holds for arbitrary gauge field $A^a_\mu(x)$. The first term vanishes from the gauge condition. The second one is rewritten as

$$- x^\rho \ G^a_{\mu\rho}(x) - x^\rho \ \frac{\partial A^a_\mu(x)}{\partial x^\rho} \ , \qquad (A\cdot 8)$$

also by the use of the gauge condition. Then we have

$$x^\rho \ G^a_{\rho\mu}(x) = A^a_\mu(x) + x^\rho \ \frac{\partial A^a_\mu(x)}{\partial x^\rho} \ . \qquad (A\cdot 9)$$

A tricky substitution of x to αx gives

$$\alpha x^\rho \ G^a_{\rho\mu}(\alpha x) = \frac{\partial}{\partial \alpha} \alpha \ A^a_\mu(\alpha x) \ . \qquad (A\cdot 10)$$

After integrating the both sides by α from zero to unity we have (A·6).

2) Expanding $A^a_\mu(x)$ in the gauge condition (A·5) at the origine by Taylor expansion, we obtain

$$x^\mu (A^a_\mu(0) + x^\alpha \partial_\alpha A^a_\mu(0) + \frac{1}{2} x^\alpha x^\beta \partial_\alpha \partial_\beta A^a_\mu(0) + \cdots\cdots) = 0 . \tag{A·11}$$

This equation holds for any x, hence each order of x must equal to zero,

$$x^\mu A^a_\mu(0) = 0 , \tag{A·12a}$$

$$x^\mu x^\alpha \partial_\alpha A^a_\mu(0) = 0 , \tag{A·12b}$$

$$x^\mu x^\alpha x^\beta \partial_\alpha \partial_\beta A^a_\mu(0) = 0 . \tag{A·12c}$$

Using these equations, covariant Taylor expansion of any quantity can be easily derived. Typically next two expansions are important. For fermions,

$$\Psi(x) = \Psi(0) + x^\alpha D_\alpha \Psi(0) + \frac{1}{2} x^\alpha x^\beta \partial_\alpha \partial_\beta \Psi(0) + \cdots\cdots \tag{A·13}$$

Combining (A·6) and covariant Taylor expansion of $G^a_{\mu\nu}(\alpha x)$ in it, we can see for gauge field $A^a_\mu(x)$ that

$$A^a_\mu(x) = \frac{1}{2\cdot 0!} x^\rho G^a_{\rho\mu}(0) + \frac{1}{3\cdot 1!} x^\alpha x^\rho (D_\alpha G^a_{\rho\mu}(0))$$

$$+ \frac{1}{4\cdot 2!} x^\alpha x^\beta x^\rho (D_\alpha D_\beta G^a_{\rho\mu}(0)) + \cdots\cdots \tag{A·14}$$

Using (A·14) and (A·13), we can extract gluon condensate and chiral condensate, respectively. We have the former in the channel which contains only heavy quarks. We can treat fermion perturbatively in this channel, though gluon is under non-perturbative effect. So we use usual perturbation for fermion propagator applying (A•14) for gluon field. Then it is straight forward to find gluon condensate in this channel. As for chiral condensate, it becomes important when light quark is concerned. In this case, the light fermion is under non-perturbative effect and we have to use expansion (A•13) to it. Easy calculation leads us to chiral condensate in such a channel.

Next we discuss the hadron side (right hand side) to complete the QCD sum rule. The imaginary part of the time ordered current-current corelator is related to the cross section of corresponding channel through the optical theorem.

The cross section can be determined by experiments. If perturbative QCD works well at all scales, the imaginary part of the time ordered current-current commutator should have almost the same structure as $\mathrm{Im}C_I^{0+1}(s)$. But $\mathrm{Im}C_I^{0+1}(s)$ has no resonances, which do exist in the real world. The vacuum expectation of local operators reflects these deviations from perturbative calculations. It is possible to use precise fit of experimental data, but we parametrize the cross section with a few characteristic parameters, i.e., resonancs masses, widths and threshold by the use of resonance continuum model.

There are several ways of actual use of the QCD sum rule. The first one is "moment" sum rule, which compairs the right and left hand sides by their derivatives with respect to

Q^2 at the origine,

$$M_n = \frac{1}{n!} \left(-\frac{1}{dQ^2}\right)^n \Pi_{QCD}(Q^2) \Big|_{Q^2=0} = \frac{1}{\pi} \int_0^{+\infty} ds \frac{Im\Pi(s)}{s^{n+1}} . \tag{A·15}$$

This type of sum rule was used by M.A.Shifman et al. in the analysis of resonance physics at T=0.

The second one is given by integrating Q^2 from zero to infinity after multiplying appropriate function. In the actual use to extract non-perturbative information in combination with OPE effectively, its form is somewhat restrected.

As QCD has asymptotically free nature, $Im\Pi$ is sufficiently satulated by $C_I^0(Q^2)$ at large q^2, which means this type of sum rule is almost trivial for $g(q^2) = 1$. The non-perturbative effects show theirselves in the small q^2 region as resonances, we must choose functions to enhance such a region and determine vacuum condensates of QCD, for example

$$g(q^2) = \exp(-\sqrt{q^2/ M^2}) , \tag{A·16}$$

whrer M^2 is a parameter.

In the case of applying the sum rule for channels including light quarks, we need further improvement called Borel transformation. The OPE method gives inverse power expansion with respect to Q^2 because the product of local operators have their own dimensions. Borel trnsformation gives one way of changing this expansion to faster convergent series. This transformation is defined by

$$\hat{f}(\lambda) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \exp(\lambda/x)\ f(x)\ x\ d(\frac{1}{x}) = L_\lambda(f(x)) , \tag{A·17}$$

where the integration contour runs to the right of all singularities of the function f(x), and $\hat{f}(\lambda)$ is called Borel transform of f(x). Its inverse transform is

$$f(x) = \int_0^{+\infty} \hat{f}(\lambda)\ \exp(-\lambda/x)\ d(\frac{\lambda}{x}) . \tag{A·18}$$

We can see this easily. From (A·18),

$$x\ f(x) = \int_0^{+\infty} \hat{f}(\lambda)\ \exp(-\lambda/x)\ d\lambda = F(\frac{1}{x}) , \tag{A·19}$$

where F(x) is Laplace transform of $\hat{f}(\lambda)$. The inverse transformation of Laplace transformation is Mellin transformation :

$$\hat{f}(\lambda) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \exp(\lambda x)\ F(x)\ dx . \tag{A·20}$$

So we have

$$\hat{f}(\lambda) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \exp(\lambda/x)\ F(\frac{1}{x})\ d(\frac{1}{x}) , \tag{A·21}$$

and the right hand side of this formula coincids with the definition of Borel transformation, which proves our assertion.

Borel transformation of f(x) is the inverse Laplace transform of x.f(x) with respect to 1/x. For example, we use the Borel transformation of $(1/Q^2)^k$ to the variable M^2 with respect to Q^2 in the improvement of the sum rule, which gives formula :

$$L_{M^2}(\ (1/Q^2)^k) = \frac{1}{(k-1)!}\ (1/M^2)^k \ . \qquad (A\cdot 22)$$

Next form is comvenient to see the relation between moment sum rule and Borel transformation. We can rewrite Borel transform of $\Pi(Q^2)$ as follows :

$$L_{M^2}(\ \Pi(Q^2)\ ) = \hat{\Pi}(M^2)$$

$$= \lim_{Q^2, n \to \infty} \frac{1}{(n-1)!}\ (Q^2)^n\ (-\ \frac{1}{dQ^2})^n\ \Pi(Q^2)\ , \qquad (A\cdot 23)$$

where $Q^2/\ n = M^2$ (fixed) .

This identity can be verified by the action on $(\ 1/Q^2)^k$, which reproduces (A·22). We notice that Borel transformation is one kind of extended moment sum rule and new series obtained after Borel transformation shows faster convergence because of the factor $1/(k-1)!$.

Finally we show the Borel transformation of dispersion relation (A·4). Noticing

$$L_{M^2}\ (\ \frac{1}{s\ +\ Q^2}\ ) = \frac{1}{M^2}\ \exp(-s/M^2)\ , \qquad (A\cdot 24)$$

we have

$$\frac{1}{\pi}\frac{1}{M^2}\int_0^{+\infty} ds\ \mathrm{Im}\Pi(s)\ \exp(-s/M^2) = \sum_{k=0}^{+\infty}\sum_i \frac{1}{(k-1)!}\left(\frac{1}{M^2}\right)^k h_k^i\ , \tag{A·25}$$

where we expanded $\Pi_{QCD}(Q^2)$ by OPE :

$$\Pi_{QCD}(Q^2) = \sum_{k=0}^{+\infty}\sum_i \frac{O_k^i}{(Q^2)^k}\ , \tag{A·26}$$

and $h_k^i = \langle 0|\ O_k^i\ |0\rangle$.

Shifman et al. investigated charmonium channel to extract gluon condensate. Heavy charm allows them to extract gluon condensate. They found

$$\langle G^a_{\mu\nu}G^a_{\mu\nu}\rangle = 0.1884\ \mathrm{GeV}^4\ , \tag{A·27}$$

by the use of moment sum rule. They applied their sum rule for ρ-meson channel and found the value

$$\langle\bar{\psi}\psi\rangle = -250\ \mathrm{MeV}^3 \tag{A·28}$$

is consistent with experimental data.

Appendix B

The basic idea of the sum rule by Bochkarev and Shaposhnikov is to write down a sum rule for spectral density of Gibbs ensemble average of current-current comutator :

$$\Pi^R_{\mu\nu}(q;T,\mu) =$$

$$i \int d^4x \exp(iq(x-y))\, \theta(x_0-y_0) \,\langle\langle \left[j_\mu(x), j_\nu(y)\right]\rangle\rangle \ , \tag{B·1}$$

where R means retarded, $\langle\langle\ \rangle\rangle$ is the Gibbs ensemble average, T and μ are temperature and chemical potrntial. $\Pi^R_{\mu\nu}(q;T,\mu)$ is an analytic function in the upper half plane of the complex energy plane and the dispersion relation for $\Pi^R_{\mu\nu}(q;T,\mu)$ is

$$\Pi^R_{\mu\nu}(q;T,\mu) = \int_{-\infty}^{+\infty} d\omega \frac{\rho_{\mu\nu}(\omega,\vec{q};T,\mu)}{\omega - q_0 - i\varepsilon} \ , \tag{B·2}$$

where $\rho_{\mu\nu}(\omega,\vec{q};T,\mu)$ is the spectral density of $\Pi^R_{\mu\nu}(q;T,\mu)$ and its concrete form can be written as

$$\rho_{\mu\nu}(\omega,\vec{q};T,\mu) = (2\pi)^3 \sum_{m,n} \exp((\Omega-E_n)/T)\, \delta^3(\vec{q}-\vec{k}_{nm})\, \delta(\omega-\omega_{nm})$$

$$\times \langle n|J_\mu(0)|m\rangle \langle m|J_\nu(0)|n\rangle\, (1-\exp(-\omega_{mn}/T)) \ , \tag{B·3}$$

where $\omega_{mn} = E_m - E_n$, $\vec{k}_{mn} = \vec{k}_m - \vec{k}_n$ and $\Omega = -T\,\mathrm{Tr}(-(H-\mu N)/T)$. Perturbative calculation of spectral density allows one to require next relation, i.e., QCD sum rule for spectral density at finite temperature :

$$\int d\omega\ g(\omega)\ \rho_{\mu\nu}(\omega,\vec{q};T,\mu) =$$

$$\int d\omega\ g(\omega)\ \mathrm{Im}\Pi^{M}_{\mu\nu}(\frac{\omega+i\varepsilon}{i},\vec{q};T,\mu) + \mathrm{N.P.} \quad , \qquad (\mathrm{B{\cdot}4})$$

where $\Pi^{M}_{\mu\nu}(\frac{\omega+i\varepsilon}{i},\vec{q};T,\mu)$ has same definition as $\Pi^{R}_{\mu\nu}(q;T,\mu)$ and is analytically continued after calculated on $q_0(n) = \frac{2\pi n i}{\beta}$ by Matsubara's method. N.P. means non-perturbative effects such as gluon condensate or chiral condensate etc.

Here we pay attention to the tensorial form of $\Pi^{R}_{\mu\nu}(q;T,\mu)$. At finite temperature there is no full Lorentz symmetry and it reduces to spacial O(3) symmetry. Because of this breaking of the Lorentz symmetry, $\Pi^{R}_{\mu\nu}(q;T,\mu)$ is decomposed into two invariant form factors, i.e., transversal one $\Pi^{R}_{t}(Q_0;T,\mu)$ and longitudinal one $\Pi^{R}_{l}(Q_0;T,\mu)$:

$$\Pi^{R}_{00}(q;T,\mu) = \vec{q}^{\,2}\ \Pi^{R}_{l}(Q_0,\vec{q};T,\mu) \ , \qquad (\mathrm{B{\cdot}5a})$$

$$\Pi^{R}_{ij}(q;T,\mu) = (\delta_{ij}q_0^2 - q_i q_j q_0^2/\vec{q}^{\,2}\ )\ \Pi^{R}_{t}(Q_0,\vec{q};T,\mu)$$

$$+\ q_i q_j q_0^2/(q_0^2-\vec{q}^{\,2})\ \Pi^{R}_{l}(Q_0,\vec{q};T,\mu) \ . \qquad (\mathrm{B{\cdot}5b})$$

where $Q_0 = -iq_0$. General argument requires that Π^{R}_{l} and Π^{R}_{t} should coincise each other at momentum rest frame $\vec{q} = \vec{0}$:

$$\Pi_{l}^{R}(Q_0,\vec{q}=0;T,\mu) = \Pi_{t}^{R}(Q_0,\vec{q}=0;T,\mu) = \Pi^{R}(Q_0;T,\mu) \ . \qquad (B\cdot 6)$$

This allows us rewrite (B.4) to the sum rule for $\Pi^R(Q_0;T,\mu)$ at momentum rest frame.

$$\int_0^\infty ds\ g(s)\ \rho(s;T,\mu) = \int_0^\infty ds\ g(s)\ \mathrm{Im}\Pi^M(s;T,\mu) + \mathrm{N.P.}\ , \qquad (B\cdot 7)$$

where $\mathrm{Im}\Pi^M(s;T,\mu)$ can be determined by Matsubara's method and $\rho(s;T,\mu)$ is the true spectral function of $\Pi^R(Q_0;T,\mu)$. We have given $\rho(s;T,\mu)$ value from experimental data at zero temperature, but at finite temperature there is no such data to input. Therefore it is natural to parametrize this cross section in terms of resonance-continuum model which contains resonance masses and threshold as parameters and they should be determined through this sum rule.

