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SOLID STATE HIGH RESOLUTION NMR OF  $^{19}\text{F}$   
IN SOME FLUORINE-CONTAINING  
COMPOUNDS

BY

YOSHICHIKA YOSHIOKA

Thesis

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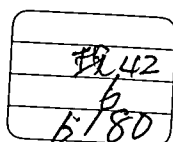
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Dedicated to  
Father and Mother,  
and  
Brothers and Sisters.

narrow is beautiful and loquacious

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## Abstract

The chemical shift data of variety of nuclei can provide important information about molecular structure, electronic structure, chemical exchange, and so on. However, traditional high-resolution NMR experiments on liquids or solutions give only the time-averaged chemical shifts due to rapid molecular motions. In order to study the structures and properties of molecules and ions in more detail, it is highly desirable to obtain the complete data of the chemical shielding tensors which are essentially anisotropic, reflecting the anisotropy of chemical bonds. Such data became available by performing variety of tricky high resolution NMR experiments on solid state which have been developed since late 1960's.

The main aim of the present work is to obtain complete chemical shielding tensor of  $^{19}\text{F}$  in some fluorine-containing materials and to relate these data to the molecular and/or ionic structure as well as to the chemical bonding in these compounds.

To perform such a kind of experiment, a spectrometer based on the multiple pulse technique was designed and constructed. The spectrometer works at 40 MHz and the optimum line width of  $^{19}\text{F}$  in  $\text{CaF}_2$  single crystal is 8.5 ppm ( 340 Hz ) which corresponds to sufficient resolution to observe the anisotropy of the  $^{19}\text{F}$  chemical shift, because the  $^{19}\text{F}$  chemical shift anisotropy extends over 150 ppm in normal substances.

The  $^{19}\text{F}$  chemical shift anisotropies were measured in an aromatic conjugated system, 1,3,5-trichloro-2,4,6-trifluorobenzene, and a series of inorganic complex compounds,  $\text{K}_2\text{MF}_6$ ; M = Si, Ge, Sn.

The values of the maximum and the minimum principal components of the  $^{19}\text{F}$  shielding tensor in 1,3,5-trichloro-2,4,6-trifluorobenzene were determined to be + 85 ppm and - 43 ppm respectively. The most shielded direction is perpendicular to the molecular plane and the least shielded direction is perpendicular to the C-F bond but lying in the molecular plane. The value of the most shielded and the least shielded components vary gradually from sym- $\text{C}_6\text{H}_3\text{F}_3$ , sym- $\text{C}_6\text{Cl}_3\text{F}_3$ , to  $\text{C}_6\text{F}_6$ , whereas the C-F bond direction components remain nearly constant in the series of the above compounds. The semi-quantitative analysis of the shielding tensors in these compounds leads to a conclusion that the substitution of the ring protons of  $\text{C}_6\text{H}_6$  by fluorine or chlorine brings about some measurable change in the  $\sigma$ -electron density but give rise to little change in the  $\pi$ -electron distribution. The high-resolution NMR of single crystal of sym- $\text{C}_6\text{Cl}_3\text{F}_3$  together with a pulsed  $^{35}\text{Cl}$  NQR measurement revealed that the molecules undergo the three-fold reorientation about its molecular  $\text{C}_3$  axis at a rate of 2 kHz at 300 K.

In a series of inorganic compounds,  $\text{K}_2\text{MF}_6$ ; M = Si, Ge, Sn, the shielding tensors are all axially symmetric and the most shielded direction is parallel to the M-F ( M = Si, Ge, Sn ) bond. A gradual change of the shielding tensor elements ( bond direction and perpendicular to the bond ) were observed on going from  $\text{K}_2\text{SiF}_6$ ,  $\text{K}_2\text{GeF}_6$ , to  $\text{K}_2\text{SnF}_6$  except for the component perpendicular to the bond in  $\text{K}_2\text{SiF}_6$ . From the analysis of shielding tensor components, it is indicated that the  $\pi$ -backbonding effect on the electronic structure in the  $\text{MF}_6^{2-}$  ions is the

strongest in the Si-F system, being consistent with the fact that the Si has the extraordinarily low electronegativity compared with C, Ge, and Sn.

## 1 Introduction

The nuclear magnetic resonance frequency of a nuclear spin  $S$  is essentially determined by an applied static magnetic field  $H_0$ . The Zeeman Hamiltonian for an isolated spin with the gyromagnetic ratio  $\gamma$  is given by

$$H_z = -\gamma \hbar H_0 S \quad (1-1)$$

and the nuclear resonance frequency ( the angular Larmor frequency ) is given by

$$\omega = -\gamma H_0. \quad (1-2)$$

But there are many interactions of  $S$  with other nuclear spins and/or with electrons surrounding it which may contribute to shifting the resonance frequency.

In a nuclear magnetic resonance experiment the resonance frequency of a nuclear spin  $S$  with the gyromagnetic ratio  $\gamma$  is determined not only by the external magnetic field but also by the interaction of the nuclear spin with the surrounding electrons. In diamagnetic materials with neither net electron spin  $n$  nor orbital angular momentum and when no other interactions such as quadrupolar interactions, spin rotation interaction etc. do not exist, the chemical shift is a second-order effect, its magnitude being proportional to the external magnetic field  $H_0$ , and can be represented by the Hamiltonian:

$$\underline{H}_{cs} = \gamma \underline{H}_0 \cdot \underline{\sigma} \cdot \underline{S} \quad (1-3)$$

where  $\underline{\sigma}$  is a dimensionless, second-rank tensor, representing the anisotropic shift of the resonance frequency in bulk matter with respect to that of a bare nucleus. In Eq.(1-3)  $H_z \gg H_{cs}$  is assumed. The nature of the  $\underline{\sigma}$  tensor is determined by the electron

distribution around the resonant nucleus and therefore its measurement provides information about the structure and properties of chemical bond. However, in gaseous, liquid, and plastic phases<sup>1)</sup> the rapid, three dimensional molecular motion averages out the anisotropic part of this chemical shift tensor, leaving only the isotropic part:

$$\sigma_i = (1/3)\text{Tr}(\underline{g}) = (1/3)(\sigma_{11} + \sigma_{22} + \sigma_{33}), \quad (1-4)$$

where  $\sigma_{11}$ ,  $\sigma_{22}$ , and  $\sigma_{33}$  are the principal components of the chemical shift tensor. Although useful information about the nature of the chemical bond or bonds can be derived from the measurement of chemical shift in these materials very important knowledge about anisotropy around the nucleus of interest may be completely lost.

In a usual solid in which no rapid molecular rotation is excited the chemical shift tensor may not be averaged out. Therefore the resonance frequency of a nuclear spin in a solid depends on the orientation of the external field with respect to the axes of chemical shift tensor in the sample. When one measures the angular dependence of the chemical shift, the anisotropic components of  $\underline{g}$  can be determined. Since the complete determination of the chemical shift tensor provides information about the local symmetry of the electron cloud around the nucleus and allows one to draw a more detailed picture of the chemical bonding of a certain atom than the isotropic chemical shift measured in solution does. It also gives information about the nature of solid such as molecular motion, intermolecular or inter ionic interactions, conductivity, ferroelectricity, etc.

From the symmetry of the chemical shift tensor, we can obtain

the data about the symmetry of molecule and crystal. If there is a molecular motion, the chemical shift tensor is partially averaged, and so the analysis of the chemical shift data gives the knowledge of the type and the correlation time of molecular motion.

Although the chemical shift in solids gives us the important informations as described above, the chemical shift may usually be obscured by strong dipolar interaction in solid state and so only limited number of investigations in this field have been made so far.

High resolution NMR spectrum in solid state was firstly obtained by Magic Angle Spinning technique (MAS). Andrew et al.<sup>2)</sup> and Lowe<sup>3)</sup> independently reported MAS method and the subject has considerably developed both theoretically<sup>4)</sup> and experimentally.<sup>5)</sup> In this technique a solid specimen is spun at an angular frequency  $\omega_p$  in a high static magnetic field about an axis inclined by an angle  $\beta$  with respect to the field direction. When  $\beta$  is equal to  $54^{\circ}44'$  and the  $\omega_p$  is larger than the dipolar line width, the motional narrowing of the dipolar width is realised and the trace of chemical shift tensor is obtained. This special angle is called the 'Magic Angle'.

Waugh et al.<sup>6)</sup> obtained the high resolution spectrum of  $^{19}\text{F}$  in  $\text{CaF}_2$  single crystal by the pulse technique in 1968. By this technique dipolar Hamiltonian is averaged out in spin space and brought to be nearly zero, and then chemical shift can be observed. This technique is applicable to abundant spins ( $^1\text{H}$ ,  $^{19}\text{F}$ , etc.) and gives full chemical shift tensor values. To improve the resolution, several pulse sequences have been developed.<sup>7)</sup>

The third technique was proposed by Pines et al.<sup>8)</sup> By this technique chemical shift tensors for rare spins ( $^{13}\text{C}$ ,  $^{15}\text{N}$ , etc.) can be obtained. Since the line width for the rare spins is usually very small the resonance of rare spins gives us high resolution spectrum without any special technique, but it is necessary to accumulate FID to obtain the signal with good S/N ratio. The problem was how to accumulate quickly where the spin-lattice relaxation times of rare spins are very long. They solved this problem by the use of abundant spins as thermal reservoir.

The multiple pulse technique was adopted in this work to obtain chemical shift tensors of abundant spins such as  $^1\text{H}$  and  $^{19}\text{F}$ , and a 40 MHz spectrometer for WAHUHA-4 and MREV-8 pulse sequences was constructed.

Measurements were made for  $^{19}\text{F}$  in an aromatic conjugated system (1,3,5-trichloro-2,4,6-trifluorobenzene) and a series of inorganic compounds ( $\text{K}_2\text{MF}_6$ ; M = Si, Ge, Sn).

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## 2 The Methods to Obtain the High Resolution NMR Spectrum in Solid

### 2-1 Introduction

There are three principal methods and their combinations to obtain the high resolution NMR spectrum in solid. A brief review will be given about the line width in solids and the methods to eliminate the dipolar line width in order to uncover the small chemical shift interactions. The secular part of homonuclear Hamiltonian, which gives rise to line broadening in solid state, can be described as follows,

$$\begin{aligned} H_d &= (1/2)\gamma_I^2 \hbar^2 \sum_{i < j} r_{ij}^{-3} (1 - 3\cos^2\theta_{ij}) (\underline{I}_i \cdot \underline{I}_j - 3I_{iz}I_{jz}) \\ &= \alpha \sum_{i < j} A \cdot B \cdot C \end{aligned}$$

where

$$\begin{aligned} \alpha &= (1/2)\hbar^2 \gamma_I^2 \\ A &= r_{ij}^{-3} \\ B &= 1 - 3\cos^2\theta_{ij} \\ C &= \underline{I}_i \cdot \underline{I}_j - 3I_{iz}I_{jz}. \end{aligned}$$

$H_d$  can be made substantially zero if one can bring the term A, B, or C to zero.

In the case of rare spin resonance the separation between the resonating spins is very large and therefore the term A becomes very small and so chemical shift can easily be observed. If the NMR signal is very weak and there are other abundant spins in the system the signal enhancement by the use of cross polarization between the rare and the abundant spins can be realized. This special technique is called Proton Enhanced NMR.

The term B can be brought to nearly zero by the magic angle

spinning method and this technique can apply to any system.

The term C is expected to become zero only when an averaging of this term is performed by the use of special pulse sequence. The method is called Multiple Pulse NMR.

In the next section the Multiple Pulse NMR method will be reviewed, which is the method that has been adopted in this Dissertation, and in the latter sections other methods will be described.

## 2-2 Line Narrowing by Multiple Pulse NMR

In this method, Hamiltonian is perturbed in spin space by the radio-frequency (RF) pulses. The repetition times of RF pulses are short compared with the spin-spin relaxation time, and so spins feel average Hamiltonian in their relaxation process. Therefore high resolution spectrum can be obtained, if dipolar Hamiltonian becomes zero by averaging Hamiltonian through the perturbation of RF pulses.

In this section, averaging Hamiltonian theory will be described first, and later the reason why high resolution spectrum can be obtained by special pulse sequences such as WAHUHA-4 and MREV-8 is described. The pulse sequences for WAHUHA-4 and MREV-8 are  $(\tau-P_x-\tau-P_y-2\tau-P_y-\tau-P_x-\tau)_n$  and  $(\tau-P_x-\tau-P_y-2\tau-P_y-\tau-P_x-2\tau-P_x-\tau-P_y-2\tau-P_y-\tau-P_x)_n$  respectively as shown in Fig. 2-1, where  $P_i$  represents the  $90^\circ$  pulse along  $i$  axis.

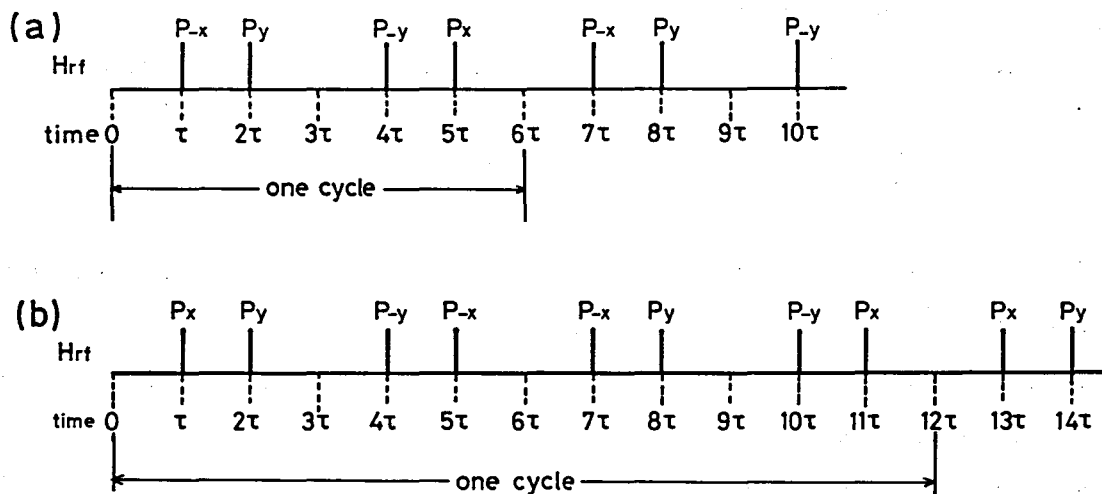


Fig. 2-1. Pulse sequences for (a) WAHUHA-4 and (b) MREV-8.

The usual detection of an NMR signal rests upon the fact that an ensemble of nuclear spins, initially polarized along the z axis as determined by the static magnetic field  $H_0$ , will produce a non-vanishing component of the magnetization in the x-y plane when disturbed from equilibrium by an RF field. This component rotates about the z axis at the Larmor frequency,  $|\omega_0| = |\gamma H_0|$  ( 40 MHz for  $^{19}\text{F}$  at a field of 1 T ), and induces a voltage in an inductor oriented with its axis perpendicular to  $H_0$ , which is part of a resonant circuit tuned at  $\omega_0$ . The observable, then, is a quantity proportional to  $\langle I_y(t) \rangle$ , the expectation value of the transverse component ( here phase sensitive detection is assumed to set to detect the y-component of the magnetization in the rotating frame ) of the spin angular momentum. In terms of the density matrix representation of the time-dependent Schrödinger equation

$$\langle I_y(t) \rangle = \text{tr } \rho(t) I_y \quad (2-1)$$

where the density matrix operator,  $\rho(t) = \Psi(t)\Psi^*(t)$ , is given as the solution of the time dependent Schrödinger equation (with  $\hbar = 1$ ),

$$i \frac{\partial \rho(t)}{\partial t} = [H, \rho]. \quad (2-2)$$

A specification of  $\rho(t)$ , i.e. the r.h.s. of eq. 2-2, will therefore specify the time evolution of transverse magnetization observed in a pulsed NMR experiment. It has been recognized that this time domain signal can be converted to the frequency domain absorption signal via Fourier transformation

$$I(\omega) \propto \text{Re} \int_0^{\infty} e^{-i\omega t} I_y(t) dt. \quad (2-3)$$

The Hamiltonian contains in general time-dependent and time-independent terms. The Hamiltonian may be separated into externally controllable terms ( the static and the radio-frequency fields ), and uncontrollable terms inherent in the system under study ( dipolar fields, quadrupolar interactions, shielding, etc. ). It is very convenient to convert the coordinate to the rotating frame in order to discuss the interactions within the system as well as between the system and the external RF field. The rotating frame is chosen so that the Zeeman interaction vanishes effectively, therefore the Hamiltonian accessible to the experimenter as  $H_{rf}(t)$ , the time-dependent ( e.g. pulsed ) radio-frequency interactions can easily be treated in this coordinate system. For an ideal delta function pulse along the x axis in the rotating frame, for example,

$$H_{rf} = -\gamma H_{1x} I_x = -\omega_{1x} I_x, \quad (2-4)$$

and the condition that the magnetic moment be rotated by  $90^\circ$  is

$$-\int_0^t H_{rf}(t') dt' = \int_0^t \omega_{1x} dt' = \int_0^{\pi/2} d\theta = \pi/2. \quad (2-5)$$

In terms of rotation operators corresponding to a "θ pulse" along an axis ν, for example, the component of angular momentum along an axis η will be transformed to

$$e^{iI\nu\theta} I_\eta e^{-iI\nu\theta} = I_\eta \cos(\theta \cdot \epsilon_{\nu\eta\lambda}) - I_\lambda \sin(\theta \cdot \epsilon_{\nu\eta\lambda}), \quad (2-6)$$

where ν, η, and λ are some permutation of x,y,z.  $\epsilon_{\nu\eta\lambda}$  is the Levi-Céivita symbol which is unity for ν, η, and λ in cyclic order ( e.g. z, x, y ), -1 for antisymmetric order ( e.g. z, y, x ),

and zero if  $v = \eta$ . So, for example,

$$e^{(\pi/2)iI_y} I_z e^{-(\pi/2)iI_y} = -I_x \quad (2-7)$$

is a new spin angular momentum operator after having been converted from the z-component of the original spin angular momentum by applying the  $90^\circ$  pulse along the y axis of the rotating frame.

The Hamiltonian not accessible to the experimenter (except by manipulation with  $H_{rf}$ ) is represented by  $H_{int}$ . Among number of components in  $H_{int}$ , we concentrate our attention to two internal Hamiltonians. One is the secular part of the homonuclear dipolar Hamiltonian within a spin pair i and j. It is given by

$$H_d = \alpha_d(3\cos^2\theta_{ij} - 1)(\underline{I}_i \cdot \underline{I}_j - 3I_{iz}I_{jz}) \quad (2-8)$$

where

$$\alpha_d = (1/2)\hbar\gamma^2/r_{ij}^3 \cdot s^{-1} \quad (2-9)$$

and  $\theta_{ij}$  specifies the angle between the inter nuclear vector  $\underline{r}_{ij}$  and external magnetic field  $\underline{H}_0$ . The second is the anisotropic chemical shift interaction

$$H_c = \alpha_c(3\cos^2\theta - 1)I_z \quad (2-10)$$

where

$$\alpha_c = \omega_o(1/3)[\sigma_{33} + (1/2)(\sigma_{11} + \sigma_{22})] \cdot s^{-1}. \quad (2-11)$$

Here  $\theta$  is the angle between the principal z axis of the shielding tensor  $\underline{\sigma}$  and  $\underline{H}_0$ .  $\sigma_{33}$  is the z component of the shielding tensor.

In terms of  $H_{rf}$  and  $H_{int}$ , eq. 2-2 becomes

$$i \frac{\partial \rho}{\partial t} = [(H_{rf} + H_{int}), \rho]. \quad (2-12)$$

A transformation to the frame of the RF is made formally as specified by the relation

$$\rho(t) = U_{rf}^{-1} \widetilde{\rho}(t) U_{rf}. \quad (2-13)$$

$U_{rf}$  is determined by  $H_{rf}$  via the differential equation (with  $h = 1$ )

$$i \frac{\partial U_{rf}}{\partial t} = H_{rf} U_{rf}. \quad (2-14)$$

In the frame of the RF the equation of motion of the density operator is

$$i \frac{\partial \widetilde{\rho}}{\partial t} = [\widetilde{H}_{int}, \widetilde{\rho}]. \quad (2-15)$$

In this frame,  $H_{int}$  is controlled by  $H_{rf}$  via the relation

$$\widetilde{H}_{int}(t) = U_{rf}^{-1} H_{int} U_{rf}. \quad (2-16)$$

Note the effect of transforming to the frame of the RF, i.e (2-13), is to remove  $H_{rf}$  from the description of  $\widetilde{\rho}$  (2-15). In addition,  $U_{rf}$  is a well known solution <sup>1)</sup> to eq. 2-14; the operator that transforms from time 0 to time t is

$$U_{rf}(t,0) = T \exp\{-i \int_0^t H_{rf}(t') dt'\} \quad (2-17)$$

where T is the Dyson time ordering operator. It suffices here to say that if

$$\int_0^t H_{rf}(t') dt' = 0, \quad (2-18)$$

then  $U_{rf}$  will be unity. Two obvious examples of causes where eq. 2-17 become unity are:

(i)  $H_{rf}$  is a  $180^\circ$  pulse at a given phase, followed by a  $180^\circ$  pulse with the same phase. This sequence give rise to a rotation by  $360^\circ$ , being equivalent to no rotation at all.

(ii) Two ideal  $90^\circ$  pulse sequence with opposite phases to each other. Again the net transformation by the two successive pulses corresponds to leaving the spin system unchanged.

It is important to note that (i) and (ii) hold only when any interactions in the spin system does not "strongly" affect  $\rho$  in the time over which  $H_{rf}(t')$  is integrated. This imposes that

$$|H_{rf}(t')| \gg |H_{int}|. \quad (2-19)$$

In other words, if the dipolar field is of the order of  $5 \times 10^{-4}$  T, then the magnitude of the RF field should be greater than  $5 \times 10^{-3}$  T. Furthermore the time over which  $H_{rf}$  is applied must be short compared with  $T_2$ , i.e. short compared with the dipolar line width  $\alpha_d$ . For protons with  $\gamma = 4.25 \text{ kHz}/10^{-4} \text{ T}$ , a value of a dipolar field of  $5 \times 10^{-4}$  T corresponds to a dipolar line width of roughly 20 kHz, so  $T_2^* \sim 16 \mu\text{s}$



If a suitable coordinate system can be chosen it is possible to eliminate any special interaction from the description of  $\rho$  as follows. Suppose a transformation

$$\tilde{\rho} = U_{\text{int}}^{-1} \tilde{\rho} U_{\text{int}}, \quad (2-20)$$

where  $U_{\text{int}}$  varies according to the Liouville equation

$$i \frac{\partial U_{\text{int}}}{\partial t} = \widetilde{H}_{\text{int}} U_{\text{int}}. \quad (2-21)$$

The condition that

$$U_{\text{int}}^{-1} \widetilde{H}_{\text{int}} U_{\text{int}} = 0$$

lead to the result

$$i \frac{\partial \rho}{\partial t} = [0, \tilde{\rho}] = 0. \quad (2-22)$$

This implies that  $\tilde{\rho}$  appears static in this frame. At time  $t = t_c$  the solution of eq. 2-21 is given either by the Dyson expression, e.g. eq. 2-17, or by the Magnus expansion <sup>1)</sup>,

$$U_{\text{int}} = \exp\{-it_c [\bar{H}_{\text{int}}^{(0)} + \bar{H}_{\text{int}}^{(1)} + \dots]\}, \quad (2-23)$$

where the zeroth-order term is given

$$\bar{H}_{\text{int}}^{(0)} = (1/t_c) \int_0^{t_c} \widetilde{H}_{\text{int}}(t) dt \quad (2-24)$$

the first order term by

$$\overline{H}_{\text{int}}^{(1)} = -(i/2t_c) \int_0^{t_c} dt \int_0^t dt' [\widetilde{H}_{\text{int}}(t), \widetilde{H}_{\text{int}}(t')]. \quad (2-25)$$

The higher term may be obtained by using higher commutators. The density operator in the frame of detection at time  $t_c$  is given by

$$\rho(t_c) = U_{\text{int}}^{-1} U_{\text{rf}} \widetilde{\rho} U_{\text{rf}} U_{\text{int}}. \quad (2-26)$$

If  $\rho$  is formally identified to  $\rho(0)$ , then eq. 2-26 formally expresses a clue to calculate the time evolution of  $\rho$ . If both  $U_{\text{int}}$  and  $U_{\text{rf}}$  become unity at time  $t_c$ , then

$$\rho(t_c) = \rho(0), \quad (2-27)$$

and a measurement of an observable, as given by eq. 2-1, at a specified time  $t_c$ , will yield the same result as at time  $t = 0$ ; i.e. it means that in the above-specified coordinate system one can measure any physical quantity under the condition that the time-dependent interaction in the system is effectively zero. In particular, if  $H_{\text{int}}$  is the homonuclear dipolar interaction (2-8), then measurement of the transverse magnetization at time  $t_c$  gives a signal which is unaffected by the homonuclear dipolar interactions and therefore not damped. If  $H_{\text{rf}}$  is both cyclic,

$$T \exp\left[-i \int_0^t H_{\text{rf}}(t) dt\right] = 1 \quad (2-28)$$

and periodic,

$$H_{\text{rf}}(t + t_c) = H_{\text{rf}}(t) \quad (2-29)$$

then, via eq. 2-16,  $\widetilde{H}_{\text{int}}$  will be periodic, such that, for example, for the zeroth-order term at time  $2t_c$ ,

$$\begin{aligned} U_{\text{int}}(2t_c, 0) &= U_{\text{int}}(2t_c, t_c) U_{\text{int}}(t_c, 0) \\ &= \exp\{-i\overline{H}_{\text{int}}^{(0)} \cdot t_c\} \exp\{-i\overline{H}_{\text{int}}^{(0)} \cdot t_c\} \\ &= \exp\{-2it_c \overline{H}_{\text{int}}^{(0)}\} \end{aligned} \quad (2-30)$$

and at a time  $nt_c$  where  $n$  is an integer,

$$U_{\text{int}}(nt_c, 0) = U_{\text{int}}^n(t_c, 0). \quad (2-31)$$

The system therefore looks as if behaved under the influence of some 'average' Hamiltonian,  $\overline{H}_{\text{int}}^{(0)}$ , over the cycle time in question.

If  $U_{\text{int}}(t_c, 0)$  is unity, so do  $U_{\text{int}}(nt_c, 0)$ , and if  $U_{\text{rf}}$  is also unity at these times, both  $\rho$  and the expectation value of  $I_y(t)$  will be unaffected by  $H_{\text{int}}$  at the 'window' at  $nt_c$ , here the window means the sampling interval between the RF pulses. A familiar example of this situation is the refocussing of a static field inhomogeneity, where the refocussing means the returning to the initial state of the static field after the perturbation of the RF pulses, with operator  $-\delta\omega I_z$ , by a sequence of two  $180^\circ_x$  pulses, at echo (cycle) time  $4\tau$ , as indicated in Fig. 2-2. Where  $H_{\text{rf}}$  and  $\widetilde{H}_{\text{int}}$  become zero as eqs. 2-32 and 2-33.

$$\int_0^{4\tau} H_{\text{rf}}(t) dt = \pi [I_z - I_z] = 0 \quad (2-32)$$

$$(1/4\tau) \int_0^{4\tau} \widetilde{H}_{\text{int}}(t) dt = (1/4\tau) [\tau I_z - 2\tau I_z + \tau I_z] = 0 \quad (2-33)$$

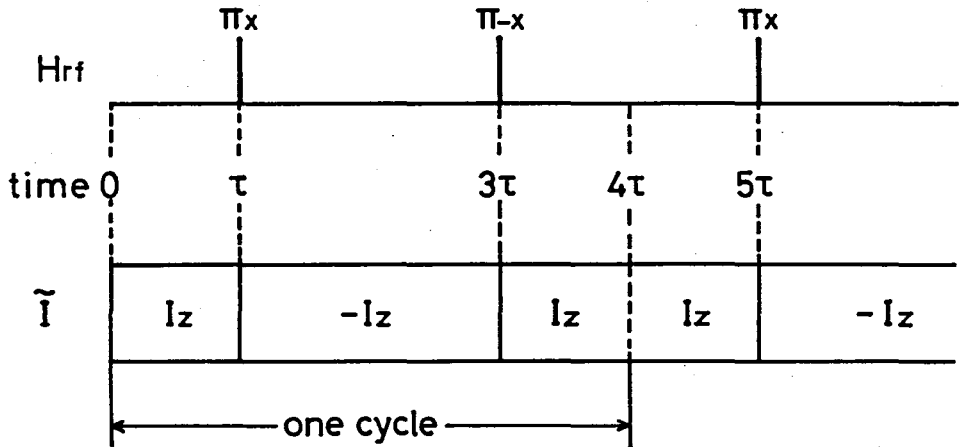


Fig. 2-2. The refocussing scheme of a static field inhomogeneity with operator  $-\delta\omega I_z$ , by a sequence of two  $\pi_x$  pulses, at echo (cycle) time  $4\tau$ .

