



Title	High precision polarimeter system for atomic polarization and tilted-foil experiments with 1.7 keV/amu 14N ⁺ beam
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Citation	Review of Scientific Instruments. 2000, 71(5), p. 2045-2049
Version Type	VoR
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Citation: *Rev. Sci. Instrum.* **71**, 2045 (2000); doi: 10.1063/1.1150576

View online: <http://dx.doi.org/10.1063/1.1150576>

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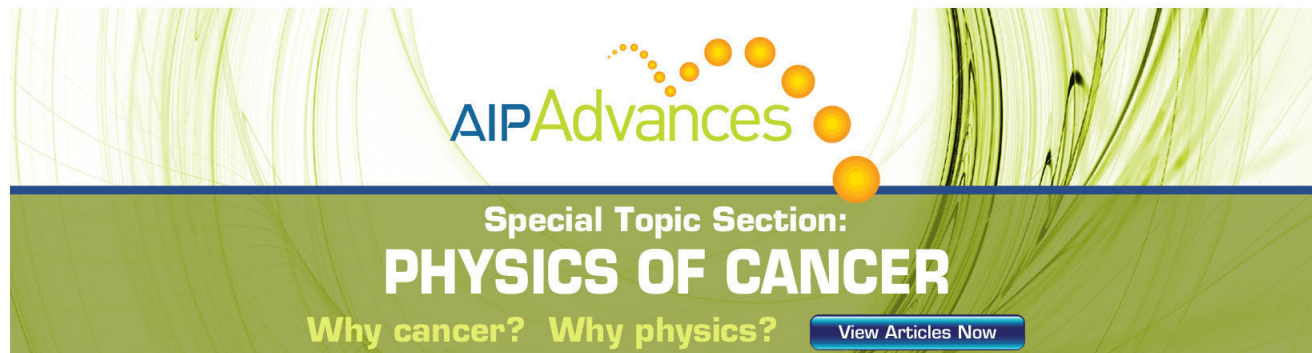
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High precision polarimeter system for atomic polarization and tilted-foil experiments with 1.7 keV/amu $^{14}\text{N}^+$ beam

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(Received 10 November 1999; accepted for publication 5 January 2000)

A high precision atomic polarimeter system for the use in beam-foil spectroscopy experiments with a few keV/amu heavy-ion beams has been developed. The polarimeter measures the circular polarization of fluorescences from the beam ion in-flight after the beam-foil interaction. The present system has two identical such polarimeters in both sides of the beam axis to reduce the systematic errors such due to the fluctuations of beam current, background and so on. A successful use of an ultrathin carbon foil ($1.5\text{ }\mu\text{g}/\text{cm}^2$), which was durable for several hours against a few hundred nA beam irradiation, enabled the beam-foil experiments with such low energy heavy-ion beams. A performance test of the polarimeter system was carried out in the tilted foil experiments with a 1.7 keV/amu $^{14}\text{N}^+$ beam. The atomic polarization was observed for the transition $1s^22s^22p3p\ ^1D \rightarrow 1s^22s^22p3p\ ^1P$, whose fluorescence wavelength is 399.5 nm, in the N^+ ion (NII). The polarization was approximately -2% for the tilt angle of -40° and showed monotone increasing with increasing tilt angle up to $+2\%$ for $+40^\circ$. The polarization at 0° was $(0.002 \pm 0.25)\%$, which is highly consistent with the expected polarization of 0%. This result indicates the high reliability of the present polarimeter system. This is the first tilted-foil experiment at such low beam energy. The present experimental technique will be very useful for studies of the polarization mechanism of the beam-foil interaction. © 2000 American Institute of Physics. [S0034-6748(00)01405-2]

