MESON EXCHANGE CURRENT

\- Meson Theoretical Derivation of Exchange Currents and Their Effects on the Magnetic Moment of Deuteron -

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Abstract:

The meson-theoretical derivation of exchange current and the effects of exchange current on the magnetic moment of deuteron are studied in part I and part II, respectively. In part I, we investigate the methods to derive exchange current from meson theory in order to solve the recent confusions in the derivation of exchange current. From the studies of non-static exchange current and nuclear potential, we clarify the reason why different methods give different static two-boson-exchange currents, while they give the same static two-boson-exchnage potentials. Next, non-uniqueness of the exchange current and nuclear potential is studied. We show that it is reduced to the arbitrariness of unitary transformation within meson-vacuum space.

In part II, the effects of static two-boson-exchange current, non--static one-boson-exchange current and the relativistic correction on the magnetic moment of deuteron are studied, where the static one-boson-exchange current does not contribute to it. The exchange currents are derived from the unitary transformation method by taking into account pion, rho and omega exchanges, and isobar intermediate state. We find the important contributions of two-boson-exchange current as well as those of one-boson-exchange current and relativistic correction, and the discrepancy between the experimental and impulse value of magnetic moment is well explained by these effects.

Contents

Part II Magnetic Moment of Deuteron

Section 1. Introduction

A nucleus is an essemblage of nucleons, isobars and mesons interacting with each other. It is, however, assumed that the nucleus consists of only nucleons interacting through nuclear force. This is justified as far as we are concerned with the nuclear properties below the threshold energy of pion production. In other words, we can eliminate degrees of freedom of nuclear constituents except for nucleons from nuclear state vector. This elimination induces the nuclear force (or nuclear potential) in the Hamiltonian. At the same time the interaction of nucleons with the external electromagnetic field is modified, i. e., we have the so-called exchange current(many-body current).

The history of the exchange current is as long as the one of the nuclear force. (See the papers by Futami et al.[1)] and by Chemtob[2)] for historical review of exchange current problem). The concept of exchange current was at first introduced by Siegert[3)] in 1937. He pointed out that if the nuclear force involves the exchange force of Majorana type, the exchange current should exist so that the current conservation law can be preserved. Generally, the exchange current can be divided into two parts. The first part is described in terms of nuclear force.[4)] This is known as the Siegert theorem, which was first applied to the sum rule of the electric dipole transition by Levinger and Bethe[5)] and also to the orbital part the magnetic moment (the Sachs moment[6)]). The second part, however, depends on the detail of the strong interaction, which cannot be expressed by the nuclear force alone, and it gives rise to effects on the intrinsic magnetic moment of nucleons.

Similar obsevation can be done for the weak interaction process in nuclei. For example, the Gamow-Teller coupling constant in the beta decay of the free netron is modified in the nucleus.

In the early time, the exchange current was studied only in the few-body systems, but not in the many-body systems, because the nuclear theory at that time was too naive to discuss the effects of exchange current by comparing the calculated results and the experimental data.

Recently owing to the progress on the nuclear theory, it becomes possible to study the effects of exchange current quantitatively. Also the exchange current operators can be obtained rather reliably owing to the development of the high-energy physics. For example, the magnetic moment and Gamow-Teller matrix element have been studied by taking into account the effects of core polarization and exchange current.[7,8]

One of the clear evidences of the exchange current is seen in the radiative capture of thermal neutron in hydrogen. The experimental value of the total cross section for the magnetic dipole transition is about 10% larger than the calculated value by the use of the so-called realistic two-nucleon wave function in the impulse approximation. This anomaly is nicely explained by the one-pion-exchange current.[9] Another example is the static magnetic moment of deuteron.[10 15] In the impulse approximation, the magnetic moment is expressed in terms of the iso-scalar part of the nucleon magnetic moment and also the probability of the d-wave component in the deuteron. The realistic deuteron wave functions predict the magnetic moment about 1.4% smaller than the experimental data. Although this anomaly seems very small when compared with the formar example, it is very difficult to explain the anomaly theoretically. The main reason is that the static one-boson-exchange (OBE) current does not contribute to the magnetic moment of deuteron. Therefore, as the effects of exchange current on the isoscalar part of magnetic moment, we must study higher order processes such as non-static correction and two-boson-exchanges (TBE). Similar example is the nuclear charge density operators.[14, 16 18] From the above example it is clear that study of two-boson-exchange processes is unavoidable if we want to understand the magnetic moment of isoscalar type or nuclear charge distribution in detail. There appeared some works in this direction.[14,16 - 18] Unfortunately, there are some confusions in defining the exchange current. In particular different approaches give different results for the two-boson-exchange currents, while they give the same results for the nuclear forces. To solve this discrepancy in the derivation of TBE current we must reinvestigate how to derive the exchange current in the meson theory. This is very important in the exchange current problem in itself, and also in the meson-nucleus physics as the first step beyond the phenomenology.

In part I, we shall study how to define and how to derive exchange current and nuclear potential in the meson theory. At first we shall briefly review methods to obtain exchange current. There are generally two approaches to define nuclear potential and exchange current. In the first approach, we divide the Hibert space for the state vector into the boson-vacuum space and boson-existing space and eliminate boson-existing space using the projection operators onto these sub-spaces. Then exchange current and nuclear potential are defined as the effective operators in the boson-vacuum space. Fukuda, Sawada and Taketani (FST),[19] and independently Okubo[20] proposed a method to eliminate boson-existing space by the unitary transformation which retain the orthogonality ans normalization conditions for the state-vector. Nishijima[21] took the canonical transformation for this purpose. In what follows, we we shall call this method as unitary transformation method, which will be reviewed in detail in sect. 2. We shal give the perturbative solution of effective operator following Hyuga and Ohtsubo,[17] and derive nuclear potential and exchange current up to the TBE processes.

The next approach is to derive exchange current and nuclear potential from the S-matrix element calculated from the meson theory. In the potential theory, the interaction must be instantaneous and hermitian, while two nucleons interact with exchanged boson at different times in the field theory. Nevertheless, the instantaneous potential must be derived so as to reproduce the same S-matrix element as in the field theory. From this point of view, Nambu[22] proposed a method to derive nuclear potential by the study of time development of system. Following Nambu's method Taketani et al.[23] derived static two-pion-exchange potential. Miyazawa[24] proposed an alternative method to derive static nuclear potential from the static S-matrix element. The static TBE potential is derived from the static fourth order S-matrix element by subtracting iterated term of static OBE potential. This method has an advantage that we can derive potential from the experimental boson-nucleon scattering amplitude and is very simple, because we need not to calculate lots of time ordered diagrams in the unitary transformation method. We shall call this method as the S-matrix method. In sect. 3, we shall review the S-matrix method of Miyazawa and apply it to the derivations of nuclear potential and exchange current.

One of the problems in these methods is that the unitary transformation method[17] and S-matrix method[15] give the different TBE currents from each other, while they are known to give the same static TBE potentials. Here, it is noticed that in the unitary transformation method Nishijima obtained the same TBE potential as Taketani et al.. The relation between the unitary transformation and S-matrix method have not been clearly discussed yet. The reason, why these two methods give the same static TBE potential but the different TBE current, is not clear. The key point to solve this problem is to understand the meaning of the static limit. It is not so obvious that the static nuclear potential and exchange current are related directly to the static S-matrix element. In sect. 4, we shall solve the discrepancy of the static TBE current by calculating the non-static exchange current and pot ntial in the unitary transformation method. As a result we shall find that the recoil effects of nucleons will play the essential role.

The another problem is non-uniqueness of the nuclear potential and exchange current. In sect. 5, we shall see that due to the different choice of the time-base in Nambu's method, in other words, due to the different forms of canonical transformation we obtain the different non-static OBE potentials. We shall show that the non-uniquenesses of the effective operators are due to the arbitrarinesses of unitary transformation which do not couple the boson-existing and boson-vacuum spaces. We shall show the consistency of our treatment of exchange current and nuclear potential by studying the conservation law of the electromagnetic current in sect. 6. The results in part I are summarized in sect. 7.

In part II, we shall study the magnetic moment of deuteron. In the isoscalar magnetic moment, the correction to the impulse value is expected to be small, since the static OBE current, which is the main contribution in the isovector magnetic moment, does not contribute to it. Therefore, we expect that the important correction to magnetic moment come from the non-static OBE current and static TBE current. From this point of view it is very interesting to study the magnetic moment of deuteron.

The magnetic moment of deuteron has been studied by taking into account exchange current contributions[10-15] and negative energy component[25] of the deuteron wave function, or nucleon resonance configuration[26]. However, we shall notice that there still remain problems in the previous works especially in the treatment of exchange current.

Within the conventional nuclear physics,[27] two-body magnetic moment operators have been studied by retaining velocity-dependent part of phenomenological nuclear potential. It is, however, dangerous unless a minimal substitution of electromagnetic field is performed at the fundamental level. Horikawa et al.[13] studied the pair current in one-boson-exchange model, which reproduce the experimental data on the two-nucleon system. They also studied the dissociation current($\rho\pi\gamma$), which was at first studied by Adler and Drell[12]. Gari and Hyuga[14] studied electromagnetic form factor for the deuteron by taking into account the pion, rho and omega-exchange pair current, and dissociation current derived by the unitary transformation method. They obtained desired value as correction to the magnetic moment of deuteron in the impulse approximation.

In these studies of exchange current in OBE model, relativistic correction to one-body-operator has not been taken into account. This relativistic correction is the same order of magnitude as the OBE current contribution, and is subtractive. Then it becomes necessary to study the other effects such as TBE current. The two-pion-exchange magnetic moment operators were studied by Sato and Itabashi[10] in the unitary transformation method, and Hatano and Kaneno[11] in the canonical transformation method. Although the model which they adopted for the strong interaction is by no means realistic, the definition of exchange current is valid.

Recently Jaus[15] have studied OBE and TBE current and relativistic correction, where the TBE current is derived by the S-matrix method. This is the only work that took into account the exchange current contribution up to TBE processes and also relativistic correction. It is noticed that he obtained the incorrect TBE current by adopting the S-matrix method.

Now it is necessary to calculate the magnetic moment of deuteron by the consistent derivation of exchange current. We shall derive relativistic correction to the one-body-current, and OBE and TBE current by the unitary transformation method. In the pion pair current, there are additional momentum-dependent currents which are neglected in the work of Gari and Hyuga. Although these current depend on the arbitrariness in the unitary transformation, there is no reason to neglect them. We shall calculate contributions of these currents. Further, we also calculate the non-static part of OBE current due to nucleon recoil, which have not been studied yet. We take into account pion, rho- and omega-exchanges. In the TBE current static limit is adopted, and nucleon and isobar (Δ_{1236}) intermediate states are included.

In sect. 8, we obtain the magnetic moment of deuteron in the impulse approximation with the Reid wave function. In order to calculate the exchange current operator, we shall specify the interaction Hamiltonian in sect. 9. The explicit forms of relativistic correction and exchange current operators are given in sect. 11 and 12, respectively. Our results and discussions will be given in sect. 14.

Part I. Consistent Definition of Nuclear Potential and Exchange Current

Section 2. Method of Unitary Transformation

We shall review in detail how to eliminate the mesonic degrees of freedom from the nuclear state-vector and to define the nuclear potential and effective operator, following the works by FST[19] and Okubo[20].

2.1. Elimination of Mesonic Degrees of Freedom

We shall start with the eigenvalue problem for the system with bosons and nucleons:

$$H \Psi = E \Psi, \tag{2.1}$$

with

$$H = H_0 + H_I, \tag{2.2}$$

where H_0 is the free Hamiltonian of bosons and nucleons, and H_I is the interaction Hamiltonian among these particles. E is an eigenvalue of the total Hamiltonian H, and Ψ is the corresponding eigen-state vector of the system.

To rewrite Eq. (2.1) by eliminating the mesonic degrees of freedom, we introduce projection operators η and Λ. The operator η projects any state Ψ onto the subspace with only nucleons, Ψ_0, and the operator Λ, onto the boson-existing subspace, Ψ_1 i.e.,

$$\Psi = \Psi_0 + \Psi_1, \tag{2.3}$$

$$\Phi_0 = \eta \Psi \;, \quad \Phi_n = \Lambda \Psi \quad \text{and} \quad \Lambda + \eta = 1. \tag{2.4}$$

From Eq. (2.1), we obtain the coupled equations for the state-vectors Φ_0 and Φ_n. as

$$(E - \eta H \eta)\Phi_0 = \eta H \Lambda \Phi_n , \tag{2.5a}$$

and

$$(E - \Lambda H \Lambda)\Phi_n = \Lambda H \eta \Phi_0 . \tag{2.5b}$$

From Eq. (2.5b), we can formally express the boson-existing state-vector Φ_n, by the boson-vacuum state-vector Φ_0,

$$\Phi_n = \frac{1}{1 - \frac{1}{E-H_0}\Lambda H_I} \Lambda H_I \eta \Phi_0 , \tag{2.6}$$

and we obtain the Schrödinger-like equation for the state-vector Φ_0, as

$$\begin{aligned} (E - H_0)\Phi_0 &= \eta H_I \frac{1}{1 - \frac{\Lambda}{E-H_0} H_I} \eta \Phi_0 \\ &= \eta H_I J(E) \eta \Phi_0 , \end{aligned} \tag{2.7}$$

where

$$J(E) = \frac{1}{1 - \frac{\Lambda}{E-H_0} H_I} \eta . \tag{2.8}$$

The true state-vector Ψ is expressed in terms of Φ_0 as

$$\Psi = J(E)\,\eta\,\Phi_0 . \qquad (2.9)$$

The right-hand side of Eq. (2.7) is called the Tamm-Dancoff potential, which is energy dependent. Thus we cannot identify it as the nuclear potential, since the energy-dependent potential does not gurantee the orthogonality of the state vectors.

$$\langle \Phi_{0f} \,|\, \Phi_{0i} \rangle \neq \delta_{if} \qquad (2.10)$$

However, we can eliminate the energy dependence of the operator J(E) as follows,

$$\begin{aligned} [\,J,\ H_0\,]\,\eta\,\Phi_0 &\equiv J\,H_0\,\eta\,\Phi_0 - H_0\,J\,\eta\,\Phi_0 \\ &= H_I\,J\,\eta\,\Phi_0 - J\,\eta\,H_I\,J\,\eta\,\Phi_0 , \end{aligned} \qquad (2.11)$$

which shows that the operator J should satisfy the equation as,

$$[\,J,\ H_0\,]\,\eta = H_I\,J\,\eta - J\,\eta\,H_I\,J\,\eta . \qquad (2.12)$$

This equation does not involve the total energy E explicitly so that we can obtain the energy-independent operator J, if there exists a solution of the above equation. In fact, as a limit of vanishing interaction, we have a solution with J = 1. Owing to the nonlinearity of this equation, we can obtain the operator J in a perturbative way, which will be shown in section 2.2.

From the above argument, we can define the Tamm-Dancoff potential in the energy-independent way. Nevertheless, the state-vector Φ_0 cannot be regarded as the state-vector in the potential theory, because the norm of the state-vector Φ_0 is not unity. If we introduce the normalized state-vector χ_i in the boson-vacuum space

$$\chi_i = (\eta J^{+} J \eta)^{-1/2} \Phi_{0i}, \tag{2.13}$$

which is related with the true state-vector Ψ as

$$\Psi_i = U \chi_i \tag{2.14}$$

with

$$U = J(\eta J^{+} J \eta)^{-1/2}, \tag{2.15}$$

then we can see that χ_i satisfies conditions of the normalization and orthogonality of the state-vectors:

$$\begin{aligned} \langle \Psi_f | \Psi_i \rangle &= \langle \chi_f | (\eta J^{+} J \eta)^{-1/2} \eta J^{+} J \eta (\eta J^{+} J \eta)^{-1/2} | \chi_i \rangle \\ &= \langle \chi_f | \chi_i \rangle = \delta_{if}. \end{aligned} \tag{2.16}$$

It is noticed here that U is the unitary transformation which transform χ_i into Ψ_i. The state-vector χ_i is just what we can regard as the nuclear state-vector in the potential theory. The probability P that we find the system in the boson-vacuum space is given by

$$\mathcal{P} = \langle \Phi_0 | \Phi_0 \rangle$$

$$= \langle \chi | (\eta J^{\dagger} J \eta)^{-1} | \chi \rangle. \tag{2.17}$$

The operator $(\eta J^{\dagger} J \eta)^{-1}$ is sometimes called the probability operator.

From Eq. (2.14), we obtain the equation

$$U^{-1} H U \chi \equiv (\eta H_0 \eta + V) \chi = E \chi, \tag{2.18}$$

from which the nuclear potential is defined as

$$V = (\eta J^{\dagger} J \eta)^{-1/2} J^{\dagger} H J (\eta J^{\dagger} J \eta)^{-1/2} - \eta H_0 \eta. \tag{2.19}$$

From the above argument, the Tamm-Dancoff potential is different from our potential obtained here. The nuclear interaction with external field, such as electromagnetic current or weak current, is also modified due to the existence of mesons in nuclei. The current operator $\mathcal{O}$ is modified in the nucleus as $\mathcal{O}_{eff}$, and they satisfiy the following equality

$$\langle \chi_f | \mathcal{O}_{eff} | \chi_i \rangle = \langle \Phi_f | \mathcal{O} | \Phi_i \rangle, \tag{2.20}$$

from which we can express $\mathcal{O}_{eff}$ as,

$$\mathcal{O}_{eff} = (\eta J^{\dagger} J \eta)^{-1/2} J^{\dagger} \mathcal{O} J (\eta J^{\dagger} J \eta)^{-1/2} \tag{2.21}$$

If we regard the operator $\mathcal{O}$ as the Hamiltonian, Eq. (2.21) define the nuclear potential. Equations (2.20) and (2.21) are the basic equations for the nuclear potential and exchange current to be obtained in the meson theory. These equations were originaly derived by FST[19] and independently Okubo.[20]

2.2. Perturbative Solution of Unitary Operator and Effective Operator

We shall find a solution of Eq. (2.12) in the perturbation following Hyuga and Ohtsubo.[17] We assume the Hamiltonian to be

$$H = H_0 + \lambda H_I, \tag{2.22}$$

where H_0 is the free Hamiltonian, H_I is the interaction Hamiltonian and λ is a parameter, whose value should be set unity after the calculation. For simplicity we assume the interaction Hamiltonian of boson and nucleon to be of the Yukawa type (See Fig. 1).

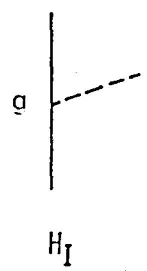


Fig. 1 Strong interaction vertex of the boson and nucleon. Solid and dashed lines show the nucleon and boson, respectively.

It is noticed that we have conditions as,

$$\Lambda H_0 \eta = 0 \qquad \text{and} \qquad \eta H_I \eta = 0. \tag{2.23}$$

To obtain the solution of J with the boundary condition,

$$J = 1 \quad \text{as} \quad \lambda \to 0 , \tag{2.24}$$

we introduce the operator F which is defined as

$$J = 1 + F, \tag{2.25}$$

and expand it in the power series of the parameter λ.

$$F = \sum_{n=1} \lambda^n F^{(n)} . \tag{2.26}$$

Inserting Eq. (2.26) together with Eqs. (2.22) and (2.23) into Eq. (2.12), and comparing term by term, we obtain the recursion relations.

$$F^{(n)} = \frac{\Lambda}{e_0 - H_0} \left[\delta_{n,1} H_I + H_I F^{(n-1)} - \sum_{m=1}^{n-2} F^{(m)} H_I F^{(n-m-1)} \right], \tag{2.27}$$

with $\quad F^{(m)} = 0 \quad \text{for} \quad m = 0, -1, -2 \cdots .$

Here e_0 is the free energy associated with η_0 ($H_0 \eta_0 = e_0 \eta_0$). For the later use, we show the resulting operator F up to the fourth order:

$$F^{(1)} = G_0 H_I \eta_0 , \tag{2.28a}$$

$$F^{(2)} = G_0 H_I G_0 H_I \eta_0 , \tag{2.28b}$$

$$F^{(3)} = G_0 H_I G_0 H_I G_0 H_I \eta_0 - G_0 G_1 H_I \eta_1 H_I G_0 H_I \eta_0 , \tag{2.28c}$$

$$F^{(4)} = G_0 H_I G_0 H_I G_0 H_I G_0 H_I \eta - G_0 H_I G_0 G_1 H_I \eta_1 H_I G_0 H_I \eta_0$$
$$- G_0 G_1 H_I \eta_1 H_I G_0 H_I \eta_0 , \tag{2.28d}$$

with

$$G_0 = \frac{\Lambda}{e_0 - H_0} \qquad \text{and} \qquad G_1 = \frac{\Lambda}{e_1 - H_0} \, . \tag{2.29}$$

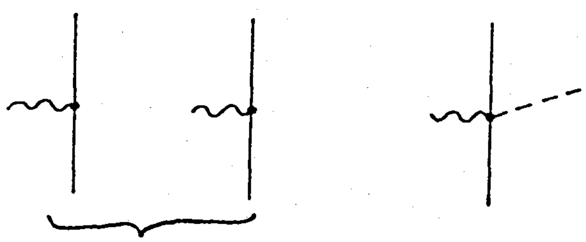
The effective current operator, $\mathcal{O}_{eff}$, defined in Eq. (2.21), is written in terms of F as

$$\mathcal{O}_{eff} = \eta \, (1 + F^\dagger F)^{-1/2} (1 + F^\dagger) \, \mathcal{O} \, (1 + F)(1 + F^\dagger F)^{-1/2} \eta . \tag{2.30}$$

We assume the operator $\mathcal{O}$ is expressed as

$$\mathcal{O} = \mathcal{O}^{(0)} + \lambda \, \mathcal{O}^{(1)} . \tag{2.31}$$

For the electromagnetic current, the term $\mathcal{O}^{(0)}$ is the sum of nucleon- and boson-currents, and $\mathcal{O}^{(1)}$ is the seagull current, as illustrated in Fig. 2.



$O^{(0)}$ $\qquad$ $O^{(1)}$

Fig. 2. Electromagnetic currents of boson and nucleon. Solid and dashed line show nucleon and boson, respectively. Wavy line shows the electromagnetic field.

The operator $\mathcal{O}^{(1)}$ satisfies the condition

$$\eta\, O^{(1)}\, \eta = 0. \tag{2.32}$$

From Eqs. (2.26) - (2.32), the perturbative expansion of the effective operator can be easily obtained as,

$$O_{eff} = O_{eff}^{(0)} + \lambda^2\, O_{eff}^{(2)} + \lambda^4\, O_{eff}^{(4)} + \cdots \tag{2.33}$$

where

$$O_{eff}^{(0)} = \eta\, O^{(0)} \eta, \tag{2.33a}$$

$$\begin{aligned} O_{eff}^{(2)} = {} & \eta\, [\, F^{(2)\dagger} O^{(0)} + F^{(1)\dagger} O^{(0)} F^{(1)} + O^{(0)} F^{(2)} - \tfrac{1}{2}\{ F^{(1)\dagger} F^{(1)}, O^{(0)} \} \,]\, \eta \\ & + \eta\, [\, F^{(1)\dagger} O^{(1)} + O^{(1)} F^{(1)} \,]\, \eta, \end{aligned} \tag{2.33b}$$

$$\begin{aligned} O_{eff}^{(4)} = {} & \eta\, [\, F^{(4)\dagger} O^{(0)} + F^{(3)\dagger} O^{(0)} F^{(1)} + F^{(2)\dagger} O^{(0)} F^{(2)} + F^{(1)\dagger} O^{(0)} F^{(3)} + O^{(0)} F^{(4)} \\ & - \tfrac{1}{2}\{ F^{(3)\dagger} F^{(1)} + F^{(2)\dagger} F^{(2)} + F^{(1)\dagger} F^{(3)}, O^{(0)} \} - \tfrac{1}{2}\{ F^{(1)\dagger} F^{(1)}, F^{(2)\dagger} O^{(0)} + F^{(1)\dagger} O^{(0)} F^{(1)} + O^{(0)} F^{(2)} \} \\ & + \tfrac{1}{4} F^{(1)\dagger} F^{(1)} O^{(0)} F^{(1)\dagger} F^{(1)} + \tfrac{3}{8} \{ F^{(1)\dagger} F^{(1)} F^{(1)\dagger} F^{(1)}, O^{(0)} \} \,]\, \eta \\ & + \eta\, [\, F^{(3)\dagger} O^{(1)} + F^{(2)\dagger} O^{(1)} F^{(1)} + F^{(1)\dagger} O^{(1)} F^{(2)} + O^{(1)} F^{(3)} \\ & - \tfrac{1}{2} \{ F^{(1)\dagger} F^{(1)}, F^{(1)\dagger} O^{(1)} + O^{(1)} F^{(1)} \} \,]\, \eta, \end{aligned} \tag{2.33c}$$

with $\{A, B\} = AB + BA$.

After these calculations, λ is put to unity. It is noticed that the terms with the anti-commutator and $\frac{1}{4}F^{(1)\dagger}F^{(1)}\mathcal{O}^{(0)}F^{(1)\dagger}F^{(1)}$ in Eq. (2.33b) and (2.33c) originate in the probability operator of Eq. (2.17). These are called terms due to wave function renormalization. The nuclear potential is obtained from Eq. (2.33), if we replace the operators $\mathcal{O}^{(0)}$ and $\mathcal{O}^{(1)}$ by H_0 and H_I, respectively.

2.3. Nuclear Potential

We shall obtain the nuclear potential up to the fourth order, i.e., the one- and two-boson-exchange potentials as illustrated in Fig. 3,

$$V = V^{OBE} + V^{TBE} . \tag{2.34}$$

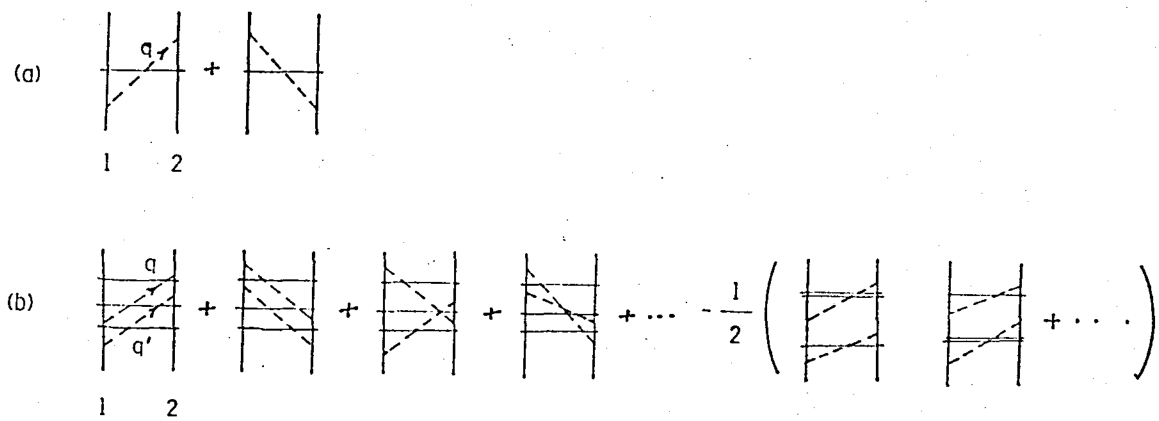


Fig. 3. The diagrams in the "time ordered perturbation". (a) and (b) show the diagrams contributing to OBE and TBE potential, respectively. The single and double horizontal lines represent G and G^2, respectively.

In what follows, to clarify the essential points of our argument we shall adopt the interaction Hamiltonian of a nucleon and a scalar boson with unit isospin.

$$H_I = g\int \bar{\psi}(\vec{x})\,\tau_j\,\psi(\vec{x})\,\phi_j(\vec{x})\,d\vec{x}. \tag{2.35}$$

Here, ψ is the field operator of nucleon with its mass M, and ϕ_j is the field operator of boson with its mass μ. The coupling constant and isospin operator of nucleon are denoted as g and τ_j. (Our results in the following discussions hold irrespective of the specific form of interaction Hamiltonian.)

i) One-Boson-Exchange (OBE) Potential

In the static limit of nucleons, the OBE potential is given as

$$V^{OBE}_{Stat} = \langle H_I \; G^2 H_I \rangle, \tag{2.36}$$

where we used abbreviations as $G = \Lambda/(-H_0)$ and $\langle A\rangle = \eta A \eta$. The word "static" means that we neglect the keinetic energy operator of nucleons in the Green function G. Then we obtain a familiar OBE potential as

$$V^{OBE}_{Stat} = -g^2(\tau^1\cdot\tau^2)\int\frac{d\vec{q}}{(2\pi)^3}\,\frac{e^{-i\vec{q}\cdot\vec{r}}}{\omega^2} \tag{2.37}$$

with $\omega = \sqrt{\vec{q}^{\,2}+\mu^2}$ and $\vec{r} = \vec{r}_1 - \vec{r}_2$.

ii) Two-Boson-Exchange (TBE) Potential

The static TBE potential is obtained by the formula

$$V^{TBE} = \langle H_I G H_I G H_I G H_I\rangle - \frac{1}{2}\left\{\langle H_I G^2 H_I\rangle, \langle H_I G H_I\rangle\right\}. \tag{2.38}$$

The first term in the right-hand side express the Tamm-Dancoff potential and, the second term, the wave function renormalization. The latter contribute to the uncrossed-boson-exchange processes, but not to the crossed-boson-exchange processes.

We obtain the potentials for the crossed and uncrossed boson-exchange processes as

$$V_{stat}^{TBE}(\text{cross}) = -(3+2\tau^1\cdot\tau^2)\, g^4 \int \frac{d\vec{q}\,d\vec{q}\,'}{(2\pi)^6}\, e^{-i(\vec{q}+\vec{q}\,')\cdot\vec{r}} \left[\frac{\omega^2+\omega\omega'+\omega'^2}{2\omega^3\omega'^3(\omega+\omega')}\right], \tag{2.39}$$

and

$$V_{stat}^{TBE}(\text{uncross}) = (3-2\tau^1\cdot\tau^2)\, g^4 \int \frac{d\vec{q}\,d\vec{q}\,'}{(2\pi)^6}\, e^{-i(\vec{q}+\vec{q}\,')\cdot\vec{r}}$$

$$\times \left[-\frac{1}{2\omega^2\omega'^2(\omega+\omega')} + \frac{1}{2\omega^2\omega'^2}\left(\frac{1}{\omega}+\frac{1}{\omega'}\right)\right]. \tag{2.40}$$

The second term in Eq. (2.40) comes from the wave function renormalization just mensioned above. Then, the static TBE potential is expressed as

$$V_{stat}^{TBE} = V_{stat}^{TBE}(\text{cross}) + V_{stat}^{TBE}(\text{uncross})$$

$$= -4(\tau^1\cdot\tau^2)\, g^4 \int \frac{d\vec{q}\,d\vec{q}\,'}{(2\pi)^6}\, e^{-i(\vec{q}+\vec{q}\,')\cdot\vec{r}} \left[\frac{\omega^2+\omega\omega'+\omega'^2}{2\omega^3\omega'^3(\omega+\omega')}\right]. \tag{2.41}$$

2.4. Exchange Charge Density Operator

We shall derive the exchange-charge-density operators associated with the nucleon-charge density (Fig. 4a), as an example, since the main source of the complication in the definition of exchange current has been concerned with the nucleon-type current. Discussions of rather

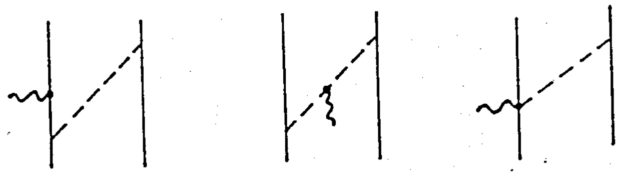


Fig. 4. Diagrams contributing to the OBE charge charge density operators. (a), (b) and (c) show the nucleon-type, boson-type and seagull currents, respectively.

well-defined boson-type current (Fig. 4b) and seagull current (Fig. 4c) will be done in the latter section.

Before going into the derivation of the charge density operator, we shall define the elctromagnetic interaction Hamiltonian and fix our notation. The interaction Hamiltonian of the system with the external electromagnetic field is given by

$$H_{em} = \int \left[\rho(\vec{x}) A_0(\vec{x}) - \vec{J}(\vec{x}) \cdot \vec{A}(\vec{x}) \right] d\vec{x}. \tag{2.42}$$

The charge density operator is expressed as

$$\rho(\vec{x}) = \hat{\rho}_N(\vec{x}) + \hat{\rho}_B(\vec{x}) \tag{2.43}$$

with

$$\hat{\rho}_N(\vec{x}) = \frac{e}{2} \psi^+(\vec{x}) (1+\tau_3) \psi(\vec{x}) \tag{2.44a}$$

and

$$\hat{\rho}_B(\vec{x}) = \frac{1}{2} e \epsilon_{ij3} : [\phi_i(\vec{x})\dot{\phi}_j(\vec{x}) - \dot{\phi}_i(\vec{x})\phi_j(\vec{x})] : , \tag{2.44b}$$

where $\hat{\rho}_N$ and $\hat{\rho}_B$ are charge density operators of nucleon and boson, respectively. The spatial current density operator is expressed as

$$\vec{J}(\vec{x}) = \vec{J}_N(\vec{x}) + \vec{J}_B(\vec{x}), \tag{2.45}$$

with

$$\vec{J}_N(\vec{x}) = \frac{e}{2} \psi^+(\vec{x}) \frac{\vec{p}+\vec{p}\,'}{2M} (1+\tau_3) \psi(\vec{x}), \tag{2.46a}$$

and

$$\vec{J}_B(\vec{x}) = \frac{e}{2} \epsilon_{ij3} : [(\vec{\nabla}\phi_i(\vec{x}))\phi_j(\vec{x}) - \phi_i(\vec{x})(\vec{\nabla}\phi_j(\vec{x}))] : . \tag{2.46b}$$

Here, $\vec{p}$ and $\vec{p}\,'$ are the momentum operators of nucleon, which operate on the initial and final states, respectively.

i) One-Boson-Exchange (OBE) current

At first, we shall study the terms due to the wave function renormalization in the static OBE operator. From Eq. (1..33b) we have

$$O^{OBE,N}_{stat} = O^{OBE}_{recoil} + O^{OBE}_{renom.} , \tag{2.47}$$

with

$$O^{OBE}_{recoil} = \langle H_I \, G \, O_N \, G \, H_I \rangle$$

and

$$O^{OBE}_{Renom} = -\frac{1}{2}\left\{\langle O_N\rangle,\ \langle H_I G^2 H_I\rangle\right\}_+ .$$

The first term of Eq. (2.47) is called the recoil current, and the second term, the wave function renormalization. They are illustrated in Fig. 5a.