In the case of WAHUHA-4 multiple pulse experiment (see Fig. 2-1), the behavior of  $I_z$ , and therefore the two spin Hamiltonian  $I_{z1}I_{z2} \equiv Z$  is indicated in Fig. 2-3. Under the assumption of ideal delta function pulses,  $\widetilde{H}_{int}$  is need not considered during the pulses as in eq. 2-34.

$$\int_0^{6\tau} H_{rf}(t) dt = (\pi/2) [-I_x + I_y - I_y + I_z] = 0 \quad (2-34)$$

The integration of the spin portion of  $\widetilde{H}_{dip}$  over the time  $0 \leq t \leq 6\tau$  gives rise to

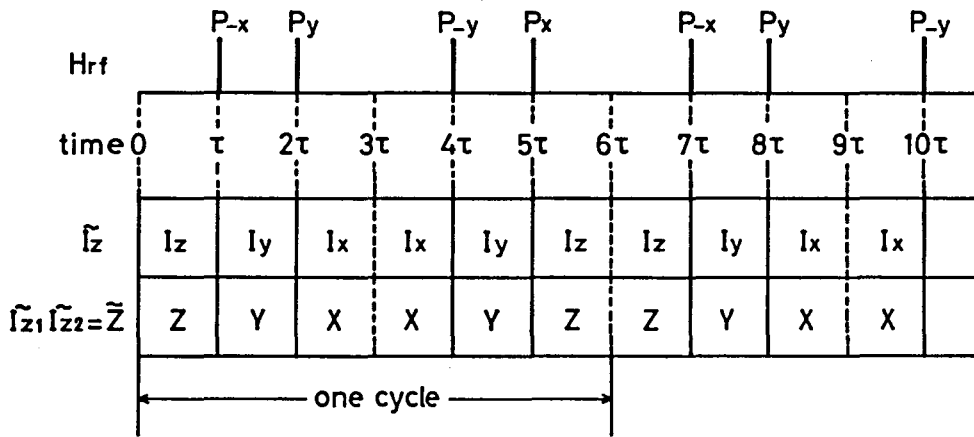


Fig. 2-3. The refocussing scheme and the behavior of  $I_z$  in WAHUHA-4 pulse sequence.

$$\begin{aligned}
 \int_0^{6\tau} \widetilde{H_{dip}}(t) dt &= \alpha_d \int_0^{6\tau} (\widetilde{I_1 \cdot I_2} - 3\widetilde{I_{z1} I_{z2}}) dt \\
 &= \alpha_d \{6\tau \widetilde{I_1 \cdot I_2} - 3\tau(Z + Y + 2X + Y + Z)\} \\
 &= 6\tau \alpha_d (\widetilde{I_1 \cdot I_2} - \widetilde{I_1 \cdot I_2}) \\
 &= 0
 \end{aligned} \tag{2-35}$$

This indicates that the homonuclear dipolar Hamiltonian vanishes under the  $(\tau, 90^\circ_{-x}, \tau, 90^\circ_y, 2\tau, 90^\circ_{-y}, \tau, 90^\circ_x, \tau)$  pulse sequence. Therefore, the free-induction decay (FID) signal sampled every  $6\tau$  time in WAHUHA-4 pulse sequence is not influenced by the homonuclear dipolar interaction, and then gives a high resolution NMR spectrum.

The chemical shift interaction is reduced a little as can be seen in eq. 2-36,

$$\begin{aligned}
 (1/6\tau) \int_0^{6\tau} H_c(t) dt &= (1/6\tau) \alpha_c (3\cos^2\theta - 1) (\tau I_z + \tau I_y + 2\tau I_x \\
 &\quad + \tau I_y + \tau I_z) \\
 &= (1/3) \alpha_c (3\cos^2\theta - 1) (I_x + I_y + I_z) \\
 &= (1/\sqrt{3}) \alpha_c (3\cos^2\theta - 1) \bar{I}_z \quad (2-36)
 \end{aligned}$$

where

$$\bar{I}_z = (1/\sqrt{3}) (I_x + I_y + I_z).$$

The standard and multiple pulse chemical shift Hamiltonians are essentially the same. They differ in two minor points only: the multiple pulse chemical shift Hamiltonian contains an extra factor (scaling factor) of  $1/\sqrt{3}$  and the spin operator is not along the z direction of the rotating frame, but along the  $[111]$  direction of the rotating frame.

The analogous scheme for MREV-8 pulse sequence is given in Fig. 2-4.

The integral of the  $H_{rf}$  and  $\widetilde{H}_{dip}$  over the time  $0 \leq t \leq 12\tau$  (one cycle) becomes zero. The treatment in this case is almost the same as that encountered in eqs. 2-34 and 2-35. The chemical shift interaction is

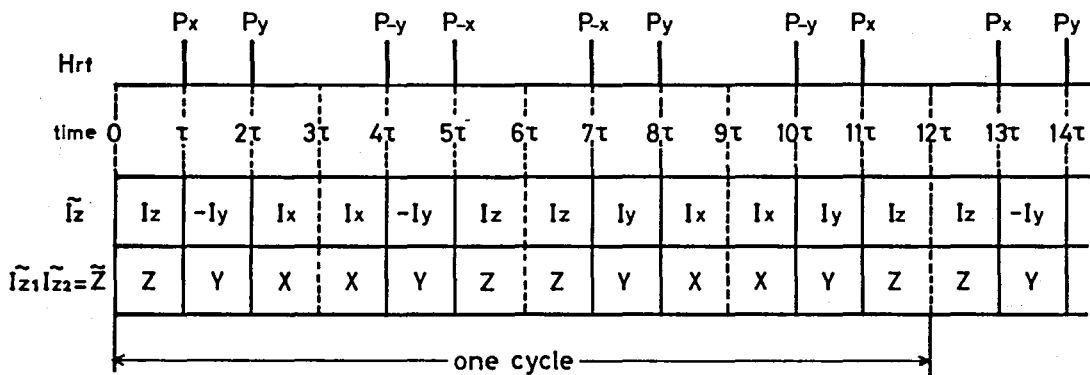


Fig. 2-4. The refocussing scheme and the behavior of  $I_z$  in MREV-8 pulse sequence.

$$\begin{aligned}
 (1/12\tau) \int_0^{12\tau} H_c(t) dt &= (1/3) \alpha_c (3 \cos^2 \theta - 1) (I_x + I_z) \\
 &= (\sqrt{2}/3) \alpha_c (3 \cos^2 \theta - 1) \bar{I}_z,
 \end{aligned}
 \tag{2-37}$$

where

$$\bar{I}_z = (1/\sqrt{2}) (I_x + I_z).$$

The multiple pulse spectrum is thus free from dipolar broadening. Scaling factor is  $\sqrt{2}/3$  and the spin operator is not along the z axis of the rotating frame, but again along a cube face diagonal.

### 2-3 Magic Angle Specimen Rotation

Magic angle spinning technique (MAS) is the simplest approach to the problem of removing dipolar interaction in solids proposed by Andrew et al.<sup>2)</sup> and Lowe.<sup>3)</sup> This subject has been developed considerably both theoretically<sup>4)</sup> and experimentally.<sup>5)</sup>

In this technique, the solid specimen is rotated at high speed about an axis tilted by  $54.74^\circ$  from the direction of the static external magnetic field; This special angle is called the magic angle. Of course, the motion of the molecule in the solid is correlated, unlike that of a liquid, and the dipolar interaction is reduced in this instance by virtue of the rotational transformation dependence of the strength of dipolar interaction on the angle between the rotation axis and the static field vector.

Rotation at the magic angle also removes chemical shift anisotropy. In this sense the absorption line shape is more akin to that of liquid. Unfortunately, the applicability of the MAS technique is limited to rather narrow lines because the rate of the sample rotation should be greater than the dipolar line width in order to attain effective line narrowing. The static dipolar line width can vary from a few kilohertz in usual phosphorus compounds to a few tens of kilohertz for fluorinated compounds to at most one hundred kilohertz for the broadest proton line in number of inorganic and organic materials. Practical problem is that the rate of sample rotation can not exceed 10 kHz because of the difficulty of mechanical device construction. Nevertheless, valuable results have been obtained by the MAS method.

Briefly theoretical examination of the MAS technique is as follows. The truncated dipolar interaction Hamiltonian, for all



nuclear pairs  $i, j$  in the solid, both like and unlike, is

$$H_d = \sum_{i < j} (1/2) \gamma_i \gamma_j \hbar^2 r_{ij}^{-3} (\underline{I}_i \cdot \underline{I}_j - 3I_{iz} I_{jz}) (3 \cos^2 \theta_{ij} - 1), \quad (2-38)$$

where  $\gamma_i, \gamma_j$  are the nuclear gyromagnetic ratios,  $r_{ij}$  is the internuclear displacement, and  $\theta_{ij}$  is the angle between  $r_{ij}$  and the Zeeman field  $H_0$ , which is directed along the  $z$  axis in the laboratory frame (see Fig. 2-4).

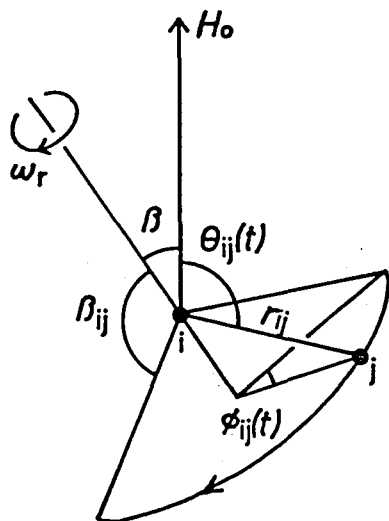


Fig. 2-4. Diagram illustrating the motion of typical internuclear vector  $r_{ij}$  when a solid is rotated with angular velocity  $\omega_r$  about an axis inclined to angle  $\beta$  to  $H_0$ .

Since the dipolar field is usually very small compared with  $H_0$ , the NMR spectrum can be calculated by treating  $H_d$  as a perturbation on the Zeeman term  $H_z$ ,

$$H_z = - \sum_i \gamma_i \hbar \underline{I}_i \cdot \underline{H}_0. \quad (2-39)$$

Since the isotropic average  $\overline{\cos^2 \theta_{ij}} = 1/3$ , it follows from eq. (2-38) that isotropic average of the dipolar Hamiltonian,  $\overline{H_d} = 0$ , and rapid isotropic motion in liquid state therefore averaged out the dipolar interaction in the NMR spectrum.

In a solid specimen, irrespective monocrystalline, polycrystalline or amorphous, being rotated uniformly with angular velocity  $\omega_p$  along the axis by an angle  $\beta$  from the static magnetic field  $H_0$ , every internuclear vector  $r_{ij}$  in the solid undergoes a motion illustrated in Fig. 2-4. Then the angle  $\theta_{ij}$  becomes time dependent, and the factor  $(3\cos^2\theta_{ij} - 1)$  in eq. (2-38) too.

According to the theorem for the spherical harmonics

$$\cos\theta_{ij} = \cos\beta \cdot \cos\beta_{ij}' + \sin\beta \cdot \sin\beta_{ij}' \cdot \cos(\omega_p t + \phi_{oij}), \quad (2-40)$$

where  $\beta_{ij}$ ,  $\phi_{ij}$ , etc. are illustrated in Fig. 2-3 and  $\phi_{oij}$  is the initial value of  $\phi_{ij}$ . Substitution of Eq.(2-40) into Eq.(2-38) gives

$$\begin{aligned} H_d = & (1/2) \sum_{i < j} (1/2) \gamma_i \gamma_j \hbar^2 r_{ij}^{-3} (\underline{I}_i \cdot \underline{I}_j - 3I_{iz} I_{jz}) \\ & \times \{ (1/2) (3\cos^2\beta - 1) (3\cos^2\beta_{ij} - 1) \\ & + (3/2) \sin 2\beta \cdot \sin 2\beta_{ij} \cdot \cos(\omega_p t + \phi_{oij}) \\ & + (3/2) \sin^2\beta \cdot \sin^2\beta_{ij} \cdot \cos 2(\omega_p t + \phi_{oij}) \}. \quad (2-41) \end{aligned}$$

The first term in the curly bracket is constant, while the second and the third terms are periodic with zero mean value. It is convenient to divide  $H_d$  into two terms, its mean value  $\overline{H_d}$  and the remainder, which is periodic with mean value:

$$H_d = \overline{H_d} + (H_d - \overline{H_d}). \quad (2-42)$$

The first term on the right hand side is constant and gives a reduced dipolar interaction and the narrowed spectrum, while the second term, which is periodic in  $\omega_p$  and  $2\omega_p$ , gives rise to rotational side bands at multiples of  $\omega_p$ . The explicit form of  $\overline{H_d}$  is

$$H_d = (1/2)(3\cos^2\beta - 1) \sum_{i < j} (1/2) \gamma_i \gamma_j \hbar^2 r_{ij}^{-3} \\ \times (\underline{I}_i \cdot \underline{I}_j - 3I_{iz} I_{jz}) (3\cos^2\beta_{ij} - 1). \quad (2-43)$$

Comparing Eq.(2-38) and Eq.(2-43), it can be seen that the spectrum for polycrystalline or amorphous material retains the identical shape, but is reduced in width by a scale factor

$$F(\beta) = [(1/2)(3\cos^2\beta - 1)]. \quad (2-44)$$

Some particular values for  $F(\beta)$  are calculated below

$$\begin{aligned} \text{For } \beta = 0, & \quad F(\beta) = 1; \\ \beta = (\pi/2), & \quad F(\beta) = 1/2; \\ \beta = \arccos(1/\sqrt{3}) = 54^\circ 44' 08'', & \quad F(\beta) = 0. \end{aligned}$$

Rotation about  $\underline{H}_0$  has no effect, rotation about an axis normal to  $\underline{H}_0$  halves the spectral width, while rotation about an axis making the magic angle,  $54^\circ 44' 08''$ , to  $\underline{H}_0$  reduces the dipolar broadning to zero.

Although line narrowing can be performed in this way, there is an inevitable disadvantage in the MAS technique that removes chemical shift anisotropy at the same time. It turns to advantage when one studies a spectrum of powdered samples in which there are many magnetically inequivalent nuclear spins that will make the spectrum too complicated and/or in which there is a large anisotropy in the chemical shift.

## 2-4 Proton Enhanced NMR

Another approach to the high resolution solid state NMR is called Proton Enhanced Nuclear Induction Spectroscopy (Proton Enhanced NMR)<sup>6)</sup> or Cross Polarization NMR. This technique is applicable only to rare spins such as <sup>2</sup>D, <sup>13</sup>C, <sup>15</sup>N, <sup>29</sup>Si, etc. The reason can easily be understood by coming back to the dipolar Hamiltonian. Secular part of dipolar Hamiltonian between homonuclear spins is

$$H_{II} = (1/2)\gamma_I^2 \hbar^2 \sum_{i,j} r_{ij}^{-3} (1 - 3\cos^2\theta_{ij}) (\underline{I}_i \cdot \underline{I}_j - 3I_{iz}I_{jz}) \quad (2-45)$$

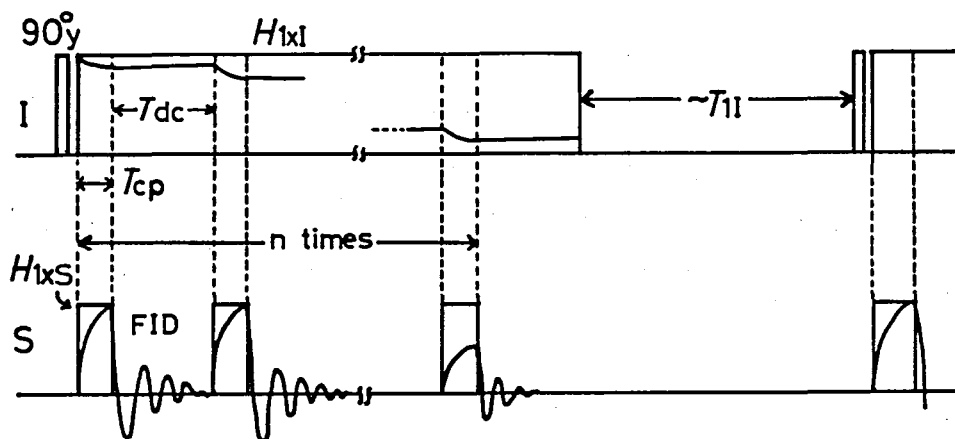
and between heteronuclear spins is

$$H_{IS} = \gamma_I \gamma_S \hbar^2 \sum_{i,j} r_{ij}^{-3} (1 - 3\cos^2\theta_{ij}) I_{iz} S_{jz}. \quad (2-46)$$

$H_{IS}$  depends on  $r_{ij}^{-3}$ ; therefore, it is negligible between rare spins. When the system contains a large number of abundant spins such as <sup>1</sup>H, or <sup>19</sup>F, so that  $H_{IS}$  is appreciably large, high resolution NMR spectrum of rare spins in solid state is expected to be observed by the decoupling technique to eliminate  $H_{IS}$ . However, in the case of rare spins a signal with good S/N ratio can hardly be obtained due to low natural abundance. Since spin-lattice relaxation time is usually very long for rare spins, the signal accumulation and averaging technique is inefficient.

A. Pines et al.<sup>6)</sup> could solve this problem by combining the ideas of double resonance<sup>7)</sup> with the decoupling scheme. In this technique abundant spin I is used to polarise the rare spins S and receive the excess energy from the rare spin as a reservoir. Since the  $T_1$  of S is determined by that of I, the recovery of magnetization is fast enough to permit rapid accumulation of the signal.

The general procedure is shown schematically in Fig. 2-5 and described below.



$T_{cp}$  cross polarization  
 $T_{dc}$  I decoupling  
 $T_{1I}$  spin lattice relaxation time of abundant spin I

Fig. 2-5. The pulse sequence for cross-polarization NMR. During the contact pulse, the Hartmann-Hahn condition,  $\gamma_I H_{1xI} = \gamma_S H_{1xS}$ , must be met.

(1) Polarize I in high external field. (2) Cool I to a low spin temperature in the rotating frame (spin lock). (3) Establish I-S contact for a time  $T_{cp}$  (under the Hartmann-Hahn cross relaxation condition,  $\gamma_I H_{1xI} = \gamma_S H_{1xS}$ ). (4) Record the S free induction decay while I is decoupled. Steps (3) and (4) are repeated  $N_I/N_S$  times while the I polarization is depleted and meanwhile the S signal is accumulated, here  $N_I$  and  $N_S$  are the number of I and S spins respectively in the system. Then the

accumulated signal of S is Fourier transformed to obtain the spectrum.

The Hartmann-Hahn condition states that the Zeeman levels of I and S have the same value in the rotating frame, and so the energy exchange can occur between I and S via dipolar interaction  $H_{IS}$ . As a result, the S spin polarization occurs efficiently in the direction of  $H_{IS}$  and the enhanced signal of S can be obtained.

This technique is applicable only to rare spins, but yields informations on the components of the shielding tensors.

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### 3 Determination of Chemical Shift Tensors

#### 3-1 Single Crystal Study

Since chemical shift is usually very small compared with the Larmor frequency ( $10^{-4} \sim 10^{-6}$ ), we may apply first-order perturbation theory to calculation of the chemical shift Hamiltonian. In the presence of the chemical shift perturbation the energy eigenvalue for the spin I can be formally written as

$$\langle m | \gamma \underline{H}_0 \cdot \underline{\sigma} \cdot \underline{I} | m \rangle = m \gamma \sigma_L \underline{H}_0, \quad (3-1)$$

where  $\sigma_L$  represents the component of  $\underline{\sigma}$  parallel to  $\underline{H}_0$ . When the chemical shift tensor in some coordinate system (x,y,z) is represented by

$$\underline{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} \quad (3-2)$$

$\sigma_L$  can be written as<sup>1)</sup>

$$\begin{aligned} \sigma_L = \underline{n} \cdot \underline{\sigma} \cdot \underline{n}^{-1} &= \sigma_{xx} \sin^2 \theta \cdot \cos^2 \phi + \sigma_{yy} \sin^2 \theta \cdot \sin^2 \phi + \sigma_{zz} \cdot \cos^2 \theta \\ &+ \sigma_{yx} \sin^2 \theta \cdot \sin 2\phi + \sigma_{zx} \sin 2\theta \cdot \cos \phi \\ &+ \sigma_{xy} \sin 2\theta \cdot \sin \phi. \end{aligned} \quad (3-3)$$

where  $\underline{n}$  is the unit vector along  $\underline{H}_0$ , defined by the polar angle  $\theta$  and the azimuthal angle  $\phi$  in the coordinate system (x,y,z).

A common procedure to find the six independent components of the chemical shielding tensor is to rotate the crystal around the three orthogonal axes and measure the shifts as a function of the rotation angle  $\rho$ . Such a rotation pattern for  $K_2GeF_6$  single crystal is shown in Fig.5-13. From the analysis of the rotational patterns, full tensor components can be determined.



### 3-2 Powder Pattern Study

For a polycrystalline or amorphous solid, the orientation of the chemical shift tensor with respect to the static magnetic field is randomly distributed and thus a characteristic 'powder pattern' is obtained.

The powder pattern is calculated as follows,<sup>2)</sup>

$$f(\sigma) = \pi^{-1}(\sigma - \sigma_{11})^{-\frac{1}{2}}(\sigma_{33} - \sigma_{22})^{-\frac{1}{2}}K(m) \quad (3-4)$$

with

$$m = (\sigma_{22} - \sigma_{11})(\sigma_{33} - \sigma) / (\sigma_{33} - \sigma_{22})(\sigma - \sigma_{11}) \quad (3-5)$$

when

$$\sigma_{33} \geq \sigma \geq \sigma_{22}$$

and

$$f(\sigma) = \pi^{-1}(\sigma_{33} - \sigma)^{-\frac{1}{2}}(\sigma_{22} - \sigma_{11})^{-\frac{1}{2}}K(m) \quad (3-6)$$

with

$$m = (\sigma - \sigma_{11})(\sigma_{33} - \sigma_{22}) / (\sigma_{33} - \sigma)(\sigma_{22} - \sigma_{11}) \quad (3-7)$$

when

$$\sigma_{22} > \sigma \geq \sigma_{11}$$

and also

$$f(\sigma) = 0 \quad (3-8)$$

when

$$\sigma > \sigma_{33} \text{ and } \sigma < \sigma_{11}$$

Here  $K(m)$  is the complete elliptic integral of the form,

$$K(m) = \int_0^{\pi/2} (1 - m \sin^2 r)^{-\frac{1}{2}} dr. \quad (3-9)$$

In equations (3-4) ~ (3-8),  $\sigma_{11}$ ,  $\sigma_{22}$ , and  $\sigma_{33}$  are the principal values and the definition  $\sigma_{33} \geq \sigma_{22} \geq \sigma_{11}$  has been adopted as a matter of convention.

In the case that  $\underline{\sigma}$  has the axial symmetry, e.g.,  $\sigma_{11} = \sigma_{22}$ , the powder pattern takes the form

$$f_{ax}(\sigma) = (1/2)(\sigma_{33} - \sigma_{11})^{-1/2}(\sigma - \sigma_{11})^{-1/2} \quad (3-10)$$

The line shapes with and without the axial symmetry are shown in Fig. 3-1.

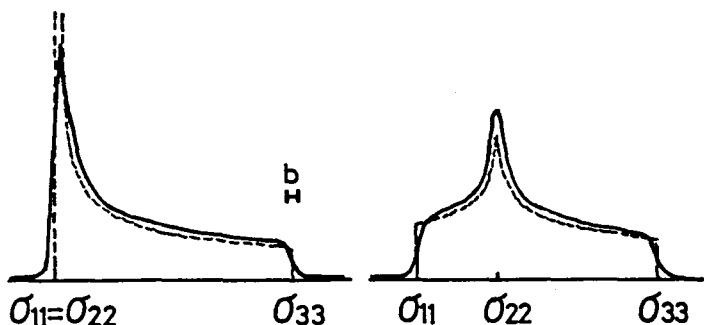


Fig. 3-1. Schematic representation of theoretical powder line shapes. Broken line: zero line width case. Solid line: the theoretical curves convoluted with Lorentzian broadening functions, where width  $b$  is indicated. (a) axially symmetric tensor, (b)  $\sigma_{11} < \sigma_{22} < \sigma_{33}$ .

It is evident from Fig. 3-1 how the magnitudes of the principal shielding components ( $\sigma_{11}$ ,  $\sigma_{22}$ ,  $\sigma_{33}$ ) can be determined directly from the powder pattern. However, the orientation of the  $\sigma$  tensor can not be obtained. Therefore it is not possible to relate the shielding tensor to the molecular bond frame from the powder pattern. Only in special case where there is a high crystal symmetry and simple molecular or ionic structure, analysis of the

chemical shift data can be used to elucidate the bond structure as will be described in § 5-2. Mehring et al.<sup>3)</sup> could determine the shielding tensor direction in the molecular frame with the aid of knowledge of the type of molecular motion. Only from the powder pattern analysis, very interested information about molecular motion has been obtained, such as jump motion in  $P_4$ <sup>4)</sup>,  $C_6(CH_3)_6$ <sup>5)</sup> and so on.

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## 4 Construction of Spectrometer for High Resolution NMR in Solid

### 4-1 Introduction

In the present work the spectrometer was designed to work at 40 MHz ( suitable to  $^{19}\text{F}$  resonance at 1 T ), whose organization is shown in Fig. 4-1.

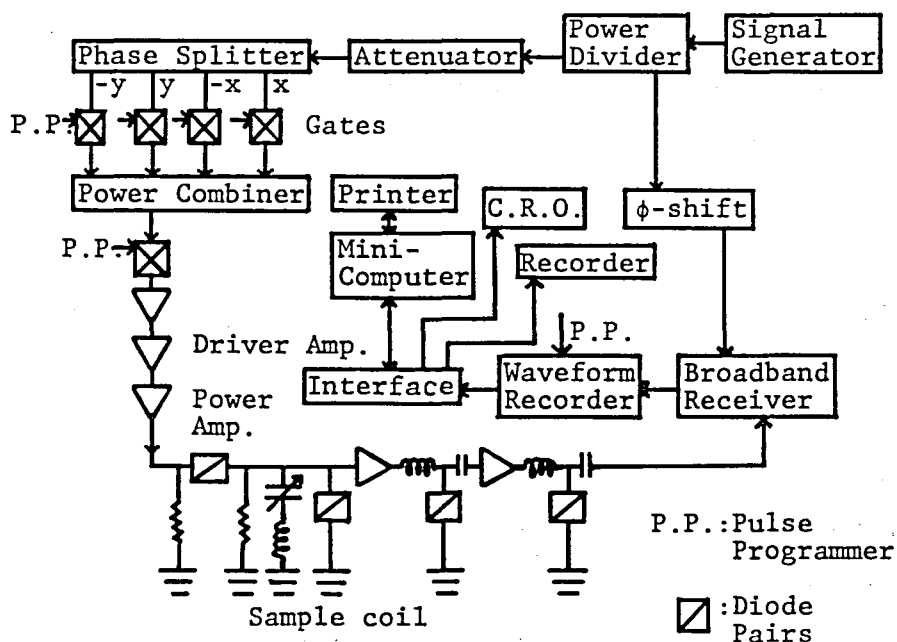


Fig. 4-1. Organization of spectrometer. This spectrometer operates at 40 MHz.

