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Introduction.

Let M be an irreducible Hermitian symmetric space of compact type and let L be a holomorphic line bundle over M. We denote by $\Omega^p(L)$ the sheaf of germs of L-valued holomorphic p-forms on M. In this paper we study the cohomology groups $H^q(M, \Omega^p(L))$. Further, applying the results so far obtained, we shall consider the hypersurfaces of M.

The paper devided into three parts. §1 is devoted to recalling basic notions and results which are necessary in the following. In §2, for the cases that M is an irreducible Hermitian symmetric space of compact type BDI, EIII or EVII, we obtain the theorems analogous to the following theorem of Bott [3] for $M = P_n(C)$.

Theorem. Let E be the hyperplane bundle over an n-dimensional complex projective space $P_n(C)$. Then the group $H^q(P_n(C), \Omega^p(E^k))$ vanishes except for the following cases: (i) $p = q$ and $k = 0$, (ii) $q = 0$ and $k > p$, (iii) $q = n$ and $k < p - n$, where $E^k = E \otimes \cdots \otimes E$ (k factors).

Further we shall discuss when the groups $H^q(M, \Omega^p(L))$ vanishes for any irreducible Hermitian symmetric space of compact type for $p = 0, 1$. These results are obtained by analyzing in detail structure of Lie algebras and their Weyl groups and applying the generalized Borel-Weil theorem.

Let V be a hypersurface of M. Denote by Θ (resp. Ω) the sheaf of germs of holomorphic vector fields (resp. holomorphic functions) on V. In §3 we study the cohomology groups $H^q(V, \Theta)$ and $H^q(V, \Omega)$ using the results in §2. And we find that if M is BDI, EIII or EVII, one has

$$H^0(V, \Theta) = 0$$

for the hypersurfaces V of M except for a certain special case (Theorem 8).

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§1. Preparations.

1.1. The generalized Borel-Weil theorem. In this section we recall the generalized Borel-Weil theorem in a form convenient for our purpose.

Let G be a simply connected complex semi-simple Lie group and let U be a parabolic Lie subgroup of G. Then the quotient manifold $M = G/U$ is a Kähler C-space, that is, a simply connected compact complex homogeneous manifold admitting a Kähler metric. Let g be the Lie algebra of G and h a Cartan subalgebra of g. We denote by Δ the root system of g with respect to h. We shall identify a linear form λ on h with the element H_λ of h defined by

$$\lambda(H) = (H_\lambda, H) \qquad \text{for } H \in h,$$

where $(,)$ is the Killing form of g. We fix a linear order on the real form $h_0 = \{ \alpha \in \Delta \}_R$ of h. Let Δ^+ (resp. Δ^-) be the set of all positive (resp. negative) roots. Let Π_1 be a subsystem of Π. We put

$$\Delta_1 = \{ \alpha \in \Delta;\ \alpha = \sum_{i=1}^{\ell} m_i\alpha_i,\ m_j = 0 \text{ for any } \alpha_j \notin \Pi_1 \}$$

$$\Delta(n^+) = \{ \beta \in \Delta;\ \beta = \sum_{i=1}^{\ell} m_i\alpha_i,\ m_j > 0 \text{ for some } \alpha_j \notin \Pi_1 \}$$

$$\Delta(u) = \Delta_1 \cup \Delta(n^+).$$

Define Lie subalgebras g_1, n^+ and u of g by

$$g_1 = h + \sum_{\alpha \in \Delta_1} g_\alpha$$

$$n^+ = \sum_{\beta \in \Delta(n^+)} g_\beta$$

$$u = h + \sum_{\alpha \in \Delta(u)} g_\alpha,$$

where g_α is the root space corresponding to $\alpha \in \Delta$. Then g_1 (resp. n^+) is a reductive (resp. nilpotent) subalgebra and $u = g_1 + n^+$ (semi-direct). We denote by U the connected Lie subgroup of G with Lie algebra u. Then U is a parabolic subgroup of G, and $M = G/U$ is a Kähler C-space.