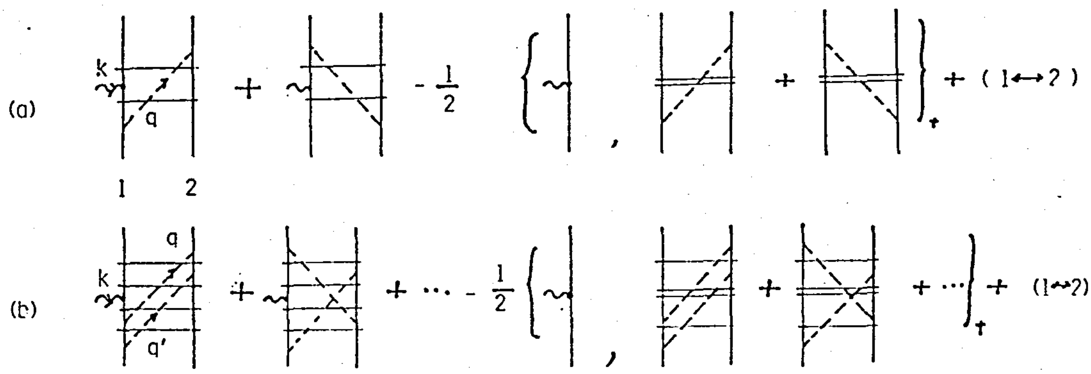


Fig. 5. Diagrams in the "time ordered perturbation". (a) and (b) show the diagrams contributing to OBE and TBE currents, respectively.

The explicit form of the static OBE charge density operator is

$$\rho^{OBE,N}_{stat} = \int \frac{d\vec{q}}{(2\pi)^3}\frac{e^{-i\vec{q}\cdot\vec{r}}}{2\omega^3}\left\{\rho_N(\vec{x}),\ g^2\tau^1\cdot\tau^2\right\}_+$$

$$-\frac{1}{2}\left\{g^2\tau^1\cdot\tau^2\int\frac{d\vec{q}}{(2\pi)^3}\frac{e^{-i\vec{q}\cdot\vec{r}}}{\omega^3},\ \rho_N(\vec{x})\right\}_+$$

$$= 0, \qquad (2.48)$$

with $$\rho_N(\vec{x}) = e\sum_{s=1}^{2}\left[\frac{1+\tau_3^s}{2}\right]\delta(\vec{x}-\vec{r}_s).$$

In generall the static OBE nucleon-type exchange current vanishes in the operator form irrespective of any detail of the operator Q_N. It is noticed that meson-type current and seagull term do not contribute to the charge density operator in this case.

In contrast to the unitary transformation method, most of people do not notice the probability operator, Eq. (2.17), or someone defines the exchange current by treating the probability operator as c-number. In the work of the former people, the elimination of mesonic degrees of freedom does not gurantee the charge conservation of nuclear system. Furthermore, the treatment of the latter people, which we shall refer to the c-number renormalization method[15, 33], is not justified, as was pointed out by some authors[29-32]. In this method the normalized nuclear state vector $\tilde{\chi}_i$ is related to the projected state vector Φ_{oi} by the c-number normalization constant Z_i as

$$\tilde{\chi}_i = Z_i^{-1/2}\,\Phi_{oi}\,, \tag{2.49}$$

The factor Z_i is given by the normalization condition for Φ_{oi} in Eq. (2.16)

$$\begin{aligned}\langle \Phi_i | \Phi_i \rangle &= \langle \Phi_{oi} | \eta J^{+} J \eta | \Phi_{oi} \rangle = \langle \tilde{\chi}_i | \eta J^{+} J \eta | \tilde{\chi}_i \rangle Z_i^{-1} \\ &= 1. \end{aligned} \tag{2.50}$$

Thus we obtain

$$Z_i = \langle \tilde{\chi}_i | \eta J^{+} J \eta | \tilde{\chi}_i \rangle . \tag{2.51}$$

The corresponding effective operator is given as

$$\widetilde{O}_{eff} = Z_f^{-1/2} \, \eta \, J^{+} O J \, \eta \, Z_i^{-1/2} \tag{2.52}$$

We notice here that these normalized state vectors χ_i are not orthogonal to each other, since the probability operator has generally off-diagonal matrix elements.

$$\begin{aligned} \langle \widetilde{\chi}_f \mid \widetilde{\chi}_i \rangle &= \sqrt{Z_i Z_f} \quad \langle \Phi_f \mid \Phi_i \rangle \\ &= \sqrt{Z_i Z_f} \quad \langle \chi_f \mid (\eta J^{+} J \eta)^{-1} \mid \chi_i \rangle \\ &\neq \delta_{if}. \end{aligned} \tag{2.53}$$

For example, the static OBE current of nucleon-type does not vanish. Thus the c-number normalization method is not valid.

ii) Two-Boson-Exchange (TBE) Current

The static TBE current of nucleon-type is

$$\begin{aligned} O_{Stat}^{TBE,N} = {} & \langle H_I G O_N G H_I G H_I G H_I \rangle + \langle H_I G H_I G O_N G H_I G H_I \rangle + \langle H_I G H_I G H_I G O_N G H_I \rangle \\ & - \langle H_I G H_I \rangle \langle H_I G^2 O_N G H_I \rangle - \langle H_I G O_N G^2 H_I \rangle \langle H_I G H_I \rangle \\ & - \frac{1}{2} \left\{ \langle H_I G^2 H_I G H_I G H_I \rangle + \langle H_I G H_I G^2 H_I G H_I \rangle + \langle H_I G H_I G H_I G^2 H_I \rangle , \langle O_N \rangle \right\}_{+} \\ & + \frac{1}{2} \left\{ \langle H_I G^3 H_I \rangle \langle H_I G H_I \rangle + \langle H_I G H_I \rangle \langle H_I G^3 H_I \rangle , \langle O_N \rangle \right\}_{+} \end{aligned}$$

$$+\frac{1}{4}\langle H_I G^2 H_I\rangle\langle O_N\rangle\langle H_I G^2 H_I\rangle+\frac{3}{8}\left\{\langle H_I G^2 H_I\rangle\langle H_I G^2 H_I\rangle,\langle O_N\rangle\right\}_+$$

$$-\frac{1}{2}\left\{\langle H_I G^2 H_I\rangle,\langle H_I G O_N G H_I\rangle\right\}_+. \tag{2.54}$$

Here, all terms except for those in the first two lines come from the wave function renormalization. As is seen in Fig. 5b, the static TBE charge density operator obtained from Eq. (2.54) can be divided into the uncrossed and crossed boson-exchange current,

$$\rho^{TBE,N}_{stat}=\rho^{TBE,N}_{stat}(\text{uncross})+\rho^{TBE,N}_{stat}(\text{cross}), \tag{2.55}$$

with

$$\rho^{TBE,N}_{stat}(\text{uncross})=g^4\int\frac{d\vec{q}\,d\vec{q}'}{(2\pi)^6}e^{-i(\vec{q}+\vec{q}')\cdot\vec{r}}\frac{1}{2\omega^2\omega'^2}\left[\frac{1}{\omega^2}+\frac{1}{\omega'^2}\right]$$

$$\times\left[\frac{1}{2}\left\{\rho_N(\vec{x}),(\tau^1\cdot\tau^2)^2\right\}_+-(\tau^1\cdot\tau^2)\rho_N(\vec{x})(\tau^1\cdot\tau^2)\right]$$

$$=2eg^4(\tau^1-\tau^2)_z\left[\delta(\vec{x}-\vec{r}_1)-\delta(\vec{x}-\vec{r}_2)\right]\int\frac{d\vec{q}\,d\vec{q}'}{(2\pi)^6}\frac{e^{-i(\vec{q}+\vec{q}')\cdot\vec{r}}}{2\omega^2\omega'^2}\left[\frac{1}{\omega^2}+\frac{1}{\omega'^2}\right], \tag{2.56}$$

and

$$\rho^{TBE,N}_{stat}(\text{cross})=g^4\int\frac{d\vec{q}\,d\vec{q}'}{(2\pi)^6}e^{-i(\vec{q}+\vec{q}')\cdot\vec{r}}\frac{1}{2\omega^2\omega'^2}\left[\frac{1}{\omega^2}+\frac{1}{\omega'^2}\right]$$

$$\times\left[\tau^1_\alpha\tau^2_\beta\,\rho_N(\vec{x})\,\tau^1_\beta\tau^2_\alpha-\frac{1}{2}\left\{\rho_N(\vec{x}),3+2\tau^1\cdot\tau^2\right\}_+\right]$$

$$=-2eg^4(\tau^1+\tau^2)_z\left[\delta(\vec{x}-\vec{r}_1)+\delta(\vec{x}-\vec{r}_2)\right]\int\frac{d\vec{q}\,d\vec{q}'}{(2\pi)^6}\frac{e^{-i(\vec{q}+\vec{q}')\cdot\vec{r}}}{2\omega^2\omega'^2}\left[\frac{1}{\omega^2}+\frac{1}{\omega'^2}\right]. \tag{2.57}$$

In addition to the above operator, we have the boson-type static TBE current. Hyuga and Ohtsubo[17)] have shown the existence of TBE charge density operator even in the static limits in this way.

Section 3. The S-Matrix Method

The basic idea of the S-matrix method is that the S-matrix is uniquely determined irrespective of any kind of representation of the original Hamiltonian. Therefore, the S-matrix given by the conventional field theory should be identical to the S-matrix obtained as iterated terms of the instantaneous and hermitian nuclear potential and possibly, exchange current. Inversely, we can define the nuclear potential and exchange current so as to reproduce the S-matrix element by the conventional field theory. The static theory along this idea was developed by Miyazawa[23] to derive TBE potential. Here, we shall apply this S-matrix method to derive static exchange current up to TBE process.

3.1. Nuclear Potential

At first, we shall try to obtain the nuclear potential by investigating the S-matrix of nucleon-nucleon scattering. The S-matrix element in the potential picture is given as illustrated in Fig. 6.

$$T_P = T_P^{OBE} + T_P^{TBE} + \cdots \tag{3.1}$$

with

$$T_P^{OBE} = V^{OBE}, \tag{3.2}$$

and

$$T_P^{TBE} = V^{TBE} + V^{OBE} G_N V^{OBE}. \tag{3.3}$$

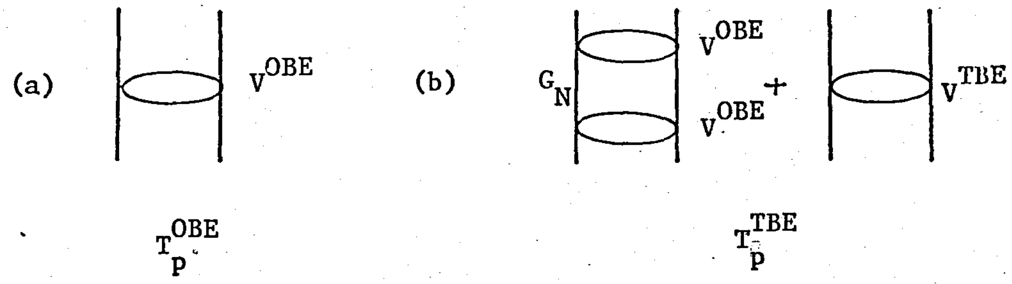


Fig. 6. S-matrix elements in the potential picture. Diagrams (a) and (b) show the OBE and TBE processes, respectively.

Here, the suffix p represents the potential picture. The Green function G_N of two free nucleons is expressed as

$$G_N = \frac{1}{(2\pi)^3} \frac{1}{E - \frac{\vec{p}_1^{\,2}}{2M} - \frac{\vec{p}_2^{\,2}}{2M} + i\epsilon} . \tag{3.4}$$

On the other hand, we obtain the same S-matrix element by the conventional technique of the field theory as,

$$T = T^{OBE} + T^{TBE} + \cdots , \tag{3.5}$$

where T^{OBE} is the second order matrix element of the OBE process (Fig. 7a), and T^{TBE} is the fourth order matrix element of the TBE process (Fig. 7b). Since the S-matrix element (3.1) should be identical to Eq. (3.5), we obtain the OBE potential V^{OBE} from the field theoretical S-matrix element T^{OBE}. If we subtract the iterated term of OBE potentials from the TBE S-matrix element T^{TBE}, we can obtain the TBE potential V^{TBE}. This is seen in the static nucleon limit, where the two-nucleon Green function

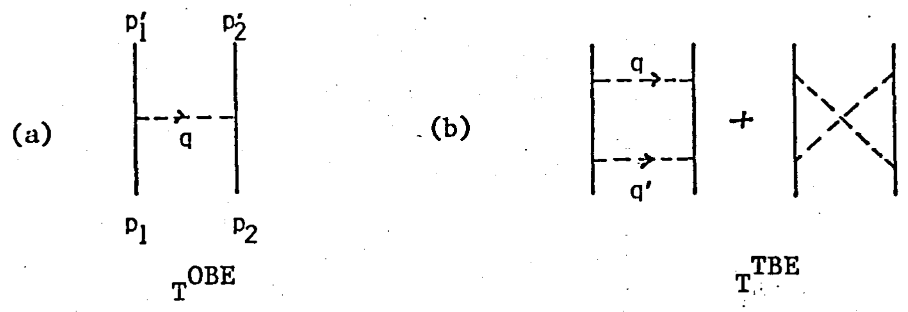


Fig. 7. Diagrams of two-nucleon scattering. Diagrams (a) and (b) represent OBE and TBE processes, respectively.

G_N diverges,

$$G_N \longrightarrow \frac{1}{(2\pi)^3} \frac{1}{i\epsilon} . \tag{3.6}$$

Therefore, the iterated term of OBE potential gives a divergent contribution to the fourth order S-matrix element. In other words, if we evaluate the fourth order S-matrix element with static nucleons and subtract the divergent term from this S-matrix, we can identify the finite term of the S-matrix element as the static TBE potential. This procedure will be shown explicitly in the following.

The nucleon propagator for the non-relativistic nucleon is expressed as

$$S(x) = \frac{-i}{(2\pi)^4} \int d^4p \, \frac{e^{-ip\cdot x}}{\frac{\vec{p}^2}{2M} - p_0 + i\epsilon} . \tag{3.7}$$

For the static nucleon, we neglect the kinetic energy so that the propagator (3.7) reduces to the form as

$$\delta(x) = \frac{i}{2\pi}\int dp_0 \; \frac{e^{ip_0 t}}{p_0 + i\epsilon} \quad \delta(\vec{x})$$

$$= \quad \theta(t) \; \delta(\vec{x}), \tag{3.8}$$

The OBE diagram is easily calculated in the scalar boson-exchange model.

$$T^{OBE}_{stat} = -g^2 \tau' \cdot \tau^2 \int \frac{d\vec{q}}{(2\pi)^3} \; \frac{e^{-i\vec{q}\cdot\vec{r}}}{\omega^2} \; . \tag{3.9}$$

From Eqs. (3.9) and (2.37), the second order S-matrix element coincides with the static OBE potential V^{OBE}_{stat}.

The fourth order matrix element is expressed as

$$T^{TBE}_{stat} = g^4 \int \frac{d\vec{q}\,d\vec{q}'}{(2\pi)^6} \; e^{-i(\vec{q}+\vec{q}')\cdot\vec{r}} \int \frac{i}{2\pi} \, dx$$

$$\times \Big[\; (3 - 2\tau' \cdot \tau^2) \; (\, (x^2 - \omega^2 + i\epsilon)(x^2 - \omega'^2 + i\epsilon)(x + i\epsilon)(-x + i\epsilon))^{-1}$$

$$+ \; (3 + 2\tau' \cdot \tau^2) \, (\, (x^2 - \omega^2 + i\epsilon)(x^2 - \omega'^2 + i\epsilon)(x + i\epsilon)^2)^{-1} \Big] \, . \tag{3.10}$$

The first and the second terms in the brackets correspond to the uncrossed and crossed boson-exchanges, respectively. Integrating over the energy parameter x, we rewrite the S-matrix element as,

$$T^{TBE}_{stat} = g^4 \int \frac{d\vec{q}\,d\vec{q}'}{(2\pi)^6} \, e^{-i(\vec{q}+\vec{q}')\cdot\vec{r}} \; \frac{\omega^2 + \omega\omega' + \omega'^2}{2\,\omega^3 \omega'^3 (\omega + \omega')} \Big[(3 - 2\tau' \cdot \tau^2) - (3 + 2\tau' \cdot \tau^2) \Big]$$

$$+ \Big[-g^2 \tau' \cdot \tau^2 \int \frac{d\vec{q}}{(2\pi)^3} \frac{e^{-i\vec{q}\cdot\vec{r}}}{\omega^2} \Big] \frac{1}{i\epsilon} \Big[-g^2 \tau' \cdot \tau^2 \int \frac{d\vec{q}'}{(2\pi)^3} \, \frac{e^{-i\vec{q}'\cdot\vec{r}}}{\omega'^2} \Big] . \tag{3.11}$$

Here, the divergent part is found to be the iterated term of static OBE potential.

$$T^{TBE}_{Stat} = V^{TBE}_{Stat} + V^{OBE}_{Stat} \frac{1}{i\epsilon} V^{OBE}_{Stat} . \qquad (3.12)$$

The finite part of the fourth order S-matrix element coincides with the static TBE potential, as we have expected. Miyazawa[24] derived the two-pion-exchange potential in this way.

3.2. Exchange Charge Density Operator

We shall derive the exchange current by investigating the S-matrix element of the radiative two-nucleon scattering. In the potential picture the S-matrix element is given by the iterated term of the exchange current and nuclear potential as illustrated in Fig. 8.

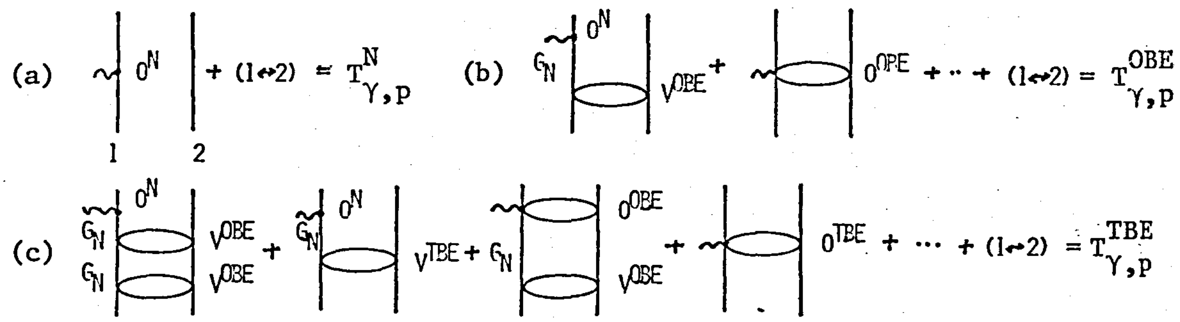


Fig. 8. S-matrix elements of radiative two-nucleon scattering in the potential picture. (a), (b) and (c) show the one-body current with uncorrelated nucleons, OBE and TBE processes, respectively.

$$T_{\gamma,p} = T^N_{\gamma,p} + T^{OBE}_{\gamma,p} + T^{TBE}_{\gamma,p} , \qquad (3.12)$$

with

$$T^{N}_{\gamma,p} = O_N , \tag{3.13}$$

$$T^{OBE}_{\gamma,p} = O_N G_N V^{OBE} + V^{OBE} G_N O_N + O^{OBE,N} , \tag{3.14}$$

and

$$T^{TBE}_{\gamma,p} = O_N G_N V^{OBE} G_N V^{OBE} + V^{OBE} G_N O_N G_N V^{OBE} + V^{OBE} G_N V^{OBE} G_N O^{N}$$
$$+ O_N G_N V^{TBE} + V^{TBE} G_N O_N + O^{OBE,N} G_N V^{OBE} + V^{OBE} G_N O^{OBE,N}$$
$$+ O^{TBE,N} . \tag{3.15}$$

If we apply the same technique as in the static nuclear potential problem, we can obtain the static exchange currents from the field theoretical S-matrix element: It is expected that the finite parts of the S-matrix element of the OBE and TBE processes in the static nucleon limit are identified with the OBE and TBE currents, respectively. First of all, the matrix element of the one-body current with uncorrelated nucleons corresponds to the S-matrix element of the first order in the field theory (Fig. 9a). Then we obtain the first order term,

$$T^{N}_{\gamma,Stat} = \rho_N . \tag{3.16}$$

The third order in the perturbation theory gives the S-matrix element as (Fig. 9b-c),

$$T^{OBE}_{\gamma,stat} = g^2 \int \frac{d\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \left[\frac{1}{\Delta + i\epsilon} \frac{1}{\Delta^2 - \omega^2} \right] \left\{ \rho_N , \tau^1 \cdot \tau^2 \right\}_+ . \tag{3.17}$$

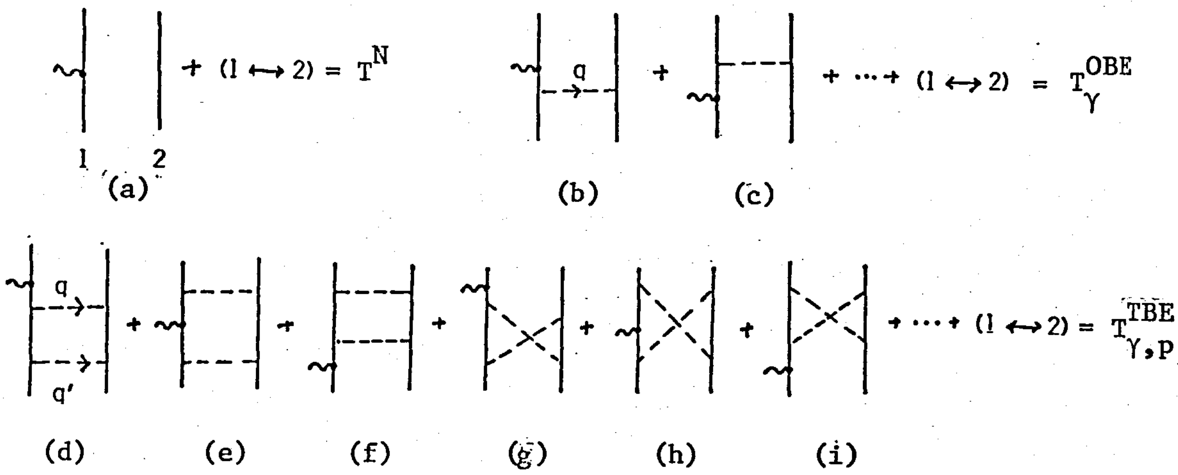


Fig. 9. Diagrams of radiative two-nucleon scattering.

Here, we introduced the infinitesimal energy transfer, Δ, supplied by the external field in order to see divergence of the S-matrix element clearly. For example, we investigate the Coulomb interaction of nucleon, as in sect. 2.5. In the limit of vanishingly small energy-transfer, Eq. (3.17) is written as

$$T^{OBE}_{\gamma,stat} = \rho_N \frac{1}{i\epsilon} V^{OBE}_{stat} + V^{OBE}_{stat} \frac{1}{i\epsilon} \rho_N , \tag{3.18}$$

where we have no finite term. This means that the static OBE current vanishes in the static limit,

$$\rho^{OBE,N}_{stat} = 0. \tag{3.19}$$

This result agrees with the one obtained by the unitary transformation method. (See Eq. (2.48))

Next we shall study the TBE matrix element. We have six types of diagrams relevant to the radiative two-nucleon scattering processes in the fifth order perturbation theory, as shown in Fig. 9. First of all, we shall study the diagram Fig. 9d in detail. The corresponding S-matrix element is written as

$$T^{TBE}_{\gamma,\,stat}\,(9d) = g^4 \int \frac{d\vec{q}\,d\vec{q}'}{(2\pi)^6}\, e^{-i(\vec{q}+\vec{q}')\cdot\vec{r}}\; \rho_N\,(3 - 2\tau^1\cdot\tau^2)$$

$$\times \left[(x^2-\omega^2+i\epsilon)(x^2-\omega'^2+i\epsilon)(x+i\epsilon)(-x-\Delta+i\epsilon)(-\Delta+i\epsilon) \right]^{-1}. \tag{3.20}$$

After integrating over the energy variable x, we obtain

$$T^{TBE}_{\gamma,\,stat}\,(9d) = \rho_N \frac{1}{\Delta+i\epsilon}\left[-g^2\tau^1\cdot\tau^2 \int \frac{d\vec{q}}{(2\pi)^3}\frac{e^{-i\vec{q}\cdot\vec{r}}}{\omega^2}\right] \frac{1}{\Delta+i\epsilon}\left[-g^2\tau^1\cdot\tau^2\int\frac{d\vec{q}'}{(2\pi)^3}\frac{e^{-i\vec{q}'\cdot\vec{r}}}{\omega'^2}\right]$$

$$+ \rho_N \frac{1}{\Delta+i\epsilon}\, g^4(3-2\tau^1\cdot\tau^2)\int\frac{d\vec{q}\,d\vec{q}'}{(2\pi)^6} e^{-i(\vec{q}+\vec{q}')\cdot\vec{r}}\,\frac{\omega^2+\omega\omega'+\omega'^2}{2\,\omega^3\omega'^3(\omega+\omega')}$$

$$+ g^4\rho_N(3-2\tau^1\cdot\tau^2)\int\frac{d\vec{q}\,d\vec{q}'}{(2\pi)^6} e^{-i(\vec{q}+\vec{q}')\cdot\vec{r}}\,\frac{1}{2\omega^2\omega'^2}\left(\frac{1}{\omega^2}+\frac{1}{\omega'^2}\right). \tag{3.21}$$

We can rewrite Eq. (3.21) by using the potential and the exchange current in the unitary tramsformation method in Eqs. (2.37), (2.40), (2.48) and (2.55), as,

$$T^{TBE}_{\gamma,\,stat} = \rho_N \frac{1}{\Delta+i\epsilon} V^{OBE}_{stat} \frac{1}{\Delta+i\epsilon} V^{OBE}_{stat} + \rho_N \frac{1}{\Delta+i\epsilon} V^{TBE}_{stat}(\text{uncross})$$

$$+ 2\,\rho^{TBE,N}_{stat}, \tag{3.22}$$

where $\rho^{TBE,N}_{stat}$ means a part of TBE current, i.e., contribution of the uncrossed diagram corresponding to Fig. 9d. It is noticed here that the finite part of the S-matrix element in the fifth order calculation is twice as large as the static TBE current obtained in sect. 2.5. In other words, if we regard the finite part of the S-matrix element as the TBE current, the S-matrix method and the unitary transformation method give different results for the TBE current to each other. The same discrepancy appears in the diagrams of Fig. 9f, 9g and 9i. As a result, the finite part of the fifth order processes is given as

$$T^{TBE}_{r,\mathrm{Stat}}(\mathrm{finite}) = g^4 \int \frac{d\vec{q}\,d\vec{q}\,'}{(2\pi)^6}\, e^{-i(\vec{q}+\vec{q}\,')\cdot\vec{r}}\, \frac{1}{2\omega^2\omega'^2}\left[\frac{1}{\omega^2}+\frac{1}{\omega'^2}\right]$$

$$\times\Big[\Big(\big\{\rho_N\,,\,(\tau^1\cdot\tau^2)^2\big\}_+ - (\tau^1\cdot\tau^2)\,\rho_N\,(\tau^1\cdot\tau^2)\Big)$$

$$+\Big(-\big\{\rho_N\,,\,3+2\tau^1\cdot\tau^2\big\}_+ + \tau^1_\alpha\tau^2_\beta\,\rho_N\,\tau^1_\beta\,\tau^2_\alpha\Big)\Big]\,, \qquad (3.23)$$

which is compared with the static TBE current in Eqs. (2.55) and (2.56). At this stage, we confront a big problem why the S-matrix method and the unitary transformation method give the different static TBE currents, while they give the same OBE- and TBE-potentials and OBE currents. We shall answer this question clearly in the next section.

Section 4. Relation between S-Matrix Method and Unitary Transformation Method

We obtained different results for the TBE charge density operator by different methods, although we obtained the same results for nuclear potential and OBE charge density operator. What is wrong in our treatment? A key point in this problem is that we completely disregarded the non-static effects in the S-matrix method. In this section we shall show that the static treatment of the S-matrix involves ambiguities due to the non-static operators. For this purpose, we shall, at first, investigate the non-static potential and charge density operator in the unitary transformation method.

4.1. Non-Static Nuclear Potential and Exchange Charge Density Operator

i) Non-Static Potential

We shall retain the recoil of nucleon in the potential,

$$V^{OBE} = {}_f\langle H_I \Delta_f (\epsilon_i + \epsilon_f - H_0) \Delta_i H_I \rangle_i - \frac{1}{2}(\epsilon_i + \epsilon_f)\, {}_f\langle H_I \Delta_f \Delta_i H_I \rangle_i , \tag{4.1}$$

with

$${}_f\langle A \rangle_i = \eta_f A \eta_i .$$

The second term of Eq. (4.1) is due to the wave function renormalization, which vanishes in the static limit. Instead of Eq. (2.37), we have

$$V^{OBE} = -g^2 \, \tau^1 \cdot \tau^2 \int \frac{d\vec{q}}{(2\pi)^3} \frac{1}{2} e^{-i\vec{q}\cdot\vec{r}} \left[\frac{1}{\omega^2 - \Delta\hat{E}_1^2} + \frac{1}{\omega^2 - \Delta\hat{E}_2^2} \right] , \tag{4.2}$$

with $\Delta\hat{E}_s = \hat{E}_s' - \hat{E}_s = \frac{1}{2M}\vec{P}_s'^{\,2} - \frac{1}{2M}\vec{P}_s^{\,2}$, $(s = 1, 2)$.

Operators $\hat{E}_s$ and $\hat{E}_s'$ are the initial and final kinetic energy operators of the s-th nucleon, respectively. The retardation of nucleon recoil is expressed by $\Delta\hat{E}_s$, and thus the nuclear force becomes momentum-dependent operator. It should be emphasized that the "momentum-dependent" potential does not mean a potential "dependent on the energy of the system". By expanding the denominators in Eq. (4.2) we rewrite the potential as

$$V^{OBE} = -g^2 \tau^1\cdot\tau^2 \int \frac{d\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \left[\frac{1}{\omega^2} + \frac{\Delta\hat{E}_1^2 + \Delta\hat{E}_2^2}{2\omega^4} + \cdots\right]. \tag{4.3}$$

The first term of Eq. (4.3) is the static OBE potential V^{OBE}_{stat}, and the second term is the first order recoil correction, $V^{OBE}_{Non-stat}$,

$$V^{OBE}_{Non-Stat} = -g^2 \tau^1\cdot\tau^2 \int \frac{d\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \frac{\Delta\hat{E}_1^2 + \Delta\hat{E}_2^2}{2\omega^4}. \tag{4.4}$$

It is noticed that this non-static OBE potential is propotional to the square of the kinetic energy operator. In what follows, we retain the retardation effect up to the first order as

$$V^{OBE} \simeq V^{OBE}_{Stat} + V^{OBE}_{Non-Stat}. \tag{4.5}$$

ii) Non-Static TBE Potential

Retarded TBE potential is also expressed as

$$V^{TBE} = \frac{1}{2}\sum_{s} \langle H_I G_f H_I G_f H_I G_f H_I + H_I G_i H_I G_i H_I G_i H_I \rangle_c$$

$$- \frac{1}{2} \left(\sum_{f} \langle H_I \omega_f H_I \rangle_m \sum_{m} \langle H_I \omega_m \omega_j H_I \rangle_i + \sum_{f} \langle H_I \omega_i \omega_m H_I \rangle_m \sum_{m} \langle H_I \omega_i H_I \rangle_i \right)$$

$$+ \frac{1}{4} \left(\epsilon_m - \frac{\epsilon_i + \epsilon_f}{2} \right) \sum_{f} \langle H_I \omega_f \omega_m H_I \rangle_m \sum_{m} \langle H_I \omega_m \omega_i H_I \rangle_i \, , \tag{4.6}$$

from which we obtain the first order recoil correction as

$$V^{TBE} \simeq V^{TBE}_{stat} + V^{TBE}_{Non-Stat} \tag{4.7}$$

where

$$V^{TBE}_{Non-Stat} = (3 + 2\tau(1)\tau(2)) g^4 \int \frac{d\vec{q}\, d\vec{q}'}{(2\pi)^6} e^{-i(\vec{q}+\vec{q}')\cdot\vec{r}} \frac{1}{2\omega^2 \omega'^2}$$

$$\times \left[\frac{\Delta \hat{e}'_{c1} + \Delta \hat{e}_{c2}}{\omega^2} + \frac{\Delta \hat{e}_{c1} + \Delta \hat{e}'_{c2}}{\omega'^2} \right], \tag{4.8}$$

with

$$\Delta \hat{e}_{cs} = \hat{e}_{cs} - \hat{E}_s \, , \qquad \Delta \hat{e}'_{cs} = \hat{e}_{cs} - \hat{E}_s' ,$$

$$\hat{e}_{c1} = \frac{1}{2M} (\vec{p}_1 - \vec{q})^2 \quad \text{and} \quad \hat{e}_{c2} = \frac{1}{2M} (\vec{p}_2 + \vec{q})^2 .$$

<u>It is noticed that the non-static correction to the uncrossed boson exchange vanishes, and that the non-static TBE potential is propotional to the kinetic energy of nucleon.</u> In the fourth order perturbation, we obtain only the non-static three-body potential, but not the static three-body potential. Since it is not essential in the following discussion , we shall not describe its derivation.

iii) Non-Static OBE Charge Density Operator

The non-static OBE current is given as

$$\mathcal{O}^{OBE,N} = \sum_f \langle H_I G_f O_N G_i H_I \rangle_i - \frac{1}{2}\Big[\sum_f \langle H_I G_f G_m H_I \rangle_m \sum \langle O_N \rangle_i + \sum_f \langle O_N \rangle_m \sum \langle H_I G_m G_i H_I \rangle_i\Big]$$

$$\simeq \mathcal{O}^{OBE,N}_{Stat} + \mathcal{O}^{OBE,N}_{Non\ Stat} , \qquad (4.9)$$

where $\mathcal{O}^{OBE,N}_{Stat}$ is the static part given in Eq. (2.47). The recoil correction to the static OBE charge density is given as

$$\rho^{OBE,N}_{Non-Stat} = e \int \frac{d\vec{q}}{(2\pi)^3} \frac{e^{-i\vec{q}\cdot\vec{r}}}{\omega^4} \Big[\Big(\frac{1+\tau_3^1}{2}\Big)\delta(x-\vec{r}_1)\,\tau^1\cdot\tau^2 \Big(\Delta\hat{E}_2 + \hat{E}_1 - \frac{(\vec{p}_1-\vec{q})^2}{2M}\Big)$$

$$- \tau^1\cdot\tau^2 \Big(\frac{1+\tau_3^1}{2}\Big)\delta(x-\vec{r}_1)\Big(\Delta\hat{E}_2 + \frac{(\vec{p}_1{}'+\vec{q})^2}{2M} - \hat{E}_1{}'\Big)\Big] + (1\leftrightarrow 2)$$

$$= \frac{e}{2M} \int \frac{d\vec{q}}{(2\pi)^3} \frac{e^{-i\vec{q}\cdot\vec{r}}}{\omega^4} \delta(x-r_1) \Big[-i\,\tau_x\,(\overleftrightarrow{p}_1 + \overleftrightarrow{p}_2)\cdot q$$

$$-k\cdot q\,(\tau^1\tau^2 + \tau_3^2)\,\Big] + (1\leftrightarrow 2) \qquad (4.10)$$

with $\qquad \tau_x = (\tau^1 \times \tau^2)_3 \qquad and \qquad \hat{P}_s = \vec{P}_c + \vec{P}_s{}' .$

<u>The non-static OBE current of nucleon type is propotional to the kinetic energy operator.</u>

4.2. Nuclear Potential and the S-Matrix Element

i) One-Boson-Exchange Potential

The second order nucleon-nucleon scattering matrix element in the momentum space is

$$T^{OBE} = \frac{g^2}{(2\pi)^3} \frac{\tau^1 \cdot \tau^2}{q_0^2 - \omega^2} \tag{4.11}$$

with
$$q_0 = E_1' - E_1 = E_2 - E_2'$$

and
$$E_s' = \frac{\vec{p}_s'^2}{2M}, \quad E_s = \frac{\vec{p}_s^2}{2M} \quad \text{for } s = 1, 2.$$

Here, $\vec{p}$ and $\vec{p}'$ denote the momentum of the initial and final state of the s-th nucleon, respectively. The S-matrix element is defined on the energy shell,

$$E_1 + E_2 = E_1' + E_2' \tag{4.12}$$

Thus the off shell extrapolation of Eq. (4.11) has some ambiguities. We shall discuss those ambiguities of off shell extrapolation in the S-matrix method in sect. 5.