A.I. Bochkarev and M.E. Shaposhnikov discussed along this line and applied this type of sum rule for ρ-meson channel in the case $\mu = 0$. They calculate $\mathrm{Im}\Pi^M(s;T,\mu)$ at 1-loop level in perturbation theory using corresponding current

$$j_\mu^\rho(x) = \frac{1}{2}\ (\ \bar{u}(x)\gamma_\mu u(x) - \bar{d}(x)\gamma_\mu d(x)\ )\ . \qquad (B\cdot 8)$$

The result is

$$\rho(s) = \rho_0(s)\ \theta(s-4m_q^2)\ \mathrm{th}(\sqrt{s}/4T)$$

$$+ \delta(s) \int_{4m_q^2}^{+\infty} ds' \ \rho_0(s') \ 2n_F(\sqrt{s'}/2T) \ , \quad \text{(B·9)}$$

with

$$\rho_0(s) = \frac{1}{8\pi^2} \left(1 + \frac{2m_q^2}{s}\right) \left(1 - \frac{4m_q^2}{s}\right)^{1/2} ,$$

where temperature dependence shows itself in the tangent hypabolic and fermionic statistical factor. As for the true spectral function $\rho(s;T)$, because ρ pole dominates in the spectrum, it is set as

$$\rho(s) = \rho_{resonance}(s) + \rho_{continuum}(s) \quad \text{(B·10a)}$$

$$\rho_{res}(s) = f_\rho m_\rho^2 \ \delta(s-m_\rho^2) \quad \text{(B·10b)}$$

$$\rho_{cont}(s) = \frac{1}{8\pi^2} \theta(s-s_0) \ th(\omega/4T) + \delta(s) \int_{s_0}^{+\infty} ds \ 2n_F(\sqrt{s}/2T) \ . \quad \text{(B·10c)}$$

The contribution from s larger than threshold cancel on both hands sides. The final form of the sum rule is

$$f \ m^2/ \ M^2 \exp(-m^2/ \ M^2) =$$

$$\frac{1}{8\pi^2} \int_0^{s_0} \frac{ds}{M^2} \left(\exp(-s/M^2) \ th(\sqrt{s}/4T) + 2n_F(\sqrt{s}/2T) \right) + \text{N.P.} \quad \text{(B·11)}$$

They discussed the hadron and quark-gluon phase transition by the use of this sum rule. They found the solution that the threshold becomes lower than squared resonance mass. In such a situation hadron will become breakable. They identified this phenomena and the phase transition, and obtaint critical temperature

$$T_C = 130 \text{ MeV}. \tag{B·12}$$

But we have a question about the treatment of non-perturbative effect. It seems possible that they have temperature dependence, as is suggested by Monte Carlo simulation at high tempeature. We think some improvement is needed to take temperature dependence of non-perturbative effects into account, which is one of our motivations of this work.

Appendix C

We can see from the same reason as in Appendix A that we must calculate diagrams in Fig.14 to obtain Wilson coefficient C_G of gluon condensate in F-meson channel. The situation is complicated because of mass inequality of quarks. We show how we calculate this coefficient in this Appendix. We use Matsubara's method and choose a frame in which the external momentum equals to zero.

After taking trace, we have next expression from (3•6).

$$\frac{2\pi}{\beta}\sum_{n=-\infty}^{+\infty}\frac{1}{(2\pi)^3}\int_0^\infty 4\pi p_s dp_s$$

$$-16\left(4m_1^4m_2^2+3m_1^2m_2^2p^2+2m_1^4r^2+5m_1^3m_2rp+m_2^2p^4+m_1m_2p^2rp+6m_1^2(rp)^2+2p^2(rp)^2\right)$$

$$/(p^2+m_1^2)^4(r^2+m_2^2)^2 - (m_1\leftrightarrow m_2,\ p\leftrightarrow r) \qquad \text{(C•1a,1b)}$$

$$+16\left(4m_1^3m_2^3+3m_1m_2^3p^2+3m_1^3m_2r^2+6m_1^2m_2^2rp+2m_1m_2p^2r^2+3m_2^2p^2rp+3m_1^2r^2rp\right.$$

$$\left.+4m_1m_2(rp)^2+4(rp)^3\right)/(p^2+m_1^2)^3(r^2+m_2^2)^3 \qquad \text{(C•1c)}$$

$$+16\left(4m_1m_2+5m_1pr+2m_1m_2p^2+2m_1p^2pr+2m_1m_2p^4+p^4pr\right)/(p^2+m_1^2)^5(r^2+m_2^2)$$

$$+\ (m_1\leftrightarrow m_2,\ p\leftrightarrow r) \qquad \text{(C•1d,1e)}$$

where $r = p-q$ and $p = \left(\frac{2\pi}{\beta}(n+\frac{1}{2})\ ,\ p\right)$, $q = \left(\frac{2\pi}{\beta}(n+\frac{1}{2}-1)\ ,\ 0\right)$

As we calculate in Euclidean metric, the inner product is taken with metric tensor $\delta_{\mu\nu}$ and $p^2 = (\frac{2\pi}{\beta}(n+\frac{1}{2}))^2 + p^2$, $r^2 = (\frac{2\pi}{\beta}(n+\frac{1}{2}-l))^2 + p^2$, $r \cdot p = \frac{1}{2}(p^2 + r^2 - (\frac{2\pi l}{\beta})^2)$. Here we introduce the notation $D_{MN}(a,b,c)$ defined as

$$D_{MN}(a,b,c) = \frac{1}{\pi} \sum_{n=-\infty}^{+\infty} \frac{(p^2)^a (r^2)^b (pr)^c}{((n+\frac{1}{2})^2+A^2)^M ((n+\frac{1}{2}-l)^2+B^2)^N} , \qquad (C\cdot 2)$$

with $A = \frac{\beta}{2\pi}\sqrt{p^2+m_1^2}$ and $B = \frac{\beta}{2\pi}\sqrt{p^2+m_2^2}$. It is obvious from (C•1) that C_G can be calculated by single integration of the summation of $D_{MN}(a,b,c)$'s with mass coefficient up to normalization. We calculate $D_{MN}(a,b,c)$ annalytically. Following reduction formula can be easily verified.

$$p^2 D_{MN} = D_{M-1\ N} - m_1^2 D_{MN} \qquad (C\cdot 3a)$$

$$r^2 D_{MN} = D_{M\ N-1} - m_2^2 D_{MN} \qquad (C\cdot 3b)$$

$$pr\ D_{MN} = \frac{1}{2}(D_{M-1\ N} + D_{M\ N-1} - (m_1^2+m_2^2+q_0^2) D_{MN} . \qquad (C\cdot 3c)$$

Iterative use of these reduction formula allows us to express $D_{MN}(a,b,c)$ by linear combination of $D_{MN}(0,0,0) = D_{MN}$. D_{MN} can be calculated through next derivative formula :

$$D_{MN} = \frac{(-)^{M-1}}{(M-1)!} \frac{(-)^{N-1}}{(N-1)!} (\frac{\partial}{\partial A^2})^{M-1} (\frac{\partial}{\partial B^2})^{N-1} D_{11} . \qquad (C\cdot 4)$$

And the mode sum in D_{MN} can be taken annalytically by the use of mathematical formula, the result is

$$D_{11} = \frac{1}{\pi} \sum_{n=-\infty}^{+\infty} \frac{1}{((n+\frac{1}{2})^2+A^2)((n+\frac{1}{2}-1)^2+B^2)}$$

$$= -\frac{1}{2B} \frac{A-B}{(A-B)^2+1^2} \frac{\tanh(\pi A)}{A} + \frac{1}{2B} \frac{A+B}{(A+B)^2+1^2} \frac{\tanh(\pi A)}{A}$$

$$+ \frac{1}{2A} \frac{A-B}{(A-B)^2+1^2} \frac{\tanh(\pi B)}{B} + \frac{1}{2A} \frac{A+B}{(A+B)^2+1^2} \frac{\tanh(\pi B)}{B} . \quad (C\cdot5)$$

In equal mass case, we have $A = B$ and D_{11} reduces to

$$D_{11} = \frac{2}{1^2+ 4A^2} \frac{\tanh(\pi A)}{A} .$$

Unfortunately inequal mass case, D_{11} has four component and this is the origine of complexity.

By the combination of (C•3) - (C•5) , we can obtain analytic form for $D_{MN}(a,b,c)$. This is the way of analytic internal mode sum. Explicit form is lengthy so we do not quote here. Finally integrating numerically with respect to p_s , we have the value of C_G .

Appendix D

The semi-relativistic potential model was proposed by Morishita et al. which gives very good mass spectrum of F, D and B-mesons. We have used this model to parametrize F-meson mass and the value of wave function at the origin ($4\pi|\Psi(0)|^2$) by string tension. We introduce brief outline of this method to help the understanding of §3. Please see reference 26 for precise contents.

Semi-relativistic potential model gives us a general method to investigate two fermion system in which one is light and the other is heavy, i.e., atom like system. In the following we identify heavy one as charm quark and light one as strenge quark. They start from relativistic two body hamiltonian introduced by Fermi and Yang. It is sufficient to take scalar and vector potential into account to our purpose. The hamiltonian is

$$H = (\vec{\alpha}_q\vec{p}_q + \beta_q m_q) + (\vec{\alpha}_Q\vec{p}_Q + \beta_Q m_Q) + \beta_q\beta_Q S + (1-\vec{\alpha}_q\vec{\alpha}_Q)V \qquad (D\cdot 1)$$

where subscripts q , Q denote s , $\bar{c}$ (or $\bar{s}$, c) in our case, respectively. If Q is sufficiently heavy for us to treat it non-relativistically, we can apply Fordy transformation to Q. Resultant eigen value equation which they find is

$$(H_0 + H_1 + H_2 + H_3 + H_4)\Phi = E\Phi \quad , \qquad (D\cdot 2)$$

with

$$H_0 = \vec{\alpha}_q \vec{p}_q + \beta_q (m_q + S) + V , \qquad (D.3\text{-}0)$$

$$H_1 = m_Q + \vec{p}^2/2m_Q - \beta_q S\, \vec{P}^2/2m_Q^2 + \nabla^2 V \ / \ 8m_Q^2 - \vec{p}^4/8m_Q^3 \qquad (D.3\text{-}1)$$

$$H_2 \Phi = \frac{1}{4m_Q^2} \frac{1}{r} \left(\frac{dS}{dr}\beta_q - \frac{dV}{dr}\right) \Phi\, \vec{\sigma}_Q \vec{I} \qquad (D.3\text{-}2)$$

$$H_3 = \frac{1}{m_Q} V\, \vec{\alpha}_q \vec{p} - \frac{iV'}{2m_Q} \vec{\alpha}_q \vec{n} , \qquad (D.3\text{-}3)$$

$$H_4 \Phi = - \frac{1}{2m_Q} \frac{dV}{dr} \vec{\alpha}_q \Phi\, \vec{\sigma}_Q \times \vec{n} , \qquad (D.3\text{-}4)$$

where $\Phi_{\alpha i}$ has two subscripts α and i , the former corresponds to four component of Dirac spinor of light quark and the latter shows the two component of the spinor state of heavy quark.

The wave function can be expanded in terms of scalar and vector spherical harmonics (Y_J^M , $\vec{X}_{JJ}^M$, $\vec{Y}_{JJ+1}^M$) as

$$\Phi = A(r) Y_J^M(\Omega) + B(r) \vec{\sigma}\vec{X}_{JJ}^M(\Omega) + C(r)\vec{\sigma}\vec{Y}_{J(-)}^M(\Omega) + D(r)\vec{\sigma}\vec{Y}_{J(+)}^M(\Omega) , \qquad (D.4)$$

where

$$\vec{Y}_{J(+)}^M = \sqrt{(J+1)/(2J+1)}\ \vec{Y}_{J\ J+1}^M + \sqrt{J/(2J+1)}\ \vec{Y}_{J\ J+1}^M .$$

But in this case, they define the rotated spherical harmonics ($\mathcal{Y}_1^{JM}$, $\mathcal{Y}_2^{JM}$, $\mathcal{Y}_+^{JM}$, $\mathcal{Y}_-^{JM}$) , which are defined as

$$\begin{pmatrix} \mathcal{Y}_1^{JM} \\ \mathcal{Y}_2^{JM} \end{pmatrix} = U \begin{pmatrix} Y_J^M \\ \vec{\sigma}\cdot\vec{X}_{JJ}^M \end{pmatrix} \quad , \quad \begin{pmatrix} \mathcal{Y}_+^{JM} \\ \mathcal{Y}_-^{JM} \end{pmatrix} = U \begin{pmatrix} \vec{\sigma}\cdot\vec{Y}_{J(-)}^M \\ \vec{\sigma}\cdot\vec{Y}_{J(+)}^M \end{pmatrix} \quad , \tag{D・5}$$

with

$$U = \frac{1}{\sqrt{2J+1}} \begin{pmatrix} \sqrt{J+1} & \sqrt{J} \\ -\sqrt{J} & \sqrt{J+1} \end{pmatrix} \quad .$$

They satisfy next two remarkable relations

$$\vec{\sigma}_q \vec{n}\, \mathcal{Y}_A^{JM} = -\mathcal{Y}_B^{JM} \; , \; \text{for } (A,B)=(B,A)=(1,+)=(2,-) \; , \tag{D・6}$$

and

$$\vec{\sigma}_q \vec{l}\, \mathcal{Y}_A^{JM} = -\,(k+1)\, \mathcal{Y}_A^{JM} \; , \tag{D・7}$$

where $k = -(J+1),\ J,\ J+1,\ -J$ for $A = 1,\ 2,\ +,\ -,$ respectively. Following relations are useful to see these relations :

$$\vec{Y}_{J(+)}^M = -i \frac{1}{\sqrt{J(J+1)}} \vec{n}\times\vec{l}\ Y_J^M \; , \tag{D・8-1}$$

$$\vec{Y}_{J(-)}^M = -\,\vec{n}\ Y_J^M \; , \tag{D・8-2}$$

$$\vec{X}_J^M = \vec{Y}_{JJ}^M = -\,\sqrt{J(J+1)}\ \vec{l} Y_J^M \; , \tag{D・8-3}$$

and

$$\vec{I}\cdot\vec{I} = \vec{I} \ , \ \vec{I}\cdot\vec{n} = 0 \ , \ \vec{I}\cdot(\vec{n}\times\vec{I}) = 0 \ ,$$

$$\vec{n}\cdot\vec{I} = 0 \ , \ \vec{n}\cdot\vec{n} = 1 \ , \ \vec{n}\cdot(\vec{n}\times\vec{I}) = 0 \ ,$$

$$(\vec{n}\times\vec{I})\cdot\vec{I} = 0 \ , \ (\vec{n}\times\vec{I})\cdot\vec{n} = 0 \ , \ (\vec{n}\times\vec{I})\cdot(\vec{n}\times\vec{I}) = \vec{I}^2 \ , \tag{D·9-1}$$

and

$$\vec{I}\times\vec{I} = i\vec{I} \ , \ \vec{I}\times\vec{n} = 2i\vec{n} - \vec{n}\times\vec{I} \ , \ \vec{I}\times(\vec{n}\times\vec{I}) = \vec{n}\ \vec{I}^2 + i\vec{n}\times\vec{I} \ ,$$

$$\vec{n}\cdot\vec{I} = 0 \ , \ \vec{n}\times\vec{n} = 0 \ , \ (\vec{n}\times\vec{I})\times\vec{I} = i\vec{n}\times\vec{I} \ ,$$

$$(\vec{n}\times\vec{I})\times\vec{I} = i\vec{n}\times\vec{I} \ , \ (\vec{n}\times\vec{I})\times\vec{n} = \vec{I} \ , \ (\vec{n}\times\vec{I})\times(\vec{n}\times\vec{I}) = -i\vec{I} \ . \tag{D·9-2}$$

(D·6) and (D·7) simplify the calculation very much and under these bases the wave function is written as

$$\Phi = \begin{pmatrix} \Psi_A \\ \Psi_B \end{pmatrix} \tag{D·10}$$

with

$$\Psi_C = \begin{pmatrix} F_C \ \mathcal{Y}_C^{JM} \\ iG_{C'} \ \mathcal{Y}_{C'}^{JM} \end{pmatrix} \ ,$$

where $(C,C') = (1,+), (2,-), (+,1), (-,2)$. $(A,B) = (1,2)$ pair corresponds to parity $(-)^J$ and $(+,-)$ to $(-)^{J+1}$. $\mathcal{Y}_+^{JM}$ and $\mathcal{Y}_-^{JM}$

are $L = J+1$, $S = 1$ and $\mathcal{Y}_1^{JM}$ and $\mathcal{Y}_2^{JM}$ are mixed states of $L = J$, $S = 0$ and $L = J$, $S = 1$. The singlet dominates in the former and the triplet in the latter, and they are called $"^1"J_J$ and $"^3"J_J$, respectively. Then the eigen value equation (D•2) is written as

$$\begin{pmatrix} H_0 + K_1 & K_2 \\ K_2 & H_0 + K_1 \end{pmatrix} \begin{pmatrix} \Psi_A \\ \Psi_B \end{pmatrix} = E \begin{pmatrix} \Psi_A \\ \Psi_B \end{pmatrix} \qquad (D\cdot 11)$$

where K_1 corresponds to H_1, H_3 and the diagonal part of H_2, H_4. If the heavy quark mass tends to infinity, only H_0 term remains except m_Q and (D•11) decompose into two usual radial Dirac equation for each Ψ .

$$\begin{pmatrix} m_q + S + V & -\frac{d}{dr} + \frac{\kappa}{r} \\ \frac{d}{dr} + \frac{\kappa}{r} & m_q + S + V \end{pmatrix} \begin{pmatrix} u_C \\ v_C \end{pmatrix} = E \begin{pmatrix} u_C \\ v_C \end{pmatrix} , \qquad (D\cdot 12)$$

where $u_C = rF_C$, $v_C = rG_C$ and $\kappa = -(J+1)$, J , $J+1$, $-J$ for $C = 1, 2, +, -$. This is the reason why they have taken rotated scalar and vector spherical harmonics.