We denote by D (resp. D_1) the set of dominant integral forms of g (resp. g_1). Let $\xi \in D_1$ and choose an irreducible representation ($\rho^1_{-\xi}$, $W_{-\xi}$) of g_1 with the lowest weight $-\xi$. We may extend it to a representation of u so that its restriction to n^+ is trivial, which will be denoted by ($\rho_{-\xi}$, $W_{-\xi}$). Since any irreducible representation of u is trivial on n^+, we may call ($\rho_{-\xi}$, $W_{-\xi}$) the irreducible representation of u with the lowest weight $-\xi$. Moreover there exists a representation of U which induces the representation $(\rho_{-\xi}, W_{-\xi})$ of u, and we denote it by $(\tilde{\rho}_{-\xi}, W_{-\xi})$. This representation $(\tilde{\rho}_{-\xi}, W_{-\xi})$ defines the holomorphic vector bundle $E_{-\xi}$ over M associated to the principal bundle $G \to M$ by the representation $\tilde{\rho}_{-\xi}$ of U.

For a holomorphic vector bundle E over a complex manifold, we denote by $\Omega(E)$ the sheaf of germs of local holomorphic sections of E. Let W be the Weyl group of g and Δ_1^+ the set of all positive roots of Δ_1. We define a subset W^1 of W by

$$W^1 = \{ \sigma \in W;\ \sigma^{-1}(\Delta_1^+) \subset \Delta^+ \},$$

For any set S, we denote by $\#S$ the cardinality of S. The index $n(\sigma)$ of $\sigma \in W$ is then defined by

$$n(\sigma) = \#(\sigma(\Delta^+) \cap \Delta^-).$$

We denote by δ the half of sum of all positive roots of g.

Theorem of Bott [3] (c.f. Kostant [7]). Under the notations defined above let $\xi \in D_1$. Then if $\xi+\delta$ is not regular,

$$H^j(M, \Omega E_{-\xi}) = 0 \quad \text{for all } j = 0, 1, \cdots .$$

If $\xi+\delta$ is regular, $\xi+\delta$ is expressed uniquely as $\xi+\delta = \sigma(\lambda+\delta)$, where $\lambda \in D$ and $\sigma \in W^1$, and

$$H^j(M, E_{-\xi}) = 0 \quad \text{for all } j \neq n(\sigma),$$

$$\dim H^{n(\sigma)}(M, E_{-\xi}) = \dim V_{-\lambda},$$

where $(\rho_{-\lambda}, V_{-\lambda})$ is the irreducible representation of G with the lowest weight $-\lambda$.

We prove the following lemmas to restate this theorem in a form suitable for our parpose.

Lemma 1. Let $\xi \in D_1$. If

$$(\xi+\delta, \beta) \neq 0 \quad \text{for} \quad \beta \in \Delta(n^+),$$

then $\xi+\delta$ is regular.

Proof. Let α be any root of Δ_1^+. Then we have $(\xi,\alpha) \geqq 0$ and $(\delta,\alpha) > 0$, so that $(\xi+\delta,\alpha) > 0$. Since $\Delta^+ = \Delta_1^+ \cup \Delta(n^+)$, we get

$$(\xi+\delta,\gamma) \neq 0 \quad \text{for} \quad \gamma \in \Delta^+.$$

q.e.d.

Lemma 2. Let $\xi \in D_1$. Assume that there are $\lambda \in D$ and $\sigma \in W^1$ such that $\xi+\delta = \sigma(\lambda+\delta)$. Then

$$n(\sigma) = \#\{ \beta \in \Delta(n^+);\ (\xi+\delta,\beta) < 0 \}.$$

Proof. Since $\sigma^{-1}(\Delta_1^+) \subset \Delta^+$, we have

$$n(\sigma) = \#\{ \beta \in \Delta(n^+);\ \sigma^{-1}(\beta) < 0 \}.$$

By the assumption

$$(\xi+\delta,\alpha) = (\lambda+\delta,\ \sigma^{-1}(\alpha)) \quad \text{for} \quad \alpha \in \Delta.$$

Since $\lambda+\delta$ is dominant and regular, $\sigma^{-1}(\alpha)$ is negative if and only if $(\lambda+\delta, \sigma^{-1}(\alpha))$ is negative. The conclusion now follows from these obserbations.

Theorem of Bott may be restated as follows by these lemmas.

