A Unified Theory of Elementary Particles with the Non-linear Spinor Field

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Abstract

Starting with a non-linear spinor interactions of Nambu-Jona-Lasinio type, we have derived in a unified way the Weinberg-Salam theory for the electromagnetic and the weak interactions of leptons and quarks and the asymptotically free gauge theory of Gross, Wilczek and Polizer for strong interactions of quarks. Here we have introduced a universal cutoff in our fermion loop calculations, and retained only divergent diagrams. All the gauge bosons and the Higgs scalars are created as composite states of fermion-antifermion pairs.As a result, all elementary particle forces are shown to be related with a single coupling strength, i.e., the fine-structure constant. The lowest order corrections to the gauge coupling constants are also considered.

§1. Introduction

Up to now, hundreds of many elementary particles, including resonances, are found by experiments. As the number of elementary particles are so huge, we want to consider that all these particles are not really "elementary particles", but "composites" built from more fundamental particles.

This idea has a long history. In 1949, Fermi and Yang[1)] have proposed a theory that the pion is composed of proton and neutron. Then, in 1959, starting from the non-linear fermion interaction, Heisenberg[2)] has developed a comprehensive theory of elementary particles which are composite states of fermion and antifermion pairs. In 1961, starting from the same Lagrangian as that of Heisenberg, Nambu and Jona-Lasinio[3)] proposed a dynamical model of elementary particles based on an analogy with superconductivity. In this model, the massless pseudoscalar composite state of nucleon-antinucleon pair, the idealized pion, appears as a Nambu-Goldstone boson when nucleon mass is generated by spontaneously breaking the chiral symmetry. Subsequently, with a nonlinear vector interaction, Bjorken[4)] and others[5)] demonstrated that the photon can be considered as a collective excitation of a fermion-antifermion pair. In 1974, in the Nambu-Jona-Lasinio model, Eguchi and Sugawara[6)] found a set of equations which describes the collective motions of fermion-antifermion pairs which is equivalent to the Higgs Lagrangian. Then, Konisi, Saito and

Shigemoto[7] examined the same model of Eguchi and Sugawara, and found that the Nielsen-Olsen[8] type theory is obtained and the type II superconductivity phase is realized in hadrons. Hadrons behave as string like objects, and this explains many experimental evidences like the duality and the linear rising trajectory.

On the other hand, the extensive theoretical works to unify all the interactions of elementary particles were performed for last several years. In 1967 and 1968, Weinberg[9] and Salam[10] proposed the theory to unify the weak and the electromagnetic interactions as a gauge theory. In this model, the weak interactions are mediated by the very heavy bosons, and the weak interactions become renormalizable. The distinct part from the former theory, among other things, is that the Weinberg and Salam theory predicts the processes mediated by the neutral currents. These processes were found experimentally at CERN in 1973. The existence of these neutral currents is taken as one of the evidence that the Weinberg and Salam theory is true. While, in the world of hadrons, the strong interactions between quarks are explained by using the colored gauge theory. The necessity of this color freedom is evident from the existence of Ω^- and from fermi statics, and the colored gauge theory is the local theory on this color freedom. One of the merits of this colored gauge theory is, of course, it is renormalizable. This theory can also explain the scaling phenomena found in 1970's at SLAC and other places by colliding high energy electrons to protons and neutrons. These scaling pheneomena tell us that in deep inside of hadrons, the spin 1/2 particles, partons, are freely moving. In 1973, Gross, Wilczek[11)] and Polizer[12)] have found, in the frame-work of the colored gauge theory, the fact that in the deep region inside hadrons the above free parton picture can be actually realized (asymptotic freedom). Therefore, the scaling phenomena are one of the aspects of the colored gauge theory. This asymptotically free gauge theory of Gross, Wilczek and Politzer, therefore, is a promising theory to explain the strong interactions of quarks. Until now, there have been many attempts to unify the Weinberg and Salam theory and the colored gauge theory.

We regard leptons and quarks as fundamental particles. Then along the above two lines of study, in this paper, starting from only the fundamental leptons and quarks, we[13)] attempt to construct the theory to unify the weak and the electromagnetic and the strong interactions, all interactions between elementary particles except gravity. Terazawa et. al.[14)] also have proposed the same unified model after our first proposal of this kind work. In our approach, we have introduced a universal cutoff and retained only divergent diagrams. In our picture, the photon and the weak vector bosons are considered as composites of lepton-antilepton or quark-antiquark pairs, while the colored gluons are considered as those of quark-antiquark pairs. As a result, the arbitrary parameters involved in the original Weinberg and Salam theory and the original colored gauge theory are largely removed. The Weinberg angle is determined to be $\sin^2\theta_W = \frac{3}{8}$ for fractionally charged quarks, which coincide with the prediction of Georgi-Glashow[15)] in their unified SU(5) gauge model of all elementary particle forces.

In §2 , starting with a Lagrangian of self-interacting leptons, we construct an effective Lagrangian of the Weinberg-Salam type, and the Weinberg angle and various coupling constants are determined. In §3 , the above model is extended to a more realistic one including quarks. In §4, the renormalization effects to our unified model is discussed. Finally, §5 is devoted to a summary and concluding remarks.

§2. Unified lepton model

In this section, we consider how to realize the Weinberg-Salam model in the framework of superconductivity model by using the functional integral technique. This method was proposed by Kikkawa[16] and Kugo[17] to obtain the collective motion of the fermion-antifermion pairs.

We begin with the nonlinear Lagrangian of the Weinberg-Salam massless leptons only:

$$\mathcal{L} = \bar{L} i\gamma\cdot\partial L + \bar{R} i\gamma\cdot\partial R + f_1 (\bar{L}\gamma_\mu L)^2 + 2 f_2 (\bar{L}\gamma^\mu L)(\bar{R}\gamma_\mu R) + f_3 (\bar{R}\gamma^\mu R)^2 , \quad (2\text{-}1)$$

where

$$L = \frac{1-\gamma_5}{2}\begin{pmatrix}\nu_e \\ e\end{pmatrix} \equiv \Lambda_L \begin{pmatrix}\nu_e \\ e\end{pmatrix} , \quad R = \frac{1+\gamma_5}{2} e \equiv \Lambda_R e , \quad (2\text{-}2)$$

i.e., the L is the iso-doublet while R is the iso-singlet. The four-fermion interactions here are of the most general form invariant under the global $SU(2)_L \otimes U(1)$ gauge group. An interaction of the type $(\bar{L}\gamma_\mu \vec{\tau} L)^2$ is reduced to $(\bar{L}\gamma_\mu L)^2$ by Fierz transformation. This Lagrangian is, of course, unrenormalizable, so that we introduce a cutoff in a familiar way.[3]

There are five possible composite states of lepton and antilepton, i.e., two vector-isoscalars U_μ and V_μ, one vector-isovector $\vec{U}_\mu$, one scalar-isospinor K and its conjugate $K^\dagger$. They will interact with leptons in the $SU(2)_L \otimes U(1)$ invariant way:

$$(\bar{L}\gamma_\mu L)\cdot U^\mu \quad , \quad (\bar{R}\gamma_\mu R)\cdot V^\mu \quad , \quad (\bar{L}\gamma_\mu \vec{\tau} L)\cdot \vec{U}^\mu \, ,$$

$$(\bar{L} R)\cdot K \quad , \quad K^\dagger\cdot(\bar{R} L) \, . \tag{2-3}$$

Therefore, we introduce such fields into our Lagrangian as auxiliary fields, and get another form of Lagrangian $\mathcal{L}'$ which is effectively equivalent to the original Lagrangian $\mathcal{L}$:

$$\mathcal{L}' = \bar{L} i\gamma^\mu(\partial_\mu + iU_\mu + i\vec{\tau}\cdot\vec{U}_\mu)L + \bar{R} i\gamma^\mu(\partial_\mu + 2iU_\mu)R$$
$$+ \bar{L}KR + \bar{R}K^\dagger L + aK^\dagger K + b\vec{U}_\mu^2 + cU_\mu^2 \, , \tag{2-4}$$

where

$$a = \frac{1}{4f_2 - 2f_3} \, , \quad b = -\frac{3}{f_3 - 4f_1} \, , \quad c = -\frac{1}{f_3} \, . \tag{2-5}$$

Here we have set $V_\mu = 2U_\mu$ in order that the brackets in (2-4) are of the Weinberg-Salam type. The equivalence of $\mathcal{L}$ and $\mathcal{L}'$ can be easily seen by taking variations with respect to lepton and auxiliary fields independently. In what follows, we start with the Lagrangian $\mathcal{L}'$ and regard the boson fields K, U_μ and $\vec{U}_\mu$ as well as L and R to be independent operators of each other.