I. INTRODUCTION

We proposed a versatile spin polarizer for low energy radioactive nuclear beams (a few keV/amu) for the spectroscopic studies of exotic nuclei far from the stability.¹ The principle of the polarizer is essentially the same as that in the optically pumped polarized ion-sources for proton.^{2,3} A polarized electron in the optically pumped alkali atom is transferred to the beam ion and the hyperfine interaction induces the nuclear polarization. In order to investigate the problems inherent in the nature of the radioactive heavy-ion beams, we are constructing a test stand, where stable nuclear beams are used, instead of the radioactive beams, at the Research Center for Nuclear Physics, Osaka University (RCNP).^{4,5} The conditions to achieve the highest polarization as well as the highest transmission efficiency of the beams will be investigated there. For the polarization measurement of the stable nuclei, we cannot use sensitive methods such as the β -nuclear magnetic resonance method.⁶ We therefore have to establish methods other than this and we decided to use the method in the beam-foil spectroscopy,⁷ where the nuclear polarization is measured through the atomic polarization.

The principle of the polarization measurement is as follows: After the spin polarizer the polarization transferred from the optically pumped alkali atom is shared by the electrons and the nucleus in the beam ion. The beam then passes through a thin foil that is placed perpendicular to the beam axis. At the moment of interaction with the foil, the atomic

polarization is completely destroyed due to the multiple atomic collisions, whereas the nuclear polarization remains the same because of the longer period of the hyperfine interaction than the beam-foil interaction time. After passing through the foil, some of the atomic excited states are populated, and then the hyperfine interaction induces the atomic polarization from the nuclear polarization. The atomic polarization is proportional to the nuclear polarization and the relation is known, if the atomic transition is identified.^{8,9} The atomic polarization causes circular polarization of the de-excitation lights, i.e., fluorescences. The nuclear polarization thus can be measured through the circular polarization of the fluorescence.

In our nuclear spin polarizer for heavy-ion beams the expected nuclear polarization is typically on the order of 10% and the atomic polarization induced by the nuclear polarization is a few percent. It is therefore necessary to install a polarimeter system which is highly precise for a few percent atomic polarization. It is essential to eliminate the spurious polarization due to the instrumental asymmetry. We have successfully realized a high precision polarimeter system by using two identical polarimeters on both sides of the beam axis. Another development was necessary for the present polarimeter system: For the heavy-ion beams of a few keV/amu, it is necessary to use a foil that is thin enough and durable for the beam bombardment. We have successfully used an ultrathin carbon foil ($1.5\text{ }\mu\text{g}/\text{cm}^2$) in the beam-foil experiments with a 1.7 keV/amu $^{14}\text{N}^+$ beam. In this article we discuss the design of the polarimeter system and

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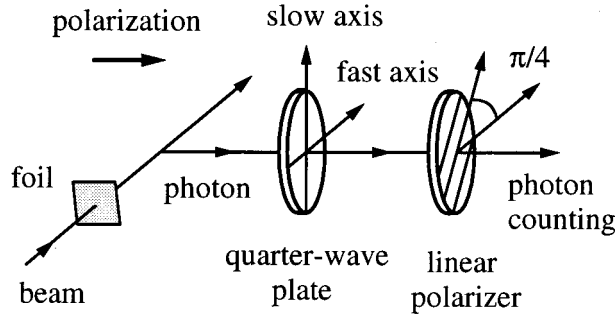


FIG. 1. Typical polarimeter system for atomic polarization in the beam-foil spectroscopy experiments.

demonstrate the performance of the polarimeter system in the tilted-foil experiments.

II. PRINCIPLE OF THE POLARIMETER

In our spin polarizer the polarization axis is perpendicular to the beam direction at the polarimeter system. The fluorescence in this direction emitted from the excited atom is circularly polarized. The circular polarization can be measured by an optical system consisting of a quarter-wave plate followed by a linear polarizer and a photon counter. Figure 1 shows a typical optical system for the beam-foil spectroscopy measurements. The quarter-wave plate converts the right-handed circularly polarized lights into the linearly polarized lights with an axis at $\theta = 45^\circ$, when the fast axis of the quarter-wave plate is set along the direction $\theta = 0^\circ$ as shown in Fig. 1. Suppose that we observe the fluorescence from the atomic polarized state described by the normalized Stokes parameter S/I , where $S = I(\sigma^-) - I(\sigma^+)$ and $I = I(\sigma^-) + I(\sigma^+)$ with $I(\sigma^-)$ and $I(\sigma^+)$ being the intensities of the right- and left-handed circularly polarized light in the optical conversion, respectively. The light intensity transmitted through the two optical plates is expressed as