A carrier signal from a signal generator is divided into two channels by a power divider. One carrier is fed into a broadband receiver for phase sensitive detection and another one through an

attenuator to a phase splitter <sup>1)</sup> in which carriers with four different phases are produced. These four independent carriers are fed into the RF gates which are controlled by DC pulses generated in the pulse programmer and connected into the RF pulses with four different phases. The four RF-outputs of the RF gates are combined by a power combiner and used to drive an RF power amplifier. This amplifier can produce the RF pulses with the power greater than 1 kW which are then fed into the sample coil via matching unit. The FID signal induced at the sample coil is led into a broadband receiver, amplified, and connected into a video signal through the phase sensitive detection. Passive dampers <sup>2)</sup> are inserted between the sample coil and the receiver to protect the receiver from the high power RF pulses and to make the dead time of the receiver system short. The FID signal after phase sensitive detection is digitally recorded by a waveform recorder. The digital data are then transferred into a minicomputer through a home-built interface, accumulated when necessary, and Fourier-transformed to give an NMR absorption spectrum. The data can be displayed on a CRO or recorded on an X-Y recorder through an interface system. The pulse intervals, the repetition period of the sequence, and the data acquisition timing are controlled by a pulse programmer.

For the readers convenience to follow the each section in this Chapter, a simplified block diagram of the spectrometer is presented in Fig. 4-2. The labels ( A - J ) in this block diagram indicate the subsection number where the interpretation of the individual components is given. The following particular cases were taken on designing the spectrometer: In multiple pulse high resolution NMR experiments, it is necessary to reduce the dipolar interaction

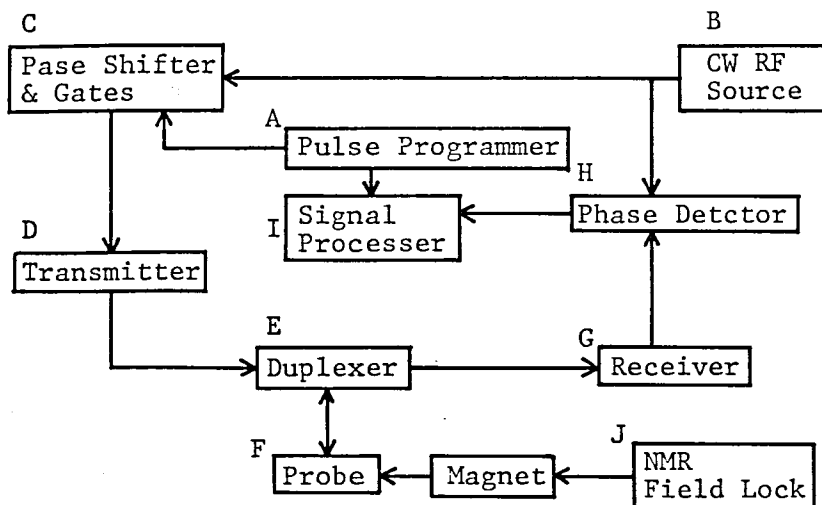


Fig. 4-2. Block diagram of the spectrometer. Each block is labeled by a character ( A - J ) to which each subsection in section 4-2 corresponds.

Hamiltonian,  $H_{int}$ , between the nuclear spins to zero or approximately zero by some means. This can be realized in the case of abundant spin system by applying a set of the RF pulses with specified phases to the system to average out the  $H_{int}$ . In order to average out the  $H_{int}$  effectively by the use of the RF pulses the following condition should be satisfied:

$$\gamma H_{rf} \gg |H_{int}|/\hbar$$

where  $\gamma$  is the gyromagnetic ratio of the nuclear spin, and  $H_{rf}$  the strength of the RF magnetic field. Since the local dipolar field of

protons ( and fluorines ) in solids may amount an order of 20 G,  $H_{rf}$  must in general be greater than 100 G, corresponding to an RF pulse power of about 1 kW. An RF power transmitter stage was designed to meet the above condition ( see 4-2-D ).

The interval of pulses should be shorter than the spin-spin relaxation time, since the multiple pulse technique can be effective only when  $H_{int}$  can be regarded as the average Hamiltonian. This requires that the interval of pulses is shorter than 10  $\mu$ s, a typical spin-spin relaxation time in proton or fluorine systems. Moreover, to observe the FID between these pulse sequences the dead time of the receiver should be less than a few microseconds. A preamplifier ( gain 40 dB ) with a recovery time of about 3  $\mu$ s together with dampers to prevent the large amplitude RF pulses to pass into the receiver system were designed ( 4-2-E ).

In multiple pulse technique, a sequence of four ( WAHUHA-4 ) or eight ( MREV-8 ) independent pulses with a certain intervals are used. The sequences consist of the combination of four pulses, whose phases differ by  $90^\circ$ ,  $180^\circ$ , or  $270^\circ$  with each other. Therefore, the four phase-shifters or splitters and the four independent RF gates are needed. The phase error should be less than  $3^\circ$ . Hence the controlable phase splitters <sup>1)</sup> were employed to achieve this condition ( 4-2-C ).

The intervals of pulses also should be precisely adjusted ( less than ten nano seconds ). The clock synchronous TTLs were adopted to produce WAHUHA-4 and MREV-8 pulse sequences and monostable multivibrators were employed to adjust the pulse widths of the four pulses independently.

It is desirable to maintain good stabilities of the RF source



and the Zeeman field to within 1 ppm for  $^{19}\text{F}$  and 0.1 ppm for  $^1\text{H}$  in order to make the signal accumulation technique effective. The Zeeman field source is a JEOL JM-310 electromagnet which is designed for wide line NMR. Its stability is not sufficiently high for high resolution NMR. Therefore, NMR static field locking system was developed ( 4-2-J ).

## 4-2 Spectrometer Design and Construction

### A. Pulse Programmer

The pulse programmer consists of a single 10 MHz clock pulse oscillator, the time multiplier, and the pulse sequence generator. The pulse sequence generator is constructed from TTL logic circuits situated on plug-in cards. The plug-in pulse programmer was designed to provide the timing for WAHUHA-4 and MREV-8 multiple pulse experiments.

The pulse programmer must provide a) the trigger pulse to initialize the whole system including the transient recorder, b) a sequence of four (WAHUHA-4) or eight (MREV-8) independent pulses,  $P_x$ ,  $P_{-x}$ , and so on, to excite the RF gates, the intervals and widths of which are precisely controllable, c) the sampling pulses for the transient recorder at the interval  $6\tau$  to collect the FID signal, d) a long pulse to hold the field-lock system during the pulse sequence, and must generate a waiting period during which the magnetization can return to the original value with the time constant  $T_1$ , the spin-lattice relaxation time. Therefore this waiting period has to be determined in relation to  $T_1$  of the spin system.

Timing chart for WAHUHA-4 method, e.g., is shown in Fig. 4-3. The MREV-8 pulse sequence is analogous to this scheme.

A pulse programmer which performs the above-mentioned functions was designed and constructed: The circuit diagram is given in Figs. 4-4 and 4-5.

A combination of 10 MHz quartz oscillator and frequency divider provide the clock pulse. The cyclic triggering pulses for

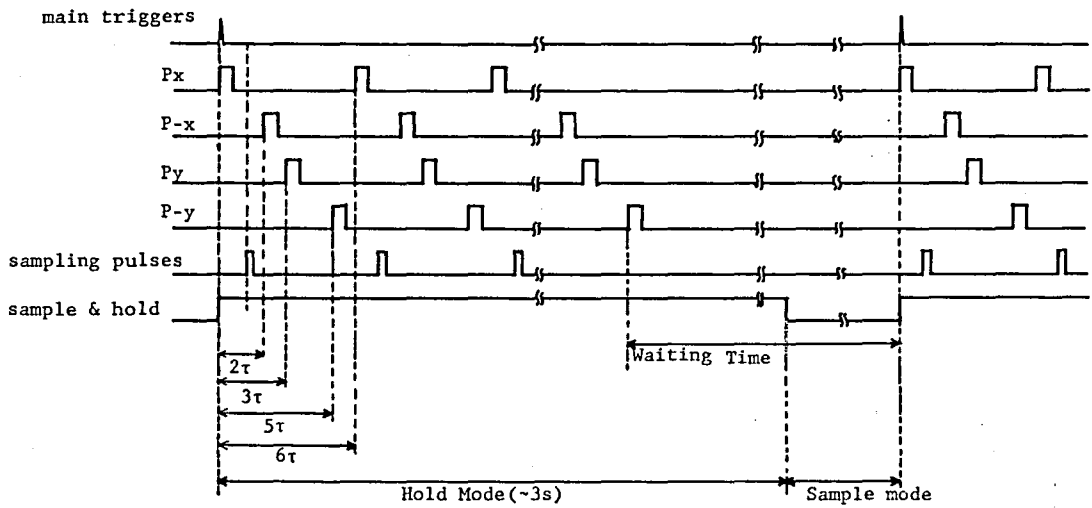


Fig. 4-3. Timing chart for WAHUHA-4.

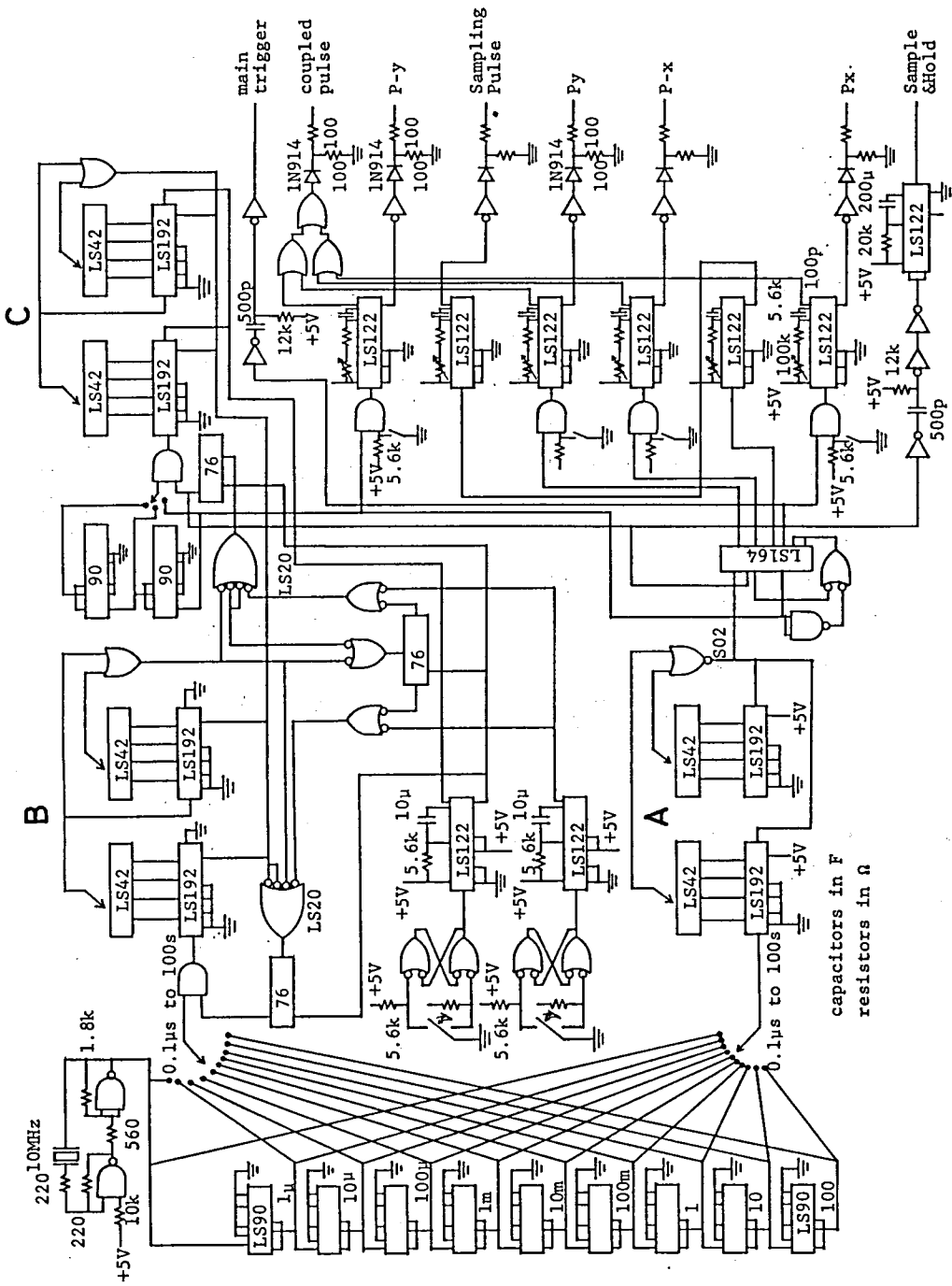


Fig. 4-4. Organization of pulse programmer for WAHUA-4.

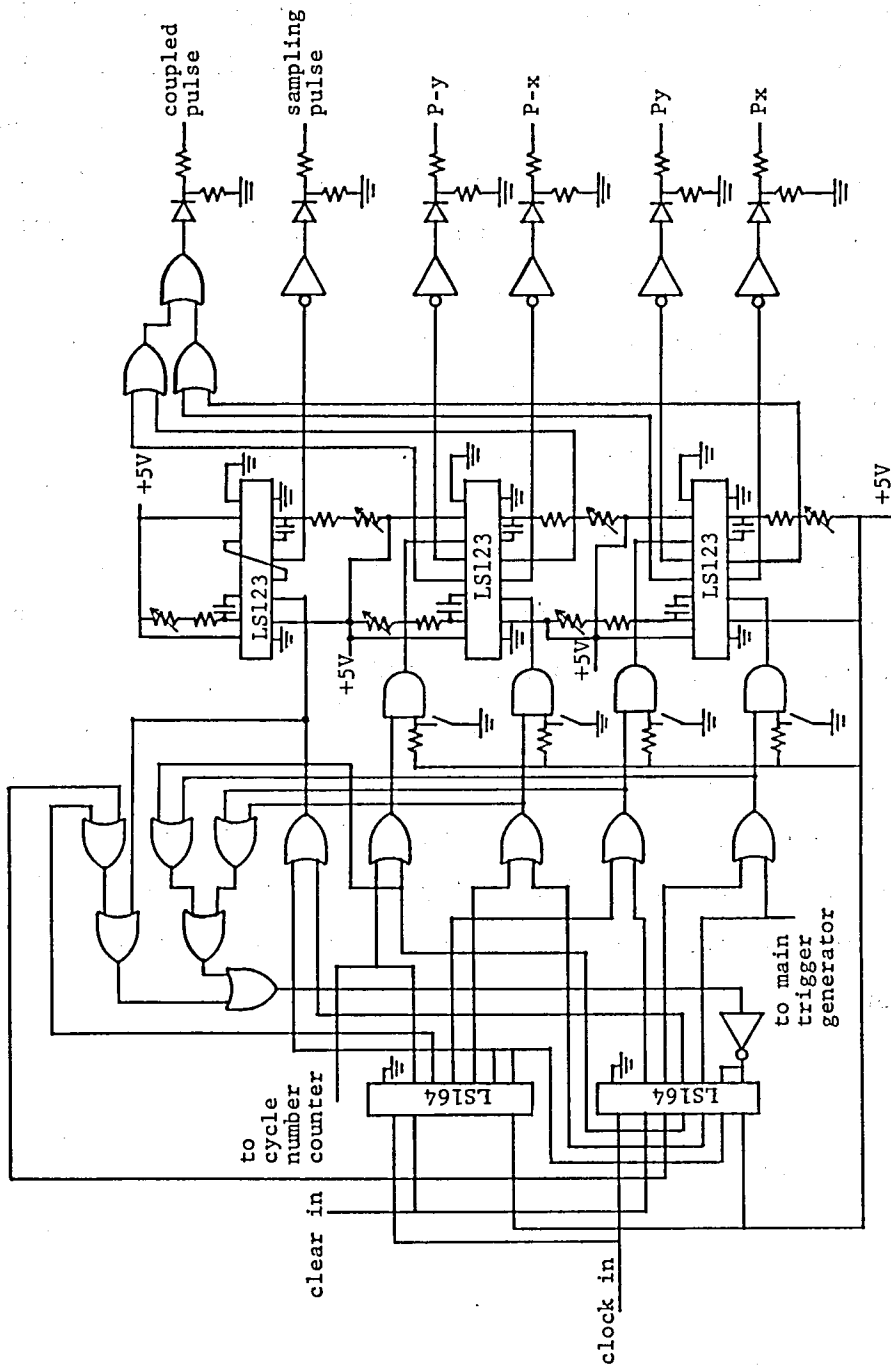


Fig. 4-5. Organization of plug-in unit for MREV-8.

$P_x \sim P_y$  in WAHUHA-4 sequence with the cycle time  $6\tau$  is generated by employing an 8 bit parallel-out serial shift register SN74LS164 as the 6-bit ring counter which is driven by the clock pulse. The pulse interval,  $\tau$ , corresponds to the cycle time of clock introduced in SN74LS164 and is set by the counter A, the waiting time by the counter B, and cycle number by the counter C. Cycle time for MREV-8 is  $12\tau$ , therefore two SN74LS164s are used.

The output of SN74LS164, used as triggering pulse, are fed into monostable multivibrators (SN74LS122, SN74LS123) which generate the pulse with finely adjustable width to drive the RF gate. A long holding pulse for the field-lock device and the sampling pulse for FID are also obtained using SN74LS122.

All adjustment operation can be done with nobs, dials, etc. on the front panel except when program card is replaced (MREV-8 or WAHUHA-4). The front view of the pulse programmer is shown in Fig. 4-6. The pulse interval and the waiting time can be changed from  $0.1 \mu\text{s}$  to  $9900 \text{ s}$ , and the cycle number from 1 to 9900 times. The pulse width can be varied from  $0.4 \mu\text{s}$  to  $7.0 \mu\text{s}$  continuously by changing the resistance of helipot connected to monostable multivibrator.

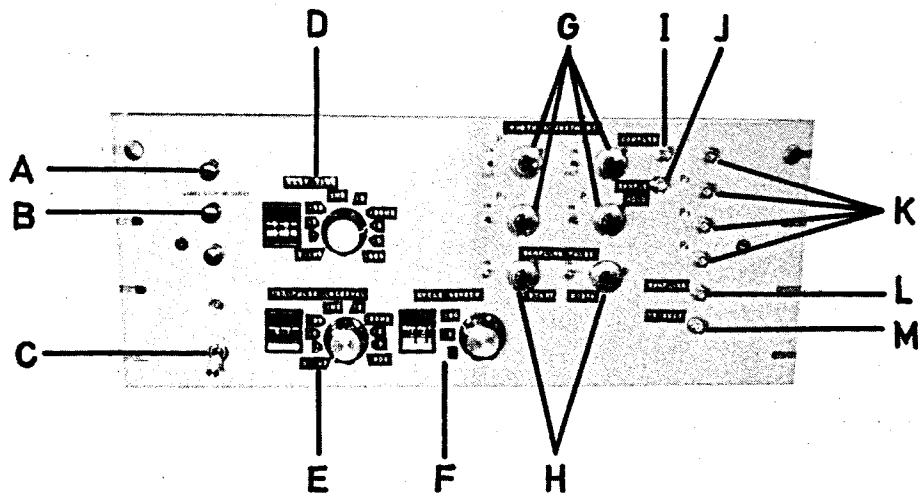


Fig. 4-6. Front panel of the pulse programmer. Functions are follows. A: reset, B: start, C: power switch, D: waiting time, E: pulse interval setting, F: cycle number setting, G: pulse width adjustment, H: sampling time setting, I: output to second gate, J: output to sample & hold circuit, K: outputs to first gates, L: output to waveform recorder for sampling, M: output to waveform recorder for trigger.

## B CW RF Source

The spectrometer is designed to work at 40 MHz to study  $^{19}\text{F}$  resonances. The frequency synthesizer (model 3325A Hewlett-Packard) was employed as 40 MHz RF source. The frequency of this synthesizer is stabilized to within 0.1 ppm when the room temperature is constant but varies about 0.4 ppm by the change of 1 °C of room temperature.

## C. Phase Splitter and Gate

Multiple pulse experiment requires  $90^\circ$  pulses along the x, -x, y, and -y directions in the rotating frame, corresponding to four different carriers with relative phases  $0^\circ$ ,  $90^\circ$ ,  $180^\circ$ , and  $270^\circ$ . To generate four  $90^\circ$  pulses with a specified exact phase relation with each other, three phase splitters (one  $0^\circ$ - $90^\circ$  and two  $0^\circ$ - $180^\circ$ ) were constructed and shown in Figs. 4-7 and 4-8. These are minor modifications of a circuit originally designed by Jeffrey.<sup>1)</sup> The fine adjustment of the phase is done by variable capacitor and the pulse-height is controlled by variable resistors equipped at the bases of transistors.

Each of four CW outputs of the phase splitter is fed into a solid state gate (model SW-1, R&K) which is driven by the pulse from the pulse programmer, and the RF pulses from the four gates are combined with a power combiner (model PD-3, R&K).



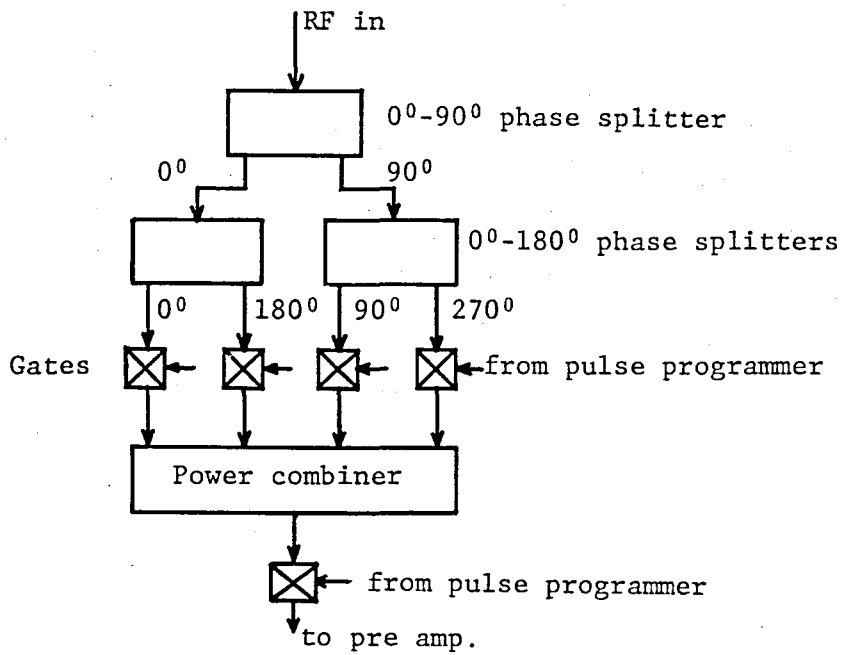


Fig. 4-7. RF pulse modulator with four different carriers.

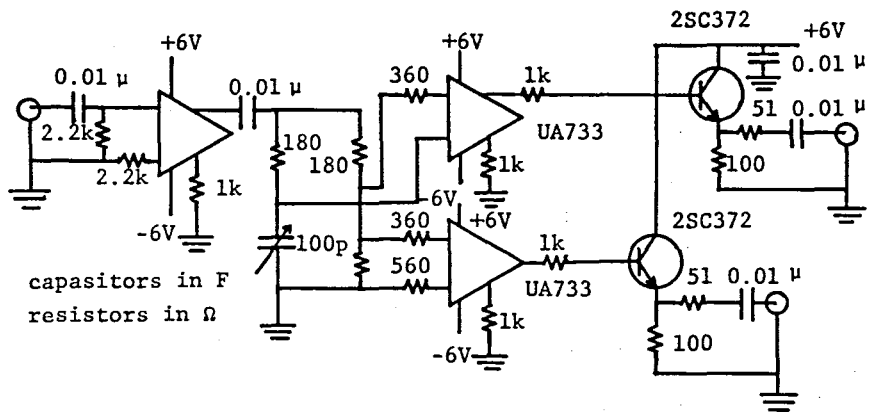


Fig. 4-8a.  $0^\circ - 90^\circ$  phase splitter.

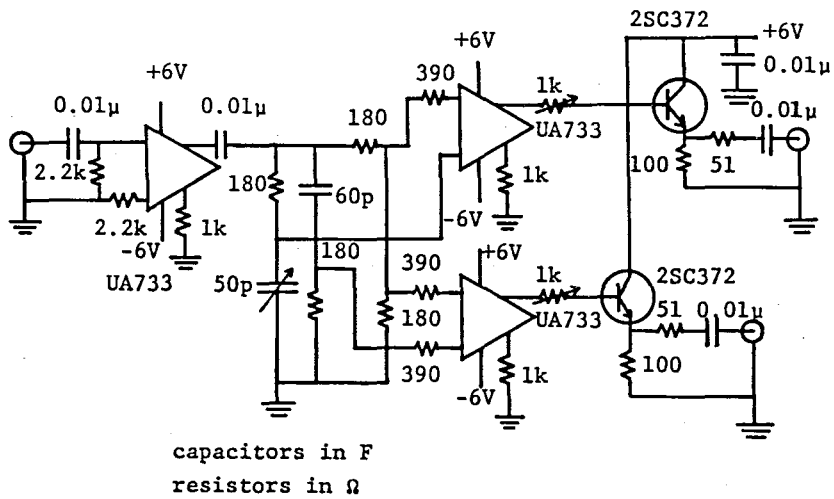


Fig. 4-8b.  $0^\circ - 180^\circ$  phase splitter.

#### D. Transmitter

Multiple pulse line narrowing experiments require that

$$\gamma H_1 \gg |H_{int}|/\hbar \quad (4-1)$$

where  $H_{int}$  is the "magnitude" of the interaction Hamiltonian between the nuclei,  $\gamma$  the gyromagnetic ratio, and  $H_1$  is the RF magnetic field strength in the rotating frame (see § 2-2). Since the local dipolar field of protons in solids is of order of 20 G,  $H_1$  fields of greater than 100 G should be applied to the sample. The  $H_1$  field in a sample coil is given by<sup>3)</sup>

$$H_1 = 3(PQ/\nu_0 V)^{\frac{1}{2}} = 3(P/\Delta\nu V)^{\frac{1}{2}} \quad (4-2)$$

where  $H_1$  is the rotating field in Oe,  $P$  the RF power in watts,  $Q = \nu_0/\Delta\nu$  the quality factor of the coil,  $\nu_0$  the resonance frequency of the circuit in MHz, and  $V$  is the volume of the coil in  $\text{cm}^3$ . The band width  $\Delta\nu$  of an RLC circuits is related to the rise time  $T_r$  of the pulsed  $H_1$  field as follows

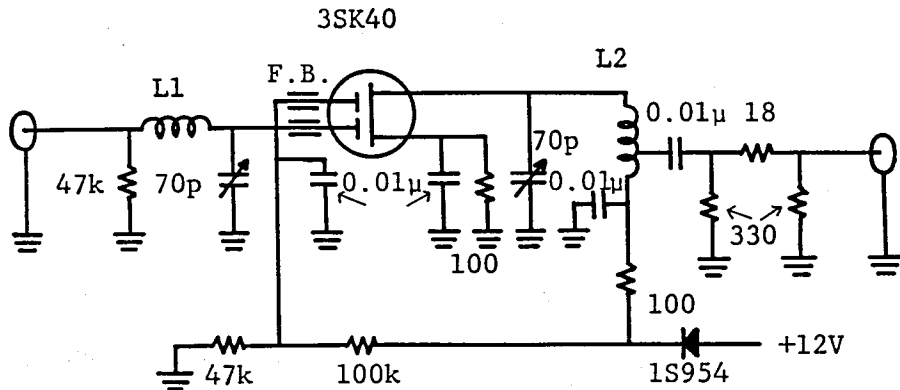
$$T_r = 2/(\pi\Delta\nu) = 2Q/(\pi\nu_0). \quad (4-3)$$

In order to reduce possible error in the phase adjustment at the transient time during the rise and fall period of the pulsed  $H_1$  field,  $T_r$  has to be very short. If  $T_r$  is  $\leq 200$  ns, i.e.,  $\Delta\nu \sim 3.3$  MHz, and so Eq.(4-3) indicates that under these conditions, even with such small volumes of the sample coil as  $0.25 \text{ cm}^3$ , an RF power of at least 1 kW is needed to produce field greater than 100 G.

