Majorana Neutrinos and the Double β Decay

by

Hiroyuki Nishiura

1981

Abstract

In order to get the information on whether neutrinos are Majorana or Dirac particles, the double β decay is examined. The effective weak charged current Hamiltonian is used which is motivated by the grand unified gauge theories. The $0^+ \to J^+$ nuclear transitions for both the two neutrino and neutrinoless modes are investigated in the two nucleon- and N^*-mechanisms.

The condition is proposed to determine whether neutrinos are Majorana particles. In order to derive this condition, it is sufficient to know only the theoretical estimate on the $(\beta\beta)_{2\nu}$ mode. By using the data on the ratio of ^{128}Te to ^{130}Te half-lives obtained by Missouri group, we conclude from this Majorana condition that neutrinos are likely to be Majorana particles.

It is found that, for the neutrinoless mode, only the $0^+ \to 0^+$ transition in the two nucleon mechanism takes place if there is no right-handed current. By using the data on the ratio of the ^{128}Te to ^{130}Te half-lives and Vergados' estimation of the nuclear matrix elements, we get the neutrino "mass" to be around 32 eV if there is no right-handed current.

The $0^+ \to 2^+$ transition for the $(\beta\beta)_{0\nu}$ mode occurs only if there is the right-handed current in addition to the left-handed current, so that the measurement of this transition gives the direct information on the right-handed current.

Contents

§1. Introduction

(1-a) *Majorana neutrinos:*

The interpretation of the so-called "negative energy states" proposed by Dirac[1] leads to a symmetric description of electrons and positrons. Majorana, however, thought that it is not satisfactory because the symmetric result is obtained only after anti-symmetrizing the Dirac fields. In 1937, Majorana[2] showed that it is possible to construct the theory without negative energy states by taking account of only the real part of spinor field which is called as the Majorana field. It is clear from the analogy of the real scalar field that there is no distinction between particle and anti-particle in the Majorana theory. Therefore, this theory can not apply to the charged spin 1/2 particle. However, it is perfectly possible to formulate a theory of the neutral spin 1/2 particle which is completely different from the ordinary Dirac theory.

Racah[3] discussed that the neutron can not be described by this Majorana theory because it has an anomalous magnetic moment. However, even at the present time, the question whether neutrinos are Majorana or Dirac particles is still open.

On the other hand, the neutrino hypothesis has been proposed by Pauli[4] in 1933. Subsequently, by taking this hypothesis, Fermi[5] has constructed the theory of β decay in 1934. The first observation of free neutrinos from a reactor was made by Reines and Cowan [6] during 1953 to 1956. Since the discovery of parity non-conservation in the single β decay was made by Columbia-NBS group,[7] following the proposal by Lee and Yang,[8] the V-A charged weak interaction theory based on massless neutrinos has been established.

Pauli,[9] Gürsey,[10] and Ryan and Okubo[11] have proved that the Majorana and two-component neutrino theories do give the same result, if neutrinos are massless and the charged weak interaction is the V-A type. Let us consider two elementary processes $n \to p + e^- + \nu_1$ and "p" $\to$ "n" $+ e^+ + \nu_2$, as examples. In the Dirac theory $\nu_1 = \overline{\nu_e}$ and $\nu_2 = \nu_e$, while in the Majorana theory ν_1 = helicity -1 state, ν_2 = helicity $+1$ state. In the Majorana theory, the helicity plays the role to distinguish the particle and the anti-particle in the Dirac theory within the framework of the massless V-A theory. This is the reason why few attention have been paid to the Majorana theory of neutrinos. However, if neutrinos are massive and/or if the V+A interaction is added, the Majorana and Dirac neutrino theories give different results.

According to the recent theoretical development of the grand unified theories, neutrinos are likely to be massive[12] because leptons and quarks are treated on the equal basis. Also, the recent experiment of tritium β decay[13] seems to show that the electron neutrino has a finite mass. Under these circumstances the following questions are revived: Are neutrinos massive? Whether are neutrinos Majorana or Dirac particles?

Let us consider how the Majorana neutrino is distinct from the Dirac neutrino: (a) By its nature of the self-conjugate field, the Majorana neutrino can not have the magnetic moment. (b) Since there is no freedom of the phase transformation for the Majorana neutrino, the lepton number is not necessarily conserved. Thus, a test for the Majorana neutrino could be performed by the observation of the total lepton number non-conserving processes such as

(i) The neutrinoless double β decay,[14),15)] $(A,Z-2) \to (A,Z) + 2e^-$,

(ii) The $\mu - e$ conversion,[16)] $\mu^- + (A,Z+2) \to (A,Z) + e^+$ etc.,

(iii) The K decay,[17),18)] $K^- \to \pi^+ + e^- + e^-$ etc.,

(iv) The antineutrino capture,[19)] $\overline{\nu_e} + n \to p + e^-$ etc..

(c) If there are a neutrino and its antineutrino in the final state, we can expect to have interference terms between them because they are indistinguishable from each other in the Majorana neutrino theory. In the Dirac neutrino theory, we have, of course, no such interference terms. The μ-decay provides this kind of test.[20)] (d) The CP-violation pattern of the charged leptonic current sector in the Majorana neutrino system is different from that in the Dirac neutrino system, as shown by Billenky, Hosek, and Petkov,[21)] by Schechter and Valle,[22)] and by Doi, Kotani, Nishiura, Okuda, and Takasugi.[18)] We shall discuss this CP-violation problem in §2. This may also serve to discriminate between Majorana and Dirac neutrinos.

In this thesis, we shall examine the double β decay as a tool to investigate whether neutrinos are Majorana or Dirac particles.

(1-b) *The double β decay:*

In the double β decay of nucleus, there are two decay modes, i. e. the neutrinoless mode, $(\beta\beta)_{0\nu}$,

$$(A,Z-2) \longrightarrow (A,Z) + e^- + e^- , \qquad (1.1)$$

and the two neutrino emitting mode, $(\beta\beta)_{2\nu}$,

$$(A,Z-2) \longrightarrow (A,Z) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e \ . \qquad (1.2)$$

The double β decay can occur in the case where the single β decay of the parent nucleus (A,Z - 2) to (A,Z - 1) is forbidden energetically or at least strongly suppressed due to a large change of spin between the initial and final nuclei. As typical examples, the level structures of ^{130}Te and ^{48}Ca are shown in Fig. 1.

The $(\beta\beta)_{0\nu}$ mode takes place only when neutrinos are Majorana particles. This is because this process changes the total lepton number by two. But if the Majorana neutrinos are massless and the weak charged current interaction is the V-A type, the $(\beta\beta)_{0\nu}$ mode does not occur because of the mismatch of neutrino helicities. On the other hand, the $(\beta\beta)_{2\nu}$ mode can occur both for Dirac and Majorana neutrinos. Therefore the observation of the $(\beta\beta)_{0\nu}$ mode gives the direct information on the property of neutrinos.

The counter (chamber) experiments have advantage to examine the $(\beta\beta)_{0\nu}$ and $(\beta\beta)_{2\nu}$ modes separately. Among this kind of experiments, in 1970, Columbia group[23)] found one event which might be from the $(\beta\beta)_{0\nu}$ mode for ^{48}Ca. Recently Moe and Lowenthal[24)] reported the observation of 23 events which are considered to be from the $(\beta\beta)_{2\nu}$ mode for ^{82}Se. The other groups have obtained the lower bounds for the half-lives, which will be discussed in §6 individually. It is worthwhile to note that the observation of the transitions to the excited states has some experimental advantage, as pointed out by Fiorini.[25] This is because those tranisitions are followed by de-exciting γ-rays, detection of which reduces the background.

The geological method has such advantage that the accumulated decay products (daughter nuclei) for a long period are observed. This method, however, is unable to discriminate the $(\beta\beta)_{0\nu}$ mode from the $(\beta\beta)_{2\nu}$ modes. In order to get some information on the $(\beta\beta)_{0\nu}$ mode, it is necessary to choose the nucleus with a small phase space so that the yield from the $(\beta\beta)_{2\nu}$ mode is suppressed relative to the $(\beta\beta)_{0\nu}$ mode. Since the half-life of the nucleus with the small pahse space is much longer than that of the double β-decay nuclei with normal-size phase space, the measurement of the ratio of its half-life to that for the nucleus with normal-size phase space is preferred. In addition, the measurement of the ratio may be free from the experimental ambiguity due to the absolute half-life measurement. Consequently the ratio of the ^{128}Te to ^{130}Te half-lives measured by Missouri group[26] may be considered to meet the condition given above. Note that in the analysis of the half-life measured geologically, the transitions to the excited states should be taken into account in addition to the ground state to ground state transition.

Let us discuss the theoretical aspects of the double β decay. Two mechanisms for the double β decay have been considered. One is the two nucleon(2n)-mechanism of successive β decays of two different neutrons (n_1 and n_2) in one nucleus are involved in the double β decay, as shown in Fig. 2. The reason why two neutrons participate in this decay process is due to the fact that two units change is required in the hadronic part. In the $(\beta\beta)_{0\nu}$ mode, a virtual neutrino emitted by n_1 is reabsorbed by another n_2 in the same nucleus. The other is the N^*-mechanism introduced by Primakoff and Rosen.[27] This mechanism is based on the fact that the nuclei may contain the admixture of the nucleon resonance with $J^{(P)} = \frac{3}{2}^+$, $I = \frac{3}{2}$ and $m = 1232$ MeV [$\Delta(1232)$]. In this mechanism, the double β decay is considered to occur through the transition either $\Delta^-(1232) \to p + e^- + e^- + (\overline{\nu_e} + \overline{\nu_e})$ or $n \to \Delta^{++}(1232) + e^- + e^- + (\overline{\nu_e} + \overline{\nu_e})$, as shown in Fig. 3.* The participation of one hadron is enough in this case, because the isospin difference between the initial and final hadrons is $2 (= \Delta I_Z)$. In the $(\beta\beta)_{0\nu}$ mode, a virtual neutrino emitted by one quark in a hadron is reabsorbed by another quark in the same hadron. Note that the average propagation distance of this virtual neutrino is much shorter in comparison with the case of the 2n-mechanism, so that the magnitude of the neutrino exchange potential is enhanced considerably. Therefore the N^*-mechanism becomes important relative to the 2n-mechanism, although the admixture probability of $\Delta(1232)$ may be small. The idea of this N^*-mechanism comes from the argument by Kerman and Kisslinger[28] that the deuteron has the possible admixture of the nucleon resonance, N(1688). The brief discussion for these two mechanisms is given in Appendix B.

Many theoretical works have been done on the double β decay. The $(\beta\beta)_{2\nu}$ mode was first discussed by Mayer[29] and later by Primakoff and Rosen[15] in the 2n-mechanism. For this mode, it is enough to consider only the V-A interaction.[30]

---

* Ejiri,[73] Ohtubo and Hosoya[74] pointed out other types of diagrams shown in Figs. 12 and 13. These possibilities are now under consideration.

Concerning the $(\beta\beta)_{0\nu}$ mode in the 2n-mechanism, Furry[14] was the first to investigate it, and later Primakoff and Rosen[15] discussed it in detail for the case where a massless Majorana neutrino couples to an electron in the leptonic current with the various tensor structures. Greuling and Whitten[31] have analyzed the case with the V-δA ($\delta \neq 1$) leptonic current for a massive neutrino. Assuming a massless neutrino, Molina and Pascual[32] investigated $0^+ \to 1^+$ and 2^+ transitions in the framework of V - δA interaction without nuclear Coulomb corrections. Concerning the N^*-mechanism, Primakoff and Rosen[17] discussed the massless neutrino case with the V+A type leptonic current in addition to the ordinary V-A current. The hadrons are treated within the SU(6) quark model. Smith, Picciotto and Bryman[33] have analyzed the $(\beta\beta)_{0\nu}$ and $(\beta\beta)_{2\nu}$ modes by using the pion core model of Δ. The extensive investigation of the double β decay was done for the $0^+ \to J^+$ transitions in the 2n- and N^*-mechanisms recently by Doi, Kotani, Nishiura, Okuda and Takasugi.[34),35]

In this thesis, we will consider the following effective weak charged current interaction Hamiltonian: (i) Massive neutrinos are considered. (ii) The mixing among neutrinos is taken into account. (iii) The right- as well as left-handed currents are considered both for leptonic and hadronic sectors.

In §2, the general form of the effective weak interaction Hamiltonian is presented in connection with the grand unified gauge theories. The CP-violating phases characteristic to the Majorana neutrino system are discussed, and, as an illustration, the case for two generations is presented in Appendix D. In §3 the $(\beta\beta)_{0\nu}$ mode is investigated for the $0^+ \to J^+$ transitions both in the

2n- and N^*-mechanisms. The brief description of the 2n- and N^*-mechanisms in the double β decay is given in Appendix B. The $(\beta\beta)_{2\nu}$ mode is analyzed in §4. The general properties of the double β decay formulae are discussed in §5. The data on the half-lives of typical double β decaying nuclei are analyzed and theoretical predictions are given in §6. The summary and the concluding remarks are given in §7.

§2. The weak interaction Hamiltonian

In the grand unified gauge theories,[36)~38)] a general form* of the effective charged current weak interaction Hamiltonian will be

$$H_W(x) = \frac{G}{\sqrt{2}} \Big[j_{L\mu}(x) J_L^{\dagger\mu}(x) + \lambda j_{R\mu}(x) J_R^{\dagger\mu}(x) + \kappa \left\{ j_{L\mu}(x) J_R^{\dagger\mu}(x) + j_{R\mu}(x) J_L^{\dagger\mu}(x) \right\} \Big] + h.c. \ , \tag{2.1}$$

where $J^{\mu}_{L(R)}(x)$ is the left(right)-handed hadronic current and the leptonic current $j_{L(R)\mu}(x)$ is expressed as follows

$$j_{L\mu}(x) = \bar{e}(x)\gamma_\mu(1-\gamma_5)\nu_{eL}(x) + \cdots \ , \tag{2.2}$$

$$j_{R\mu}(x) = \bar{e}(x)\gamma_\mu(1+\gamma_5)\nu'_{eR}(x) + \cdots \ . \tag{2.3}$$

Here $e(x)$ is an electron field and $\nu_{eL}(x)$ ($\nu'_{eR}(x)$) is a left(right)-handed electron-neutrino field appeared in the leptonic current. This neutrino field is referred to as the current neutrino.

(2-a) *The right-handed current*

This effective Hamiltonian is derived from the underlying fundamental interaciton among quarks and leptons. The appearance of λ-term is due to the additional gauge boson W^{μ}_{R} which couples to the right-handed current and the κ-term comes from the mixing between W^{μ}_{L} and W^{μ}_{R} . This mixing is expressed in terms of the mass eigenstate gauge bosons W_1 and W_2 with masses M_1 and M_2 as follows,[39),40)]

---

* The effects which come from Higgs particles are not taken into account.

$$W_L = W_1 \cos\zeta + W_2 \sin\zeta , \tag{2.4}$$

$$W_R = -W_1 \sin\zeta + W_2 \cos\zeta . \tag{2.5}$$

In this notation, the parameters G, λ and κ in Eq.(2.1) are expressed in the left-right symmetric model[40] as

$$G/\sqrt{2} = (g^2/8)\cos^2\zeta\,(M_1^2\tan^2\zeta + M_2^2)/(M_1 M_2)^2 , \tag{2.6}$$

$$\lambda = (M_1^2 + M_2^2\tan^2\zeta)/(M_1^2\tan^2\zeta + M_2^2) , \tag{2.7}$$

$$\kappa = (M_1^2 - M_2^2)\tan\zeta/(M_1^2\tan^2\zeta + M_2^2) . \tag{2.8}$$

By eliminating ζ from Eqs.(2.7) and (2.8), two parameters λ and κ satisfy the relation,

$$\frac{\kappa^2}{(\sqrt{\lambda_c} - 1/\sqrt{\lambda_c})^2} + \frac{\{\lambda - \frac{1}{2}(\lambda_c + 1/\lambda_c)\}^2}{(\lambda_c - 1/\lambda_c)^2} = \frac{1}{4} . \tag{2.9}$$

where $\lambda_c = (M_1/M_2)^2$. We assume $M_1 \ll M_2$ for simplicity. This constraint is shown in Fig. 4 schematically. Note the following remarks: (i) The physically expected λ-value is around $\lambda \simeq \lambda_c \ll 1$, i. e., $\lambda = \lambda_c + \tan^2\zeta$ and $\kappa = -\tan\zeta$ in the limit $\lambda_c \to 0$. (ii) If we write $\lambda = \lambda_c + \delta$, κ behaves like $\kappa \sim \pm\sqrt{\delta}$ for $\delta \ll 1$. If $\delta \ll \sqrt{\lambda_c}$, $\lambda \gg |\kappa|$ (iii) On the other hand there is a possibility that the magnitude of κ dominates over λ, e. g. if $\delta \simeq \lambda_c$, $|\kappa| \simeq \sqrt{\lambda_c} \gg \lambda = 2\lambda_c$. It is of interest in this situation to observe that $|\kappa|$ is the order of (M_1/M_2), while λ is order of $(M_1/M_2)^2$.

In the following sections, we shall treat λ and κ as the small free parameters (λ, $|\kappa| \ll 1$).

(2-b) *The hadronic current*

In the 2n-mechanism, the hadronic currents are written as

$$J_L^{+\mu}(x) = \bar{\psi}_N(x)\tau^+\gamma^\mu(g_V - g_A\gamma_5)\psi_N(x) , \tag{2.10}$$

$$J_R^{+\mu}(x) = \bar{\psi}_N(x)\tau^+\gamma^\mu(g_V' + g_A'\gamma_5)\psi_N(x) , \tag{2.11}$$

where $\psi_N^T = (p, n)$ and τ^+ is the isospin raising matrix. Here $g_V = \cos\theta$ and $g_V' = \cos\theta'$ where $\theta(\theta')$ is the left(right)-mixing angle between u and d quarks. We expect $\theta \simeq \theta_c$ where θ_c is the Cabbibo angle. Also

$$g_A / g_V = g_A' / g_V' \simeq 1.24 . \tag{2.12}$$

is taken. This deviation from unity is due to the strong interaction renormalization.

In the N^*-mechanism, the hadronic currents are considered to act on quarks in a hadron and are obtained by replacing ψ_N^T with $\psi_q^T = (u, d)$ and taking $g_A/g_V = g_A'/g_V' = 1$. The effect from strong interaction will be taken into account by evaluating the matrix element in the SU(6) quark model.

(2-c) *The neutrino mixing and CP-violating effect*

Let us discuss the diagonalization of mass terms of neutrino fields.[18),21),22),41),42)] Three types of mass terms are considered; case (i), a Dirac-type mass term is

$$\mathcal{L}_m = - \overline{\nu_R'} M \nu_L + h.c. . \tag{2.13}$$

case (ii), a Majorana-type mass term is

$$\mathcal{L}_m = -\frac{1}{2} \overline{(\nu_L)^c} M_L \nu_L + h.c. , \tag{2.14}$$

case (iii), a mixed-type mass term is

$$\mathcal{L}_m = -\overline{\nu_R'} M \nu_L - \frac{1}{2}\{\overline{(\nu_L)^c} M_L \nu_L + \overline{(\nu_R')^c} M_R \nu_R'\} + h.c. , \quad (2.15)$$

where $\nu^T_{L(R)} = (\nu_{1L(R)}, \nu_{2L(R)}, \cdots, \nu_{nL(R)})$, n is the number of generations, and $(\nu_{L(R)})^c = C\bar{\nu}^T_{L(R)}$ with the charge conjugation matrix C. The Majorana mass matrices M_i (i=L, R) are in general complex symmetric $n \times n$ matrices, i. e., $M_i^T = M_i$ due to the property of $\overline{\nu^c_\alpha}\nu_\beta = \overline{\nu^c_\beta}\nu_\alpha$.

Case (i): The Dirac-type mass term becomes

$$\mathcal{L}_m = \overline{N}(V^{(1)\dagger} M U^{(1)}) N + h.c., \quad (2.16)$$

after taking the unitary transformations,

$$\nu_L = U^{(1)} N_L , \quad (2.17)$$

$$\nu_R' = V^{(1)} N_R . \quad (2.18)$$

The mass matrix M can be diagonalized by choosing some appropriate unitary matrices U and V, i. e.

$$V^{(1)\dagger} M U^{(1)} = (\text{real diagonal}). \quad (2.19)$$

Note that we have n mass eigenstate Dirac neutrinos. Since $U^{(1)}$ and $V^{(1)}$ are unitary matrices, respectively, there are $n(n-1)/2$ rotational angles and $n(n+1)/2$ phases in each matrices. By the redefinition of n charged lepton and n neutrino fields appeared in the leptonic currents in Eqs. (2.2) and (2.3), $(2n-1)$ phases out of total $n(n+1)$ phases in $U^{(1)}$ and $V^{(1)}$ can be absorbed. This freedom is used to reduce the number of phases in the left-handed current sector so that $U^{(1)}$ includes $(n-1)(n-2)/2$ CP-violating phases (the Kobayashi-Maskawa scheme[43]) and $V^{(1)}$ has the remaining $n(n+1)/2$ phases.

Case (ii): The Majorana-type mass term becomes

$$\mathcal{L}_m = \frac{1}{2}\,\overline{N}\,(U^{(1)T} M_L U^{(1)})\,N \quad + \text{h.c.}, \tag{2.20}$$

after the unitary transformations,

$$\nu_L = U^{(1)} N_L \,, \tag{2.21}$$

$$(\nu_L)^c = U^{(1)T} N_R \,. \tag{2.22}$$

Note that $U^{(1)T}$ appeared instead of $V^{(1)\dagger}$ in Case (i). Since M_L is the symmetric complex mass matrix, we can diagonalize it by choosing some appropriate unitary matrix $U^{(1)}$. In this case there are n mass eigenstate Majorana neutrinos, $N_1, \cdots, N_n$. Since there is no freedom of the phase transformation for the Majorana fields, only n charged leptons can absorb n phases. That is, $n(n-1)/2$ CP-violating phases remain in $U^{(1)}$.[18),21),22)] It is worthwhile to point out that there is no right-handed current $(\lambda = \kappa = 0)$ in this case.

Case (iii): The mixed-type of mass term becomes[42)]

$$\mathcal{L}_m = (\overline{N^{(1)}}, \overline{N^{(2)}}) \begin{pmatrix} U^{(1)}, & U^{(2)} \\ V^{(1)*}, & V^{(2)*} \end{pmatrix}^T \begin{pmatrix} M_L, & M^T \\ M, & M_R \end{pmatrix} \begin{pmatrix} U^{(1)}, & U^{(2)} \\ V^{(1)*}, & V^{(2)*} \end{pmatrix} \begin{pmatrix} N^{(1)} \\ N^{(2)} \end{pmatrix} + \text{h.c.}, \tag{2.23}$$

after the transformations,

$$\nu_L = U^{(1)} N_L^{(1)} + U^{(2)} N_L^{(2)} \,, \tag{2.24}$$

$$\nu_R' = V^{(1)} N_R^{(1)} + V^{(1)} N_R^{(2)} \,. \tag{2.25}$$

The $2n \times 2n$ mass matrix $\begin{pmatrix} M_L, & M^T \\ M, & M_R \end{pmatrix}$ is a complex symmetric matrix. Therefore, similarly to Case (ii) we can diagonalize the mass matrix by choosing some appropriate unitary matrix $\begin{pmatrix} U^{(1)} & , U^{(2)} \\ V^{(1)*} & , V^{(2)*} \end{pmatrix}$, i. e.

$$\begin{pmatrix} U^{(1)}, U^{(2)} \\ V^{(1)*}, V^{(2)*} \end{pmatrix}^T \begin{pmatrix} M_L, M^T \\ M \ , M_R \end{pmatrix} \begin{pmatrix} U^{(1)}, U^{(2)} \\ V^{(1)*}, V^{(2)*} \end{pmatrix} = (\text{real diagonal}). \tag{2.26}$$

In this case there are 2n mass eigenstate Majorana neutrinos ; $N^{(1)}$ (L-type) $= (N_1, \cdots, N_n)^T$ and $N^{(2)}$ (R-type) $= (N_{n+1}, \cdots, N_{2n})^T$. Since $\begin{pmatrix} U^{(1)} & , U^{(2)} \\ V^{(1)*} & , V^{(2)*} \end{pmatrix}$ is a $2n \times 2n$ unitary matrix, there are $n(2n-1)$ rotational angles and $n(2n+1)$ phases. Only n charged leptons can absorb n phases so that $2n^2$ CP-violating phases remain in $U^{(1)}$, $U^{(2)}$, $V^{(1)}$ and $V^{(2)}$. We use this phase freedom of leptons to reduce the number of phases in $U^{(1)}$ and $V^{(1)}$. Thus, $n(3n-1)/2$ CP-violating phases are in $U^{(1)}$ and $V^{(1)}$, and the remaining $n(n+1)/2$ phases are in $U^{(2)}$ and $V^{(2)}$.[22] In Appendix D, we shall show an explicit illustration for $n = 2$ case.

In order to treat the three cases simultaneously, we shall use the following relations among 2n current neutrino fields $\nu_{\alpha L,R}$ and 2n mass eitenstate neutrino fields N_j with mass m_j,

$$\nu_L = \sum_{j=1}^{n} \left(U^{(1)}_{\alpha j} N^{(1)}_{jL} + U^{(2)}_{\alpha j} N^{(2)}_{jL} \right) \equiv \sum_{j=1}^{2n} U_{\alpha j} N_{jL}, \tag{2.27}$$

$$\nu'_R = \sum_{j=1}^{n} \left(V^{(1)}_{\alpha j} N^{(1)}_{jR} + V^{(2)}_{\alpha j} N^{(2)}_{jR} \right) \equiv \sum_{j=1}^{2n} V_{\alpha j} N_{jR}. \tag{2.28}$$

It should be understood that $U^{(1)} \neq 0$, $V^{(1)} \neq 0$ and others $= 0$ for Case (i), $U^{(1)} \neq 0$ and others $= 0$ for Case (ii).

Generally speaking, the mixing between neutrinos turns off in their massless limit. In the Dirac neutrino Case (i), the mixing matrices should be taken as $U^{(1)}_{\alpha j} = V^{(1)}_{\alpha j} = \delta_{\alpha j}$. While, in the Majorana neutrino case, we have $U^{(1)}_{\alpha j} = \delta_{\alpha j}$ in Case (ii), and $U^{(1)}_{\alpha j} = V^{(2)}_{\alpha j} = \delta_{\alpha j}$ and others $= 0$ in Case (iii). However, we keep the possibility that mixings still remain in $U^{(1)}$ and $V^{(1)}$ on the phenomenological basis even in the massless limit.

(2-d) *The properties of various gauge models:*

Let us discuss briefly the magnitudes of parameters appeared in the weak Hamiltonian Eq.(2.1) in the various typical gauge models.

(I) *The $SU(2)_L \times U(1)$ models:*

In the standard Glashow, Salam and Weinberg model,[44] only the massless left-handed neutrinos are present ($m_j = 0$). Resultantly there is no neutrino mixing, $U^{(1)} = 1$ and $U^{(2)} = 0$ and no right-handed current, $\lambda = \kappa = 0$ ($V = 0$). It is not impossible to generate neutrino massess by introducing a Higgs triplet. In this case, neutrinos are Majorana particles (corresponding to Case (ii) in the subsection (2-c)). A natural way to generate neutrino masses is to introduce a right-handed neutrino. If singlet right-handed neutrinos are added, neutrinos become massive Dirac particles naturally (Case (i)). By introducing the Higgs triplet in addition to the above singlet right-handed neutrinos, Barger et al.[45] showed it possible that neutrinos are massive Majorana particles (Case (iii)). Note that there is no right-handed charged current, $\lambda = \kappa = 0$, in these theories because the weak charged bosons do not couple to the right-handed neutrinos.

(II) *The $SU(2)_L \times SU(2)_R \times U(1)$ models:*

In the typical model of this type,[35] neutrinos are massive Dirac particles since there exist the left-and right-handed neutrinos (Case (i)). An interesting alternative model has been proposed by Mohapatra and Senjanovic[46] where both Dirac and Majorana mass terms exist (Case (iii)). Now, neutrinos become massive Majorana particles. They obtained the values of parameters; $m_{N^{(1)}(\text{L-type})} \approx m_e M_{W_L}/M_{W_R} \approx O(\text{eV})$, $m_{N^{(2)}(\text{R-type})} \approx M_{W_R} \approx 10^7$ GeV, $U^{(1)}$ and $V^{(2)} \approx O(1)$, $U^{(2)}$ and $V^{(1)} \approx O(M_{W_L}/M_{W_R}) \approx 10^{-5}$ and λ, $|\kappa| \sim 10^{-10}$. Presently available data on the charged-current interaction give the ristriction,[40] λ, $|\kappa| \lesssim 10^{-1}$.

(III) *The SU(5) models:*

In the standard model[37] with a Higgs scalar in a 5 and/or 45 dimensional representation, there is a global symmetry which leads to the B-L number conservation. Moreover in the $\bar{5}$ and 10 quark-lepton multiples, there is no room for the right-handed neutrino. Therefore neutrinos are massless. If the B-L global symmetry is violated by introducing a two-fermion interaction with two 5-representation of Higgs,[47] neutrinos become massive Majorana particles (Case (ii)). In this case, $m_{N^{(1)}(\text{L-type})} \approx 10^{-5}$ eV, $\lambda = \kappa = 0$ and $U^{(2)} = V = 0$.

(IV) *The SO(10) models:*

In this kind of models,[38] a left-and a right-handed neutrinos are assigned in a same 16-plet representation. Therefore the mixed type of mass matrix (Case (iii)) is generally obtained and neutrinos become Majorana particles. In this scheme, the small mass of the "left-handed" neutrino can be explained in the following manner. Suppose that the SO(10) symmetry breaks down to SU(5) at the grand unification mass scale $M_{GUT} \simeq 10^{15}$ GeV . Since a right-handed neutrino is assigned to a singlet representation of SU(5), it can have a mass at this mass scale. Therefore, the right-handed Majorana mass term M_R in Eq.(2.15) is order of M_{GUT}. The mass scale of the Dirac mass term should be of the order of quark mass, $m_q \simeq 1$ GeV. The left-handed mass term M_L is considered to be much smaller than m_q . After diagonalizing the mass matrix, Eq.(2.26), we obtain $m_{N^{(1)}\,(L\text{-type})} \simeq m_q^2/M_{GUT} \simeq O(10^{-6}$ eV), $m_{N^{(2)}\,(R\text{-type})} \approx M_{GUT}$ or $\approx 10^{-8}\, M_{GUT}$. The order of magnitude of mixing parameters are $U^{(1)} \sim V^{(2)} \sim O(1)$, $U^{(2)} \sim V^{(1)} \approx m_{N^{(1)}\,(L\text{-type})}/m_{N^{(2)}\,(R\text{-type})} \simeq 10^{-30}$ and $\lambda \approx |\kappa| \approx (M_{W_L}/M_{W_R})^2 \approx (M_{W_L}/M_{GUT})^2 \simeq 10^{-26}$. Witten[48] has proposed a model where the neutrino masses are generated by loop diagrams. In this model, the neutrino can get a larger mass in comparison with the above mentioned values, that is, $m_{N^{(1)}\,(L\text{-type})} = m_q M_W/(\alpha^2 M_{GUT}) \approx 10^{-2}$ eV .