Theorem 1. Let $\xi \in D_1$. Then if there exists a root α of $\Delta(n^+)$ such that $(\xi+\delta,\alpha) = 0$, we have

$$H^j(M, \Omega E_{-\xi}) = 0 \quad \text{for} \quad j = 0, 1, \cdots.$$

If there exists no root β of $\Delta(n^+)$ such that $(\xi+\delta,\beta) = 0$, we have

$$H^j(M, \Omega E_{-\xi}) = 0 \quad \text{for} \quad j \neq q,$$

and

$$H^q(M, \Omega E_{-\xi}) \neq 0,$$

where $q = \#\{ \beta \in \Delta(n^+);\ (\xi+\delta,\beta) < 0 \}$.

1.2. Kostant's results. We denote by $T(M)$ the holomorphic tangent bundle of M and denote by $T(M)^*$ its dual bundle. Let L be a holomorphic line bundle over M. Then it is easy to see that $\Omega^p(L)$ coincides with $\Omega(\overset{p}{\Lambda}T(M)^*\otimes L)$, where $\overset{p}{\Lambda}T(M)^*$ is p-th exterior product of $T(M)^*$. Since any holomorphic line bundle over a Kähler C-space M is associated to the principal bundle $G\longrightarrow M$ by a representation of U (Murakami [8]), we may put $L = E_{-\xi}$ for $\xi \in D_1$. It is known n^+ is invariant by the adjoint representation of U on g. Hence p-th exterior product of n^+ has a U-module structure. Since n^+ may be identified with the cotangent space of M at U, $\overset{p}{\Lambda}T(M)^*\longrightarrow M$ coincides with the holomorphic vector bundle associated to the principal bundle $G\longrightarrow M$ by the representation of U on $\overset{p}{\Lambda}n^+$.

From now on we assume $M = G/U$ is a Hermitian symmetric space of compact type. Then n^+ is abelian. For any integer $p \geqq 0$, put

$$W^1(p) = \{ \sigma \in W^1;\ n(\sigma) = p \}.$$

Kostant [7] has proved that $\overset{p}{\Lambda}n^+$ is decomposed into direct sum:

$$(1) \qquad \overset{p}{\Lambda}n^+ = \sum_{\sigma\in W^1(p)} (\Lambda n^+)_{-(\sigma\delta-\delta)} \qquad (\text{as U-module}),$$

where $(n^+)_{-(\sigma\delta-\delta)}$ denotes an irreducible U-module with the lowest weight $-(\sigma\delta-\delta)$. The following theorem follows easily from (1) and theorems of Bott [3].

Let W be a holomorphic U-module represented as follows:

$$W = W_{-\xi_1} + \cdots + W_{-\xi_\ell} \qquad \text{for} \quad \xi_i \in D_1.$$

Denote by E_W the holomorphic vector bundle over M associated to the principal bundle $G \longrightarrow M$ by the representation of U on W.

Proposition 1. Under the notations introduced above we have

$$\dim H^j(M, \Omega E_W) = \sum_{i=1}^{\ell} \dim H^j(M, \Omega E_{-\xi_i}) \qquad \text{for} \quad j = 0, 1, \cdots.$$

We recall the results of Bott [3] which are necessary to proof the above proposition.

Theorem A. Let S be a holomorphic U-module, and let V be a holomorphic G-module. If E_S is the holomorphic vector bundle over M associated to the principal bundle $G \longrightarrow M$ by the representation of U on S, then

$$\text{multiplicity of } V \text{ in } H^j(M, \Omega E_S) = \dim H^j(u, g_1, \mathrm{Hom}(V,S)) \qquad \text{for} \quad j = 0, 1, \cdots,$$

where $H^j(u, g_1, \mathrm{Hom}(V,S))$ denotes the j-th relative cohomology group of Lie algebras u, g_1, with coefficients in the u-module Hom(V,S).

For g_1-module T, T^{g_1} donotes the subspace of S annihilated by all $X \in g_1$.