In order to obtain the effective Lagrangian for those bose fields, we make use of the path integral technique. Define the effective Lagrangian $\mathcal{L}_{eff}$ by

$$\exp\left\{i\int d^4x\, \mathcal{L}_{eff}\right\} \equiv \int [d(\text{independent fermion fields})] \exp\left\{i\int d^4x\, \mathcal{L}'\right\} . \tag{2-6}$$

Carrying out the path integral over the independent lepton fields, we get

$$\int d^4x\, \mathcal{L}_{eff} = \int d^4x \left\{a\, K^\dagger K + b\, \vec{U}_\mu^2 + c\, U_\mu^2\right\}$$
$$-i\,\mathrm{Tr}\log\begin{bmatrix} 1-\frac{1}{i\gamma\partial}\gamma\cdot(U+\vec{\tau}\cdot\vec{U})\Lambda_L, & \frac{1}{i\gamma\partial}K\Lambda_R \\ \frac{1}{i\gamma\partial}K^\dagger\Lambda_L, & 1-\frac{1}{i\gamma\partial}2\gamma\cdot U\Lambda_R \end{bmatrix} \tag{2-7}$$

Here, in the logarithmic term, the fields appear in the projected form. For details, see Appendix. The logarithmic term corresponds to a series of lepton loop diagrams if it is expanded into Taylor series in bose fields. We retain only divergent terms proportional to a cutoff parameter Λ^2 or $\log\Lambda^2$.[6),7)], because, as it is easily seen by a simple dimensional analysis, the other higher derivative terms vanish in an infinite cutoff limit.

$$
\begin{aligned}
\mathcal{L}_{eff} \;=\; & \beta \left| (\partial_\mu - i U_\mu + i \vec{\tau}\cdot\vec{U}_\mu) K \right|^2 + (2\alpha + a) |K|^2 \\
& - \beta |K|^4 \qquad - \frac{\beta}{3} (\vec{U}_{\mu\nu})^2 \;-\; \beta\, U_{\mu\nu}^2 \qquad \text{(2-8)} \\
& - (\alpha - b)\, \vec{U}_\mu^2 \quad - (3\alpha - c)\, U_\mu^2
\end{aligned}
$$

where ,

$$\alpha = \frac{\Lambda^2}{(4\pi)^2} \quad , \quad \beta = \frac{1}{(4\pi)^2} \log \frac{\Lambda^2}{m_e^2} \quad , \qquad \text{(2-9)}$$

$$\vec{U}_{\mu\nu} = \partial_\mu \vec{U}_\nu - \partial_\nu \vec{U}_\mu - 2 \vec{U}_\mu \times \vec{U}_\nu \quad , \qquad \text{(2-10)}$$

$$U_{\mu\nu} = \partial_\mu U_\nu - \partial_\nu U_\mu \quad . \qquad \text{(2-11)}$$

Here m_e is an arbitrary dimensional constant, and is chosen to be the electron mass.

The created vector fields U_μ and $\vec{U}_\mu$ enter of the Weinberg-Salam type in (2-4) and (2-8). This suggests to us that the created vector fields can play the role of local gauge fields, if they happen to be massless. In the following this will be shown to be the case: We require that the fields U_μ and $\vec{U}_\mu$ are massless, i.e.,

$$\alpha - b = 0 \qquad \text{and} \qquad 3\alpha - c = 0 \qquad \text{(2-12)}$$

and that the mass term of K is finite, i.e.,

$$\frac{2\alpha}{\beta} + \frac{a}{\beta} = -\mu^2 \; (>0) \quad . \tag{2-13}$$

Solutions to Eqs. (2-12) and (2-13) are

$$f_1 = \frac{2}{3\alpha} \;, \; f_3 = -\frac{1}{3\alpha} \;, \; f_2 = -\frac{7}{24\alpha} + O\left(\frac{\beta}{\alpha^2}\right) \quad . \tag{2-14}$$

Then, after some rescalings such as

$$U_\mu = \frac{1}{2\sqrt{\beta}} B_\mu \;, \quad \vec{U}_\mu = -\frac{\sqrt{3}}{2\sqrt{\beta}} \vec{A}_\mu \;, \quad K = -\frac{1}{\sqrt{\beta}} \phi \;, \tag{2-15}$$

we have the Higgs type Lagrangian

$$\begin{aligned} \mathcal{L}_{eff} \equiv \mathcal{L}_{Higgs} &= \left| (\partial_\mu - \tfrac{i}{2} g' B_\mu - \tfrac{i}{2} g \vec{\tau} \cdot \vec{A}_\mu) \phi \right|^2 \\ &\quad - \mu^2 |\phi|^2 - \lambda |\phi|^4 \\ &\quad - \tfrac{1}{4} \vec{A}_{\mu\nu}^{\,2} - \tfrac{1}{4} B_{\mu\nu}^2 \quad , \end{aligned} \tag{2-16}$$

where

$$\lambda = \frac{1}{\beta} \;, \quad g' = \frac{g}{\sqrt{3}} = \frac{1}{\sqrt{\beta}} \quad . \tag{2-17}$$

This shows that at this stage the created vector fields $\vec{A}_\mu$ and B_μ can be regarded as the gauge fields, though the original Lagrangian $\mathcal{L}'$ does not have the local gauge symmetry.

Finally we take into account the interactions between $(L\,,\,R)$ and $(\phi, \vec{A}_\mu, B_\mu)$, which are obtained from the starting

Lagrangian $\mathcal{L}'$, and add them together with the lepton kinetic parts to the $\mathcal{L}_{Higgs}$ after rescaling (2-15). Then what we get is the following Lagrangian of the Weinberg-Salam type:

$$\begin{aligned}\mathcal{L}_{W.S.} = \mathcal{L}_{Higgs} & - G_e(\bar{L}\phi R + \bar{R}\phi^{+} L) \\ & + \bar{L}\, i\gamma^{\mu}(\partial_{\mu} + \tfrac{i}{2} g' B_{\mu} - \tfrac{i}{2} g \vec{\tau}\cdot\vec{A}_{\mu}) L \\ & + \bar{R}\, i\gamma^{\mu}(\partial_{\mu} + i g' B_{\mu}) R\end{aligned} \qquad (2\text{-}18)$$

where

$$G_e = \frac{1}{\sqrt{\beta}} .$$

Here it should be noted that in any calculations based on the above Lagrangian we should not take into account of one-fermion loop diagrams where bosonic fields are attached as external ones, because such diagrams have already been considered. This rule will be formulated in the following way[16].

The starting Lagrangian $\mathcal{L}$ can be written as

$$\mathcal{L} = \mathcal{L}_{Higgs} + \mathcal{L} - \mathcal{L}_{Higgs} = \mathcal{L}_{total} + \mathcal{L}_{counter\ terms} , \qquad (2\text{-}19)$$

where $\mathcal{L}_{counter\ terms} = \frac{a}{\beta}|\phi|^2 + \frac{3b}{4\beta}\vec{A}_{\mu}^{\,2} + \frac{c}{4\beta}B_{\mu}^{\,2} - \mathcal{L}_{Higgs}$.

The one-fermion-loop diagrams can be seen to be always cancelled by $\mathcal{L}_{\text{counter terms}}$. Therefore, the above rule is satisfied if we adopt the Lagrangian of the form (2-19).

It may be surprising that the model with global $SU(2)_L \otimes U(1)$ symmetry should be equivalent to one with local $SU(2)_L \otimes U(1)$ symmetry. This occurred due to massless conditions (2-12) for U_μ and $\vec{U}_\mu$.

From our Lagrangian (2-18) we can see that arbitrary parameters involved in the original Weinberg-Salam model are largely removed here. We summarize in the following the main results which are drawn from (2-18):

i) The Weinberg angle θ_W is fixed to be $\theta_W = 30°$,[18)] because of the definition $\tan\theta_W = g'/g$ together with our result $g'/g = 1/\sqrt{3}$.

ii) This leads to $M_W = (\sqrt{3}/2)\, M_Z = e/(\sqrt{2}G_W)^{1/2} = 76\,(\text{GeV})$.[18)]

iii) If the gauge symmetry is broken by the vacuum expectation value $\langle\phi\rangle_o = (-\frac{\mu^2}{2\lambda})^{1/2}$, then the electron acquires a mass $m_e = \langle\phi_o\rangle Ge = (-\frac{\mu^2}{2})^{1/2}$, or $2m_e^2 = -\mu^2 (>0)$, because of $G_e = g/\sqrt{3}$. The equation $2m_e^2 = -\mu^2$ coupled with (2-13) gives

$$-\frac{1}{2a} = \frac{i}{(2\pi)^4}\int^{\Lambda} \frac{d^4P}{(p^2 - m_e^2 + i\epsilon)} , \tag{2-20}$$

or

$$-\frac{1}{2(4f_2 - 2f_3)} = \frac{1}{16\pi^2}\left(\Lambda^2 - m_e^2 \log\left(1 + \frac{\Lambda^2}{m_e^2}\right)\right) , \tag{2-21}$$

which is essentially nothing but the Nambu-Jona-Lasinio self-consistent equation for the electron mass m_e.[3)]

iv) After the spontaneous symmetry breaking, the Higgs boson has a mass $m_H^2 = -2\mu^2 = 4m_e^2$ or $m_H = 2m_e$, because $2m_e^2 = -\mu^2$. These results are of course based on the bare coupling constants λ, g', g and G_e. Some of them could be modified if we take into account the renormalization effects. This problem is discussed in §4.

