Exchange Current and Core Polarization Effect on Time Component of Axial Vector Current in Nuclear β Decay

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Abstract

The time component of weak axtial vector current is investigated in the β-ray angular distribution from oriented nuclei for the A=12 and A=13 systems. New formalism of β decay is adopted, which introduces the nuclear form factors and treats the lepton wave functions with no approximations. The analysis in the impulse approximation with the 0p shell nuclear wave functions shows a good agreement between the experiments and the theoretical calculations, if we assume the C.V.C. hypothesis and no existence of the second-class current. However, the exchange current contribution to the time component is calculated and it enhances the matrix element of the time component by about 30%. This discrepancy is solved by introducing a more realistic nuclear model through the first-order core polarization effects. Although the first-order core polarization does not affect the space component of the axial vector current, it has an appreciable effect on time component due to the momentum dependent nature of the operator. In this case, the core polarization by the tensor force has a crucial effect and it is dominated by the intermediate states with $2\hbar\omega$ excitation. The calculation with the tensor force of the Hamada-Johnston type reduces the matrix element of the time component by about 30%. This almost cancels the exchange current contribution. Thus the total value of the time component by taking with these contributions is nearly equal to that of the impulse term. It is also found that the situation is the same in the A=13 system. Finally we conclude that the individual values of the exchange current and the core polarization effects on the time component are considerably large for the asymmetry parameter of β-ray angular distribution, however, they almost cancel each other.

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Contents

§1 Introduction

The study of weak nuclear processes [1] has two main purposes. One is the understanding of the nature of the weak interaction, and the other is to obtain nuclear structure informations from the weak processes.

Nucleus has been a testing ground for fundamental interactions, especially for the weak interaction. Important properties of the weak interaction were discovered mostly from the study of atomic nuclei. These are the discovery of the parity nonconservation [2], determination of the V-A structure of the weak interaction [3] and the test of C.V.C.(conserved vector current) hypothesis [4,5]. Although the elementary particle physics and the low energy nuclear physics are to a large extent now going on their separate ways, the study of weak interaction in nuclei still provides the significant and important informations about some aspects of the fundamental interactions.

On the other hand, the nucleus appears now an excellent testing ground and generator of many-body theories. Once the fundamental nature of the weak interaction is understood, it can be used as a well known probe for testing our theoretical ideas on nuclear structure. The electromagnetic interaction also provides an excellent tool to study the nuclear structure, because it is very well known and we can extract informations without much disturbing the nuclear structure. Owing to the C.V.C. hypothesis, the half of the weak current (the weak vector current) is related to the electromagnetic current, and the other half (the weak axial vector current) brings new excitation modes of nucleus which electromagnetic interaction cannot excite, and new informations. Thus both of these interactions work complementary.

Recent developement of intermediate energy nuclear physics concerns on the new many-body aspects of nuclei: the exchange current or the extra nucleonic degrees of freedom in nuclei. In this stage, weak interaction again plays an important role. The P.C.A.C.(partially conserved axial vector current) hypothesis [6] tells us that the divergence of the axial vector current is proportional to the pion field. Therefore the strong interaction and the weak interaction in the nucleus are connected by the P.C.A.C. hypothesis, and the pionic or the other extra nucleonic degrees of freedom are strongly related to the weak interaction. [7]

The study of weak interaction now stands on the intersection of the elementary particle physics, conventional nuclear physics and intermediate energy nuclear physics.

Recent study of the structure of weak nuclear current attracted attention by a possible evidence of the S.C.C.(second-class current) pointed out by Wilkinson in 1970 [8]. The S.C.C. was first introduced by Weinberg [9], and it is defined by the G transformation properties of the currents, where G transformation is the product of the charge conjugation and the 180° rotation around the iso y-axis. The first-class current transforms as $GV_\lambda^{(1)}G^{-1} = V_\lambda^{(1)}$ and $GA_\lambda^{(1)}G^{-1} = -A_\lambda^{(1)}$, for vector and axial vector currents, respectively, while the second-class current has the opposite sign under the G transformation. The ordinary nuclear currents belongs to the first-class current. Wilkinson and his co-workers have made an extensive search for the asymmetries of the ft values in the mirror β decays [10]. The ft-value asymmetry can be written as

$$\frac{(ft)_+}{(ft)_-} - 1 = \delta_{exp} = \delta_{scc} + \delta_{nucl} \qquad (1.1)$$

Here δ_{scc} comes from the S.C.C. term and δ_{nucl} arise from the mirror asymmetry of the Gamow-Teller matrix elements, which is the nuclear structure effect induced by the electrmagnetic interaction. Using the parametrization of Kubodera, Delorme and Rho [11], the systematic analysis of δ was done in the mass number A=8-30 region. However, the effect of δ_{nucl} is strongly dependent on the nuclear model and it was difficult to obtain a definite conclusion about δ_{scc}..

Less ambiguous informations about S.C.C. can be obtained from the investigation of angular correlation coefficients in nuclear β decay [12], [13] such as the β asymmetry from polarized nucleus or β-γ directional correlation. As is shown in the later section, the S.C.C. effect appears with the ratios of the nuclear matrix elements for each case of β^- and β^+ decays, hence ambiguities due to nuclear structure become smaller. The β asymmetry of the A=12 system provides us one of the best candidate to investigate the problem. Energy releases are very large (up to 15 MeV) and the structure of nuclei is well known. Furthermore, geometrical factors favor decays of this system. These types of experiments determine a linear combination of the " weak magnetism " term and S.C.C. term. Thus the test of C.V.C. hypothesis again excited attension so as to fix the " weak magnetism " term more precisely.

In 1976, Calaprice and Holstein [14] noticed that a systematic error found in the numerical table of positron wave function may change the results of Lee, Mo and Wu [5]. Their reexamination of the spectral shape factor of the A=12 system with the latest value of the end point energy of β rays shows that the C.V.C. hypothesis is not supported strongly as was previously thought. Subsequently in 1977, Wu, Lee and Mo [15] found again a consistency of experiments with C.V.C. hypothesis in their repeated analysis of their data by adopting the latest value of the branching ratios of the inner β groups. New experiments [16] and theoretical analysis with higher precision [17], [18] have been made for this problem and confirmed the conclusion of Wu, Lee and Mo. The other candidates are also investigated and they are consistent with the C.V.C. hypothesis [18].

The β-ray asymmetry from ^{12}B and ^{12}N are investigated experimentally[19),20)] and theoretically[21)]. Assuming the C.V.C. hypothesis, the S.C.C. term is found to be small and it is consistent with no existence of S.C.C.. A systematic analysis with the other experiments also supported no existence of S.C.C. [22)]. Thus the problem of the S.C.C. and C.V.C. in nuclear weak interaction seemed to be solved, but there still remained the question about the exchange current.

In 1978, Kubodera, Delorme and Rho [23)] pointed out that a clear evidence of exchange current can be obtained not in the space component but in the time component in the case of axial vector current. Although there has been an extensive study of the exchange current for the space component of the axial vector [24)] (that is, the correction to the allowed Gamow-Teller matrix element in β decay and μ capture), there are many ambiguities remained to draw a definite conclusion. Using soft pion theorems and current algebra, they showed that the exchange current effects on the time component of axial vector current is given with less model dependence, and it appears in the β-ray asymmetry of the A=12 system. It is also suggested that the 0^+-0^- transition (as $^{16}N(0^-) \rightarrow {}^{16}O(0^+) + e^- + \bar{\nu}_e$ or $^{16}O(0^+) + \mu^- \rightarrow {}^{16}N(0^-) + \nu_\mu$) is strongly affected by the large exchange current for the time component.

In 1978, Guichon, Giffon and Samour [25)] made an analysis of the ratio, W_μ/W_β , of the μ-capture to β-decay rate, based on the 1p-1h Tamm-Dancoff approximation. They found that the inclusion of the exchange current for the time component reduces W_μ/W_β about factor 2 and it falls into the present experimental value. However, it was pointed out that the nuclear structure effect cannot be eliminated by taking the ratio of μ capture and β decay [26], and there still remains a strong nuclear structure dependence. Due to the complicated nuclear structure of the A=16 system and the experimental uncertainties, it is difficult to obtain a definite conclusion about the exchange current for the time component at this stage.

The analysis of β-ray asymmetry [21] was performed in the impulse approximation by adopting the Cohen-Kurath wave function [27] which is able to reproduce the electromagnetic and weak transitions in the 0p shell region quite well, and it was consistent with no large exchange current for the time component. Therefore, the question is " Where have the large exchange current for time component gone ? ".

The exchange current problem is usually followed by the nuclear structure problem called " configuration mixing " or " core polarization ". They are extensively studied in the magnetic properties of nuclei [24], and revealed the interplay between the configuration mixing (core polarization) and the exchange current. For example, the magnetic moment and the Gamow-Teller β decay rates in the LS closed ± 1 nuclei, or the inelastic electron scattering. Recently, in connection with the study of precritical phenomena of pion condensation [28], M1 form factors of the A=12 and A=13 systems are investigated. [29] It is shown that the first-order core polarization effect with tensor force can reproduce the experiments up to the second maximum of the form factors.

While in the study of β decays in the A=12 system, the core polarization effect has been neglected, since the Gamow-Teller matrix elements cannot be renormalized by the first-order core polarization, and other operators are expected to be renormalized in the same way. [30] But we find that it is not the case. Since the time component operator of the axial vector is momentum dependent, it is renormalized in the first-order core polarization and the large reduction occurs if we adopt the effective interaction used in the analysis of the electron scattering. Thus the experimental data are explained as a cancellation of the exchange current and core polarization contributions, and a new aspect of the interplay between the exchange current and the nuclear structure effect is revealed in nuclear weak processes.

It should be noted here that since we deal with the induced terms in the weak interaction Hamiltonian, that is, we concern with the quantity $O(E/M)$ where E is the typical electron energy and M is the nucleon mass, it is important to take into account the higher order corrections such as the Coulomb correction of finite size nucleus or leptons with higher partial waves. Thus we make a new formalism of β decay where we treat the lepton wave functions exactly and take into account the higher order corrections properly.

In §2, we briefly summarize the theory of nuclear β decay. The weak interaction Hamiltonian of the V-A type is presented, and the nucleon form factors appearing in the expression for the vector and axial vector currents are summarized. The difficulties in the conventional theory of nuclear β decay and the advantages of our formalism are discussed. In §3, we derive a new formalism of β decay. The effective Hamiltonian is derived, which is written as a sum of operators with a definite angular momentum and a parity. The nuclear form factors in the impulse approximaiton are represented explicitly. We also comment on the nuclear recoil correction. As is discussed now, the A=12 system is the best candidate and the experimental data are accumulated. We found that the A=13 system also has a similar property as the A=12 system. Therefore, we investigate both of these systems. The explicit formulas for these cases are given in §4, and the relation of the present formalism with the conventional theory is discussed.

In §5, the nuclear models are discussed. The 0p shell formula: the matrix elements of one-body and two-body operators in the two-orbit states are given. We briefly summarize the formal theory of effective operator, and derive the explicit formula for the first-order core polarization. The details of the effective interactions used in the analysis of core polarization are shown. The properties of the core polarization effects are qualitatively investigated in the simple j-j model. Core polarization effects on the time component operator are compared with those on the M1 operator, and the differences between two cases are clarified.

The exchange current operator for the time component of axial vector current is derived in §6. The operator in the momentum space is derived using low enegy theorems and current algebra. It is transformed into the coordinate space operator and expanded into multipoles. The two-body part of the nuclear form factor for the time component of axial vector current is obtained from the coordinate space exchange current operator.

Numerical results are presented in §7. The analysis in the impulse approximation is performed and confirmed the previous results for β-ray asymmetry coefficients in our new formalism. We calculated the exchange current and core polarization effects on the time component of axial vector current in the A=12 and A=13 systems. The core polarization effects are also calculated with the different effective interactions given in §5, and their effect on the higher order correctins in nuclear β decay are discussed. Discussions and summary are given in §8.

§2 Nuclear β decay

We shall briefly review the weak interaction Hamiltonian and the conventional theories for nuclear β decay.

2.1 Weak interaction Hamiltonian

The nuclear β decays are introduced by the V-A current-current interaction as follows,

$$\mathcal{H}_{\beta}(x) = \frac{G}{\sqrt{2}} J_{\lambda}(x)\, \ell_{\lambda}(x) + h.c. \quad , \tag{2.1}$$

with

$$J_{\lambda}(x) = V_{\lambda}(x) + A_{\lambda}(x) \quad , \tag{2.2}$$

and

$$\ell_{\lambda}(x) = -i\, \bar{\psi}_e(x) \gamma_{\lambda} (1+\gamma_5) \psi_{\nu}(x) \ . \tag{2.3}$$

Here G is the effective vector coupling constant for nuclear β decay and it is related to the μ decay coupling constant G_0 through the relation

$$G = G_0 \cos\theta_c \quad , \tag{2.4}$$

where θ_c is the Cabibbo angle[31]. The systematic analyses of $0^+ - 0^+$ super allowed Fermi β decays have been performed[32],[33] and have yielded the latest value of G

$$G = (2.9960 \pm 0.0009) \times 10^{-12} \quad (\hbar^3/m_e c^2) \,.^{33)} \tag{2.5}$$

In the framework of the Lorentz covariance, the matrix elements of the vector current V_λ and the axial vector current A_λ between nucleon states can be written as,

$$\langle p | V_\lambda | n \rangle = i \bar{\psi}_p (f_V \gamma_\lambda + f_W \sigma_{\lambda\rho} k_\rho + i f_S k_\lambda) \psi_n \quad , \tag{2.6}$$

$$\langle p | A_\lambda | n \rangle = i \bar{\psi}_p \gamma_5 (f_A \gamma_\lambda + f_T \sigma_{\lambda\rho} k_\rho + i f_P k_\lambda) \psi_n \quad , \tag{2.7}$$

$$\bar{\psi}_p = \psi_p^{\dagger} \gamma_4 \quad , \qquad k = k_p - k_n \quad , \quad \sigma_{\lambda\rho} = [\gamma_\lambda , \gamma_\rho]/2i \ . \tag{2.8}$$

The six terms in eqs.(2.6) and (2.7) are called the vector, weak magnetism, induced scalar, axial vector, induced tensor and induced pseudoscalar couplings, respectively. These nucleon form factors are the functions of k^2 and are real if time reversal invariance holds. The f_T and f_S terms are the second-class currents.

If the conserved vector current hypothesis (C.V.C.) holds, the structure of the weak vector current is determined. The electromagnetic current can be written as,

$$\langle p_f | J_\lambda^{em} | p_i \rangle = \frac{i}{2} \bar{\psi}_{p_f} [(F_1^S + \tau_3 F_1^V) \gamma_\lambda + (F_2^S + \tau_3 F_2^V) \sigma_{\lambda\rho} k_\rho] \psi_{p_i} \quad , \tag{2.9}$$

with

$$F_1^S = F_1^V = \left\{ 1 + \frac{k^2}{4M^2} (1 + \mu_p) \right\} F \quad , \qquad F_2^S = - \frac{1}{2M} \left\{ \mu_p + \mu_n \left(1 - \frac{k^2}{4M^2}\right) \right\} F \, ,$$

$$F_2^V = -\frac{1}{2M}\left\{\mu_p-\mu_n\left(1+\frac{K^2}{4M^2}\right)\right\}F \quad , \qquad F=\left\{\left(1+\frac{K^2}{4M^2}\right)\left(1+\frac{K^2}{m_V^2}\right)^2\right\}^{-1} , \quad K=P_f-P_i , \tag{2.10}$$

and

$$\mu_p=1.793 , \qquad \mu_n=-1.913 , \qquad M=939\,MeV \quad \text{and} \quad m_V^2=0.71GeV^2 . \tag{2.11}$$

C.V.C. implies $f_V=F_1^V$ and $f_W=F_2^V$. Since the four momentum transfer K^2 is small in nuclear β decay, the following values are adopted,

$$f_V=F_1^V(0)=1 \quad , \quad f_W=F_2^V(0)=-\frac{\mu_p-\mu_n}{2M} = -\frac{3.706}{2M} . \tag{2.12}$$

Furthermore, the second-class vector current (induced scalar term f_S) vanishes, if C.V.C. hypothesis holds. The experimental upper limits for f_S is derived from the analysis of super allowed Fermi β decays as,

$$f_S=(-0.17\pm 0.80)\times 10^{-3} . \text{ [34)]} \tag{2.13}$$

It is consistent with the C.V.C. prediction $f_S=0$.

Axial vector coupling f_A can be determined from the free neutron β-decay experiments, such as the life time of the neutron decay, β asymmetry in the decay of polarized neutron or the electron-neutrino angular correlation. The latest value is as follows,

$$f_A=-1.254\pm 0.007 . \text{ [35)]} \tag{2.14}$$

The P.C.A.C. hypothesis gives the pseudoscalar form factor f_P as[6)]

$$f_P = 2M f_A / (m_\pi^2 + K^2) \quad . \tag{2.15}$$

Here m_π is the pion mass. For $K^2=0$, this gives

$$f_P \approx -0.06 \quad . \tag{2.16}$$

In the β-decay processes, it is difficult to determine the induced pseudo-scalar coupling f_P, because of the smallness of the momentum transfer. Thus most of experimental tests are concerned with the μ capture processes [1),36)], such as the partial capture rate, average polarization of the recoil nucleus or photon spectrum from radiative μ capture. The upper limits to the induced tensor coupling f_T are determined through the systematic analysis of all available β-decay data,

$$|f_T| \lesssim 0.31 \times 10^{-3} \quad . \tag{2.17}$$ [22)]

We use the following values as canonical values of the β-decay coupling constants,

$$f_V = 1 \ , \ f_W = -\frac{3.706}{2M} \ , \ f_S = 0 \ , \quad f_A = -1.25 \ , \ f_T = 0 \ , \ f_P = -0.06 \ . \tag{2.18}$$

The units $\hbar = c = m_e = 1$ is adopted, unless we state otherwise.

2.2 Theory of nuclear β decay

The theory of nuclear β decay is complicated by the fact that the electron wave functions are not plane waves but are distorted by the Coulomb potential of the daughter nucleus. The transition matrix element M_{fi} for β decay can be written schematically as

$$M_{fi} \approx \int \Psi_f^{+}(\Psi_e^{+}\Psi_\nu)\Psi_i \, d\tau \quad , \tag{2.19}$$

where Ψ_f and Ψ_i represent the final and initial nuclear states, Ψ_e and Ψ_ν are the lepton wave functions. The lepton wave length is large compared to the nuclear dimensions, thus Ψ_e and Ψ_ν vary only slowly through the nuclear volume. It is, therefore, usual to factor out the lepton wave functions from M_{fi} . For example, the transition rate for the allowed transition can be written as [1],

$$W_\beta = \frac{G^2}{2\pi^3}\left(\int_1^{E_0} F(Z,E)PE(E_0-E)^2 dE\right)\left(|f_V \textstyle\int 1|^2 + |f_A \textstyle\int \sigma|^2\right) . \tag{2.20}$$

Here P,E and E_0 are the electron momentum, energy (including rest mass)and its maximum value, and $\int 1$, $\int \sigma$ are called the Fermi and Gamow-Teller matrix elements, respectively. The effects of Coulomb corrections for the electron wave functions are factorized in the conventional Fermi function F(Z,E).

The more precise treatment which includes the higher lepton partial waves and the retardation effects is required if we want to know the induced terms of weak interaction Hamiltonian or the detailed information for the nuclear structure. Two methods are usually adopted to include these higher order corrections. One is to estimate the electron wave functions at the nuclear surface r=R [1),37)] such as,

$$\int \Psi_f^{+} \Psi_e \Psi_i \, d\tau = [\Psi_e(R)/R^\ell] \int \Psi_f^{+} r^\ell \Psi_i \, d\tau . \tag{2.21}$$

Here, ψ_ℓ denotes the ℓ-th partial wave of the electron. While the neutrino wave functions are plane waves and they are expanded in the power series of qr [38], where q is the momentum of the neutrino. For example,

$$\int \psi_f^+ j_0(qr)\psi_i \, d\tau = \int \psi_f^+ \psi_i \, d\tau - \frac{q^2}{6}\int \psi_f^+ r^2 \psi_i \, d\tau \; . \tag{2.22}$$

The second term in the right hand side of eq. (2.22) represents the typical higher order matrix elements. Thus, by factorizing out the lepton wave functions; M_{fi} can be written as the linear combination of the products of the energy independent nuclear matrix elements and lepton combinations.

The other method is to expand both the electron and neutrino wave functions around the center of the nucleus r=0 [39],

$$\psi_\ell(r) = \alpha_\ell \, r^\ell \{ a_0 + a_2 r^2 + a_4 r^4 + \cdots \} \; . \tag{2.23}$$

Each term in the right-hand side of the above equation introduces a new higher order matrix elements. Althouth the latter method seems more rigorous than the former, the power series expansion is valid only in the domain of convergence $r \leqslant R_c$, where R_c is finite. This difficulty disappears if a well behaved nuclear charge distribution like a Gaussian type is used, however, the β-decay formula do not converge at all in this case. To avoid these difficulties, electron wave functions are expressed in the Neumann series and are expanded in combined powers of PR and αZ [40]. But these conventional formulas introduce many types of nuclear matrix elements and it is rather tedious to calculate each term individually if the higher precision of the theory is required.

Therefore we develop a new formalism of the β decay, where the lepton wave functions are treated with no approximation. The exact lepton wave functions are not factorized out of the integral of eq. (2.19). The nuclear wave functions are combined into nuclear form factors, and the integrand in eq. (2.19) is expressed as the products of the lepton wave functions and the nuclear form factors. Thus the transition matrix elements become energy dependent. The present formalism has the following two advantages:

(1) The higher order corrections due to the retardations and higher partial waves of the lepton wave functions can be treated as exactly as possible. These corrections become important when the energy transfer to the electron is large, and a precise analysis of experimental data is required or when cancellations of nuclear matrix elements take place.

(2) In the present formulation, exchange current contributions are easily taken into account, through the nuclear form factors. Since the same nuclear form factors can also be used in the other weak or electromagnetic transitions, the relations among these processes become clear.

§3 Formulation

In this section, we develope general formalism of β decay[17] which are analogously obtained as those of the muon capture in complex nuclei[41]. The lepton wave functions are expanded into multipole series to satisfy the spin and parity selection rules in the transitions. We introduce the concept of the nuclear form factors, and the transition matrix elements are expressed as the integral of the products of the lepton wave functions and the nuclear form factors. The radial integrations are left for the numerical calculations.

3.1 Effective Hamiltonian

The effective Hamiltonian which operates on the nuclear states is derived from the interaction Hamiltonian density of eq. (2.1), by replacing the lepton part by its matrix element $L_\lambda(\mathbf{r})$

$$H_I = \frac{G}{\sqrt{2}} \int J_\lambda(\mathbf{r}) L_\lambda(\mathbf{r})\, d\mathbf{r} \quad , \tag{3.1}$$

with

$$L_\lambda(\mathbf{r}) = \langle e\bar{\nu} | \ell_\lambda(\mathbf{r}) | 0 \rangle = -i\, \overline{\Psi}_{s_e}(\mathbf{r}) \gamma_\lambda (1+\gamma_5) \phi^c_{s_\nu}(\mathbf{r}) \quad , \tag{3.2}$$

for the negatron decay. Here $\Psi_{s_e}(\mathbf{r})$ and $\phi^c_{s_\nu}(\mathbf{r})$ are the electron and antineutrino wave functions, respectively, and S_e and S_ν represent the Z components of their spins. We expand these lepton wave functions into series of the partial waves.

$$\Psi_{s_e}(\mathbf{r}) = 4\pi \sum_{\kappa_e \mu_e m_e} i^{\ell_e} (\ell_e \tfrac{1}{2} m_e s_e | j_e \mu_e) Y^*_{\ell_e m_e}(\hat{\mathbf{p}}) \exp(-i\Delta_{\kappa_e}) \begin{pmatrix} G_{\kappa_e} \chi_{\kappa_e \mu_e} \\ i F_{\kappa_e} \chi_{-\kappa_e \mu_e} \end{pmatrix} \quad , \tag{3.3}$$

and

$$\phi^{c}_{s_\nu}(\mathbf{r}) = \frac{4\pi}{\sqrt{2}} \sum_{\kappa_\nu \mu_\nu m_\nu} i^{-\ell_\nu} (\ell_\nu \tfrac{1}{2} m_\nu - s_\nu | j_\nu \mu_\nu)\, Y^{*}_{\ell_\nu m_\nu}(\hat{q}) \begin{pmatrix} -i f_{\kappa_\nu} \chi_{-\kappa_\nu \mu_\nu} \\ g_{\kappa_\nu} \chi_{\kappa_\nu \mu_\nu} \end{pmatrix} , \tag{3.4}$$

where $\mathbb{P}$ and $\mathbb{q}$ are the momenta of the electron and antineutrino. G_{κ_e} and F_{κ_e} are the large and small components of the radial Coulomb wave functions for the electron and Δ_{κ_e} is Coulomb phase shift given in appendix A. g_{κ_ν} and f_{κ_ν} are the radial wave functions for the neutrino, which is given by the spherical Bessel functions,

$$g_{\kappa_\nu} = j_{\ell_\nu}(qr) , \qquad f_{\kappa_\nu} = S_{\kappa_\nu} j_{\bar{\ell}_\nu}(qr) . \tag{3.5}$$

The total and orbital angular momenta of the leptons are expressed by the quantum number κ, respectively, as follows:

$$j_\kappa = |\kappa| - \tfrac{1}{2} \quad \text{and} \quad \ell_\kappa = \begin{cases} \kappa & (\kappa > 0) \\ -(\kappa+1) & (\kappa < 0) \end{cases} , \tag{3.6}$$

while the sign of κ is defined $S_\kappa = \kappa/|\kappa|$ so that

$$\bar{\ell}_\kappa = \ell_{-\kappa} = \ell_\kappa - S_\kappa . \tag{3.7}$$

We use the notations $\ell_\nu = \ell_{\kappa_\nu}$ and $\bar{\ell}_\nu = \bar{\ell}_{\kappa_\nu}$ etc.
The spin angular function is defined by

$$\chi_{\kappa\mu} = \sum_{ms} (\ell_\kappa \tfrac{1}{2} m s | j_\kappa \mu)\, Y_{\ell_\kappa m}(\hat{r})\, \chi_s , \tag{3.8}$$

with

$$\chi_{1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad , \quad \chi_{-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad . \tag{3.9}$$

Using the partial wave expansion in eqs. (3.3) and (3.4), and the following relation

$$\gamma_5 \phi^c_{s_\nu}(\mathbf{r}) = \frac{4\pi}{\sqrt{2}} \sum_{\kappa_\nu m_\nu \mu_\nu} i^{-\ell_\nu} (\bar{\ell}_\nu \tfrac{1}{2} m_\nu - s_\nu | j_\nu m_\nu) Y^*_{\bar{\ell}_\nu m_\nu}(\hat{q}) \begin{pmatrix} -i f_{\kappa_\nu} \chi_{-\kappa_\nu \mu_\nu} \\ g_{\kappa_\nu} \chi_{\kappa_\nu \mu_\nu} \end{pmatrix}, \tag{3.10}$$

we have an explicit form of $L_\lambda(\mathbf{r})$,

$$L_\lambda(\mathbf{r}) = -i \frac{(4\pi)^2}{\sqrt{2}} \sum_{\substack{\kappa_e m_e \mu_e \\ \kappa_\nu m_\nu \mu_\nu}} (-i)^{\ell_e + \ell_\nu} (\ell_e \tfrac{1}{2} m_e s_e | j_e \mu_e) Y_{\ell_e m_e}(\hat{\mathbf{p}}) e^{i\Delta_{\kappa_e}}$$

$$\cdot [(\ell_\nu \tfrac{1}{2} m_\nu - s_\nu | j_\nu \mu_\nu) Y^*_{\ell_\nu}(\hat{q}) \pm (\bar{\ell}_\nu \tfrac{1}{2} m_\nu - s_\nu | j_\nu \mu_\nu) Y^*_{\bar{\ell}_\nu}(\hat{q})] (\bar{\Psi}_{\kappa_e \mu_e} \gamma_\lambda \psi_{\kappa_\nu \mu_\nu}) \quad , \tag{3.11}$$

with

$$\bar{\Psi}_{\kappa_e \mu_e} = (G_{\kappa_e} \chi^\dagger_{\kappa_e \mu_e} , \, i F_{\kappa_e} \chi^\dagger_{-\kappa_e \mu_e}) \quad , \quad \psi_{\kappa_\nu \mu_\nu} = \begin{pmatrix} -i f_{\kappa_\nu} \chi_{-\kappa_\nu \mu_\nu} \\ g_{\kappa_\nu} \chi_{\kappa_\nu \mu_\nu} \end{pmatrix} \quad . \tag{3.12}$$

In eq. (3.11), the upper (lower) sign refers to the negatron (positron) decay, and this convention is adopted throughout this thesis. The positron formula is obtained from the negatron formula by using the charge conjugation relation, and is shown in appendix B.

To calculate ($\bar{\Psi}_{\kappa_e \mu_e} \gamma_\lambda \psi_{\kappa_\nu \mu_\nu}$), we use the following formula,

$$\chi^\dagger_{\kappa_e \mu_e} (\mathbb{e}^{(\kappa)} \cdot \sigma^{(\kappa)}) \chi_{\kappa_\nu \mu_\nu} = \sum_{JML} (-)^{\mu_e + 1/2} (j_e j_\nu - \mu_e \mu_\nu | JM) S_{JLk}(\kappa_e, \kappa_\nu)$$

$$\cdot \frac{1}{\sqrt{4\pi}} \sum_{m\mu} (L k m \mu | JM) Y_{LM} \mathbb{e}^{(k)}_\mu \quad , \tag{3.13}$$

with

$$S_{JLK}(\kappa_e, \kappa_\nu) = S_{\kappa_e}(-)^{K+1}\sqrt{2(2\ell_e+1)(2\ell_\nu+1)(2j_e+1)(2j_\nu+1)}\,(\ell_e\ell_\nu 00|L0)\begin{Bmatrix}\ell_e & 1/2 & j_e\\ \ell_\nu & 1/2 & j_\nu\\ L & K & J\end{Bmatrix} \tag{3.14}$$

Here K is either 0 or 1, and $\mathbb{e}^{(0)}=1$, $\mathbb{e}^{(1)}$ is the unit vector, $\sigma^{(0)}$ is the 2×2 unit matrix and $\sigma^{(1)}$ is the Pauli spin matrix.