Theorem B. Let F be a u-module which, considered as g_1-module, is completely reducible. Then

$$\dim H^j(u, g_1, F) = \dim H^j(n^+, F)^{g_1}.$$

Proof of Proposition 1. Let V be a holomorphic G-module. Then by Theorems A and B, we have

multiplicity of V in $H^j(M, \Omega E_W) = \dim H^j(n^+, \mathrm{Hom}(V,W))^{g_1}$

for $j = 0, 1, \cdots$. Since $W_{-\xi_i}$, $1 \leqq i \leqq \ell$, are irreducible u-modules, the restrictions to n^+ of the representations of u on $W_{-\xi_i}$ and W are both trivial. Hence

$$\begin{aligned}
&\dim H^j(n^+, \mathrm{Hom}(V,W))^{g_1} \\
&= \dim (H^j(n^+, \mathrm{Hom}(V,C)) \otimes W)^{g_1} \\
&= \sum_{i=1}^{\ell} \dim (H^j(n^+, \mathrm{Hom}(V,C)) \otimes W_{-\xi_i})^{g_1} \\
&= \sum_{i=1}^{\ell} \dim H^j(n^+, \mathrm{Hom}(V,W_{-\xi_i}))^{g_1}.
\end{aligned}$$

By Theorems A and B

$$\dim H^j(n^+, \mathrm{Hom}(V,W_{-\xi_i}))^{g_1} = \sum_{i=1}^{\ell} \text{multiplicity of } V \text{ in } H^j(M, E_{-\xi_i})$$

for $j = 0, 1, \cdots$.

q.e.d.

The following theorem follows immediately from (1) and the above proposition.

Theorem 2. Let M be a Hermitian symmetric space of compact type. Assume that $E_{-\xi}$, $\xi \in D_1$, is a line bundle over M. Then

$$\dim H^q(\,M,\ \Omega^p(E_{-\xi})\,) = \sum_{\sigma \in W^1(p)} \dim H^q(\,M,\ \Omega(E_{-(\sigma\delta-\delta+\xi)})\,)$$

for $q = 0, 1, \cdots$.

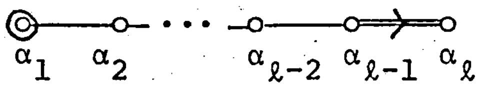
Theorem 2 shows us the importance of the study of the structure of W^1 for our porpose.

2. Vanishing of $H^q(\,M,\ \Omega^p(L)\,)$.

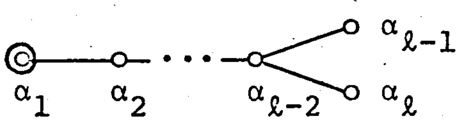
We retain the notations and assumptions introduced in the previous section.

Assume that M is an irreducible Hermitian symmetric space of compact type. Then G is simple and there exists $\alpha_j \in \Pi$ such that $\Pi_1 = \Pi - \{\,\alpha_j\,\}$. Let $\{\,\omega_1, \cdots, \omega_\ell\,\}$ be fundamental weights with respect to $\Pi = \{\,\alpha_1, \cdots, \alpha_\ell\}$. Then any holomorphic line bundle L over M is isomorphic to $E_{-k\omega_j}$ for some integer k, since any 1-dimensional representation of g_1 is induced by a representation of the centor of g_1.

2.1. The case that M is of type BD I i.e. a complex quadric Put $\dim M = n$. The Dinkin diagram of Π is as follows:



if $n = 2\ell - 1$ ($\ell \geq 2$),



if $n = 2\ell - 2$ ($\ell \geqq 3$),

where α_1 ◎ shows that $\alpha_j = \alpha_1$. Let $\{ \varepsilon_i ;\ i = 1, \cdots, \ell \}$ be a basis of h_0 which satisfies $(\varepsilon_i, \varepsilon_j) = \delta_{ij}$. Then, we have:

$$\Delta = \begin{cases} \{ \pm(\varepsilon_i \pm \varepsilon_j);\ 1 \leqq i < j \leqq \ell,\ \ \varepsilon_i;\ 1 \leqq i \leqq \ell \}, & \text{if } n = 2\ell - 1, \\ \{ \pm(\varepsilon_i \pm \varepsilon_j);\ 1 \leqq i < j \leqq \ell \}, & \text{if } n = 2\ell - 2, \end{cases}$$

$$\Pi = \begin{cases} \{ \alpha_i = \varepsilon_i - \varepsilon_{i+1};\ 1 \leqq i \leqq \ell - 1,\ \alpha_\ell = \varepsilon_\ell \}, & \text{if } n = 2\ell - 1, \\ \{ \alpha_i = \varepsilon_i - \varepsilon_{i+1};\ 1 \leqq i \leqq \ell - 1,\ \alpha_\ell = \varepsilon_{\ell-1} + \varepsilon_\ell \}, & \text{if } n = 2\ell - 2, \end{cases}$$

