Vol.57 No.4 OSAKA ECONOMIC PAPERS March 2008

Market Maker's Price–correction Behavior towards Synchronization Risk[*]

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Abstract

This paper presents a model that explains why asset bubbles collapse despite any delay in arbitrage. An endogenous mechanism for bubble bursting that persists under synchronization risk is examined. Previous studies have shown that bubbles under synchronization risk collapse when a market impact is supplied. This analysis argues that the bubble may collapse before this time when a market maker who maximizes expected profits is considered. In other words, the bubble collapses because of the market maker's price–correction behavior before full arbitrage. Accordingly, the market maker's behavior in maximizing his or her own expected profit contributes to the bubble bursting earlier than expected.

Keywords: Bubble, Delayed Arbitrage, Synchronization Risk
JEL Classification: G10

1. Introduction

Bubbles persist for long periods. During this time, many market participants argue that stock prices are too highly valued and they share the opinion that stock prices have deviated from their fundamentals. Nevertheless, many traders still do not sell their stock holdings. In other words, bubbles persist because the market impact is not supplied for long periods. The object of this paper is to analyze the mechanism behind the bubble bursting. Put more concretely, we analyze the way a market maker finds the occurrence of a bubble before the time when the market impact is not supplied.

In general, positive bubbles persist when arbitrage is limited. The efficient market hypothesis suggests that bubbles do not exist in a market with rational arbitrageurs because arbitrage is unlimited. Under the efficient market hypothesis, all arbitrageurs receive fundamental information at the same

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* The author is grateful to Yoshiro Tsutsui, Shinsuke Ikeda, Masamitsu Ohnishi, Wataru Ohta, Katu Hata, Yuichi Fukuta, Masao Nakagawa, and Naoki Watanabel for their valuable comments. A Japanese language version of this paper was presented at the 2005 Autumn Meeting of the Japanese Economic Association at Chuo University on 18 September 2005, and the author gratefully acknowledges Katsumi Matsuura for his remarks as discussant. This paper was also presented at Monetary Economic Workshop (MEW) at Osaka University on 22 October 2005, and the author gratefully acknowledges Yuichi Abiko, Atsuo Fukuda, Hirofumi Uchida, and all other participants for their valuable comments. All errors are solely mine.

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time. As result, arbitrageurs wish to sell when the market stock price exceeds the fundamental stock price: the stock price will fall towards the fundamental price.

On the other hand, Abreu and Brunnermeier (2003) prove that bubbles can exist in markets where rational arbitrageurs exist. (Hereafter, Abreu and Brunnermeier's (2002) and (2003) models are referred to as AB (2002) and AB (2003), respectively.) AB (2003) consider a model in which accurate fundamental information is sequentially received by the arbitrageurs. Their model assumes that each arbitrageur cannot know how many arbitrageurs have already received this fundamental information. Because of this, each arbitrageur cannot know whether the bubble will collapse or not through their own selling order.

AB (2003) define this arbitrage strategy as 'timing the bubble'. The arbitrageur also has the possibility of obtaining capital gain by a delay in selling. There is also the prospect of a capital loss should the bubble collapse before selling. Therefore, each arbitrageur calculates their subjective probability of whether the bubble collapses before the next time period. The arbitrageur then calculates the expected value of the capital gain and loss using this subjective probability and does not sell out of the stock while the expected profit remains positive: that is, the difference between the expected value of the capital gain and loss. In other words, arbitrageurs cannot synchronize their sales because the fundamental information is not common knowledge and bubbles persist because arbitrageurs time the bubble. AB (2002) define this risk of a bubble's persistence as 'synchronization risk'. Their model assumes that bubbles collapse when selling orders reach a threshold. As the threshold is exogenously determined, it does not depend on any endogenous pricing mechanism, such as the market maker's pricing rule or each trader's behavior.

There are two extensions in our model. First, this paper explicitly considers the process of trading. The trading players comprise one market maker, arbitrageurs, and noise traders. We also explicitly consider the market maker's price–setting rule. It is assumed that the market maker cannot receive any fundamental information. In addition, we assume that the market maker calculates the subjective probability of whether the bubble exists and sequentially updates her belief at each time using Bayes Rule. At the time when the expected profit of the market maker becomes negative, she corrects the stock price in favor of the fundamental price and the bubble collapses. Second, we consider three types of noise traders: a market where pessimists and arbitrageurs exist; a market where neutral noise traders and arbitrageurs exist; and one where optimists and arbitrageurs exist. We then compare the difference in timing when the market maker corrects the mispricing in each type of market.

The conclusions of this paper are summarized by the following three points. First, the market maker corrects the price when the market impact is supplied, and this is fastest in the market where arbitrageurs and optimists exist. Second, the market maker may correct the price when not enough market impact is supplied. Third, the market maker's price–correction behavior does not occur when there is no market impact.

