# ON THE HYPERSURFACES OF HERMITIAN SYMMETRIC SPACES OF COMPACT TYPE

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Introduction.

Let $M$ be an irreducible Hermitian symmetric space of compact type and let $L$ be a holomorphic line bundle over $M$. We denote by $\Omega^p(L)$ the sheaf of germs of L-valued holomorphic p-forms on $M$. In this paper we study the cohomology groups $H^q(M, \Omega^p(L))$. Further, applying the results so far obtained, we shall consider the hypersurfaces of $M$.

The paper devided into three parts. §1 is devoted to recalling basic notions and results which are necessary in the following. In §2, for the cases that $M$ is an irreducible Hermitian symmetric space of compact type BDI, EIII or EVII, we obtain the theorems analogous to the following theorem of Bott [3] for $M = P_n(C)$.

Theorem. Let $E$ be the hyperplane bundle over an n-dimensional complex projective space $P_n(C)$. Then the group $H^q(P_n(C), \Omega^p(E^k))$ vanishes except for the following cases: (i) $p = q$ and $k = 0$, (ii) $q = 0$ and $k > p$, (iii) $q = n$ and $k < p - n$, where $E^k = E \otimes \cdots \otimes E$ ( k factors ).

Further we shall discuss when the groups $H^q(M, \Omega^p(L))$ vanishes for any irreducible Hermitian symmetric space of compact type for $p = 0, 1$. These results are obtained by analyzing in detail structure of Lie algebras and their Weyl groups and applying the generalized Borel-Weil theorem.

Let $V$ be a hypersurface of $M$. Denote by $\Theta$ ( resp. $\Omega$ ) the sheaf of germs of holomorphic vector fields ( resp. holomorphic functions ) on $V$. In §3 we study the cohomology groups $H^q( V, \Theta )$ and $H^q( V, \Omega )$ using the results in §2. And we find that if $M$ is BDI, EIII or EVII, one has

$$H^0( V, \Theta ) = 0$$

for the hypersurfaces $V$ of $M$ except for a certain special case (Theorem 8 ).

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§1. Preparations.

1.1. The generalized Borel-Weil theorem. In this section we recall the generalized Borel-Weil theorem in a form convenient for our purpose.

Let $G$ be a simply connected complex semi-simple Lie group and let $U$ be a parabolic Lie subgroup of $G$. Then the quotient manifold $M = G/U$ is a Kähler C-space, that is, a simply connected compact complex homogeneous manifold admitting a Kähler metric. Let $g$ be the Lie algebra of $G$ and $h$ a Cartan subalgebra of $g$. We denote by $\Delta$ the root system of $g$ with respect to $h$. We shall identify a linear form $\lambda$ on $h$ with the element $H_\lambda$ of $h$ defined by

$$\lambda( H ) = ( H_\lambda, H ) \qquad \text{for } H \in h,$$

where $( , )$ is the Killing form of $g$. We fix a linear order on the real form $h_0 = \{ \alpha \in \Delta \}_R$ of $h$. Let $\Delta^+$ ( resp. $\Delta^-$ ) be the set of all positive ( resp. negative ) roots. Let $\Pi_1$ be a subsystem of $\Pi$. We put

$$\Delta_1 = \{ \alpha \in \Delta ; \ \alpha = \sum_{i=1}^{\ell} m_i \alpha_i , \ m_j = 0 \text{ for any } \alpha_j \notin \Pi_1 \}$$

$$\Delta(n^+) = \{ \beta \in \Delta ; \ \beta = \sum_{i=1}^{\ell} m_i \alpha_i , \ m_j > 0 \text{ for some } \alpha_j \notin \Pi_1 \}$$

$$\Delta(u) = \Delta_1 \cup \Delta(n^+) .$$

Define Lie subalgebras $g_1$, $n^+$ and $u$ of $g$ by

$$g_1 = h + \sum_{\alpha \in \Delta_1} g_\alpha$$

$$n^+ = \sum_{\beta \in \Delta(n^+)} g_\beta$$

$$u = h + \sum_{\alpha \in \Delta(u)} g_\alpha ,$$

where $g_\alpha$ is the root space corresponding to $\alpha \in \Delta$. Then $g_1$ ( resp. $n^+$ ) is a reductive ( resp. nilpotent ) subalgebra and $u = g_1 + n^+$ ( semi-direct ). We denote by $U$ the connected Lie subgroup of $G$ with Lie algebra $u$. Then $U$ is a parabolic subgroup of $G$, and $M = G/U$ is a Kähler C-space.

We denote by $D$ ( resp. $D_1$ ) the set of dominant integral forms of $g$ ( resp. $g_1$ ). Let $\xi \in D_1$ and choose an irreducible representation ( $\rho^1_{-\xi}$, $W_{-\xi}$ ) of $g_1$ with the lowest weight $-\xi$. We may extend it to a representation of $u$ so that its restriction to $n^+$ is trivial, which will be denoted by ( $\rho_{-\xi}$, $W_{-\xi}$ ). Since any irreducible representation of $u$ is trivial on $n^+$, we may call ( $\rho_{-\xi}$, $W_{-\xi}$ ) the irreducible representation of $u$ with the lowest weight $-\xi$. Moreover there exists a representation of $U$ which induces the representation $( \rho_{-\xi}, W_{-\xi} )$ of $u$, and we denote it by $( \tilde{\rho}_{-\xi}, W_{-\xi} )$. This representation $( \tilde{\rho}_{-\xi}, W_{-\xi} )$ defines the holomorphic vector bundle $E_{-\xi}$ over $M$ associated to the principal bundle $G \longrightarrow M$ by the representation $\tilde{\rho}_{-\xi}$ of $U$.

For a holomorphic vector bundle $E$ over a complex manifold, we denote by $\Omega(E)$ the sheaf of germs of local holomorphic sections of $E$. Let $W$ be the Weyl group of $g$ and $\Delta_1^+$ the set of all positive roots of $\Delta_1$. We define a subset $W^1$ of $W$ by

$$W^1 = \{ \sigma \in W;\ \sigma^{-1}(\Delta_1^+) \subset \Delta^+ \},$$

For any set $S$, we denote by $\#S$ the cardinality of $S$. The index $n(\sigma)$ of $\sigma \in W$ is then defined by

$$n(\sigma) = \#( \sigma(\Delta^+) \cap \Delta^- ).$$

We denote by $\delta$ the half of sum of all positive roots of $g$.

Theorem of Bott [3] ( c.f. Kostant [7] ). Under the notations defined above let $\xi \in D_1$. Then if $\xi+\delta$ is not regular,

$$H^j( M, \Omega E_{-\xi} ) = 0 \quad \text{for all } j = 0, 1, \cdots .$$

If $\xi+\delta$ is regular, $\xi+\delta$ is expressed uniquely as $\xi+\delta = \sigma(\lambda+\delta)$, where $\lambda \in D$ and $\sigma \in W^1$, and

$$H^j( M, E_{-\xi} ) = 0 \quad \text{for all } j \neq n(\sigma),$$

$$\dim H^{n(\sigma)}( M, E_{-\xi} ) = \dim V_{-\lambda},$$

where $( \rho_{-\lambda}, V_{-\lambda} )$ is the irreducible representation of $G$ with the lowest weight $-\lambda$.

We prove the following lemmas to restate this theorem in a form suitable for our parpose.

Lemma 1. Let $\xi \in D_1$. If

$$( \xi+\delta, \beta ) \neq 0 \quad \text{for} \quad \beta \in \Delta(n^+),$$

then $\xi+\delta$ is regular.

Proof. Let $\alpha$ be any root of $\Delta_1^+$. Then we have $( \xi,\alpha ) \geqq 0$ and $( \delta,\alpha ) > 0$, so that $( \xi+\delta,\alpha ) > 0$. Since $\Delta^+ = \Delta_1^+ \cup \Delta(n^+)$, we get

$$( \xi+\delta,\gamma ) \neq 0 \quad \text{for} \quad \gamma \in \Delta^+.$$

q.e.d.

Lemma 2. Let $\xi \in D_1$. Assume that there are $\lambda \in D$ and $\sigma \in W^1$ such that $\xi+\delta = \sigma(\lambda+\delta)$. Then

$$n(\sigma) = \#\{ \beta \in \Delta(n^+);\ ( \xi+\delta,\beta ) < 0 \}.$$

Proof. Since $\sigma^{-1}(\Delta_1^+) \subset \Delta^+$, we have

$$n(\sigma) = \#\{ \beta \in \Delta(n^+);\ \sigma^{-1}(\beta) < 0 \}.$$

By the assumption

$$( \xi+\delta,\alpha ) = ( \lambda+\delta, \sigma^{-1}(\alpha) ) \quad \text{for} \quad \alpha \in \Delta.$$

Since $\lambda+\delta$ is dominant and regular, $\sigma^{-1}(\alpha)$ is negative if and only if $( \lambda+\delta, \sigma^{-1}(\alpha) )$ is negative. The conclusion now follows from these obserbations.

Theorem of Bott may be restated as follows by these lemmas.

Theorem 1. Let $\xi \in D_1$. Then if there exists a root $\alpha$ of $\Delta(n^+)$ such that $( \xi+\delta,\alpha ) = 0$, we have

$$H^j( M, \Omega E_{-\xi} ) = 0 \quad \text{for} \quad j = 0, 1, \cdots.$$

If there exists no root $\beta$ of $\Delta(n^+)$ such that $( \xi+\delta,\beta ) = 0$, we have

$$H^j( M, \Omega E_{-\xi} ) = 0 \quad \text{for} \quad j \neq q,$$

and

$$H^q( M, \Omega E_{-\xi} ) \neq 0,$$

where $q = \#\{ \beta \in \Delta(n^+);\ ( \xi+\delta,\beta ) < 0 \}$.

1.2. Kostant's results. We denote by $T(M)$ the holomorphic tangent bundle of $M$ and denote by $T(M)^*$ its dual bundle. Let $L$ be a holomorphic line bundle over $M$. Then it is easy to see that $\Omega^p(L)$ coincides with $\Omega( \overset{p}{\Lambda} T(M)^* \otimes L )$, where $\overset{p}{\Lambda} T(M)^*$ is p-th exterior product of $T(M)^*$. Since any holomorphic line bundle over a Kähler C-space $M$ is associated to the principal bundle $G \longrightarrow M$ by a representation of $U$ ( Murakami [8] ), we may put $L = E_{-\xi}$ for $\xi \in D_1$. It is known $n^+$ is invariant by the adjoint representation of $U$ on $g$. Hence p-th exterior product of $n^+$ has a U-module structure. Since $n^+$ may be identified with the cotangent space of $M$ at $U$, $\overset{p}{\Lambda} T(M)^* \longrightarrow M$ coincides with the holomorphic vector bundle associated to the principal bundle $G \longrightarrow M$ by the representation of $U$ on $\overset{p}{\Lambda} n^+$.

From now on we assume $M = G/U$ is a Hermitian symmetric space of compact type. Then $n^+$ is abelian. For any integer $p \geqq 0$, put

$$W^1(p) = \{ \sigma \in W^1;\ n(\sigma) = p \}.$$

Kostant [7] has proved that $\overset{p}{\Lambda} n^+$ is decomposed into direct sum:

$$(1) \qquad \overset{p}{\Lambda} n^+ = \sum_{\sigma \in W^1(p)} (\Lambda n^+)_{-(\sigma\delta-\delta)} \qquad ( \text{ as U-module } ),$$

where $(n^+)_{-(\sigma\delta-\delta)}$ denotes an irreducible U-module with the lowest weight $-(\sigma\delta-\delta)$. The following theorem follows easily from (1) and theorems of Bott [3].

Let $W$ be a holomorphic $U$-module represented as follows:

$$W = W_{-\xi_1} + \cdots + W_{-\xi_\ell} \qquad \text{for} \quad \xi_i \in D_1.$$

Denote by $E_W$ the holomorphic vector bundle over $M$ associated to the principal bundle $G \longrightarrow M$ by the representation of $U$ on $W$.

Proposition 1. Under the notations introduced above we have

$$\dim H^j(\, M,\ \Omega E_W\,) = \sum_{i=1}^{\ell} \dim H^j(\, M,\ \Omega E_{-\xi_i}\,) \qquad \text{for} \quad j = 0,\ 1,\ \cdots.$$

We recall the results of Bott [3] which are necessary to proof the above proposition.

Theorem A. Let $S$ be a holomorphic $U$-module, and let $V$ be a holomorphic $G$-module. If $E_S$ is the holomorphic vector bundle over $M$ associated to the principal bundle $G \longrightarrow M$ by the representation of $U$ on $S$, then

$$\text{multiplicity of } V \text{ in } H^j(\, M,\ \Omega E_S\,) = \dim H^j(\, u,\ g_1,\ \mathrm{Hom}(V,S)\,) \qquad \text{for} \quad j = 0,\ 1,\ \cdots,$$

where $H^j(\, u,\ g_1,\ \mathrm{Hom}(V,S)\,)$ denotes the $j$-th relative cohomology group of Lie algebras $u$, $g_1$, with coefficients in the $u$-module $\mathrm{Hom}(V,S)$.

For $g_1$-module $T$, $T^{g_1}$ donotes the subspace of $S$ annihilated by all $X \in g_1$.

Theorem B. Let $F$ be a $u$-module which, considered as $g_1$-module, is completely reducible. Then

$$\dim H^j(\, u,\ g_1,\ F\,) = \dim H^j(\, n^+,\ F\,)^{g_1}.$$

Proof of Proposition 1. Let $V$ be a holomorphic $G$-module. Then by Theorems A and B, we have

multiplicity of $V$ in $H^j(M, \Omega E_W) = \dim H^j(n^+, \mathrm{Hom}(V,W))^{g_1}$

for $j = 0, 1, \cdots$. Since $W_{-\xi_i}$, $1 \leqq i \leqq \ell$, are irreducible $u$-modules, the restrictions to $n^+$ of the representations of $u$ on $W_{-\xi_i}$ and $W$ are both trivial. Hence

$$\begin{aligned}
&\dim H^j(n^+, \mathrm{Hom}(V,W))^{g_1} \\
&= \dim (H^j(n^+, \mathrm{Hom}(V,C)) \otimes W)^{g_1} \\
&= \sum_{i=1}^{\ell} \dim (H^j(n^+, \mathrm{Hom}(V,C)) \otimes W_{-\xi_i})^{g_1} \\
&= \sum_{i=1}^{\ell} \dim H^j(n^+, \mathrm{Hom}(V,W_{-\xi_i}))^{g_1}.
\end{aligned}$$

By Theorems A and B

$$\dim H^j(n^+, \mathrm{Hom}(V,W_{-\xi_i}))^{g_1} = \sum_{i=1}^{\ell} \text{multiplicity of } V \text{ in } H^j(M, E_{-\xi_i})$$

for $j = 0, 1, \cdots$.

q.e.d.

The following theorem follows immediately from (1) and the above proposition.

Theorem 2. Let $M$ be a Hermitian symmetric space of compact type. Assume that $E_{-\xi}$, $\xi \in D_1$, is a line bundle over $M$. Then

$$\dim H^q(\, M,\ \Omega^p(E_{-\xi})\, ) = \sum_{\sigma \in W^1(p)} \dim H^q(\, M,\ \Omega(E_{-(\sigma\delta-\delta+\xi)})\, )$$

for $q = 0, 1, \cdots$.

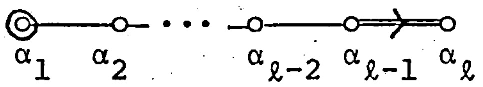
Theorem 2 shows us the importance of the study of the structure of $W^1$ for our porpose.

## 2. Vanishing of $H^q(\, M,\ \Omega^p(L)\, )$.

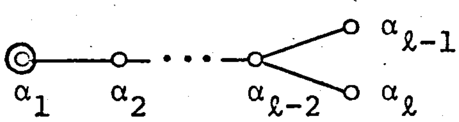
We retain the notations and assumptions introduced in the previous section.

Assume that $M$ is an irreducible Hermitian symmetric space of compact type. Then $G$ is simple and there exists $\alpha_j \in \Pi$ such that $\Pi_1 = \Pi - \{\ \alpha_j\ \}$. Let $\{\ \omega_1, \cdots, \omega_\ell\ \}$ be fundamental weights with respect to $\Pi = \{\ \alpha_1, \cdots, \alpha_\ell \}$. Then any holomorphic line bundle $L$ over $M$ is isomorphic to $E_{-k\omega_j}$ for some integer $k$, since any 1-dimensional representation of $g_1$ is induced by a representation of the centor of $g_1$.