The transmitter consists of three stages as follows (Fig. 4-2).

- 1) Tuned preamplifier
- 2) Wideband driver stage
- 3) Tuned power amplifier

Tuned preamplifier is composed of two stages of 20 dB amplifier each using 3SK40 as shown in Fig. 4-9.<sup>4)</sup>



resistors in  $\Omega$   
 capacitors in F

Fig. 4-9. Tuned preamplifier.

2) The driver stage amplifier is a commercial, solid-state, broadband amplifier (model 4020L, Electronic Navigation Industries) which generate 20 W RF pulses.

3) Tuned power amplifier is a home-built tuned class C amplifier using 2B46P and 2B29P shown in Fig. 4-10. The 2B46P is first driven by the broadband amplifier described in 2). And then RF pulses are amplified by a 2B29P. In multiple pulse experiments, the complete pulse burst length might range from 10 ms to seconds. The RF power supplied to the probe was kept constant over the whole burst by using a capacitor with large capacity in the power supply. Although this final amplifier can produce pulses of more than 1 kW, it was operated at less than the maximum power to avoid the droop of pulse heights.

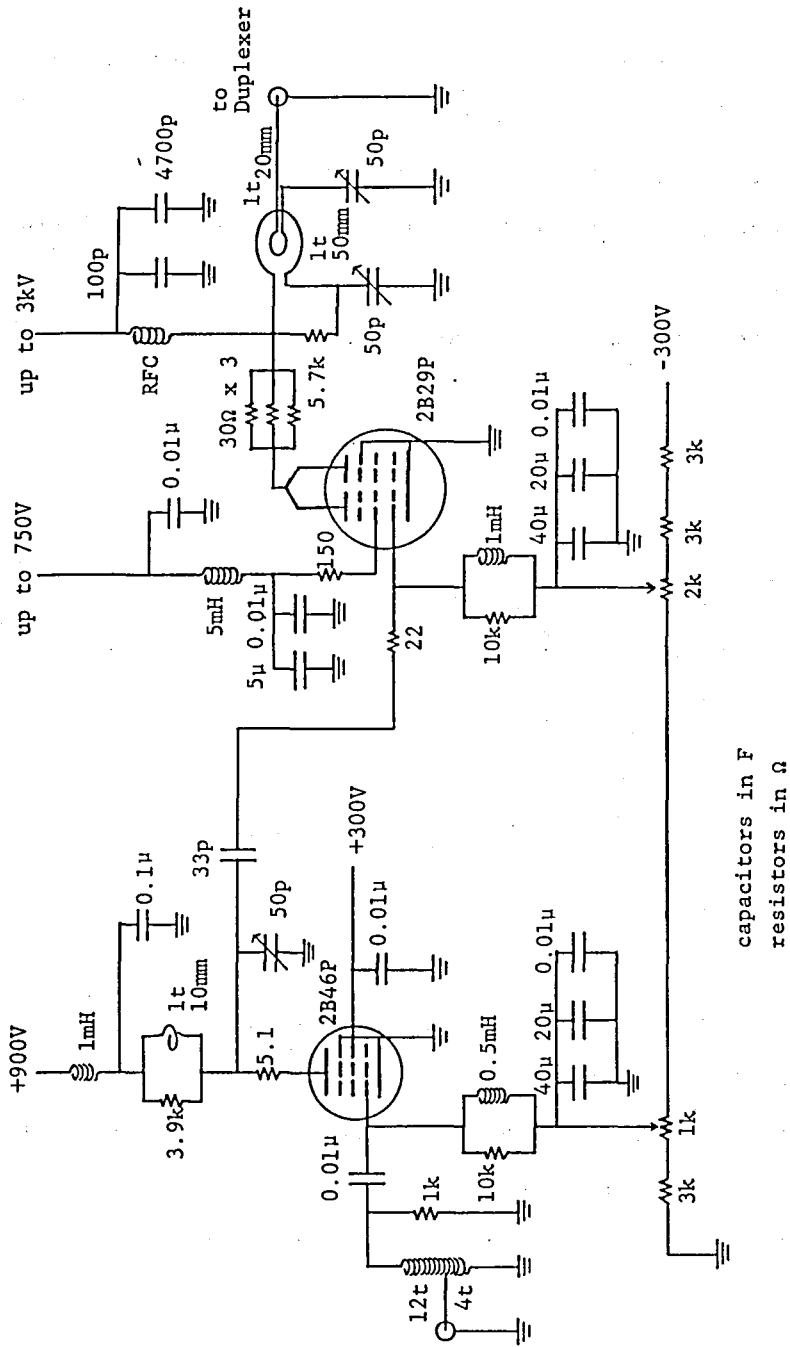


Fig. 4-10. 40 MHz tuned amplifier providing more than 1 kW of RF pulse power.

## E. Duplexer

The ideal duplexer is a fast single-pole, double-throw switch which connects the probe to the transmitter and disconnects the receiver in the transmitter mode, whereas during signal reception the probe is connected to the receiver and completely disconnected from the transmitter.

Duplexer and preamplifier are shown in Fig. 4-11.

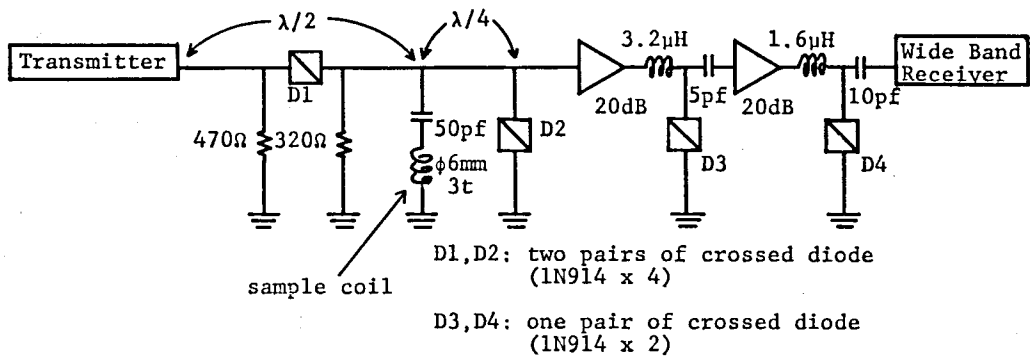


Fig. 4-11. Duplexer and preamplifier.

In order to minimize the power loss in the transmitter stage the  $\lambda/2$  cable was employed to connect the transmitter with the probe.<sup>5)</sup> The  $\lambda/4$  cable with diode short<sup>5)</sup> and passive dampers<sup>6)</sup> were adopted between receiver amplifiers to make the dead time of the receiver system short. The dead time of this spectrometer is about 5  $\mu$ s.

## F. Probe

The probe consists of a single coil in a series tuned circuit which can be tuned easily and can provide RF power to the sample efficiently. The position and the form of the probe are shown in Fig. 4-11.

## G. Receiver

The chief requirements for the receiver are to have a low noise figure of a few dB, sufficient band width of several tens kilohertz to pass the nuclear signals without distortion, and the short recovery time of a few microseconds from the 1 V overload that may appear at the output of the duplexer.

Two homebuilt tuned preamplifiers and a commercial wideband receiver (model 625, Matec) were employed in this stage. Passive dampers are placed between them. Preamplifiers are the same as that shown in Fig. 4-9. Noise figure of the preamplifier is 2.0 dB and its gain is 20 dB. A white noise is suppressed by this tuned amplifier. Although there is a little loss due to passive dampers, the total gain is more than 100 dB, and so NMR signal can be easily detected.

## H. Phase Detection

A commercial double balanced mixer (model MC-13, R&K) was employed for the phase detection circuit.

## I. Signal Processing

Multiple pulse NMR experiments require sampling of FID in a certain time interval. A typical output from the spectrometer's phase detector and the timing of sampling are shown in Fig. 4-12.

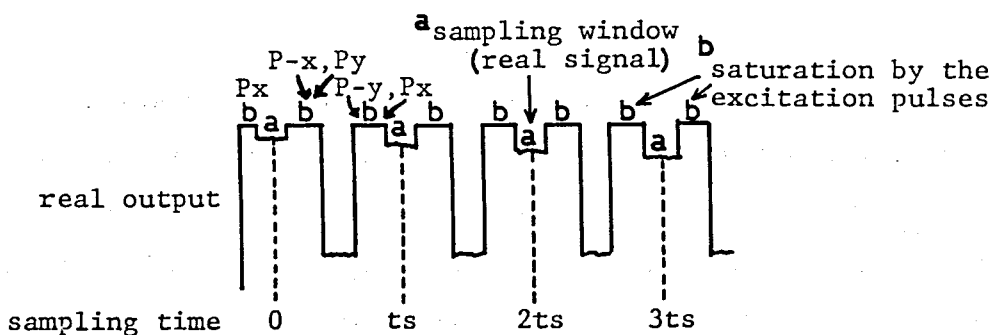


Fig. 4-12. Actual receiver output during a multiple pulse experiment.

Real signal appears at time intervals labeled by a. b represents the saturated parts due to RF pulses. Signals must be sampled in multiple pulse sequence, that is, real signal parts are recorded digitally with an interval of  $t_s$ . Therefore short recovery time is also required for digital memory since sampling window (part a



in Fig. 4-12) lasts only a few microseconds after pulses. Commercial waveform recorder (model 805, Biomation) was employed, which is gated by the series of the sampling pulses at  $\tau, 7\tau, \dots$  in Fig. 4-3, where  $6\tau$  is equal to  $t_s$  in Fig. 4-12, and collects the nuclear signal there. This waveform recorder can collect the data up to 2048 points, and converts analog data to 8-bit digital ones. Digital data memorized in the waveform recorder are then transformed into a minicomputer (NOVA-01) through a home-built interface and accumulated when necessary, and Fourier-transformed to give an NMR absorption spectrum.

The data acquisition process is controlled using ASSEMBLY program of NOVA-01 and processing afterwards, such as Fourier transform, is performed with BASIC program.<sup>6)</sup> Data can be displayed on CRO and recorded on an X-Y recorder through an interface.

When the sampling interval is  $40.2 \mu\text{s}$  and there are 1024 data points, the spectrum resolution is 24.29 Hz and covers the frequency range of 24.88 kHz. In multiple pulse experiments, above values are to be multiplied by about  $\sqrt{3}$  due to scaling factor (see § 2-2). The BASIC program for data processing is listed in the appendix.

#### J. NMR Field Lock

When signal averaging is necessary, of major importance is the stability of the DC field for a long time. The DC field source is a JEOL JM-310 electromagnet which is designed for wide

line NMR: Its stability is not sufficiently high for the high resolution spectra and therefore a unique NMR static field lock system is employed to keep the  $H_0$  constant. The principle is as follows.  $^1\text{H}$  signal in  $\text{H}_2\text{O}$  doped with  $\text{CuSO}_4$  is detected in a CW mode as the reference signal, fed into a lock-in amplifier. Its output is the derivative of the resonance line as shown in Fig. 4-13 and so between the points a and b it is proportional roughly to the deviations of the  $H_0$  from the central position 0.

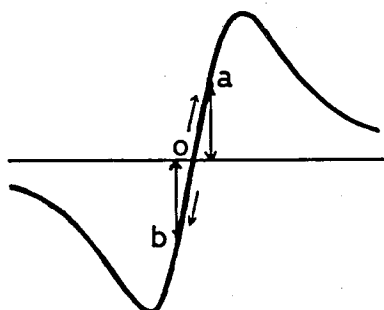


Fig. 4-13.

The lock point in NMR signal.

This output voltage (i.e. the error voltage) is amplified and used through a power amplifier to drive the modulation coils wound around the pole-pieces of the magnet in a negative-feedback manner, so that  $H_0$  is pulled back to the exact resonance point 0.

Fig. 4-14 is the block diagram of the field stabilizer. Usual NMR field lock systems adopt the Zeeman modulation technique to detect the reference signal but it is unsuitable for the high resolution multiple pulse experiment because even a small modulation of the  $H_0$  affect the nuclear signal collected over a long time. Therefore the method of frequency modulation was

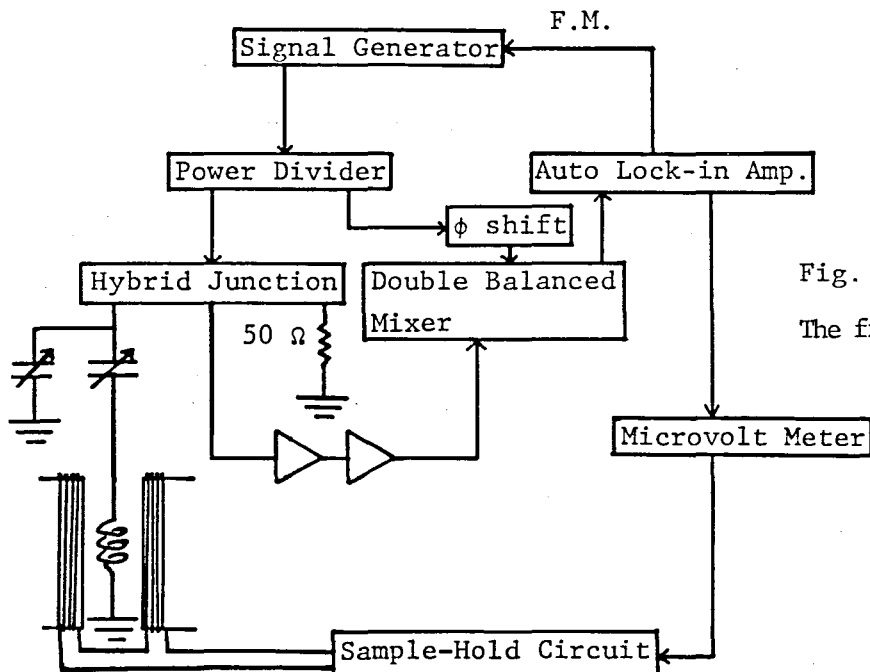


Fig. 4-14.  
The field stabilizer

adopted. In order to make the system to be able to respond to the temperature change of the sample and to avoid possible interference between the field lock system and the pulse spectrometer assembly, the external lock technique is employed.

Signal generator (model 3000, Wavetek) frequency-modulated by auto lock-in amp. provides the carrier frequency at 42.538 MHz (suitable for  $^1\text{H}$  resonance at 1 T) with the stability of  $\pm 0.1$  ppm over a day. The carrier signal is divided into two channels by a power divider (model PD-1, R&K). One carrier is fed into a double balanced mixer (model M-13, R&K) and another one through a hybrid junction (model HYB-1, R&K) to the probe coil (with water) which is

shielded in a copper can. The output of the hybrid junction is amplified by two stage tuned amplifiers (gain 40 dB, the same as in Fig. 4-9) and conducted to double balanced mixer in which the phase sensitive detection is performed. The output voltage of lock-in amp. supplies current to the modulation coil through a D.C. amplifier and a sample-hold circuit. The sample-hold circuit is needed to avoid the effects of high power RF pulses, the details of which are shown in Fig. 4-15. IH5040 and IH5042 (Intersil) are analog switches.

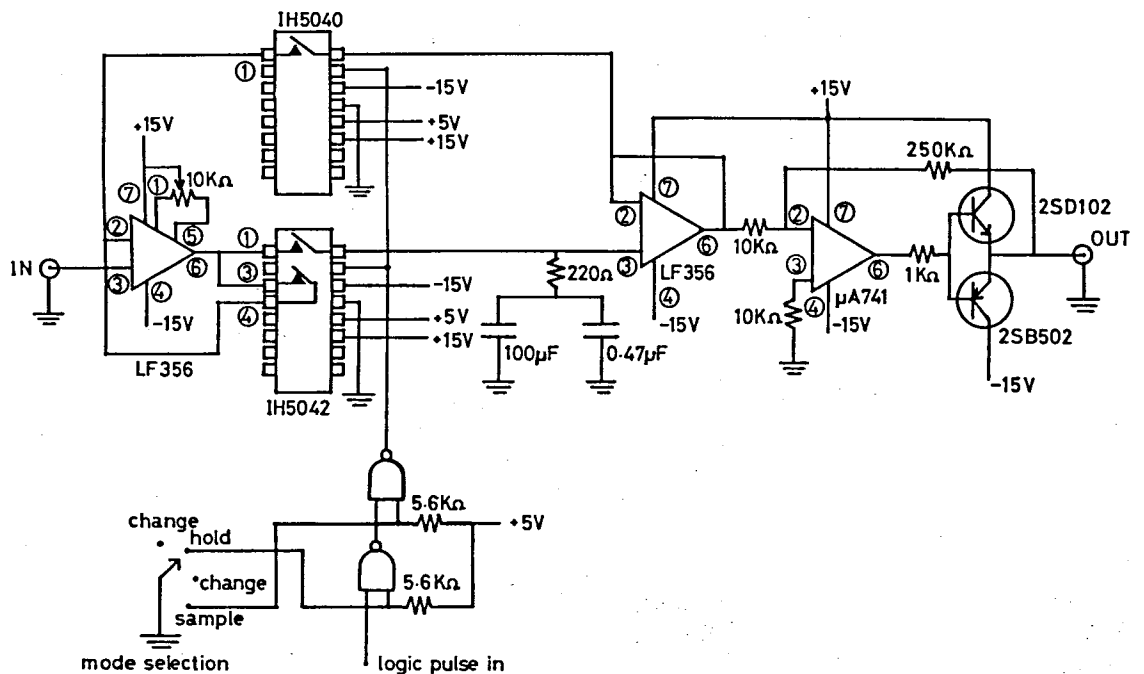


Fig. 4-15. Sample—hold circuit.

Another simpler sample-hold circuit was later designed as shown in Fig. 4-16 by using an analog memory (model EUW-A1C, National).

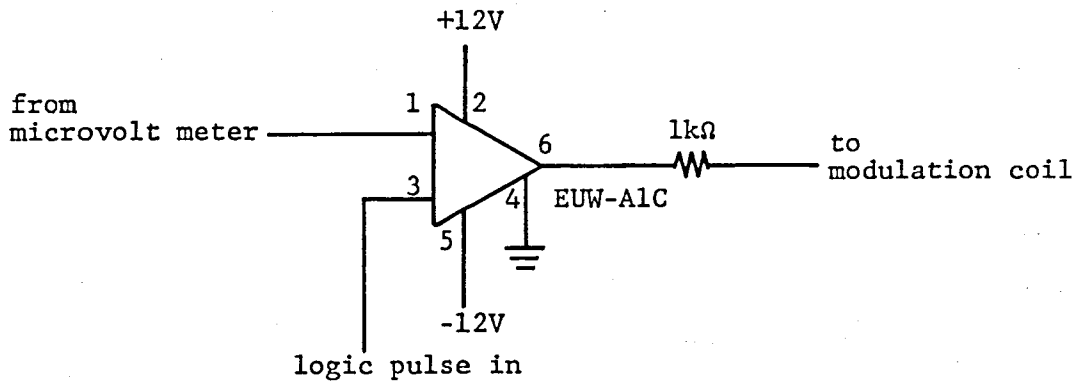


Fig. 4-16. Sample-hold circuit.

Both apparatus can be used successfully to control the static high field  $H_0$ . The overall stability of the  $H_0$  with the present field-lock system is better than  $\pm 1$  ppm ( $\pm 40$  Hz).

### 4-3 Operation

This section presents a brief example of the operation of the spectrometer.

The multiple-pulse line-narrowing experiments involve application of a series of  $90^\circ$  pulses with exactly defined phases between which the sampling points of the magnetization lie. As the number of sampling points is usually 512 or 1024, even small error in the pulse width or the phase, or the phase error at rise and fall transients of pulses will accumulate quickly to produce a fatal error for the experiment. Hence, in order to obtain high resolution NMR spectra careful adjustment of those quantities is necessary. A description of the adjustment procedure to obtain the optimum condition of the spectrometer will be given here. The test sample is protons in  $\text{CuSO}_4$  doped water.

#### (1) pulse width adjustment

A string of simple kind of pulses  $(90^\circ_x - t - 90^\circ_x - t - 90^\circ_x - t -)_n$  is applied, where  $t$  is about  $100 \mu\text{s}$ . The signal at the sampling window appears at the points "+, 0, -, 0, +, 0, -, ..." as shown in Fig. 4-17, but due to imperfect nature of the pulse shape and/or due to the deviation from the exact  $90^\circ$ , the signal usually decays as  $t$  increases.

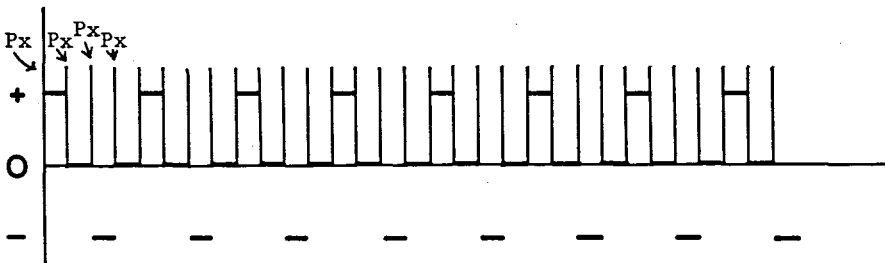


Fig. 4-17. Receiver output in  $(90^\circ_x - t - 90^\circ_x - t -)_n$  pulse train.

Optimum condition for this  $P_x$  pulse can be found by changing the pulse width and looking for the point for which the decay rate of the signal becomes minimum. The same procedure is repeated for each of the pulses with different phases.

(2) phase adjustment of  $x$  against  $-x$

A flip flop sequence  $(90^\circ_x -t - 90^\circ_{-x} -t - 90^\circ_x -t - 90^\circ_{-x} -t)_n$  is employed as shown in Fig. 4-18, where  $t$  is about 100  $\mu$ s. The signals labelled by "+, 0, +, 0, +, 0, ..." decay with time and the optimum point will be obtained by adjusting the relative phases of  $P_x$  and  $P_{-x}$  so that the decay time constant becomes maximum.

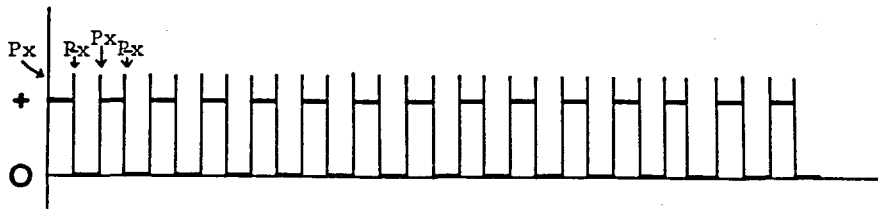


Fig. 4-18. Receiver output in  $(90^\circ_x -t - 90^\circ_{-x} -t)_n$  pulse train.

(3) phase adjustment of  $x$  against  $y$

A pulse sequence  $(90^\circ_x -t - 90^\circ_y -t - 90^\circ_y -t - 90^\circ_x -t)_n$  is employed?<sup>7)</sup> The signal appears as "+, +, +, 0, 0, 0, -, -, -, 0, 0, 0, +, +, +, ...". The optimum point will be obtained by adjusting the relative phases of  $P_x$  and  $P_y$  so that the decay time constant becomes maximum.

(4) final pulse width adjustment

Finally fine pulse width adjustment is done by applying multiple pulse sequence (WAHUA-4 or MREV-8) on  $\text{CaF}_2$  single crystal.

Adjustment of the pulse width of each of four pulses from  $P_x$  to  $P_y$  is carried out by a trial and error iterative procedure so that the decay rate of the FID becomes minimum. The half width of the most narrow spectrum obtained for  $\text{CaF}_2$  single crystal after the above procedure is 8.5 ppm (scaling factor is 0.5). FID and spectrum are shown in Fig. 4-19.

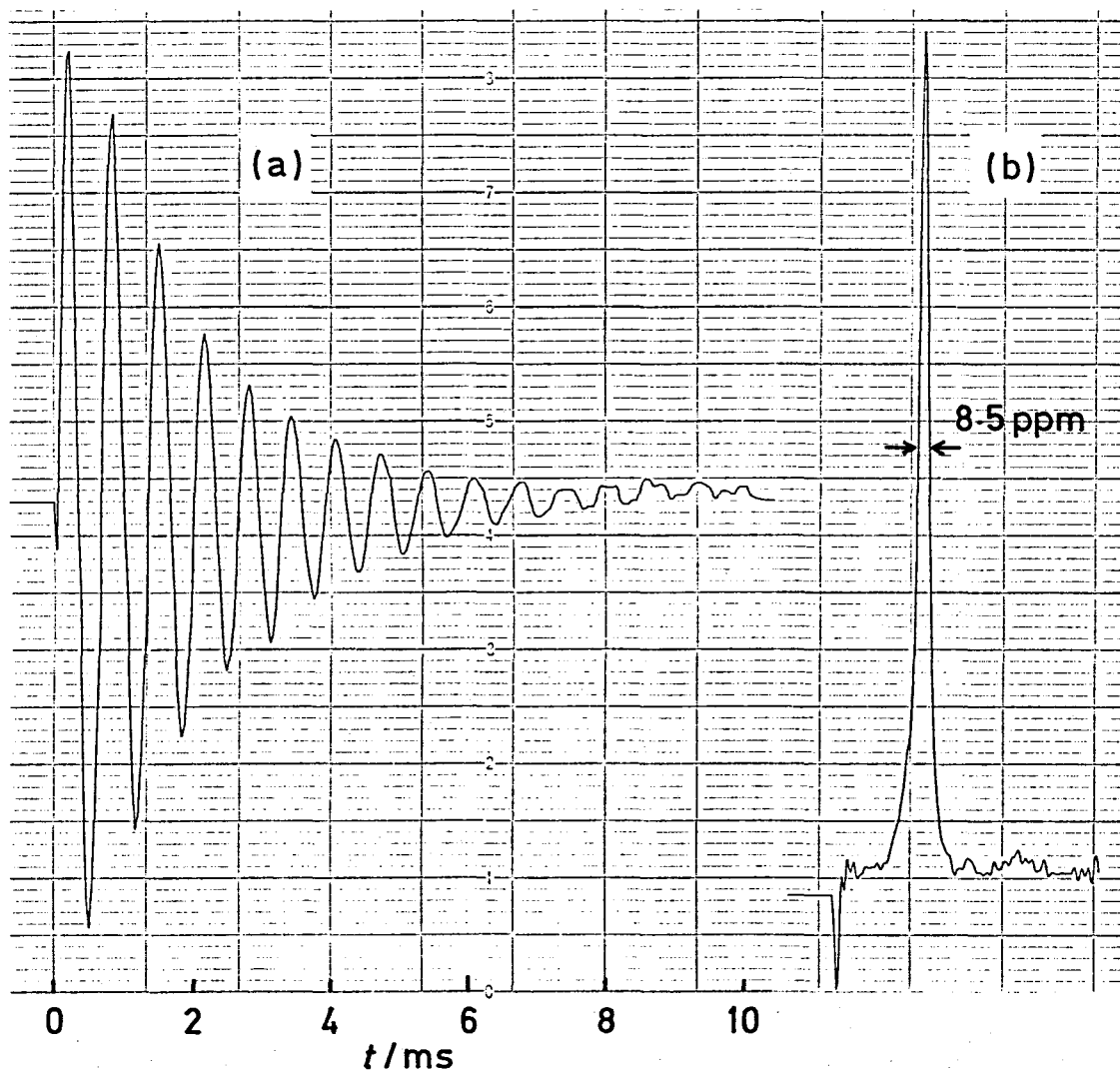


Fig. 4-19. (a) FID of  $\text{CaF}_2$  single crystal with MREV-8 pulse sequence. (b) FFT spectrum of FID in (a).