The remainder of the paper is organized as follows. Section 2 introduces the extant literature relating to our paper. Section 3 describes our model. In Section 4, we analyze the model equilibrium. Section 5 concludes.

2. Related Literature

Bubbles persist because of limits to arbitrage. One possible explanation for these limits to arbitrage is that noise traders take opposing positions to arbitrageurs. De Long et al. (1990) and Shleifer and Vishny (1997) consider this mechanism. De Long et al.'s (1990) model assumes that arbitrageurs' short horizon towards noise traders and risk aversion causes the limits to arbitrage. Alternatively, Shleifer and Vishny (1997) refer to a model where the bubble is caused by noise traders. Another explanation of limits to arbitrage is that it is caused by synchronization risk. AB (2002) and AB (2003)[1] consider this mechanism in a continuous time setting. However, Sakawa and Watanabel (2006) provide a sufficient condition of the existence of synchronization risk and its existence is empirically shown by Brunnermeier and Nagel (2004).

A number of previous studies have considered why bubbles collapse. Blanchard (1979, 1982), for example, examines stochastic bubbles. Fukuta (1998) considers incompletely bursting bubbles, and shows that these include stochastic bubbles. In terms of market maker behavior, O'Hara (1997) and Biais et al. (2005) provide detailed surveys. Typically, there are two kinds of models that analyze market maker behavior. Initially, Kyle (1985) introduces a batch–clearing market where market makers, arbitrageurs, and noise traders coexist. Kyle's model analyzes how risk–neutral market makers extract the price information from the total net order flow. Kyle's model shows that the efficient price is achieved in the terminal trading period. On the other hand, Glosten and Milgrom (1985) introduce a continuous auction market. In their model, arbitrageurs are defined as traders who only receive fundamental information. In that market, the market maker sequentially changes the price by the order of arrival and whether it is a selling or buying order.

3. Model

In this section, we describe a model setting that extends AB (2003). In this model, there are arbitrageurs, noise traders, and a market maker. Subsection 3.1 introduces the discrete–time approximation model of AB (2003), and the arbitrageurs' optimal strategy is defined. Subsection 3.2 analyzes the information structure. Subsection 3.3 describes the differences between AB (2003) and the current model. Subsection 3.4 introduces the noise traders' behavior. Subsection 3.5 specifies the market maker's behavior.

3. 1. The Discrete–time Approximation Model of AB (2003)

The discrete–time approximation of AB's (2003) model is as follows. We represent discrete time as $0 = t_0 < t_1 < \dots < t_n$, and a length of one trading period as $\Delta(\Delta = t_{i+1} - t_i, i \in n)$. We consider a market where m risk–neutral arbitrageurs exist (m is a natural number). There is only one risky stock in the market. Each arbitrageur has $(1/m)$ units of stock. We define the sequence of stock prices as

[1] Chamley (2004) provides a useful survey of this work.

Ψ_i, and the sequence of fundamental prices as v_i. The interest rate on the risk–free (or safe) asset is r.

At time t_b, a positive shock occurs and a bubble emerges. Prior to time t_b, the price of the stock is equal to its fundamental price: $\Psi_i = v_i = \left(1+g\right)^{\{(i-b)\Delta\}} (i \le b)$, and the interest rate is $g(>r)$. In other words, the fundamental price is higher than the safe rate before time t_b. At time t_b, the shock occurs and the fundamental's growth rate is adjusted to the safe interest rate. Accordingly, the bubble emerges after time t_b: $\Psi_i = \left(1+g\right)^{\{(i-b)\Delta\}} > v_i = (1+r)^{\{(i-b)\Delta\}}$. This model treats time t_b as a random variable. The probability distribution function of t_b is a discrete–exponential function: $F(t_b) = 1 - \left(\frac{1}{2}\right)^b$.

It is assumed that there are only rational arbitrageurs in the market. After the shock at time t_b, the stock price deviates from the fundamental value. Between t_{b+1} and t_{b+m}, rational arbitrageurs become sequentially aware of the new fundamental value at a uniform rate. An individual arbitrageur who becomes aware of the change in fundamentals at time t_j believes that t_b is distributed between t_{j-m} and t_{j-1}. We denote this type of arbitrageur by j. If $b = j-1$, this arbitrageur is the first to realize that the fundamental value has deviated from the stock price. If $b = j-m$, all other traders have already received this information. Arbitrageur j does not know the precise timing of the shock. Therefore, the shock's information is not common knowledge among arbitrageurs. Arbitrageur j sells out their stock for an arbitrage profit after learning the shock's information. As the shock's information arrives at a uniform rate, the amount of arbitrageur selling orders gradually increases, but the selling is not synchronized.