§3. The $(\beta\beta)_{0\nu}$ mode

We shall discuss the $(\beta\beta)_{0\nu}$ mode for the $0^+ \to J^+$ transition. In this section we will make a unified treatment both for the 2n- and N^*-mechanisms. To do this, the effective interaction Hamiltonian, $H_{int} = H_W + H_S$ is considered as in Appendix B , where H_S represents the effective strong interaction for the transition $N + N \leftrightarrow \Delta + N$ by the exchange of π, ρ and so on. In the 2n-mechanism, the double β decay takes place through the second order perturbation in H_W and the 0th order in H_S as shown in Fig. 2a. In the N^*-mechanism, the double β decay occurs through the 2nd order in H_W and the 1st order in H_S as shown in Figs. 3a and 3b. Note that the intermediate nuclear state $N_{\Delta^-(\Delta^{++})}$ which includes $\Delta^-(\Delta^{++})$ has the same J^P as $N_A(N_B)$ has, because the strong interaction operates as an internal force. In the following discussion, the second order weak interaction parts of these figures are singled out and treated collectively by using the expression of $N_\alpha \leftrightarrow N_\beta + e^- + e^-$ transition.

The $(\beta\beta)_{0\nu}$ mode takes place only if neutrinos are Majorana particles. And it is necessary that at least one of three parameters, m_j, λ and κ, does not vanish in the framework of the weak interaction given in §2. This comes from the following reasons. If the two lepton vertices in the Feynman diagrams for the $(\beta\beta)_{0\nu}$ mode are either combination (L, L) or (R, R) as shown in Fig. 5a, the contribution from this diagram is proportional to the neutrino mass m_j of the intermediate Majorana neutrino. When two lepton vertices are (L, R) or (R, L) as in Fig. 5b, these contributions are proportional to the neutrino four momentum q and depend on the relative strength λ or κ as seen from Eq.(2.1). These situations can be easily seen from the neutrino propagators given in Appendix A.

Thus the R-matrix element for $N_\alpha \to N_\beta + e^-(p_1) + e^-(p_2)$ is expressed by

$$
\begin{aligned}
R_w = \frac{1}{\sqrt{2}}\left(\frac{G}{\sqrt{2}}\right)^2 (2\pi)^{-3}(p_1^\circ p_2^\circ)^{-1/2} \int d\vec{x}\, d\vec{y} \\
\cdot 2\sum_j \Big\{ m_j \left[U_{ej}^2 t_{\mu\nu}^L \widetilde{K}_{LL}^{\mu\nu} + V_{ej}^2 t_{\mu\nu}^R \widetilde{K}_{RR}^{\mu\nu} \right] \\
+ U_{ej}V_{ej} \left[u_{\mu\nu\rho}^L \widetilde{L}_{LR}^{\mu\nu\rho} + u_{\mu\nu\rho}^R \widetilde{L}_{RL}^{\mu\nu\rho} \right] \Big\},
\end{aligned}
\tag{3.1}
$$

where the first $1/\sqrt{2}$ is the statistical factor for the emitted two electrons and

$$t_{\mu\nu}^{L,R} = \overline{\phi}(\hat{p}_1, p_1^\circ, \vec{x})\gamma_\mu(1 \mp \gamma_5)\gamma_\nu \phi^c(\hat{p}_2, p_2^\circ, \vec{y}), \tag{3.2}$$

$$u_{\mu\nu\rho}^{L,R} = \overline{\phi}(\hat{p}_1, p_1^\circ, \vec{x})\gamma_\mu(1 \mp \gamma_5)\gamma_\rho\gamma_\nu \phi^c(\hat{p}_2, p_2^\circ, \vec{y}). \tag{3.3}$$

Here the $\phi(\hat{p}, p^0, \vec{x})$ is the Coulomb distorted wave function for an electron, $\hat{p}$ means the direction at the observation point and $\phi^c = C\overline{\phi}^T$ with the charge conjugation matrix C. The other terms are

$$\widetilde{K}_{LL}^{\mu\nu} = K_{LL}^{\mu\nu} + \kappa\left(K_{LR}^{\mu\nu} + K_{RL}^{\mu\nu}\right) + \kappa^2 K_{RR}^{\mu\nu}, \tag{3.4}$$

$$\widetilde{K}_{RR}^{\mu\nu} = \lambda^2 K_{RR}^{\mu\nu} + \lambda\kappa\left(K_{LR}^{\mu\nu} + K_{RL}^{\mu\nu}\right) + \kappa^2 K_{LL}^{\mu\nu}, \tag{3.5}$$

$$\widetilde{L}_{LR}^{\mu\nu\rho} = \lambda L_{LR}^{\mu\nu\rho} + \kappa L_{LL}^{\mu\nu\rho} + \lambda\kappa L_{RR}^{\mu\nu\rho} + \kappa^2 L_{RL}^{\mu\nu\rho}, \tag{3.6}$$

$$\widetilde{L}_{RL}^{\mu\nu\rho} = \lambda L_{RL}^{\mu\nu\rho} + \kappa L_{LL}^{\mu\nu\rho} + \lambda\kappa L_{RR}^{\mu\nu\rho} + \kappa^2 L_{LR}^{\mu\nu\rho}, \tag{3.7}$$

where

$$K_{ab}^{\mu\nu} = \int \frac{d\vec{q}}{2(2\pi)^2 q^0} e^{i\vec{q}\cdot(\vec{x}-\vec{y})}$$

$$\cdot \langle N_\beta | \sum_n \left\{ \frac{J_a^{\dagger\mu}(\vec{x})|N_n\rangle\langle N_n| J_b^{\dagger\nu}(\vec{y})}{q^0 + E_n - E_\alpha + p_2^0} + \frac{J_b^{\dagger\nu}(\vec{y})|N_n\rangle\langle N_n| J_a^{\dagger\mu}(\vec{x})}{q^0 + E_n - E_\alpha + p_1^0} \right\} |N_\alpha\rangle ,$$

$$L_{ab}^{\mu\nu\rho} = \int \frac{d\vec{q}}{2(2\pi)^2 q^0} \cdot q^\rho$$

$$\cdot \langle N_\beta | \sum_n \left\{ e^{i\vec{q}\cdot(\vec{x}-\vec{y})} \frac{J_a^{\dagger\mu}(\vec{x})|N_n\rangle\langle N_n| J_b^{\dagger\nu}(\vec{y})}{q^0 + E_n - E_\alpha + p_2^0} - e^{-i\vec{q}\cdot(\vec{x}-\vec{y})} \frac{J_b^{\dagger\nu}(\vec{y})|N_n\rangle\langle N_n| J_a^{\dagger\mu}(\vec{x})}{q^0 + E_n - E_\alpha + p_1^0} \right\} |N_\alpha\rangle . \tag{3.9}$$

Here a(b) takes L and R, and N_n is the intermediate nuclear state with energy E_n. The nuclear Coulomb effect on the emitted electrons is taken into account through the Fermi factor $F(Z, p^0)$ by using the following approximation

$$\phi(\hat{p}, p^0, \vec{x}) \approx \sqrt{F(Z,p^0)}\, u(p)\, e^{i\vec{p}\cdot\vec{x}} , \tag{3.10}$$

where u(p) is a free electron spinor and $F(Z, P^0)$ is[49)]

$$F(Z,p^0) = 4(2pR)^{2\gamma-2} e^{\pi\eta} |\Gamma(\gamma+i\eta)|^2 [\Gamma(2\gamma+1)]^{-2} . \tag{3.11}$$

Here R is nuclear radius and

$$\gamma = \sqrt{1-(\alpha Z)^2} , \tag{3.12}$$

$$\eta = \alpha Z p^0/p . \tag{3.13}$$

The factor $(2pR)^{2\gamma-2}$ in Eq.(3.11) plays an important role for the heavy nuclei because γ deviates largely from 1. In this thesis, we do not take into account the other relatively small corrections such as the finite de Broglie wavelength effect, the finite nuclear size correction, etc.

Now we adopt the following further approximations: (i) The energy of the intermediate nucleus is replaced by the average value $\langle E_n \rangle$. (ii) The non-relativistic impulse approximation is used for the hadronic currents $J_L^{+\mu}(x)$ and $J_R^{+\mu}(x)$ including the recoil correction term (v/c term) for the 2n-mechanism. (iii) The first two terms of the multipole expansion for the leptonic wave function are kept; $\exp[-i(\vec{p}_1\cdot\vec{x}+\vec{p}_2\cdot\vec{y})] \simeq 1 - i(\vec{p}_1\cdot\vec{x}+\vec{p}_2\cdot\vec{y})$.

Under the approximation (i), the intermediate nuclear states can be summed by closure.[66] By the approximation (ii), the hadronic currents may be expressed as follows,

$$J_L^{+\mu}(x) = \sum_n \tau_n^+ (g_V g^{\mu 0} 1_n + g_A g^{\mu j} \sigma_n^j) \, \delta(\vec{x}-\vec{r}_n), \tag{3.13}$$

where the subscript n implies that the operators act on the n-th nucleon in the 2n-mechanism or the n-th quark in the N^*-mechanism. The v/c terms are not written explicitly in Eq. (3.13) for simplicity.[50] The similar expression is obtained for $J_R^{+\mu}$.

With these approximations, the $\vec{q}$-integrations in $\tilde{K}_{ab}^{\mu\nu}$ and $\tilde{L}_{ab}^{\mu\nu\rho}$ can be performed and the results are

$$\tilde{K}_{LL}^{\mu\nu} = \frac{1}{8\pi}\left[\langle H_1(r,m_j)\rangle + \langle H_2(r,m_j)\rangle\right] \langle N_\beta | \sum_{n,m} \tau_n^+ \tau_m^+ \, \delta(\vec{x}-\vec{r}_n)\,\delta(\vec{y}-\vec{r}_m) \cdot (G_V g^{\mu 0} + G_A \sigma_n^j g^{\mu j})(G_V g^{\nu 0} + G_A \sigma_m^k g^{\nu k}) | N_\alpha \rangle, \tag{3.14}$$

$$\tilde{L}_{LR}^{\mu\nu 0} = \frac{1}{8\pi}\left[A_1\langle H_1(r,m_j)\rangle - A_2\langle H_2(r,m_j)\rangle\right] \langle N_\beta | \sum_{n,m} \tau_n^+ \tau_m^+ \, \delta(\vec{x}-\vec{r}_n)\,\delta(\vec{y}-\vec{r}_m) \cdot (G_V g^{\mu 0} + G_A \sigma_n^j g^{\mu j})(\mathcal{E}_V g^{\nu 0} - \mathcal{E}_A \sigma_m^k g^{\nu k}) | N_\alpha \rangle, \tag{3.15}$$

$$\tilde{L}^{\mu\nu k}_{LR} = \frac{-1}{8\pi}\left[\langle rH_1'(r,m_j)\rangle + \langle rH_2'(r,m_j)\rangle\right]\langle N_\beta | \sum_{n,m} \tau_n^+ \tau_m^+ \delta(\vec{x}-\vec{r}_n)\delta(\vec{y}-\vec{r}_m)$$
$$\cdot \frac{1}{2}\left[(\vec{p}_1-\vec{p}_2)\cdot\hat{r}_{nm} + (\vec{p}_1+\vec{p}_2)\cdot\hat{r}_{+nm}\right]\hat{r}_{nm}$$
$$\cdot (G_V g^{\mu 0} + G_A \sigma_n^p g^{\mu p})(\varepsilon_V g^{\nu 0} - \varepsilon_A \sigma_m^q g^{\nu q}) | N_\alpha\rangle$$

$$+ \frac{1}{8\pi}\left[\langle H_1'(r,m_j)\rangle + \langle H_2'(r,m_j)\rangle\right]\langle N_\beta | \sum_{n,m} \tau_n^+ \tau_m^+ \delta(\vec{x}-\vec{r}_n)\delta(\vec{y}-\vec{r}_m)$$
$$\cdot\Big\{ (G_V g^{\mu p} D_n^p + G_A g^{\mu 0} C_n)(\varepsilon_V g^{\nu 0} - \varepsilon_A \sigma_m^q g^{\nu q})$$
$$+ (G_V g^{\mu 0} + G_A \sigma_n^p g^{\mu p})\cdot(\varepsilon_V g^{\nu q} D_m^q - \varepsilon_A g^{\nu 0} C_m)\Big\} | N_\alpha\rangle, \qquad (3.16)$$

where

$$G_V = g_A + \kappa g_V', \qquad G_A = g_A - \kappa g_A', \qquad (3.17)$$

$$\varepsilon_V = \lambda g_V' + \kappa g_V, \qquad \varepsilon_A = \lambda g_A' - \kappa g_A. \qquad (3.18)$$

We note that only the first term of the multipole expansion is taken into account for $\tilde{K}^{\mu\nu}_{LL}$ and $\tilde{L}^{\mu\nu 0}_{LR}$, and the dipole term is used to obtain the first term for $\tilde{L}^{\mu\nu k}_{LR}$. In order to maintain the consistency of the approximation, the v/c term of the hadronic current should be included. The second term of $\tilde{L}^{\mu\nu k}_{LR}$ in Eq.(3.16) is due to this correction. Here C_n and $\vec{D}_n$ are defined by

$$C_n = \vec{\sigma}_n \cdot (\vec{Q}_n - 2\vec{P}_n)/(2M), \qquad (3.19)$$

$$\vec{D}_n = [(\vec{Q}_n - 2\vec{P}_n) + i(\vec{\sigma}_n \times \vec{Q}_n)]/(2M), \qquad (3.20)$$

where $\vec{P}$, $\vec{Q}$ and M are the momentum, the momentum transfer and the mass of the nucleon, respectively. In Eqs.(3.14)-(3.16), $H(r, m_j)$ is the potential-like term due to the exchange of a neutrino and is defined as

$$H_i(|\vec{r}_n-\vec{r}_m|, m_j) = \int \frac{d\vec{q}}{2\pi^2} \cdot \frac{e^{i\vec{q}\cdot(\vec{r}_n-\vec{r}_m)}}{q^0(q^0+A_i)} , \qquad (3.21)$$

where $q^0 = \sqrt{|\vec{q}|^2 + m_j^2}$ and $A_i = \langle E_n\rangle - M_A + p_i^0$. Also, $H_i' = dH_i/dr$, $\vec{r}_{nm} = \vec{r}_n - \vec{r}_m$, $\hat{r}_{nm} = \vec{r}_{nm}/|\vec{r}_{nm}|$ and $\hat{r}_{+nm} = (\vec{r}_n + \vec{r}_m)/|\vec{r}_{nm}|$. The terms like $\langle H_i\rangle$ and $\langle rH_i'\rangle$ mean the average values of "potentials" with the weight of nuclear tensor operators.* Note that the potential $H_i(r, m_j)$ behaves like $1/r$ for $m_j \lesssim O(\text{MeV})$ and $e^{-m_j r}/r$ for $m_j \gtrsim O(\text{GeV})$. The replacement E_n by $\langle E_n\rangle$(the approximation (i)) is not crucial because the main contribution to the potentials comes from $|\vec{q}| \gtrsim 20$ MeV which is much larger than A_i (≃ a few MeV). The other terms, $\tilde{K}_{RR}^{\mu\nu}$, $\tilde{L}_{RL}^{\mu\nu 0}$ and $\tilde{L}_{RL}^{\mu\nu k}$, are obtained by taking the interchanges $(G_V \leftrightarrow \varepsilon_V)$ and $(G_A \leftrightarrow -\varepsilon_A)$ in the expressions of $\tilde{K}_{LL}^{\mu\nu}$, $\tilde{L}_{LR}^{\mu\nu 0}$ and $\tilde{L}_{LR}^{\mu\nu k}$, respectively.

The product of the leptonic and hadronic parts can be easily calculated and the results are as follows;

$$\int d\vec{x}\, d\vec{y} \begin{Bmatrix} t^L_{\mu\nu} \tilde{K}^{\mu\nu}_{LL} \\ t^R_{\mu\nu} \tilde{K}^{\mu\nu}_{RR} \end{Bmatrix} = \frac{1}{8\pi}\left[\langle H_1\rangle + \langle H_2\rangle\right] \overline{u}(p_1)(1\pm\gamma_5)u^c(p_2) \begin{Bmatrix} G_V^2 M_F - G_A^2 M_{GT} \\ \varepsilon_V^2 M_F - \varepsilon_A^2 M_{GT} \end{Bmatrix}, \qquad (3.22)$$

$$\int d\vec{x}\, d\vec{y} \left(u^L_{\mu\nu 0} \tilde{L}^{\mu\nu 0}_{LR} + u^R_{\mu\nu 0} \tilde{L}^{\mu\nu 0}_{RL}\right) = \frac{1}{4\pi}\left[A_1\langle H_1\rangle - A_2\langle H_2\rangle\right]$$

$$\cdot\, \overline{u}(p_1)\left\{\gamma^0 (G_A \varepsilon_V M_F - G_A \varepsilon_A M_{GT}) - \gamma^k 2 G_- M^k\right\} u^c(p_2), \qquad (3.23)$$

---

* The average is defined as $\langle f\rangle = \langle N_B|\Sigma_{n,m}\, f\, O_{nm}|N_A\rangle / \langle N_B|\Sigma_{n,m}\, O_{nm}|N_A\rangle$, where O_{nm} is the nuclear tensor operator.

$$\int d\vec{x}\, d\vec{y} \left(U^L_{\mu\nu k} \tilde{L}^{\mu\nu k}_{LR} + U^R_{\mu\nu k} \hat{L}^{\mu\nu k}_{RL} \right) = \frac{-1}{8\pi} \left[\langle r H_1' \rangle + \langle r H_2' \rangle \right]$$

$$\cdot\, \bar{u}(p_1) \left[(p_1 - p_2)^\ell \left\{ \gamma^k P^{\ell k} + \gamma^0 Q^\ell \right\} + (p_1 + p_2)^\ell \gamma_5 \left\{ \gamma^k Q^{\ell k} + \gamma^0 Q^\ell \right\} \right] u^c(p_2)$$

$$+ \frac{i}{4\pi} \left[\langle H_1' \rangle + \langle H_2' \rangle \right] \bar{u}(p_1) \gamma_5 \left\{ \gamma^k R^k + \gamma^0 S \right\} u^c(p_2) \,, \qquad (3.24)$$

where $G_\pm = (G_V \varepsilon_A \pm G_A \varepsilon_V)/2$ and

$$M_F = \langle 1_n 1_m \rangle \,, \qquad M_{GT} = \langle \vec{\sigma}_n \cdot \vec{\sigma}_m \rangle \,, \qquad (3.25)$$

$$M^k = \langle \sigma_n^k \rangle \,, \qquad (3.26)$$

$$P^{\ell k} = \langle \hat{r}^\ell_{nm} \{ \hat{r}^k_{nm} (G_V \varepsilon_V + G_A \varepsilon_A \vec{\sigma}_n \cdot \vec{\sigma}_m)$$

$$+ i (G_V \varepsilon_A + G_A \varepsilon_V)(\hat{r}_{nm} \times \vec{\sigma}_n)^k - 2 G_A \varepsilon_A (\hat{r}_{nm} \cdot \vec{\sigma}_n) \sigma_m^k \rangle \,, \qquad (3.27)$$

$$Q^\ell = \langle (G_V \varepsilon_A + G_A \varepsilon_V)(\hat{r}_{nm} \cdot \vec{\sigma}_n) - i\, G_A \varepsilon_A \hat{r}_{nm} \cdot (\vec{\sigma}_n \times \vec{\sigma}_m)^\ell \rangle \,, \qquad (3.28)$$

$$P^\ell = -2 G_- \langle \hat{r}^\ell_{nm} (\hat{r}_{nm} \cdot \vec{\sigma}_n) \rangle \,, \qquad (3.29)$$

$$Q^{\ell k} = -2 G_- \langle \hat{r}_{+nm}\, i (\hat{r}_{nm} \times \vec{\sigma}_n)^k \rangle \,, \qquad (3.30)$$

$$R^k = \langle (G_V \varepsilon_A + G_A \varepsilon_V) \{ \hat{r}^k_{nm} (C_n + \vec{D}_n \cdot \vec{\sigma}_m) - (\hat{r}_{nm} \cdot \vec{\sigma}_m) D_n^k - (\hat{r}_{nm} \cdot \vec{D}_n) \sigma_m^k \}$$

$$+ 2i\, G_V \varepsilon_V (\hat{r}_{nm} \times \vec{D}_n)^k + 2i\, G_A \varepsilon_A (\hat{r}_{nm} \times \vec{\sigma}_m)^k C_n \rangle \,, \qquad (3.31)$$

$$S = -2 G_- \langle \hat{r}_{nm} \cdot (\vec{D}_n \times \vec{\sigma}_m) \rangle \,. \qquad (3.32)$$

Here we used the following definition,*

$$\langle O_{nm} \rangle = \langle N_\beta | \sum_{n,m} \tau_n^+ \tau_m^+ O_{nm} | N_\alpha \rangle \,. \qquad (3.33)$$

---

* The better parameterization would be $\langle rH \rangle \langle \frac{1}{r} \vec{\sigma}_n \cdot \vec{\sigma}_m \rangle$ instead of $\langle H \rangle \langle \vec{\sigma}_n \cdot \vec{\sigma}_m \rangle$ as given in Eq. (3.25).

It is clear from Eqs.(3.25)-(3.32) that the $0^+ \to J^+$ ($J \geqq 3$) transitions are forbidden within our approxiamtions.

In the following, a further simplification is made by replacing A_i in the potentials with their average value, i. e.,

$$\mu_0 = (A_1 + A_2)/2m_e = \left[\langle E_n \rangle - (M_A + M_B)/2 \right] / m_e \quad . \tag{3.34}$$

As a consequence, we can write H_i's in the single form;

$$H_1(r, m_j) = H_2(r, m_j) = H(r, m_j, \mu_0) \; . \tag{3.35}$$

(3-a) *The 2n-mechanism*

The R-matrix is obtained from R_W in Eq.(3.1) by replacing N_α and N_β with N_A and N_B, respectively. It should be noted that the M_F and M_{GT} terms are of rank 0 with respect to the angular momentum and the Q^ℓ, M^ℓ, C^ℓ and R^ℓ terms are of rank 1. On the other hand, the $P^{\ell k}$ and $Q^{\ell k}$ terms consist of irreducible tensor operators of rank 0, 1 and 2. Consequently, the terms $t^L_{\mu\nu} \tilde{K}^{\mu\nu}_{LL}$, $t^R_{\mu\nu} \tilde{K}^{\mu\nu}_{RR}$ contribute to the $0^+ \to 0^+$ transition and $u^L_{\mu\nu 0} \tilde{L}^{\mu\nu 0}_{LR} + u^R_{\mu\nu 0} \tilde{L}^{\mu\nu 0}_{RL}$ to the $0^+ \to 0^+$ and 1^+ transitions. While, $u^L_{\mu\nu k} \tilde{L}^{\mu\nu k}_{LR} + u^R_{\mu\nu k} \tilde{L}^{\mu\nu k}_{RL}$ contributes to the $0^+ \to 0^+$, 1^+ and 2^+ transitions. These features and the relative order of magnitudes are listed in Table I.

(i) *The $0^+ \to 0^+$ transition*

The nuclear matrix elements M_F , M_{GT} , $P^{ii}/3$, $Q^{ii}/3$ and S contribute to this transition. We obtain

$$d\Gamma^{2n}_{0\nu}(0^+\to 0^+) = (a_{0\nu}/m_e^5)(m_e R)^{-2}$$
$$\cdot\Big[f_{01}\,(|X_1+X_4-X_5|^2+|X_2+X_4+X_5|^2) + f_{02}\,|X_3-X_4|^2$$
$$+ f_{03}\,|X_5|^2 - f_{04}\,\mathrm{Re}(X_5 X_6^*) + f_{05}\,|X_6|^2$$
$$+ f_{06}\,\mathrm{Re}\{(X_1+X_2+\tfrac{1}{2}X_3+\tfrac{3}{2}X_4)(X_3^*-X_4^*)\}$$
$$+ f_{07}\,\mathrm{Re}\{(X_1-X_2-\tfrac{3}{2}X_5)X_5^*\} - f_{08}\,\mathrm{Re}\{(X_1-X_2-X_5)X_6^*\}$$
$$- f_{09}\,\mathrm{Re}\{(X_1+X_4-X_5)(X_2^*+X_4^*+X_5^*)\}\Big]\, d\Omega_{0\nu}\,, \tag{3.36}$$

where
$$a_{0\nu} = \frac{G^4 m_e^9}{2(2\pi)^5}\,, \tag{3.37}$$
$$d\Omega_{0\nu} = F(Z,p_1^0)F(Z,p_2^0)|\vec{p}_1||\vec{p}_2|\,\delta(p_1^0+p_2^0+M_B-M_A)\,d\cos\theta\, dp_1^0\, dp_2^0\,. \tag{3.38}$$

Here θ is the angle between two emitted electrons, and the kinematic factors which show the momentum dependence are

$$f_{01} = (p_1^0 p_2^0 - \vec{p}_1\cdot\vec{p}_2)\,, \tag{3.39}$$
$$f_{02} = (p_1^0 p_2^0 + \vec{p}_1\cdot\vec{p}_2)(p_1^0 - p_2^0)^2 m_e^{-2}/2\,, \tag{3.40}$$
$$f_{03} = (p_1^0 p_2^0 + \vec{p}_1\cdot\vec{p}_2)(p_1^0 + p_2^0)^2 m_e^{-2}/2\,, \tag{3.41}$$
$$f_{04} = (p_1^0 p_2^0 + \vec{p}_1\cdot\vec{p}_2)(p_1^0 + p_2^0) m_e^{-1}\,, \tag{3.42}$$
$$f_{05} = (p_1^0 p_2^0 + \vec{p}_1\cdot\vec{p}_2 + m_e^2)/2\,, \tag{3.43}$$

$$f_{06} = (p_1^0 - p_2^0)^2, \tag{3.44}$$

$$f_{07} = (p_1^0 + p_2^0)^2, \tag{3.45}$$

$$f_{08} = (p_1^0 + p_2^0) m_e , \tag{3.46}$$

$$f_{09} = 2 m_e^2 , \tag{3.47}$$

and the nuclear matrix elements are

$$X_1 = \sum_j (m_j/m_e) U_{ej}^2 \langle H \rangle R [G_V^2 M_F - G_A^2 M_{GT}], \tag{3.48}$$

$$X_2 = \sum_j (m_j/m_e) V_{ej}^2 \langle H \rangle R [\varepsilon_V^2 M_F - \varepsilon_A^2 M_{GT}], \tag{3.49}$$

$$X_3 = \sum_j U_{ej} V_{ej} \langle H \rangle R [G_V \varepsilon_V M_F - G_A \varepsilon_A M_{GT}], \tag{3.50}$$

$$X_4 = \sum_j U_{ej} V_{ej} \langle r H' \rangle R (1/3) [G_V \varepsilon_V M_F + G_A \varepsilon_A (M_{GT}/3 - 2M_T)], \tag{3.51}$$

$$X_5 = \sum_j U_{ej} V_{ej} \langle r H' \rangle R (1/3)\, 2 G_- M_Q , \tag{3.52}$$

$$X_6 = \sum_j U_{ej} V_{ej} (\langle H' \rangle R / m_e) \cdot 2 G_- M_S , \tag{3.53}$$

where M_F and M_{GT} are defined in Eq. (3.25), and

$$M_T = \langle (\hat{r}_{nm} \cdot \vec{\sigma}_n)(\hat{r}_{nm} \cdot \vec{\sigma}_m) - \tfrac{1}{3} \vec{\sigma}_n \cdot \vec{\sigma}_m \rangle, \tag{3.54}$$

$$M_D = \langle i \, \hat{r}_{nm} \cdot (\hat{r}_{nm} \times \vec{\sigma}_n) \rangle, \tag{3.55}$$

$$M_S = \langle \hat{r}_{nm} \cdot (\vec{D}_n \times \vec{\sigma}_m) \rangle . \tag{3.56}$$

The inverse of the half-life for the $0^+ \to 0^+$ transition in the 2n-mechanism is

$$\left[T^{2n}_{0\nu}(0^+\to 0^+)\right]^{-1} = G_{01}\ (|X_1+X_4-X_5|^2+|X_2+X_4+X_5|^2)$$

$$+\ G_{02}\ |X_3-X_4|^2\ +\ G_{03}|X_5|^2\ -\ G_{04}\ \mathrm{Re}(X_5X_6^*)$$

$$+\ G_{05}\ |X_6|^2\ -\ G_{06}\ \mathrm{Re}\{(X_1+X_2+\tfrac{1}{2}X_3+\tfrac{3}{2}X_4)(X_3^*-X_4^*)\}$$

$$+\ G_{07}\ \mathrm{Re}\{(X_1-X_2-\tfrac{3}{2}X_5)X_5^*\}$$

$$-\ G_{08}\ \mathrm{Re}\{(X_1-X_2-X_5)X_6^*\}$$

$$-\ G_{09}\ \mathrm{Re}\{(X_1+X_4-X_5)(X_2^*+X_4^*+X_5^*)\}, \qquad (3.57)$$

where

$$G_{0i} = \int d\Omega_{0\nu}\ (a_{0\nu}/m_e^5)(m_eR)^{-2} f_{0i}/\ln 2\ . \qquad (3.58)$$

Let us compare our results with those obtained previously. In the limit of $\lambda = \kappa = 0$ and $U_{ej} = \delta_{ej}$, we found that the overall normalization by Greuling and Whitten[31)] is twice larger than ours.* In the other limit of $m_j = 0$, our results can be compared with those by Primakoff and Rosen[15)] who used the quite general form for H_W.** They assumed $M_S=M_Q=M_T=0$ and their result is twice larger than ours.*

---

* It seems that they did not take into account of the statistical factor. See, e. g., Eq.(20) of Ref. 31) and Eq.(33) of Ref. 15).

** The correspondence between our notation and theirs in Ref. 15) is as follows; $C_V = D_V = G[(g_V + \kappa g_V')U_{ej} + (\lambda g_V' + \kappa g_V)V_{ej}]/2$, $C_A = D_A = G[(g_A - \kappa g_A')U_{ej} + (\lambda g_A' - \kappa g_A)V_{ej}]/2$, $C_V\delta_V = D_V\eta_V = G[-(g_V + \kappa g_V')U_{ej} + (\lambda g_V'+\kappa g_V)V_{ej}]/2$, and $C_A\delta_A = D_A\eta_A = G[-(g_A-\kappa g_A')U_{ej}+(\lambda g_A'-\kappa g_A)V_{ej}]/2$. We remind also that the result by Molina and Pascual[32)] is four times larger than ours.