From the potential picture the S-matrix element is given as

$$T_P^{OBE} = -\frac{g^2}{(2\pi)^3} \tau^1 \cdot \tau^2 \frac{1}{2} \left[\frac{1}{\omega^2 - \Delta E_1^2} + \frac{1}{\omega^2 - \Delta E_2^2} \right]$$

$$= \frac{g^2}{(2\pi)^3} \tau^1 \cdot \tau^2 \frac{1}{q_0^2 - \omega^2} . \tag{4.13}$$

From Eq. (4.13), we can see the S-matrix element is equivalent irrespective to the method of calculation. In the static limit ($q_0 \to 0$), we can see the static potential is unambiguously defined from the field theoretical S-matrix element, while its non-static correction has some arbitrariness.

ii) Two-Boson-Exchange Potential

The fourth order S-matrix element with the kinematics of Fig. 7 is given as

$$T^{TBE}(\text{Uncross}) = g^4(3-2\tau^1\cdot\tau^2)\int\frac{d\vec{q}}{(2\pi)^6}\int\frac{i}{2\pi}dx$$

$$\times\left[(x-e_1+i\epsilon)(-x-e_2+i\epsilon)(x^2-\omega^2+i\epsilon)(x^2-\omega'^2+i\epsilon)\right]^{-1}, \tag{4.14}$$

and

$$T^{TBE}(\text{cross}) = g^4(3+2\tau^1\cdot\tau^2)\int\frac{d\vec{q}}{(2\pi)^6}\int\frac{i}{2\pi}dx$$

$$\times\left[(x-e_{1c}+i\epsilon)(x-e_{2c}+i\epsilon)(x^2-\omega^2+i\epsilon)(x^2-\omega'^2+i\epsilon)\right]^{-1}, \tag{4.15}$$

with $\quad e_1 = \frac{1}{2M}(\vec{p}_1-\vec{q})^2 \quad$ and $\quad e_2 = \frac{1}{2M}(\vec{p}_2+\vec{q})^2.$

Eqs. (4.14) and (4.15) correspond to the uncrossed- and crossed-boson exchanges, respectively. Performing the integral of the energy variable x, we obtain

$$T^{TBE}(\text{uncross}) \simeq g^4(3-2\tau^1\cdot\tau^2)\int\frac{d\vec{q}}{(2\pi)^6}\left[\frac{\omega^2+\omega\omega'+\omega'^2}{\omega^3\omega'^3(\omega+\omega')}\right.$$

$$+\frac{1}{\omega^2}\frac{1}{E-e+i\epsilon}\frac{1}{\omega'^2}+\frac{1}{\omega^2}\frac{1}{E-e+i\epsilon}\frac{(e_1-E_1)^2+(e_2-E_2)^2}{2\omega^4}+\frac{(E_1'-e_1)^2+(E_2'-e_2)^2}{2\omega^4}\frac{1}{E-e+i\epsilon}\frac{1}{\omega'^2}\Big],$$

and

$$T^{TBE}(cross) = g^4(3+2\tau'\cdot\tau^2)'\int\frac{d\vec{q}}{(2\pi)^4}\cdot\Big[-\frac{\omega^2+\omega\omega'+\omega'^2}{\omega^2\omega'^3(\omega+\omega')}$$

$$-\frac{1}{2\omega^2\omega'^2}\Big(\frac{E_1+E_2'-e_c}{\omega^2}+\frac{E_1'+E_2-e_c}{\omega'^2}\Big)\Big], \tag{4.16}$$

with $\quad E = E_1+E_2\ ,\ e = e_1+e_2$ and $e_c = e_{1c}+e_{2c}$.

Here, we have used the energy consevation relation of Eq. (4.12), and neglected the higher order terms with respect to the kinetic energy. We can rewrite the TBE matrix element in terms of nuclear potential in Eqs. (2.37), (2.41), (4.4) and (4.8) as

$$T^{TBE} = T^{TBE}(uncross) + T^{TBE}(cross)$$

$$= V^{TBE}_{Stat} + V^{TBE}_{Non-Stat} + V^{OBE}_{Stat}G_N V^{OBE}_{Stat} + V^{OBE}_{Stat}G_N V^{OBE}_{Non-Stat} + V^{OBE}_{Non-Stat}G_N V^{OBE}_{Stat}. \tag{4.17}$$

The fourth order S-matrix in the potential theory coincides with that of the field theory. In the static limit the contribution of non-static OBE and TBE potential vanishes, since their contributions are linear to the kinetic energy of the nucleons,

$$V^{OBE}_{Non-Stat}\ G_N\ V^{OBE}_{Stat} \propto (\Delta E)^2\frac{1}{(\Delta E)} \longrightarrow 0 \tag{4.18}$$

and

$$V^{TBE}_{Non\text{-}Stat} \propto (\Delta E) \to 0 \quad \text{as} \quad \Delta E \to 0, \tag{4.19}$$

where ΔE shows the kinetic energy operator for nucleons. Besides, the iterated term of static OBE potential diverges. Thus the static TBE potential is unambiguously defined from the field theoretical S-matrix element irrespective of any form in the non-static OBE potential. It is, however, noticed here that the vanishing contribution of the non-static operator plays an essential role in the derivation of the static TBE potential.

4.3. Exchange Current and Radiative S-Matrix Element

i) One-Boson-Exchange Charge Density Operators

The S-matrix element of radiative two-nucleon scattering as shown in Fig. 9b is given as

$$T^{OBE}_{\gamma}(9b) = \frac{g^2}{(2\pi)^{\mu}}\left[\frac{e(1+\tau_z^1)}{2} e^{ik\cdot n}\tau^1\cdot\tau^2\right]\frac{1}{\hat{e}_1-(E_1-q_0)-i\epsilon}\,\frac{1}{\omega^2-q_0^2-i\epsilon}, \tag{4.20}$$

with $\quad q_0 = E_2' - E_2 = E_1 - E_1' - \Delta \quad$ and $\quad \hat{e}_1 = \frac{1}{2M}(\vec{p}_1 - q)^2.$

Neglecting the higher-order recoil corrections in Eq. (4.20), we have,

$$T^{OBE}_{\gamma}(9b) \simeq \frac{g^2}{(2\pi)^{\mu}}\left[\frac{e(1+\tau_z^1)}{2} e^{ik\cdot n}\tau^1\cdot\tau^2\right]$$

$$\times\left[-\frac{1}{E-\tilde{e}+i\epsilon}\left(\frac{1}{\omega^2}+\frac{(E_2'-E_2)^2+(E_1-\hat{e}_1)^2}{2\omega^4}\right)+\frac{E_2'-E_2+E_1-\hat{e}_1}{2\omega^4}\right], \tag{4.21}$$

with $\quad \tilde{e} = \hat{e}_1 + E_2' + \Delta \quad$ and $\quad E = E_1 + E_2 + \Delta$.

If we add the matrix element corresponding to Fig. 9c to Eq. (4.21), the field theoretical S-matrix element which is identical to the one in the potential picture is given as

$$T_{\gamma}^{OBE} = \rho_N G_N [V_{Stat}^{OBE} + V_{Non-Stat}^{OBE}] + [V_{Stat}^{OBE} + V_{Non-Stat}^{OBE}] G_N \rho_N$$

$$+ \rho_{Non-Stat}^{OBE,N} , \qquad (4.22)$$

where nuclear potentials and charge density operators are given in Eq. (2.37), (2.48) and (4.10). In the static limit the contribution if the non-static effective operator vanishes,

$$\rho_N \; G_N \; V_{Non-Stat}^{OBE} \propto \frac{1}{(\Delta E)} (\Delta E)^2 \longrightarrow 0 \qquad (4.23)$$

and

$$\rho_{Non-Stat}^{OBE,N} \propto (\Delta E) \longrightarrow 0 , \qquad (4.24)$$

while the iterated term of the static OBE potential diverges as

$$\rho_N \; G_N \; V_{Stat}^{OBE} \propto \frac{1}{(\Delta E)} \longrightarrow \frac{1}{i\epsilon} . \qquad (4.25)$$

Thus the finite part of T^{OBE} does not exist, and this fact shows the vanishing of the static OBE current, as we have seen in the unitary transformation method in sect. 2.4.

ii) Two-Boson-Exchange Charge Density Operator

For the TBE processes in the radiative two-nucleon scattering, we shall study the diagram Fig. 9d in detail. The other processes as in Fig. 9e–i can be discussed in the similar way. By using the standard method of the field theory we have

$$T_{\gamma}^{TBE}(9d) = \frac{g^4}{(2\pi)^6} \int d\vec{q} \int \frac{i}{2\pi} dx \; \frac{e}{2}(1+\tau_z^1)\, e^{ik\cdot r_1} \left[3 - 2\tau^1 \tau^2\right]$$

$$\times \left[(E_1' - \Delta - e_1' + i\epsilon)(E_2 + x - e_2 + i\epsilon)(-x + E_1 - e_1 + i\epsilon)(x^2 - \omega^2 + i\epsilon)(x^2 - \omega'^2 + i\epsilon)\right]^{-1} \qquad (4.26)$$

with $\quad e_1' = \frac{1}{2M}(\vec{p}_1' - \vec{k})^2.$

We perform the integration over x in Eq. (4.26) and expand it by the nucleon kinetic energy,

$$T_{\gamma}^{TBE}(9d) = \frac{g^4}{(2\pi)^6} \int d\vec{q} \left[\frac{e}{2}(1+\tau_z^1)\, e^{ik\cdot r_1} (3 - 2\tau^1 \tau^2)\right]$$

$$\times \left[\frac{1}{E-e'+i\epsilon}\left(-\frac{1}{\omega^2}\right)\frac{1}{E-e+i\epsilon}\left(-\frac{1}{\omega'^2}\right) + \frac{1}{E-e'+i\epsilon}\left(-\frac{1}{\omega^2}\right)\frac{1}{E-e+i\epsilon}\left(-\frac{(E_2-e_2)^2+(e_1-E_1)^2}{2\omega'^4}\right)\right.$$

$$+ \frac{1}{E-e'+i\epsilon}\left(-\frac{(E_2'-e_2)^2+(e_1'-e_1)^2}{2\omega^4}\right)\frac{1}{E-e+i\epsilon}\left(-\frac{1}{\omega'^2}\right) + \frac{1}{E-e'+i\epsilon}\left(\frac{\omega^2+\omega\omega'+\omega'^2}{2\omega^3\omega'^3(\omega+\omega')}\right)$$

$$\left. + \frac{E_2'+e_1-e_2-e_1'}{2\omega^4}\frac{1}{E-e+i\epsilon}\left(-\frac{1}{\omega'^2}\right) + \frac{1}{4\omega^2\omega'^2}\left(\frac{1}{\omega^2}+\frac{1}{\omega'^2}\right)\right]. \qquad (4.27)$$

Comparing Eq. (4.27) with Eqs. (2.37), (2.41), (2.48), (2.55), (2.56), (4.4), (4.8) and (4.10), we can easily show

$$T_{\gamma}^{TBE}(9d) = \rho_N G_N \left[V_{Stat}^{OBE} G_N V_{Stat}^{OBE} + V_{Stat}^{OBE} G_N V_{Non\text{-}Stat}^{OBE} + V_{Non\text{-}Stat}^{OBE} G_N V_{Stat}^{OBE} + V_{Stat}^{TBE}\right]$$

$$+ \int^{OBE,N}_{Non\text{-}Stat} G_N V^{OBE}_{Stat} + \int^{TBE,N}_{Stat} . \qquad (4.28)$$

This shows that the S-matrix is unique irrespective to the method of calculation. In the static limit, the iterated terms of the static effective operators diverges as is expected,

$$\int_N G_N V^{OBE}_{Stat} G_N V^{OBE}_{Stat} \propto \left(\frac{1}{\Delta E}\right)\left(\frac{1}{\Delta E}\right) \longrightarrow \left(\frac{1}{i\epsilon}\right)^2, \qquad (4.29)$$

and

$$\int_N G_N V^{TBE}_{Stat} \propto \left(\frac{1}{\Delta E}\right) \longrightarrow \left(\frac{1}{i\epsilon}\right), \qquad (4.30)$$

while the non-static operaotrs have the non-vanishing finite contribution as,

$$\int^{OBE,N}_{Non\text{-}Stat} G_N V^{OBE}_{Stat} \sim (\Delta E)\left(\frac{1}{\Delta E}\right) \rightarrow \text{finite} \qquad (4.31)$$

and

$$\int_N G_N V^{OBE}_{Stat} G_N V^{OBE}_{Non\text{-}Stat} \propto \left(\frac{1}{\Delta E}\right)\left(\frac{1}{\Delta E}\right)(\Delta E)^2 \rightarrow \text{finite} . \qquad (4.32)$$

Thus the finite part of the S-matrix element with static nucleon contains not only the contribution of static TBE current but also the contributions of the non-static operators as Eqs. (4.31) and (4.32). Now it is clear why the S-matrix method gives the static TBE current different from the one by the unitary transformation method. In the S-matrix method it is essentially important to subtract the iterates part of the non-static operators in order to obtain the static TBE current. It is noticed that the limiting procedure in Eqs. (4.31) and (4.32) is not unique, so that the static treatment of the S-matrix element cannot give us even the static charge density operators of multi-boson exchanges.

From the above investigation we can see that success of S-matrix method for the static OBE and TBE potential and OBE charge density operator is exceptional. In generall, unambiguous extraction of the non-static operator from the S-matrix element is basically impossible, so that the multiple boson exchange operator cannot be defined in the S-matrix method. This has not been recoginzed in the S-matrix treatment in the previous publications.[15,24]

Section 5. Arbitrariness of Unitary Transformation

5.1. Nambu's Method

We can derive the nuclear potential from studies of the time development of the system, which is closely related to th- S-matrix approach. In the one-boson-exchange process as illustrated in Fig. 10-a, the time t_1 at which boson interact with nucleon 1 is different from time t_2 when boson interacts with another nucleon 2 due to the propagation of the exchanged boson and nucleon recoil. On the other hand, in the potential picture, the interaction between two-nucleons must be instantaneous and of coirse hemitian (Fig. 10-b). Thus, in order to define the nuclear

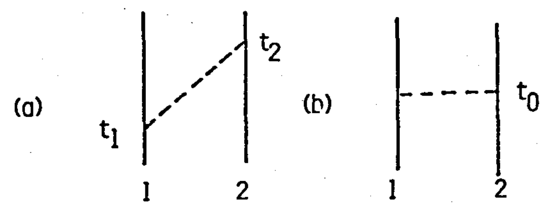


Fig. 10 Interaction of two nucleons due to OBE.

potential from the field theoretical boson-exchange picture, we must describe the process in Fig. 10-a by the single time, that is, average average time of two-nucleons. The OBE potential is defined at the time-base t_0 which describe the motion of the system, and average over the relative time between t_1 and t_2:

$$V^{OBE}(t_0) = \langle T^{OBE}(t_1, t_2) \rangle . \tag{5.1}$$

The bracket means the average over the relative time, which is shown in detail in the following. The two-boson-exchange processes include the iterated term of OBE potential. The TBE potential is derived from the fourth order S-matrix element by the subtraction of iterated term of OBE potentials and description by the time-base t_0

$$V^{TBE}(t_0) = \langle T^{TBE}(t_1, t_2, t_3, t_4) - V^{OBE}(t_0') G_N(t_0' - t_0'') V^{OBE}(t_0'') \rangle . \tag{5.2}$$

Here G_N is the Green function of free two-nucleons. In this way Nambu[22] derived the formula for the nuclear potential. And Taketani et al.[23] calculated two-pion-exchange potential by this method. This static TBE potential agrees with that of the unitary transformation method and the S-matrix method, which we have discussed in the previous section. But there is arbitrariness in the non-static part of nuclear potential. We shall see this arbitrariness in detail by the folded diagram method of Johnson[34], whose basic idea was given by Nambu.

The S-matrix element of the the OBE process (Fig. 10-a) given in the momentum space is

$$S = g^2 \tau^1 \cdot \tau^2 \int dt_1 dt_2 \int dq_0 \frac{i}{(2\pi)^4} \frac{e^{-iq_0(t_1 - t_2)}}{\omega^2 - q_0^2 - i\epsilon} e^{i(E_1' - E_1)t_1} e^{i(E_2' - E_2)t_2} . \tag{5.3}$$

On the other hand, the same S-matrix element is given in the potential picture as,

$$S = -2\pi i \, \delta(E' - E) \langle p_1', p_2' | V^{OBE} | p_1, p_2 \rangle$$

$$= -i \int dt_0 \, e^{-it_0(E' - E)} \langle p_1', p_2' | V^{OBE} | p_1, p_2 \rangle , \tag{5.4}$$

where $\quad E' = E_1' + E_2' \quad \text{and} \quad E = E_1 + E_2 .$

Here we shall define the time-base t_0 as

$$t_0 = \left(\frac{1+\lambda}{2}\right) t_1 + \left(\frac{1-\lambda}{2}\right) t_2 \tag{5.5}$$

where λ is the arbitrary parameter and relative time is defined as

$$T = t_1 - t_2 . \tag{5.6}$$

We shall give the S-matrix element at the time-base t_0 and integrate over the relative time T:

$$S = 1\, g^2 \tau^1 \cdot \tau^2 \int dt_0 \, e^{i(E'-E)t_0}$$

$$\times \int \frac{dq_0}{(2\pi)^3} \frac{1}{\omega^2 - q_0^2} \delta\left(q_0 - \frac{1-\lambda}{2} \Delta E_1 + \frac{1+\lambda}{2} \Delta E_2\right), \tag{5.7}$$

where $\quad \Delta E_s = E_s' - E_s .$

From Eqs. (5.4) and (5.7) we define the OBE potential with non-static correction as

$$V^{OBE}(\lambda) = -g^2 \tau^1 \cdot \tau^2 \int \frac{d\vec{q}}{(2\pi)^3} \frac{e^{-i\vec{q}\cdot\vec{r}}}{\omega^2 - \omega^2(\lambda)} \tag{5.8}$$

with

$$\omega(\lambda) = \Delta E_1 \left(\frac{1-\lambda}{2}\right) - \Delta E_2 \left(\frac{1+\lambda}{2}\right). \tag{5.9}$$

Here the OBE potential has arbitrary parameter λ , which correspond to the choice of the time-base. We define the OBE potential as the superposition of the potential (5.8) with the weight function $f(\lambda)$ as

$$V^{OBE} = \int f(\lambda)\, V^{OBE}(\lambda)\, d\lambda . \tag{5.10}$$

Here $f(\lambda)$ is normalized as

$$\int f(\lambda)\, d\lambda = 1 , \tag{5.11}$$

and it is symmetric with respective to the past and future,

$$f(\lambda) = f(-\lambda). \tag{5.12}$$

The OBE potential which we have derived by the unitary transformation method corresponds to the choice

$$f(\lambda) = \frac{1}{2}\left[\delta(\lambda-1) + \delta(\lambda+1)\right]. \tag{5.13}$$

Here we see that the non-static part of potential has the arbitrariness due to the choice of the time-base, while the static part of the potential is unique. We shall show in the next section, what this arbitrariness is in the unitary transformation method.

5.2. Arbitrariness of Unitary Transformation in Nuclear Potential and Exchange Current

We have shown that the nuclear potential and exchange current can be derived consistently by eliminating the bosons degrees of freedom from the original Hamiltonian. As an exapmle, we have derived explicit forms of the nuclear potential and exchange charge density operator in the system of nucleons and charged scalar bosons.

These operators are, however, not unique, since there still remains an ambiguity in the unitary transformation in the boson-vacuum sub-space. We shall study this arbitrariness of unitary transformation in the canonical transformation method of Nishijima[21], which is essentially the same as the unitary transformation method. Before going into the problem of arbitrariness of unitary transformation, we shall briefly summarize the method of the canonical transformation. Nuclear Hamiltonian is obtained from the total Hamiltonian of the system by eliminating order by order the off diagonal part, that is the interaction Hamiltonian which couples the boson-vacuum and boson-existing space. By the unitary transformation

$$U = \exp[iS] \tag{5.14}$$

the Hamiltonian H is transformaed into H' as

$$\begin{aligned} H' &= U^{-1} H U \\ &= H + i[H, S] + \frac{i^2}{2!}[[H, S], S] + \cdots \end{aligned} \tag{5.15}$$

with $H = H_0 + H_I,$

where S is hermitian. At first, in order to diagonalize the Hamiltonian up to the second order we must choose the operator S_1 which satisfies the following condition,

$$i\ [H_0, S_1] + H_I = 0\ . \tag{5.16}$$

Then the tranformed Hamiltonian is written as

$$H^{(1)} = H_0 + \frac{i}{2}[H_0, S_1] + \frac{i^2}{3}[[H_I, S_1], S_1] + \frac{i^3}{8}[[[H_I, S_1], S_1], S_1] + \cdots\ . \tag{5.17}$$

The second order potential is given as the diagonal part of the second term in Eq. (5.17)

$$V^{OBE} = \frac{i}{2}\ \eta\ [\ H_I, S_1\ ]\ \eta. \tag{5.18}$$

The explicit formula is,

$$V^{OBE} = -g^2 \tau^1 \cdot \tau^2 \int \frac{d\vec{q}}{(2\pi)^3} \frac{e^{-i\vec{q}\cdot\vec{r}}}{2} \left[\frac{1}{\omega^2 - \Delta\hat{E}_1^2} + \frac{1}{\omega^2 - \Delta\hat{E}_2^2} \right], \tag{5.19}$$

which coincide with Eq. (4.4). In the next step we eliminate the off-diagonal part of the second term in Eq. (5.18) by the transformation

$$U_2 = e^{iS_2} \tag{5.20}$$

with

$$i\ [H_0, S_2] + \frac{i}{2}\ [H_I, S_1]\ o.d.\ , \tag{5.21}$$

and $$\eta S_2 \eta = 0 .\tag{5.22}$$

The off diagonal element of the operator is denoted as o.d.. We obtain the transformed Hamiltonian $H^{(2)}$

$$H^{(2)} = H_0 + \frac{i}{2}\eta [H_I, S_1]\eta + \frac{i^3}{8}[[[H_I,S_1],S_1],S_1] + \frac{1}{2}[[H_0,S_2],S_2] + \cdots .\tag{5.23}$$

Here we have dropped the third term in Eq. (5.17), for that term has odd number of boson-nucleon interaction and thus it does not contribute to nuclear force. The TBE potential is defined as

$$V^{TBE} = \frac{i^3}{8}\eta [[[H_I,S_1],S_1],S_1]\eta + \frac{1}{2}\eta [[H_0,S_2],S_2]\eta .\tag{5.24}$$

From which we obtian the same potential as in sect. 2. This was first carried out by Nishijima.[21] By thse succesive transformation to eliminate the off diagonal part, we can diagonalize the Hamiltonian up to the desired order and obtain the nuclear potential in principle.

It is noticed here that the transformation is not unique. We shall derive the second order OBE potential by the canonical transformation different from the one in Eq. (5.16). We divide the free Hamiltonian of the system into two parts,

$$H_0 = T + K ,\tag{5.25}$$

where

$$T = \int \psi^+(\vec{x}) \left[-\frac{\vec{\nabla}^2}{2M}\right] \psi(\vec{x})\, d\vec{x}\tag{5.26}$$

and

$$K = \int \varphi_\alpha^{+}(\vec{x})(-\vec{\nabla}^2+\mu^2)\,\varphi(\vec{x})\,d\vec{x} \, . \tag{5.27}$$

Here, T and K are the free Hamiltonian of nucleon and boson fields, respectively. In the first unitary transformation U_1', we shall define the static OBE potential

$$U_1' = \exp\,[\,i\,S_1'\,]\,, \tag{5.28}$$

here U_1' is determied by

$$i\,[K\,,\,S_1'] + H_I = 0\,. \tag{5.29}$$

Here we take only the boson free energy in Eq. (5.29). The transformed Hamiltonian $H^{(1)}$ is

$$H^{(1)} = T + K + \frac{i}{2}[H_I, S_1'] + i\,[T, S_1'] + O\,(g^3) \tag{5.30}$$

The third term gives the static OBE potential as

$$\begin{aligned} V^{OBE}_{Stat} &= \frac{i}{2}\,\eta\,[\,H_I,\,S_1'\,]\,\eta \\ &= -\,g^2\,\tau^1\cdot\tau^2 \int \frac{d\vec{q}}{(2\pi)^3}\,\frac{e^{-i\vec{q}\cdot\vec{r}}}{\omega^2}\,. \end{aligned} \tag{5.31}$$

The Eq. (5.30) still have the off diagonal part with the first order in the boson-nucleon coupling constant and the nucleon kinetic energy. We shall eliminate this term by the second transformation U_2':

$$U_2' = \exp [i S_2'] \tag{5.32}$$

where

$$i [K, S_2'] + i [T, S_1'] = 0. \tag{5.33}$$

The resulting Hamiltonian is

$$H^{(2)} = T + K + \frac{i}{2}[V, S_1'] + \frac{i^2}{2}[[T, S_1'], S_2']$$

$$+ i [T, S_2'] + O(g^3). \tag{5.34}$$

The fourth term in Eq. (5.34) is the non-static correction to the static OBE potential.

$$V^{OBE}_{Non\text{-}Stat} = \frac{i^2}{2} \eta [[T, S_1'], S_2'] \eta$$

$$= g^2 \tau^1 \cdot \tau^2 \int \frac{d\vec{q}}{(2\pi)^3} \frac{e^{-i\vec{q}\cdot\vec{r}}}{\omega^4} [\Delta \hat{E}_1 \cdot \Delta \hat{E}_2] . \tag{5.35}$$

There still remains the off diagonal Hamiltonian in the first order of coupling constant, which depends upon the nucleon recoil operator. They contribute to the further higher order non-static correction to the OBE potential. At this stage we have different OBE potential due to different elimination procedure of off diagonal Hamiltonian. As we shall see from Eq. (5.19)

$$V^{OBE} \simeq -g^2 \tau^1 \cdot \tau^2 \int \frac{d\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \left[\frac{1}{\omega^2} + \frac{\Delta\hat{E}_1^2 + \Delta\hat{E}_2^2}{2\omega^4} \right], \tag{5.36}$$

which is obtained from the original Hamiltonian by the transformation

$$U_1 = e^{iS_1}. \tag{5.37}$$

And from Eqs. (5.31) and (5.35) we obtain

$$V^{OBE} \simeq -g^2 \tau^1 \cdot \tau^2 \int \frac{d\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \left[\frac{1}{\omega^2} - \frac{\Delta\hat{E}_1 \Delta\hat{E}_2}{\omega^4} \right], \tag{5.38}$$

which is given by the transformation

$$U_1' U_2' = e^{iS_1'} e^{iS_2'} \tag{5.39}$$

The static potential is uniquely derived but its non-static correction is not. Eq. (5.36) is transformed into Eq. (5.38) by the unitary transformation

$$\hat{U} = e^{-iS_1} e^{iS_1'} e^{iS_2'} \tag{5.40}$$

Clearly the above unitary operator transforms the boson-vacuum state into the boson-vacuum state. From this example, we can conclude there exist arbitrary unitary transformations. It is noticed here, so far as we define the potential and exchange current consistently in any fixed unitary transformation, the calculated observable remains unchanged.

These OBE potentials of Eqs. (5.36) and (5.38) are also obtained from point of view in the Nambu's method of sect. 4.1. That is the potential in Eq. (5.36) is obtained by the choice

$$f(\lambda) = \frac{1}{2} \left[\delta(\lambda+1) + \delta(\lambda-1) \right], \tag{5.41}$$

and the potential in Eq. (5.38) is obtained from

$$f(\lambda) = 2\delta(\lambda) - \frac{1}{2} \left[\delta(\lambda+1) + \delta(\lambda-1) \right]. \tag{5.42}$$

The physical meaning of arbitrariness of unitary transformation in the elimination of mesonic degrees of freedom corresponds to the arbitrariness of choosing the time-base in the field theoretical picture. These kinds of arbitrariness also occur in the other physical problems, where we trancate the Hilbert space and obtain the effective operators. One example is the Foldy-Wouthuysen-Tani transformation[35) (F-W-T) in the elimination of the negative energy component from the relativistic wave function.[36,37) This will be discussed in Appendix A.