Chemical shift values obtained in multiple pulse measurement is small compared with the true shift obtained in usual high-resolution technique. The reduction factor called scaling factor must therefore be evaluated to discuss the multiple pulse NMR spectra on the same ground as the common high resolution NMR. Two scaling techniques employed are described below.

(1) In Fig. 4-20, the position of the center of  $^{19}\text{F}$  NMR spectrum in  $\text{K}_2\text{SiF}_6$  powder is plotted against the carrier frequency.

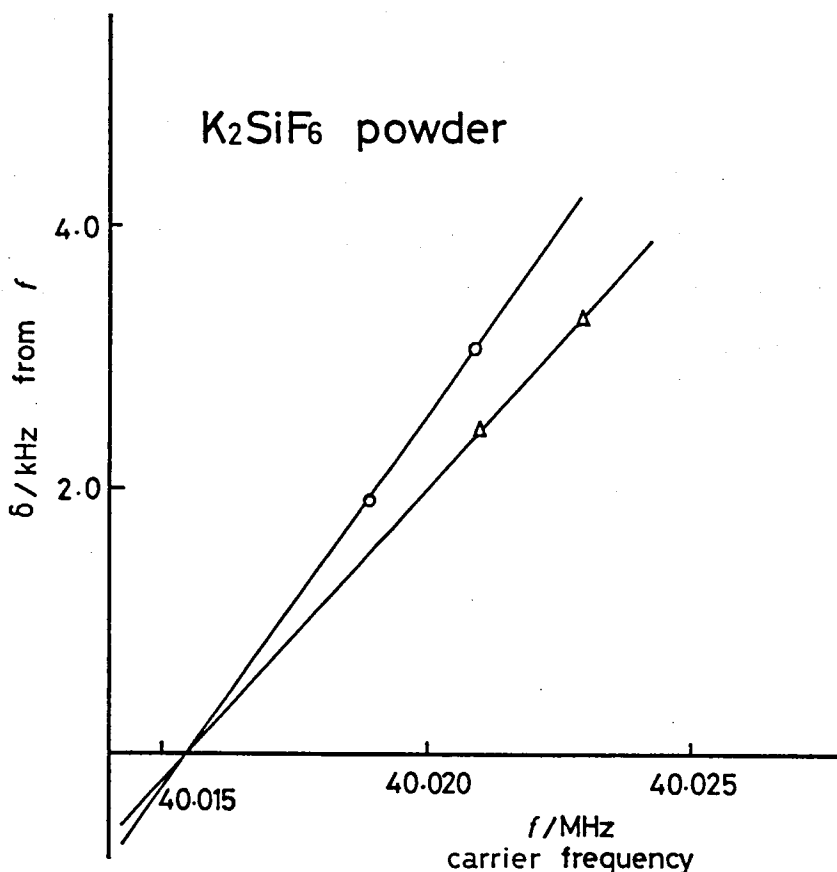


Fig. 4-20.  $^{19}\text{F}$  resonance positions at different carrier frequencies in  $\text{K}_2\text{SiF}_6$  powder. Circles and triangles have different scaling factors from each other due to a little pulse width difference.

We can estimate the scaling factor by comparing the change of the carrier frequency (real change) with apparent shift of  $^{19}\text{F}$  NMR spectrum. The scaling factors for circles and triangles are different due to a small difference in pulse width. The point at which the frequency axis is crossed is the resonance frequency of  $^{19}\text{F}$  in  $\text{K}_2\text{SiF}_6$  at the DC field.

(2) Fig. 4-21(a) shows the  $^{19}\text{F}$  NMR spectra in p- $\text{CF}_3\text{C}_6\text{H}_4\text{F}$  neat liquid in capillary with MREV-8 pulse sequence. Left hand side peak is due to F in  $-\text{CF}_3$ , and the right hand side due to F in  $\text{F}-\text{C}_6\text{H}_4-$ . Fig. 4-21(b) was obtained for the sample containing both p- $\text{CF}_3\text{C}_6\text{H}_4\text{F}$  neat liquid and powdered  $\text{K}_2\text{SiF}_6$ . A fairly broad spectrum at the right hand side is from  $^{19}\text{F}$  in the  $\text{K}_2\text{SiF}_6$  powder. Since the position of  $^{19}\text{F}$  NMR in p- $\text{CF}_3\text{C}_6\text{H}_4\text{F}$  are known, Scaling factor can be determined and so chemical shift of  $^{19}\text{F}$  in the  $\text{K}_2\text{SiF}_6$  powder is obtained immediately.

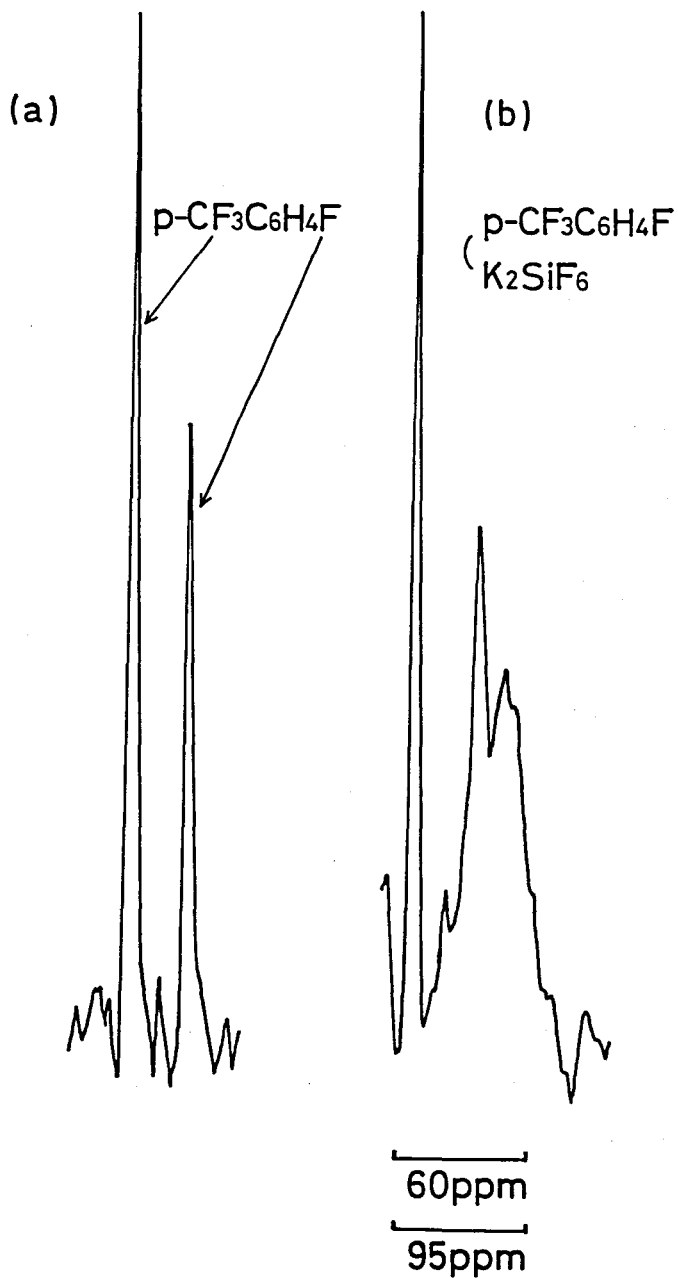


Fig. 4-21.  $^{19}\text{F}$  NMR spectra of (a)  $p\text{-CF}_3\text{C}_6\text{H}_4\text{F}$  neat liquid and (b)  $p\text{-CF}_3\text{C}_6\text{H}_4\text{F}$  neat liquid and  $\text{K}_2\text{SiF}_6$  powder. The scale is apparently 60 ppm, but really 95 ppm.

#### 4-4 Discussion

This spectrometer gives a high resolution signal of  $^{19}\text{F}$  in  $\text{CaF}_2$  single crystal with the optimum line width of 8.5 ppm ( 340 Hz ). This resolution is sufficiently high for most studies of  $^{19}\text{F}$  resonance, but insufficient for usual  $^1\text{H}$  resonance which gives the chemical shift of only 10 ppm.

The origin of the degradation of the resolution will be discussed below: In current experiments the RF power is decreased to about 500 W in order to avoid the droop of the RF pulse power in the pulse train. Since the volume of the sample coil is about  $0.22 \text{ cm}^3$  in the present work, the 500 W RF pulses produce the RF field of about 70 G. Therefore, the power of the RF pulses is still high enough to obtain the high-resolution spectra of  $^1\text{H}$  system.

The dead time of this spectrometer is about  $5 \mu\text{s}$ , then the longest pulse interval had to be set longer than  $10 \mu\text{s}$  in order to sample the FID signal effectively. This pulse interval is slightly too long to average the dipolar interaction between  $^1\text{H}$  nuclei. One of the fatal sources that degradate the resolution of the spectrometer comes from the fact that the pulse interval cannot be shorten due to relatively long dead time of the receiver system.

Among many other sources for the line broadning in the present spectrometer system the error in the phases of the RF pulses, the stabilities in the pulse timing, the RF carrier frequency, and the Zeeman static field, the inhomogeneity of the static field should be checked: The phases of carriers can be adjusted precisely. The error is seemed within  $3^\circ$  which does not produce any artificial broadning of the spectral line.

The timing of the pulses is controlled by a clock generator using a quartz oscillator, therefore the error in the timing is within ten nanoseconds which also doesn't cause any line broadning.

The stability of the RF source is better than  $\pm 0.2$  ppm and that of the Zeeman field is controlled to within  $\pm 1$  ppm for several hours which also does not effect the line width of the order of few ppm.

The optimum line width of liquid sample was found to be 2.3 ppm. This line width may be brought about by some inhomogeneity of the static field. This inhomogeneity effect is another fatal source to degradate the resolution of the spectrometer.

#### References to Chapter 4

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- 6) The FFT program was firstly developed by H. Nakayama ( M.D. thesis, Osaka University, 1981). The program was modified for multiple pulse high resolution NMR method by the author.
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## 5 Chemical Shift Anisotropy of $^{19}\text{F}$ in Some Fluorine-containing Compounds

### 5-1 Introduction

Three dimensional information about the electronic structure of the molecules can in principle be obtained by high resolution NMR in solid state. The aim of the present work is to obtain  $^{19}\text{F}$  high resolution spectra of several fluorine-containing materials in order to discuss the results in relation to the nature of the chemical bond.

As the object of the present work an aromatic conjugated system, 1,3,5-trichloro-2,4,6-trifluorobenzene ( sym- $\text{C}_6\text{Cl}_3\text{F}_3$  ), and a series of inorganic compounds (  $\text{K}_2\text{MF}_6$ ; M = Si, Ge, Sn ) were selected. The chemical shift tensors in these compounds were determined by the high resolution spectra in their crystalline state and the bond nature, such as  $\pi$ -bonding and ionic character, in the above compounds will be discussed in relation to the chemical shift data.

High resolution spectrum can also provide fruitful information about molecular motions in solids. In the present study the molecular motions in sym- $\text{C}_6\text{Cl}_3\text{F}_3$  and  $\text{K}_2\text{SiF}_6$  were discussed on the point of view of the high-resolution NMR.

## 5-2 1,3,5-trichloro-2,4,6-trifluorobenzene (sym-C<sub>6</sub>Cl<sub>3</sub>F<sub>3</sub>)

### 5-2-1 Sample Preparation and Crystal Symmetry

Sym-C<sub>6</sub>Cl<sub>3</sub>F<sub>3</sub> was purchased from Alfa Chemical Co. Ltd. Single crystals were grown at 30 °C or 5 °C by slow evaporation of a heptane solution. The crystal was colorless and transparent with a shape of hexagonal pyramid. Single crystal with the size of about 3 mm x 3 mm x 2 mm was used for the high resolution NMR study. Sym-C<sub>6</sub>Cl<sub>3</sub>F<sub>3</sub> is a molecule having the D<sub>3d</sub> symmetry. An early NQR study found a single resonance line at 39.312 MHz at 77 K,<sup>1)</sup> showing that all chlorines in the crystal are crystallographically equivalent. This shows that all fluorines in the crystal are crystallographically equivalent at the same time.

### 5-2-2 Measurements of Chemical Shift Anisotropy of <sup>19</sup>F in Sym-C<sub>6</sub>Cl<sub>3</sub>F<sub>3</sub>

The single crystal was placed between two polystyrene plates and set into the sample coil. The MREV-8 pulse method was used to obtain the high resolution spectra at 20 °C. The short pulse interval,  $\tau$ , was 6.7  $\mu$ s and the sampling interval was 40.2  $\mu$ s. FID signal was accumulated 10 or 20 times every 20 min, and FFT was done to obtain the absorption spectra using 1024 data points. In most cases, initial 100 actual data points were used for FFT and the rest was set to be zero. One of the spectra obtained is shown in Fig. 5-1. Rotational patterns were obtained by rotating the probe about an axis perpendicular to the Zeeman field. The



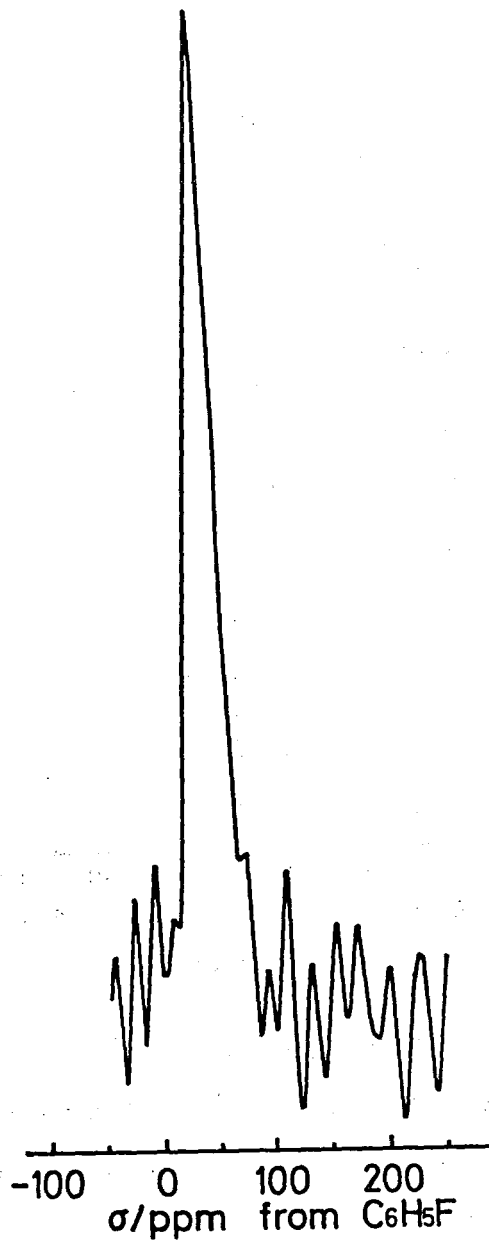


Fig. 5-1.  $^{19}\text{F}$  NMR spectrum of  $\text{sym-C}_6\text{Cl}_3\text{F}_3$  single crystal.  
This is the spectrum at  $a^*$  in Fig. 5-2.

results are shown in Fig. 5-2.

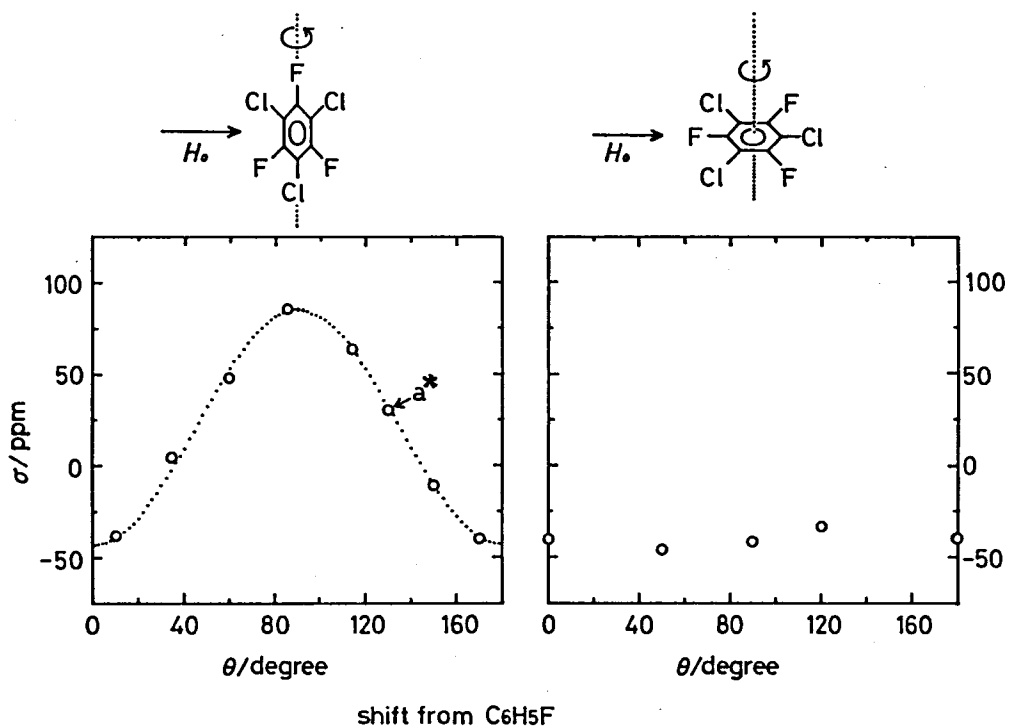


Fig. 5-2. Angular dependence of the  $^{19}\text{F}$  chemical shifts in  $\text{sym-C}_6\text{Cl}_3\text{F}_3$  single crystal at  $20^\circ\text{C}$ . The rotational pattern on the right was obtained when the Zeeman field was perpendicular to the apparent hexagonal C axis of the crystal. The pattern on the left was observed when the Zeeman field was rotated in a plane which contains the C axis. The spectrum in Fig.5-1 corresponds to the point marked a\*.

The rotational pattern on the right was obtained when the Zeeman field was perpendicular to the apparent hexagonal C axis of the crystal. There is the only one absorption line with a line width of about 30 ppm (1.2 kHz) at any rotational angle. The  $^{19}\text{F}$  chemical shift does not show the angular dependence and has the value of -43 ppm from liquid  $\text{C}_6\text{H}_5\text{F}$ . The left one was obtained when the Zeeman field was rotated in a plane which contains the C axis. Although there is the only one absorption line with a line width of about 30 ppm in spite of the change of the rotational angle, the angular dependence of the  $^{19}\text{F}$  chemical shift is observed. The chemical shift takes the maximum value of +83 ppm from liquid  $\text{C}_6\text{H}_5\text{F}$  when the Zeeman field is in the direction of the C axis.

### 5-2-3 Discussion

The  $^{19}\text{F}$  high resolution NMR spectrum on single crystal of sym- $\text{C}_6\text{Cl}_3\text{F}_3$  gave only one absorption line with a linewidth of about 30 ppm (1.2 kHz), indicating not only that all the fluorines are crystallographically equivalent but also that all the C-F bond directions are effectively averaged by some molecular motion. When the external field  $H_0$  is rotated about the apparent hexagonal C axis of the crystal, the chemical shielding tensor assumes an almost constant value (-43 ppm from liquid  $\text{C}_6\text{H}_5\text{F}$ ). On the other hand when  $H_0$  is rotated in a plane which contains the C axis, the chemical shift takes the maximum value (+85 ppm) when  $H_0$  is in the direction of the C axis. Such characteristic rotational patterns indicate that the direction of the molecular  $\text{C}_3$  axis coincides with that of the crystalline C axis and that the molecules undergo a reorientation about their  $\text{C}_3$  axes at room temperature.

The most shielded direction is perpendicular to the benzene ring as in the other fluorinated benzenes that have been reported.<sup>2)</sup> Thus, Raber and Mehring measured the shielding tensor values in analogous compounds.<sup>3)</sup> Their values as well as ours are listed in Table 5-1 and shown in Fig. 5-3.

According to Raber et al.,<sup>3)</sup> the least shielded direction is perpendicular to the C-F bond and lies in the benzene ring plane, and  $\sigma_{22}$ , which assumes a medium value, is almost constant in spite of the changes of the functional groups.  $(\sigma_{11} + \sigma_{22})/2$  in Table 5-1 and Fig. 5-3 indicates the motionally averaged value of the two directions in the molecular plane.

	$\sigma_{11}$	$\sigma_{22}$	$\sigma_{33}$	$\sigma_{av}$
1,3,5-C <sub>6</sub> H <sub>3</sub> F <sub>3</sub> <sup>3)</sup>	-79	2	63	-5
1,3,5-C <sub>6</sub> Cl <sub>3</sub> F <sub>3</sub>	$(\sigma_{11} + \sigma_{22})/2 = -43$		85	0
C <sub>6</sub> F <sub>6</sub> <sup>3)</sup>	-3	-3	155	50

Table 5-1. Chemical shift tensor elements ( $\sigma_{11} < \sigma_{22} < \sigma_{33}$ ) in ppm. Values are quoted from C<sub>6</sub>H<sub>5</sub>F. The 1-axis is perpendicular to the C-F bond, but in the molecular plane. The 2-axis is parallel to the C-F bond. The 3-axis is perpendicular to the benzene ring.

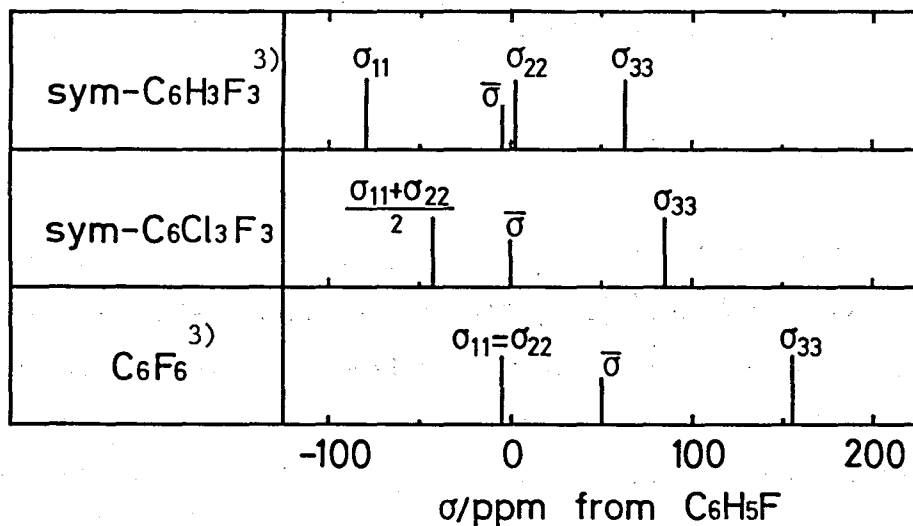


Fig. 5-3. Pictorial representation of the chemical shift tensor elements  $\sigma_{11}$ ,  $\sigma_{22}$  and  $\sigma_{33}$  of <sup>19</sup>F in fluorobenzenes.

The chemical shift tensor can be separated into two different parts according to Ramsey<sup>4)</sup>

$$\underline{\sigma} = \underline{\sigma}^d + \underline{\sigma}^p, \quad (5-1)$$

where  $\underline{\sigma}^d$  and  $\underline{\sigma}^p$  are obtained by first-order and second-order perturbation theory, respectively. A variational treatment gives similar results.<sup>5)</sup>  $\underline{\sigma}^d$  is the diamagnetic part of the chemical shift tensor and  $\underline{\sigma}^p$  is the paramagnetic part. Gierke and Flygare<sup>6)</sup> showed that  $\underline{\sigma}^d$  and  $\underline{\sigma}^p$  expanded into six terms as follows;

$$\sigma_{\alpha\beta}^d = I + II + III + IV, \quad \alpha, \beta = x, y, z \quad (5-2)$$

where

$$I = \sigma_{\text{atom}}, \quad (5-3)$$

$$II = (e^2/2mc^2) \sum_i Z_i r_i^{-3} (r_i^2 \delta_{\alpha\beta} - \alpha_i \beta_i), \quad (5-4)$$

$$III = (e^2/2mc^2) \sum_i \{ r_i^{-3} [ 2 \underline{r}_i \cdot \langle \underline{\rho} \rangle \delta_{\alpha\beta} - (\alpha_i \langle \beta_i \rangle + \beta_i \langle \alpha_i \rangle) ] - 3 r_i^{-5} (r_i^2 \delta_{\alpha\beta} - \alpha_i \beta_i) \underline{r}_i \cdot \langle \underline{\rho} \rangle_i \}, \quad (5-5)$$

$$IV = (e^2/2mc^2) \sum_i r_i^{-3} \langle \rho^2 \rangle_i (3 r_i^{-2} \alpha_i \beta_i - \delta_{\alpha\beta}), \quad (5-6)$$

and

$$\sigma_{\alpha\beta}^p = V + VI, \quad (5-7)$$

where

$$V = (e^2/2mc^2) (C_{\alpha\beta} I_{\alpha\beta} e) / (e \hbar \mu_0 g_I), \quad (5-8)$$

$$VI = -(e^2/2mc^2) \sum_i Z_i r_i^{-3} (r_i^2 \delta_{\alpha\beta} - \alpha_i \beta_i) = -II. \quad (5-9)$$

Here  $\underline{r}_i = (x_i, y_i, z_i)$  is the vector from the shielded nucleus to a neighbouring atom  $i$ , with the atom dipole moment  $\langle \underline{\sigma} \rangle_i$  and the atom electronic second moment  $\langle \rho^2 \rangle_i$ , and  $\delta_{\alpha\beta}$  is the Kronecker's symbol.  $Z_i$  is the atomic number of nucleus  $i$  and the sum  $\sum_i$  runs over all nuclei with the shielded nucleus being omitted. All terms I - IV can be calculated once the molecular structure and the atomic dipole moment and electronic second moment are known. The term

V, however, is related to a different molecular quantity, namely  $C_{\alpha\beta}$ , which is the spin rotation interaction tensor, where  $I_{\alpha\beta}$  is the corresponding moment of inertia,  $\mu_0$  is the nuclear magneton and  $g_I$  is the nuclear  $g$  value of the shielded nucleus. All other parameters have their usual meaning. The tensor  $\sigma_{\alpha\beta}$  can be diagonalized and the principal axes and the principal elements are readily evaluated, once all the molecular parameters are known.