(ii) *The $0^+ \to 2^+$ transition*

Only the rank 2 part of the $p^{\ell k}$ and $Q^{\ell k}$ terms contribute to this transition. The final result is

$$d\Gamma^{2n}_{0\nu}(0^+\to 2^+) = (a_{0\nu}/m_e^7)(m_e R)^{-2}(1/30)\left|\sum_j U_{ej}V_{ej}\langle rH'\rangle R\right|^2 \cdot\left\{f_{2+}(N_2^{pq},N_2^{pq}) + f_{2-}(N_3^{pq},N_3^{pq})\right\} d\Omega_{0\nu}, \tag{3.59}$$

where $a_{0\nu}$ is defined in Eq.(3.37) and

$$f_{2\pm} = \pm 3(\vec{p}_1\cdot\vec{p}_2)^2 \mp \vec{p}_1\cdot\vec{p}_2\left[10(p_1^0p_2^0+m_e^2) \pm (|\vec{p}_1|^2+|\vec{p}_2|^2)\right] + 5(p_1^0p_2^0 \pm m_e^2)(|\vec{p}_1|^2+|\vec{p}_2|^2) \mp |\vec{p}_1|^2|\vec{p}_2|^2, \tag{3.60}$$

and $(N^{pq}_{2(3)}, N^{pq}_{2(3)}) \equiv \sum_{J_z}\sum_{p,q=1}^{3} N^{pq*}_{2(3)} N^{pq}_{2(3)}$ with

$$N_2^{pq} = G_V\varepsilon_V N^{pq}_{2(1)} + G_A\varepsilon_A N^{pq}_{2(2)} + 2G_+ N^{pq}_{2(3)}, \tag{3.61}$$

$$N_3^{pq} = 2G_- N^{pq}_{3(1)}. \tag{3.62}$$

Here

$$N^{pq}_{2(1)} = \langle \hat{r}^p_{nm}\hat{r}^q_{nm}\rangle, \quad N^{pq}_{2(2)} = \langle \hat{r}^p_{nm}\{\hat{r}^q_{nm}(\vec{\sigma}_n\cdot\vec{\sigma}_m) - 2(\hat{r}_{nm}\cdot\vec{\sigma}_n)\sigma^q_m\}\rangle, \tag{3.63}$$

$$N^{pq}_{2(3)} = \langle i\,\hat{r}^p_{nm}(\hat{r}_{nm}\times\vec{\sigma}_n)^q\rangle, \quad N^{pq}_{3(1)} = \langle i\,\hat{r}^p_{+nm}(\hat{r}_{nm}\times\vec{\sigma}_m)^q\rangle. \tag{3.64}$$

The inverse of the half-life is

$$\left[T^{2n}_{0\nu}(0^+\to 2^+)\right]^{-1} = \left|\sum_j U_{ej}V_{ej}\langle rH'\rangle R\right|^2 \left\{G_{2+}(N_2^{pq},N_2^{pq}) + G_{2-}(N_2^{pq},N_2^{pq})\right\}, \tag{3.65}$$

where

$$G_{2\pm} = \int d\Omega_{0\nu}\,(a_{0\nu}/m_e^7)(m_e R)^{-2}(1/30)\, f_{2\pm}/\ln 2. \tag{3.66}$$

(iii) *The $0^+ \to 1^+$ transition*

The terms Q^{ℓ}, P^{ℓ}, M^k, $\varepsilon_{\ell kj}P^{kj}$, $\varepsilon_{\ell kj}Q^{kj}$ and R^{ℓ} contribute to this transition. After small calculations, the decay formula is

$$
\begin{aligned}
d\Gamma^{2n}_{0\nu}(0^+\to 1^+) &= (a_{0\nu}/m_e^5)(m_eR)^{-2}(1/6)\,|\Sigma U_{ej}V_{ej}|^2 \\
&\cdot\Big[\, f_{01}\, 16(R^q,R^q)m_e^{-2}(\langle H\rangle R)^2 \\
&+ f_{02}\, 2\{(N_3^q,N_3^q)(\langle rH'\rangle R)^2 + 2\,\mathrm{Re}(N_3^q,N_5^q)\langle rH'\rangle R\langle H\rangle R + (N_5^q,N_5^q)(\langle H\rangle R)^2\} \\
&+ f_{03}\, 2(N_1^q,N_1^q)(\langle rH'\rangle R)^2 - f_{08}\, 8\,\mathrm{Im}(N_1^q,R^q)m_e^{-1}(\langle rH'\rangle R)(\langle H\rangle R) \\
&+ f_{06}\Big[\{(N_3^q,N_3^q)+4(N_4^q,N_4^q)\}(\langle rH'\rangle R)^2 + \{3(N_5^q,N_5^q)+16(R^q,R^q)m_e^{-2}\}(\langle H\rangle R)^2 \\
&\qquad + 2\{\mathrm{Re}(N_3^q,N_5^q) - 2\,\mathrm{Im}(N_4^q,N_5^q)\}\langle rH'\rangle R\langle H\rangle R\Big] \\
&+ f_{07}\Big[16(R^q,R^q)m_e^{-2}(\langle H\rangle R)^2 - \{(N_1^q,N_1^q)+(N_2^q,N_2^q)\}(\langle rH'\rangle R)^2\Big] \\
&+ f_{08}\, 16\{\mathrm{Im}(N_1^q,R^q) + \mathrm{Re}(N_2^q,R^q)\}\langle rH'\rangle R\langle H\rangle R \\
&- f_{09}\, 24(R^q,R^q)m_e^{-2}(\langle H\rangle R)^2 \\
&+ g_{01}\, 2\{(N_3^q,N_3^q)+(N_4^q,N_4^q)-(N_1^q,N_1^q)-2\,\mathrm{Im}(N_1^q,N_2^q)+2\,\mathrm{Im}(N_3^q,N_4^q)\}(\langle rH'\rangle R)^2 \\
&+ g_{02}\, 4(N_5^q,N_5^q)(\langle H\rangle R)^2 + g_{03}\Big[4\{(N_2^q,N_2^q)+(N_4^q,N_4^q)\}(\langle rH'\rangle R)^2 \\
&- 8\,\mathrm{Im}(N_4^q,N_5^q)\langle rH'\rangle R\langle H\rangle R\Big] - g_{04}\, 16\,\mathrm{Re}(N_2^q,R^q)m_e^{-1}\langle rH'\rangle R\langle H\rangle R \\
&- g_{05}\, 2\,\mathrm{Re}(N_1^q,N_3^q)(\langle rH'\rangle R)^2 + g_{06}\, 2\,\mathrm{Re}(N_1^q,N_3^q)(\langle rH'\rangle R)^2\,\Big]\, d\Omega_{0\nu},
\end{aligned}
\tag{3.67}
$$

where f_{0i} $(i=1\sim 9)$ are defined in Eqs.(3.39)-(3.47) and

$$g_{01} = (\vec{p}_1 \times \vec{p}_2)^2 m_e^{-2}, \tag{3.68}$$

$$g_{02} = (p_1^0 p_2^0 - \vec{p}_1 \cdot \vec{p}_2)(p_1^0 - p_2^0)^2 m_e^{-2}/2, \tag{3.69}$$

$$g_{03} = (p_1^0 p_2^0 - \vec{p}_1 \cdot \vec{p}_2)(p_1^0 + p_2^0)^2 m_e^{-2}/2, \tag{3.70}$$

$$g_{04} = (p_1^0 p_2^0 - \vec{p}_1 \cdot \vec{p}_2)(p_1^0 + p_2^0) m_e^{-1}, \tag{3.71}$$

$$g_{05} = (p_1^0 - p_2^0), \tag{3.72}$$

$$g_{06} = (p_1^0 p_2^0 + \vec{p}_1 \cdot \vec{p}_2)(p_1^{0\,2} - p_2^{0\,2}), \tag{3.73}$$

and $(N_i^q, N_i^q) = \sum_{J_z} \sum_{q=1}^{3} N_i^{q*} N_i^q$ with

$$N_1^q = \langle \hat{r}_{+nm}^{\,q} \{ 2G_+ (\hat{r}_{nm} \cdot \vec{\sigma}_n) - i G_A \varepsilon_A \hat{r}_{nm} \cdot (\vec{\sigma}_n \times \vec{\sigma}_m) \} \rangle, \tag{3.74}$$

$$N_2^q = -2 G_A \varepsilon_A \langle (\hat{r}_{nm} \cdot \vec{\sigma}_n)(\hat{r}_{nm} \times \vec{\sigma}_m)^q \rangle, \tag{3.75}$$

$$N_3^q = -2 G_- \langle \hat{r}_{nm}^{\,q} (\hat{r}_{nm} \cdot \vec{\sigma}_m) \rangle, \tag{3.76}$$

$$N_4^q = -2 G_- \langle [\hat{r}_{+nm} \times (\vec{r}_{nm} \times \vec{\sigma}_n)]^q \rangle, \tag{3.77}$$

$$N_5^q = \langle \sigma_n^q \rangle. \tag{3.78}$$

The terms N_1^q, N_2^q, N_3^q, N_4^q and N_5^q come from Q^q, $\varepsilon_{q\ell k} P^{\ell k}$, P^q, $\varepsilon_{q\ell k} Q^{\ell k}$ and M^q, respectively.

(3-b) *The N^*-mechanism*

The R-matrix for the N^*-mechanism is obtained by substituting the R_W-matrices corresponding to the $N_{\Delta^-} \to N_B + 2e^-$ and $N_A \to N_{\Delta^{++}} + 2e^-$

transitions into Eq. (B.3) in Appendix B.

The nuclear matrix elements in the R_W-matrix are given by Eqs. (3.25)-(3.32). Since there is no standard method to treat the v/c correction terms in the quark model, these corrections (the R^k and S terms in Eq. (3.24)) are discarded. It should be noted that the operators* $\tau^+_{n(m)}$, $\sigma^j_{n(m)}$, $\hat{r}'_{nm}$, $\hat{r}'_{+nm}$ act on quarks and change the intrinsic part of the hadron. The nucleon $N(\frac{1}{2}^+)$ and $\Delta(\frac{3}{2}^+)$ are assigned in the SU(6) quark model to the ground state i. e., the zero orbital angular momentum states around the center of hadron. From these considerations, we conclude that M_F, M_{GT}, Q^{ℓ} and $Q^{\ell k}$ defined in Eqs. (3.25), (3.28) and (3.30) turn out to be zero, and the terms $P^{\ell k}$, M^{ℓ} and P^{ℓ} contribute. (See Appendix B for the detailed discussion.) Consequently, the terms $u^L_{\mu\nu k}\tilde{L}^{\mu\nu k}_{LR} + u^R_{\mu\nu k}\tilde{L}^{\mu\nu k}_{RL}$ and $u^L_{\mu\nu 0}\tilde{L}^{\mu\nu 0}_{LR} + u^R_{\mu\nu 0}\tilde{L}^{\mu\nu 0}_{RL}$ contribute to the $(\beta\beta)_{0\nu}$ mode in the N^*-mechanism, while $t^L_{\mu\nu}\tilde{K}^{\mu\nu}_{LL}$, $t^R_{\mu\nu}\tilde{K}^{\mu\nu}_{RR}$ do not.

In summarizing the above discussion, the $(\beta\beta)_{0\nu}$ mode in the N^*-mechanism takes place only when $\lambda \neq 0$ or $\kappa \neq 0$ whether neutrinos are massive or massless. These results are listed in Table I. Halprin et al.[51)] have derived the bounds of the neutrino mass both in the 2n- and N^*-mechanisms. However, the bounds they obtained in the N^*-mechanism seem to be meaningless, because there is no contribution from the m_ν-term within the N^*-mechanism adopted in the present thesis. We would like to emphasize here that the above discussions are independent of the "factorization hypothesis" which will be used later.

---

* $\hat{r}'_{nm}$ and $\hat{r}'_{+nm}$ are defined in terms of the position operators of quarks measured from the center of the hadron.

The non-vanishing product of the leptonic and hadronic parts for the $N_{\Delta^-} \to N_B + 2e^-$ transition is

$$\int d\vec{x} d\vec{y} \left(U^L_{\mu\nu\rho} \tilde{L}^{\mu\nu\rho}_{LR} + U^R_{\mu\nu\rho} \tilde{L}^{\mu\nu\rho}_{RL} \right) = \frac{1}{6\pi} \langle rH' \rangle_\Delta$$

$$\cdot \overline{U}(p_1) \Big\{ \gamma^k \Big[\{ G_+ i\epsilon^{\ell kj} (p_1 - p_2)^\ell + G_- (p_1^0 - p_2^0) \delta^{jk} \} M^j_\Delta + G_A \varepsilon_A (p_1 - p_2)^\ell_\Delta M^{\ell k}_\Delta \Big]$$

$$- \gamma^0 G_- (p_1 - p_2)^\ell M^\ell_\Delta \Big\} U^c(p_2) , \tag{3.79}$$

where

$$M^j_\Delta = \langle \sigma^j_n \rangle_\Delta , \tag{3.80}$$

$$M^{jk}_\Delta = \langle \sigma^j_n \sigma^k_m \rangle_\Delta . \tag{3.81}$$

The nuclear tensor operators in Eqs.(3.80) and (3.81) only change the spin and isospin of Δ^- and leave the remainder unchanged. Therefore, M^j_Δ and M^{jk}_Δ represent essentially the matrix elements between Δ^- and p. Also, the expectation value of the "potential" $\langle rH' \rangle$ should be taken between Δ^- and p. Note that $g_A = g_V$ and $g'_A = g'_V$ should be taken in the N^*-mechanism as explained in §2.

Now we use the "factorization hypothesis"[27] (see Eq.(B.15)) and evaluate the decay formula,*

---

* Here we have used the following results; $\Sigma \tilde{M}^{j*} \tilde{M}^k = (16/3)\delta_{jk}$, $\Sigma \tilde{M}^{j*} \tilde{M}^{k\ell} = 0$, $\Sigma \tilde{M}^{jk*} \tilde{M}^{\ell m} = 16(\delta_{j\ell}\delta_{km} + \delta_{jm}\delta_{k\ell} - \frac{2}{3}\delta_{jk}\delta_{\ell m})$, where $\tilde{M}^j$ and $\tilde{M}^{jk}$ are defined from M^j_Δ and M^{jk}_Δ by replacing N_{Δ^-} and N_B with Δ^- and p, respectively. The same relations hold for the matrix elements between n and Δ^{++}.

$$d\Gamma_{0\nu}^{N*} = (a_{0\nu}/m_e^9)(m_e a)^{-2}(32/27)\left|\sum_j U_{ej}V_{ej}\right|^2 P(\Delta)\left|\langle\Phi_f|\Phi_i\rangle\right|^2$$

$$\cdot\Big[\left\{G_+^2 f_1^{N*} + (G_A\varepsilon_A)^2 f_2^{N*}\right\}(\langle rH'\rangle_\Delta a)^2$$

$$+ G_-^2\left\{f_3^{N*}(\langle rH'\rangle_\Delta a)^2 - f_4^{N*}(\langle H\rangle_\Delta a)(\langle rH'\rangle_\Delta a) + f_5^{N*}(\langle H\rangle_\Delta a)^2\right.\Big] d\Omega_{0\nu}, \tag{3.82}$$

where a (= 0.7 fm) is the range of quarks in Δ and

$$f_1^{N*} = (\vec{p}_1\cdot\vec{p}_2)^2 - \vec{p}_1\cdot\vec{p}_2\left[2(p_1^0p_2^0+m_e^2)+|\vec{p}_1|^2+|\vec{p}_2|^2\right] + (p_1^0p_2^0+m_e^2)(|\vec{p}_1|^2+|\vec{p}_2|^2) + |\vec{p}_1|^2|\vec{p}_2|^2, \tag{3.83}$$

$$f_2^{N*} = 3(\vec{p}_1\cdot\vec{p}_2)^2 - \vec{p}_1\cdot\vec{p}_2\left[10(p_1^0p_2^0+m_e^2)+|\vec{p}_1|^2+|\vec{p}_2|^2\right] + 5(p_1^0p_2^0+m_e^2)(|\vec{p}_1|^2+|\vec{p}_2|^2) - |\vec{p}_1|^2|\vec{p}_2|^2, \tag{3.84}$$

$$f_3^{N*} = \left\{-2(\vec{p}_1\cdot\vec{p}_2)^2 - \vec{p}_1\cdot\vec{p}_2\left[2(p_1^0p_2^0-m_e^2)-|\vec{p}_1|^2-|\vec{p}_2|^2\right] + (p_1^0p_2^0-m_e^2)(|\vec{p}_1|^2+|\vec{p}_2|^2)\right\}/2, \tag{3.85}$$

$$f_4^{N*} = 3(p_1^0-p_2^0)^2(p_1^0p_2^0+m_e^2+\vec{p}_1\cdot\vec{p}_2), \tag{3.86}$$

$$f_5^{N*} = \frac{9}{2}(p_1^0-p_2^0)^2\left\{3(p_1^0p_2^0+m_e^2)-\vec{p}_1\cdot\vec{p}_2\right\}. \tag{3.87}$$

Here $P(\Delta)$ is the probability of producing Δ per nucleon inside the nucleus, and $\langle\Phi_f|\Phi_i\rangle$ means the overlap between the initial and final nuclei. The $N_A \to N_{\Delta^{++}} + 2e^-$ transition is also included in the above formula. We refer to Appendix B for the detailed discussion.

The inverse of the half-life is

$$\left[T_{0\nu}^{N*}(0^+\to J^+)\right]^{-1} = \left|\sum_j U_{ej}V_{ej}\right|^2 P(\Delta)\left|\langle\Phi_f|\Phi_i\rangle\right|^2$$

$$\cdot\Big[\left\{G_+^2 G_1^{N*} + (G_A\varepsilon_A)^2 G_2^{N*}\right\}(\langle rH'\rangle_\Delta a)^2$$

$$+ G_-^2\left\{G_3^{N*}(\langle rH\rangle_\Delta a)^2 - G_4^{N*}\langle rH\rangle_\Delta a\langle H\rangle_\Delta a + G_5^{N*}(\langle H\rangle_\Delta a)^2\right.\Big], \tag{3.88}$$

where

$$G_i^{N*} = \int d\Omega_{o\nu} (a_{o\nu}/m_e^9)(m_e a)^{-2}(32/27) f_i^{N*}/\ln 2 . \qquad (3.89)$$

The above formula is applicable to the $0^+ \to 0^+$, 1^+ and 2^+ transitions by the appropriate choice of $P(\Delta)|\langle\Phi_f|\Phi_i\rangle|^2$.

§4. The $(\beta\beta)_{2\nu}$ mode

Similarly to the $(\beta\beta)_{0\nu}$ mode, the $0^+ \to J^+$ transitions are investigated. In our Hamiltonian in Eq.(2.1), the $(\beta\beta)_{2\nu}$ mode takes place through the process,

$$N_A(p_A) \to N_B(p_B) + e^-(p_1) + e^-(p_2) + \overline{N_i}(k_1) + \overline{N_j}(k_2). \tag{4.1}$$

The contribution from the right-handed current is suppressed by λ and κ (λ, $|\kappa| \ll 1$) so that this is neglected here.

The R_W-matrix due to the V-A interaction for the $N_\alpha \to N_\beta + 2e^- + \overline{N_i} + \overline{N_j}$ transition is expressed by

$$R_{Wij} = \frac{\epsilon_{ij}}{\sqrt{2}} \left(\frac{G}{\sqrt{2}}\right)^2 U_{ei} U_{ej} \int d\vec{x}\, d\vec{y}\, (2\pi)^{-6} (p_1^0 p_2^0 k_1^0 k_2^0)^{-1/2} \cdot \left\{ E_{\mu\nu} J^{\mu\nu} - (p_1 \leftrightarrow p_2) \right\}, \tag{4.2}$$

where

$$E_{\mu\nu} = \overline{\Phi}(\hat{p}_1, p_1^0, \vec{x}) \gamma_\mu (1-\gamma_5) U^c(k_1) e^{-i\vec{k}_1 \cdot \vec{x}} \cdot \overline{\Phi}(\hat{p}_2, p_2^0, \vec{y}) \gamma_\nu (1-\gamma_5) U^c(k_2) e^{-i\vec{k}_2 \cdot \vec{y}}, \tag{4.3}$$

$$J^{\mu\nu} = \langle N_\beta | \sum_n \left\{ \frac{J_L^{+\mu}(\vec{x}) |N_n\rangle\langle N_n| J_L^{+\nu}(\vec{y})}{E_n - E_\alpha + p_2^0 + k_2^0} + \frac{J_L^{+\nu}(\vec{y}) |N_n\rangle\langle N_n| J_L^{+\mu}(\vec{x})}{E_n - E_\alpha + p_1^0 + k_1^0} \right\} |N_\alpha\rangle. \tag{4.4}$$

Here the term $\varepsilon_{ij}/\sqrt{2}$ is the statistical factor for the final two electrons and two neutrinos, i. e., $\varepsilon_{ij} = 1/\sqrt{2}$ for $i = j$ and $\varepsilon_{ij} = 1$ for $i \neq j$. The full R-matrix for the $(\beta\beta)_{2\nu}$ mode can be readily obtained in the same manner as for the $(\beta\beta)_{0\nu}$ mode.

Now we use the approximations (i) and (ii) introduced in §3 and take only the S-wave contributions from the electron and neutrino wave functions. Under these assumptions, the nuclear part $J^{\mu\nu}$ can be simplified. It is easy from the similar discussion in §3 to confirm that the $0^+ \to J^+$ $(J \geqq 3)$ transitions are forbidden.

(4-a) *The 2n-mechanism*

The R-matrix is obtained from the R_W-matrix by the replacements, $N_\alpha \to N_A$ and $N_\beta \to N_B$.

(i) *The $0^+ \to 0^+$ transition*

After straightforward calculations, we obtain

$$d\Gamma_{2\nu}^{2n}(0^+ \to 0^+) = (a_{2\nu}/m_e^9)(1/8)[p_1^0 p_2^0 C - \vec{p}_1 \cdot \vec{p}_2 D]\, d\Omega_{2\nu}' , \tag{4.5}$$

where

$$a_{2\nu} = \Bigl(2 \sum_{i \geq j}{}' \epsilon_{ij}^2 |U_{ei} U_{ej}|^2\Bigr) \frac{1}{4} \frac{G^4 m_e^9}{2\pi^7} , \tag{4.6}$$

$$\begin{aligned} d\Omega_{2\nu}' = {} & F(Z, p_1^0) F(Z, p_2^0) |\vec{p}_1| |\vec{p}_2| |\vec{k}_1| |\vec{k}_2| k_1^0 k_2^0 \\ & \cdot \delta(p_1^0 + p_2^0 + k_1^0 + k_2^0 + M_B - M_A)\, d\cos\theta\, dp_1^0 dp_2^0 dk_1^0 dk_2^0 , \end{aligned} \tag{4.7}$$

$$C = g_V^4 (K^2 - KL + L^2) |M_F'|^2 - 2 g_V^2 g_A^2 KL\, \mathrm{Re}(M_F' M_{GT}'^*) + \frac{1}{3} g_A^4 (K^2 + KL + L^2) |M_{GT}'|^2 , \tag{4.8}$$

$$D = g_V^4 KL |M_F'|^2 - \frac{2}{3} g_V^2 g_A^2 (K^2 + KL + L^2)\, \mathrm{Re}(M_F' M_{GT}'^*) + \frac{1}{9} g_A^4 (2K^2 + 5KL + 2L^2) |M_F'|^2 . \tag{4.9}$$

Here $F(Z, p^0)$ is defined in Eq.(3.11), and M_F' and M_{GT}' are defined similarly to Eq.(3.25) and

$$K = [\langle E_n\rangle - M_A + p_1^0 + k_1^0]^{-1} + [\langle E_n\rangle - M_A + p_2^0 + k_2^0]^{-1}, \tag{4.10}$$

$$L = [\langle E_n\rangle - M_A + p_2^0 + k_1^0]^{-1} + [\langle E_n\rangle - M_A + p_1^0 + k_2^0]^{-1}. \tag{4.11}$$

The primed sum in Eq.(4.6) should extend over all energetically allowed neutrinos in the final state. Rigorously speaking, the neutrino masses m_j in k_1^0 and k_2^0 should be taken into account in this primed sum. If all neutrinos are allowed to contribute and the replacement of k_i^0 by $|\vec{k}_i|$ is permissible, then $(2\sum'_{i\leq j} \varepsilon_{ij}{}^2 |U_{ei}|^2 |U_{ej}|^2) = 1$. The factor 1/4 in $a_{2\nu}$ is to represent the statistical factor for the case of $U_{ej} = \delta_{ej}$.

To perform the phase space integration, we neglect the masses of neutrinos and assume the following replacement (within a few % errors); $p_i^0 + k_j^0 \rightarrow \langle p_i^0 + k_j^0\rangle = (M_A - M_B)/2$. Then $K \simeq L \simeq 2(\mu_0 m_e)^{-1}$.

Now the straightforward calculations lead to

$$d\Gamma_{2\nu}^{2n}(0^+\rightarrow 0^+) = (a_{2\nu}/m_e^{11})(g_A^4/60)\,|(g_V/g_A)^2 M_F' - M_{GT}'|^2 \mu_0^{-2} \cdot (p_1^0 p_2^0 - \vec{p}_1\cdot\vec{p}_2)(M_A - M_B - p_1^0 - p_2^0)^5 \, d\Omega_{2\nu}, \tag{4.12}$$

where

$$d\Omega_{2\nu} = F(Z, p_1^0) F(Z, p_2^0) |\vec{p}_1||\vec{p}_2| \, d\cos\theta \, dp_1^0 dp_2^0 . \tag{4.13}$$

The inverse of the half-life is

$$[T_{2\nu}^{2n}(0^+\rightarrow 0^+)]^{-1} = |(g_V/g_A)^2 M_F' - M_{GT}'|^2 \mu_0^{-2} F_0(T), \tag{4.14}$$

where

$$F_0(T) = \int d\Omega_{2\nu}(a_{2\nu} g_A^4)/(60 m_e^{11})(p_1^0 p_2^0 - \vec{p}_1 \cdot \vec{p}_2)(M_A - M_B - p_1^0 - p_2^0)^5 . \quad (4.15)$$

T being the maximum kinetic energy release in units of m_e, i.e., $T = (M_A - M_B - 2m_e)/m_e$.

Primakoff and Rosen[15] have derived Eq. (4.5) in the limit of $U_{ej} = \delta_{ej}$. Note that their result has a few misprints and also is four times larger than ours.* Concerning this overall normalization, our result in Eq. (4.12) agrees in this limit with that by Konopinski.[30]

(ii) *The $0^+ \to 2^+$ transition*

For this transition, we have

$$d\Gamma_{2\nu}^{2n}(0^+ \to 2^+) = (a_{2\nu}/m_e^9)(g_A^4/8)(M_2^{pq}, M_2^{pq})(K-L)^2(p_1^0 p_2^0 + \tfrac{1}{3}\vec{p}_1 \cdot \vec{p}_2)\, d\Omega_{2\nu}' , \quad (4.16)$$

where $(M_2^{pq}, M_2^{pq}) \equiv \sum_{J_z} \sum_{p,q=1}^{3} M_2^{pq*} M_2^{pq}$ with

$$M_2^{pq} = \langle \sigma_n^p \sigma_m^q \rangle . \quad (4.17)$$

To evaluate the term $(K-L)^2$, we use the approximation $p_i^0 + k_j^0 \simeq \langle p_i^0 + k_j^0 \rangle = (M_A - M_B)/2$ only in the denominator of $K-L$ and obtain $K - L = 2(p_1^0 - p_2^0)(k_1^0 - k_2^0)(\mu_0 m_e)^{-3}$. This approximation is valid within several % errors for $\mu_0 \geq 4$. After the phase space integration, we get

---

* See footnotes on page 28 and Eq. (60) of Ref. 15). Greuling and Whitten[31] give results four times larger than ours.

$$d\Gamma^{2n}_{2\nu}(0^+\to 2^+) = (a_{2\nu}/m_e^{15})(g_A^4/420)(M_2^{pq}, M_2^{pq})\mu_0^{-6}$$
$$\cdot(p_1^0-p_2^0)^2(p_1^0p_2^0+\tfrac{1}{3}\vec{p}_1\cdot\vec{p}_2)(M_A-M_B-p_1^0-p_2^0)^7\, d\Omega_{2\nu}\,. \tag{4.18}$$

The inverse of the half-life is given by

$$\left[T^{2n}_{2\nu}(0^+\to 2^+)\right]^{-1} = (M_2^{pq}, M_2^{pq})\mu_0^{-6}F_2(T)\;, \tag{4.19}$$

where

$$F_2(T) = \int d\Omega_{2\nu}(p_1^0-p_2^0)^2(p_1^0p_2^0+\tfrac{1}{3}\vec{p}_1\cdot\vec{p}_2)(a_{2\nu}g_A^4)/(420m_e^{15})\,. \tag{4.20}$$

(iii) *The $0^+\to 1^+$ transition*

Similarly, we obtain

$$d\Gamma^{2n}_{2\nu}(0^+\to 1^+) = (a_{2\nu}/m_e^9)(g_A^2g_V^2/4)(M_1^p, M_1^p)$$
$$\cdot(K-L)^2(p_1^0p_2^0+\tfrac{1}{3}\vec{p}_1\cdot\vec{p}_2)\, d\Omega'_{2\nu}\;, \tag{4.21}$$

where $(M_1^p, M_1^p) \equiv \sum_{J_z}\sum_{p=1}^{3} M_1^{p*}M_1^p$ with

$$M_1^p = \langle\, \sigma_n^p \,\rangle\,. \tag{4.22}$$

Note that the transition formula given above is exactly the same as the one for the $0^+\to 2^+$ transition given in Eq.(4.16), aside from the overall normalization. Therefore, the decay rate can be read off from the one for the $0^+\to 2^+$ transition.

Before closing this subsection, we would like to mention the work by Molina and Pascual[32)] who estimated the $0^+ \to J^+$ transitions. We found several errors in their foumulae: (i) They showed that $\Gamma_{2\nu}^{2n}(0^+ \to 1^+) = 0$, while we get the non-vanishing rate as given in Eq.(4.21). (ii) Their decay rates of the $0^+ \to 0^+, 2^+$ transitions for the $(\beta\beta)_{2\nu}$ mode are twice larger than ours.

(4-b) *The N^*-mechanism*

The R-matrix element for the N^*-mechanism is obtained from Eq.(B.3) in Appendix B by substituting the R_W-matrices corresponding to the $N_{\Delta^-} \to N_B$ and $N_A \to N_{\Delta^{++}}$ transitions shown in Figs. 3c and 3d. The hadronic part of the amplitude for $N_{\Delta^-} \to N_B + 2e^- + \overline{N_i} + \overline{N_j}$ is expressed as follows;

$$\int d\vec{x}\, d\vec{y}\, (J^{\mu\nu}_{N^*}) = K\left\{g_V g_A (g^{\mu 0} g^{\nu j} + g^{\mu j} g^{\nu 0}) M^j_\Delta + \frac{1}{2} g_A^2 (g^{\mu j} g^{\nu k} + g^{\mu k} g^{\nu j}) M^{jk}_\Delta\right\}, \tag{4.23}$$

where K, M^j_Δ and M^{jk}_Δ are defined in Eqs.(B.12), (3.80) and (3.81). In order to express the origin of the M^j_Δ and M^{jk}_Δ terms clearly, we retain g_V and g_A explicitly, but we take $g_A = g_V$ hereafter.