Section 6. Conservation of Electromagnetic Current

To see the consistency of our treatment of effective operators, we shall investigate the conservation of nuclear electromagnetic current within the two-boson-exchange model. The conservation of the electromagnetic current is

$$\vec{\nabla}\cdot\vec{j}(\vec{x}) = -i\,[H, \rho(\vec{x})] \qquad (6.1)$$

where, $\vec{j}$, ρ and H are the nuclear current density, charge density and Hamiltonian, respectively. We shall intergrated form of current conservation relation as,

$$\vec{J} = -\int \vec{x}\,(\vec{\nabla}\cdot\vec{j}(\vec{x}))\,d\vec{x} \qquad (6.2a)$$

$$= i\,[H, \int \vec{x}\,\rho(\vec{x})]\,d\vec{x} \qquad (6.2b)$$

$$= i\,[H, \vec{D}], \qquad (6.2c)$$

with

$$\vec{D} = \int \vec{x}\,\rho(\vec{x})\,d\vec{x}. \qquad (6.2d)$$

Here, we shall show the consistency of the nuclear potential and the current operators in the unitary transformation method in the two-nucleons system, and show the importance of the non-static operaotrs. We have obtained the nuclear potential in the charged scalar boson-exchange model in sect. 2 and 4. We summarize the results here,

$$H = T + V \,, \tag{6.3}$$

with

$$T = \frac{\vec{p}_1^{\,2}}{2M} + \frac{\vec{p}_2^{\,2}}{2M} \,, \tag{6.3a}$$

and

$$V = V^{OBE}_{Stat} + V^{OBE}_{Non\text{-}Stat} + V^{TBE}_{Stat} + V^{TBE}_{Non\text{-}Stat} \,, \tag{6.3b}$$

where

$$V^{OBE}_{Stat} = -g^2 \tau^1 \cdot \tau^2 \int \frac{d\vec{q}}{(2\pi)^3} \frac{e^{-i\vec{q}\cdot\vec{r}}}{\omega^2} \,, \tag{6.3c}$$

$$V^{OBE}_{Non\text{-}Stat} = -\frac{g^2 \tau^1 \cdot \tau^2}{8M^2} \int \frac{d\vec{q}}{(2\pi)^3} \frac{e^{-i\vec{q}\cdot\vec{r}}}{\omega^4} \left[(q \cdot \overleftrightarrow{p}_1)^2 + (q \cdot \overleftrightarrow{p}_2)^2 \right] , \tag{6.3d}$$

$$V^{TBE}_{Stat} = -4 g^4 \tau^1 \cdot \tau^2 \int \frac{d\vec{q}\, d\vec{q}'}{(2\pi)^6} e^{-i(\vec{q}+\vec{q}')\cdot\vec{r}} \left[\frac{\omega^2 + \omega\omega' + \omega'^2}{2\omega^3 \omega'^3 (\omega + \omega')} \right] , \tag{6.3e}$$

and

$$V^{TBE}_{Non\text{-}Stat} = -\frac{g^4}{M} (3 + 2\tau^1 \cdot \tau^2) \int \frac{d\vec{q}\, d\vec{q}'}{(2\pi)^6} e^{-i(\vec{q}+\vec{q}')\cdot\vec{r}} \frac{(\vec{q}\cdot\vec{q}')}{2\omega^2 \omega'^2} \left[\frac{1}{\omega^2} + \frac{1}{\omega'^2} \right] . \tag{6.3f}$$

Here $\overleftrightarrow{p} = \vec{p} + \vec{p}\,'$

$\vec{p}$ and $\vec{p}\,'$ are the momentum operators operating the initial and final state vector.

i) One-Body-Current

In the impulse approximation, the one-body-current and dipole operator are

$$\vec{J}^N = e\sum_{s=1}^{2}\frac{1+\tau_z^s}{2}\frac{\overleftrightarrow{p_s}}{2M} \tag{6.4}$$

and

$$\vec{D}^N = e\sum_{s=1}^{2}\frac{1+\tau_z^s}{2}\vec{r}_s. \tag{6.5}$$

From Eqs. (6.3a), (6.4) and (6.5) we can show the charge conservation for non-interacting nucleons:

$$\vec{J}^N = i\,[T, \vec{D}^N]. \tag{6.6}$$

ii) One-Boson-Exchange Current

We have the static boson-type current in OBE model as

$$\vec{J}^{OBE,B}_{Stat} = eg^2\tau_x\vec{r}\int\frac{d\vec{q}}{(2\pi)^3}\frac{e^{-i\vec{q}\cdot\vec{r}}}{\omega^2}. \tag{6.7}$$

This current is related to the static OBE potential as

$$\vec{J}^{OBE,B}_{Stat} = i\,[V^{OBE}_{Stat}, \vec{D}^N]. \tag{6.8}$$

In addition to the static current, we have the non-static nucleon- and boson-type currents

$$\vec{J}^{OBE,B}_{Non\text{-}Stat} = \frac{ieg^2}{4M^2}\tau_x\int\frac{d\vec{q}}{(2\pi)^3}e^{-i\vec{q}\cdot\vec{r}}\frac{2\vec{q}}{\omega^4}\left[(\hat{p}_1\cdot q)^2+(\hat{p}_2\cdot q)^2\right], \tag{6.9}$$

and

$$\vec{J}^{OBE,N}_{Non\text{-}Stat} = -\frac{eg^2}{4M^2}\int\frac{d\vec{q}}{(2\pi)^3}\,\frac{e^{-i\vec{q}\cdot\vec{r}}}{\omega^4}\left[\frac{(\overleftrightarrow{p}_1+\overleftrightarrow{p}_2)}{2}\cdot q\right]$$

$$\cdot\left[i\,\tau_\times(\hat{p}_1+\hat{p}_2)+(\tau^1+\tau^2)_z\,\vec{q}+2\,\tau^1\cdot\tau^2\,\vec{q}\right]. \qquad (6.10)$$

They are related to the non-static OBE potential and non-static OBE charge density operator:

$$\vec{J}^{OBE,B}_{Non\text{-}Stat}+\vec{J}^{OBE,N}_{Non\text{-}Stat} = i\left[V^{OBE}_{Non\text{-}Stat},\vec{D}^N\right]+i\left[T,\vec{D}^{OBE,B}_{Non\text{-}Stat}+\vec{D}^{OBE,N}_{Non\text{-}Stat}\right], \qquad (6.11)$$

where

$$\vec{D}^{OBE,B}_{Non\text{-}Stat} = \frac{eg^2}{4M}\tau_\times\int\frac{d\vec{q}}{(2\pi)^3}\,\frac{e^{-i\vec{q}\cdot\vec{r}}}{\omega^4}\left[-i\,(\hat{p}_1-\hat{p}_2)-(\vec{r}_1+\vec{r}_2)\,\vec{q}\cdot(\hat{p}_1+\hat{p}_2)\right], \qquad (6.12)$$

and

$$\vec{D}^{OBE,N}_{Non\text{-}Stat} = \frac{eg^2}{4M}\int\frac{d\vec{q}}{(2\pi)^3}\,\frac{e^{i\vec{q}\cdot\vec{r}}}{\omega^4}\left[(\tau^1-\tau^2)_z\,\vec{q}+\tau_\times(\vec{r}_1+\vec{r}_2)\,\vec{q}\cdot(\hat{p}_1+\hat{p}_2)\right]. \qquad (6.13)$$

Here it is noticed that the static charge density operator does not exist in the OBE model.

ii) Two-Boson-Exchange Current

As in the case of OBE current, we have the static boson-type current,

$$\vec{J}^{TBE,B}_{Stat} = 4\,eg^4\,\tau_\times\,\vec{r}\int\frac{d\vec{q}\,d\vec{q}'}{(2\pi)^6}\,e^{-i(\vec{q}+\vec{q}')\cdot\vec{r}}\,\frac{\omega^2+\omega\omega'+\omega'^2}{2\omega^3\omega'^3(\omega+\omega')}, \qquad (6.14)$$

which is related to the static TBE potential as

$$\vec{J}^{TBE,B}_{Stat} = i\,[\,V^{TBE}_{Stat}\,,\vec{D}^{N}]. \tag{6.15}$$

The non-static boson-type and nucleon-type currents are

$$\vec{J}^{TBE,B}_{Non-Stat} = \frac{eg^4}{M}\int\frac{d\vec{q}\,d\vec{q}'}{(2\pi)^6}\,\frac{e^{-i(\vec{q}+\vec{q}')\cdot\vec{r}}}{2\omega^2\omega'^2}\left[\frac{1}{\omega^2}+\frac{1}{\omega'^2}\right]$$

$$\times\Big\{(\tau^1+\tau^2)_3\,(\vec{p}_1+\vec{p}_2)+2\,(\tau^1-\tau^2)_3\,(\vec{p}_1-\vec{p}_2+i\,\vec{r}\,(q+q')\cdot(\vec{p}_1-\vec{p}_2))$$

$$+\,i\,\tau_\times\,(\vec{q}+\vec{q}'-2i\,\vec{r}\,(\vec{q}\cdot\vec{q}'))\Big\}, \tag{6.16}$$

and

$$\vec{J}^{TBE,N}_{Stat} = \frac{eg^4}{M}\int\frac{d\vec{q}\,d\vec{q}'}{(2\pi)^6}\,\frac{e^{-i(\vec{q}+\vec{q}')\cdot\vec{r}}}{2\omega^2\omega'^2}\left(\frac{1}{\omega^2}+\frac{1}{\omega'^2}\right)$$

$$\times\Big\{-(\tau^1+\tau^2)_3\,(\hat{p}_1+\hat{p}_2)+(\tau^1-\tau^2)_3\,(\hat{p}-\hat{p}_2)-2i\,\tau_\times(\vec{q}+\vec{q}')\Big\}. \tag{6.17}$$

They are related to the static TBE dipole operator, non-static OBE dipole operator and non-static TBE potential in the following:

$$\vec{J}^{TBE,B}_{Non-Stat}+\vec{J}^{TBE,N}_{Stat} = i\,[\,V^{TBE}_{Non-Stat},\vec{D}^{N}]+i\,[V^{OBE}_{Stat}\,,\,\vec{D}^{OBE,N}_{Non-Stat}+\vec{D}^{OBE,B}_{Non-Stat}]$$

$$+\,i\,[T,\vec{D}^{TBE,N}_{Stat}+\vec{D}^{TBE,B}_{Stat}], \tag{6.18}$$

where

$$\vec{D}^{TBE,N}_{Stat} = 2eg^4\,[(\tau^1-\tau^2)_3\,\vec{r}-(\tau^1+\tau^2)_3\,(\vec{r}_1+\vec{r}_2)]\int\frac{d\vec{q}\,d\vec{q}'}{(2\pi)^6}$$

$$\times \frac{e^{-i(\vec{q}+\vec{q}')\cdot\vec{r}}}{2\omega^2\omega'^2}\left(\frac{1}{\omega^2}+\frac{1}{\omega'^2}\right), \tag{6.19}$$

and

$$\vec{D}^{TBE,B}_{Stat} = 2eg^4(\tau+\tau^2)_z(\vec{r}_1+\vec{r}_2)\int\frac{d\vec{q}\,d\vec{q}'}{(2\pi)^6}$$

$$\times \frac{e^{-i(\vec{q}+\vec{q}')\cdot\vec{r}}}{2\omega^2\omega'^2}\left(\frac{1}{\omega^2}+\frac{1}{\omega'^2}\right). \tag{6.20}$$

Eqs. (6.19) and (6.20) show the non-vanishing static charge density operator in the TBE model. This was pointed out by Hyuga and Ohtsubo[17] as the break down of Siegert theorem even in the static nucleon limit. The currents in Eqs. (6.6), (6.8) and (6.15) are given from the Siegert theorem[3], i.e.,

$$\vec{J}^N + \vec{J}^{OBE,B}_{Stat} + \vec{J}^{TBE,B}_{Stat} = i\,[T_T\; V^{OBE}_{Stat} + V^{TBE}_{Stat}\;,\; \vec{D}^N] \tag{6.21}$$

Summarizing the above discussions, we have the current conservation as follows:

$$\vec{J} = i\;[H\,,\,\vec{D}] \tag{6.22}$$

where

$$\vec{J} = \vec{J}^N + \vec{J}^{OBE,B}_{Stat} + \vec{J}^{OBE,B}_{Non\text{-}Stat} + \vec{J}^{OBE,N}_{Non\text{-}Stat} + \vec{J}^{TBE,B}_{Stat} + \vec{J}^{TBE,B}_{Non\text{-}Stat} + \vec{J}^{TBE,N}_{Stat}, \tag{6.23}$$

and

$$\vec{D} = \vec{D}^{U} + \vec{D}^{OBE,B}_{Non\text{-}Stat} + \vec{D}^{OBE,N}_{Non\text{-}Stat} + \vec{D}^{TBE,B}_{Stat} + \vec{D}^{TBE,N}_{Stat} . \tag{6.24}$$

To construct a consistent model for the nuclear Hamiltonian and current operators, it is necessary to include the non-static operators. For example, to satisfy the conservation relation corresponding to the static charge density operator we need the non-static spatial current and non-static potential. Here, the non-static operators we have derived in part I are due to the kinematical nucleon recoil, but not the effect of the negative enrgy states. Since the kinetic energy operator of nucleon is one of the non-static operators in this sense, which is already included in the Schrödinger equation, it is naturally understood that the non-static effective operators are required in a consistent description of the nuclear system.

Section 7. Summary of Part I

We have shown the relation between the various methods which define the effective operators in boson-vacuum space by eliminating the mesonic degrees of freedom from the field theoretical Hamiltonian of the system. We shall summarize the results obtained in Part I.

The discrepancy in the static two-boson-exchange currents derived from the unitary transformation method[19,20] and the S-matrix method[23] is due to the insufficient subtruction of the iterated terms of one-boson-exchange non-static potential and exchange current in the S-matrix method. The contributions to the finite part of the static S-matrix element are not only the contribution of the static current, but also include the contribution of the iterated terms of the non-static potential and exchange currents. Of course, the S-matrix element is uniquely given irrespective of any method of calculation, if the method is selfconsistent. It is exceptional that the same static TBE potential is obtained by the S-matrix method, since the contribution of the non-static OBE potential vanishes in the static limit. In general, the finite part of S-matrix elementwith static nucleon does not give a correct multiple-boson exchange potential. Inversely, if we define these non-static effective operators by any principle and subtract them from the S-matrix element with full nucleon recoil correctly, we can obtain the static TBE current. This can be achieved by the folded diagram method[34].

Both the Nambu's method[22] and the unitary transformation method have the arbitrariness. For example, the different non-static OBE potential is obtained by the different elimination of the mesonic degrees of freedom in the unitary transformation method and also by the different choice of the time-base in the Nambu's method. As we have shown, these arbitrarinesses are related with each other. The arbitrariness of unitary transformation in the boson-vacuum space, which does not couple the boson-vacuum space to boson-existing space, corresponding to the arbitrary choice of the time-base, when we reduce the field theoretical S-matrix element into the single time of the system, in other words, when we are dealing with an instantaneous interaction. These kinds of arbitrariness of unitary transformation also appear when we eliminate the negative energy component of Dirac field. This is the arbitrariness of the F-W-T transformation.[36,37] It is noticed that this arbitrariness does not affect the matrix element of the observables. If we fix the unitary transformation , we should use the effective operators and the state vectors obtained by the same unitary transformation.

The consistency of the exchange current and the nuclear potential in the unitary transformation method is shown by proving charge conservation within the TBE model. We include here both boson- and nucleon-type currents in the charged scalar boson-exchange model. To verify this charge conservation law, the non-static exchange current and potential cannot be neglected. For example, even the static TBE charge density operator is related to the non-static exchange current and potential. The role of the non-static effective operator is also essential in the derivation of the static TBE current in the S-amtrix method. A consistent derivation of the nuclear potential and exchange current is thus important. Althouth there are many methods to define the effective operators which are equivalent to each other, we believe the unitary transformation method discussed in sect. 1, is practically the most useful method, while the canonical transformation method[21] is convenient to see the transformation property of the theorem, and the Nambu's method is convenient to see the physical or the graphical meaning of the effective operators.

Part II. Magnetic Moment of Deuteron

Section 8. Magnetic Moment of Deuteron

We shall investigate the magnetic moment of deuteron. The operator of magnetic moment is expressed in terms of spatial part of nuclear current density operator $\vec{J}(\vec{x})$ as

$$\vec{\mu} = \frac{1}{2} \int \vec{x} \times \vec{J}(\vec{x})\, d\vec{x} \qquad (8.1)$$

and the nuclear magnetic moment is defined as

$$\mu = \langle J, J_z = J \mid \vec{\mu}_z \mid J, J_z = J \rangle . \qquad (8.2)$$

Since the deuteron has spin J = 1 and isospin T = 0, only the isoscalar part of magnetic moment operator contributes to the magnetic moment of deuteron. The deuteron wave function is written as

$$\Psi = \frac{1}{\sqrt{4\pi}} \left[\frac{u(r)}{r} + \frac{3}{\sqrt{8}} S_{12} \frac{w(r)}{r} \right] \chi(\sigma)\, \chi(\tau), \qquad (8.3)$$

where $$S_{12} = (\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - \frac{1}{3} \sigma_1 \cdot \sigma_2 .$$

$\chi(\sigma)$ is the spin wave function with S = 1, and $\chi(\tau)$ is the isospin one with T = 0. The radial wave functions of S- and D-state are denoted as u(r) and w(r), respectively, and they are normalized as

$$\int_0^\infty \left[u^2(r) + w^2(r) \right] dr = 1 . \qquad (8.4)$$

In the impulse approximation, the magnetic moment μ_{IA} is given with Eq. (8.3) by the magnetic moments of nucleons and the D-state probability P_D as

$$\mu_{IA} = (\mu_p + \mu_p)(1 - \frac{3}{2} P_D) + \frac{3}{4} P_D, \tag{8.5}$$

with $$P_D = \int_0^\infty w^2(r)\, dr$$

and $$\mu_p + \mu_n = 0.8798$$

in units of nuclear magneton.

We shall adopt the wave function of the deuteron in the Reid-potential[28], which reproduces experimental data on the two-nucleon system satisfactorily and is widely used in nuclear physics. Although the Reid-potential is not derived meson-theoretically, but is the phenomenological one, we assume that our model of the interaction Hamiltonian will reproduce the Reid-potential. If we use the following values

$$P_D = 6.497\% \text{ for the hard core potential (H. C.)} \tag{8.6}$$

$$P_D = 6.470\% \text{ for the soft core potential (S. C.)} \tag{8.7}$$

given by Reid, we obtain

$$\mu_{IA} = 0.8428 \text{ (H. C.)}, \tag{8.8}$$

$$\mu_{IA} = 0.8429 \text{ (S. C.)}, \tag{8.9}$$

while the measured deuteron magnetic moment [38] is

$$\mu_d = 0.857406 \pm 0.000001 \text{ (n.m.)} \tag{8.10}$$

Thus we find a discrepancy between the calculation of impulse approximation and the experiment by about 0.014 n.m. It is very interesting to resolve this discrepancy:

$$\Delta\mu_{exp} = \mu_d - \mu_{IA} = 0.0146 \text{ (H. C.)} \tag{8.11}$$

$$= 0.0145 \text{ (S. C.)} \tag{8.12}$$

We shall take into account relativistic correction to the one-body-operator, $\Delta\mu_{\text{rel. cor.}}$, OBE current μ_{OBE}, and TBE current μ_{TBE},

$$\Delta\mu_{theor.} = \Delta\mu_{rel.cor.} + \mu_{OBE} + \mu_{TBE} \; . \tag{8.13}$$

We study the dissociation current, pair current and also the first-order recoil correction to the nucleon-type current in the OBE model. We adopt the static limit in the TBE current: We retain the nucleon-type current due to the intrinsic magnetic moment and orbital motion of nucleon, and dissociation current of boson-type. For the realistic description of boson-nucleon system, we take into account pi-, rho- and omega-meson exchange currents. Further we include the isobar (Δ_{1236}) in the TBE current, with which we treat approximately an important part of virtual p-wave pion-nucleon scattering. These exchange magnetic moment operators are derived from the unitary transformation method in sect. 2, where we take into account the isobar by modifying the projection operator Λ and η as the projection on the boson- and isobar-existing space, and boson- and isobar-vacuum space, respectively.

Section 9. Interaction Hamiltonian

9.1. Strong Interaction Hamiltonian

Now, we take the following Hamiltonian of boson-nucleon interactions:

$$\langle N(p')|H_{\pi NN}|N(p)\pi^j(q)\rangle = -ig_\pi \bar{u}(p')\tau_j\gamma_5 u(p), \quad \text{for pion,} \tag{9.1}$$

$$\langle N(p')|H_{\rho NN}|N(p)\rho_\mu^j(q)\rangle = ig_\rho \bar{u}(p')\tau_j [\gamma_\mu - \frac{\kappa_\rho}{2M}\sigma_{\mu\nu}q_\nu]u(p), \quad \text{for rho,} \tag{9.2}$$

and

$$\langle N(p')|H_{\omega NN}|N(p)\omega_\mu(q)\rangle = ig_\omega \bar{u}(p')[\gamma_\mu - \frac{\kappa_\omega}{2M}\sigma_{\mu\nu}q_\nu]u(p), \quad \text{for omega,} \tag{9.3}$$

with $\quad q_\lambda = p'_\lambda - p_\lambda$,

where the index j shows the isospin component of boson, and we adopted the hermite Pauli matrix for γ_μ.

The interaction Hamiltonian of nucleon (N) and isobar (Δ) are written as

$$\langle \Delta_\mu(p')|H_{\pi N\Delta}|N(p)\pi^j(q)\rangle = i\frac{f_{\pi N\Delta}}{m_\pi} q_\mu \bar{u}_{\mu,j}(p')u(p) \tag{9.4}$$

for pion and

$$\langle \Delta_\nu(p')|H_{\rho N\Delta}|N(p)\rho_\mu^j(q)\rangle = i\frac{f_{\rho N\Delta}}{m_\rho} q_\nu \bar{u}_{\mu j}(p')\gamma_5\gamma_\mu u(p) \tag{9.5}$$

for rho-meson.

Here, $\mathcal{U}_\mu$ is the vector-spinor for a particle with spin 3/2. Omega-meson does not couple to the N-Δ vertex.

9.2. Electromagnetic Interaction Hamiltonian

We shall show only the isoscalar current relevant to the deuteron magnetic moment. The electromagnetic interaction Hamiltonian is given as

$$H_{em} = \int J_\mu^{em}(\vec{x})\, A_\mu(\vec{x})\, d\vec{x}. \tag{9.6}$$

Here, the nucleon current is expressed as

$$\langle N(p')\,|\, J_\mu^{em,s}\,|\, N(p)\rangle = \frac{ie}{2}\,\bar{u}(p')\left[\gamma_\mu - \frac{\kappa_s}{2M}\,\sigma_{\mu\nu} q_\nu\right] u(p), \tag{9.7}$$

and the isobar current as

$$\langle \Delta_{\nu j}(p')\,|\, J_\mu^{em,s}\,|\, \Delta_{\nu j}(p)\rangle = i\frac{e}{2}\,\bar{u}_{\nu j}(p')\left[\gamma_\mu - \frac{\kappa_s^\Delta}{2M_\Delta}\,\sigma_{\mu\nu} q_\nu\right] u_{\nu j}(p). \tag{9.8}$$

The dissociation current of rho-meson into pion is given by

$$\langle \pi^j(p')\,|\, J_\mu^{em,s}\,|\, \rho_\nu^{\ell}(p)\rangle = -i\,\frac{e\, g_{\rho\pi\gamma}}{m_\rho}\,\epsilon_{\mu\nu\sigma\lambda}\, p'_\lambda\, p_\sigma\, \delta_{j\ell} \tag{9.10}$$

Here the boson-type currents of pion and rho-meson do not contribute , since they are purely of the isovector type.

Section 10. Elimination of Component with Negative Energy

We shall eliminate at first the negative energy component of the nucleon, and then the mesonic degrees of freedom from the nuclear wave function. We shall reduce the covariant expression of the interaction Hamiltonian into the two-component effective Hamiltonian of the non-relativistic nucleon. The lowest order static Hamiltonians are directly obtained by the non-relativistic reduction of the Dirac spinors. The strong interaction Hamiltonian of N-Δ and the electromagnetic current of Δ are reduced in this way. On the other hand, for the strong interaction Hamiltonian of the nucleon and the electromagnetic current of the nucleon, we need the relativistic correction to the one-body-current and OBE current. They are obtained by eliminating the negative energy component of the nucleon by using F-W-T transformation.

The one-body Direc Hamiltonian for nucleon is

$$h = \beta M + \mathcal{E} + \mathcal{O}, \tag{10.1}$$

where

$$\mathcal{E} = \mathcal{E}_{\gamma NN} + \mathcal{E}_{\rho NN} + \mathcal{E}_{\omega NN} \tag{10.2}$$

and

$$\mathcal{O} = \vec{\alpha}\cdot\vec{p} + \mathcal{O}_{\gamma NN} + \mathcal{O}_{\pi NN} + \mathcal{O}_{\rho NN} + \mathcal{O}_{\omega NN} \tag{10.3}$$

with

$$\mathcal{E}_{\gamma NN} = \frac{e}{2}\phi - \frac{e}{4M}\kappa_s \beta \vec{\Sigma}\cdot\vec{B}, \tag{10.4}$$

$$\mathcal{E}_{\rho NN} = g_\rho \underset{\sim}{\tau} [\underset{\sim}{\phi}_0^\rho - \frac{\kappa_\rho}{2M} \beta \vec{\Sigma} \times \vec{\nabla}_\rho \cdot \underset{\sim}{\vec{\phi}}^\rho], \tag{10.5}$$

$$\mathcal{E}_{\omega NN} = g_\omega [\phi_0^\omega - \frac{\kappa_\omega}{2M} \beta \vec{\Sigma} \times \vec{\nabla}_\omega \cdot \vec{\phi}^\omega], \tag{10.6}$$

$$\mathcal{O}_{\gamma NN} = -\frac{e}{2} \vec{\alpha} \cdot \vec{A} + \frac{i\kappa_S}{4M} \beta \vec{\alpha} \cdot \vec{E}, \tag{10.7}$$

$$\mathcal{O}_{\pi NN} = i g_\pi \gamma_4 \gamma_5 \underset{\sim}{\tau} \underset{\sim}{\phi}^\pi, \tag{10.8}$$

$$\mathcal{O}_{\rho NN} = - g_\rho \underset{\sim}{\tau} [\vec{\alpha} \cdot \underset{\sim}{\vec{\phi}}^\rho + \frac{i\kappa_\rho}{2M} \beta \vec{\alpha} \cdot (\vec{\nabla}_\rho \underset{\sim}{\phi}_0^\rho + \dot{\underset{\sim}{\vec{\phi}}}^\rho)], \tag{10.9}$$

$$\mathcal{O}_{\omega NN} = - g_\omega [\vec{\alpha} \cdot \vec{\phi}^\omega + \frac{i\kappa_\omega}{2M} \beta \vec{\alpha} \cdot (\vec{\nabla}_\omega \phi_0^\omega + \dot{\vec{\phi}}^\omega)], \tag{10.10}$$

where $\vec{B} = \vec{\nabla}_\gamma \times \vec{A}$, $\vec{E} = -\vec{\nabla}_\sigma \phi - \dot{\vec{A}}$ and $\vec{\Sigma} = (\vec{\sigma} \; {}_{\vec{\sigma}})$.

$(\vec{A}, \phi)$ is the electromagnetic field, $\underset{\sim}{\phi}^\pi$, $\underset{\sim}{\phi}_\mu^\rho$, ϕ_μ^ω are the field operators of pion, rho- and omega-meson, respectively. We have replaced the momentum transfer q_μ by the derivative operators on the boson field in the equations of sect. 9.

The F-W-T transformation has the arbitrariness of unitary transformation as shown in Appendix A. To see this explicitly, we divide the odd operator into two parts A and B.

$$\mathcal{O} = \mathcal{A} + \mathcal{B} \tag{10.11}$$

with

$$\mathcal{A} = \lambda_1 \vec{\alpha} \cdot \vec{P} - \lambda_2 \frac{e}{2} \vec{\alpha} \cdot \vec{A} + \lambda_3 i g_\pi \gamma_4 \gamma_5 \underset{\sim}{\tau} \cdot \underset{\sim}{\phi}^\pi + \mathcal{O}_{\rho NN} + \mathcal{O}_{\omega NN}, \tag{10.12}$$

$$B = (1-\lambda_1)\,\vec{\alpha}\cdot\vec{p} - (1-\lambda_2)\frac{e}{2}\vec{\alpha}\cdot\vec{A} + (1-\lambda_3)\, ig_\pi \gamma_4\gamma_5 \underset{\sim}{\tau}\underset{\sim}{\phi}^{\pi}. \tag{10.13}$$

Here, λ_1, λ_2 and λ_3 are the arbitrary constants. We also define constants a, b and c as Ref. 37).

$$a = \lambda_1 - \lambda_2,\quad b = \lambda_2 - \lambda_3 \quad \text{and} \quad c = \lambda_3 - \lambda_1, \tag{10.14}$$

where
$$a + b + c = 0. \tag{10.15}$$

In what follows we shall keep the terms linear to the boson-field operators. We have fixed the transformation relevant to the rho- and omega-meson-fields as in Eq. (10.12), since their contributions are uniquely given within our approximation.

10.1. Electromagnetic Interaction of Nucleon and Isobar

The electromagnetic interaction of the nucleon is written as

$$H_{\gamma NN} = H_{\gamma NN}\,(\text{static}) + H_{\gamma NN}\,(\text{rel. cor.}), \tag{10.16}$$

where

$$H_{\gamma NN}\,(\text{static}) = -\frac{e}{4M}\left[\vec{p}\cdot\vec{A} + \vec{A}\cdot\vec{p} + (1+\kappa_s)\,\vec{\sigma}\cdot\vec{B}\right]. \tag{10.17}$$

$$H_{\gamma NN}\,(\text{rel. cor.}) = \frac{e}{16M^3}\left(\{\vec{p}^{\,2}, \vec{p}\cdot\vec{A} + \vec{A}\cdot\vec{p} + \vec{\sigma}\cdot\vec{B}\}_+ + \frac{i}{2}\,[\vec{p}^{\,2}, \vec{\sigma}\times\vec{p}\cdot\vec{A} + \vec{A}\cdot\vec{\sigma}\times\vec{p}]\right)$$
$$+\frac{e\kappa_s}{16M^3}\left(\vec{\sigma}\cdot\vec{p}\,\vec{\sigma}\cdot\vec{B}\,\vec{\sigma}\cdot\vec{p} + \frac{1}{2}\{\vec{p}^{\,2}, \vec{B}\cdot\vec{\sigma}\}_+\right). \tag{10.18}$$

Here, we only show a part of Hamiltonian relevant to the magnetic moment operator. $H_{\gamma NN}$(rel. cor.) gives the relativistic correction to the one-body-current due to Zitterbewegung.

The static operator of isobar current is

$$H_{\gamma\Delta\Delta} = -\frac{e}{4M_\Delta}\left(\vec{P}_\Delta\cdot\vec{A} + \vec{A}\cdot\vec{P}_\Delta + (1+\kappa_S^\Delta)\,\vec{\sigma}_\Delta\cdot\vec{B}\right). \tag{10.19}$$

Here, $\vec{p}_\Delta$ is the momentum operator of isobar. σ_Δ is the Pauli-spin matrix.

10.2. Electromagnetic Interaction of Seagull Type

Here, we show the contact interaction of boson-photon ($B\gamma N$), which gives rise to a relativistic correction to the magnetic moment operator. The $\pi\gamma NN$ interaction in the ps-ps coupling scheme is given as

$$\begin{aligned} H_{\pi\gamma NN} = {} & \frac{e g_\pi}{8M^3}(1+\kappa_s)\,\underset{\sim}{\tau}\,\vec{\nabla}_\pi\cdot\vec{B}\,\underset{\sim}{\phi}^\pi \\ & + \frac{e g_\pi}{16M^3}\left[(1-(1+\kappa_s)c)\,\epsilon_{ijk}\sigma_k\{\vec{P}_i,\vec{B}_j\underset{\sim}{\phi}^\pi\cdot\underset{\sim}{\tau}\}_+ + ib\,[\vec{P}^2,\vec{\sigma}\cdot\vec{A}\,\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}^\pi]_- + \{\vec{P},\vec{X}\}_+\right] \\ & - \frac{e g_\pi}{8M^2}(1-b)\,\vec{\sigma}\cdot\vec{A}\,\underset{\sim}{\tau}\,\dot{\underset{\sim}{\phi}}^\pi, \end{aligned} \tag{10.20}$$

with

$$\vec{X} = [2\vec{A}\,(\vec{\sigma}\cdot\vec{\nabla}_\pi) + b\,\vec{\sigma}\,(\vec{A}\cdot\vec{\nabla}_\pi) + a\,\vec{\nabla}_\pi\,(\vec{\sigma}\cdot\vec{A}) - c\,\vec{A}\,(\vec{\sigma}\cdot\vec{\nabla}_\pi)]\,\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}^\pi.$$

The first line in Eq. (10.19) gives the local operator adopted in Ref. 14). There,however, remain the other momentum dependent terms, which cannot be neglected in the general case. The non-local terms depend on the parameters in the unitary transformation.

The $\rho\gamma$NN and $\omega\gamma$NN interactions are given as,

$$H_{\rho\gamma NN} = -\frac{e}{2M} g_\rho \underset{\sim}{\tau} \left[\vec{\underset{\sim}{\phi}}^\rho \cdot \vec{A} + \frac{(1+\kappa_\rho)}{2M} \vec{A} \times \vec{\nabla}_\rho \cdot \vec{\sigma} \underset{\sim}{\phi}_0^\rho \right], \tag{10.21}$$

$$H_{\omega\gamma NN} = -\frac{e}{2M} g_\omega \left[\vec{\phi}^\omega \cdot \vec{A} + \frac{(1+\kappa_\omega)}{2M} \vec{A} \times \vec{\nabla}_\omega \cdot \vec{\sigma} \phi_0^\omega \right]. \tag{10.22}$$

These interactions in Eqs. (10.21) and (10.22) are uniquely given irrespective to the arbitrariness of F-W-T transformation.

10.3. Strong Interaction Hamiltonian

We need only the lowest order static boson-nucleon Hamiltonian in Eqs. (9.1-3). The relativistic correction of these boson-nucleon vertex gives the higher order relativistic correction to the non-static OBE current and we shall neglect these terms. Thus the boson-nucleon interaction Hamiltonians read

$$H_{\pi NN} = \frac{g_\pi}{2M} \vec{\sigma} \cdot \vec{\nabla}_\pi \underset{\sim}{\tau} \underset{\sim}{\phi}^\pi, \tag{10.23}$$

$$H_{\rho NN} = -\frac{g_\rho}{2M} \underset{\sim}{\tau} \left[\vec{p} \cdot \vec{\underset{\sim}{\phi}}^\rho + \vec{\underset{\sim}{\phi}}^\rho \cdot \vec{p} + (1+\kappa_\rho) \vec{\sigma} \times \vec{\nabla}_\rho \cdot \vec{\underset{\sim}{\phi}}^\rho \right] + g_\rho \underset{\sim}{\tau} \underset{\sim}{\phi}_0^\rho, \tag{10.24}$$

$$H_{\omega NN} = -\frac{g_\omega}{2M} \left[\vec{p} \cdot \vec{\phi}^\omega + \vec{\phi}^\omega \cdot \vec{p} + (1+\kappa_\omega) \vec{\sigma} \times \vec{\nabla}_\omega \cdot \vec{\phi}^\omega \right] + g_\omega \phi_0^\omega . \tag{10.25}$$

Eqs. (10.23), (10.24) and (10.25) correspond to the pi-, rho- and omega-meson nucleon interactions, respectively. We also have the rho-pion interaction of seagull-type

$$H_{\pi\rho NN} = -\frac{g_\pi g_\rho}{M}\ \vec{\sigma}\cdot\vec{\phi}^{\rho}\times\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}^{\pi}\ . \tag{10.26}$$

The isobar-boson interactions are given in the static approximation as,

$$H_{\pi N\Delta} = \frac{f_{\pi N\Delta}}{m_\pi}\ \vec{S}\cdot\vec{\nabla}_\pi\ \underset{\sim}{T}\,\underset{\sim}{\phi}^{\pi} + \text{h. c.}\ , \tag{10.27}$$

$$H_{\rho N\Delta} = -\frac{f_{\rho N\Delta}}{m_\rho}\ \vec{S}\times\vec{\nabla}_\rho\cdot\underset{\sim}{\phi}^{\rho}\,\underset{\sim}{T} + h.\,c.\ , \tag{10.28}$$

where S and T are the transition spin and isospin operators which couple the four component isobar to the two component nucleon. Their reduced matrix elements are defined as

$$\langle 3/2 \,\|\, S \,\|\, 1/2 \rangle = 2\ , \tag{10.29}$$

$$\langle 3/2 \,\|\, T \,\|\, 1/2 \rangle = 2\ . \tag{10.30}$$

Section 11. One-Body Magnetic Moment Operator

From Eqs. (10.16) and (10.17), we obtian the one-body-magnetic moment operator as

$$\vec{\mu}^N = \vec{\mu}^N(\text{stat}) + \vec{\mu}^N(\text{rel. cor.}) \tag{11.1}$$

with
$$\vec{\mu}^N(\text{stat}) = \frac{e}{4M}\left[\frac{1}{2}\overleftrightarrow{L}_1 + \frac{1}{2}\overleftrightarrow{L}_2 + (1+\kappa_s)\vec{\sigma}_+\right] \tag{11.2}$$

and

$$\vec{\mu}^N(\text{rel. cor.}) = \sum_{s=1}^{2}\frac{e}{16M^3}\left[-\overleftrightarrow{L}_s\vec{P}_s^{\,2} - (2+a)\vec{\sigma}_s\vec{P}_s^{\,2} + (a-2\kappa_s)(\vec{\sigma}_s\cdot\vec{P}_s)\vec{P}_s\right], \tag{11.3}$$

where
$$\overleftrightarrow{L}_s = \vec{r}_s \times \vec{P}_s + \vec{r}_s \times \vec{P}_s{}',$$

$$\vec{\sigma}_+ = \vec{\sigma}_1 + \vec{\sigma}_2 .$$

Eq. (11.2) is a familiar static magnetic moment operator in the impulse apprximation, and Eq. (11.3) is the relativistic correction to the above operator.