2.1. The case that $M$ is of type BD I i.e. a complex quadric Put $\dim M = n$. The Dinkin diagram of $\Pi$ is as follows:




if $n = 2\ell - 1$ ( $\ell \geq 2$ ),




if $n = 2\ell - 2$ ( $\ell \geqq 3$ ),

where $\alpha_1$ ◎ shows that $\alpha_j = \alpha_1$. Let $\{ \varepsilon_i ; i = 1, \cdots, \ell \}$ be a basis of $h_0$ which satisfies $( \varepsilon_i , \varepsilon_j ) = \delta_{ij}$. Then, we have:

$$\Delta = \begin{cases} \{ \pm(\varepsilon_i \pm \varepsilon_j) ; 1 \leqq i < j \leqq \ell, \ \varepsilon_i ; 1 \leqq i \leqq \ell \}, & \text{if } n = 2\ell - 1, \\ \{ \pm(\varepsilon_i \pm \varepsilon_j) ; 1 \leqq i < j \leqq \ell \}, & \text{if } n = 2\ell - 2, \end{cases}$$

$$\Pi = \begin{cases} \{ \alpha_i = \varepsilon_i - \varepsilon_{i+1} ; 1 \leqq i \leqq \ell - 1, \ \alpha_\ell = \varepsilon_\ell \}, & \text{if } n = 2\ell - 1, \\ \{ \alpha_i = \varepsilon_i - \varepsilon_{i+1} ; 1 \leqq i \leqq \ell - 1, \ \alpha_\ell = \varepsilon_{\ell-1} + \varepsilon_\ell \}, & \text{if } n = 2\ell - 2, \end{cases}$$

$$\Delta(n^+) = \begin{cases} \{ \varepsilon_1 \pm \varepsilon_j ; 2 \leqq j \leqq \ell, \ \varepsilon_1 \}, & \text{if } n = 2\ell - 1, \\ \{ \varepsilon_1 \pm \varepsilon_j ; 2 \leqq j \leqq \ell \}, & \text{if } n = 2\ell - 2, \end{cases}$$

$$\omega_i = \varepsilon_1,$$

$$2\delta = \begin{cases} (2\ell - 1)\varepsilon_1 + (2\ell - 3)\varepsilon_2 + \cdots + \varepsilon_\ell & \text{if } n = 2\ell - 1, \\ 2(\ell - 1)\varepsilon_1 + 2(\ell - 2)\varepsilon_2 + \cdots + 2\varepsilon_{\ell-1} & \text{if } n = 2\ell - 2. \end{cases}$$

An element $\sigma \in W$ acts in $h_0$ by $\sigma\varepsilon_i = \pm\varepsilon_{\sigma(i)}$ for $1 \leqq i \leqq \ell$, where $\sigma$ in the index is a permutation of $\{ 1, 2, \cdots, \ell \}$. We represent this element $\sigma \in W$ by

$$\begin{pmatrix} 1 & 2 & \cdots & \ell \\ \pm\sigma(1) & \pm\sigma(2) & \cdots & \pm\sigma(\ell) \end{pmatrix}.$$

Then

$$W^1 = \begin{cases} \left\{ \sigma \in W ; \sigma^{-1} = \begin{pmatrix} 1 & 2 & \cdots & \ell \\ s(\sigma)i & i_2 & \cdots & i_\ell \end{pmatrix}, \begin{matrix} 0 < i_2 < \cdots < i_\ell \leqq \ell \\ s(\sigma) = \pm 1 \end{matrix} \right\}, & \text{if } n = 2\ell - 1 \\ \left\{ \sigma \in W ; \sigma^{-1} = \begin{pmatrix} 1 & 2 & \cdots & \ell \\ s(\sigma)i & i_2 & \cdots & s(\sigma)i_\ell \end{pmatrix}, \begin{matrix} 0 < i_2 < \cdots < i_\ell \leqq \ell \\ s(\sigma) = \pm 1 \end{matrix} \right\} & \text{if } n = 2\ell - 2. \end{cases}$$

An element $\sigma \in W^1$ is determined by $i$ and $s(\sigma)$, and its index $n(\sigma)$ of $\sigma$ is as follows:

$$n(\sigma) = \begin{cases} i - 1 & \text{if } s(\sigma) = 1, \\ n - (i - 1) & \text{if } s(\sigma) = -1. \end{cases}$$

Furthermore for $\sigma \in W^1$, the values of $\left( \sigma\delta, \frac{2\beta}{(\beta,\beta)} \right)$ $(\,\beta \in \Delta(n^+)\,)$ are as follows.

If $n = 2\ell - 1$,

$s(\sigma) = 1$

| $\Delta(n^+)$ | $\left( \sigma\delta, \frac{2\beta}{(\beta,\beta)} \right)$ |
|---|---|
| $\varepsilon_1 - \varepsilon_2$ | $-(i - 1)$ |
| $\varepsilon_1 - \varepsilon_3$ | $-(i - 2)$ |
| $\cdots$ | $\cdots$ |
| $\varepsilon_1 - \varepsilon_i$ | $-1$ |
| $\varepsilon_1 - \varepsilon_{i+1}$ | $1$ |
| $\varepsilon_1 - \varepsilon_{i+2}$ | $2$ |
| $\cdots$ | $\cdots$ |
| $\varepsilon_1 - \varepsilon_\ell$ | $\ell - i$ |
| $\varepsilon_1 + \varepsilon_\ell$ | $\ell - i + 1$ |
| $\varepsilon_1 + \varepsilon_{\ell-1}$ | $\ell - i + 2$ |
| $\cdots$ | $\cdots$ |
| $\varepsilon_1 + \varepsilon_{i+1}$ | $2\ell - 2i$ |
| $\varepsilon_1$ | $2\ell - 2i + 1$ |
| $\varepsilon_1 + \varepsilon_i$ | $2\ell - 2i + 2$ |
| $\cdots$ | $\cdots$ |
| $\varepsilon_1 + \varepsilon_2$ | $2\ell - i$ |

$s(\sigma) = -1$

| $\Delta(n^+)$ | $\left( \sigma\delta, \frac{2\beta}{(\beta,\beta)} \right)$ |
|---|---|
| $\varepsilon_1 - \varepsilon_2$ | $-2\ell + i$ |
| $\varepsilon_1 - \varepsilon_3$ | $-2\ell + i + 1$ |
| $\cdots$ | $\cdots$ |
| $\varepsilon_1 - \varepsilon_i$ | $-2\ell + 2i - 2$ |
| $\varepsilon_1$ | $-2\ell + 2i - 1$ |
| $\varepsilon_1 - \varepsilon_{i+1}$ | $-2\ell + 2i$ |
| $\cdots$ | $\cdots$ |
| $\varepsilon_1 - \varepsilon_\ell$ | $-\ell + i - 1$ |
| $\varepsilon_1 + \varepsilon_\ell$ | $-\ell + i$ |
| $\varepsilon_1 + \varepsilon_{\ell-1}$ | $-\ell + i + 1$ |
| $\cdots$ | $\cdots$ |
| $\varepsilon_1 + \varepsilon_{i+1}$ | $-1$ |
| $\varepsilon_1 + \varepsilon_i$ | $1$ |
| $\varepsilon_1 + \varepsilon_{i-1}$ | $2$ |
| $\cdots$ | $\cdots$ |
| $\varepsilon_1 + \varepsilon_2$ | $i - 1$ |

If $n = 2\ell - 2$,

$s(\sigma) = 1$

| $\Delta(n^+)$ | $(\sigma\delta, \frac{2\beta}{(\beta,\beta)})$ |
|---|---|
| $\varepsilon_1 - \varepsilon_2$ | $-(i-1)$ |
| $\varepsilon_1 - \varepsilon_3$ | $-(i-2)$ |
| $\cdots$ | $\cdots$ |
| $\varepsilon_1 - \varepsilon_i$ | $-1$ |
| $\varepsilon_1 - \varepsilon_{i+1}$ | $1$ |
| $\varepsilon_1 - \varepsilon_{i+2}$ | $2$ |
| $\cdots$ | $\cdots$ |
| $\varepsilon_1 - \varepsilon_\ell$ | $\ell - i$ |
| $\varepsilon_1 + \varepsilon_\ell$ | $\ell - i$ |
| $\varepsilon_1 + \varepsilon_{\ell-1}$ | $\ell - i + 1$ |
| $\cdots$ | $\cdots$ |
| $\varepsilon_1 + \varepsilon_{i+1}$ | $2\ell - 2i - 1$ |
| $\varepsilon_1 + \varepsilon_i$ | $2\ell - 2i + 1$ |
| $\varepsilon_1 + \varepsilon_{i-1}$ | $2\ell - 2i + 2$ |
| $\cdots$ | $\cdots$ |
| $\varepsilon_1 + \varepsilon_2$ | $2\ell - i - 1$ |

$s(\sigma) = -1$

$1 \leqq i < \ell$

| $\Delta(n^+)$ | $(\sigma\delta, \frac{2\beta}{(\beta,\beta)})$ |
|---|---|
| $\varepsilon_1 - \varepsilon_2$ | $-2\ell + i + 1$ |
| $\varepsilon_1 - \varepsilon_3$ | $-2\ell + i + 2$ |
| $\cdots$ | $\cdots$ |
| $\varepsilon_1 - \varepsilon_i$ | $-2\ell + 2i - 1$ |
| $\varepsilon_1 - \varepsilon_{i+1}$ | $-2\ell + 2i + 1$ |
| $\varepsilon_1 - \varepsilon_{i+2}$ | $-2\ell + 2i + 2$ |
| $\cdots$ | $\cdots$ |
| $\varepsilon_1 - \varepsilon_\ell$ | $-(\ell - i)$ |
| $\varepsilon_1 + \varepsilon_\ell$ | $-(\ell - i)$ |
| $\varepsilon_1 + \varepsilon_{\ell-1}$ | $-(\ell - i - 1)$ |
| $\cdots$ | $\cdots$ |
| $\varepsilon_1 + \varepsilon_{i+1}$ | $-1$ |
| $\varepsilon_1 + \varepsilon_i$ | $1$ |
| $\varepsilon_1 + \varepsilon_{i-1}$ | $2$ |
| $\cdots$ | $\cdots$ |
| $\varepsilon_1 + \varepsilon_2$ | $i - 1$ |

$i = \ell$

| $\Delta(n^+)$ | $(\sigma\delta, \frac{2\beta}{(\beta,\beta)})$ |
|---|---|
| $\varepsilon_1 - \varepsilon_2$ | $-(\ell - 1)$ |
| $\varepsilon_1 - \varepsilon_3$ | $-(\ell - 2)$ |
| $\cdots$ | $\cdots$ |
| $\varepsilon_1 - \varepsilon_{\ell-1}$ | $-2$ |
| $\varepsilon_1 - \varepsilon_\ell$ | $1$ |
| $\varepsilon_1 + \varepsilon_\ell$ | $-1$ |
| $\varepsilon_1 + \varepsilon_{\ell-1}$ | $2$ |
| $\varepsilon_1 + \varepsilon_{\ell-2}$ | $3$ |
| $\cdots$ | $\cdots$ |
| $\varepsilon_1 + \varepsilon_2$ | $-1$ |

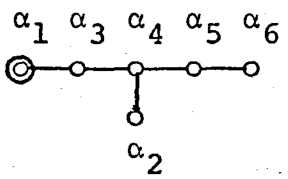
Furthermore we have

$$(k\omega_1, \frac{2\beta}{(\beta,\beta)}) = \begin{cases} k & \text{if } \beta = \varepsilon_1 \pm \varepsilon_j,\ 2 \leqq j \leqq \ell, \\ 2k & \text{if } \beta = \varepsilon_1. \end{cases}$$

Then, we obtain the following by Theorems 1 and 2,

Theorem 3. Let $M$ be a complex quadric of dimension $n$, $n \geqq 3$. Then the group $H^q(M, \Omega^p(E_{-k\omega_1}))=0$ except for the following cases: (i) $q=0$ and $k>p$, (ii) $p=q$ and $k=0$, (iii) $p+q=n$ and $k=2p-n$, (iv) $q=n$ and $k<p-n$.

2.2. The case $M$ is of type EIII. The Dinkin diagram is:




, where $\alpha_1$ ◎ shows that $\alpha_j=\alpha_1$ in this case. We have $\#\Delta(n^+)=16$ and $\#W^1=27$. We express $\beta=\sum_{i=1}^{6} m_i\alpha_i \in \Delta(n^+)$ by $(\, m_1\ m_2\ m_3\ m_4\ m_5\ m_6\,)$. For $\sigma$ of $W^1$, we put $\sigma\delta=(\, n_1\ n_2\ n_3\ n_4\ n_5\ n_6\,)$ if $\sigma\delta=\sum_{i=1}^{6} n_i\alpha_i$. Then we give the values $\left(\sigma\delta, \frac{2\beta}{(\beta,\beta)}\right)$ for $\sigma \in W^1$ and $\beta \in \Delta(n^+)$ by Table 1. From Table 1, Theorems 1 and 2, we obtain the following theorem.

Theorem 4. Let $M$ be of type EIII. Then the group $H^q(M, \Omega^p(E_{-k\omega_j}))$ vanishes except for $(\, p, q, k\,)$ listed in Table 2.

Table 1

Values $\left( \sigma\delta, \frac{2\beta}{(\beta,\beta)} \right)$ for $\sigma \in W^1$ and $\beta \in \Delta(n^+)$ ( type E III )

| $\sigma\delta$ \ $\beta$ | $\beta_1$ | $\beta_2$ | $\beta_3$ | $\beta_4$ | $\beta_5$ | $\beta_6$ | $\beta_7$ | $\beta_8$ | $\beta_9$ | $\beta_{10}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{13}$ | $\beta_{14}$ | $\beta_{15}$ | $\beta_{16}$ | the number which does not appear in the sequence |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\sigma_0\delta$ | 1 | 2 | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 7 | 7 | 8 | 8 | 9 | 10 | 11 | |
| $\sigma_1\delta$ | -1 | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 | 7 | 8 | 7 | 9 | 10 | 11 | 0 |
| $\sigma_2\delta$ | -2 | -1 | 1 | 3 | 2 | 4 | 3 | 4 | 5 | 6 | 6 | 7 | 7 | 8 | 10 | 11 | 0, 9 |
| $\sigma_3\delta$ | -3 | -2 | -1 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 5 | 6 | 7 | 8 | 9 | 11 | 0, 10 |
| $\sigma_4\delta$ | -4 | -3 | -2 | -1 | 1 | 1 | 2 | 4 | 3 | 5 | 4 | 6 | 6 | 7 | 8 | 11 | 0, 9, 10 |
| $\sigma_4'\delta$ | -4 | -3 | -2 | 1 | -1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 7 | 8 | 9 | 10 | 0, 6 |
| $\sigma_5\delta$ | -5 | -4 | -3 | -2 | 1 | -1 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 | 7 | 11 | 0, 8, 9, 10 |
| $\sigma_5'\delta$ | -5 | -4 | -3 | -1 | -1 | 1 | 1 | 3 | 2 | 4 | 3 | 5 | 6 | 7 | 8 | 10 | -2, 0, 9 |
| $\sigma_6\delta$ | -6 | -5 | -4 | -2 | -1 | -1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 10 | -3, 0, 8, 9 |
| $\sigma_6'\delta$ | -6 | -5 | -3 | -2 | -2 | 1 | -1 | 2 | 1 | 4 | 2 | 5 | 5 | 6 | 8 | 9 | -4, 0, 3, 7 |
| $\sigma_7\delta$ | -7 | -6 | -4 | -3 | -2 | -1 | -1 | 1 | 1 | 3 | 2 | 4 | 4 | 5 | 7 | 9 | -5, 0, 6, 8 |
| $\sigma_7'\delta$ | -7 | -5 | -4 | -3 | -3 | 1 | -2 | 2 | -1 | 3 | 1 | 5 | 4 | 6 | 7 | 8 | -6, 0 |
| $\sigma_8\delta$ | -8 | -7 | -4 | -3 | -3 | -2 | -2 | -1 | 1 | 2 | 2 | 3 | 3 | 4 | 7 | 8 | -6, -5, 0, 5, 6 |
| $\sigma_8'\delta$ | -8 | -6 | -5 | -4 | -3 | -1 | -2 | 1 | -1 | 2 | 1 | 4 | 3 | 5 | 6 | 8 | -7, 0, 7 |
| $\sigma_8''\delta$ | -7 | -6 | -5 | -4 | -4 | 1 | -3 | 2 | -2 | 3 | -1 | 4 | 4 | 5 | 6 | 7 | 0 |
| $\sigma_9\delta$ | -9 | -7 | -5 | -4 | -4 | -2 | -3 | -1 | -1 | 1 | 1 | 3 | 2 | 4 | 6 | 7 | -8, -6, 0, 5 |
| $\sigma_9'\delta$ | -8 | -7 | -6 | -5 | -4 | -1 | -3 | 1 | -2 | 2 | -1 | 3 | 3 | 4 | 5 | 7 | 0, 6 |
| $\sigma_{10}\delta$ | -9 | -8 | -6 | -5 | -5 | -2 | -4 | -1 | -2 | 1 | -1 | 2 | 2 | 3 | 5 | 6 | -7, -3, 0, 4 |
| $\sigma_{10}'\delta$ | -10 | -7 | -6 | -4 | -5 | -3 | -3 | -2 | -2 | -1 | 1 | 2 | 1 | 4 | 5 | 6 | -9, -8, 0, 3 |
| $\sigma_{11}\delta$ | -11 | -7 | -6 | -5 | -5 | -4 | -4 | -3 | -3 | -2 | 1 | 2 | -1 | 3 | 4 | 5 | -10, -9, -8, 0 |
| $\sigma_{11}'\delta$ | -10 | -8 | -7 | -5 | -6 | -3 | -4 | -2 | -3 | -1 | -1 | 1 | 1 | 3 | 4 | 5 | -9, 0, 2 |
| $\sigma_{12}\delta$ | -11 | -8 | -7 | -6 | -6 | -4 | -5 | -3 | -4 | -2 | -1 | 1 | -1 | 2 | 3 | 4 | -9, -10, 0 |
| $\sigma_{12}'\delta$ | -10 | -9 | -8 | -5 | -7 | -4 | -4 | -3 | -3 | -2 | -2 | -1 | 1 | 2 | 3 | 4 | -6, 0 |
| $\sigma_{13}\delta$ | -11 | -9 | -8 | -6 | -7 | -5 | -5 | -4 | -4 | -3 | -2 | -1 | -1 | 1 | 2 | 3 | -10, 0 |
| $\sigma_{14}\delta$ | -11 | -10 | -8 | -7 | -7 | -6 | -6 | -5 | -4 | -3 | -3 | -2 | -2 | -1 | 1 | 2 | -9, 0 |
| $\sigma_{15}\delta$ | -11 | -10 | -9 | -8 | -7 | -7 | -6 | -5 | -5 | -4 | -4 | -3 | -3 | -2 | -1 | 1 | 0 |
| $\sigma_{16}\delta$ | -11 | -10 | -9 | -8 | -8 | -7 | -7 | -6 | -5 | -5 | -4 | -4 | -3 | -3 | -2 | -1 | |

, where $\sigma\delta$, $\sigma \in W^1$, and $\beta \in \Delta(n^+)$ are expressed as follows:

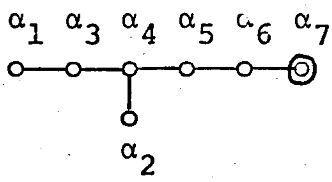
| | |
|---|---|
| $\sigma_0\delta$ | ( 1 1 1 1 1 1 ) |
| $\sigma_1\delta$ | ( -1 1 2 1 1 1 ) |
| $\sigma_2\delta$ | ( -2 1 1 2 1 1 ) |
| $\sigma_3\delta$ | ( -3 2 1 1 2 1 ) |
| $\sigma_4\delta$ | ( -4 3 1 1 1 2 ) |
| $\sigma_4{}'\delta$ | ( -4 1 1 1 3 1 ) |
| $\sigma_5\delta$ | ( -5 4 1 1 1 1 ) |
| $\sigma_5{}'\delta$ | ( -5 2 1 1 2 2 ) |
| $\sigma_6\delta$ | ( -6 3 1 1 2 1 ) |
| $\sigma_6{}'\delta$ | ( -6 1 1 2 1 3 ) |
| $\sigma_7\delta$ | ( -7 2 1 2 1 2 ) |
| $\sigma_7{}'\delta$ | ( -7 1 2 1 1 4 ) |
| $\sigma_8\delta$ | ( -8 1 1 3 1 1 ) |
| $\sigma_8{}'\delta$ | ( -8 2 2 1 1 3 ) |
| $\sigma_8{}''\delta$ | ( -7 1 1 1 1 5 ) |
| $\sigma_9\delta$ | ( -9 1 2 2 1 2 ) |
| $\sigma_9{}'\delta$ | ( -8 2 1 1 1 4 ) |
| $\sigma_{10}\delta$ | ( -9 1 1 2 1 3 ) |
| $\sigma_{10}{}'\delta$ | ( -10 1 3 1 2 1 ) |
| $\sigma_{11}\delta$ | ( -11 1 4 1 1 1 ) |
| $\delta_{11}{}'\delta$ | ( -10 1 2 1 2 2 ) |
| $\sigma_{12}\delta$ | ( -11 1 3 111 2 ) |
| $\sigma_{12}{}'\delta$ | ( -10 1 1 1 3 1 ) |
| $\sigma_{13}\delta$ | ( -11 1 2 1 2 1 ) |
| $\sigma_{14}\delta$ | ( -11 1 1 2 1 1 ) |
| $\sigma_{15}\delta$ | ( -11 2 1 1 1 1 ) |
| $\sigma_{16}\delta$ | ( -11 1 1 1 1 1 ) |

| | |
|---|---|
| $\beta_1$ | ( 1 0 0 0 0 0 ) |
| $\beta_2$ | ( 1 0 1 0 0 0 ) |
| $\beta_3$ | ( 1 0 1 1 0 0 ) |
| $\beta_4$ | ( 1 0 1 1 1 0 ) |
| $\beta_5$ | ( 1 1 1 1 0 0 ) |
| $\beta_6$ | ( 1 0 1 1 1 1 ) |
| $\beta_7$ | ( 1 1 1 1 1 0 ) |
| $\beta_8$ | ( 1 1 1 1 1 1 ) |
| $\beta_9$ | ( 1 1 1 2 1 0 ) |
| $\beta_{10}$ | ( 1 1 1 2 1 1 ) |
| $\beta_{11}$ | ( 1 1 2 2 1 0 ) |
| $\beta_{12}$ | ( 1 1 2 2 1 1 ) |
| $\beta_{13}$ | ( 1 1 1 2 2 1 ) |
| $\beta_{14}$ | ( 1 1 2 2 2 1 ) |
| $\beta_{15}$ | ( 1 1 2 3 2 1 ) |
| $\beta_{16}$ | ( 1 2 2 3 2 1 ) |

Table 2

| p | q = 0 | $1 \leqq q \leqq 15$, (a,b) shows q = a and k = b | q = 16 |
|---|---|---|---|
| 0 | $k > -1$ | | $k < -11$ |
| 1 | $k > 1$ | (1,0) | $k < -11$ |
| 2 | $k > 2$ | (2,0), (14,-9) | $k < -11$ |
| 3 | $k > 3$ | (3,0), (15,-10) | $k < -11$ |
| 4 | $k > 4$ | (4,0), (12,-6), (15,-9), (15,-10) | $k < -11$ |
| 5 | $k > 5$ | (5,0), (3,2), (15,-8), (15,-9), (15,-10) | $k < -11$ |
| 6 | $k > 6$ | (6,0), (3,3), (2,4), (10,-3), (14,-7), (15,-8) (15,-9) | $k < -10$ |
| 7 | $k > 7$ | (7,0), (1,6), (2,5), (14,-6), (15,-8) | $k < -9$ |
| 8 | $k > 8$ | (8,0), (1,7), (2,5), (2,6), (14,-5), (14,-6) (14,-7) | $k < -8$ |
| 9 | $k > 9$ | (9,0), (1,8), (2,6), (14,-5), (15,-6) | $k < -7$ |
| 10 | $k > 10$ | (10,0), (1,8), (1,9), (2,7), (6,3), (13,-3) (14,-4) | $k < -6$ |
| 11 | $k > 11$ | (11,0), (1,8), (1,9), (1,10), (13,-2) | $k < -5$ |
| 12 | $k > 11$ | (12,0), (1,9), (1,10), (4,6) | $k < -4$ |
| 13 | $k > 11$ | (13,0), (1,10) | $k < -3$ |
| 14 | $k > 11$ | (14,0), (2,9) | $k < -2$ |
| 15 | $k > 11$ | (15,0) | $k < -1$ |
| 16 | $k > 11$ | | $k < 1$ |

2.3. The case $M$ is of type E VII. The Dinkin diagram of $\Pi$ is:




,where $\alpha_7$ ◎ shows that $\alpha_j = \alpha_7$ in this case. We have $\#\Delta(n^+) = 27$ and $\#W^1 = 56$. We express $\beta$ of $\Delta(n^+)$ and $\sigma\delta$ for $\sigma \in W^1$ in a similar way as in 2.2. Then the values $( \sigma\delta, \frac{2\beta}{(\beta,\beta)} )$ for $\sigma \in W^1$ and $\beta \in \Delta(n^+)$ are as in Table 3. From Table 3, Theorems 1 and 2, we obtain the following.

Theorem 5. Let $M$ be of type E VII. Then the group $H^q( M, \Omega^p(E_{-k\omega_j}) ) = 0$ except for $( p, q, k )$ listed in Table 4.

Table 3

Values $\left(\sigma\delta, \frac{2\beta}{(\beta,\beta)}\right)$ for $\sigma \in W^1$ and $\beta \in \Delta(n^+)$ ( type E VII )

| $\sigma\delta$ \ $\beta$ | $\beta_1$ | $\beta_2$ | $\beta_3$ | $\beta_4$ | $\beta_5$ | $\beta_6$ | $\beta_7$ | $\beta_8$ | $\beta_9$ | $\beta_{10}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{13}$ | $\beta_{14}$ | $\beta_{15}$ | $\beta_{16}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\sigma_0\delta$ | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 6 | 7 | 7 | 8 | 8 | 9 | 9 | 9 | 10 |
| $\sigma_1\delta$ | -1 | 1 | 2 | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 7 | 7 | 9 | 9 | 8 | 10 |
| $\sigma_2\delta$ | -2 | -1 | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 7 | 6 | 8 | 8 | 7 | 9 |
| $\sigma_3\delta$ | -3 | -2 | -1 | 1 | 2 | 2 | 3 | 3 | 5 | 4 | 7 | 6 | 8 | 7 | 7 | 8 |
| $\sigma_4\delta$ | -4 | -3 | -2 | -1 | 1 | 1 | 3 | 2 | 3 | 4 | 4 | 5 | 5 | 6 | 7 | 7 |
| $\sigma_5\delta$ | -5 | -4 | -3 | -2 | 1 | -1 | 2 | 1 | 2 | 4 | 3 | 5 | 4 | 6 | 6 | 7 |
| $\sigma_5'\delta$ | -5 | -4 | -3 | -2 | -1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 7 | 6 |
| $\sigma_6\delta$ | -6 | -5 | -4 | -3 | -1 | -1 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 6 | 6 |
| $\sigma_6'\delta$ | -6 | -5 | -4 | -3 | 1 | -2 | 2 | -1 | 3 | 3 | 4 | 4 | 5 | 5 | 5 | 6 |
| $\sigma_7\delta$ | -7 | -6 | -5 | -4 | -1 | -2 | 1 | -1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 |
| $\sigma_7'\delta$ | -7 | -6 | -5 | -3 | -2 | -2 | -1 | 1 | 1 | 2 | 2 | 4 | 3 | 5 | 5 | 6 |
| $\sigma_8\delta$ | -8 | -7 | -6 | -4 | -2 | -3 | -1 | -1 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 5 |
| $\sigma_8'\delta$ | -8 | -7 | -5 | -4 | -3 | -3 | -2 | 1 | -1 | 2 | 1 | 3 | 2 | 5 | 4 | 6 |
| $\sigma_9\delta$ | -9 | -8 | -7 | -4 | -3 | -3 | -2 | -2 | 1 | -1 | 2 | 2 | 3 | 3 | 3 | 4 |
| $\sigma_9'\delta$ | -9 | -8 | -6 | -5 | -3 | -4 | -2 | -1 | -1 | 1 | 1 | 2 | 2 | 4 | 3 | 5 |
| $\sigma_9''\delta$ | -9 | -7 | -6 | -5 | -4 | -4 | -3 | 1 | -2 | 2 | -1 | 3 | 1 | 4 | 4 | 6 |
| $\sigma_{10}\delta$ | -10 | -9 | -7 | -5 | -4 | -4 | -3 | -2 | -1 | -1 | 1 | 1 | 2 | 3 | 2 | 4 |
| $\sigma_{10}'\delta$ | -10 | -8 | -7 | -6 | -4 | -5 | -3 | -1 | -2 | 1 | -1 | 2 | 1 | 3 | 3 | 5 |
| $\sigma_{10}''\delta$ | -9 | -8 | -7 | -6 | -5 | -5 | -4 | 1 | -3 | 2 | -2 | 3 | -1 | 4 | 4 | 5 |
| $\sigma_{11}\delta$ | -11 | -10 | -7 | -6 | -5 | -4 | -3 | -3 | 2 | -2 | 11 | -1 | 2 | 2 | 1 | 3 |
| $\sigma_{11}'\delta$ | -11 | -9 | -8 | -6 | -5 | -5 | -4 | -2 | -2 | -1 | -1 | 1 | 1 | 2 | 2 | 4 |
| $\sigma_{11}''\delta$ | -10 | -9 | -8 | -7 | -5 | -6 | -4 | -1 | -3 | 1 | -2 | 2 | -1 | 3 | 3 | 4 |
| $\sigma_{12}\delta$ | -12 | -11 | -7 | -6 | -5 | -5 | -4 | -4 | -3 | -3 | 1 | -2 | 2 | 2 | -1 | 3 |
| $\sigma_{12}'\delta$ | -12 | -10 | -8 | -7 | -6 | -5 | -4 | -3 | -3 | -2 | -1 | -1 | 1 | 1 | 1 | 3 |
| $\sigma_{12}''\delta$ | -11 | -10 | -9 | -7 | -6 | -6 | -5 | -2 | -3 | -1 | -2 | 1 | -1 | 2 | 2 | 3 |
| $\sigma_{13}\delta$ | -13 | -11 | -8 | -7 | -6 | -6 | -5 | -4 | -4 | -3 | -1 | -2 | 1 | 1 | -1 | 3 |
| $\sigma_{13}'\delta$ | -13 | -10 | -9 | -8 | -7 | -5 | -4 | -4 | -3 | -3 | -2 | -2 | 1 | -1 | 1 | 2 |
| $\sigma_{13}''\delta$ | -12 | -11 | -9 | -8 | -7 | -6 | -5 | -3 | -4 | -2 | -2 | -1 | -1 | 1 | 1 | 2 |
| $\sigma_{14}\delta$ | -14 | -11 | -9 | -8 | -7 | -6 | -5 | -5 | -4 | -4 | -2 | -3 | 1 | -1 | -1 | 2 |
| $\sigma_{14}'\delta$ | -13 | -12 | -9 | -8 | -7 | -7 | -6 | -4 | -5 | -3 | -2 | -2 | -1 | 1 | -1 | 1 |
| $\sigma_{14}''\delta$ | -13 | -11 | -10 | -9 | -8 | -6 | -5 | -4 | -4 | -3 | -3 | -2 | -1 | -1 | 1 | 1 |
| $\sigma_{15}\delta$ | -13 | -12 | -11 | -10 | -9 | -6 | -5 | -5 | -4 | -4 | -3 | -3 | -2 | -2 | 1 | -1 |
| $\sigma_{15}'\delta$ | -14 | -12 | -10 | -9 | -8 | -7 | -6 | -5 | -5 | -4 | -3 | -3 | -1 | -1 | -1 | 1 |
| $\sigma_{15}''\delta$ | -15 | -11 | -10 | -8 | -7 | -7 | -6 | -6 | -4 | -5 | -3 | -3 | 1 | -2 | -2 | 2 |
| $\sigma_{16}\delta$ | -14 | -13 | -11 | -10 | -9 | -7 | -6 | -6 | -5 | -5 | -3 | -4 | -2 | -2 | -1 | -1 |
| $\sigma_{16}'\delta$ | -15 | -12 | -11 | -9 | -8 | -8 | -7 | -6 | -5 | -5 | -4 | -3 | -1 | -2 | -2 | 1 |
| $\sigma_{16}''\delta$ | -16 | -11 | -10 | -9 | -7 | -8 | -6 | -7 | -5 | -5 | -4 | -4 | 1 | -3 | -3 | 2 |
| $\sigma_{17}\delta$ | -15 | -13 | -12 | -10 | -9 | -8 | -7 | -7 | -5 | -6 | -4 | -4 | -2 | -3 | -2 | -1 |
| $\sigma_{17}'\delta$ | -16 | -12 | -11 | -10 | -8 | -9 | -7 | -7 | -6 | -5 | -5 | -4 | -1 | -3 | -3 | 1 |
| $\sigma_{17}''\delta$ | -17 | -11 | -10 | -9 | -8 | -8 | -7 | -7 | -6 | -6 | -5 | -5 | 1 | -4 | -4 | 2 |
| $\sigma_{18}\delta$ | -15 | -14 | -13 | -10 | -9 | -9 | -8 | -8 | -5 | -7 | -4 | -4 | -3 | -3 | -3 | -2 |
| $\sigma_{18}'\delta$ | -16 | -13 | -12 | -11 | -9 | -9 | -7 | -8 | -6 | -6 | -5 | -5 | -2 | -4 | -3 | -1 |
| $\sigma_{18}''\delta$ | -17 | -12 | -11 | -10 | -9 | -9 | -8 | -7 | -7 | -6 | -6 | -5 | -1 | -4 | -4 | 1 |
| $\sigma_{19}\delta$ | -16 | -14 | -13 | -11 | -9 | -10 | -8 | -9 | -6 | -7 | -5 | -5 | -3 | -4 | -4 | -2 |
| $\sigma_{19}'\delta$ | -17 | -13 | -12 | -11 | -10 | -9 | -8 | -8 | -7 | -7 | -6 | -6 | -2 | -5 | -4 | -1 |
| $\sigma_{20}\delta$ | -16 | -15 | -13 | -12 | -9 | -11 | -8 | -10 | -7 | -7 | -5 | -6 | -4 | -4 | -5 | -3 |
| $\sigma_{20}'\delta$ | -17 | -14 | -13 | -11 | -10 | -10 | -9 | -9 | -7 | -8 | -6 | -6 | -3 | -5 | -5 | -2 |
| $\sigma_{21}\delta$ | -16 | -15 | -14 | -13 | -9 | -12 | -8 | -11 | -7 | -7 | -6 | -6 | -5 | -5 | -5 | -4 |
| $\sigma_{21}'\delta$ | -17 | -15 | -13 | -12 | -10 | -11 | -9 | -10 | -8 | -8 | -6 | -7 | -4 | -5 | -6 | -3 |
| $\sigma_{22}\delta$ | -17 | -15 | -14 | -13 | -10 | -12 | -9 | -11 | -8 | -8 | -7 | -7 | 5 | -6 | -6 | -4 |
| $\sigma_{22}'\delta$ | -17 | -16 | -13 | -12 | -11 | -11 | -10 | -10 | -9 | -9 | -6 | -8 | -5 | -5 | -7 | -4 |
| $\sigma_{23}\delta$ | -17 | -16 | -14 | -13 | -11 | -12 | -10 | -11 | -9 | -9 | -7 | -8 | -6 | -6 | -7 | -5 |
| $\sigma_{24}\delta$ | -17 | -16 | -15 | -13 | -12 | -12 | -11 | -11 | -9 | -10 | -8 | -8 | -7 | -7 | -7 | -6 |
| $\sigma_{25}\delta$ | -17 | -16 | -15 | -14 | -13 | -12 | -11 | -11 | -10 | -10 | -9 | -9 | -8 | -8 | -7 | -7 |
| $\sigma_{26}\delta$ | -17 | -16 | -15 | -14 | -13 | -13 | -12 | -11 | -11 | -10 | -10 | -9 | -9 | -8 | -8 | -7 |
| $\sigma_{27}\delta$ | -17 | -16 | -15 | -14 | -13 | -13 | -12 | -12 | -11 | -11 | -10 | -10 | -9 | -9 | -8 | -8 |