$$\Delta(n^+) = \begin{cases} \{ \varepsilon_1 \pm \varepsilon_j;\ 2 \leqq j \leqq \ell,\ \varepsilon_1 \}, & \text{if } n = 2\ell - 1, \\ \{ \varepsilon_1 \pm \varepsilon_j;\ 2 \leqq j \leqq \ell \}, & \text{if } n = 2\ell - 2, \end{cases}$$

$$\omega_i = \varepsilon_1,$$

$$2\delta = \begin{cases} (2\ell - 1)\varepsilon_1 + (2\ell - 3)\varepsilon_2 + \cdots + \varepsilon_\ell & \text{if } n = 2\ell - 1, \\ 2(\ell - 1)\varepsilon_1 + 2(\ell - 2)\varepsilon_2 + \cdots + 2\varepsilon_{\ell-1} & \text{if } n = 2\ell - 2. \end{cases}$$

An element $\sigma \in W$ acts in h_0 by $\sigma\varepsilon_i = \pm\varepsilon_{\sigma(i)}$ for $1 \leqq i \leqq \ell$, where σ in the index is a permutation of $\{ 1, 2, \cdots, \ell \}$. We represent this element $\sigma \in W$ by

$$\begin{pmatrix} 1 & 2 & \cdots & \ell \\ \pm\sigma(1) & \pm\sigma(2) & \cdots & \pm\sigma(\ell) \end{pmatrix}.$$

Then

$$W^1 = \begin{cases} \left\{ \sigma \in W;\ \sigma^{-1} = \begin{pmatrix} 1 & 2 & \cdots & \ell \\ s(\sigma)i & i_2 & \cdots & i_\ell \end{pmatrix},\ \begin{matrix} 0 < i_2 < \cdots < i_\ell \leqq \ell \\ s(\sigma) = \pm 1 \end{matrix} \right\}, & \text{if } n = 2\ell - 1 \\ \left\{ \sigma \in W;\ \sigma^{-1} = \begin{pmatrix} 1 & 2 & \cdots & \ell \\ s(\sigma)i & i_2 & \cdots & s(\sigma)i_\ell \end{pmatrix},\ \begin{matrix} 0 < i_2 < \cdots < i_\ell \leqq \ell \\ s(\sigma) = \pm 1 \end{matrix} \right\} & \text{if } n = 2\ell - 2. \end{cases}$$

An element $\sigma \in W^1$ is determined by i and $s(\sigma)$, and its index $n(\sigma)$ of σ is as follows:

$$n(\sigma) = \begin{cases} i - 1 & \text{if } s(\sigma) = 1, \\ n - (i - 1) & \text{if } s(\sigma) = -1. \end{cases}$$

Furthermore for $\sigma \in W^1$, the values of $(\sigma\delta, \frac{2\beta}{(\beta,\beta)})$ $(\beta \in \Delta(n^+))$ are as follows.

If $n = 2\ell - 1$,

$s(\sigma) = 1$

$\Delta(n^+)$	$(\sigma\delta, \frac{2\beta}{(\beta,\beta)})$
$\varepsilon_1 - \varepsilon_2$	$-(i - 1)$
$\varepsilon_1 - \varepsilon_3$	$-(i - 2)$
...	...
$\varepsilon_1 - \varepsilon_i$	-1
$\varepsilon_1 - \varepsilon_{i+1}$	1
$\varepsilon_1 - \varepsilon_{i+2}$	2
...	...
$\varepsilon_1 - \varepsilon_\ell$	$\ell - i$
$\varepsilon_1 + \varepsilon_\ell$	$\ell - i + 1$
$\varepsilon_1 + \varepsilon_{\ell-1}$	$\ell - i + 2$
...	...
$\varepsilon_1 + \varepsilon_{i+1}$	$2\ell - 2i$
ε_1	$2\ell - 2i + 1$
$\varepsilon_1 + \varepsilon_i$	$2\ell - 2i + 2$
...	...
$\varepsilon_1 + \varepsilon_2$	$2\ell - i$

