Structure of ^{28}Mg studied by β-decay spectroscopy of spin-polarized ^{28}Na: The first step of systematic studies on neutron-rich Mg isotopes

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As the first step of systematic studies on neutron-rich Mg isotopes, the structure of ^{28}Mg has been investigated by β-decay spectroscopy of spin-polarized ^{28}Na at TRIUMF. Spin-parity assignments have been successfully performed for levels in ^{28}Mg by taking advantage of the anisotropic β decay. Previous assignments for three levels were reconfirmed. New assignments were performed for four levels including a newly found 2^+ level at 7.461 MeV. The experimental results were compared, on a level-by-level basis, with the shell-model calculation in the model space restricted to the *sd* shell. Overall good agreement between the experimental and theoretical results indicates dominance of the normal configurations in most of the ^{28}Mg levels. High performance of the present method promises successful application to heavier Mg isotopes to elucidate the shell evolution in the region of the $N = 20$ island of inversion.

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I. INTRODUCTION

Shell evolution as a function of asymmetry in neutron and proton numbers is one of the most interesting phenomena in nuclei far from the β-stability line. Much effort has been devoted to reveal the disappearance of the $N = 20$ shell gap in neutron-rich nuclei [1–3]. In nuclei with $20 \leqslant N \leqslant 22$ and $10 \leqslant Z \leqslant 12$, shell-model calculations indicated that the states with intruder configurations, where neutrons are excited across the $N = 20$ shell gap, are lower in energy than that with normal configurations. This region of the nuclear chart is referred to as the "island of inversion" [4]. However, the border of the island seems not so clear as shown by the systematics in Fig. 1, which compares experimental reduced transition probabilities $B(E2; 0_1^+ \rightarrow 2_1^+)$ in neutron-rich Mg isotopes to the shell-model calculations with the restricted model space in the *sd* shell. The theoretical value indeed underestimates significantly at $N = 20$, but the discrepancy is observed even at $N = 18$. This suggests effects of the intruder configurations in nucleus outside of the island of inversion. It is also to be noted that many of the discussions have focused on the ground states (g.s.) or a few low-lying levels. However, in order to investigate the shell evolution, it is also important to examine the significance of the intruder configurations for the excited states in a wide energy range.

Recent measurements of the single-neutron knockout reaction clearly demonstrated that the intruder configurations are prevailing in $^{32}\mathrm{Mg}_{\mathrm{g.s.}}$, whereas $^{30}\mathrm{Mg}_{\mathrm{g.s.}}$ is well described by the normal configurations [9]. On the other hand, for the second 0^+ states, other experiments suggested the inverse situation; normal configurations in ^{32}Mg [10] and intruder configurations in ^{30}Mg [11,12]. These observations indicate lowering of the intruder levels with increasing neutron number. Detailed information on the effects of intruder configurations in other excited states of neutron-rich Mg isotopes will shed light on the shell evolution.

The spin-parity (I^π) of the excited states is the key information to discuss the intruder configurations. However, the present status of the spin-parity assignments are not sufficient in Mg isotopes with $A \geqslant 29$. Spin-parity assignments have been proposed for the following levels: ^{29}Mg: g.s. ($3/2^+$) [13]; ^{30}Mg: 1.482 MeV (2_1^+) [14], 1.789 MeV (0_2^+) [11,12], 2.467 MeV [(2_2^+)] [11], 2.541 MeV [($2^-, 3^-$)] [15], 3.302 MeV [(4_1^+)] [16], 3.379 MeV (4^+) [15], 3.455 MeV (4^+) [15], and 4.181 MeV (5) [15]; ^{31}Mg: g.s. ($1/2^+$) [17]; ^{32}Mg: 0.885 MeV (2_1^+) [18] and 1.058 MeV (0_2^+) [10]; ^{33}Mg: g.s. [$3/2^{(-)}$] [19]; and ^{34}Mg: 0.656 MeV (2_1^+) [8].

We have started systematic studies of the neutron-rich Mg isotopes by the β-decay spectroscopy of spin-polarized Na isotopes at TRIUMF, placing an emphasis on spin-parity assignments of the levels in Mg isotopes. The polarized nucleus exhibits anisotropic β decay and the asymmetry with respect to polarization is very effective in allowing us to unambiguously assign the spins of the daughter states, as shown in the preceding experiment, in which spins and parities of seven levels in ^{11}Be were assigned for the first time [20]. As the first step of a series of studies on heavy Mg isotopes, we have applied the same method to the β decay of polarized ^{28}Na. The experiment aimed at a thorough understanding of the structure of ^{28}Mg by using the effective method. The present paper describes the details of the experiment and the results.

The level structure of ^{28}Mg has been investigated by the ^{26}Mg$(t, p)^{28}$Mg reaction [21–24] and the β decay of ^{28}Na [14,25]. Middleton and Pullen [22] assigned the spins and parities of nine levels by the angular distribution of the proton.

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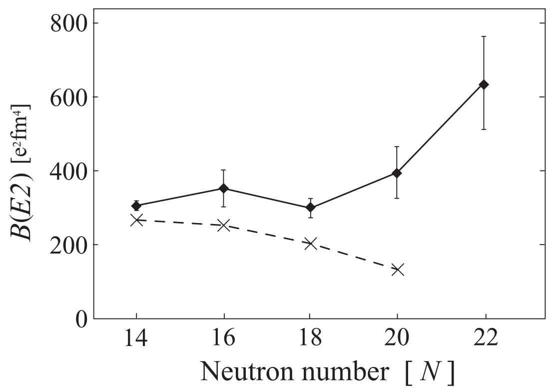

FIG. 1. Systematics of $B(E2; 0_1^+ \rightarrow 2_1^+)$ values in neutron-rich Mg isotopes, ^{26,28}Mg [5], ^{30}Mg [6], ^{32}Mg [7], and ^{34}Mg [8]. The experimental values and shell-model predictions with the *sd*-shell model space are displayed by closed diamonds and crosses, respectively. The lines are to guide the eye.

Rastegar *et al*. [24] reported additional assignments for another five levels from the angular correlation between protons and γ rays. They revised the previous 0^+ assignment [22] for the 5.272-MeV level to 1^-. Later, the level structure of ^{28}Mg was also investigated in the β decay of ^{28}Na by Détraz *et al*. [25] and Guillemaud-Mueller *et al*. [14]. Confirmed were the allowed β transitions leading to the g.s. as well as the known levels including the 5.272-MeV level [14]. By taking into account the positive parity of ^{28}Na$_{\text{g.s.}}$ (1^+ [26]), Endt concluded that the level at 5.272 MeV is of 1^+ [27].