$$I_R = \varepsilon_R(0^\circ) \times I_0 \times \frac{1}{2}(1 + S/I), \quad (1)$$

where $\varepsilon_R(\theta)$ and I_0 are the overall photon counting efficiency when the quarter-wave plate axis is set at an angle θ and the beam current, respectively.¹⁰ The subscript R denotes that the polarimeter system is set in the right-hand side with respect to the beam direction, and it is meaningless for a single-arm system such shown in Fig. 1. The factor $\frac{1}{2}(1 + S/I)$ is the transmission efficiency of the two optical plates. In order to obtain the absolute value of the polarization S/I , we have to know the absolute values of $\varepsilon_R(0^\circ)$ and I_0 , or we have to measure the light intensity when $S/I = 0$ as far as $\varepsilon_R(0^\circ) \times I_0$ is the same. The latter condition is achieved, if the optical pumping laser is turned off in our specific system. In a usual beam-foil system the quarter-wave plate is instead rotated by 90° : Now the quarter-wave plate and the linear polarizer transmit the left-handed circularly polarized light. The light intensity becomes

$$I'_R = \varepsilon_R(90^\circ) \times I'_0 \times \frac{1}{2}(1 - S/I). \quad (2)$$

The intensity ratio is

$$\frac{I_R}{I'_R} = \frac{\varepsilon_R(0^\circ)}{\varepsilon_R(90^\circ)} \times \frac{I_0}{I'_0} \times \left(\frac{1 + S/I}{1 - S/I} \right) \sim \frac{\varepsilon_R(0^\circ)}{\varepsilon_R(90^\circ)} \times \frac{I_0}{I'_0} \times (1 + 2S/I). \quad (3)$$

Here, if we can assume that (i) the beam currents I_0 and I'_0 are the same in the separate measurements of I_R and I'_R ; (ii) the backgrounds are also the same; and (iii) the photon counting efficiency $\varepsilon_R(\theta)$ of the quarter-wave plate does not depend on the angle θ , we can obtain S/I . The last assumption seems to be reasonable. However, the beam currents are not stable in general. The beam current measured downstream from the foil depends on the foil condition. In particular in the energy region of a few keV/amu, most of the beam atoms are in the neutral charge state after the foil¹¹ and the beam current therefore must be measured as the current on the foil. The current may be affected by the foil condition to a large extent. An alternative method is to monitor the intensity of the light from the foil region as a measure of the beam current. It is still affected by the fluctuation. Furthermore the backgrounds are not necessarily stable in general. This single-arm method therefore has inherent problems of spurious polarization due to the fluctuations.

In our system we have installed another identical polarimeter on the opposite side to the above polarimeter with respect to the beam axis and the measurement was performed *at the same time*. The light intensity in the opposite side polarimeter (denoted by L) is expressed as

$$I_L = \varepsilon_L(0^\circ) \times I_0 \times \frac{1}{2}(1 - S/I), \quad (4)$$

since the helicity of the photon is opposite that of the right-hand side system. Note that the beam current I_0 is exactly the same as that in Eq. (1). By taking the intensity ratio the influence due to the beam current can be eliminated with a high accuracy

$$I_R/I_L = \frac{\varepsilon_R(0^\circ)}{\varepsilon_L(0^\circ)} \times \left(\frac{1 + S/I}{1 - S/I} \right) \sim \frac{\varepsilon_R(0^\circ)}{\varepsilon_L(0^\circ)} \times (1 + 2S/I). \quad (5)$$