The effective Hamiltonian can be expanded into a series of nuclear multipole operators $\Xi_{JM}(\kappa_e, \kappa_\nu)$, each of which corresponds to the lepton system of the total angular momentum J, its Z component M, with parity $(-)^{\ell_e+\ell_\nu}$. We write down the space and time components of nuclear current explicitly as,

$$V_\lambda(\mathbf{r}) = (\mathbb{J}^V(\mathbf{r}),\ i\rho^V(\mathbf{r})), \quad A_\lambda(\mathbf{r}) = (\mathbb{J}^A(\mathbf{r}),\ i\rho^A(\mathbf{r})). \tag{3.15}$$

Thus the effective Hamiltonian can be derived from eq.(3.1) with eqs. (3.11)-(3.15),

$$H_I = \frac{G}{2}(4\pi)\sum_{\substack{\kappa_e m_e \mu_e\\ \kappa_\nu m_\nu \mu_\nu}} (-i)^{\ell_e+\ell_\nu}(\ell_e \tfrac{1}{2} m_e s_e|j_e\mu_e)\,Y_{\ell_e m_e}(\hat{\mathbf{p}})\,e^{i\Delta_{\kappa_e}}(-)^{\mu_e+1/2}$$

$$\cdot\left[(\ell_\nu \tfrac{1}{2} m_\nu - s_\nu|j_\nu\mu_\nu)\,Y^*_{\ell_\nu m_\nu}(\hat{q}) \pm (\bar{\ell}_\nu \tfrac{1}{2} m_\nu - s_\nu|j_\nu\mu_\nu)\,Y^*_{\bar{\ell}_\nu m_\nu}(\hat{q})\right]$$

$$\cdot\sum_{JM}(j_e j_\nu - \mu_e\mu_\nu|JM)\,\Xi_{JM}(\kappa_e, \kappa_\nu), \tag{3.16}$$

where

$$\Xi_{JM}(\kappa_e,\kappa_\nu) = \sqrt{4\pi}\sum_L \int d\mathbf{r}$$

$$\{ \rho^{\mp V}(\mathbf{r})\, Y_{JM}(\hat{\mathbf{r}})[G_{\kappa_e} g_{\kappa_\nu} S_{JL0}(\kappa_e,\kappa_\nu) - F_{\kappa_e} f_{\kappa_\nu} S_{JL0}(-\kappa_e,-\kappa_\nu)]\,\delta_{JL}$$

$$\mp i\,\mathbb{J}^V(\mathbf{r})\cdot \mathbb{Y}_{JLM}(\hat{\mathbf{r}})[G_{\kappa_e} f_{\kappa_\nu} S_{JL1}(\kappa_e,-\kappa_\nu) + F_{\kappa_e} g_{\kappa_\nu} S_{JL1}(-\kappa_e,\kappa_\nu)]$$

$$-\mathbb{J}^A(\mathbf{r})\cdot \mathbb{Y}_{JLM}(\hat{\mathbf{r}})[G_{\kappa_e} g_{\kappa_\nu} S_{JL1}(\kappa_e,\kappa_\nu) - F_{\kappa_e} f_{\kappa_\nu} S_{JL1}(-\kappa_e,-\kappa_\nu)]$$

$$-i\rho^A(\mathbf{r})\, Y_J(\hat{\mathbf{r}})[G_{\kappa_e} f_{\kappa_\nu} S_{JL0}(\kappa_e,-\kappa_\nu) + F_{\kappa_e} g_{\kappa_\nu} S_{JL0}(-\kappa_e,\kappa_\nu)]\,\delta_{JL} \}, \qquad (3.17)$$

with

$$\mathbb{Y}_{JLM}(\hat{\mathbf{r}}) = \sum_{m\mu} (L1m\mu|JM)\, Y_{LM}(\mathbf{r})\, \mathbf{e}_\mu \quad . \qquad (3.18)$$

3.2 Nuclear form factors

By using the more explicit expression for eq. (3.14), we can rewrite the operator $\Xi_J(\kappa_e,\kappa_\nu)$ as,

$$\Xi_{JM}(\kappa_e,\kappa_\nu) = \sqrt{4\pi(2j_e+1)(2j_\nu+1)/(2J+1)}\,(j_e j_\nu \tfrac{1}{2} -\tfrac{1}{2}|J0) \sum_L \left[\frac{1+(-)^{\ell_e+\ell_\nu+L}}{2}\right]$$

$$\cdot\int d\mathbf{r} \{ \mp\rho^V(\mathbf{r}) Y_{JM}(\hat{\mathbf{r}}) L^V(\kappa_e,\kappa_\nu)\delta_{JL} \mp i\,\mathbb{J}^V(\mathbf{r})\cdot \mathbb{Y}_{JLM}(\hat{\mathbf{r}})\,\mathbb{L}^V_{JL}(\kappa_e,\kappa_\nu)$$

$$-\mathbb{J}^A(\mathbf{r})\cdot \mathbb{Y}_{JLM}(\hat{\mathbf{r}})\,\mathbb{L}^A_{JL}(\kappa_e,\kappa_\nu) - i\rho^A(\mathbf{r}) Y_{JM}(\mathbf{r}) L^A(\kappa_e,\kappa_\nu)\,\delta_{JL} \} . \qquad (3.19)$$

where $L^V(\kappa_e,\kappa_\nu)$ etc. are the combinations of the radial parts of the lepton wave functions and they are tabulated in table 1. If we define the nuclear form factors as,

$$F_J^V(r) = \langle J_f \| \int d\hat{\mathbf{r}}\, \rho^V(\mathbf{r})\, Y_J(\hat{\mathbf{r}}) \| J_i \rangle \ ,$$

$$\mathbb{F}_{JL}^V(r) = \langle J_f \| \int d\hat{\mathbf{r}}\, i\mathbb{J}^V(\mathbf{r}) \cdot \mathbb{Y}_{JL}(\hat{\mathbf{r}}) \| J_i \rangle \ ,$$

$$\mathbb{F}_{JL}^A(r) = \langle J_f \| \int d\hat{\mathbf{r}}\, \mathbb{J}^A(\mathbf{r}) \cdot \mathbb{Y}_{JL}(\hat{\mathbf{r}}) \| J_i \rangle \ ,$$

$$F_J^A(r) = \langle J_f \| \int d\hat{\mathbf{r}}\, i\rho^A(\mathbf{r})\, Y_J(\hat{\mathbf{r}}) \| J_i \rangle \ , \tag{3.20}$$

we can express the reduced nuclear matrix elements of Ξ_{JM} in a simple form as,

$$\begin{aligned} \langle J_f \| \Xi_J(\kappa_e,\kappa_\nu) \| J_i \rangle = {} & \sqrt{4\pi(2j_e+1)(2j_\nu+1)/(2J+1)}\ (j_e j_\nu \tfrac{1}{2} -\tfrac{1}{2} | J 0) \\ & \sum_L \int r^2 dr \Big\{ \mp F_J^V(r)\, L^V(\kappa_e,\kappa_\nu) \mp \mathbb{F}_{JL}^V(r)\, \mathbb{L}_{JL}^V(\kappa_e,\kappa_\nu) \\ & \qquad - \mathbb{F}_{JL}^A(r)\, \mathbb{L}_{JL}^A(\kappa_e,\kappa_\nu) - F_J^A(r)\, L^A(\kappa_e,\kappa_\nu) \Big\} \ . \end{aligned} \tag{3.21}$$

Here the reduced nuclear matrix element is defined by

$$\langle J_f M_f | \Xi_{JM} | J_i M_i \rangle = (J_i J M_i M | J_f M_f)\, \frac{1}{\sqrt{2J_f+1}}\, \langle J_f \| \Xi_J \| J_i \rangle \ . \tag{3.22}$$

Thus the nuclear matrix element is expressed by the radial integral of the products of the lepton wave functions and the nuclear form factors. If we replace the lepton combination with its numerical value at a certain point (e.g. at the nuclear surface) and factorize it out of the integrand, we have the integral of the type of $\int r^{\lambda+2} dr\, F_J^V(r)$ etc., we call these the nuclear matrix elements for β decay in the conventional theories.

We shall show a few simple examples for the relation between the nuclear form factors and the conventional nuclear matrix elements. If ρ^V and $\mathbb{J}^A$ have the following forms,

$$\rho^V(\mathbf{r}) = \sum_{j=1}^{A} \tau_j^{\pm} \delta(\mathbf{r}-\mathbf{r}_j) , \quad \mathbb{J}^A(\mathbf{r}) = \sum_{j=1}^{A} \tau_j^{\pm} \sigma_j \delta(\mathbf{r}-\mathbf{r}_j) , \tag{3.23}$$

we have

$$\sqrt{4\pi} \int F_0^V(r)\, r^2 dr = \langle J_f \| \sum_{j=1}^{A} \tau_j^{\pm} \| J_i \rangle = \sqrt{2J_i+1} \int 1 ,$$

$$\sqrt{4\pi} \int F_{10}^A(r)\, r^2 dr = \langle J_f \| \sum_{j=1}^{A} \tau_j^{\pm} \sigma \| J_i \rangle = \sqrt{2J_i+1} \int \sigma . \tag{3.24}$$

Here $\int 1$ and $\int \sigma$ are the same as in eq. (2.20).
Isospin operators are defined as,

$$\tau^{\pm} = \mp \frac{1}{\sqrt{2}} \tau_{\pm 1} = \frac{1}{2}(\tau_x \pm i \tau_y) . \tag{3.25}$$

3.3 Transition probabilities of β decay

The probability of β-ray emission with energy between E and E+dE in the solid angle $d\Omega_e$ is given by

$$\frac{dW}{dE d\Omega_e} = \frac{1}{(2\pi)^5} pE(E_0-E)^2 \sum_{M_i} \rho_{M_i M_i} a_{M_i} , \tag{3.26}$$

with the density matrix which is defined by

$$\rho_{M_i'M_i} = \sum_{S_e S_\nu M_f} \int d\Omega_\nu \, \langle J_f M_f | H_I | J_i M_i \rangle \langle J_f M_f | H_I | J_i M_i' \rangle^* . \tag{3.27}$$

Here $|J_i M_i\rangle$ and $|J_f M_f\rangle$ stands for the initial and final nuclear states with spins J_i and J_f, and their Z components M_i and M_f, respectively. a_{M_i} is the population of the intitial magnetic substates M_i with a normalization $\sum_{M_i} a_{M_i} = 1$.

From eqs. (3.16), (3.26) and (3.27), we have an explicit form for the β-ray angular distribution from oriented nuclei as follows:

$$\frac{dW}{dE d\Omega_e} = \frac{G^2}{(2\pi)^4} PE(E_0 - E)^2 \cdot \sum_\ell \sum_{JJ'} \hat{f}_\ell(J_i) \frac{1}{\sqrt{2J_i+1}} (-)^{J_f - J_i + J + J' + \ell} W(J_i J_i J J'; \ell J_f) b^{(\ell)}_{JJ'} P_\ell(\cos\theta) , \tag{3.28}$$

with

$$\hat{f}_\ell(J_i) = \sum_{M_i} \sqrt{2J_i+1} \, (-)^{J_i - M_i} (J_i J_i M_i - M_i | \ell 0) \, a_{M_i} , \tag{3.29}$$

$$b^{(\ell)}_{JJ'} = \sum_{\kappa_e \kappa_e' \kappa_\nu \kappa_\nu'} \left(\frac{2}{1+\delta_{JJ'}}\right) (-)^{j_\nu - 1/2} \sqrt{(2j_e+1)(2j_e'+1)(2J+1)(2J'+1)} \cdot (j_e j_e' \tfrac{1}{2} - \tfrac{1}{2} | \ell 0)$$
$$\cdot W(j_e j_e' J J'; \ell j_\nu) \langle J_f \| \Xi_J(\kappa_e, \kappa_\nu) \| J_i \rangle \langle J_f \| \Xi_{J'}(\kappa_e', \kappa_\nu') \| J_i \rangle^*$$
$$\cdot [\delta_{\kappa_\nu \kappa_\nu'} \cos(\delta_{\kappa_e} - \delta_{\kappa_e'}) \pm S_{\kappa_\nu} \delta_{\kappa_\nu, -\kappa_\nu'} \sin(\delta_{\kappa_e} - \delta_{\kappa_e'})] , \tag{3.30}$$

and

$$\delta_{\kappa_e} = \Delta_{\kappa_e} - \frac{\pi}{2}(\ell_e + 1) . \tag{3.31}$$

Here the condition $l_e + l_e' + l$ =even should be satisfied, the angle θ denotes the direction of the momentum of the electron with respect to the nuclear orientation axis. The summations over κ_e, κ_ν and J should be performed so as to satisfy the spin and parity selection rules for the related transitions.

The β-decay spectrum is given by integrating eq. (3.28) over the solid angle for the electron, and we define the shape correction factor C(E) as follows:

$$\frac{dW}{dE} = \frac{G^2}{4\pi^3} F_0 PE (E_0 - E)^2 C(E) \quad , \tag{3.32}$$

with

$$C(E) = \{ \sum_J (-)^J \frac{1}{\sqrt{2J+1}} b_{JJ}^{(0)} \} / F_0 \quad , \tag{3.33}$$

and

$$F_0 = 4(2PR)^{2(\gamma-1)} \exp(\pm \pi y) \, | \Gamma(\gamma \pm iy) / \Gamma(2\gamma+1) |^2 ,$$

$$\gamma = \sqrt{1-(\alpha Z)^2} \quad , \qquad y = \frac{\alpha Z E}{P} \quad . \tag{3.34}$$

3.4 Impulse approximations

Until now, we have made no restrictions for weak nuclear current densities $\rho^V(\mathbf{r})$, $\mathbf{J}^V(\mathbf{r})$, $\mathbf{J}^A(\mathbf{r})$ and $\rho^A(\mathbf{r})$. In the following, we assume that the nuclear currents can be expressed by the sum of individual nucleon currents as in eqs. (2.6) and (2.7) of the free nucleon. We also assume the non-relativistic description for nucleons. In this case, the nuclear current densities are expressed as,

$$\rho^V(\mathbf{r}) = \sum_{j=1}^{A} \delta(\mathbf{r}-\mathbf{r}_j)\tau_j^{\pm}[f_V \pm E_0 f_S]_j \quad ,$$

$$\mathbf{J}^V(\mathbf{r}) = \sum_{j=1}^{A} \delta(\mathbf{r}-\mathbf{r}_j)\tau_j^{\pm}[f_V \frac{\mathbf{P}}{2M} + (f_V - 2Mf_W)\frac{i\boldsymbol{\sigma}\times\mathbf{k}}{2M} \mp f_S \mathbf{k}]_j \; ,$$

$$\mathbf{J}^A(\mathbf{r}) = \sum_{j=1}^{A} \delta(\mathbf{r}-\mathbf{r}_j)\tau_j^{\pm}[(f_A \mp E_0 f_T)\boldsymbol{\sigma} - f_P \frac{(\boldsymbol{\sigma}\cdot\mathbf{k})\mathbf{k}}{2M}]_j \quad ,$$

$$\rho^A(\mathbf{r}) = \sum_{j=1}^{A} \delta(\mathbf{r}-\mathbf{r}_j)\tau_j^{\pm}[f_A \frac{\boldsymbol{\sigma}\cdot\mathbf{P}}{2M} + (\pm 2Mf_T + E_0 f_P)\frac{\boldsymbol{\sigma}\cdot\mathbf{k}}{2M}]_j \quad , \tag{3.35}$$

with

$$\mathbf{P} = \mathbf{P}_i + \mathbf{P}_f \quad , \quad \mathbf{k} = \mathbf{P}_f - \mathbf{P}_i \tag{3.36}$$

Here the momenta $\mathbf{P}_i$ and $\mathbf{P}_f$ operate on the nuclear wave functions in the initial and final nucleon states, respectively.

From eqs. (3.20) and (3.35), we obtain the explicit expression for the nuclear form factors with one-body currents. They are summarized in table2. Note that the induced pseudoscalar and scalar terms are treated separately from the other space and time components. Induced pseudoscalar term (P.S.) is proportional to

$$P.S. \propto E_0(\bar{\Psi}_e \gamma_4 \gamma_5 \psi_\nu) + i\mathbf{k}(\bar{\Psi}_e \boldsymbol{\gamma} \gamma_5 \psi_\nu)$$

$$= (E+q)(\bar{\Psi}_e \gamma_4 \gamma_5 \psi_\nu) - i(\mathbf{P}_e + \mathbf{P}_\nu)(\bar{\Psi}_e \boldsymbol{\gamma} \gamma_5 \psi_\nu) \quad . \tag{3.37}$$

By the help of the Dirac equations for the electron and the neutrino:

$$\bar{\Psi}_e (i\gamma_\mu P_\mu^e + m_e) = 0 \quad , \qquad i\gamma_\mu P_\mu^\nu \Psi_\nu = 0 \; , \tag{3.38}$$

eq. (3.37) can be expressed by a single term which is multiplied with electron mass m_e ,

$$P.S. \;\propto\; m_e \bar{\Psi}_e \gamma_5 \Psi_\nu \quad . \tag{3.39}$$

As a consequence, many observables in nuclear β decay are insensitive to the induced pseudoscalar coupling[42]. In the case of the μ capture, this multiplication factor m_e is replaced by the muon mass m_μ which is 207 times larger[41]. Because of this fact, the pseudoscalar coupling becoms important in the muon capture. Thus the additional nuclear form factors and lepton combinations appear, and they are also tabulated in table 1 and table 2. Therefore, the matrix elements of $\Xi_J(\kappa_e, \kappa_\nu)$ should read as

$$\begin{aligned} \langle J_f \| \Xi_J(\kappa_e,\kappa_\nu) \| J_i \rangle &= \sqrt{4\pi(2j_e+1)(2j_\nu+1)/(2J+1)} \; (j_e j_\nu \tfrac{1}{2} -\tfrac{1}{2} | J 0) \\ &\cdot \sum_L \int r^2 dr \Big\{ \mp F_J^V(r) L^V(\kappa_e,\kappa_\nu) \mp \mathbb{F}_{JL}^V(r) \mathbb{L}_{JL}^V(\kappa_e,\kappa_\nu) \\ &\qquad - \mathbb{F}_{JL}^A(r) \mathbb{L}_{JL}^A(\kappa_e,\kappa_\nu) - F_J^A(r) L^A(\kappa_e,\kappa_\nu) \\ &\qquad + F_J^P(r) L^P(\kappa_e,\kappa_\nu) + F_J^S(r) L^S(\kappa_e,\kappa_\nu) \Big\} \; . \end{aligned} \tag{3.40}$$

3.5 Nuclear recoil corrections to β decay rate

The phase space volume should change by taking into account the recoil of the final state nucleus [13] so that,

$$\frac{d\rho}{dE_f} = \frac{1}{(2\pi)^6} PE(E_0 - E)^2 dE\, d\Omega_e d\Omega_\nu \left(1 + \frac{3E - E_0 - 3\mathbb{P}\cdot\hat{q}}{M_f}\right), \tag{3.41}$$

where $\hat{q}$ is the unit vector of the neutrino momentum, and M_f is the mass of the daughter nucleus. The correction term with $O(\frac{1}{M_f})$ may not be neglected for light nuclei, since we are to concern ourselves with the $O(\frac{1}{M})$ terms in this work. This nuclear recoil correction should be done for the integrand of eq. (3.27), which should be multiplied with the following factor:

$$\sum_{k\mu} R_K Y^*_{k\mu}(\hat{\mathbb{P}}) Y_{K\mu}(\hat{q}) \left(\frac{4\pi}{2K+1}\right), \tag{3.42}$$

with

$$R_0 = 1 + \frac{3E - E_0}{M_f}, \qquad R_1 = -\frac{3P}{M_f}. \tag{3.43}$$

The term with K=0 gives the same result as eq. (3.30), except that it is multiplied with R_0 . The expression for the K=1 term, and some examples are given in appendix B.

§4 β decays in the A=12 and A=13 systems

In this section, we shall write down the explicit formulas for β-ray angular distribution from oriented nuclei in the case of the $(1^+, 1 \to 0^+, 0)$ transition for A=12 and the $(3/2^-, 3/2 \to 1/2^-, 1/2)$ for A=13. The level schemes for these transitions are shown in fig 1 and fig 2. The transition operator $\Xi_J(\kappa_e, \kappa_\nu)$ with J=1 contributes to both cases, while the operator with J=2 contributes only to the transition in the A=13 system. We shall show that we can derive the conventional formulas[21] by making some approximations for the lepton wave functions.

4.1 Explicit formulas for the A=12 and A=13 transitions

From eq. (3.28), the β-ray angular distribution in the β decay of the A=12 and A=13 systems can be written as,

$$\frac{dW}{d\Omega_e dE} = \frac{G^2}{(2\pi)^4} PE(E_0-E)^2 \frac{2}{2J_i+1} \left\{ B_0 + B_1 \mathbb{P} P_1(\cos\theta) + B_2 \mathbb{A} P_2(\cos\theta) + B_3 \mathbb{T} P_3(\cos\theta) \right\}, \quad (4.1)$$

with

$$B_0 = -\frac{1}{2\sqrt{3}} b_{11}^{(0)} + \frac{1}{2\sqrt{5}} b_{22}^{(0)},$$

$$B_1 = -\frac{1}{2\sqrt{2}} b_{11}^{(1)} - \frac{\sqrt{3}}{10\sqrt{2}} b_{12}^{(1)} + \frac{9}{10\sqrt{10}} b_{22}^{(1)},$$

$$B_2 = -\frac{1}{2\sqrt{6}} b_{11}^{(2)} - \frac{1}{2\sqrt{10}} b_{12}^{(2)} + \frac{\sqrt{7}}{10\sqrt{2}} b_{22}^{(2)},$$

$$B_3 = -\frac{3}{20} b_{12}^{(3)} + \frac{3}{10\sqrt{10}} b_{22}^{(3)}, \quad (4.2)$$

where the polarizations of the initial state $\mathbb{P}$, $\mathbb{A}$ and $\mathbb{T}$ are given as

$$\mathbb{P} = a_1 - a_{-1}\,, \quad \mathbb{A} = 1 - 3a_0 \text{ and } \mathbb{T} = 0 \qquad \text{for the } A=12 \text{ system},$$

and

$$\mathbb{P} = a_{3/2} + \tfrac{1}{3}a_{1/2} - \tfrac{1}{3}a_{-1/2} - a_{-3/2}\;, \quad \mathbb{A} = a_{3/2} - a_{1/2} - a_{-1/2} + a_{-3/2}$$

$$\text{and} \quad \mathbb{T} = \tfrac{1}{3}a_{3/2} - a_{1/2} + a_{-1/2} - \tfrac{1}{3}a_{-3/2} \qquad \text{for the } A=13 \text{ system.} \tag{4.3}$$

In the case of the A=12 formula, we have only $b_{11}^{(\ell)}$'s with ℓ =0,1 and 2 in eq. (4.2). Nuclear matrix elements $\langle J_f \| \hat{\Xi}_J(\kappa_e,\kappa_\nu) \| J_i \rangle$ with J=1 and J=2 in are given by

$$\langle J_f \| \hat{\Xi}_1(\kappa_e,\kappa_\nu) \| J_i \rangle = \sqrt{\frac{4\pi(2j_e+1)(2j_\nu+1)}{3}}\,(j_e j_\nu \tfrac{1}{2} -\tfrac{1}{2} | 1 0)\int r^2 dr\,\cdot$$

$$\left\{ -\frac{1}{\sqrt{3}}\mathbb{F}_{10}^{A}(\kappa^+ L^+ - L^-) - \frac{1}{\sqrt{6}}\mathbb{F}_{12}^{A}(\kappa^+ L^+ + 2L^-) - F_1^A \mathcal{L}^+ \mp \frac{1}{\sqrt{2}}\mathbb{F}_{11}^{V}\kappa^+ \mathcal{L}^- + F_1^P \mathcal{L}^- \right\}, \tag{4.4}$$

and

$$\langle J_f \| \hat{\Xi}_2(\kappa_e,\kappa_\nu) \| J_i \rangle = \sqrt{\frac{4\pi(2j_e+1)(2j_\nu+1)}{5}}\,(j_e j_\nu \tfrac{1}{2} -\tfrac{1}{2} | 2 0)\int r^2 dr\,\cdot$$

$$\left\{ \mp F_2^V L^- \mp \frac{1}{\sqrt{10}}\mathbb{F}_{21}^{V}(\kappa^- \mathcal{L}^- - 2\mathcal{L}^+) \mp \frac{1}{\sqrt{15}}\mathbb{F}_{23}^{V}(\kappa^- \mathcal{L}^- + 3\mathcal{L}^+) - \frac{1}{\sqrt{6}}\mathbb{F}_{22}^{A}\kappa^- L^+ + F_2^S L^+ \right\}. \tag{4.5}$$

4.2 Relation of the present formulas with the conventional theory.

The formulas in the conventional theories of β decay can be derived from the present formalism under some approximations. For example, the electron wave functions (or those multiplied with an appropriate power of r) are replaced by their values at the nuclear surface r =R, and the neutrino wave functions are expanded into a power series in qr . If we neglect the terms with r^2 and higher powers in r, and the partial waves with $|\kappa| \geqslant 3$, the lepton combinations are approximated as those in table 3. Lepton combinations with (κ_e, κ_ν)=(-1,-1) and (1,1) are excluded by the spin selection rule for $\Xi_2(\kappa_e, \kappa_\nu)$.

Factorization of the lepton wave functions in table 3 introduces the following nuclear matrix elements,

$$\sqrt{4\pi}\int \mathbb{F}_{10}^{A}(r)\, r^2 dr = g_A \langle \sigma \rangle ,$$

$$\sqrt{4\pi}\int F_1^{A}(r)\, r^3 dr = \sqrt{3}\, f_A \langle i\gamma_5 \mathbf{r} \rangle ,$$

$$\sqrt{4\pi}\int \mathbb{F}_{11}^{V}(r)\, r^3 dr = -\sqrt{\tfrac{3}{2}}\, f_V \langle \boldsymbol{\alpha} \times \mathbf{r} \rangle ,$$

$$\sqrt{4\pi}\int \mathbb{F}_{21}^{V}(r)\, r^3 dr = -\frac{\sqrt{3}}{2} f_V \langle iA_{ij} \rangle , \tag{4.6}$$

with

$$g_A = f_A \mp E_0 f_T . \tag{4.7}$$

In the impulse approximation, these matrix elements are given by

$$\langle i\gamma_5 \mathbb{r}\rangle = \frac{1}{2M}\langle \sigma\rangle + \frac{1}{M}\langle \mathbb{r}(\sigma\cdot\nabla)\rangle \pm \frac{f_T}{f_A}\langle\sigma\rangle ,$$

$$\langle \alpha\times \mathbb{r}\rangle = \frac{1}{M}\{ \langle \mathbb{L}\rangle + (1-2Mf_W/f_V)\langle\sigma\rangle \} , \tag{4.8}$$

$$\langle iA_{ij}\rangle = -\frac{2}{\sqrt{3}M}\left\{ \langle r\mathcal{Y}_{21}\nabla\rangle - \frac{1}{\sqrt{10}}\langle r \mathbb{D}^2 Y_2\rangle + \frac{\sqrt{15}}{2}(1-2Mf_W/f_V)\langle r D_-^2 \mathcal{Y}_{22}\sigma\rangle\right\} ,$$

with

$$\langle r D_\pm^L O\rangle = \int r^3 dr\, D_\pm^L \langle J_f \| \sum_{j=1}^{A} \tau_j^\pm \frac{\delta(r-r_j)}{r^2} O_j \| J_i\rangle . \tag{4.9}$$

From eqs. (4.1)-(4.7), we have an approximate formula for the β-ray angular distribution for A=12(1^+, 1 → 0^+, 0) and A=13($3/2^-$, 3/2 → $1/2^-$, 1/2), as follows:

$$\frac{dW}{d\Omega_e dE} = \frac{G^2}{(2\pi)^4} F_0 PE(E_0-E)^2 \frac{2}{2J_i+1} g_A^2 \langle\sigma\rangle^2 \{\beta_0 \mp \mathbb{P}\beta_1 P_1(\cos\theta) + \mathbb{A}\beta_2 P_2(\cos\theta)\} , \tag{4.10}$$

with

$$\beta_0 = L_0 \mp 4a\left(\frac{9}{3}L_0 + N_0\right) \pm 2b\left(\frac{9}{3}L_0 - N_0\right) ,$$

$$\beta_1 = 2\Lambda_1 \mp 4a\left(\frac{29}{3}\Lambda_1 + \mathbb{N}_{11} - \frac{1}{2}\mathbb{L}_{12}\right) \mp 2b\left(-\frac{29}{3}\Lambda_1 + \mathbb{N}_{11} + \mathbb{L}_{12}\right) \pm \frac{3}{\sqrt{10}} c\,\mathbb{L}_{12} ,$$

$$\beta_2 = \mp 2(a-b) L_{12} \mp \frac{3}{\sqrt{10}} c\, L_{12} . \tag{4.11}$$

Here parameters a, b and c are defined by

$$a = -\frac{1}{2} f_V \langle \alpha \times \mathbf{r} \rangle / g_A \langle \sigma \rangle ,$$

$$b = \pm f_A \langle i \gamma_5 \mathbf{r} \rangle / g_A \langle \sigma \rangle , \tag{4.12}$$

$$c = - f_V \langle i A_{ij} \rangle / g_A \langle \sigma \rangle .$$

Combinations of electron wave functions, L_0 , N_0 , etc., are given in table 4. In the limit of the point nuclear charge, we have the well known expression for β-ray angular distribution from oriented nuclei [21],

$$\frac{dW}{d\Omega_e dE} = \frac{G^2}{(2\pi)^4} F_0 P E (E_0 - E)^2 C(E)$$
$$\cdot \left\{ 1 \mp \mathbb{P} \frac{P}{E} (1 \pm d_\mp E) P_1(\cos\theta) + \mathbb{A}\, d_\mp E P_2(\cos\theta) \right\} , \tag{4.13}$$

with

$$C(E) = \frac{2}{2J_i + 1} g_A^2 \langle \sigma \rangle^2 \left(1 \pm \frac{8}{3} a E \right) , \tag{4.14}$$

and

$$d_\mp = \pm \frac{2}{3} (a - b) \mp \frac{3}{\sqrt{10}} c . \tag{4.15}$$

In the case of the A=12, we set c equal to zero. It is interesting to note that the same geometrical factor 2/3 appears as a coefficient for the time component b in both cases of A=12 and A=13, though the individual values of J_i , $\mathbb{P}$, $\mathbb{A}$ and the nuclear matrix elements are different.

The time component matrix element $f_A \langle i\gamma_5 \| r \rangle$ contains the induced tensor coupling as

$$f_A \langle i\gamma_5 \| r \rangle = f_A \langle i\gamma_5 \| r \rangle_I \pm f_T \langle i\gamma_5 \| r \rangle_{II} \quad . \tag{4.16}$$

Here $\langle i\gamma_5 \| r \rangle_I$ and $\langle i\gamma_5 \| r \rangle_{II}$ are contributions of the first and second-class currents, and their expressions in the impulse approximation are given in eq. (4.8). From the definition of $\alpha_\mp$ in eq. (4.15), we can single out the first-class and the second-class time component matrix elements as follows,

$$\alpha_- - \alpha_+ = -\left[\tfrac{2}{3}\left\{ f_V \langle \alpha \times \| r \rangle - f_T \langle i\gamma_5 \| r \rangle_{II} \right\} - \tfrac{6}{\sqrt{10}} f_V \langle i A_{ij} \rangle \right] / f_A \langle \sigma \rangle ,$$

$$\alpha_- + \alpha_+ = -\tfrac{4}{3} \langle i\gamma_5 \| r \rangle_I / \langle \sigma \rangle \, . \tag{4.17}$$

Here we neglect $E_0 f_T$ in g_A , and $\langle iA_{ij} \rangle$ equals to zero in A=12.
If C.V.C. hypothesis holds, the magnitude of $\langle \alpha \times \| r \rangle$ is determined and the strength of S.C.C. can be discussed from the difference of α_- and α_+ . On the other hand, the space components of vector current, $\langle \alpha \times \| r \rangle$ and $\langle iA_{ij} \rangle$, are canceled in the sum of α_- and α_+ , which contains only the time component matrix element for the first-class current. Here we define the parameter y as usual [21]:

$$y = 2M \frac{\langle i\gamma_5 \| r \rangle_I}{\langle \sigma \rangle} \, . \tag{4.18}$$

In the impulse approximation, this is written as

$$y_{IA} = 1 + 2 \langle r(\sigma \cdot \nabla) \rangle / \langle \sigma \rangle \quad . \tag{4.19}$$

This is essentially the ratio of matrix elements of the time component and space component of the axial vector current,and are related to $\alpha_- + \alpha_+$ as in eq. (4.17) in the approximate formula.