II. EXPERIMENT

A. Principle of spin-parity assignment

The levels of ^{28}Mg are populated by the β decay of ^{28}Na and the decay scheme is constructed from the β-γ and γ-γ coincidence measurements. Our uniqueness is in the use of the spin polarization of ^{28}Na. The spins and parities of the levels in ^{28}Mg are unambiguously assigned from the β-decay asymmetry as follows. The β-ray angular distribution from a polarized nucleus is given by

$$W(\theta) \approx 1 + AP\cos\theta, \tag{1}$$

for an allowed transition, where A, P, and θ are the asymmetry parameter, polarization of the parent nucleus, and the β-ray emission angle with respect to the polarization direction, respectively [28]. The asymmetry parameter A takes one of three different values according to the possible spin I_{f} of the final state for a given spin I_{i} of the initial state as

$$A = \begin{cases} -1 & (I_{\text{f}} = I_{\text{i}} - 1), \\ \dfrac{-1/(I_{\text{i}}+1) - 2\tau\sqrt{I_{\text{i}}/(I_{\text{i}}+1)}}{1+\tau^2} & (I_{\text{f}} = I_{\text{i}}), \\ \dfrac{I_{\text{i}}}{I_{\text{i}}+1} & (I_{\text{f}} = I_{\text{i}} + 1). \end{cases} \tag{2}$$

Here, τ is the mixing ratio defined by

$$\tau = C_{\text{V}}\langle 1\rangle/(C_{\text{A}}\langle\sigma\rangle), \tag{3}$$

TABLE I. Asymmetry parameters A of the β decay of ^{28}Na$_{\text{g.s.}}$($I_{\text{i}}^{\pi} = 1^+$) to the ^{28}Mg levels with possible spins.

I_{i}^{π} (^{28}Na$_{\text{g.s.}}$)	I_{f}^{π} (^{28}Mg)	A
	0^+	-1.0
1^+	1^+	-0.5
	2^+	$+0.5$

where C_{V} and C_{A} are the Fermi and the Gamow-Teller coupling constants, respectively, and $\langle 1\rangle$ and $\langle\sigma\rangle$ are the corresponding nuclear matrix elements [29]. In the case of the neutron-rich nucleus ^{28}Na, the β decays to the levels below the neutron threshold are mostly the Gamow-Teller (GT) transitions. Therefore, it is plausible to approximate $\tau \approx 0$ in Eq. (2), and the asymmetry parameters for ^{28}Na$_{\text{g.s.}}$($I_{\text{i}}^{\pi} = 1^+$) $\rightarrow$ ^{28}Mg($I_{\text{f}}^{\pi} = 0^+$, 1^+, or 2^+) are evaluated as listed in Table I. Note the very discrete values of A. It is possible to firmly assign the spins and parities of the levels in ^{28}Mg based on the experimental A.

Figure 2 illustrates the idea of measurement of the asymmetry parameter. The β-ray counts N_{R}^+ and N_{L}^+ are obtained by the detectors placed in the same (R: $\theta = 0^\circ$) and opposite (L: $\theta = 180^\circ$) directions with respect to the polarization, respectively. They are related to the asymmetry parameter A as

$$N_{\text{R}}^+ = \epsilon_{\text{R}} N(1 + AP), \tag{4}$$
$$N_{\text{L}}^+ = \epsilon_{\text{L}} N(1 - AP), \tag{5}$$

where N is the number of disintegrations of the parent nucleus, and ϵ_{R} and ϵ_{L} are the efficiencies of the respective detectors. To cancel out the spurious asymmetry due to differences in the efficiency, the spin orientation is rotated by 180°. Then the counts are expressed as

$$N_{\text{R}}^- = \epsilon_{\text{R}} N(1 - AP), \tag{6}$$
$$N_{\text{L}}^- = \epsilon_{\text{L}} N(1 + AP). \tag{7}$$

By taking a double ratio of the counts, the product AP is deduced, independently of the efficiency, as

$$AP = \frac{\sqrt{R} - 1}{\sqrt{R} + 1}\left(R = \frac{N_{\text{R}}^+/N_{\text{L}}^+}{N_{\text{R}}^-/N_{\text{L}}^-}\right). \tag{8}$$

The asymmetry parameter A can be obtained, if the polarization P is known.

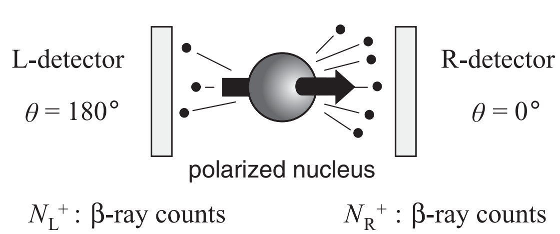

FIG. 2. Schematic illustration of β-decay asymmetry measurement.

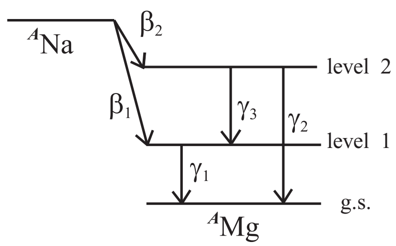

FIG. 3. Schematic decay scheme of a neutron-rich ^{A}Na isotope.

The relative error of AP is expressed as

$$\left|\frac{\Delta(AP)}{AP}\right| = \left|\frac{1-(AP)^2}{4AP}\right| \left(\frac{1}{N_R^+} + \frac{1}{N_R^-} + \frac{1}{N_L^+} + \frac{1}{N_L^-}\right)^{1/2}$$
$$= \left|\frac{1-(AP)^2}{AP}\right| \frac{1}{\sqrt{N_{\text{total}}}}, \tag{9}$$

where $N_{\text{R}}^+ = N_{\text{L}}^+ = N_{\text{R}}^- = N_{\text{L}}^- \approx N_{\text{total}}/4$. It is understood that high polarization is important. In the case of high polarization of $P = 0.5$, only 200 counts for N_{total} are necessary to distinguish $A = -0.5$ from -1.0 with 2σ accuracy. Note that this estimation assumes $\theta = 0^\circ$ or 180°, even though the actual detectors have finite opening angles. This fact increases the required statistics to some extent in practice.