This, however, is now subject to the spurious asymmetry on the left-right counting efficiency. This problem is solved simply by rotating the quarter-wave plate by 90° in both polarimeters. The intensity ratio in this geometry is expressed as

$$I'_R/I'_L = \frac{\varepsilon_R(90^\circ)}{\varepsilon_L(90^\circ)} \times \left(\frac{1 - S/I}{1 + S/I} \right) \sim \frac{\varepsilon_R(90^\circ)}{\varepsilon_L(90^\circ)} \times (1 - 2S/I), \quad (6)$$

and in the double ratio the left-right difference in the counting efficiency is canceled out by reasonably assuming that $[\varepsilon_R(0^\circ)/\varepsilon_R(90^\circ)] \times [\varepsilon_L(90^\circ)/\varepsilon_L(0^\circ)] = 1$

$$(I_R/I_L)/(I'_R/I'_L) = \left(\frac{1 + S/I}{1 - S/I} \right)^2 \sim 1 + 4S/I. \quad (7)$$

This is free from the spurious asymmetry due to fluctuations. It should be noted that the polarization can be measured with a higher sensitivity by a factor of 2 than in a single-arm system, as shown in Eq. (3).

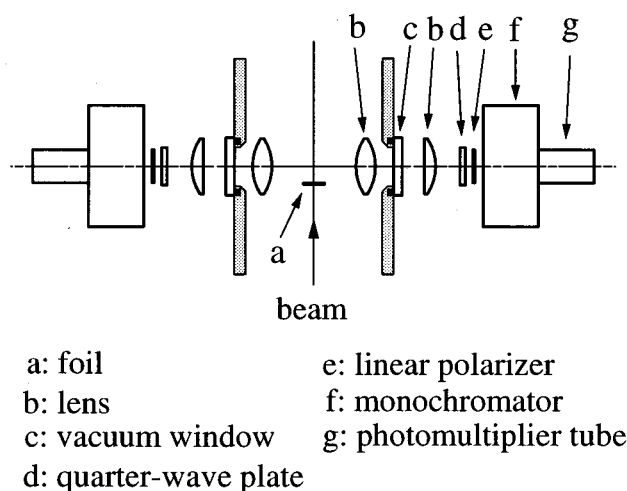
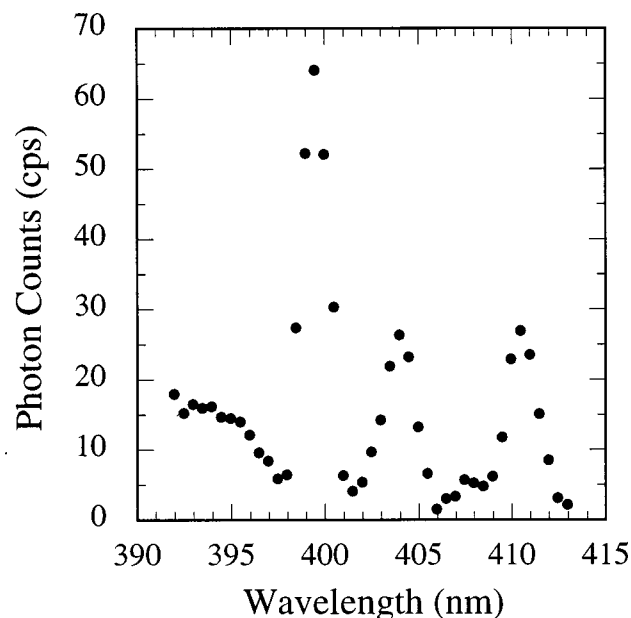


FIG. 2. The polarimeter system in the present work.