Finally we make a remark on the choice of auxiliary fields. In the path-integral approach we first face the question of how to choose auxiliary fields. The Weinberg-Salam model has been created from the Lagrangian (2-1) by choosing U_μ, $\vec{U}_\mu$ and K as auxiliary fields and for special values (2-14) of coupling constants f_i. One can easily see that another choice of auxiliary fields leads to another composite theory which is realized by another values of coupling constants f_i. It should be noted that the special coupling constants (2-14) make only the special channels U_μ, $\vec{U}_\mu$ and K excited and thus the Lagrangian (2-1) turns out to be of the Weinberg-Salam type.

§3. Unified lepton quark model

Following the same method of §2, in this section, we propose a further unified model including leptons and quarks. It is shown that a Lagrangian of self-interacting leptons and quarks generates the unified model of all the elementary particle interactions, i.e., the Weinberg-Salam model and the asymptotically free colored gauge model of Gross, Wilczek and Politzer.

We begin with the following Lagrangian which includes leptons and quarks:

$$\begin{aligned}
\mathcal{L} = \sum_{j=1,2} \{ & \bar{\ell}_j i\gamma(\partial - iY_{\ell_j} U - i\vec{\tau}\vec{U})\ell_j \\
& + \bar{r}_j i\gamma(\partial - iY_{r_j} U)r_j \\
& + a_j(\bar{\ell}_j K r_j + \text{h.c.}) \}
\end{aligned}$$

$$\begin{aligned}
+ \sum_{j=1,2} \{ & \bar{L}_j i\gamma(\partial - iY_{L_j} U - i\vec{\tau}\cdot\vec{U} - i\lambda^a V^a)L_j \\
& + \bar{R}_j i\gamma(\partial - iY_{R_j} U - i\lambda^a V^a)R_j \\
& + \bar{R}^p_j i\gamma(\partial - iY_{p_j} U - i\lambda^a V^a)R^p_j \\
& + b_j(\bar{L}_j K R_j + \text{h.c.}) + c_j(\bar{L}_j \tilde{K} R^p_j + \text{h.c.}) \} \\
& + k|K|^2 + m U_\mu^2 + n\vec{U}_\mu^2 + p V_\mu^{a\,2} ,
\end{aligned} \qquad (3\text{-}1)$$

where

$$\ell_1 = \Lambda_L \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \; r_1 = \Lambda_R e \quad ; \quad \ell_2 = \Lambda_L \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \; r_2 = \Lambda_R \mu \quad ,$$

$$L_1 = \Lambda_L \begin{pmatrix} p \\ n_\theta \end{pmatrix}, \; R_1 = \Lambda_R n_\theta \quad ; \quad L_2 = \Lambda_L \begin{pmatrix} c \\ \lambda_\theta \end{pmatrix}, \; R_2 = \Lambda_R \lambda_\theta ,$$

$$R_1^p = \Lambda_R p \quad , \quad R_2^p = \Lambda_R c \quad ; \quad \tilde{K} = i \tau_2 K^+ \quad ,$$

and

$$n_\theta = n \cos\theta + \lambda \sin\theta$$

$$\lambda_\theta = -n \sin\theta + \lambda \cos\theta$$

The quark fields L_j, R_j and R_j^p are all color triplets, while the leptonic fields l_j and r_j are color singlets. The L_j and l_j are weak iso-doublets, while R_j and r_j are iso-singlets. The vector V_μ^a is color octets. The Y_j's are the weak hyper-charges of corresponding leptons or quarks. The Lagrangian (3-1) is invariant under the global $SU(3)^{color} \otimes SU(2)_L \otimes U(1)$ group.

The bosonic fields K, U_μ, $\vec{U}_\mu$ and V_μ^a play the role of auxiliary fields, because they do not have kinetic terms. The Lagrangian (3-1) is effectively equivalent to the purely self-interacting fermionic system. This can be easily seen by taking variations with respect to bosonic and fermionic fields independently. The above choice of auxiliary fields is the necessary and sufficiant one for our purpose.

The effective Lagrangian $\mathcal{L}_{\text{eff}}$ for those bosonic fields is defined by

$$\exp\left\{ i\int d^4x\, \mathcal{L}_{eff} \right\}$$
$$\equiv \int [d(\text{independent fermion fields})] \exp\left\{ i\int d^4x\, \mathcal{L} \right\} \qquad (3\text{-}2)$$

Carrying out the path-integrals over fermionic fields, we get

$$\int d^4x\, \mathcal{L}_{eff} = -i \sum_{j=1,2} \mathrm{Tr}\log \begin{bmatrix} 1 + \frac{1}{i\gamma\partial}\gamma\cdot(Y_{l_j}U + \vec{\tau}\cdot\vec{U})\Lambda_L , & \frac{1}{i\gamma\partial} a_j K \Lambda_R \\ \frac{1}{i\gamma\partial} a_j K^{+}\Lambda_L , & 1 + \frac{1}{i\gamma\partial} Y_{r_j}\gamma\cdot U\, \Lambda_R \end{bmatrix}$$

$$-i\sum_{j=1,2}\mathrm{Tr}\log \begin{bmatrix} 1 + \frac{1}{i\gamma\partial}\gamma\cdot(Y_{L_j}U + \vec{\tau}\cdot\vec{U} + \lambda^a V^a)\Lambda_L , & \frac{1}{i\gamma\partial} b_j K\Lambda_R , & \frac{1}{i\gamma\partial} c_j \tilde{K}\Lambda_R \\ \frac{1}{i\gamma\partial} b_j K^{+}\Lambda_L , & 1 + \frac{1}{i\gamma\partial}\gamma\cdot(Y_{R_j}U + \lambda^a V^a)\Lambda_R , & 0 \\ \frac{1}{i\gamma\partial} c_j \tilde{K}^{+}\Lambda_L , & 0 , & 1 + \frac{1}{i\gamma\partial}\gamma\cdot(Y_{P_j}U + \lambda^a V^a)\Lambda_R \end{bmatrix}$$

$$+ \int d^4x \left\{ k|K|^2 + m\, U_\mu^2 + n\, \vec{U}_\mu^2 + p\, V_\mu^{a\,2} \right\} \qquad (3\text{-}3)$$

We retain only divergent terms proportional to a cutoff parameter Λ^2 or $\log \Lambda^2$. After some trace calculations we get an effective Lagrangian

$$\mathcal{L}_{eff} = -\frac{1}{3}(\beta_1+\beta_2+3\beta_3+3\beta_4)\,\vec{U}_{\mu\nu}^{\,2} \quad -\frac{1}{9}(9\beta_1+9\beta_2+11\beta_3+11\beta_4)\,U_{\mu\nu}^{2}$$

$$-(\alpha_1+\alpha_2+3\alpha_3+3\alpha_4-n)\,\vec{U}_\mu^{\,2} \quad -(3\alpha_1+3\alpha_2+\frac{11}{3}\alpha_3+\frac{11}{3}\alpha_4-m)\,U_\mu^2$$

$$-\frac{4}{3}(\beta_3+\beta_4)\,G_{\mu\nu}^{a\,2} \quad -(4\alpha_3+4\alpha_4-p)\,V_\mu^{a\,2} \tag{3-4}$$

$$+(a_1^2\beta_1+a_2^2\beta_2+3(b_1^2+c_1^2)\beta_3+3(b_2^2+c_2^2)\beta_4)\,|(\partial-iU-i\vec{\tau}\vec{U})K|^2$$

$$+k|K|^2 \quad + \quad I_1(|K|^2) \quad + I_2(|K|^2),$$

where

$$I_1 = -2i\int\frac{d^4p}{(2\pi)^4}\log\left(1-\frac{a_1^2|K|^2}{p^2}\right) - 2i\int\frac{d^4p}{(2\pi)^4}\log\left(1-\frac{a_2^2|K|^2}{p^2}\right),$$

$$I_2 = -6i\int\frac{d^4p}{(2\pi)^4}\log\left(1-\frac{(b_1^2+c_1^2)}{p^2}|K|^2\right) \tag{3-5}$$

$$-6i\int\frac{d^4p}{(2\pi)^4}\log\left(1-\frac{(b_2^2+c_2^2)}{p^2}|K|^2\right).$$