The levels in a neutron-rich ^{A}Mg isotope are identified by performing a β-γ coincidence measurement. The asymmetry parameter A of the β transition to a specific level can be simply deduced from the β-γ counts, if no γ transition populates this level, such as the case of level 2 in Fig. 3. However, in many cases the levels, such as level 1, are populated by γ transitions deexciting higher levels. In this case, the asymmetry parameter, A_1^γ, determined from β-γ_1 counts is affected by the asymmetry of β transitions to the higher levels, as

$$A_1^\gamma = A_1 \times \frac{I_{\beta_1}}{I_{\gamma_1}} + A_2 \times \frac{I_{\gamma_3}}{I_{\gamma_1}}, \tag{10}$$

where I_γ and I_β are the γ-ray intensity and the β-decay branching ratio, respectively. The asymmetry parameter A_1 is deduced as

$$A_1 = A_1^\gamma \times \frac{I_{\gamma_1}}{I_{\beta_1}} - A_2 \times \frac{I_{\gamma_3}}{I_{\beta_1}}. \tag{11}$$

Accordingly, the spin-parity of level 1 can be assigned.

B. Experimental setup

The experiment was carried out at the radioactive beam facility ISAC in TRIUMF. The world-highest nuclear polarization for alkali atoms is available there. The ^{28}Na ($I^\pi = 1^+$, $T_{1/2} = 30.5$ ms) beam was produced by the 500-MeV-proton-induced target-fragmentation reaction on a stack of Ta targets (21.8 g/cm^2 in total thickness). The isotope-separated ^{28}Na$^+$ beam with an energy of 30.6 keV was delivered to the polarized beam line [30,31]. The beam was neutralized in colliding with the Na vapor, and the nuclear spin was polarized ($\approx$50%) by collinear optical pumping. Then, the polarized ^{28}Na beam was ionized through low-temperature He gas. The beam was transported to the OSAKA beam line where the polarization direction was perpendicular to the beam direction. The spin orientation was flipped by changing the laser helicity in every 5 min.

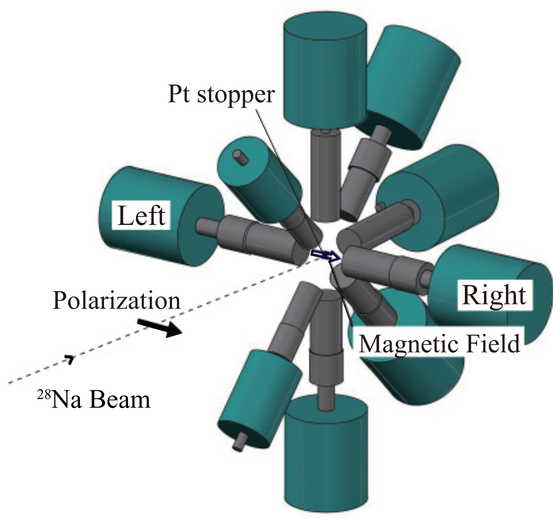

FIG. 4. (Color online) Layout of the Ge detectors. The ^{28}Na beam with transverse polarization is stopped on the Pt stopper in vacuum. Two detectors labeled "Left" and "Right" are used to measure the β-decay asymmetry. Seven other detectors are placed on a plane perpendicular to the polarization direction.

The beam was focused and stopped on a Pt foil (10 μm thick) placed in a cylindrical vacuum chamber made of fiber-reinforced plastic (FRP) with 36 and 38 mm inner and outer diameters, respectively. Such a thin FRP chamber wall allowed β- and γ-ray detections in the atmosphere with the least disturbance. To preserve the ^{28}Na polarization in the stopper, a static magnetic field of about 83 mT was applied along the polarization direction by a pair of small permanent magnets in the atmosphere. With the proton beam intensity of 70 μA, the beam intensity of ^{28}Na was about 450 pps.

The β and γ rays associated with the β decay of ^{28}Na were detected by nine high-purity germanium (Ge) detectors surrounding the Pt stopper, as shown in Fig. 4. A pair of plastic scintillators (1.5 mm thick) was placed in front of each Ge detector to distinguish β and γ rays. The detector telescopes placed at 0° and 180° with respect to the polarization direction were used to measure the β-decay asymmetry. The solid angle of the two detectors was about 14% of 4π. The other seven telescopes were placed in a plane perpendicular to the polarization direction. The total efficiency for the 1.3-MeV γ-ray detection was 1.6%. Triggers for data acquisition were β singles, γ singles, γ-γ coincidence, and β-γ coincidence events. In total, 3.0×10^7 events were accumulated in 42.8 h.

III. RESULTS

A. Construction of ^{28}Na decay scheme

Figure 5 shows the projection spectrum of the γ-γ coincidence data. It is to be noted that small peaks such as the 3995-keV peak could be observed because of the

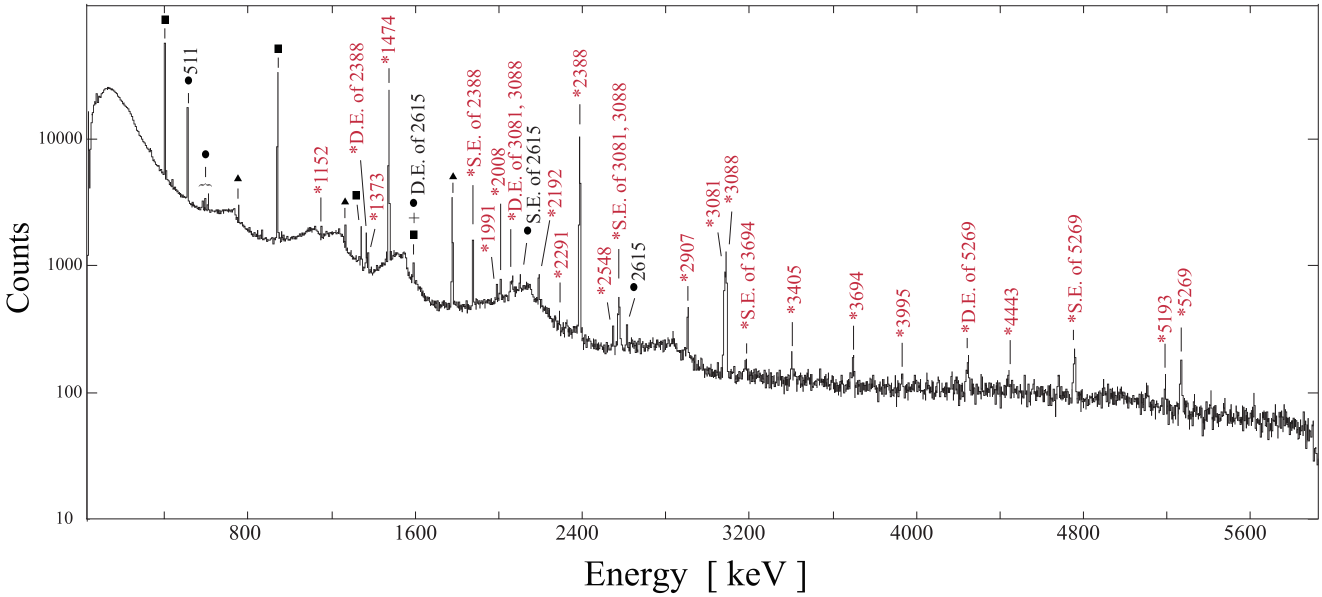

FIG. 5. (Color online) Total projection spectrum of γ-γ coincidence measurement. The numbers are γ-ray energies in keV. Peaks are assigned as due to the γ rays in the daughter ^{28}Mg (denoted by asterisks), granddaughter ^{28}Al (closed squares), great-granddaughter ^{28}Si (closed triangles), and background (closed circles).