Table 3 —— continued 

| $\sigma\delta$ \ $\beta$ | $\beta_{17}$ | $\beta_{18}$ | $\beta_{19}$ | $\beta_{20}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{23}$ | $\beta_{24}$ | $\beta_{25}$ | $\beta_{26}$ | $\beta_{27}$ | the number which does not appear in the sequence |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\sigma_0\delta$ | 10 | 11 | 11 | 12 | 12 | 13 | 13 | 14 | 15 | 16 | 17 | |
| $\sigma_1\delta$ | 9 | 11 | 10 | 12 | 11 | 11 | 13 | 14 | 15 | 16 | 17 | 0 |
| $\sigma_2\delta$ | 9 | 10 | 10 | 11 | 11 | 13 | 12 | 14 | 15 | 16 | 17 | 0 |
| $\sigma_3\delta$ | 8 | 9 | 10 | 11 | 11 | 12 | 12 | 13 | 15 | 16 | 17 | 0, 14 |
| $\sigma_4\delta$ | 8 | 9 | 9 | 10 | 11 | 11 | 12 | 13 | 14 | 16 | 17 | 0, 15 |
| $\sigma_5\delta$ | 7 | 8 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 17 | 0, 16 |
| $\sigma_5'\delta$ | 8 | 9 | 9 | 10 | 10 | 11 | 11 | 12 | 13 | 16 | 17 | 0, 14, 15 |
| $\sigma_6\delta$ | 7 | 8 | 8 | 9 | 10 | 10 | 11 | 12 | 13 | 15 | 17 | -2, 0, 14, 16 |
| $\sigma_6'\delta$ | 6 | 7 | 7 | 8 | 11 | 9 | 12 | 13 | 14 | 15 | 16 | 0, 10 |
| $\sigma_7\delta$ | 6 | 7 | 7 | 8 | 10 | 9 | 11 | 12 | 13 | 15 | 16 | -3, 0, 14 |
| $\sigma_7'\delta$ | 6 | 7 | 8 | 9 | 9 | 10 | 10 | 11 | 13 | 14 | 17 | -4, 0, 12, 15, 16 |
| $\sigma_8\delta$ | 5 | 6 | 7 | 8 | 9 | 9 | 10 | 11 | 13 | 14 | 16 | -5, 0, 12, 15 |
| $\sigma_8'\delta$ | 6 | 7 | 7 | 8 | 8 | 10 | 9 | 11 | 12 | 13 | 17 | -6, 0, 14, 15, 16 |
| $\sigma_9\delta$ | 4 | 5 | 7 | 8 | 8 | 9 | 9 | 10 | 13 | 14 | 15 | -5, -6, 0, 6, 11, 12 |
| $\sigma_9'\delta$ | 5 | 6 | 6 | 7 | 8 | 9 | 9 | 11 | 12 | 13 | 16 | -7, 0, 10, 14, 15 |
| $\sigma_9''\delta$ | 5 | 7 | 6 | 8 | 7 | 9 | 9 | 10 | 11 | 12 | 17 | -8, 0, 13, 14, 15, 16 |
| $\sigma_{10}\delta$ | 4 | 5 | 6 | 7 | 7 | 9 | 8 | 10 | 12 | 13 | 15 | -8, -6, 0, 11, 14 |
| $\sigma_{10}'\delta$ | 4 | 6 | 5 | 7 | 7 | 8 | 9 | 10 | 11 | 12 | 16 | -9, 0, 13, 14, 15 |
| $\sigma_{10}''\delta$ | 5 | 6 | 6 | 7 | 7 | 8 | 8 | 9 | 10 | 11 | 17 | 0, 12, 13, 14, 15, 16 |
| $\sigma_{11}\delta$ | 4 | 5 | 5 | 6 | 6 | 9 | 7 | 10 | 11 | 13 | 14 | -8, -9, o, 8, 12 |
| $\sigma_{11}'\delta$ | 3 | 5 | 5 | 7 | 6 | 8 | 8 | 9 | 11 | 12 | 15 | -10, -7, -3, 0, 10, 13,14 |
| $\sigma_{11}''\delta$ | 4 | 5 | 5 | 6 | 7 | 7 | 8 | 9 | 10 | 11 | 16 | 0, 12, 13, 14, 15 |
| $\sigma_{12}\delta$ | 3 | 4 | 4 | 5 | 5 | 9 | 6 | 10 | 11 | 12 | 13 | -10, -9, -8, 0, 7, 8 |
| $\sigma_{12}'\delta$ | 3 | 5 | 4 | 6 | 5 | 8 | 7 | 9 | 10 | 12 | 14 | -11, -9, 0, 2, 11, 13 |
| $\sigma_{12}''\delta$ | 3 | 4 | 5 | 6 | 6 | 7 | 7 | 8 | 10 | 11 | 15 | -8, -4, 0, 9, 12, 13, 14 |
| $\sigma_{13}\delta$ | 2 | 4 | 3 | 5 | 4 | 8 | 6 | 9 | 10 | 11 | 13 | -12, -10, -9, 0, 7, 12 |
| $\sigma_{13}'\delta$ | 2 | 5 | 3 | 6 | 4 | 7 | 7 | 8 | 9 | 12 | 13 | -12, -11, -6, 0, 10, 11 |
| $\sigma_{13}''\delta$ | 3 | 4 | 4 | 5 | 5 | 7 | 6 | 8 | 9 | 11 | 14 | -10, 0, 10, 12, 13 |
| $\sigma_{14}\delta$ | 1 | 4 | 2 | 5 | 3 | 7 | 6 | 8 | 9 | 11 | 12 | -13, -12, -10, 0, 10 |
| $\sigma_{14}'\delta$ | 2 | 3 | 3 | 4 | 4 | 7 | 5 | 8 | 9 | 10 | 13 | -11, -10, 0, 6, 11, 12 |
| $\sigma_{14}''\delta$ | 2 | 4 | 3 | 5 | 4 | 6 | 6 | 7 | 8 | 11 | 13 | -12, -7, 0, 9, 10, 12 |
| $\sigma_{15}\delta$ | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 | 7 | 11 | 12 | -8, -7, 0, 8, 9, 10 |
| $\sigma_{15}'\delta$ | 1 | 3 | 2 | 4 | 3 | 6 | 5 | 7 | 8 | 10 | 12 | -13, -11, -2, 0, 9, 11 |
| $\sigma_{15}''\delta$ | -1 | 3 | 1 | 5 | 2 | 6 | 6 | 7 | 9 | 10 | 11 | -14,-13,-12,-9, 0, 4, 9 |
| $\sigma_{16}\delta$ | 1 | 2 | 2 | 3 | 3 | 5 | 4 | 6 | 7 | 10 | 11 | -12, -8, 0, 8, 9 |
| $\sigma_{16}'\delta$ | -1 | 2 | 1 | 4 | 2 | 5 | 5 | 6 | 8 | 9 | 11 | -14,-13,-10, 0, 3, 7, 10 |
| $\sigma_{16}''\delta$ | -2 | 3 | -1 | 4 | 1 | 5 | 6 | 7 | 8 | 9 | 10 | -15, -14, -13, -12, 0 |
| $\sigma_{17}\delta$ | -1 | 1 | 1 | 3 | 2 | 4 | 4 | 5 | 7 | 9 | 10 | -14, -11 0, 6, 8 |
| $\sigma_{17}'\delta$ | -2 | 2 | -1 | 3 | 1 | 4 | 5 | 6 | 7 | 8 | 10 | -15, -14, -13, 0, 9 |
| $\sigma_{17}''\delta$ | -3 | 3 | -2 | 4 | -1 | 5 | 5 | 6 | 7 | 8 | 9 | -16, -15, -14, -13, -12,0 |
| $\sigma_{18}\delta$ | -2 | -1 | -1 | 2 | 2 | 3 | 3 | 4 | 7 | 8 | 9 | -12, -11, -6, 0, 5, 6 |
| $\sigma_{18}'\delta$ | -2 | 1 | -1 | 2 | 1 | 3 | 4 | 5 | 6 | 8 | 9 | -15, -14, -10, 0, 7 |
| $\sigma_{18}''\delta$ | -3 | 2 | -2 | 3 | -1 | 4 | 4 | 5 | 6 | 7 | 9 | -16, -15, -14, -13, 0, 8 |
| $\sigma_{19}\delta$ | -3 | -1 | -1 | 1 | 1 | 2 | 3 | 4 | 6 | 7 | 8 | -15, -12, 0, 5 |
| $\sigma_{19}'\delta$ | -3 | 1 | -2 | 2 | -1 | 3 | 3 | 4 | 5 | 7 | 8 | -16, -15, -14, 0, 6 |
| $\sigma_{20}\delta$ | -3 | -2 | -2 | -1 | 1 | 1 | 2 | 4 | 5 | 6 | 7 | -14, 0, 3 |
| $\sigma_{20}'\delta$ | -4 | -1 | -2 | 1 | -1 | 2 | 2 | 3 | 5 | 6 | 7 | -16,-15, -12, 0, 4 |
| $\sigma_{21}\delta$ | -4 | -3 | -3 | -2 | 1 | -1 | 2 | 3 | 4 | 5 | 6 | -10, 0 |
| $\sigma_{21}'\delta$ | -4 | -2 | -3 | -1 | -1 | 1 | 1 | 3 | 4 | 5 | 6 | -16, -14, 0, 2 |
| $\sigma_{22}\delta$ | -5 | -3 | -4 | -2 | -1 | -1 | 1 | 2 | 3 | 4 | 5 | -16,0 |
| $\sigma_{22}'\delta$ | -4 | -3 | -3 | -2 | -2 | 1 | -1 | 2 | 3 | 4 | 5 | -15, -14, 0 |
| $\sigma_{23}\delta$ | -5 | -4 | -4 | -3 | -2 | -1 | -1 | 1 | 2 | 3 | 4 | -15, 0 |
| $\sigma_{24}\delta$ | -6 | -5 | -4 | -3 | -3 | -2 | -2 | -1 | 1 | 2 | 3 | -14, 0 |
| $\sigma_{25}\delta$ | -6 | -5 | -5 | -4 | -4 | -3 | -3 | -2 | -1 | 1 | 2 | 0 |
| $\sigma_{26}\delta$ | -7 | -6 | -6 | -5 | -5 | -4 | -4 | -3 | -2 | -1 | 1 | 0 |
| $\sigma_{27}\delta$ | -8 | -7 | -7 | -6 | -6 | -5 | -5 | -4 | -3 | -2 | -1 | |

, where $\sigma\delta$, $\sigma \in W^1$, and $\beta \in \Delta(n^+)$ are expressed as follows:

| | |
|---|---|
| $\sigma_0\delta$ | ( 1 1 1 1 1 1 1 ) |
| $\sigma_1\delta$ | ( 1 1 1 1 1 2 -1 ) |
| $\sigma_2\delta$ | ( 1 1 1 1 2 1 -2 ) |
| $\sigma_3\delta$ | ( 1 1 1 2 1 1 -3 ) |
| $\sigma_4\delta$ | ( 1 2 2 1 1 1 -4 ) |
| $\sigma_5\delta$ | ( 2 3 1 1 1 1 -5 ) |
| $\sigma_5'\delta$ | ( 1 1 3 1 1 1 -5 ) |
| $\sigma_6\delta$ | ( 2 2 2 1 1 1 -6 ) |
| $\sigma_6'\delta$ | ( 1 4 1 1 1 1 -6 ) |
| $\sigma_7\delta$ | ( 1 3 2 1 1 1 -7 ) |
| $\sigma_7'\delta$ | ( 3 1 1 2 1 1 -7 ) |
| $\sigma_8\delta$ | ( 2 2 1 2 1 1 -8 ) |
| $\sigma_8'\delta$ | ( 4 1 1 1 2 1 -8 ) |
| $\sigma_9\delta$ | ( 1 1 1 3 1 1 -9 ) |
| $\sigma_9'\delta$ | ( 3 2 1 1 2 1 -9 ) |
| $\sigma_9''\delta$ | ( 5 1 1 1 1 2 -9 ) |
| $\sigma_{10}\delta$ | ( 2 1 1 2 2 1 -10 ) |
| $\sigma_{10}'\delta$ | ( 4 2 1 1 1 2 -10 ) |
| $\sigma_{10}''\delta$ | ( 6 1 1 1 1 1 -9 ) |
| $\sigma_{11}\delta$ | ( 1 1 2 1 3 1 -11 ) |
| $\sigma_{11}'\delta$ | ( 3 1 1 2 1 2 -11 ) |
| $\sigma_{11}''\delta$ | ( 5 2 1 1 1 1 -10 ) |
| $\sigma_{12}\delta$ | ( 1 1 1 1 4 1 -12 ) |
| $\sigma_{12}'\delta$ | ( 2 1 2 1 2 2 -12 ) |
| $\sigma_{12}''\delta$ | ( 4 1 1 2 1 1 -11 ) |
| $\sigma_{13}\delta$ | ( 2 1 1 1 3 2 -13 ) |
| $\sigma_{13}'\delta$ | ( 1 1 3 1 1 3 -13 ) |
| $\sigma_{13}''\delta$ | ( 3 1 2 1 2 1 -12 ) |
| $\sigma_{14}\delta$ | ( 1 1 2 1 2 3 -14 ) |
| $\sigma_{14}'\delta$ | ( 3 1 1 1 3 1 -13 ) |
| $\sigma_{14}''\delta$ | ( 2 1 3 1 1 2 -13 ) |
| $\sigma_{15}\delta$ | ( 1 1 4 1 1 1 -13 ) |
| $\sigma_{15}'\delta$ | ( 2 1 2 1 2 2 -14 ) |
| $\sigma_{15}''\delta$ | ( 1 1 1 2 1 4 -15 ) |
| $\sigma_{16}\delta$ | ( 1 1 3 1 2 1 -14 ) |
| $\sigma_{16}'\delta$ | ( 2 1 1 2 1 3 -15 ) |
| $\sigma_{16}''\delta$ | ( 1 2 1 1 1 5 -16 ) |
| $\sigma_{17}\delta$ | ( 2 1 2 2 1 2 -15 ) |
| $\sigma_{17}'\delta$ | ( 2 2 1 1 1 4 -16 ) |
| $\sigma_{17}''\delta$ | ( 1 1 1 1 1 6 -17 ) |
| $\sigma_{18}\delta$ | ( 1 1 1 3 1 1 -15 ) |
| $\sigma_{18}'\delta$ | ( 1 2 2 1 1 3 -16 ) |
| $\sigma_{18}''\delta$ | ( 2 1 1 1 1 5 -17 ) |
| $\sigma_{19}\delta$ | ( 1 2 1 2 1 2 -16 ) |
| $\sigma_{19}'\delta$ | ( 1 1 2 1 1 4 -17 ) |
| $\sigma_{20}\delta$ | ( 1 3 1 1 2 1 -16 ) |
| $\sigma_{20}'\delta$ | ( 1 1 1 2 1 3 -17 ) |
| $\sigma_{21}\delta$ | ( 1 4 1 1 1 1 -16 ) |
| $\sigma_{21}'\delta$ | ( 1 2 1 1 2 2 -17 ) |
| $\sigma_{22}\delta$ | ( 1 3 1 1 1 2 -17 ) |
| $\sigma_{22}'\delta$ | ( 1 1 1 1 3 1 -17 ) |
| $\sigma_{23}\delta$ | ( 1 2 1 1 2 1 -17 ) |
| $\sigma_{24}\delta$ | ( 1 1 1 2 1 1 -17 ) |
| $\sigma_{25}\delta$ | ( 1 1 2 1 1 1 -17 ) |
| $\sigma_{26}\delta$ | ( 2 1 1 1 1 1 -17 ) |
| $\sigma_{27}\delta$ | ( 1 1 1 1 1 1 -17 ) |

| | |
|---|---|
| $\beta_1$ | ( 0 0 0 0 0 0 1 ) |
| $\beta_2$ | ( 0 0 0 0 0 111 ) |
| $\beta_3$ | ( 0 0 0 0 1 1 1 ) |
| $\beta_4$ | ( 0 0 0 1 1 1 1 ) |
| $\beta_5$ | ( 0 1 0 1 1 1 1 ) |
| $\beta_6$ | ( 0 0 1 1 1 1 1 ) |
| $\beta_7$ | ( 0 1 1 1 1 1 1 ) |
| $\beta_8$ | ( 1 0 1 1 1 1 1 ) |
| $\beta_9$ | ( 0 1 1 2 1 1 1 ) |
| $\beta_{10}$ | ( 1 1 1 1 1 1 1 ) |
| $\beta_{11}$ | ( 0 1 1 2 2 1 1 ) |
| $\beta_{12}$ | ( 1 1 1 2 1 1 1 ) |
| $\beta_{13}$ | ( 0 1 1 2 2 2 1 ) |
| $\beta_{14}$ | ( 1 1 1 2 2 1 1 ) |
| $\beta_{15}$ | ( 1 1 2 2 1 1 1 ) |
| $\beta_{16}$ | ( 1 1 1 2 2 2 1 ) |
| $\beta_{17}$ | ( 1 1 2 2 2 1 1 ) |
| $\beta_{18}$ | ( 1 1 2 2 2 2 1 ) |
| $\beta_{19}$ | ( 1 1 2 3 2 1 1 ) |
| $\beta_{20}$ | ( 1 1 2 3 2 2 1 ) |
| $\beta_{21}$ | ( 1 2 2 3 2 1 1 ) |
| $\beta_{22}$ | ( 1 1 2 3 3 2 1 ) |
| $\beta_{23}$ | ( 1 2 2 3 2 2 1 ) |
| $\beta_{24}$ | ( 1 2 2 3 3 2 1 ) |
| $\beta_{25}$ | ( 1 2 2 4 3 2 1 ) |
| $\beta_{26}$ | ( 1 2 3 4 3 2 1 ) |
| $\beta_{27}$ | ( 2 2 3 4 3 2 1 ) |

Table 4

| p | q = 0 | 1 ≦ q ≦ 26, (a,b) shows q = a and k = b | q = 27 |
|---|---|---|---|
| 0 | k > -1 | | k < -17 |
| 1 | k > 1 | (1,0) | k < -17 |
| 2 | k > 2 | (2,0) | k < -17 |
| 3 | k > 3 | (3,0), (24, -14) | k < -17 |
| 4 | k > 4 | (4,0), (25,-15) | k < -17 |
| 5 | k > 5 | (5,0), (25,-14∿-15) | k < -17 |
| 6 | k > 6 | (6,0), (4,2), (21,-10), (25,-14), (26,-16) | k < -17 |
| 7 | k > 7 | (7,0), (3,4), (4,3), (24,-12), (25,-14), (26,-15∿-16) | k < -17 |
| 8 | k > 8 | (8,0), (2,6), (3,5), (24,-12), (26,-14∿-16) | k < -17 |
| 9 | k > 9<br>> | (9,0), (1,8), (2,7), (3,5 6), (18,-6), (23,-10),<br>(24,-11∿-12), (26,-13∿-16) | k < -17 |
| 10 | k > 10 | (10,0), (1,9), (2,8), (3,6), (24,-11), (26,-12∿-16) | k < -17 |
| 11 | k > 11 | (11,0), (1,10), (2,8∿9), (3,7), (7,3), (22,-8),<br>(24,-10), (25,-12), (26,-12∿-15) | k < -16 |
| 12 | k > 12 | (12,0), (1,11), (2,8∿10), (3,8), (7,4), (15,-2),<br>(22,-7∿-8), (24,-9), (25,-11), (26,-12∿-14) | k < -15 |
| 13 | k > 13 | (13,0), (1,11∿12), (2,9∿10), (5,6), (22,-7),<br>(25,-10∿-11), (26,-12∿-13) | k < -14 |
| 14 | k > 14 | (14,0), (1,12∿13), (2,10∿11), (5,7), (22,-6),<br>(25,-9∿-10), (26,-11∿-12) | k < -13 |
| 15 | k > 15 | (15,0), (1,12∿14), (2,11), (5,7∿8), (3,9), (12,2),<br>(20,-4), (24,-8), (25,-8∿-10), (26,-11) | k < -12 |
| 16 | k > 16 | (16,0), (1,12∿15), (2,12), (3,10), (5,8), (20,-3),<br>(24,-7), (25,-8∿-9), (26,-10) | k < -11 |
| 17 | k > 17 | (17,0), (1,12∿16), (3,11), (24,-6), (25,-8), (26,-9) | k < -10 |
| 18 | k > 17 | (18,0), (1,13∿16), (3,11∿12), (4,10), (9,6), | k < -9 |
| 19 | k > 17 | (19,0), (1,14∿16), (3,12), (24,-5), (25,-6) | k < -8 |
| 20 | k > 17 | (20,0), (1,15∿16), (2,14), (3,12), (23,-3), (24,-4) | k < -7 |
| 21 | k > 17 | (21,0), (1,16), (2,14) (6,10), (23,-2) | k < -6 |
| 22 | k > 17 | (22,0), (1,16), (2,14∿15) | k < -5 |
| 23 | k > 17 | (23,0), (2,15) | k < -4 |
| 24 | k > 17 | (24,0), (3,14) | k < -3 |
| 25 | k > 17 | (25,0) | k < -2 |
| 26 | k > 17 | (26,0) | k < -1 |
| 27 | k > 17 | | k < 1 |

2.4. Other cases. If $M$ is of type A III, D III or C I, it is not known completely when the groups $H^q(M, \Omega^p(E))$ vanish. in this section we consider for the case when $p$ is equal to $0$ or $1$.