By using the factorization hypothesis, the decay formula is obtained in the following form;

$$d\Gamma^{N^*}_{2\nu} = (a_{2\nu}/m_e^9) 12\, g_V^4\, P(\Delta) |\langle \Phi_f | \Phi_i \rangle|^2 \cdot (K-L)^2 (p_1^0 p_2^0 + \frac{1}{3} \vec{p}_1 \cdot \vec{p}_2)\, d\Omega'_{2\nu}, \tag{4.24}$$

where both the $N_{\Delta^-} \to N_B$ and $N_A \to N_{\Delta^{++}}$ transitions are included according to the argument given in Appendix B. It is amusing to observe that the above formula is exactly the same as the one for the $0^+ \to 2^+$ transition in the 2n-mechanism aside from the overall normalization.

The inverse of the half-life is

$$[T_{2\nu}^{N^*}]^{-1} = 96\, P(\Delta)|\langle\Phi_f|\Phi_i\rangle|^2 \cdot \mu_0^{-6} F_2(T), \tag{4.25}$$

where $F_2(T)$ is defined in Eq.(4.20) and $a_{2\nu}$ is in Eq.(4.6).

We would like to note that this formula is completely different from the one obtained by Picciotto.[52] This is due to the fact that he used a crucial approximation* for the R-matrix, instead of taking the spin sum explicitly. However, his approximation can not be ragarded as reasonable. Note also that he has neglected the M_Δ^{jk} term (g_A^4 term) in Eq.(4.23) which turns out to be dominant in the N^*-mechanism for the $(\beta\beta)_{2\nu}$ mode.

---

* Picciotto[52] used the approximation $\{E_{\mu\nu}J^{\mu\nu} - (p_1 \leftrightarrow p_2)\} \propto E_{\mu\nu}g_V g_A(g^{\mu 0}g^{\nu k} + g^{\mu k}g^{\nu 0})M^k \mu_0^{-1}$. We have evaluated the spin sum exactly.

§5. The general properties of various transitions

In the previous sections, we presented the formulae of the $0^+ \to J^+$ transitions both for the $(\beta\beta)_{0\nu}$ and $(\beta\beta)_{2\nu}$ modes. The $0^+ \to 0^+$, 1^+ and 2^+ transitions are only allowed in the double β decay within our approximations introduced in §3. We also found several interesting "selection rules" as given in Table I: (i) If $\lambda = \kappa = 0$ (no right-handed interaction), the N^*-mechanism does not contribute to the $(\beta\beta)_{0\nu}$ mode, whether neutrinos are massive or massless. (ii) If $\lambda = \kappa = 0$, the $0^+ \to 1^+$ and 2^+ transitions of the $(\beta\beta)_{0\nu}$ mode are forbidden and the $0^+ \to 0^+$ transition is only allowed in the 2n-mechanism. We emphasize that these selection rules for N^*-mechanism do not depend on the factorization hypothesis. The selection rule (i) seems to nullify the neutrino mass bounds derived by Halprin et al. in the N^*-mechanism.[51)]

In the following we shall investigate the general properties of various measurable quantities in some details.

(5-a) *The decay rate*

We first discuss the relative order of magnitudes of the decay rates for various processes in both two modes.

(5-a-1) *The $(\beta\beta)_{2\nu}$ mode*

Let us compare the yields from the N^*- and 2n-mechanisms to the $0^+ \to 0^+$, 1^+ and 2^+ transitions. We obtain the half-life formula in the following parameterized forms from Eqs.(4.14), (4.19) and (4.25);

$$[T_{2\nu}(0^+\to 0^+)]^{-1} = B_1\cdot 10\cdot|(g_V/g_A)^2 M_F' - M_{GT}'|^2 (10/\mu_0)^2$$
$$+ B_2\cdot 10^3\cdot P(\Delta)|\langle\Phi_f|\Phi_i\rangle|^2(10/\mu_0)^6 \quad , \qquad (5.1)$$

$$[T_{2\nu}(0^+\to 2^+)]^{-1} = B_3\cdot 10\,(M_2^{pq}, M_2^{pq})\,(10/\mu_0)^6$$
$$+ B_4\cdot 10^3\, P(\Delta)'|\langle\Phi_f|\Phi_i\rangle|^2(10/\mu_0)^6 \,. \qquad (5.2)$$

Here unknown quantities which are related to the nuclear structure are normalized so that the coefficients B_i's give us a rough idea about the contribution from each term. The numerical values of B_i's for some typical nuclei are listed in Table II. The first and second terms in Eqs.(5.1) and (5.2) come from the 2n- and N^*-mechanisms, respectively. Concerning the quantities in the N^*-mechanism, $P(\Delta)$ is the probability of producing $\Delta(1232)$ per neutron inside the nucleus and $\langle\Phi_f|\Phi_i\rangle$ means the overlap between the initial and final nuclear states. (See Appendix B for the detailed discussions.)

According to Primakoff and Rosen who introduced the idea of N^*-mechanism,[27] we shall consider that $P(\Delta)$ and $|\langle\Phi_f|\Phi_i\rangle|^2$ for the N^*-mechanism are order of 10^{-2} and 10^{-1}, respectively.

The comparison of the yields from N^*- and 2n-mechanism can be easily made from Eqs.(5.1) and (5.2). By assuming $|M_F'| \ll |M_{GT}'|$ and taking $|M_{GT}'| \gtrsim 0.01$, we conclude the followings: (i) The contribution from the N^*-mechanism to the $0^+ \to 0^+$ transition is at most 0.1 %. (ii) As for the $0^+ \to 2^+$ transition, both mechanisms have equally important in contributions if we assume $P'(\Delta)|\langle\Phi_f|\Phi_i\rangle'|^2 \simeq 10^{-3}$ and $(M_2^{pq}, M_2^{pq}) \simeq 10^{-1}$. (iii) The $0^+ \to 2^+$ transition is not important in comparison with the $0^+ \to 0^+$ transition. (iv) As for the $0^+ \to 1^+$ transition, we shall not consider it because there is no 1^+ level near the ground state of the daughter nucleus for the some typical nuclei which we shall deal with later.

In summary, we conclude that in the $(\beta\beta)_{2\nu}$ mode, the $0^+ \to 0^+$ transition in the 2n-mechanism dominates over all other transitions. It is expected that this fact simplifies the analysis of data, especially those obtained by the geological method.

(5-a-2) *The $(\beta\beta)_{0\nu}$ mode*

The half-life formula for the $0^+ \to 0^+$ transition is given in Eqs. (3.57) and (3.88). The numerical values of the G_{0i} and G_i^{N*} for some typical nuclei are listed in Table III(a). In the following, the next simplifications are employed: (i) Only the second order terms of m_j, λ and κ are retained, i. e. $m_j G_A^2 \simeq m_j g_A^2$, $G_A \varepsilon_A' \simeq \lambda(g_A'/g_A)\, g_A^2$ and neglecting X_2 term. (ii) The nuclear matrix elements M_F, M_T, M_Q and M_S appeared in the $0^+ \to 0^+$ transition are neglected in order to simplify the discussions. This approximation for M_T is good for the spherically symmetric nuclei.[15)] (iii) The potential terms $\langle H(r, m_\nu, \mu_0)\rangle$ and $\langle rH'(r, m_\nu, \mu_0)\rangle$ are replaced by their values at $m_\nu = 0$ and are taken to be real quantities. This replacement is valid for the terms like $\Sigma U_{ej} V_{ej} \langle H\rangle$, because the potential behaves roughly as a constant for $m_j \lesssim 1$ MeV and decreases rapidly from $m_j \gtrsim 10$ MeV as shown in Figs.11a and 11b. The above replacement for the term $\Sigma m_j U_{ej}^2 \langle H(r, m_j, \mu_0)\rangle$ corresponds to assuming that the mixing angles between the light mass neutrinos ($m_j \lesssim 1$ MeV) and the heavy mass neutrinos ($m_j \gtrsim 10$ MeV) are sufficiently small so that the contribution from the heavy mass neutrinos can be safely neglected. Here we consider only this case and a detailed discussion is given in Appendix C.

According to the above simplifications, we shall express the half-life in the following parameterized forms,

$$
\begin{aligned}
[T_{0\nu}(0^+ \to 0^+)]^{-1} = \Big\{ & C_1 \left|\Sigma (m_j/m_e) U_{ej}^2\right|^2 \\
& - C_2 (\lambda' - \kappa) \left|\Sigma (m_j/m_e) U_{ej}^2\right| \cdot \left|\Sigma U_{ej} V_{ej}\right| \cos\psi \\
& + C_3 (\lambda' - \kappa)^2 \left|\Sigma U_{ej} V_{ej}\right|^2 \Big\} \cdot 10 \cdot |M_{GT}|^2 \\
& + \left\{ C_4 \lambda'^2 + C_5 (\lambda' - \kappa)^2 + C_6 \kappa^2 \right\} \left|\Sigma U_{ej} V_{ej}\right| \cdot 10^3 P(\Delta) \left|\langle \Phi_f | \Phi_i \rangle\right|^2,
\end{aligned}
\tag{5.3}
$$

where $\lambda' = \lambda(g_V'/g_V)$ and

$$\psi = \arg\left(\Sigma m_j U_{ej}^{*2} \cdot \Sigma U_{ej} V_{ej}\right) \tag{5.4}$$

The coefficients C_i are defined by

$$C_1 = G_{01} (\langle H\rangle R)^2 g_A^4 \cdot 10^{-1}, \tag{5.5}$$

$$C_2 = \left\{ G_{06} (\langle H\rangle R)^2 + (2G_{01} + G_{06} - G_{09}) \langle H\rangle R \langle rH'\rangle R/9 \right\} g_A^4 \cdot 10^{-1}, \tag{5.6}$$

$$
\begin{aligned}
C_3 = \Big\{ & (2G_{01} + G_{02} + \tfrac{3}{2} G_{06} - G_{09}) (\langle rH'\rangle R)^2/9^2 \\
& + (2G_{02} + G_{06}) \langle rH\rangle R \langle H\rangle R/9 + (G_{02} - \tfrac{1}{2} G_{06}) (\langle H\rangle R)^2 \Big\} g_A^4 \cdot 10^{-1},
\end{aligned}
\tag{5.7}
$$

$$C_4 = G_1^{N*}(\langle rH'\rangle_\Delta a)^2 g_V^4 \cdot 10^{-3}, \tag{5.8}$$

$$C_5 = G_2^{N*}(\langle rH'\rangle_\Delta a)^2 g_V^4 \cdot 10^{-3}, \tag{5.9}$$

$$C_6 = \left\{ G_3^{N*}(\langle rH'\rangle_\Delta a)^2 - G_4^{N*}(\langle rH'\rangle_\Delta a)(\langle H\rangle_\Delta a) + G_5^{N*}(\langle H\rangle_\Delta a)^2 \right\} g_V^4 \cdot 10^{-3}, \tag{5.10}$$

where G_{0i} and G_i^{N*} are defined in Eqs.(3.58) and (3.89).

The numerical values of $C_1 - C_6$ for some typical nuclei are listed in Table IV. In obtaining these coefficients, we used the numerical values of the average potentials as follows. (See Appendix C for the detailed discussions)

	^{48}Ca	^{76}Ge	^{82}Se	^{128}Te	^{130}Te	^{150}Nd
$\langle H\rangle R$	0.65	0.68	0.68	0.55	0.53	0.67
$-\langle rH'\rangle R$	0.78	0.80	0.80	0.72	0.71	0.81
$\langle H\rangle_\Delta a$	0.94	0.95	0.95	0.91	0.90	0.95
$-\langle rH'\rangle_\Delta a$	0.99	0.99	0.99	0.97	0.97	0.99

It is easy from Eq.(5.3) to conclude that if $\lambda \neq 0$ and/or $\kappa \neq 0$, the N^*-mechanism for the $0^+ \to 0^+$ transition gives 10 ~ 200 times larger contribution in comparison with λ- or κ-term in 2n-mechanism, provided $P(\Delta)|\langle\Phi_f|\Phi_i\rangle|^2 = 10^{-3}$. This dominance may come from two reasons: (i) The neutrino exchange potential in the N^*-mechanism ($-\langle rH'\rangle \sim 1/a$ where a is the size of Δ) is enhanced by the order of magnitude relative to that in the 2n-mechanism ($\langle H\rangle \sim 1/R$). (ii) In the N^*-mechanism, the transition of the single constituent of nucleus triggers the double β decay like a single β decay so that the nuclear matrix elements are expected to be enhanced by the order of magnitude.

Once the half-life $(T_{1/2})_{0\nu}$ is given, it restricts three parameters $|\Sigma m_j U_{ej}^2|$, $\lambda|(g_V'/g_V)\Sigma U_{ej} V_{ej}|$ and $|\kappa \Sigma U_{ej} V_{ej}|$ to the domain which is determined by the ellipsoids as seen from Eq. (5.3). We consider the typical cases; (i) $\lambda \gg |\kappa|$ and (ii) $\lambda \ll |\kappa|$. In these cases the allowed domain is, in principle, surrounded by two ellipses in $m_\nu - \lambda$ (or κ) plane, where these ellipses correspond to the no-CP-violation case ($\cos\psi = \pm 1$). If CP-violation phases are known ($|\cos\psi| < 1$), the allowed values should be restricted to be on one ellipse. If the lower limit of the half-life is only known from the experiment, the allowed domain becomes the indide of the ellipse. Since the contribution from the N^*-mechanism dominates over the one from the 2n-mechanism and no interference between these two mechanisms is assumed, these two ellipses overlap practically. Later we shall present some examples which will be shown in Fig. 9.

As for the $0^+ \to 2^+$ transition, the half-life is given in Eqs. (3.65) and (3.88). The numerical values of $G_{\pm 2}$ and G_i^{N*} are listed in Table III(b). If the nuclear matrix elements and $P'(\Delta)|\langle\Phi_f|\Phi_i\rangle'|^2$ are assumed to be some appropriate values given in §6-d, the following are concluded: The contribution of N^*-mechanism in the $0^+ \to 2^+$ transition is as important as that of the λ- or κ-part in the 2n-mechanism of the $0^+ \to 0^+$ transition. Note also that the half-life of the $0^+ \to 2^+$ transition depends only on λ and κ.

The $0^+ \to 1^+$ transitions are not considered here because for the nuclei listed in Table VI there are no 1^+ levels near the ground state of the daughter nucleus.

(5-b) *The angular correlation*

For the $(\beta\beta)_{2\nu}$ mode, the angular correlations are given from Eqs.(4.5), (4.16), (4.21) and (4.24). The $0^+ \to 0^+$ transition in the 2n-mechanism dominates over others as discussed in §5-a-1. Therefore the angular correlations are governed by $1 - \cos\theta$. Those behaviors are shown in Table V.

For the $(\beta\beta)_{0\nu}$ mode, the angular correlations are given from Eqs.(3.36), (3.59), (3.67) and (3.82). The behaviors of the angular correlations are somewhat complicated because of the existences of many nuclear matrix elements. We shall only consider the $0^+ \to 0^+$ transition by using the same simplifications stated in §5-a-2. The angular correlations of the $0^+ \to 0^+$ transition are shown in Table V. Here we only consider the case where $|p_1^0 - p_2^0| > m_e$.

(5-c) *The single electron kinetic energy spectrum*

The typical single electron kinetic energy spectra for the $(\beta\beta)_{2\nu}$ mode are plotted in Figs.6a and 6b. For the $(\beta\beta)_{0\nu}$ mode, the $0^+ \to 0^+$ spectra are plotted in Fig. 7 for three cases;(a) $\lambda = \kappa = 0$ and $m_\nu \neq 0$, (b) $\lambda \neq 0$ and $\kappa = m_\nu = 0$ in the 2n-mechanism, (c) $\lambda \neq 0$ and $\kappa = 0$ in the N^*-mechanism.

(5-d) *The sum energy spectrum*

The spectra for the kinetic energy sum of two electrons are shown in Fig. 8 both for the $(\beta\beta)_{2\nu}$ and $(\beta\beta)_{0\nu}$ modes. Note that as the energy sum tends to its maximum energy value T, the spectra for the $(\beta\beta)_{2\nu}$ mode die away rapidly in contrast to the $(\beta\beta)_{0\nu}$ mode where the yield appears only at T as shown in Fig. 8.[15),31)]

§6. The data analysis

There are two different approaches to measure the half-life, the geological method and the counter (chamber) experiment. In the geological method, the half-life is estimated by measuring the abandance of the decay product (the daughter nucleus) in the geologically old ores with the use of the mass spectrometer. The geological experiment has some advantage because the decay products are accumulated for a long period. However, there may be the ambiguity of measuring half-lives mainly due to the evaporation of the decay product (noble gas). In addition, it is inherently unable to distinguish directly the $(\beta\beta)_{2\nu}$ and $(\beta\beta)_{0\nu}$ modes. The transitions to the final excited states also contribute to the half-life measured in this method, too. This type of measurements has been made for the total half-lives of ^{82}Se,[54),55)] ^{128}Te,[26)] and ^{130}Te.[56)~59)] On the other hand, the counter experiment can distinguish not only the two decay modes but also the various transitions in principle. The lower limits of $(T_{1/2})_{0\nu}$ for ^{48}Ca,[23)] ^{76}Ge,[60),61)] ^{82}Se,[62)] ^{150}Nd,[63)] and of $(T_{1/2})_{2\nu}$ for ^{48}Ca[23)] have been measured. Recently, Moe and Lowenthal reported the observation of the $(\beta\beta)_{2\nu}$ mode for ^{82}Se by using the cloud chamber.[24)] All those data are listed in Table VI .

(6-a) *The nature of the neutrino:*

We shall analyze the data on the ratio of the ^{128}Te to ^{130}Te half-lives measured geologically by Hennecke et al.[26)] Their result is

$$(R_T)_{exp} = [T_{1/2}(^{128}Te)/T_{1/2}(^{130}Te)]_{exp} = (1.59 \pm 0.06)\cdot 10^3. \qquad (6.1)$$

The much longer half-life of ^{128}Te than that of ^{130}Te comes from the fact that the available phase space is considerably smaller for ^{128}Te than for ^{130}Te, because the maximum kinetic energy releases are 1.7 and 5.0 for ^{128}Te and ^{130}Te in units of m_e, respectively. This smaller phase space gives us the following expectations: The branching ratio of the $(\beta\beta)_{0\nu}$ mode for ^{128}Te is considerably enhanced relative to the case of ^{130}Te because of the phase space difference between three-body and five-body decays if there is the $(\beta\beta)_{0\nu}$ mode. In other words, if the half-lives of the $(\beta\beta)_{0\nu}$ and $(\beta\beta)_{2\nu}$ modes are denoted by $T_{0\nu}$ and $T_{2\nu}$, we can expect,

$$^{128}(T_{2\nu}/T_{0\nu}) > {}^{130}(T_{2\nu}/T_{0\nu}) . \qquad (6.2)$$

Let us rewrite the ratio R_T in terms of the half-lives for the $(\beta\beta)_{0\nu}$ and $(\beta\beta)_{2\nu}$ modes,

$$R_T = \frac{^{128}T_{2\nu}}{^{130}T_{2\nu}} \cdot \frac{1 + {}^{130}(T_{2\nu}/T_{0\nu})}{1 + {}^{128}(T_{2\nu}/T_{0\nu})} . \qquad (6.3)$$

Note that when there is no $(\beta\beta)_{0\nu}$ mode $(T_{0\nu} = \infty)$, Eq. (6.3) reduces to the trivial relation $R_T = {}^{128}T_{2\nu}/{}^{130}T_{2\nu}$. If there exists the $(\beta\beta)_{0\nu}$ mode at all, the inequality

$$R_T < {}^{128}T_{2\nu}/{}^{130}T_{2\nu} \qquad (6.4)$$

should be satisfied as we can see from Eqs.(6.2) and (6.3). Therefore, the accurate theoretical evaluation of the $(\beta\beta)_{2\nu}$ mode is essential to answer whether there exists the $(\beta\beta)_{0\nu}$ mode or not. Fortunately enough, we have demonstrated in §5 that as for the $(\beta\beta)_{2\nu}$ mode the $0^+ \to 0^+$ transition in the 2n-mechanism dominates over the other transitions as well as the contribution from the N^*-mechanism. Thus we obtain from Eqs.(4.14) and (5.1).

$$\frac{{}^{128}T_{2\nu}}{{}^{130}T_{2\nu}} = r_{2\nu} \frac{\left|{}^{130}\left(\left[(g_V/g_A)^2 M_F' - M_{GT}'\right]/\mu_0\right)\right|^2}{\left|{}^{128}\left(\left[(g_V/g_A)^2 M_F' - M_{GT}'\right]/\mu_0\right)\right|^2}, \tag{6.5}$$

where*

$$r_{2\nu} = F_0(5.0)/F_0(1.7) = 5.66 \cdot 10^3. \tag{6.6}$$

Here $F_0(T)$ is the kinematical factor for the $0^+ \to 0^+$ transition in the 2n-mechanism which is defined in Eq.(4.15).

Following the above argument and using Eqs.(6.1),(6.4)~(6.6), we obtain the (sufficient) condition for the existence of the $(\beta\beta)_{0\nu}$ mode for ^{128}Te.

$$\frac{\left|{}^{130}\left(\left[(g_V/g_A)^2 M_F' - M_{GT}'\right]/\mu_0\right)\right|^2}{\left|{}^{128}\left(\left[(g_V/g_A)^2 M_F' - M_{GT}'\right]/\mu_0\right)\right|^2} > \frac{(R_T)_{exp}}{r_{2\nu}} \simeq 0.28 . \tag{6.7}$$

Since ^{128}Te and ^{130}Te are neighboring isotopes and their nuclear matrix elements are expected to be similar, the above inequality is considered to be well satisfied. Thus the existence of the $(\beta\beta)_{0\nu}$ mode for ^{128}Te is suggested and neutrinos are likely to be Majorana particles.

---

* See next page.

The above argument is supported by the theoretical estimate of the nuclear matrix elements by Vergados[64]; $|M'_{GT}|^2 = 0.32$ and $\mu_0 = 23.4$ for ^{128}Te, and $|M'_{GT}|^2 = 0.25$ and $\mu_0 = 25.0$ for ^{130}Te. The M'_F is neglected in comparison with M'_{GT}.[15),65),67)]

Thus it is concluded that the experimental data on the ratio of the ^{128}Te to ^{130}Te half-lives by Hennecke et al. strongly suggest that neutrinos are likely to be Majorana particles. It should be noted that the above conclusion depends on both the reliability on the data by Henneke et al., the closure approximation[66] introduced in §3 and the evaluation of nuclear matrix element as seen form Eq. (6.7).

---

* Althouth the estimate of $r_{2\nu}$ given in Eq. (6.6) is considered to be very good, we shall argue that this value is "at least" the minimum value. Let us consider the following two effects which might change the value of $r_{2\nu}$: (i) The phase space integrations for the $(\beta\beta)_{2\nu}$ mode are carried out by ignoring the neutrino masses. If there were neutrinos with masses around 1 MeV, the kinematical factors should be modified. However, this inclusion only increases $r_{2\nu}$ because the available phase space for ^{128}Te is much smaller than for ^{130}Te. (ii) If we take account of the contributions from $0^+ \rightarrow 1^+$ and 2^+ as well as the transitions due to the N^*-mechanism, they also increase $r_{2\nu}$ because their common kinematical factor $F_2(T)$ defined in Eq. (4.20) gives $F_2(5.0)/F_2(1.7) = 8.6 \cdot 10^7$ which is much larger than $F_0(5.0)/F_0(1.7) = 5.6 \cdot 10^3$.

The fact that the value of $r_{2\nu}$ is the smallest gives a further support to our conclusion that neutrinos may be Majorana. That is, the above effects work only to loosen the condition given in Eq. (6.7).

(6-2) *The neutrino mass and the right-handed interaction:*

Before going into the detailed discussion, the following remarks are in order:

For the $(\beta\beta)_{2\nu}$ mode, it is sufficient to consider only the $0^+ \to 0^+$ transition in the 2n-mechanism as discussed in §5.

For the $(\beta\beta)_{0\nu}$ mode: (i) When there are no right-handed interactions ($\lambda = \kappa = 0$), there is only the $0^+ \to 0^+$ transition in the 2n-mechanism and its decay rate depends on m_j, M_{GT} and M_F. (ii) When there are right-handed interactions ($\lambda \neq 0$ or $\kappa \neq 0$), the $0^+ \to 1^+$ and 2^+ transitions in the 2n-mechanism are allowed in addition to the $0^+ \to 0^+$ transition, and also the $0^+ \to 0^+$, 1^+ and 2^+ transitions in the N^*-mechanism should be taken into account. (iii) The $0^+ \to 1^+$ transitions are not considered here because for the nuclei listed in Table VI there are no 1^+ levels near the ground state of the daughter nucleus. (iv) The $0^+ \to 2^+$ transition is not also taken into account in the data analysis because nuclear matrix elements are not well known. (v) For the $0^+ \to 0^+$ transition, the matrix elements M_F, M_Q, M_S and M_T are neglected in evaluating λ and κ. Thus the estimates of λ and κ give only the order of magnitudes, although the estimate of the neutrino mass is more reliable. (vi) As for the N^*-mechanism, $P(\Delta)|\langle\Phi_f|\Phi_i\rangle|^2$ is taken to be 10^{-3}. (vii) The nuclear matrix elements M_F and M_{GT} for the $(\beta\beta)_{0\nu}$ mode are assumed to be equal to M_F' and M_{GT}' for the $(\beta\beta)_{2\nu}$ mode, respectively.

Let us derive the constraints on the neutrino masses m_j and the relative strength of the right-handed interaction λ and κ by using the Vergados' estimation of nuclear matrix elements for ^{128}Te and ^{130}Te. We obtain from Eqs.(6.1), (6.3)~(6.6),

$$^{128}(T_{2\nu}/T_{0\nu}) = 1.44 + 2.44\,^{130}(T_{2\nu}/T_{0\nu}) \geq 1.44. \tag{6.8}$$

It should be noted that the yield of the $(\beta\beta)_{0\nu}$ mode is more than 58% of the double β decay for ^{128}Te.

It is clear from Eqs.(3.57) and (4.14) that when $\lambda = \kappa = 0$, $T_{2\nu}/T_{0\nu}$ depends only on μ_0 because the nuclear matrix elements $|g_V^2 M_F - g_A^2 M_{GT}|$ and $|g_V^2 M_F' - g_A^2 M_{GT}'|$ are canceled out, if we assume these nuclear matrix elements have the same value. Thus we obtain*

$$|\sum_j m_j U_{ej}^2| \simeq 32 \text{ eV} \tag{6.9}$$

for $\lambda = \kappa = 0$. If there is no mixing among neutrinos, the above limit on neutrino masses is the constraint on the mass of the electron neutrino.

When $\lambda \neq 0$ or $\kappa \neq 0$, Eq.(6.8) restricts three parameters $|\Sigma m_j U_{ej}^2|$, $\lambda|(g_V'/g_V)\Sigma U_{ej} V_{ej}|$ and $|\kappa \Sigma U_{ej} V_{ej}|$ to the domain which is determind by the ellipsoid. We derive the relations among them as shown in Figs. 9a and 9b for some special cases.

From these figures, we get the following values.

$$\lambda|(g_V'/g_V)\sum_j U_{ej} V_{ej}| = 1.6 \cdot 10^{-5} \tag{6.10}$$

for $\lambda \gg |\kappa|$ and $m_\nu \ll 1$ eV,

$$|\kappa \sum_j U_{ej} V_{ej}| = 1.5 \cdot 10^{-5} \tag{6.11}$$

for $\lambda \ll |\kappa|$ and $m_\nu \ll 1$ eV. If there is no mixing among neutrinos,

---

* We assumed that $2\sum_{i\leq j} \varepsilon_{ij}^2 |U_{ei}|^2 |U_{ej}|^2 \simeq 1$ for the $(\beta\beta)_{2\nu}$ mode.

i. e., $U_{ej}^{(1)} = \delta_{jl}$, $V_{ej+n}^{(2)} = \delta_{jl}$, and $U_{ej+n}^{(2)} = V_{ej}^{(1)} = 0$, the limits on λ and κ become meaningless because $\Sigma U_{ej} V_{ej}$ becomes zero, but one may still assume that mixings remain in $V_{ej}^{(1)}$ from the phenomenological point of view.

If the contribution from the $0^+ \to 2^+$ transition to the $(\beta\beta)_{0\nu}$ mode is taken into account, the smaller values of λ and $|\kappa|$ would be obtained. Note that the above estimates of λ and $|\kappa|$ are much smaller than the bounds obtained from various weak interaction processes,[40] but they are much larger than the ones expected from the grand unified theories discussed in §2.

So far, we only discussed the data on the ratio $^{128}T_{1/2}/^{130}T_{1/2}$ by Hennecke et al..[26] In order to see the consistency of our argument, we evaluated the half-life of ^{130}Te where the $(\beta\beta)_{2\nu}$ mode is expected to dominate over the $(\beta\beta)_{0\nu}$ mode. In Table VI , we give the predictions for ^{130}Te, which are a little smaller than the measured half-lives.[56)-59]

(6-c) *The analysis on other double* β *decaying nuclei:* *

(6-c-1) $^{48}Ca \to {}^{48}Ti + 2e^- (+2\overline{\nu}_e)$

By using the experimental lower limit of $(T_{1/2})_{2\nu}$ by Bardin et al.[23] in Table VI and the numerical values in Table II, the inequality $|M_{GT}/\mu_0|^2 < 7.3 \cdot 10^{-4}$ is obtained. In the following we adopt the estimates by Vergados,[64] $|M_{GT}|^2 \simeq 0.012$ and $\mu_0 \simeq 12.7$, which well satisfy the above inequality. Note that this value of $|M_{GT}|^2$ is considerably smaller than the other theoretical estimates.[65),67] Bardin et al.[23] reported the lower limit of $(T_{1/2})_{0\nu}$ for the $0^+ \to 0^+$ transition. This gives the restriction

---

* Hereafter we assume $M'_{GT} = M_{GT}$.

on the neutrino masses, λ and κ from Eq.(5.3). The allowed domain is the shaded area in Fig. 9. From these figures, we obtain $|\Sigma m_j U_{ej}^2| < 640$ eV for $\lambda = \kappa = 0$. We also get $\lambda|(g_V'/g_V)\,\Sigma U_{ej} V_{ej}| < 3.3 \cdot 10^{-5}$ for $\lambda \gg |\kappa|$, $|\kappa\, \Sigma U_{ej} V_{ej}| < 2.8 \cdot 10^{-5}$ for $\lambda \ll |\kappa|$.