§5 Nuclear models

The original nuclear shell model is based on the assumption that each nucleon in the nucleus moves independently in the average field produced by the other nucleons. This can be understood qualitatively from the Pauli exclusion principle and the weakness of the nuclear force at large distance. Thus the nucleus is described as an ensemble of non-interacting nucleons in a common potential well. But, in fact, the nucleons interact each other and there still remains the residual interaction except for the part which is used to construct the average field. Then the problem is reduced to solving the secular equation for a Hamiltonian which contains the residual interaction. Since there are $10^1 \sim 10^2$ nucleons in the nucleus, it is a very difficult task to solve the complete problem. Owing to the existence of the magic numbers, we can describe the nucleus near the closed shell as a system of a few valence nucleons moving around the inert core, which greatly simplifies the problem. In other words, we can truncate the nuclear Hilbert space. These procedures have brought a great success in nuclear physics[43].

Of course, there still remains the question "How is this truncation justified?". The answer to the question have been investigated in the context of "effective interaction" and "effective operator"[44]. We are not in the place to discuss these problems in detail,but give some short comments on the "core polarization". Core polarization is a revival of the inert core problem which was once discarded. Since the interactions between the valence nucleons and the core nucleus cause the excitation of the core, the excited core also has non-zero spin and electromagnetic moments, and contributes to the transitions.

Concept of the core polarization was first introduced by Arima and Horie[45] as the "configuration mixing" in the study of magnetic moments. The gap between the Schmidt value and the experimental data for the magnetic moment is mostly explained by them. The importance of the core polarization in the effective interaction is also shown by Kuo and Brown[46]. The E2 operator for nuclei with the closed shell + one nucleon also provides a possible test for core polarization effect, and, in fact, many works have been done[44]. Their effects are also seen in the other electromagnetic or weak transitions in the different region of nuclei[47].

In what follows we shall briefly summarize the derivation of the effective operators[48], for the purpose of the later discussion. Then the explicit formula for the first order core polarization is derived. The qualitative feature of the core polarization effects will be investigated in the simple j-j model. Notations, symbols and some definitions used in this section is summarized in appendix D.

5.1 0p shell formula

Before going into the discussion of the core polarization, we shall summarize the formula which is needed to describe the nuclear model within the truncated 0p shell. We assume that the nucleus of ^{16}O forms the core, and A=12 and 13 systems are described as hole states. The n-hole basis state with the set of quantum number Γ is written as,

$$|\lambda^n \Gamma\rangle = |\lambda_1^{n_1}\Gamma_1 \otimes \lambda_2^{n_2}\Gamma_2 ; \Gamma\rangle . \tag{5.1}$$

Here $\lambda_i^{n_i}\Gamma_i$ represents the n_i -hole states for the i -th orbit with Γ_i, and λ_1, λ_2 stand for the $0p_{3/2}$ and $0p_{1/2}$ orbit, respectively.

The matrix elements of the one-body operator F^{ω} and the two-body operator G^{ω} are obtained from the formulas in appendix D,

$$\langle \lambda^n \Gamma ||| F^\omega ||| \lambda^{n'} \Gamma' \rangle = - n \delta_{nn'} [\Gamma \Gamma']^{1/2} \sum_{\alpha\beta} \langle \beta ||| F^\omega ||| \alpha \rangle (-)^{\alpha-\beta-\omega}$$

$$\cdot \sum_{\Delta} W(\Gamma \Delta \omega \beta ; \alpha \Gamma') \langle \lambda^n \Gamma \{| \lambda^{n-1} \Delta ; \alpha \rangle \langle \lambda^n \Gamma' \{| \lambda^{n-1} \Delta ; \beta \rangle \ , \qquad (5.2)$$

$$\langle \lambda^n \Gamma ||| G^\omega ||| \lambda^{n'} \Gamma' \rangle = \frac{n(n-1)}{2} \delta_{nn'} [\Gamma \Gamma']^{1/2} \sum_{\substack{\alpha\leq\beta\,\gamma\leq\delta \\ xy}} \langle \gamma\delta ; y ||| G^\omega ||| \alpha\beta ; x \rangle_{AN} (-)^{x-y+\omega}$$

$$\cdot \sum_{\Delta} W(\Gamma \Delta \omega y ; x \Gamma') \langle \lambda^n \Gamma \{| \lambda^{n-2} \Delta ; (\alpha\beta) x \rangle \langle \lambda^n \Gamma' \{| \lambda^{n-2} \Delta ; (\gamma\delta) y \rangle$$

$$- n \delta_{nn'} [\Gamma \Gamma']^{1/2} \sum_{\alpha\beta c x y} \langle c\beta ; y ||| G^\omega ||| c\alpha ; x \rangle_{AN} (-)^{\alpha-\beta-\omega} W(y c \omega \alpha ; \beta x) [x y]^{1/2}$$

$$\cdot \sum_{\Delta} W(\Gamma \Delta \omega \beta ; \alpha \Gamma') \langle \lambda^n \Gamma \{| \lambda^{n-1} \Delta ; \alpha \rangle \langle \lambda^n \Gamma' \{| \lambda^{n-1} \Delta ; \beta \rangle \ . \qquad (5.3)$$

Here the summation on α, β, γ and δ runs over $0p_{3/2}$ and $0p_{1/2}$, and c over the orbits within the core. The first and the second line in eq.(5.3) represent the two-body and the one-body part of G^ω, respectively.

From eq. (5.2) and (5.3), the matrix elements of Hamiltonian between the two basis states are obtained by setting ω =0.

$$\langle \lambda^n \Gamma | H^0 + V | \lambda^n \Gamma' \rangle = (-n_1 \varepsilon_{0p3/2} - n_2 \varepsilon_{0p1/2}) \delta_{\Gamma\Gamma'}$$

$$+ \frac{n(n-1)}{2} \sum_{\alpha\leq\beta\,\gamma\leq\delta\, x} \langle \gamma\delta ; x | V | \alpha\beta ; x \rangle_{AN} \cdot \sum_{\Delta} \langle \lambda^n \Gamma \{| \lambda^{n-2} \Delta ; (\alpha\beta) x \rangle \langle \lambda^n \Gamma' \{| \lambda^{n-2} \Delta ; (\gamma\delta) x \rangle \delta'_{\Gamma\Gamma'} , \qquad (5.4)$$

where H^0 is the one-body part of the Hamiltonian and V is the residual interaction, $\varepsilon_{0p3/2}$ and $\varepsilon_{0p1/2}$ are the single particle energies of each orbit. The matrix element of the one-body operator can be written in the form

$$\langle \lambda^n \Gamma_f ||| F^\omega ||| \lambda^n \Gamma_i \rangle = \sum_{\alpha\beta} C^\omega_{\alpha\beta} \langle \alpha ||| F^\omega ||| \beta \rangle \quad , \tag{5.5}$$

where $|\lambda^n \Gamma_f\rangle$ and $|\lambda^n \Gamma_i\rangle$ are the final and initial nuclear states which are the linear combination of the basis states of eq. (5.1). The coefficients $C^\omega_{\alpha\beta}$ for some nuclear models used in this thesis are summarized in table 5.

5.2 Formal theory of the effecive operator

The Schrödinger equation for the stationary state of the nucleus is written,

$$H \Psi_\alpha = E_\alpha \Psi_\alpha \quad , \tag{5.6}$$

with

$$H = H^0 + V \quad , \quad H^0 \phi_i = \varepsilon_i \phi_i \quad , \tag{5.7}$$

where H^0 is the model Hamiltonian, ϕ_i is its eigenfunction and V defines the residual interaction. The true eigenfunction Ψ_α of the system can be expanded into the series of ϕ_i ,

$$\Psi_\alpha = \sum_i C_{\alpha i} \phi_i \quad . \tag{5.8}$$

Since the index runs from one to infinity, it is necessary to truncate the series in eq. (5.8),

$$\Psi_\alpha = \sum_{i \in P} C_{\alpha i} \phi_i \tag{5.9}$$

Here we introduce the model space $\mathcal{P}$ with the truncated configuration. The division of the full Hilbert space into the model space ($\mathcal{P}$ space) and the other excluded space ($\mathcal{Q}$ space) can be accomplished by the use of the projection operators,

$$P = \sum_{i \in \mathcal{P}} |\phi_i\rangle\langle\phi_i| \quad , \quad Q = \sum_{i \notin \mathcal{P}} |\phi_i\rangle\langle\phi_i| \quad . \tag{5.10}$$

They satisfy the following properties of the projection operator,

$$P+Q = I \quad , \quad P^2 = P \; , \quad Q^2 = Q \quad , \quad PQ = QP = 0 \; . \tag{5.11}$$

Since H^0 commute with P and Q, H^0 cannot couple the $\mathcal{P}$ space to $\mathcal{Q}$ space

$$H^0_{PQ} = H^0_{QP} = 0 \; . \tag{5.12}$$

Here we have

$$H^0_{PQ} = PH^0Q \quad , \quad etc. \tag{5.13}$$

By applying P and Q to eq. (5.6) from the left hand side, we obtain the two coupled equations,

$$[H^0_{PP} + V_{PP} + V_{PQ}]\Psi_\alpha = E_\alpha P\Psi_\alpha \quad , \tag{5.14}$$

$$[H^0_{QQ} + V_{QP} + V_{QQ}]\Psi_\alpha = E_\alpha Q\Psi_\alpha \quad . \tag{5.15}$$

Eq. (5.15) can be solved for $Q\Psi_\alpha$ to give

$$Q\Psi_\alpha = \frac{1}{E_\alpha - H^0_{QQ} - V_{QQ}} V_{QP}\Psi_\alpha = \frac{1}{E_\alpha - H^0_{QQ}} \mathcal{V}_{QP}\Psi_\alpha \ , \tag{5.16}$$

with

$$\mathcal{V}_{QP} = V_{QP} + V_{QQ}\Gamma^0_{QQ}\mathcal{V}_{QP} \ , \quad \Gamma^0_{QQ} = \frac{1}{E_\alpha - H^0_{QQ}} \ . \tag{5.17}$$

By substituting eq. (5.16) into eq. (5.14), we find

$$H_{eff}\, P\Psi_\alpha = E_\alpha P\Psi_\alpha \ , \tag{5.18}$$

with

$$H_{eff} = H^0_{PP} + \mathcal{V}_{PP} \ , \tag{5.19}$$

and

$$\mathcal{V}_{PP} = V_{PP} + V_{PQ}\Gamma^0_{QQ}\mathcal{V}_{QP} \ . \tag{5.20}$$

Thus Heff is the effective Hamiltonian which acts only the model space and gives the same eigenvalue E_α as the original equation (5.6). The effective Hamiltonian depends on the true energy E_α of the system through Γ^0_{QQ} , therefore the solutions of eq. (5.18) with different E_α are not orthogonal to each other. In the more sophisticated treatments, the energy dependence of the effective Hamiltonian can be eliminated by introducing the folded diagrams[50)], and Heff is expressed as the linked valence diagram expansion with the unperturbed energy denominaters. The eigenfunctions with different energies are not orthogonal even in this case, because of the non-Hermitian nature of the folded diagrams. On the other hand, the FST-Okubo method[57] ensures both the use of the unperturbed energy denominator and the orthogonality of the model wave functions. In the first order in V, these methods gives the same results and we assume that the orthogonality of the model wave function holds.

The normalized projected wave functions are defined as

$$\Psi_\alpha^P = \alpha_P^{-1} P \Psi_\alpha \tag{5.21}$$

where α_P is the normalization constant obtained by

$$\langle \Psi_\alpha | Q | \Psi_\alpha \rangle = 1 - \alpha_P^2 = \alpha_P^2 \langle \Psi_\alpha^P | N_{PP} | \Psi_\alpha^P \rangle \quad , \tag{5.22}$$

with

$$N_{PP} = V_{PQ}\, \Gamma_{QQ}^0\, \Gamma_{QQ}^0\, V_{QP} \quad . \tag{5.23}$$

We now consider the matrix elements $\langle \Psi_\alpha | T | \Psi_\beta \rangle$ of any physical operator T. This matrix element can be also described in terms of the effective operator Teff which operates within the model space,

$$\langle \Psi_\alpha | T | \Psi_\beta \rangle = \langle \Psi_\alpha | (P+Q) T (P+Q) | \Psi_\beta \rangle = \alpha_P \beta_P \langle \Psi_\alpha^P | T_{eff} | \Psi_\beta^P \rangle \ , \tag{5.24}$$

where the effective operator Teff is given by

$$(T_{eff})_{PP} = T_{PP} + T_{PQ}\Gamma^{0}_{QQ}\mathcal{V}_{QP} + \mathcal{V}_{PQ}\Gamma^{0}_{QQ}T_{QP} + \mathcal{V}_{PQ}\Gamma^{0}_{QQ}T_{QQ}\Gamma^{0}_{QQ}\mathcal{V}_{QP} \quad . \tag{5.25}$$

Eq. (5.24) shows that the true matrix elements of operator T are equal to those of Teff in the model space. If we know the model space wave functions, normalization constants and the effective operator Teff from eqs. (5.18), (5.22) and (5.25), the true matrix elements are obtained in the truncated space. Thus the problem is to obtain $\mathcal{V}_{QP}$, $\mathcal{V}_{P}$ and $\mathcal{V}_{PP}$ by solving eqs. (5.17) and (5.20), and then we have the consistent description of the effective interaction and the effective operator.

Any model can be used for H^0 and $\mathcal{P}$ space, if the correct renormalization through the above procedure is done. Unfortunately, the correct renormalization requires the complex knowledge of the whole excluded space, and it is a very difficult task. The problem in nuclear theory is thus to choose a model Hamiltonian H^0 and a model space $\mathcal{P}$, which enables us to solve the problem easily. For example, the renormalization due to $\mathcal{Q}$ space can be ignored ($\mathcal{V}_{QP}$ =0) or the $\mathcal{Q}$ space can be included by the perturbation theory ($\mathcal{V}_{QP} = V_{QP}$).

Here we assume that $\mathcal{V}_{PQ}$ and $\mathcal{V}_{PP}$ are given in certain functional forms $\hat{V}_{PQ}$ and $\hat{V}_{PP}$, which lead to the reasonable results for the energy levels and the transition probabilities in the relevant region. Then eqs. (5.18) and (5.24) are written as,

$$(H^{0}_{PP} + \hat{V}_{PP})\Psi^{P}_{\alpha} = E_{\alpha}\Psi^{P}_{\alpha} \quad , \tag{5.26}$$

$$\langle\Psi_{\alpha}|T|\Psi_{\beta}\rangle = \langle\Psi^{P}_{\alpha}|\,T + T\frac{Q}{E_{\alpha}-H^{0}}\hat{V}_{QP} + \hat{V}_{PQ}\frac{Q}{E_{\beta}-H^{0}}T\,|\Psi^{P}_{\beta}\rangle \, . \tag{5.27}$$

Here we neglect the second order in $\hat{V}_{PQ}$, and the normalization α_P and β_P are reduced to unities. For the lowlying states of the 0p shell nuclei, 0p shell are chosen as the model space.

Furthermore, we regard the interaction $H^0_{PP} + \hat{V}_{PP}$ as the Cohen-Kurath effective interaction[27] which is empirically derived from the χ^2-fitting with experimental energy levels of the 0p shell nuclei. Thus the Cohen-Kurath wave functions are adopted for Ψ^P_α and Ψ^P_β . H^0 is assumed to be the harmonic oscillator Hamiltonian, and the upperturbed energy difference of H^0 is inserted in the denominator of eq. (5.27).

5.3 Explicit formula for first-order core polarization

We start from eq. (5.27) and Q is replaced by its explicit expression in eq. (5.10). Since the nuclear states have definite spins and parities, we deal with the reduced nuclear matrix elements of the following form,

$$\langle \Gamma_f ||| T^{\omega}_{eff} ||| \Gamma_i \rangle = \langle \Gamma_f ||| T^{\omega} ||| \Gamma_i \rangle$$

$$+ \left(\sum_m \langle \Gamma_f ||| T^{\omega} ||| \Gamma_m \rangle \frac{1}{E_m} \langle \Gamma_m | V | \Gamma_i \rangle + \sum_m \langle \Gamma_f | V | \Gamma_m \rangle \frac{1}{E_m} \langle \Gamma_m ||| T^{\omega} ||| \Gamma_i \rangle \right) . \qquad (5.28)$$

Here T^{ω} is the transition operator with rank ω , and V is the effective interaction ($\hat{V}_{PQ}$ in §5.2). Γ_f , Γ_i and Γ_m denote the quantum numbers of the relevant states and summation runs over the states Γ_m in the Q space.

Γ_f and Γ_i are the n-hole states in the 0p shell. For the one-body operator T^{ω} , Γ_m becomes the one-particle (n+1)-hole states. The particle states are $0d_{5/2}$, $1s_{1/2}$, $0d_{3/2}$, $1p_{1/2}$, ... , and they are denoted by ρ , and the hole states $0s_{1/2}$, $0p_{3/2}$, $0p_{1/2}$ are denoted by λ . E_m is the energy difference between Γ_m and Γ_i or Γ_f . The terms in the curly brackets of eq. (5.28) are called the core polarization terms (C.P.).

The final, initial and intermediate nuclear states are expressed as the following linear combinations of the basis states,

$$|\Gamma_f\rangle = \sum_\alpha C_\alpha(\Gamma_f)\,|\lambda^n\Gamma_f(\alpha)\rangle \quad , \qquad |\Gamma_i\rangle = \sum_\beta C_\beta(\Gamma_i)\,|\lambda^n\Gamma_i(\beta)\rangle \, ,$$

$$|\Gamma_m\rangle = |\,\rho\otimes\Delta d\,;\Gamma_m\rangle = \Big[\,|\rho\rangle\otimes\sum_\gamma C_\gamma(\Delta d)\,|\lambda^{n+1}\Delta(\gamma)\rangle\,\Big]^{\Gamma_m} \, , \tag{5.29}$$

where λ^n denotes the n-hole states. α , β and γ stand for the quantum numbers which specify the basis states, and $C_\alpha(\Gamma_f)$, $C_\beta(\Gamma_i)$ and $C_\gamma(\Delta d)$ are their amplitudes determined by the diagonalization of the Cohen-Kurath effective Hamiltonian. The symbol d in eq. (5.29) is an extra equantum number to specify the individual energy eigenstates with the same quantum number Δ.

The C.P. term in eq. (5.28) can be written as

$$\begin{aligned} C.P. = {} & \sum_{\alpha\beta} C_\alpha(\Gamma_f)\,C_\beta(\Gamma_i) \sum_{\rho\Delta d\gamma\delta} C_\gamma(\Delta d)\,C_\delta(\Delta d) \\ & \cdot\Big\{ \langle\lambda^n\Gamma_f(\alpha)|||T^\omega|||\rho\otimes\lambda^{n+1}\Delta(\gamma);\Gamma_m\rangle \frac{1}{E_m} \langle\rho\otimes\lambda^{n+1}\Delta(\delta);\Gamma_m|V|\lambda^n\Gamma_i(\beta)\rangle \\ & + \langle\lambda^n\Gamma_f(\alpha)|V|\rho\otimes\lambda^{n+1}\Delta(\delta);\Gamma_m\rangle \frac{1}{E_m} \langle\rho\otimes\lambda^{n+1}\Delta(\gamma);\Gamma_m|||T^\omega|||\lambda^n\Gamma_i(\beta)\rangle \Big\} \, . \end{aligned} \tag{5.30}$$

Here the spin isospin selection rule for Δ should be independently applied to the first and the second term in the bracket. We assume that the energy denominator E_m is independent of d, then the summation over d can be performed independently,

$$\sum_{d} C_{\gamma}(\Delta, d)\, C_{\delta}(\Delta, d) = \delta_{\gamma\delta} \,. \tag{5.31}$$

With the help of the 2-orbit and 3-orbit c.f.p. defined in appendix D, the following formula is obtained,

$$\begin{aligned}
C.P. = \sum_{\alpha\beta} C_{\alpha}(\Gamma_f)\, C_{\beta}(\Gamma_i) \sum_{p\Delta\gamma} \frac{n(n+1)}{\sqrt{2}} \frac{1}{E_m} \cdot \\
\Big\{ \cdot \sum_{h} [\Gamma_i \Delta]^{1/2}\, W(\Gamma_i \Delta \omega h ; p \Gamma_f)\, \langle \lambda^{n+1} \Delta \gamma \{| \lambda^{n} \Gamma_f \alpha ; h \rangle \langle h ||| T^{\omega} ||| p \rangle \\
\cdot \Big[\frac{\sqrt{2}}{n} \sum_{cqy} \left[\frac{\Delta}{\Gamma_i}\right]^{1/2} \left[\frac{y}{p}\right] \langle pc ; y | V | qc ; y \rangle_A \langle \lambda^{n+1} \Delta \gamma \{| \lambda^{n} \Gamma_i \beta ; q \rangle \delta'_{pq} \\
+ \sum_{jk\kappa\ell x\theta} [x\Delta]^{1/2}\, W(\Delta \Gamma_i x j ; p\theta) \langle pj ; x | V | k\ell ; x \rangle_{AN} \\
\cdot \langle \lambda^{n+1} \Delta \gamma \{| \lambda^{n-1} \theta (k\ell) x \rangle \langle \lambda^{n} \Gamma_i \beta \{| \lambda^{n-1} \theta ; j \rangle (-)^{\Gamma_i + j - \theta} \Big] \Big\} \\
+ (-)^{\Gamma_i + p - h - \Gamma_f} \Big((\Gamma_f \alpha) \longleftrightarrow (\Gamma_i \beta) \quad , \langle h ||| T^{\omega} ||| p \rangle \rightarrow \langle p ||| T^{\omega} ||| h \rangle \Big) \,.
\end{aligned} \tag{5.32}$$

The derivation of the formula is summarized in appendix E. The summation of c, h, q, k, ℓ and j runs over the hole states which are allowed by the selection rules for spin and the particle-hole number selection rules in c.f.p.'s. The first term in the square brackets comes from the one-body part of the effective interaction, and the second term from the two-body part. Note that the first term does not vanish since we did not adopt the Hartree-Fock condition. The last term in the curly bracket in eq. (5.32) comes from the second term in eq. (5.30).

5.4 Effective interactions

We shall give the detail of the effective interactions which will be adopted for the core polarization calculation in this thesis. As the effective interactions, we adopt the following four models: (1) Rosenfeld[52], (2) Suzuki-Hyuga-Arima-Yazaki[29], (3) Millener-Kurath[53] and (4) Sussex[54].
Models (1)∿(3) assume the effective interactions with the simple functional form, and the central part of the effective interaction is given

$$V(r) = V_c \{ a_0 + a_\sigma \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + a_\tau \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + a_{\sigma\tau} \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\, \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \} f_c(r) , \tag{5.33}$$

with $\quad r = |\mathbf{r}_1 - \mathbf{r}_2|$.

The exchange character of this interaction can be presented also in a different expression

$$V(r) = V_c \{ P_{11} \mathbb{P}_{11} + P_{13} \mathbb{P}_{13} + P_{31} \mathbb{P}_{31} + P_{33} \mathbb{P}_{33} \} f_c(r) , \tag{5.34}$$

with the spin isospin projection operators

$$\mathbb{P}_{11} = \tfrac{1}{16}(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(1-\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) , \quad \mathbb{P}_{13} = \tfrac{1}{16}(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(3+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) ,$$

$$\mathbb{P}_{31} = \tfrac{1}{16}(3+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(1-\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) , \quad \mathbb{P}_{33} = \tfrac{1}{16}(3+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(3+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) , \tag{5.35}$$

and their cofficients P , which are related to a in eq. (5.33) as

$$\begin{pmatrix} P_{11} \\ P_{13} \\ P_{31} \\ P_{33} \end{pmatrix} = \begin{pmatrix} 1 & -3 & -3 & 9 \\ 1 & -3 & 1 & -3 \\ 1 & 1 & -3 & -3 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a_0 \\ a_\sigma \\ a_\tau \\ a_{\sigma\tau} \end{pmatrix} . \qquad (5.36)$$

They are normalized as follows:

$$a_0 + a_\sigma - 3a_\tau - 3a_{\sigma\tau} = P_{31} = 1 . \qquad (5.37)$$

Numerical values of the potential parameters are summarized in table 6.

Now we shall go into the details of the four models.

Model (1) and (2): These are used in the analysis of the inelastic electron scattering in the A=12 ($0^+,0 \to 1^+,1$) and A=13 ($1/2^-$, $1/2^- \to 1/2^-$, 1/2) transitions by Arima et al.[29] The potential parameters of Model (2) is obtained by fitting both the experimental data on the magnetic form factors of the inelastic electron scattering for the A=12 and A=13 systems. As is seen from table 6, its triple odd component (P_{33}) is fairly strong. The authors of Ref. 29) adopted the tensor force of Hamada-Johnston potential[55] with a radial cut off at $r_c = 0.7 fm$.

$$V_T S_{12} f_T(r) = v_0(\tau_1\tau_2)\left(1+\frac{3}{x}+\frac{3}{x^2}\right)\frac{e^{-x}}{x}\left[1+a_\tau\frac{e^{-x}}{x}+b_\tau\frac{e^{-2x}}{x^2}\right]S_{12}\,\theta(r-r_c) , \qquad (5.38)$$

with

$$x = \mu r, \quad S_{12} = 3(\sigma_1 \cdot r)(\sigma_2 \cdot r) - \sigma_1\sigma_2 , \quad \theta(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases} . \qquad (5.39)$$

Suffix τ (=0 or 1) of a and b indicates the total isospin of the two nucleon system. The parameters adopted here are V_0 =3.7 MeV, μ =1.41fm^{-1}, a_0 =-0.5, b_0 =0.2, a_1 =-1.29 and b_1 =0.55. The M1 form factor of the inelastic electron scattering is well reproduced up to the second maximum (momentum transfer $q \sim 2fm^{-1}$) by Model (2).

Model (3): Millener and Kurath obtained the particle-hole interaction which gives a good account of the non-normal parity states of a number of nuclei from ^{11}Be to ^{16}O . It is noticed that this effective interaction is just the particle-hole interaction that connects the model space and the excluded space, and that it incorporates the non-central components. The additional non-central components to eq. (5.33) are given by

$$V_T(b+b_\tau \tau_1\tau_2)S_{12}\frac{e^{-x_T}}{x_T} + V_{LS}(c+c_\tau \tau_1\tau_2)\mathbb{L}\cdot\mathbb{S}\frac{e^{-x_{LS}}}{x_{LS}} , \qquad (5.40)$$

with

$$x_T=\mu_T r , \; x_{LS}=\mu_{LS} r , \quad \mathbb{L}\cdot\mathbb{S}=\frac{1}{2}(\mathbb{r}_1-\mathbb{r}_2)\times(\mathbb{P}_1-\mathbb{P}_2)\cdot(\mathbb{S}_1+\mathbb{S}_2) . \qquad (5.41)$$

The parameters adopted here are V_T =-16.25 MeV, V_{LS} =-26 MeV, μ_T =1.4fm^{-1}, μ_{LS} =0.7fm^{-1}, b =-0.035, b_τ =-0.345, c =2.875 and c_τ =0.625. This interaction is shown to give a good description of the allowed β decay of ^{14}B to the lowest 1^-, 2^- and 3^- level of ^{14}C .

The matrix elements of the effective interactions (1)-(3) are derived with the harmonic ascillator nuclear wave functions with b =1.65fm.

Model (4): Elliot et al. used the experimental nucleon-nucleon phase shifts to deduce matrix elements of the nucleon-nucleon potential in a basis of the harmonic oscillator wave functions of the internucleon distance. The matrix elements are given in the form

$$\langle n\, {}^{2S+1}L_J \,|\, V \,|\, n'\, {}^{2S+1}L'_J \rangle \tag{5.42}$$

up to G wave and $|n-n'| \leqslant 2$. They are tabulated for several values of the oscillator parameter b, and we adopt those for b =1.7fm. Hauge and Maripuu[51] used them for the effective interaction calculation of the 0p shell nuclei. Although non-central components of this interaction are different from those of the Cohen and Kurath[56], both of them obtained a fairly good agreements with the electromagnetic and weak transitions in the 0p shell.

5.5 Qualitative features of core polarization effects

To see the core polarization effects qualitatively, we shall investigate $(1^+, 1 - 0^+, 0)$transitions in the A=12 system by adopting simple j-j coupling model. The ground state of ^{12}C is assumed to be closed shell of the $0P_{3/2}$ orbit. Then the excited state $^{12}C(1^+, 1)$ is the one-particle one-hole state (1p-1h) written as $[\, 0P_{1/2} \otimes 0P_{3/2}^{-1} \,]^{(1,1)}$. Thus in eq. (5.28), Γ_f and Γ_i stand for the closed shell and the 1p-1h state and the state with Γ_m of the first and the second terms are 1p-1h states and 1p-1h or 2p-2h states, respectively. Therefore, C.P. term can be written as

$$
\begin{aligned}
C.P. = &\sum_{Ph\Gamma} \langle 0 \| T^{\omega} \| Ph;\Gamma \rangle \frac{1}{E_m} \langle Ph;\Gamma | V | P_0 h_0 ; \Gamma_0 \rangle \\
&+ \sum_{Ph\Gamma} \langle 0 \| T^{\omega} \| Ph;\Gamma \rangle \frac{1}{E_m} \langle Ph;\Gamma | U | P_0 h_0 ; \Gamma_0 \rangle \\
&+ \sum_{\alpha\leqslant\beta\,\mu\leqslant\nu\,\Gamma} \langle 0 | V | (\alpha\beta)\pi \otimes (\mu\nu)\theta ; \Gamma \rangle \frac{1}{E_m} \langle (\alpha\beta)\pi \otimes (\mu\nu)\theta ; \Gamma \| T^{\omega} \| P_0 h_0 ; \Gamma_0 \rangle \\
&+ \sum_{Ph\Gamma} \langle 0 | U | Ph;\Gamma \rangle \frac{1}{E_m} \langle Ph;\Gamma \| T^{\omega} \| P_0 h_0 ; \Gamma_0 \rangle ,
\end{aligned} \tag{5.43}
$$

where U denotes the one-body part of the effective interaction, and it is removed from V in the above expression. $|ph;\Gamma\rangle$ and $|(\alpha\beta)\pi\varpi(\mu\nu)\theta;\Gamma\rangle$ denote the 1p-1h and normalized 2p-2h states, respectively.