The discrete version of AB (2003) assumes that mispricing is corrected at the time when the arbitrageurs' selling orders cross a threshold $k(<1)$; this time is $t_{b+\tau^*}$. The bubble persists as long as the arbitrageurs' selling pressure remains below k. We consider the symmetric equilibrium that all arbitrageurs take a symmetric strategy. Each arbitrageur j estimates the crash probability after noticing the mispricing. The arbitrageur can then calculate the expected profit of the capital gain when the bubble does not burst during the next time period, and the expected loss of holding a stock when the bubble does burst during the next time period. The optimal strategy of each arbitrageur j is to sell the stock when the expected profit of capital gain becomes equal to the expected loss of holding stock.

In this setting, two propositions are established as in AB (2003).

Proposition 1 *Trigger strategy: In equilibrium, arbitrageur j sells out the stock at time $t_{j+\tau^1}$ after noticing the shock and arbitrageur j does not buy back the stock. Arbitrageur j takes the optimal strategy when delaying selling the stock.*

Proposition 2 *Perfect Bayesian Nash Equilibrium: Each arbitrageur j holds a stock during the periods when the expected profit of capital gain is higher than the expected loss due to crash (arbitrage profit). They sell out stock at the time when their arbitrage profit becomes zero. Mispricing is corrected when an arbitrageur $j\left(j = b + \tau^* - \tau^1\right)$ sells out.*

The proofs of above two propositions are given as in Sakawa and Watanabel (2006).

3. 2. Information Structure

There are information asymmetries between arbitrageurs and the other players. In other words, fundamental information that concerns the accurate fundamental level is sequentially spread to arbitrageurs. The fundamental information is sequentially informed after time t_b.

The shock's occurrence at time t_b is commonly known to arbitrageurs, noise traders, and the market maker. Because fundamental information is not received by the noise traders and the market maker, they cannot judge whether the shock really falls to the fundamental price. Arbitrageurs who received the fundamental information cannot know that the stock really falls to the fundamental price.

Under the above information structure, the bubble does not burst at time t_b. The market maker does not know that the bubble bursting at time t_b is incorrect. Thus, the market maker has a belief that the price level is equal to the fundamental level. Table 1 shows each agent's information structure. ✓ signifies that the player has that information.

Table 1: Information Structure

Player	Fundamentals Information	Rule
Market maker		✓
Arbitrageurs	✓	
Noise traders		

Table 1 shows that the market maker and noise traders cannot receive the fundamental information. This model refers to a situation where arbitrageurs do not know about the market maker's price–setting rule. In reality, the market maker monopolizes book information in major markets such as the New York Stock Exchange. On this basis, asymmetric information between the market maker and other traders is a realistic assumption. As the market maker computes the relevant price based on book information, arbitrageurs do not know how the market maker sets the price.

3. 3. The Features of This Model

There are four unique settings in this paper not considered by AB (2003). First, noise traders are introduced into this model. It is assumed that one noise trader enters the market during one trading interval and buys the stock, sells the stock, or does not trade during the same trading interval.

Second, we explicitly consider the market maker's behavior. AB (2003) implicitly considers the market maker, and mispricing is corrected when the arbitrageur's selling orders cross a threshold k. On the other hand, this paper assumes that the market maker corrects the price upon noticing the arbitrageurs selling. Because the market maker knows that arbitrageurs have fundamental information, the market maker notices the mispricing through their selling behavior. The market maker who finds the arbitrageurs' selling recalculates the relevant price, $v_i = (1+r)^{\{(i-b)\Delta\}}$, and they notice that the previous price is too high. Therefore, the mispricing is corrected as early as possible to avoid the additional loss of carrying inventory. Noise traders do not discover the mispricing until the market maker corrects the price. The mispricing then becomes common knowledge. In addition, the market maker's revenue is the service charge for traders at each time, as in the following definition.

Definition 1 *The market maker observes an order sequence and corrects the price upon noticing the* *arbitrageurs selling. During periods where it is not noticed, mispricing is corrected when the expected loss of accepting the orders is higher than the expected profit. The market maker's revenue is the service charge for traders at each time. She receives a transaction cost at a charge of C per* $\left(\frac{1}{m}\right)$ *units of trading, and this transaction cost becomes her gain. Even if traders do not order, they have to pay the charge for participating in the market.*

Third, arbitrageurs have incorrect beliefs about the market maker's price–setting rule. The next definition represents the arbitrageurs' beliefs.

Definition 2 *Arbitrageurs believe that the market maker corrects the price when the total selling orders cross a threshold* k *after time* t_b.