In the actual use of this model, they take

scalar potential : $S(r) = kr + b$

and vector potential : $V(r) = -\frac{4}{3}\frac{\alpha}{r}$.

To investigate the mass spectrum from (D•11), dimensional variational method is used. They consider diagonal part up to 1st inverse power of heavy quark mass,

$$(H_0 + m_Q + \vec{p}^2/2m_Q)\Phi = E\,\Phi \quad , \qquad (D\cdot 13)$$

which has similar form as (D·12). They treat the remainder of the hamiltonian perturbatively. They consider n dimensional functional space composed of next trial functions

$$\sum_{k=1}^{n} a_k r^{k+\alpha} \exp(-\frac{1}{2}(\frac{r}{\lambda})^2) \quad , \qquad (D\cdot 14)$$

for each u and v (α may have different value for u and v). The expansion parameter λ is the variational parameter. Taking the matrix element of the hamiltonian in this functional space and diagonalizing this n×n matrix, they have n energy levels correspond to quantum number κ . The essential point here is that the hamiltonian is still relativistic so that the usual variational criterion of energy minimum does not work in this case. Another criterion to fix the variational parameter is needed. The quantum virial theorem is used for this purpose. The statment is that the virial should vanish for stationaly states, i.e.,

$$\langle\ [rp,H]\ \rangle = 0 \quad . \qquad (D\cdot 15)$$

We vary the variational parameter observing the virial and the value is determined when the virial is nearest to zero. Because of this lack of positivity, these n eigen values contain negative ones. We choose only positive eigen values. At this stage, the states which have same value of κ are degenerate. For example $^1S_0$ (J=0) and $^3s_1$ (J=1) gives same $\kappa = -1$ and they are spritted by the perturbation of remaining hamiltonian. We estimated the lowest level variation of string tension. The result is shown in Fig.15a.

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$k(GeV^2)$	λ (fm)	$^1S_0$(MeV)	$4\pi\lvert\Psi(0)\rvert^2$ $\times 10^{-2}$
0.17	2.16	1967	4.768
0.16	2.25	1957	4.661
0.15	2.19	1943	4.177
0.13	2.35	1924	3.803
0.11	2.50	1903	3.364
0.10	2.53	1890	3.100
0.07	2.84	1853	2.399
0.05	3.35	1825	1.970
0.03	4.04	1791	1.453
0.01	5.50	1746	0.877

Table : The extension parameter, lowest mass and value of wave function at the origin for v various values of string tension.

Figure Captions

Fig.1a The diagram which represents invariant form factor for vector current.

1b The diagram which represents invariant form factor for pseudo scalar current.

Fig.2 The diagram of C_I^0 in charmonium channel.

Fig.3 The diagrams of C_I^1 in charmonium channel.

Fig.4 The diagrams of C_G in charmonium channel.

Fig.5a Masses of J/ψ and ψ' as functions of string tension and gluon condensate (linear in k).

5b Decay widths of J/ψ and ψ' as functions of string tension and gluon condensate (linear in k).

Fig.6a Contributions form terms in QCD side at zero temperature in charmonium channel.

6b Contributions form terms in hadron side at zero temperature in charmonium channel.

Fig.7 The effect of gluon condensate to L/R-ratio at zero temperature in charmonium channel.

Fig.8 The break down of sum rule for zero temperature parameters at finite temperature in charmonium channel.

Fig.9 Restoration of sum rule by the variation of gluon condensate in charmonium channel.

Fig.10 Temperature dependence of gluon condensate in charmonium channel.

Fig.11a Temperature dependence of masses of J/ψ and ψ'.

11b Temperature dependence of leptonic decay widths of J/ψ and ψ' normalized at zero temperature.

Fig.12 The diagram of C_I^0 in F-meson channel.

Fig.13 The diagrams of C_I^1 in F-meson channel.

Fig.14 The diagrams of C_G in F-meson channel.

Fig.15a Masses of F-meson as function of string tension and gluon condensate (linear in k).

15b Decay constant of F-meson as function of string tension and gluon condensate (linear in k).

Fig.16a Contributions form terms in QCD side at zero temperature in F-meson channel.

16b Contributions form terms in hadron side at zero temperature in F-meson channel.

Fig.17 L/R-ratio for various values of f_F at zero temperature.

Fig.18 The break down of sum rule for zero temperature parameters at finite temperature in F-meson channel.

Fig.19 Restoration of sum rule by the variation of gluon condensate in F-meson channel.

Fig.20 Temperature dependence of gluon condensate in F-meson channel.

Fig.21 Temperature dependence of effective threshold in F-meson channel.

Fig.22a Temperature dependence of F-meson mass.

Fig.22b Temperature dependence of decay constant f_F noemalized at zero temperature.

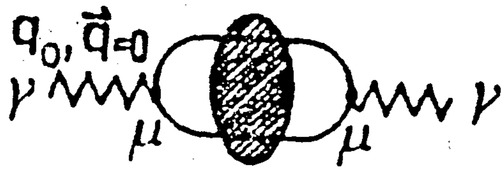


Fig. 1a

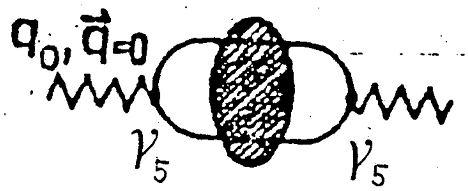


Fig. 1b

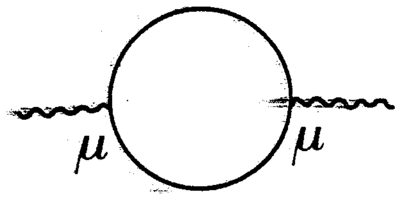


Fig.2

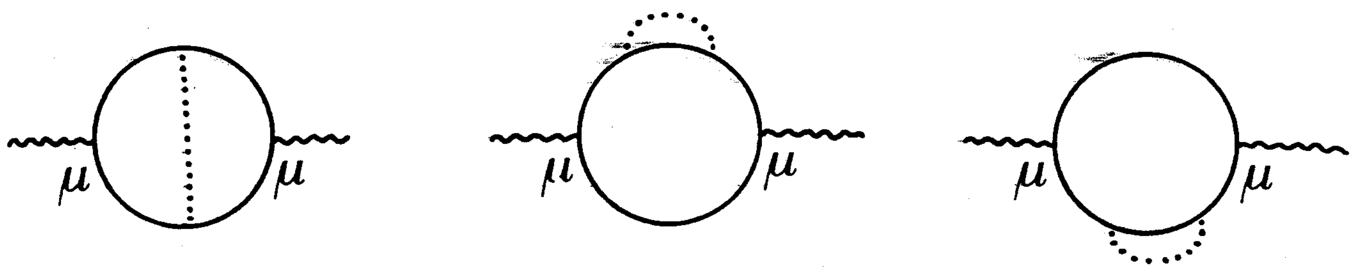


Fig.3

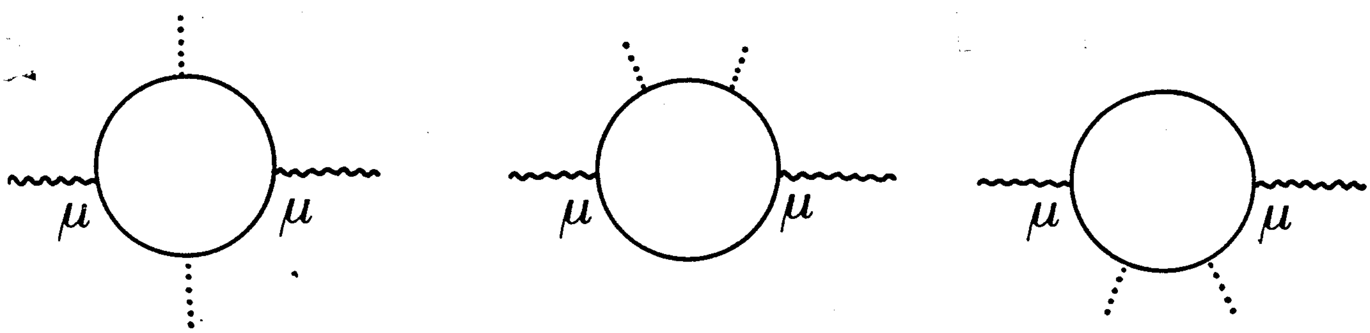


Fig.4

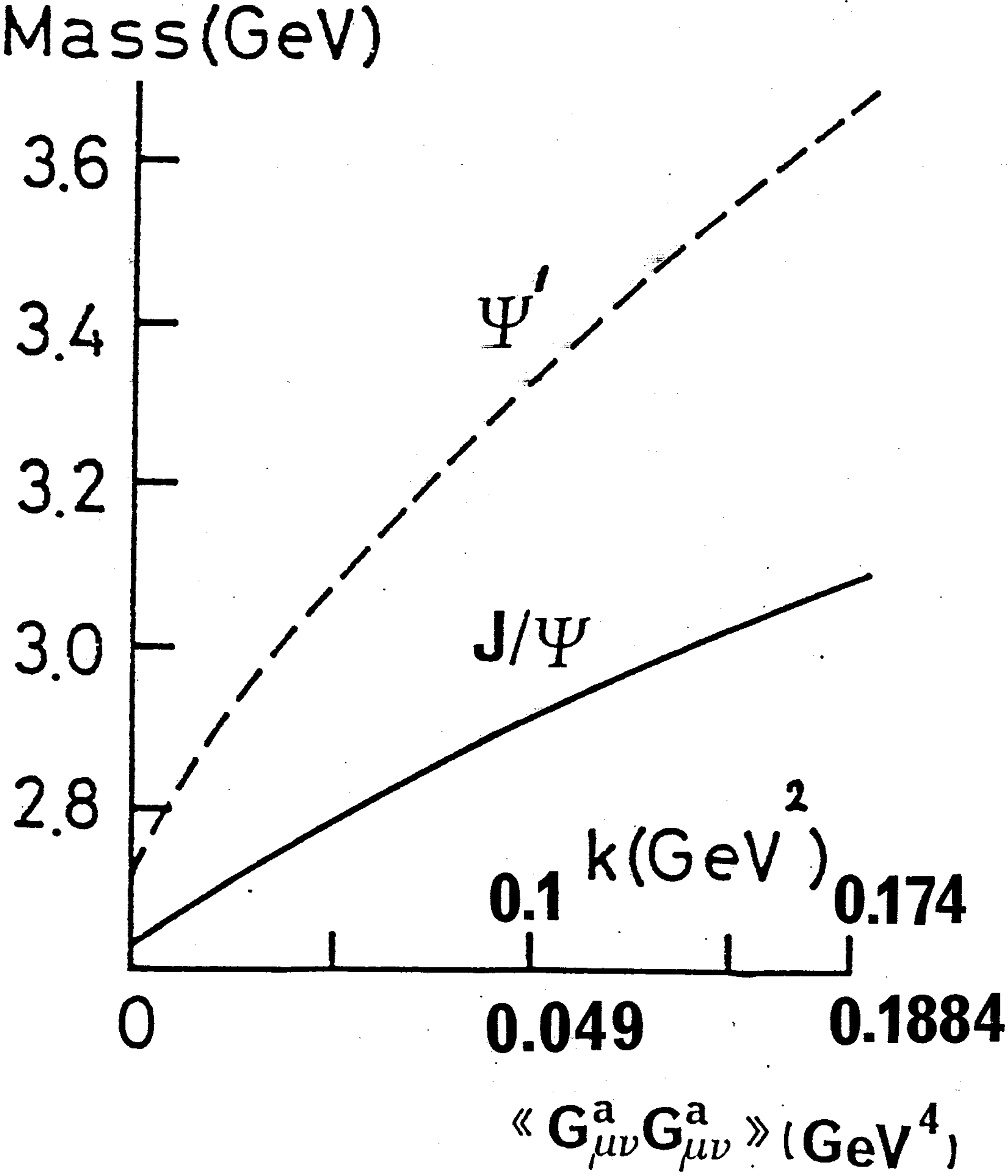
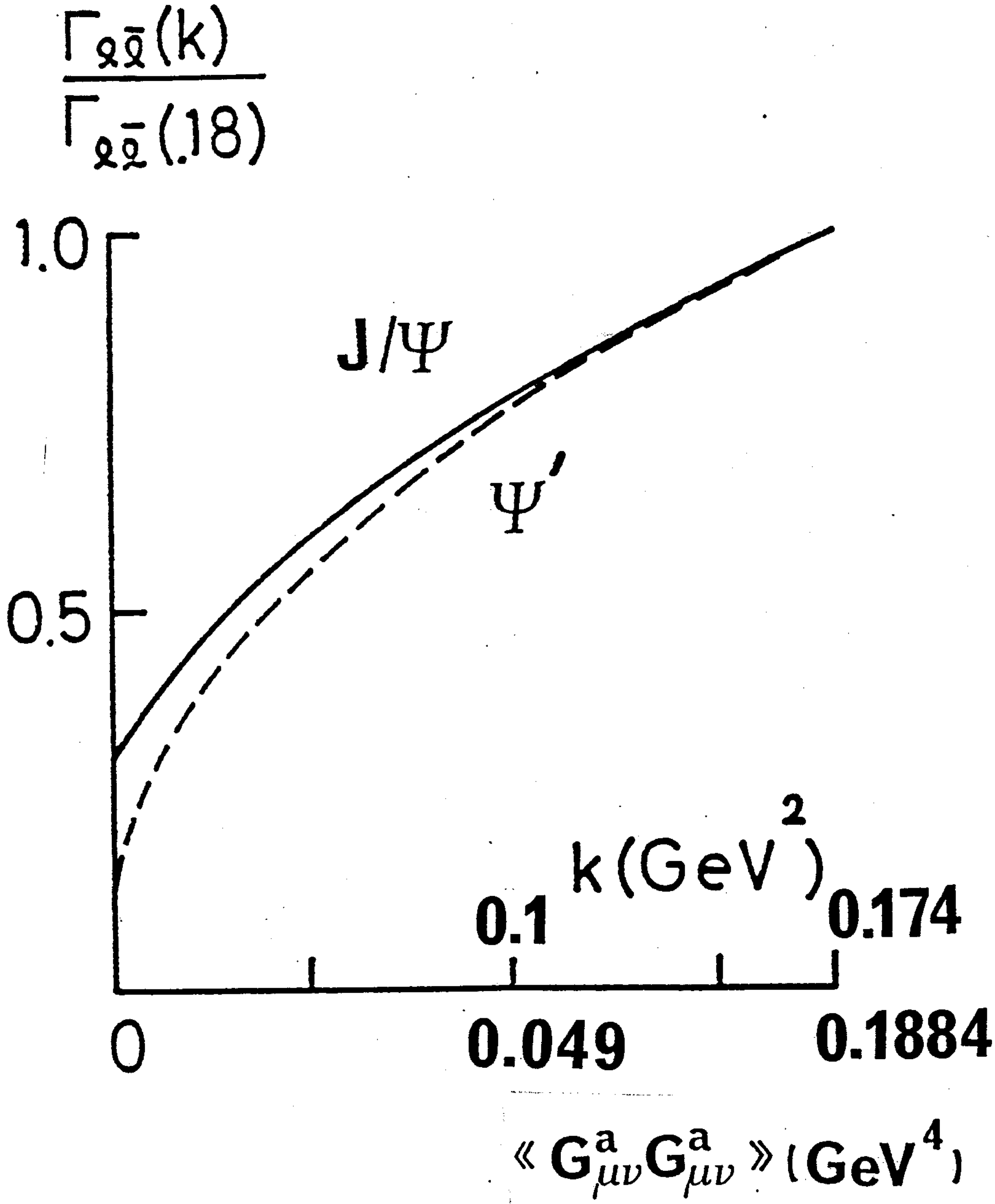