The terms II - IV are calculated according to Eqs. (5-4) and (5-6) for sym-C<sub>6</sub>Cl<sub>3</sub>F<sub>3</sub> discussed in this work by using the appropriate dipole moments,<sup>6)</sup> second moments<sup>6)</sup> and the molecular structure deduced from analogous compounds such as C<sub>6</sub>F<sub>6</sub> and C<sub>6</sub>Cl<sub>6</sub><sup>7)</sup>. All angles and bond lengths have been taken to be standard values. The so-called dipolar contribution (term III) amounts to about 1 ppm and may be neglected in an analysis of this kind. Term II and IV cancel each other and reflect the gauge dependence of  $\sigma^d$  and  $\sigma^p$ . The spin-rotation interaction term V can then be estimated by means of the experimental shift tensor and term I, III, and IV as

$$\begin{aligned} V = \sigma^{p'} &= \sigma_{exp} - (I + III + IV) = \sigma_{exp} - (I + IV) \\ &= \sigma_{exp} - \sigma^{d'} \end{aligned} \quad (5-10)$$

The "gauge independent" terms  $\sigma^{d'}$  and  $\sigma^{p'}$  were calculated according to Eqs. (5-6)-(5-10). In this calculation the  $\sigma_{22}$  component of <sup>19</sup>F in sym-C<sub>6</sub>Cl<sub>3</sub>F<sub>3</sub> is assumed to be the same as that in sym-C<sub>6</sub>H<sub>3</sub>F<sub>3</sub> since the  $\sigma_{22}$  components in several fluorinated benzenes have almost the same value. The calculated values are listed in Table 5-2 together with Raber's<sup>3)</sup> value for analogous compounds.

	$\sigma_{11}^{d'}$	$\sigma_{22}^{d'}$	$\sigma_{33}^{d'}$	$\sigma_{11}^{p'}$	$\sigma_{22}^{p'}$	$\sigma_{33}^{p'}$
1,3,5-C <sub>6</sub> H <sub>3</sub> F <sub>3</sub> <sup>3)</sup>	161	191	157	-240	-189	-94
1,3,5-C <sub>6</sub> Cl <sub>3</sub> F <sub>3</sub>	162	191	157	-250	-189	-72
C <sub>6</sub> F <sub>6</sub> <sup>3)</sup>	162	192	156	-165	-194	-1

Table 5-2. Calculated values of  $\sigma^{d'}$  and  $\sigma^{p'}$ .

The  $\sigma^{d'}$  value is not very sensitive to substitution on the benzene ring, as can be seen in Table 5-2. This is due to the fact, that the quadrupolar term (IV), Eq. 5-6, does not depend very much on substitution "far away" from the nucleus considered, since this interaction falls off as  $r^{-3}$ . Then, all the experimentally observed variations in the chemical shift tensor are attributed to  $\sigma^{p'}$ . The change in  $\sigma_{22}^{p'}$  is very small and gradual change can be seen in  $\sigma_{11}^{p'}$  and  $\sigma_{33}^{p'}$  components.  $\sigma^{p'}$  can be related to the spin-rotation coupling constant through Eq. 5-8. The knowledge of the spin-rotation interaction tensor is generally of considerable interest in molecular physics, but in the case of large molecules like fluorobenzenes the extraction of the spin-rotation interaction tensor from the molecular beam experiments is extremely difficult and even in the simplest case, namely mono-fluorobenzene, no reliable data can be obtained. Spin-lattice relaxation of <sup>19</sup>F or <sup>1</sup>H via chemical shift anisotropy and spin-rotation interaction 8) has also attracted much attention.



Here, the relation between the reorientational correlation time is of considerable interest, concerning different models for molecular motion each other. A better knowledge of the chemical shift tensor and the spin-rotation interaction tensor will help to elucidate the feature of the molecular motion.

Substitution from Eq. 5-8 into Eq. 5-10 gives

$$c_{\alpha\beta}/h = (me\mu_o g_I) / (\pi e I_{\alpha\beta}) \sigma_{\alpha\beta} P'. \quad (5-11)$$

The values used in calculating the spin-rotation interaction tensor are as follows:

$$\begin{aligned} m &= 9.108 \times 10^{-28} \text{ g} \\ e &= 2.998 \times 10^{10} \text{ cm/s} \\ e &= 4.803 \times 10^{-10} \text{ e.s.u.} \\ \mu_o &= 5.050 \times 10^{-24} \text{ erg/gauss} \\ g_F &= 5.257 \\ R(\text{C-F}) &= 1.327 \times 10^{-8} \text{ cm} \\ R(\text{C-C}) &= 1.394 \times 10^{-8} \text{ cm} \\ R(\text{C-Cl}) &= 1.715 \times 10^{-8} \text{ cm.} \end{aligned}$$

The calculated values are as follows,

$$\begin{aligned} c_{11}/h &= -910 \text{ Hz} \\ c_{22}/h &= -687 \text{ Hz} \\ c_{33}/h &= -131 \text{ Hz} \\ \bar{c}_{av}/h &= -576 \text{ Hz} \\ \sqrt{c^2}/h &= [(c_{11}^2 + c_{22}^2 + c_{33}^2)/3]^{1/2}/h = 662 \text{ Hz.} \end{aligned}$$

To obtain some insight into the electronic structure of the molecule, the theory of Karplus and Das<sup>9)</sup> will be used. According to them, one of the paramagnetic components is given by

$$\sigma_{xx}^{(p)} = (3/2) \sigma_o (P_{yy} + P_{zz} - P_{yy} P_{zz}). \quad (5-12)$$

Here,  $y$  and  $z$  components are given by cyclic permutation for the indices for the  $y$ - and  $z$ - components in Eq. 5-12, where the  $P_{ii}$  represent populations of the fluorine p orbitals and the cross terms  $P_{ij}$  are neglected.  $\sigma_o$  is a semiempirical parameter as defined by Karplus and Das.<sup>9)</sup> For a closed shell ion all the  $P_{ii}$  should be two, leading to zero paramagnetic shift. Thus a deviation from a closed shell form can lead to a nonvanishing paramagnetic shift.

Since experimentally observed variations in the chemical shifts are loaded onto the paramagnetic contribution as mentioned above in relation to  $\sigma^P$ , only the paramagnetic part may be considered to investigate the electronic structure.

A deviation from the closed shell ion state will be defined by the following parameters:

$$\begin{aligned} P_{xx} &= 2 - \rho_x, \\ P_{yy} &= 2 - \rho_y, \\ P_{zz} &= 2 - \epsilon, \end{aligned} \quad (5-13)$$

where  $\rho_x$  and  $\rho_y$  may be taken as the double bond characters and

$$\epsilon = 1 - (I + S - IS), \quad (5-14)$$

where  $I$  is the ionic character and  $S$  the degree of sp hybridization.

Eq. (5-15) is derived from Eqs. (5-13) and (5-14).

$$\sigma_{xx}^P = (3/2)\sigma_o[\epsilon + \rho_y(1 - \epsilon)] \quad (5-15a)$$

$$\sigma_{yy}^P = (3/2)\sigma_o[\epsilon + \rho_x(1 - \epsilon)] \quad (5-15b)$$

$$\sigma_{zz}^P = (3/2)\sigma_o(\rho_x + \rho_y - \rho_x\rho_y). \quad (5-15c)$$

Here, the  $x$  axis is perpendicular to the benzene ring, the  $y$  axis lies in the plane and is perpendicular to the C-F bond, and the  $z$  axis is parallel to the C-F bond. Assuming that the double bond

character of a C-F bond within the plane ( $\rho_y$ ) is zero, simpler relations are obtained as follows,

$$\sigma_{xx}^P = (3/2)\sigma_o \epsilon, \quad (5-16a)$$

$$\sigma_{yy}^P = (3/2)\sigma_o [\epsilon + \rho_x(1 - \epsilon)], \quad (5-16b)$$

$$\sigma_{zz}^P = (3/2)\sigma_o \rho_x. \quad (5-16c)$$

Since the most shielded direction is in the  $z$  axis direction,  $\sigma_{xx}$  is equal to  $\sigma_{33}$ . The bond direction component ( $\sigma_{zz}$ ) is  $\sigma_{22}$ , and  $\sigma_{yy}$  is  $\sigma_{11}$ . Comparing the chemical shift tensor components of the three compounds listed in Table 5-1 and Table 5-2 or shown in Fig. 5-3, the  $z$  component  $\sigma_{zz}$  ( $\sigma_{22}$ ) is almost insensitive to substitution. This indicates little change of  $\rho_x$ , that is,  $\pi$  electron density is not affected by the substitution. The component  $\sigma_{xx}$  is gradually increased from sym-C<sub>6</sub>H<sub>3</sub>F<sub>3</sub> to C<sub>6</sub>F<sub>6</sub>. This indicates a gradual change of  $\epsilon$  in Eq. (5-16a).  $\epsilon$  becomes small since  $\sigma_{xx}^P$  is negative. The decrease in  $\epsilon$  is attributed to an increase of the ionic character  $I$  of Eq. (5-14). This shows that the ionicity in a C-F bond is gradually increased on going from sym-C<sub>6</sub>H<sub>3</sub>F<sub>3</sub>, sym-C<sub>6</sub>Cl<sub>3</sub>F<sub>3</sub>, to C<sub>6</sub>F<sub>6</sub>. The gradual change of  $\sigma_{xx}$  and the insensitivity of  $\sigma_{zz}$  by substitution indicate that in these aromatic conjugated system  $\pi$  electron density is changed little and  $\sigma$  electron density varies gradually by substitution. Above conclusion agrees with the <sup>13</sup>C NMR measurement by Miyajima.<sup>10)</sup> He showed that  $\pi$  electron density on <sup>13</sup>C is not affected by substitution, on the other hand  $\sigma$  electron density is affected easily by substitution.

#### 5-2-4 Molecular Motion and Phase Transition

As is mentioned in Section 5-2-3 it was found during the analysis of  $^{19}\text{F}$  chemical shift rotational patterns that the molecules undergo reorientation about its  $C_3$  axis at room temperature. In order to obtain more detailed information about the molecular motion in this compound, the spin-lattice relaxation time ( $T_1$ ) and the spin-spin relaxation time ( $T_2$ ) of  $^{35}\text{Cl}$  NQR were measured by the use of MATEC 5800 pulsed NQR spectrometer system. The results are given in Fig. 5-4. The gross feature concerning  $T_1$  agrees with a previous result <sup>11)</sup>. The  $T_1$  can be reproduced by the relation

$$T_1^{-1} = 3.27 \times 10^{-4} T^2 + 3.90 \times 10^{-6} \exp(-18.2 \text{ kJmol}^{-1}/RT).$$

The activation energy of  $18.2 \pm 0.5 \text{ kJmol}^{-1}$  is slightly larger than that by Alymov et al. <sup>11)</sup> ( $17.7 \text{ kJmol}^{-1}$ ). The decrease in  $T_2$  above 200 K is an evidence of marked line broadening above this temperature.

The correlation time,  $\tau_c$ , for the three-fold reorientation is calculated by using the relation <sup>12)</sup>

$$\tau_c = (3/4)T_1$$

and is plotted in Fig. 5-5. This figure shows that  $\tau_c$  at 300 K is about  $5 \times 10^{-4}$  s, which is just the order of magnitude to average out the parallel components of the  $^{19}\text{F}$  shielding tensor being consistent with the conclusion of the  $^{19}\text{F}$  high-resolution NMR study mentioned above.

Heat capacity measurements <sup>13)</sup> found a higher order phase transition in this substance at  $T_c = 296.7 \text{ K}$  with an enthalpy change of only  $18 \text{ Jmol}^{-1}$ .  $^{35}\text{Cl}$  NQR frequency up to the melting

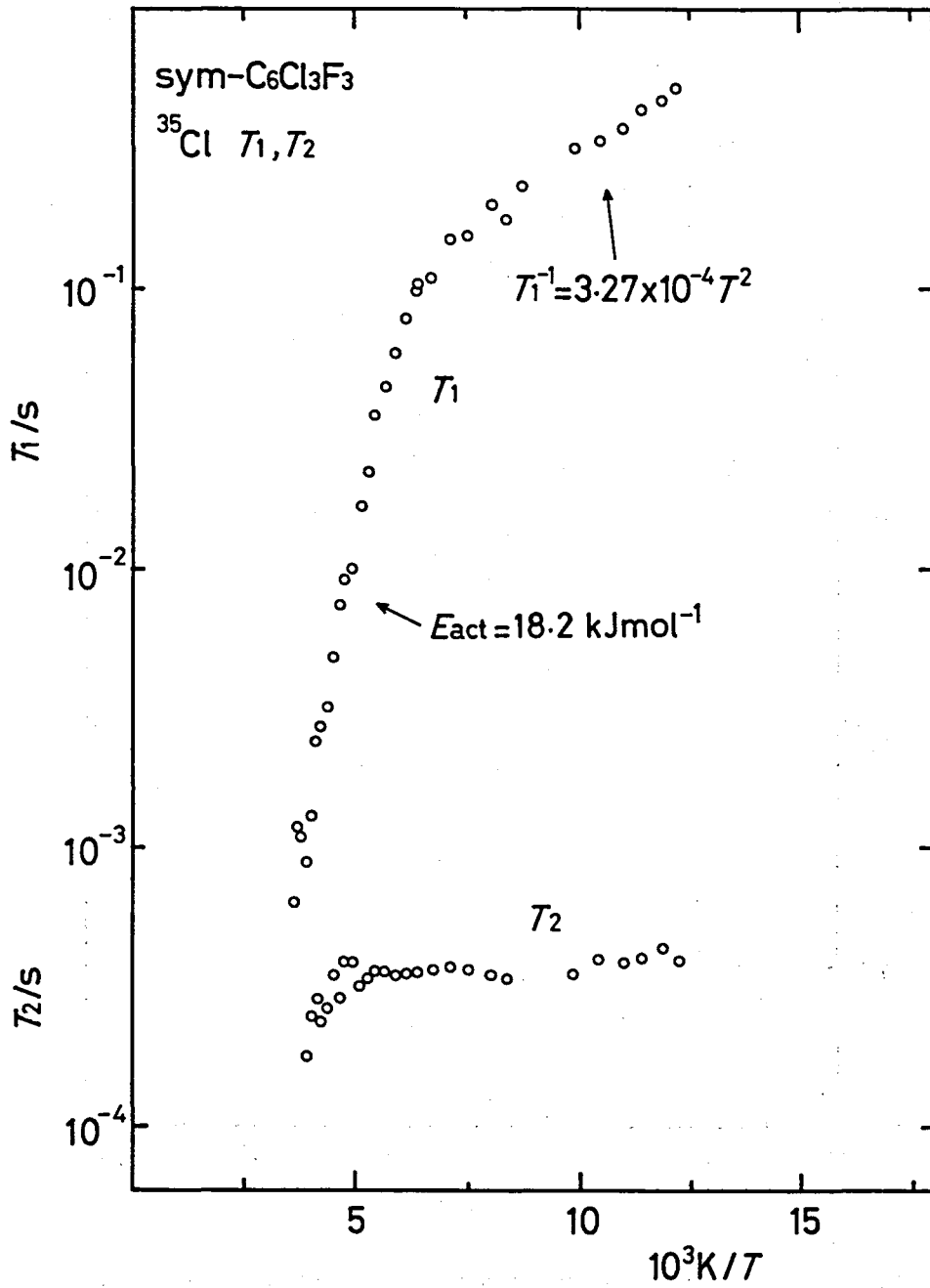


Fig. 5-4. The spin-lattice and spin-spin relaxation times of <sup>35</sup>Cl.

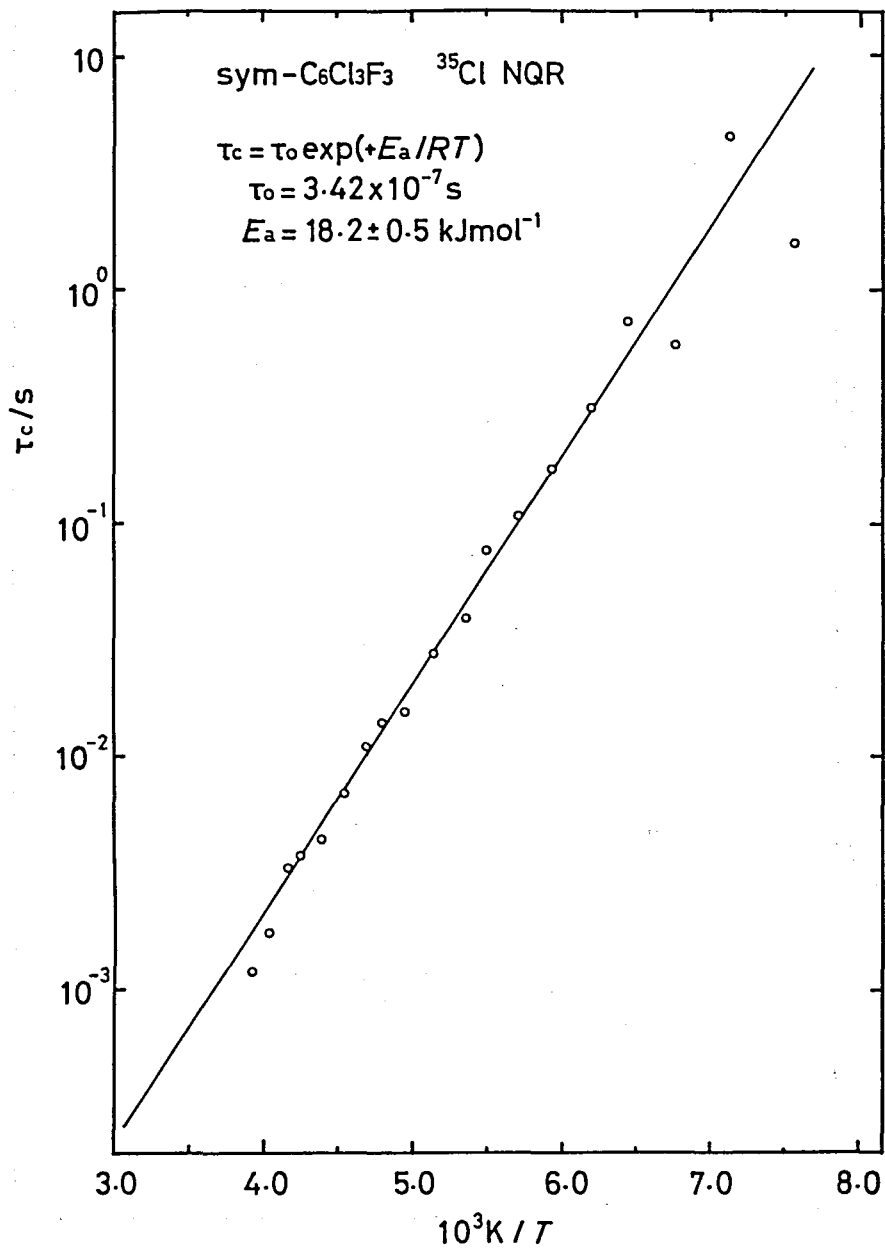


Fig. 5-5. The correlation time for the three-fold reorientation of molecules.

point (334.9 K)<sup>13)</sup> was measured as shown in Fig. 5-6. The NQR frequency shows only small deflection at  $T_c$ , suggesting the absence of any large structural change at the phase transition.

$^{19}\text{F}$  high resolution NMR spectra in powdered specimen were observed from 293 K to 337 K ( up to just above the melting point ). The spectra show no discernible change from 293 K up to the melting point. This indicates that no other kind of molecular motion than the three-fold molecular reorientation is excited in the high-temperature phase in the time scale of  $^{19}\text{F}$  high resolution spectra ( 130 ppm = 5.2 kHz ).

Raman spectra were also collected on a single crystal at 289 K and 308 K. No detectable change in the spectra was observed in the frequency range between  $0\text{ cm}^{-1}$  and  $1000\text{ cm}^{-1}$ .

Hence the mechanism of the phase transition in this material will remain unsolved.

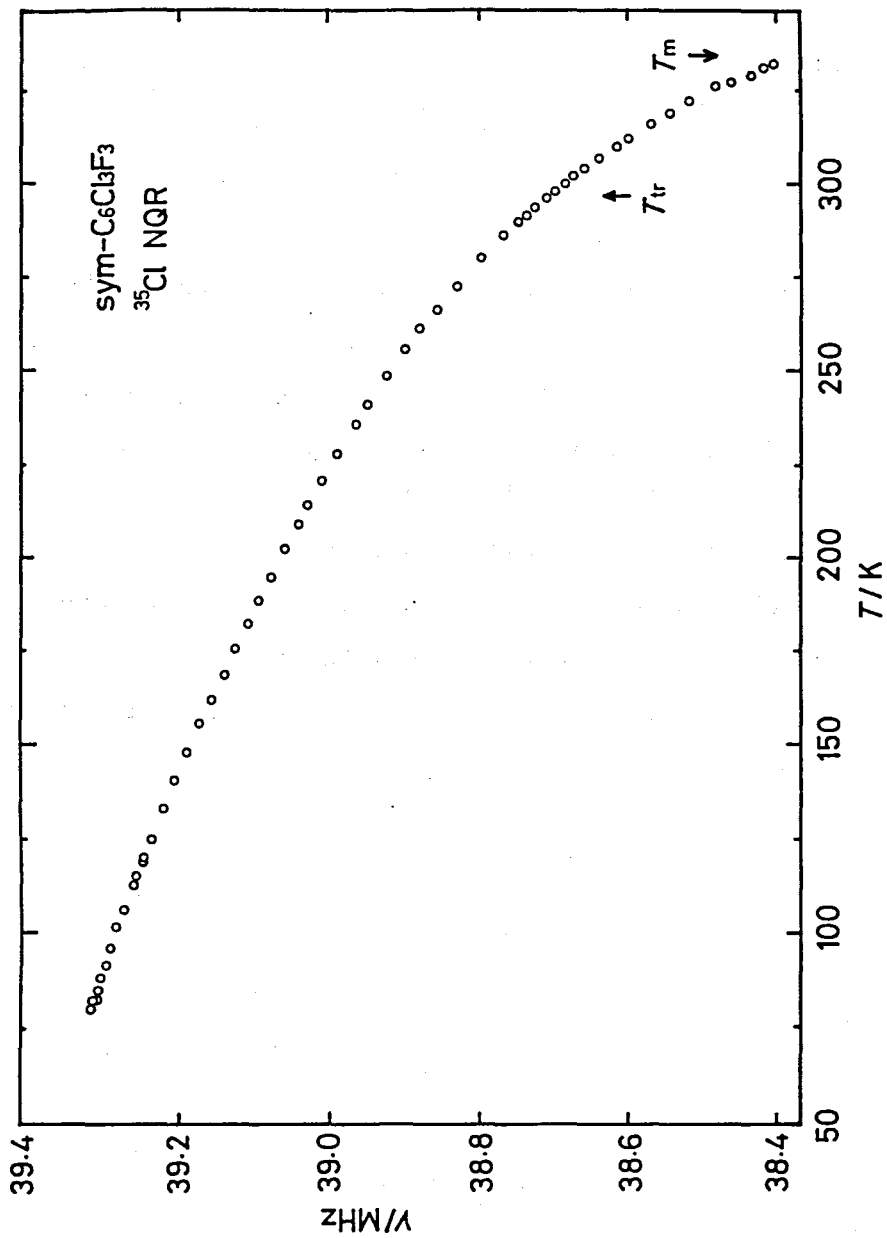


Fig. 5-6. Temperature dependence of <sup>35</sup>Cl NQR frequency.



## 5-2-5 Conclusion

High resolution  $^{19}\text{F}$  NMR spectrum was observed in a single crystal of  $\text{sym-C}_6\text{Cl}_3\text{F}_3$  and chemical shift shielding tensor was obtained. The most shielded direction is perpendicular to the molecular plane and the least shielded direction is perpendicular to the C-F bond but lying in the molecular plane. The most shielded component ( $\sigma_{33}$ ) and the least shielded component ( $\sigma_{11}$ ) change gradually from  $\text{sym-C}_6\text{H}_3\text{F}_3$  to  $\text{C}_6\text{F}_6$ , whereas the C-F bond direction component ( $\sigma_{22}$ ) changes little. From the analysis of components of shielding tensors in  $\text{sym-C}_6\text{H}_3\text{F}_3$ ,  $\text{sym-C}_6\text{Cl}_3\text{F}_3$ , and  $\text{C}_6\text{F}_6$ , it is indicated that  $\pi$ -electron density distribution varies little and  $\sigma$ -electron density is changed when other substituents are introduced.

At 20  $^\circ\text{C}$ ,  $\text{sym-C}_6\text{Cl}_3\text{F}_3$  molecule rotates about its figure axis and its motion averages out the shielding tensor components within the molecular plane.

### 5-3 $K_2MF_6$ type compounds ( M = Si, Ge, Sn )

#### 5-3-1 Lattice types and site symmetry of $K_2MF_6$ type compounds

At room temperature the lattice type of  $K_2SiF_6$  is cubic<sup>14)</sup> and others are trigonal<sup>15)</sup> as shown in Figs. 5-7 and 5-8. The site symmetry of Si in the former compound is  $O_h$  and that of the central atoms in the other compounds is  $D_{3d}$ . Although the latter two anions have  $D_{3d}$  site symmetry in the solid state, each  $(M)^{4+}$  ion is surrounded by six  $F^-$  ions which form a slightly distorted regular octahedron. All the anions in the crystal are equivalent. Since  $MF_6^{2-}$  ions are of approximately regular octahedron, the electron distributions in the M-F bond may be close to the axial symmetry. Therefore, axially symmetric chemical shift tensors will be expected.

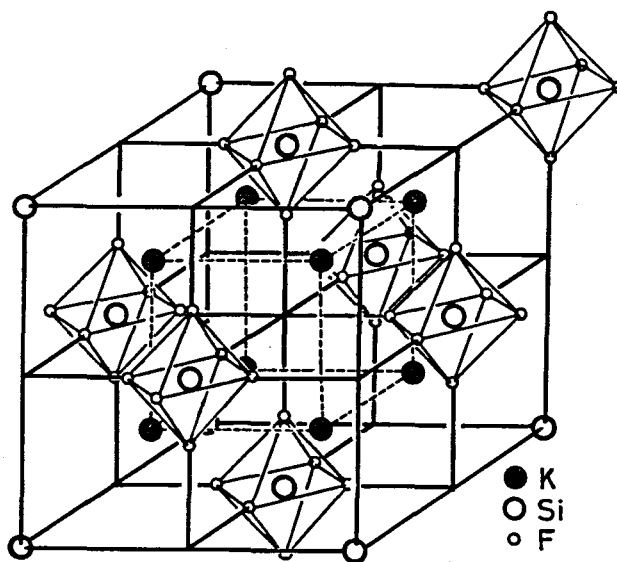


Fig. 5-7. Structure of  $K_2SiF_6$ .

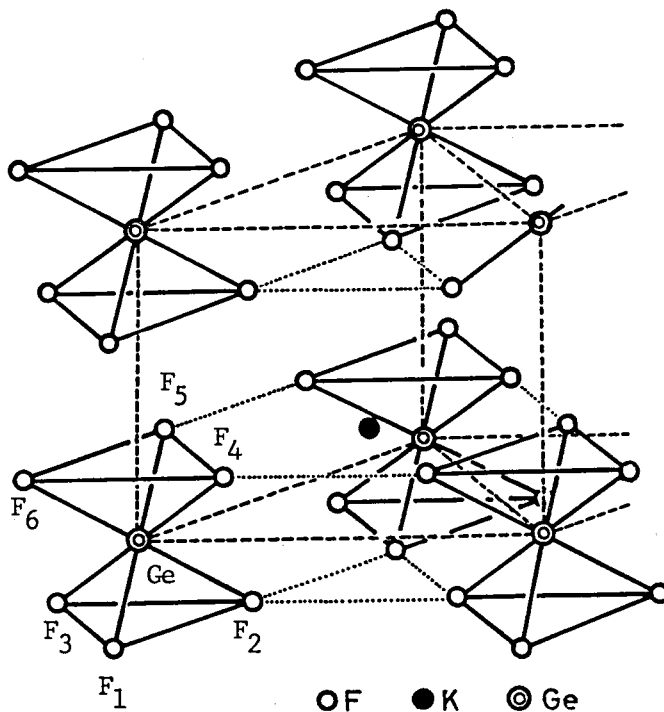


Fig. 5-8. Structure of  $K_2GeF_6$ .  $K_2SnF_6$  is isomorphous with it.