Finally, it is assumed that agents who order at each time in this market include one arbitrageur and one noise trader. Traders (including both arbitrageurs and noise traders) who trade once exit the market. In this market, orders at each time are executed when an arbitrageur orders the opposite side against a noise trader or market maker's inventory. If the order at one time is not executed, the order becomes the market maker's inventory the next time.

3. 4. Noise Traders' Behavior

This model defines noise traders as follows. It is assumed that the population of noise traders at each time is one. Each noise trader has a $\left(\frac{1}{m}\right)$ unit of stocks, and noise traders who order exit the market.

There are three types of noise trader, namely, optimists, neutral noise traders, and pessimists. This paper analyzes differences in the time of price correction among the markets where the different types of noise traders exist. A noise trader's behavior is as follows: $p_S^N(k)$ represents the probability that noise trader sells at time t_k, and $p_B^N(k)$ is the probability that the noise trader buys at time t_k.

$$\left(p_S^N(k), p_B^N(k)\right) = \left(\frac{2}{3} - p, p\right), \left(p \in \left(0, \frac{2}{3}\right)\right)$$

The optimist has a probability that satisfies $p_B^N(k) > p_S^N(k)$. The pessimist has a probability that satisfies $p_B^N(k) < p_S^N(k)$. The neutral noise trader has a probability that satisfies $p_B^N(k) = p_S^N(k) = \frac{1}{3}$. In other words, neutral noise traders place orders randomly.

3. 5. The Market Maker's Behavior

In this model, we assume the batch auction systems in each time interval and a monopolistic market maker suspects the mispricing at time t_{b+1} when the first arbitrageur $b+1$ receives the fundamental information. The market maker can know the shock at time t_b, but cannot know whether the fundamental level is pushed down by the shock or not. The market maker has an incentive to know the difference between the mispricing and fundamental price because she cannot receive the fundamental information. The market maker observes an order sequence at each time, and extracts the information whether the arbitrageur sells or not. In each time interval, the market maker gathers the aggregate orders in the market. In other words, this market adopts batch auction systems and consists of discrete and $(n+1)$ periods.

In this market, arbitrageurs take the symmetric trigger strategy defined in Proposition 1. We define binary state space $\partial \in (0,1)$. $\partial = 1$ means that each arbitrageur j sells out at time, and $\partial = 0$ means that arbitrageur j does not sell out at time t_j. This model defines a net trading selling order's quantity at time t_k as a signal $s_k (0 < t_k < t_n)$.

The market maker updates her belief, observing the net trading quantity at each trading time. We define the market maker's subjective probability as whether the arbitrageurs sell as per their belief. The market maker updates this belief with Bayes Rule. The market maker has a belief $(Pr_{k+1}(\partial = 1 \mid s_k))$ at each next time t_{k+1} after arbitrageurs receive the signal s_k. The market maker notices the mispricing when her belief reaches 1. This binary model is expressed in Table 2.

Table 2: Binary Model

$Pr_k(s_k \mid \partial)$	$s_k = 1$	$s_k = 0$
$\partial = 1$	p	$1-p$
$\partial = 0$	$1-q$	q

The market maker's belief is updated as below.[2]

$$\frac{Pr_{k+1}(\partial = 1 \mid s_k)}{Pr_{k+1}(\partial = 0 \mid s_k)} = \frac{Pr_k(\partial = 1)}{Pr_k(\partial = 0)} \frac{Pr_k(s_k \mid \partial = 1)}{Pr_k(s_k \mid \partial = 0)} \tag{1}$$

Bayes Rule is the same as the Log Likelihood Ratio (LLR) updating method when the number of states is finite.[3] We define the LLR at time t_k as λ_k. λ_k is defined as the LLR at the time before receiving a signal s_k.

$$\lambda_k = \ln \frac{Pr_k(\partial = 1)}{Pr_k(\partial = 0)}$$

λ_{k+1} is similarly defined as the LLR at time after receiving a signal s_k.

$$\lambda_{k+1} = \ln \frac{Pr_{k+1}(\partial = 1 \mid s_k)}{Pr_{k+1}(\partial = 0 \mid s_k)}$$

The LLR's updating equation is as follows: the LLR's multiplier $\ln\left(\frac{p}{1-q}\right)$ represents the LLR's variation.

$$\lambda_{k+1} = \lambda_k = \ln \frac{Pr_k(s_k = 1 \mid \partial = 1)}{Pr_k(s_k = 1 \mid \partial = 0)} = \lambda_k + \ln\left(\frac{p}{1-q}\right) \tag{2}$$

