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Breakup processes in heavy-ion induced reactions

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Cross sections for breakup of ^{20}Ne into ^{16}O and α during scattering from ^{40}Ca were calculated in terms of the distorted wave Born approximation. The inclusive ^{16}O cross section observed in the ^{40}Ca(^{20}Ne, ^{16}O) reaction was then found to be fitted very well by the sum of this breakup contribution and that of the α-transfer reaction calculated in our previous work.

[NUCLEAR REACTIONS ^{40}Ca(^{20}Ne,^{16}O) breakup reactions, calculated $d^2\sigma/d\Omega\, dE$; $E(^{20}\text{Ne}) = 149$–262 MeV, direct reaction mechanism for heavy-ion reaction.]

In a recent publication,[1] we reported on measurements of continuum cross sections of α-transfer (like) reactions with a ^{40}Ca target, induced by ^{20}Ne, ^{14}N, and ^{13}C ions with incident energies E_{lab}, respectively, equal to 262, 153, and 149 MeV. The continuum spectra were taken at several angles between $\theta_{\text{lab}} = 5°$–$20°$, and the results were analyzed in terms of the distorted wave Born approximation (DWBA) assuming that the reaction proceeded as a one-step α-transfer process. It was found[1] that the calculation reproduced very well the experimental spectra at all angles for two of the three reactions, i.e., for (^{14}N, ^{10}B) and (^{13}C, ^{9}Be) reactions. On the other hand, for the (^{20}Ne, ^{16}O) reaction, it was found that, although we were able to fit very well the experimental spectra at $\theta_{\text{lab}} \geq 16°$, the theoretical cross sections became progressively too small compared with experiment, as θ_{lab} was decreased. A very similar situation was again experienced in the analysis of more recent data taken at E_{lab} (^{20}Ne) = 149 MeV.

The purpose of the present article is to discuss ways this discrepancy can be removed, i.e., to explain the part of the experimental cross sections that were left *unexplained* within the framework of the DWBA calculations reported in Ref. 1. This unexplained part of the cross section has two distinct features, as can be seen in Fig. 1 of Ref. 1 [and by the dotted curves in Fig. 2(a) of the present work]. The first is that it is forward peaked, as we already remarked above. The second is that the cross section, seen as a function of E_{lab} (^{16}O), is peaked at 205 MeV, which is $\frac{4}{5}$ of E_{lab} (^{20}Ne), i.e., just the kinetic energy which the ^{16}O cluster had in the incident ^{20}Ne ion. These two features indicate that the excess cross section can be explained as the cross section associated with the breakup of the incident ^{20}Ne. We shall show below that such a supposition can indeed be proved correct.

To our knowledge not many calculations of the breakup cross section have been done for heavy-ion induced reactions. In the regime of light-ion induced reactions, however, it has been one of the classic problems, and a number of investigations have in fact been reported, see, e.g., a recent review by Baur and Trautmann.[2] Consider, as an example, the DWBA treatment of deuteron breakup, the amplitude being described in the post

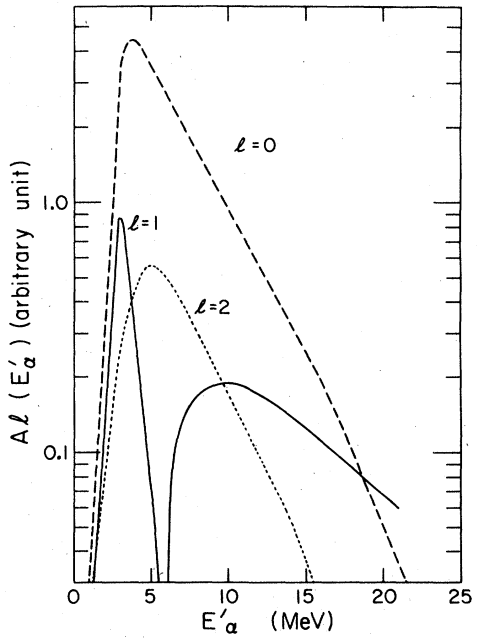

FIG. 1. Calculated $A_l(E'_\alpha)$ values for $l = 0$–2 as a function of E'_α.