Mass(GeV)
3.6
3.4
3.2
3.0
2.8
Ψ′
J/Ψ
k(GeV²)
0.1
0.174
0
0.049
0.1884
« $G^a_{\mu\nu}G^a_{\mu\nu}$ » (GeV⁴)

Fig.5a

Fig 5a



$\frac{\Gamma_{\ell\bar{\ell}}(k)}{\Gamma_{\ell\bar{\ell}}(.18)}$
1.0
0.5
J/Ψ
Ψ'
k (GeV^2)
0.1
0.174
0
0.049
0.1884
« $G^a_{\mu\nu}G^a_{\mu\nu}$ » (GeV^4)

Fig. 5b

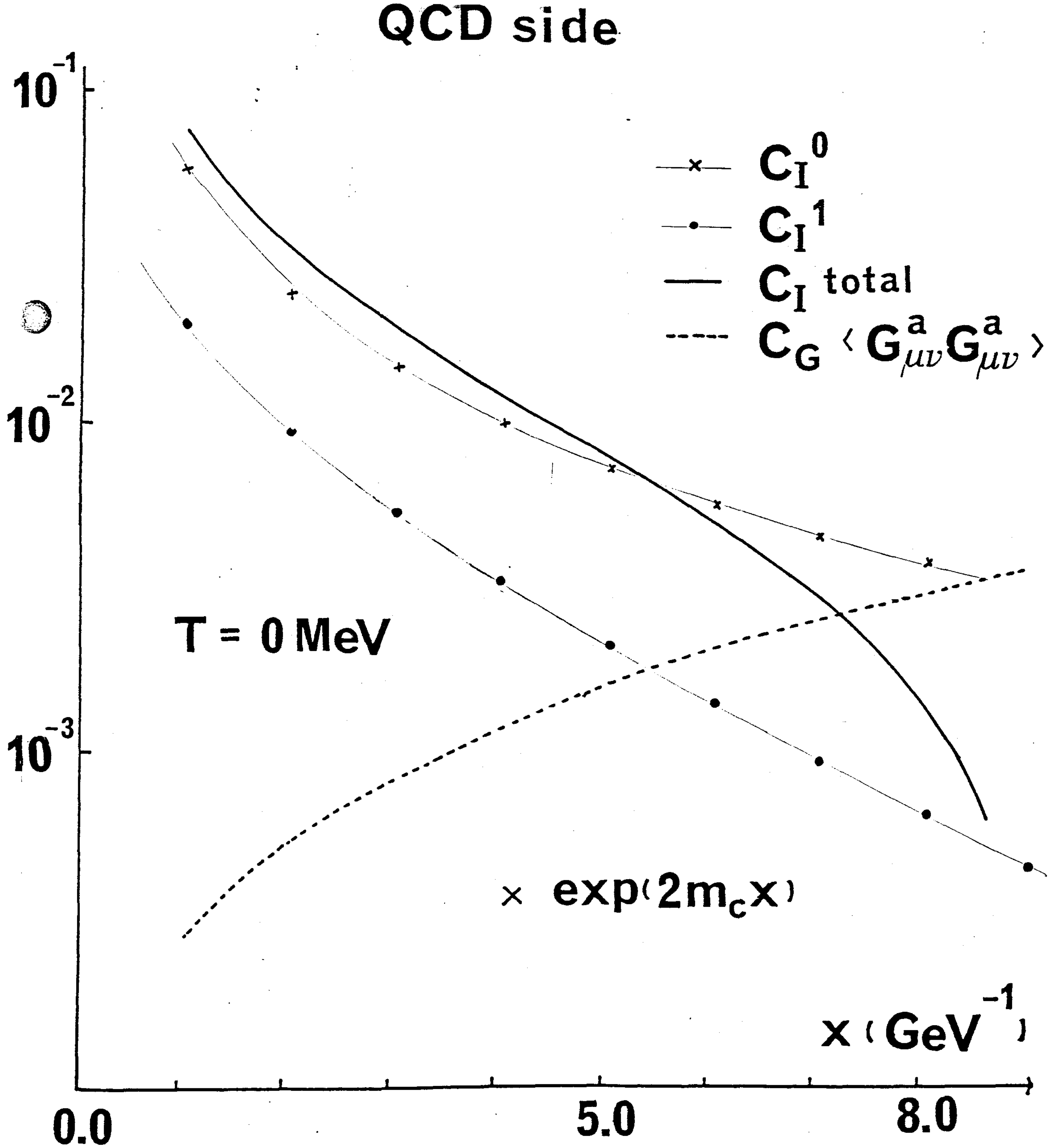

QCD side
10^{-1}
10^{-2}
10^{-3}
C_I^0
C_I^1
C_I total
$C_G \langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle$
T = 0 MeV
× exp(2m_c x)
x (GeV^{-1})
0.0
5.0
8.0

Fig. 6a

Fig 6a

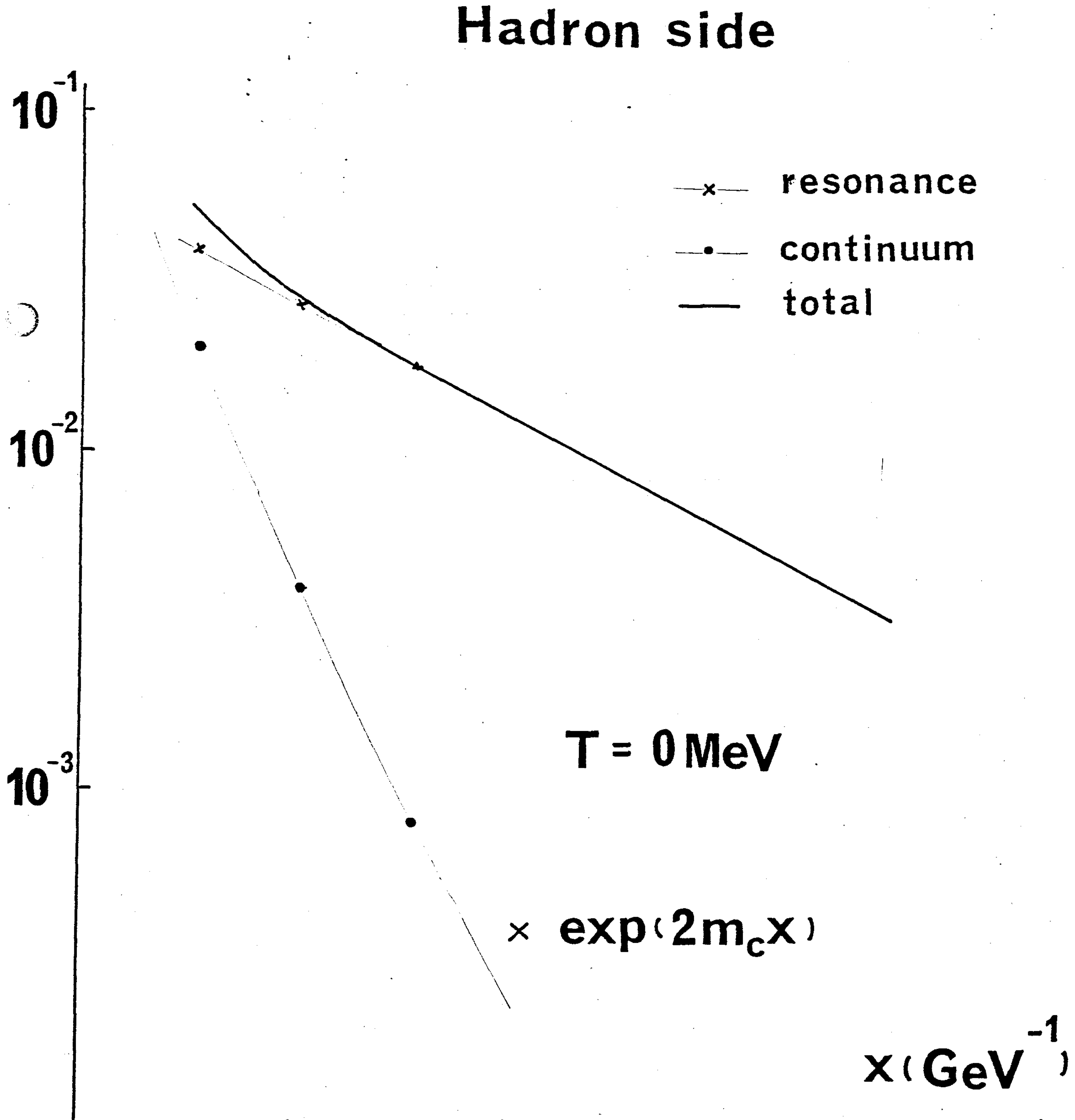

Hadron side
10^{-1}
10^{-2}
10^{-3}
resonance
continuum
total
T = 0 MeV
× exp(2m_cx)
x (GeV^{-1})
0.0
5.0
8.0