high efficiency of the measurement. By the β-γ and γ-γ coincidence analyses, 18 γ rays specified by asterisks in the figure were found to be attributed to the transitions in ^{28}Mg. Many γ rays were observed for the first time in the β decay of ^{28}Na. The γ-ray peak energies and intensities are listed in Table II. The level energies were re-evaluated from the γ-ray energies in the present work. The cascade relations between γ transitions were examined by γ-γ coincidence analysis with a time window of $\pm$400 ns. A new decay scheme of ^{28}Na has been established, as shown in Fig. 6. The physical quantities and decay paths marked by asterisks are newly found ones in the present work. The known spins and parities [23,24] labeled by daggers were confirmed in the present work. The β-decay branching ratios and log ft values are listed in Table III. Note that small γ-ray intensities of less than 0.1% were not taken into account in estimating the β-decay branching ratios by the sum of γ-ray intensities for the levels, e.g., the 7.461-MeV level.

It is to be noted that seven γ rays (1373, 1991, 2008, 2192, 2291, 2907, and 4443 keV) have been newly observed in the present work (see the γ transitions marked with asterisks in Fig. 6). Six γ rays of 1152, 2548, 3405, 3694, 3995, and 5193 keV, which were observed in the ^{26}Mg$(t, p)^{28}$Mg reaction [23,24], were also newly found as the transitions after the β decay of ^{28}Na. Five γ rays (1474, 2388, 3081, 3088, and 5269 keV) depopulating the levels at 1.474, 3.862, 4.555, 4.562, and 5.269 MeV [26] were confirmed. The 3405-keV γ ray depopulating the 4.878-MeV level was reported in the β-decay experiment [25], but it was not adopted in Ref. [26].

The absolute intensities would have been deduced by normalizing the intensity of the 1342-keV γ transition in the granddaughter nucleus ^{28}Al, if the ^{28}Na beam intensity had been stable enough during a time span of the order of the ^{28}Mg half-life (20.9 h). In the present work, the intensities were estimated by normalizing the ^{28}Mg 1474-keV γ-ray intensity to the previously reported value of 37(5)% [26].

The γ-ray intensities were evaluated based on the counts in the Ge detector (Right) placed along the polarization direction, by assuming isotropic γ-ray emission. However,

TABLE II. Gamma transitions observed in the present work. E_i and E_f are the energies of the initial and final levels, respectively. Intensities are shown in percent of ^{28}Na β decay.

E_γ [a] (keV)	E_i [a] $\rightarrow$ E_f [a] (MeV)$\rightarrow$(MeV)	I_γ [a] (%)	E_γ [b] (MeV)	I_γ [b] (%)
1151.6(11)	5.171 → 4.021	<0.1	1150.3(4)	—
1373.4(2)	6.545 → 5.171	<0.1	—	—
1473.5(1)	1.474 → g.s.	37(5)[b]	1474.0(2)	37(5)
1990.7(5)	6.545 → 4.555	0.2(1)	—	—
2007.7(4)	7.200 → 5.193	0.5(1)	—	—
2191.7(3)	7.461 → 5.269	0.8(1)	—	—
2290.9(6)	7.461 → 5.171	<0.1	—	—
2388.4(1)	3.862 → 1.474	22(3)	2389.0(4)	18.7(25)
2547.8(7)	4.021 → 1.474	<0.1	2547.2(15)	—
2906.9(6)	7.461 → 4.555	0.6(1)	—	—
3081.3(3)	4.555 → 1.474	2.7(4)	3083.5(4)	1.3(3)
3088.3(3)	4.562 → 1.474	4.0(6)	3086.0(7)	2.6(5)
3404.9(13)	4.878 → 1.474	0.2(1)	3403(4)	—
3694.2(13)	5.171 → 1.474	0.3(1)	3697.5(7)	—
3994.9(15)	5.468 → 1.474	<0.1	3996.5(5)	—
4443.0(11)	5.917 → 1.474	0.3(1)	—	—
5192.6(5)	5.193 → g.s.	0.4(1)	5190(3)	—
5269.1(5)	5.269 → g.s.	2.3(4)	5271.7(10)	0.50(15)

[a]Present work.
[b]Reference [26].