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[2] Using the property of Bayes Rule, the following two equations are established:

$$\frac{Pr_{k+1}(\partial = 1, s_k)}{Pr_{k+1}(\partial = 0, s_k)} = \frac{Pr_{k+1}(\partial = 1 \mid s_k)}{Pr_{k+1}(\partial = 0 \mid s_k)} \frac{Pr(s_k)}{Pr(s_k)} = \frac{Pr_{k+1}(\partial = 1 \mid s_k)}{Pr_{k+1}(\partial = 0 \mid s_k)}$$

$$\frac{Pr_{k+1}(\partial = 1, s_k)}{Pr_{k+1}(\partial = 0, s_k)} = \frac{Pr_k(s_k \mid \partial = 1)}{Pr_k(s_k \mid \partial = 0)} \frac{Pr_k(\partial = 1)}{Pr_k(\partial = 0)}$$

[3] Chamley (2004) refers to this method.

The exponential of the LLR above means the ratio of the market maker's belief.

$$\exp(\lambda_k) = \frac{Pr_k\,(\partial = 1)}{Pr_k\,(\partial = 0)} \tag{3}$$

The equation above is equivalent to the following equation.

$$Pr_k\,(\partial = 1) = \exp(\lambda_k)\,Pr_k\,(\partial = 0) = \exp(\lambda_k)\big(1 - Pr_k\,(\partial = 1)\big)$$

We can calculate the market maker's belief at time t_k by solving the above equation.

$$Pr_k\,(\partial = 1) = \frac{\exp(\lambda_k)}{1 + \exp(\lambda_k)}$$

Now, we explain the market maker's belief at time t_k.

$$\big(Pr_k\,(\partial = 1), Pr_k\,(\partial = 0)\big) = \left(\frac{\exp(\lambda_k)}{1 + \exp(\lambda_k)}, \frac{1}{1 + \exp(\lambda_k)}\right) \tag{4}$$

4. Equilibrium Analysis

This section analyzes the market maker's price–correction time in the market where both arbitrageurs and noise traders exist. Subsection 4.1 analyzes a time when two selling orders ($\left(\frac{2}{m}\right)$ units of selling order) arrive. Subsection 4.2 analyzes a time when one selling order ($\left(\frac{1}{m}\right)$ units of selling order) arrives. Subsection 4.3 analyzes a time when no selling order arrives.

We assume both arbitrageurs and noise traders exist in this market. Arbitrageurs follow the trigger strategy explained in Proposition 1. They hold their stock before time $t_{b+\tau^1}$, and they sell out at the time after $t_{b+\tau^1+1}$. Before time $t_{b+\tau^1}$, only noise traders trade, and after time $t_{b+\tau^1+1}$, noise traders trade and arbitrageurs sell out. The number of orders that arrive before time $t_{b+\tau^1}$ can be (1,0,–1), and the number of orders that arrive after time $t_{b+\tau^1+1}$ can be (2,1,0). Therefore, the market's number of selling orders at each time becomes $s \in \{2, 1, 0, -1\}$.

It is assumed that the market maker's prior belief $Pr_b\,(\partial = 1)$ is equal to some positive value. As the market maker has no prior information at time t_b, a signal's probability is expressed in Table 3.

Table 3: Probability Structure of a Signal

$Pr_k\,(s_k \mid \partial)$	$s_k = 2$	$s_k = 1$	$s_k = 0$	$s_k = -1$
$\partial = 1$	$\frac{2}{3} - p$	$\frac{1}{3}$	p	0
$\partial = 0$	0	$\frac{2}{3} - p$	$\frac{1}{3}$	p

There is the possibility of price correction at time t_k when the signal s_k is 2, 1, or 0, because the market maker corrects the price when she finds that ∂ equals 1. We show in the next subsection that the market maker corrects the mispricing when two selling orders arrive. In the period when two selling orders do not arrive, the market maker corrects the mispricing when the expected loss of accepting the orders is higher than the expected profit of doing so. The market maker may correct the mispricing at time t_k when one or no selling orders arrive.[4] In other words, she corrects the mispricing when not enough market impact is supplied.

We also consider two cases where the market maker corrects the mispricing at time t_k when one or no selling orders arrive.

4. 1. Two Selling Orders at Time t_k

In this market, the following proposition exists.

Proposition 3 *The market maker corrects the price when two selling orders arrive in the market where arbitrageurs and noise traders trade.*

Proof

If the market maker at time t_k receives the signal $s_k = 2$, Equation (3) becomes:

$$\lambda_{k+1} = \ln\frac{pr_{k+1}(\partial = 1 \mid s_k = 2)}{pr_{k+1}(\partial = 0 \mid s_k = 2)} = \infty\text{: the LLR's multiplier diverges to infinity.}$$

By probability's property, $pr_{k+1}(\partial = 1 \mid s_k = 2) + pr_{k+1}(\partial = 0 \mid s_k = 2) = 1$.