$s(\sigma) = -1$

$\Delta(n^+)$	$(\sigma\delta, \frac{2\beta}{(\beta,\beta)})$
$\varepsilon_1 - \varepsilon_2$	$-2\ell + i$
$\varepsilon_1 - \varepsilon_3$	$-2\ell + i + 1$
...	...
$\varepsilon_1 - \varepsilon_i$	$-2\ell + 2i - 2$
ε_1	$-2\ell + 2i - 1$
$\varepsilon_1 - \varepsilon_{i+1}$	$-2\ell + 2i$
...	...
$\varepsilon_1 - \varepsilon_\ell$	$-\ell + i - 1$
$\varepsilon_1 + \varepsilon_\ell$	$-\ell + i$
$\varepsilon_1 + \varepsilon_{\ell-1}$	$-\ell + i + 1$
...	...
$\varepsilon_1 + \varepsilon_{i+1}$	-1
$\varepsilon_1 + \varepsilon_i$	1
$\varepsilon_1 + \varepsilon_{i-1}$	2
...	...
$\varepsilon_1 + \varepsilon_2$	$i - 1$

If $n = 2\ell - 2$,

$s(\sigma) = 1$

$\Delta(n^+)$	$(\sigma\delta, \frac{2\beta}{(\beta,\beta)})$
$\varepsilon_1 - \varepsilon_2$	$-(i-1)$
$\varepsilon_1 - \varepsilon_3$	$-(i-2)$
...	...
$\varepsilon_1 - \varepsilon_i$	-1
$\varepsilon_1 - \varepsilon_{i+1}$	1
$\varepsilon_1 - \varepsilon_{i+2}$	2
...	...
$\varepsilon_1 - \varepsilon_\ell$	$\ell - i$
$\varepsilon_1 + \varepsilon_\ell$	$\ell - i$
$\varepsilon_1 + \varepsilon_{\ell-1}$	$\ell - i + 1$
...	...
$\varepsilon_1 + \varepsilon_{i+1}$	$2\ell - 2i - 1$
$\varepsilon_1 + \varepsilon_i$	$2\ell - 2i + 1$
$\varepsilon_1 + \varepsilon_{i-1}$	$2\ell - 2i + 2$
...	...
$\varepsilon_1 + \varepsilon_2$	$2\ell - i - 1$

$s(\sigma) = -1$

$1 \leqq i < \ell$

$\Delta(n^+)$	$(\sigma\delta, \frac{2\beta}{(\beta,\beta)})$
$\varepsilon_1 - \varepsilon_2$	$-2\ell + i + 1$
$\varepsilon_1 - \varepsilon_3$	$-2\ell + i + 2$
...	...
$\varepsilon_1 - \varepsilon_i$	$-2\ell + 2i - 1$
$\varepsilon_1 - \varepsilon_{i+1}$	$-2\ell + 2i + 1$
$\varepsilon_1 - \varepsilon_{i+2}$	$-2\ell + 2i + 2$
...	...
$\varepsilon_1 - \varepsilon_\ell$	$-(\ell - i)$
$\varepsilon_1 + \varepsilon_\ell$	$-(\ell - i)$
$\varepsilon_1 + \varepsilon_{\ell-1}$	$-(\ell - i - 1)$
...	...
$\varepsilon_1 + \varepsilon_{i+1}$	-1
$\varepsilon_1 + \varepsilon_i$	1
$\varepsilon_1 + \varepsilon_{i-1}$	2
...	...
$\varepsilon_1 + \varepsilon_2$	$i - 1$

$i = \ell$

$\Delta(n^+)$	$(\sigma\delta, \frac{2\beta}{(\beta,\beta)})$
$\varepsilon_1 - \varepsilon_2$	$-(\ell - 1)$
$\varepsilon_1 - \varepsilon_3$	$-(\ell - 2)$
...	...
$\varepsilon_1 - \varepsilon_{\ell-1}$	-2
$\varepsilon_1 - \varepsilon_\ell$	1
$\varepsilon_1 + \varepsilon_\ell$	-1
$\varepsilon_1 + \varepsilon_{\ell-1}$	2
$\varepsilon_1 + \varepsilon_{\ell-2}$	3
...	...
$\varepsilon_1 + \varepsilon_2$	-1