We denote by $K_N$ the canonical line bundle of a complex manifold $N$. There exists an integer $\lambda$ such that $K_N = E_{\lambda\omega_j}$. Further we know

$$\lambda = 2\Big(\sum_{\beta\in\Delta(n^+)} \beta,\ \alpha_j\Big)/(\alpha_j,\ \alpha_j)$$

( Borel-Hirzebruch [2] ). Applying this formula, we may calculate $\lambda$ for each type and get the following table.

| | | |
|---|---|---|
| A III | $SU(m+n)/S(U(m)\times U(n))$, | $\lambda = m + n$, |
| D III | $SO(2n)/U(n)$, | $\lambda = 2n - 2$, |
| C I | $Sp(n)/U(n)$, | $\lambda = n + 1$, |
| BD I | $SO(n+2)/SO(2)\times SO(n)$, | $\lambda = n$, |
| E III | $E_6/Spin(10)\times T^1$, | $\lambda = 12$, |
| E VII | $E_7/E_6\times T^1$ | $\lambda = 18$. |

Theorem 6. Let $M$ be an n-dimensional irreducible Hermitian symmetric space of compact type. Then the group $H^q(M, \Omega E_{-k\omega_j}) = 0$ except for the following cases: (i) $q = 1$ and $k \geqq 0$, (ii) $q = n$ and $k \leqq -\lambda$.

Proof. By the theorem of Bott, we get

$$H^0(M, \Omega E_{-k\omega_j}) \neq 0 \qquad \text{if } k \geqq 0, \tag{2.1}$$

$$H^j(M, \Omega E_{-k\omega_j}) = 0 \quad \text{for } j > 0, \text{ if } k \geqq 0, \tag{2.2}$$

$$H^0(M, \Omega E_{-k\omega_j}) = 0 \qquad \text{if } k < 0. \tag{2.3}$$

By serre's duality theorem, we have

$$\dim H^q(\, M,\ \Omega E_{-k\omega_j}\,) = \dim H^{n-q}(\, M,\ \Omega(K_M \otimes E_{k\omega_j})\,).$$

Hence we obtain, from (2.1) and (2.2)

(2.4) $\qquad H^n(\, M,\ \Omega E_{-k\omega_j}\,) \neq 0 \qquad$ if $k \leqq -\lambda$,

(2.5) $\qquad H^j(\, M,\ \Omega E_{-k\omega_j}\,) = 0 \qquad$ for $j < n$, if $k \leq -\lambda$.

We note $E_{-k\omega_j}$ is positive if $k > 0$. Then by Kodaira's vanishing theorem, we see

(2.6) $\qquad H^j(\, M,\ \Omega E_{-k\omega_j}\,) = 0 \qquad$ for $j > 0$, if $k > -\lambda$.

The conclusion follows from (2.1), (2.3), (2.4), (2.5) and (2.6).

Remark. If $M$ is a Kahler C-space whose 2nd Betti number is 1, we get the same conclusion in the same way as above.

Theorem 7. Let $M$ be an irreducible Hermitian symmetric space of compact type. Assume that $M$ is not $P_n(C)$, $Sp(2)/U(2)$, $SO(6)/U(3)$ or $SO(8)/U(4)$. Then the group $H^q(\, M,\ \Omega^1(E_{-k\omega_j})\,) = 0$ except for the following cases: (i) $q = 0$ and $k > 1$, (ii) $q = 1$ and $k = 0$, (iii) $q = n$ and $k < -\lambda + 1$.

Proof. We may assume that $M$ is of type A III, C I or D III by Theorems 3, 4 and 5.

It is known

$$n(\sigma) = \min\{\, k;\ \sigma = \tau_{\alpha_{i_1}} \cdots \tau_{\alpha_{i_k}},\ \alpha_{i_j} \in \Pi \,\} \qquad \text{for } \sigma \in W$$

, where $\tau_\alpha$ denotes the symmetry with respect to $\alpha \in \Delta$. Therefore by the definition of $W^1(1)$, we have

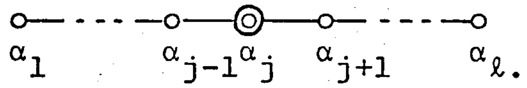
$$W^1(1) = \{\, \tau_{\alpha_j} \,\}.$$

Since $\tau_{\alpha_j}\delta = \delta - \alpha_j$, we have by Theorem 2

(2.7) $\dim H^q(\,M,\ \Omega^1(E_{-k\omega_j})\,) = \dim H^q(\,M,\ \Omega(E_{-(k\omega_j-\alpha_j)})\,)$

for $q = 0, 1, \cdots$.

1. The case $M = SU(\ell+1)/S(U(j)\times U(\ell+1-j))$ for $1 < j < \ell$. The Dinkin diagram of $\Pi$ is:




We may assume that $h_0$ is the set of points $(x_i) \in R^{\ell+1}$ such that $\sum_{i=1}^{\ell+1} x_i = 0$. Let $\{\varepsilon_i\}_{i=1}^{+1}$ be the natural basis of $R^{\ell+1}$. Then

$$\alpha_j = \varepsilon_j - \varepsilon_{j+1},$$

$$\delta = \ell\varepsilon_1 + (\ell-1)\varepsilon_2 + \cdots + 2\varepsilon_{\ell-1} + \varepsilon_\ell,$$

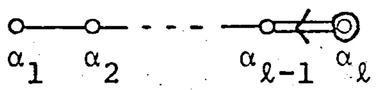
$$\Delta(n^+) = \{\ \varepsilon_s - \varepsilon_t;\ 1 \leqq s \leqq j < t \leqq \ell+1\ \}.$$

It follows that

$$\{\ (\delta-\alpha_j, \frac{2\beta}{(\beta,\beta)});\ \beta \in \Delta(n^+)\ \} = \{\ -1,\ 1,\ 2,\ \cdots,\ \ell\}.$$

Further if $\beta \in \Delta(n^+)$ satisfies $(\delta-\alpha_j, \frac{2\beta}{(\beta,\beta)}) = -1$, then $\beta = \alpha_j$. Therefore the conclusion follows from Theorem 1.

2. The case $M = Sp(\ell)/U(\ell)$ for $\ell \geqq 3$. The Dinkin diagram of $\Pi$ is:




,where $\alpha_\ell$ ◎ means $\alpha_j = \alpha_\ell$. Let $\{\varepsilon_i\}_{i=1}$ be the basis of $h_0$ which satisfies $(\varepsilon_i,\varepsilon_j) = \delta_{ij}$. Then

$\alpha_\ell = 2\varepsilon_\ell,$

$\omega_\ell = \varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_\ell,$

$\delta = \ell\varepsilon_1 + (\ell-1)\varepsilon_2 + \cdots + 2\varepsilon_{\ell-1} + \varepsilon_\ell,$

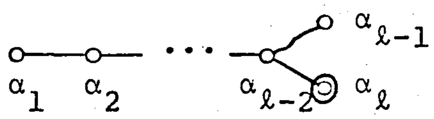
$\Delta(n^+) = \{ \varepsilon_i + \varepsilon_j ;\ 1 \leqq i < j \leqq \ell,\ 2\varepsilon_i ;\ 1 \leqq i \leqq \ell \}.$

Hence we have

$$\{ (\delta-\alpha_\ell, \beta);\ \beta \in \Delta(n^+) \} = \begin{cases} \{-2,1,2,\cdots,2\ -1,2\} & \text{if } \ell > 3, \\ \{-2,\ 1,\ 2,\ 4,\ 5,\ 6\} & \text{if } \ell = 3. \end{cases}$$

Further if $\beta \in \Delta(n^+)$ satisfies $(\delta-\alpha_\ell, \beta) = -2$, then $\beta = \alpha_\ell$. Since $(k\omega_\ell, \beta) = 2k$, $\beta \in \Delta(n^+)$, the conclusion follows then from Theorem 1.

3. $M = SO(2\ell)/U(\ell)$ for $\ell \geqq 5$. The Dinkin diagram of is:




, where $\alpha_\ell$◎ means $\alpha_j = \alpha_\ell$. Let $\{\varepsilon_i\}_{i=1}$ be a basis of $h_0$ such that $(\varepsilon_i, \varepsilon_j) = \delta_{ij}$. Then

$\alpha_\ell = \varepsilon_{\ell-1} + \varepsilon_\ell,$

$\omega_\ell = 1/2(\varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_\ell),$

$\delta = (\ell-1)\varepsilon_1 + (\ell-2)\varepsilon_2 + \cdots + \varepsilon_{\ell-1},$

$\Delta(n^+) = \{ \varepsilon_i + \varepsilon_j ;\ 1 \leqq i < j \leqq \ell \}.$

Hence we have

$\{(\delta-\alpha_\ell, \beta);\ \beta \in \Delta(n^+) \} = \{ -1,\ 1,\ 2,\ \cdots,\ 2\ell-3\},$

$(k\omega_\ell, \beta) = k \qquad$ for any $\beta \in \Delta(n^+)$.

Further, if $\beta \in \Delta(n^+)$ satisfies $(\delta-\alpha_\ell, \beta) = -1$, then $\beta = \alpha_\ell$. The conclusion follows then from Theorem 1.

q.e.d.

Remark. Assume that $M$ is one of the following Hermitian symmetric spacesof compact type. Then the group $H^q(M, \Omega^p(E_{-k\omega_j}))$ $= 0$ except for the following cases:

$Sp(2)/U(2)$ (i) $q = 0$ and $k > 1$, (ii) $q = 1$ and $k = 0$, (iii) $q = 2$ and $k = -1$, (iv) $q = 3$ and $k < -2$,

$SO(6)/U(3)$ (i) $q = 0$ and $k > 1$, (ii) $q = 1$ and $k = 0$, (iii) $q = 3$ and $k < -2$,

$SO(8)/U(4)$ (i) $q = 0$ and $k > 1$, (ii) $q = 1$ and $k = 0$, (iii) $q = 5$ and $k = -4$, (iv) $q = 6$ and $k < -5$.

## 3 Hypersurfaces of Hermitian symmetric spaces of compact type.

We retain the notations and assumptions introduced in the previous sections.

Let $V$ be a hypersurface, that is, closed codimension 1 complex submanifold in a Kähler C-space $M$. Taking a sufficiently fine finite covering $\{U_j\}$ of $V$, $V$ is defined in each $U_j$ by a holomorphic equation $s_j = 0$. We associate with $V$ the complex line bumdle $\{V\}$ over $M$ determined by the system $\{s_{jk}\}$ of non-vanising holomorphic functions $s_{jk} = s_j/s_k$ on $U_j \cap U_k$. There is an integer $d$ such that $\{V\} = E_{-d\omega_j}$. Since $\{V\}$ has a holomorphic section, $d > 0$. We call $d$ the degree of $V$. If $M = P_n(C)$, this definition coinsides with the usual definition of degree. We denote by $\Theta$ ( resp. $\Omega$ ) the sheaf of germs of holomorphic vector fields ( resp. holomorphic functions ) on $V$. we spall compute the dimensions of $H^q(V, \Theta)$ and $H^q(V, \Omega)$.

By Serre's duality theorem, we have

$$\dim H^0(\, V,\ \Theta\,) = \dim H^n(\, V,\ \Omega^1(K_V)\,).$$

Denote by $E|_V$ the restriction to $V$ of a holomorphic vector bundle over $M$. Since $K_V = (K_M \otimes \{V\})|_V$, we have

$$\dim H^0(\, V,\ \Theta\,) = \dim H^n(\, V,\ \Omega^1(E_{-(d-\lambda)\omega_j}|_V)\,). \tag{3.1}$$

Let us recall the following vanishing theorem of Akizuki-Nakano [1]. If $L$ is a ~~positive~~ holomorphic line bundle over a compact complex manifold $N$. Then we have

$$H^q(\, N,\ \Omega^p(L)\,) = 0 \qquad \text{for} \quad p+q \geqq n+1, \quad \text{if} \quad L \text{ is positive.} \tag{3.2}$$

Therefore we get

$$H^0(\, V,\ \Theta\,) = 0 \qquad\qquad \text{if} \quad d > \lambda,$$

by (3.1).

Theorem 8. Let $M$ be an irreducible Hermitian symmetric space of compact type BD I, E III or E VII, and let $V$ be a hypersurface of $M$ whose degree is $d$. Then we have

$$H^0(\, V,\ \Theta\,) = 0 \qquad\qquad \text{if} \quad d \geqq 2.$$

The following lemma follows from Theorems 3,4 and 5.

Lemma 3. Let $M$ be an n-dimensional irreducible Hermitian symmetric space of compact type BD I, E III or E VII. Then we have

$$H^q(\, M,\ \Omega^p(E_{-k\omega_j})\,) = 0, \quad H^{q+1}(\, M,\ \Omega^p(E_{-(k-d)\omega_j})\,) = 0$$

for $p + q = n + 2$, $k = pd - \lambda$ if $2 \leqq p \leqq n$ and $d \geqq 2$.

Proof of Theorem 8. Recall the pair of exact sequences ( Kodaira and Spencer [6] )

$$\cdots \longrightarrow H^{q-1}(\ V,\ \Omega^{p}(E_{-k\omega_j}|_V)\ ) \longrightarrow H^{q}(\ M,\ \Omega''^{p}(E_{-k\omega_j})\ ) \longrightarrow$$

$$H^{q}(\ M,\ \Omega^{p}(E_{-k\omega_j})\ ) \longrightarrow \cdots,$$

$$\cdots \longrightarrow H^{q}(\ M,\ \Omega''^{p}(E_{-k\omega_j})\ ) \longrightarrow H^{q}(\ V,\ \Omega^{p-1}(E_{-(k-d)\omega_j}|_V)\ ) \longrightarrow$$

$$H^{q+1}(\ M,\ \Omega^{p}(E_{-(k-d)\omega_j})\ ) \longrightarrow \cdots$$

, where $\Omega''^{p}(L)$ is the kernel of the canonical map of $\Omega^{p}(L)$ onto $\Omega^{p}(L|_V)$ for a holomorphic line bundle $L$ over $M$.

We see from the above pair of exact sequences and Lemma 3 that

$$H^{\overset{n}{\chi}-p-1}(\ V,\ \Omega^{p}(E_{-(pd-\lambda)\omega_j}|_V)\ ) \longrightarrow H^{\overset{n}{\chi}-p+2}(\ M,\ \Omega''^{p}(E_{-(pd-\lambda)\omega_j})) \longrightarrow 0,$$

$$H^{\overset{n}{\chi}-p+2}(\ M,\ \Omega''^{p}(E_{-(pd-\lambda)\omega_j})) \longrightarrow H^{\overset{n}{\chi}-p+2}(\ V,\ \Omega^{p-1}(E_{-((p-1)d-\lambda)\omega_j}|_V)) \longrightarrow$$

$0.$

Thus $H^{\overset{n}{\chi}-p+1}(\ V,\ \Omega^{p}(E_{-(pd-\lambda)\omega_j}|_V)\ ) = 0$ implies

$H^{\overset{n}{\chi}-p+2}(\ V,\ \Omega^{p-1}(E_{-((p-1)d-\lambda)\omega_j}|_V)\ ) = 0$, while we have

$H^{1}(\ V, \Omega^{n}(E_{-(nd-\lambda)\omega_j}|_V)\ ) = 0$ by (3.2). Hence we obtain

$H^{n}(\ V,\ \Omega^{1}(E_{-(d-\lambda)\omega_j}|_V)\ ) = 0.$

q.e.d.

Remark. The above proof is motivated by Kodaira and Spencer [5].

Let $N$ be a complex manifold and let $W \longrightarrow N$ be a holomorphic vector bundle over $N$. Assume that $V$ is a hypersurface of $N$.

We denote by $\hat{\Omega}(W|_V)$ the trivial extension of $\Omega(W|_V)$ to N. Then we have the following exact sequence ( Kodoira and Spencer [6] )

(3.3) $0 \longrightarrow \Omega(W\times\{V\}^{-1}) \longrightarrow \Omega(W) \longrightarrow \hat{\Omega}(W|_V) \longrightarrow 0$ .

Assume that V is a hypersurface of M with degree d. It is easy to see that the normal bundle of V is equivaliant to $\{V\}|_V$. Hence, by Kimura [4], the nullity of V as a minimal submanifold of M is given as follows:

(3.4) $$n(V) = \dim_R H^0(\ V,\ \Omega(\{V\}|_V)\ ).$$

Denote by C the trivial line bundle over M. Then, by (3.3), we have the exact sequence:

$$0 \longrightarrow \Omega(C) \longrightarrow \Omega(\{V\}) \longrightarrow \hat{\Omega}(\{V\}|_V) \longrightarrow 0.$$

Since $H^1(\ M,\ \Omega(C)\ ) = 0$,

$$\dim H^0(\ V,\ \Omega(\{V\}|_V)\ ) = \dim H^0(\ M,\ \Omega(\{V\})\ ) - 1.$$

Since $\{V\} = E_{-d\omega_j}$, we get

$$\dim H^0(\ M,\ \Omega(\{V\})\ ) = \dim V_{-d\omega_j}$$

by the theorem of Bott. Therefore,

(3.5) $$\dim H^0(\ V, \Omega(\{V\}|_V)\ ) = \dim V_{-d\omega_j} - 1,$$

and by (3.4)

$$n(V) = 2(\dim V_{-d\omega_j} - 1).$$

We prove the following lemma.