(6-c-2) $^{76}Ge \to {}^{76}Se + 2e^- (+2\overline{\nu}_e)$

Only the data on the lower limit of $(T_{1/2})_{0\nu}$ by Fiorini et al.[60] is available. By assuming $|M_{GT}|^2 \simeq 0.1$ and $\mu_0 \simeq 10$, we obtain the outer boundary ellipse in Fig. 9. From these figures, we obtain $|\sum_j m_j U_{ej}^2| < 430$ eV for $\lambda = \kappa = 0$, $\lambda|(g_V'/g_V)\sum_j U_{ej} V_{ej}| < 9.7\cdot 10^{-5}$ for $\lambda \gg |\kappa|$, $|\kappa\, \Sigma U_{ej} V_{ej}| < 8.4 \cdot 10^{-5}$ for $\lambda \ll |\kappa|$.

(6-c-3) $^{82}Se \to {}^{82}Kr + 2e^- (+2\overline{\nu}_e)$

By comparing the total half-life[54),55)] with the lower limit of the half-life for the $(\beta\beta)_{0\nu}$ mode[62)] in Table VI, we conclude that the $(\beta\beta)_{0\nu}$ yield is at most 10%, i. e., $T_{1/2} \simeq (T_{1/2})_{2\nu}$. By using the data on $T_{1/2}$ by Srinivasan et al.[55)] and the above relation, we obtain*

$$|M_{GT}/\mu_0| \simeq 8.5 \cdot 10^{-4}. \tag{6.12}$$

By assuming $\mu_0 \simeq 10$, $|M_{GT}|^2 \simeq 0.085$ is obtained. Then the lower limit of $(T_{1/2})_{0\nu}$ by Cleveland et al.[62)] leads to the outer

---

* We obtain $|M_{GT}/\mu_0|^2 \simeq 1.7 \cdot 10^{-3}$ and $2.3 \cdot 10^{-2}$ from the data by Kirsten et al.[54)] and Moe and Lowenthal,[24)] respectively.

boundary ellipse shown in Fig. 9 which gives*

$$\left|\sum_j m_j U_{ej}^2\right| < 280 \quad eV \qquad \text{for } \lambda = \kappa = 0\,, \tag{6.13}$$

$$\lambda\left|(g_V'/g_V)\sum_j U_{ej}V_{ej}\right| < 4.3\cdot 10^{-5} \qquad \text{for } \lambda \gg |\kappa| \tag{6.14}$$

and

$$\left|\kappa\sum_j U_{ej}V_{ej}\right| < 3.6\cdot 10^{-5} \qquad \text{for } \lambda \ll |\kappa|. \tag{6.15}$$

---

* If the nuclear matrix element derived from the data by Moe and Lowenthal[24] is used, $|\sum_j m_j U_{ej}^2| < 54$ eV is obtained. However, their half-life for the $(\beta\beta)_{2\nu}$ mode is 10 ~ 20 times larger than the others[54),55)] and the much larger nuclear matrix element $|M_{GT}|^2 \simeq 2.3$ is required. Recently, Haxton et al.[66] estimated the values of nuclear matrix elements; $|M_F| < 0.02$, $\mu_0(F) = 21.9$ (20.2), $|M_{GT}| = 1.88$ (2.56), $\mu_0(GT) = 19.7$ (18.4) for the $(\beta\beta)_{2\nu}$ mode in ^{82}Se (^{76}Ge), respectively. They showed that these estimates are consistent with the data of ^{82}Se by Moe and Lowenthal.

(6-d) *The predictions:*

To estimate the half-lives of the $0^+ \to 0^+$ and $0^+ \to 2^+$ transitions for the $(\beta\beta)_{2\nu}$ and $(\beta\beta)_{0\nu}$ modes, we used the following values:*

	^{48}Ca	^{76}Ge	^{82}Se	^{128}Te	^{130}Te	^{150}Nd
$\lvert M_{GT}\rvert^2$	0.012	0.1	0.085	0.32	0.25	0.1
μ_0	12.7	10	10	23.4	25.0	10

The nuclear matrix element M_{GT} for the $(\beta\beta)_{0\nu}$ mode is assumed to be equal to M'_{GT} for the $(\beta\beta)_{2\nu}$ mode.
For the $0^+ \to 0^+$ transition, $P(\Delta)\lvert\langle\Phi_f\lvert\Phi_i\rangle\rvert^2 \simeq 10^{-3}$ is used.
For the $0^+ \to 2^+$ transition, we assume the following values:
$(M_2^{pq}\ ,\ M_2^{pq}) \simeq 0.1$, $(N_2^{pq}\ ,\ N_2^{pq})(\lambda'^2\ g_A^4)^{-1} \simeq 0.1$ for the $\lambda \gg \lvert\kappa\rvert$ case, $(N_2^{pq}\ ,\ N_2^{pq})(\kappa^2\ g_A^4)^{-1} \simeq 0.1$ and $(N_3^{pq}\ ,\ N_3^{pq})(4\kappa^2\ g_V^2\ g_A^2)^{-1} \simeq 0.1$ for the $\lambda \ll \lvert\kappa\rvert$ case,

$P'(\Delta)\lvert\langle\Phi_f\lvert\Phi_i\rangle'\rvert^2 \simeq 10^{-3}$ and $\mu_0(0^+ \to 2^+) \simeq \mu_0(0^+ \to 0^+)$. The predictions for the $(\beta\beta)_{0\nu}$ mode are presented in three cases; (i) $\lambda = \kappa = 0$ and $\lvert\Sigma m_j U_{ej}^2\rvert = 32$ eV, (ii) $\lvert\kappa\rvert \ll \lambda \neq 0$ and $m_j \simeq 0$, (iii) $\lambda \ll \lvert\kappa\rvert \neq 0$ and $m_j \simeq 0$. In Cases (ii) and (iii), the values in Eqs.(6.10) and (6.11) are used for λ and κ, respectively.

---

* The numerical values for ^{76}Ge and ^{150}Nd, and μ_0 for ^{82}Se are the assumed ones in this table. M_F, M_T, M_Q and M_S are neglected.

§7. The summary and discussion

The $0^+ \to J^+$ transitions for both $(\beta\beta)_{0\nu}$ and $(\beta\beta)_{2\nu}$ modes in the 2n- and N^*-mechanisms are investigated by using the general effective charged current interaction Hamiltonian which is motivated by the grand unified theories.

The condition is proposed to determine whether neutrinos are Majorana particles. This Majorana condition Eq.(6.7) is derived through the following steps in order:

(i) If there exists the $(\beta\beta)_{0\nu}$ mode at all, the branching ratio of the $(\beta\beta)_{0\nu}$ mode for ^{128}Te is larger than that in ^{130}Te. In other words, the inequality $^{128}(T_{2\nu}/T_{0\nu}) > {}^{130}(T_{2\nu}/T_{0\nu})$, Eq.(6.2), should be satisfied.

(ii) From the inequality in (i), the ratio of the total half-lives, R_T, should satisfy $R_T = \left[T_{1/2}(^{128}\mathrm{Te})/T_{1/2}(^{130}\mathrm{Te})\right] < {}^{128}T_{2\nu}/{}^{130}T_{2\nu}$, Eq.(6.4). This is the condition for neutrinos being Majorana particles. In order to evaluate this condition, it is sufficient to know only the theoretical estimate on the $(\beta\beta)_{2\nu}$ mode, once the data on R_T is given.

(iii) Concerning the $(\beta\beta)_{2\nu}$ mode, the $0^+ \to 0^+$ transition in the 2n-mechanism is proved to dominate over all other transitions in both the 2n- and N^*-mechanisms. In evaluating the half-life of this $0^+ \to 0^+$ transition, there is no ambiguity except for the nuclear matrix elements $|(g_V/g_A)^2 M_F' - M_{GT}'|^2/\mu_0^2$, Eq.(5.1).*

(iv) Thus, the Majorana condition is expressed as the inequality between the nuclear matrix elements and the experimental data on R_T as given in Eq.(6.7).

---

* The information on $\langle E_n \rangle$ and M_{GT} may be obtained experimentally by examining giant resonances, as pointed out by H. Ejiri.

The data on the ratio of the ^{128}Te to ^{130}Te half-lives have been reported by Missouri group.[26] If we take this value of the ratio seriously, we conclude that neutrinos are likely to be Majorana particles. This conclusion comes from the observation that, since ^{128}Te and ^{130}Te are neighboring isotopes, their nuclear matrix elements are expected to take similar values and thus the Majorana condition is satisfied. In fact, Vergados' estimates[64] of these nuclear matrix elements in the shell model confirm our conclusion. As seen from the Majorana condition Eq.(6.7), our conclusion is not altered even if the experimental value on R_T is changed by factor 2.

The data measured by the geological method contain the uncertainty which is due to the evapolation of the daughter nucleus (noble gas) from the ore. It is desirable to compare the results obtained from samples of various ages and at different locations.

So far, only the theoretical analysis on the $(\beta\beta)_{2\nu}$ mode has been used. Now let us consider the $(\beta\beta)_{0\nu}$ mode. From this mode it is possible to obtain the information on the neutrino mass and the magnitudes of λ and κ, if the experimental data on the half-lives of this mode are given.

If $\lambda = \kappa = 0$, only the $0^+ \to 0^+$ transition in the 2n-mechanism takes place in the $(\beta\beta)_{0\nu}$ mode. Its decay formula is proportional to the square of the neutrino mass in the form of $|\Sigma m_j U_{ej}^2|^2$ and the square of the nuclear matrix elements $|g_V^2 M_F - g_A^2 M_{GT}|^2$ as seen from Eq.(5.3). We have used the assumption that M_F and M_{GT} in the $(\beta\beta)_{0\nu}$ mode are equal to M_F' and M_{GT}' in the $(\beta\beta)_{2\nu}$ mode, respectively, and then taken Vergados' estimates for them.[64] By using the data on the ratio of the ^{128}Te to ^{130}Te half-lives,[26] the mass of neutrino is estimated to be $|\Sigma m_j U_{ej}^2| \simeq 32\,\mathrm{eV}$. If there is no mixing among neutrinos, the above neutrino mass value should be interpreted as that for the Majorana electron-neutrino. As an example of the finite mixing, we consider the maximal mixing case discussed in Appendix D. As shown in Eq.(D.12), this result can be expressed as

$$\frac{1}{2}\left| m_1^2 + m_2^2 + 2 m_1 m_2 \cos 2\beta_1 \right|^{1/2} \simeq 32 \ \mathrm{eV}, \tag{7.1}$$

where β_1 stands for the CP-violating phase in the leptonic sector of the Majorana neutrino system. If $\beta_1 = 0$, the obtained mass value means the average of two masses $\frac{1}{2}(m_1 + m_2)$, while if $\beta_1 = \pi/2$, it corresponds to the half of the mass difference, $\frac{1}{2}|m_1 - m_2|$. It is interesting to compare this value with the recent experimental results for the (antineutrino) mass $m_{\bar{\nu}}$ obtained from the ^3H decay. The result obtained by Lubimov et al.[13] is

$$14 \le m_{\bar{\nu}} \le 46 \ \mathrm{eV} \ \ (99\%\ \mathrm{C.L.}), \tag{7.2}$$

and the other by Bergkvist et al.[68] is

$$m_{\bar{\nu}} < 60 \ \mathrm{eV} \ \ (90\%\ \mathrm{C.L.}). \tag{7.3}$$

In the single β decay, the neutrino mass effect appears in the phase space part $F(m_j)$. If there is the neutrino mixing, we have the combination $\sum_j |U_{ej}|^2 F(m_j)$ instead of the simple form like $|\sum_j m_j U_{ej}^2|$ for the double β decay.[69] Thus, it should be careful to compare $m_{\bar{\nu}}$ with $|\Sigma m_j U_{ej}^2|$ directly, if there is the mixing.[66] Therefore, $m_{\bar{\nu}}$ can not be expressed in a simple form of m_j's and the mixing angles. Note that the CP-violating phases characteristic to Majorana neutrino system do not appear in the single β decay.

When $\lambda \neq 0$ and $\kappa \neq 0$, the constraints on m_j, λ and κ are obtained from the data on the half-life of the $(\beta\beta)_{0\nu}$ mode as shown in Fig. 9. The values of λ and κ are estimated only by the order of magnitudes, because there are various kinds of nuclear matrix elements in this case and some of them are not well-known. When we estimated λ and κ, we neglected all other nuclear matrix elements except M_{GT}. We obtain the values of λ and κ as $\lambda|(g_V'/g_V)\Sigma U_{ej}V_{ej}| \simeq 1.6 \cdot 10^{-5}$ for $\lambda \gg |\kappa|$ and $m_\nu = 0$, and $|\kappa \Sigma U_{ej} V_{ej}| \simeq 1.5 \cdot 10^{-5}$ for $|\kappa| \gg \lambda$ and $m_\nu = 0$. These values are considered to be the upper bounds if $m_\nu \neq 0$. We would like to mention that the above values of λ and $|\kappa|$ are much larger than the predicted ones in most of the grand unified theories as discussed in §2.

The $0^+ \to 2^+$ transition occurs only if $\lambda \neq 0$ and/or $\kappa \neq 0$, so that the measurement of this transition gives the direct information on λ and κ. Note that in the $\lambda \neq 0$ and/or $\kappa \neq 0$ case, the $0^+ \to 2^+$ transition may give the comparable contribution with the $0^+ \to 0^+$ transition.

Predictions for the $0^+ \to 0^+$ and $0^+ \to 2^+$ transitions of the $(\beta\beta)_{2\nu}$ and $(\beta\beta)_{0\nu}$ modes are made by using some appropriate values given in §6-d. For the $(\beta\beta)_{0\nu}$ mode, three extreme limits are considered: $m_\nu \neq 0$ and $\lambda = \kappa = 0$; $m_\nu = 0$ and $\lambda \gg |\kappa|$; and $m_\nu = 0$ and $|\kappa| \gg \lambda$. The results are given in Table VI. Note that these predictions give us only the order of magnitudes, because the theoretical estimates of the nuclear matrix elements in the 2n-mechanism and the parameters in the N^*-mechanism are not yet well-known.

Acknowledgement

The author would like to express his deep appreciation to Professor Tsuneyuki Kotani and Dr. Eiichi Takasugi of Osaka University, and Professor Masaru Doi of Osaka College of Pharmacy for their painstaking guidance, valuable advices and discussions as well as their careful reading of the manuscript. Many parts of this work were done in the collabolation with them and Miss Kazuko Okuda. Sincere thanks are due to Professor Masakatsu Kenmoku of Nara Women's University for his helpful discussions espetially on the quantization of the Majorana field. The author thanks Professor H. Ejiri, Professor H. Ohtubo and Professor A. Hosoya of Osaka University for their pointing out the additional contributing diagrams. Thanks are also due to all members of the high energy theoretical group at Osaka University and at Research Institute for Fundamental Physics of Kyoto University for their kind encouragement and discussions.

The author would like to thank Iue-Kinenkai and Soryushi-Shogakukai for their financial supports.

The author is indepted to Miss Yoshiko Hinohara, Miss Mizue Tanaka, Miss Yoshiko Tanaka and Miss Hiroko Matsumura for typing this thesis carefully and patiently.

Appendix A : The quantization of the massive Majorana field

Let us review shortly the quantum theory of the massive Majorana field $N(x)$ following the treatment by Case.[70] The free Lagrangian is given by

$$\mathcal{L}(x) = \frac{1}{2}\,\overline{N}(x)\,(i\gamma^{\mu}\partial_{\mu} - m)\,N(x)\,, \tag{A.1}$$

with the constraint of

$$N^{c}(x) = N(x)\,. \tag{A.2}$$

Because of this constraint $\bar{N}(x)$ and $N(x)$ can not be treated as the independent quantity in contrast to the Dirac field case. Therefore, it is useful to express the Majorana field in the Weyl representation by using the two-component spinor field $\eta(x)$ as follows,

$$N(x) = \begin{bmatrix} -i\sigma^{2}\eta^{*}(x) \\ \eta(x) \end{bmatrix}, \tag{A.3}$$

where σ^2 is 2×2 Pauli matrix. This $N(x)$ satisfies the condition (A-2) automatically.

In terms of $\eta(x)$ the Lagrangian becomes

$$\mathcal{L}(x) = \eta^{\dagger} i(\partial_0 + \sigma^k\partial^k)\eta + \frac{i}{2}m\,(\eta^{\dagger}\sigma^2\eta^{*} - \eta^{T}\sigma^2\eta)\,. \tag{A.4}$$

The field equations for $\eta(x)$ are given by

$$(\partial_0 - \sigma^k\partial_k)\,\eta(x) = -m\,\sigma^2\eta^{*}(x)\,, \tag{A.5}$$

$$(\partial_0 + \sigma^k\partial_k)\,\sigma_y\eta^{*}(x) = m\,\eta(x)\,. \tag{A.6}$$

Note that from Eqs. (A.5) and (A.6) the two-component spinor field $\eta(x)$ satisfies the Klein-Gordon equation,

$$(\Box + m^2)\eta(x) = 0 \, . \tag{A.7}$$

From the above Lagrangian, the canonical conjugate field of $\eta(x)$ is $i\eta^+(x)$. The quantization of the field is performed by imposing the equal time cannonical anticommutation relations,

$$\{\eta(x), \eta^+(y)\}_{x^0=y^0} = \delta(\vec{x}-\vec{y}) \, , \tag{A.8}$$

$$\{\eta(x), \eta(y)\}_{x^0=y^0} = \{\eta^+(x), \eta^+(y)\}_{x^0=y^0} = 0 \, . \tag{A.9}$$

The quantized form of $\eta(x)$ is

$$\eta(x) = \frac{1}{(2\pi)^{3/2}} \int d\vec{p} \sqrt{\frac{m}{p_0}} \sum_s \{ a(p,s) u_\eta(p,s) e^{-ipx} + a^+(p,s) v_\eta(p,s) e^{ipx} \} . \tag{A.10}$$

where the creation and annihilation operators $a^+(p, s)$ and $a(p, s)$ satisfy the cannonical anti-commutation relation,

$$\{ a(p,s) , a^+(p',s') \} = \delta_{ss'} \delta(p-p') . \tag{A.11}$$

Then, the quantized form of $N(x)$ is obtained from Eqs. (A.3) and (A.10) as follows

$$N(x) = \frac{1}{(2\pi)^{3/2}} \int d\vec{p} \sqrt{\frac{m}{p_0}} \sum_s \{ a(p,s) u(p,s) e^{-ipx} + a^+(p,s) u^c(p,s) e^{ipx} \} . \tag{A.12}$$

where $u^c = C\bar{u}^T$ with the charge conjugation matrix C.

The propagator of the Majorana field can be calculated straightforwardly by using Eqs. (A.11) and (A.12),

$$\langle 0 | T[N(x), \bar{N}(y)] | 0 \rangle = i S_F(x-y) . \tag{A.13}$$

Note also that we obtain

$$\langle 0 | T[N(x), N(y)^T] | 0 \rangle = i\, S_F(x-y) C^T. \tag{A.14}$$

The latter is only possible for the (self-conjugate) Majorana field and makes the $(\beta\beta)_{0\nu}$ mode possible.

As defined in Eqs. (2.27) and (2.28), current neutrinos ν_{eL} and ν'_{eR} are the superpositions of massive Majorana neutrinos N_j. We obtain from Eqs. (2.27), (2.28) and (A.14),

$$\begin{aligned} \langle 0 | T[\nu_{eL}(x), \nu_{eL}^T(y)] | 0 \rangle &= i \sum_j U_{ej}^2 \frac{1-\gamma_5}{2} S_F(x-y) C^T \left(\frac{1-\gamma_5}{2}\right)^T \\ &= i \sum_j m_j U_{ej}^2 \int \frac{d^4 q}{(2\pi)^4} \frac{e^{-iq(x-y)}}{q^2 - m_j^2 + i\epsilon} \left(\frac{1-\gamma_5}{2}\right) C^T, \end{aligned} \tag{A.15}$$

$$\begin{aligned} \langle 0 | T[\nu_{eL}(x), \nu_{eR}'^T(y)] | 0 \rangle &= i \sum_j U_{ej}^2 \frac{1-\gamma_5}{2} S_F(x-y) C^T \left(\frac{1+\gamma_5}{2}\right)^T \\ &= i \sum_j U_{ej} V_{ej} \int \frac{d^4 q}{(2\pi)^4} \frac{\not{q}\, e^{-iq(x-y)}}{q^2 - m_j^2 + i\epsilon} \left(\frac{1+\gamma_5}{2}\right) C^T. \end{aligned} \tag{A.16}$$

The other propagators are obtained similarly.

Appendix B : The brief description of the 2n- and N^*-mechanisms

(a) *The general description*

Let us consider the $N_A \to N_B + \ell$ transition, where $N_A (N_B)$ is the parant (daughter) nucleus, and ℓ stands for either $2e^-$ or $2e^- + 2\bar{\nu}_e$ depending on the $(\beta\beta)_{0\nu}$ or $(\beta\beta)_{2\nu}$ modes, respectively.

In order to deal with the N^*-mechanism, the following effective Hamiltonian is considered,

$$H_{int} = H_W + H_S \,, \tag{B.1}$$

where H_S represents the effective strong interaction for the transition $N + N \leftrightarrow \Delta + N$ by the exchange of π, ρ, $\cdots$.

In the 2n-mechanism, the double β decay takes place through the 2nd order perturbation in H_W and the 0th order in H_S as shown in Fig. 3 and the R-matrix is

$$R^{2n} = \langle N_B , \ell \,|\, R_W \,|\, N_A \rangle \,, \tag{B.2}$$

where R_W represents the R-matrix due to the 2nd order weak interaction.

In the N^*-mechanism, the double β decay occurs through the 2nd order in H_W and the 1st order in H_S as shown in Fig. 3.* Then the R-matrix element for the N^*-mechanism may be expressed in the following form;

---

* There may be the third possible combination of H_W and H_S such as the sequence $H_W - H_S - H_W$ in contrast to the $H_W - H_W - H_S$ in Fig. 3.. Since this contribution is expected to be small, this is not considered in this thesis.

$$R^{N^*} = \sum_{N_\Delta} \left\{ \langle N_B, \ell | R_W | N_{\Delta^-} \rangle \frac{1}{M_A - E_{\Delta^-}} \langle N_{\Delta^-} | H_S | N_A \rangle \right.$$

$$\left. + \langle N_B | H_S | N_{\Delta^{++}} \rangle \frac{1}{M_B - E_{\Delta^{++}}} \langle N_{\Delta^{++}} | R_W | N_A \rangle \right\}, \qquad \text{(B.3)}$$

where $M_A (M_B)$ is the mass of $N_A (N_B)$, and $E_{\Delta^-} (E_{\Delta^{++}})$ is the energy of the intermediate nucleus $N_{\Delta^-} (N_{\Delta^{++}})$ which includes $\Delta^- (\Delta^{++})$. Note that the nuclear state $N_{\Delta^-} (N_{\Delta^{++}})$ has the same J^P as $N_A (N_B)$ has.

Let us consider, for definiteness, the $N_{\Delta^-} \rightarrow N_B + 2e^-$ transition. The nuclear states N_{Δ^-} and N_B are expressed in the following forms;

$$|N_{\Delta^-}\rangle = |\Delta^-\rangle_S \otimes |\Delta^-\rangle_L \otimes |R_{\Delta^-}\rangle, \qquad \text{(B.4)}$$

$$|N_B\rangle = |p\rangle_S \otimes |p\rangle_L \otimes |R_B\rangle, \qquad \text{(B.5)}$$

where s and L in the hadronic states stand for the intrinsic (spin and isospin) part and the orbital angular momentum part with respect to the center of the nucleus. Here $|R_{\Delta^-}\rangle$ and $|R_B\rangle$ represent the remainders of the nuclear states. It should be understood that there is some appropriate sum with respect to the angular momenta. The nuclear matrix elements in Eqs. (3.25)-(3.32) can be written in this notation as follows;

$$\langle N_B | O | N_{\Delta^-} \rangle = \langle R_B | \otimes {}_L\langle p | \otimes {}_S\langle p | \; O \; |\Delta^-\rangle_S \otimes |\Delta^-\rangle_L \otimes |R_{\Delta^-}\rangle, \qquad \text{(B.6)}$$

where O represents one of the nuclear tensor operators appeared in M_F, M_{GT}, M^ℓ, Q^ℓ, P^ℓ, $P^{\ell k}$ and $Q^{\ell k}$.

Let us discuss what kinds of nuclear tensor operators change quark states inside the hadron. Obviously, the operators $\tau^+_{n(m)}$ and $\sigma^j_{n(m)}$ act on quarks. As for $\hat{r}_{nm}$ and $\hat{r}_{+nm}$, some cautions are necessary. Consider the following decomposition of the position operator for the n-th quark; $\vec{r}_n = \vec{r}_G + \vec{r}'_n$ where $\vec{r}_G$ is the position operator of Δ^- measured from the center of N_{Δ^-}. The relative coordinate $\vec{r}'_n$ changes the orbital angular momentum of quarks around the center of the hadron. Thus we conclude that the relevant operators for quarks in Δ^- are $\tau^+_{n(m)}$, $\sigma^j_{n(m)}$, $\vec{r}'_{nm}$ and $\vec{r}'_{+nm}$. With this caution, the nuclear matrix elements are calculated in the SU(6) quark model where $\Delta(\frac{3}{2}^+)$ and the nucleon $N(\frac{1}{2}^+)$ are assigned to their S-state. The nuclear tensor operators contributing to the transition $\Delta(\frac{3}{2}^+) \to N(\frac{1}{2}^+)$ should be of rank 0 with respect to $\vec{r}'_{nm}$ and $\vec{r}'_{+nm}$, and of rank 1 or 2 with respect to the spin part. We conclude from Eqs. (3.25)-(3.32) that M_F and M_{GT} do not contribute to this transition. The Q^ℓ, P^ℓ, $P^{\ell k}$, $Q^{\ell k}$ and M^ℓ take the following forms;

$$Q^\ell = \frac{2}{3} \langle \{ G_+ (\sigma_n^\ell - \sigma_m^\ell) - i G_A \varepsilon_A (\vec{\sigma}_n \times \vec{\sigma}_m)^\ell \} (\hat{r}'_{nm} \cdot \hat{r}'_{nm}) \rangle_S \cdot \langle P | \Delta^- \rangle_L \langle R_B | R_{\Delta^-} \rangle , \tag{B.7}$$

$$P^\ell = -\frac{1}{3} G_- \langle \sigma_n^\ell + \sigma_m^\ell \rangle_S \langle P | \Delta^- \rangle_L \langle R_B | R_{\Delta^-} \rangle , \tag{B.8}$$

$$P^{\ell k} = -\frac{4}{3} \langle i G_+ \epsilon_{\ell k j} \sigma_n^j + G_A \varepsilon_A \sigma_n^\ell \sigma_m^k \rangle_S \langle P | \Delta^- \rangle_L \langle R_B | R_{\Delta^-} \rangle , \tag{B.9}$$

$$Q^{\ell k} = \frac{4}{3} \langle i G_- \epsilon_{\ell k j} \sigma_n^j (\hat{r}'_{nm} \cdot \hat{r}'_{+nm}) \rangle_S \langle P | \Delta^- \rangle_L \langle R_B | R_{\Delta^-} \rangle , \tag{B.10}$$

$$M^\ell = \frac{1}{2} \langle \sigma_n^\ell + \sigma_m^\ell \rangle_S \langle P | \Delta^- \rangle_L \langle R_B | R_{\Delta^-} \rangle . \tag{B.11}$$

In the SU(6) quark model, we get $Q^\ell = Q^{\ell k} = 0$ which may be understood from the following argument. Note that the spin operators in Q^ℓ

and $Q^{\ell k}$ are antisymmetric under the interchange of quarks so that it is expected that the spin part of ${}_s\langle p|\sum_{nm}\tau_n^+\tau_m^+[(\sigma_n^{\ell}-\sigma_m^{\ell})-i(\vec{\sigma}_n\times\vec{\sigma}_m)^{\ell}]$ etc. is also antisymmetric. Since the spin wave function of $|\Delta^-\rangle_s$ is symmetric, $Q^{\ell}=Q^{\ell k}=0$ is concluded. Therefore, $P^{\ell k}$, M^{ℓ} and P^{ℓ} contribute to the $(\beta\beta)_{0\nu}$ mode. The same argument also holds for the $N_A\rightarrow N_{\Delta^{++}}+2e^-$ transition.

The similar argument applies to the $(\beta\beta)_{2\nu}$ mode. Of course, it should be noted that K in Eq.(4.10) should be modified as follows;

$$K=[\langle E_n\rangle-E_{\Delta^-}+p_1^0+k_1^0]^{-1}+[\langle E_n\rangle-E_{\Delta^-}+p_2^0+k_2^0]^{-1}, \tag{B.12}$$

and similarly for L .

(b) *The factorization hypothesis*

As we have seen in the previous subsection (a), the R_W-matrix may be written in the following form;

$$\langle N_B,\ell\,|R_W|\,N_{\Delta^-}\rangle={}_S\langle p,\ell\,|R_W|\Delta^-\rangle_S\;{}_L\langle p|\Delta^-\rangle_L\,\langle R_B|R_{\Delta^-}\rangle. \tag{B.13}$$

This is valid under the approximations (i), (ii), (iii) introduced in §3 and in the framework of the SU(6) quark model. The "factorization hypothesis" means the approximation that the amplitude ${}_s\langle p,\ell|R_W|\Delta^-\rangle_s$ is modified by the following replacement,

$${}_S\langle p,\ell\,|R_W|\Delta^-\rangle_S\;\Longrightarrow\;e^{i\varphi}\Big[\sum_{S_{\Delta z},S_{pz}}{}'\;|{}_S\langle p,\ell\,|R_W|\Delta^-\rangle_S|^2\Big]^{1/2}, \tag{B.14}$$

where the primed sum means the spin average with respect to Δ . Under this replacement, the R-matrix is rewritten as follows;

$$R^{N^*}=\Big[\sum_{S_{\Delta z},S_{pz}}{}'\;|{}_S\langle p,\ell\,|R_W|\Delta^-\rangle_S|^2\Big]^{1/2}\sqrt{2}\;P(\Delta)^{1/2}\langle\Phi_f|\Phi_i\rangle, \tag{B.15}$$

where

$$\sqrt{2}\, P(\Delta)^{1/2}\langle\Phi_f|\Phi_i\rangle = {}_L\langle p|\Delta^-\rangle_L \langle R_B|R_{\Delta^-}\rangle e^{i\varphi}\, \frac{\langle N_{\Delta^-}|H_S|N_A\rangle}{M_A - E_{\Delta^-}} + \frac{\langle N_B|H_S|N_{\Delta^{++}}\rangle}{M_B - E_{\Delta^{++}}} e^{i\varphi'} \langle R_{\Delta^{++}}|R_A\rangle\, {}_L\langle\Delta^{++}|n\rangle_L \,. \tag{B.16}$$

Here we used the relation

$$\sum_{S_{\Delta_z}, S_{P_z}}{}' \left| {}_S\langle p, \ell | R_W | \Delta^-\rangle_S \right|^2 = \sum_{S_{\Delta_z}, S_{P_z}}{}' \left| {}_S\langle \Delta^{++}, \ell^+ | R_W | n\rangle_S \right|^2 . \tag{B.17}$$

The factors $P(\Delta)$ and $\langle\Phi_f|\Phi_i\rangle$ are introduced to give some physical images of the N^*-mechanism. Let us assume the decomposition

$$\langle N_{\Delta^-}|H_S|N_A\rangle \simeq \langle\Delta^-|H_S|n\rangle\langle R_{\Delta^-}|R_A\rangle . \tag{B.18}$$

Now the probability admixture $P(\Delta)$ may be defined as

$$P(\Delta) = \frac{1}{N_n}\sum_{n,\Delta^-}\left|\frac{\langle\Delta^-|H_S|n\rangle}{E_{\Delta^-} - M_A}\right|^2 , \tag{B.19}$$

where N_n is the number of neutrons which can contribute to the double β decay and the sum of n extends over all those neutrons. In other words, $P(\Delta)$ is the probability to make Δ^- per neutron. Now $\langle\Phi_f|\Phi_i\rangle$ may be written as

$$\langle\Phi_f|\Phi_i\rangle \simeq {}_L\langle p|\Delta^-\rangle_L \langle R_B|R_{\Delta^-}\rangle\langle R_{\Delta^-}|R_A\rangle \simeq {}_L\langle p|\Delta^-\rangle_L \langle R_B|R_A\rangle . \tag{B.20}$$

Here we used $\langle R_{\Delta^-}|R_A\rangle \simeq \delta_{R_{\Delta^-}R_A}$. In this way, the $\langle\Phi_f|\Phi_i\rangle$ may be interpreted as the overlap between the initial and final nuclear wave functions.