Here we have used

$$Y_{\ell_1}=Y_{\ell_2}=-1, \quad Y_{L_1}=Y_{L_2}=\frac{1}{3}, \quad Y_{r_1}=Y_{r_2}=-2,$$

$$Y_{n_1}=Y_{n_1}=-\frac{2}{3}, \quad Y_{p_1}=Y_{p_2}=\frac{4}{3},$$

$$G_{\mu\nu}^a = \partial_\mu V_\nu^a - \partial_\nu V_\mu^a + f^{abc}V_\mu^b V_\nu^c,$$

$$\vec{U}_{\mu\nu} = \partial_\mu\vec{U}_\nu - \partial_\nu\vec{U}_\mu + 2\,\vec{U}_\mu\times\vec{U}_\nu,$$

$$U_{\mu\nu} = \partial_\mu U_\nu - \partial_\nu U_\mu,$$

$$\alpha_1 = \frac{i}{(2\pi)^4}\int^{\Lambda}\frac{d^4p}{p^2-a_1^2\langle|K|^2\rangle} - a_1^2\langle|K|^2\rangle\frac{i}{(2\pi)^4}\int^{\Lambda}\frac{d^4p}{(p^2-a_1^2\langle|K|^2\rangle)^2},$$

$$\alpha_2 = \frac{i}{(2\pi)^4}\int^{\Lambda}\frac{d^4p}{p^2-a_2^2\langle|K|^2\rangle} - a_2^2\langle|K|^2\rangle\frac{i}{(2\pi)^4}\int^{\Lambda}\frac{d^4p}{(p^2-a_2^2\langle|K|^2\rangle)^2},$$

$$\alpha_3 = \frac{i}{(2\pi)^4}\int^{\Lambda}\frac{d^4p}{p^2-(b_1^2+c_1^2)\langle|K|^2\rangle} - (b_1^2+c_1^2)\langle|K|^2\rangle\frac{i}{(2\pi)^4}\int^{\Lambda}\frac{d^4p}{(p^2-(b_1^2+c_1^2)\langle|K|^2\rangle)^2},$$

$$\alpha_4 = \frac{i}{(2\pi)^4}\int^{\Lambda}\frac{d^4p}{p^2-(b_2^2+c_2^2)\langle|K|^2\rangle} - (b_2^2+c_2^2)\langle|K|^2\rangle\frac{i}{(2\pi)^4}\int^{\Lambda}\frac{d^4p}{(p^2-(b_2^2+c_2^2)\langle|K|^2\rangle)^2},$$

$$\beta_1 = -\frac{i}{(2\pi)^4}\int^{\Lambda}\frac{d^4p}{(p^2-a_1^2\langle|K|^2\rangle)^2}, \quad \beta_2 = -\frac{i}{(2\pi)^4}\int^{\Lambda}\frac{d^4p}{(p^2-a_2^2\langle|K|^2\rangle)^2}$$

$$\beta_3 = -\frac{i}{(2\pi)^4}\int^{\Lambda}\frac{d^4p}{(p^2-(b_1^2+c_1^2)\langle|K|^2\rangle)^2}, \quad \beta_4 = -\frac{i}{(2\pi)^4}\int^{\Lambda}\frac{d^4p}{(p^2-(b_2^2+c_2^2)\langle|K|^2\rangle)^2},$$

where $\langle|K|^2\rangle$ is a vacuum expectation value of $|K|^2$.

The created vector fields can play the role of local gauge fields if they happen to be massless. In the following we require that they are massless, i.e.,

$$\begin{aligned} m &= 3\alpha_1 + 3\alpha_2 + \tfrac{11}{3}\alpha_3 + \tfrac{11}{3}\alpha_4, \\ n &= \alpha_1 + \alpha_2 + 3\alpha_3 + 3\alpha_4, \\ p &= 4\alpha_3 + 4\alpha_4 . \end{aligned} \tag{3-6}$$

Then, after some rescaling such as

$$\vec{U}_\mu \equiv \left(\frac{3}{4\beta_1+4\beta_2+12\beta_3+12\beta_4} \right)^{1/2} \vec{A}_\mu \equiv \frac{g}{2}\vec{A}_\mu ,$$

$$U_\mu \equiv \left(\frac{9}{36\beta_1+36\beta_2+44\beta_3+44\beta_4} \right)^{1/2} B_\mu \equiv \frac{g'}{2} B_\mu ,$$

$$V_\mu^a \equiv \left(\frac{3}{16\beta_3+16\beta_4} \right)^{1/2} C_\mu^a \equiv \frac{f}{2} C_\mu^a , \tag{3-7}$$

$$K \equiv -\left(\frac{1}{a_1^2\beta_1 + a_2^2\beta_2 + 3(b_1^2+c_1^2)\beta_3 + 3(b_2^2+c_2^2)\beta_4} \right)^{1/2} \phi$$
$$\equiv -\frac{1}{\xi}\phi ,$$

we have the Higgs type Lagrangian

$$\mathcal{L}_{eff} \equiv \mathcal{L}_{Higgs} = -\frac{1}{4}\vec{A}_{\mu\nu}^{\,2} - \frac{1}{4}B_{\mu\nu}^2 \tag{3-8}$$
$$-\frac{1}{4}C_{\mu\nu}^{a\,2} + \left|\left(\partial - \frac{i}{2}g'B - \frac{i}{2}g\vec{\tau}\cdot\vec{A}\right)\phi\right|^2 - V(|\phi|^2) ,$$

where

$$V(|\phi|^2) = \Big\{ -\frac{k}{\xi^2}|\phi|^2 - 2i\int\frac{d^4p}{(2\pi)^4}\log\left(1-\frac{a_1^2|\phi|^2}{\xi^2p^2}\right)$$
$$-2i\int\frac{d^4p}{(2\pi)^4}\log\left(1-\frac{a_2^2|\phi|^2}{\xi^2p^2}\right) - 6i\int\frac{d^4p}{(2\pi)^4}\log\left(1-\frac{(b_1^2+c_1^2)}{\xi^2p^2}|\phi|^2\right)$$
$$-6i\int\frac{d^4p}{(2\pi)^4}\log\left(1-\frac{(b_2^2+c_2^2)}{\xi^2p^2}|\phi|^2\right) \Big\} . \tag{3-9}$$

Finally we take into account of interactions between (l_j, r_j, L_j, R_j) and $(\phi, \vec{A}_\mu, B_\mu, C_\mu^a)$, which are given by the starting Lagrangian (3-1), and add them togather with the fermion kinetic parts to the $\mathcal{L}_{Higgs}$ after scaling (3-7).

Then what we get is the following Lagrangian:

$$
\begin{aligned}
\mathcal{L}_{\text{total}} &= \mathcal{L}_{\text{Higgs}} \\
&+ \sum_{j=1,2} \Big\{ \bar{\ell}_j\, i\gamma \Big(\partial - \frac{ig'}{2} Y_{\ell_j} B - \frac{ig}{2}\vec{\tau}\cdot\vec{A}\Big) \ell_j + \bar{r}_j\, i\gamma\Big(\partial - \frac{ig'}{2} Y_{r_j} B\Big) r_j \\
&\qquad - G_j^{(1)} \big(\bar{\ell}_j \phi r_j + h.c.\big) \Big\} \\
&+ \sum_{j=1,2} \Big\{ \bar{L}_j\, i\gamma\Big(\partial - \frac{ig'}{2} Y_{L_j} B - \frac{ig}{2}\vec{\tau}\vec{A} - \frac{if}{2}\lambda_a C^a\Big) L \\
&\qquad + \bar{R}_j\, i\gamma\Big(\partial - \frac{ig'}{2} Y_{R_j} B - \frac{if}{2}\lambda_a C^a\Big) R_j \\
&\qquad + \bar{R}_j^P\, i\gamma\Big(\partial - \frac{ig'}{2} Y_{P_j} B - \frac{if}{2}\lambda_a C^a\Big) R_j^P \\
&\qquad - G_j^{(2)} \big(\bar{L}_j \phi R_j + h.c.\big) - G_j^{(3)} \big(\bar{L}_j \tilde{\phi} R_j^P + h.c.\big) \Big\},
\end{aligned}
\tag{3-10}
$$

where

$$
G_j^{(1)} = \frac{a_j}{\xi}, \quad G_j^{(2)} = \frac{b_j}{\xi}, \quad G_j^{(3)} = \frac{c_j}{\xi}. \tag{3-11}
$$