Fig. 6b

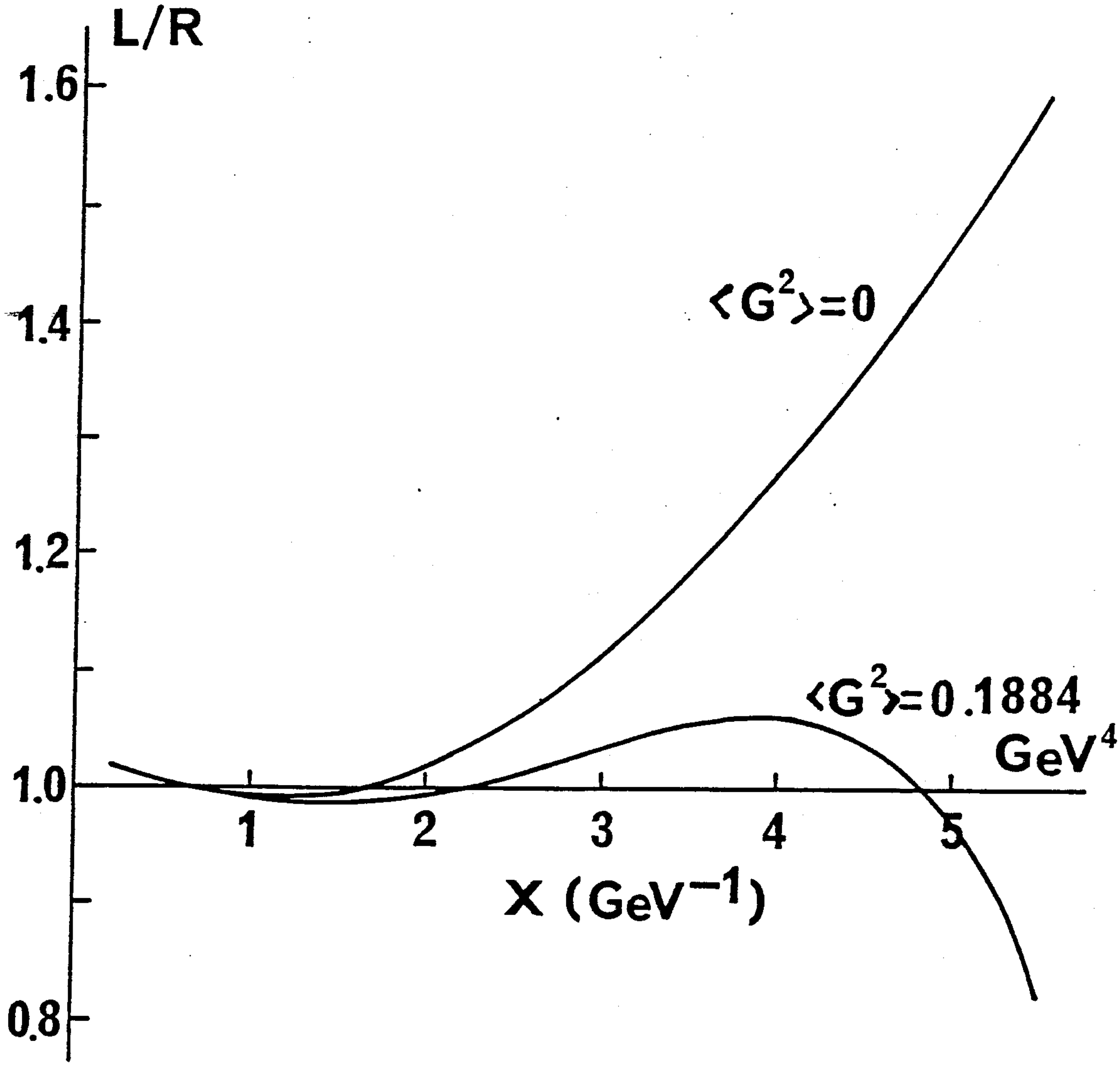


Fig. 7

Fig 7

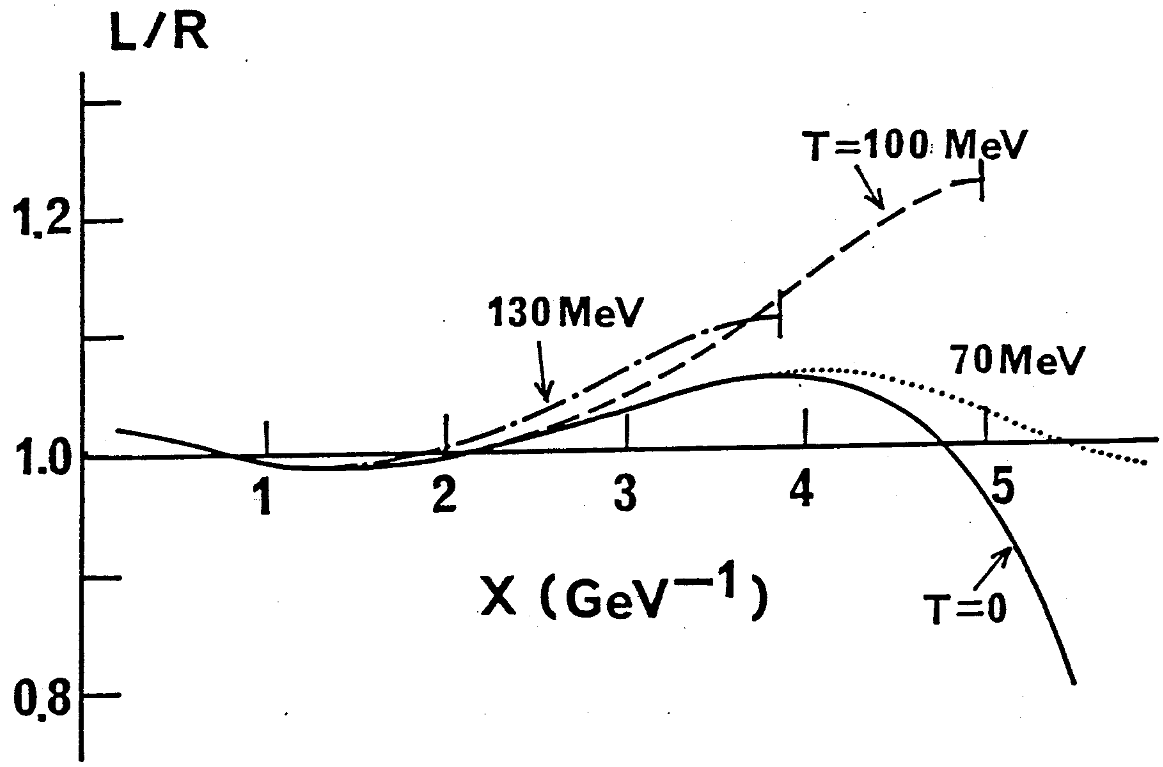


Fig. 8

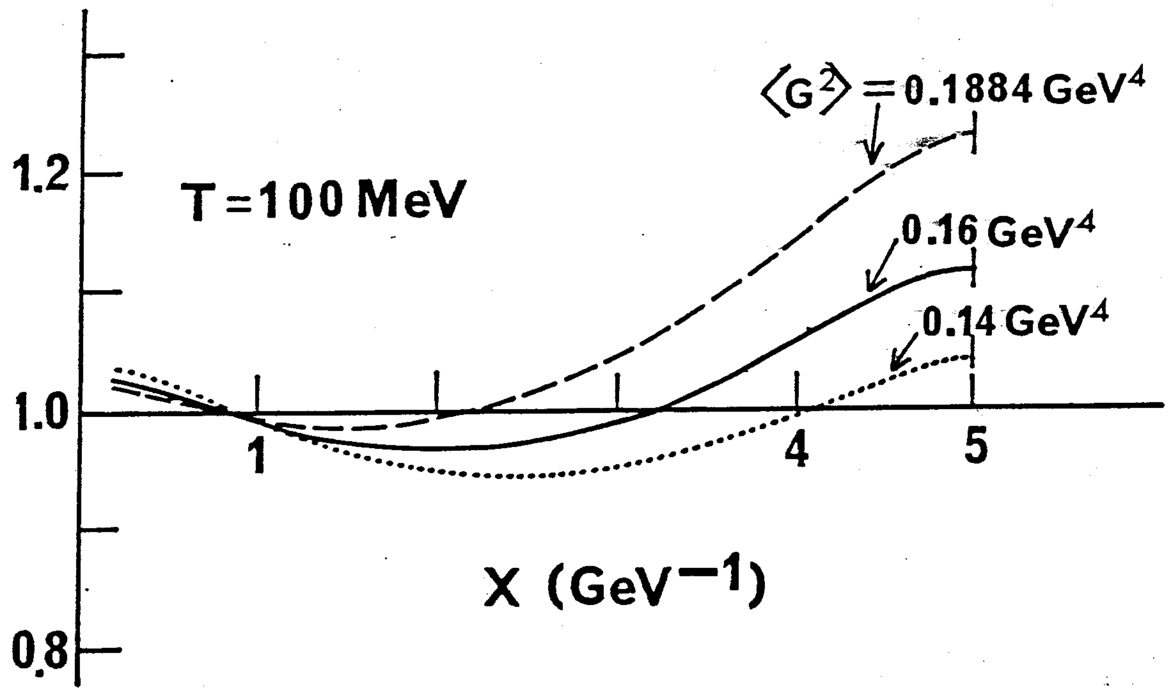
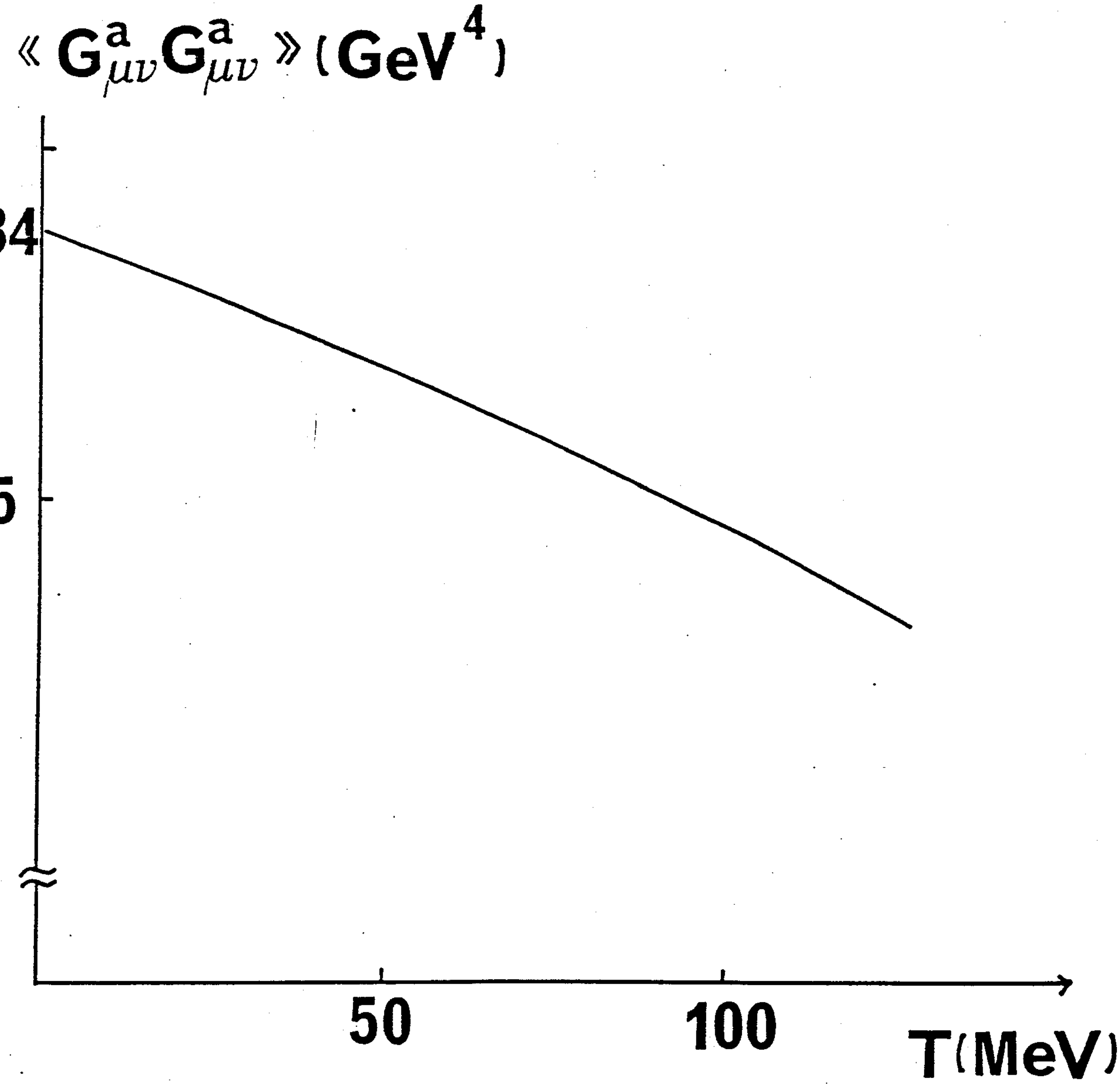


Fig. 9



$\langle\langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle\rangle$ (GeV^4)
0.1884
0.15
50
100
T(MeV)

Fig.10

Fig 10.