Lemma 4. Let $M$ be an irreducible Hermitian symmetric space of compact type of dimension > 3. Assume that $M$ is not $P_n(C)$, $Sp(2)/U(2)$, $SO(6)/U(3)$ or $SO(8)/U(4)$. Then for a hypersurface $V$ of $M$, we have

$$\dim H^0(\ V,\ \Omega(T(M)|_V)\ ) = \dim H^0(\ M,\ \Omega T(M)\ ),$$

$$H^1(\ V,\ \Omega(T(M)|_V)\ ) = 0.$$

Proof. We have the exact sequence:

$$\cdots \longrightarrow H^q(\ M,\ \Omega(T(M)\otimes E_{d\omega_j})\ ) \longrightarrow H^q(\ M,\ \Omega T(M)\ )$$

$$\longrightarrow H^q(\ V,\ \Omega(T(M)|_V)\ ) \longrightarrow \cdots$$

by (3.3). On the other hand, by Serre's duality theorem

$$\dim H^j(\ M,\ \Omega(T(M)\otimes E_{d\omega_j})\ ) = \dim H^{n-j}(\ M,\ \Omega^1(E_{-(d-\lambda)\omega_j})\ ).$$

Hence, since $H^j(\ M,\ \Omega T(M)\ ) = 0$, $j = 1, 2$, the lemma follows from Theorem 7.

From this lemma we get the following.

Theorem 9. Let $M$ be an irreducible Hermitian symmetric space of compact type of dimension > 3. Assume that $M$ is not $P_n(C)$, $Sp(2)/U(2)$, $SO(6)/U(3)$ or $SO(8)/U(4)$. Then for a hypersurface $V$ of $M$, we have

$$\dim H^1(\ V,\ \Theta\ ) = \dim H^0(\ V,\ \{V\}|_V\ ) + \dim H^0(\ V,\ \Theta\ )$$

$$-\dim H^0(\ M,\ \Omega T(M)\ ).$$

By Theorems 8, 9 and (3.5), we obtain the following.

Theorem 10. Let $M$ be an irreducible Hermitian symmetric space of compact type: BD I, E III or E VII, and let $V$ be a hypersurface of $M$. Assume that $\dim M > 3$ and the degree of $V \geqq 2$. Then we have

$$\dim H^1(V, \Theta) = \dim V_{-d\omega_j} - \dim H^0(M, \Omega T(M)) - 1.$$

Finally the following theorem follows from Theorem 6.

Theorem 11. Let $M$ be an n-dimensional irreducible Hermitian symmetric space of compact type, and let $V$ be a hypersurface of $M$ with degree $d$. Then the group $H^q(V, \Omega)$ vanishes except for the following cases:

$$q = 0 \text{ or } n-1 \qquad \text{if } d \geqq \lambda,$$

$$q = 0 \qquad \text{if } d < \lambda.$$

Proof. By Serre's duality theorem we have

$$\text{(3.7)} \quad \dim H^q(V, \Omega) = \dim H^{n-1-q}(V, \Omega(E_{-(d-\lambda)\omega_j}|_V))$$

for $q = 0, \cdots, n-1$. On the other hand, by applying (3.3), we obtain the exact sequence:

$$\text{(3.8)} \quad \cdots \longrightarrow H^j(M, \Omega(E_{\lambda\omega_j})) \longrightarrow H^j(M, \Omega(E_{-(d-\lambda)\omega_j}))$$

$$\longrightarrow H^j(V, \Omega(E_{-(d-\lambda)\omega_j}|_V)) \longrightarrow \cdots.$$

It follows from Theorem 6 that:

$$H^q(M, \Omega(E_{\lambda\omega_j})) = 0, \qquad \text{for } q = 0, 1, \cdots, n-1,$$

$$H^n(M, \Omega(E_{\lambda\omega_j})) \neq 0,$$

$$H^q( M, \Omega(E_{-(d-\lambda)\omega_j}) ) = 0, \quad \text{for any } q, \text{ if } d < \lambda,$$

$$H^q( M, \Omega(E_{-(d-\lambda)\omega_j}) ) = 0, \quad \text{for } q > 0, \text{ if } d \geqq \lambda,$$

$$H^0( M, \Omega(E_{-(d-\lambda)\omega_j}) ) \neq 0, \quad \text{if } d \geqq \lambda.$$

Hence the theorem is obtained by (3.7) and (3.8).

Osaka University

References

[1] Y. Akizuki and S. Nakano: Note on Kodaira-Spencer's proof of Lefschetz theorems, Proc. of the Jap. Acad., 30(1954), 226-272.

[2] A. Borel and F. Hirzebruch: Characteristic classes and homogeneous spaces, I., Am. J. Math., 80(1958), 458-538.

[3] R. Bott: Homogeneous vector bundles, Ann. of Math., 66(1957), 203-248.

[4] Y. Kimura: The nullity of compact Kahler submanifolds in a complex projective space, J. Math. Soc. Japan, 29(1977) 561-580.

[5] K. Kodaira and D. C. Spencer: On deformations of complex analytic structures, I-II., Ann. of Math., 67(1958), 328 - 466.

[6] K. Kodaira and D.C. Spencer: On a theorem of Lefschez and the Lemma of Enriques-Severi-Zariski, Proc. Nat. Acad. Sci. U.S.A., 39(1953), 1273-1278.

[7] B. Kostant: Lie algebra cohomology and the generalized Borel-Weil theorem, Ann. of Math., 74(1961), 329-387.

[8] S. Murakami: Sur certains espaces fibres principaux differentiables et holomorphes, Nagoya Math. J., 15(1959), 171-199.

[9] H. C. Wang: Closed manifolds with homogeneous complex structure, Amer. J. Math., 76(1954), 1-32.

# ON THE HYPERSURFACES OF HERMITIAN SYMMETRIC SPACES OF COMPACT TYPE II

YOSHIO KIMURA

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## 1. Introduction.

Let $M$ be an irreducible Hermitian symmetric space of compact type and let $L$ be a holomorphic line bundle over $M$. Denote by $\Omega^p(L)$ the sheaf of germs of $L$-valued holomorphic p-forms on $M$. In the previous paper [1] we have studied the cohomology groups $H^q(M, \Omega^p(L))$ of $M$ if $M$ is of type BDI, EIII or EVII. This note is the continuation of [1], and we retain the notations introduced in [1]. In this note we study the cohomology groups $H^q(M, \Omega^p(L))$ of $M$ of type AIII, CI or DIII and show the following theorem.

**Theorem.** Let イタ $M$ be an irreducible Hermitian symmetric space of compact type but not a complex projective space or a complex quadric of even dimension. Let $V$ be a hypersurface of $M$ whose degree $\geq 2$. Then

$$H^0(V, \Theta) = (0)$$

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where $\Theta$ is the sheaf of germs of holomorphic vector fields on $V$.

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## 2. Proof of the theorem.

Theorem 8 and Lemma 3 in the previous paper [1] is incorrect. The followings are true.

**Theorem 8.** Let $M$ be an irreducible Hermitian symmetric space

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of type EIII, EVII or a complex quadoric of odd dimension ( resp. a complex quadoric of even dimension ), and let $V$ be a hypersurface of $M$ whose degree is $d$. Then

$$H^0(V, \Theta) = (0) \quad \text{if } d \geqq 2 \text{ ( resp. 3 ).}$$

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**Lemma 3.** Let $M$ be an $n$-dimensional irreducible Hermitian

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symmetric space of compact type EIII, EVII or a complex quadoric of odd dimension ( resp. a complex quadoric of even dimension ). Then

$$H^q(M, \Omega^p(E_{-k\omega_j})) = (0), \quad H^{q+1}(M, \Omega^p(E_{-(k-d)\omega_j})) = (0)$$

for $p+q = n+1$, $k = pd-\lambda$ if $2 \leqq p \leqq n-1$ and $d \geqq 2$ (resp. 3 ).

From the above theorem we may assume that $M$ is of type AIII, CI or DIII but not a complex projective space or a complex quadoric. If we prove the following proposition, we get the above theorem in the same way as in the proof of Theorem 8 in [1].

**Proposition 1.** If $d \geqq 2$

$$H^q(M, \Omega^p(E_{-k\omega_j})) = (0), \quad H^{q+1}(M, \Omega^p(E_{-(k-d)\omega_j})) = (0),$$

for $p+q \geqq n+1$, $k = pd-\lambda$.

By Theorems 1 and 2 in [1], we get Proposition 1 if we prove the following inequalities:

$$\#\{\beta \in \Delta(\mathfrak{n}^+)\,;\ (\sigma\delta + (dn(\sigma) - \lambda)\omega_j, \beta) < 0\} < n+1-n(\sigma),$$
$$\#\{\beta \in \Delta(\mathfrak{n}^+)\,;\ (\sigma\delta + (dn(\sigma) - d - \lambda)\omega_j, \beta) < 0\} < n+2-n(\sigma),$$

for $\sigma \in W^1$ and $d \geqq 2$.

Since $(\omega_j, \beta) > 0$ for $\beta \in \Delta(\mathfrak{n}^+)$, we only have to prove the inequalities in the case of $d = 2$. Recall that $\#\Delta(\mathfrak{n}^+) = n$. We can restate the inequalities, in the case of d = 2, sa follows:

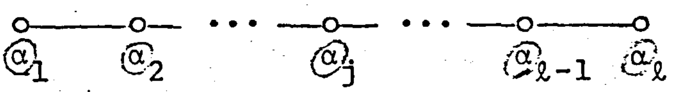
Proposition 2. For $\sigma \in W^1$

$$\#\{\beta \in \Delta(\mathfrak{n}^+)\,;\ (\sigma\delta, \beta) \geqq ((\lambda - 2n(\sigma))\omega_j, \beta)\} > n(\sigma)-1,$$

$$\#\{\beta \in \Delta(\mathfrak{n}^+)\,;\ (\sigma\delta, \beta) \geqq ((\lambda + 2 - 2n(\sigma))\omega_j, \beta)\} > n(\sigma)-2.$$

In the following we shall prove Proposition 2 in each case.

2.1. The case that $M$ is of type AIII but not a complex projective space, that is $M = SU(\ell+1)/S(U(j)\times U(\ell+1-j))$, $\ell \geqq 3$ and $2 \leqq j \leqq \ell-1$. We immedietely see that $n = j(\ell+1-j)$ and $\lambda = \ell+1$. The Dynkin diagram of $\Pi$ is as follows:




Let $\{\varepsilon_i\,;\ 1 \leqq i \leqq \ell+1\}$ be a usual basis of $R^{\ell+1}$. Then we have:

$$\mathfrak{h}_0 = \{ \sum_{i=1}^{\ell+1} a_i \varepsilon_i \in R^{\ell+1};\ \sum_{i=1}^{\ell+1} a_i = 0 \},$$

$$\Delta = \{ \varepsilon_i - \varepsilon_k;\ 1 \leqq i,\ k \leqq \ell+1,\ i \neq k \},$$

$$\Pi = \{ \alpha_1 = \varepsilon_1 - \varepsilon_2,\ \alpha_2 = \varepsilon_2 - \varepsilon_3,\ \cdots,\ \alpha_\ell = \varepsilon_\ell - \varepsilon_{\ell+1} \},$$

$$\Delta(\mathfrak{n}^+) = \{ \varepsilon_i - \varepsilon_k;\ 1 \leqq i \leqq j < k \leqq \ell + 1 \},$$

$$2\delta = \ell\varepsilon_1 + (\ell - 2)\varepsilon_2 + (\ell - 4)\varepsilon_3 + \cdots - (\ell - 2)\varepsilon_\ell - \ell\varepsilon_{\ell+1},$$

$$\omega_j = \varepsilon_1 + \cdots + \varepsilon_j - \frac{j}{\ell+1} \sum_{i=1}^{\ell+1} \varepsilon_i .$$

An element $\sigma \in W$ acts on $R^{\ell+1}$ by $\sigma\varepsilon_i = \varepsilon_{\sigma(i)}$ for $1 \leqq i \leqq \ell + 1$, where $\sigma$ in the index is a permutation of $\{ 1, 2, \cdots, \ell + 1 \}$. We represent $\sigma$ by 次頁へ続く

$$\begin{pmatrix} 1 & 2 & \cdots & \ell+1 \\ \sigma(1) & \sigma(2) & \cdots & \sigma(\ell+1) \end{pmatrix}.$$

Then

$$W^1 = \left\{ \sigma \in W;\ \sigma^{-1} = \begin{pmatrix} 1 & \cdots & \ell+1 \\ \sigma^{-1}(1) & \cdots & \sigma^{-1}(\ell+1) \end{pmatrix},\ \begin{matrix} \sigma^{-1}(1) < \cdots < \sigma^{-1}(j) \\ \sigma^{-1}(j+1) < \cdots < \sigma^{-1}(\ell+1) \end{matrix} \right\}.$$

The index $n(\sigma)$ of $\sigma \in W^1$ is given by

$$n(\sigma) = \sum_{i=1}^{j} ( \sigma^{-1}(i) - i )$$

( Takeuchi [2] ). We see easily that

$$( \omega_j, \beta ) = 1 \quad \text{for any } \beta \in \Delta(n^+),$$

$$( \sigma\delta, \varepsilon_i - \varepsilon_k ) = \sigma^{-1}(k) - \sigma^{-1}(i) \quad \text{for } 1 \leqq i, k \leqq \ell+1.$$

Therefore we have to prove that the following two inequalities are true for any $\sigma \in W^1$

(1.1) $\#\{ (i,k);\ 1 \leqq i \leqq j < k \leqq \ell+1,\ \sigma^{-1}(k) - \sigma^{-1}(i) \geqq \ell+1-2n(\sigma) \} > n(\sigma)-1,$

(1.2) $\#\{ (i,k);\ 1 \leqq i \leqq j < k < \ell+1,\ \sigma^{-1}(k) - \sigma^{-1}(i) \geqq \ell+3-2n(\sigma) \} > n(\sigma)-2.$

First we prove the inequality (1.1).

Lemma 1.1. Let $\sigma \in W^1$. If $n(\sigma) \geqq \ell+1$, the inequality (1.1) is true.

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Proof. Since $n(\sigma) \geqq \ell+1$, $\ell+1-2n(\sigma) \geqq -(\ell+1)$. There existe no pair $( i, k )$, $i \neq k$, which satisfies

$$\sigma^{-1}(k) - \sigma^{-1}(i) < -(\ell+1).$$

Therefore

$$\#\{(i,k);\ 1 \leqq i \leqq j < k \leqq \ell+1,\ \sigma^{-1}(k) - \sigma^{-1}(i) \geqq \ell+1-2n(\sigma)\} = n.$$

From the definition of the index $n(\sigma) \leq n$, it follows that $n > n(\sigma)-1$.

Q.E.D.

Lemma 1.2. Let $\sigma \in W^1$. Assume that $\sigma(1) \neq 1$ and $\sigma(\ell+1) \neq \ell+1$. Then $n(\sigma) \geqq \ell$.

Proof. By the assumption $\sigma^{-1}(j) = \ell+1$ and $\sigma^{-1}(i)-i \geqq 1$, $1 \leqq i \leqq j$. Therefore

$$\begin{aligned} n(\sigma) &= \sum_{i=1}^{j} (\sigma^{-1}(i)-i) \\ &= \sigma^{-1}(j) - j + \sum_{i=1}^{j-1} (\sigma^{-1}(i)-i) \\ &\geqq (\ell+1-j) + (j-1) \\ &= \ell. \end{aligned}$$

Q.E.D.

Lemma 1.3. Let $\sigma \in W^1$. Assume that $\sigma(1) \neq 1$ and $\sigma(\ell+1) \neq \ell+1$. Then the inequality (1.1) is true for $\sigma$.

Proof. By Lemmas 1.1 and 1.2 we assume that $n(\sigma) = \ell$. Then such an element $\sigma$ is unique and given by

$$\sigma^{-1} = \begin{pmatrix} 1 & \cdots & j-1 & j & j+1 & j+2 & \cdots & \ell+1 \\ 2 & \cdots & j & \ell+1 & 1 & j+1 & \cdots & \ell \end{pmatrix}.$$

The pair $(i, k)$, $1 \leqq i \leqq j < k \leqq \ell+1$, which satisfies

$$\sigma^{-1}(k) - \sigma^{-1}(i) < \ell + 1 - 2n(\sigma) = 1 - \ell$$

is $(j, j+1)$. Hence

$\#\{(i,k);\ 1 \leqq i \leqq j < k \leqq \ell+1,\ \sigma^{-1}(k) - \sigma^{-1}(i) \geqq 1-\ell\} = n-1 > n(\sigma)-1.$

Q.E.D.

Lemma 1.4. If $j = 2$, the inequality (1.1) is true for any $\sigma \in W^1$.

Proof. From the definition of $n(\sigma)$

$$(1.3) \qquad n(\sigma) = \sigma^{-1}(1) + \sigma^{-1}(2) - 3.$$

If $n(\sigma) = 0$, the inequation (1.1) is clearly true. Let $n(\sigma) = 1$. Then $\sigma^{-1}(1) = 1$, $\sigma^{-1}(2) = 3$ and

$$\sigma^{-1}(\ell+1) - \sigma^{-1}(1) = \ell > \ell + 1 - 2n(\sigma).$$

It follows that the inequality (1.1) is true. Let $n(\sigma) = 2$. It is easy to see that the inequality (1.1) is true.

By Lemma 1.1 we have already seen that if $n(\sigma) \geqq \ell + 1$ the inequality is true. Hence we only have to show that (1.1) is true under the following condition:

$$(1.4) \qquad 5 < \sigma^{-1}(1) + \sigma^{-1}(2) < \ell + 4.$$

By (1.3)

$$\ell + 1 - 2n(\sigma) = \ell + 7 - 2(\sigma^{-1}(1) + \sigma^{-1}(2)).$$

Since $\sigma^{-1}(k) \geqq k - 2$ for $2 < k \leqq \ell + 1$,

$$\#\{\ k;\ 2<k\leqq\ell+1,\ \sigma^{-1}(k) - \sigma^{-1}(1) \geqq \ell+7-2(\sigma^{-1}(1)+\sigma^{-1}(2))\ \}$$

$$\geqq \min\{\sigma^{-1}(1) + 2\sigma^{-1}(2) - 7,\ \ell - 1\ \}.$$

Similarly

$$\#\{\ k;\ 2<k\leqq \ell+1,\ \sigma^{-1}(k)-\sigma^{-1}(2)\geqq \ell+7-2(\sigma^{-1}(1)+\sigma^{-1}(2))\}$$
$$\geqq \min\{\ 2\sigma^{-1}(1)+\sigma^{-1}(2)-7,\ \ell-1\ \}.$$

Therefore

$$\#\{(i,k);\ 1\leqq i\leqq 2<k\leqq \ell+1,\ \sigma^{-1}(k)-\sigma^{-1}(i)\geqq \ell+1-2n(\sigma)\}$$
$$\geqq \min\{\ 3(\sigma^{-1}(1)+\sigma^{-1}(2))-14,\ \ell+2\sigma^{-1}(1)+\sigma^{-1}(2)-8,\ 2\ell-2\ \}.$$

It is easy to see that $3(\sigma^{-1}(1)+\sigma^{-1}(2))-14$, $\ell+2\sigma^{-1}(1)+\sigma^{-1}(2)-8$ and $2\ell-2$ are both larger than $n(\sigma)-1=\sigma^{-1}(1)+\sigma^{-1}(2)-4$ under the condition (1.4).