Appendix C: The properties of the "potentials" due to the neutrino propagation

Let us discuss the porperties of the "potential"-like function due to the neutrino propagation defined in Eqs.(3.21) and (3.35),

$$H(r, m_\nu, \mu_0) = \frac{2}{\pi r}\int_0^\infty dq \, \frac{q \sin(qr)}{q^0(q^0+\mu_0 m_e)}, \tag{C.1}$$

where the parameter r is the distance between two neutrons (quarks) which participate in the double β decay. In this Appendix, we shall evaluate the expectation values <H> and <rH'> by using the nucleon-nucleon (or quark-quark) correlation function $\rho(\vec{r})$ as the weight function:[35)]

$$\langle H \rangle \equiv \int d\vec{r}\, \rho(\vec{r})\, H(r, m_\nu, \mu_0), \tag{C.2}$$

$$\langle rH' \rangle \equiv \int d\vec{r}\, \rho(\vec{r})\, r \, dH(r, m_\nu, \mu_0)/dr. \tag{C.3}$$

(a) *The averaging scheme*

Two types of correlation functions are considered.

Case S (the spherical shell distribution in 2n-mechanism):

This case corresponds to the assumption that both neutrons are located on the surface of the nucleus as shown in Fig. 10. This scheme is condidered reasonable because neutrons which actively participate in the double β decay carry the large principal quantum numbers so that they are considered to be located in the outer shell. We remind that the weak decay must occur at two different neutrons because $\tau_n^+ \tau_n^+ = 0$. This is taken into account by imposing a cut for θ, i. e., $\theta < \pi - \theta_c$ with $\theta_c \simeq 1/R\,m_\pi$. Thus, we assume the following form,

$$\rho_S(\vec{r}) = [4\pi r^2(1-\xi^2)]^{-1}\delta(r-2R\cos(\theta/2))\,\theta(\pi-\theta_c-\theta), \qquad \text{(C.4)}$$

where $\theta(x)$ is the step function, $R = 1.2\,A^{1/3} \cdot 10^{-13}$cm and $\xi = \sin(\theta_c/2)$. For $A = 48 \sim 130$, $R = (1.1 \sim 1.6) \cdot 10^{-2}/m_e$ and $\xi = 0.11 \sim 0.16$, respectively.

Case Δ (the quark correlation in the N^*-mechanism):

Suppose that there is a potential which confines quarks within distance $(2m_\pi)^{-1}$ and does not provide any hard core repulsion. According to Halprin, Minkowski, Primakoff and Rosen,[51)] the following form of $\rho_\Delta(r)$ is considered,

$$\rho_\Delta(\vec{r}) = (\pi a^3)^{-1}\exp(-2r/a), \qquad \text{(C.5)}$$

where $a = 0.7 \cdot 10^{-13}$cm.

(b) *The global features of the average potentials*

The numerical integrations are made and the m_ν, μ_0 and A dependences of $\langle H\rangle_i$ and $\langle rH'\rangle_i$ with $i = S$ and Δ are shown in Fig. 11 for some typical values of m_ν, μ_0 and A.

(b-1) *The μ_0 and A dependences*

The A dependence of $\langle H\rangle_S$ and $\langle rH'\rangle_S$ comes in through the nuclear radius R ($\propto A^{1/3}$). As we can see from Fig. 11, the dimensionless quantities $R\langle H\rangle_S$ and $R\langle rH'\rangle_S$ are almost independent of A. Of course, $\langle H\rangle_\Delta$ and $\langle rH'\rangle_\Delta$ are independent of A.

The μ_0 dependence of $\langle H\rangle$ and $\langle rH'\rangle$ is not important either, because the average of nuclear energy level difference μ_0 seems to be in the range of $5\,\mathrm{MeV} \lesssim \mu_0 m_e \lesssim 15\,\mathrm{MeV}$ for typical nuclei. As we can see from Fig. 11, $\langle H \rangle$ and $\langle rH' \rangle$ are almost independent* of μ_0 in this region.

(b-2) *The* m_ν *dependence*

Contrary to A and μ_0 dependences, the neutrino mass dependence is much complicated. As shown in Fig. 11, there are essentially two distinct regions: One is the light neutrino mass region ($m_\nu \lesssim 1\,\mathrm{MeV}$) where $\langle H \rangle$ and $\langle rH' \rangle$ behave like constant.* The other is the heavy neutrino mass region ($m_\nu \gtrsim 1\,\mathrm{GeV}$), where $\langle H \rangle$ and $\langle rH' \rangle$ are rapidly decreasing functions of m_ν.[35] More precisely, for $m_\nu \lesssim 1\,\mathrm{MeV}$, they behave essentially like Coulomb potential, $\langle H \rangle_S$ and $\langle rH' \rangle_S \simeq \chi_S/R$, $\langle H \rangle_\Delta \simeq (1-\chi_{\Delta A})/a$ and $\langle rH' \rangle_\Delta \simeq -(1+\chi_{\Delta B})/a$ where χ_i's are independent of m_ν and depend weakly on μ_0 and A.

For $m_\nu \gtrsim 1\,\mathrm{GeV}$, the average potentials behave like Yukawa potential, i. e., $\langle H \rangle_S \simeq \xi \exp(-2\xi m_\nu R)/R$ and $\langle rH' \rangle_S \simeq \exp(-2\xi m_\nu R)(2 m_\nu R^2)^{-1}$ and $\langle H \rangle_\Delta \simeq -\langle rH' \rangle_\Delta \simeq 4\, m_\nu^{-2}\, a^{-3}$.

The following remarks are in order; (i) From Fig. 11 we observe that $-a\langle rH' \rangle_\Delta$ is greater than $R\langle H \rangle_S$ in the light neutrino mass region ($m_\nu \lesssim 1\,\mathrm{MeV}$). This difference is due to the average distance which neutrino propagates. We remind $a/R \sim 1/10$. (ii) The milder damping on m_ν (m_ν^{-2} dependence) for the case Δ is traced to the assumption of no core repulsion among quarks.

---

* This may be understood by the fact that the momentum of the virtual neutrino which mainly contributes to the potential is larger than 20 MeV so that $\mu_0 m_e$ ($\lesssim 15\,\mathrm{MeV}$) and m_ν ($\lesssim 1\,\mathrm{MeV}$) may be neglected.

(c) *Application to the decay formulae*

In the decay formulae of the $(\beta\beta)_{0\nu}$ mode, the average potentials appear in the forms of $\Sigma\, m_j U_{ej}^2 \langle H\rangle_S$, $\Sigma\, U_{ej} V_{ej} \langle H\rangle_{S,\Delta}$ and $\Sigma\, U_{ej} V_{ej} \langle rH'\rangle_{S,\Delta}$.

Since $\langle H\rangle$ and $-\langle rH'\rangle$ behave like almost constants at the region $m_\nu \lesssim 1\,\text{MeV}$ and the mixing among light and heavy neutrinos is considered to be small, the following approximation is allowed to us;

$$[\sum_j U_{ej} V_{ej} \langle H(r, m_j, \mu_0)\rangle] \simeq \langle H(r, 0, \mu_0)\rangle [\sum_j U_{ej} V_{ej}], \qquad (C.6)$$

and the corresponding approximation for $\Sigma\, U_{ej} V_{ej} \langle rH'\rangle$.

For the term $\Sigma\, m_j U_{ej}^2 \langle H\rangle_S$, the situation becomes complicated. In the analysis in §5 and §6, the contributions from the heavy neutrinos ($m_\nu \gtrsim 10\,\text{MeV}$) are neglected and the equality

$$[\sum_j m_j U_{ej}^2 \langle H(r, m_j, \mu_0)\rangle] \simeq \langle H(r, 0, \mu_0)\rangle [\sum_j m_j U_{ej}^2] \qquad (C.7)$$

is used. Rigorously speaking, the values of $|\Sigma\, m_j U_{ej}^2|$ obtained in §6 should be understood as those of $|\Sigma\, m_j U_{ej}^2 \langle H(r, m_j, \mu_0)\rangle / \langle H(r, 0, \mu_0)\rangle|$.

In the following, we discuss some cases where the equality (C.7) becomes invalid. We present the approximate formula,[35]

$$\sum_j m_j U_{ej}^2 R\langle H\rangle_S \simeq \sum_{m_j \lesssim 1\,\text{MeV}} m_j U_{ej}^2 X_S(2\mu_0 m_e R) + m_h U_{eh}^2 \frac{e^{-(2\xi m_h R)}}{2 m_h R}, \qquad (C.8)$$

where

$$X_S(\rho) = -2[\pi(1-\xi^2)\rho]^{-1} \{ [-2 \ln \xi + [2\cos(\xi\rho) + \xi\rho \sin(\xi\rho)]\, Ci(\xi\rho)$$
$$+ [2\sin(\xi\rho) - \xi\rho\cos(\xi\rho)]\, Si(\xi\rho) - [\xi = 1 \text{ terms}] \}. \qquad (C.9)$$

Note that among heavy neutrinos, the lightest one (denoted by h) will mainly contribute as a result of the presence of the exponential damping factor. For definiteness, we restrict our attention to the data analysis of tellurium. Note that when $\lambda = \kappa = 0$, $|\Sigma_{light}\, m_j\, U_{ej}^2| \simeq 32$ eV is obtained. In order that the heavy neutrino contributes equally, $m_h \simeq 2.3$ GeV is required, even if $U_{eh} = 1$ is taken. Therefore, if $m_h > 3$ GeV, the heavy neutrino contribution may well be neglected. If the masses of some neutrinos happen to be in the range of $1\,\text{MeV} < m_\nu < 1\,\text{GeV}$, a careful analysis must be made. Here we shall not discuss this case to avoid the complexity. We would like to mention that the masses of heavy neutrinos are much larger than 1 GeV in most grand unified theories as discussed in §2.

Appendix D: The CP-violation in the leptonic sector

Let us first consider the case where only two generations of Majorana neutrinos mix strongly. We assumed Case (iii) where both the left- and right-handed neutrinos, the Majorana mass terms and the Dirac mass term exist.[20] Then the mixing matrix is described by a 4×4 unitary matrix. We start from the following 4×4 unitary matrix,[71]

$$X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_4 & -s_4 \\ 0 & 0 & s_4 & c_4 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_2 & -s_2 & 0 \\ 0 & s_2 & c_2 & 0 \\ 0 & 0 & 0 & e^{i\delta_2} \end{pmatrix} \begin{pmatrix} c_1 & s_1 & 0 & 0 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & e^{i\delta_1} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_6 & s_6 \\ 0 & 0 & -s_6 & c_6 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_3 & s_3 & 0 \\ 0 & -s_3 & c_3 & 0 \\ 0 & 0 & 0 & e^{i\delta_3} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_5 & s_5 \\ 0 & 0 & -s_5 & c_5 \end{pmatrix}, \qquad \text{(D.1)}$$

where $s_i = \sin\theta_i$ and $c_i = \cos\theta_i$. Note that there are three phases, δ_1, δ_2 and δ_3. The angles θ_2 and θ_3 give the mixings between the L-type neutrinos (N_1, N_2) and R-type neutrinos (N_3, N_4). The most general form of a 4×4 unitary matrix is obtained from Eq. (D.1) by the replacement

$$X_{jk} \longrightarrow e^{i(\rho_j + \sigma_k)} X_{jk}, \qquad \text{(D.2)}$$

which supplies seven nontrivial additional phases. Among total ten phases, two phases can be absorbed by the redefinition of two charged lepton fields. We choose X_{11} and X_{22} to be real and the matrix becomes

$$e^{i(\rho_j - \rho_1 + \sigma_k - \sigma_1)} X_{jk}, \qquad \text{for } j = 1, 3,$$

$$e^{i(\rho_j - \rho_2 + \sigma_k - \sigma_2)} X_{jk}, \qquad \text{for } j = 2, 4. \qquad \text{(D.3)}$$

Thus, there remain eight CP-violating phases, δ_1, δ_2, δ_3, $\beta_1 = \delta_2 - \delta_1$, $\beta_2 = \rho_3 - \rho_1$, $\beta_3 = \rho_4 - \rho_2$, $\beta_4 = \sigma_3 - \sigma_1$ and $\beta_5 = \sigma_4 - \sigma_1$.

As an illustration, we consider the case where the R-type Majorana neutrinos N_3 and N_4 are heavy enough so that they do not contribute to the $\beta\beta$ decay. In this case, we only need to consider the 2×2 submatrices $U^{(1)}$ and $V^{(1)*}$ which are defined in Eq.(2.23) as

$$\begin{pmatrix} U \\ V^* \end{pmatrix} = \begin{pmatrix} U^{(1)} & U^{(2)} \\ V^{(1)*} & V^{(2)*} \end{pmatrix} . \tag{D.4}$$

Since the mixings between the L-type and R-type neutrinos would be weak, we keep only the first order term with respect to θ_2 and θ_3. Now the matrices $U^{(1)}$ and $V^{(1)*}$ take the forms

$$U^{(1)} \simeq \begin{pmatrix} c_1 & s_1 e^{i\beta_1} \\ -s_1 e^{-i\beta_1} & c_1 \end{pmatrix}, \tag{D.5}$$

$$V^{(1)*} \simeq \theta_2 \begin{pmatrix} -s_1 c_4 e^{i\beta_2} & \left[c_1 c_4 - \frac{\theta_3}{\theta_2}\left(c_4 c_6 e^{i\delta_1} + s_4 s_6 e^{i\delta_2}\right)\right] \times e^{i(\beta_1+\beta_2)} \\ -s_1 s_4 e^{i(\beta_3-\beta_1)} & \left[c_1 s_4 - \frac{\theta_3}{\theta_2}\left(s_4 c_6 e^{i\delta_1} - c_4 s_6 e^{i\delta_2}\right)\right] \times e^{i\beta_3} \end{pmatrix}, \tag{D.6}$$

where $s_2 \cong \theta_2 \simeq 0$ and $s_3 \cong \theta_3 \simeq 0$. As a typical example for the maximal mixings , we take $\theta_1 = \theta_4 = \theta_5 = \theta_6 \simeq \pi/4$ and assume $\theta_2 = \theta_3$ for simplicity. Then we obtain

$$U^{(1)} \simeq \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{i\beta_1} \\ -e^{-i\beta_1} & 1 \end{pmatrix}, \tag{D.7}$$

and

$$V^{(1)*} \simeq \frac{\theta_2}{2} \begin{pmatrix} -e^{i\beta_2} & (1-e^{i\delta_1}-e^{i\delta_2})e^{i(\beta_1+\beta_2)} \\ -e^{i(\beta_3-\beta_1)} & (1-e^{i\delta_1}+e^{i\delta_2})e^{i\beta_3} \end{pmatrix}. \tag{D.8}$$

Note that there are five complex phases (δ_1, δ_2, β_1, β_2 and β_3) in agreement with the case of $n = 2$ in the general formula $n(3n-1)/2$. The remaining three phases are included in $U^{(2)}$ and $V^{(2)*}$.

For Dirac neutrinos (Case(i)), $U^{(2)} = V^{(2)*} = 0$ and also $U^{(1)}$ and $V^{(1)*}$ should be unitary. If we consider the case of two generations, three complex phases in $U^{(1)}$ can be removed by the redefinition of two charged lepton and two neutrino fields, but three phases in $V^{(1)}$ remain. Thus we obtain

$$U^{(1)} = \begin{pmatrix} c_1 & s_1 \\ -s_1 & c_1 \end{pmatrix}, \tag{D.9}$$

and

$$V^{(1)*} = e^{i\alpha_1} \begin{pmatrix} c_2 e^{i\alpha_2} & s_2 e^{i\alpha_3} \\ -s_2 e^{-i\alpha_3} & c_2 e^{-i\alpha_2} \end{pmatrix}. \tag{D.10}$$

Let us discuss the combinations of the mixing matrices appeared in the double β decay.

For the $(\beta\beta)_{2\nu}$ mode, the term $2\sum_{i\leq j}\varepsilon_{ij}^2|U_{ej}|^2|U_{ej}|^2$ comes in. From Eq.(D.7) or (D.9), we obtain

$$2\sum_{i\geq j}\varepsilon_{ij}^2\,|U_{ei}|^2|U_{ej}|^2 = 1\ , \tag{D.11}$$

both for Dirac and Majorana neutrino cases.

For the $(\beta\beta)_{0\nu}$ mode, two terms $|\sum_j m_j U_{ej}^2|$ for Case (ii) and (iii), and $|\sum_j U_{ej} V_{ej}|$ for Case (iii) appear. By using Eqs.(D.7) and (D.8), we obtain

$$\left|\sum_j m_j U_{ej}^2\right| = \frac{1}{2}\left|m_1^2+m_2^2+2m_1m_2\cos 2\beta_1\right|^{1/2}, \tag{D.12}$$

$$\left|\sum_j U_{ej}V_{ej}\right| = \frac{\theta_2}{\sqrt{2}}\left|\cos\left(\frac{\delta_1-\delta_2}{2}\right)\right|\ . \tag{D.13}$$

As seen from Eqs.(D.12) and (D.13), $|\sum_j m_j U_{ej}^2|$ becomes $|m_1+m_2|/2$ or $|m_1-m_2|/2$ depending on $\beta_1 = 0$ (no CP-violation) or $\beta_1 = \pi/2$, and $|\sum U_{ej} V_{ej}| = \theta_2/\sqrt{2}$ or 0 depending on $\delta_1 - \delta_2 = 2n\pi$ or $\delta_1-\delta_2 = (2n+1)\pi$. Thus the CP-violating phases play important roles.

References

1) P. A. M. Dirac, Proc. Roy. Soc., A126 (1930), 360; Proc. Camb. Phil. Soc., 30 (1934), 150.

2) E. Majorana, Nuovo Cimento, 14 (1937), 171.

3) G. Racah, Nuovo Cimento, 14 (1937), 322.

4) W. Pauli, Noyaux atomiques, Proc. Solvay Congr., Brussels (1933), P. 324.

5) E. Fermi, Z. Phys., 88 (1934), 161.

6) F. Reines and C. L. Cowan, Phys. Rev., 92 (1953), 830; 113 (1959), 273; C. L. Cowan, F. Reines, F. B. Harrison, H. W. Kruse, and A. D. Mcquire, Science, 3213 (1956), 103.

7) C. S. Wu, E. Ambler, R. Hayward, D. Hoppers and R. Hudson, Phys. Rev., 104 (1957), 1413; 107 (1957), 641.

8) T. D. Lee and C. N. Yang, Phys. Rev., 104 (1956), 254.

9) W. Pauli, Nuovo Cimento, 6 (1957), 204.

10) F. Gürsey, Nuovo Cimento, 7 (1958), 411.

11) C. Ryan and S. Okubo, Nuovo Cimento Suppl., 2 (1964), 234.

12) H. Georgi and D. V. Nanopoulos, Nucl. Phys., B155 (1979), 52; M. Gell-Mann, P. Ramond, and R. Slansky, Supergravity, ed. by P. Van Nieuwenheuzen and D. Z. Freedman (North-Holland, 1979, Amsterdam); T. Yanagida, in Proc. Workshop on the Unified Theory and the Baryon Number in the Universe (KEK, Japan, Feb. 1979); R. Barbieri, D. V. Nanopoulos, G. M. Morchio and F. Strocchi, Phys. Lett., 90B (1980), 91; E. Witten, Phys. Lett. 91B (1980), 81.

13) V. A. Lubimov, V. Z. Nozsik, E. G. Novikov, E. F. Tertyakov, and V. S. Kosik, Phys. Lett., 94B (1980), 266.

14) W. H. Furry, Phys. Rev., 56 (1939), 1184.

15) H. Primakoff and S. P. Rosen, Rep. Prog. Phys., 22 (1959), 121; Proc. Phys. Soc., 78 (1961), 464.

16) L. Kisslinger, Phys. Rev. Lett., 26 (1971), 998.
B. Pontecorvo, Zurn Eksp. Theor. Fiz., 53 (1967), 1717
(Sov. Phys. JETP., 26 (1968), 784.)

17) H. Primakoff and S. P. Rosen, Phys. Rev., D5 (1972), 1784.

18) M. Doi, T. Kotani, H. Nishiura, K. Okuda and E. Takasugi, Phys. Lett., 102B (1981), 323.

19) R. Davis Jr, Phys. Rev., 97 (1955), 766.

20) M. Doi, T. Kotani, H. Nishiura, K. Okuda, and E. Takasugi, Osaka University preprint OS-GE 81-34 (1981).

21) S. M. Bilenky, J. Hosek and S. T. Petcov, Phys. Lett., 94B (1980), 495.

22) J. Schechter and J. W. F. Valle, Phys. Rev., D22 (1980), 2227.

23) R. K. Bardin, P. J. Gollon, J. D. Ullman and C. S. Wu, Nucl. Phys. A158 (1970), 337.

24) M. K. Moe and D. D. Lowenthal, Phys. Rev., C22 (1980), 2186.

25) E. Fiorini, CERN/EP/Phys 78-33.

26) E. W. Hennecke, O. K. Manuel, and D. D. Sabu, Phys. Rev., C11 (1975), 1378.

27) H. Primakoff and S. P. Rosen, Phys. Rev., 184 (1969), 1925.

28) A. K. Kerman and L. S. Kisslinger, Phys. Rev., 180 (1969), 1483; See also H. J. Weber and H. Arenhövel, Phys. Rep., 36C (1978), 277.

29) M. G. Mayer, Phys. Rev., 48 (1935), 512.

30) E. J. Konopinski, *Theory of Beta Radioactivity* (Oxford Univ. Press. London, England, 1966).

31) E. Greuling and R. C. Whitten, Ann. Phys., 11 (1960), 510.

32) A. Molina and P. Pascual, Nuovo Cimento, 41 (1977), 756.

33) D. Smith, C. Picciotto, and D. Bryman, Phys. Lett., 46B (1973), 157. ; Nucl. Phys., B64 (1973), 525 ; Phys. Lett., 46B (1973), 157 ; Phys. Rev., D8 (1973), 342.
D. Bryman and C. Picciotto, Rev. Mod. Phys., 50 (1978), 11.

34) M. Doi, T. Kotani, H. Nishiura, K. Okuda, and E. Takasugi, Phys. Lett., 103B (1981), 219.

35) M. Doi, T. Kotani, H. Nishiura, K. Okuda, E. Takasugi, Prog. Theor. Phys., 66 (1981), No. 5 in press.

36) J. C. Pati and A. Salam, Phys. Rev., D10 (1974), 275.

37) H. Georgi and S. L. Glashow, Phys. Rev. Lett., 32 (1974), 438; A. J. Buras, J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys., B135 (1978), 66.

38) H. Georgi, in Particle and Fields, ed. by C. E. Carlson, AIP Conf. No. 23, New York (1975); H. Fritzsch and P. Minkowski, Ann. Phys., (N. Y.) 93 (1975), 193. ; M. S. Chanowitz, J. Ellis and M. K. Gaillard, Nucl. Phys., B128 (1977), 506. ; H. Georgi and D. V. Nanopoulos, Nucl. Phys., B155 (1979), 52.

39) R. N. Mohapatra and J. C. Pati, Phys. Rev., D11 (1975), 2559; G. Senjanovic and R. N. Mohapatra, Phys. Rev., D12 (1975), 1502.

40) M. A. B. Beg, R. V. Budny, R. Mohapatra, and A. Sirlin, Phys. Rev. Lett., 38 (1977), 1252.

41) S. M. Bilenky and B. Pontecorvo, Phys. Reports, 41 (1978), 226. Lett. Nuovo Cim., 17 (1976), 569.

42) T. Yanagida and M. Yoshimura, Prog. Theor. Phys., 64 (1980), 1870.

43) M. Kobayashi and T. Maskawa , Prog. Theor, Phys., 49 (1973), 652.

44) S. Weinberg, Phys. Rev. Lett., 19 (1967), 1264 ; A. Salam, *Elementary Particle Theory* ed. by N. Swartholom, Stockholm (1968) ; S. L. Glashow, Nucl. Phys., 22 (1961), 579.

45) V. Barger, P. Langacker, J. P. Leveille, and S. Pakvasa, Phys. Rev. Lett., 45 (1980), 692.

46) R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett., 44 (1980), 912; Phys. Rev., D23 (1981), 165.

47) R. Barbieri, J. Ellis and M. K. Gaillard, Phys. Lett., 90B (1980), 249.

48) E. Witten, Phys. Lett., 91B (1980), 81.

49) R. H. Hulme, Proc. Roy. Soc. London (A) 133 (1931), 381; See also H. Behrens and J. Jänecke, *Numerical Tables for Beta-Decay and Electron Capture*, ed. by H. Schopper (Landolt-Börnstein, Group I, Vol. 4), (Springer-Verlag, Berlin, 1969).

50) R. J. Blin-Stoyle, *Fundamental Interactions and the Nucleus* (North-Holland, Amsterdam, 1973).

51) A. Halprin, P. Minkowski, H. Primakoff, and S. P. Rosen, Phys. Rev., D13 (1976), 2567.

52) C. Picciotto, Can. J. Phys., 56 (1978), 399.

53) S. P. Rosen, Proc. Phys. Soc., 74 (1959), 350.

54) T. Kirsten and H. W. Muller, Earth Planet Sci. Lett., 6 (1969), 271.

55) B. Srinivasan, E. C. Alexander Jr., R. D. Beaty, D. E. Sinclair and O. K. Manuel, Econ. Geol., 68 (1973), 252.

56) M. G. Inghram and J. H. Reynolds, Phys. Rev., 78 (1950), 822.

57) N. Takaoka and K. Ogata, Z. Naturforschg, 21a (1966), 84.

58) T. Kirsten, O. A. Schaeffer, E. Norton and R. W. Stoenner, Phys. Rev. Lett., 20 (1968), 1300.

59) B. Srinivasan, E. C. Alexander Jr., and O. K. Manuel, Econ. Geol., 67 (1972), 592.

60) E. Fiorini, A. Pullia, G. Bertolini, F. Cappellani and G. Restelli, Nuovo Cimento, 13A (1973), 747.

61) E. Fiorini, Proceedings of the 1977 International Conference on Neutrino Physics and Astrophysics, Baksam Valley (Nauka, Moscow, 1978), Vol. 2, p. 315.

62) B. T. Cleveland, W. R. Leo, C. S. Wu, L. R. Kasday, A. M. Rushton, P. J. Gollon and J. D. Ullman, Phys. Rev. Lett., 35 (1975), 757.

63) E. Fiorini, private communication.

64) J. D. Vergados, Phys. Rev., C13 (1976), 865.

65) V. A. Khodel, Phys. Lett., 32B (1970), 583.
S. A. Fayans and V. A. Khodel, J. Phys., G3 (1977), 359.

66) W. C. Haxton, G. J. Stephenson Jr. and D. Strottman, Phys. Rev. Lett., 47 (1981), 153.

67) A. H. Huffman, Phys. Rev., C2 (1970), 742.

68) K. E. Bergkvist, Nucl. Phys., B39 (1972), 317. See also J. J. Simpson, Phys. Rev., D23 (1981), 649.

69) T. Kotani, Soryshiron Kenkyu (Kyoto), 62 (1980), B60; H. Nishiura, ibid., 62 (1980), B50.

70) K. M. Case, Phys. Rev., 107 (1957), 307.

71) V. Barger, K. Whisnat and R. J. N. Philips, Phys. Rev., D23 (1981), 2773.

72) A. H. Wapstra and K. Bos, Atomic DATA and Nuclear Data Tables, 19 (1977), 177.

73) H. Ejiri, private communication.

74) H. Ohtubo and A. Hosoya, private communication.

Table Captions

Table I. Allowed transitions and relative orders of magnitudes. Each parenthesis under the interactions indicates the possible transition, and m_ν, λ and κ represent the typical neutrino mass and the relative strengths of the right-handed currents, respectively.

Table II. The numerical values of B_i in $(T_{1/2})_{2\nu}$. B_i's are the coefficients of various terms in the half-life formulae of the $0^+ \to 0^+$ and $0^+ \to 2^+$ transitions for the $(\beta\beta)_{2\nu}$ mode which are given in Eqs. (5.1) and (5.2). T is the maximum kinetic energy release in units of m_e and their values are deduced from the data on nuclear masses by Wapstra and Bos.[72] These B_i's are related to $F_0(T)$ in Eq. (4.15) and $F_2(T)$ in Eq. (4.20) as follows: $B_1=F_0(T)\cdot 10^{-3}$, $B_3=F_2(T)\cdot 10^{-9}$, $B_2=B_4=F_2(T)\cdot 10^{-9}/96$.

Table III. The numerical values of G_{0i}, $G_{2\pm}$ and G_i^{N*} in $(T_{1/2})_{0\nu}$. G_{0i} in Eq. (3.57) and $G_{2\pm}$ in Eq. (3.65) are the coefficients of various terms in the half-life formulae of the $0^+ \to 0^+$ and $0^+ \to 2^+$ transitions in the 2n-mechanism for the $(\beta\beta)_{0\nu}$ mode. G_i^{N*} in Eq. (3.88) are those in the N^*-mechanism.

Table IV. The numerical values of C_i in $T(_{1/2})_{0\nu}$. C_i in Eq. (5.3) are the coefficients of various terms in the half-life formula of the $0^+ \to 0^+$ transition for the $(\beta\beta)_{0\nu}$ mode.

Table V. The angular correlations of the $0^+ \to 0^+$, 1^+ and 2^+ tranisitons for the $(\beta\beta)_{2\nu}$ mode (Table (a)), and of the $0^+ \to 0^+$ transition for $(\beta\beta)_{0\nu}$ mode (Table (b)).