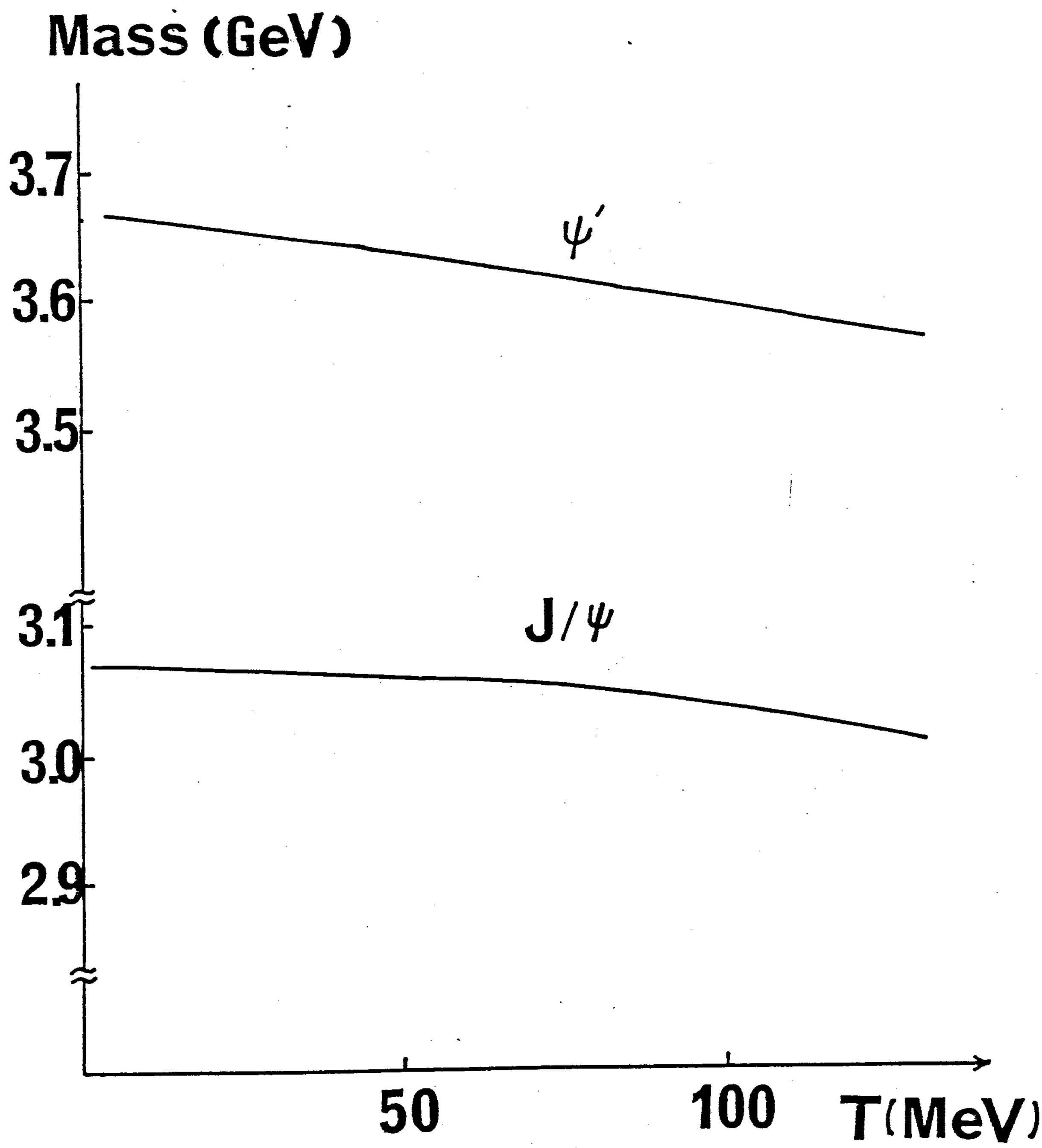


Fig.11a

Fig 11a

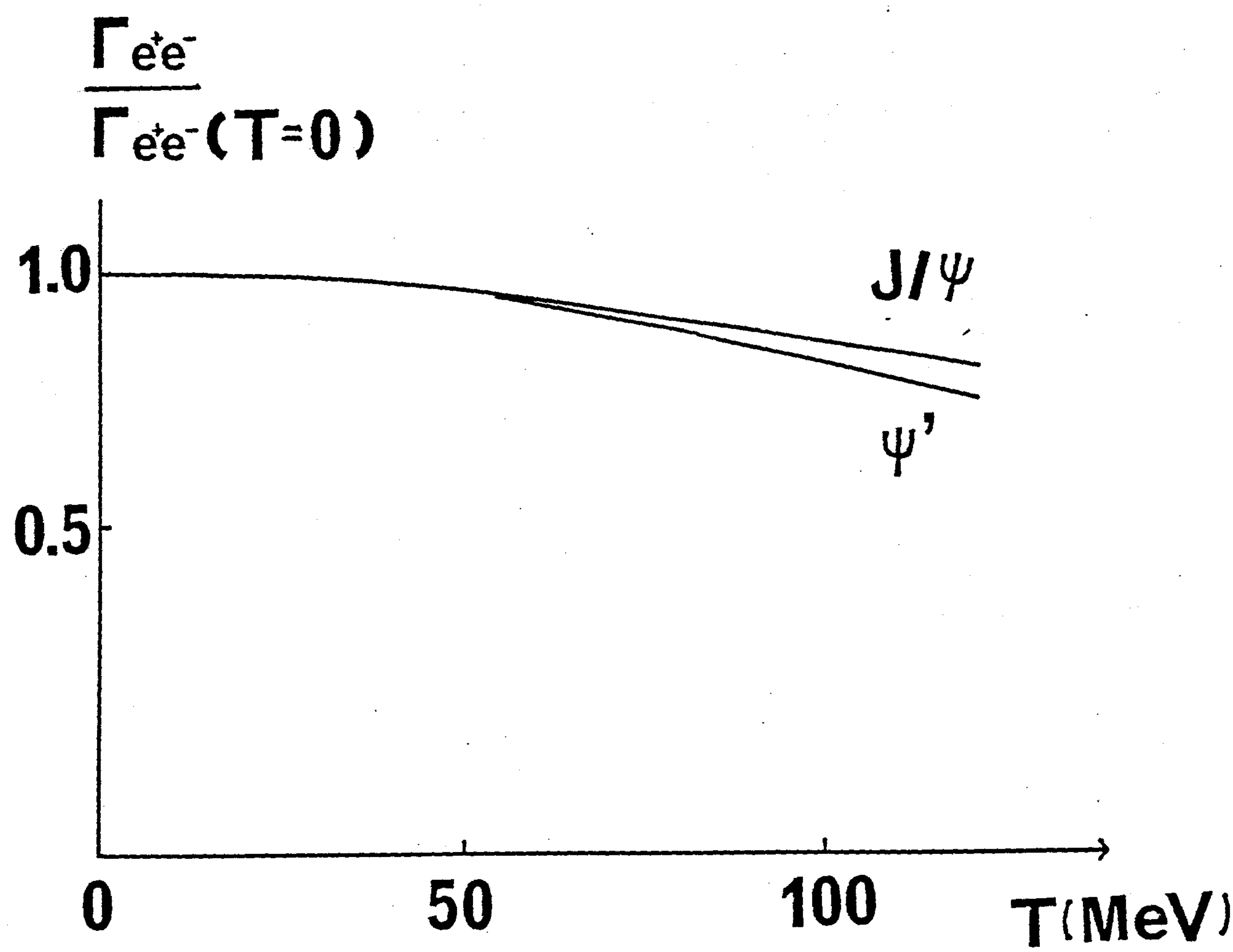


Fig.11b

Fig 11b

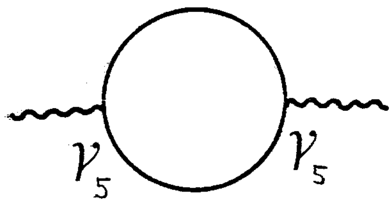


Fig. 12

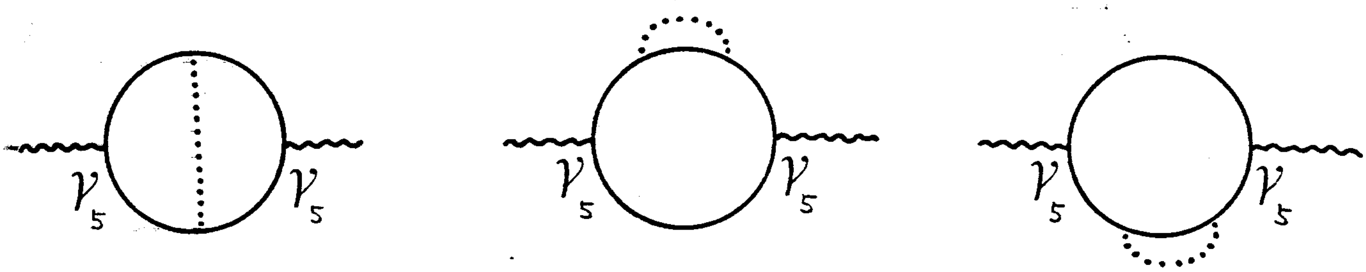


Fig. 13

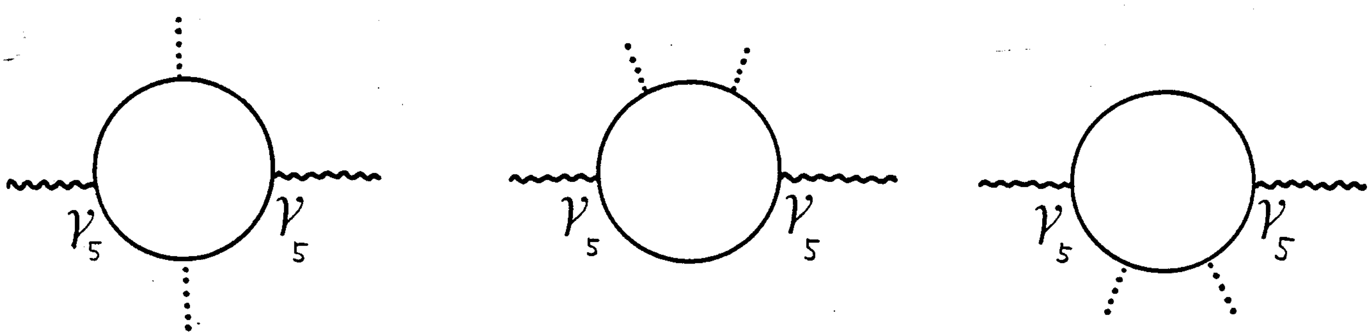


Fig. 14

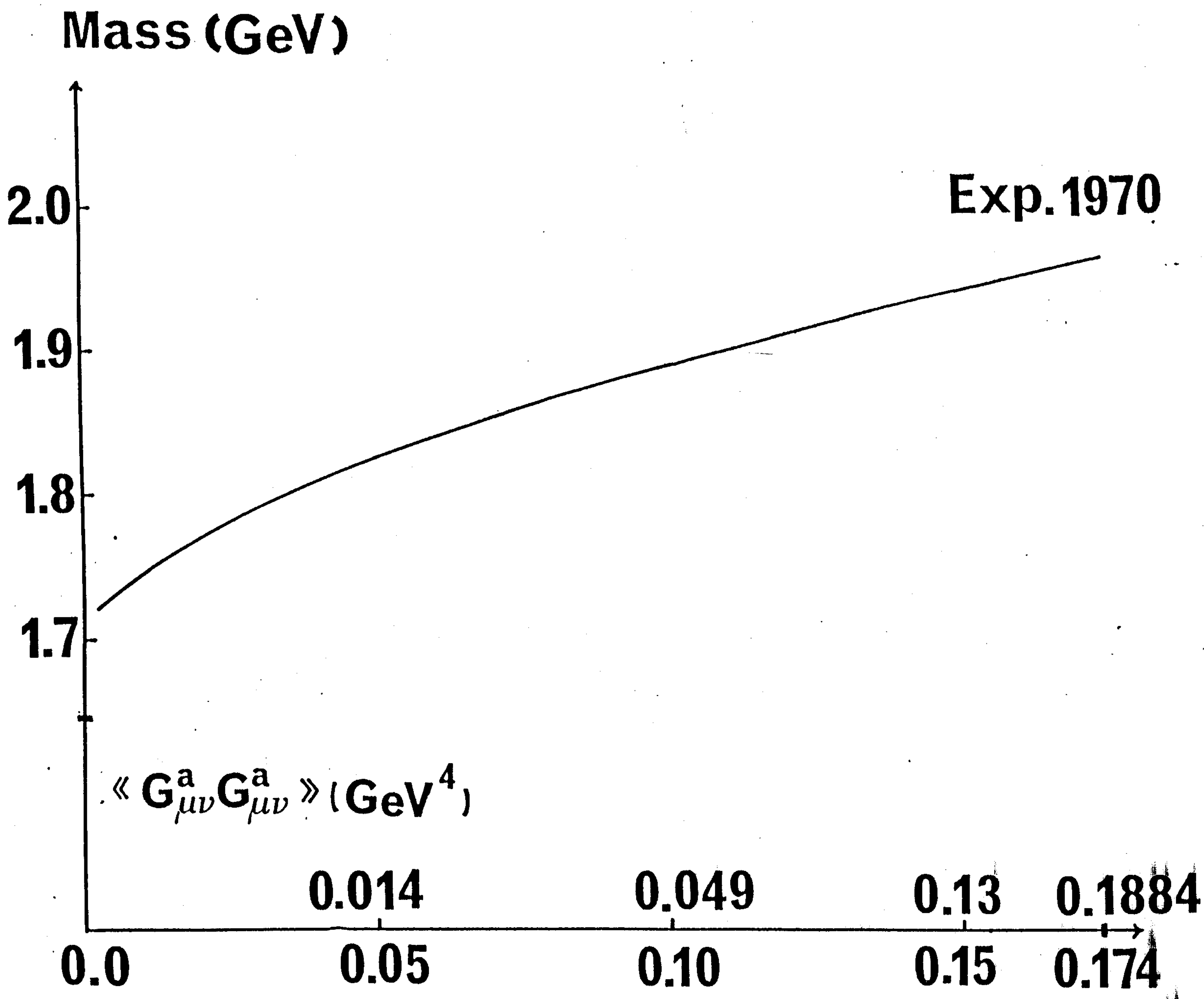

Mass (GeV)
2.0
1.9
1.8
1.7
Exp. 1970
« $G^a_{\mu\nu}G^a_{\mu\nu}$ » (GeV^4)
0.014
0.049
0.13
0.1884
0.0
0.05
0.10
0.15
0.174
k (GeV^2)

Fig. 15a

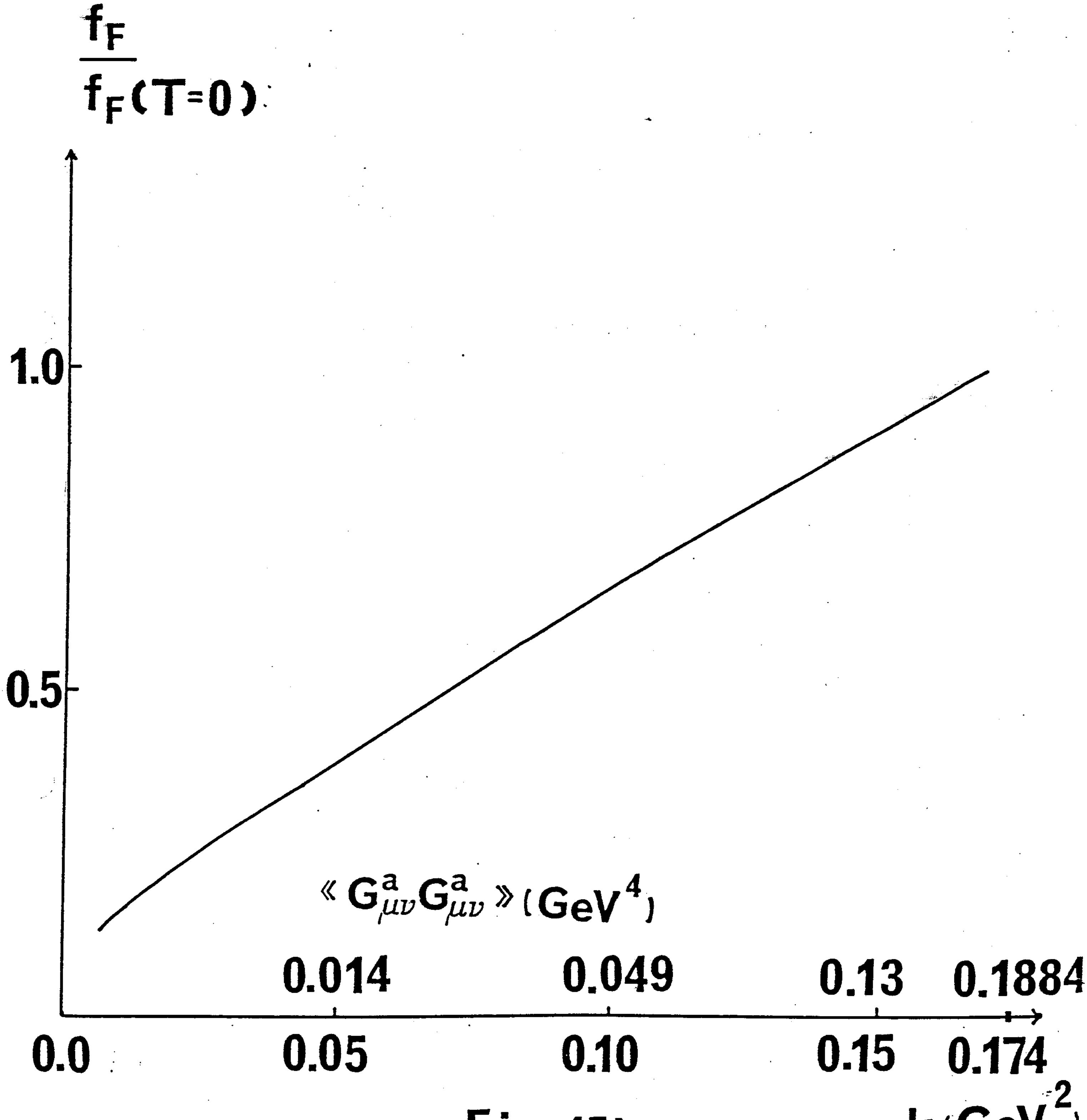
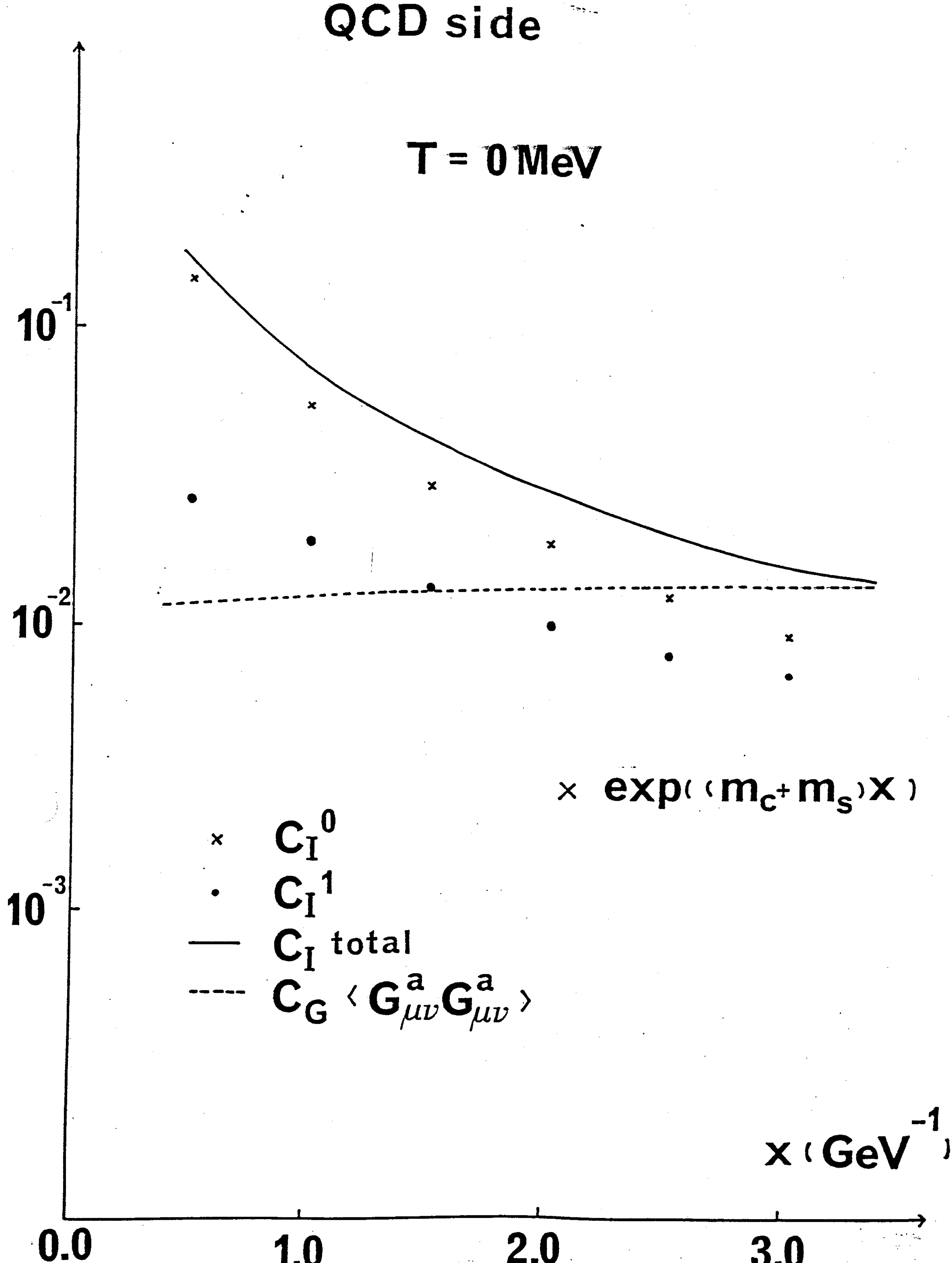


Fig.15b

Fig 15b



QCD side
T = 0 MeV
10^{-1}
10^{-2}
10^{-3}
× exp((m_c+m_s)x)
C_I^0
C_I^1
C_I total
$C_G \langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle$
x (GeV^{-1})
0.0
1.0
2.0
3.0