Q.E.D.

We get the following lemma in the similar way as above.

<u>Lemma 1.5.</u> <u>If $j=\ell-1$, the inequality (1.1) is true for any $\sigma\in W^1$.</u> 1.7

We shall prove that the inequality (1.1) is true for any $\sigma\in W^1$ by using induction on $\ell$. If $\ell=3$ so that $j=2$, it follows, by Lemma 1.4, our assertion is true.

Let $\ell=\ell_0\geqq 4$. We can assume that $3\leqq j=j_0\leqq \ell_0-2$ and whether $\sigma(1)=1$ or $\sigma(\ell_0+1)=\ell_0+1$ by Lemmas 1.3, 1.4 and 1.5.

Case 1: $\sigma(1)=1$. Define the element $\tau$ of $W^1$, which is considered as an element of $W^1$ for $\ell=\ell_0-1$ and $j=j_0-1$, by

$$\tau^{-1}=\begin{pmatrix} 1 & 2 & \cdots & \ell_0 \\ \sigma^{-1}(2)-1 & \sigma^{-1}(3)-1 & \cdots & \sigma^{-1}(\ell_0+1)-1 \end{pmatrix}.$$

We immedietly see that $2\leqq j\leqq \ell-2$ and $n(\tau)=n(\sigma)$. By the assumption of the induction,

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$$\#\{\ (i,k);\ 1 \leqq i \leqq j_0 - 1 < k \leqq \ell_0,\ \tau^{-1}(k) - \tau^{-1}(i) \geqq \ell_0 - 2n(\tau)\} > n(\tau) - 1.$$

Hence

$$(1.5)\quad \#\{(i,k);\ 2 \leqq i \leqq j_0 < k \leqq \ell_0 + 1,\ \sigma^{-1}(k) - \sigma^{-1}(i) \geqq \ell_0 - 2n(\sigma)\} > n(\sigma) - 1.$$

For any k, $j_0 \leqq k \leqq \ell_0 + 1$, if there existes i, $2 \leqq i \leqq j_0$, which satisfies the following:

$$\sigma^{-1}(k) - \sigma^{-1}(i) = \ell_0 - 2n(\sigma),$$

such an integer $i$ is unique and

$$\sigma^{-1}(k) - \sigma^{-1}(1) \geqq \ell_0 + 1 - 2n(\sigma).$$

Hence (1.5) leads to (1.1).

Case 2: $\sigma(\ell_0 + 1) = \ell_0 + 1$. Define the element $\tau \in W^1$, which is considered as an element of $W^1$ for $\ell = \ell_0 - 1$ and $j = j_0$, by

$$\tau^{-1} = \begin{pmatrix} 1 & \cdots & \ell_0 \\ \sigma^{-1}(1) & \cdots & \sigma^{-1}(\ell_0) \end{pmatrix}.$$

Then $n(\tau) = n(\sigma)$. By the assumption of the induction, $3 \leq j \leq \ell - 1$ and

$$(1.6)\quad \#\{\ (i,k);\ 1 \leqq i \leqq j_0 < k \leqq \ell_0,\ \sigma^{-1}(k) - \sigma^{-1}(i) \geqq \ell_0 - 2n(\sigma)\} > n(\sigma) - 2.$$

For any $i$, $1 \leqq i \leqq j_0$, if there existes $k$, $j_0 < k \leqq \ell_0$, which satisfies the following:

$$\sigma^{-1}(k) - \sigma^{-1}(i) = \ell_0 - 2n(\sigma),$$

such an integer k is unique and

$$\sigma^{-1}(\ell_0 + 1) - \sigma^{-1}(i) \geqq \ell_0 + 1 - 2n(\sigma).$$

Hence (1.5) leads to (1.1).

Thus we proved that the inequality (1.1) is true for any $\sigma \in W^1$. have

In the following we shall prove that the inequality (1.2) is true for any $\sigma \in W^1$.

Lemma 1.6. Let $\sigma \in W^1$. If $n(\sigma) \geqq \ell+1$, the inequality (1.2) is true.
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Proof. Since $n(\sigma) \geqq \ell+1$

$$\ell + 3 - 2n(\sigma) \leqq 1 - \ell .$$

If there existe a pair $(i, k)$, $i \neq k$, which satisfies

$$\sigma^{-1}(k) - \sigma^{-1}(i) < 1 - \ell ,$$

such a pair is unique. Therefore

$$\#\{ (i,k) ;\ 1 \leqq i \leqq j < k \leqq \ell+1,\ \sigma^{-1}(k) - \sigma^{-1}(i) \geqq \ell+1-2n(\sigma) \} \geqq n-1 > n(\sigma)-2.$$

Q.E.D.

Lemma 1.7. Let $\sigma \in W^1$. Assume that $\sigma(1) \neq 1$ and $\sigma(\ell+1) \neq \ell+1$. Then the inequality (1.2) is true.
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Proof. By Lemmas 1.2 and 1.6 we may assume that $n(\sigma) = \ell$. Such an element $\sigma$ is unique and represented by

$$\sigma^{-1} = \begin{pmatrix} 1 & \cdots & j-1 & j & j+1 & j+2 & \cdots & \ell+1 \\ 2 & \cdots & j & \ell+1 & 1 & j+1 & \cdots & \ell \end{pmatrix} .$$

The pairs $(i,k)$, $1 \leqq i \leqq j < k \leqq \ell+1$, which satisfy

$$\sigma^{-1}(k) - \sigma^{-1}(i) < \ell + 3 - 2n(\sigma) = 3 - \ell$$

are at most 2. Therefore

$$\#\{ (i,k);\ 1 \leqq i \leqq j < k \leqq \ell+1,\ \sigma^{-1}(k) - \sigma^{-1}(i) \geqq \ell+3-2n(\sigma) \} \geqq n - 2.$$

Since $n(\sigma) = \ell$, $n(\sigma) < n$. It follows that (1.2) is true.

Q.E.D.

Lemma 1.8. If $j = 2$, the inequality (1.2) is true for any $\sigma \in W^1$.
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Proof. It is easy to see that (1.2) is true if $n(\sigma) \leqq 3$. By Lemma 1.6 and (1.3), we only have to show that (1.2) is true under the condition:

$$(1.7) \qquad 6 < \sigma^{-1}(1) + \sigma^{-1}(2) < \ell + 4.$$

We get the following inequality in the same way as in the proof of Lemma 1.4.

$$\#\{ (i,k);\ 1 \leqq i \leqq 2 < k \leqq \ell+1,\ \sigma^{-1}(k) - \sigma^{-1}(i) \leqq \ell+3-2n(\sigma) \}$$
$$\geqq \min\{ 3(\sigma^{-1}(1)+\sigma^{-1}(2))-18,\ \ell+2\sigma^{-1}(1)+\sigma^{-1}(2)-10,\ 2\ell-2 \}.$$

It is easy to see that $3(\sigma^{-1}(1)+\sigma^{-1}(2))-18$, $\ell+2\sigma^{-1}(1)+\sigma^{-1}(2)-10$ and $2\ell-2$ are both larger than $n(\sigma)-2 = \sigma^{-1}(1)+\sigma^{-1}(2)-4$ under the condition (1.7).

Q.E.D.

We get the following lemma in the similar way as above.

Lemma 1.9. If $j = \ell - 1$, the inequality (1.2) is true for any $\sigma \in W^1$.
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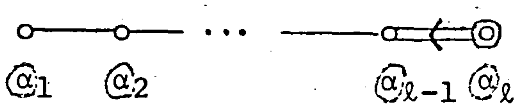
From Lemmas 1.7, 1.8 and 1.9, we can prove that the inequality (1.2) is true for any $\sigma \in W^1$ in the same way as in the proof of the inequality (1.1).

2.2. The case that $M$ is of type CI, that is $M = Sp(\ell)/U(\ell)$. If $\ell = 1$, $M = P_1(C)$. If $\ell = 2$, $M$ is a complex quadoric of dimension 3. Hence we assume that $\ell \geqq 3$. 次頁へ続く.

In this case $n = \frac{1}{2}\ell(\ell+1)$ and $\lambda = \ell + 1$. The Dynkin diagram of $\Pi$ is as follows:




where $\alpha_\ell$◎ shows ~~that~~ $\alpha_j = \alpha_\ell$. Let $\{ \varepsilon_i ;\ 1 \leqq i \leqq \ell \}$ be the basis of $\mathfrak{h}_0$ which satisfies ~~that~~ $( \varepsilon_i , \varepsilon_j ) = \delta_{ij}$. Then we have:

$$\Delta = \{ \pm 2\varepsilon_i ;\ 1 \leqq i \leqq \ell,\ \pm \varepsilon_i \pm \varepsilon_j ;\ 1 \leqq i < j \leqq \ell \},$$

$$\Pi = \{ \alpha_1 = \varepsilon_1 - \varepsilon_2 ,\ \cdots ,\ \alpha_{\ell-1} = \varepsilon_{\ell-1} - \varepsilon_\ell ,\ \alpha_\ell = 2\varepsilon_\ell \},$$

$$\Delta(\mathfrak{n}^+) = \{ 2\varepsilon_i ;\ 1 \leqq i \leqq \ell,\ \varepsilon_i + \varepsilon_j ;\ 1 \leqq i < j \leqq \ell \},$$

$$\delta = \ell\varepsilon_1 + (\ell-1)\varepsilon_2 + \cdots + \varepsilon_\ell ,$$

$$\omega_\ell = \varepsilon_1 + \cdots + \varepsilon_\ell .$$

An element $\sigma \in W$ acts on $\mathfrak{h}_0$ by $\sigma\varepsilon_i = \pm \varepsilon_{\bar{\sigma}(i)}$ for $1 \leqq i \leqq \ell$, where $\bar{\sigma}$ in the index is a permutation of $\{ 1, 2, \cdots , \ell \}$. We denote the element $\sigma \in W$ by the symbol

$$\begin{pmatrix} 1 & 2 & \cdots & \ell \\ \pm\bar{\sigma}(1) & \pm\bar{\sigma}(2) & \cdots & \pm\bar{\sigma}(\ell) \end{pmatrix}$$

Then

$$W^1 = \left\{ \sigma \in W;\ \bar{\sigma}^{-1} = \begin{pmatrix} 1 & \cdots & r & r+1 & \cdots & \ell \\ \bar{\sigma}^{-1}(1) & \cdots & \bar{\sigma}^{-1}(r) & -\bar{\sigma}^{-1}(r+1) & \cdots & -\bar{\sigma}^{-1}(\ell) \end{pmatrix} \right.$$

$$\left. \text{for} \quad 0 \leqq r \leqq \ell,\ \bar{\sigma}^{-1}(1) < \cdots < \bar{\sigma}^{-1}(r),\ \bar{\sigma}^{-1}(r+1) > \cdots > \bar{\sigma}^{-1}(\ell) \right\}.$$

The index $n(\sigma)$ of $\sigma \in W^1$ is given by

$$n(\sigma) = \sum_{i=1}^{r} (\sigma^{-1}(i) - i) + {}_{\ell+1-r}C_2$$

( Takeuchi [2] ). We see easily that

$$(\omega_\ell, \beta) = 2 \qquad \text{for any } \beta \in \Delta(n^+),$$

$$(\sigma\delta, \varepsilon_i) = \begin{cases} (\ \ell + 1 - \hat{\sigma}^{-1}(i)\ ) & \text{if } 1 \leqq i \leqq r \\ -(\ \ell + 1 - \hat{\sigma}^{-1}(i)\ ) & \text{if } r < i \leqq \ell. \end{cases}$$

Therefore we have to prove that the following inequalities are true for any $\sigma \in W^1$

(2.1) $\#\{\ \beta \in \Delta(n^+) ; (\sigma\delta, \beta) \geqq 2(\ell+1) - 4n(\sigma)\ \} > n(\sigma) - 1,$

(2.2) $\#\{\ \beta \in \Delta(n^+) ; (\sigma\delta, \beta) \geqq 2(\ell+3) - 4n(\sigma)\ \} > n(\sigma) - 2.$

Since $(\sigma\delta, \beta) \geqq -2\ell$, $\beta \in \Delta(n^+)$, we immedietely see that if $n(\sigma) \geqq \ell + 1$ ( resp. $\ell + 2$ ), the inequality (2.1) (resp.(2.2)) is true for any $\sigma \in W^1$.

Lemma 2.1. *Let $\sigma \in W^1$. If $n(\sigma) \geqq \ell$, the inequality (2.1)*
イタ.
*is true.*

Proof. From the above notice we can assume that $n(\sigma) = \ell$. In this case

$$2(\ell+1) - 4n(\sigma) = 2 - 2\ell.$$

It is easy to see that

$$\#\{\ \beta \in \Delta(n^+) ;\ (\sigma\delta, \beta) < 2 - 2\ell\ \} \leqq 2.$$

Hence

$$\#\{\beta \in \Delta(\mathfrak{n}^+);\ (\sigma\delta, \beta) \geqq 2 - 2\ell\} \leqq {}_{\ell+1}C_2 - 2 > \ell - 1 = n(\sigma) - 1.$$

Q.E.D.

Lemma 2.2. Let $\sigma \in W^1$. If $n(\sigma) \geq \ell$, the inequality (2.2) is true.

Proof. If $n(\sigma) \geqq \ell + 1$, the inequality is ture in the same way as above. Therefore we may assume that $n(\sigma) = \ell$.

Case 1 : $\ell = 3$. If $r = 0$, $n(\sigma) = 6 \neq 3$. Hence $r > 0$, and $\sigma$ is one of the following elements:

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & -3 & -2 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & -1 \end{pmatrix}.$$

In each case (2.2) is true.

Case 2 : $\ell = 4$. If $r \leqq 1$, $n(\sigma) \geqq 6 > 4$. Hence $r \geqq 2$

It follows that $(\sigma\delta, 2\varepsilon_1)$, $(\sigma\delta, \varepsilon_1 + \varepsilon_2)$ and $(\sigma\delta, 2\varepsilon_2)$ are larger than $2(\ell+3) - 4n(\sigma) = -2$.

On the other hand $n(\sigma) - 2 = 2$. Therefore (2.2) is true.

Case 3 : $\ell \geqq 5$. If $\beta \in \Delta(\mathfrak{n}^+)$ satisfies

$$(\sigma\delta, \beta) < 2(\ell+3) - 4n(\sigma) = 6 - 2\ell,$$

$\beta$ is one of the following 12 elements:

$$2\varepsilon_\ell,\ \varepsilon_\ell + \varepsilon_{\ell-1},\ \varepsilon_\ell + \varepsilon_{\ell-2},\ \varepsilon_\ell + \varepsilon_{\ell-3},\ \varepsilon_\ell + \varepsilon_{\ell-4},\ \varepsilon_\ell + \varepsilon_{\ell-5},$$

$$2\varepsilon_{\ell-1},\ \varepsilon_{\ell-1} + \varepsilon_{\ell-2},\ \varepsilon_{\ell-1} + \varepsilon_{\ell-3},\ \varepsilon_{\ell-1} + \varepsilon_{\ell-4},\ 2\varepsilon_{\ell-2},\ \varepsilon_{\ell-2} + \varepsilon_{\ell-3}.$$

On the other hand

$$
\begin{aligned}
&{}_{\ell+1}C_2 - 12\ (\ell-2) \\
&= \tfrac{1}{2}\{\ell(\ell+1) - 20 - 2\ell\} \\
&= \tfrac{1}{2}(\ell^2 - \ell - 20) \\
&= \tfrac{1}{2}(\ell+4)(\ell-5) \geqq 0.
\end{aligned}
$$

The equality holds only in the case $\ell = 5$. But if $\ell = 5$, $\beta \neq \varepsilon_\ell + \varepsilon_{\ell-5}$ for $\beta \in \Delta(\mathfrak{n}^+)$. Therefore the inequality is true.

Q.E.D.

Lemma 2.3. Let $\sigma \in W$. If $\sigma(1) \neq 1$, $n(\sigma) \geqq \ell$.

Proof. By the assumption,

$$\boxed{\sum_{i=1}^{r}} (\sigma(i) - i) \geqq r.$$

Hence

$$
\begin{aligned}
&n(\sigma) - \ell \\
&\geqq r + {}_{\ell+1-r}C_2 - \ell \\
&= \tfrac{1}{2}(\ell-r-1)(\ell-r) \geqq 0.
\end{aligned}
$$

Q.E.D.

We shall prove that the inequality (2.1) is true for any $\sigma \in W^1$ by using induction on $\ell$. Let $\ell = 3$. If $n(\sigma) \geqq 3$. the inequatlity is true by Lemma 2.1. If $n(\sigma) = 0$, the inequality is also true for $n(\sigma) - 1 < 0$. If $n(\sigma) = 1$ (resp.2), $\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & -3 \end{pmatrix}$ $\left(\text{resp. } \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & -2 \end{pmatrix}\right)$, and (2.1) is true.

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Let $\ell = \ell_0 > 3$. By Lemmas 2.1 and 2.3, we may assume that $\sigma(1) = 1$. Define the element $\tau \in W^1$, which is considered for $\ell = \ell_0 - 1$, by

$$\tau^{-1} = \begin{pmatrix} 1 & \cdots & r-1 & r & \cdots & \ell_0-1 \\ \hat{\sigma}^{-1}(2)-1 & \cdots & \hat{\sigma}^{-1}(r)-1 & -(\hat{\sigma}^{-1}(r+1)-1) & \cdots & -(\hat{\sigma}^{-1}(\ell_0)-1) \end{pmatrix}.$$

We easily see that $n(\tau) = n(\hat{\sigma})$. By the assumption of the induction,
$\#\{\varepsilon_i + \varepsilon_j;\ 1 \leqq i,j \leqq \ell_0 - 1,\ (\tau\delta', \varepsilon_i + \varepsilon_j) \geqq 2 - 4n(\tau)\} > n(\tau) - 1$,
where $\delta' = (\ell_0-1)\varepsilon_1 + (\ell_0-2)\varepsilon_2 + \cdots + \varepsilon_{\ell_0-1}$. It follows, by the fact that $(\tau\delta', \varepsilon_{i-1}) = (\sigma\delta, \varepsilon_i)$ for $2 \leqq i \leqq \ell_0$, that

(2.3) $\#\{\varepsilon_i + \varepsilon_j;\ 2 \leqq i,j \leqq \ell_0,\ (\sigma\delta, \varepsilon_i + \varepsilon_j) \geqq 2\ell - 4n(\hat{\sigma})\} > n(\hat{\sigma}) - 1.$

**Lemma 2.4.** Let

$s = \#\{\varepsilon_i;\ 2 \leqq i \leqq \ell_0,\ \exists\varepsilon_j,\ 2 \leqq j \leqq \ell_0,\ j \neq i$, such that
$(\sigma\delta, \varepsilon_i + \varepsilon_j) = 2\ell - 4n(\hat{\sigma})$ or $2\ell + 1 - 4n(\hat{\sigma})\}$.