III. EXPERIMENT

A. Experimental setup

Prior to completion of the test stand for the nuclear spin polarizer, the performance of the present atomic polarimeter system has been tested. The $^{14}\text{N}^+$ beam at a few keV/amu was supplied from an ultracompact 2.45 GHz electron cyclotron resonance (ECR) ion source incorporating permanent magnets for the confinement field.¹² After the mass/charge-state analysis, the beam was focused directly on the polarimeter system. Figure 2 shows the polarimeter system from a more realistic viewpoint than that in Fig. 1. A carbon foil with $1.5\ \mu\text{g}/\text{cm}^2$ thickness was placed on the beam axis. This foil was prepared by Dr. Sugai at the High Energy Accelerator Research Organization, Japan (former INS and KEK), based on the prescription developed for the long-lived stripper foils ($\geq 10\ \mu\text{g}/\text{cm}^2$).¹³ The uncertainty of the foil thickness was conservatively estimated to be $\pm 0.5\ \mu\text{g}/\text{cm}^2$ at present. Establishing the method to precisely measure the thickness of such thin foils is an important subject. We have also tested the foils of 5.0 and $1.0\ \mu\text{g}/\text{cm}^2$. The former foil was so thick that the beam stopped in the foil and the latter was too short-lived. The foil could be moved along the beam direction in a range of 25 mm in order to change the distance between the foil and the optical axis of the polarimeter system. The beam spot size at the foil was approximately 5 mm in diameter. This somewhat large size was helpful for increasing the foil lifetime. It was also found to be important (for the lifetime) to irradiate the foil with a beam whose intensity was spatially uniform. The electric quadrupole lens system in the beam line was used to control the beam uniformity as well as the beam spot size. The foil was insulated and the current on the foil was monitored as the measure of the beam current. The fluorescence from the nitrogen ion in-flight was observed downstream from the foil by two identical polarimeters. The fluorescence was focused by two fused silica biconvex lenses, one in vacuum and the other in atmosphere, to a monochromator followed by a photomultiplier tube. The monochromator (NIKON P-250 with a blazed grating of 1200 lines/mm optimized for 300 nm wavelength)

FIG. 3. Fluorescence spectrum after the 1.7 keV/amu N^+ ion interacted with a thin carbon foil.

was used to select the wavelength region of the fluorescence. The vacuum window was made of calcium fluoride CaF_2 . The photon counting was made by a photomultiplier of a very low noise type with a bialkali photocathode (Hamamatsu R464S), which has the highest sensitivity for the 420 nm region. The noise rate was 0.5–1 counts/s. The lens geometry was determined so that not only the highest figure of merit $I_i \times (S/I)^2$ ($i=R$ or L) is obtained but also the light from the foil is invisible as much as possible in close proximity of the foil. It is estimated that about 1.3% of the photons emitted from a point source on the beam axis can be accepted by a typical monochromator slit aperture (1 mm wide and 10 mm high: $\Delta\lambda = 1.5$ nm). The photon counting was made for 100 s and the quarter-wave plates were rotated subsequently in a second by a stepping motor system (SIGMA MINI-60). This cycle was repeated to accumulate sufficient counting statistics. The counting rate was several tens counts/s with a typical beam intensity of 300 nA. The foil was durable for several hours at this beam intensity.

B. Search for fluorescences for the beam–foil spectroscopy

Figure 3 shows the fluorescence spectrum at 2 mm downstream from the foil. The wavelength resolution of the monochromator was set at 0.7 nm. A prominent peak around 399.5 nm corresponds to the transition in the N^+ ion (NII) from the $1s^2 2s^2 2p^3 p^1 D$ state to the $1s^2 2s^2 2p^3 p^1 P$ state. A broad peak around 410 nm corresponds to the transition in a neutral nitrogen (NI) from the 2D state to the 2P state. A peak around 404 nm is a multiplet in NII. It was found that the fluorescence intensity from NII showed increase with increasing beam energy, whereas that from NI showed decrease, as far as an energy range of $\pm 40\%$. This may be due to the energy dependence of the charge state distribution and/or the dynamics of the beam–foil interaction. The fluo-