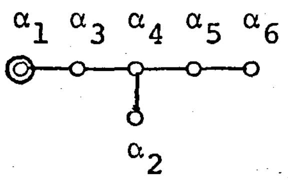
Furthermore we have

$$\left(k\omega_1, \frac{2\beta}{(\beta,\beta)}\right) = \begin{cases} k & \text{if } \beta = \varepsilon_1 \pm \varepsilon_j,\ 2 \leqq j \leqq \ell, \\ 2k & \text{if } \beta = \varepsilon_1. \end{cases}$$

Then, we obtain the following by Theorems 1 and 2,

Theorem 3. Let M be a complex quadric of dimension n, $n \geqq 3$. Then the group $H^q(M, \Omega^p(E_{-k\omega_1}))=0$ except for the following cases: (i) $q = 0$ and $k > p$, (ii) $p = q$ and $k = 0$, (iii) $p + q = n$ and $k = 2p - n$, (iv) $q = n$ and $k < p - n$.

2.2. The case M is of type EIII. The Dinkin diagram is:



, where α_1 ◎ shows that $\alpha_j = \alpha_1$ in this case. We have $\#\Delta(n^+) = 16$ and $\#W^1 = 27$. We express $\beta = \sum_{i=1}^{6} m_i\alpha_i \in \Delta(n^+)$ by $(\,m_1\ m_2\ m_3\ m_4\ m_5\ m_6\,)$. For σ of W^1, we put $\sigma\delta = (\,n_1\ n_2\ n_3\ n_4\ n_5\ n_6\,)$ if $\sigma\delta = \sum_{i=1}^{6} n_i\alpha_i$. Then we give the values $\left(\sigma\delta, \frac{2\beta}{(\beta,\beta)}\right)$ for $\sigma \in W^1$ and $\beta \in \Delta(n^+)$ by Table 1. From Table 1, Theorems 1 and 2, we obtain the following theorem.

Theorem 4. Let M be of type EIII. Then the group $H^q(M, \Omega^p(E_{-k\omega_j}))$ vanishes except for $(\,p, q, k\,)$ listed in Table 2.

Table 1

Values $\left(\sigma\delta, \frac{2\beta}{(\beta,\beta)} \right)$ for $\sigma \in W^1$ and $\beta \in \Delta(n^+)$ (type E III)