Fig. 16a

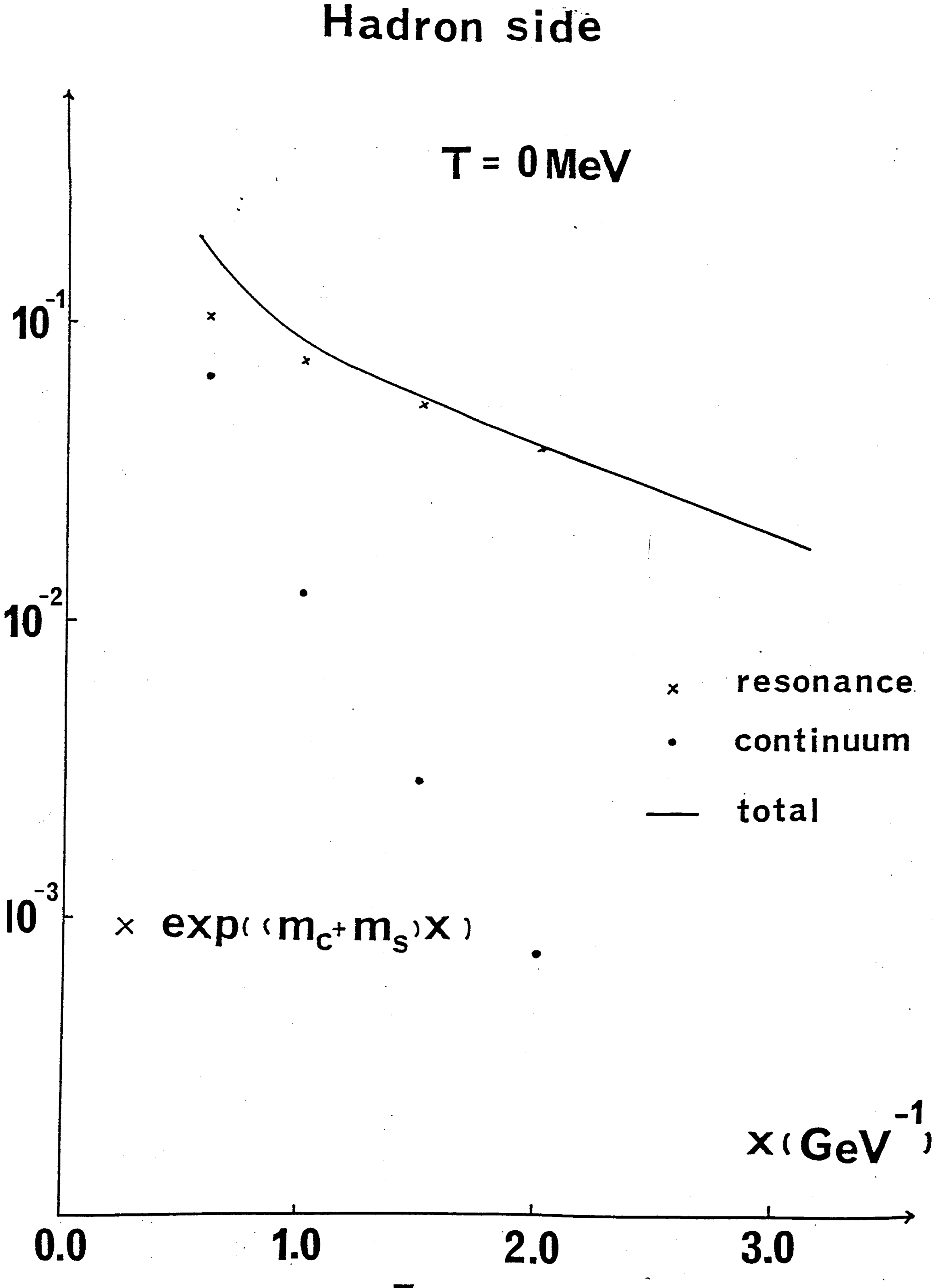

Hadron side
T = 0 MeV
10^-1
10^-2
10^-3
resonance
continuum
total
× exp((m_c+m_s)x)
x (GeV^-1)
0.0
1.0
2.0
3.0

Fig. 16b

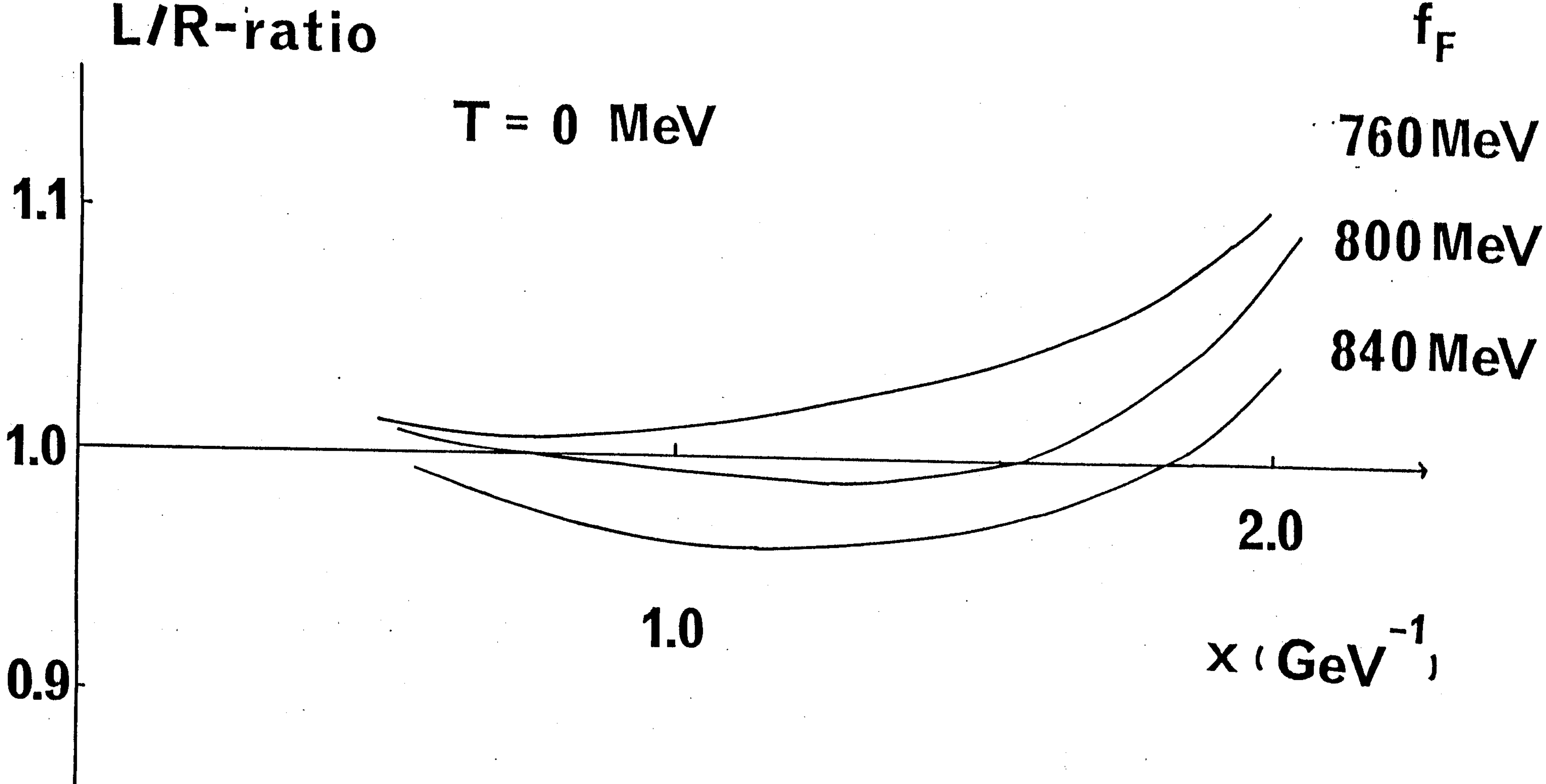


Fig.17

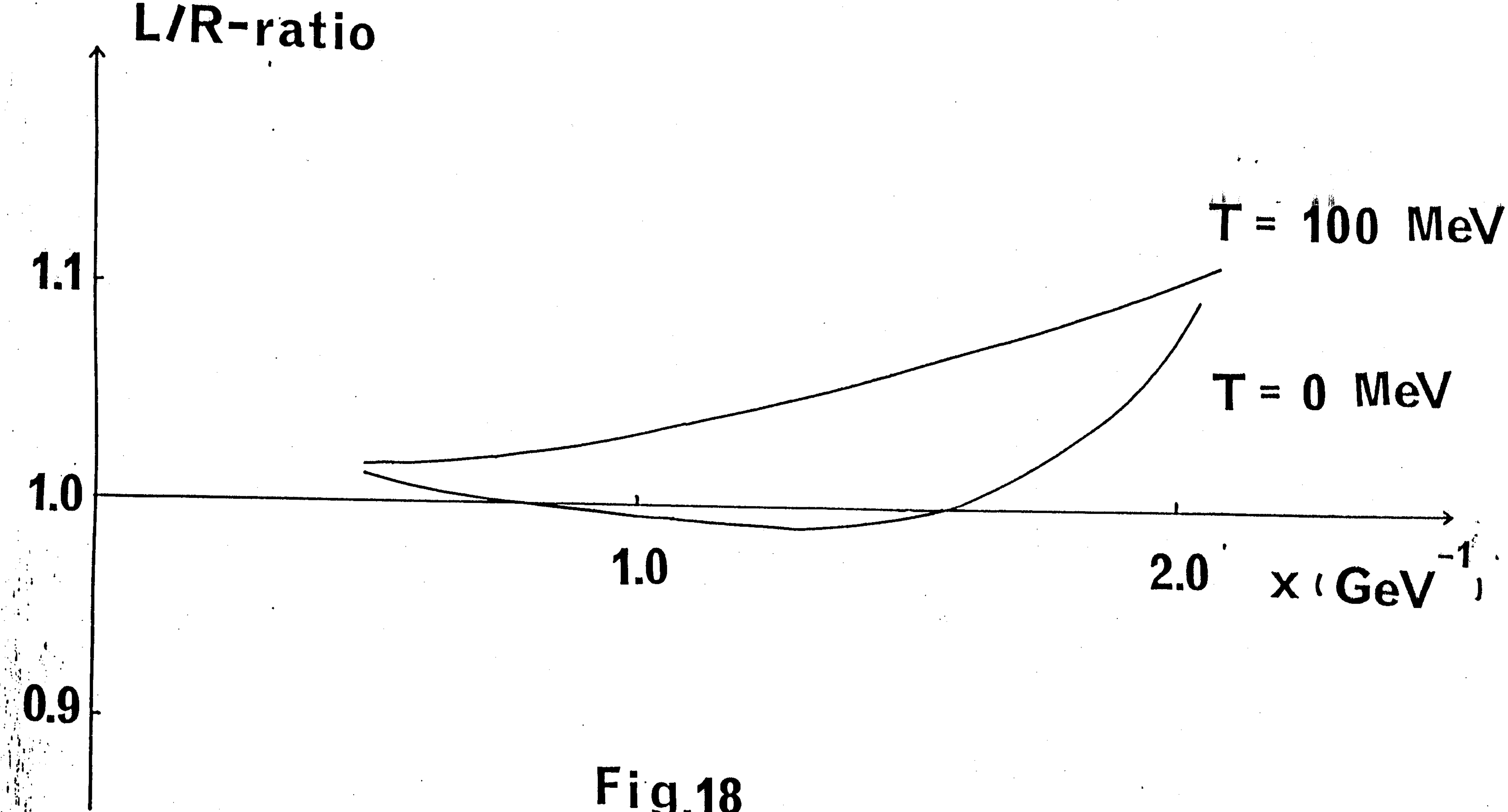


Fig.18

Fig 18

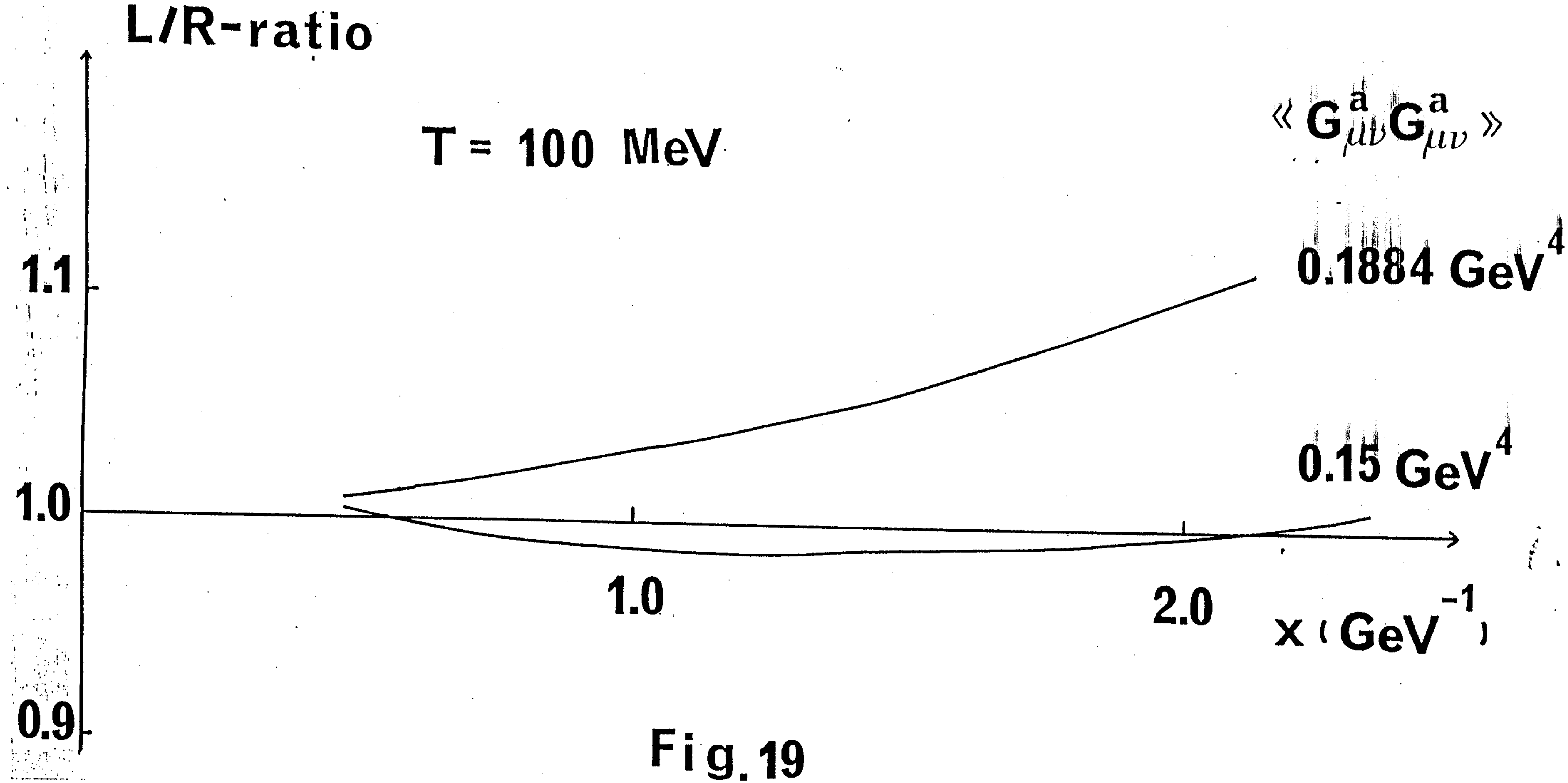

L/R-ratio
T = 100 MeV
$\langle\langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle\rangle$
0.1884 GeV^4
0.15 GeV^4
1.1
1.0
0.9
1.0
2.0
x (GeV^{-1})