Then

$\#\{\varepsilon_i + \varepsilon_j;\ 2 \leqq i < j \leqq \ell_0,\ (\sigma\delta, \varepsilon_i + \varepsilon_j) = 2\ell - 4n(\hat{\sigma})$ or
$2\ell + 1 - 4n(\hat{\sigma})\} \leqq s - 1.$

**Proof.** Let $\varepsilon_i$, $2 \leqq i \leqq \ell_0$, satisfy the condition that there exists $\varepsilon_j$, $2 \leqq j \leqq \ell_0$, $j \neq i$, such that $(\sigma\delta, \varepsilon_i + \varepsilon_j)$ $= 2\ell - n(\hat{\sigma})$ or $2\ell + 1 - n(\hat{\sigma})$. For the element $\varepsilon_i$,

(2.4) $\#\{\varepsilon_i + \varepsilon_j;\ 2 \leqq j \leqq \ell_0,\ j \neq i,\ (\sigma\delta, \varepsilon_i + \varepsilon_j) = 2\ell - 4n(\hat{\sigma})$
or $2\ell + 1 - 4n(\hat{\sigma})\} \leqq 2.$

In this way we find at most $2s$ ordered pairs $(i, j)$, $2 \leqq i \leqq \ell_0$, $j \neq i$, which satisfies $(\sigma\delta, \varepsilon_i + \varepsilon_j) = 2\ell - 4n(\hat{\sigma})$ or $2\ell + 1 - 4n(\hat{\sigma})$. On the other hand the distinct pairs $(i, j)$ and $(j, i)$ induce the same element $\varepsilon_i + \varepsilon_j$. Therefore

(2.5) $\#\{ \varepsilon_i + \varepsilon_j ;\ 2 \leqq i < j \leqq \ell_0,\ (\sigma\delta, \varepsilon_i + \varepsilon_j) = 2\ell - 4n(\sigma)$ or $2\ell + 1 - 4n(\sigma) \} \leqq s$,

and the equality holds if and only if the equality in (2.4) holds for any $\varepsilon_i$, $2 \leqq i \leqq \ell_0$.

Define the integer $i_0$ ( resp. $i_m$) by

$$\min \text{ ( resp. } \max \text{ ) } \{ i;\ 2 \leqq i \leqq \ell_0,\ \exists j,\ 2 \leqq j \leqq \ell_0,\ j \neq i \text{ such that } (\sigma\delta, \varepsilon_i + \varepsilon_j) = \ell - 2n(\sigma) \text{ or } \ell + 1 - 2n(\sigma) \},$$

If the equality in (2.4) holds for $\varepsilon_{i_0}$ and $\varepsilon_{i_m}$, there exist e the integers $i$ and $j$ such that

$$(\sigma\delta, \varepsilon_{i_0} + \varepsilon_j) = \ell - 2n(\sigma) \text{ or } \ell + 1 - 2n(\sigma),$$

$$(\sigma\delta, \varepsilon_i + \varepsilon_{i_m}) = \ell - 2n(\sigma) \text{ or } \ell + 1 - 2n(\sigma),$$

$$i_0 < i \text{ and } j < i_m.$$

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Hence

$$(\sigma\delta, \varepsilon_i + \varepsilon_{i_m}) \leqq (\sigma\delta, \varepsilon_{i_0} + \varepsilon_j) - 2.$$

This is impossible, and therefore the equality does not hold.

Q.E.D.

Let $\varepsilon_i$, $2 \leqq i \leqq \ell_0$, satisfy that there exists $\varepsilon_j$, $2 \leqq j \leqq \ell_0$, $j \neq i$, such that

$$(\sigma\delta, \varepsilon_i + \varepsilon_j) = 2\ell - 4n(\sigma) \text{ or } 2\ell + 1 - 4n(\sigma).$$

For this element $\varepsilon_i$,

$$(\sigma\delta, \varepsilon_i + \varepsilon_1) \geqq 2\ell + 2 - 4n(\sigma),$$

in all but the following case:

$$(\sigma\delta, \varepsilon_i + \varepsilon_2) = 2\ell - 4n(\sigma).$$

Therefore

$$\#\{\varepsilon_i + \varepsilon_j;\ 1 \leqq i < j \leqq \ell_0,\ (\sigma\delta, \varepsilon_i + \varepsilon_j) \leqq 2\ell + 2 - 4n(\sigma)\}$$

$$\geqq \#\{\varepsilon_i + \varepsilon_j;\ 2 \leqq i < j \leqq \ell_0,\ (\sigma\delta, \varepsilon_i + \varepsilon_j) \leqq 2\ell - 4n(\sigma)\}.$$

There exists at most one element $\varepsilon_i$, $2 \leqq i \leqq \ell_0$, such that $(\sigma\delta, 2\varepsilon_i) = 2\ell - 4n(\sigma)$ or $2\ell + 1 - 4n(\sigma)$.

If such $\varepsilon_i$ exists,

$$(\sigma\delta, 2\varepsilon_1) \geqq 2\ell + 2 - 4n(\sigma).$$

Therefore the inequality (2.1) is true.

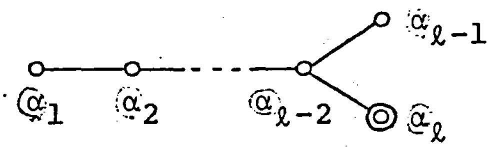
Thus we have proved that the inequality (2.1) is true for any $\sigma \in W^1$.

From Lemmas 2.2 and 2.3, we can prove that the inequality (2.2) is true for any $\sigma \in W^1$ in the same way as above.

If $\ell = 4$, M is a complex quadoric of dimension 6.

2.3. The case that M is of type DIII, that is $M = SO(2\ell)/U(\ell)$

If $\ell = 3$, $M = P_3(\mathbb{C})$. Hence we assume that $\ell \geqq 5$. In this case $n = \frac{1}{2}\ell(\ell-1)$ and $\lambda = 2\ell - 2$. The Dinkin diagram of $\Pi$ is as follows:



where $\alpha_\ell$ ◎ shows ~~that~~ $\alpha_j = \alpha_\ell$. Let $\{ \varepsilon_i ;\ 1 \leqq i \leqq \ell \}$ be the basis of $\mathfrak{h}_0$ which satisfies that $(\varepsilon_i, \varepsilon_j) = \delta_{ij}$. Then we have:

$$\Delta = \{ \pm\varepsilon_i \pm \varepsilon_j ;\ 1 \leqq i < j \leqq \ell \},$$

$$\Pi = \{ \alpha_1 = \varepsilon_1 - \varepsilon_2, \cdots, \alpha_{\ell-1} = \varepsilon_{\ell-1} - \varepsilon_\ell, \alpha_\ell = \varepsilon_{\ell-1} + \varepsilon_\ell \},$$

$$\Delta(\mathfrak{n}^+) = \{ \varepsilon_i + \varepsilon_j ;\ 1 \leqq i < j \leqq \ell \},$$

$$\delta = (\ell-1)\varepsilon_1 + (\ell-2)\varepsilon_2 + \cdots + \varepsilon_{\ell-1},$$

$$\omega = \tfrac{1}{2}(\varepsilon_1 + \cdots + \varepsilon_\ell).$$

An element $\sigma \in W$ acts on h by $\sigma\varepsilon_i = \pm\varepsilon_{\bar\sigma(i)}$ for $1 \leqq i \leqq \ell$, where $\bar\sigma$ ~~in the index~~ is a permutation of $\{ 1, 2, \cdots, \ell \}$. We denote the element $\sigma \in W$ by the symbol

$$\begin{pmatrix} 1 & 2 & \cdots & \ell \\ \pm\bar\sigma(1) & \pm\bar\sigma(2) & \cdots & \pm\bar\sigma(\ell) \end{pmatrix}.$$

Then

$$W^1 = \left\{ \sigma \in W;\ \sigma^{-1} = \begin{pmatrix} 1 & \cdots & r & r+1 & \cdots & \ell \\ \bar\sigma^{-1}(1) & \cdots & \bar\sigma^{-1}(r) & -\bar\sigma^{-1}(r+1) & \cdots & -\bar\sigma^{-1}(\ell) \end{pmatrix}, \right.$$
$$\left. \ell - r \text{ is even}, \bar\sigma^{-1}(1) < \cdots < \bar\sigma^{-1}(r), \bar\sigma^{-1}(r+1) > \cdots > \bar\sigma^{-1}(\ell) \right\}.$$

$\bar\sigma$

The index $n(\sigma)$ of $\sigma \in W^1$ is given by

$$n(\sigma) = \sum_{i=1}^{r} (\bar{\sigma}^{-1}(i) - i) + {}_{\ell-r}C_2$$

( Takeuchi [2] ). We see easily that

$$(\omega_\ell, \beta) = 1 \quad \text{for any } \beta \in \Delta(\mathfrak{n}^+),$$

$$(\sigma\delta, \varepsilon_i) = \begin{cases} \ell - \bar{\sigma}^{-1}(i) & \text{if } 1 \leqq i \leqq r \\ -(\ell - \bar{\sigma}^{-1}(i)) & \text{if } r < i \leqq \ell. \end{cases}$$

Therefore we have to prove that the following inequalities are true for any $\sigma \in W^1$.

(3.1) $\#\{\beta \in \Delta(\mathfrak{n}^+);\ (\sigma\delta, \beta) \geqq 2\ell-2-2n(\sigma)\} > n(\sigma)-1,$

(3.2) $\#\{\beta \in \Delta(\mathfrak{n}^+);\ (\sigma\delta, \beta) \geqq 2\ell-2n(\sigma)\} > n(\sigma)-2.$

Lemma 3.1. Let $\sigma \in W^1$. If $n(\sigma) \geqq 2\ell-3$, the inequality (3.1) is true.

Proof. By the assumption $2\ell-2-2n(\sigma) \leqq 4-2\ell$. Let $\beta$ be an element of $\Delta(\mathfrak{n}^+)$ which satisfies that

$$(\sigma\delta, \beta) < 4-2\ell,$$

then $\beta = \varepsilon_{\ell-1} + \varepsilon_\ell$. Therefore

$$\#\{\beta \in \Delta(\mathfrak{n}^+);\ (\sigma\delta, \beta) \geqq 2\ell-2-2n(\sigma)\} \geqq n-1.$$

If the equality holds, $n(\sigma) = 2\ell-3$ and $n - n(\sigma) = \frac{1}{2}(\ell-2)(\ell-3) > 0.$

Q.E.D.

Lemma 3.2. Let $\sigma \in W^1$. If $n(\sigma) \geqq 2\ell-3$, the inequality (3.2) is true.

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Proof. If $n(\sigma) \geqq 2\ell-2$, the inequality is true in the same way as above. Therefore we assume that $n(\sigma) = 2\ell-3$. The number of the elements $\beta \in \Delta(n^+)$ such that

$$(\sigma\delta, \beta) < 2\ell - 2n(\sigma) = 6-2\ell$$

is at most 4. Since $\ell \geqq 5$,

$$(n - 4) - (n(\sigma) - 2) = \frac{1}{2}\ell(\ell - 1) - 4 - 2\ell - 1 = \frac{1}{2}\ell(\ell - 5) + 1 > 0.$$

Q.E.D.

Lemma 3.3. If $\sigma^{-1}(1) \geqq 3$, then $n(\sigma) \geqq 2\ell-3$.

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Proof By the assumption

$$\sum_{i=1}^{r}(\sigma^{-1}(i) - i) \geqq 2r.$$

It follows that

$$\begin{aligned} & n(\sigma) - (2\ell - 3) \\ \geqq\ & 2r + {}_{\ell-r}C_2 - (2\ell - 3) \\ =\ & \frac{1}{2}(\ell - r - 2)(\ell - r - 3) \geqq 0. \end{aligned}$$

Q.E.D.

We prove that the inequality (3.1) is true for all $\sigma \in W^1$ by using induction on $\ell$. If $\ell = 5$, we easily see that the inequation is true.

Let $\ell = \ell_0 > 5$. By Lemmas 3.1 and 3.3, we can assume that $\sigma^{-1}(1) = 1$ or $2$.

Case 1 : $\bar{\sigma}^{-1}(1) = 1$. Define the element $\tau \in W^1$, which is considered for $\ell = \ell_0 - 1$, by

$$\tau^{-1} = \begin{pmatrix} 1 & \cdots & r-1 & r & \cdots & \ell-1 \\ \bar{\sigma}^{-1}(2)-1 & \cdots & \bar{\sigma}^{-1}(r)-1 & -(\bar{\sigma}^{-1}(r+1)-1) & \cdots & -(\bar{\sigma}^{-1}(\ell)-1) \end{pmatrix}.$$

Then $n(\tau) = n(\sigma)$. By the assumption of the induction,

$$\#\{ \varepsilon_i + \varepsilon_j ;\ 2 \leqq i < j \leqq \ell_0,\ (\sigma\delta, \varepsilon_i + \varepsilon_j) \geqq 2\ell - 4 - 2n(\sigma) \}$$

$$> n(\sigma) - 1.$$

Let

$$s = \#\{\varepsilon_i ;\ 2 \leqq i \leqq \ell_0,\ \exists\, \varepsilon_j,\ 2 \leqq j \neq i \leqq \ell_0, \text{ such that } (\sigma\delta, \varepsilon_i + \varepsilon_j)$$

$$= 2\ell - 4 - 2n(\sigma) \text{ or } 2\ell - 3 - 2n(\sigma) \}.$$

Then, in the same way as in Lemma 2.4,we see that

$$\#\{\varepsilon_i + \varepsilon_j ;\ 2 \leqq i < j \leqq \ell_0,\ (\sigma\delta, \varepsilon_i + \varepsilon_j) = 2\ell - 4 - 2n(\sigma) \text{ or }$$

$$2\ell - 3 - 2n(\sigma) \} \leqq s - 1.$$

Let $\varepsilon_i$ satisfy that there exists $\varepsilon_j$, $2 \leqq j \leqq \ell_0$, $j \neq i$, such that

$$(\sigma\delta, \varepsilon_i + \varepsilon_j) = 2\ell - 4 - 2n(\sigma) \text{ or } 2\ell - 3 - 2n(\sigma).$$

Then

$$(\sigma\delta, \varepsilon_i + \varepsilon_1) \geqq 2\ell - 2 - 2n(\sigma)$$

in all but the following case:

$$(\sigma\delta, \varepsilon_i + \varepsilon_2) = 2\ell - 4 - 2n(\sigma) \quad \text{and} \quad \bar{\sigma}^{-1}(2) = 2.$$

Therefore the inequality is true.

Case 2: $\bar{\sigma}^{-1}(1) = 2$. By the definition of $W^1$

$$\bar{\sigma}^{-1} = \begin{pmatrix} 1 & 2 & \cdots & r & r+1 & \cdots & \ell_0 \\ 2 & \bar{\sigma}^{-1}(2) & \cdots & \bar{\sigma}^{-1}(r) & -\bar{\sigma}^{-1}(r+1) & \cdots & -1 \end{pmatrix}.$$

Define the element $\bar{\sigma}' \in W^1$ by

$$(\bar{\sigma}')^{-1} = \begin{pmatrix} 1 & 2 & \cdots & r & r+1 & \cdots & \ell_0-1 & \ell_0 \\ 1 & \bar{\sigma}^{-1}(2) & \cdots & \bar{\sigma}^{-1}(r) & -\bar{\sigma}^{-1}(r+1) & \cdots & -\bar{\sigma}^{-1}(\ell_0-1) & -2 \end{pmatrix}.$$

Then $n(\bar{\sigma}') = n(\sigma) - 1$. Define another element $\tau \in W^1$, which is considered for $\ell = \ell_0 - 1$, by

$$\tau^{-1} = \begin{pmatrix} 1 & \cdots & r & r+1 & \cdots & \ell_0-1 \\ \bar{\sigma}^{-1}(2)-1 & \cdots & \bar{\sigma}^{-1}(r)-1 & -(\bar{\sigma}^{-1}(r+1)-1) & \cdots & -1 \end{pmatrix}.$$

Then $n(\tau) = n(\sigma')$.

Assume that the inequality (3.2) is true for $\tau$. If we notice that $(\bar{\sigma}')^{-1}(2) > 2$, we get the following inequality in the same way as in case 1.

$$\#\{\beta \in \Delta(n^+);\ (\sigma'\delta, \beta) \geqq 2\ell - 2 - 2n(\sigma')\} > n(\sigma').$$

Clearly

$$(\sigma\delta, \beta) \geqq (\sigma'\delta, \beta\beta) - 2 \quad \text{for any } \beta \in \Delta(n^+).$$

Hence if $\beta \in \Delta(n^+)$ satisfies that

$$(\sigma'\delta, \beta) \geqq 2\ell - 2 - 2n(\sigma'),$$

then

$$(\sigma\delta, \beta) \geqq 2\ell - 2 - 2n(\sigma).$$

Therefore

$$\#\{\beta \in \Delta(n^+);\ (\sigma\delta, \beta) \geqq 2\ell - 2 - 2n(\sigma)\} > n(\sigma) - 1.$$

Thus we have proved that the inequality (3.1) is true for any $\sigma \in W^1$. We can prove that the inequality (3.2) is true in the same way as above.

Nagasaki Institute of Applied Science

## References

[1] Y. Kimura: On the hypersurfaces of Hermitian symmetric spaces of compact type, Osaka J. of Math., to appear.

[2] M. Takeuchi: Cell decompositions and Morse equalities on certain symmetric spaces, J. Fac. Sci. Uni. Tokyo, 12 (1965), 81-192.