The Lagrangian (3-10) apparently contains the Weinberg-Salam Lagrangian for leptons and quarks. It contains furthermore the vector color-gluon theory in the following form:

$$
\mathcal{L}_{\text{gluon}} = -\frac{1}{4} C_{\mu\nu}^{a\,2} + \bar{q}\, i\gamma\Big(\partial - \frac{if}{2}\lambda^a V^a\Big) q, \tag{3-12}
$$

where $q=(p, n, \lambda, c)$ and V_μ^a is a vector color-gluon. The vector coupling constants g, g' and f are given by (3-7), i.e.,

$$g^2 = \frac{3}{\beta_1 + \beta_2 + 3\beta_3 + 3\beta_4} \cong \frac{3}{8\beta}, \quad g'^2 = \frac{9}{9\beta_1 + 9\beta_2 + 11\beta_3 + 11\beta_4} \cong \frac{9}{40\beta},$$

$$f^2 = \frac{3}{4\beta_3 + 4\beta_4} \cong \frac{3}{8\beta} . \tag{3-13}$$

Here we have set $\beta \equiv \beta_1 \cong \beta_2 \cong \beta_3 \cong \beta_4$, since β_j's do not so strongly depend on mass terms in β_j's for large cutoff Λ. This shows that the Weinberg angle is fixed to be

$$\sin^2\theta_W = \frac{g'^2}{g^2 + g'^2} \cong \frac{3}{8} , \tag{3-14}$$

and g, g' and f are related to the fine-structure constant e^2 through

$$e^2 = g^2 \sin^2\theta_W \cong \frac{3}{8} g^2 = \frac{5}{8} g'^2 = \frac{3}{8} f^2 = \left(\frac{3}{8}\right)^2 \frac{1}{\beta} . \tag{3-15}$$

Our Lagrangian (3-10) is invariant under the local $SU(3)^{color} \otimes SU(2)_L \otimes U(1)$ gauge group. If the Higgs scalars develop vacuum-expectation values

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} , \tag{3-16}$$

both $SU(2)_L$ and $U(1)$ gauge symmetries are spontaneously broken. Then, by choosing the U-gauge such that

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+\eta \end{pmatrix} , \qquad (3\text{-}17)$$

relevant fermion acquire masses, i.e.,

$$m_e = \frac{v}{\sqrt{2}}\frac{a_1}{\xi} , \quad m_\mu = \frac{v}{\sqrt{2}}\frac{a_2}{\xi} ,$$

and

$$m_p = \frac{v}{\sqrt{2}}\frac{c_1}{\xi} , \quad m_c = \frac{v}{\sqrt{2}}\frac{c_2}{\xi} ,$$

$$m_n = \frac{v}{\sqrt{2}}\frac{b_1}{\xi} , \quad m_\lambda = \frac{v}{\sqrt{2}}\frac{b_2}{\xi} . \qquad (3\text{-}18)$$

The W and Z boson masses are given by

$$M_W^2 = \frac{g^2 v^2}{4} = \left(\frac{\pi \alpha}{\sqrt{2} G_W}\right)\frac{1}{\sin^2\theta_W} \cong \frac{38^2}{3/8}(\mathrm{GeV})^2 = (62.1\ \mathrm{GeV})^2 , \qquad (3\text{-}19)$$

$$M_Z^2 = \frac{M_W^2}{\cos^2\theta_W} \cong (78.5\ \mathrm{GeV})^2 . \qquad (3\text{-}20)$$

These equations together with (3-18) and (3-19) give us relations

$$m_e^2 + m_\mu^2 + 3(m_n^2 + m_p^2 + m_\lambda^2 + m_c^2) = \frac{v^2}{2\beta} \cong 2\times\left(\frac{8}{3}\times 38\right)^2 (\mathrm{GeV})^2 . \qquad (3\text{-}21)$$

The Higgs scalar η generate the following mass

$$m_\eta^2 = \frac{2v^2}{\beta}\frac{a_1^4 + a_2^4 + 3(b_1^2+c_1^2)^2 + 3(b_2^2+c_2^2)^2}{(a_1^2 + a_2^2 + 3(b_1^2+c_1^2) + 3(b_2^2+c_2^2))^2} . \qquad (3\text{-}22)$$

In the approximation of m_e, m_μ, m_p, m_n, $m_\lambda \ll m_c$, the mass of the Higgs field is approximately given by $m_\eta \cong 2m_c$.

The vacuum-expectation values $\langle\phi\rangle$ are given by equation:

$$\left.\frac{\partial V(|\phi|)}{\partial|\phi|}\right|_{|\phi|=\langle|\phi|\rangle}=0 \quad ,$$

from which, together with (3-18), it follows that

$$\frac{k}{2}=-i\int^{\Lambda}\frac{d^4p}{(2\pi)^4}\left\{\frac{a_1^2}{p^2-m_e^2}+\frac{a_2^2}{p^2-m_\mu^2}+\frac{3(b_1^2+c_1^2)}{p^2-m_p^2-m_n^2}+\frac{3(b_2^2+c_2^2)}{p^2-m_c^2-m_\lambda^2}\right\} . \qquad (3\text{-}23)$$

This is nothing but Nambu and Jona-Lasinio's self-consistent equations for fermion masses.

Finally the β_j's defined before can be written, in terms of fermion masses, as

$$\beta_1=-\frac{i}{(2\pi)^4}\int^{\Lambda}\frac{d^4p}{(p^2-m_e^2)^2} \quad , \quad \beta_2=-\frac{i}{(2\pi)^4}\int^{\Lambda}\frac{d^4p}{(p^2-m_\mu^2)^2}$$

$$\beta_3=-\frac{i}{(2\pi)^4}\int^{\Lambda}\frac{d^4p}{(p^2-m_p^2-m_n^2)^2} \quad , \quad \beta_4=-\frac{i}{(2\pi)^4}\int^{\Lambda}\frac{d^4p}{(p^2-m_\lambda^2-m_c^2)^2} . \qquad (3\text{-}24)$$

They do not so strongly depend on fermion masses for large cutoff Λ. From (3-24) one can see that the typical cutoff Λ is given by $(\Lambda/m)^2 \sim \exp(3042) \gg 1$. This permits us to set $\beta\equiv\beta_1\cong\beta_2\cong\beta_3\cong\beta_4$.

In this way, we can construct the unified theory of the weak and the electromagnetic and the strong interactions starting from fundamental leptons and quarks only.

Our main results from our composite unified theory are:

i) The Weinberg angle is fixed to be $\sin^2\theta_W=3/8$, and created gauge-field coupling constants g, g' and f are related to the fine-strucute constant e^2 as $e^2 \cong \frac{3}{8}g^2 = \frac{5}{8}g'^2 = \frac{3}{8}f^2$. This coincide with the result of Terazawa et. al.[14] and hence with that of Georgi and Glashow[15] based on the SU(5) gauge model.

ii) The W and Z bosons acquire masses $M_W = 62.1$ GeV and $M_Z = 78.5$ GeV.

iii) The mass of Higge scalar is given by $m_\eta \cong 2m_c$ when charm quark is heavy.

Some of these results, of course, could be modified if we take into account of further renormalizations. The way of this renormalization is discussed in the next section.

§4. Renormalization effects to the unified model

The unified theory, thus derived, is a "bare" theory in the sense that it is taken into account only of fermion loop diagrams. The aim of this section is to calculate the other renormalization effects to the "bare" theory.

We consider that all the leptons and quarks is sequentially obtained if (ν_e, e, u, d) are replaced by (ν_μ, μ, c, s), (ν_τ, τ, t, b),$\cdots$. By introducing auxiliary bosonic fields the non-linear spinor Lagrangian for this system can be written as

$$
\begin{aligned}
\mathcal{L} = \sum_{j=1}^{n} \Big\{ & \bar{l}_j i\gamma\left(\partial - \frac{ig'}{2} Y_{l_j} B - \frac{ig}{2} \vec{\tau}\vec{A}\right) l_j \\
& + \bar{r}_j i\gamma\left(\partial - \frac{ig'}{2} Y_{r_j} B\right) r_j \\
& - a_j \left(\bar{l}_j \phi r_j + h.c.\right) \\
& + \bar{L}_j i\gamma\left(\partial - ig' \frac{Y_{L_j}}{2} B - \frac{ig}{2} \vec{\tau}\vec{A} - \frac{if}{2} \lambda^a C^a\right) L_j \\
& + \bar{R}_j i\gamma\left(\partial - \frac{ig'}{2} Y_{R_j} B - \frac{if}{2} \lambda^a C^a\right) R_j \\
& + \bar{R}_j^P i\gamma\left(\partial - \frac{ig'}{2} Y_{P_j} B - \frac{if}{2} \lambda^a C^a\right) R_j^P \\
& - b_j \left(\bar{L}_j \phi R_j + h.c.\right) - c_j \left(\bar{L}_j \tilde{\phi} R_j^P + h.c.\right) \qquad \text{(4-1)} \\
& + k_i |\phi|^2 + m_j B^2 + h_j \vec{A}_\mu^2 + p_j C_\mu^2 \Big\}
\end{aligned}
$$