If two equations exist, then $pr_{k+1}(\partial = 1 \mid s_k = 2) = 1$: the market maker finds the mispricing at time t_{k+1}.□

The intuition of this proposition is that the market maker extracts the fundamental information from two selling orders. It is interpreted that two selling orders supply a market impact. After time t_k when two selling orders arrive, the market maker corrects the mispricing to avoid the expected loss of future execution.

Proposition 3 shows that the price–correction time is when two selling orders arrive, but this time is different for each type of noise trader in the market. As $pr_{k+1}(s_k = 2 \mid \partial = 1) = 1$ is not equal to one, the time when two selling orders arrive cannot be uniquely determined. This time depends on the type of noise trader.

Simulations of price–correction times in three markets are represented in Figures 1, 2, and 3. Figure 1 corresponds to the market where pessimists $(p = 0.1)$ and arbitrageurs exist. Figure 2 corresponds to the market where neutral noise traders $\left(p = \frac{1}{3}\right)$ and arbitrageurs exist. Figure 3 corresponds to the market where optimists $(p = 0.6)$ and arbitrageurs exist. The numerical value is given as follows: $t_{b+1} = t_{11}$, $g = 0.1$, $r = 0.05$. The noise traders' orders at each time are generated by random sampling numbers following discrete–uniform distribution.[5]

Figures 1, 2, and 3 give a sample path of mispricing persistence. Mispricing occurs at time $t_{11}(= t_{b+1})$. However, the price–correction time differs among the three markets. These figures show that it takes the longest time to arrive at two selling orders in the market where optimists and arbitrageurs exist. The interpretation of this result is that it takes a long time for the market maker to extract fundamental information in this market because of the optimists' tendency to buy the stock.

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[4] −1 order does not come after the time $t_{b+\tau^1}$ because the arbitrageur sells one stock at each time.

[5] Each noise trader buys when a random sampling number is included in the interval $[0, p)$, does not trade when $\left[p, \frac{1}{3}+p\right)$, and sells when $\left[\frac{1}{3}+p, 1\right]$.

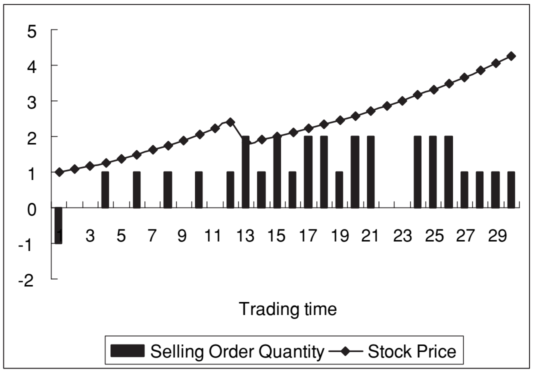

Figure 1: Market where pessimists and arbitrageurs coexist.

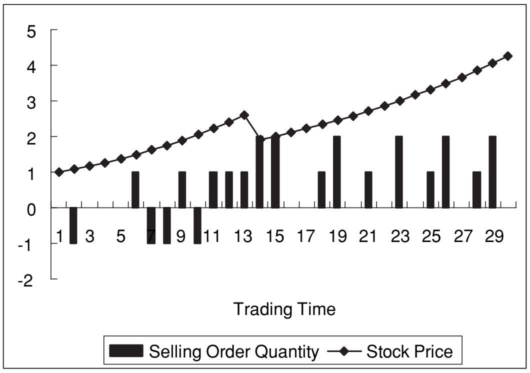

Figure 2: Market where neutral noise traders and arbitrageurs coexist.

Figure 3: Market where optimists and arbitrageurs coexist.

4. 2. One Selling Order at Time $\mathbf{t_k}$

Equation (3) means that when one selling order arrives, the LLR increases by $\ln\left(\frac{1/3}{2/3-p}\right)$. When $p > \frac{1}{3}$, the LLR goes up. On the other hand, the LLR goes down in the market where pessimists exist $\left(p < \frac{1}{3}\right)$. Thus, the market maker's belief increases in the market where pessimists exist. On the other hand, the market maker's belief falls in a market with optimists.