Section 12. Two-Body Magnetic Moment Operator

12.1. One-Boson-Exchange Magnetic Moment Operators

The nucleon-type and pair-currents are expressed as

$$\vec{J}(\vec{x}) = \int \frac{d\vec{q}\,d\vec{k}}{(2\pi)^6} e^{-i\vec{q}\cdot\vec{r}} e^{i\vec{k}\cdot(\vec{r}_1-\vec{x})} \vec{J}_1(\vec{q},\vec{k}) + (1 \leftrightarrow 2), \tag{12.1}$$

from which we obtian the magnetic moment operator as

$$\vec{\mu} = \frac{1}{2} \int \vec{x} \times \vec{J}(\vec{x})\, d\vec{x}$$

$$= \int \frac{d\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \vec{\mu}(\vec{q}). \tag{12.2}$$

The boson-type dissociation current is expressed as

$$\vec{J}^{\rho\pi\gamma}(\vec{x}) = \int \frac{d\vec{q}\,d\vec{k}}{(2\pi)^6} e^{-i\vec{q}\cdot\vec{r}} e^{i\vec{k}\cdot(\vec{R}-\vec{x})} \vec{J}^{\rho\pi\gamma}(\vec{q},\vec{k}) + (1 \leftrightarrow 2), \tag{12.3}$$

and the exchange magnetic moment operator is

$$\vec{\mu}^{\rho\pi\gamma} = \frac{1}{2} \int \vec{x} \times \vec{J}^{\rho\pi\gamma}(\vec{x})\, d\vec{x}$$

$$= \int \frac{d\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \vec{\mu}^{\rho\pi\gamma}(\vec{q}). \tag{12.4}$$

i) Recoil Current

We have shown in sect. 2.4 that OBE nucleon-type current vanishes in the static limit. Therefore we have non-static correction due to nucleon recoil. These non-static operators have not been studied yet in the pervious paper. The OBE current of nucleon-type is given as

$$\vec{J}_1^{\pi}(\vec{q},\vec{k}) = -\frac{e}{4M^2}\left(\frac{g_\pi}{2M}\right)^2 \frac{\tau^1\cdot\tau^2}{\omega_\pi^4}\Big[\frac{1}{2}\,(\vec{q}\ \overleftrightarrow{P}\cdot q + \overleftrightarrow{p}_1\, q\cdot k)(\sigma_1\cdot q)$$

$$+\ ((q\cdot\overleftrightarrow{P})\, q\times\sigma_1 + i(k\cdot q)\,\vec{q}\,)\times\vec{k}\ \Big]\,(\sigma_2\cdot q), \tag{12.5}$$

$$\vec{J}_1^{\rho}(\vec{q},\vec{k}) = -\frac{e}{4M}\left(\frac{g_\rho}{2M}\right)^2\frac{\tau^1\cdot\tau^2}{\omega_\rho^4}\Big(\Big[\left(\frac{1+\kappa_\rho}{2M}\right)^2(\sigma_1\times q)\cdot(\sigma_2\times q) - 1\Big]\frac{(\overleftrightarrow{P}\cdot q)\,\vec{q}}{2}$$

$$+\left(\frac{1+\kappa_\rho}{2M}\right)^2\Big[(q\cdot\overleftrightarrow{P})\,(q\times(q\times\sigma_2))\times\sigma_1 + i(k\cdot q)(q\times(q\times\sigma_2)\Big]\times k\Big), \tag{12.6}$$

where $\overleftrightarrow{P} = \overleftrightarrow{p}_1 + \overleftrightarrow{p}_2$.

Eqs. (12.5) and (12.6) correspond to pion and rho-meson exchange currents, respectively. The exchange current of omega-meson can be obtained by the following replacement in Eq. (12.6)

$$g_\rho \to g_\omega\,,\quad \kappa_\rho \to \kappa_\omega\,,\quad m_\rho \to m_\omega \quad \text{and} \quad \tau^1\cdot\tau^2 \to 1. \tag{12.7}$$

The magnetic moment operators associated with the intrinsic magnetic moment are

$$\vec{\mu}_\pi(q) = \frac{e}{4M}(1+\kappa_s)\left(\frac{g_\pi}{2M}\right)^2\frac{\tau^1\cdot\tau^2}{\omega_\pi^4}\,[(\sigma_1\times\sigma_2)\times q]\times q\,(q\cdot\overleftrightarrow{P}), \tag{12.8}$$

$$\vec{\mu}_\rho(q) = -\frac{e}{4M}(1+\kappa_s)\left(\frac{g_\rho}{2M}(1+\kappa_\rho)\right)^2\frac{\tau^1\cdot\tau^2}{\omega_\rho^4}\Big[((\sigma_1\times\sigma_2)\times q)\times q + \sigma_1\times\sigma_2\, q^2\Big](q\cdot\overleftrightarrow{P}). \tag{12.9}$$

The omega-meson exchange current is obtained by the replacement of Eq. (12.7) in Eq. (12.9). The magnetic moment operator associated with orbital motion of nucleon is

$$\vec{\mu}_\pi(q) = \frac{\vec{R}}{2} \times \tilde{J}_\pi + \frac{ie}{16M^2}\left(\frac{g_\pi}{2M}\right)^2 \frac{\tau^1 \cdot \tau^2}{\omega_\pi^4} (\sigma_1 \cdot q)(\sigma_2 \cdot q)\; q \times \overleftrightarrow{Q}, \tag{12.10}$$

$$\vec{\mu}_\rho(\vec{q}) = \frac{\vec{R}}{2} \times \tilde{J}_\rho + \frac{ie}{16M^2} g_\rho^2 \frac{\tau^1 \cdot \tau^2}{\omega_\rho^2}\left[\left(\frac{1+K_\rho}{2M}\right)^2 (\sigma_1 \times q)\cdot(\sigma_2 \times q) - 1\right] q \times \overleftrightarrow{Q}, \tag{12.11}$$

where $\int \frac{d\vec{q}}{(2\pi)^3} \tilde{J}_B e^{-i\vec{q}\cdot\vec{r}} = \int \vec{J}_B(\vec{x})\, dx$ and $\overleftrightarrow{Q} = \frac{1}{2}(\overleftrightarrow{P}_1 - \overleftrightarrow{P}_2)$.

The omega-meson exchange current is obtained by the replacement of Eq. (12.7) in Eq. (12.11). The first terms of Eqs. (12.10) and (12.11) are the isoscalar magnetic moment operators associated with non-static OBE potential as

$$\int \frac{d\vec{q}}{(2\pi)^3} \tilde{\mu}(q)\, e^{-i\vec{q}\cdot\vec{r}} = \frac{\vec{R}}{2} \times i\,[\,V^{OBE}_{Non\text{-}Stat}, \vec{D}^N\,]. \tag{12.12}$$

These terms and the operators in Eqs. (12.8) and (12.9) do not contribute to the deuteron magnetic moment, since they depend on the center of mass motion.

ii) Dissociation Current

The current associated with the dissociation of rho-meson into pion is given by

$$\vec{J}^{\rho\pi\gamma}(\vec{q}, \vec{k}) = \frac{i e\, g_{\rho\pi\gamma}\, g_\rho\, g_\pi}{2M\, m_\rho}\, \tau^1 \cdot \tau^2\, \frac{\sigma^2 \cdot q_+ \;\; q \times \vec{k}}{\omega_{\rho+}^2\, \omega_{\pi-}^2} \tag{12.13}$$

with $\vec{q}_\pm = \vec{q} \pm \vec{k}/2$, $\omega_{\rho+} = \sqrt{m_\rho^2 + q_+^2}$ and $\omega_{\pi-} = \sqrt{\mu_\pi^2 + q_-^2}$,

and the corresponding magnetic moment operator is given as

$$\vec{\mu}^{\rho\pi\gamma}(q) = \frac{e}{2M}\left(\frac{g_{\rho\pi\gamma} g_\rho g_\pi}{m_\rho}\right)(\tau^1\cdot\tau^2)\frac{\sigma_+\cdot q\ \vec{q}}{\omega_\pi^2\ \omega_\rho^2}, \tag{12.14}$$

with $\omega_\rho = \sqrt{m_\rho^2 + q^2}$ and $\omega_\pi = \sqrt{m_\pi^2 + q^2}$.

iii) Pair Current

Elimination of negative-energy component introduce the contact interaction of boson, nucleon and electromagnetic field, and gives rise to the pair current. The pion pair current with the strong interaction of ps-ps coupling is given as

$$\vec{J}_1^\pi(q,k) = \frac{e g_\pi^2}{32M^4}(\tau^1\cdot\tau^2)(\sigma^2\cdot q)\Big[2i\,(1+\kappa_s)\,\vec{k}\times\vec{q}$$

$$-\,(\kappa_s + c\,(1+\kappa_s))\,(\vec{\sigma}_1\times\overleftrightarrow{p}_1)\times\vec{k} + a\,(\overleftrightarrow{p}_1\cdot q)\,\vec{\sigma}_1 + 2(1-b)\,(\overleftrightarrow{p}_2\cdot q)\,\vec{\sigma}_1$$

$$+\,b\,(\overleftrightarrow{p}_1\cdot\sigma_1)\,\vec{q} - 2\,\overleftrightarrow{p}_1\,(\sigma_1\cdot q) + c\,\overleftrightarrow{p}_1\,(\sigma_1\cdot k)\Big]. \tag{12.15}$$

The rho- and omega-meson exchange currents are written as

$$\vec{J}_1^\rho(q,k) = -\frac{e g_\rho^2}{4M^2}\,\frac{\tau_1\cdot\tau_2}{\omega_\rho^2}\left[\overleftrightarrow{p}_1 - i\,(1+\kappa_\rho)\,q\times\sigma_+\right], \tag{12.16}$$

$$\vec{J}_1^\omega(q,k) = -\frac{e g_\omega}{4M^2}\,\frac{1}{\omega_\omega^2}\left[\overleftrightarrow{p}_1 - i\,(1+\kappa_\omega)\,q\times\sigma_+\right]. \tag{12.17}$$

The OBE magnetic moment operators corresponding to Eqs. (12.15), (12.16) and (12.17) are

$$\vec{\mu}_\pi(q) = \frac{e}{4M^2}\left(\frac{g_\pi}{2M}\right)^2\frac{\tau^1\cdot\tau^2}{\omega_\pi^2}\Big[(1+\kappa_s)\,\vec{q}\ \sigma_2\cdot\vec{q}$$

$$+\frac{1}{4}\Big(2\overleftrightarrow{L}_1\,\sigma_1\cdot q\,\sigma_2\cdot q-i\,(2(1+c)\kappa_S-b+c)\,\sigma_1\times\overleftrightarrow{p}_1\,\sigma_2\cdot q$$

$$+b\,\overleftrightarrow{p}_1\cdot\sigma_1\,r_1\times q\,\sigma_2\cdot q-(c\,\overleftrightarrow{p}_1\cdot q+(b-1)\,\overleftrightarrow{p}_2\cdot q)\,r_1\times\sigma_1\,\sigma_2\cdot q\Big)\Big]+(1\leftrightarrow 2), \qquad (12.18)$$

$$\vec{\mu}_\rho(\vec{q})=-\frac{eg_\rho^2}{8M^2}\,\frac{\tau^1\cdot\tau^2}{\omega_\rho^2}\left[\overleftrightarrow{L}_1+\overleftrightarrow{L}_2-i\,(1+\kappa_\rho)\,\vec{r}\times(q\times\sigma_+)\right], \qquad (12.19)$$

$$\vec{\mu}_\omega(q)=-\frac{eg_\omega^2}{8M^2}\,\frac{1}{\omega_\omega^2}\left[\overleftrightarrow{L}_1+\overleftrightarrow{L}_2-i\,(1+\kappa_\omega)\,\vec{r}\times(q\times\sigma_+)\right]. \qquad (12.20)$$

The first term in Eq. (12.18), which is local operator and free from the arbitrariness of unitary transformation, is taken into account in Ref. 14). However, ther is no reason to neglect the other momentum-dependent terms, although they depend on the unitary transformation. We shall take into account these non-local terms in the numerical calculation. The pair currents due to rho- and omega-meson are given irrespective of arbitrariness in the F-W-T transformation.

12.2. Two-Boson-Exchange Magnetic Moment Operators

We shall show the isoscalar part of TBE static magnetic moment operators. We obtain the nucleon-type current and also the boson-type currents, as shown in Fig. 11. We shall express the spatical current density of the nucleon and the corresponding magnetic moment operator, respectively, as

$$\vec{J}(\vec{x}) = \sum_{\alpha,\beta} \int \frac{d\vec{q}\,d\vec{p}\,d\vec{k}}{(2\pi)^9} e^{-i(\vec{p}+\vec{q})\cdot\vec{r}} e^{i\vec{k}\cdot(\vec{r}_1-\vec{x})} \vec{J}_1^{\alpha\beta}(p,q,k) + (1\leftrightarrow 2), \tag{12.21}$$

and

$$\vec{\mu} = \sum_{\alpha,\beta} \int \frac{d\vec{q}\,d\vec{p}}{(2\pi)^6} e^{-i(\vec{p}+\vec{q})\cdot\vec{r}} \vec{\mu}^{\alpha,\beta}(p,q), \tag{12.22}$$

where p abd q are the momenta of the exchanged two bosons (α,β), such as (π,π), (ρ,π) and (ρ,ρ). The dissociation current in momentum space is given as

$$\vec{J}^{\rho\pi\gamma}(\vec{x}) = \sum_{\beta} \frac{d\vec{p}\,d\vec{q}\,d\vec{k}}{(2\pi)^9} e^{-i(\vec{p}+\vec{q})\cdot\vec{r}} e^{i\vec{k}\cdot(\vec{R}-\vec{x})} \vec{J}^{\rho\pi\gamma}(p,q,k) + (1\leftrightarrow 2), \tag{12.23}$$

which leads to the magnetic moment operator as

$$\vec{\mu}^{\rho\pi\gamma} = \sum_{\beta} \int \frac{d\vec{p}\,d\vec{q}}{(2\pi)^6} e^{-i(\vec{p}+\vec{q})\cdot\vec{r}} \vec{\mu}^{\rho\pi\gamma}(p,q). \tag{12.24}$$

where q is the momentum of the exchanged boson β.

We shall explain the nontation by which we shall express the TBE magnetic moment operators hereafter. The upper (lower) sign of the operators correspond to the uncrossed (crossed) boson-exchange diagrams.

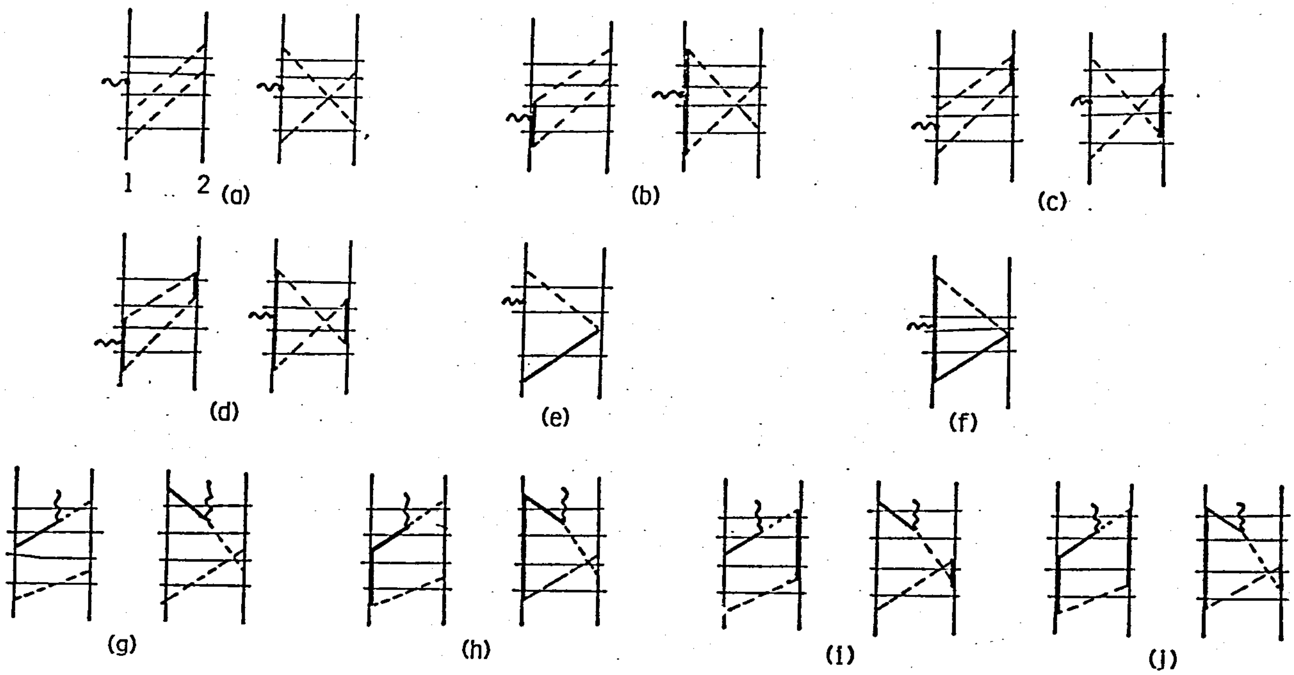


Fig. 11 Typical diagrams in the time ordered perturbation which contribute to TBE magnetic moment operators.

The boson-nucleon coupling constants are denoted as f and g, and they are summarized in table 1. The contributions of the energy denominator, i. e., G in Eq. (2.54), are included in the function I(p,q) and their explicit forms are given in Appendix B. It is noticed that the contributions from lots of time ordered diagrams in the unitary transformation method are expressed by a simple integral form, as will be seen in the following. This is easily understood in the S-matrix method. The vertices are denoted as Γ_{Jx}, Γ_x, $\Gamma_{Jx}^{\rho\pi\gamma}$ and $\Gamma_x^{\rho\pi\gamma}$, and they are summarized in table 2. We also used the following symbols:

$$\omega_p = \sqrt{M_\delta^2 + p^2}, \quad \omega_q = \sqrt{M_\beta^2 + q^2}, \quad \omega_+ = \sqrt{m_\pi^2 + p_+^2}, \quad \omega_- = \sqrt{m_\beta^2 + p_-^2} \qquad (12.25)$$

where $\vec{p}_\pm = \vec{p} \pm \frac{\vec{k}}{2}$ and $\Delta = M_\Delta - M$.

Here M and M_Δ are the masses of nucleon and isobar, respectively. In the following, we shall show only the expression for the spatical current density, magnetic moment operator and function I(p,q).

i) TBE Currents due to Intrinsic Magnetic Moments of Nucleon and Isobar

a) N-N Intermediate State

The currents associated with the diagrams in Fig. 11-a are written as,

$$\vec{J}_1^{\alpha\beta}(p,q,k) = \pm\mu_s f^{NN}(3 \mp 2\tau^1\cdot\tau^2)\, I^{NN}(p,q)\left[2\vec{\Gamma}_{JA} - \vec{\Gamma}_{JB} \mp 2\vec{\Gamma}_{JC}\right], \tag{12.26}$$

$$\vec{\mu}^{\alpha\beta}(p,q) = \pm\mu_s f^{NN}(3 \mp 2\tau^1\cdot\tau^2)\, I^{NN}(p,q)\left[2\vec{\Gamma}_A - \vec{\Gamma}_B \mp 2\vec{\Gamma}_C\right], \tag{12.27}$$

with

$$I^{NN}(p,q) = \frac{i}{2\pi}\int dx\left[(x^2-\omega_p^2+i\epsilon)(x^2-\omega_q^2+i\epsilon)(x+i\epsilon)^3\right]^{-1}. \tag{12.28}$$

b) N-Δ Intermediate State

The currents associated with the diagrams in Fig. 11b are written as,

$$\vec{J}_1^{\alpha\beta}(p,q,k) = \frac{2}{9} f^{N\Delta}(3 \pm \tau^1\cdot\tau^2)\, I^{N\Delta}_{A,\pm}(p,q)\left[\frac{\mu_s^\Delta}{3}(4\vec{\Gamma}_{JA} - \vec{\Gamma}_{JB} \pm 5\vec{\Gamma}_{JC})\right.$$

$$\left. + \mu_s(-2\vec{\Gamma}_{JA} \mp \vec{\Gamma}_{JC})\right], \tag{12.29}$$

$$\vec{\mu}^{\alpha\beta}(p,q) = \frac{2}{9} f^{N\Delta}(3 \pm \tau^1\cdot\tau^2)\, I^{N\Delta}_{A,\pm}(p,q)\left[\frac{\mu_s^\Delta}{3}(4\vec{\Gamma}_A - \vec{\Gamma}_B \pm 5\vec{\Gamma}_C)\right.$$

$$\left. + \mu_s(-2\vec{\Gamma}_A \mp \vec{\Gamma}_C)\right]. \tag{12.30}$$

with

$$I^{N\Delta}_{A,\pm}(p,q) = \frac{i}{2\pi}\int dx\left[(x^2-\omega_p^2+i\epsilon)(x^2-\omega_q^2+i\epsilon)(x+i\epsilon)(\mp x-\Delta+i\epsilon)^2\right]^{-1}. \qquad (12.31)$$

The currents associated with the diagrams in Fig. 11-c are written as,

$$\vec{J}_1^{\alpha\beta}(p,q,k) = \frac{2}{9} f^{N\Delta}(3\pm\tau'\cdot\tau^2)\, I^{N\Delta}_{B,\pm}(p,q)\, \mu_S\left[-4\vec{T}_{JA}+2\vec{T}_{JB}\mp 2\vec{T}_{JC}\right], \qquad (12.32)$$

$$\vec{\mu}^{\alpha\beta}(p,q) = \frac{2}{9} f^{N\Delta}(3\pm\tau'\cdot\tau^2)\, I^{N\Delta}_{B,\pm}(p,q)\, \mu_S\left[-4\vec{T}_A+2\vec{T}_B\mp 2\vec{T}_C\right], \qquad (12.33)$$

with

$$I^{N\Delta}_{B,\pm}(p,q) = \frac{i}{2\pi}\int dx\left[(x^2-\omega_p^2+i\epsilon)(x^2-\omega_q^2+i\epsilon)(x-\Delta+i\epsilon)(\mp x+i\epsilon)^2\right]^{-1}. \qquad (12.34)$$

c) Δ-Δ Intermediate State

The currents associated with the diagrams in Fig. 11-d are written as,

$$\vec{J}_1^{\alpha\beta}(p,q,k) = \frac{2}{81} f^{\Delta\Delta}(6\mp\tau'\cdot\tau^2)\, I^{\Delta\Delta}_{\pm}(p,q)\left[\frac{\mu_S^{\Delta}}{3}(8\vec{T}_{JA}-2\vec{T}_{JB}\mp 5\vec{T}_{JC})\right.$$

$$\left. - \mu_S(4\vec{T}_{JA}\mp\vec{T}_{JC})\right], \qquad (12.35)$$

$$\vec{\mu}^{\alpha\beta}(p,q) = \frac{2}{81} f^{\Delta\Delta}(6\mp\tau'\cdot\tau^2)\, I^{\Delta\Delta}_{\pm}(p,q)\left[\frac{\mu_S^{\Delta}}{3}(8\vec{T}_A-2\vec{T}_B\mp 5\vec{T}_C)\right.$$

$$\left. - \mu_S(4\vec{T}_A\mp\vec{T}_C)\right], \qquad (12.36)$$

with

$$I^{\Delta\Delta}_{\pm}(p,q) = \frac{i}{2\pi}\int dx\left[(x^2-\omega_p^2+i\epsilon)(x^2-\omega_q^2+i\epsilon)(x-\Delta+i\epsilon)^2(\mp x-\Delta+i\epsilon)\right]^{-1}. \qquad (12.37)$$

d) ρ-π Exchange Currents due to Seagull Interaction

The currents associated with the diagrams in Fig. 11-e are written as,

$$\vec{J}_1^{\rho\pi}(p,q,k) = 16 i \mu_s f^N \tau^1\cdot\tau^2 \left[q (p\cdot\sigma_2) - (p\cdot q)\sigma_2 \right] \times k \, I^N(p,q), \tag{12.38}$$

$$\vec{\mu}^{\rho\pi}(p,q) = 16 \mu_s f^N \tau^1\cdot\tau^2 \left[q (p\cdot\sigma_+) - (p\cdot q)\sigma_+ \right] I^N(p,q), \tag{12.39}$$

with

$$I^N(p,q) = \frac{-i}{2\pi} \int dx \left[(x^2-\omega_p^2+i\epsilon)(x^2-\omega_q^2+i\epsilon)(x+i\epsilon)^2 \right]^{-1}. \tag{12.40}$$

The currents associated with the diagrams in Fig. 11-f are written as,

$$\vec{J}_1^{\rho\pi}(p,q,k) = \frac{8}{9} f^\Delta \left[\mu_s - \frac{5}{3}\mu_s^\Delta \right] \tau^1\cdot\tau^2 \left[q (p\cdot\sigma_2) - (p\cdot q)\sigma_2 \right] \times k \, I^\Delta(p,q), \tag{12.41}$$

$$\vec{\mu}^{\rho\pi}(p,q) = \frac{8}{9} f^\Delta \left[\mu_s - \frac{5}{3}\mu_s^\Delta \right] \tau^1\cdot\tau^2 \left[q (p\cdot\sigma_+) - (p\cdot q)\sigma_+ \right] I^\Delta(p,q), \tag{12.42}$$

with

$$I^\Delta(p,q) = \frac{-i}{2\pi} \int dx \left[(x^2-\omega_p^2+i\epsilon)(x^2-\omega_q^2+i\epsilon)(x-\Delta+i\epsilon)^2 \right]^{-1}. \tag{12.43}$$

ii) TBE Magnetic Moment Operator due to Convection Currents of Nucleon and Isobar

a) N-N Intermediate State

The currents associated with the diagrams in Fig. 11-a are written as,

$$\vec{J}_1^{c\ell}(p,q,k) = \mp \frac{e}{4M} f^{NN} (3 \mp 2\tau^1\cdot\tau^2) I^{NN}(p,q) \, \vec{T}_p^{\pm}, \tag{12.44}$$

$$\vec{\mu}^{dP}(p,q) = \frac{e}{8M} f^{NN} I^{NN}(p,q) \begin{pmatrix} -(3-2\tau^1\cdot\tau^2)\ \vec{r}\times\vec{T}_D^{+} \\ 2(3+2\tau^1\cdot\tau^2)\vec{R}\times\vec{T}_D^{-} \end{pmatrix} . \tag{12.45}$$

b) N-Δ Intermediate State

The currents associated with the diagrams in Fig. 11-b are written as,

$$\vec{J}_1^{dP}(p,q,k) = \frac{e}{36M_\Delta} f^{N\Delta}(3\pm\tau^1\cdot\tau^2) I^{N\Delta}_{A,\pm} \left[\binom{1}{-3} \vec{T}_D^{+} + \binom{-3}{1} \vec{T}_D^{-} + 2\left(-\frac{\Delta}{M}\right) \vec{P}_1 \ (2T_E \pm T_F) \right], \tag{12.46}$$

$$\vec{\mu}^{dP}(p,q) = \frac{e}{72M_\Delta} f^{N\Delta}(3\pm\tau^1\cdot\tau^2) I^{N\Delta}_{A,\pm}(p,q) \left[\binom{1}{-3} \vec{r}\times\vec{T}_D^{+} + \binom{-6}{2}\vec{R}\times\vec{T}_D^{-} - \frac{2\Delta}{M}(\vec{L}_1+\vec{L}_2)\ (2T_E \pm T_F) \right]. \tag{12.47}$$

The currents associated with the diagrams in Fig. 11-c are written as,

$$\vec{J}_1^{dP}(p,q,k) = \frac{e}{36M_\Delta} f^{N\Delta}(3\pm\tau^1\cdot\tau^2) I^{N\Delta}_{B,\pm}(p,q) \left[\binom{1}{3} \vec{T}_D^{+} + \binom{3}{1} \vec{T}_D^{-} \right], \tag{12.48}$$

$$\vec{\mu}^{dP}(p,q) = \frac{e}{72M} f^{N\Delta}(3\pm\tau^1\cdot\tau^2) I^{N\Delta}_{B,\pm}(p,q) \left[\binom{1}{3} \vec{r}\times\vec{T}_D^{+} + \binom{6}{2}\vec{R}\times\vec{T}_D^{-} \right]. \tag{12.49}$$

c) Δ-Δ Intermediate State

The currents associated with diagrams in Fig. 11-d are written as,

$$\vec{J}_1^{\Delta\beta}(p,q,k)=\frac{e}{81M_\Delta}f^{\Delta\Delta}(6\mp\tau^1\cdot\tau^2)\,I_\pm^{\Delta\Delta}(p,q)\Big[-\vec{T}_D^\pm-\frac{\Delta}{2M}\overset{\leftrightarrow}{p}_1$$

$$\times(4T_E\mp T_F)\Big],\tag{12.50}$$

$$\vec{\mu}^{\Delta\beta}(p,q)=\frac{e}{162M_\Delta}f^{\Delta\Delta}(6\mp\tau^1\cdot\tau^2)\left[-\begin{pmatrix}\vec{r}\times\vec{T}_D^+\\ 2\vec{R}\times\vec{T}_D^-\end{pmatrix}\right.$$

$$\left.-\frac{\Delta}{M}(\overset{\leftrightarrow}{L}_1+\overset{\leftrightarrow}{L}_2)(4T_E\mp T_F)\right].\tag{12.51}$$

d) ρ-π Exchange Currents due to Seagull Interaction

The currents associated with the diagrams in Fig. 11-e are written as,

$$\vec{J}_1^{\rho\pi}(p,q,k)=\frac{e}{2M}f^N\tau^1\cdot\tau^2\,I^N(p,q)\,4i\,(\vec{p}-\vec{q})\;p\times q\cdot\sigma_2,\tag{12.52}$$

$$\vec{\mu}^{\rho\pi}(p,q)=i\frac{e}{2M}f^N\tau^1\tau^2\,I^N(p,q)\left[\vec{r}\times(\vec{p}-\vec{q})\,p\times q\cdot\sigma_+-2\vec{R}\times(\vec{p}-\vec{q})\,\vec{p}\times\vec{q}\cdot\sigma_-\right].\tag{12.53}$$

The currents associated with the diagrams in Fig. 11-f are written as,

$$\vec{J}_1^{\rho\pi}(p,q,k)=-\frac{e}{2M_\Delta}f^\Delta\tau^1\cdot\tau^2\;\frac{4}{9}I^\Delta(p,q)\left[2i(\vec{p}-\vec{q})\,p\times q\cdot\sigma_2+\frac{\Delta}{M}\overset{\leftrightarrow}{p}_1(\sigma^1\times p)\cdot(\sigma^2\times q)\right],\tag{12.54}$$

$$\vec{\mu}^{\rho\pi}(p,q)=\frac{e}{2M_\Delta}f^\Delta\tau^1\cdot\tau^2\frac{2}{9}I^\Delta(p,q)\Big[-\vec{r}\times(\vec{p}-\vec{q})\;p\times q\cdot\sigma_+\;+2\vec{R}\times(\vec{p}-\vec{q})\;p\times q\cdot\sigma_-$$

$$-\frac{\Delta}{M}\left[\vec{r}_1\times\vec{p}_1\,(\sigma^1\times p)\cdot(\sigma^2\times q)+(1\leftrightarrow 2)\right]\Big].\tag{12.55}$$

iii) TBE Magnetic Moment Operator due to Dissociation Current of Rho-Meson into Pion

a) N-N Intermediate State

The currents associated with the diagrams in Fig. 11-g are written as,

$$\vec{J}_1^{(\beta)}(p,q,k) = \pm 2 f_{\rho\pi\delta}^{NN} (3 \mp 2\tau^1\cdot\tau^2)\, I_{\rho\pi\delta}^{NN1}(p,q,k)\, \vec{\Gamma}_{JA}^{\rho\pi\delta}$$

$$+ 2 g_{\rho\pi\delta}^{NN} (3 + 2\tau^1\cdot\tau^2)\, I_{\rho\pi\delta}^{NN2}(p,q,k) \left[\vec{\Gamma}_{JB}^{\rho\pi\delta} - \vec{\Gamma}_{JC}^{\rho\pi\delta} \right], \tag{12.56}$$

$$\vec{\mu}^{(\beta)}(p,q) = \pm 2 f_{\rho\pi\delta}^{NN} (3 \mp 2\tau^1\cdot\tau^2)\, I_{\rho\pi\delta}^{NN1}(p,q,0)\, \vec{\Gamma}_{A}^{\rho\pi\delta}$$

$$+ 2 g_{\rho\pi\delta}^{NN} (3 + 2\tau^1\cdot\tau^2)\, I_{\rho\pi\delta}^{NN2}(p,q,0) \left[\vec{\Gamma}_{B}^{\rho\pi\delta} - \vec{\Gamma}_{C}^{\rho\pi\delta} \right], \tag{12.57}$$

with

$$I_{\rho\pi\delta}^{NN1}(p,q,k) = \int \frac{i}{2\pi} dx \left[(x^2 - \omega_+^2 + i\epsilon)(x^2 - \omega_-^2 + i\epsilon)(x^2 - \omega_k^2 + i\epsilon)(x + i\epsilon)^2 \right]^{-1}, \tag{12.58}$$

$$I_{\rho\pi\delta}^{NN2}(p,q,k) = -\int \frac{i}{2\pi} dx\, x \left[(x^2 - \omega_+^2 + i\epsilon)(x^2 - \omega_-^2 + i\epsilon)(x^2 - \omega_k^2 + i\epsilon)(x + i\epsilon)^2 \right]^{-1}. \tag{12.59}$$

b) N-Δ Intermediate State

The currents associated with the diagrams in Fig. 11-h and Fig. 11-i are written as

$$\vec{J}_1^{(\beta)}(p,q,k) = \frac{8}{9} f_{\rho\pi\delta}^{N\Delta} (3 \pm \tau^1\cdot\tau^2)\, I_{\rho\pi\delta(\pm)}^{N\Delta 1}(p,q,k)\, \vec{\Gamma}_{JA}^{\rho\pi\delta}$$

$$+ \frac{4}{9} (3 \pm \tau^1\cdot\tau^2)\, I_{\rho\pi\delta}^{N\Delta 2}(p,q,k) \left[g_{\rho\pi\delta}^{\Delta N} \left(2\vec{\Gamma}_{JB}^{\rho\pi\delta} \mp \vec{\Gamma}_{JC}^{\rho\pi\delta} \right) \right.$$

$$\left. + g_{\rho\pi\delta}^{N\Delta} \left(\mp \vec{\Gamma}_{JB}^{\rho\pi\delta} + 2\vec{\Gamma}_{JC}^{\rho\pi\delta} \right) \right], \tag{12.60}$$

$$\vec{\mu}^{(\beta)}(p,q) = \frac{8}{9} f_{p\pi\gamma}^{N\Delta}\,(3 \pm \tau^1\cdot\tau^2)\, I_{p\pi\gamma,\pm}^{N\Delta 1}(p,q,k)\, \vec{\Gamma}_A^{p\pi\gamma}$$

$$+ \frac{4}{9}(3 \pm \tau^1\cdot\tau^2)\, I_{p\pi\gamma}^{N\Delta 2}(p,q,k)\left[g_{p\pi\gamma}^{\Delta N}(2\vec{\Gamma}_B^{p\pi\gamma} \mp \vec{\Gamma}_C^{p\pi\gamma}) \right.$$

$$\left. + g_{p\pi\gamma}^{N\Delta}(\mp \vec{\Gamma}_B^{p\pi\gamma} + 2\vec{\Gamma}_C^{p\pi\gamma}) \right], \qquad (12.61)$$

with

$$I_{p\pi\gamma,\pm}^{N\Delta 1}(p,q,k) = \int \frac{i}{2\pi} dx\, \left[(x^2 - \omega_\pi^2 + i\epsilon)(x^2 - \omega^2 + i\epsilon)(x^2 - \omega_q^2 + i\epsilon) \right.$$

$$\left. \times (x - \Delta + i\epsilon)(\mp x + i\epsilon) \right]^{-1}, \qquad (12.62)$$

$$I_{p\pi\gamma}^{N\Delta 2}(p,q,k) = \int \frac{i}{2\pi} dx\; x\left[(x^2 - \omega_\pi^2 + i\epsilon)(x^2 - \omega^2 + i\epsilon)(x^2 - \omega_q^2 + i\epsilon) \right.$$

$$\left. \times (x - \Delta + i\epsilon)(-x + i\epsilon) \right]^{-1}. \qquad (12.63)$$

c) $\Delta - \Delta$ Intermediate State

The currents associated with the diagrams in Fig. 11-j are written as,

$$J_1^{(\varphi)}(p,q,k) = \frac{8}{81} g_{p\pi\gamma}^{\Delta\Delta}\,(6 + \tau^1\cdot\tau^2)\, I_{p\pi\gamma}^{\Delta\Delta}(p,q,k)\left[\vec{\Gamma}_{JB}^{p\pi\gamma} - \vec{\Gamma}_{JC}^{p\pi\gamma} \right], \qquad (12.64)$$

$$\vec{\mu}^{(\varphi)}(p,q) = \frac{8}{81} g_{p\pi\gamma}^{\Delta\Delta}\,(6 + \tau^1\cdot\tau^2)\, I_{p\pi\gamma}^{\Delta\Delta}(p,q,0)\left[\vec{\Gamma}_B^{p\pi\gamma} - \vec{\Gamma}_C^{p\pi\gamma} \right], \qquad (12.65)$$

with

$$I_{p\pi\gamma}^{\Delta\Delta}(p,q,k) = \int \frac{i}{2\pi} dx\; x\left[(x^2 - \omega_\pi^2 + i\epsilon)(x^2 - \omega^2 + i\epsilon)(x^2 - \omega_q^2 + i\epsilon) \right.$$

$$\left. \times (x - \Delta + i\epsilon)^2 \right]^{-1}. \qquad (12.66)$$

Section 13. Numerical Results and Discussions

Here, the exchange current operators are expressed in the configuration space, as shown in Appendix C. We have evaluated the magnetic moment of deuteron using the wave function in the Reid-potential, and the explicit forms of magnetic moment are shown in Appendix D.