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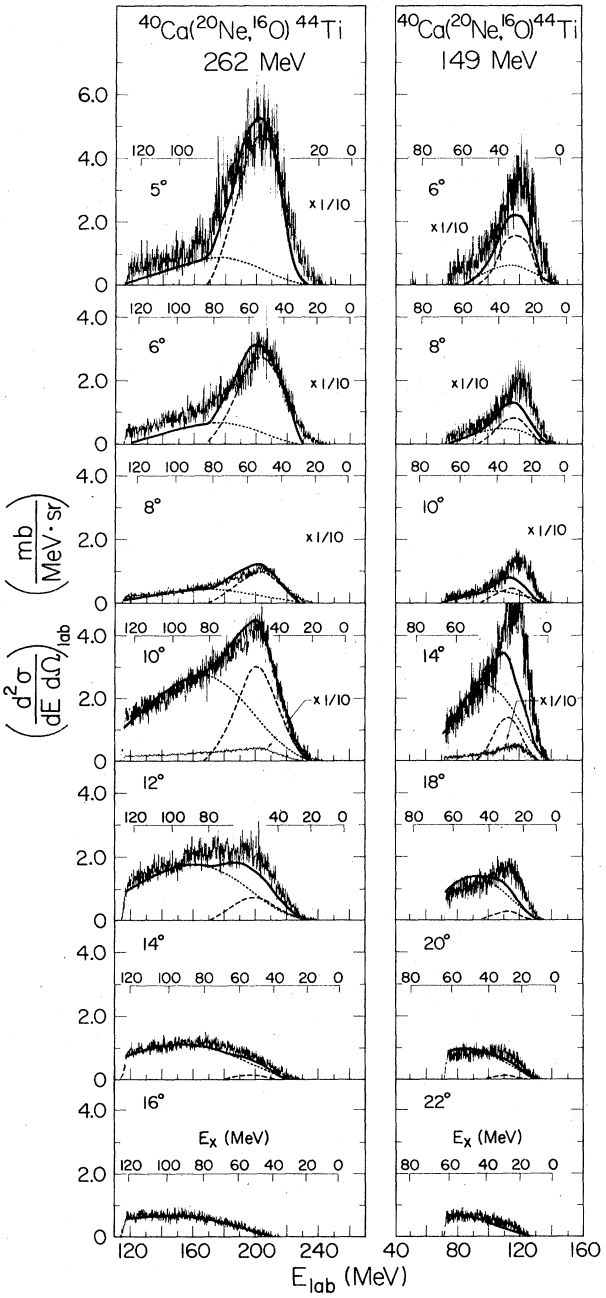

FIG. 2. Energy spectra in the laboratory systems. Each spectrum is labeled by the laboratory angle, and the excitation energy in the residual ^{44}Ti system is indicated by the E_X scale inserted. The dashed and dotted lines represent theoretical cross sections obtained assuming only the breakup and the transfer process, respectively, while the solid lines represent the sum of these two.

form. Contrary to the case, e.g., of a (d,p) reaction, there is ambiguity (or freedom) in replacing the exact bra-vector by a distorted wave, and Baur and Trautmann discussed two possibilities. The first, which is originally due to Landau and Lifschitz,[3] is to use a product of proton and neutron distorted waves, their positions being measured from the target nucleus A. The second, which is originally due to Serber[4] and was studied in further detail by Rybicki and Austern,[5] is to take the approximate bra-vector as a product of the deuteron distorted wave (with respect to A), and the wave function describing the relative motion of the proton and the neutron in the continuum. It was shown in Ref. 2 that, so long as the deuteron breakup data considered there are concerned, the first choice worked excellently, while the second failed. In our calculation, however, the second choice is made. The reason for this choice will be discussed after presenting the results of the calculation.

The DWBA amplitude used to start our calculations may thus be written as[2,5]

$$T=\langle\chi_a^{(-)}(\vec{k}'_a,\vec{r}_a)\chi_\alpha^{(-)}(\vec{k}'_\alpha,\vec{r}_\alpha)|U_{bA}+U_{\alpha A}| \times\phi_\alpha(\vec{r}_\alpha)\chi_a^{(+)}(\vec{k}_a,\vec{r}_a)\rangle\,, \quad (1)$$

where $\chi_a^{(+)}$ and $\chi_a^{(-)}$ are the distorted waves of ^{20}Ne (denoted by the symbol a) with respect to the target A (= ^{40}Ca), in the incident and exit channels, respectively, while ϕ_α and $\chi_\alpha^{(-)}$ stand for bound and continuum wave functions, respectively, of the motion between α and ^{16}O (denoted by b). Note that the T of Eq. (1) can also be interpreted as a DWBA amplitude, describing an inelastic excitation of ^{20}Ne into the continuum.