The Lagrangian of j=1 is that of (ν_e, e, u, d). Similar sequential Lagrangians are those of sequential multiplets. In (4-1), n is the number of sequential multiplets (ν_e, e, u, d), (ν_μ, μ, c, s), (ν_τ, τ, t, b), $\cdots$. Carrying out the path-integrals over fermionic field, the effective Lagrangian is given by

$$\int d^4x\, \mathcal{L}_{eff}$$

$$= -i\sum_{j=1}^{n} \mathrm{Tr}\,\log \begin{bmatrix} 1+\frac{1}{i\gamma\partial}\gamma\cdot\left(\frac{g}{2}\vec{\tau}\cdot\vec{A}+\frac{g'}{2}Y_{\ell_j}B\right)\Lambda_L , & \frac{-1}{i\gamma\partial}a_j\phi\Lambda_R \\ \frac{-1}{i\gamma\partial}a_j\phi^{+}\Lambda_L , & 1+\frac{1}{i\gamma\partial}\frac{g'}{2}Y_{e_j}\gamma\cdot B\Lambda_R \end{bmatrix} \tag{4-2}$$

$$-i\sum_{j=1}^{n} \mathrm{Tr}\,\log \begin{bmatrix} 1+\frac{1}{i\gamma\partial}\gamma\cdot\left(\frac{g}{2}\vec{\tau}\cdot\vec{A}+\frac{g'}{2}Y_{q_j}+\frac{f}{2}\lambda^a C^a\right)\Lambda_L , & \frac{-1}{i\gamma\partial}b_j\phi\Lambda_R , & \frac{-1}{i\gamma\partial}c_j\tilde{\phi}\Lambda_R \\ \frac{-1}{i\gamma\partial}b_j\phi^{+}\Lambda_L , & 1+\frac{1}{i\gamma\partial}\gamma\cdot\left(\frac{g'}{2}Y_{R_j}B+\frac{f}{2}\lambda^a C^a\right)\Lambda_R , & 0 \\ \frac{-1}{i\gamma\partial}c_j\tilde{\phi}^{+}\Lambda_L , & 0 , & 1+\frac{1}{i\gamma\partial}\gamma\cdot\left(\frac{g'}{2}Y_{P_j}B+\frac{f}{2}\lambda^a C^a\right)\Lambda_R \end{bmatrix}$$

$$+\sum_{j=1}^{n}\int d^4x\left\{ k_j|\phi|^2 + m_j\vec{A}_\mu^{\,2} + n_j B_\mu^2 + P_j C_\mu^{a\,2}\right\} .$$

We retain only divergent terms proportional to a cutoff parameter Λ^2 or $\ln\Lambda^2$. After some trace calculations we get the effective Lagrangian

$$\mathcal{L}_{eff} = -\frac{1}{4}(Z_3^A)^{-1}\vec{A}_{\mu\nu}^{\,2} - \frac{1}{4}(Z_3^B)^{-1}B_{\mu\nu}^2 - \frac{1}{4}(Z_3^C)^{-1}C_{\mu\nu}^{a\,2} + Z_4^{-1}\left|\left(\partial_\mu - \frac{ig}{2}\vec{\tau}\vec{A}_\mu - \frac{ig'}{2}B_\mu\right)\phi\right|^2 - V(|\phi|^2) , \tag{4-3}$$

where

$$(Z_3^A)^{-1} = g^2 \sum_{j=1}^{n} \frac{1}{3}(\beta_j^l + 3\beta_j^q) \quad , \quad (Z_3^B)^{-1} = g'^2 \sum_{j=1}^{n} \frac{1}{9}(9\beta_j^l + 11\beta_j^q) \; ,$$

$$(Z_3^c)^{-1} = f^2 \sum_{j=1}^{n} \frac{4}{3}\beta_j^q \quad , \quad Z_4^{-1} = \sum_{j=1}^{n}(a_j^2\beta_j^l + 3b_j^2\beta_j^q + 3c_j^2\beta_j^q) \; , \tag{4-4}$$

and

$$\beta_j^l = -\frac{i}{(2\pi)^4}\int^{\Lambda} \frac{d^4p}{(p^2 - a_j^2\langle|\phi|^2\rangle)^2} \quad , \quad \beta_j^q = -\frac{i}{(2\pi)^4}\int^{\Lambda} \frac{d^4p}{(p^2 - (b_j^2 + c_j^2)\langle|\phi|^2\rangle)^2} \; ,$$

$V(|\phi|^2) =$ (effective potential of $|\phi|$) .

The massless conditions for the vector fields $\vec{A}_\mu$, B_μ and C_μ^a have been imposed, i.e., the constants m_j, n_j and p_j in (4-2) have been so chosen as to be cancelled by Λ^2- divergent terms of A_μ^2, B_μ^2 and C_μ^a, respectively.

The kinetic terms in (4-3) will be normalized by

$$(Z_3^A)^{-1} = (Z_3^B)^{-1} = (Z_3^c)^{-1} = Z_4^{-1} = 1 \; ,$$

so that

$$g^2 = \frac{3}{\sum_{j=1}^{n}(\beta_j^l + 3\beta_j^q)} \cong \frac{3}{4n\beta} \quad , \quad g'^2 = \frac{9}{\sum_{j=1}^{n}(9\beta_j^l + 11\beta_j^q)} \cong \frac{9}{20n\beta} \tag{4-5}$$

$$f^2 = \frac{3}{4\sum_{j=1}^{n}\beta_j^q} \cong \frac{3}{4n\beta} \; .$$

Here we have set $\beta^l_j = \beta^q_j = \beta$, because the β_j's do not so strongly depend on mass terms in β_j's for large cutoff Λ. If we add the fermion parts in $\mathcal{L}$ to $\mathcal{L}_{eff}$, then what we get is the following Lagrangian:

$$\mathcal{L} = \mathcal{L}_{W.S.} + \mathcal{L}_{Q.C.D.} \; , \tag{4-6}$$

where

$$
\begin{aligned}
\mathcal{L}_{W.S.} = \sum_{j=1}^{n} \Big\{ & \bar{l}_j\, i\gamma^\mu \left(\partial_\mu - \frac{ig}{2}\vec{\tau}\cdot\vec{A}_\mu - \frac{ig'}{2} Y_{l_j} B_\mu\right) l_j \\
& + \bar{r}_j\, i\gamma^\mu \left(\partial_\mu - \frac{ig'}{2} Y_{r_j} B_\mu\right) r_j \quad - a_j\left(\bar{l}_j \phi r_j + h.c.\right) \\
& + \bar{L}_j\, i\gamma^\mu \left(\partial_\mu - \frac{ig}{2}\vec{\tau}\cdot\vec{A}_\mu - \frac{ig'}{2} Y_{L_j} B_\mu\right) L_j \\
& + \bar{R}_j\, i\gamma^\mu \left(\partial_\mu - \frac{ig'}{2} Y_{R_j} B_\mu\right) R_j \quad + \bar{R}^P_j\, i\gamma^\mu \left(\partial_\mu - \frac{ig'}{2} Y_{P_j} B_\mu\right) R^P_j \\
& - b_j\left(\bar{L}_j \phi R_j + h.c.\right) \quad - c_j\left(\bar{L}_j \tilde{\phi} R^P_j + h.c.\right) \\
& - \frac{1}{4} \vec{A}_{\mu\nu}^{\,2} \quad - \frac{1}{4} B_{\mu\nu}^{2} \\
& + \left| \left(\partial_\mu - \frac{i}{2} g \vec{\tau}\cdot\vec{A}_\mu - \frac{ig'}{2} B_\mu\right)\phi \right|^2 - V\left(|\phi|^2\right)
\end{aligned} \qquad , \qquad (4\text{-}7)
$$

and

$$
\mathcal{L}_{Q.C.D.} = \sum_{q=u,d,\ldots} \left\{ \bar{q}\, i\gamma^\mu \left(\partial_\mu - \frac{if}{2}\lambda^a C^a_\mu\right) q \right\} - \frac{1}{4} C^{a\,2}_{\mu\nu} \qquad . \qquad (4\text{-}8)
$$

The first Lagrangian $\mathcal{L}_{W.-S.}$ is just of the Weinberg-Salam type for leptons and quarks, while the second one $\mathcal{L}_{Q.C.D.}$ is of the quantum chromodynamics type without quark mass. Here the "bare" coupling constants g, g', and f are given by (4-5), so that the "bare" Weinberg angle is fixed to be

$$
\sin^2\theta_W = \frac{g'^2}{g^2 + g'^2} = \frac{3}{8} \qquad . \qquad (4\text{-}9)
$$

Next, we consider the renormalization effects to these bare coupling constant.