The core polarization formula in the j-j model is given by

$$
\begin{aligned}
C.P. = & \sum_{ph\chi} \langle h|||T^{\omega}|||p\rangle \frac{1}{E_m}(-)^{h+h_0-\chi-\Gamma_0}[\chi]\, W(ph\,h_0p_0;\Gamma_0\chi)\langle ph_0;\chi|V|hp_0;\chi\rangle_A \\
& + \sum_{pc\chi} \langle h_0|||T^{\omega}|||p\rangle \frac{1}{E_m}(-)^{p-h_0}\left[\frac{\chi}{p}\right]\langle pc;\chi|V|p_0c;\chi\rangle_A\, \delta'_{pp_0} \\
& + \sum_{ph\chi} \langle p|||T^{\omega}|||h\rangle \frac{1}{E_m}[\chi]\, W(h_0\omega\chi p;p_0h)\langle h_0h;\chi|V|p_0p;\chi\rangle_A \\
& + \sum_{pc\chi} \langle p|||T^{\omega}|||p_0\rangle \frac{1}{E_m}(-)^{p-h_0-\Gamma_0}\left[\frac{\chi}{p}\right]\langle h_0c;\chi|V|pc;\chi\rangle_A\, \delta'_{ph}\delta'_{ph_0}\ ,
\end{aligned}
\tag{5.44}
$$

with $p_0 = 0p_{1/2}$, $h_0 = 0p_{3/2}$, $\Gamma_0 = 1^+, 1$. (see appendix E) Here the first and the second terms correspond to the first term, and the third and the fourth, to the second term in eq. (5.28), respectively. The second and the fourth terms come from the one-body part of the effective interaction. The summation on p and h runs over the (p-h) pairs as ($1p_{1/2}$, $0p_{3/2}$), ($1p_{3/2}$, $0p_{3/2}$), ($0f_{5/2}$, $0p_{3/2}$), ($1s_{1/2}$, $0s_{1/2}$) and ($0d_{3/2}$, $0s_{1/2}$), and that for c runs over $0s_{1/2}$ and $0p_{3/2}$. Due to the Kronecker delta functions, p is restricted to $1p_{1/2}$ and $1p_{3/2}$ in the second and the fourth terms, respectively. Here the intermediate states are restricted to $2\hbar\omega$ states for simplicity.

Introducing the parameters, X_{ph} , Y_{ph} , U_X and U_Y , we rewrite eq. (5.44) as,

$$
\begin{aligned}
C.P. = & \sum_{ph}\langle h|||T^{\omega}|||p\rangle X_{ph} + \sum_{ph}\langle p|||T^{\omega}|||h\rangle Y_{ph} \\
& + \langle h_0|||T^{\omega}|||1p_{1/2}\rangle U_X + \langle 1p_{3/2}|||T^{\omega}|||p_0\rangle U_Y\ .
\end{aligned}
\tag{5.45}
$$

The definition of the parameters is clear from eqs. (5.44) and (5.45).

We put our particular interest on the distinct features of the core polarization effects on the time component of axial vector current T_{PA} and the M1 electron scattering operator T_{M1} defined as,

$$T_{PA} = \frac{f_A}{M}\,[\sigma + 2\, r(\sigma\cdot\nabla)]\,\tau^{\pm} \;, \tag{5.46}$$

$$T_{M1} = \frac{q}{4M}\,\Big[\,\mu\Big\{\sqrt{\tfrac{2}{3}}\,Y_{10}\,\sigma\, j_0(qr) - \sqrt{\tfrac{1}{3}}\,Y_{12}\,\sigma\, j_2(qr)\Big\} + \sqrt{6}\,Y_{10}\,\ell\,\frac{j_1(qr)}{qr}\Big]\,\tau_z \;. \tag{5.47}$$

with μ =4.706. Here q is the momentum transfer in the electron scattering. If the Hermitian property of the operator T^{ω}_{q} (q is the z component of rank ω) is given by

$$T^{\omega\dagger}_{q} = (-)^{K+q}\, T^{\omega}_{-q} \;, \tag{5.48}$$

the reduced matrix elements of this operator has the following property

$$\langle a|||T^{\omega}|||b\rangle = (-)^{a-b-K}\,\langle b|||T^{\omega}|||a\rangle^{*} . \tag{5.49}$$

Here k is determined by the property of each operator, and for the time component operator T_{PA} and M1 operator T_{M1} , k takes the values 1 and 0, respectively.

From the time reversal property of these operators, both of their reduced matrix elements are real. Thus the first two terms in eq. (5.45) are combined into a single expression

$$\sum_{ph} C_{ph}\,\langle h|||T^{\omega}|||p\rangle \;, \quad \text{with} \qquad C_{ph} = X_{ph} + (-)^{p-h-K}\, Y_{ph} \;. \tag{5.50}$$

Using the effective interactions discussed in §5.4, the numerical values of these parameters are obtained and they are summarized in table 7. The results are shown individually for the central and tensor parts of the effective interaction. For the central force, the parameters X_{ph} and Y_{ph} appear with the same absolute values for the four components of the $p-h$ pairs, and the relative phase of them is given by $(-)^{p-h}$. Therefore, a strong cancellation occurs for the case of the time component operator, but not for the M1 operator. On the other hand, this is not the case for the tensor force, except for ($1s_{1/2}$, $0s_{1/2}$). The core polarization for the time component operator has large amplitude in the $0f_{5/2}$ - $0p_{3/2}$ and $0d_{3/2}$ - $0s_{1/2}$ transitions, which are the tensor type transitions with $\Delta \ell$ =2. One-body part of the effective interaction has no effect for the tensor force ($U_X = U_Y = 0$), and again the cancellation of the central force effects occurs for the time component operator ($U_X = U_Y = -0.0961$). Thus both the central and the tensor parts of the effective interaction play an important role for the M1 operator. On the other hand, only the tensor part is important for the time component operator of the axial vector.

Another difference between the M1 operator and the time component operator lies in the contribution of the intermediate states. The one-body matrix elements of the time component operator is given by,

$$\langle n_1 \ell_1 j_1 \| r(\sigma \cdot \nabla) \| n_2 \ell_2 j_2 \rangle = \sqrt{(2j_1+1)(2j_2+1)}\,(-)^{j_1-1/2} (j_1 \tfrac{1}{2} j_2 -\tfrac{1}{2} | 10)$$

$$\cdot \left[\frac{1+S_{\kappa_2}}{2} \int r^3 dr\, R_{n_1\ell_1} D_-^{\ell_2} R_{n_2\ell_2} + \frac{1-S_{\kappa_2}}{2} \int r^3 dr\, R_{n_1\ell_1} D_+^{\ell_2} R_{n_2\ell_2} \right], \qquad (5.51)$$

with the derivertive operators $D_\pm^{\ell}$ and S_κ defined in §3. Here only the relevant part of the time component operator is written. The matrix elements vanish for $|n_1-n_2| \geqslant 2$ for the harmonic oscillator single particle radial wave function R_{nl} , due to the ladder nature of the derivertive operator $D_\pm^l$. This means that only the intermediate states with $2\hbar\omega$ contribute, in the case of T_{PA} . On the other hand, T_{M1} contains $j_l(qr)$, and $\langle n_1 l_1 j_1 \| T_{M1} \| n_2 l_2 j_2 \rangle$ has non zero values for arbitrary $n_1 - n_2$.

The tensor force has also important contributions to the magnetic moment and β decay transition probabilities in the nuclei with LS doubly closed shell plus or minus one nucleon [57]. The core polarization appears as the second-order effect of the configuration mixing, and it is shown that the mixing of the highly excited intermediate states ($\sim 12\hbar\omega$) by the tensor force is essential to explain the reduction of G-T type β decay. On the other hand, for the time component operator, the core polarization appears in the first order of V due to the momentum dependent nature of the operator, and mixing of $2\hbar\omega$ intermediate states by the tensor force is important.

§6 Exchange current effect in axial vector time component.

In describing nuclear weak and electromagnetic processes, it is usually assumed that the transition operator T^{ω} is given as a sum of one-body operators $t^{\omega}(i)$ of the free nucleon

$$T^{\omega} = \sum_{i=1}^{A} t^{\omega}(i) \quad . \tag{6.1}$$

This assumption is called "impulse approximation", and many electromagnetic and weak processes show that this is a fairly good assumption. However, it is evident that the nucleons in the nucleus are no more the free nucleons but they are interacting with each other by the strong interaction. The deviation of the transition operator from the impulse approximation is usually called "exchange current", and it is written as the two-body and the other many-body operators for nucleus

$$T^{\omega}_{ex} = \sum_{i<j=1}^{A} t^{\omega}_{ex}(i,j) + (\text{3 body and higher terms}) \quad . \tag{6.2}$$

To test the existence of the exchange current has been a subject of intensive studies in nuclear physics[7,24], but apart from a few exceptional cases, conclusions are not clear cut. Two positive evidences for the exchange current exist in the thermal nutron capture by proton $(n + p \rightarrow d + \gamma)$ [58] and the electro-disintegration of deuteron $(e + d \rightarrow e' + n + p)$. [59] In these cases, the nuclear wave function is well known, and the one-pion exchange current can explain the main part of the deviation from the impulse approximation.

However, no such clear evidence exist for the axial vector current. Most efforts were concerned on the allowed transition of β decay and μ capture [60], which is dominated by the space component of the axial vector current.

Kubodera, Delorme and Rho [23] pointed out that, owing to the soft pion theorems, one-pion exchange current processes can be obtained almost model independently and the successful theory of exchange current is that one-pion exchange process be dominant over other short-ranged processes such as multi-pion or heavy-meson exchanges. They showed that such a situation occurs not for the space component but for the time component of axial vector currents, and it could give rise to large effects.

In this section, we derive the explicit formula for the time component exchange current operator, by taking into account one-pion exchange between two nucleons. The momentum space operator is obtained from the Adler and Dothan's result [61] and then it is transformed into the coordinate space. They are expanded into multipoles and the nuclear form factor for the exchange current for the time component of axial vector current is given.

6.1 Momentum space operator

The one-pion exchange contribution to the two nucleon current J_λ^α shown in fig. 3 is given by the following transition matrix element $\tilde{J}_\lambda^\alpha$,

$$
\begin{aligned}
\tilde{J}_\lambda^\alpha &= (2\pi)^3\delta(\mathbb{P}_1+\mathbb{P}_2+\mathbb{K}-\mathbb{P}_1'-\mathbb{P}_2')\langle P_1'P_2'|J_\lambda|P_1P_2\rangle \\
&= (2\pi)^3\delta(\mathbb{P}_1+\mathbb{P}_2+\mathbb{K}-\mathbb{P}_1'-\mathbb{P}_2') \\
&\quad\cdot\left\{\langle\pi^\beta(q)N(P_1')|J_\lambda^\alpha|N(P_1)\rangle\frac{1}{q^2+m_\pi^2}\langle N(P_2')|J_\pi^\beta|N(P_2)\rangle\right\} .
\end{aligned}
\tag{6.3}
$$

Here P_i , P_i' , q and K are the four momentum of the initial nucleon, final nucleon and the pion, and the momentum transfer induced by the external field. ($\mathbb{P}_i$, $\mathbb{P}_i'$, $\mathbb{q}$ and $\mathbb{K}$ are the three momentum of them). α and β denotes the isospin indices of the current and the pion. J_π^β is the pion source current.

The pion production amplitude by the external field in eq. (6.3) is expressed as follows, by the help of the soft pion theorems [61],

$$\langle \pi^{\beta}(q) N(P_1') | J_\lambda^{\alpha} | N(P_1) \rangle = i \bar{u}(P_1') \Big[\frac{g_r}{M f_A} \tilde{J}_\lambda^{\beta\alpha}(k-q) - \frac{g_r}{2M} \{ \gamma_5 \tau_\beta , J_\lambda^{\alpha}(k) \}_+$$

$$+ g_r \gamma_5 \tau_\beta \frac{-i\gamma(P_1'+q)+M}{(P_1'+q)^2+M^2} J_\lambda^{\alpha}(k) + J_\lambda^{\alpha}(k) \frac{-i\gamma(P_1-q)+M}{(P_1-q)^2+M^2} g_r \gamma_5 \tau_\beta$$

$$+ \text{pion pole terms} \Big] u(P_1) \psi_\beta^{*} \quad . \qquad (6.4)$$

Here, M is the nucleon mass, g_r is the pion nucleon coupling constant given by $\frac{g_r^2}{4\pi}$ = 14.6 and f_A is the axial vector coupling constant given in §2. ψ_β^* is the isospin wave function of pion

$$\psi_j = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \pm i \\ 0 \end{pmatrix} \qquad \text{for } \pi^{\pm} \quad ,$$

$$= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \text{for } \pi^{0} \quad . \qquad (6.5)$$

$J_\lambda^{\alpha}(k)$ and $\tilde{J}_\lambda^{\beta\alpha}(k)$ are defined as follows:

$$\langle N(P_2) | J_\lambda^{\alpha} | N(P_1) \rangle = \bar{u}(P_2) J_\lambda^{\alpha}(P_2 - P_1) u(P_1) \quad , \qquad (6.6)$$

$$\langle N(P_2) | [Q_\beta^5 , J_\lambda^5] | N(P_1) \rangle = \bar{u}(P_2) \tilde{J}_\lambda^{\alpha}(P_2 - P_1) u(P_1) \quad , \qquad (6.7)$$

with the axial charge $Q_\beta = \int d\mathbf{x}\, A_0^{\beta}(\mathbf{x}, 0)$.

The first term in eq. (6.4) is known as the "commutator term", the second as the "P.C.A.C. consistency term", the third and the fourth are the Born terms. The last pion pole term does not contribute in the case of axial vector current.

The propagator of the Born term is rewritten as a sum of the positive energy part and the negative energy part as,

$$\frac{-i\gamma P+M}{P^2+M^2} = -\frac{1}{2E_P}\left[\frac{\gamma_4 E_P - i\vec{\gamma}\vec{P}+M}{P_0-E_P+i\varepsilon} + \frac{\gamma_4 E_P + i\vec{\gamma}\vec{P}-M}{P_0+E_P-i\varepsilon}\right], \tag{6.8}$$

with $E_P = \sqrt{P^2+M^2}$.

Each term corresponds to the diagrams in fig. (4.a) and (4.b). The first term (positive energy part) can be considered that it is already incorporated in the nuclear wave function. The second term is the so called "pair current". In the non-relativistic approximation, the pair current term reduces to $\frac{1}{2M}$ and is canceled exactly with the P.C.A.C. consistency term. Thus, only the commutator term remains

$$\langle \pi^{\beta}(q) N(P_1')| J_\lambda^{\alpha} | N(P_1)\rangle = i\frac{g_r}{M f_A}\langle N(P_1)|[Q_\beta^5, J_\lambda^{\alpha}]|N(P_2)\rangle . \tag{6.9}$$

Current algebra shows us that the commutator in eq. (6.9) for the axial vector current A_λ^{α} is given by

$$[Q_\beta^5, A_\lambda^{\alpha}] = i\epsilon_{\alpha\beta\gamma} V_\lambda^{\gamma} . \tag{6.10}$$

Here V_λ^{γ} is the vector current. The leading term of V_λ^{γ} in eq. (6.10) is proportional to γ_λ and this implies that the time component and space component can have very different magnitude, $\gamma_\lambda \sim O(1)$ for $\lambda = 4$ and $\gamma_\lambda \sim O(\frac{\vec{P}}{M})$ for $\lambda = 1,2,3$. On the other hand, the impulse term is proportional to $\gamma_\lambda\gamma_5$, which dominates in the space component. Thus the exchange current contribution is relatively large in the time component of axial vector current. From eqs. (6.9) and (6.10), the pion production amplitude is given as,

$$\langle \pi^{\beta}(q) N(P_1') | J_4^{\alpha} | N(P_1) \rangle = \frac{g_r}{M f_A} i \varepsilon_{\alpha\beta\gamma} \tau_1^{\gamma} , \tag{6.11}$$

and the pion source current,

$$\langle N(P_2') | J_\pi^{\beta} | N(P_2) \rangle = i g_r \frac{\sigma \cdot q}{2M} \tau_2^{\beta} . \tag{6.12}$$

From eqs. (6.3), (6.11) and (6.12), the transition matrix element in the non-relativistic limit is obtained,

$$\tilde{J}_4^{\alpha} = \frac{1}{(2\pi)^3} \delta^3(\mathbb{P}_1 + \mathbb{P}_1' + \mathbb{K} - \mathbb{P}_2 - \mathbb{P}_2') \frac{g_r}{M f_A} \frac{1}{q^2 + m_\pi^2} \frac{\sigma_2 \cdot q}{2M} (\tau_1 \times \tau_2)^{\alpha} . \tag{6.13}$$

6.2 Multipole expansion in the coordinate space

The exchange current operator in the coordinate space $J_\lambda^{\alpha}(\mathbb{X}_1, \mathbb{X}_2)$ is defined by the Fourier transform[62] of the momentum space operator $\tilde{J}_\lambda^{\alpha}$ as

$$\langle \mathbb{X}_1' \mathbb{X}_2' | J_\lambda^{\alpha} | \mathbb{X}_1 \mathbb{X}_2 \rangle = \delta(\mathbb{X}_1 - \mathbb{X}_1') \delta(\mathbb{X}_2 - \mathbb{X}_2') J_\lambda^{\alpha}(\mathbb{X}_1 - \mathbb{X}_2)$$

$$= \frac{1}{(2\pi)^6} \int d\mathbb{P}_1 \, d\mathbb{P}_2 \, d\mathbb{P}_1' \, d\mathbb{P}_2' \exp[-i(\mathbb{P}_1 \mathbb{X}_1 + \mathbb{P}_2 \mathbb{X}_2 - \mathbb{P}_1' \mathbb{X}_1' - \mathbb{P}_2' \mathbb{X}_2')] \, \tilde{J}_\lambda^{\alpha} . \tag{6.14}$$

Substituting $\tilde{J}_\lambda^{\alpha}$ in eq. (6.14) from eq. (6.12), we obtain

$$J_4^{\alpha}(\mathbb{X}_1, \mathbb{X}_2) = \frac{1}{(2\pi)^3} \int dq \left\{ \frac{g_r^2}{M f_A} \frac{e^{-iqr}}{q^2 + m_\pi^2} \cdot \frac{\sigma_2 \cdot q}{2M} \cdot (\tau_1 \times \tau_2)^{\alpha} \right\} . \tag{6.15}$$

Here, $\mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2$, $\mathbf{q} = \mathbf{p}_2' - \mathbf{p}_2$ and we set the momentum transfer by the external field $k \to 0$. Replacing $\sigma_2 \cdot \mathbf{q}$ by $i\sigma_2 \cdot \nabla$ and using the formula

$$\int d\mathbf{q} \frac{e^{-i\mathbf{q}\mathbf{r}}}{q^2 + m_\pi^2} = 2\pi^2 m_\pi \, y_0(m_\pi r) \quad , \tag{6.16}$$

$$-\frac{d}{dr} y_0(m_\pi r) = m_\pi \cdot y_1(m_\pi r) \quad , \tag{6.17}$$

with

$$y_0(x) = \frac{e^{-x}}{x} \quad , \qquad y_1(x) = \left(1 + \frac{1}{x}\right) \frac{e^{-x}}{x} \quad , \tag{6.18}$$

then, $J_4^\alpha(\mathbf{x}_1, \mathbf{x}_2)$ is given as

$$J_4^\alpha(\mathbf{x}_1, \mathbf{x}_2) = i c \, \sigma_2 \cdot \hat{\mathbf{r}} \; y_1(m_\pi r) \; (\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)^\alpha \quad , \tag{6.19}$$

with

$$C = -\frac{1}{2f_A} \frac{g_r^2}{4\pi} \left(\frac{m_\pi}{M}\right)^2 = 0.13 \quad , \qquad \hat{\mathbf{r}} = \mathbf{r}/|\mathbf{r}| \tag{6.20}$$

By symmetrizing $J_4^\alpha(\mathbf{x}_1, \mathbf{x}_2)$, with its indices $\mathbf{x}_1$ and $\mathbf{x}_2$, the two-body operator for the nucleus is written as,

$$\rho^A(\mathbf{x}) = C \sum_{i<j} (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)^{\pm} [\sigma_i \cdot \hat{\mathbf{r}} \; \delta(\mathbf{x} - \mathbf{x}_j) + \sigma_j \cdot \hat{\mathbf{r}} \; \delta(\mathbf{x} - \mathbf{x}_i)] \; y_1(m_\pi r) \, . \tag{6.21}$$

The time component operator $\rho^A(\mathbf{x})$ is expanded into multipoles as follows. We define the relative and center of mass coordinate as

$$\mathbf{r} = \mathbf{x}_i - \mathbf{x}_j \quad , \qquad \mathbf{R} = (\mathbf{x}_i + \mathbf{x}_j)/2 \ . \tag{6.22}$$

The δ function in eq. (6.21) can be written as

$$\delta(\mathbf{x}-\mathbf{x}_k) = \delta(\mathbf{x}-\mathbf{R} \mp \tfrac{\mathbf{r}}{2}) = \frac{1}{(2\pi)^3}\int d\mathbf{k}\, e^{i\mathbf{k}\mathbf{x}} e^{-i\mathbf{k}\mathbf{R}} e^{\mp\frac{i}{2}\mathbf{k}\mathbf{r}} \quad , \qquad k = \binom{i}{j} \ , \tag{6.23}$$

Using the Rayleigh expansion of these exponential factors, $\rho^A(\mathbf{x})$ is written as

$$\rho^A(\mathbf{x}) = \sum_{i<j} C[\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j]^{\pm} \frac{1}{(2\pi)^3}\int d\mathbf{k}\,(4\pi)^3 \sum_{\substack{\lambda L \ell \\ \mu M m}} i^{\lambda - L + \ell}\, \mathbf{S}_\ell \cdot \hat{\mathbf{r}}$$

$$\cdot\, Y_{\lambda\mu}(\hat{\mathbf{k}})\, Y^*_{LM}(\hat{\mathbf{k}})\, Y^*_{\ell m}(\hat{\mathbf{k}})\, Y_{\ell m}(\hat{\mathbf{r}})\, Y_{LM}(\hat{\mathbf{R}})\, Y^*_{\lambda\mu}(\hat{\mathbf{x}})$$

$$\cdot\, j_\ell(\tfrac{kr}{2})\, \mathcal{Y}_1(m_\pi r)\, j_L(kR)\, j_\lambda(kx) \quad , \tag{6.24}$$

with

$$\mathbf{S}_\ell = \boldsymbol{\sigma}_i + (-)^\ell \boldsymbol{\sigma}_j \ . \tag{6.25}$$

Integrating over the angular variables of the momentum $\mathbf{k}$, and recoupling the angular momenta, we have

$$\rho^A(x) = \sum_{i<j} \frac{8c}{\sqrt{4\pi}} [\tau_i \times \tau_j]^{\pm} \int_0^\infty k^2 dk \sum_{\substack{\lambda \ell L f g \\ \mu}} i^{\ell - L + \lambda} (-)^{g-\lambda} (\ell L 00|\lambda 0)$$

$$\cdot (\ell 100|f0)\, W(1f\lambda L;\ell g)\, [[Y_f(\hat{r}) \otimes Y_L(\hat{R})]^g \otimes \mathcal{S}_\ell]^{\lambda\mu}$$

$$\cdot j_\ell\left(\frac{kr}{2}\right) \mathcal{Y}_1(m_\pi r)\, j_L(kR)\, j_\lambda(kx)\, Y^*_{\lambda\mu}(\hat{x}) \,. \tag{6.26}$$

The nuclear form factor for the exchange current in the time component of axial vector current is defined as in §3, which is given as follows,

$$F^A_J(x) = \langle J_f \| \int d\hat{x}\; i\rho^A(\hat{x}) Y_J(\hat{x}) \| J_i \rangle$$

$$= \frac{8ci}{\sqrt{4\pi}} \int_0^\infty k^2 dk\, j_J(kx) \sum_{\ell L f g} i^{\ell - L + J} (-)^{g-J} \left[\frac{\ell^2 L g}{J}\right]^{1/2}$$

$$\cdot (\ell L 00|J0)(\ell 100|f0)\, W(1fJL;\ell g)$$

$$\cdot \langle J_f \| \sum_{i<j} [\tau_1 \times \tau_2]^{\pm} [[j_\ell(\tfrac{kr}{2}) \mathcal{Y}_1(m_\pi r) Y^*_f(\hat{r}) \otimes j_L(kR) Y_L(\hat{R})]^g \otimes \mathcal{S}_\ell]^J \| J_i \rangle \,. \tag{6.27}$$

Total time component form factor is the sum of eq. (6.27) and the impulse form factor obtained in §3. The calculation of the two-body matrix element is given in appendix F.

So far we have assumed to start from the Adler and Dothan's result, however, it was pointed out that the explicit introduction of ρ meson may change the results [63]. The pion production vertex in fig. 3 consists of some pole diagrams such as nucleon, nucleon isobars, ρ meson etc.. In the limit $K \to 0$, the ρ exchange diagram shown in fig. 5 dominates over other diagrams [64], and the propagator should be modefied as

$$\frac{1}{m_\pi^2+q^2} \rightarrow \frac{m_\rho^2}{m_\rho^2+q^2} \cdot \frac{1}{m_\pi^2+q^2} = \frac{m_\rho^2}{m_\rho^2-m_\pi^2}\left(\frac{1}{m_\pi^2+q^2} - \frac{1}{m_\rho^2+q^2}\right) \tag{6.28}$$

If we retain the q^2 dependence of the ρ meson propagator, the radial dependence of the time component operator in eq. (6.27) changes as follows,

$$\mathcal{Y}_1(m_\pi r) \longrightarrow \frac{m_\rho^2}{m_\rho^2-m_\pi^2}\left[\mathcal{Y}_1(m_\pi r) - \left(\frac{m_\rho}{m_\pi}\right)^2 \mathcal{Y}_1(m_\rho r)\right]. \tag{6.29}$$

This change has the effect to remove the singularity of the $\mathcal{Y}_1(m_\pi r)$, and may bring the similar effect as the introduction of the short range correlation of two nucleons. The comparison of both of these calculation is given in §7.

§7 Numerical results

In order to analyze the experimental data, we must take into account the higher order corrections in nuclear β decay by the formalism which we have given in §3. Experimental data are derived by the following procedure [20]. The ratio R is obtained by the measured intensities, $W(\theta, \mathbb{A})$, of β rays at the angle θ with the alignments $\mathbb{A}_1$ and $\mathbb{A}_2$,

$$R = \frac{W(\theta, \mathbb{A}_1)}{W(\theta, \mathbb{A}_2)} . \qquad (7.1)$$

Here the angle θ is either 0 or π, and the coefficients $\alpha_{\mp}$ are defined by

$$R - 1 = (\mathbb{A}_1 - \mathbb{A}_2)\, \alpha_{\mp} E . \qquad (7.2)$$

We derive theoretical values of $\alpha_{\mp}$ in the same way as in eq. (7.2), this corresponds to the average slope of B_2/B_0 in eq. (4.2) in the energy region of experiments. If the experiments are done for the coefficient of polarization $\mathbb{P}$, the asymmetry coefficient is defined as the average slope of $\mp(B_1/B_0)\cdot(E/P)-1$. These coefficients in aligned and polarized nuclei are the same in the simplest formula. By taking into account the exact formula, this is, however, not the case. Formerly y is defined as the ratio of the matrix elements for time and space components without higher order corrections in §3. Now we introduce $\tilde{y}$ which includes higher order corrections, and the quantity can be compared directly to experimental data.

$$\tilde{y} = -\frac{3M}{2}(\alpha_- + \alpha_+) . \qquad (7.3)$$

The average slope of spectral shape factor C(E) is defined as follows

$$a_{\mp} = [C(E_1) - C(E_2)] / [C(\bar{E})(E_1 - E_2)] \quad , \tag{7.4}$$

with $\bar{E} = (E_1 + E_2)/2$.
Here, E_1 and E_2 are the upper and lower values of the electron energy in experiments. Since the higher order corrections introduce the E^2 dependence in B_i , eq. (4.2), we must be carefull about the averaging procedure [17].

7.1 Impulse approximation

At first, we show the results in the impulse approximation within the 0p shell configurations. The experimental values of $\alpha_{\mp}$ and $a_{\mp}$ in the A=12 system are summarized in table 8 together with the theoretical values. The calculation of β decay is performed with the formalism derived in §3. with the Cohen-Kurath 8-16 2BME model. The harmonic oscillator strengh, b=1.64fm, is adopted for the single particle wave functions. In conformity with the shell-model wave functions, nuclear charge distribution of the harmonic oscillator is adopted

$$\rho(r) = \left(\frac{4}{3}\frac{1}{\sqrt{\pi}} b^{-3}\right) [\, 1 + (4r^2/3b^2)\,] \exp(-r^2/b^2) . \tag{7.5}$$

The electron wave functions are numerically obtained by solving the Dirac equation with a Coulomb potential of the nuclear charge distribution in eq. (7.5). The maximum energies of the electron are E_0 =13.873MeV for β^- and E_0 =16.819 MeV for β^+ [14]. The weak coupling constants in eq. (2.18) are adopted as the canonical values. The results show a good agreement between the theory and experiments, although there still remains a slight difference for the coefficients $a_{\mp}$ of the Heidelberg group [16].

Using the asymmetry coefficients $\alpha_\mp$ of the Osaka group [20], we can show the limits of the strength of S.C.C. and the validity of C.V.C. hypothesis. In fig.6, $\alpha_- - \alpha_+$ is shown as a function of f_T/f_W where $f_W = -\frac{3.706}{2M}$ ($\equiv f_W^{CVC}$). This impose the following limits on f_T

$$f_T/f_W = -0.020 \pm 0.164 \quad \text{or} \quad 2Mf_T/f_A = -0.059 \pm 0.486 \; . \tag{7.6}$$

Thus the experimental data of the asymmetry coefficients $\alpha_\mp$ in the A=12 system is consistent with no existence of S.C.C., and it is also consistent with the result of the other analysis in nuclear β decay [22].

Now we assume that $f_T = 0$, and $\alpha_- - \alpha_+$ is given as a function of f_W. The result is shown in fig. 7, and it gives the following limits,

$$f_W / f_W^{CVC} = 1.03 \pm 0.21 \; . \tag{7.7}$$

If we use the coefficient of the spectral shape factor which is derived from the experiment of Lee, Mo and Wu [5], the result is

$$f_W / f_W^{CVC} = 1.05 \pm 0.30 \; , \tag{7.8}$$

and is shown in fig. 8. The experimental result of the Heidelberg group is also shown in this figure. In the case of Lee, Mo and Wu, the errors are not so small, but are consistent with the C.V.C. prediction within 30% ambiguity.

On the other hand, the sum of the asymmetry parameter $\alpha_- + \alpha_+$ is shown in fig.9, as a function of χ where χ is defined as the muliplication factor of the time component matrix element which is derived in the impulse approximation within the 0p shell. This shows the limits on χ

$$x = 1.10 \pm 0.18 , \tag{7.9}$$

which seems to be consistent with no large exchange current effects and this is the starting point of our discussion.