The operator U_{bA} appearing in (1) is the interaction between ^{16}O and ^{40}Ca, while $U_{\alpha A}$ is that between α and ^{40}Ca. We choose a real Woods-Saxon form for both of them, with the parameters V, a and r_0 equal to 25.3 MeV, 0.399 fm, and 1.38 fm, and 240 MeV, 0.67 fm, and 1.21 fm, respectively.[6,7] In obtaining ϕ_α and $\chi_\alpha^{(-)}$, we also introduced a real Woods-Saxon potential with the above three parameters equal to 130 MeV, 0.65 fm, and 1.25 fm. This gave a correct separation energy of 4.7 MeV for ^{20}Ne into α and ^{16}O. The use of a real potential for obtaining a continuum wave function $\chi_\alpha^{(-)}$ is permissible because, as will be shown soon below, only $\chi_a^{(-)}$ corresponding to very low relative kinetic energy between α and ^{16}O contributes to our breakup process.

The matrix element of the operator in (1) with respect to ϕ_α and $\chi_\alpha^{(-)}$ can be brought into the form

$$\langle\chi_\alpha^{(-)}(\vec{k}'_\alpha,\vec{r}_\alpha)|U_{bA}+U_{\alpha A}|\phi_\alpha(\vec{r}_\alpha)\rangle = \sum_{lm} i^{-l}Y_{lm}^*(\hat{r}_a)Y_{lm}(\hat{k}'_\alpha)F_l(E'_\alpha,r_a)e^{i\sigma_l}\,, \quad (2)$$

where E'_α, l, and σ_l are, respectively, the kinetic energy, the angular momentum, and the

phase shift of the relative motion between α and ^{16}O. Obviously, the function $F_l(E'_\alpha, r_a)$ plays the role of the radial form factor for the inelastic excitation of ^{20}Ne when (2) is used in (1). The dependence of this function on E'_α, as well as r_a, can make its treatment somewhat involved in general. In practice, however, the inelastic process we are considering is highly peripheral, the reaction taking place predominantly within a very narrow radial range of $r_a = 9-11$ fm. We found that, within this range, it is a good approximation to use for F_l the following factorized form:

$$F_l(E'_\alpha, r_a) = A_l(E'_\alpha) f_l(r_a) . \tag{3}$$

The function $A_l(E'_\alpha)$ can be regarded as the spectroscopic strength for the inelastic excitation of ^{20}Ne, l being the angular momentum transferred in this process. Its dependence on l and E'_α is thus of great interest, and in Fig. 1 we show it for $l = 0-2$. It was found that the $A_l(E'_\alpha)$ for $l > 2$ were very small, so that they can safely be ignored. This fact greatly simplifies the calculation.

It is seen in Fig. 1 that $A_l(E'_\alpha)$ are peaked very sharply at around $E'_\alpha \approx 4$ MeV, in accordance with the normally accepted picture of the breakup process; a very small amount of the incident energy is transferred to the relative kinetic energy of the broken-up pair. The sharp decrease of $A_l(E'_\alpha)$ for $E'_\alpha < 4$ MeV is due to the Coulomb and the centrifugal barriers, while that for $E'_\alpha > 4$ MeV is due to the decreased overlap between ϕ_α and $\chi_\alpha^{(-)}$. The large difference in the E'_α dependences of $A_l(E'_\alpha)$ with $l = 0$ and 2 and that with $l = 1$ originated not only from the centrifugal force, but also from the fact that the contributions from U_{bA} and $U_{\alpha A}$ are constructive for $l = 0$ and 2, while they are destructive for $l = 1$. We further found that the radial function $f_l(r_a)$ can be expressed very closely, for all l, by a Woods-Saxon derivative form with $a = 0.8$ fm and $r_0 = 1.4$ fm. We use this form henceforth and thus suppress the suffix l from $f_l(r_a)$.