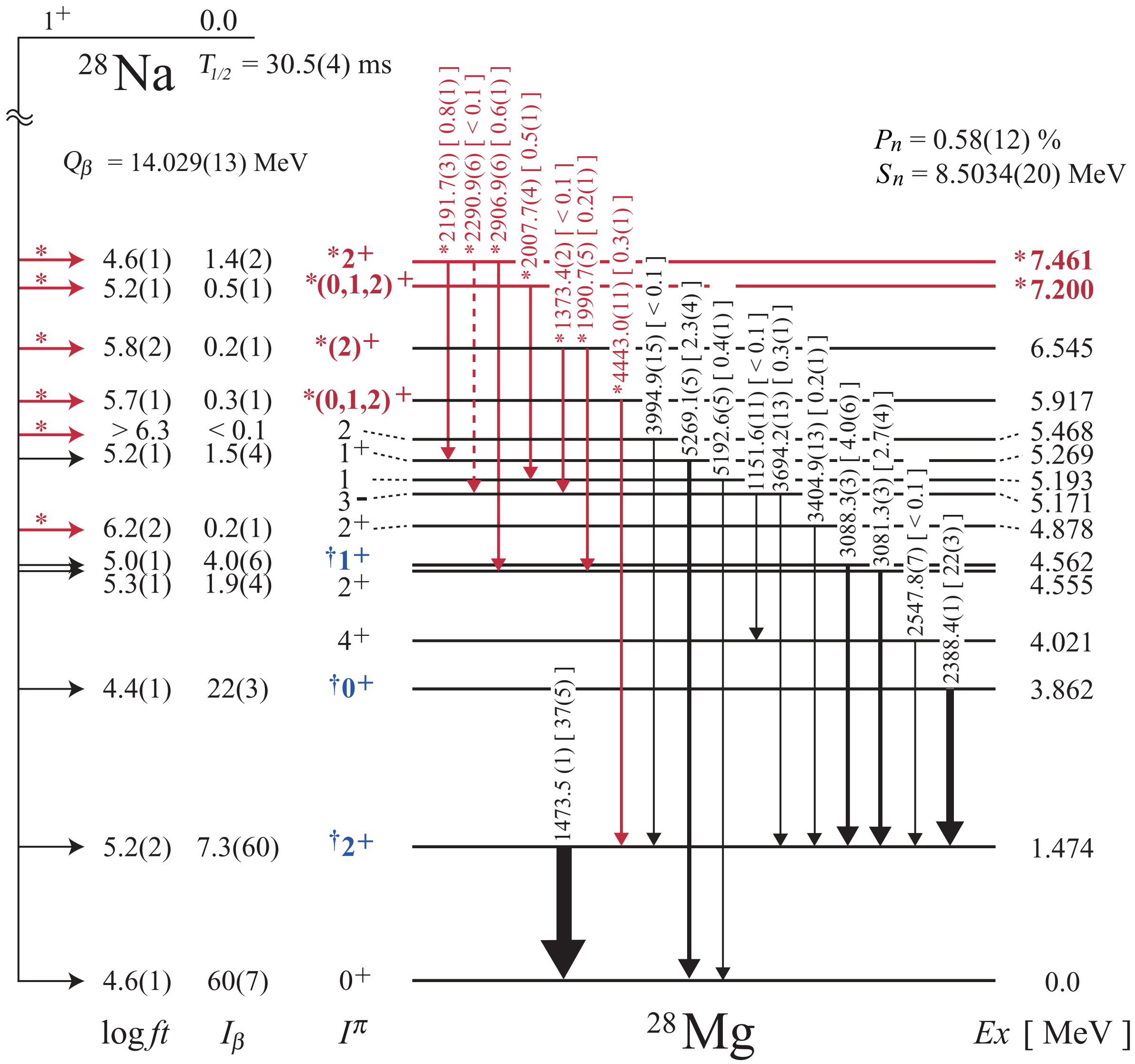

FIG. 6. (Color online) Revised decay scheme of ^{28}Na. The level energies are re-evaluated. Newly found transitions and energy levels as well as newly assigned spins and parities are labeled by asterisks. The spins and parities labeled by daggers denote that these assignments are reconfirmed in the present work. Reported values of $T_{1/2}$ [33], Q_β [34], P_n [33], and S_n [34] are also shown.

possible residual polarization in the ^{28}Mg levels may cause anisotropic γ-ray emission [32]. From the observed 90°/0° anisotropy for the most polarization-affected $2^+ \rightarrow 0^+$ transition of the 1474-keV γ ray, it was found that the isotropic assumption underestimates the γ-ray intensities by 7%. The effect in other transitions should be less than this amount. The intensities listed in Table II do not include this ambiguity.

The details of the decay-scheme construction will be discussed below.

1. Levels observed for the first time in the β decay of ^{28}Na

The high efficiency of γ-γ coincidence measurements in the present work enabled the new finding of three γ rays from the known levels: the 4443-keV transition depopulating the 5.917-MeV level and 1373- and 1991-keV γ rays emitted from the 6.545-MeV level (see Table II and Fig. 6). These levels have been found in the ^{26}Mg$(t, p)^{28}$Mg reaction [21], but not observed in the β decay of ^{28}Na. The present work established the first observation of the β decays to these levels, as shown by the β-decay paths with asterisks in Fig. 6. Since it was confirmed that there were no γ rays populating these levels, the branching ratios of β decay to the respective levels were determined directly from the intensities of the newly found γ rays, as listed in Table II and Table III.

Small peaks in Fig. 5 at 1152, 2548, 3694, and 5193 keV are due to transitions depopulating the known levels found in the ^{26}Mg$(t, p)^{28}$Mg reaction [23,24]. These levels have not

TABLE III. ^{28}Mg levels observed in the β decay of ^{28}Na. Asterisks indicate that the β-decay branching ratios (I_β) to these levels are less than the sensitivity of the present measurement.

E_x (MeV)	I_β (%)		log ft	
	Present work	Ref. [26]	Present work	Ref. [26]
g.s.	60(7)	63(6)	4.6(1)	4.57(5)
1.4735(1)	7.3(60)	14(6)	5.2(2)	4.99(19)
3.8619(1)	22(3)	19(3)	4.4(1)	4.42(8)
4.0213(7)	*	—	*	—
4.5548(3)	1.9(4)	1.3(3)	5.3(1)	5.43(11)
4.5618(3)	4.0(6)	2.6(5)	5.0(1)	5.13(9)
4.878(1)	0.2(1)	—	6.2(2)	—
5.1713(7)	*	—	*	—
5.1926(5)	*	—	*	—
5.2691(5)	1.5(4)	0.50(15)	5.2(1)	5.68(14)
5.468(1)	<0.1	—	>6.3	—
5.917(1)	0.3(1)	—	5.7(1)	—
6.5452(5)	0.2(1)	—	5.8(2)	—
7.2003(6)	0.5(1)	—	5.2(1)	—
7.4611(4)	1.4(2)	—	4.6(1)	—

been observed in the β decay of ^{28}Na.[1] It was found that the 5.193-MeV level ($I = 1$) is populated by the 2008-keV γ ray, which depopulates a new level at 7.200 MeV. (The new levels found in the present work will be discussed in Sec. III A2.) From the γ-ray intensity balance, the branching ratio of the β decay to the 5.193-MeV level was estimated to be negligibly small. This forbidden nature suggests negative parity of this level and agrees with the first assignment of 1^- [22], whereas the later assignments removed the parity assignment [14,26]. The 1152- and 3694-keV γ rays have their origins in the 5.171-MeV level ($I^\pi = 3^-$). Also the 2548-keV γ ray is assigned as the transition from the 4.021-MeV level (4^+) to the first 2^+ level at 1.474 MeV. The resultant negligibly small β-decay branching ratios to these levels support the previous spin-parity assignments.