Table VI. The experimental data and the theoretical predictions for half-lives.

The predictions are made by taking the values of the average neutrino mass, λ and κ which are determined from the data on tellurium and also by using the parameters related to the nuclear matrix elements given in §6-d. The indices (a) and (b) correspond to two cases, namely Case(a) where $\lambda \gg |\kappa|$ and Case(b) where $\lambda \ll |\kappa|$. The data in Refs. 23), 24), 60), 61), 62) and 63) have been obtained by the counter (chanmber) experiments and others by the geological method.

Figure Captions

Fig. 1. The level structures of ^{130}Te (figure (a)) and ^{48}Ca (figure (b)).

Fig. 2. The schematic diagrams for the $(\beta\beta)_{0\nu}$ mode (figure (a)) and for the $(\beta\beta)_{2\nu}$ mode (figure (b)) in the 2n-mechanism. The N_A, N_B and N_n are the parent, the daughter, and the intermediate nuclei, respectively.

Fig. 3. The diagrams for the $(\beta\beta)_{0\nu}$ mode (figures (a) and (b)) and for the $(\beta\beta)_{2\nu}$ mode (figures (c) and (d)) in the N^*-mechanism. The N_{Δ^-} and $N_{\Delta^{++}}$ mean the intermediate nuclear states including Δ^- and Δ^{++}, respectively.

Fig. 4. The constraint on λ and κ in the left-right symmetric model. The λ_c represents $(M_1/M_2)^2$. The M_1 and M_2 are masses of the mass eigenstate gauge bosons, W_1 and W_2, respectively.

Fig. 5. The diagrams for the $(\beta\beta)_{0\nu}$ mode corresponding to the second order weak interactions. The wavy line represents the weak intermediate boson which controls the left- or right-handed weak interaction.

Fig. 6. The single electron kinetic energy spectra for the $(\beta\beta)_{2\nu}$ mode. ε_1 and T are the electron kinetic energy and the maximum kinetic energy release in units of m_e, respectively. Note that for the $0^+ \rightarrow 2^+$ transition, the 2n- and N^*-mechanisms give the identical spectrum.

Fig. 7. The single electron kinetic energy spectra for the $(\beta\beta)_{0\nu}$ mode. Note that the N^*-mechanism contributes only through the λ- or κ- terms.

Fig. 8. The sum-energy spectra for the $(\beta\beta)_{2\nu}$ and $(\beta\beta)_{0\nu}$ modes. ε is the kinetic energy sum of two electrons in units of m_e.

Fig. 9. The allowed domain for the neutrino mass and the relative strengths of the right-handed currents λ and κ from the data on $(T_{1/2})_{0\nu}$. The allowed domains imposed by the data on ^{48}Ca, ^{76}Ge and ^{82}Se are inside the solid line ellipses (the shaded areas). From the data on $T_{1/2}(^{128}Te)/T_{1/2}(^{130}Te)$, the allowed domain is restricted to the narrow region surrounded by two ellipses, corresponding to $\cos\psi = \pm 1$, though its difference is too small to be shown. Figures (a) and (b) correspond to the cases of $\lambda >> |\kappa|$ and $\lambda << |\kappa|$, respectively.

Fig. 10. The spherical shell distribution in the 2n-mechanism (Case S). The neutrino propagates the distance r from the first decaying neutron n_1 to the second one n_2, both of which are located on the surface of nucleus with radius R. The angle cut θ_c is due to the size of the neutron n_1.

Fig. 11. The m_ν and μ_0 dependences of the average potentials. The subscripts S and Δ indicate two different tupes of average described in Appendix C.

Fig. 12. The diagrams for the $(\beta\beta)_{2\nu}$ mode pointed out by Ejiri.[73)]

Fig. 13. The diagrams for the $(\beta\beta)_{2\nu}$ mode (figure (a)) and for the $(\beta\beta)_{0\nu}$ mode (figure (b)) which appear in the additional mechanism pointed out by Ohtubo and Hosoya.[74)]

$(\beta\beta)_{0\nu}$ mode					
interaction patterns	2n-mechanism		N^*-mechanism		Feynman diagrams
	$m_\nu \lesssim O(eV)$	$m_\nu \gtrsim O(GeV)$	$m_\nu \lesssim O(eV)$	$m_\nu \gtrsim O(GeV)$	
$m_\nu\ \tilde{K}^{\mu\nu}_{LL}$ $(0^+ \to 0^+)$	m_ν $\left\langle \frac{1}{r} \right\rangle$	m_ν $\left\langle \frac{e^{-m_\nu r}}{r} \right\rangle$	no contribution		Fig. 5a
$m_\nu\ \tilde{K}^{\mu\nu}_{RR}$ $(0^+ \to 0^+)$	$\lambda^2 m_\nu, \lambda\kappa m_\nu, \kappa^2 m_\nu$ $\left\langle \frac{1}{r} \right\rangle$	$\lambda^2 m_\nu, \lambda\kappa m_\nu, \kappa^2 m_\nu$ $\left\langle \frac{e^{-m_\nu r}}{r} \right\rangle$	no contribution		
$\tilde{L}^{\mu\nu 0}_{ab}$ $(0^+ \to 0^+, 1^+)$	λ, κ $(p_1^0 - p_2^0) \left\langle \frac{1}{r} \right\rangle$	λ, κ $(p_1^0 - p_2^0) \left\langle \frac{e^{-m_\nu r}}{r} \right\rangle$	κ $(p_1^0 - p_2^0) \left\langle \frac{1}{r} \right\rangle_\Delta$	κ $(p_1^0 - p_2^0) \left\langle \frac{e^{-m_\nu r}}{r} \right\rangle_\Delta$	Fig. 5b
$\tilde{L}^{\mu\nu k}_{ab}$ $(0^+ \to 0^+, 1^+, 2^+)$	λ, κ $\lvert \vec{p}_1 \pm \vec{p}_2 \rvert \left\langle \frac{1}{r} \right\rangle$	λ, κ $\lvert \vec{p}_1 \pm \vec{p}_2 \rvert \left\langle \frac{e^{-m_\nu r}}{r} \right\rangle$	λ, κ $\lvert \vec{p}_1 \pm \vec{p}_2 \rvert \left\langle \frac{1}{r} \right\rangle_\Delta$	λ, κ $\lvert \vec{p}_1 \pm \vec{p}_2 \rvert \left\langle \frac{e^{-m_\nu r}}{r} \right\rangle_\Delta$	

Table I

$(\beta\beta)_{2\nu}$ mode						
	$0^+ \to 0^+$ (1/yr)			$0^+ \to 2^+$ (1/yr)		
	T (m_e)	B_1	B_2	T (m_e)	B_3	B_4
^{48}Ca	8.359	$3.784 \cdot 10^{-20}$	$1.259 \cdot 10^{-23}$	6.435	$1.001 \cdot 10^{-24}$	$4.063 \cdot 10^{-25}$
^{76}Ge	4.003	$1.279 \cdot 10^{-22}$	$2.461 \cdot 10^{-27}$	2.909	$1.166 \cdot 10^{-28}$	$4.736 \cdot 10^{-29}$
^{82}Se	5.881	$4.283 \cdot 10^{-21}$	$3.759 \cdot 10^{-25}$	4.361	$2.094 \cdot 10^{-26}$	$8.504 \cdot 10^{-27}$
^{128}Te	1.700	$8.014 \cdot 10^{-25}$	$4.831 \cdot 10^{-31}$	0.834	$3.103 \cdot 10^{-34}$	$1.260 \cdot 10^{-34}$
^{130}Te	4.957	$4.538 \cdot 10^{-21}$	$2.060 \cdot 10^{-25}$	3.908	$2.654 \cdot 10^{-26}$	$1.077 \cdot 10^{-26}$
^{150}Nd	6.589	$1.123 \cdot 10^{-19}$	$1.569 \cdot 10^{-23}$	5.936	$1.036 \cdot 10^{-23}$	$4.206 \cdot 10^{-24}$

Table II

$(\beta\beta)_{0\nu}$ mode $0^+ \rightarrow 0^+$						
	^{48}Ca	^{76}Ge	^{82}Se	^{128}Te	^{130}Te	^{150}Nd
	$\cdot 10^{-13}$	$\cdot 10^{-15}$	$\cdot 10^{-14}$	$\cdot 10^{-16}$	$\cdot 10^{-14}$	$\cdot 10^{-13}$
G_{01}	0.2932	2.936	1.291	8.139	2.024	0.9763
G_{02}	2.028	5.666	4.934	3.495	5.958	4.822
G_{03}	15.73	52.90	40.08	55.71	48.99	36.01
G_{04}	3.038	17.63	10.17	30.11	14.08	8.386
G_{05}	0.1531	1.654	0.6942	5.344	1.110	0.5199
G_{06}	0.2260	1.664	0.9060	2.331	1.373	0.7742
G_{07}	1.399	13.41	6.068	34.89	9.470	4.680
G_{08}	0.1350	2.234	0.7700	9.429	1.361	0.5448
G_{09}	0.02607	0.7442	0.1954	5.097	0.3913	0.1269
	$\cdot 10^{-10}$	$\cdot 10^{-11}$	$\cdot 10^{-10}$	$\cdot 10^{-12}$	$\cdot 10^{-10}$	$\cdot 10^{-9}$
G_1^{N*}	1.198	0.5089	0.4249	0.6178	0.6940	0.5774
G_2^{N*}	4.132	1.859	1.503	2.496	2.499	2.039
G_3^{N*}	0.2842	0.1092	0.09758	0.1005	0.1565	0.1360
G_4^{N*}	0.5910	0.2436	0.2125	0.2474	0.3564	0.3073
G_5^{N*}	2.660	1.096	0.9563	1.113	1.604	1.383

Table III (a)

$(\beta\beta)_{0\nu}$ mode $0^+ \rightarrow 2^+$						
	^{48}Ca	^{76}Ge	^{82}Se	^{128}Te	^{130}Te	^{150}Nd
	$\cdot 10^{-13}$	$\cdot 10^{-14}$	$\cdot 10^{-13}$	$\cdot 10^{-16}$	$\cdot 10^{-13}$	$\cdot 10^{-12}$
G_{2+}	0.6938	0.2234	0.1658	0.8587	0.2992	0.4072
G_{2-}	0.6059	0.1531	0.1314	0.2762	0.2286	0.3486
	$\cdot 10^{-11}$	$\cdot 10^{-12}$	$\cdot 10^{-11}$	$\cdot 10^{-13}$	$\cdot 10^{-10}$	$\cdot 10^{-9}$
$G_1^{N^*}$	2.735	1.109	0.9020	0.5215	0.2180	0.3377
$G_2^{N^*}$	9.575	4.189	3.270	2.279	0.8022	1.201
$G_3^{N^*}$	0.6312	0.2192	0.1971	0.05689	0.04690	0.7848
$G_4^{N^*}$	1.329	0.5019	0.4390	0.1444	0.1089	0.1787
$G_5^{N^*}$	5.908	2.258	1.975	0.6497	0.4898	0.8040

Table III (b)

$(\beta\beta)_{0\nu}$ mode						
	$0^+ \to 0^+$					
	C_1	C_2	C_3	C_4	C_5	C_6
^{48}Ca	$2.633\cdot10^{-15}$	$1.088\cdot10^{-15}$	$1.254\cdot10^{-14}$	$1.055\cdot10^{-13}$	$3.640\cdot10^{-13}$	$2.857\cdot10^{-13}$
^{76}Ge	$2.885\cdot10^{-16}$	$7.627\cdot10^{-17}$	$3.304\cdot10^{-16}$	$4.483\cdot10^{-15}$	$1.638\cdot10^{-14}$	$1.191\cdot10^{-14}$
^{82}Se	$1.269\cdot10^{-15}$	$4.674\cdot10^{-16}$	$3.165\cdot10^{-15}$	$3.743\cdot10^{-14}$	$1.324\cdot10^{-13}$	$1.041\cdot10^{-13}$
^{128}Te	$5.232\cdot10^{-17}$	$2.350\cdot10^{-18}$	$8.731\cdot10^{-16}$	$5.225\cdot10^{-16}$	$2.111\cdot10^{-15}$	$1.110\cdot10^{-15}$
^{130}Te	$1.206\cdot10^{-15}$	$3.727\cdot10^{-16}$	$2.120\cdot10^{-15}$	$5.870\cdot10^{-14}$	$2.114\cdot10^{-13}$	$1.580\cdot10^{-13}$
^{150}Nd	$9.314\cdot10^{-15}$	$4.054\cdot10^{-15}$	$3.030\cdot10^{-14}$	$5.087\cdot10^{-13}$	$1.796\cdot10^{-12}$	$1.502\cdot10^{-12}$

Table IV

mechanisms	$(\beta\beta)_{2\nu}$ mode			$(\beta\beta)_{0\nu}$ mode	
	$0^+ \to 0^+$	$0^+ \to 1^+$	$0^+ \to 2^+$	$0^+ \to 0^+$	
				$m_\nu \neq 0$	$m_\nu = 0$
2n	$1-\cos\theta$			$1-\cos\theta$	$1+\cos\theta$
N^*	$1 + \frac{1}{3}\cos\theta$			$(1+\cos\theta)(1-\frac{1}{3}\cos\theta)$ $(\lambda >> \lvert\kappa\rvert)$ $(1+\cos\theta)(1+\frac{1}{5}\cos\theta)$ $(\lvert\kappa\rvert >> \lambda)$	

Table V

	$(T_{1/2})^{Th.}$ (yr)					$(T_{1/2})^{Exp.}$ (yr)	(Ref.)
	$(\beta\beta)_{2\nu}$ mode		$(\beta\beta)_{0\nu}$ mode				
	$0^+ \to 0^+$	$0^+ \to 2^+$	$0^+ \to 0^+$		$0^+ \to 2^+$		
			$\lambda = \kappa = 0,\ m_\nu \neq 0$	$\lambda,\ \kappa \neq 0,\ m_\nu = 0.$	$\lambda,\ \kappa \neq 0$		
^{48}Ca	$3.5 \cdot 10^{20}$	$3.0 \cdot 10^{24}$	$8.1 \cdot 10^{23}$	$8.3 \cdot 10^{21}$ (a) $6.8 \cdot 10^{21}$ (b)	$3.3 \cdot 10^{22}$ (a) $2.5 \cdot 10^{22}$ (b)	$>2 \cdot 10^{21}$ $(0\nu,\ 0^+ \to 0^+)$ $>3.6 \cdot 10^{19}$ (2ν)	(23) (23)
^{76}Ge	$7.8 \cdot 10^{21}$	$6.5 \cdot 10^{27}$	$8.8 \cdot 10^{23}$	$1.8 \cdot 10^{23}$ (a) $1.6 \cdot 10^{22}$ (b)	$7.9 \cdot 10^{23}$ (a) $6.4 \cdot 10^{23}$ (b)	$>5 \cdot 10^{21}$ $(0\nu,\ 0^+ \to 0^+)$ $>3 \cdot 10^{21}$ $(0\nu,\ 0^+ \to 2^+)$	(60) (61)
^{82}Se	$2.7 \cdot 10^{20}$	$6.2 \cdot 10^{25}$	$2.4 \cdot 10^{23}$	$2.3 \cdot 10^{22}$ (a) $1.8 \cdot 10^{22}$ (b)	$1.0 \cdot 10^{23}$ (a) $7.8 \cdot 10^{22}$ (b)	$>3.1 \cdot 10^{21}$ $(0\nu,\ 0^+ \to 0^+)$ $(1.0 \pm 0.4) \cdot 10^{19}$ (2ν) $(1.4 \pm 0.3) \cdot 10^{20}$ $(2.76 \pm 0.88) \cdot 10^{20}$	(62) (24) (54) (55)
^{128}Te	$2.1 \cdot 10^{24}$	$3.8 \cdot 10^{35}$	$1.5 \cdot 10^{24}$	$1.5 \cdot 10^{24}$ (a) $1.4 \cdot 10^{24}$ (b)	$1.6 \cdot 10^{25}$ (a) $1.6 \cdot 10^{25}$ (b)	$T_{1/2}(^{128}Te)/T_{1/2}(^{130}Te)$ $= (1.59 \pm 0.05) \cdot 10^3$	(26)
^{130}Te	$5.5 \cdot 10^{20}$	$6.5 \cdot 10^{27}$	$8.5 \cdot 10^{22}$	$1.4 \cdot 10^{22}$ (a) $1.2 \cdot 10^{22}$ (b)	$4.4 \cdot 10^{22}$ (a) $3.5 \cdot 10^{22}$ (b)	$1.4 \cdot 10^{21}$ $(8.20 \pm 0.68) \cdot 10^{20}$ $(2.2 \pm 0.7) \cdot 10^{21}$ $(2.83 \pm 0.30) \cdot 10^{21}$	(56) (57) (58) (59)
^{150}Nd	$8.9 \cdot 10^{18}$	$6.8 \cdot 10^{22}$	$2.7 \cdot 10^{22}$	$1.7 \cdot 10^{21}$ (a) $1.4 \cdot 10^{21}$ (b)	$2.8 \cdot 10^{21}$ (a) $2.1 \cdot 10^{21}$ (b)	$>2 \cdot 10^{18}$ $(0\nu,\ 0^+ \to 2^+)$	(63)

Table VI

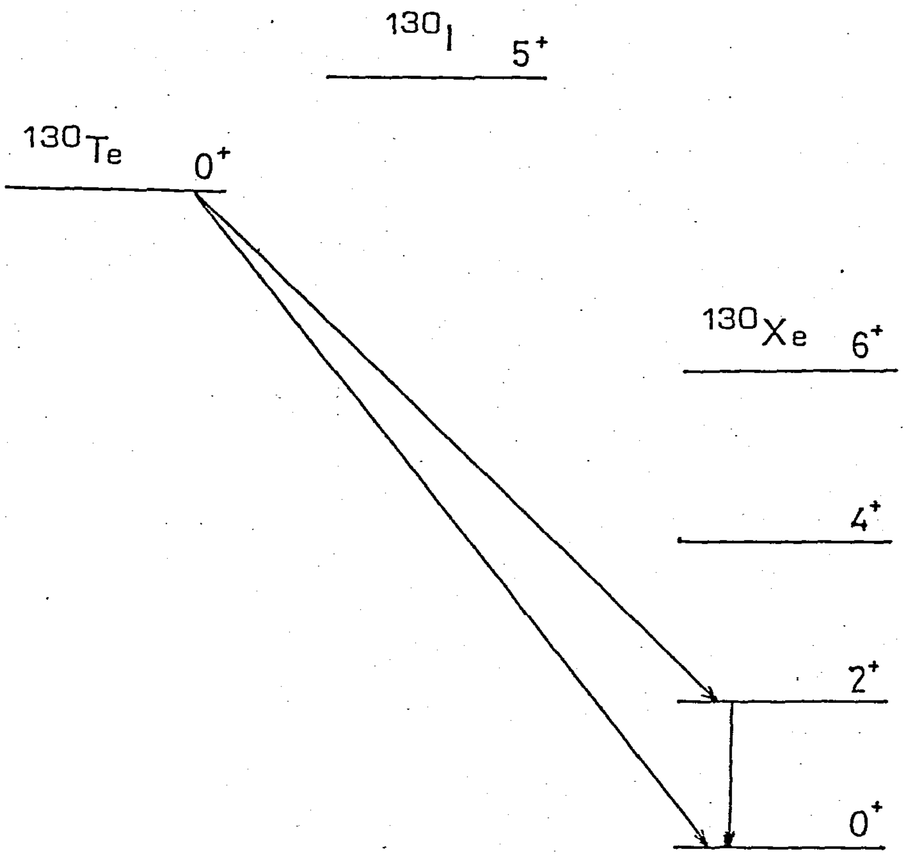


Fig. 1a

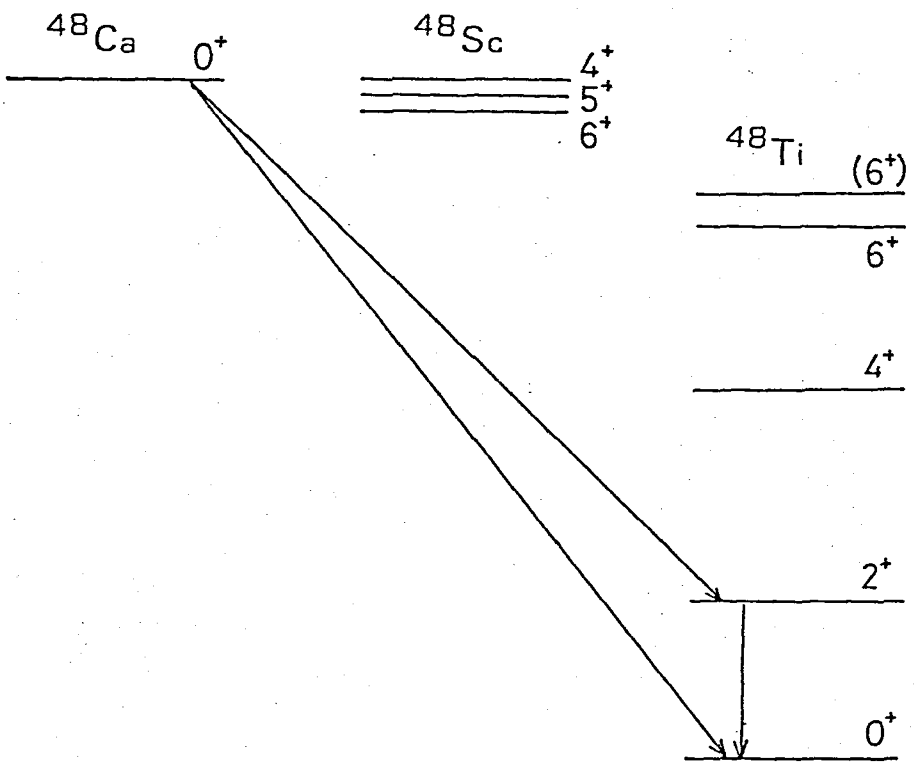
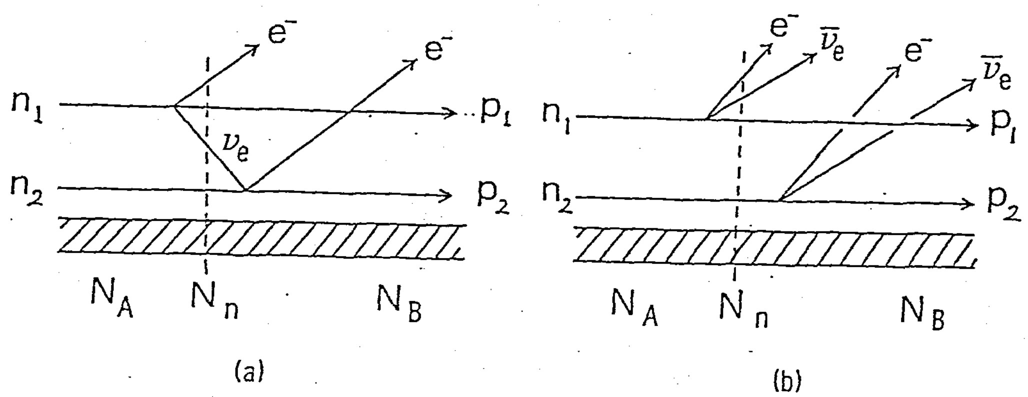
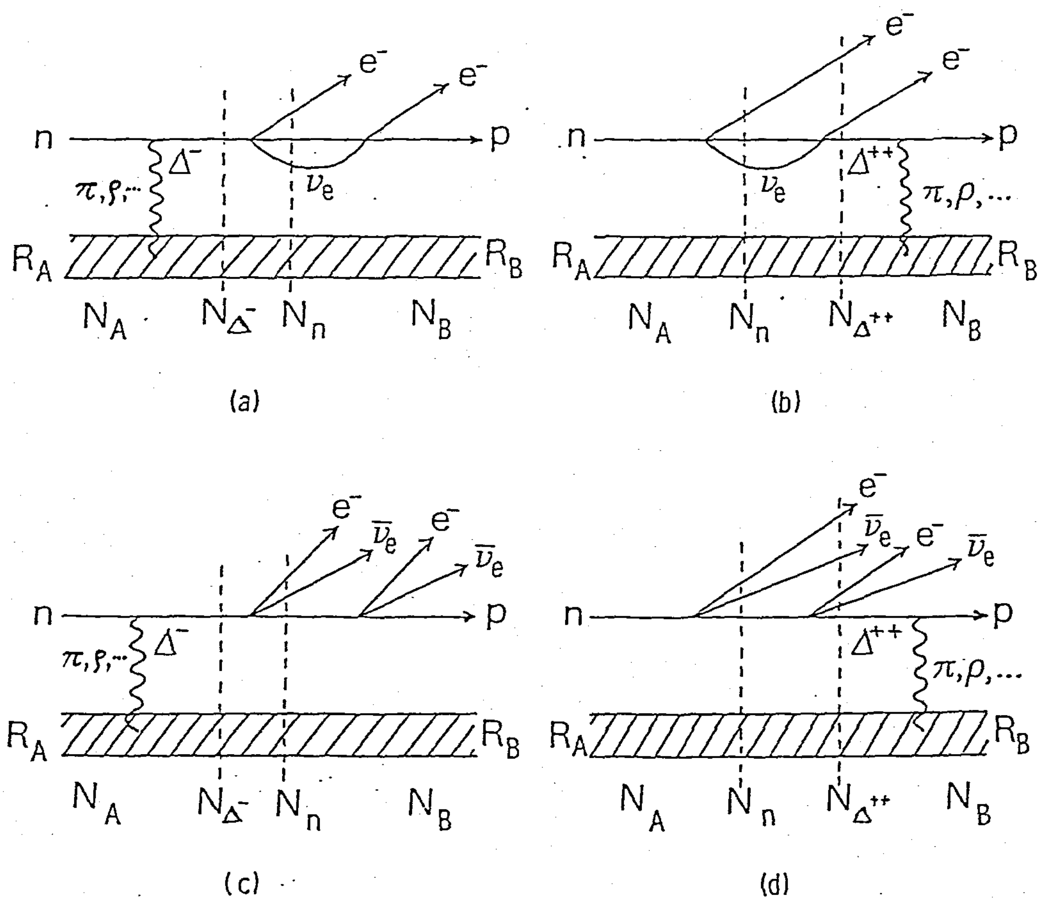


Fig. 1b



e⁻
ν̄e
n1
n2
p1
p2
νe
NA
Nn
NB
(a)
(b)

Fig. 2



e⁻
ν̄e
n
p
Δ⁻
Δ⁺⁺
π, ρ, ...
νe
RA
RB
NA
NΔ⁻
Nn
NΔ⁺⁺
NB
(a)
(b)
(c)
(d)

Fig. 3

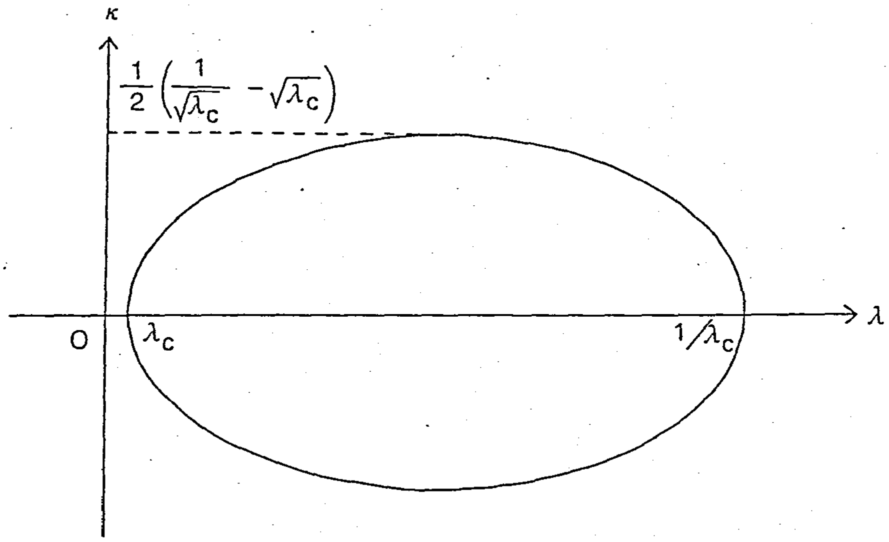

κ
$\frac{1}{2}\left(\frac{1}{\sqrt{\lambda_c}} - \sqrt{\lambda_c}\right)$
O
λ_c
$1/\lambda_c$
λ

Fig.4

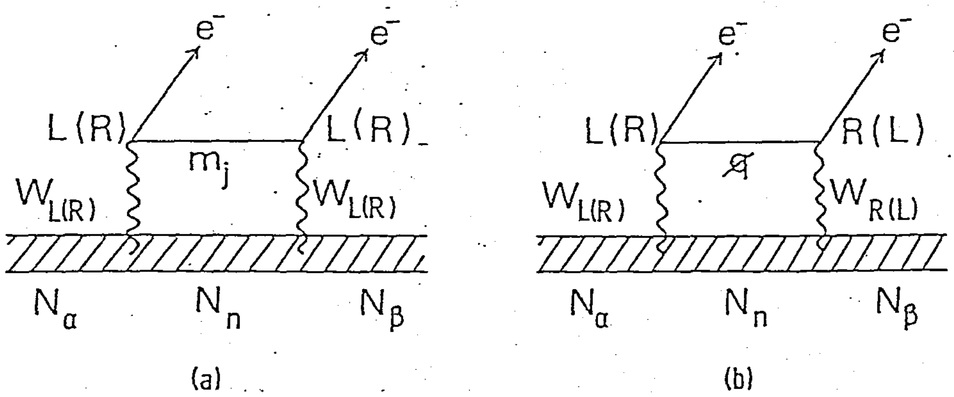

e⁻
e⁻
L(R)
L(R)
m_j
$W_{L(R)}$
$W_{L(R)}$
N_α
N_n
N_β
(a)
e⁻
e⁻
L(R)
R(L)
$\not{q}$
$W_{L(R)}$
$W_{R(L)}$
N_α
N_n
N_β
(b)