Fig. 19

Fig 19

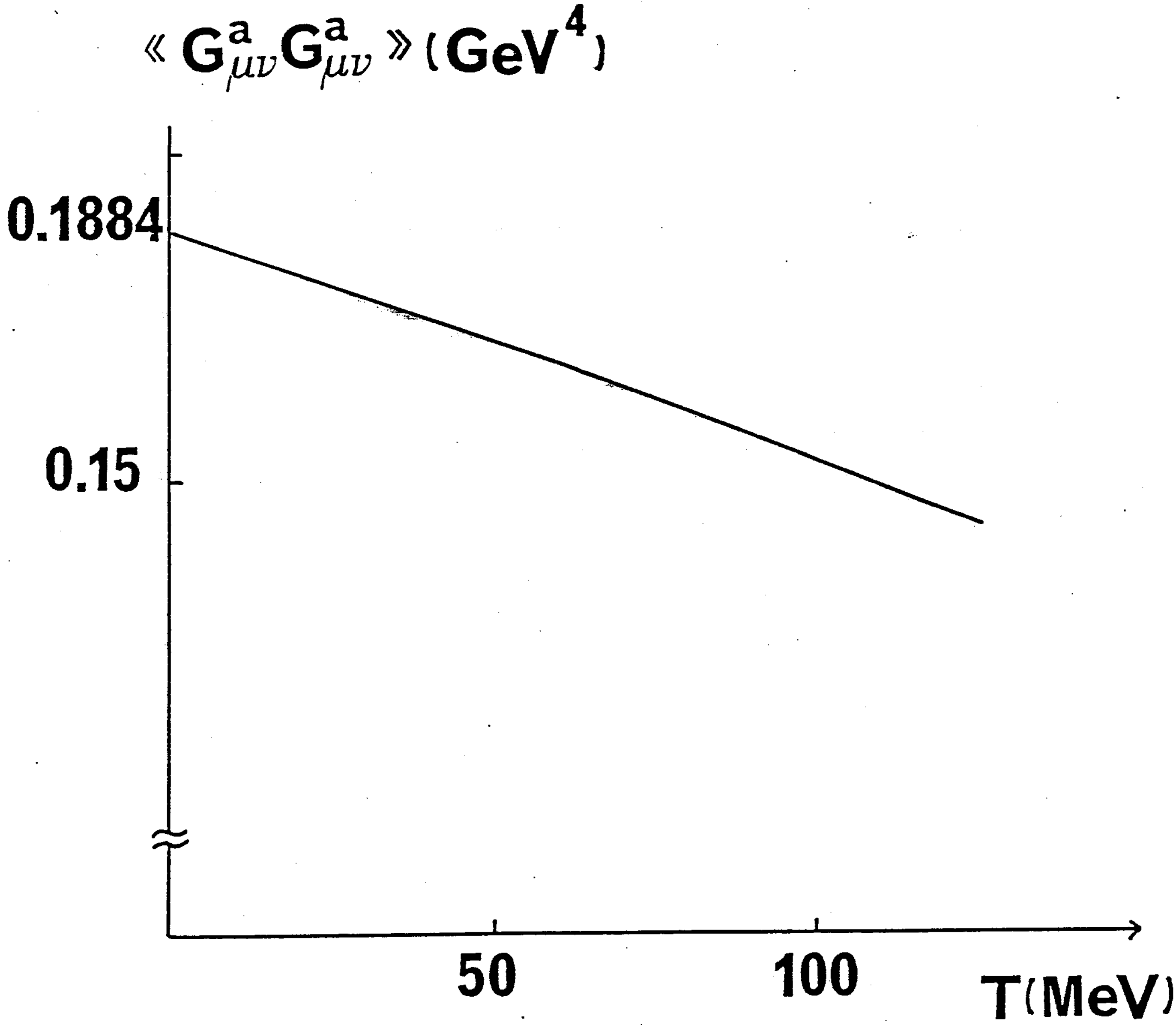


Fig. 20

Fig 20

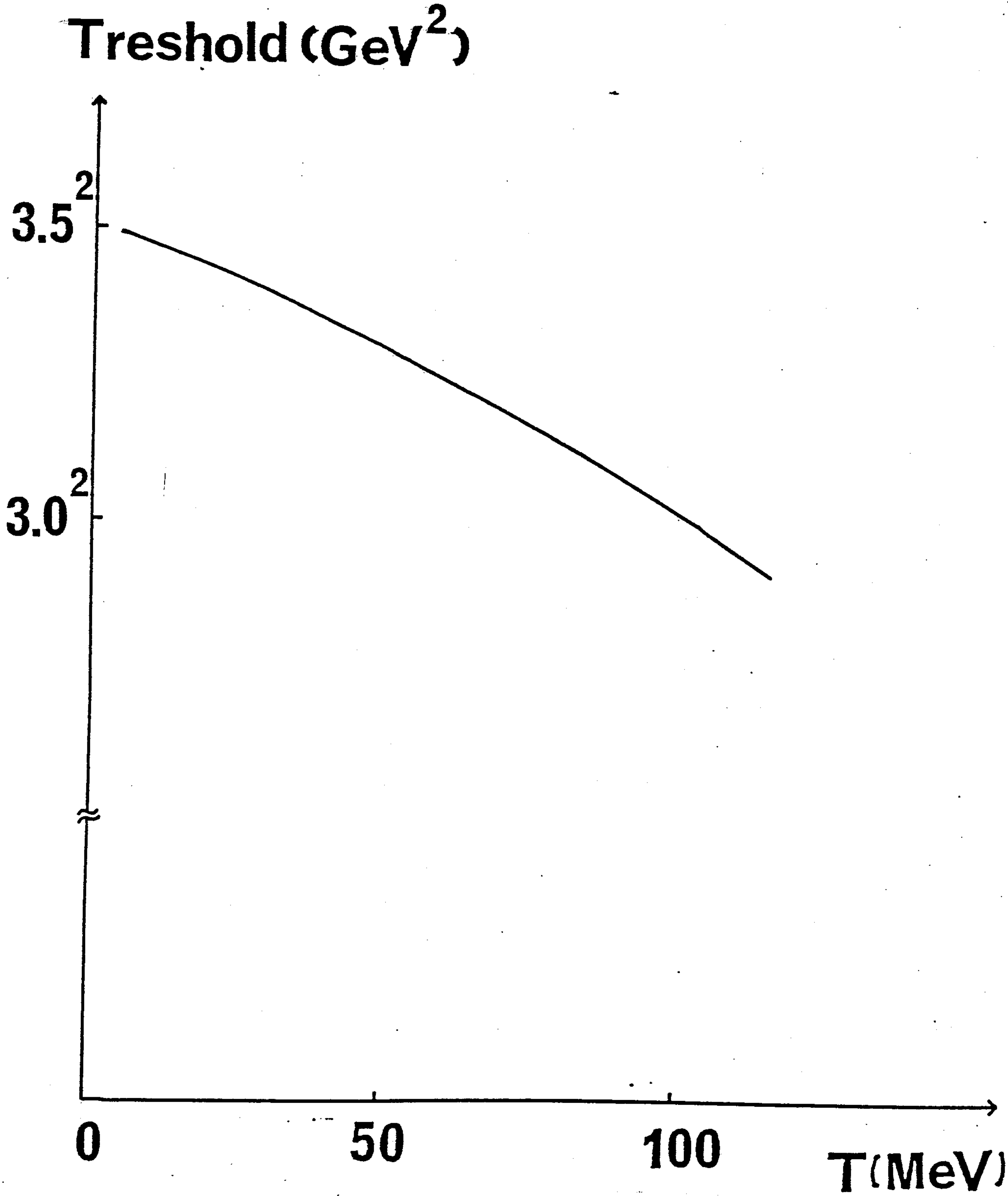


Fig. 21

Fig 21

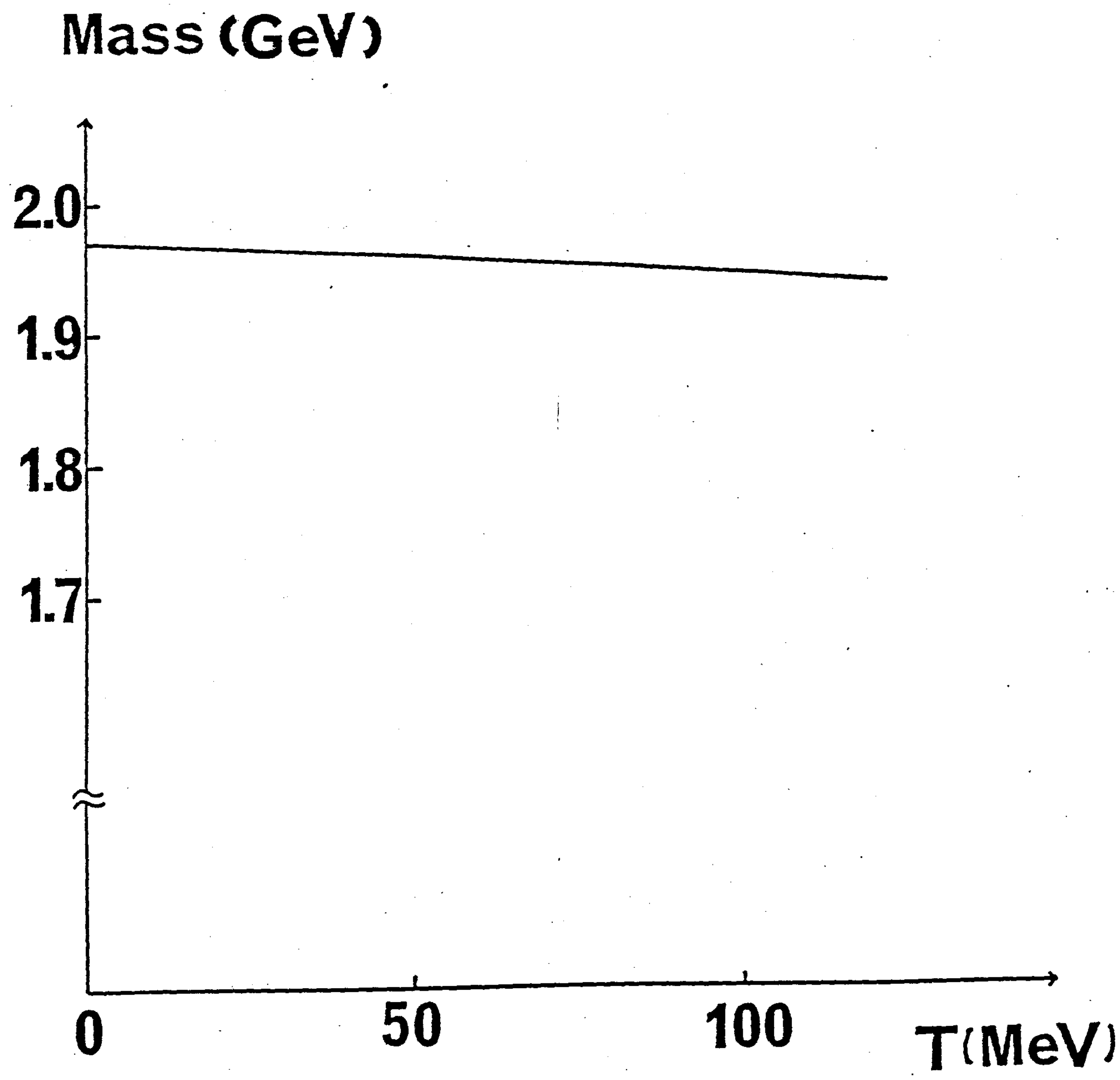


Fig. 22a

Fig 22a

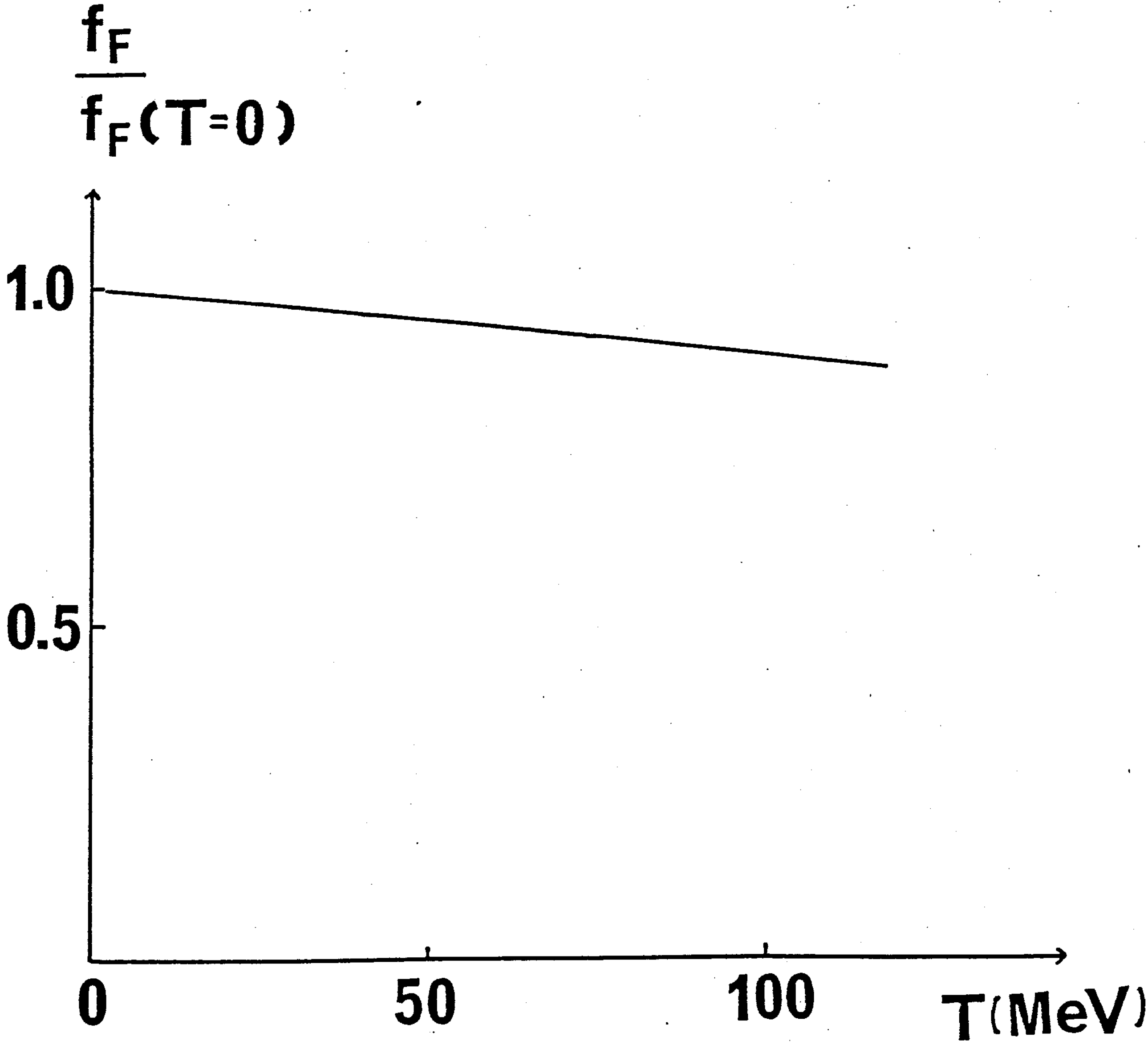


Fig.22b

Fig.22b