13.1. Parameters in Operators of Exchange Magnetic Moment

We adopt following values of the coupling constants for boson-baryon interactions in our numerical calculations.

i) Electromagnetic Interaction

a) Magnetic Moment of Nucleon and Isobar

The isoscalar magnetic moments of nucleon and isobar are given as follows:

$$\mu_S = \frac{e}{4M}(1+\kappa_S) = \frac{1}{2}(\mu_p+\mu_s), \tag{13.1}$$

$$\mu_S^{\Delta} = \frac{e}{4M_\Delta}(1+\kappa_S^{\Delta}) = \frac{1}{2}\mu_p \; . \tag{13.2}$$

with
$$\mu_p = 2.7928 \text{ (n.m.)} \quad \text{and} \quad \mu_n = -1.9130 \text{ (n.m.)}.$$

Here the quark model prediction is used for the magnetic moement of the isobar.

b) $\rho\pi\gamma$ Coupling Constant

The $\rho\pi\gamma$ coupling is determined from the decay width[40] as

$$g_{\rho\pi\gamma} = 0.408 , \tag{13.3}$$

where we adopt the relative phase of the quark model prediction[39);

ii) Strong Interaction Hamiltonian

a) πNN Coupling Constant

$$\left(\frac{m_\pi}{2M}\right)^2\frac{g_\pi^2}{4\pi} = 0.08. \tag{13.4}$$

b) ρNN Coupling Constant

The ρNN coupling constant is given from the vector dominance model as

$$g_\rho^2/4\pi = 0.52 \quad \text{and} \quad K_\rho = 3.7, \tag{13.5}$$

while from Ref. 41) as

$$g_\rho^2/4\pi = 0.55 \quad \text{and} \quad K_\rho = 6.6. \tag{13.6}$$

In our numerical calculations, we give the results for a fixed value of $g_\rho^2/4\pi = 0.52$ with an adjustable parameter K_ρ.

c) ωNN Coupling Constant

$$g_\omega^2/4\pi = 9\, g_\rho^2/4\pi \quad \text{and} \quad K_\omega = K_s. \tag{13.7}$$

Here, the quark model prediction is used for g_ω.

d) πNΔ Coupling Constant

The coupling constant of πNΔ vertex is predicted as

$$f_{\pi N\Delta}^2/4\pi = 0.23, \tag{13.8}$$

in the quark model, while it is determined from the decay width [42] of isobar as

$$f_{\pi N\Delta}^2/4\pi = 0.38 , \tag{13.9}$$

We show our numerical results with an adjustable paeameter $f_{\pi N\Delta}$.

e) $\rho N\Delta$ Coupling Constant

The quark model predicts :

$$f_{\rho N\Delta}^2/4\pi = \frac{72}{25}\left[\frac{g_\rho}{2M}(1+\kappa_\rho)\right]^2/4\pi . \tag{13.10}$$

iii) Vertex Functions

a) πNN Vertex Functions

We adopt the πNN vertex function below

$$K_{\pi NN}(\vec{q}^2) = \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 + \vec{q}^2} , \tag{13.11}$$

with $\Lambda_\pi^2 \approx 71\, m_\pi^2$.[43]

b) ρNN Vertex Function

The ρNN vertex function is not well known yet. Two types of vertex function is usually adopted

$$K_{\rho NN}(\vec{q}^2) = \frac{\Lambda_\rho^2}{\Lambda_\rho^2 + \vec{q}^2} \tag{13.12}$$

with $\Lambda_\rho \approx 1450$ MeV[44] and

$$K_{\rho NN}(\vec{q}^{\,2}) = \frac{\Lambda_\rho^2 - m_\rho^2}{\Lambda_\rho^2 + \vec{q}^{\,2}} \tag{13.13}$$

with $\Lambda_\rho \approx 1 \sim 2$ GeV[45]. Here we have fixed $\Lambda_\rho = 1450$ MeV and studied the above two different forms of vetex function in Eqs. (13.12) and (13.13).

We have assumed the same vertex functions for N-Δ vertex as those for N-N. The introduction of vertex functions is phenomenological one. Therefore the current conservation law which we have shown in sect. 6 do not hold exactly, if we introduce the vertex functions. Additional electromagnetic interactions in the structure of vertices are required. However, we shall neglect these currents, where we need the model of vetex functions to take into account these currents.

13.2. Numerical Results and Discussions

Our numerical results are given in table 3$\sim$7.

i) Relativistic Correction of One-Body-Current

The relativistic correction of one-body-current is shown in table 3. Here, we fixed the arbitriness of F-W-T transformation as a = 0, which is called gauge invariant reduction. The gauge invariance is always satisfied at the each step of F-W-T transformation, that is the combination, $\vec{p} - e\vec{A}$, holds at each step. The relativistic correction is about - 0.01 n.m., which has an opposite sign to that of $\Delta\mu_{exp}$ in Eqs. (8.11) and (8.12). This correction is rather dependent on the choice of nuclear wave function. Since the relativistic correction is essentially the expectation value of kinetic energy operator (see Eqs. (D.8) - (D.10)), the dependence of relativistic corrections in the soft-core and hard-core wave functions is due to the different kinetic energies between them, in other words, due to different potential energies.

ii) Contribution of Exchange Current

At first we shall study the model dependence of the exchange magnetic moment contributions due to K_ρ and the choice of ρNN vertex function, where we shall fix the parameter $f_{\pi N\Delta}$ and the F-W-T transformation as $f_{\pi N\Delta} = 0.23$. a = 0 and c = -1. Two choices of the ρNN vertex functions, (13.12) and (13.13), change the effective coupling constant of rho-meson. From table 4, we can see that the total contribution of exchange magnetic moment is rather stable, however, the relative importance of OBE and TBE currents varies with the magnitude of K_ρ . In either case, the contribution of TBE current is non-negligible in the magnetic moment of deuteron. In what follows, we shall fix K_ρ = 3.7 and take the vertex function of Eq. (13.13).

a) OBE Current

The contribution of OBE current is shown in table 5. Non-static correction of OBE recoil current is small as we see in table 5.1. The pion pair current depends on the parameter c of unitary transformation. This parameter also affects the correponding relativistic correction of one-pion-exchange potential. The choice of c = 0 means that $\vec{\sigma}\cdot\vec{p}$ and $O_{\pi NN}$ are eliminated at the same time, and the choice of c = -1 means that $\vec{\sigma}\cdot\vec{p}$ is eliminated at first and then $O_{\pi NN}$ is eliminated, which correspond to the Foldy interaction picture. In table 5.3, contribution of pion-pair current is shown, where the local term given by Gari and Hyuga[14], and the non-local term with the different choices of unitary transformation are shown. There is non-negligible contribution of non-local terms. We shall fix c = -1 in the follwing discussion. It is noticed that if we construct the nuclear potential to obtain the deuteron wave function and the exchange current by the same model, the arbitrariness of unitary transformation does not appear in the matrix element.

The sum of OBE current contributions, i. e. , recoil, dissociation and pair current is about 0.016 n.m. (0.015) using H. C. (S. C.) wave function. Here, the S-D matrix element gives the most important contribution reflecting the tensor character of OBE current. It is noticed that the exchange current contribution is rather independent on the choice of deuteron wave function compared with the case of relativistic correction. This is also seen in the TBE current.

b) TBE Current

The contributions of TBE currents are shown in table 6. The most important effects of TBE current are those of the N-N intermediate state. The contribution of N-Δ intermediate state is as important as that of the Δ-Δ intermediate state, and these two contributions are destructive to each other. Here, only the diagrams of crossed-boson exchanges with the N-Δ intermeidate state contribute to the isoscalar magnetic moment of deuteron. The (ρ,π) exchange processes reduce the contribution of (π,π) exchanges. This cancellation makes the TBE contribution smaller when the ρNN coupling becomes stronger. Here, we have neglected the (ω,π) exchange current, since their contribution is negligible. We show the contribution of (ω,π) exchange current in table 6.4, which is evaluated with $m_\omega = m_\rho$ (= 770 MeV). We cannot neglect the destructive contributions of S-D and D-D matrix elements, which are neglected in Ref. 15). It is noticed that many TBE processes have equally important contributions and they tend to cancel to each other. Therefore, it is quite dangerous to evaluate TBE current by taking only a special type of diagrams among many boson-exchange processes. We obtained the total contribution of TBE currents to be 0.006 n.m. (0.007) using H.C. (S.C.) wave function.

iii) Magnetic Moment of Deuteron

Our results are summarized in table 7. $\Delta\mu$ can be explained so far by the OBE current which coincides with the result of Ref. 14). However, the relativistic correction tends to cancel the OBE contribution, and the sum of the OBE current and relativistic correction cannot explain the discrepancy $\Delta\mu$. If we take into account the contribution of TBE current together with the relativistic correction and OBE current, the discrepancy between the experimental magnetic moment and the calculation in the impulse approximation is solved.

We shall discuss some problems in the other approaches to study mesonic and isobar degrees of freedom. At first we shall point out the problem in the S-matrix method of TBE current bt Jaus[15]. The main difference between the unitary transformation method and S-matrix method of Jaus occurs in the diagram in Fig. 9-d, which is the uncrossed TBE current with the N-N intermediate state. In Ref. 15), the TBE current is derived by subtructing the iterated term of static OBE potential from the S-matrix element as

$$T_{\gamma}^{TBE}(9d) = V_{Stat}^{OBE}\, G_N O_N G_N\, V_{Stat}^{OBE} + \tilde{O}^{TBE,N}(9d). \tag{13.14}$$

Then the limit is adopted as

$$\tilde{O}^{TBE,N}(9d) \longrightarrow \tilde{O}^{TBE,N}_{Stat}(9d). \tag{13.15}$$

This TBE current is a half of the one from the unitary transformation method,

$$\tilde{O}^{TBE,N}_{Stat}(9d) = \frac{1}{2} O^{TBE,N}_{Stat}(9d). \tag{13.16}$$

Since the non-static potential and exchange current are not defined in Ref. 15), and they are not subtracted from the S-matrix element, we cannot regard $\tilde{O}^{TBE,N}_{Stat}$ as the true TBE current. In Ref. 15, c-number normalization method is also adopted. It is noticed, however, that the S-matrix method is incompatible with the c-number normalization method, because if we define OBE potential, OBE current and TBE potential, and subtract their contributions from the S-matrix element, the true TBE current is automatically obtained without taking into account the normalization correction.

Some authors[26)] treat the isobar as the explicit constituent of nucleus by the coupled channel method. This method cannot take into account the crossed-boson-exchange diagrams, while we have seen that the crossed diagrams of N-Δ intermediate state is as important as the Δ-Δ intermediate state. Then the coupled channel approach to study the effects of isobar will miss to take into account the important effects from the crossed-boson-exchange diagrams.

Section 14. Conclusion and Remarks

We have obtained a consistent explanation of the difference between the experimental vale of the magnetic moment of the deuteron and that of the impulse approximation by taking into account the TBE current together with OBE current and relativistic correction. We found that the contribution of the TBE current is important to solve this discrepancy, in which the TBE current with two-nucleon intermediate state has the most important contribution. It is noticed that the correct derivation of exchange current is especially important in the TBE current with the two-nucleon intermediate state. For example, naive S-matrix method give the incorrect TBE current[15]. We found many types of TBE processes are equally important. Therefore it is very dangerous to evaluate multi-boson-exchange currents by taking into account the special types of diagrams. In the coupled channel approach to include the isobar degrees of freedom, the crossed diagrams are not taken into account consistently as the uncrossed diagrams.

In the OBE current, the pion-pair current has the most important contribution. The non-local part of pion-pair current depend on the choice of F-W-T transformation. To resolve this arbitrariness, we should use the deuteron wave function which is obtained from the nuclear potential derived by the same F-W-T transformation as in the exchange current.

The individual contributions of OBE and TBE currents depend on the ambiguity of coupling constants, however, the similar model-dependence also appears in the nuclear potential derived from the boson-exchange model. The model-dependence of potential turns back to the magnetic moment in mainly the relativistic correction ans D-state probability.

Then the consistent treatment of nuclear potential and exchange current will solve a part of these ambiguities. In the next step of our study, we shall investigate the nuclear potential in the same model of boson-nucleon system as the exchange current, and study the properties of two-nucleon system as the testing ground of realistic model of exchange current..

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The numerical calculations were performed using TOSBAC 5600 at the Research Center of Nuclear Physics, Osaka University.

Appendix A. Elimination of Negative Energy Component

We shall show the elimination of the negative energy component of nucleon by the Foldy-Wouthuysen-Tani transformation.[35] In this case there appear ambiguities due to unitary transformation as the terms with order M^{-2} or higher order in the reduced Hamiltonian. We shall show them along the work of Hyuga and Gari[37] for the purpose of the clear discussion in part II. The one-body-nucleon Hamiltonian is

$$h = \beta M + \mathcal{E} + \mathcal{O}, \tag{A.1}$$

where $\mathcal{E}$ and $\mathcal{O}$ are the operators associated with even and odd Dirac matrices, respectively. By the usual method of the F-W-T transformation the transformed Hamiltonian h_{FWT} is

$$h_{FWT} = \beta M + \mathcal{E} + \frac{\beta}{2M}\mathcal{O}^2 - \frac{1}{8M^2}[\mathcal{O}, [\mathcal{O}, \mathcal{E}] + i\dot{\mathcal{O}}]$$

$$- \frac{\beta}{8M^3}\mathcal{O}^4 - \frac{\beta}{8M^3}([\mathcal{O}, \mathcal{E}] + i\dot{\mathcal{O}})^2 + \cdots . \tag{A.2}$$

This Hamiltonian is obtained when we eliminated all the odd operators at the same time. If we eliminate a part of the odd operator (A) at the first time and then eliminate the rest (B), we obtain a different reduced Hamiltonian $h_{FWT}(A,B)$ as

$$h_{FWT}(A, B) = h_{FWT} + \frac{i}{8M^2}\frac{d}{dt}[A, B] + \frac{1}{8M^2}[[A, B], \mathcal{E}]$$

$$+ \frac{\beta}{16M^3}[[A, B], \mathcal{O}^2] + \cdots , \tag{A.3}$$

where $\mathcal{O} = A + B$

They are related by the unitary transformaiton within the positive energy space.

$$h_{FWT}(A,B) = e^{iS} h_{FWT} e^{-iS} \tag{A.4}$$

with

$$S = -\frac{i}{8M^2}[A,B] + \cdots .$$

It is noticed here that this reduced Hamiltonian by the F-W-T transformation is also obtained from the unitary transformation method in sect. 1. In this case the η and Λ spaces are regarded as the positive and negative energy components, respectively. And the free Hamiltonian H_0 corresponds to βM. $\mathcal{E}$ and $\mathcal{O}$ are the interaction Hamiltonians.

Appendix B. Energy Denominator Functions for TBE Magnetic Moment Operators

At first we shall define the following functions $I^N_{\alpha\beta}(n)$ and $I^{\Delta}_{\alpha\beta}(n,m)$:

$$I^{N}_{\alpha\beta}(n) = \frac{1}{\omega_p^2 - \omega_q^2}\left[\frac{1}{2\omega_p^n} - \frac{1}{2\omega_q^n}\right], \tag{B.1}$$

and

$$I^{\Delta}_{\alpha\beta}(n,m) = \frac{1}{\omega_p^2 - \omega_q^2}\left[\frac{1}{2\omega_p^n(\omega_p+\Delta)^m} - \frac{1}{2\omega_q^n(\omega_q+\Delta)^m}\right], \tag{B.2}$$

with $\qquad \omega_p = \sqrt{m_\alpha^2 + \vec{p}^2} \qquad$ and $\qquad \omega_q = \sqrt{m_\beta^2 + \vec{q}^2}$.

The following relations between I^{Δ} and I^N are easily proved.

$$I^{\Delta}_{\alpha\beta}(n,0) = I^{N}_{\alpha\beta}(n), \tag{B.3}$$

$$I^{\Delta}_{\alpha\beta}(n,m) = \frac{1}{\Delta}\left[I^{\Delta}_{\alpha\beta}(n,m-1) - I^{\Delta}_{\alpha\beta}(n-1,m)\right], \tag{B.4}$$

$$I^{N}_{\alpha\beta}(2n+3) = -\frac{2}{2n+1}\left[\frac{d}{d(\omega_p^2)} I^{N}_{\alpha\beta}(2n+1) + \frac{d}{d(\omega_q^2)} I^{N}_{\alpha\beta}(2n+1)\right]. \tag{B.5}$$

The energy denominator functions of TBE magnetic moment operators are given by using functions $I^N_{\alpha\beta}(n)$ and $I^{\Delta}_{\alpha\beta}(n,m)$.

$$I^{NN}(p,q) = -I^{N}_{\alpha\beta}(4), \tag{B.6}$$

$$I^{N\Delta}_{A\pm}(p,q) = \begin{pmatrix} I^{\Delta}_{\alpha\beta}(2,2) - \frac{2}{\Delta^2} I^{N}_{\alpha\beta}(2) \\ - I^{\Delta}_{\alpha\beta}(2,2) \end{pmatrix}, \tag{B.7}$$

$$I^{N\Delta}_{B\pm}(p,q) = \begin{pmatrix} -I^{\Delta}_{\alpha\beta}(3,1) - \frac{2}{\Delta^2} I^{N}_{\alpha\beta}(3) \\ -I^{\Delta}_{\alpha\beta}(3,1) \end{pmatrix}, \tag{B.8}$$

$$I^{\Delta\Delta}_{\pm}(p,q) = \begin{pmatrix} -\frac{1}{2\Delta^2} I^{\Delta}_{\alpha\beta}(1,1) - \frac{1}{2\Delta} I^{\Delta}_{\alpha\beta}(1,2) \\ -I^{\Delta}_{\alpha\beta}(1,3) \end{pmatrix}, \tag{B.9}$$

$$I^{N}(p,q) = -I^{N}_{\alpha\beta}(3), \tag{B.10}$$

$$I^{\Delta}(p,q) = -I^{\Delta}_{\alpha\beta}(1,2), \tag{B.11}$$

$$I^{NN1}_{\rho\pi\sigma}(p,q,0) = \frac{1}{m_\rho^2 - m_\pi^2}\left[-I^{N}_{\rho\beta}(3) + I^{N}_{\pi\beta}(3)\right], \tag{B.12}$$

$$I^{NN2}_{\rho\pi\sigma}(p,q,0) = \frac{1}{m_\rho^2 - m_\pi^2}\left[I^{N}_{\rho\beta}(2) - I^{N}_{\pi\beta}(2)\right], \tag{B.13}$$

$$I^{N\Delta 1}_{\rho\pi\sigma,\pm}(p,q,0) = \frac{1}{m_\rho^2 - m_\pi^2}\begin{bmatrix} I^{\Delta}_{\rho\beta}(2,1) - I^{\Delta}_{\pi\beta}(2,1) \\ -I^{\Delta}_{\rho\beta}(2,1) + I^{\Delta}_{\pi\beta}(2,1) + \frac{2}{\Delta} I^{N}_{\rho\alpha}(2) - \frac{2}{\Delta} I^{N}_{\pi\alpha}(2) \end{bmatrix}, \tag{B.14}$$

$$I^{N\Delta 2}_{\rho\pi\sigma}(p,q,0) = \frac{1}{m_\rho^2 - m_\pi^2}\left[I^{\Delta}_{\rho\beta}(1,1) - I^{\Delta}_{\pi\beta}(1,1)\right], \tag{B.15}$$

$$I^{\Delta\Delta}_{\rho\pi\sigma}(p,q,0) = \frac{1}{m_\rho^2 - m_\pi^2}\left[I^{\Delta}_{\rho\beta}(0,2) - I^{\Delta}_{\pi\beta}(0,2)\right]. \tag{B.16}$$

Appendix C. Configuration Space Representation of Exchange Current

C.1. Fourier Transformation of Energy Denominator Functions

i) OBE Current

We shall define the following integrals,

$$Y_{\ell,n}(x) = \frac{1}{2\pi^2}\int_0^\infty dq \, \frac{j_\ell(qr)}{\omega^2} q^{n+2}, \tag{C.1}$$

$$\bar{Y}_{\ell,n}(x) = \frac{1}{2\pi^2}\int_0^\infty dq \, \frac{j_\ell(qr)}{\omega^4} q^{n+2}, \tag{C.2}$$

with $\quad x = \mu r \quad$ and $\quad \omega = \sqrt{\vec{q}^{\,2}+\mu^2}.$

The explicit forms of these functions are written as

$$Y_{0,2}(x) = \frac{-\mu^3}{4\pi}\frac{e^{-x}}{x}, \tag{C.3}$$

$$Y_{0,0}(x) = \frac{\mu}{4\pi}\frac{e^{-x}}{x}, \tag{C.4}$$

$$Y_{1,1}(x) = \frac{\mu^2}{4\pi}\left(1+\frac{1}{x}\right)\frac{e^{-x}}{x}, \tag{C.5}$$

$$Y_{2,2}(x) = \frac{\mu^3}{4\pi}\left(1+\frac{3}{x}+\frac{3}{x^2}\right)\frac{e^{-x}}{x}, \tag{C.6}$$

$$\bar{Y}_{0,0}(x) = \frac{1}{4\pi}\frac{e^{-x}}{2\mu}, \tag{C.7}$$

$$\bar{Y}_{0,2}(x) = \frac{\mu}{4\pi}\left(-\frac{1}{2}+\frac{1}{x}\right)e^{-x}, \tag{C.8}$$

$$\bar{Y}_{1,1}(x) = \frac{1}{8\pi}e^{-x}, \tag{C.9}$$

$$\bar{Y}_{1,3}(x) = \frac{\mu^2}{4\pi}\left(-\frac{1}{2} + \frac{1}{x} + \frac{1}{x^2}\right)e^{-x}, \tag{C.10}$$

$$\bar{Y}_{2,2}(x) = \frac{\mu}{8\pi}\left(1+\frac{1}{x}\right)e^{-x}, \tag{C.11}$$

$$\bar{Y}_{3,3}(x) = \frac{\mu^2}{8\pi}\left(1+\frac{3}{x} + \frac{3}{x^2}\right)e^{-x}. \tag{C.12}$$

The monopole type vertex function is taken into account by the modification of Eqs. (C.1) and (C.2) as

$$Y_{\ell,n}^{N}(r) = \frac{1}{2\pi^2}\int_0^\infty dq \left[\frac{\Lambda^2-\mu^2}{\Lambda^2+q^2}\right]^2 \frac{j_\ell(qr)}{\omega^2}\, q^{n+2}, \tag{C.13}$$

$$\bar{Y}_{\ell,n}^{N}(r) = \frac{1}{2\pi^2}\int_0^\infty dq \left[\frac{\Lambda^2-\mu^2}{\Lambda^2+q^2}\right]^2 \frac{j_\ell(qr)}{\omega^4}\, q^{n+2}, \tag{C.14}$$

for the nucleon-type current. And for the dissociation current, the function

$$\frac{1}{\omega_\pi^2\,\omega_\rho^2},$$

where $\omega_\pi = \sqrt{q^2+m_\pi^2}$ and $\omega_\rho = \sqrt{q^2+m_\rho^2}$,

is replaced to include vertex function as

$$Y_{\ell,n}^{\rho\pi d}(r) = \int_0^\infty dq \left[\frac{\Lambda_\rho^2-m_\rho^2}{\Lambda_\rho^2+q^2}\right]\left[\frac{\Lambda_\pi^2-m_\pi^2}{\Lambda_\pi^2+q^2}\right] \frac{j_\ell(qr)}{\omega_\pi^2\omega_\rho^2}\, q^{n+2}. \tag{C.15}$$

Eqs. (C.13) and (C.15) are written by using Eqs. (C.1) and (C.2) as,

$$Y_{\ell,N}^{N}(r) = Y_{\ell,n}(\mu r) - Y_{\ell,n}(\Lambda r) - (\Lambda^2-\mu^2)\,\bar{Y}_{\ell,n}(\Lambda r), \tag{C.16}$$

$$\bar{Y}^{N}_{\ell,n}(r) = \bar{Y}_{\ell,n}(\mu r) + \bar{Y}_{\ell,n}(\Lambda r) - \frac{2}{\Lambda^2-\mu^2}\left[Y_{\ell,n}(\mu r) - Y_{\ell,n}(\Lambda r)\right], \tag{C.17}$$

$$Y^{\rho\pi\gamma}_{\ell,n}(r) = \Big[\frac{1}{m_\rho^2-m_\pi^2}\Big(\frac{Y_{\ell,n}(m_\pi r)}{(\Lambda_\pi^2-m_\pi^2)(\Lambda_\rho^2-m_\pi^2)} - \frac{Y_{\ell,n}(m_\rho r)}{(\Lambda_\pi^2-m_\rho^2)(\Lambda_\rho^2-m_\rho^2)}\Big)$$

$$+ \frac{1}{\Lambda_\rho^2-\Lambda_\pi^2}\Big(\frac{Y_{\ell,n}(\Lambda_\pi r)}{(\Lambda_\pi^2-m_\pi^2)(\Lambda_\pi^2-m_\rho^2)} - \frac{Y_{\ell,n}(\Lambda_\rho r)}{(\Lambda_\rho^2-m_\pi^2)(\Lambda_\rho^2-m_\rho^2)}\Big)\Big]$$

$$\times (\Lambda_\rho^2 - m_\rho^2)(\Lambda_\pi^2 - m_\pi^2). \tag{C.18}$$

ii) TBE Current

The configuration space representation of TBE current is obtained by the following type of integral.

$$J_{\ell\ell',nn'}(r) = \int \frac{dp\,dq}{(2\pi^2)^2}\, I(p,q)\, j_\ell(pr)\, j_{\ell'}(qr)\, p^{n+2} q^{n'+2}. \tag{C.19}$$

Here I(p,q) represents the function I given in Eqs. (B.6) - (B.16). To perform the integrals over p and q, we shall rewrite Eqs. (B.1) and (B.2) as the following forms.

$$I_{\alpha\beta}(2) = -\left[2\omega_\alpha^2\omega_\beta^2\right]^{-1}, \tag{C.20}$$

$$I_{\alpha\beta}(3) = -\frac{2}{\pi}\int_0^\infty dz\left[\frac{1}{z^2+\omega_\alpha^2} + \frac{1}{z^2+\omega_\beta^2}\right]\cdot\left[(z^2+\omega_\alpha^2)(z^2+\omega_\beta^2)\right]^{-1}, \tag{C.21}$$

$$I_{\alpha\beta}(4) = -\frac{1}{2\omega_\alpha^2\omega_\beta^2}\left[\frac{1}{\omega_\alpha^2} + \frac{1}{\omega_\beta^2}\right] \tag{C.22}$$

$$I^{\Delta}_{\alpha\beta}(1,1) = -\frac{\Delta}{\pi}\int_0^\infty dz\left[(z^2+\Delta^2)(z^2+\omega_\alpha^2)(z^2+\omega_\beta^2)\right]^{-1}, \tag{C.23}$$

$$I^{\Delta}_{\alpha\beta}(1,2) = \frac{1}{\pi}\int_0^\infty dz\left(1-\frac{2\Delta^2}{z^2+\Delta^2}\right)\left[(z^2+\Delta^2)(z^2+\omega_\alpha^2)(z^2+\omega_\beta^2)\right]^{-1}, \tag{C.24}$$

$$I^{\Delta}_{\alpha\beta}(1,3) = \frac{\Delta}{\pi}\int_0^\infty \left(3 - \frac{4\Delta^2}{z^2+\Delta^2}\right)\left[(z^2+\Delta^2)^2(z^2+\omega_\alpha^2)(z^2+\omega_\beta^2)\right]^{-1}, \tag{C.25}$$

$$I^{\Delta}_{\alpha\beta}(2,1) = \frac{1}{\Delta} I^{\Delta}(2,0) - \frac{1}{\Delta} I^{\Delta}(1,1), \tag{C.26}$$

$$I^{\Delta}_{\alpha\beta}(2,2) = -\frac{1}{\Delta} I^{\Delta}(1,2) + \frac{1}{\Delta^2} I^{\Delta}(1,1) - \frac{1}{\Delta^2} I^{\Delta}(2,0), \tag{C.27}$$

$$I^{\Delta}_{\alpha\beta}(3,1) = -\frac{1}{\Delta} I^{\Delta}(3,0) + \frac{1}{\Delta^2} I^{\Delta}(2,0) - \frac{1}{\Delta^2} I^{\Delta}(1,1), \tag{C.28}$$

$$I^{\Delta}_{\alpha\beta}(0,2) = -\frac{2\Delta}{\pi}\int_0^\infty \left(1 - \frac{\Delta^2}{(z^2+\Delta^2)}\right)\left[(z^2+\Delta^2)(z^2+\omega_\alpha^2)(z^2+\omega_\beta^2)\right]^{-1}, \tag{C.29}$$

where $\omega_\alpha = \sqrt{p^2+m_\alpha^2}$ and $\omega_\beta = \sqrt{q^2+m_\beta^2}$.

Using Eqs. (C.3) - (C.12) and (C.20) and (C.29), the TBE currents are transformaed into configuration space representation. For example, we have

$$J^{NN}_{\ell\ell',nn'}(r) = -\int \frac{dp\,dq}{(2\pi^2)^2}\, I_{\alpha\beta}(4)\, j_\ell(pr)\, j_{\ell'}(qr)\, p^{n+2} q^{n'+2}$$

$$= \frac{1}{2}\left[\bar{Y}_{\ell,n}(m_\alpha r)\, Y_{\ell',n'}(m_\beta r) + Y_{\ell,n}(m_\alpha r)\, \bar{Y}_{\ell',n'}(m_\beta r)\right], \tag{C.30}$$

and

$$\tilde{J}_{\ell\ell',nn'}(r) = \int \frac{dp\,dq}{(2\pi^2)^2}\, I^{\Delta}_{\alpha\beta}(1,1)\, j_\ell(pr)\, j_{\ell'}(qr)\, p^{n+2} q^{n'+2}$$

$$= -\frac{\Delta}{\pi}\int_0^\infty dz \left[\frac{1}{z^2+\Delta^2}\, Y_{\ell,n}(m_{\alpha z} r)\, Y_{\ell',n'}(m_{\beta z} r)\right], \tag{C.31}$$

where $m_{\alpha z} = \sqrt{z^2+m_\alpha^2}$ and $m_{\beta z} = \sqrt{z^2+m_\beta^2}$.