We consider that the expected loss of accepting the orders is higher than the expected profit of doing so. The capital loss of one inventory at $\partial = 1$ is $\left(1+g\right)^{\Delta(k-b)} - (1+r)^{\Delta(k-b)}$ when the market maker corrects the mispricing. The income gain in both states is 2C. Therefore, the expected profit of filling one selling order at time t_k becomes $2C - Pr_k(\partial = 1)\left\{\left(1+g\right)^{\Delta(k-b)} - (1+r)^{\Delta(k-b)}\right\}$. The market maker corrects the mispricing at the time when the expected profit becomes negative. When she corrects the mispricing at time t_{k+1}, Equations (5) and (6) are established.

$$2C - Pr_k(\partial = 1)\left\{\left(1+g\right)^{\Delta(k-b)} - (1+r)^{\Delta(k-b)}\right\} \geq 0 \tag{5}$$

$$2C - Pr_k(\partial = 1)\left\{\left(1+g\right)^{\Delta(k-b)} - (1+r)^{\Delta(k-b)}\right\} < 0 \tag{6}$$

Equation (5) means that at time t_k, the market maker does not correct the mispricing because the expected profit is positive. Equation (6) means that at time t_{k+1}, the market maker corrects the mispricing because the expected profit is equal to or lower than the expected loss from price correction. Equations (2) and (4) can be equivalent to the relation below.

$$\exp(\lambda_{k+1}) = \exp\left(\lambda_k + \ln\frac{1/3}{2/3-p}\right) = (2/3-p)\exp(\lambda_k)$$

$$Pr_{k+1}(\partial = 1) = \frac{\exp(\lambda_{k+1})}{1+\exp(\lambda_{k+1})} = \frac{(2/3-p)\exp(\lambda_k)}{1+(2/3-p)\exp(\lambda_k)} \tag{7}$$

Equations (5) and (6) are calculated by Equation (7) as follows.

$$\frac{2C(2/3-p)}{(1+g)^{\Delta(k-b)} - (1+r)^{\Delta(k-b)} + 2C(1-3p)} < Pr_k(\partial = 1) \leq \frac{2C}{(1+g)^{\Delta(k-b)} - (1+r)^{\Delta(k-b)}} \tag{8}$$

$$Pr_k(\partial = 1) \leq \frac{2C(2/3-p)}{(1+g)^{\Delta(k-b)} - (1+r)^{\Delta(k-b)} + 2C(1-3p)} \tag{9}$$

Equation (8) shows the interval of the market maker's belief at time t_k in the market where arbitrageurs and optimists exist. The left–hand side of the inequality is the lower bound value of the market maker's belief. When this inequality is not satisfied, the market maker does not correct the mispricing at the next time t_{k+1}.

Equation (9) shows the semi–interval of the market maker's belief at time t_k in the markets where pessimists $\left(p < \frac{1}{3}\right)$ or neutral noise traders $\left(p = \frac{1}{3}\right)$ exist. If this inequality is satisfied, the market maker corrects the mispricing at time t_k. In this market, the market maker's belief is a decreasing function of time. As a result, the threshold of the market maker's belief also becomes a decreasing function of time.

These relations are expressed as a numerical example in Figures 4, 5, and 6. In these numerical calculations, the numerical value is given as follows: $\Delta = 0.1$, $t_b = t_{11}$, $g = 0.1$, $r = 0.05$, $C = 0.1$. The dotted line represents the lower bound of the market maker's belief when the market maker corrects the mispricing at time t_{k+1}. The solid line is the upper bound of the market maker's belief when the market maker does not correct the mispricing at time t_k. These figures suggest that the two upper bounds of the market maker's belief are a decreasing function of trading time t_k.

Figure 4 corresponds to the market where pessimists $(p = 0.6)$ and arbitrageurs exist. Figure 5 corresponds to the market where neutral noise traders $\left(p = \frac{1}{3}\right)$ and arbitrageurs exist. Figure 6 corresponds to the market where optimists $(p = 0.1)$ and arbitrageurs exist.

Area I in Figure 4 shows the market maker corrects the mispricing at time t_k. Area II shows that the market maker corrects the mispricing at time t_{k+1}. Area III shows that the market maker does not correct the mispricing at time t_{k+1}.

Figure 5 shows that the two upper bounds of the market maker are the same because the neutral noise trader's belief is invalid to time.[6] This area shows that the market maker does not correct the

[6] This is easily checked by substituting equation (8) into $p = \frac{1}{3}$.

mispricing at time t_{k+1}. Area II is where the market maker corrects the mispricing at time t_k.

In Figure 6, Area I is where the market maker corrects the mispricing at time t_{k+1}. Area II shows that market maker corrects the mispricing at time t_k. Therefore, the threshold of the solid line is not binding. Areas I and II are where the market maker corrects the mispricing at time t_k. Area III is where the market maker does not correct the mispricing at time t_k.

These intervals show that the upper bound of the market maker's belief to correct the mispricing at time t_k has its smallest value in the market where optimists exist $\left(p > \frac{1}{3}\right)$. In other words, the market maker tends to correct the mispricing earlier. This is the opposite implication of the case of two selling orders. When one selling order arrives, the market maker may correct the mispricing at the time when either Equation (8) or (9) is satisfied.