Fig.5

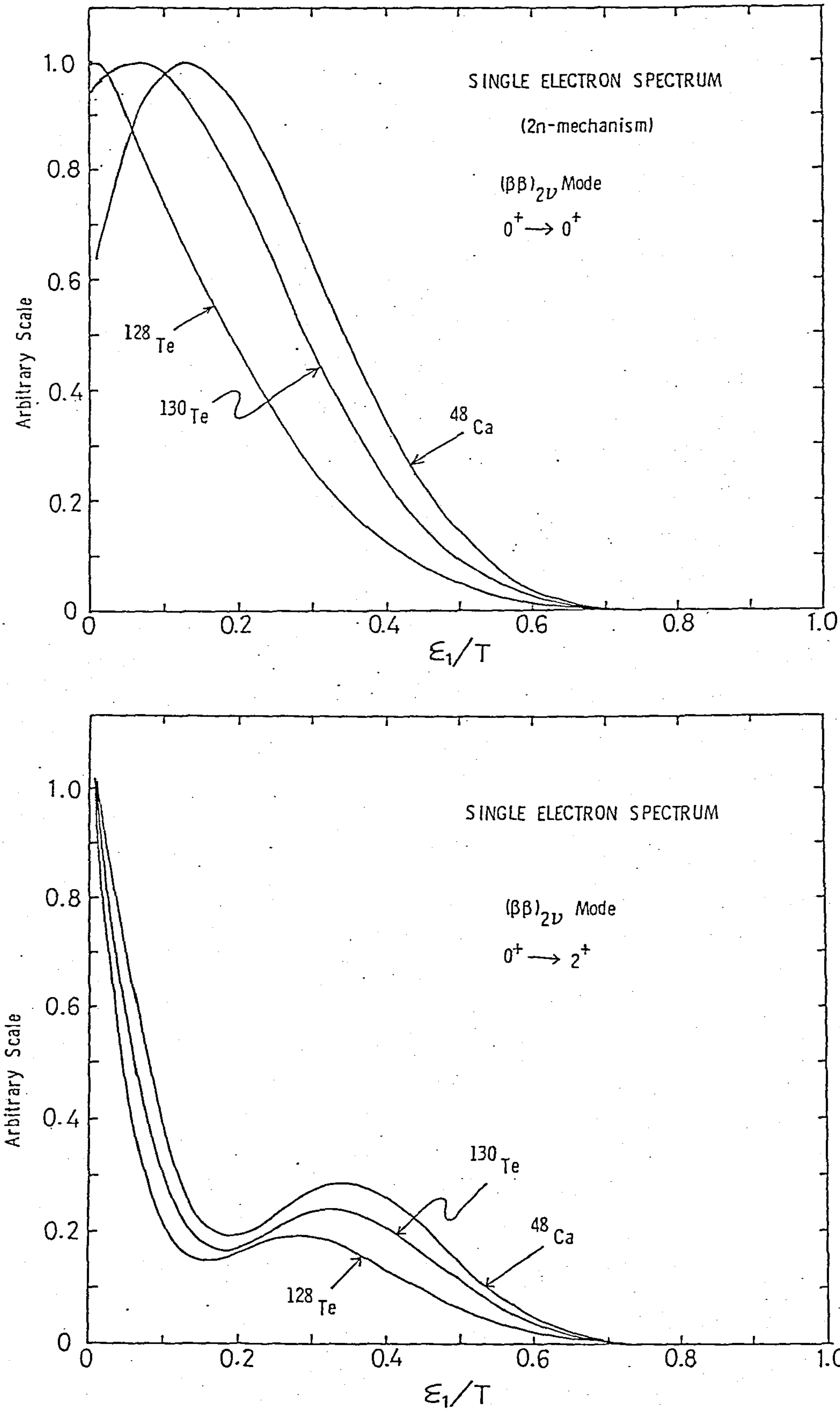


Fig.6a

Fig.6b

Fig.6

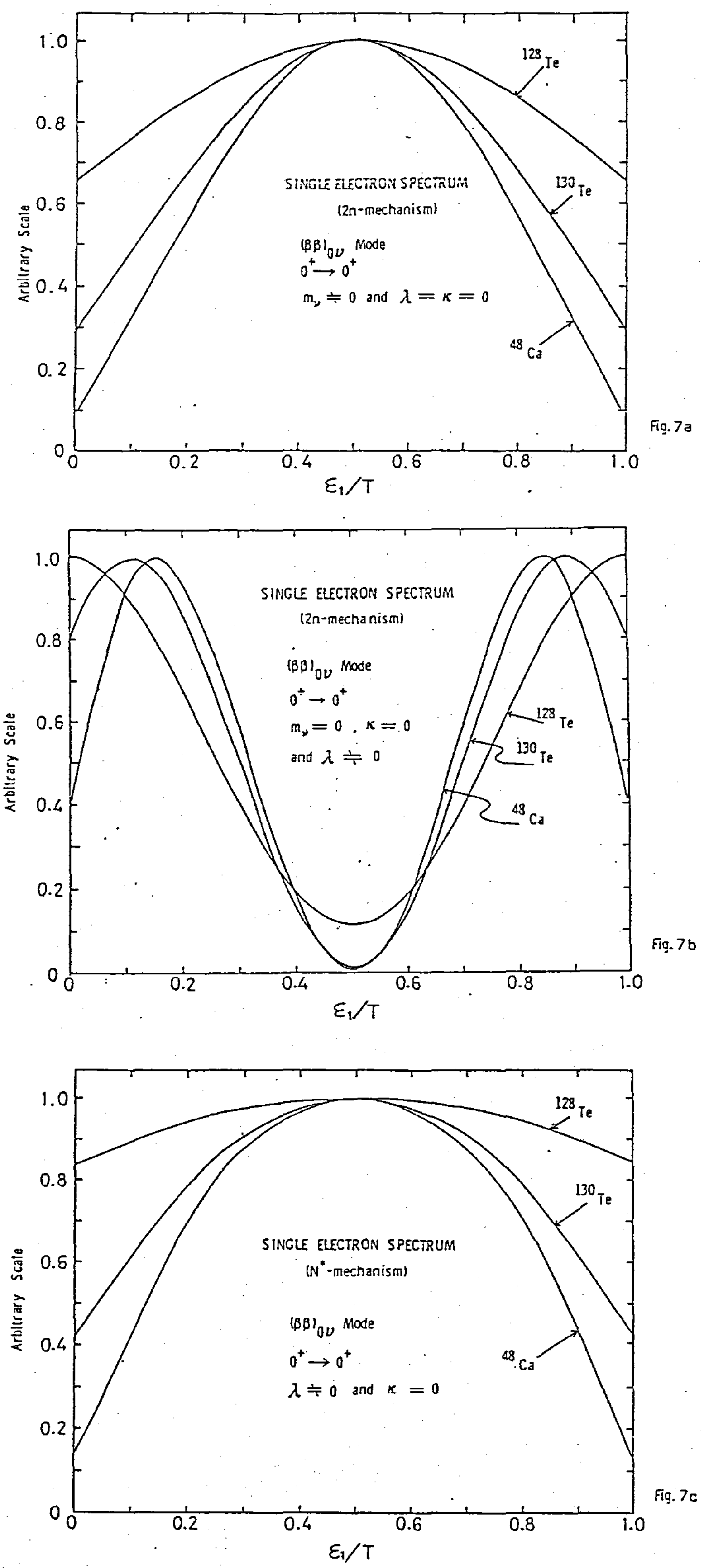

SINGLE ELECTRON SPECTRUM
(2n-mechanism)
(ββ)0ν Mode
0+ → 0+
mν ≑ 0 and λ = κ = 0
128Te
130Te
48Ca
Arbitrary Scale
ε1/T
Fig.7a
SINGLE ELECTRON SPECTRUM
(2n-mechanism)
(ββ)0ν Mode
0+ → 0+
mν = 0 , κ = 0
and λ ≑ 0
128Te
130Te
48Ca
Arbitrary Scale
ε1/T
Fig.7b
SINGLE ELECTRON SPECTRUM
(N*-mechanism)
(ββ)0ν Mode
0+ → 0+
λ ≑ 0 and κ = 0
128Te
130Te
48Ca
Arbitrary Scale
ε1/T
Fig.7c

Fig.7

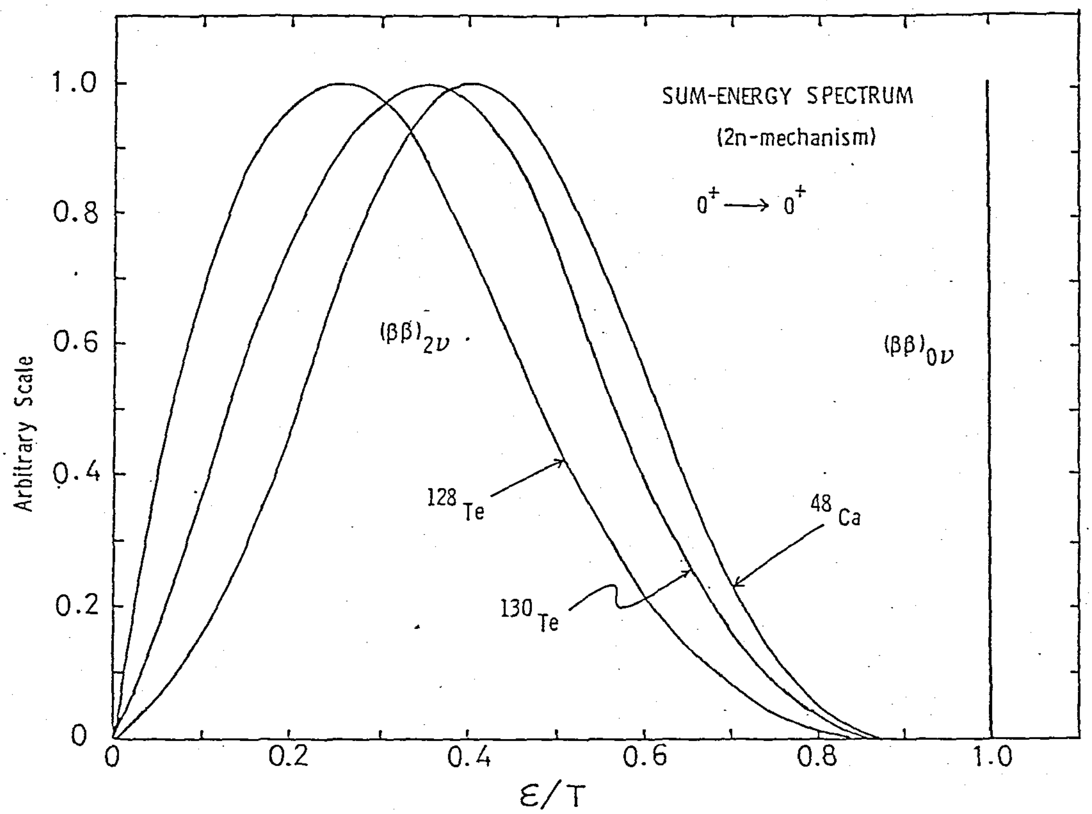


Fig. 8a

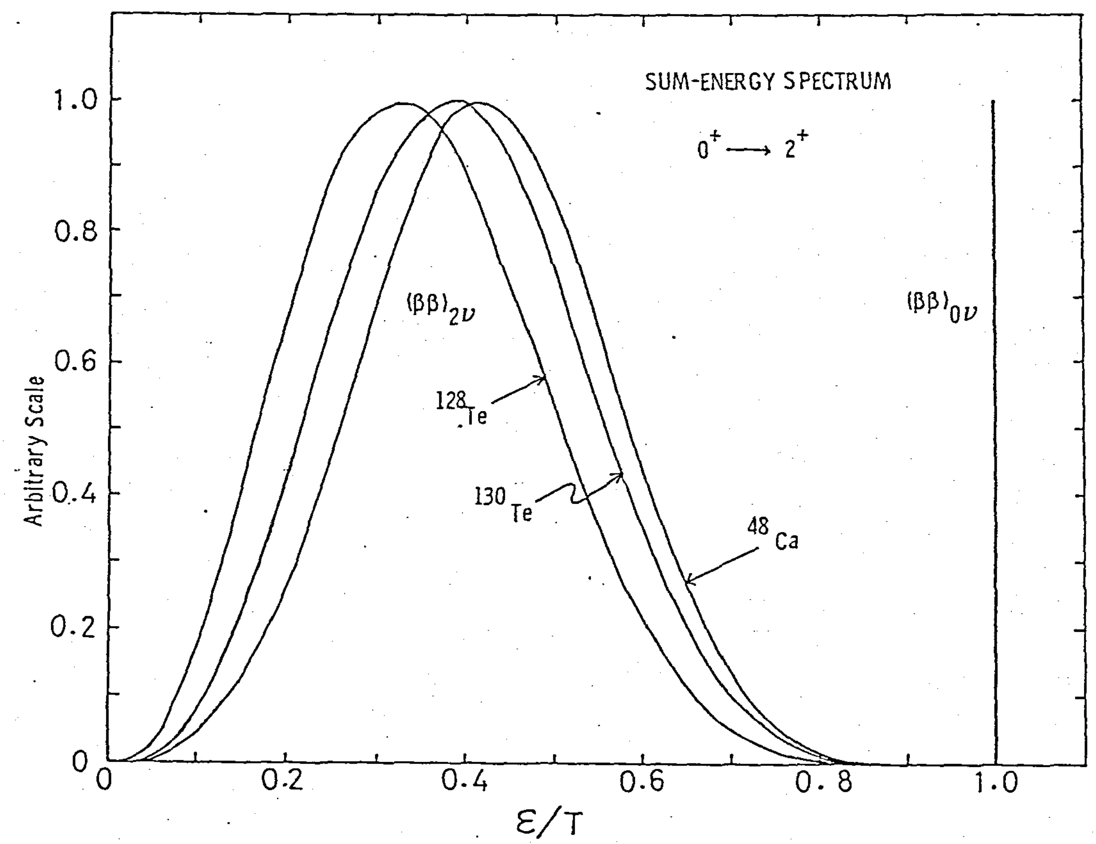


Fig. 8b

Fig. 8

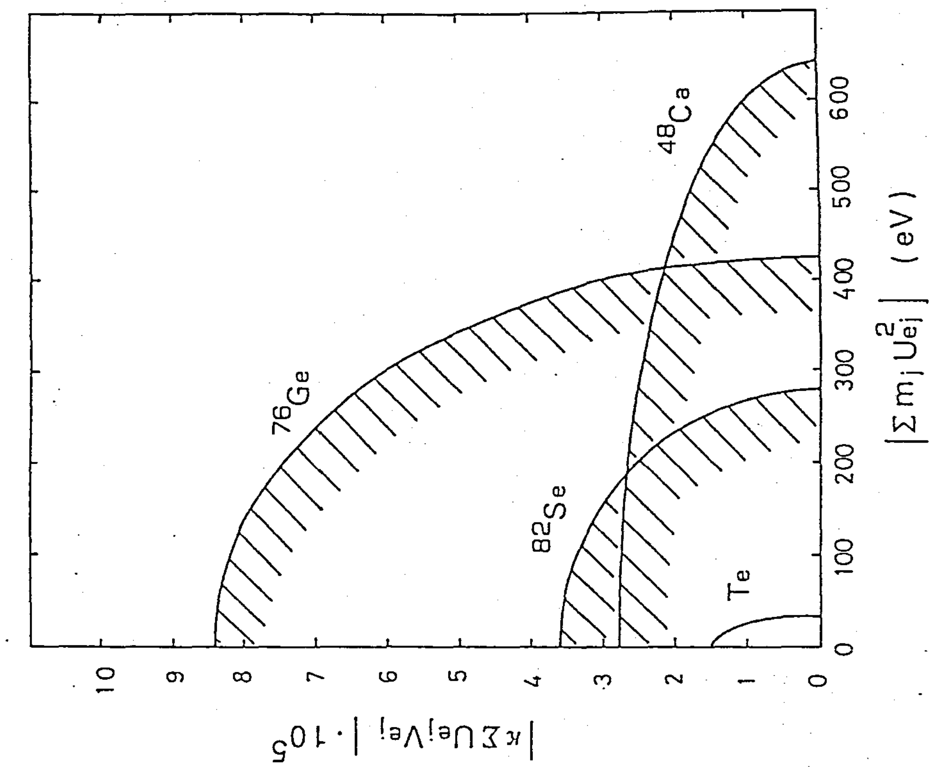


Fig.9b

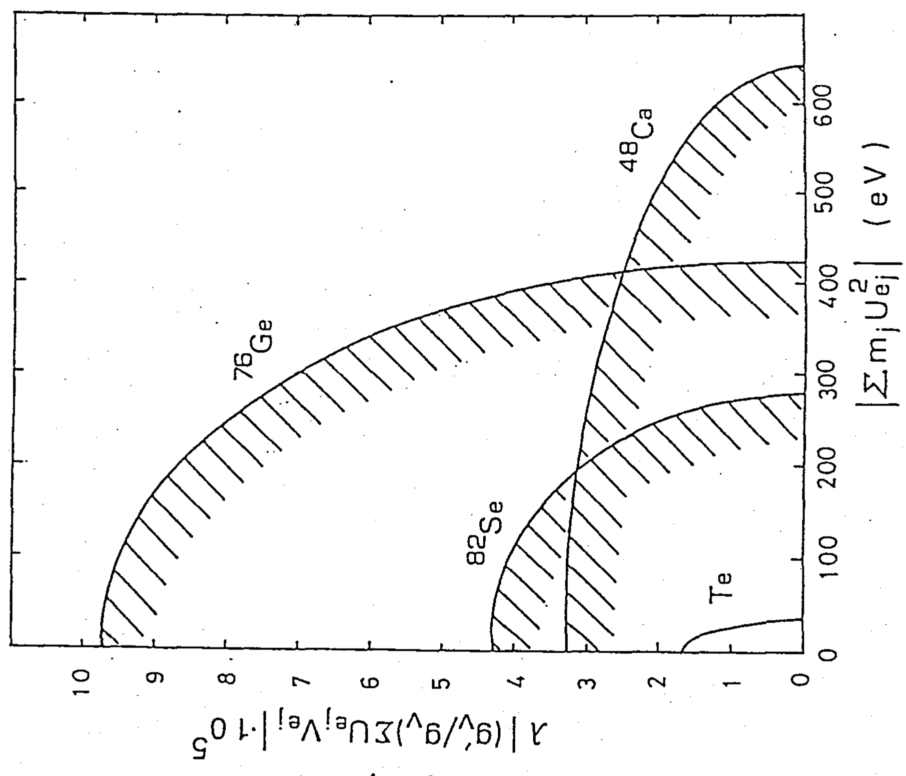


Fig.9a

Fig.9

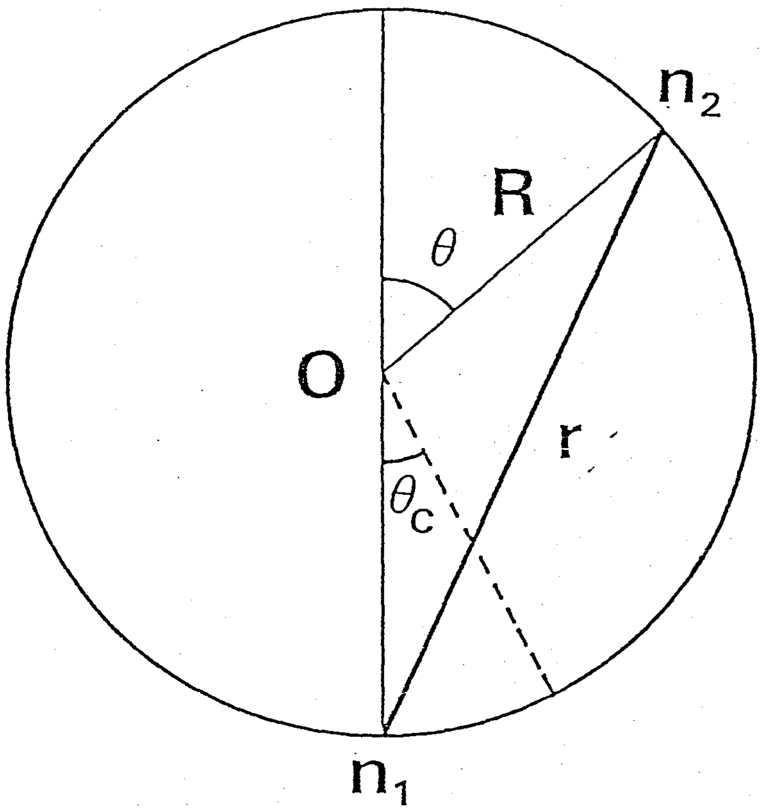


Fig.10

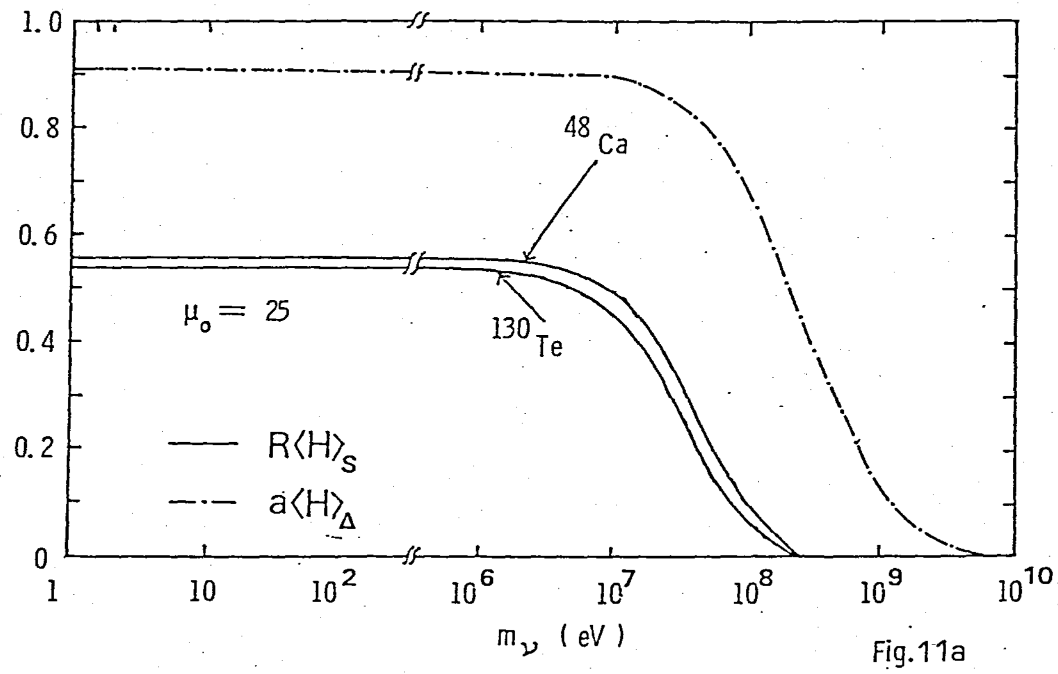


Fig.11a

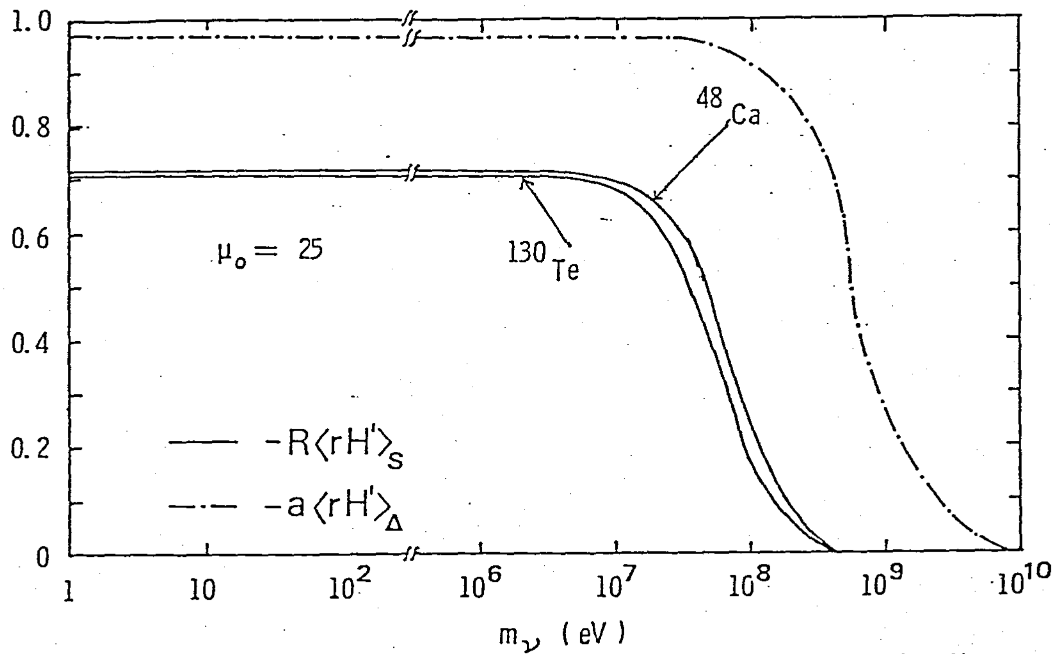


Fig.11b

Figs.11a and 11b

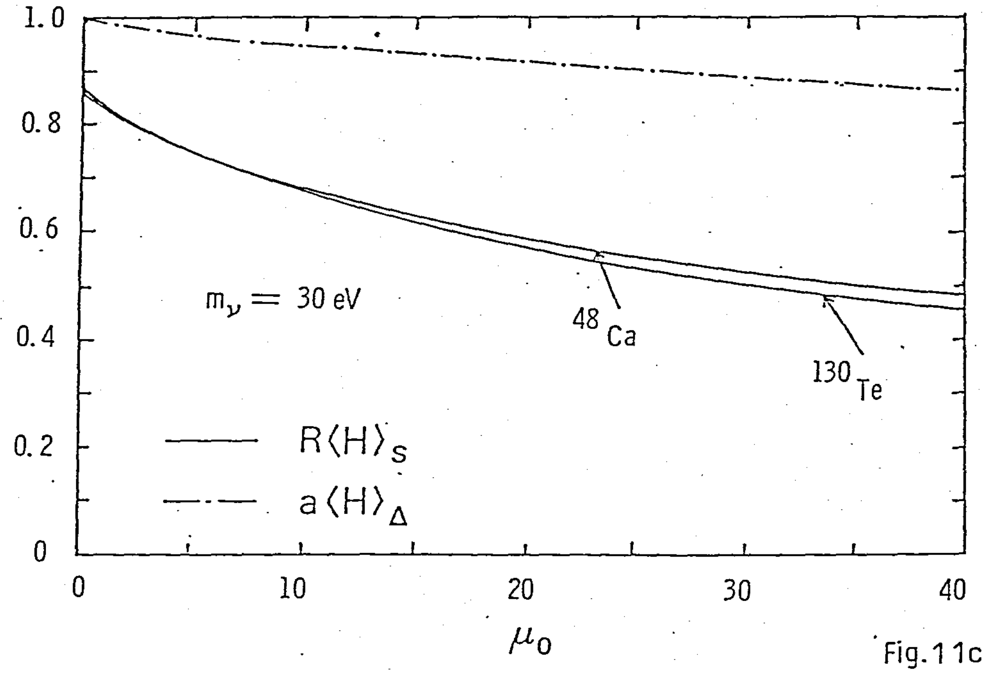


Fig.11c

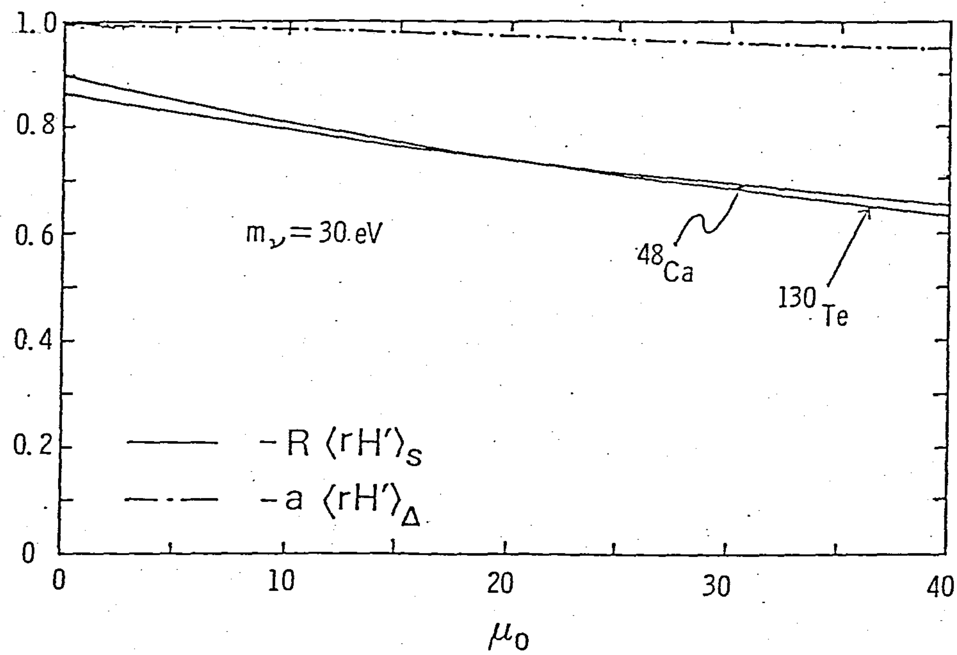
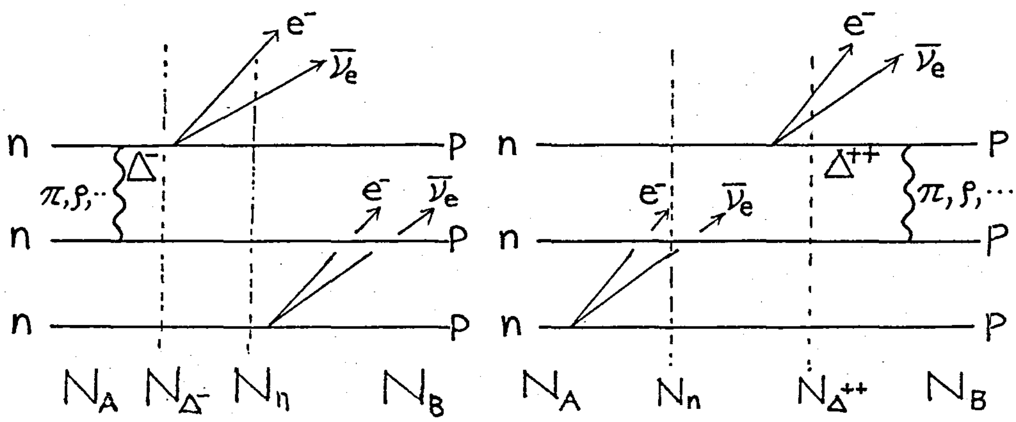


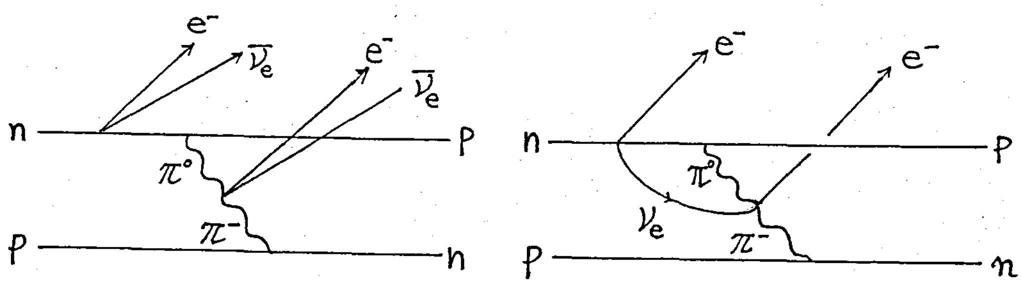
Fig. 11d

Figs.11c and 11d



(a) (b)

Fig. 12



(a) (b)

Fig. 13