Here the integral over z is performed numerically. The vertex functions are easily taken into account by the similar way as in the OBE currents.

C.2. Configuration Space Representation of Exchange Current

We show only the operaotrs which contribute to the magnetic moment of deuteron.

i) Notation

At first we shall summarize the symbols used hereafter.

$$\sigma_\pm = \sigma_1 \pm \sigma_2 \;, \quad \sigma_x = \sigma_1 \times \sigma_2 \;, \quad r = r_1 - r_2 \;, \quad R = \tfrac{1}{2}(r_1 + r_2) \;,$$

$$\overleftrightarrow{P} = \vec{p}_1 + \vec{p}_1{}' + \vec{p}_2 + \vec{p}_2{}' \;, \quad \overleftrightarrow{Q} = \tfrac{1}{2}(\vec{p}_1 + \vec{p}_1{}' \cdot \vec{p}_2 - \vec{p}_2{}') \;,$$

$$\overleftrightarrow{L}_i = \vec{r}_i \times (\vec{p}_i + \vec{p}_i{}') \;, \quad \overleftrightarrow{L} = \vec{r} \times \overleftrightarrow{Q}$$

$$S_{12} = (\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - \tfrac{1}{3}(\sigma^1 \cdot \sigma^2) \;,$$

$$\overrightarrow{T}_{12}^{\odot} = (\sigma_1 \odot \sigma^2) \cdot \hat{r}\, \hat{r} - \tfrac{1}{3}(\sigma^1 \odot \sigma^2) \;, \tag{C.32}$$

with $\quad \odot = \pm \;, \; \times \;.$

ii) OBE Current

a) Recoil Current

Non-static correction of recoil current due to convection current is given in pion exchange as

$$\vec{\mu}_\pi = \frac{e}{8M^2}\left[\frac{g_\pi}{2M}\right]^2 (\tau^1 \cdot \tau^2) \left[\overleftrightarrow{L} \left(S_{12} f_I(r) + \sigma^1 \cdot \sigma^2 f_{II}(r) \right. \right.$$

$$+ \left(\sigma_1 (\sigma_2 \cdot \hat{r}) + \sigma^2 (\sigma^1 \cdot \hat{r}) \right) \times \overleftrightarrow{Q} f_{III}(r) \Big], \tag{C.33}$$

where

$$f_I(r) = \frac{1}{r} \bar{Y}_{3,3}(m_\pi r), \tag{C.34}$$

$$f_{II}(r) = \frac{1}{r^2} \bar{Y}_{2,2}(m_\pi r) - \frac{1}{3r} \bar{Y}_{3,3}(m_\pi r), \tag{C.35}$$

$$f_{III}(r) = \frac{1}{r} \bar{Y}_{2,2}(m_\pi r), \tag{C.36}$$

and the rho-exchange current is

$$\vec{\mu}_\rho = -\frac{e}{8M^2} \left[\frac{g_\rho}{2M} (1+K_\rho) \right]^2 (\tau^1 \cdot \tau^2) \Big[\overleftrightarrow{L} \left(S_{12} f_I(r) + \sigma^1 \cdot \sigma^2 f_{II}(r) \right)$$

$$+ \left(\sigma_1 (\sigma_2 \cdot \hat{r}) + \sigma^2 (\sigma^1 \cdot \hat{r}) \right) \times \overleftrightarrow{Q} f_{III}(r) \Big] - \frac{e}{8M^2} g_\rho^2 \tau^1 \cdot \tau^2 \overleftrightarrow{L} f_{IV}(r), \tag{C.37}$$

where

$$f_I(r) = \frac{1}{r} \bar{Y}_{3,3}(m_\rho r), \tag{C.38}$$

$$f_{II}(r) = \frac{1}{r^2} \bar{Y}_{2,2}(m_\rho r) - \frac{1}{3r} \bar{Y}_{3,3}(m_\rho r) - \frac{1}{r} \bar{Y}_{1,3}(m_\rho r), \tag{C.39}$$

$$f_{III}(r) = \frac{1}{r} \bar{Y}_{2,2}(m_\rho r), \tag{C.40}$$

$$f_{IV}(r) = \frac{1}{r} \bar{Y}_{1,1}(m_\rho r). \tag{C.41}$$

The omega-exchange current is obtained by the following replacement in Eq. (C.37) ,

$$g_\rho \to g_\omega \, , \; \kappa_\rho \to \kappa_\omega \, , \; m_\rho \to m_\omega \quad \text{and} \quad \tau^1 \cdot \tau^2 \to 1 \tag{C.42}$$

b) Dissociation Current

The OBE current due to dissociation of rho-meson into pion is given as,

$$\vec{\mu}^{\rho\pi\gamma} = \frac{e}{2M} \frac{g_{\rho\pi\gamma} g_\rho g_\pi}{m_\rho (m_\rho^2 - m_\pi^2)} \tau^1 \cdot \tau^2 \left[\frac{1}{3} \sigma_+ f_I(r) + \vec{T}_{12}^{(+)} f_{II}(r) \right] , \tag{C.43}$$

where

$$f_I(r) = Y_{0,2}(m_\pi r) - Y_{0,2}(m_\rho r) , \tag{C.44}$$

$$f_{II}(r) = - \left[Y_{2,2}(m_\pi r) - Y_{2,2}(m_\rho r) \right] . \tag{C.45}$$

c) Pair Current

Pion-pair current is written as

$$\begin{aligned}
\vec{\mu}_\pi = \frac{e}{4M^2} \left[\frac{g_\pi}{2M} \right]^2 \tau^1 \cdot \tau^2 \Big\{ & (1+\kappa_S) \left[\frac{1}{3} \sigma_+ Y_{0,2}(x_\pi) - \vec{T}_{12}^{(+)} Y_{2,2}(x_\pi) \right] \\
& + \overleftrightarrow{L} \left[\frac{1}{6} \sigma^1 \cdot \sigma^2 Y_{0,2}(x_\pi) - \frac{1}{2} S_{12} \; Y_{2,2}(x_\pi) \right] \\
& + \Big[\left(\frac{b-c}{4} - \frac{1+c}{2} \kappa_S \right) \left[\sigma^1 (\sigma^2 \cdot \hat{r}) + \sigma^2 (\sigma^1 \cdot \hat{r}) \right] \times \overleftrightarrow{Q} \\
& \quad + \frac{1}{8} (c - 2b + 1) \left[\sigma^1 (\sigma^2 \cdot \overleftrightarrow{Q}) + \sigma^2 (\sigma^1 \cdot \overleftrightarrow{Q}) \right] \times \hat{r} \; \Big] Y_{11}(x_\pi) \\
& + \frac{1}{8} (b - c - 1) \left[\sigma_1 (\sigma^2 \cdot \hat{r}) + \sigma^2 (\sigma^1 \cdot \hat{r}) \right] \times \hat{r} \; \overleftrightarrow{Q} \cdot \hat{r} \; Y_{22}(x_\pi) \Big\} .
\end{aligned} \tag{C.46}$$

The rho-pair current is written as

$$\vec{\mu}_\rho = -\frac{e g_\rho^2}{8M^2}\,\tau^1\cdot\tau^2\left[-\vec{L}\,Y_{00}(x_\rho) + (1+\kappa_\rho)\left(\frac{2}{3}\sigma_+ - \vec{T}_{12}^{(n)}\right)Y_{11}(x_\rho)\,r\right]. \tag{C.47}$$

The omega-pair current is obtained from Eq. (C.47) by the replacement of Eq. (C.42).

iii) TBE Current

We shall define J and $\bar{J}$ from Eq. (C.19) as

$$J_{\ell,\ell'} = J_{\ell,\ell',22}(r) \tag{C.48}$$

$$\bar{J}_{\ell,\ell'} = J_{\ell\ell',11}(r) \tag{C.49}$$

The TBE currents are given by using the configuration representation of radial functions J and $\bar{J}$. In what follows, we shall drop the suffix in energy denominator functions due to different intermediate states in Eqs. (B.6) - (B.16). The upper (lower) part of the following equations show the contributions of uncrossed (crossed) diagrams. In the expressions in sub-section iii.1. a)- c) and iii.2. a) - c), only the expressions for the two-pion-exchange currents are shown. For the (ρ,π) and (ρ,ρ) exchange currents , function J should be replaced as follows. J_{00} should be replaced as

$4J_{00}$ for (ρ,π) exchanges

and

$4J_{00}$ for (ρ,ρ) exchanges.

$J_{02} + J_{20}$ should be replaced as

$$4J_{02} - 2J_{20} \qquad \text{for } (\rho,\pi) \text{ exchanges,}$$

and

$$-2J_{02} - 2J_{20} \qquad \text{for } (\rho,\rho) \text{ exchanges.}$$

J_{22} should be replaced as

$$-2J_{22} \qquad \text{for } (\rho,\pi) \text{ exchanges.}$$

and

$$J_{22} \qquad \text{for } (\rho,\rho) \text{ exchanges.}$$

iii.1. TBE Current due to Intrinsic Magnetic Moment of Nucleon and Isobar

a) N-N Intermediate State

$$\vec{\mu} = \pm(3 \mp 2\tau^{1}\cdot\tau^{2})\,\mu^{s} f^{NN}\left[\sigma_{+} f_{I} + \vec{T}_{12}^{(+)} f_{II}\right] \tag{C.50}$$

with

$$f_{I} = \frac{1}{9}\left[\begin{pmatrix}0\\8\end{pmatrix}J_{00} + \begin{pmatrix}12\\4\end{pmatrix}J_{22}\right], \tag{C.51}$$

$$f_{II} = \frac{1}{3}\left[\begin{pmatrix}0\\4\end{pmatrix}(J_{02}+J_{20}) + \begin{pmatrix}-6\\2\end{pmatrix}J_{22}\right]. \tag{C.52}$$

b) N-Δ Intermediate State

Operators corresponding to the diagrams in Fig. 11-b are written as,

$$\vec{\mu} = (3 \pm \tau^1 \cdot \tau^2) f^{N\Delta} \{ \mu_S \, [\sigma_+ f_I + \vec{T}_{12}^{(+)} f_{II}]$$

$$+ \mu_S^{\Delta} \, [\sigma_+ f_I^{\Delta} + \vec{T}_{12}^{(+)} f_{II}^{\Delta}] \}, \tag{C.53}$$

with

$$f_I = \frac{1}{81} \left[\binom{-16}{-8} J_{00} + \binom{-20}{-24} J_{22} \right], \tag{C.54}$$

$$f_{II} = \frac{1}{27} \left[\binom{-2}{2} (J_{02} + J_{20}) + \binom{-4}{4} J_{22} \right], \tag{C.55}$$

$$f_I^{\Delta} = \frac{1}{243} \left[\binom{40}{0} J_{00} + \binom{20}{60} J_{22} \right], \tag{C.56}$$

$$f_{II}^{\Delta} = \frac{1}{81} \left[\binom{7}{-3} (J_{02} + J_{20}) + \binom{16}{-24} J_{22} \right]. \tag{C.57}$$

Operators corresponding to diagrams in Fig. 11-c are written as

$$\vec{\mu} = (3 \pm \tau^1 \cdot \tau^2) f^{N\Delta} \mu_S \, [\sigma_+ f_I + \vec{T}_{12}^{(+)} f_{II}], \tag{C.58}$$

with

$$f_I = \frac{1}{81} \left[\binom{-24}{-8} J_{00} + \binom{-24}{-40} J_{22} \right], \tag{C.59}$$

$$f_{II} = \frac{1}{27} \left[\binom{-12}{-4} J_{00} + \binom{0}{16} J_{22} \right]. \tag{C.60}$$

c) Δ-Δ Intermediate State

$$\vec{\mu}^{\Delta\Delta} = (6 \mp \tau^1 \cdot \tau^2) f^{\Delta\Delta} \{ \mu_S [\sigma_+ f_I + \vec{T}_{12}^{(+)} f_{II}]$$

$$+ \mu_S^{\Delta} [\sigma_+ f_I^{\Delta} + \vec{T}_{12}^{(+)} f_{II}^{\Delta}] \}, \tag{C.61}$$

with

$$f_I = \frac{1}{729} \left[\binom{-20}{-28} J_{00} + \binom{-52}{-44} J_{22} \right], \tag{C.62}$$

$$f_{II} = \frac{1}{243} \left[\binom{2}{-2} (J_{02} + J_{20}) + \binom{4}{-2} J_{22} \right], \tag{C.63}$$

$$f_I^{\Delta} = \frac{1}{2187} \left[\binom{20}{60} J_{00} + \binom{100}{60} J_{22} \right], \tag{C.64}$$

$$f_{II}^{\Delta} = \frac{1}{729} \left[\binom{-2}{18} (J_{02} + J_{20}) + \binom{-28}{12} J_{22} \right]. \tag{C.65}$$

d) ρ-π Exchange Current due to Seagull Interaction

Operators corresponding to diagrams in Fig. 11-e are written as

$$\vec{\mu}^{N} = 16 f^{N} \mu_S \tau^1 \cdot \tau^2 \left[\frac{2}{3} \sigma_+ - \vec{T}_{12}^{(+)} \right] \mathcal{J}_{11}, \tag{C.66}$$

and the operators corresponding to diagrams in Fig. 11-f are written as,

$$\vec{\mu}^{\Delta} = \frac{8}{9} f^{\Delta} \left[\mu_S - \frac{5}{3} \mu_S^{\Delta} \right] \tau^1 \cdot \tau^2 \left[\frac{2}{3} \sigma_+ - \vec{T}_{12}^{(+)} \right] \mathcal{J}_{11}. \tag{C.67}$$

iii.2. TBE Current due to Convection Current

a) N- N Intermediate State

$$\vec{\mu}^{NN} = \frac{e}{2M} f^{NN}(3-2\tau^1\tau^2)[2\vec{\sigma}_+ - 3\vec{T}_{12}]\cdot\left[-\frac{1}{6}J_{22}\right]. \tag{C.68}$$

Here, only uncrossed diagrams contribute to magnetic moment of deuteron.

b) N-Δ Intermediate State

$$\vec{\mu}^{N\Delta} = \frac{e}{2M}\left(\frac{M}{M_\Delta}\right) f^{N\Delta}(3\pm\tau^1\tau^2)[2\vec{\sigma}_+ - 3\vec{T}_{12}]\left[\frac{1}{54}\begin{bmatrix}1\\-3\end{bmatrix} J_{22}^{(\pm)}\right]$$

$$-\frac{e}{2M}\frac{\Delta}{M_\Delta} f^{N\Delta}(3\pm\tau^1\tau^2)\overleftrightarrow{L}\left[f_I + \sigma^1\cdot\sigma^2 f_{II} + S_{12} f_{III}\right], \tag{C.69}$$

with

$$f_I = \frac{1}{27}\left(\tilde{J}_{00}^{(\pm)} + 2\tilde{J}_{22}^{(\pm)}\right), \tag{C.70}$$

$$f_{II} = \pm\frac{1}{81}\left(\tilde{J}_{00}^{(\pm)} - \tilde{J}_{22}^{(\pm)}\right), \tag{C.71}$$

$$f_{III} = \pm\frac{1}{54}\left(\tilde{J}_{02}^{(\pm)} + \tilde{J}_{20}^{(\pm)} + 2\tilde{J}_{22}^{(\pm)}\right), \tag{C.72}$$

$$J_{22}^{(\pm)} = \int\frac{dp\,dq}{(2\pi^2)^2} p^4 q^4 j_2(pr) j_2(qr)\frac{1}{2}\left[I_{A(\pm)}^{N\Delta} \pm I_{B(\pm)}^{N\Delta}\right], \tag{C.73}$$

$$\tilde{J}_{\ell,\ell'}^{(\pm)} = \int\frac{dp\,dq}{(2\pi^2)^2} p^4 q^4 j_\ell(pr) j_{\ell'}(qr)\, I_{A\pm}^{N\Delta}. \tag{C.74}$$

c) Δ-Δ Intermediate State

$$\vec{\mu}^{\Delta\Delta} = \frac{e}{2M}\left(\frac{M}{M_\Delta}\right) f^{\Delta\Delta}(6-\tau^1\tau^2)[2\vec{\sigma}_+ - 3\vec{T}_{12}^{(\pm)}]\left[-\frac{1}{243}\begin{pmatrix}1\\0\end{pmatrix} J_{22}\right]$$

$$-\frac{e}{2M}\frac{\Delta}{M_\Delta} f^{\Delta\Delta}(6\mp\tau^1\cdot\tau^2)\overleftrightarrow{L}\left[f_I + f_{II}\,\sigma^1\cdot\sigma^2 + f_{III}\,S_{12}\right], \tag{C.75}$$

with

$$f_I = \frac{2}{243}\left(J_{00} + 2J_{22}\right), \tag{C.76}$$

$$f_{II} = \mp \frac{1}{729} (J_{00} - J_{22}), \tag{C.77}$$

$$f_{III} = \mp \frac{1}{486} (J_{02} + J_{20} + 2J_{22}) \tag{C.78}$$

d) ρ-π Exchange Current due to Seagull Interaction

Operator corresponding to diagrams in Fig. 11-e is written as

$$\vec{\mu}^{N} = \frac{e}{2M} f^{N} \tau^1 \cdot \tau^2 \left[-\frac{4}{3} \sigma_+ + 2 \vec{T}_{12}^{(+)} \right] \bar{J}_{11}, \tag{C.79}$$

and the operator corresponding to diagrams in Fig. 11-d is written as

$$\vec{\mu}^{\Delta} = \frac{e}{2M_\Delta} f^{\Delta} \tau^1 \cdot \tau^2 \left(\frac{8}{27} \sigma_+ - \frac{4}{9} \vec{T}_{12}^{(+)} \right) \bar{J}_{11}$$

$$- \frac{e}{2M} \frac{\Delta}{9M_\Delta} f^{\Delta} \tau^1 \cdot \tau^2 \vec{L} \left[\frac{4}{3} \sigma^1 \cdot \sigma^2 - 2S_{12} \right] \bar{J}_{11}. \tag{C.80}$$

iii.3. Dissociation Current

Here, the radial functions of exchange currents are defined as

$$J_{\ell\ell'}^{\rho\pi\gamma} = \frac{1}{m_\rho^2 - m_\pi^2} \left[J_{\ell\ell'}^{\rho\beta} - J_{\ell\ell'}^{\pi\beta} \right] \tag{C.81}$$

For example, the radial function of N-N intermediate state $J_{\ell\ell'}^{\rho\pi\gamma,NN1}$ is given as

$$J_{\ell\ell'}^{\rho\pi\gamma,NN1} = \frac{1}{m_\rho^2 - m_\pi^2} \left[J_{\ell\ell'}^{\rho\beta,NN1} - J_{\ell\ell'}^{\pi\rho,NN1} \right], \tag{C.82}$$

with

$$J_{\ell\ell'}^{\rho\beta,NN1} = \int \frac{dp\, dq}{(2\pi^2)^2}\, p^4 q^4\, j_\ell(pr)\, j_{\ell'}(qr)\, [-I_{\rho\beta}^{N}(3)]. \tag{C.83}$$

In what follows we shall show the $\rho\pi-\pi$ exchange currents. The $\rho\pi-\rho$ exchange currents are obtained by the replacement in the $\rho\pi-\pi$ exchange currents as follows. J_{00} should be replaced as

$2\, J_{00}$ for $\rho\pi-\rho$ exchange currents .....

$J_{02} + J_{20}$ should be replaced as

$2\, J_{02} - J_{20}$ for $\rho\pi-\rho$ exchange currents

J_{22} should be replaced as

$-J_{22}$ for $\rho\pi-\rho$ exchange currents.

a) N-N Intermediate State

$$\vec{\mu}^{NN} = \pm(3 \mp 2\tau^1\cdot\tau^2) f_{\rho\pi\gamma}^{NN}\, \vec{\gamma}_{A,X} - 2(3+2\tau^1\cdot\tau^2) g_{\rho\pi\gamma}^{NN} (\vec{\gamma}_{B,Y} - \vec{\gamma}_{C,Y}), \tag{C.84}$$

where

$$\vec{\gamma}_{A,X} = \frac{2}{9}\, \vec{\sigma}_+ (J_{00}^{\rho\pi\gamma,X} + 2J_{22}^{\rho\pi\gamma,X}) - \frac{1}{3}\vec{T}_{12}^{(+)} (J_{02}^{\rho\pi\gamma,X} + J_{20}^{\rho\pi\gamma,X} - J_{22}^{\rho\pi\gamma,X}), \tag{C.85}$$

$$\vec{\gamma}_{B,Y} = -\frac{2}{9}\, \sigma_+ [J_{00}^{\rho\pi\gamma Y} - J_{22}^{\rho\pi\gamma,Y}] - \frac{1}{3}\vec{T}_{12}^{(+)} [J_{02}^{\rho\pi\gamma,Y} + J_{20}^{\rho\pi\gamma,Y} + 2\, J_{22}^{\rho\pi\gamma,Y}], \tag{C.86}$$

$$\vec{\gamma}_{C,Y} = \frac{1}{9}\, \sigma_+ [2J_{00}^{\rho\pi\gamma,Y} + 4J_{22}^{\rho\pi\gamma,Y}] + \frac{1}{3}\vec{T}_{12}^{(+)} [J_{02}^{\rho\pi\gamma,Y} + J_{20}^{\rho\pi\gamma,Y} - J_{22}^{\rho\pi\gamma,Y}], \tag{C.87}$$

with $X = NN1$ and $Y = NN2$.

b) N-Δ Intermediate State

$$\vec{\mu}^{N\Delta} = \frac{8}{9}(3 \pm \tau^1 \cdot \tau^2) f_{\rho\pi\sigma}^{N\Delta} \vec{\gamma}_{A,X} + \frac{4}{9} g_{\rho\pi\sigma}^{\Delta N} (3 \pm \tau^1 \cdot \tau^2)(2\vec{\gamma}_{B,Y} \mp \vec{\gamma}_{C,Y})$$

$$+ \frac{4}{9} g_{\rho\pi\sigma}^{N\Delta} (3 \pm \tau^1 \cdot \tau^2)(\mp \vec{\gamma}_{B,Y} - 2\vec{\gamma}_{C,Y}) \qquad \text{(C.88)}$$

where $X = N\Delta 1$ and $Y = N\Delta 2$.

c) Δ-Δ Intermediate State

$$\vec{\mu}^{\Delta\Delta} = -\frac{8}{81} g_{\rho\pi\sigma}^{\Delta\Delta} (6 + \tau^1 \cdot \tau^2)(\vec{\gamma}_{B,X} - \vec{\gamma}_{C,X}) \qquad \text{(C.89)}$$

with $X = \Delta\Delta$.

Appendix D. Matrix Elements of Magnetic Moment Operators

We summarize the matrix elements of magnetic moment operators by using the deuteron wave function in Eq. (8.3). At first we shall explain the notation to express the matrix elements in the following. The contributions of the magnetic moment of deuteron are divided into S-S. S-D and D-D matrix elements as $\langle M_{SS}\rangle$, $\langle M_{SD}\rangle$, $\langle M_{DD}\rangle$, respectively:

$$\langle M_{SS}\rangle = \int_0^\infty \left(\frac{u}{r}\right)^2 M_{SS}\, r^2 dr,$$

$$\langle M_{SD}\rangle = \int_0^\infty \left(\frac{u w}{r^2}\right) M_{SD}\, r^2 dr,$$

$$\langle M_{DD}\rangle = \int_0^\infty \left(\frac{w}{r}\right)^2 M_{DD}\, r^2 dr. \tag{D.1}$$

We denote the derivative operator on the radial wave functions of S- and D-waves as

$$\frac{d}{dr_S} \quad \text{and} \quad \frac{d}{dr_D}, \tag{D.2}$$

and we also define

$$\overleftrightarrow{D} = \frac{d}{dr_D} - \frac{d}{dr_S} + \frac{3}{r} \tag{D.3}$$

Here, following equations are useful to calculate the magnetic moment due to two-body operators. From the magnetic moment operator as the following form,

$$\vec{\mu} = \sigma_+ f_1 + T_{12}^{(+)} f_2 + \vec{L}\,(g_1 + \sigma^1\cdot\sigma^2 g_2 + S_{12}\, g_3), \tag{D.4}$$

we obtained the contributions to the magnetic moment of deuteron as,

$$\mu_{SS} = 2 f_1, \tag{D.5}$$

$$\mu_{SD} = 2\sqrt{2}\ \left(\frac{1}{3} f_2 + g_3\right), \tag{D.6}$$

$$\mu_{DD} = - f_1 + \frac{2}{3} f_2 + 3 (g_1 + g_2) - 2 g_3 \tag{D.7}$$

In what follows, we shall show the contributions of relativistic correction of one-body-current ($\Delta\mu_{\mathrm{rel.cor.}}$), one-boson-exchange current (μ_{OBE}) and two-boson-exchange current(μ_{TBE}) to the magnetic moment of deuteron.

D.1. Relativistic Correction of One-Body-Current

$$\mu_{SS} = \frac{e}{2M} \left[-\frac{1}{6M^2}\right] \left[(3 + a) + \kappa_S \right] \vec{p}^{\,2}, \tag{D.8}$$

$$\mu_{SD} = \frac{e}{2M} \frac{2\kappa_S - a}{6\sqrt{2}\, M^2} \left[\frac{d^2}{dr_D^2} + \frac{5}{r} \frac{d}{dr_D} + \frac{3}{r^2}\right], \tag{D.9}$$

$$\mu_{DD} = \frac{e}{2M} \frac{1}{24 M^2} \left[-2\kappa_S + (-3 + 2a) \right] \vec{p}^{\,2}, \tag{D.10}$$

where

$$\langle \vec{p}^{\,2} \rangle_{SS} = \int_0^\infty \left(\frac{du}{dr}\right)^2 dr, \tag{D.11}$$

$$\langle \vec{p}^{\,2} \rangle_{DD} = \int_0^\infty \left[\left(\frac{dw}{dr}\right)^2 + \frac{6}{r^2} w^2 \right] dr. \tag{D.12}$$

D.2. One-Boson-Exchange Current

i) Recoil Current

a) Pion-Exchange Current

$$M_{SS} = 0, \tag{D.13}$$

$$M_{SD} = -\frac{3e}{8M^2}\left[\frac{g_\pi}{2M}\right]^2\left[2\sqrt{2}\,\overleftrightarrow{D} f_{III} + f_I\right], \tag{D.14}$$

$$M_{DD} = -\frac{3e}{8M^2}\left[\frac{g_\pi}{2M}\right]^2\left[3 f_{II} - 2 f_I\right]. \tag{D.15}$$

b) Rho-Exchange Current

$$M_{SS} = 0, \tag{D.16}$$

$$M_{SD} = -\frac{3e}{8M^3}\left[\frac{g_\rho}{2M}(1+K_\rho)\right]^2\left[2\sqrt{2}\,\overleftrightarrow{D} f_{III} + f_I\right], \tag{D.17}$$

$$M_{DD} = -\frac{3e}{8M^3}\left[\frac{g_\rho}{2M}(1+K_\rho)\right]^2\left[3 f_{II} - 2 f_I\right] + \frac{9e}{8M^2} g_\rho^2 f_{IV}. \tag{D.18}$$

c) Omega-Exchange Current

$$M_{SS} = 0, \tag{D.19}$$

$$M_{SD} = \frac{e}{8M^2}\left[\frac{g_\omega}{2M}(1+K_\omega)\right]^2\left[2\sqrt{2}\,\overleftrightarrow{D} f_{III} + f_I\right], \tag{D.20}$$

$$M_{DD} = \frac{e}{8M^2}\left[\frac{g_\omega}{2M}(1+K_\omega)\right]^2\left[3 f_{II} - 2 f_I\right] - \frac{3e}{8M^2} g_\omega^2 f_{IV}. \tag{D.21}$$

ii) Dissociation Current

$$M_{SS} = \frac{e}{2M}\left[-\frac{g_{\rho\pi\gamma}\, g_\rho g_\pi}{m_\rho (m_\rho^2 - m_\pi^2)}\right][2 f_I], \tag{D.22}$$

$$M_{SD} = \frac{e}{2M}\left[-\frac{g_{\rho\pi\gamma}\, g_\rho g_\pi}{m_\rho (m_\rho^2 - m_\pi^2)}\right][2\sqrt{2} f_{II}], \tag{D.23}$$

$$M_{DD} = \frac{e}{2M}\left[-\frac{g_{\rho\pi\gamma}\, g_\rho g_\pi}{m_\rho (m_\rho^2 - m_\pi^2)}\right][-f_I + 2 f_{II}]. \tag{D.24}$$

iii) Pair Current

a) Pion-Exchange Current

Here, we define $\widetilde{M}_x$ as

$$M_x = \left[\frac{e}{2M}\right]\left[-\frac{3}{2M}\left(\frac{g_\pi}{2M}\right)^2\right]\widetilde{M}_x . \tag{D.25}$$

The matrix element of local current is

$$\widetilde{M}_{SS} = \frac{2}{3}(1 + \kappa_S)\, Y_{0,2} \tag{D.26}$$

$$\widetilde{M}_{SD} = -\frac{2\sqrt{2}}{3}(1 + \kappa_S)\, Y_{2,2} \tag{D.27}$$

$$\widetilde{M}_{DD} = -\frac{1}{3}(1 + \kappa_S)\, [\, Y_{0,2} + 2 Y_{2,2}] \tag{D.28}$$

The matrix element of non-local current is

$$\widetilde{M}_{SD} = \sqrt{2}\left[-\kappa_S (1+c) + \frac{b-c+1}{4}\right]\overleftrightarrow{D}\, Y_{1,1}$$

$$+\sqrt{2}\,\frac{b-c-1}{4}\left[\overleftrightarrow{D} - \frac{3}{r}\right] Y_{2,2}\, r \;-\; \sqrt{2}\; Y_{2,2} \tag{D.29}$$

$$\widehat{M}_{DD} = \frac{1}{2} Y_{0,2} + Y_{2,2} \tag{D.30}$$

b) Rho-Exchange Current

$$M_{SS} = \frac{e}{2M}\left[\frac{g_\rho^2}{4M}\right]\left[4(1+\kappa_\rho)\, r\, Y_{1,1}\right], \tag{D.31}$$

$$M_{SD} = \frac{e}{2M}\left[\frac{g_\rho^2}{4M}\right]\left[-2\sqrt{2}(1+\kappa_\rho)\, r\, Y_{1,1}\right], \tag{D.32}$$

$$M_{DD} = \frac{e}{2M}\left[\frac{g_\rho^2}{4M}\right]\left[9 Y_{0,0} - 4(1+\kappa_\rho)\, r\, Y_{1,1}\right]. \tag{D.33}$$

c) Omega-Exchange Current

$$M_{SS} = \frac{e}{2M}\left[\frac{g_\omega^2}{12M}\right]\left[-4(1+\kappa_\omega)\, r\, Y_{1,1}\right], \tag{D.34}$$

$$M_{SD} = \frac{e}{2M}\left[\frac{g_\omega^2}{12M}\right]\left[2\sqrt{2}(1+\kappa_\omega)\, r\, Y_{1,1}\right], \tag{D.35}$$

$$M_{DD} = \frac{e}{2M}\left[\frac{g_\omega^2}{12M}\right]\left[-9 Y_{0,0} + 4(1+\kappa_\omega)\, r\, Y_{1,1}\right]. \tag{D.36}$$

D.3. Two-Boson-Exchange Current

Here, we shall show the contribution of two-pion-exchange current due to the nucleon-type currents and dissociation current, except the currents due to the $\rho\pi$ seagull-type interaction. The rho-exchange is taken into account by performing the replacement which is shown in Appendix C.2. iii).

i) TBE Current due to Intrinsic Magnetic Moment of Nucleon and Isobar

a) N-N Intermediate State

$$M_{SS} = \mu_s f^{NN}\left[\begin{bmatrix} 0 \\ \frac{16}{3} \end{bmatrix} J_{00} + \begin{bmatrix} 24 \\ \frac{8}{3} \end{bmatrix} J_{22} \right], \tag{D.37}$$

$$M_{SD} = \mu_s f^{NN}\sqrt{2}\left[\begin{bmatrix} 0 \\ \frac{8}{3} \end{bmatrix} (J_{02}+J_{20}) + \begin{bmatrix} -12 \\ \frac{4}{3} \end{bmatrix} J_{22} \right], \tag{D.38}$$

$$M_{DD} = \mu_s f^{NN}\left[\begin{bmatrix} 0 \\ -\frac{8}{3} \end{bmatrix} J_{00} + \begin{bmatrix} 0 \\ \frac{8}{3} \end{bmatrix} (J_{02}+J_{20}) + \begin{bmatrix} -24 \\ 0 \end{bmatrix} J_{22} \right]. \tag{D.39}$$

b) N-Δ Intermediate State

Here, only the crossed diagrams contribute to the magnetic moment of deuteron. Contributions due to diagrams in Fig. 11-b are

$$M_{SS} = f^{N\Delta}\left[-\frac{8}{27}\mu_S \left[4J_{00} + 14J_{22}\right] + \frac{80}{27}\mu_S^{\Delta} J_{22} \right], \tag{D.40}$$

$$M_{SD} = f^{N\Delta}\sqrt{2}\left[\frac{8}{27}\mu_S \left[J_{02} + J_{20} + 2J_{22}\right] - \frac{8}{27}\mu_S^{\Delta}\left[J_{02} + J_{20} + 4J_{22}\right] \right], \tag{D.41}$$

$$M_{DD} = f^{\Delta}\left[\frac{8}{27}\mu_S \left[2J_{00} + J_{02} + J_{20} + 9J_{22}\right] - \frac{8}{27}\mu_S^{\Delta}\left[J_{02} + J_{20} + 9J_{22}\right] \right]. \tag{D.42}$$