### 5-3-2 Potassium Hexafluorosilicate

#### (1) Sample preparation and measurements

Powdered  $K_2SiF_6$  was purchased from Nakarai Chemical Co. Ltd. An attempt to grow a single crystal by slow evaporation of 47 % hydrofluoric acid solution was unsuccessful.

Using powdered specimen, multiple pulse NMR measurements were made with MREV-8 pulse method at 300 K and 77 K. The spectra

obtained are shown in Fig. 5-9. The spectrum at 300 K has the half height width of 30 ppm and seems to have no anisotropy. The line width at 77 K is larger than that at 300 K and a small hump appears on the high field side.

## (2) Determination of Chemical Shift Tensor

The spin-lattice relaxation time of  $^{19}\text{F}$  was measured in the temperature range between 77 K and 973 K by Moskvich et al.<sup>16)</sup> According to their data, the reorientational correlation time  $\tau_c$  is about  $10^{-6}$  s at 300 K and so it is expected that the motion fully contributes to the averaging the shielding anisotropy. The spectrum observed at 300 K in this work shows a single peak as shown in Fig. 5-9, being explained by the above averaging effect according to Moskvich et al. The value for  $\sigma_{iso}$  (isotropic averaged value) obtained is 16 ppm from liquid  $\text{C}_6\text{H}_5\text{F}$ .

The spectra at 77 K does not show the rigid lattice powder pattern (usual powder pattern of rigid lattice is shown in Fig.3-1), as shown in Fig. 5-9. There is a small hump on the high field side. The reorientational motion at this temperature is thought to affect the line shape. Spiess et al.<sup>17)</sup> calculated the powder pattern for trigonal bipyramidal, tetrahedral, and octahedral molecules with axially symmetric shielding tensor in the presence of some characteristic molecular motion in each case. They showed that a small hump may appear when the reorientational correlation time of molecule approaches the inverse of the chemical shift anisotropy under the hypothesis of jump model. The powder pattern

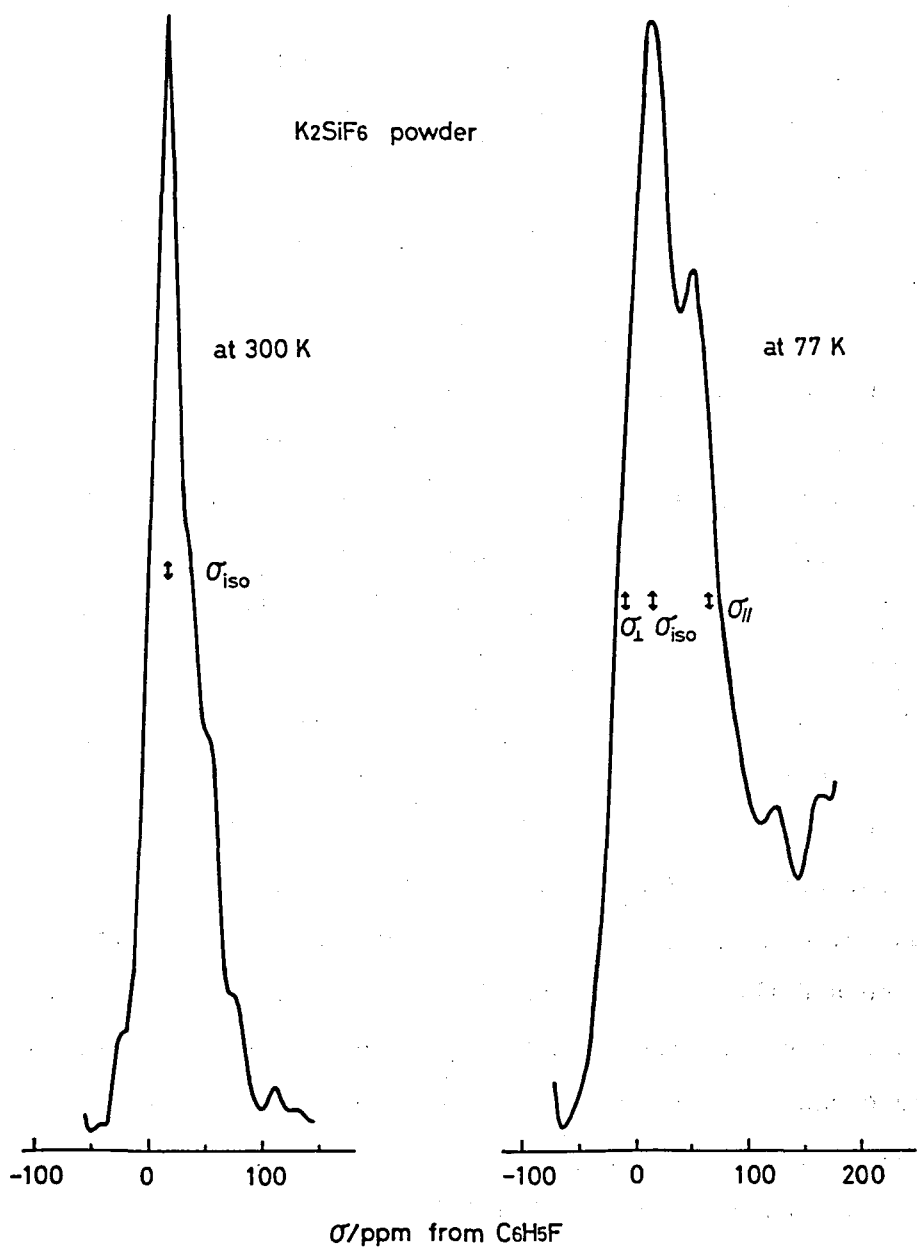


Fig. 5-9. High resolution NMR spectra of <sup>19</sup>F in powdered K<sub>2</sub>SiF<sub>6</sub> at 300 K and 77 K. The reference material for the chemical shift is liquid C<sub>6</sub>H<sub>5</sub>F.

of  $K_2SiF_6$  at 77 K can be reasonably explained by this isotropical jump model. From the line shape analysis, the jump frequency is estimated to be about 3 kHz and correlation time is about  $3.3 \times 10^{-3}$  s, and  $\sigma_{33}$  is 66 ppm and  $\sigma_{22} = \sigma_{11}$  is -9 ppm from liquid  $C_6H_5F$ .

$SiF_6^{2-}$  ion is considered to form regular octahedron in crystal, and so chemical shielding tensor is expected to be axially symmetric. This expectation agrees well with the spectral analysis. The symmetry axis of axially symmetric shielding tensor coincides with the Si-F bond. Therefore one can consider two types of axially symmetric shielding tensors as shown in Fig. 5-10. In type (a) the maximum shielding direction is along the Si-F bond, and in this case the powder pattern of type (a)' is obtained. In the case of type (b) the least shielded direction is along the Si-F bond, and the powder pattern should be of the type (b)'. The analysis of the powder pattern of  $K_2SiF_6$  indicates that the shielding tensor in this compound belongs to type (a). Its most shielded direction is along Si-F bond and the value of the chemical shift is 60 ppm. Its least shielded direction, on the other hand, is perpendicular to the Si-F bond and its chemical shift is -9 ppm.

Chemical shift of  $SiF_6^{2-}$  in solution<sup>18)</sup> is 12 ppm, a little smaller than that in solid (16 ppm).

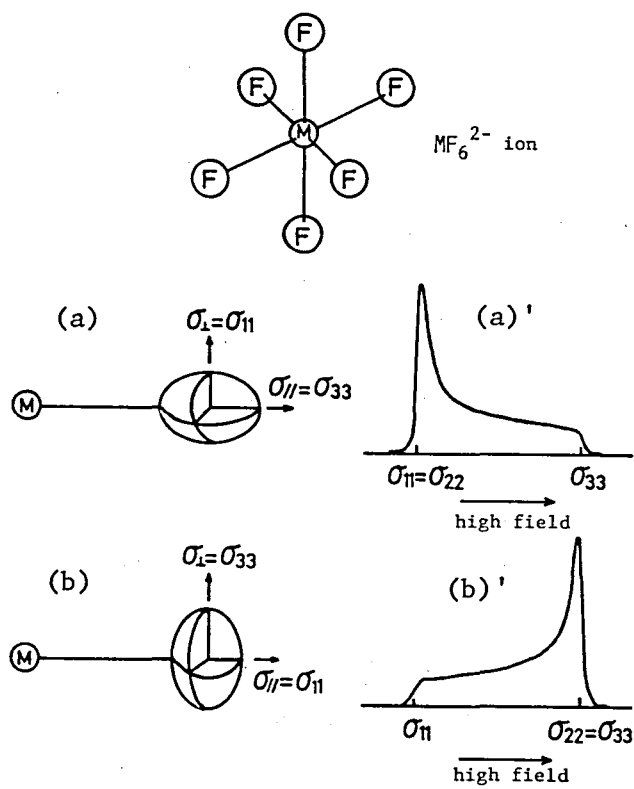


Fig. 5-10. Axially symmetric shielding tensor models and their powder pattern.

### 5-3-3 Potassium Hexafluorogermanate

#### (1) Sample Preparation and Measurements

A colorless, transparent single crystal of the shape of hexagonal plate was obtained by slow evaporation at 30 °C of solution prepared by dissolving stoichiometric  $\text{GeO}_2$  and  $\text{KF}$  in 47 % hydrofluoric acid. This substance is trigonal,  $D_{3d}^3$ , with  $Z=1$  and the  $C_3$  axis of  $\text{GeF}_6^{2-}$  ion is parallel to the crystallographic  $C_3$  axis <sup>15)</sup> (see Fig. 5-8).

$\text{GeF}_6^{2-}$  ion is of the shape of nearly regular octahedron and so all  $\text{F}^-$  ions have the same axially symmetric shielding tensors. Therefore a pair of lines will be observed if the Zeeman field is rotated in the plane on which  $\text{F}_5$ ,  $\text{Ge}$ , and  $\text{F}_1$  atoms and the  $C_3$  axis sit (see Fig. 5-8). If the Zeeman field is rotated in the plane, the rotational patterns will consists of two curves and their intensity ratio will be 2:1. One typical example of the FID signals is shown in Fig. 5-11, and its FFT spectrum in Fig. 5-12. FID signals were accumulated 20 times every 1800 s, and then Fourier-transformed. The Zeeman field was rotated in the above mentioned plane of the crystal and the rotational patterns in Fig. 5-13 were obtained. The intensity of the line labelled by A is stronger than that of B. The intensity ratio is approximately 2:1.

#### (2) Determination of Chemical Shift Tensor

Sergienko et al. <sup>19)</sup> observed the second moment of  $^{19}\text{F}$  as a function of temperature and concluded that neither the isotropic



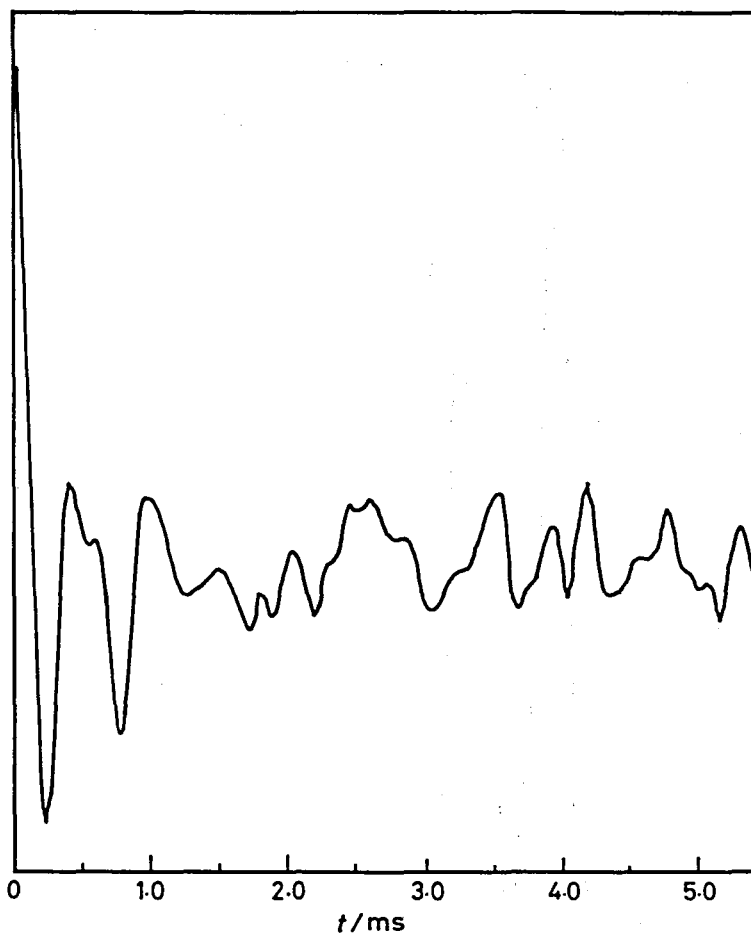


Fig. 5-11. FID signal of  $^{19}\text{F}$  in  $\text{K}_2\text{GeF}_6$  single crystal.

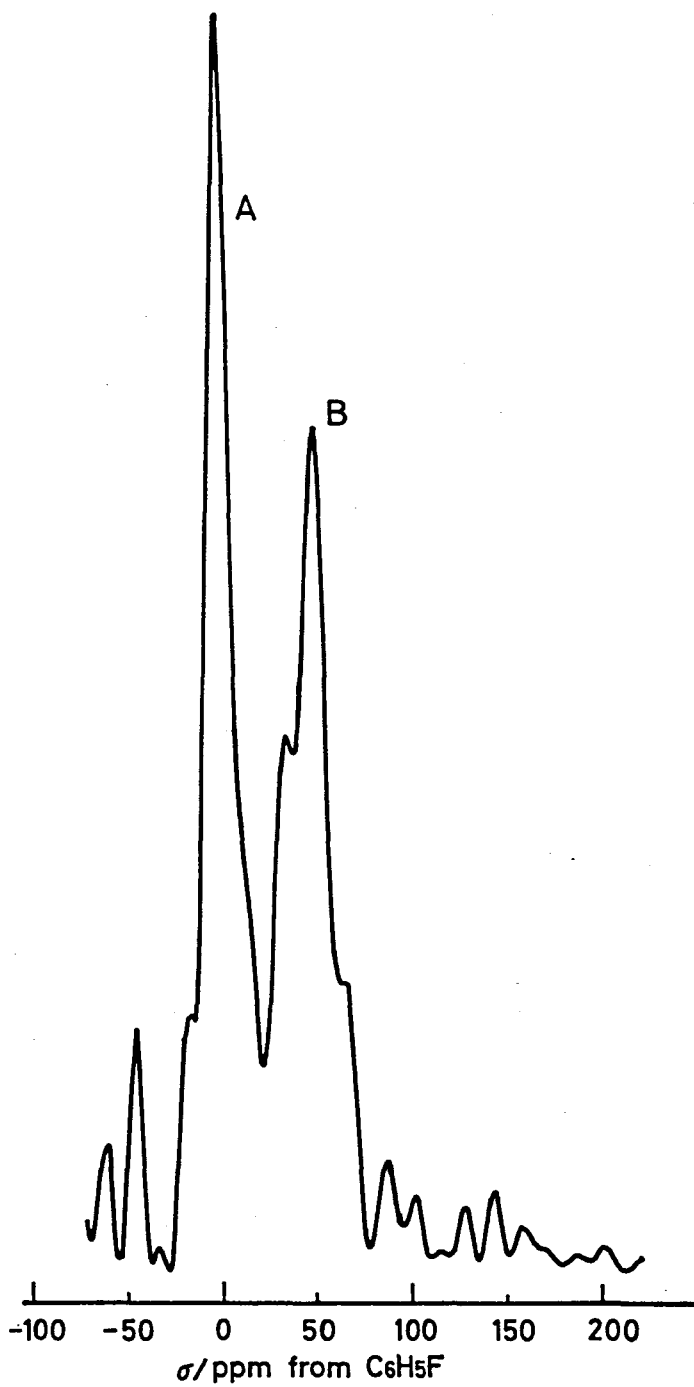


Fig. 5-12.  $^{19}\text{F}$  NMR spectrum obtained by FFT of FID in Fig. 5-11. The spectrum corresponds to the orientation,  $a^*$ , in Fig. 5-13.

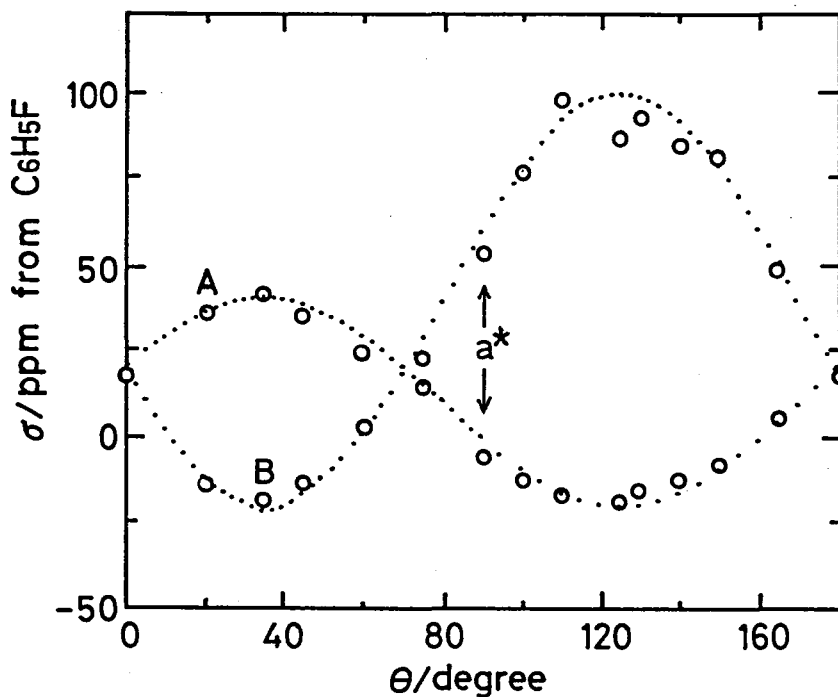


Fig. 5-13. Angular dependence of the  $^{19}\text{F}$  chemical shifts of  $\text{K}_2\text{GeF}_6$  single crystal. This pattern was obtained when the Zeeman field was rotated in the plane on which  $\text{F}_5$ ,  $\text{Ge}$ , and  $\text{F}_1$  atoms and the  $\text{C}_3$  axis sit (see Fig. 5-8). The  $\text{C}_3$  axis and the Zeeman field become parallel at  $0^\circ$ .

rotation nor the free rotation of the anion about an axis was "seen" even at 300 K. The above mentioned high resolution spectra also indicated no anion motion. Therefore, the shielding tensor can easily be determined from the rotational pattern of Fig. 5-13. The dotted lines in Fig. 5-13 are the calculated ones on the assumption of axially symmetric shielding tensor. The maximum point of the curve B corresponds to a Ge-F bond direction, that is, at this orientation the  $F_1$ -Ge- $F_5$  bond (see Fig. 5-8) is parallel to the Zeeman field, where the curve A has a minimum value and the Zeeman field is perpendicular to both  $F_2$ -Ge- $F_6$  and  $F_3$ -Ge- $F_4$  bonds. Therefore, similarly to the case of  $K_2SiF_6$ , the most shielded direction is along Ge-F bond, and the least shielded direction is perpendicular to the Ge-F bond. Their shift values are 99 ppm and -20 ppm from the resonance of liquid  $C_6H_5F$ , respectively. The mean value of the chemical shift is 19 ppm, being fairly larger than that in solution (7 ppm).<sup>18)</sup>

#### 5-3-4 Potassium Hexafluorostannate

##### (1) Sample Preparation and Measurements

A  $K_2SnF_6$  single crystal (about 10 mm x 10 mm x 10 mm) was obtained by slow evaporation at 30 °C of its solution prepared by dissolving  $K_2Sn(OH)_6$  into 47 % hydrofluoric acid. The crystal belongs to a trigonal system and the site symmetry of Sn is  $D_{3d}$ .

Multiplu pulse measurements were made with the same expectation as in the case of  $K_2GeF_6$  single crystal. Although several attempts were made at 300K, an angular dependence of the chemical shift was not observed. Then the powder patterns at 77 K and 300 K were obtained due to make sure that there is a motional effect to the chemical shift at 300 K. The powder patterns are shown in Fig. 5-14. The spectrum at 300 K shows a single peak, whereas at 77 K, the spectrum gives an asymmetric line shape.

##### (2) Determination of Chemical Shielding Tensor

The powder pattern at 300 K shows a single peak. It is considered that the correlation time of the reorientational motion of  $SnF_6^{2-}$  ion is so short compared with the magnitude of the chemical shift anisotropy that the anisotropic part of the chemical shift may be averaged out. This agrees with the result that the rotational pattern showed no angular dependence. The correlation time at 300 K could be similar to that in  $K_2SiF_6$ . From the powder pattern at 77 K,  $\sigma_{//} = 183$  ppm and

$\sigma_{\perp} = 33$  ppm were obtained on the assumption of axially symmetric shielding tensor. Isotropic average value  $\sigma_{iso}$  is 83 ppm. To compare with this value, the value in solution  $\sigma_{sol}$  is 40 ppm. The fact that  $\sigma_{iso}$  is much larger than  $\sigma_{sol}$  suggests that the interaction in solid is stronger than in solution.

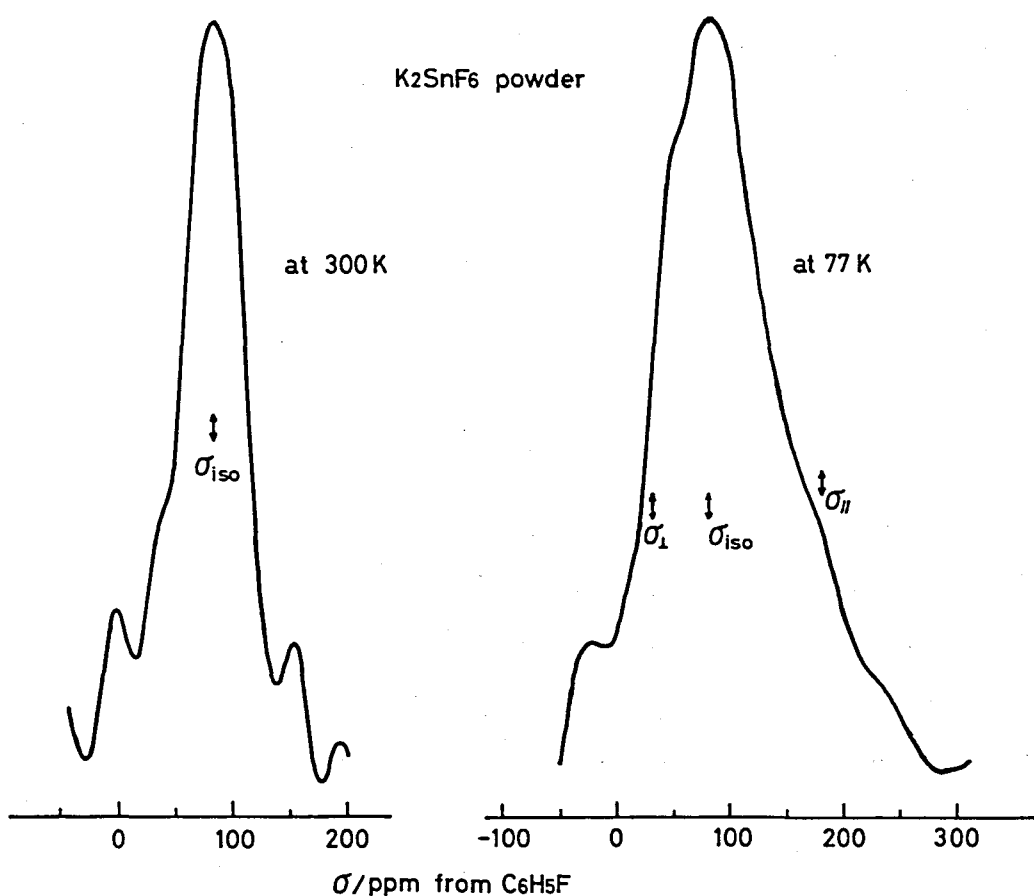


Fig. 5-14. Powder pattern of  $^{19}\text{F}$  NMR in  $\text{K}_2\text{SnF}_6$ .

### 5-3-5 Discussion

Silicon compounds show an outstanding behavior in the several type of measurements on the compounds of IVb sub-group elements ( C, Si, Ge, Sn, Pb ). Some of the results will be reviewed here briefly.

( i ) nuclear quadrupole resonance of  $^{35}\text{Cl}$ ,  $^{63}\text{Br}$ ,  $^{127}\text{I}$  in  $\text{MX}_4$  ( M = C, Si, Ge, Sn; X = Cl, Br, I )<sup>20)</sup>

The nuclear quadrupole resonance frequencies in a series of  $\text{MX}_4$  ( M = C, Si, Ge, Sn; X = Cl, Br, I ) compounds does not follow the law of corresponding state: anomalous decrease of the nuclear quadrupole resonance frequency in Si-compounds relative to C- and Ge- compounds has been observed so far.

( ii )  $^{35}\text{Cl}$  NMR in  $\text{MCl}_4$  ( M = C, Si, Ge, Sn )<sup>21)</sup>

Although  $^{35}\text{Cl}$  chemical shifts in  $\text{MCl}_4$  ( M = C, Si, Ge, Sn ) is expected to increase from  $\text{CCl}_4$  to  $\text{SnCl}_4$ , the shift in  $\text{SiCl}_4$  is anomalously high comparative to that in  $\text{GeCl}_4$ .

( iii )  $^{19}\text{F}$  NMR in  $\text{MF}_4$  and  $\text{NH}_4\text{MF}_6$  in solutions ( M = C, Si, Ge, Sn )<sup>18,22)</sup>

$^{19}\text{F}$  chemical shifts in  $\text{MF}_4$  as well as in  $\text{NH}_4\text{MF}_6$  in solutions ( M = C, Si, Ge, Sn ) are expected to shift gradually to high field side from C-, Si-, Ge- to Sn-compounds. However, the shifts in  $\text{SiF}_4$  and  $\text{NH}_4\text{SiF}_6$  are anomalously high and nearly equal to that in germanium compounds.

( iv )  $^{13}\text{C}$  NMR in  $(\text{CH}_3)_4\text{M}$  solution ( M = C, Si, Sn, Pb )<sup>23)</sup>

As a whole,  $^{13}\text{C}$  chemical shifts in  $(\text{CH}_3)_4\text{M}$  in solution ( M = C, Si, Ge, Pb ) gradually increase from  $(\text{CH}_3)_4\text{C}$  to  $(\text{CH}_3)_4\text{Pb}$  except  $(\text{CH}_3)_4\text{Si}$  which gives anomalously high  $^{13}\text{C}$  chemical shift.

( v )  $^{59}\text{Co}$  NMR in  $\text{X}_3\text{MCo}(\text{CO})_4$  single crystal ( M = Si, Ge, Sn, Pb; X =  $\text{C}_6\text{H}_5$ , Cl, Br, I )<sup>24)</sup>.

The chemical shift of the  $^{59}\text{Co}$  were observed in  $\text{X}_3\text{MCo}(\text{CO})_4$  single crystal (  $\text{M} = \text{Si}, \text{Ge}, \text{Sn}, \text{Pb}$ ;  $\text{X} = \text{C}_6\text{H}_5, \text{Cl}, \text{Br}, \text{I}$  ). The chemical shift components along the Co-M bond (  $\sigma_{//}$  ) vary little in above compounds. On the other hand, the components perpendicular to the Co-M bond (  $\sigma_{\perp}$  ) vary with M and X.  $\sigma_{\perp}$  in silicon and tin compounds lie at high field side than that in germanium compounds. From the results of  $\sigma_{\perp}$ , Spiess et al.<sup>24)</sup> concluded that the  $\pi$ -backbondings in M-Co bonds in silicon and tin compounds are stronger than that in germanium compounds.