7.2 Exchange current and Core polarization

Before going into our discussion, we show the difference of the two models of exchange current operator given in §6. The models (a) and (b) are those without and with the explicit ρ meson propagator, respectively. The ratio of the matrix elements for the time and the space components, $\mathcal{Y}$, is calculated with the Cohen-Kurath 8-16 POT model. The results are as follows:

$$\begin{aligned} &\text{(a)} \quad \mathcal{Y}_{IA+EC} = 5.134 \qquad \delta\mathcal{Y}_{EC} = 38.1\%, \\ &\text{(b)} \quad \mathcal{Y}_{IA+EC} = 4.975 \qquad \delta\mathcal{Y}_{EC} = 33.8\%, \end{aligned} \tag{7.10}$$

with

$$\delta\mathcal{Y}_{EC} = (\mathcal{Y}_{IA+EC} - \mathcal{Y}_{IA})/\mathcal{Y}_{IA} , \qquad \mathcal{Y}_{IA} = 3.716 . \tag{7.11}$$

Here $\mathcal{Y}_{IA+EC}$ is given by taking into account the impulse term with the exchange current for the time component, but not with the core polarization. $\mathcal{Y}_{IA}$ corresponds to the calculation in the impulse approximation. As was expected before, the inclusion of ρ meson propagator suppresses the short distance singularity of the Yukawa function. Thus the matrix element for the exchange current is reduced by about 13%, and this reduces $\mathcal{Y}$ by about 4%.

As this is a minor change, it is supposed that the inclusion of a short range correlation factor doesn't change the results considerably. The model (b) is adopted throughout the following calculations. In the calculation of core polarization effect, we adopt the energy denominator in eq. (5.32) as follows: $E_m = -2\hbar\omega = -30.74\,MeV$.

In fig. 10, the results for different nuclear models are shown in the case of the A=12 system. The symbol ● in fig. 10 denotes $\tilde{y}$ which includes all the effects, but the other symbols O and - correspond to y , the calculation without higher order corrections. The core polarization is calculated by the effective interaction of Arima et al.[29] with the tensor force of Hamada-Johonston type given in §5.4. The three models of the Cohen-Kurath wave functions give the similar results, not only for the impulse term but also for the exchange current and the core polarization. As is seen in table 9, the contribution of the exchange current for the time component is about +34% and the core polarization is about -30% of the impulse value of y . Thus, both effects cancel each other and the final result is only slightly larger than the impulse value. The difference between y_{TOT} and $\tilde{y}$ in table 9 comes from the higher order corrections and it seems to be not large. These points are discussed later. The Hauge-Maripuu wave function gives a smaller value of y_{IA} , but the exchange current and the core polarization contributions are nearly the same as those of the Cohen-Kurath wave functions.

The results for the A=13 system is also shown in fig. 9, where we derived only the ratio of the matrix elements, because no experimental data is available now. As was expected in §4, the magnitude of y is the same order as in the A=12 system and the contributions of the exchange current and the core polarization are found to have the same tendency as in A=12, and they almost cancel each other. The study of the A=13 system is as useful as that of the A=12 system.

7.3 Effective interactions and the core polarization

The core polarization effects for various effective interactions discussed in §5.4 are shown in fig. 11 and table 10. Here we used the Cohen-Kurath 8-16 POT model. As was discussed in §5.5, the core polarization effect is strongly dependent on the tensor force, and less sensitive to the central force. The three models of the left side of fig. 11 has the same tensor force, that is, the Hamada-Johnston tensor force with cut off at r_c =0.7fm. Therefore, they show nearly the same results. On the other hand, the Millener-Kurath interaction and the Sussex interaction show the smaller core polarization effects. This is due to the weakness of the tensor force in these interactions.

In order to see the differences of these interactions more explicitly, we calculate the 0p shell matrix elements of the tensor force and compare them with the other models which we used to obtain the 0p shell wave functions. The matrix elements of the tensor component of different interactions are shown in table 11. The Cohen-Kurath 8-16 POT and Hauge-Maripuu interactions are similar, but they have some large matrix elements which are different from each other even their phases. The Hauge-Maripuu and Hamada-Johnston interactions agree quite well, but the Millener-Kurath interaction is very different from other interactions and it is the weakest among these interactions. Even though these interactions explain the energy levels of nuclei in the 0p shell region, they have different tensor components and it is observed from the analysis of the transitions where the tensor force plays an important role as in the case of the time component matrix element.

The Hauge-Maripuu interaction is obtained from the Sussex interaction by the second order perturbation theory, but they seem quite different in their tensor forces. That is, the tensor force of the Hauge-Maripuu interaction is nearly equal to that of the Hamada-Johnston which is stronger than the Sussex matrix element, as is seen from the core polarization effect on time component in fig. 11. It may indicate that the higher order correction terms to the bare G matrix elements amplify the tensor part of the effective interaction.

7.4 Higher order corrections

Introduction of the core polarization changes also the other matrix elements, for example, $\langle \sigma r^2 \rangle$ and $\langle [\sigma \otimes Y_2]^{(1)} r^2 \rangle$, in addition to those for the time component. In table 12, we show these higher order matrix elements. The effect of the core polarization is large for these higher order matrix elements, particularly for the tensor-type operator, $\langle [\sigma \otimes Y_2]^{(1)} r^2 \rangle$. Since both of the matrix elements $\langle \sigma r^2 \rangle$ and $\langle [\sigma \otimes Y_2]^{(1)} r^2 \rangle$ have the same transformation properties as the M1 operator, both the tensor and central parts of the effective interaction are important for the core polarization effect, and indeed it is seen from table 12. These changes of higher order matrix elements may affect the coefficients, $a_{\mp}$ and $\alpha_{\mp}$.

For the spectral shape factors, this does not bring a considerable change. In fact, the change is at most 0.01%/MeV for $a_- - a_+$. For the asymmetry coefficients $\alpha_{\mp}$, it is noticed in §7.2 that the effect of higher order corrections seems to be very small. In table 13, we show the difference of y and $\tilde{y}$ for some cases. This difference mainly comes from the higher order matrix elements. It is clear from this table that the higher order corrections many affect the results nearly 4% in some cases and it is non-negligible.

The another reason why the higher order corrections are important is as follows. As is seen from fig. 12, the higher order corrections introduce the terms proportional to E^2 and the line has a curvature, while in the simple approximation, it appears as the straight line. The exchange current and core polarization change the curvature, thus we have to be careful to derive the slope $\alpha_{\mp}$ of these lines and compare them with the experimental values.

Finally, we comment on the nuclear recoil corrections given in appendix C. The results with and without recoil corrections are shown in table 14. The recoil corrections are almost cancel each other in the differences $a_{-}-a_{+}$ and $\alpha_{-}-\alpha_{+}$, but they remain in the sum $\alpha_{-}+\alpha_{+}$. These corrections are not large, but cannot be neglected completely.

§8 Discussions and Summary

We investigated the time component of axial vector current in the A=12 and A=13 systems. The new formalism of β decay is adopted, where the lepton wave functions are treated exactly and the nuclear form factors are introduced. The interplay between the exchange current and the core polarization for the time component of axial vector current is clarified. the results are summarized as follows.

(1) From the analysis of experimental data on the β-ray asymmetry coefficients $\alpha_{\mp}$ in the A=12 system by the impulse approximation within 0p shell, the following features are found. The difference $\alpha_{-}-\alpha_{+}$ indicates that the induced tensor coupling constant f_T is small and it is consistent with no existence of the second-class current. With the analysis of the spectral shape factors, the validity of the C.V.C. hypothesis is confirmed. The sum $\alpha_{-}+\alpha_{+}$ which singles out the time component of axial vector current indicates that the calculation by the impulse approximation within the 0p shell can reproduce the experiments very well. On the other hand, as was pointed out by Kubodera, Delorme and Rho, the exchange current contribution to the time component of axial vector current is large, in fact, our calculation shows that the exchange current enhances the matrix element of the time component by about 30%. This is independent of the nuclear models used. Therefore, introduction of the exchange current breaks the agreement between the theory and experiments.

(2) The problem is solved by using the realistic nuclear model, that is, by incorporating the first-order core polarization. The Cohen-Kurath wave function is commonly used for the analysis of the electromagnetic and weak transitions in the 0p shell region. Because it succeeded in reproducing

the magnetic moments, M1 gamma transitions and allowed Gamow-Teller β decay rates in the 0p shell nuclei. One of the reason why the Cohen-Kurath wave function succeeded in explaining these quantities is as follows. As is seen in §5, the truncation of the nuclear Hilbert space introduces the correction term $\langle f|T\frac{Q}{e}V+V\frac{Q}{e}T|i\rangle$ to $\langle f|T|i\rangle$ in the first order in the residual interaction V. If the transition operater T does not have matrix elements between the model space and the excluded space projected by Q, these first-order correction terms vanish. Indeed, the magnetic dipole operator and the allowed Gamow-Teller operator have this property. Then the corrections appear in the second and higher order in V, and they are supposed to be not large. Therefore the trancation in the 0p shell is justified for these operators. On the other hand, time component operator for the axial vector current is of the momentum dependent, and it has matrix elements between the model space and the excluded space. Thus the first-order correction terms in V can survive as in the case of the inelastic M1 electron scattering.

The core polarization effect reduces the time component matrix by about 30% and almost cancels the exchange current contribution. In other words, the existence of the large exchange current effect on the time component of axial vector current is shown indirectly but clearly in the analysis of the β-ray asymmetry in the A=12 system. Introduction of the exchange current and core polarization does not change the conclusion for the second-class current and the C.V.C. bypothesis, since the total value of the time component by taking with these contribution is nealy equal to that of the impulse value.

(3) The importance of the interplay between the exchange current and the core polarization has been discussed in the magnetic moment, allowed β decay and inelastic electron scattering. In the former two cases, the contribution

of the highly excited configurations through the tensor force is important in the second-order effects. For the latter case, the core polarization appears in the first-order effect and is dominated not only by the tensor force but also by the central force of the effective interaction. The difference of core polarization effects between the M1 operator in inelastic electron scattering and the time component operator is investigated in the simple j-j model. Only the tensor part of the effective interaction is essential to reduce the time component matrix element and the intermediate state with $2\hbar\omega$ excitation dominate the effect. The result is dependent on the model of the tensor force.

In the study of the magnetic form factors in the A=12 and A=13 systems, Arima et al. used the tensor force of Hamada-Johnston type with radial cut off and they succeeded in reproducing the experimental data up to the second maximum of the magnetic form factors. The tensor force is important to obtain the agreement around the second maximum of the form factors. Therefore it is reasonable to consider the Hamada-Johnston tensor force is well reproducing the character of the tensor part of the effective interaction for the nuclei in this region. Both the electromagnetic and the weak process in this region can be explained by this tensor force.

(4) We have been concentrating on the time component of the axial vector current, and the space component of the axial vector current and vector current are calculated in the impulse approximation. The exchange current contributions to these quantities are calculated 60), 65) and their ratios to the impulse matrix elements are given by

$$\langle \sigma \rangle_{E.C} \;/\; \langle \sigma \rangle_{IA} = -6\,\% \,, \tag{8.1}$$

$$\langle \sigma \times r \rangle_{EC} \;/\; \langle \sigma \times r \rangle_{IA} = 4\,\% \,. \tag{8.2}$$

These are the minor effects as compared to the case of time component matrix element and we did not take them into account. So far, the non-relativistic description of the nucleus has been assumed. The relativistic correction [66] which comes from the small components of the nuclear wave function also changes the result. There may be 10% enhancement of the time component matrix element, but it is strongly dependent on the models of the relativistic correction. The second-order effects in V and the interference term of the first-order core polarization and the exchange current may also give some corrections. The above corrections compete each other. The agreement between experiments and the theory including the first-order core polarization and the exchange current for the time component of the exial vector current may imply that these additional corrections are not large, or they are canceled among each other. The problem is left to the future investigation.

(5) In addition to the A=12 system, we also showed the results for the A=13 system in the simple approximation (we neglect the higher order corrections in nuclear β decay). The calculation supported our first observation that the A=13system also provides a testing ground for the exchange current for the time component of the axial vector current. The situation is nearly the same as in the A=12 system. The individual values of the exchange current and core polarization effects are considerably large, but they almost cancel each other. The analyses of the angular correlations in nuclear β decay for

the A=8 and A=20 system [30] and the μ capture to the β decay rate in the A=16 system will also provide fruitful informations about the exchange current and nucleon structure problem.

Appendix A. Electron wave functions

The electron radial wave functions G_{κ_e} and F_{κ_e} in eq.(3.3) are the solutions of the Dirac equation,

$$\frac{d}{dr}\begin{pmatrix} G_{\kappa_e} \\ F_{\kappa_e} \end{pmatrix} = \begin{pmatrix} -\frac{\kappa_e+1}{r} & E+1-V(r) \\ -E+1+V(r) & \frac{\kappa_e-1}{r} \end{pmatrix}\begin{pmatrix} G_{\kappa_e} \\ F_{\kappa_e} \end{pmatrix}, \tag{A.1}$$

where V(r) is the Coulomb potential of the daughter nucleus.

(1) Plane wave solution

The plane wave solution $\psi^{p}_{s_e,\mathbf{p}}(\mathbf{r})$ is obtained if we set V(r)=0 in eq.(A.1),

$$\begin{aligned} \psi^{p}_{s_e,\mathbf{p}}(\mathbf{r}) &= \sqrt{\frac{E+1}{2E}}\begin{pmatrix} \chi_{s_e} \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+1}\chi_{s_e} \end{pmatrix} e^{i\mathbf{p}\mathbf{r}} \\ &= 4\pi \sum_{\kappa_e m_e \mu_e} i^{l_e}(l_e \tfrac{1}{2} m_e s_e | j_e \mu_e)\, Y^{*}_{l_e m_e}(\hat{\mathbf{p}}) \begin{pmatrix} \sqrt{\frac{E+1}{2E}}\, j_{l_e}(pr)\, \chi_{\kappa_e \mu_e} \\ i\sqrt{\frac{E-1}{2E}}\, S_{\kappa_e} j_{\bar{l}_e}(pr)\, \chi_{-\kappa_e \mu_e} \end{pmatrix}. \end{aligned} \tag{A.2}$$

It is normalized as,

$$\int \psi^{p\dagger}_{s_e\mathbf{p}}(\mathbf{r})\, \psi^{p}_{s_e'\mathbf{p}'}(\mathbf{r})\, d\mathbf{r} = (2\pi)^3\delta(\mathbf{p}-\mathbf{p}')\delta_{s_e s_e'} . \tag{A.3}$$

(2) Solution for the point charge distribution

For the point charge distribution $V(r)= -\frac{\alpha Z}{r}$, G_{κ_e} and F_{κ_e} are given with their asymptotic forms as,

$$\begin{pmatrix} G_{\kappa_e} \\ F_{\kappa_e} \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{E+1}{2E}}\, R_e \\ -\sqrt{\frac{E-1}{2E}}\, I_m \end{pmatrix}\left[2(\gamma+iy)\frac{|\Gamma(\gamma+iy)|}{\Gamma(2\gamma+1)}(2pr)^{\gamma-1} e^{\frac{\pi y}{2}-ipr+i\eta_\kappa}\right] F(\gamma+1+iy, 2\gamma+1, 2ipr) \tag{A.4}$$

$$\xrightarrow[r\to\infty]{} \begin{pmatrix} \sqrt{\frac{E+1}{2E}}\cdot\frac{1}{pr}\cdot\cos[pr+y\log 2pr-\frac{\pi}{2}(l_e+1)+\Delta_{\kappa_e}] \\ -\sqrt{\frac{E-1}{2E}}\cdot\frac{1}{pr}\cdot\sin[pr+y\log 2pr-\frac{\pi}{2}(l_e+1)+\Delta_{\kappa_e}] \end{pmatrix}, \tag{A.5}$$

with

$$\Delta_{\kappa_e} = \frac{\pi}{2}(l_e+1) + \eta_{\kappa_e} - \arg\Gamma(\gamma+iy) - \frac{\pi\gamma}{2} \quad , \tag{A.6}$$

and

$$\gamma = \sqrt{\kappa_e^2 - (\alpha Z)^2} \quad , \qquad y = \frac{\alpha Z E}{p} \quad , \qquad e^{2i\eta_{\kappa_e}} = -\left(\frac{\kappa_e - \frac{iy}{E}}{\gamma + iy}\right),$$

$$F(a,c,x) = \frac{\Gamma(c)}{\Gamma(a)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)}{\Gamma(c+n)} \cdot \frac{x^n}{n!} \quad . \tag{A.7}$$

The wave function with the proper asymptotic form is given by

$$\psi^{(\pm)}_{s_e,\mathbf{p}} = 4\pi \sum_{\kappa_e m_e \mu_e} i^{l_e} (l_e \tfrac{1}{2} m_e s_e | j_e \mu_e)\, Y^{*}_{l_e m_e}(\hat{\mathbf{p}})\, e^{\pm i\Delta_{\kappa_e}} \begin{pmatrix} G_{\kappa_e} \chi_{\kappa_e \mu_e} \\ i F_{\kappa_e} \chi_{-\kappa_e \mu_e} \end{pmatrix}$$

$$\xrightarrow[r\to\infty]{} \psi^{P}_{s_e,\mathbf{p}} + \frac{e^{\pm ipr}}{r} \cdot f(\theta) \quad . \tag{A.8}$$

$\psi^{(+)}_{s_e,\mathbf{p}}$ and $\psi^{(-)}_{s_e,\mathbf{p}}$ denotes the solutions with outgoing and incoming boundary conditions, and $\psi^{P}_{s_e,\mathbf{p}}$ is the plane wave solution in eq.(A.2), except the phase factor proportional to ylog2pr. In the theory of β decay, the solution with the incoming boundary condition $\psi^{(-)}_{s_e,\mathbf{p}}$ is adopted.[67)]

(3) Solution for the finite charge distribution

Inside the nuclear charge distribution, we put the inner solution of the following form,

$$\begin{pmatrix} G_\kappa^{in} \\ F_\kappa^{in} \end{pmatrix} = N\, r^{|\kappa|-1} \begin{pmatrix} u_\kappa \\ v_\kappa \end{pmatrix} \tag{A.9}$$

Here N is the normalization constant and u_κ and v_κ satisfy the equation,

$$\frac{d}{dr}\begin{pmatrix} u_\kappa \\ v_\kappa \end{pmatrix} = \begin{pmatrix} -\frac{\kappa+|\kappa|}{r} & E+1-V(r) \\ -E+1+V(r) & \frac{\kappa-|\kappa|}{r} \end{pmatrix}\begin{pmatrix} u_\kappa \\ v_\kappa \end{pmatrix} . \tag{A.10}$$

u_κ and v_κ are solved numerically with the initial conditions,

$$\begin{aligned} &u_\kappa(0)=1 , \quad v_\kappa(0)=0 \qquad \text{for } \kappa<0 , \\ &u_\kappa(0)=0 , \quad v_\kappa(0)=1 \qquad \text{for } \kappa>0 . \end{aligned} \tag{A.11}$$

The outer solution is represented as the linear combination of the regular and irregular solution for the point charge distribution.

$$\begin{pmatrix} G_\kappa^{out} \\ F_\kappa^{out} \end{pmatrix} = A\begin{pmatrix} G_\kappa \\ F_\kappa \end{pmatrix} + B\begin{pmatrix} \bar{G}_\kappa \\ \bar{F}_\kappa \end{pmatrix} , \tag{A.12}$$

where $\bar{G}_\kappa$ and $\bar{F}_\kappa$ are the irregular solutions. The regular solution is given in eq.(A.4) and the irregular solution is obtained if we replace $\gamma \to -\gamma$ in eq.(A.4). The inner and the outer solutions are connected at a point of suitably large r where the Coulomb potential of the finite size nucleus shows the 1/r dependence.

A, B and N are determined as follows:

$$A/B = (\bar{G}_\kappa/G_\kappa)[(\bar{F}_\kappa/\bar{G}_\kappa) - (v_\kappa/u_\kappa)]/[(v_\kappa/u_\kappa) - (F_\kappa/G_\kappa)]\Big|_{r=R_c} , \tag{A.13}$$

$$B = [1 + 2(A/B)\cos(\delta_\kappa - \bar{\delta}_\kappa) + (A/B)^2]^{-1/2} , \tag{A.14}$$

$$N = B \cdot (G_\kappa \bar{F}_\kappa - F_\kappa \bar{G}_\kappa) / [(G_\kappa v_\kappa - F_\kappa u_\kappa) \cdot r^{|\kappa|-1}] \Big|_{r=R_c} , \tag{A.15}$$

with

$$\delta_\kappa - \bar{\delta}_\kappa = - \arg \frac{\Gamma(\gamma + iy)}{\Gamma(-\gamma + iy)} + (\eta_\kappa - \bar{\eta}_\kappa) - \pi\gamma . \tag{A.16}$$

Here R_c is the point of the connection.

The Coulomb phase shift for the finite charge distribution Δ_κ^f is given by,

$$\Delta_\kappa^f = \Delta_\kappa + \tan^{-1} [\sin(\bar{\delta}_\kappa - \delta_\kappa) / (A/B + \cos(\bar{\delta}_\kappa - \delta_\kappa))] , \tag{A.17}$$

where Δ_κ is that of the point charge given in eq.(A.6), and δ_κ^f is related Δ_κ^f by eq.(3.31).

Appendix B. Formula for positron decay

For positron decay, the Hermitian conjugate in eq.(2.1) contributes, and it is written as,

$$h.c. = \frac{G}{\sqrt{2}} J_\lambda^*(x)\, \ell_\lambda^*(x) \quad , \tag{B.1}$$

with

$$J_\lambda^*(x) = V_\lambda^*(x) + A_\lambda^*(x) \quad , \tag{B.2}$$

and

$$\ell_\lambda^*(x) = -i\, \overline{\psi}_\nu(x)\, \gamma_\lambda (1+\gamma_5)\, \psi_e(x) \, . \tag{B.3}$$

The matrix elements of the vector current V_λ and the axial vector current A_λ between the nucleon states can be written as,

$$\langle n | V_\lambda^* | P \rangle = i\, \overline{\psi}_n \left(f_V^* \gamma_\lambda + f_W^* \sigma_{\lambda\rho} K_\rho' - i f_S^* K_\lambda' \right) \psi_p \quad , \tag{B.4}$$

$$\langle n | A_\lambda^* | P \rangle = i\, \overline{\psi}_n \gamma_5 \left(f_A^* \gamma_\lambda - f_T^* \sigma_{\lambda\rho} K_\rho' + i f_P^* K_\lambda' \right) \psi_p \quad , \tag{B.5}$$

with

$$K' = K_n - K_p \, . \tag{B.6}$$

If time reversal invariance holds, these nucleon form factors are real and the change from the negatron formula appears in the sign change of S.C.C.

terms f_S and f_T.

The lepton matrix element $L_\lambda(\mathbf{r})$ is given by,

$$L_\lambda(\mathbf{r}) = \langle \bar{e}\nu | \ell_\lambda^*(\mathbf{r}) | 0 \rangle = -i\, \bar{\Phi}_{s_\nu}(\mathbf{r}) \gamma_\lambda (1+\gamma_5)\, \psi_{s_e}^c(\mathbf{r}) \, . \tag{B.7}$$

Here $\phi_{s_\nu}(\mathbf{r})$ and $\psi_{s_e}^c(\mathbf{r})$ are the neutrino and the positron wave functions, respectively. The particle wave function ψ and the antiparticle wave function ψ^c are related by the following relation

$$\psi^c = C\,\psi^* \;, \tag{B.8}$$

where C is the charge conjugation operater which is given by

$$C = \gamma_2 \;, \tag{B.9}$$

in our metric (Pauli metric). Using eqs.(B.8) and (B.9), eq.(B.7) can be rewritten as

$$
\begin{aligned}
L_\lambda(\mathbf{r}) &= -i\, [\gamma_2 \phi_{s_\nu}^{c\,*}(\mathbf{r})]^\dagger \gamma_4 \gamma_\lambda (1+\gamma_5)\, [\gamma_2 \psi_{s_e}^*(\mathbf{r})] \\
&= -i\, \bar{\psi}_{s_e}(\mathbf{r}) \gamma_4 \{ \gamma_2 \gamma_4 \gamma_\lambda (1+\gamma_5) \gamma_2 \}^t \phi_{s_\nu}^c(\mathbf{r}) \\
&= -i\, \bar{\psi}_{s_e}(\mathbf{r}) \gamma_\lambda (1-\gamma_5)\, \phi_{s_\nu}^c(\mathbf{r}) \;,
\end{aligned}
\tag{B.10}
$$

where t denotes the transpose of γ matrix.

Thus the positron formula is obtained by replacing $1+\gamma_5$ in eq.(3.2) by $1-\gamma_5$, for the lepton part. Furthermore, the radial wave functions for the electron should be replaced by the charge conjugated solutions which are obtained from eq.(A.1) by replacing $V(r) \to -V(r)$.

Appendix C. Nuclear recoil correction

The density matrix which incorporate the nuclear recoil correction is given by

$$
\begin{aligned}
\sum_{M_i} \rho_{M_i M_i} a_{M_i} = {} & 2\pi G^2 \sum_{\substack{\kappa_e \kappa_e' \kappa_\nu \kappa_\nu' \\ J J' J_e K \ell}} R_K \, i^{\ell_e' - \ell_e} \, i^{\ell_\nu' - \ell_\nu} (-)^{J_f + J_i + J' + K} (-)^{j_\nu - j_e'} e^{i(\Delta_{\kappa_e} - \Delta_{\kappa_e'})} \\
& \cdot \left[(2j_e+1)(2j_e'+1)(2j_\nu+1)(2j_\nu'+1)(2J+1)(2J'+1)(2J_e+1)/(2K+1)(2J_i+1) \right]^{1/2} \\
& \cdot (j_e j_e' \tfrac{1}{2} -\tfrac{1}{2} | J_e 0)(j_\nu j_\nu' \tfrac{1}{2} -\tfrac{1}{2} | K 0)(K J_e 0 0 | \ell 0) \, \hat{f}_\ell(J_i) \, P_\ell(\cos\theta) \\
& \cdot W(J_i J_i J J'; \ell J_f) \begin{Bmatrix} j_e & j_\nu & J \\ j_e' & j_\nu' & J' \\ J_e & K & \ell \end{Bmatrix} \langle J_f \| \Xi_J(\kappa_e, \kappa_\nu) \| J_i \rangle \langle J_f \| \Xi_{J'}(\kappa_e', \kappa_\nu') \| J_i \rangle^* \\
& \cdot \left[(\ell_\nu + \ell_\nu' + K = \text{even}) \pm (\ell_\nu + \ell_\nu' + K = \text{odd}) \right] \cdot \left[\frac{1 + (-)^{\ell_e + \ell_e' + J_e}}{2} \right] .
\end{aligned}
\tag{C.1}
$$

In the case of the A=12(1^+,1 0^+,0) transition, the particle parameters $b_{11}^{(\ell)}$ in eq.(3.30) should be multiplied with R_0, and the additional terms appear in the following manner,

$$
\begin{aligned}
b_{11}^{(0)} & \longrightarrow R_0 b_{11}^{(0)} - \frac{2}{3\sqrt{3}} R_1 \sin(\delta_{-1} - \delta_1) \xi_1 \xi_2 , \\
b_{11}^{(1)} & \longrightarrow R_0 b_{11}^{(1)} \mp \frac{\sqrt{2}}{3} R_1 (\xi_1^2 + \xi_2^2) , \\
b_{11}^{(2)} & \longrightarrow R_0 b_{11}^{(2)} - \frac{4\sqrt{2}}{3\sqrt{3}} R_1 \sin(\delta_{-1} - \delta_1) \xi_1 \xi_2 ,
\end{aligned}
\tag{C.2}
$$

with

$$
\xi_1 = \langle J_f \| \Xi_1(-1,-1) \| J_i \rangle , \quad \xi_2 = \langle J_f \| \Xi_1(1,1) \| J_i \rangle . \tag{C.3}
$$

The effects of the recoil correction are discussed in §7.4.

Appendix D. Shell model calculations

D.1 Notations

The isospin formalism is used throughout §5, and the alphabet which specifies the individual nuclear states is an abbreviation of the spin and isospin as follows:

$$|a\rangle = |j_a t_a\rangle \ , \quad (-)^a = (-)^{j_a + t_a} \ , \quad [a] = (2j_a+1)(2t_a+1) \ ,$$

$$W(abcd\,;ef) = W(j_a j_b j_c j_d\,; j_e j_f)\,W(t_a t_b t_c t_d\,; t_e t_f) \quad \text{etc.} \tag{D.1}$$

The reduced matrix element is defined by

$$\langle f|O^{\omega}_{q}|i\rangle = (-)^{2(J_\omega+T_\omega)}(J_i J_\omega M_i M_q|J_f M_f)(T_i T_\omega N_i N_q|T_f N_f)\frac{\langle f|||O^{\omega}|||i\rangle}{\sqrt{(2J_f+1)(2T_f+1)}} \ , \tag{D.2}$$

where the phase factor vanishes for the ordinary operater with integer rank, and it is consistent with the definition in §3, if the isospin indices are dropped.

An irreducible tensor operater made of the tensor product is defined by

$$[A^{\alpha}\otimes B^{\beta}]^{\omega} = \sum_{M_\alpha M_\beta N_\alpha N_\beta}(J_\alpha J_\beta M_\alpha M_\beta|J_\omega M_\omega)(T_\alpha T_\beta N_\alpha N_\beta|T_\omega N_\omega)\,A^{J_\alpha T_\alpha}_{M_\alpha N_\alpha}\,B^{J_\beta T_\beta}_{M_\beta N_\beta} \ . \tag{D.3}$$

Antisymmetrized(A) and normalized antisymmetrized(AN) two-body matrix element between the angular momentum coupled states ($|\alpha\beta;\Gamma\rangle = |[\alpha\otimes\beta]^{\Gamma}\rangle$) are given by

$$\langle\alpha\beta\,;\Gamma|G|\gamma\delta\,;\Gamma'\rangle_{AN} = \frac{1}{\sqrt{(1+\delta_{\alpha\beta})(1+\delta_{\gamma\delta})}}\langle\alpha\beta\,;\Gamma|G|\gamma\delta\,;\Gamma'\rangle_{A}$$

$$= \frac{1}{\sqrt{(1+\delta_{\alpha\beta})(1+\delta_{\gamma\delta})}}\left\{\langle\alpha\beta;\Gamma|G|\gamma\delta;\Gamma'\rangle - (-)^{\gamma+\delta-\Gamma'}\langle\alpha\beta;\Gamma|G|\delta\gamma;\Gamma'\rangle\right\} \ , \tag{D.4}$$

with $\delta_{\alpha\beta} = \delta_{j_\alpha j_\beta}\delta_{\ell_\alpha \ell_\beta}\delta_{n_\alpha n_\beta}$. Note that the Kronecker's delta is an abbreviation of the product of the Kronecker's delta functions for all necessary quantum numbers, while δ' represents the product of the Kronecker's deltas for spin and isospin.