Contributions due to diagrams in Fig. 11-c are

$$M_{SS} = \mu_S f^{N\Delta}\left(-\frac{32}{27}\right)\left[J_{00} + 5J_{22}\right], \tag{D.43}$$

$$M_{SD} = \mu_S f^{N\Delta}\left(-\frac{16\sqrt{2}}{27}\right)\left[J_{02} + J_{20} - 4J_{22}\right], \tag{D.44}$$

$$M_{DD} = \mu_S f^{N\Delta}\left[-\frac{16}{27}\right]\left[-J_{00} + J_{02} + J_{20} - 9J_{22}\right]. \tag{D.45}$$

c) Δ-Δ Intermediate State

$$M_{SS}=\frac{4}{243}\mu^{s}f^{\Delta\Delta}\left[\begin{bmatrix}-30\\-14\end{bmatrix}J_{00}+\begin{bmatrix}-78\\-22\end{bmatrix}J_{22}\right]$$

$$+\frac{4}{243}\mu_{s}^{\Delta}f^{\Delta\Delta}\left[\begin{bmatrix}10\\10\end{bmatrix}J_{00}+\begin{bmatrix}50\\10\end{bmatrix}J_{22}\right], \tag{D.46}$$

$$M_{SD}=\frac{4\sqrt{2}}{243}\mu^{s}f^{\Delta\Delta}\left[\begin{bmatrix}3\\-1\end{bmatrix}(J_{02}+J_{20})+\begin{bmatrix}6\\2\end{bmatrix}J_{22}\right]$$

$$+\frac{4\sqrt{2}}{243}\mu_{s}^{\Delta}f^{\Delta\Delta}\left[\begin{bmatrix}-1\\3\end{bmatrix}(J_{02}+J_{20})+\begin{bmatrix}-14\\2\end{bmatrix}J_{22}\right], \tag{D.47}$$

$$M_{DD}=\frac{2}{243}\mu^{s}f^{\Delta\Delta}\left[\begin{bmatrix}30\\14\end{bmatrix}J_{00}+\begin{bmatrix}6\\-2\end{bmatrix}(J_{02}+J_{20})+\begin{bmatrix}90\\18\end{bmatrix}J_{22}\right]$$

$$+\frac{2}{243}\mu_{s}^{\Delta}f^{\Delta\Delta}\left[\begin{bmatrix}-10\\-10\end{bmatrix}J_{00}+\begin{bmatrix}-2\\6\end{bmatrix}(J_{02}+J_{20})+\begin{bmatrix}-78\\-6\end{bmatrix}J_{22}\right]. \tag{D.48}$$

d) ρ-π Exchange Current due to Seagull Interaction

Contributions due to diagrams in Fig. 11-e are

$$M_{SS} = -64 f^{N}\mu_{S}J_{11}, \tag{D.49}$$

$$M_{SD} = 32\sqrt{2}\, f^{N}\mu_{S}J_{11}, \tag{D.50}$$

$$M_{DD} = 64 f^{N}\mu_{S}J_{11}. \tag{D.51}$$

contributions due to diagrams in Fig. 11-f are

$$M_{SS} = -\frac{32}{9}f^{\Delta}(\mu_{S}-\frac{5}{3}\mu_{S}^{\Delta})J_{11} \tag{D.52}$$

$$M_{SD} = \frac{16}{9}\sqrt{2}\, f^{\Delta}\left(\mu_S - \frac{5}{3}\mu_S^{\Delta}\right) J_{11}, \tag{D.53}$$

$$M_{DD} = \frac{32}{9} f^{\Delta}\left(\mu_S - \frac{5}{3}\mu_S^{\Delta}\right) J_{11}. \tag{D.54}$$

ii) TBE Current due to Convection Current

Here, only the uncrossed diagrams in N-N intermediate state and crossed diagrams in N-Δ intermediate state have the contribution in deuteron magnetic moment.

a) N-N Intermediate State

$$M_{SS} = \frac{e}{2M} f^{NN}\left(-6 J_{22}\right), \tag{D.55}$$

$$M_{SD} = \frac{e}{2M} f^{NN}\left(3\sqrt{2} J_{22}\right), \tag{D.56}$$

$$M_{DD} = \frac{e}{2M} f^{NN}\, 6 J_{22}. \tag{D.57}$$

b) N-Δ Intermediate State

The contributions due to diagrams in Fig. 11-b are

$$M_{SS} = \frac{e}{2M_\Delta} f^{N\Delta}\left[-\frac{4}{3}J_{22}\right], \tag{D.58}$$

$$M_{SD} = \frac{e}{2M_\Delta} f^{N\Delta}\left[\frac{2\sqrt{2}}{3}J_{22} + \frac{\Delta}{M}\frac{2\sqrt{2}}{9}\left(J_{02}+J_{20}+2J_{22}\right)\right], \tag{D.59}$$

$$M_{DD} = \frac{e}{2M_\Delta} f^{N\Delta}\left[\frac{4}{3}J_{22} - \frac{\Delta}{M}\frac{2}{9}\left(2J_{00}+J_{02}+J_{20}+9J_{22}\right)\right]. \tag{D.60}$$

The contributions due to diagrams in Fig. 11-c are

$$M_{SS} = \frac{e}{2M} f^{N\Delta} \left(\frac{4}{3} J_{22}\right), \tag{D.61}$$

$$M_{SD} = \frac{e}{2M} f^{N\Delta} \left(-\frac{2\sqrt{2}}{3} J_{22}\right), \tag{D.62}$$

$$M_{DD} = \frac{e}{2M} f^{N\Delta} \left(-\frac{4}{3} J_{22}\right). \tag{D.63}$$

c) Δ-Δ Intermediate State

$$M_{SS} = \frac{e}{2M_\Delta} f^{\Delta\Delta} \begin{bmatrix} -\frac{8}{27} \\ 0 \end{bmatrix} J_{22}, \tag{D.64}$$

$$M_{SD} = \frac{e}{2M_\Delta} f^{\Delta\Delta} \left[\begin{bmatrix} \frac{4\sqrt{2}}{27} \\ 0 \end{bmatrix} J_{22} + \frac{\sqrt{2}\,\Delta}{27 M} \begin{bmatrix} J_{02} + J_{20} + 2 J_{22} \\ -\frac{1}{3}(J_{02} + J_{20} + 2 J_{22}) \end{bmatrix} \right], \tag{D.65}$$

$$M_{DD} = \frac{e}{2M_\Delta} f^{\Delta\Delta} \left[\begin{bmatrix} \frac{8}{27} \\ 0 \end{bmatrix} J_{22} - \frac{\Delta}{81 M} \begin{bmatrix} 15 J_{00} + 3(J_{02} + J_{20}) + 45 J_{22} \\ 7 J_{00} - J_{02} - J_{20} + 9 J_{22} \end{bmatrix} \right]. \tag{D.66}$$

d) ρ-π Exchange Current due to Seagull Interaction

The contributions due to diagrams in Fig. 11-e are

$$M_{SS} = \frac{e}{2M} f^{N} \; 8 J_{11}, \tag{D.67}$$

$$M_{SD} = \frac{e}{2M} f^{N} (-4\sqrt{2}) J_{11}, \tag{D.68}$$

$$M_{DD} = \frac{e}{2M} f^{N} (-8) J_{11}. \tag{D.69}$$

The contributions due to diagrams in Fig. 11-f are

$$M_{SS} = \frac{1}{2M_\Delta} f^\Delta \left(-\frac{16}{9}\right) J_{11}, \tag{D.70}$$

$$M_{SD} = \frac{\sqrt{2}}{2M_\Delta} f^\Delta \left[\frac{8}{9} + \frac{4\Delta}{3M}\right] J_{11}, \tag{D.71}$$

$$M_{DD} = \frac{1}{2M_\Delta} f^\Delta \left[\frac{16}{9} - \frac{8\Delta}{3M}\right] J_{11}. \tag{D.72}$$

iii) Dissociation Current

We shall define the matrix element of operators $\vec{\gamma}_{Ax}$, $\vec{\gamma}_{Bx}$ and $\vec{\gamma}_{cx}$ as the following.

$$\gamma^{SS}_{A,x} = \frac{4}{9}\left(J_{00}^{\rho\pi\gamma,x} + 2J_{22}^{\rho\pi\gamma,x}\right), \tag{D.73}$$

$$\gamma^{SD}_{A,x} = -\frac{2\sqrt{2}}{9}\left(J_{02}^{\rho\pi\gamma,x} + J_{20}^{\rho\pi\gamma,x} - J_{22}^{\rho\pi\gamma,x}\right), \tag{D.74}$$

$$\gamma^{DD}_{A,x} = -\frac{2}{9}\left(J_{00}^{\rho\pi\gamma,x} + J_{02}^{\rho\pi\gamma,x} + J_{20}^{\rho\pi\gamma,x} + J_{22}^{\rho\pi\gamma,x}\right), \tag{D.75}$$

$$\gamma^{SS}_{B,x} = -\frac{4}{9}\left(J_{00}^{\rho\pi\gamma,x} - J_{22}^{\rho\pi\gamma,x}\right), \tag{D.76}$$

$$\gamma^{SD}_{B,x} = -\frac{2\sqrt{2}}{9}\left(J_{02}^{\rho\pi\gamma,x} + J_{20}^{\rho\pi\gamma,x} + 2J_{22}^{\rho\pi\gamma,x}\right), \tag{D.77}$$

$$\gamma^{DD}_{B,x} = \frac{2}{9}\left(J_{00}^{\rho\pi\gamma,x} - J_{02}^{\rho\pi\gamma,x} - J_{20}^{\rho\pi\gamma,x} - 3J_{22}^{\rho\pi\gamma,x}\right), \tag{D.78}$$

$$\gamma^{SS}_{C,x} = \frac{2}{9}\left(2J_{00}^{\rho\pi\gamma,x} + 4J_{22}^{\rho\pi\gamma,x}\right), \tag{D.79}$$

$$\gamma^{SD}_{C,x} = \frac{2\sqrt{2}}{9}\left(J_{02}^{\rho\pi\gamma,x} + J_{20}^{\rho\pi\gamma,x} - J_{22}^{\rho\pi\gamma,x}\right), \tag{D.80}$$

$$\gamma^{DD}_{C,x} = \frac{2}{9}\left(-J_{00}^{\rho\pi\gamma,x} + J_{02}^{\rho\pi\gamma,x} + J_{20}^{\rho\pi\gamma,x} - 3J_{22}^{\rho\pi\gamma,x}\right). \tag{D.81}$$

In what follows we shall drop the suffix SS, SD and DD.

a) N-N Intermeidate State

$$M = f^{NN}_{\rho\pi\gamma} \, \gamma_{A,NN1} \begin{bmatrix} 9 \\ -3 \end{bmatrix} + g^{NN}_{\rho\pi\gamma} (\gamma_{B,NN2} - \gamma_{C,NN2}) \begin{bmatrix} 0 \\ 6 \end{bmatrix} . \tag{D.82}$$

b) N-Δ Intermediate State

$$M = \frac{16}{3} f^{N\Delta}_{\rho\pi\gamma} \, \gamma_{A,N\Delta 1} + \frac{8}{3} g^{\Delta N}_{\rho\pi\gamma} (2\gamma_{B,N\Delta 2} + \gamma_{C,N\Delta 2})$$

$$+ \frac{8}{3} g^{N\Delta}_{\rho\pi\gamma} (\gamma_{B,N\Delta 2} - 2\gamma_{C,N\Delta 2}) . \tag{D.83}$$

c) Δ - Δ Intermediate State

$$M = -\frac{8}{27} g^{\Delta\Delta}_{\rho\pi\gamma} (\gamma_{B,\Delta\Delta} - \gamma_{C,\Delta\Delta}) . \tag{D.84}$$

References

1) Y. Futami, J.-I. Fujita and H. Ohtsubo, Prog. Theor. Phys. Suppl. 60 (1976), 83; Prog. Theor. Phys. 58 (1977), 1077.

2) M. Chemtob, Mesons in Nuclei edited by M. Rho and D. Wilkinson (North-Holland Pub. Co., 1979), Chap. 13, p. 495.

3) A. F. Siegert, Phys. Rev. 52 (1937), 787.

4) J I. Fujita and M. Ichimura, Mesons in Nuclei, Chap. 15, p.595.

5) F. S. Levinger and H. A. Bethe, Phys. Rev. 78 (1950), 115.

6) R. G. Sachs, Phys. Rev. 74 (1948), 433; 75 (1949), 1605..

7) R. J. Blin-Stoyle, Progress in Particle and Nuclear Physics, 1 (1978), 5.

8) A. Arima and H. Hyuga, Mesons in Nuclei, Chap. 17, p.651.

9) D. O. Riska and G. E. Brown, Phys. Lett. 38B (1972), 193.

10) I. Sato and K. Itabashi, Prog. Theor. Phys. 13 (1955), 341.

11) S. Hatano and T. Kaneno, Prog. Theor. Phys. 15 (1956), 63.

12) R. J. Adler and S. D. Drell, Phys. Rev. Lett. 13 (1964), 349.
R. J. Adler, Phys. Rev. 141 (1966), 1499.

13) Y. Horikawa, T. Fujita and K. Yazaki, Phys. Lett. 42B (1972), 173.

14) M. Gari and H. Hyuga, Nucl. Phys. A264 (1976), 409.

15) W. Jaus, Helv. Phys. Acta 49 (1976), 479.
W. Jaus, Nucl. Phys. A314 (1979), 419.

16) W. Kloet and J. Tjon, Phys. Lett. 49B (1974), 419.
M. Chemtob, E. J. Moniz and M. Rho, Phys. Rev. C10 (1974), 344.
A. D. Jackson, A. Lande and D. O. Riska, Phys. Lett. 55B (1975), 23.
J. L. Friar, Phys. Lett. 59B (1975), 145; Ann. of Phys. 104 (1977), 380.
D. O. Riska and M. Radomski, Phys. Rev. C16 (1977), 2105.

17) H. Hyuga and H. Ohtsubo, Nucl. Phys. A294 (1978), 348.

18) H. Hyuga, H. Ohtsubo and T. Sato, Nucl. Phys. A300 (1978), 242.
T. Sato, H. Ohtsubo and H. Hyuga, Prog. Theor. Phys. 63 (1980) in press.

19) N. Fukuda, K. Sawada and M. Taketani, Prog. Theor. Phys. 12 (1954), 150.

20) S. Okubo, Prog. Theor. Phys. 12 (1954), 102; 603.

21) K. Nishijima, Prog. Theor. Phys. 6 (1951), 815; 911.

22) Y. Nambu, Prog. Theor. Phys. 5 (1950), 614.

23) M. Taketani, S. Machida and S. Onuma, Prog. Theor. Phys. 7 (1952), 45.

24) H. Miyazawa, Phys. Rev. 104 (1956), 1941.

25) F. Gross, Phys. Rev. 142 (1966), 1025.
R. Faustov, Nuovo Cim. A69 (1970), 37.
J. L. Friar, Mesons in Nuclei, Chap. 14, p. 599; Proceedings of International Conference on Nuclear Physics with Electromagnetic Interactions, Mainz 1979, edited by H. Arenhovel and D. Drechsel, 455.

26) L. S. Kisslinger, Phys. Lett. 29B (1969), 211.
H. Arenhövel, M. Danos and H. Williams, Phys. Lett. 31B (1970), 1091.
P. Haapakoski and H. Saarela, Phys. Lett. 53B (1974), 333.
E. Hadjimichael, Nucl. Phys. A312 (1978), 341.

27) H. Feshbach, Phys. Rev. 107 (1957), 1626.
J. C. Bergström, Phys. Rev. C9 (1974), 2435.
R. R. Scheerbaum, Phys. Rev. C11 (1975), 255.
D. W. Sprung, Nucl. Phys. A242 (1975), 141.

28) R. Reid, Ann. of Phys. 50 (1968), 411.

29) J. Blomqvist, Phys. Lett. 32B (1970), 1.

30) H. Ohtsubo, J. I. Fujita and G. Takeda, Prog. Theor. Phys. 44 (1970), 1596.

31) K. Ohta and M. Wakamatsu, Prog. Theor. Phys. 55 (1976), 131.

32) M. Gari and H. Hyuga, Z. Phys. A277 (1976), 291.

33) M. Chemtob and M. Rho, Nucl. Phys. A163 (1971), 1.

34) M. Johnson, Ann. of Phys. 97 (1976), 400.

35) L. L. Foldy and S. A. Wouthuysen, Phys. Rev. 98 (1950), 29.
S. Tani, Prog. Theor. Phys. 6 (1951), 267.

36) D. L. Pursey, Nucl. Phys. 8 (1958), 595.
M. Barnhill III, Nucl. Phys. A131 (1969), 106.

37) H. Hyuga and M. Gari, Nucl. Phys. A274 (1976), 333.

38) N. F. Ramsey, Nuclear Moments (Wiley and Sons, 1953) .

39) J. J. J. Kokkedee, The Quark Model (W. A. Benjamin, 1969).
G. E. Brown and W. Weise, Phys. Rep. 22C (1975), 279.

40) B. Gobbi et al. ,Phys. Rev. Lett. 33 (1974), 1450.

41) G. Höhler and E. Pieterinen, Nucl. Phys. B95 (1975), 210.

42) M. M. Nagels et al., Nucl. Phys. B147 (1979), 189.

43) J. W. Durso, A. D. Jackson and B. J. Verwest, Nucl. Phys. A282 (1977), 404.

44) F. Iachello, A. D. Jackson and A. Lande, Phys. Lett. 43B (1973), 101.

45) M. Brack, D. O. Riska and W. Weise, Nucl. Phys. A287 (1977), 425.

Table 1.

Boson-Nucleon Coupling Constants f and g

1.1. TBE Currents of Nucleon-Type

(α,β)	f^{NN}	$f^{N\Delta}$	$f^{\Delta\Delta}$	f^{N}	f^{Δ}
(π,π)	$\left(\frac{g_\pi}{2M}\right)^4$	$\left[\frac{g_\pi}{2M}\right]^2\left[\frac{f_{\pi N\Delta}}{m_\pi}\right]^2$	$\left[\frac{f_{\pi N\Delta}}{m_\pi}\right]^4$	—	—
(ρ,π)	$\left[\frac{g_\pi}{2M}\right]^2\left[\frac{g_\rho}{2M}(1+K_\rho)\right]^2$	$\frac{g_\pi g_\rho}{(2M)^2}(1+K_\rho)\frac{f_{\pi N\Delta}f_{\rho N\Delta}}{m_\pi m_\rho}$	$\left[\frac{f_{\pi N\Delta}}{m_\pi}\right]^2\left[\frac{f_{\rho N\Delta}}{m_\rho}\right]^2$	$\frac{g_\pi^2 g_\rho^2}{4M^3}(1+K_\rho)$	$\frac{f_{\pi N\Delta}f_{\rho N\Delta}g_\pi g_\rho}{m_\pi m_\rho M}$
(ρ,ρ)	$\left[\frac{g_\rho}{2M}(1+K_\rho)\right]^4$	$\left[\frac{g_\rho}{2M}(1+K_\rho)\right]^2\left[\frac{f_{\rho N\Delta}}{m_\rho}\right]^2$	$\left[\frac{f_{\rho N\Delta}}{m_\rho}\right]^4$	—	—

1.2. TBE Currents due to Dissociation of Rho-Meson into Pion

(β)	$f^{NN}_{\rho\pi\gamma}$	$f^{N\Delta}_{\rho\pi\gamma}$	$g^{NN}_{\rho\pi\gamma}$
(π)	$\frac{e\, g_{\rho\pi\gamma}\, g_\pi g_\rho}{2M\, m_\rho}\left[\frac{g_\pi}{2M}\right]^2$	$\frac{e\, g_{\rho\pi\gamma}\, g_\rho f_{\pi N\Delta}\, g_\pi f_{\pi N\Delta}}{2m_\rho m_\pi^2 M}$	$\frac{e\, g_{\rho\pi\gamma}\, g_\rho g_\pi (1+K_\rho)}{4M^2 m_\rho}\left[\frac{g_\pi}{2M}\right]^2$
(ρ)	$\frac{e\, g_{\rho\pi\gamma}\, g_\pi g_\rho}{2M\, m_\rho}\left[\frac{g_\rho}{2M}(1+K_\rho)\right]^2$	$\frac{e\, g_{\rho\pi\gamma}\, g_\rho f_{\pi N\Delta}\, g_\rho (1+K_\rho) f_{\rho N\Delta}}{2m_\rho^2 m_\pi M}$	$\frac{e\, g_{\rho\pi\gamma}\, g_\rho g_\pi (1+K_\rho)}{4M^2 m_\rho}\left[\frac{g_\rho}{2M}(1+K_\rho)\right]^2$

Table 1.2. (cont.)

(β)	$g^{N\Delta}_{\rho\pi\gamma}$	$g^{\Delta N}_{\rho\pi\gamma}$	$g^{\Delta\Delta}_{\rho\pi\gamma}$
(π)	$\frac{e\, g_{\rho\pi\gamma}\, g_\pi\, g_\rho\, f^2_{\pi N\Delta}\,(1+K_\rho)}{4M^2\, m_\pi^2\, m_\rho}$	$\frac{e\, g_{\rho\pi\gamma}\, g^2_\pi\, f_{\pi N\Delta}\, f_{\rho N\Delta}}{4M^2\, m_\pi\, m_\rho^3}$	$\frac{e\, g_{\rho\pi\gamma}\, f^3_{\pi N\Delta}\, f_{\rho N\Delta}}{m_\rho^2\, m_\pi^3}$
(ρ)	$\frac{e\, g_{\rho\pi\gamma}\, g_\rho^2\, f_{\pi N\Delta}\, f_{\rho N\Delta}\,(1+K_\rho)^2}{4M^2\, m_\rho^2\, m_\pi}$	$\frac{e\, g_{\rho\pi\gamma}\, g_\pi\, g_\rho\, f^2_{\rho N\Delta}\,(1+K_\rho)}{4M^2\, m_\rho^3}$	$\frac{e\, g_{\rho\pi\gamma}\, f^3_{\rho N\Delta}\, f_{\pi N\Delta}}{m_\pi\, m_\rho^4}$

Table 2. Vertices for the TBE Magnetic Moment Operators

2.1. Vertices for the TBE Currents of Nucleon-Type

(α,β)	Γ_{JA}	Γ_{JB}	Γ_{JC}
(π,π)	$i\,(p\cdot q)^2\ \vec{\sigma}_1 \times \vec{k}$	$i\,(p\cdot q)\ \vec{\Gamma}^{1}_{J\sigma} \times \vec{k}$	$i\,(\vec{p}\times\vec{q})\times\vec{k}\quad \sigma^2\cdot(p\times q)$
(ρ,π)	$2i\,(p\times q)\cdot(p\times q)\ \vec{\sigma}_1 \times \vec{k}$	$-2i\,(p\cdot q)\ \vec{\Gamma}^{1}_{J\sigma}\times\vec{k}$ $+4i\ p^2\,(q\cdot\sigma_1)\ \vec{q}\times\vec{k}$	$-2i\,(p\cdot q)\ \vec{\Gamma}^{2}_{J\sigma}\times\vec{k}$ $+2i\,[(p\cdot q)^2\,\vec{\sigma}^2 + q^2\,(p\cdot\sigma^2)\,\vec{p}\,]\times\vec{k}$
(ρ,ρ)	$i\,[(p\cdot q)^2 + p^2q^2]\ \vec{\sigma}^1\times\vec{k}$	$i\,(p\cdot q)\ \vec{\Gamma}^{1}_{J\sigma}\times\vec{k} + 2i\,p^2q^2\ \vec{\sigma}^1\times\vec{k}$ $-2i\,[\,p^2\,(q\cdot\sigma^1)\,\vec{q} + q^2\,(p\cdot\sigma^1)\,\vec{p}\,]\times\vec{k}$	$i\,(p\cdot q)\ \vec{\Gamma}^{2}_{J\sigma}\times\vec{k}$ $+i\,[\,p^2q^2-(p\cdot q)^2]\ \vec{\sigma}^2\times\vec{k}$

(α,β)	Γ_{A}	Γ_{B}	Γ_{C}
(π,π)	$(p\cdot q)^2\ \vec{\sigma}_+$	$(p\cdot q)\ \vec{\Gamma}_\sigma$	$\vec{p}\times\vec{q}\quad \sigma_+\cdot p\times q$
(ρ,π)	$-2\,[(p\cdot q)^2 - p^2q^2]\ \vec{\sigma}_+$	$-2\,(p\cdot q)\,\vec{\Gamma}_\sigma + 4\,p^2\,(q\cdot\sigma_+)\,\vec{q}$	$-2\,(p\cdot q)\,\vec{\Gamma}_\sigma + 2(p\cdot q)^2\,\vec{\sigma}_+ + 2q^2\,(p\cdot\sigma_+)\,\vec{p}$
(ρ,ρ)	$[\,(p\cdot q)^2 + p^2q^2]\ \vec{\sigma}_+$	$(p\cdot q)\,\vec{\Gamma}_\sigma - 2\,[\,p^2\,(q\cdot\sigma_+)\,\vec{q} + q^2\,(p\cdot\sigma_+)\,\vec{p}\,]$ $+2p^2q^2\,\sigma_+$	$(p\cdot q)\,\vec{\Gamma}_\sigma + [\,p^2q^2-(p\cdot q)^2]\ \vec{\sigma}_+$

Table 2. (cont.)

2.1. (cont.)

(α,β)	$\Gamma_D^{(\pm)}$	Γ_E	Γ_F
(π,π)	$i(p\cdot q)(p\times q\cdot\sigma_\pm)(\vec{p}-\vec{q})$	$(p\cdot q)^2$	$(p\times q)\cdot\sigma^1\,(p\times q)\cdot\sigma^2$
(ρ,π)	$-2i(p\cdot q)(p\times q\cdot\sigma_\pm)(\vec{p}-\vec{q})$	$2[p^2q^2-(p\cdot q)^2]$	$-2(p\times q)\cdot\sigma^1(p\times q)\cdot\sigma^2$ $+2p^2(q\times\sigma^1)\cdot(q\times\sigma^2)$
(ρ,ρ)	$(p\cdot q)(p\times q\cdot\sigma_\pm)(\vec{p}-\vec{q})$	$(p\cdot q)^2+p^2q^2$	$(p\times q)\cdot\sigma^1(p\times q)\cdot\sigma^2+p^2(q\cdot\sigma^1)(q\cdot\sigma^2)$ $+q^2(p\cdot\sigma^1)(p\cdot\sigma^2)$

Operatros $\vec{T}^i_{J\sigma}$ and $\vec{T}_\sigma$ are given as follows: $\vec{T}^i_{J\sigma}=[(\sigma^i\cdot p)\vec{q}+(\sigma^i\cdot q)\vec{p}\,]\times k$ and $\vec{T}_\sigma=(\sigma_+\cdot p)\vec{q}+(\sigma_+\cdot q)\vec{p}$

2.2. Vertices for TBE Currents due to Dissociation of Rho-Meson into Pion

(β)	$\Gamma^{\rho\pi\gamma}_{JA}$	$\Gamma^{\rho\pi\gamma}_{JB}$	$\Gamma^{\rho\pi\gamma}_{JC}$
(π)	$i\,\vec{p}\times\vec{k}\;\sigma^1\cdot q\;p_+\cdot q$	$i(\vec{p}_-\times\vec{q})\times\vec{k}\;\sigma_2\cdot(p_+\times q)$	$i[\vec{p}_-\times(\vec{q}\times\vec{\sigma}_q)]\times\vec{b}\;(p_+\cdot q)$
(ρ)	$i\,\vec{p}\times\vec{h}\;(\sigma^1\times q)\cdot(p_+\times q)$	$i[\vec{p}_-\times(\vec{\sigma}^2\times\vec{p}_+)]\times\vec{k}\;q^2$ $-i[\vec{p}_-\times\vec{q})\times\vec{k}\;\sigma^2\cdot(p_+\times q)$	$i[\vec{p}_-\times(\vec{p}_+\times\vec{\sigma}_1)]\times\vec{k}\;q^2$ $-i[\vec{p}_-\times(\vec{q}\times\vec{\sigma}_1)]\times\vec{k}\;(p_+\cdot q)$

Table 2.2 (cont.)

(β)	$\Gamma_A^{\rho\pi\gamma}$	$\Gamma_B^{\rho\pi\gamma}$	$\Gamma_C^{\rho\pi\gamma}$
(π)	$\vec{p}\ \sigma_+\cdot q\ p\cdot q$	$\vec{p}\times\vec{q}\ \ \sigma_+\cdot(p\times q)$	$\vec{p}\times(\vec{q}\times\vec{\sigma}_+)\ (p\cdot q)$
(ρ)	$\vec{p}\ (\sigma_+\times q)\cdot(p\times q)$	$\vec{p}\times(\vec{\sigma}_+\times\vec{p})\ q^2$ $-\vec{p}\times\vec{q}\ \ \sigma_+\cdot(p\times q)$	$\vec{p}\times(\vec{p}\times\vec{\sigma}_+)\ q^2$ $-\vec{p}\times(\vec{q}\times\vec{\sigma}_+)(p\cdot q)$

Table 3. Contribution of Relativistic Correction to Magnetic Moment of Deuteron

	S - S	S - D	D - D	SUM
H.C.	-0.0110	-0.0002	-0.0014	-0.0127
S.C.	-0.0064	-0.0000	-0.0012	-0.0077

Table 4. Contribution of OBE and TBE Magnetic Moment Operators

Here we have used wave function from H.C. potential and the following parameters: $f^2_{\pi N\Delta}/4\pi = 0.23$, $a = 0$ and $c = -1$.

4.1. Contribution of Exchange Current for Different κ_ρ

κ_ρ	3.7	5.15	6.6
OBE	0.0164	0.0172	0.0180
TBE	0.0062	0.0044	0.0025
SUM	0.0226	0.0216	0.0205

4.2. Contribution of Exchange Current for Different Choice of Vertex Function.

A and B correspond to the choice of vertex functions of Eqs.(13.13) and (13.12), respectively.

	A	B
OBE	0.0164	0.0195
TBE	0.0062	0.0033
SUM	0.0226	0.0228

Table 5. Contribution of OBE Current to Magnetic Moment of Deuteron

The upper (lower) values show the contribution of OBE current using H. C. (S. C.) wave function. We have used $\kappa_\rho = 3.7$ and vertex functions of Eqs. (13.11) and (13.13).

5.1. Contributions of Recoil, Pair and Dissociation Currents to μ_{OBE}.

In the recoil and pair currents, individual contributions of π, ρ and ω exchanges and their summed contributions are shown.

	π	ρ	ω	SUM
Recoil	-0.0008	-0.0001	-0.0001	-0.0009
	-0.0011	-0.0000	-0.0001	-0.0012
Pair	0.0108	0.0030	-0.0018	0.0120
	0.0095	0.0033	-0.0020	0.0109
Dissociation		0.0053		
		0.0049		
μ_{OBE}		0.0164		
		0.0146		

5.2. μ_{OBE} due to S-S, S-D and D-D Matrix Elements

	S - S	S - D	D - D
Recoil	0	0.0001	-0.0010
	0	-0.0001	-0.0010
Pair	0.0031	0.0108	-0.0018
	0.0029	0.0099	-0.0018
Dissociation	-0.0005	0.0045	0.0013
	-0.0008	0.0045	0.0013
SUM	0.0026	0.0154	-0.0016
	0.0021	0.0142	-0.0016

5.3. Contribution of Pion-Pair Current to μ_{OBE}.

A and B show the choice c = 0 and - 1, respectively.

Local		Non-Local	SUM
0.0087	(A)	0.0033	0.0120
0.0084		0.0035	0.0119
	(B)	0.0021	0.0108
		0.0012	0.0095

Table 6. Contribution of TBE Current to Magnetic Moment of Deuteron

We denote the contributions of TBE currents in sect. 12.1. i), ii) and iii) as Intrinsic, Orbital and Dissociation, respectively. The upper (lower) values show the contributions of TBE currents using H.C. (S.C.) wave function. We have adopted $\kappa_\rho = 3.7$, $f^2_{\pi N\Delta}/4\pi = 0.23$ and the vertex functions of Eqs. (13.11) and (13.13).

6.1. μ_{TBE} due to N-N, N-Δ , Δ-Δ, N and Δ Intermediate States

N and Δ represent the contributions of diagrams in Fig. 11-e and 11-f, respectively.

	N - N	N - Δ	Δ - Δ	N	Δ	SUM
Intrinsic	0.0163	-0.0037	0.0033	-0.0015	0.0003	0.0148
	0.0180	-0.0043	0.0040	-0.0021	0.0005	0.0161
Orbital	-0.0068	0.0014	-0.0013	0.0004	-0.0001	-0.0064
	-0.0072	0.0015	-0.0015	0.0006	-0.0001	-0.0068
Dissociation	-0.0038	0.0017	-0.0001			-0.0022
	-0.0041	0.0019	-0.0001			-0.0023
SUM	0.0057	-0.0006	0.0019	-0.0011	0.0003	0.0062
	0.0067	-0.0010	0.0023	-0.0015	0.0004	0.0070

6.2. μ_{TBE} due to π-π, π-ρ and ρ-ρ Exchange Currents

	π − π	π − ρ	ρ − ρ
Intrinsic	0.0207	−0.0062	0.0003
	0.0240	−0.0086	0.0007
Orbital	−0.0085	0.0022	−0.0001
	−0.0097	0.0032	−0.0003
Dissociation	−0.0029	0.0007	
	−0.0032	0.0008	
SUM	0.0094	−0.0033	0.0002
	0.0112	−0.0046	0.0004

6.3. μ_{TBE} due to S-S, S-D and D-D Matrix Elements.

	S − S	S − D	D − D
Intrinsic	0.0226	−0.0047	−0.0031
	0.0236	−0.0046	−0.0030
Orbital	−0.0110	0.0033	0.0014
	−0.0113	0.0033	0.0013
Dissociation	−0.0023	−0.0001	0.0002
	−0.0026	0.0000	0.0002
SUM	0.0093	−0.0015	−0.0016
	0.0097	−0.0012	−0.0015

6.4. Contribution of (π-ω) Exchange Current to μ_{TBE}

Intrinsic	Orbital	Dissociation	SUM
−0.0004	−0.0002	0.0003	−0.0003
−0.0004	−0.0002	0.0004	−0.0002

Table 7. Magnetic Moment of Deuteron

We have adopted $\kappa_\rho = 3.7$ and the vertex functions of Eqs. (13.11) and (13.13)

	H. C.			S. C.		
$\Delta\mu = \mu_{exp.} - \mu_{IA}$		0.0146			0.0145	
$\Delta\mu_{rel.cor.}$		-0.0127			-0.0077	
μ_{OBE}		0.0164			0.0146	
$f^2_{\pi N\Delta}/4\pi$	0.230	0.305	0.380	0.230	0.305	0.380
μ_{TBE}	0.0062	0.0079	0.0102	0.0070	0.0091	0.0121
$\Delta\mu_{theor.}$	0.0099	0.0116	0.0140	0.0140	0.0161	0.0191