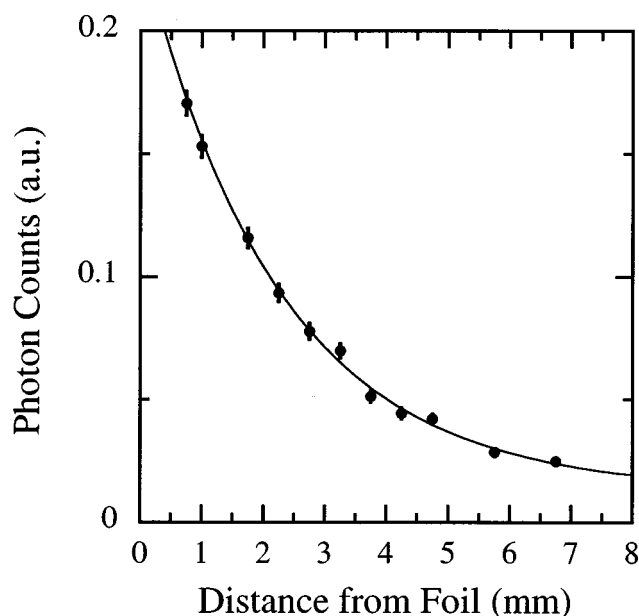


FIG. 4. The light intensity of the transition $^1D \rightarrow ^1P$ in N II (399.5 nm) as a function of the distance from the foil.

rescence intensity was measured as a function of the distance from the foil with the monochromator wavelength being set at 399.5 ± 0.75 nm. In this setup the fluorescence is integrated at a distance range of 1 mm. Figure 4 shows the result. The solid line in Fig. 4 shows the least-square fit by assuming that the ions decay in-flight with their mean lifetime of the initial state $1s^2 2s^2 2p 3p \ ^1D$ in N II of 6.3 ns,¹⁴ and the velocity of the ions after the foil was a free parameter. The excellent fit implies that the fluorescence from the foil was invisible after 1.0 mm from the foil and the fluorescence in Fig. 3 was certainly that from the ions in-flight. The ion velocity after the foil was determined to be $(0.354^{+0.018}_{-0.015})$ mm/ns or equivalently $(0.162^{+0.008}_{-0.007})$ AU. It should be noted that the energy loss of the ions in the foil can be determined in this analysis. In the above case the energy loss was estimated to be $(14.7^{+0.8}_{-1.0})$ keV. Note that this is concerned only with the N^+ ions, which remain in the $1+$ charge state after their passage through the $1.5 \mu\text{g}/\text{cm}^2$ carbon foil. Namely, the present experimental technique enables the energy loss measurements for a specific ionic charge state. This method is therefore effective for investigations of the energy loss mechanism when the charge state equilibrium is not achieved.

C. Performance test of the polarimeter system in the tilted-foil experiments

In order to test the performance of the polarimeter system, the tilted-foil technique has been used: When the foil is tilted as shown in Fig. 5 atomic polarization will be induced in the direction $\mathbf{n} \times \mathbf{v}$, where \mathbf{n} and \mathbf{v} denote the normal vector to the foil surface and the beam velocity, respectively. The foil was so tilted that the polarization axis was the same as that in the polarimeter system. The circular polarization was measured for the $^1D \rightarrow ^1P$ transition (the monochromators were set at 399.5 ± 0.75 nm) at 3 mm from the foil with the $1.7 \text{ keV}/\text{amu}$ $^{14}\text{N}^+$ beam. The zero-order quartz wave-

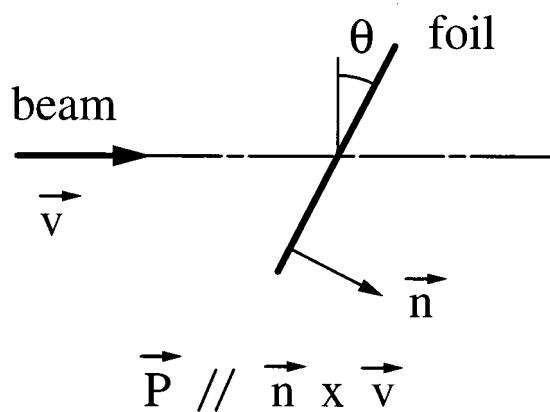


FIG. 5. Polarization in the tilted-foil experiments.