The known γ rays of 3405 and 3995 keV, which depopulate the levels at 4.878 ($I^\pi = 2^+$) and 5.468 ($I = 2$) MeV, respectively, were also observed in the present work. These levels were reported in the β decay of ^{28}Na [14,25], but the details are not clear. Accordingly, the β decays to these levels were not adopted in Ref. [26]. In the present work, the β decays were established for the first time and the β-decay branching ratios were determined to be 0.2(1) and <0.1% for the 4.878- and 5.468-MeV levels, respectively.

2. New levels at 7.200 and 7.461 MeV

In the present work, four more γ-ray peaks were newly found at 2008, 2192, 2291, and 2907 keV. Their origins could not be assigned to the known levels in ^{28}Mg. From the γ-ray coincidence relations, two levels with small log ft values were newly established in the excitation-energy region between the reported highest 6.759-MeV level [21] and the neutron-threshold energy (around 8.5 MeV): a higher one at 7.461 MeV [log $ft = 4.6(1)$] and a lower one at 7.200 MeV [5.2(1)] (see Fig. 6). The new level assignment will be discussed below.

The newly observed 2907-keV γ ray was coincident with the 1474- and 3081-keV γ rays. Similarly, the new 2192-keV γ ray was found in coincidence with the 5269-keV γ ray. The sum energies of these cascade γ rays, $1473.5(1) + 2906.9(6) + 3081.3(3) = 7461.7(7)$ and $2191.7(3) + 5269.1(5) = 7460.8(6)$ keV, were consistent with each other. Therefore, the energy of the new level was determined to be 7.4611(4) MeV.

The newly found 2291-keV γ ray was observed in coincidence only with the 3694-keV γ ray, which depopulates the 5.171-MeV level. We propose a placement of the 2291-keV γ transition as shown by the dashed arrow in Fig. 6, because the energy difference between the levels, 2289.8(8) keV [$=7461.1(4) - 5171.3(7)$], was consistent with this γ-ray energy of 2290.9(6) keV.

Another newly found 2008-keV γ ray was coincident only with the 5193-keV γ ray (5.193 MeV $\rightarrow$ g.s.). This observation required another new level and the excitation energy was reasonably determined to be 7.2003(6) MeV [$=5.1926(5) + 2.0077(4)$].

B. Polarization of ^{28}Na

To obtain the β-decay asymmetry parameter A for each β transition from Eq. (8), the polarization P has to be known. The polarization P can be evaluated by using the β transition which populates the level with known spin. We selected the $^{28}\mathrm{Na}_{\mathrm{g.s.}}(1^+) \rightarrow {}^{28}\mathrm{Mg}_{\mathrm{g.s.}}(0^+)$ transition by setting a β-ray energy gate on the range 9.7–11.4 MeV to cut off the transitions to the excited levels in ^{28}Mg. These gate energies were determined by taking into account the β-ray energy losses in the materials between the Pt stopper and the Ge detector. The AP value was obtained to be $-0.283(5)$. Since the asymmetry parameter of this transition is $A = -1$, the polarization P was obtained to be 28.3(5)%. This value is smaller than the reported one [35]. This reduction may be caused by spin relaxation in the Pt foil, because the external magnetic field was lower than in the reported experiment.

C. Spin-parity assignment

In the present work, the β decay to the 2^+ level at 4.878 MeV [22] was observed for the first time. Since its log ft value was determined to be 6.2(2), all the decays with log $ft \leqslant 6.2$ can be regarded as GT transitions. As a result, the levels associated with log $ft \leqslant 6.2$ are assigned to be of positive parity. Spins of these levels were assigned in the present work as discussed in this section. The assigned spins and parities for the levels at 1.474, 3.862, and 4.562 MeV were consistent with the reported ones [27]. This fact demonstrates the effectiveness of the present method. It is to be emphasized that the newly found level at 7.461 MeV was assigned to be 2^+.

[1] The β-decay branch to the 5.193-MeV level was reported in Ref. [14], but it was not adopted in their decay scheme of ^{28}Na.

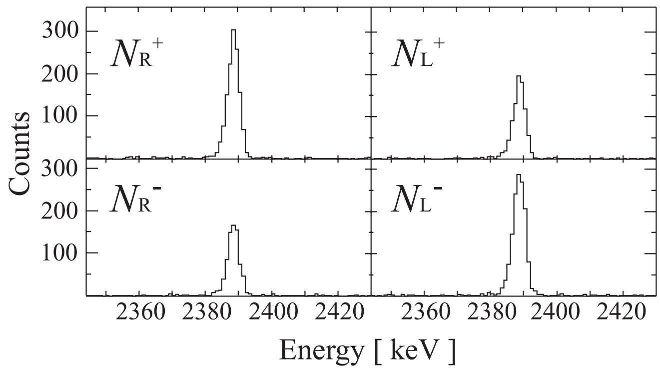

FIG. 7. Gamma-ray energy spectra around the 2388-keV peak in coincidence with the β rays detected in the Left (L) or Right (R) detector when polarization is in the direction to the R detector (+) or L detector (−).

1. Spin-parity of the 1.474-, 3.862-, and 4.562-MeV levels

The details of spin assignment are discussed here in the order of simplicity. Since the levels at 3.862 and 4.562 MeV are not populated by the γ transitions from the higher levels, the asymmetry parameters were obtained by simply counting the γ rays of 2388 and 3088 keV in coincidence with the β rays (see Fig. 6). Figure 7 shows the γ-ray energy spectra around the 2388-keV peak in four patterns according to the direction of polarization (+ or −) and the β-ray detector (L or R). Larger peak counts of N_R^+ and N_L^- than those of N_R^- and N_L^+ indicate a positive and sizable asymmetry [see Eq. (8)]. From the peak counts in Fig. 7, the value of $A_{3.862}$ was obtained to be $-0.85(6)$.[2] Among the expected values of A in Table I, $A = -1(I^\pi = 0^+)$ is mostly consistent with the experimental result. Thus the 3.862-MeV level was assigned to be 0^+. This assignment agrees with the previous one [22].

In the same way, $A_{4.562} = -0.54(19)$ was obtained from the 3088-keV γ-ray counts. This result enabled a 1^+ assignment for the 4.562-MeV level. Note that the spin-parity assignment is again consistent with the reported one [27].

The first excited level of ^{28}Mg at 1.474 MeV is populated not only by the direct β transition but also by eight γ transitions from higher levels (see Fig. 6). In order to select the direct β transition, we set the β-ray energy gate with $E_\beta = 7.7-9.7$ MeV so that the β transitions to the levels higher than the 3.862-MeV level were rejected. From the 1474-keV γ-ray counts in coincidence with the gated β rays, $A_{1.474} = 0.48(3)$ was obtained, indicating very reasonable assignment of 2^+ for the first excited state [27].