The results of this subsection are as follows. First, a market maker may correct the mispricing at the time when not enough market impact is supplied. Second, the market maker tends to correct the mispricing earlier in the market where arbitrageurs and optimists exist because she knows that two selling orders arrive earlier in this market. We can interpret this as meaning that, in this market, the market maker has to be more cautious about the possibility of a bubble.

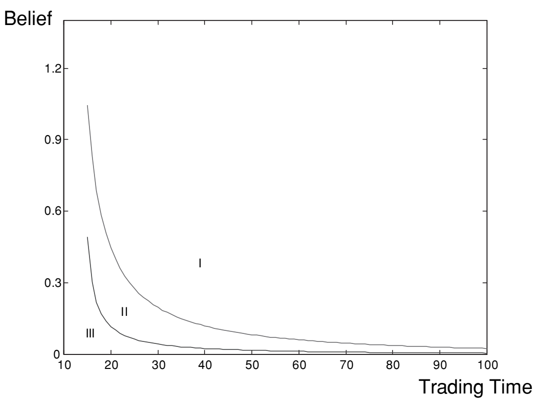

Figure 4: Market where optimists and arbitrageurs coexist.

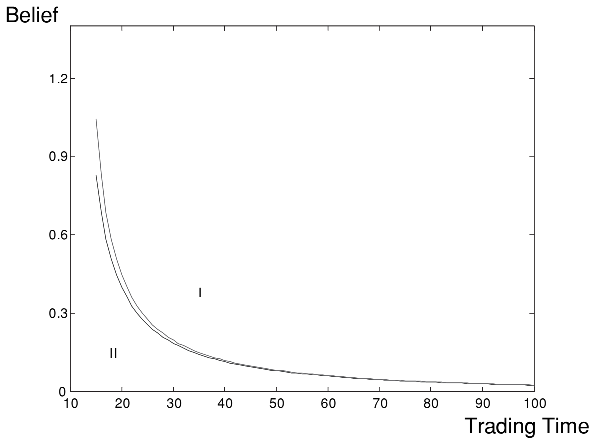

Figure 5: Market where neutral noise traders and arbitrageurs coexist.

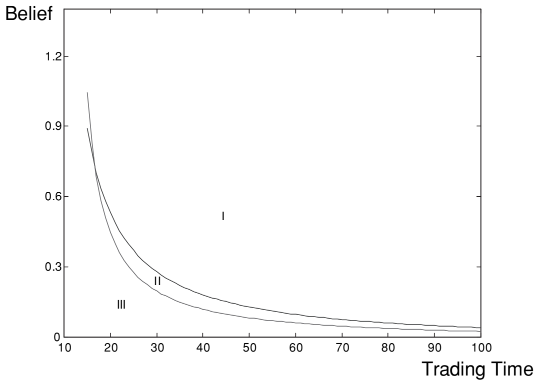

Figure 6: Market where pessimists and arbitrageurs coexist.

4. 3. No Selling Order at Time t_k

In this case, one buying order from a noise trader and one selling order from an arbitrageur are matched. No order comes to the market maker and she obtains a charge for participation from one arbitrageur and one noise trader at each trading time. The expected loss of filling no selling orders at time t_k becomes 0. The expected profit of filling no selling orders at time t_k becomes 2C. Therefore, the market maker does not correct the mispricing at time t_k because the expected profit is higher than expected loss of price correction. In other words, the market maker does not correct the price when no selling order arrives.

5. Concluding Remarks

This paper analyzes the market maker's price–correction behavior towards synchronization risk. In this model, the market maker extracts information whether or not arbitrageurs sell from an order sequence using Bayes Rule. The result of the current analysis easily explains the process of price correction towards synchronization risk. Price correction occurs when the number of selling orders crosses the threshold κ in AB (2003). However, their paper assumes that the threshold κ is an exogenous variable. In our model, the price–correction time is endogenously determined by endogenous variables, including the type of noise trader and the market maker's beliefs.

There are three main findings in this paper. First, the market maker corrects the price when market impacts are supplied; this time is the longest in a market where arbitrageurs and optimists exist. Second, the market maker may correct the price when one selling order arrives. In other words, she may correct the price when not enough market impact is supplied. The numerical examples depicted in Figures 4, 5, and 6 show that this time is the fastest in a market where arbitrageurs and optimists exist. This can be interpreted such that the market maker has to be more cautious of the possibility of a bubble in the optimists' market because she knows that two selling orders arrive earlier in this market. Third, the market maker's price–correction behavior does not occur if no orders arrive.

This model can be extended to a more general analysis of an order sequence by following a normal distribution. By using a normal distribution, we can describe the market where n (an arbitrary natural number) orders arrive in one trading interval. This extension remains a challenge for future research.

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