D.2 Second quantized formalism for shell model calculation[43),68)]

One-body operator F^ω and two-body operator G^ω are generally expressed as

$$F^\omega = \sum_{mn} \langle m | F^\omega | n \rangle a_m^+ a_n \quad , \tag{D.5}$$

$$G^\omega = \frac{1}{4} \sum_{mnpq} \langle m n | G^\omega | p q \rangle_A \, a_m^+ a_n^+ a_q a_p \quad , \tag{D.6}$$

in the second quantized formalism. Then we define the core, and the hole operator b_α^+ is introduced as follows:

$$(-)^{j+m+t+n} a_{j-m\,t-n} = \begin{cases} b^+_{jmtn} & (\text{ states inside the core}) \\ \tilde{a}_{jmtn} & (\text{ states outside the core}) \end{cases} \quad , \tag{D.7}$$

where $\tilde{a}$ is the particle annihilation operator with the correct transformation property for angular momentum, and $\tilde{b}$ is defined in the same manner. Eqs. (D.5) and (D.6) are rewritten by the particle and hole operators, and they are rearranged in normal order. Then we reduce the matrix elements by eq.(D.2) and sum over the magnetic quantum numbers. Finally we obtain five terms of irreducible tensor operators for F^ω, and fourteen terms for G^ω. We give the explicit expressions for the relevant terms in our analysis,

$$F^{\omega} = -\sum_{ij}\langle i|||F^{\omega}|||j\rangle [\omega]^{-1/2} B_{ji}^{\omega} (-)^{i-j+\omega} \tag{D.8.a}$$

$$+\sum_{mj}\langle m|||F^{\omega}|||j\rangle [\omega]^{-1/2} C_{mj}^{+\omega} \tag{D.8.b}$$

$$+\sum_{ni}\langle i|||F^{\omega}|||n\rangle [\omega]^{-1/2} \tilde{C}_{ni}^{\omega} (-)^{i-n+\omega} , \tag{D.8.c}$$

$$G^{\omega} = \frac{1}{4}\sum_{ijk\ell xy}\langle ij;x|||G^{\omega}|||k\ell;y\rangle_A [\omega]^{-1/2} [B_{k\ell}^{+y}\otimes \tilde{B}_{ij}^{x}]^{\omega} (-)^{x-y+\omega} \tag{D.9.a}$$

$$+\frac{1}{2}\sum_{jkm\ell xy}\langle mj;x|||G^{\omega}|||k\ell;y\rangle_A [\omega]^{-1/2} [B_{k\ell}^{+y}\otimes \tilde{D}_{mj}^{x}]^{\omega} (-)^{x-y+\omega} \tag{D.9.b}$$

$$+\frac{1}{2}\sum_{ij\ell pxy}\langle ij;x|||G^{\omega}|||p\ell;y\rangle_A [\omega]^{-1/2} [D_{p\ell}^{+y}\otimes \tilde{B}_{ij}^{x}]^{\omega} (-)^{x-y+\omega} \tag{D.9.c}$$

$$-\frac{1}{4}\sum_{k\ell mnxy}\langle mn;x|||G^{\omega}|||pq;y\rangle_A [\omega]^{-1/2} [A_{mn}^{+x}\otimes B_{k\ell}^{+y}]^{\omega} \tag{D.9.d}$$

$$-\frac{1}{4}\sum_{ijpqxy}\langle ij;x|||G^{\omega}|||pq;y\rangle_A [\omega]^{-1/2} [\tilde{B}_{ij}^{x}\otimes \tilde{A}_{pq}^{y}]^{\omega} (-)^{2x} \tag{D.9.e}$$

$$+\sum_{c\ell nxy}\langle cn;x|||G^{\omega}|||c\ell;y\rangle_A \left[\frac{xy}{\omega}\right]^{1/2} W(xc\omega\ell;ny)\, C_{n\ell}^{+\omega} \tag{D.9.f}$$

$$+\sum_{cjqxy}\langle cj;x|||G^{\omega}|||cq;y\rangle_A \left[\frac{xy}{\omega}\right]^{1/2} W(xc\omega q;jy)\, \tilde{C}_{qj}^{\omega} (-)^{j-q+\omega} , \tag{D.9.g}$$

with

$$B_{ji}^{\omega} = [b_j^{+}\otimes \tilde{b}_i]^{\omega}, \quad B_{k\ell}^{+y} = [b_k^{+}\otimes b_\ell^{+}]^{y} , \quad \tilde{B}_{ij}^{x} = -[\tilde{b}_i\otimes \tilde{b}_j]^{x},$$

$$C_{mj}^{+\omega} = [a_m^{+}\otimes b_j^{+}]^{\omega}, \quad \tilde{C}_{ni}^{\omega} = -[\tilde{a}_n\otimes \tilde{b}_i]^{\omega}, \quad D_{p\ell}^{+y} = [\tilde{a}_p\otimes b_\ell^{+}]^{y}, \quad \tilde{D}_{mj}^{x} = [\tilde{b}_j\otimes a_m^{+}]^{x} (-)^{m-j+x},$$

$$A_{mn}^{+x} = [a_m^{+}\otimes a_n^{+}]^{x} , \quad \tilde{A}_{pq}^{y} = -[\tilde{a}_p\otimes \tilde{a}_q]^{y}. \tag{D.10}$$

Here the summation runs over particle states for the suffices of a^{+} and $\tilde{a}$, hole states for those of b^{+} and $\tilde{b}$ and all the states in the core for c. x and y are restricted by the spin and isospin selection rules.

Matrix elements of these operators between the particle-hole states are reduced by the formula,

$$\langle X^{\Gamma_p'} \otimes Y^{\Gamma_h'}; \Gamma' ||| [A^{\Omega_p} \otimes B^{\Omega_h}]^{\Omega} ||| X^{\Gamma_p} \otimes Y^{\Gamma_h}; \Gamma \rangle$$

$$= (-)^{n_p \nu_h} [\Gamma \Gamma' \Omega]^{1/2} \begin{Bmatrix} \Gamma_p' & \Gamma_h' & \Gamma' \\ \Gamma_p & \Gamma_h & \Gamma \\ \Omega_p & \Omega_h & \Omega \end{Bmatrix} \langle X^{\Gamma_p'} ||| A^{\Omega_p} ||| X^{\Gamma_p} \rangle \langle Y^{\Gamma_h'} ||| B^{\Omega_h} ||| Y^{\Gamma_h} \rangle \quad . \tag{D.11}$$

Here $X^{\Gamma_p'}$, X^{Γ_p} and $Y^{\Gamma_h'}$, Y^{Γ_h} are the particle and hole states, respectively. A^{Ω_p} (B^{Ω_h}) is constructed from the product of particle (hole) operators a^+ and $\tilde{a}$ (b^+ and $\tilde{b}$). n_p is the particle number of X^{Γ_p} and ν_h is the number of hole operators in B^{Ω_h}, and the phase factor $(-)^{n_p \nu_h}$ appears by the anti-commutation properties of the particle hole operators.

D.3 Definition of the coefficient of fractional parentage (c.f.p.)

The problem is now reduced to obtain the matrix elements of a^+ and $\tilde{a}$ (b^+ and $\tilde{b}$) between the many particle (hole) states. The task is accomplished with the help of c.f.p.. In the following, we give the formulas for the particle operators, and the situation is completely equivalent for the hole operators.

(1) One-orbit formula

One-particle c.f.p. is defined by

$$|\rho^n \Gamma\rangle = \sum_{\Gamma'} \langle \rho^n \Gamma \{| \rho^{n-1} \Gamma'; \rho \rangle \, | \rho^{n-1} \Gamma' \otimes \rho ; \Gamma \rangle \quad , \tag{D.12}$$

and the reduced matrix elements of a^+ and $\tilde{a}$ can be written as

$$\begin{aligned} \langle \rho^n \Gamma ||| a_\rho^+ ||| \rho^{n'} \Gamma' \rangle &= \delta_{n' n-1} (-)^{n-1} (-)^{2\rho} \sqrt{n} [\Gamma]^{1/2} \langle \rho^n \Gamma \{| \rho^{n-1} \Gamma'; \rho \rangle \quad , \\ \langle \rho^{n'} \Gamma' ||| \tilde{a}_\rho ||| \rho^n \Gamma \rangle &= (-)^{\Gamma' + \rho - \Gamma} \langle \rho^n \Gamma ||| a_\rho^+ ||| \rho^{n'} \Gamma' \rangle \quad . \end{aligned} \tag{D.13}$$

Here $|\rho^n\Gamma\rangle$ is the n-particle orthonormal state with definite spin and isospin. Two-particle c.f.p. is defined by

$$|\rho^n\Gamma\rangle = \sum_{\Gamma'\theta} \langle \rho^n\Gamma \{| \rho^{n-2}\Gamma'; (\rho_\alpha\rho_\beta)\theta\rangle\langle(\rho_\alpha\rho_\beta)\theta\{|\rho_\beta;\rho_\alpha\rangle \,|\, \rho^{n-2}\Gamma' \otimes [\rho_\alpha \otimes \rho_\beta]^\theta;\Gamma\rangle \quad , \tag{D.14}$$

where two particles α and β are distinguished for later convenience. From eqs.(D.12) and (D.14), the relation between one-particle c.f.p. and two-particle c.f.p. is obtained,

$$\langle \rho^n\Gamma\{|\rho^{n-2}\Gamma';(\rho_\alpha\rho_\beta)\theta\rangle\langle(\rho_\alpha\rho_\beta)\theta\{|\rho_\beta;\rho_\alpha\rangle = \sum_{\Gamma''} \langle \rho^n\Gamma\{|\rho^{n-1}\Gamma'';\rho_\alpha\rangle\langle\rho^{n-1}\Gamma''\{|\rho^{n-2}\Gamma';\rho_\beta\rangle \cdot [\Gamma''\theta]^{1/2} W(\Gamma'\rho_\beta\Gamma\rho_\alpha;\Gamma''\theta) \, . \tag{D.15}$$

It is also defined as the matrix element of the second quantized operator,

$$\langle \rho^n\Gamma ||| [a_\alpha^\dagger \otimes a_\beta^\dagger]^\theta ||| \rho^{n'}\Gamma'\rangle = (-)^{2\theta}\sqrt{n(n-1)}\,\delta_{n'n-2}\, \langle \rho^n\Gamma\{|\rho^{n-2}\Gamma';(\rho_\alpha\rho_\beta)\theta\rangle\langle(\rho_\alpha\rho_\beta)\{|\rho_\alpha;\rho_\beta\rangle \, . \tag{D.16}$$

The one-body operator F^ω and two-body operator G^ω can be expanded in the following nine types of operators: 1, $a^\dagger$, $\tilde{a}$, $[a^\dagger\otimes\tilde{a}]$, $[a^\dagger\otimes a^\dagger]$, $[\tilde{a}\otimes\tilde{a}]$, $[(a^\dagger\otimes a^\dagger)\otimes\tilde{a}]$, $[a^\dagger\otimes(\tilde{a}\otimes\tilde{a})]$ and $[(a^\dagger\otimes a^\dagger)\otimes(\tilde{a}\otimes\tilde{a})]$.

With the help of eqs.(D.13) and (D.16), they are reduced to the product of c.f.p.'s and Racah coefficients with proper factors.

(2) Two-orbit formula

In the two-orbit formula, $|\rho^n\Gamma\rangle$ in the one-orbit formulas should be replaced by $|[\rho_1^{n_1}\otimes\rho_2^{n_2}]^\Gamma\rangle$. It is accomplished by replacing the one-orbit c.f.p.'s with the two-orbit c.f.p.'s in eq.(D.13) and (D.16).

The two-orbit c.f.p. can be expressed in terms of the one orbit c.f.p. as,

$$
\begin{aligned}
&\langle \rho_1^{n_1}\Gamma_1 \otimes \rho_2^{n_2}\Gamma_2 ; \Gamma \{\!| \rho_1^{n_1'}\Gamma_1' \otimes \rho_2^{n_2'}\Gamma_2' ; \Gamma' ; \rho_\lambda \rangle \\
&= \delta_{\lambda 1}\delta_{n_2' n_2}\delta_{n_1' n_1-1}(-)^{n_2}\sqrt{\frac{n_1}{N}}\,[\Gamma_1 \Gamma']^{1/2} W(\Gamma' \Gamma_2 \rho_\lambda \Gamma_1 ; \Gamma_1' \Gamma) \langle \rho_1^{n_1}\Gamma_1 \{\!| \rho_1^{n_1-1}\Gamma_1' ; \rho_1 \rangle \delta_{\Gamma_2 \Gamma_2'} \\
&+ \delta_{\lambda 2}\delta_{n_1' n_1}\delta_{n_2' n_2-1}\sqrt{\frac{n_2}{N}}\,[\Gamma_2 \Gamma']^{1/2} W(\Gamma \Gamma_1 \rho_\lambda \Gamma_2' ; \Gamma_2 \Gamma') \langle \rho_2^{n_2}\Gamma_2 \{\!| \rho_2^{n_2-1}\Gamma_2' ; \rho_2 \rangle \delta_{\Gamma_1 \Gamma_1'} \quad ,
\end{aligned}
\tag{D.17}
$$

$$
\begin{aligned}
&\langle \rho_1^{n_1}\Gamma_1 \otimes \rho_2^{n_2}\Gamma_2 ; \Gamma \{\!| \rho_1^{n_1'}\Gamma_1' \otimes \rho_2^{n_2'}\Gamma_2' ; \Gamma' ; (\rho_\alpha \rho_\beta)\theta \rangle \\
&= \delta_{\alpha 1}\delta_{\beta 1}\delta_{n_1' n_1-2}\sqrt{\frac{n_1(n_1-1)}{N(N-1)}}\,[\Gamma_1 \Gamma']^{1/2} W(\Gamma' \Gamma_2 \theta \Gamma_1 ; \Gamma_1' \Gamma) \langle \rho_1^{n_1}\Gamma_1 \{\!| \rho_1^{n_1-2}\Gamma_1' (\rho_\alpha \rho_\beta)\theta \rangle \delta_{\Gamma_2 \Gamma_2'} \\
&+ \delta_{\alpha 2}\delta_{\beta 2}\delta_{n_2' n_2-2}\sqrt{\frac{n_2(n_2-1)}{N(N-1)}}\,[\Gamma_2 \Gamma']^{1/2} W(\Gamma \Gamma_1 \theta \Gamma_2' ; \Gamma_2 \Gamma') \langle \rho_2^{n_2}\Gamma_2 \{\!| \rho_2^{n_2-2}\Gamma_2' (\rho_\alpha \rho_\beta)\theta \rangle \delta_{\Gamma_1 \Gamma_1'} \\
&+ \delta_{\alpha 1}\delta_{\beta 2}\delta_{n_1' n_1-1}\delta_{n_2' n_2-1}(-)^{n_2-1}\sqrt{\frac{2n_1 n_2}{N(N-1)}}\,[\Gamma_1 \Gamma_2 \Gamma' \theta]^{1/2} \begin{Bmatrix} \Gamma_1 & \Gamma_2 & \Gamma \\ \Gamma_1' & \Gamma_2' & \Gamma' \\ \rho_\alpha & \rho_\beta & \theta \end{Bmatrix} \langle \rho_1^{n_1}\Gamma_1 \{\!| \rho_1^{n_1-1}\Gamma_1' ; \rho_\alpha \rangle \langle \rho_2^{n_2}\Gamma_2 \{\!| \rho_2^{n_2-1}\Gamma_2' ; \rho_\beta \rangle ,
\end{aligned}
\tag{D.18}
$$

with $N=n_1+n_2$.

The three-orbit c.f.p. can be defined in the same manner as eqs.(D.17) and (D.18). In §5.3, the three-orbit states appears in the following special form with $0s_{1/2}$ (ρ_s),

$$
|\rho^{n+1}\Delta\rangle = |\,[\rho_1^{n_1}\Gamma_1 \otimes \rho_2^{n_2}\Gamma_2]^{\Gamma} \otimes \rho_s \; ; \Delta\rangle \quad . \tag{D.19}
$$

Then the three-orbit c.f.p. can be reduced as follows:

$$
\langle \rho^{n+1}\Delta \{\!| \rho^{n}\Gamma' ; \rho_\lambda \rangle = \sqrt{\frac{1}{n+1}}\,\delta_{\Gamma\Gamma'}\delta_{\lambda s} \quad , \tag{D.20}
$$

$$
\begin{aligned}
&\langle \rho^{n+1}\Delta \{\!| \rho^{n-1}\Gamma' ; (\rho_\alpha \rho_\beta)\theta \rangle \\
&= \delta_{\alpha 1}\delta_{\beta s}\delta_{n_1' n_1-1}\delta_{\Gamma_2' \Gamma_2}(-)^{n_2}\sqrt{\frac{2n_1}{n(n+1)}}\,[\Gamma \Gamma_1 \Gamma' \theta]^{1/2} W(\Gamma' \Gamma_2 \rho_\alpha \Gamma_1 ; \Gamma_1' \Gamma) W(\Delta \Gamma' \rho_s \rho_\alpha ; \theta \Gamma) \langle \rho_1^{n_1}\Gamma_1 \{\!| \rho_1^{n_1-1}\Gamma_1' ; \rho_\alpha \rangle \\
&+ \delta_{\alpha 2}\delta_{\beta s}\delta_{n_2' n_2-1}\delta_{\Gamma_1' \Gamma_1}\sqrt{\frac{2n_2}{n(n+1)}}\,[\Gamma \Gamma_2 \Gamma' \theta]^{1/2} W(\Gamma \Gamma_1 \rho_\alpha \Gamma_2' ; \Gamma_2 \Gamma') W(\Delta \Gamma' \rho_s \rho_\alpha ; \theta \Gamma) \langle \rho_2^{n_2}\Gamma_2 \{\!| \rho_2^{n_2-1}\Gamma_2' ; \rho_\alpha \rangle \\
&= \sqrt{\frac{2}{n+1}}\,W(\Delta \Gamma' \rho_s \rho_\alpha ; \theta \Gamma)[\Gamma \theta]^{1/2} \langle \rho_1^{n_1}\Gamma_1 \otimes \rho_2^{n_2}\Gamma_2 ; \Gamma \{\!| \rho_1^{n_1'}\Gamma_1' \otimes \rho_2^{n_2'}\Gamma_2' ; \Gamma' ; \rho_\alpha \rangle \delta_{\beta s}
\end{aligned}
\tag{D.21}
$$

The c.f.p.'s in §5 should be read as the two-orbit or three-orbit c.f.p.'s.

Appendix E. Derivation of the core polarization formulas

E.1 Derivation of eq.(5.32)

As is seen from eq.(5.30), we need to calculate $\langle \lambda^n \Gamma_f \| T^\omega \| \rho\otimes\lambda^{n+1}\Delta ; \Gamma_m \rangle$ and $\langle \rho\otimes\lambda^{n+1}\Delta ; \Gamma_m | V | \lambda^n \Gamma_i \rangle$. The contribution to the reduced matrix elements of the one-body operator T^ω comes from eq.(D.8.c), as

$$\langle \lambda^n \Gamma_f \| T^\omega \| \rho\otimes\lambda^{n+1}\Delta ; \Gamma_m \rangle = \sum_{ni} \langle i \| T^\omega \| n \rangle [\omega]^{-1/2} (-)^{i-n+\omega} \langle \lambda^n \Gamma_f \| \tilde{C}^\omega_{ni} \| \rho\otimes\lambda^{n+1}\Delta ; \Gamma_m \rangle . \qquad \text{(E.1)}$$

Then, eq.(D.11) is applied and we obtain

$$= \sum_{ni} \langle i \| T^\omega \| n \rangle [\omega]^{-1/2} (-)^{i-n+\omega} [\Gamma_f \Gamma_m \omega]^{1/2} \begin{Bmatrix} 0 & \Gamma_f & \Gamma_f \\ \rho & \Delta & \Gamma_m \\ n & i & \omega \end{Bmatrix} \langle 0 \| \tilde{a}_n \| \rho \rangle \langle \lambda^n \Gamma_f \| \tilde{b}_i \| \lambda^{n+1}\Delta \rangle . \qquad \text{(E.2)}$$

Using the definition of c.f.p. in eq.(D.13),

$$= \sum_{i} (-)^n \sqrt{n+1}\, [\Gamma_m \Delta]^{1/2} W(\Gamma_m \Delta \omega i ; \rho \Gamma_f) \langle \lambda^{n+1}\Delta \{| \lambda^n \Gamma_f ; i \rangle \langle i \| T^\omega \| \rho \rangle . \qquad \text{(E.3)}$$

The contributions to the matrix elements of the effective interaction come from eqs.(D.9.b) and (D.9.f) with rank ω=0, which correspond to the two-body and one-body part, respectively. The one-body part of V can be calculated in almost the same way as eqs.(E.1)-(E.3).

$$\langle \rho\otimes\lambda^{n+1}\Delta ; \Gamma_m | V | \lambda^n \Gamma_i \rangle = \sum_{cjx} (-)^n \sqrt{n+1} \left[\frac{\Delta}{\Gamma_i}\right]^{1/2} \left[\frac{x}{\rho}\right] \langle \rho c ; x | V | j c ; x \rangle_A \langle \lambda^{n+1}\Delta \{| \lambda^n \Gamma_i ; j \rangle \delta'_{\rho j} \delta'_{\Gamma_m \Gamma_i} . \qquad \text{(E.4)}$$

While the two-body part is derived as follows:

$$\langle \rho\otimes\lambda^{n+1}\Delta ; \Gamma_m | V | \lambda^n \Gamma_i \rangle = \frac{1}{2} \sum_{jkl mx} \langle mj ; x | V | kl ; x \rangle_A [x]^{1/2} \langle \rho\otimes\lambda^{n+1}\Delta ; \Gamma_m | [B^{\dagger x}_{kl} \otimes \tilde{D}^{x}_{mj}]^0 | \lambda^n \Gamma_i \rangle . \qquad \text{(E.5)}$$

Recoupling the operator $[B^{+x}_{k\ell} \otimes \tilde{D}^{x}_{mj}]^{0}$ and applying eq.(D.11). we obtain

$$\langle \rho\otimes\lambda^{n+1}\Delta;\Gamma_m | [B^{+x}_{k\ell} \otimes \tilde{D}^{x}_{mj}]^{0} | \lambda^{n}\Gamma_i \rangle = -[\Gamma_i m x]^{1/2} \begin{Bmatrix} \rho & \Delta & \Gamma_i \\ 0 & \Gamma_i & \Gamma_i \\ m & \beta & 0 \end{Bmatrix} (-)^{m+j-x} \cdot$$

$$\cdot \langle \rho ||| a^{+}_{m} ||| 0 \rangle \langle \lambda^{n+1}\Delta ||| [B^{+x}_{k\ell} \otimes \tilde{b}_j]^{m} ||| \lambda^{n}\Gamma_i \rangle \delta_{\Gamma_i \Gamma_m}' \quad . \tag{E.6}$$

The matrix element of the hole operator is given as

$$\langle \lambda^{n+1}\Delta ||| [B^{+x}_{k\ell} \otimes \tilde{b}_j]^{m} ||| \lambda^{n}\Gamma_i \rangle = (-)^{n+1} (-)^{2m} n\sqrt{n+1}\, [\Delta \Gamma_i m]^{1/2}$$

$$\cdot \sum_{\theta} W(\Delta\theta m j; x \Gamma_i) \langle \lambda^{n}\Gamma_i \{| \lambda^{n-1}\theta ; j \rangle \langle \lambda^{n+1}\Delta \{| \lambda^{n-1}\theta ; (k\ell)x \rangle \langle (k\ell)x \{| k,\ell \rangle . \tag{E.7}$$

Finally we obtain,

$$\langle \rho\otimes\lambda^{n+1}\Delta;\Gamma_m | V | \lambda^{n}\Gamma_i \rangle = (-)^{n} \frac{n\sqrt{n+1}}{\sqrt{2}} \sum_{j k \leq \ell x \theta} \langle \rho j; x | V | k\ell ; x \rangle_{AN} [\Delta x]^{1/2} W(\Delta \Gamma x j; \rho\theta) \cdot (-)^{\Gamma+j-\theta}$$

$$\cdot \langle \lambda^{n+1}\Delta \{| \lambda^{n-1}\theta ; (k\ell)x \rangle \langle \lambda^{n}\Gamma \{| \lambda^{n-1}\theta ; j \rangle \quad . \tag{E.8}$$

Here we used the relations

$$\sum_{k\ell} \sim \; = \sum_{k\leq\ell} \left(\frac{2}{1+\delta_{k\ell}} \right) \sim \quad , \qquad \langle (k\ell)x \{| k,\ell \rangle = \sqrt{\frac{1+\delta_{k\ell}}{2}} \; ,$$

$$\langle \rho j; x | V | k\ell ; x \rangle_{AN} = \sqrt{1+\delta_{k\ell}}\, \langle \rho j; x | V | k\ell ; x \rangle_{AN} . \tag{E.9}$$

The second term in eq.(5.30) can be calculated in the same manner.

E.2 Derivation of eq.(5.44)

Since the calculation in this case is simpler than that of eq.(5.32), we give only the final results for each term.

$$\langle 0 \| T^{\omega} \| Ph; \Gamma \rangle = (-)^{P-h-\Gamma} \langle h \| T^{\omega} \| P \rangle \delta_{\Gamma\omega} \quad , \tag{E.10}$$

$$\begin{aligned} \langle (\alpha\beta)\pi \otimes (\mu\nu)\theta; \Gamma \| T^{\omega} \| P_0 h_0; \Gamma_0 \rangle = & - \langle \beta \| T^{\omega} \| \nu \rangle [\Gamma \Gamma_0 \pi \theta]^{1/2} \begin{Bmatrix} \pi & \theta & \Gamma \\ P_0 & h_0 & \Gamma_0 \\ \beta & \nu & \omega \end{Bmatrix} \delta_{\alpha P_0} \delta_{\mu h_0} \\ & \cdot \frac{1}{\sqrt{(1+\delta_{\alpha\beta})(1+\delta_{\mu\nu})}} \{ [1 - (-)^{\alpha+\beta-\pi} (\alpha \leftrightarrow \beta)] [1 - (-)^{\mu+\nu-\theta} (\mu \leftrightarrow \nu)] \quad , \end{aligned} \tag{E.11}$$

$$\langle Ph; \Gamma \| T^{\omega} \| P_0 h_0; \Gamma_0 \rangle = \langle P \| T^{\omega} \| P_0 \rangle [\Gamma \Gamma_0]^{1/2} W(P h_0 \omega \Gamma_0; \Gamma P_0) \delta_{h h_0} \quad , \tag{E.12}$$

$$\langle Ph; \Gamma | V | P_0 h_0; \Gamma_0 \rangle = \sum_{x} \langle P h_0; x | V | h P_0; x \rangle_A [x] W(P h h_0 P_0; \Gamma x) (-)^{P+h_0-x} \delta_{\Gamma\Gamma_0} , \tag{E.13}$$

$$\langle 0 | V | (\alpha\beta)\pi \otimes (\mu\nu)\theta; \Gamma \rangle = - [\pi]^{1/2} \langle \mu\nu; \theta | V | \alpha\beta; \pi \rangle_{AN} \delta_{\pi\theta} \delta_{\Gamma 0} \quad , \tag{E.14}$$

$$\langle Ph; \Gamma | U | P_0 h_0; \Gamma \rangle = \sum_{cx} \left[\frac{x}{P}\right] \langle cP; x | V | cP_0; x \rangle_A \delta_{h h_0} \delta'_{P P_0} \quad , \tag{E.15}$$

$$\langle 0 | U | Ph; \Gamma \rangle = \sum_{cx} \left[\frac{x^2}{P}\right]^{1/2} \langle ch; x | V | cP; x \rangle_A \delta'_{Ph} \delta_{\Gamma 0} . \tag{E.16}$$

These terms are combined into eq.(5.43) to give eq.(5.44). Note that the third line in eq.(5.44) is obtained with the following relation,

$$\sum_{\alpha \leq \beta\, \mu \leq \mu} \sim \; = \sum_{\alpha\beta\mu\nu} \frac{(1+\delta_{\alpha\beta})(1+\delta_{\mu\nu})}{4} \sim \quad , \tag{E.17}$$

then the exchange terms in $[1-(-)^{\alpha+\beta-\pi}(\alpha \leftrightarrow \beta)]\,[1-(-)^{\mu+\nu-\theta}(\mu \leftrightarrow \nu)]$ gives the same contribution as the main term, and they give factor 4. These are canceled with $[(1+\delta_{\alpha\beta})(1+\delta_{\mu\nu})]^{-1}$ which comes from the normalization factor of the 2p-2h state and from $\langle \mu\nu; \theta | V | \alpha\beta; \pi \rangle_{AN} = [(1+\delta_{\alpha\beta})(1+\delta_{\mu\nu})]^{-1} \langle \mu\nu; \theta | V | \alpha\beta; \pi \rangle_A$.

Appendix F. Two-body matrix elements

The formula to calculate the matrix elements of the two-body operator between the antisymmetrized two-particle states with the harmonic oscillator single particle wave function is given for general two-body operator and for the effective interaction.

F.1 General two-body operator

We assume that the two-body operator is given by

$$\mathbb{G}^{\omega} = \left[\{\mathbb{V}^{v}(\mathbb{r})\otimes\mathbb{U}^{u}(\mathbb{R})\}^{x}\otimes\mathbb{S}^{s}\right]^{\omega}\mathbb{T}^{t} , \tag{F.1}$$

with

$$\mathbb{V}^{v}(\mathbb{r}) = v(r)Y_{v}(\hat{r}) , \qquad \mathbb{U}^{u}(\mathbb{R}) = u(R)Y_{u}(\hat{R}) , \tag{F.2}$$

and

$$\mathbb{r} = \mathbb{r}_1 - \mathbb{r}_2 , \qquad \mathbb{R} = (\mathbb{r}_1 + \mathbb{r}_2)/2 . \tag{F.3}$$

Here $\mathbb{S}^{s}$ and $\mathbb{T}^{t}$ are the spin and isospin operator for two-nucleon system with rank s and t.