Figure 8 compares the experimental asymmetry parameters of $A_{1.474}$, $A_{3.862}$, and $A_{4,562}$ with the three expected values. It is seen that the value of $A_{3.862}$ is somewhat beyond the 1σ error range from the expected value of $A = -1.0$. This must be due to the assumption on the β-ray emission angle θ, as mentioned in Sec. II A.

[2]Hereafter, the asymmetry parameter of the β decay to the daughter level with excitation energy E is expressed as A_E.

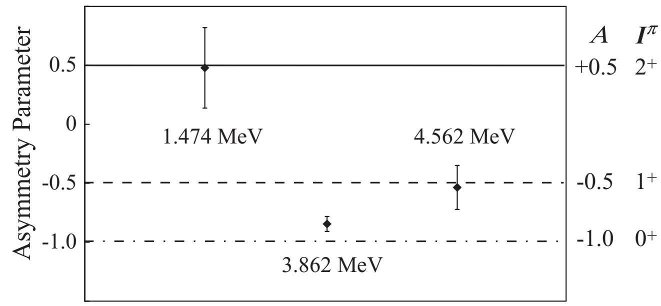

FIG. 8. Asymmetry parameters of the β transitions leading to the 1.474-, 3.862-, and 4.562-MeV levels.

2. Spin-parity of the new level at 7.461 MeV

The intensities of γ rays deexciting the new level at 7.461 MeV were too weak to determine the asymmetry parameter of $A_{7.461}$. Therefore, the following indirect method of spin assignment was used, based on the discussion in Sec. II A. The key is the known 1^+ assignment for the 5.269-MeV level. Because of the cascade relation of the 2192-keV (7.461 MeV → 5.269 MeV) and 5269-keV (5.269 MeV → g.s.) transitions, the relation in Eq. (10) is applied as

$$A^\gamma_{5269} = A_{5.269} \times \frac{I_\beta^{5.269}}{I_\gamma^{5269}} + A_{7.461} \times \frac{I_\gamma^{2192}}{I_\gamma^{5269}}, \qquad (12)$$

where the asymmetry parameter A^γ_{5269} is deduced from the β-decay asymmetry in coincidence with the 5269-keV γ ray. By taking into account $A_{5.269}(1^+) = -0.5$ and the γ-ray intensity I_γ and the β-decay branching ratio I_β in Tables II and III, respectively, the values of A^γ_{5269} were calculated for the three possible values of $A_{7.461}$. The results are compared in Fig. 9 with the experimental A^γ_{5269}. It is found that only the 2^+ case is consistent with the experimental result. Thus the spin-parity of the 7.461-MeV level was assigned to be 2^+.

3. Spin-parity of the 6.545-MeV level

The level at 6.545 MeV was identified in the ^{26}Mg$(t,p)^{28}$Mg reaction [21]. In the present work the β decay to this level was observed for the first time. The log ft value of 5.8(2) indicates that the transition is an allowed one.

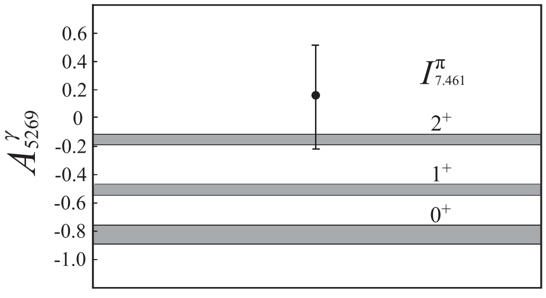

FIG. 9. Asymmetry parameter A^γ_{5269} of β decay in coincidence with the 5269-keV γ ray. The experimental result is shown by the closed circle and the shaded area corresponds to the expected A^γ_{5269} values for the assumed spin-parity of the 7.461-MeV level.

TABLE IV. Experimental intensities and Weisskopf estimates of the probabilities of γ transitions depopulating the 6.545-MeV level.

γ transition (keV)	I_γ^{exp} (%)	I^π(6.545 MeV)	$\sigma\lambda$	$T_W(\sigma\lambda)$ (s^{-1})
		0^+	$E3$	2.5×10^5
1373	<0.1	1^+	$M2$	1.0×10^9
		2^+	$E1$	2.4×10^{15}
		0^+	$E2$	2.0×10^{11}
1991	0.2	1^+	$M1$	2.4×10^{14}
		2^+	$M1$	2.4×10^{14}

The asymmetry parameter of the β decay leading to the 6.545-MeV level could not be determined because of weak intensities of the 1373- and 1991-keV γ rays which deexcite the 6.545-MeV level. Therefore, the spin-parity of this level was determined based on the γ-transition probabilities as follows. The 1373- and 1991-keV γ rays populate the 5.171-MeV (3^- [24]) and the 4.555-MeV (2^+ [24]) levels, respectively, as shown in Fig. 6. The experimental intensities (I_γ^{exp}) and transition probability of the Weisskopf estimate, $T_W(\sigma\lambda)$, for the transitions from the 6.545-MeV level are compared for the possible spin-parity of 0^+, 1^+, or 2^+ of this level in Table IV. It is found that the ratio of intensities can be explained by neither the 0^+ nor the 1^+ case even with a hindrance or enhancement factor for the transition probabilities. In order to consider whether the 2^+ assignment is reasonable, the hindrance factors of $E1$ and $M1$ transitions were examined as follows. The known half-life of the 5.171-MeV level ($T_{1/2} = 120$ fs [27]) enables us to deduce the hindrance factors of $E1$ transitions for the transitions of 5.171 MeV(3^-) $\rightarrow$ 4.021 MeV(4^+) and 5.171 MeV(3^-) $\rightarrow$ 1.474 MeV(2^+) to be 10^{-3}–10^{-4}. Since the 1991-keV transition is most likely an ℓ-forbidden $M1$ transition, a hindrance factor of 10^{-2} is reasonably estimated [32]. Given these hindrance factors, the experimental data are consistent with the 2^+ assignment. Thus the spin-parity of the 6.545-MeV level is proposed to be $(2)^+$.

4. *Other levels*

Three other levels at 4.878, 5.468, and 5.917 MeV were also found as the final states of the ^{28}Na β decay for the first time. These levels have been identified in the ^{26}Mg$(t,p)^{28}$Mg reaction [21]. The spins and parities of the 4.878- and 5.468-MeV levels were reported to be 2^+ and 2, respectively. The former assignment is consistent with the presently measured log ft values of 6.2(2), as mentioned before. The spins and parities of the 5.917- and 7.200-MeV levels are newly proposed to be both $(0, 1, 2)^+$ based on log ft values of 5.7(1) and 5.2(1), respectively.