A further simplification of our calculation can be achieved by parametrizing the DWBA overlap integral in the form

$$I_{l'_a l_a} = [\sqrt{4\pi}/(k_a k'_a)] \int \chi_{l'_a}(k'_a, r_a) f(r_a) \chi_{l_a}(k_a, r_a) dr_a$$
$$= N_0 w(l_d) \exp[-(l'_a - l'^{(0)}_a)^2/\Gamma^2 - i(l'_a - l'^{(0)}_a)\psi] , \tag{4a}$$

with

$$w(l_d) = \exp[-(Q - q_0 - q_1 l_d)^2/\Gamma_Q] \quad (l_d \equiv l'_a - l_a) . \tag{4b}$$

The validity of this parametrized form has been explored and discussed on several earlier occasions.[1,8] It will be obvious in (4) that l_a and l'_a are the orbital angular momenta denoting partial waves of ^{20}Ne in the incident and exit channels, $l'^{(0)}_a$ being the grazing value of l'_a. As was the case before,[1,8] we have carried out accurate DWBA calculations of the overlap integral [i.e., the second version of Eq. (4a), for a large but selected number of sets of values of l_a, l'_a, and reaction Q values] and fixed the parameters in the last version of (4a), so that it reproduces best, in the sense of χ^2, the exact results with the second version. The parameters Γ, ψ, $l'^{(0)}_a$, C_0, Γ_Q, q_0, and q_1 thus fixed had values equal, respectively, to 7.7, 0°, 85, 3.06, 13.5, 0.0, and 3.5 for $E_{lab}(^{20}Ne) = 262$ MeV, and 6.3, 5°, 55, 3.08, 9.2, 0.0, and 2.0 for $E_{lab}(^{20}Ne) = 149$ MeV.

Using the last version of (4) and the asymptotic form[9]

$$Y_{lm}(\theta, 0) \simeq \left(\frac{2l+1}{4\pi} \frac{\theta}{\sin\theta}\right)^{1/2} (-)^m J_m[(l+\tfrac{1}{2})\theta] , \tag{5}$$

J_m being the Bessel function of order m, and also the asymptotic form for the Clebsch-Gordan coefficients involved, it is possible to rewrite the DWBA amplitude as[10]

$$T \simeq N_0 \frac{\Gamma}{4}(2l'^{(0)}_a + 1)\left(\frac{\theta'_a}{\sin\theta'_a}\right)^{1/2} \{\exp[-\Gamma^2(\theta'_a - \psi)^2/4] + \exp[-\Gamma^2(\theta'_a + \psi)^2/4]\}$$
$$\times \{[R_{00}e^{i\sigma_0} + R_{10}e^{i\sigma_1}\cos\theta'_\alpha + R_{20}e^{i\sigma_2}(3\cos^2\theta'_\alpha - 1)]J_0$$
$$- (R_{11}e^{i\sigma_1} + R_{21}e^{i\sigma_2}\cos\theta'_\alpha)\sin\theta'_\alpha\cos(\phi'_\alpha - \phi'_a)J_1 + R_{22}e^{i\sigma_2}\sin^2\theta'_\alpha\cos 2(\phi'_\alpha - \phi'_a)J_2\} , \tag{6a}$$

where

$$R_{00} = A_0 w(0) ,$$
$$R_{10} = 3A_1[w(-1) - w(1)]/2 ,$$
$$R_{11} = 3A_1[w(-1) + w(1)]/2 , \tag{6b}$$
$$R_{20} = 5A_2[3w(-2) - 2w(0) + 3w(2)]/16 ,$$
$$R_{21} = 15A_2[w(-2) - w(2)]/4 ,$$
$$R_{20} = 15A_2[w(-2) + 2w(0) + w(2)]/16 .$$

In Eq. (6a), $(\theta'_a\phi'_a)$ and $(\theta'_\alpha\phi'_\alpha)$ are polar angles of the vectors $\vec{k}'_a$ and $\vec{k}'_\alpha$.

The triple differential cross section is then given by[5]

$$\frac{d^3\sigma}{dE_b d\Omega_b d\Omega_\alpha} = \frac{m_b m_\alpha k_b k_\alpha}{(2\pi\hbar)^5 v_a} |T|^2 . \tag{7}$$

The double differential cross section $d^2\sigma/dE_b d\Omega_b$ can of course be obtained by integrating (7) over $d\Omega_\alpha$.

The cross sections (energy spectra) thus obtained are presented in Fig. 2 as functions of $E_{\rm lab}(^{16}{\rm O})$ at several $\theta_{\rm lab}(^{16}{\rm O})$ by dashed curves. In this figure, presented by dotted curves, are also the cross sections corresponding to the α-transfer process, as obtained in the work of Ref. 1. [The cross sections represented by dotted curves in Fig. 2(a) are exactly the same as those given by solid lines in Fig. 1 of Ref. 1.] The incoherent sum of these two cross sections is also shown in Fig. 2 by solid curves, and it is seen to agree very well with the experiment, particularly for the case with $E_{\rm lab}(^{20}{\rm Ne}) = 262$ MeV. This fact confirms that the cross sections left *unexplained* in Ref. 1 can indeed be explained as largely due to the breakup processes considered here.