The antisymmetrized normalized matrix element is given by

$$\langle ab;\Gamma_f|||G^{\omega}|||cd;\Gamma_i\rangle_{AN} = \frac{1}{\sqrt{(1+\delta_{ab})(1+\delta_{cd})}} \cdot\left\{\langle ab;\Gamma_f|||G^{\omega}|||cd;\Gamma_i\rangle - (-)^{c+d-\Gamma_i}\langle ab;\Gamma_f|||G^{\omega}|||dc;\Gamma_i\rangle\right\} . \tag{F.4}$$

Each term in the brackets is calculated by the help of jj-LS recoupling coefficients,

$$\langle ab;\Gamma_f \| G^{\omega} \| cd;\Gamma_i\rangle = \sum_{L_iS_iL_fS_f} \begin{pmatrix} l_a & 1/2 & j_a \\ l_b & 1/2 & j_b \\ L_f & S_f & J_f \end{pmatrix} \begin{pmatrix} l_c & 1/2 & j_c \\ l_d & 1/2 & j_d \\ L_i & S_i & J_i \end{pmatrix} \begin{Bmatrix} L_f & S_f & J_f \\ L_i & S_i & J_i \\ x & s & \omega \end{Bmatrix}$$
$$\cdot \langle l_a l_b; L_f \| [\mathbb{V}^{v}(\mathbb{r}) \otimes \mathbb{U}^{u}(\mathbb{R})]^{x} \| l_c l_d; L_i\rangle \langle S_f \| \mathbb{S}^{s} \| S_i\rangle \langle T_f \| \mathbb{T}^{t} \| T_i\rangle \quad , \tag{F.5}$$

with

$$\begin{pmatrix} l_a & 1/2 & j_a \\ l_b & 1/2 & j_b \\ L_f & S_f & J_f \end{pmatrix} = [j_a j_b L_f S_f]^{1/2} \begin{Bmatrix} l_a & 1/2 & j_a \\ l_b & 1/2 & j_b \\ L_f & S_f & J_f \end{Bmatrix} \quad . \tag{F.6}$$

Using the Moshinsky bracket $\langle nl NL;\lambda | n_a l_a n_b l_b;\lambda\rangle$, the reduced matrix element of the product of the relative and center of mass coordinate operator is given as

$$\langle l_a l_b; L_f \| [\mathbb{V}^{v}(\mathbb{r}) \otimes \mathbb{U}^{u}(\mathbb{R})]^{x} \| l_c l_d; L_i\rangle$$
$$= \sum_{\substack{n_1 l_1 n_2 l_2 \\ N_1 L_1 N_2 L_2}} \langle n_1 l_1 N_1 L_1; L_f | n_a l_a n_b l_b; L_f\rangle \langle n_2 l_2 N_2 L_2; L_i | n_c l_c n_d l_d; L_i\rangle [L_i L_f x]^{1/2}$$
$$\begin{Bmatrix} l_1 & L_1 & L_f \\ l_2 & L_2 & L_i \\ v & u & x \end{Bmatrix} \langle n_1 l_1 \| \mathbb{V}^{v}(\mathbb{r}) \| n_2 l_2\rangle \langle N_1 L_1 \| \mathbb{U}^{u}(\mathbb{R}) \| N_2 L_2\rangle \quad . \tag{F.7}$$

The reduced matrix elements are written as

$$\langle n_1 l_1 \| \mathbb{V}^{v}(\mathbb{r}) \| n_2 l_2\rangle = (-)^{l_2} \frac{1}{\sqrt{4\pi}} [l_1 l_2]^{1/2} (l_1 l_2 00 | v0) \int r^2 dr\, \phi_{n_1 l_1}(r) V(\sqrt{2}r) \phi_{n_2 l_2}(r) \quad , \tag{F.8}$$

$$\langle N_1 L_1 \| \mathbb{U}^{u}(\mathbb{R}) \| N_2 L_2\rangle = (-)^{L_2} \frac{1}{\sqrt{4\pi}} [L_1 L_2]^{1/2} (L_1 L_2 00 | u0) \int R^2 dR\, \phi_{N_1 L_1}(R) U(\tfrac{R}{\sqrt{2}}) \phi_{N_2 L_2}(R) \quad . \tag{F.9}$$

Here the factor $\sqrt{2}$ comes from the definition of $\mathbb{r}$ and $\mathbb{R}$ in the Moshinsky brackets ($\mathbb{r} = \frac{\mathbb{r}_1 - \mathbb{r}_2}{\sqrt{2}}$, $\mathbb{R} = \frac{\mathbb{r}_1 + \mathbb{r}_2}{\sqrt{2}}$), and $\phi_{nl}(r)$ is the radial part of the harmonic osillator wave function. The exchange term in eq.(F.4) is calculated in a similar way, together with the following phase relation

$$\begin{pmatrix} \ell_d & 1/2 & j_d \\ \ell_c & 1/2 & j_c \\ L_i & S_i & J_i \end{pmatrix} = (-)^{\ell_c+\ell_d+L_i+1+S_i+j_c+j_d+J_i} \begin{pmatrix} \ell_c & 1/2 & j_c \\ \ell_d & 1/2 & j_d \\ L_i & S_i & J_i \end{pmatrix} , \tag{F.10}$$

$$\langle n_2\ell_2 N_2L_2 ; L_i | n_d\ell_d n_c\ell_c ; L_i \rangle = (-)^{L_2-L_i} \langle n_2\ell_2 N_2L_2 ; L_i | n_c\ell_c n_d\ell_d ; L_i \rangle , \tag{F.11}$$

with $(-)^{c+d-\Gamma_i}$, the total phase for the exchange part is given by $(-)^{\ell_2+S_i+T_i}$.

Finally we obtain the antisymmetrized normalized reduced matrix element of G^{ω} in eq.(F.1) as

$$\langle ab;\Gamma_f \| G^{\omega} \| cd;\Gamma_i \rangle_{AN} = \frac{1}{\sqrt{(1+\delta_{ab})(1+\delta_{cd})}} \sum_{\substack{L_i S_i \\ L_f S_f}} \begin{pmatrix} \ell_a & 1/2 & j_a \\ \ell_b & 1/2 & j_b \\ L_f & S_f & J_f \end{pmatrix} \begin{pmatrix} \ell_c & 1/2 & j_c \\ \ell_d & 1/2 & j_d \\ L_i & S_i & J_i \end{pmatrix} \begin{Bmatrix} L_f & S_f & J_f \\ L_i & S_i & J_i \\ \chi & s & \omega \end{Bmatrix} [L_i L_f \chi]^{1/2}$$

$$\cdot \langle S_f \| \mathbb{S}^s \| S_i \rangle \langle T_f \| \mathbb{T}^t \| T_i \rangle \sum_{\substack{n_1\ell_1 n_2\ell_2 \\ N_1L_1N_2L_2}} \langle n_1\ell_1 N_1L_1 ; L_f | n_a\ell_a n_b\ell_b ; L_f \rangle \langle n_2\ell_2 N_2L_2 ; L_i | n_c\ell_c n_d\ell_d ; L_i \rangle$$

$$\cdot \begin{Bmatrix} \ell_1 & L_1 & L_f \\ \ell_2 & L_2 & L_i \\ v & u & \chi \end{Bmatrix} \langle n_1\ell_1 \| \mathbb{V}^v \| n_2\ell_2 \rangle \langle N_1L_1 \| \mathbb{U}^u \| N_2L_2 \rangle [1-(-)^{\ell_2+S_i+T_i}] . \tag{F.12}$$

F.2 Effective interaction

The matrix element of effective interaction may be derived from eq.(F.12) by substituting $U^u(\mathbb{R})=1$ and rank $\omega=0$. We make, however, another formula which is more useful in wider applications.

The effective interaction has a form of

$$V = [\mathbb{V}^v(\mathbb{r}) \otimes \mathbb{S}^s]^{(0)} \mathbb{T}^{(0)} . \tag{F.13}$$

The matrix element of V is, therefore, given by

$$\langle ab;\Gamma | V | cd;\Gamma \rangle = \sum_{L_i S_i L_f S_f} \begin{pmatrix} \ell_a & 1/2 & j_a \\ \ell_b & 1/2 & j_b \\ L_f & S_f & J \end{pmatrix} \begin{pmatrix} \ell_c & 1/2 & j_c \\ \ell_d & 1/2 & j_d \\ L_i & S_i & J \end{pmatrix}$$
$$\cdot \langle [\ell_a \otimes \ell_b]^{L_f} \otimes S_f ; JT | V | [\ell_c \otimes \ell_d]^{L_i} \otimes S_i ; JT \rangle . \tag{F.14}$$

Using the Moshinsky brackets and recoupling the angular momenta, we have

$$\langle ab;\Gamma|V|cd;\Gamma\rangle = \sum_{L_iS_iL_fS_f} \begin{pmatrix} l_a & 1/2 & j_a \\ l_b & 1/2 & j_b \\ L_f & S_f & J \end{pmatrix} \begin{pmatrix} l_c & 1/2 & j_c \\ l_d & 1/2 & j_d \\ L_i & S_i & J \end{pmatrix}$$

$$\cdot \sum_{\substack{nln'l' \\ \nu\lambda\nu'\lambda'}} \langle nl\nu\lambda;L_f|n_al_an_bl_b;L_f\rangle\langle n'l'\nu'\lambda';L_i|n_cl_cn_dl_d;L_i\rangle$$

$$\cdot \sum_{LL'} W(\lambda lJS_f;L_fL)W(\lambda'l'JS_i;L_iL')[LL'L_iL_f]^{1/2}(-)^{L_i+L_f}$$

$$\cdot \quad \langle[nl\otimes S_f]^L\otimes\nu\lambda;JT|V|[n'l'\otimes S_i]^{L'}\otimes\nu'\lambda';JT\rangle \quad . \tag{F.15}$$

The exchange term can be incorporated as the same in F.1, and we finally obtain

$$\langle ab;\Gamma|V|cd;\Gamma\rangle_{AN} = \frac{1}{\sqrt{(1+\delta_{ab})(1+\delta_{cd})}} \sum_{L_iS_iL_fS_f} \begin{pmatrix} l_a & 1/2 & j_a \\ l_b & 1/2 & j_b \\ L_f & S_f & J \end{pmatrix} \begin{pmatrix} l_c & 1/2 & j_c \\ l_d & 1/2 & j_d \\ L_i & S_i & J \end{pmatrix}$$

$$\cdot \sum_{nn'l'\nu\lambda} \langle nl\nu\lambda;L_f|n_al_an_bl_b;L_f\rangle\langle n'l'\nu\lambda;L_i|n_cl_cn_dl_d;L_i\rangle$$

$$\cdot \sum_{L} W(\lambda lJS_f;L_fL)W(\lambda l'JS_i;L_iL)[L^2L_iL_f]^{1/2}(-)^{L_i+L_f}$$

$$\langle nl\otimes S_f;LT|[V^\nu(r)\otimes S^s]^{(0)}\mathbb{T}^{(0)}|n'l'\otimes S_i;LT\rangle\,[1-(-)^{l'+S_i+T_i}] \quad . \tag{F.16}$$

The matrix element of relative coordinate and spin isospin operator are calculated for some examples. The matrix elements for isospin is given by

$$\langle[1/2\otimes 1/2]^T|\begin{pmatrix}1\\ \tau_1\cdot\tau_2\end{pmatrix}|[1/2\otimes 1/2]^{T'}\rangle = \begin{cases}\delta_{TT'} \\ (-)^{T+1}6\cdot W(1/2\,1/2\,1/2\,1/2;T1)\,\delta_{TT'}\end{cases} \quad . \tag{F.17}$$

The matrix elements for the space and spin are given as follows:

(1) central force

$$\langle nl\otimes S_f;L|V_c|n'l'\otimes S_i;L\rangle = \delta_{ll'}I_c(nl\,n'l') \quad . \tag{F.18}$$

(2) spin-spin force

$$\langle nl\otimes S_f;L|\sigma_1\cdot\sigma_2\,V_\sigma|n'l'\otimes S_i;L\rangle = \delta_{ll'}(\delta_{S1}-3\delta_{S0})I_\sigma(nl\,n'l') \quad . \tag{F.19}$$

(3) spin-orbit force

$$\langle nl \otimes S_f; L | \mathbf{L}\cdot\mathbf{S}\, V_{LS} | n'l' \otimes S_i; L \rangle = \delta_{ll'}\delta_{S1}\frac{1}{2}[L(L+1) - l(l+1) - 2]\, I_{LS}(nl\, n'l') \quad . \tag{F.20}$$

(4) tensor force

$$\langle nl \otimes S_f; L | S_{12} V_T | n'l' \otimes S_i; L \rangle = \delta_{S1}(-)^{L+1}\sqrt{24}\,[ll']^{1/2}(ll'00|20)$$
$$W(ll'11;2L)\, I_T(nl\, n'l') \quad . \tag{F.21}$$

Here the radial integral $I_x(nl\ n'l')$ is defined by

$$I_x(nl\, n'l') = \int r^2 dr\, \phi_{nl}(r)\, v_x(\sqrt{2}r)\, \phi_{n'l'}(r) \quad , \tag{F.22}$$

and it is calculated with the help of the Talmi integral. Another type of effective interaction is given in the form $\langle nl \otimes S; LT | V | n'l' \otimes S; LT \rangle$ as in eq.(F.16), and the calculation proceeds from eq.(F.16).

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Table 1. Combinations of the lepton wave functions

$L^V(\kappa_e, \kappa_\nu)$	L^-
$\mathbb{L}^V_{JL}(\kappa_e, \kappa_\nu)$	$\left\{ \frac{\kappa^- \mathcal{L}^-}{\sqrt{J(2J+1)}} - \sqrt{\frac{J}{2J+1}}\, \mathcal{L}^+ \right\} \delta_{J,L+1} + \frac{\kappa^+ \mathcal{L}^-}{\sqrt{J(J+1)}} \delta_{J,L} + \left\{ \frac{\kappa^- \mathcal{L}^-}{\sqrt{(J+1)(2J+1)}} + \sqrt{\frac{J+1}{2J+1}}\, \mathcal{L}^+ \right\} \delta_{J,L-1}$
$\mathbb{L}^A_{JL}(\kappa_e, \kappa_\nu)$	$\left\{ \frac{\kappa^+ L^+}{\sqrt{J(2J+1)}} - \sqrt{\frac{J}{2J+1}}\, L^- \right\} \delta_{J,L+1} + \frac{\kappa^- L^+}{\sqrt{J(J+1)}} \delta_{J,L} + \left\{ \frac{\kappa^+ L^+}{\sqrt{(J+1)(2J+1)}} + \sqrt{\frac{J+1}{2J+1}}\, L^- \right\} \delta_{J,L-1}$
$L^A(\kappa_e, \kappa_\nu)$	$\mathcal{L}^+$
$L^P(\kappa_e, \kappa_\nu)$	$\mathcal{L}^-$
$L^S(\kappa_e, \kappa_\nu)$	L^+

Lepton conbinations $L^\pm$ and $\mathcal{L}^\pm$ are defined as, $L^\pm = G_{\kappa_e} g_{\kappa_\nu} \pm F_{\kappa_e} f_{\kappa_\nu}$, and $\mathcal{L}^\pm = G_{\kappa_e} f_{\kappa_\nu} \pm F_{\kappa_e} g_{\kappa_\nu}$, with $\kappa^\pm = \kappa_e \pm \kappa_\nu$.

Table 2. Nuclear form factors in the impulse approximation

$F_J^V(r)$	$f_V\,[\,Y_J\,]$
$F_{JL}^V(r)$	$\frac{f_V}{M}[\,\mathcal{Y}_{JL}\cdot\nabla\,] + \frac{f_V}{2M}\left\{\delta_{J,L-1}\sqrt{\frac{L}{2L-1}}\,D_+^{L-1}[\,Y_J\,] - \delta_{J,L+1}\sqrt{\frac{L+1}{2L+3}}\,D_-^{L+1}[\,Y_J\,]\right\}$ $+\left(\frac{f_V}{2M} - f_W\right)\left\{\sqrt{6(L+1)}\,W(JL11,1L+1)\,D_-^{L+1}[\,\mathcal{Y}_{JL+1}\cdot\sigma\,] - \sqrt{6L}\,W(JL11,1L-1)\,D_+^{L-1}[\,\mathcal{Y}_{JL-1}\cdot\sigma\,]\right\}$
$F_J^A(r)$	$(f_A \mp E_0 f_T)\,[\,\mathcal{Y}_{JL}\cdot\sigma\,]$
$F_J^A(r)$	$\frac{f_A}{M}[\,Y_J\sigma\cdot\nabla\,] + \left(\frac{f_A}{2M} \pm f_T\right)\left\{\sqrt{\frac{J+1}{2J+1}}\,D_-^{J+1}[\,\mathcal{Y}_{JJ+1}\cdot\sigma\,] - \sqrt{\frac{J}{2J+1}}\,D_+^{J-1}[\,\mathcal{Y}_{JJ-1}\cdot\sigma\,]\right\}$
$F_J^P(r)$	$-\frac{f_P}{2M}\left\{\sqrt{\frac{J+1}{2J+1}}\,D_-^{J+1}[\,\mathcal{Y}_{JJ+1}\cdot\sigma\,] - \sqrt{\frac{J}{2J+1}}\,[\,\mathcal{Y}_{JJ-1}\cdot\sigma\,]\right\}$
$F_J^S(r)$	$-f_S\,[\,Y_J\,]$

Nuclear form factors $[\mathcal{O}]$ are defined as, $[\mathcal{O}] = \langle J_f \| \sum_{j=1}^{A} \delta(r-r_j)/r^2\,\tau_j^{\pm}\,\mathcal{O}_j \| J_i \rangle$.
∇ operates on the nuclear initial states. The derivative operators D_+^L and D_-^L are defined as, $D_+^L = \frac{d}{dr} - \frac{L}{r}$, and $D_-^L = \frac{d}{dr} + \frac{L+1}{r}$, which operate on $[\mathcal{O}]$.

Table 3. Approximated lepton combinations

κ_e, κ_ν	$L^{\pm}$	$\mathcal{L}^{\pm}$
-1, -1	$G_{-1}(R)$	$r\left\{\pm \frac{F_{-1}(R)}{R} - \frac{q}{3} G_{-1}(R)\right\}$
1, 1	$\pm F_1(R)$	$r\left\{\frac{G_1(R)}{R} \pm \frac{q}{3} F_1(R)\right\}$
-1, 2	0	$r\left\{\frac{q}{3} G_{-1}(R)\right\}$
1, -2	0	$r\left\{\pm \frac{q}{3} F_1(R)\right\}$
-2, 1	0	$r\left\{\frac{G_{-2}(R)}{R}\right\}$
2, -1	0	$r\left\{\pm \frac{F_2(R)}{R}\right\}$

Table 4. Combinations of electron wave functions

L_0	$\{G_{-1}^2 + F_1^2\}/F_0$	1
N_0	$\{G_{-1}F_{-1} - F_1 G_1\}/F_0 R$	$-\frac{p^2}{3E}$
Λ_1	$G_{-1} F_1 \sin(\delta_{-1}-\delta_1)/F_0$	$\frac{p}{2E}$
$\mathbb{N}_{11}$	$\{F_{-1}F_1 - G_{-1}G_1\}\sin(\delta_{-1}-\delta_1)/F_0$	$-\frac{p}{3} - \frac{p}{E}V$
$\mathbb{L}_{12}$	$\{F_1F_2 \sin(\delta_1-\delta_2) + G_{-1}G_{-2}\sin(\delta_{-1}-\delta_{-2})\}/F_0R$	$\frac{p}{3}$
L_{12}	$\{G_{-1}F_2 \cos(\delta_{-1}-\delta_2) - F_1 G_{-2}\cos(\delta_1-\delta_2)\}/F_0R$	$-\frac{p^2}{3E}$

The expressions in the right column are obtained for the point nuclear charge with $(\alpha Z)^2 \ll 1$. $V=\alpha Z/2R$ is the half of the Coulomb energy at the nuclear surface. ($G_{-1}=G_{-1}(R)$ etc.)

Table 5. Coefficients $C^{\omega}_{\alpha\beta}$ in eq. (5.5) for the A=12 and A=13 systems

α	β	A=12 $\omega=(1,1)$ 8-16POT	8-162BME	6-162BME	H-M	A=13 (8-16POT) $\omega=(1,1)$	$\omega=(2,1)$
3	3	-0.0764	-0.0825	-0.0924	-0.0991	0.0065	-0.0027
3	1	0.6902	0.6906	0.6679	0.6727	1.0076	-1.0044
1	3	0.3394	0.3270	0.3225	0.2969	0.3084	0.0830
1	1	-0.0581	-0.0733	-0.0857	-0.0801	-0.0107	0.0

8-16POT, 8-162BME and 6-162BME are the three models of Cohen-Kurath[27)], and H-M is the model of Hauge-Maripuu[51)]. In the case of the A=13 system, coefficients are given for both $(J_\omega, T_\omega)=(1,1)$ and $(2,1)$. ($3=0p_{3/2}$, $1=0p_{1/2}$)

Table 6. Parameters of the model wave functions which we have adopted. Those for the other two models are also shown for coparison purpose.

Model	a_0	a_σ	a_τ	$a_{\sigma\tau}$
Rosenfeld	-0.0025	-0.0025	-0.1025	-0.2325
Arima et al.	-0.3000	-0.1250	-0.2750	-0.2000
Millener-Kurath	0.0945	-0.0590	-0.1590	-0.1625
Gillet	0.0	-0.2	-0.3	-0.1
Ferrell-Visscher	0.1005	-0.1250	-0.2165	-0.1250

Model	P_{11}	P_{13}	P_{31}	P_{33}
Rosenfeld	-1.78	0.6	1.0	-0.34
Arima et al.	-0.9	0.4	1.0	-0.9
Millener-Kurath	-0.714	0.6	1.0	-0.286
Gillet	0.6	0.6	1.0	-0.6
Ferrell-Visscher	0.0	0.634	1.0	-0.366

Model	V_0(MeV)	f(r)	r_0(fm)
Rosenfeld	-60	$e^{-(r/r_0)^2}$	1.60
Arima et al.	-60	$e^{-(r/r_0)^2}$	1.60
Millener-Kurath	-45	$e^{-r/r_0}/(r/r_0)$	1.40

Table 7. Amplitudes of single particle matrix elements of the core polarization in the j-j model.

p	h	central X_{ph}	central Y_{ph}	tensor X_{ph}	tensor Y_{ph}
$1p_{1/2}$	$0p_{3/2}$	0.0282	0.0509	-0.0147	-0.0087
$1p_{3/2}$	$0p_{3/2}$	0.0489	0.0489	0.0257	0.0106
$0f_{5/2}$	$0p_{3/2}$	0.0166	-0.0166	0.0590	-0.0065
$1s_{1/2}$	$0s_{1/2}$	0.0494	0.0494	0.0272	0.0272
$0d_{3/2}$	$0s_{1/2}$	0.0106	-0.0106	0.0443	0.0427

p	h	central $C_{ph}(T_{\rho A})$	central $C_{ph}(T_{M1})$	tensor $C_{ph}(T_{\rho A})$	tensor $C_{ph}(T_{M1})$
$1p_{1/2}$	$0p_{3/2}$	0.0791	-0.0227	-0.0234	-0.0060
$1p_{3/2}$	$0p_{3/2}$	0.0	0.0978	0.0151	0.0363
$0f_{5/2}$	$0p_{3/2}$	0.0	0.0332	0.0525	0.0655
$1s_{1/2}$	$0s_{1/2}$	0.0	0.0988	0.0	0.0544
$0d_{3/2}$	$0s_{1/2}$	0.0	0.0212	0.0870	0.0016

The central and tensor forces are those of the model (2) in §5.4, and $E_m = -2\hbar\omega = 30.74$ MeV is adopted.

Table 8. Comprison of the experiments and the theory (impulse approximation within 0p shell) for $\alpha_{\mp}$ and $a_{\mp}$.

(%/MeV)	Osaka[20]	Louvain, Zurich[19]	Theory
α_-	0.006±0.018	-0.007±0.020	0.016
α_+	-0.273±0.041	-0.273±0.039	-0.257
$\alpha_- - \alpha_+$	0.279±0.045	0.267±0.044	0.273
$\alpha_- + \alpha_+$	-0.267±0.045	-0.280±0.044	-0.241

(%/MeV)	Wu et al.[5]	Heidelberg[16]	Theory
a_-	0.41±0.10	0.64±0.11	0.405
a_+	-0.45±0.09	-0.31±0.09	-0.417
$a_- - a_+$	0.86±0.24	0.95±0.09	0.822

Theoretical values are derived in the following energy region: β^- : E_1=5.1MeV, E_2=11.2MeV. β^+ : E_1=7.2MeV, E_2=13.3MeV. The coefficients $a_{\mp}$ of Heidelberg group are derived from the original data with the corrections $\delta_{\mp} = \mp 0.07$%/MeV, which comes from the use of the different Fermi functions.

Table 9. Parameters y and $\tilde{y}$ and asymmetry coefficients $\alpha_{\mp}$ for different nuclear models of the A=12 system and for the A=13 system (8-16POT).

Model	8-16POT	8-162BME	6-162BME	H-M	A=13 (8-16POT)
$y_{I.A.}$	3.716	3.550	3.551	3.173	2.868
$y_{I.A.+E.C.}$	4.975	4.763	4.769	4.259	3.882
$\delta y_{E.C.}$	33.8%	34.2%	34.3%	34.2%	35.4%
$y_{I.A.+C.P.}$	2.579	2.500	2.505	2.192	2.089
$\delta y_{C.P.}$	-30.6%	-29.6%	-29.5%	-30.9%	-27.2%
$y_{TOT.}$	3.838	3.713	3.723	3.278	3.108
α_- (%/MeV)	0.000	0.005	0.007	0.021	
α_+ (%/MeV)	-0.273	-0.268	-0.270	-0.254	
$\tilde{y}$	3.845	3.704	3.704	3.282	

Table 10. Parameters y and $\tilde{y}$ and asymmetry coefficients $\alpha_{\mp}$ for different effective interactions. ($y_{I.A.} = 3.716$)

Model	Rosenfeld	Arima et al.	H-J tensor	Millener-Kurath	Sussex
$y_{I.A.+C.P.}$	2.500	2.579	2.507	3.643	3.249
$\delta y_{C.P.}$	-32.7%	-30.6%	-32.5%	-2.0%	-12.6%
$y_{TOT.}$	3.759	3.838	3.766	4.902	4.508
α_- (%/MeV)	0.001	0.000	0.007	-0.031	-0.024
α_+ (%/MeV)	-0.271	-0.273	-0.265	-0.310	-0.296
$\tilde{y}$	3.803	3.845	3.643	4.803	4.507

Table 11. Matrix elements of tensor force $\langle ab;JT|V_T|cd;JT\rangle_{AN}$ in the 0p shell.

a b c d	J T	Cohen-Kurath*	Hauge-Maripuu*	Hamada-Johnston	Millener-Kurath
3 3 3 3	1 0	0.15	0.46	0.63	0.14
	3 0	-0.37	0.31	0.32	0.14
	0 1	0.63	0.57	0.55	0.27
	2 1	0.13	0.11	0.11	0.05
3 3 3 1	1 0	-0.11	0.19	0.22	0.08
	2 1	-0.09	-0.08	-0.08	-0.04
3 3 1 1	1 0	-0.12	-0.92	-1.22	-0.30
	0 1	-0.88	-0.80	-0.78	-0.38
3 1 3 1	1 0	-0.73	-0.56	-0.89	-0.12
	2 0	1.29	-1.10	-1.13	-0.50
	1 1	-0.94	-0.85	-0.82	-0.41
	2 1	0.06	0.06	0.05	0.03
3 1 1 1	1 0	0.91	0.26	0.54	0.00
1 1 1 1	1 0	-0.71	1.13	1.39	0.48
	0 1	1.25	1.20	1.10	0.54

* The numerical values of matrix elements for the Cohen-Kurath (8-16POT) and Hauge-Maripuu are taken from ref. 56).

($3=0p_{3/2}$, $1=0p_{1/2}$)

Table 12. Effect of the core polarization on higher order matrix elements for different effective interactions.

(fm^2)	I.A. (8–16POT)	I.A.+C.P. Rosenfeld	Arima et al.	H–J tensor
$\langle \sigma r^2 \rangle$	6.454	6.076	7.683	4.402
δ		-6%	19%	-32%
$\langle [Y_2 \otimes \sigma]^{(1)} r^2 \rangle$	-2.454	-0.913	-1.642	-0.703
δ		-63%	-33%	-71%

The deviation δ is defined as, $\delta = \{ |\langle O \rangle_{I.A.+C.P.}| - |\langle O \rangle_{I.A.}| \} / |\langle O \rangle_{I.A.}|$.

Table 13. Contribution of higher order corrections to y for different effective interactions.

	I.A.	I.A.+C.P.		
	(8-16POT)	Rosenfeld	Arima et al.	H-J tensor
y	3.716	3.759	3.838	3.766
$\tilde{y}$	3.578	3.803	3.845	3.634
δ	-3.7%	1.2%	0.2%	-3.5%

The deviation δ is defined as, $\delta = (\tilde{y} - y)/y$.

Table 14. Recoil corrections to $a_{\mp}$ and $\alpha_{\mp}$.

(%/MeV)	with recoil	without recoil
a_-	0.385	0.355
a_+	-0.414	-0.443
$a_- - a_+$	0.799	0.798
α_-	0.000	-0.006
α_+	-0.273	-0.279
$\alpha_- - \alpha_+$	0.273	0.273
$\alpha_- + \alpha_+$	-0.273	-0.285

The calculation includes the exchange current and core polarization. The Cohen-Kurath 8-16POT is adopted.

Figure Captions

Fig. 1 Decay scheme of the 1^+, 1 states in the A=12 system.

Fig. 2 Decay scheme of the $3/2^-$, 3/2 states in the A=13 system.

Fig. 3 The one-pion exchange contribution to the two nucleon current.

Fig. 4 Time ordered diagrams for nucleon Born contributions. (a) The part contained in the nuclear wave functions. (b) The pair current.

Fig. 5 The ρ-π exchange diagram.

Fig. 6 $\alpha_- - \alpha_+$ as a function of f_T/f_W. The experimental data indicated by the cross hatched area are those in ref. 20).

Fig. 7 $\alpha_- - \alpha_+$ as a function of f_W/f_W^{cvc}. The experimental data are the same as in fig. 6.

Fig. 8 $a_- - a_+$ as a function of f_W/f_W^{cvc}. The experimental data indicated by the cross hatched area (a) and (b) correspond to those of refs. 5) and 15), and ref. 16), respectively.

Fig. 9 $\alpha_- + \alpha_+$ as a function of x. The experimental data are the same as in fig. 6.

Fig.10 $\tilde{y}$(•) and y(–, o) for different nuclear models. The upper, middle and lower bars (–) correspond to the calculations with, impulse + exchange, impulse and impulse + core polarization, respectively. The circles (•) include all the effects, while the open circle (o) does not include the higher order corrections in β decay. The core polarization is calculated with the effective interaction of Arima et al.. The experimental data indicated by the cross hatched area are those in ref. 20).

Fig.11 $\tilde{y}$(•) and y(–) for different effective interactions. The symbols have the same meanings as in fig. 10), except that we do not show the calculation with impulse + exchange current. They are calculated for the A=12 system with the 8-16POT model of the Cohen-Kurath wave function. The experimental data are the same as in fig. 10).

Fig.12 B_2/B_0 as a function of the electron enegy E. The solid curves include the contribution of the exchange current and core polarization, while the dashed curves are calculated with the impulse approximation.