plates optimized for 441.6 nm were used. It is estimated that this wavelength mismatch causes 2% error in the absolute magnitude of S/I , which is much smaller than the statistical error. Figure 6 shows the results of the polarization as a function of the tilt angle. Here the absolute magnitude of polarization has been corrected for the reduction due to the finite angular acceptance of the lens system. The correction was an approximately 5% increase of the measured S/I value, according to the computer simulation. The data in Fig. 6 are the first results of the tilted-foil experiment in this extremely low energy region. It is seen in Fig. 6 that the polarization smoothly depends on the tilt angle. The polarization is approximately -2% for the tilt angle of -40° and shows monotone increasing with increasing tilt angle up to $+2\%$ for $+40^\circ$. This behavior is in accordance with the prediction by the ‘torque model.’¹⁵ The magnitude of the polarization is somewhat smaller than those observed in experiments at higher beam energy, typically a few hundred keV.^{15,16} Such an energy dependence is also reported in the experiments of the ion beam surface interaction at glazing incidence.¹⁷ It should be noted that at the tilt angle of 0° the polarization is $(0.002 \pm 0.25)\%$, which is very consistent with the expectation that the polarization must be zero at zero tilt angle, because the polarization is due to the asymmetric interaction between the electrons in the beam atom

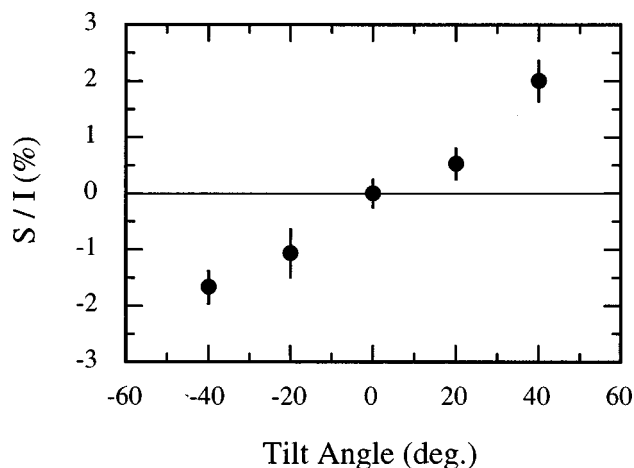


FIG. 6. Atomic polarization as a function of the tilt angle for the $^1D \rightarrow ^1P$ transition in N II (399.5 nm).

and the electrons in the foil material at the very exit of the foil. This result indicates the high reliability of the present polarimeter system.

IV. DISCUSSION

The present double-arm polarimeter system is free from the systematic errors such as those due to the fluctuations of beam current, background, and so on and this system therefore ensures highly precise polarization measurements. The success of the long-lived thin carbon foils also makes it possible to perform the tilted-foil experiments with ultralow energy heavy-ion beams. These progresses enable us to systematically investigate the polarization mechanism in the tilted foil experiments, in particular the energy dependence of the polarization mechanism. It should also be noted that the observation of the fluorescences in-flight, such as shown in Fig. 4, provides information on the beam ion energy loss in a thin foil for a *specific ionic charge state*. This will give a unique tool to investigate the energy loss mechanism. In the tilted-foil experiments at such a low beam energy of 1.7 keV/amu, the atomic polarization on the order of a few percent was unambiguously observed. This is the first tilted-foil experiment in this energy region. The present results encourage us to apply the tilted-foil method to produce the nuclear polarization for the beams from an on-line isotope separator, which provides unstable nuclear beams of a few keV/amu energy.

ACKNOWLEDGMENTS

The authors are grateful to the staff at RCNP for their warm encouragement throughout our R&D project of the spin polarizer. They would also like to thank Dr. E. Tojyo and Dr. I. Sugai at the High Energy Accelerator Research Organization for providing us the ECR ion source and the ultrathin carbon foils, respectively. Discussions with Dr. M. Tanaka, Dr. M. Adachi, Dr. Y. Nojiri, Dr. M. Kimura, and

Dr. S. Momota were very valuable. A critical reading of the manuscript by Dr. Nojiri is gratefully acknowledged. This work was supported in part by Grant-in-Aid for Scientific Research of the Japan Ministry of Education, Science and Culture and by the Yamada Science Foundation. The KEK support program for R&D was also very helpful.

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