To be more precise, a factor $N_{bu} = 1.5$ was multiplied onto the theoretical breakup cross sections, before they were plotted in Fig. 2, while a factor $N_{\rm tr} = 6.8$ was multiplied onto the theoretical transfer cross sections. In other words, the agreement shown in Fig. 2 was obtained to within these two normalization factors. It should be noted, however, that the fit obtained in Fig. 2 still has a significant meaning in that the same pair of N_{bu} and $N_{\rm tr}$ values was used for all the theoretical spectra shown in Figs. 2(a) and 2(b). Also, it is clear that a slight modification of, e.g., the radius parameter r_0 used for the potential binding the α particle to ^{16}O and/or ^{40}Ca, and also the parameters involved in the spectroscopic density, will easily result in cross sections that make $N_{bu} = N_{\rm tr} = 1$. Thus introducing the above normalization factors, which are of the order of unity, does not prevent us from concluding that we have succeeded in fitting the experiment, not only as regards the spectral shape and the angular distributions, but also as regards the absolute magnitude of the cross sections. [A somewhat poorer agreement obtained for $E_{\rm lab}(^{20}{\rm Ne}) = 149$ MeV would not force us to withdraw this conclusion. We believe this discrepancy is largely removed by adding the Coulomb interactions to $U_{bA} + U_{\alpha A}$ of Eq. (1), i.e., by taking into account the Coulomb breakup as well, which is expected to become more important as $E_{\rm lab}(^{20}{\rm Ne})$ decreases.] It may be interesting to remark here that the contributions from $l = 1$ and 2 change the magnitude by about 10–40%, depending on the scattering angles considered, but affect very little the shape of the spectra.

A subtle problem remains, however. In obtaining the above fit, we simply added the two types of cross sections together. The α particle captured by ^{40}Ca may eventually be reemitted and if its continuum wave function is combined with that of ^{16}O, the resultant bra-vector will essentially be the same as the one appearing in Eq. (1). In other words, there is a danger that double counting can be committed. In the present work, however, this double counting has been avoided in practice, because for the two types of processes considered the important values of l_a', for a given pair of l_a and Q, were quite different. This fact may also be seen in Fig. 2 which shows that the two processes contribute to quite different portions of the energy spectra.

The possibility of choosing two different forms for the bra-vector is presumably like that found in describing deuteron breakup, as we remarked in the beginning. In the terminology introduced there, we have used the Serber form[4] to describe the ^{20}Ne breakup. On the other hand, Baur and Trautmann[2] found that the use of the Landau-Lifschitz[3] form was appropriate. We believe that the source of these different conclusions lies in the difference in the energy region considered. The examples considered in Ref. 2 were mostly for cases with rather low energy deuterons. Had they might have found that the Serber form worked equally well or was preferable, as was in fact experienced by Serber himself.[4] In this sense, we believe that the ^{20}Ne breakup reaction we considered here belongs to the high energy regime.

We finally have to explain why the breakup contributions were so small in the (^{14}N, ^{10}B) and (^{13}C, ^{9}Be) reactions, in spite of the rather large spectroscopic amplitude for dissociating ^{14}N into $^{10}{\rm B} + \alpha$ and ^{13}C into $^{9}{\rm Be} + \alpha$; see Ref. 1. The major reason for this is that the Q value for these dissociations to take place is 11.6 and 10.6 MeV respectively, much larger than for dissociating ^{20}Ne into $^{16}{\rm O} + \alpha$. Also, the α particle in $^{14}{\rm N}(^{13}{\rm C})$ is mainly in the $l = 4$ (2) state, which means that in order to break up $^{14}{\rm N}(^{13}{\rm C})$ into $^{10}{\rm B} + \alpha(^{9}{\rm Be} + \alpha)$ in the relative s state, at least 4 (2) units of angular momentum must be transferred. This also reduces dramatically the breakup cross section. The discussion we gave above in relation to Fig. 1 should then convince the reader that the breakup contribution in the above two reactions is indeed small.

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