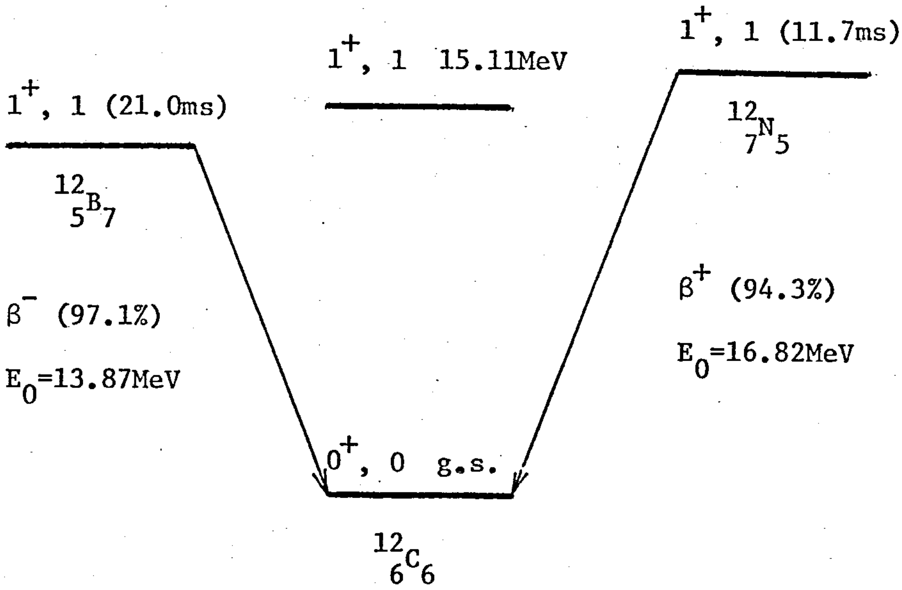
1+, 1 (21.0ms)
1+, 1 15.11MeV
1+, 1 (11.7ms)
12 5 B 7
12 7 N 5
β− (97.1%)
E0=13.87MeV
β+ (94.3%)
E0=16.82MeV
0+, 0 g.s.
12 6 C 6

Fig. 1

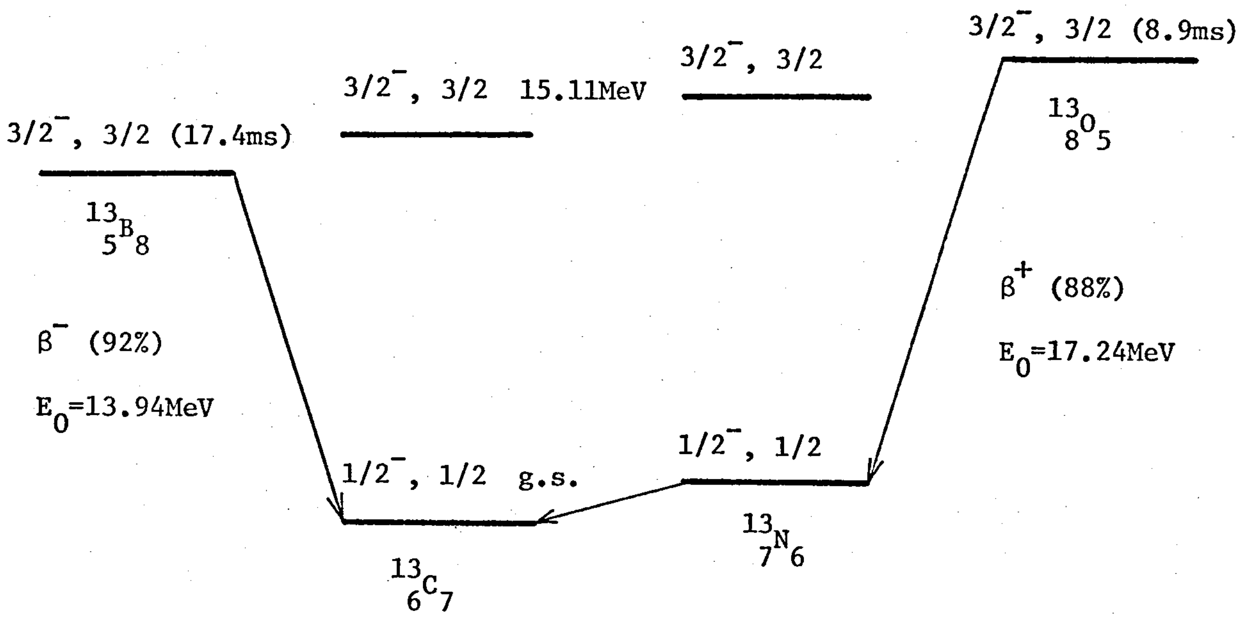
3/2−, 3/2 (17.4ms)
3/2−, 3/2 15.11MeV
3/2−, 3/2
3/2−, 3/2 (8.9ms)
13 5 B 8
13 8 O 5
β− (92%)
E0=13.94MeV
β+ (88%)
E0=17.24MeV
1/2−, 1/2 g.s.
1/2−, 1/2
13 6 C 7
13 7 N 6

Fig. 2

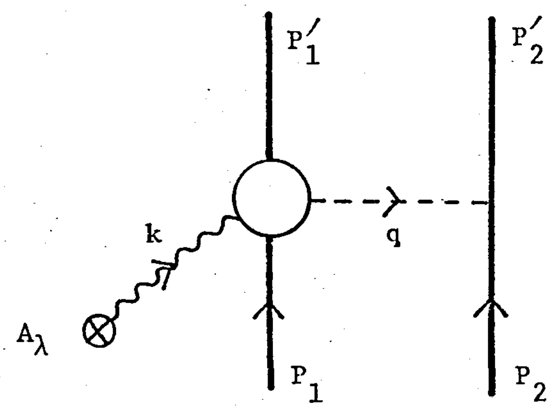

Fig. 3

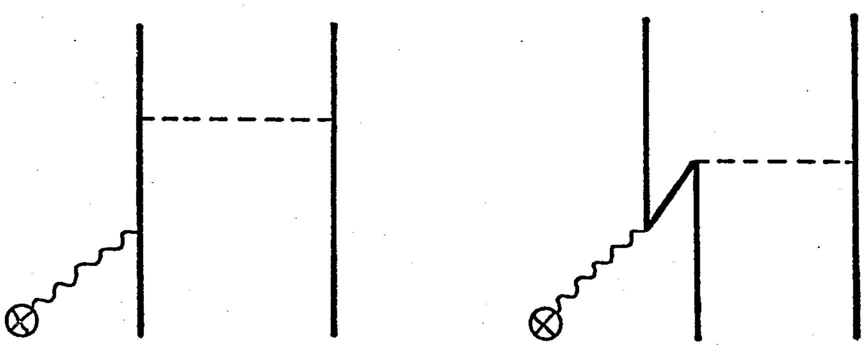

Fig. 4 (a) (b)

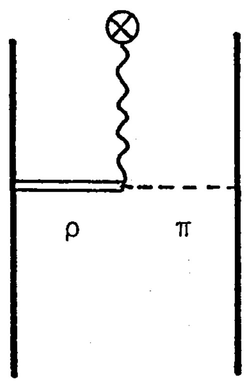

Fig. 5

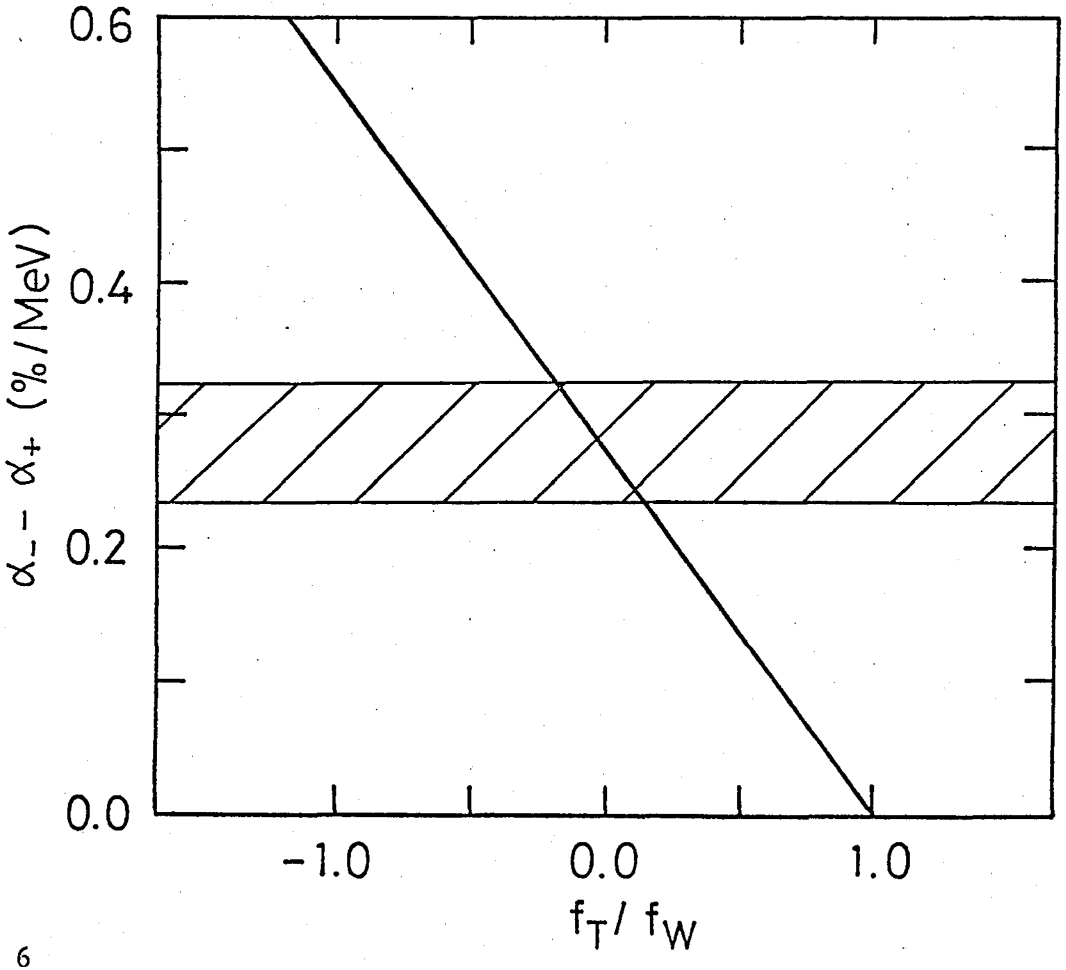

Fig. 6

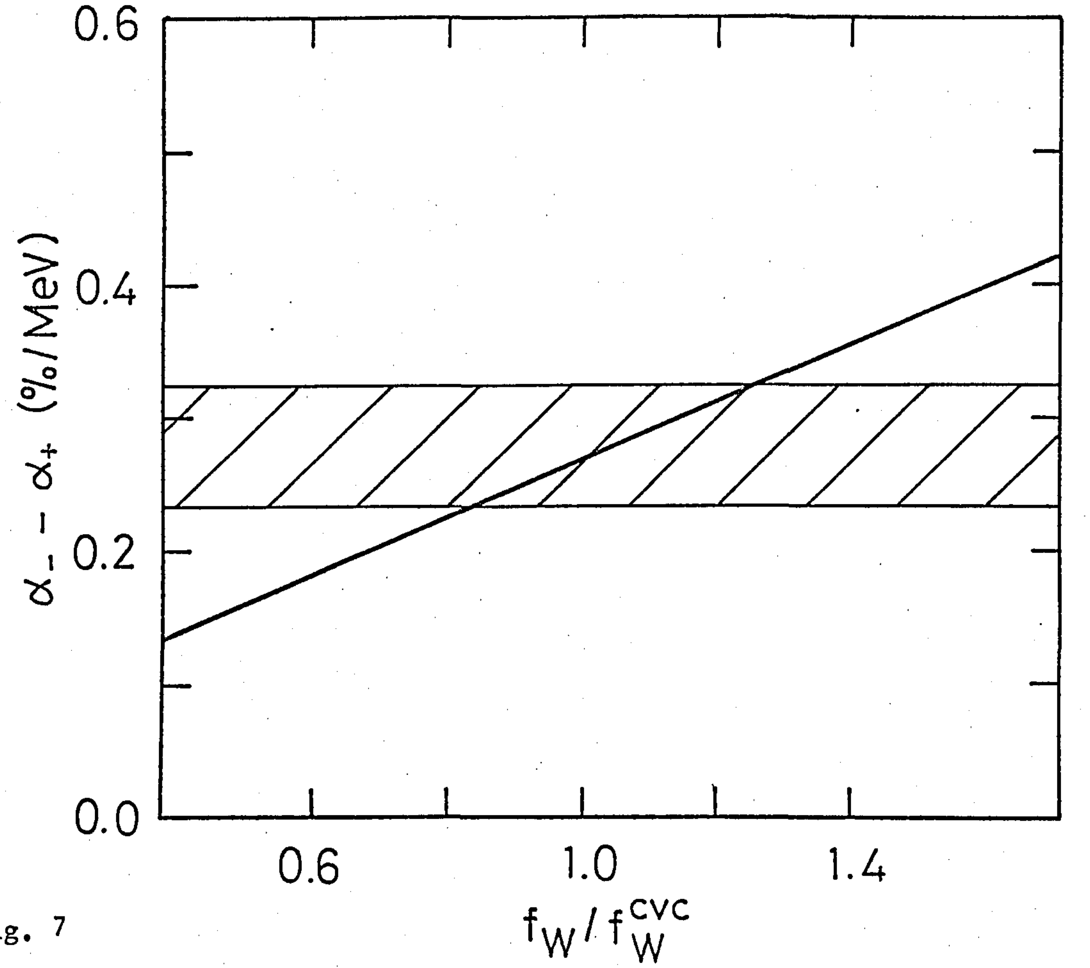

Fig. 7

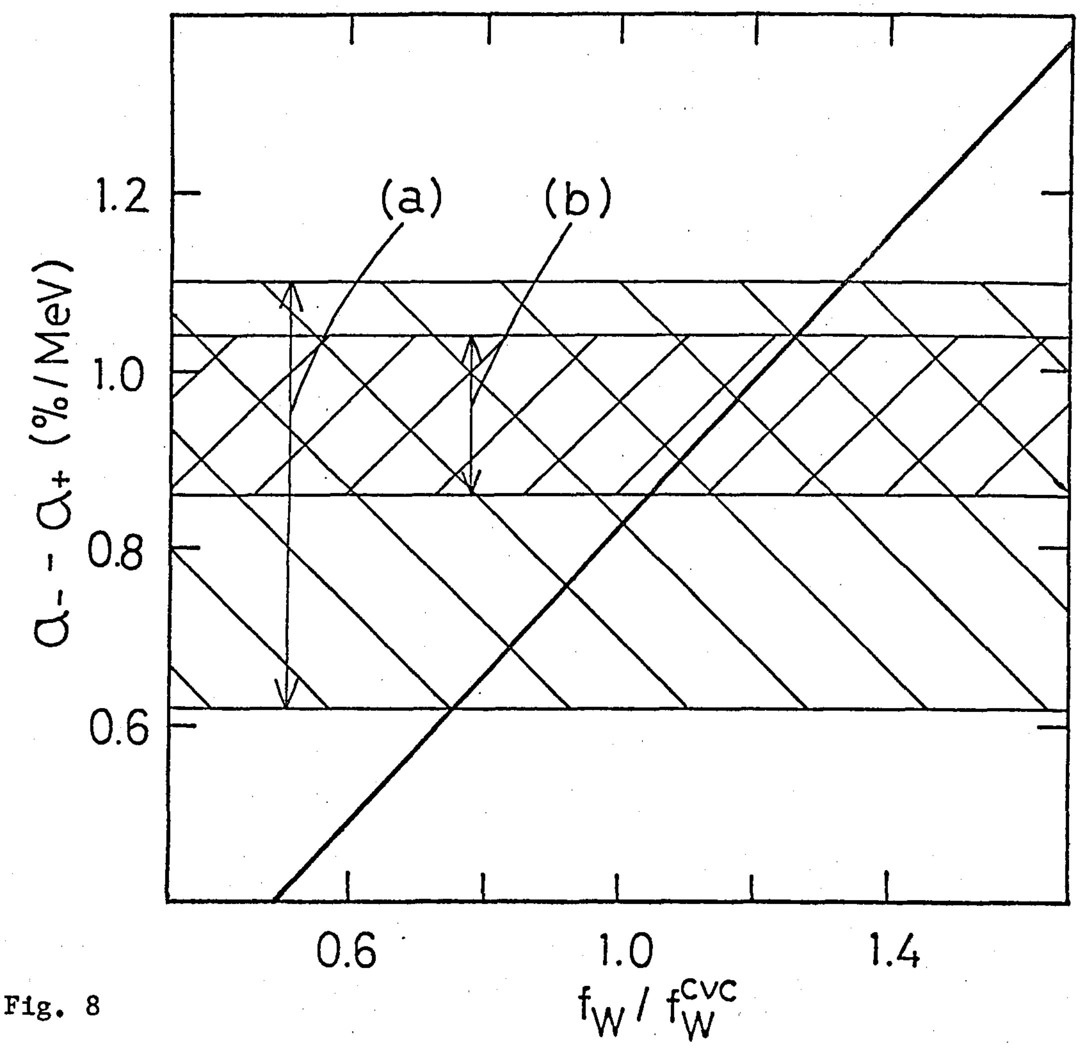

Fig. 8

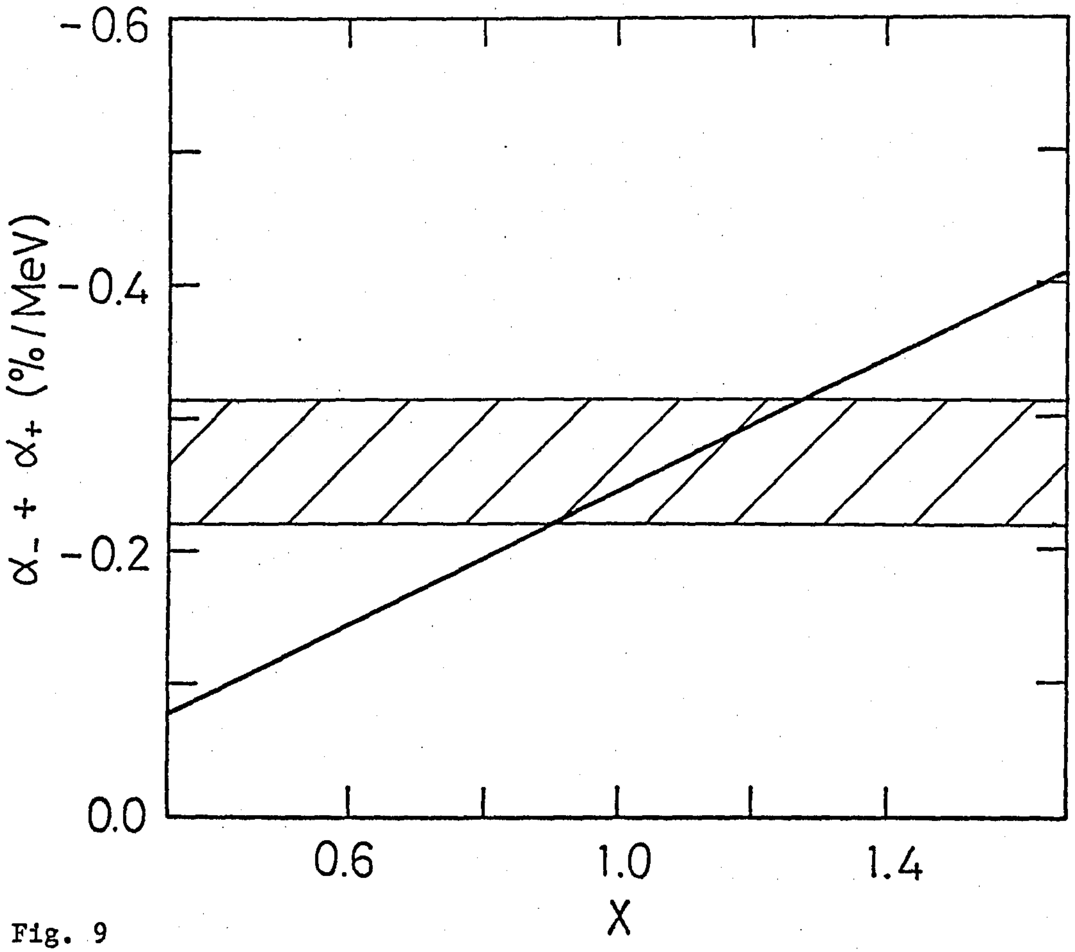

Fig. 9

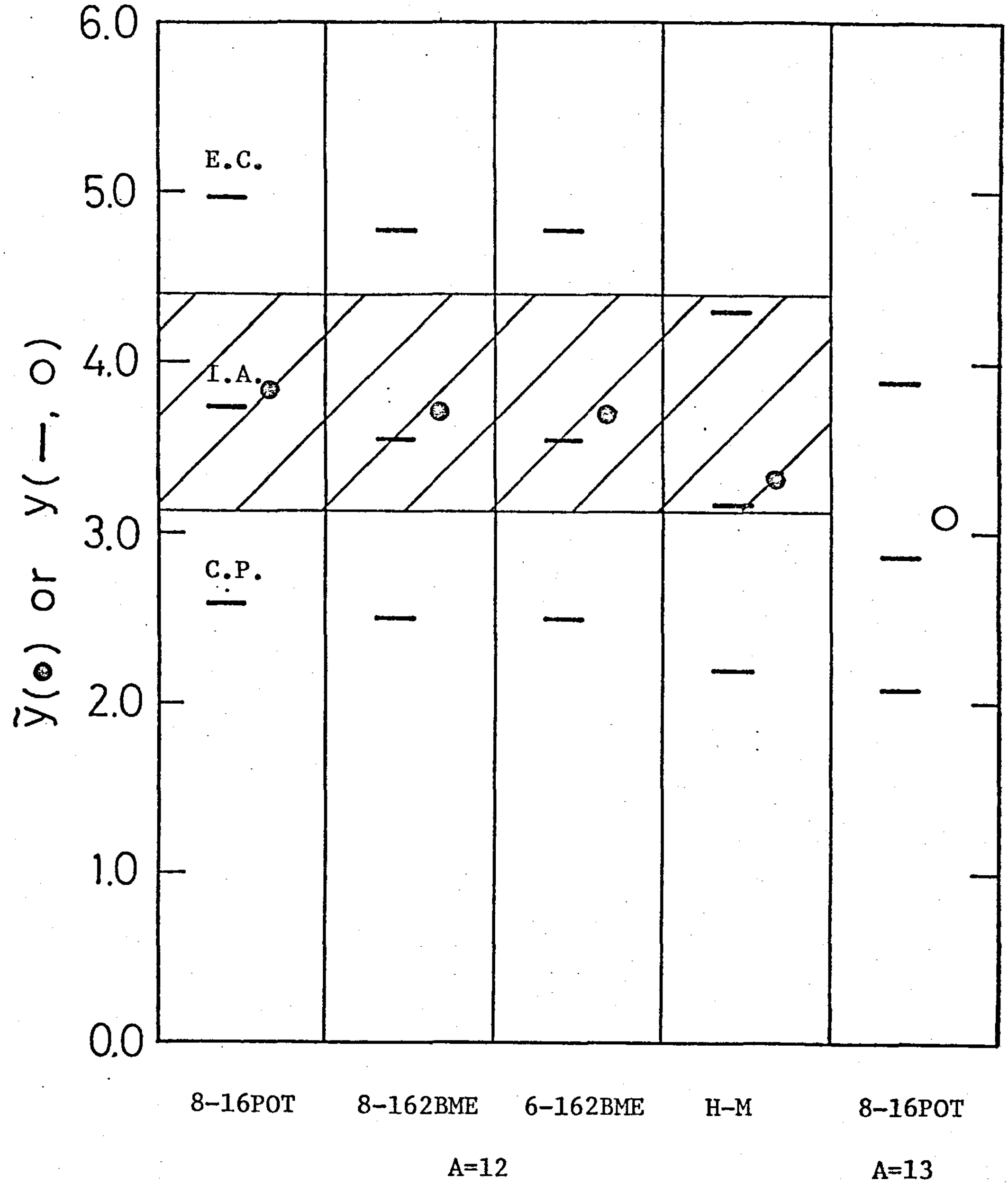
6.0
5.0
4.0
3.0
2.0
1.0
0.0
$\tilde{y}(\bullet)$ or $y(—, \circ)$
E.C.
I.A.
C.P.
8-16POT
8-162BME
6-162BME
H-M
8-16POT
A=12
A=13

Fig. 10

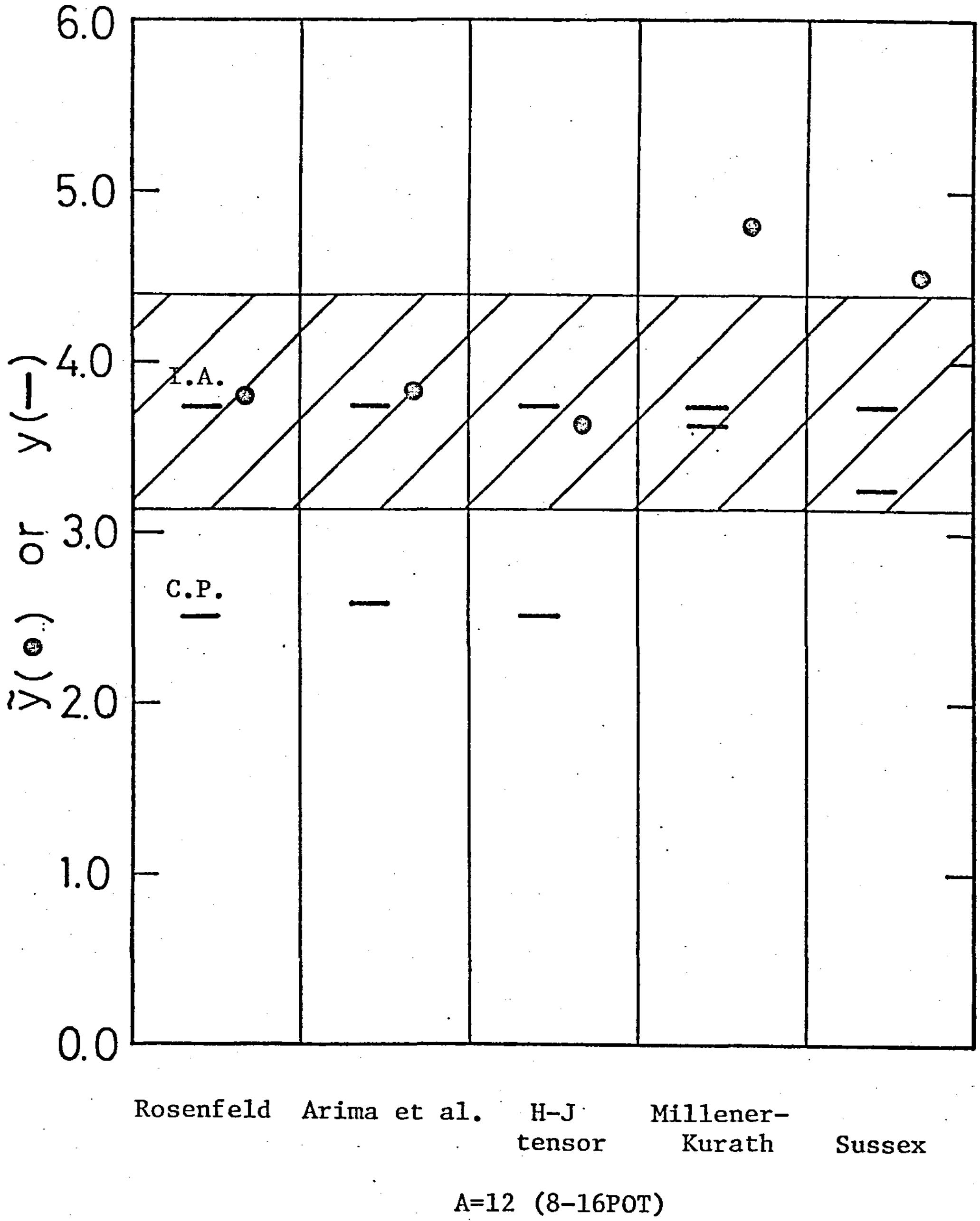
6.0
5.0
4.0
3.0
2.0
1.0
0.0
$\tilde{y}$(•) or y(—)
I.A.
C.P.
Rosenfeld
Arima et al.
H-J tensor
Millener-Kurath
Sussex
A=12 (8-16POT)

Fig. 11

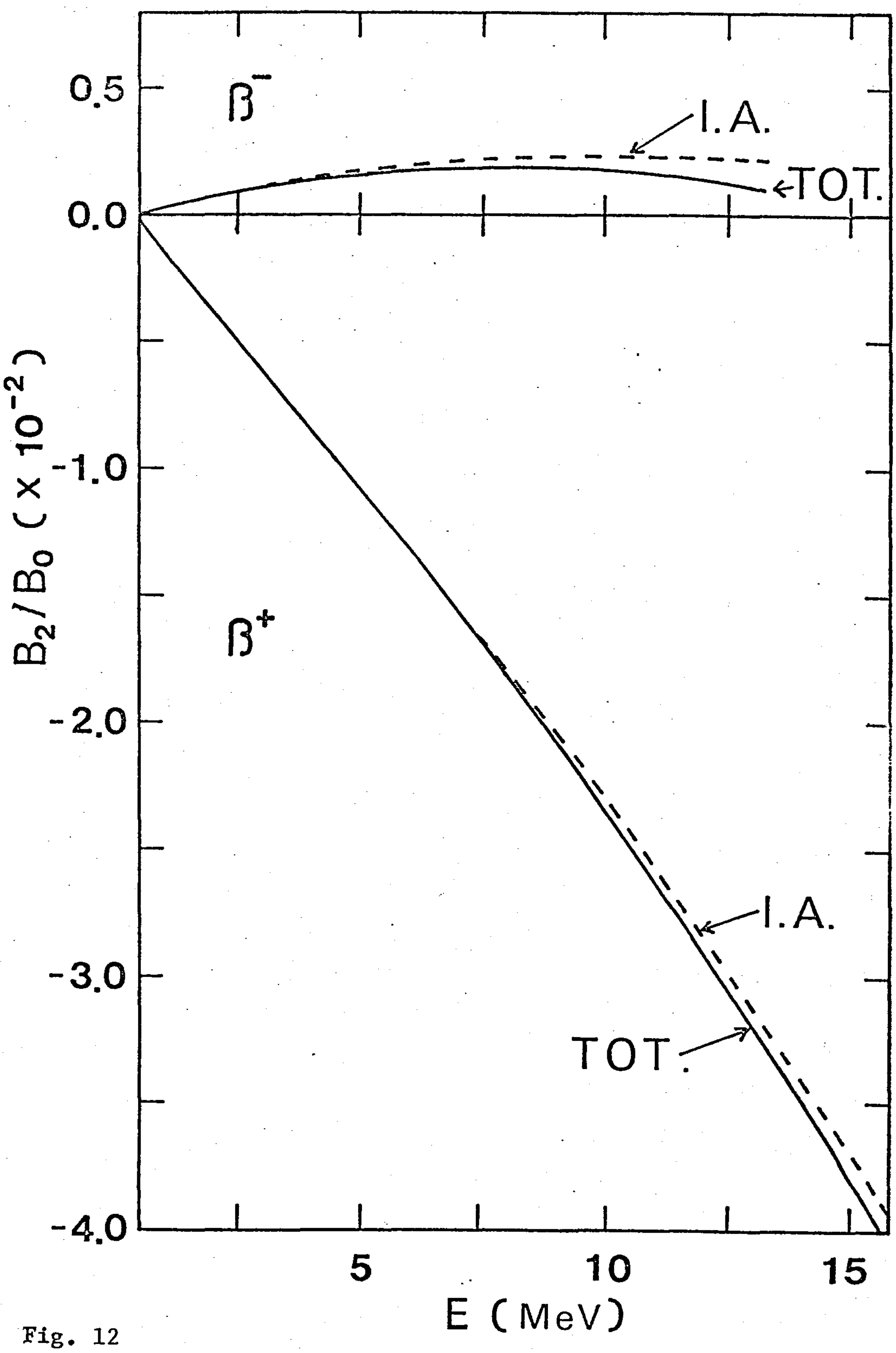

Fig. 12