First, we calculate the wave function renormalization constant Z_3^B of B_μ to order g'^2, other than fermion-loop contribution. In this case, the vector-meson self-energy graph is considered only of the Higgs-scalar loop. Thus we have

$$\left(Z_3^B(\text{Higgs})\right)^{-1} = 1 + \frac{g'^2}{6}\beta \quad , \tag{4-10}$$

where use is made of the same β as defined in (4-5). The renormalized coupling constant g'_R is, therefore,

$$g_R'^2 = (Z_1^B)^{-2}(Z_2^B)^2 Z_3^B(\text{Higgs})\, g'^2 = Z_3^B(\text{Higgs})\, g'^2$$

$$= \frac{g'^2}{1 + \frac{\beta}{6}g'^2} = \frac{1}{\left(\frac{20n}{9} + \frac{1}{6}\right)\beta} \equiv \frac{1}{b_1\beta} \tag{4-11}$$

Here we have used $g'^2 = 9/(20n\beta)$ of (4-5) and $Z_1^B = Z_2^B$, Z_1^B being the vertex renormalization constant of B_μ-fermion-fermion and Z_2^B the fermion wave function renormalization constant.

For the A_μ field the charge renormalization constant Z_1^A and the wave function renormalization constant Z_3^A have contributions from A_μ itself plus Faddeev-Popov ghosts and also from the Higgs scalar. These are known to be [11), 12)]

$$(Z_1^A(A))^{-1} = 1 - g^2\left(\frac{17}{6} - \frac{3\alpha}{2}\right)\beta \quad ,$$
$$(Z_1^A(\text{Higgs}))^{-1} = 1 + \frac{g^2}{6}\beta \quad ,$$
$$(Z_3^A(A))^{-1} = 1 - g^2\left(\frac{13}{3} - \alpha\right)\beta \quad , \qquad (4\text{-}12)$$
$$(Z_3^A(\text{Higgs}))^{-1} = (Z_1^A(\text{Higgs}))^{-1} \quad ,$$

where α is the gauge parameter. The renormalized coupling constant g_R is, therfore, given by

$$g_R^2 = (Z_3^A(A))^3 (Z_3^A(\text{Higgs}))^3 (Z_1^A(A))^{-2} (Z_1^A(\text{Higgs}))^{-2} g^2$$
$$= \frac{g^2}{1 - \frac{43}{6}g^2\beta} = \frac{1}{\left(\frac{4n}{3} - \frac{43}{6}\right)\beta} \equiv \frac{1}{b_2\beta} \quad , \qquad (4\text{-}13)$$

where to the second order the gauge dependent terms have been cancelled out and use is made of $g^2 = 3/(4n\beta)$ of (4-5).

In the same way, renormalization constants Z_1^c and Z_3^c of the gluon field C_μ^a have contributions only from C_μ^a itself plus Faddeev-Popov ghosts. The results is also known to be [11),12)]

$$(Z_1^c(\text{gluon}))^{-1} = 1 - f^2\left(\frac{17}{4} - \frac{9\alpha}{4}\right)\beta \quad ,$$
$$(Z_3^c(\text{gluon}))^{-1} = 1 - f^2\left(\frac{13}{2} - \frac{3\alpha}{2}\right)\beta \quad . \qquad (4\text{-}14)$$

The renormalized coupling constant f_R is, therefore, given by

$$f_R^2 = (Z_3^c(\text{gluon}))^3 (Z_1^c(\text{gluon}))^{-2} f^2$$
$$= \frac{f^2}{1 - 11f^2\beta} = \frac{1}{\left(\frac{4n}{3} - 11\right)\beta} \equiv \frac{1}{b_3\beta} \quad , \qquad (4\text{-}15)$$

where the relation $f^2 = 3/(4n\beta)$ of (4-5) has been used.

Recaling the relation

$$\frac{1}{e^2} = \frac{1}{g_R^2} + \frac{1}{g'^2_R} , \tag{4-16}$$

for the electric charge e, and substituting (4-11), (4-13) into (4-16), we have

$$e^2 = \frac{1}{(b_1+b_2)\beta} , \tag{4-17}$$

or

$$\beta = \frac{1}{(b_1+b_2)e^2} = \frac{1}{(\frac{32n}{9} - 7)} \cdot \frac{137}{4\pi} . \tag{4-18}$$

Eliminating β from g_R, g'_R, f_R and e, we get

$$\frac{f_R^2}{4\pi} = \frac{b_1+b_2}{b_3}\left(\frac{e^2}{4\pi}\right) = \frac{\frac{32n}{9} - 7}{\frac{4n}{3} - 11} \cdot \frac{1}{137} , \tag{4-19}$$

$$\frac{g_R^2}{4\pi} = \frac{b_1+b_2}{b_2}\left(\frac{e^2}{4\pi}\right) = \frac{1}{\sin^2\theta_W^R} \cdot \frac{e^2}{4\pi} , \tag{4-20}$$

$$\frac{g'^2_R}{4\pi} = \frac{b_1+b_2}{b_1}\left(\frac{e^2}{4\pi}\right) = \frac{1}{\cos^2\theta_W^R} \cdot \frac{e^2}{4\pi} , \tag{4-21}$$

where the renormalized Weinberg angle $\sin^2\theta_W^R$ is given by

$$\sin^2\theta_W^R = \frac{g'^2_R}{g_R^2 + g'^2_R} = \frac{b_2}{b_1+b_2}$$

$$= \frac{3}{8} - \frac{327}{256n - 504} . \tag{4-22}$$

Those lowest order corrections to coupling constants are of the same order as those from fermion loop diagrams. But, as the number n of the sequential multiplets becomes large, the more the correction parts become small. The rough estimate of the region when the perturbation becomes reliable is $n \gtrsim 9$. As $n \to \infty$, the Weinberg angle $\sin^2\theta_W^R$ tends to the previous value 3/8. The numerical values of $\sin^2\theta_W^R$, $f_R^2/4\pi$ and β against n are shown in Table 1 through the formula (4-22), (4-19) and (4-18), respectively. One can see from this table the typical cutoff parameter to be $\Lambda/m \sim 10^{6.9}$ for $\beta \sim 0.2$. To be compared with the experimental Weinberg angle, n becomes relatively large number $n = 20 \sim 30$. The number n of sequential multiplets has been restricted in our theory to be $n \gtrsim 9$, because of positivity of g'^2_R, g_R^2 and f_R^2. Then, the asymptotic free theory is not realized to this order. Our conjecture on this point is the following: The β-function of the renormalization-group equation for the C_μ^a field is positive to the lowest order of the coupling constant f and for $n \gtrsim 9$. If, however, higher order corrections to the β-function make it negative for large f, and if its fixed point f_0 be of order of electromagnetic coupling constant e, then the asymptotic freedom will be approximately satisfied in our case.

Finally, other similar work by Georgi, Quinn and Weinberg[19] should be compared with our result. The sharp difference is that they start from the SU(5) symmetric limit of the coupling constants g, g' and f, and calculate renormalization effects to them, whereas we never use such a symmetry group. They leave the gauge coupling constants in the symmetric limit to be free parameters, while our gauge coupling constants are all completely determined by fixing the model, that is, by fixing the number of the sequential multiplets.

§5. Summary and concluding remarks

We regard leptons and quarks as fundamental spinor particles. Then, starting from the non-linear spinor interactions of Nambu-Jona-Lasinio type, we have approximately constructed the Weinberg-Salam theory for the electromagnetic and the weak interactions of leptons and quarks and the asymptotically free color-gauge theory of Gross, Wilczek and Polizer for strong interactions of quarks. All the gauge bosons and the Higgs scalars are created as composite states of fermion-antifermion pairs. Arbitrary parameters involved in the unified theory are all determined by the physical masses and the cutoff parameter. As a result, the gauge coupling constants are all related and can be written by the fine-structure constant. We thus obtained the unified model of the strong, electromagnetic and weak interactions by dynamically creating the boson fields. This is the quite different point from the unified model of Georgi and Glashow based on SU(5) group. We further have calculated the lowest order corrections to our unified model.

There is a further attempts[20] to unify not only the strong, electromagnetic and weak interactions but also the gravity from the Nambu-Jona-Lasinio type Lagrangian. In that theory, the gravitational constant is connected with the fine-structure constant. But there are some difficulties to understand such created spin 2 field to be the Einstein's gravitational field. Until now, there is no evidence that the gravity play the important role in the elementary particle physics. Therefore, our unified model is sufficient to explain the interactions between elementary particles.

There still remains the important and difficult problem. We assume that leptons and quarks are fundamental. Then there arises the question why these quarks are not found by experiment. This quark confinement problem will be solved in future.