IV. DISCUSSION

The experimental results were compared with the shell-model calculations with the USD Hamiltonian [36,37] and its revised versions of the USDA and USDB Hamiltonians [38]. The model space is restricted to the *sd* shell. The calculations were carried out by using the NuShell code [39,40]. Prior to estimating the β-decay probability of $^{28}\mathrm{Na}_{\mathrm{g.s.}}$, the wave function of $^{28}\mathrm{Na}_{\mathrm{g.s.}}$ was calculated, and it was confirmed that the experimental value of the $^{28}\mathrm{Na}_{\mathrm{g.s.}}$ magnetic moment, $\mu = +2.420(2)\mu_N$ [41], was reasonably reproduced ($\mu_{\mathrm{calc}} = +2.164\mu_N$). The log ft values were evaluated by assuming a quenching factor of 0.6 for the Gamow-Teller matrix element B(GT) [38].

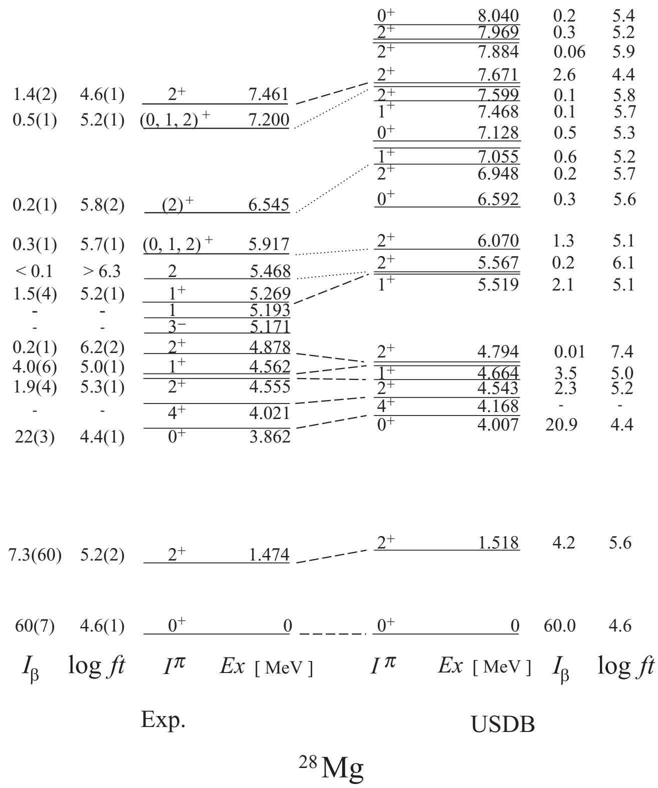

FIG. 10. Experimental and theoretical energy levels. The shell-model calculations were performed with the USDB Hamiltonian. The dashed and dotted lines denote reasonable and possible correspondences, respectively, between the experimental and theoretical levels.

Figure 10 compares the experimental results to the shell-model calculations with the USDB Hamiltonian. Because of the detailed information obtained in the present work, it is possible to compare, on a level-by-level basis, the level energy (E_x), spin-parity (I^π), β-decay branching ratio (I_β), and log ft value. It is to be noted that all the experimental levels lower than 5 MeV (1.474-, 3.862-, 4.021-, 4.555-, 4.562-, and 4.878-MeV levels) show reasonable correspondence with the shell-model calculation, as shown by the dashed lines in Fig. 10. These correspondences are also supported by the fact that both the observed decay pattern and intensities of γ rays deexciting these levels are in good agreement with the prediction.

There are four predicted 2^+ levels above 7.5 MeV. Among them, the 7.671-MeV level associated with the smallest log ft value (4.4) corresponds most likely to the newly found 2^+ level at 7.461 MeV [log $ft = 4.6(1)$]. The experimental 1^+ level at 5.269 MeV [5.2(2)] populates only the ground state (see Fig. 6). This decay pattern is in accord with the theoretical

1^+ level at 5.519 MeV (1^+, 5.1). By taking into account also the $\log ft$ value, the 5.269-MeV level is likely the predicted 1^+ one, as shown by the dashed line in Fig. 10.

Also suggested from the $\log ft$ values and/or decay patterns are the correspondences between the experimental and predicted levels, as shown by dotted lines in Fig. 10: (experimental and theoretical levels) = [5.468 MeV (2, >6.3) and 5.567 MeV (2^+, 6.1)], {5.917 MeV [$(0, 1, 2)^+$, 5.7(1)] and 6.070 MeV [2^+, 5.1]}, {6.545 MeV [$(2)^+$, 5.8(2)] and 6.948 MeV [2^+, 5.7]}, and {7.200 MeV [$(0, 1, 2)^+$, 5.2(1)] and 7.599 MeV [2^+, 5.8]}. Therefore, 2^+ spin-parity assignments are suggested for the 5.468-, 5.917-, 6.545- and 7.200-MeV levels. There are no predicted levels for the 5.171-MeV ($I^\pi = 3^-$) and 5.193-MeV ($I = 1$) levels. The latter level has been assigned to be 1^- for the first time [22] and later work reassigned it to be 1 [23,26]. The present experimental and theoretical results seem to support the original 1^- assignment.

The overall good reproduction by the shell-model calculations with the sd-shell configurations indicates a negligible contribution of the intruder configurations for the positive-parity levels in ^{28}Mg. It is concluded that ^{28}Mg is located outside the region of the island of inversion.

V. SUMMARY

The level structure of ^{28}Mg has been investigated by the β-decay spectroscopy of spin-polarized ^{28}Na. Detailed measurements of β-γ and γ-γ coincidence and β-decay asymmetry enabled (i) the finding of seven new γ transitions in ^{28}Mg and, consequently, (ii) the finding of two new levels at 7.200 and 7.461 MeV and (iii) spin-parity assignments for seven levels. The validity of the method to assign spin and parity was confirmed by the successful reproduction of the previous assignments for three levels. It is to be emphasized that a firm 2^+ assignment for the newly found 7.461-MeV level was performed. Another three levels were also newly assigned in the present work.

The detailed level scheme established in the present work enabled level-by-level comparison between the experimental results and the theoretical predictions. Overall good agreement with the shell-model calculations in the sd-shell model space indicates that the structure of ^{28}Mg is understood mostly based on the normal configurations.

The present work is the first step of a series of investigations on neutron-rich Mg isotopes at TRIUMF. The present achievement promises successful application of the method to heavier Mg isotopes. The results on ^{29}Mg and ^{30}Mg will be published soon.

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