Three dimensional information about the electronic structure can be obtained by high resolution NMR in solid state and so this technique is thought to be useful in investigating the anomalous behavior of these compounds, in which the law of corresponding state is destroyed. Si, Ge, and Sn have  $sp^3d^2$  hybridization in  $\text{MX}_6$  type compounds or ions (  $\text{M} = \text{Si}, \text{Ge}, \text{Sn}$  ), and vacant d-orbitals of M and p-orbitals of X constitute the  $\pi$ -backbonding as shown in Fig. 5-15. Through this back bonding, some electrons are transferred from X to M.

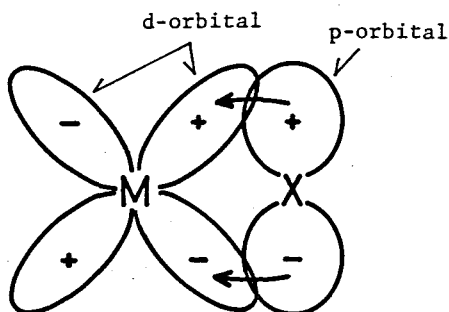


Fig. 5-15. The system of  $\pi$ -backbonding in  $\text{MX}_6$  (  $\text{M} = \text{Si}, \text{Ge}, \text{Sn}$  ) compounds.



The origin of the anomalous behavior in Si compounds have been attributed to such a  $\pi$ -backbonding<sup>20)</sup>. The data to be obtained by high resolution NMR in solid is thought to help elucidate the nature of bonding in these compounds. The paramagnetic part, which is responsible for the variation of  $^{19}\text{F}$  chemical shift, has already been given in eq. 5-15, and can be rewritten as follows under the assumption of axial symmetry of  $\underline{\sigma}$ ;

( i ) bond direction component (  $\sigma_{//}^{\text{P}}$  )

$$\sigma_{//}^{\text{P}} = (3/2)\sigma_0(2\rho_{\perp} - \rho_{\perp}^2) \quad (5-17a)$$

( ii ) component perpendicular to the M-F ( M = Si, Ge, Sn ) bond (  $\sigma_{\perp}^{\text{P}}$  )

$$\sigma_{\perp}^{\text{P}} = (3/2)\sigma_0[\epsilon + \rho_{\perp}(1 - \epsilon)] \quad (5-17b)$$

The  $\sigma_{//}$  component increases gradually from  $\text{K}_2\text{SiF}_6$  to  $\text{K}_2\text{SnF}_6$  as shown in Fig. 5-16.

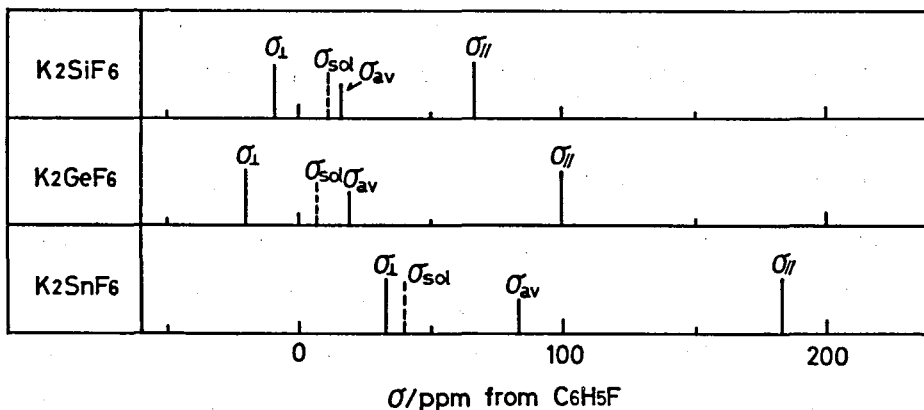


Fig. 5-16. Pictorial representation of the chemical shift tensor elements in  $\text{K}_2\text{MF}_6$  ( M = Si, Ge, Sn ) compounds.

The  $\sigma_{\perp}$  component of  $K_2SiF_6$  is abnormally large. The anomaly in  $K_2SiF_6$  is attributed to the anomalous character in the Si-F bond. The gradual change in  $\sigma_{//}$  component indicates that there is a corresponding change in  $\sigma_{//}^P$ . Since  $\sigma_{//}$  increases on going from  $K_2SiF_6$  to  $K_2SnF_6$ ,  $|\sigma_{//}^P|$  must decrease in this order. According to eq. 5-17a, it is concluded that the double bond character,  $\rho_{\perp}$ , decreases gradually from  $K_2SiF_6$  to  $K_2SnF_6$ . This shows that  $\pi$ -backbonding is the strongest in  $K_2SiF_6$ . Since  $\sigma_{\perp}^P$  is given by eq. 5-17b,  $\sigma_{\perp}^P$  must decrease from  $K_2SnF_6$  to  $K_2SiF_6$  if  $\epsilon$  does not change. In order to explain the anomalous behavior in  $\sigma_{\perp}$ , a decrease in  $\epsilon$  must be large enough to counter-balance the increase in  $\rho_{\perp}$ . The smallness of  $\epsilon$  shows a high ionicity of Si-F bond. This means that the electronegativity of Si must be smaller than that of Ge. In fact, recently, the periodicity of the electronegativity is reported to be destroyed in the third row of the periodic table.<sup>25)</sup> Sanderson<sup>25)</sup> shows that the electronegativity decreases in the order, C > Ge > Sn > Pb > Si. Such anomalous behavior in Si compounds is therefore attributed to the extraordinarily small electronegativity of Si.

$\sigma_{av}$  is the averaged value of components of the chemical shift tensor in the solid state.  $\sigma_{sol}$  is the corresponding averaged value in solution. The value ( $\sigma_{av} - \sigma_{sol}$ ) is the largest in  $K_2SnF_6$ . It is interesting to note that the solubility of  $K_2MF_6$  compounds in water increases from  $K_2SiF_6$  to  $K_2SnF_6$ . The large value of ( $\sigma_{av} - \sigma_{sol}$ ) may be attributed to the large difference of environment of  $MF_6^{2-}$  ion between solid and liquid state. It is considered that  $SnF_6^{2-}$  is surrounded by  $H_2O$  molecules more

strongly since the solubility of  $K_2SnF_6$  is the largest, and that this interaction causes a large change of electron distribution.

### 5-3-6 Conclusion

High resolution  $^{19}F$  NMR spectra were observed in  $K_2SiF_6$ ,  $K_2GeF_6$ , and  $K_2SnF_6$  and chemical shift shielding tensors were obtained. Shielding tensors are all axially symmetric and the most shielded direction is parallel to the MF ( M = Si, Ge, Sn ) bond. Gradual change of shielding tensor elements (  $\sigma_{//}$  and  $\sigma_{\perp}$  ) were observed on going from  $K_2SiF_6$  to  $K_2SnF_6$  except for the  $\sigma_{\perp}$  component of  $K_2SiF_6$ . From the analysis of shielding tensor components, it is indicated that  $\pi$ -backbonding is the strongest in the Si-F system and anomalous behavior of Si compounds is attributed to the extraordinarily low electronegativity of Si.

At room temperature, reorientational motion of  $MF_6$  ( M = Si and Sn ) ions are rapid enough to average out the shielding tensor components. On the other hand, ionic reorientation does not affect the chemical shift tensor in  $K_2GeF_6$  at room temperature. At 77 K, the correlation time of reorientation of  $SiF_6^{2-}$  ion is evaluated as  $3.3 \times 10^{-3}$  s from the powder pattern analysis. In  $K_2SnF_6$ , reorientational motion of  $SnF_6^{2-}$  does not affect the powder pattern at 77 K.

The magnitude of the difference of chemical shifts in solid and in solution is considered to serve as a measure of the magnitude of interaction between  $MF_6^{2-}$  ion and surrounding ions or  $H_2O$  molecules.

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## Appendix

### The BASIC Program for Data Processing

```
0010 PRINT "FT PROGRAM"
0020 PRINT
0030 PRINT
0040 DIM A$(3)
0050 DIM X(3,255),U(3,255)
0060 DIM Y(3,255),V(3,255)
0070 PRINT "TYPE-IN G OF N=2*G,WHERE N IS NUMBER OF DATA,";
0080 PRINT "YOU NEED"
0090 INPUT G
0100 LET N=2*G
0110 PRINT "TYPE-IN FREQUENCY/MHZ, SAMPLING INTERVAL/MICRO SEC."
0120 INPUT F,S
0130 PRINT
0140 REM DATA INPUT ROUTINE
0170 FOR J=0 TO 3
0180   FOR K=0 TO 255
0190     LET X(J,K)=0
0200     LET U(J,K)=0
0210     LET Y(J,K)=0
0220     LET V(J,K)=0
0230   NEXT K
0240 NEXT J
0250 LET C=0
0260 LET N5=INT((N-1)/1024)
0270 LET N6=INT((N-1-1024*N5)/256)
0287 LET A=0
0288 PRINT "TYPE IN NUMBER OF ACCUMULATION "
0289 INPUT W
0290 FOR H=1 TO W
0300   CALL 2,C
0310   FOR I=0 TO N5
0320     FOR J=0 TO N6
0330       FOR K=0 TO 255
0340         CALL 3,A
0350         IF I=1 THEN GOTO 0380
0360         LET X(J,K)=X(J,K)+A/W
0370         GOTO 0390
0380         LET U(J,K)=U(J,K)+A/W
0390       NEXT K
0400     NEXT J
0410   NEXT I
0420   CALL 1
0430 NEXT H
0440 PRINT "DO YOU NEED DATA OUTPUT,YES OR NO ?"
0450 INPUT AS
```

```

0460 IF A$(1,1)="N" THEN GO TO 0690
0470 GOSUB 1800
0690 PRINT " DO YOU NEED PRINT-OUT DATA ,YES OR NO ?"
0700 INPUT A$
0710 IF A$(1,1)="N" THEN GO TO 0820
0720 PRINT "TYPE-IN THE INITIAL AND FINAL NUMBER OF PRINT-OUT";
0730 PRINT " DATA, YOU NEED"
0740 INPUT Q1, Q2
0750 PRINT "FREQUENCY="; F; "MHZ   NUMBER OF ACCUMULATION="; W;
0760 PRINT " N="; N; "SAMPLING INTERVAL="; S; "MICRO SEC."
0770 PRINT
0780 PRINT "TIME/S"; TAB(8); "REAL X"; TAB(21); "IMAGINARY Y"
0790 GOSUB 3000
0800 PRINT "DO YOU NEED MORE DATA, YES OR NO ?"
0810 INPUT A$
0815 IF A$(1,1)="Y" THEN GO TO 0720
0820 PRINT "DO YOU NEED NEW ACCUMULATION, YES OR NO?"
0823 INPUT A$
0827 IF A$(1,1)="Y" THEN GO TO 0145
0830 PRINT "TYPE-IN THE FIRST NUMBER OF DATA, YOU NEED"
0840 INPUT Q
0850 FOR T=0 TO 1023
0860   LET T1=INT(T/256)
0870   LET T2=T-256*T1
0880   LET T3=INT((T+Q)/1024)
0890   LET T4=INT((T+Q-1024*T3)/256)
0900   LET T5=T+Q-1024*T3-256*T4
0910   IF T=>1024-Q THEN GO TO 0940
0920   LET X(T1, T2)=X(T4, T5)
0930   GO TO 0950
0940   LET X(T1, T2)=U(T4, T5)
0950 NEXT T
0960 FOR T=1024 TO 2047
0970   LET T1=INT((T-1024)/256)
0980   LET T2=T-1024-256*T1
0990   LET T3=INT((T+Q-1024)/256)
1000   LET T4=T+Q-1024-256*T3
1010   IF T=>2048-Q THEN GO TO 1040
1020   LET U(T1, T2)=U(T3, T4)
1030   GO TO 1042
1040   LET U(T1, T2)=0
1042 NEXT T
1053 PRINT "DO YOU NEED BASE LINE COMPENSATION ,YES OR NO?"
1055 INPUT A$
1057 IF A$(1,1)="Y" THEN GOSUB 7000
1060 PRINT
1070 REM FFT ROUTINE
1080 PRINT "DO YOU NEED FFT, YES OR NO ?"
1090 INPUT A$
1100 IF A$(1,1)="N" THEN GO TO 1710
1110 GOSUB 4000
1120 PRINT
1130 REM RESULTS OUTPUT ROUTINE

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1140 PRINT "DO YOU NEED RESULTS OUTPUT,YES OR NO ?"
1150 INPUT AS
1160 IF AS[1,1]="N" THEN GO TO 1480
1410 GOSUB 1800
1480 PRINT "DO YOU NEED PRINT-OUT RESULTS,YES OR NO ?"
1490 INPUT AS
1500 IF AS[1,1]="N" THEN GO TO 1610
1510 PRINT "TYPE-IN THE INITIAL AND FINAL NUMBER OF PRINT-OUT";
1520 PRINT " RESULTS, YOU NEED"
1530 INPUT Q1,Q2
1540 PRINT "FREQUENCY=";F;"MHZ   NUMBER OF ACCUMULATION=";W;
1550 PRINT " N=";N
1560 PRINT "SAMPLING INTERVAL=";S;"MICRO SEC.";
1570 PRINT " RESOLUTION=";10+3/(S*N);"KHZ"
1580 PRINT
1590 PRINT "SHIFT/R"; TAB(8);"ABSORPTION"; TAB(21);"DISPERSION"
1600 GOSUB 3000
1610 PRINT "DO YOU NEED AMPLITUDE SPECTRUM,YES OR NO ?"
1620 INPUT AS
1630 IF AS[1,1]="N" THEN GO TO 1710
1640 FOR J=0 TO 3
1650   FOR K=0 TO 255
1660     LET X[J,K]=X[J,K]+2+Y[J,K]+2
1670     LET U[J,K]=U[J,K]+2+V[J,K]+2
1680   NEXT K
1690 NEXT J
1700 GO TO 1140
1710 PRINT "WANT YOU COMPENSATE SAME FID DATA AGAIN,YES OR NO?"
1711 INPUT AS
1712 IF AS[1,1]="Y" THEN GOSUB 7900
1713 END
1800 REM
1810 REM SUBROUTINE OUTPUT
1820 LET O=0
1822 LET P=0
1824 FOR L=0 TO 1
1826   FOR J=0 TO 3
1828     FOR K=0 TO 255
1830       IF L=1 THEN GO TO 1842
1832       IF O<X[J,K] THEN GO TO 1836
1834       LET O=X[J,K]
1836       IF P>X[J,K] THEN GO TO 1840
1838       LET P=X[J,K]
1840       GO TO 1850
1842       IF O<U[J,K] THEN GO TO 1846
1844       LET O=U[J,K]
1846       IF P>U[J,K] THEN GO TO 1850
1848       LET P=U[J,K]
1850     NEXT K
1852   NEXT J
1854 NEXT L
1856 FOR J=0 TO 3
1858   FOR K=0 TO 255

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1860     LET X[J,K]=(X[J,K]-0)*2000/(P-0)
1862     LET U[J,K]=(U[J,K]-0)*2000/(P-0)
1864     NEXT K
1866 NEXT J
2000 REM
2010 REM SUBROUTINE OUTPUT
2020 FOR I=0 TO 1
2030     FOR J=0 TO 3
2040         FOR K=0 TO 255
2050             IF I=1 THEN GO TO 2090
2060             LET B=X[J,K]
2070             CALL 4,B
2080             GO TO 2110
2090             LET B=U[J,K]
2100             CALL 4,B
2110         NEXT K
2120     NEXT J
2130 NEXT I
2140 CALL 1
2150 PRINT "DO YOU NEED CRO DISPLAY,YES OR NO ?"
2160 INPUT A$
2170 IF A$(1,1)="N" THEN GO TO 2310
2230 PRINT "TYPE-IN NUMBER OF DISPLAYS"
2240 INPUT D
2250 CALL 5,D
2260 LET C=1
2270 CALL 2,C
2280 PRINT "DO YOU NEED ONE MORE DISPLAY,YES OR NO ?"
2290 INPUT A$
2300 IF A$(1,1)="Y" THEN GO TO 2230
2310 PRINT "DO YOU NEED RECORDER OUT,YES OR NO ?"
2320 INPUT A$
2330 IF A$(1,1)="N" THEN GO TO 2441
2370 PRINT "TYPE IN X TO RECORDER!"
2390 INPUT Z
2400 LET C=2
2410 CALL 2,C
2420 PRINT "DO YOU NEED ONE MORE RECORDING,YES OR NO ?"
2430 INPUT A$
2440 IF A$(1,1)="Y" THEN GO TO 2370
2441 FOR J=0 TO 3
2442     FOR K=0 TO 255
2443         LET X[J,K]=X[J,K]*(P-0)/2000+0
2444         LET U[J,K]=U[J,K]*(P-0)/2000+0
2445     NEXT K
2446 NEXT J
2450 RETURN
3000 REM
3010 REM SUBROUTINE PRINT-OUT
3020 FOR Q=Q1 TO Q2
3030     LET Q3=INT(Q/1024)
3040     LET Q4=INT((Q-1024*Q3)/256)
3050     LET Q5=Q-1024*Q3-256*Q4

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3060 IF Q3=1 THEN GO TO 3090
3070 PRINT Q; TAB(8);X[Q4,Q5]; TAB(21);Y[Q4,Q5]
3080 GO TO 3100
3090 PRINT Q; TAB(8);U[Q4,Q5]; TAB(21);V[Q4,Q5]
3100 NEXT Q
3110 RETURN
4000 REM
4010 REM FFT SUBROUTINE 1
4020 LET P=8*ATN(1)/N
4030 FOR L=0 TO G-1
4040 LET G1=2*(G-L-1)
4050 LET M=0
4060 FOR I=1 TO 2*L
4070 LET K1=INT(M/G1)
4080 GOSUB 4710
4090 LET Y1=COS(P*K2)
4100 LET Y2=-SIN(P*K2)
4110 FOR J=1 TO G1
4120 LET M1=INT((M+G1)/1024)
4130 LET M2=INT((M+G1-1024*M1)/256)
4140 LET M3=M+G1-1024*M1-256*M2
4150 LET M4=INT(M/1024)
4160 LET M5=INT((M-1024*M4)/256)
4170 LET M6=M-1024*M4-256*M5
4180 LET E1=(1-M1)*Y[M2,M3]+M1*V[M2,M3]
4190 LET D1=(1-M1)*X[M2,M3]+M1*U[M2,M3]
4200 LET D2=(1-M4)*X[M5,M6]+M4*U[M5,M6]
4210 LET E2=(1-M4)*Y[M5,M6]+M4*V[M5,M6]
4220 LET Y3=D1*Y1-E1*Y2
4230 LET Y4=D1*Y2+E1*Y1
4240 IF M1=1 THEN GO TO 4300
4250 LET X[M2,M3]=D2-Y3
4260 LET Y[M2,M3]=E2-Y4
4270 LET X[M5,M6]=D2+Y3
4280 LET Y[M5,M6]=E2+Y4
4290 GO TO 4380
4300 LET U[M2,M3]=D2-Y3
4310 LET V[M2,M3]=E2-Y4
4320 IF M4=1 THEN GO TO 4360
4330 LET X[M5,M6]=D2+Y3
4340 LET Y[M5,M6]=E2+Y4
4350 GO TO 4380
4360 LET U[M5,M6]=D2+Y3
4370 LET V[M5,M6]=E2+Y4
4380 LET M=M+1
4390 NEXT J
4400 LET M=M+G1
4410 NEXT I
4420 NEXT L
4430 FOR I=0 TO N-1
4440 LET K1=I
4450 GOSUB 4710
4460 IF K2=>I THEN GO TO 4690

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```

4470 LET I1=INT(I/1024)
4480 LET I2=INT((I-1024*I1)/256)
4490 LET I3=I-1024*I1-256*I2
4500 LET I4=INT(K2/1024)
4510 LET I5=INT((K2-1024*I4)/256)
4520 LET I6=K2-1024*I4-256*I5
4530 LET K3=(1-I1)*X[I2,I3]+I1*U[I2,I3]
4540 LET K4=(1-I1)*Y[I2,I3]+I1*V[I2,I3]
4550 IF I1=1 THEN GO TO 4610
4560 LET X[I2,I3]=X[I5,I6]
4570 LET Y[I2,I3]=Y[I5,I6]
4580 LET X[I5,I6]=K3
4590 LET Y[I5,I6]=K4
4600 GO TO 4690
4610 LET U[I2,I3]=(1-I4)*X[I5,I6]+I4*U[I5,I6]
4620 LET V[I2,I3]=(1-I4)*Y[I5,I6]+I4*V[I5,I6]
4630 IF I4=1 THEN GO TO 4670
4640 LET X[I5,I6]=K3
4650 LET Y[I5,I6]=K4
4660 GO TO 4690
4670 LET U[I5,I6]=K3
4680 LET V[I5,I6]=K4
4690 NEXT I
4700 RETURN
4710 REM FFT SUBROUTINE 2
4720 LET K2=0
4730 FOR K=1 TO G
4740 LET K3=K1-2*INT(K1/2)
4750 LET K1=INT(K1/2)
4760 IF K3=0 THEN GO TO 4780
4770 LET K2=K2+2*(G-K)
4780 NEXT K
4790 RETURN
6000 FOR J=250 TO 255
6100 PRINT X[0,J]
6200 NEXT J
7000 REM
7010 REM SUBROUTINE BASE LINE COMPENSATION
7012 DIM R[255]
7014 FOR K=0 TO 255
7016 LET R[K]=0
7018 NEXT K
7020 FOR K=0 TO 255
7022 LET R[K]=X[0,K]
7024 NEXT K
7040 LET X1=0
7041 LET X2=0
7042 LET X3=0
7043 LET X4=0
7044 LET X5=0
7045 LET X6=0
7050 LET X7=0
7053 LET X8=0

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7054 LET X9=0
7055 LET Z1=0
7056 LET Z2=0
7057 PRINT "TYPE-IN TIME CONSTANT AND CUT OFF NUMBER!"
7058 INPUT Z1,Z2
7060 FOR K=Z2 TO 255
7061   LET X[0,K]=0
7064 NEXT K
7069 FOR K=0 TO Z2-1
7070   LET X1=EXP(-K/Z1)
7080   LET X2=X2+X[0,K]
7090   LET X3=X3+X1
7100   LET X4=X4+X1*X[0,K]
7110   LET X5=X5+X1*X1
7120   LET X6=X6*X[0,K]+2
7130 NEXT K
7140 LET X7=(Z2*X4-X2*X3)/(Z2*X5-X3+2)
7150 LET X8=(X2-X7*X3)/Z2
7160 FOR K=0 TO Z2-1
7170   LET X9=X8+X7*EXP(-K/Z1)
7180   LET X[0,K]=X[0,K]-X9
7190 NEXT K
7192 PRINT "BASE LINE FUNCTION"
7194 PRINT "      Y =";X8;"+";X7;"* EXP ( - K /";Z1;" )"
7200 PRINT "DO YOU WANT DATA OUTPUT,YES OR NO?"
7210 INPUT A$
7220 IF A$(1,1)="Y" THEN GOSUB 1800
7500 PRINT "DO YOU WANT PRINT-OUT DATA,YES OR NO?"
7510 INPUT A$
7520 IF A$(1,1)="N" THEN GO TO 7700
7530 PRINT "TYPE-IN INITIAL AND FINAL NUMBER OF DATA YOU WANT"
7540 INPUT Q1,Q2
7550 PRINT "FREQUENCY=";F;"MHZ   NUMBER OF ACCUMULATION=";W;
7560 PRINT " N=";N;"SAMPING INTERVAL=";S;"MICRO SEC."
7570 PRINT
7580 PRINT "TIME/S"; TAB(8);"REAL X"; TAB(21);"IMAGINARY Y"
7590 GOSUB 3000
7600 PRINT "DO YOU WANT MORE DATA,YES OR NO?"
7610 INPUT A$
7620 IF A$(1,1)="Y" THEN GO TO 7530
7700 PRINT "DO YOU NEED NEW BASE LINE COMPENSATION,YES OR NO?"
7710 INPUT A$
7720 IF A$(1,1)="N" THEN GO TO 1080
7730 REM ONCE MORE DATA READ
7740 FOR K=0 TO 255
7750   LET X[0,K]=0
7760 NEXT K
7770 FOR K=0 TO 255
7780   LET X[0,K]=R[K]
7790 NEXT K
7800 GO TO 7040
7900 REM SUBROUTINE SECOND COMPENSATION AFTER FFT
7902 FOR J=0 TO 3

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7904   FOR K=0 TO 255
7906     LET X[J,K]=0
7908     LET U[J,K]=0
7910     LET Y[J,K]=0
7912     LET V[J,K]=0
7914   NEXT K
7916 NEXT J
7918 GO TO 7770
8000 REM BASE SHIFT FOR ONE PULSE
8010 LET R1=0
8020 FOR K=0 TO 255
8030   LET R1=R1+U[3,K]/256
8040 NEXT K
8045 PRINT R1
8050 PRINT "PRINT Z IF YOU NEED BASE SHIFT!"
8060 INPUT Z
8080 FOR J=0 TO 3
8090   FOR K=0 TO 255
8100     LET X[J,K]=X[J,K]-R1
8110     LET U[J,K]=U[J,K]-R1
8120   NEXT K
8130 NEXT J
8140 END
9000 REM COMPENSATION FOR MREV-8
9010 FOR K=0 TO Z2/2-1
9020   LET X[0,2*K+1]=(X[0,2*K]+X[0,2*K+2])/2
9030 NEXT K
9040 GO TO 7200
9100 LET S3=0
9102 LET S4=0
9110 FOR K=Z2/2-31 TO Z2/2-1
9120   LET S3=S3+X[0,2*K]/31
9130   LET S4=S4+X[0,2*K+1]/31
9140 NEXT K
9150 FOR K=0 TO Z2/2-1
9160   LET X[0,2*K]=X[0,2*K]-S3
9170   LET X[0,2*K+1]=X[0,2*K+1]-S4
9180 NEXT K
9190 PRINT "EVEN SHIFT"; S3; "ODD SHIFT"; S4
9200 GO TO 7200
9400 LET S5=0
9405 FOR K=150 TO 250
9410   LET S5=S5+X[0,K]/101
9415 NEXT K
9416 PRINT S5
9420 FOR K=0 TO 255
9425   LET X[0,K]=X[0,K]-S5
9430 NEXT K
9435 END
9500 FOR J=1 TO 3
9510   FOR K=0 TO 255
9520     LET X[J,K]=0
9530     LET U[J,K]=0

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```

9540 NEXT K
9550 NEXT J
9560 FOR K=0 TO 255
9570 LET U[0,K]=0
9580 NEXT K
9590 END
9600 FOR J=2 TO 3
9610 FOR K=0 TO 255
9620 LET X[J,K]=0
9630 NEXT K
9640 NEXT J
9650 FOR J=0 TO 3
9660 FOR K=0 TO 255
9670 LET U[J,K]=0
9680 NEXT K
9690 NEXT J
9695 END
9700 FOR J=0 TO 3
9710 FOR K=0 TO 126
9720 LET X[J,2*K+1]=(X[J,2*K]+X[J,2*K+2])/2
9730 NEXT K
9740 NEXT J
9760 FOR J=0 TO 2
9770 LET X[J,255]=(X[J,244]+X[J+1,0])/2
9780 NEXT J
9790 END
9800 LET S6=0
9810 FOR K=0 TO 255
9820 LET S6=S6+X[1,K]/256
9830 NEXT K
9840 PRINT S6
9850 FOR J=0 TO 1
9860 FOR K=0 TO 255
9870 LET X[J,K]=X[J,K]-S6
9880 NEXT K
9890 NEXT J
9895 END

```