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Appendix

Proof of (2-7)

Using the proper representation of the γ-matrices. we can write

$$\gamma^\mu \Lambda_L = \begin{pmatrix} 0, & -\bar{\sigma}^\mu \\ 0, & 0 \end{pmatrix} , \quad \gamma^\mu \Lambda_R = \begin{pmatrix} 0 & , 0 \\ -\sigma^\mu, & 0 \end{pmatrix} ,$$

$$\Lambda_L = \begin{pmatrix} 0, & 0 \\ 0, & 1 \end{pmatrix} , \quad \Lambda_R = \begin{pmatrix} 1, & 0 \\ 0, & 0 \end{pmatrix} ,$$

where $\sigma^\mu = (1, \sigma^i)$, $\bar{\sigma}^\mu = (1, -\sigma^i)$.

We define the two-component spinor R_1, R_2, L_1, L_2 in the following way,

$$R = \begin{pmatrix} R_1 \\ R_2 \end{pmatrix} \quad , \quad L = \begin{pmatrix} L_1 \\ L_2 \end{pmatrix} \quad ,$$

where both L_1 and L_2 are iso-doublet. Using these two component spinors, the Lagrangian (2-4) is written as

$$\mathcal{L}' = (\bar{R}_2, \bar{L}_1) \begin{pmatrix} -i\sigma^\mu(\partial_\mu + iU_\mu) & , & K^+ \\ K & , & -i\bar{\sigma}^\mu(\partial_\mu + iU_\mu + i\vec{\tau}\vec{U}) \end{pmatrix} \begin{pmatrix} R_1 \\ L_2 \end{pmatrix} \tag{A-1}$$

$$+ aK^+K + b\vec{U}_\mu^2 + cU_\mu^2 \quad .$$

The functional integral is performed over the independent fermion fields $\bar{R}_2$, $\bar{L}_1$, R_1 and L_2. After performing the path integral, the effective action is given in the following form:

$$\int d^4x \, \mathcal{L}_{eff} = \int d^4x \left\{ aK^+K + b\vec{U}_\mu^2 + cU_\mu^2 \right\}$$

$$-i \, \mathrm{tr} \log \begin{bmatrix} 1 + \frac{1}{-i\sigma\cdot\partial} 2\sigma\cdot U & , & \frac{1}{-i\sigma\cdot\partial} K^+ \\ \frac{1}{-i\bar{\sigma}\cdot\partial} K & , & 1 + \frac{1}{-i\bar{\sigma}\cdot\partial}(\bar{\sigma}\cdot U + \bar{\sigma}\vec{\tau}\cdot\vec{U}) \end{bmatrix} \tag{A-2}$$

$$= \int d^4x \left\{ a K^+ K + b \vec{U}_\mu^2 + c U_\mu^2 \right\}$$

$$-i \, tr \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \begin{bmatrix} \frac{i\bar{\sigma}\cdot\partial}{\partial^2} 2\sigma\cdot U & , \frac{i\bar{\sigma}\cdot\partial}{\partial^2} K^+ \\ \frac{i\sigma\cdot\partial}{\partial^2} K , & \frac{i\sigma\cdot\partial}{\partial^2}(\bar{\sigma}\cdot U + \bar{\sigma}\cdot\vec{\tau}\cdot\vec{U}) \end{bmatrix}^n , \qquad \text{(A-3)}$$

where tr means the trace of 2x2 matrices for the spinor index.

Well, let's consider the following quantity

$$-i \, Tr \log \begin{bmatrix} 1 - \frac{1}{i\gamma\partial} 2\gamma U \Lambda_R , & \frac{1}{i\gamma\partial} K^+ \Lambda_L \\ \frac{1}{i\gamma\partial} K \Lambda_R , & 1 - \frac{1}{i\gamma\partial} \gamma\cdot(U + \vec{\tau}\cdot\vec{U})\Lambda_L \end{bmatrix} \qquad \text{(A-4)}$$

$$= -i \, Tr \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \begin{bmatrix} \frac{i\gamma\cdot\partial}{\partial^2} 2\gamma\cdot U \Lambda_R & , -\frac{i\gamma\cdot\partial}{\partial^2} K^+ \Lambda_L \\ -\frac{i\gamma\cdot\partial}{\partial^2} K \Lambda_R , & \frac{i\gamma\cdot\partial}{\partial^2} \gamma\cdot(U + \vec{\tau}\cdot\vec{U})\Lambda_L \end{bmatrix}^n \qquad \text{(A-5)}$$

$$= -i \, Tr \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \begin{bmatrix} \frac{2i\partial_\mu \bar{\sigma}^\mu \sigma^\nu}{\partial^2} \otimes \frac{1}{2}\begin{pmatrix} 1, & 1 \\ 1, & 1 \end{pmatrix} U_\nu , & -\frac{i\partial_\mu \bar{\sigma}^\mu}{\partial^2} \otimes \frac{1}{2}\begin{pmatrix} 1, & -1 \\ 1, & -1 \end{pmatrix} K^+ \\ -\frac{i\partial_\mu \sigma^\mu}{\partial^2} \otimes \frac{1}{2}\begin{pmatrix} 1, & 1 \\ -1, & -1 \end{pmatrix} K , & \frac{i\partial_\mu}{\partial^2} \sigma^\mu \bar{\sigma}^\nu \otimes \frac{1}{2}\begin{pmatrix} 1, & -1 \\ -1, & 1 \end{pmatrix} (U_\nu + \vec{\tau}\vec{U}_\nu) \end{bmatrix}^n , \qquad \text{(A-6)}$$

where the each element of

$$\frac{1}{2}\begin{pmatrix} 1, & 1 \\ 1, & 1 \end{pmatrix} , \frac{1}{2}\begin{pmatrix} 1, & -1 \\ 1, & -1 \end{pmatrix} , \frac{1}{2}\begin{pmatrix} 1, & 1 \\ -1, & -1 \end{pmatrix} , \frac{1}{2}\begin{pmatrix} 1, & -1 \\ -1, & 1 \end{pmatrix} \qquad \text{(A-7)}$$

is a 2x2 unit matrix, and T_r means the trace of 4x4 matrices for the spinor index. Each (1,1), (1,2), (2,1) and (2,2) component of (A-6) is always proportional to each matrix of (A-7). Then, using the relation

$$T_r\left\{(\bar{\sigma}\sigma)^n \otimes \frac{1}{2}\begin{pmatrix}1, & 1\\ 1, & 1\end{pmatrix}\right\} = tr(\bar{\sigma}\sigma)^n \quad , \quad T_r\left\{(\sigma\bar{\sigma})^n \otimes \frac{1}{2}\begin{pmatrix}1, & -1\\ -1, & 1\end{pmatrix}\right\} = tr(\sigma\bar{\sigma})^n \quad , \qquad \text{(A-8)}$$

and noticing that K and $K^{\dagger}$ always appears as bilinear form in the diagonal element, we obtain the relation

$$-i \; T_r \log \begin{bmatrix} 1 - \frac{1}{i\gamma\partial} 2\gamma\cdot U \Lambda_R \, , & \frac{1}{i\gamma\partial} K^{\dagger} \Lambda_L \\ \frac{1}{i\gamma\partial} K \Lambda_R \, , & 1 - \frac{1}{i\gamma\partial} \gamma\cdot(U + \vec{\tau}\cdot\vec{U}) \Lambda_L \end{bmatrix}$$

$$= -i \; tr \log \begin{bmatrix} 1 + \frac{1}{-i\sigma\cdot\partial} 2\sigma\cdot U \, , & \frac{1}{-i\sigma\cdot\partial} K^{\dagger} \\ \frac{1}{-i\bar{\sigma}\cdot\partial} K \, , & 1 + \frac{1}{-i\bar{\sigma}\cdot\partial} (\bar{\sigma}\cdot U + \bar{\sigma}\cdot\vec{\tau}\cdot\vec{U}) \end{bmatrix}$$

Therefore, the effective Lagrangian takes the form of (2-7).

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Table Caption

Calculated values of the Weinberg angle $\sin^2\theta_W^R$, the colored gluon coupling constant $f_R^2/4\pi$ and cutoff parameter $\beta=(4\pi)^{-2}\ell n(\Lambda/m)^2$ against the number n of sequential multiplets.

n	$\sin^2\theta_W^R$	$f_R^2/4\pi$	β
9	0.193	0.183	0.436
10	0.216	0.089	0.382
11	0.234	0.064	0.340
12	0.248	0.052	0.306
13	0.259	0.045	0.278
14	0.269	0.041	0.255
15	0.277	0.038	0.235
16	0.284	0.035	0.219
17	0.290	0.033	0.204
18	0.295	0.032	0.191
19	0.300	0.031	0.180
20	0.304	0.030	0.170
30	0.329	0.025	0.109
40	0.341	0.023	0.081

Table 1