$\sigma\delta$ \ β	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}	β_{11}	β_{12}	β_{13}	β_{14}	β_{15}	β_{16}	the number which does not appear in the sequence
$\sigma_0\delta$	1	2	3	4	4	5	5	6	6	7	7	8	8	9	10	11	
$\sigma_1\delta$	-1	1	2	3	3	4	4	5	5	6	7	8	7	9	10	11	0
$\sigma_2\delta$	-2	-1	1	3	2	4	3	4	5	6	6	7	7	8	10	11	0, 9
$\sigma_3\delta$	-3	-2	-1	1	1	2	3	4	4	5	5	6	7	8	9	11	0, 10
$\sigma_4\delta$	-4	-3	-2	-1	1	1	2	4	3	5	4	6	6	7	8	11	0, 9, 10
$\sigma_4'\delta$	-4	-3	-2	1	-1	2	2	3	3	4	4	5	7	8	9	10	0, 6
$\sigma_5\delta$	-5	-4	-3	-2	1	-1	2	3	3	4	4	5	5	6	7	11	0, 8, 9, 10
$\sigma_5'\delta$	-5	-4	-3	-1	-1	1	1	3	2	4	3	5	6	7	8	10	-2, 0, 9
$\sigma_6\delta$	-6	-5	-4	-2	-1	-1	1	2	2	3	3	4	5	6	7	10	-3, 0, 8, 9
$\sigma_6'\delta$	-6	-5	-3	-2	-2	1	-1	2	1	4	2	5	5	6	8	9	-4, 0, 3, 7
$\sigma_7\delta$	-7	-6	-4	-3	-2	-1	-1	1	1	3	2	4	4	5	7	9	-5, 0, 6, 8
$\sigma_7'\delta$	-7	-5	-4	-3	-3	1	-2	2	-1	3	1	5	4	6	7	8	-6, 0
$\sigma_8\delta$	-8	-7	-4	-3	-3	-2	-2	-1	1	2	2	3	3	4	7	8	-6, -5, 0, 5, 6
$\sigma_8'\delta$	-8	-6	-5	-4	-3	-1	-2	1	-1	2	1	4	3	5	6	8	-7, 0, 7
$\sigma_8''\delta$	-7	-6	-5	-4	-4	1	-3	2	-2	3	-1	4	4	5	6	7	0
$\sigma_9\delta$	-9	-7	-5	-4	-4	-2	-3	-1	-1	1	1	3	2	4	6	7	-8, -6, 0, 5
$\sigma_9'\delta$	-8	-7	-6	-5	-4	-1	-3	1	-2	2	-1	3	3	4	5	7	0, 6
$\sigma_{10}\delta$	-9	-8	-6	-5	-5	-2	-4	-1	-2	1	-1	2	2	3	5	6	-7, -3, 0, 4
$\sigma_{10}'\delta$	-10	-7	-6	-4	-5	-3	-3	-2	-2	-1	1	2	1	4	5	6	-9, -8, 0, 3
$\sigma_{11}\delta$	-11	-7	-6	-5	-5	-4	-4	-3	-3	-2	1	2	-1	3	4	5	-10, -9, -8, 0
$\sigma_{11}'\delta$	-10	-8	-7	-5	-6	-3	-4	-2	-3	-1	-1	1	1	3	4	5	-9, 0, 2
$\sigma_{12}\delta$	-11	-8	-7	-6	-6	-4	-5	-3	-4	-2	-1	1	-1	2	3	4	-9, -10, 0
$\sigma_{12}'\delta$	-10	-9	-8	-5	-7	-4	-4	-3	-3	-2	-2	-1	1	2	3	4	-6, 0
$\sigma_{13}\delta$	-11	-9	-8	-6	-7	-5	-5	-4	-4	-3	-2	-1	-1	1	2	3	-10, 0
$\sigma_{14}\delta$	-11	-10	-8	-7	-7	-6	-6	-5	-4	-3	-3	-2	-2	-1	1	2	-9, 0
$\sigma_{15}\delta$	-11	-10	-9	-8	-7	-7	-6	-5	-5	-4	-4	-3	-3	-2	-1	1	0
$\sigma_{16}\delta$	-11	-10	-9	-8	-8	-7	-7	-6	-5	-5	-4	-4	-3	-3	-2	-1	

, where $\sigma\delta$, $\sigma \in W^1$, and $\beta \in \Delta(n^+)$ are expressed as follows:

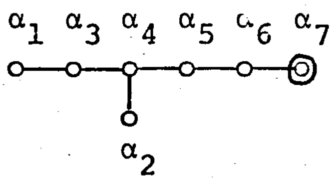
$\sigma_0\delta$	(1 1 1 1 1 1)
$\sigma_1\delta$	(-1 1 2 1 1 1)
$\sigma_2\delta$	(-2 1 1 2 1 1)
$\sigma_3\delta$	(-3 2 1 1 2 1)
$\sigma_4\delta$	(-4 3 1 1 1 2)
$\sigma_4'\delta$	(-4 1 1 1 3 1)
$\sigma_5\delta$	(-5 4 1 1 1 1)
$\sigma_5'\delta$	(-5 2 1 1 2 2)
$\sigma_6\delta$	(-6 3 1 1 2 1)
$\sigma_6'\delta$	(-6 1 1 2 1 3)
$\sigma_7\delta$	(-7 2 1 2 1 2)
$\sigma_7'\delta$	(-7 1 2 1 1 4)
$\sigma_8\delta$	(-8 1 1 3 1 1)
$\sigma_8'\delta$	(-8 2 2 1 1 3)
$\sigma_8''\delta$	(-7 1 1 1 1 5)
$\sigma_9\delta$	(-9 1 2 2 1 2)
$\sigma_9'\delta$	(-8 2 1 1 1 4)
$\sigma_{10}\delta$	(-9 1 1 2 1 3)
$\sigma_{10}'\delta$	(-10 1 3 1 2 1)
$\sigma_{11}\delta$	(-11 1 4 1 1 1)
$\delta_{11}'\delta$	(-10 1 2 1 2 2)
$\sigma_{12}\delta$	(-11 1 3 111 2)
$\sigma_{12}'\delta$	(-10 1 1 1 3 1)
$\sigma_{13}\delta$	(-11 1 2 1 2 1)
$\sigma_{14}\delta$	(-11 1 1 2 1 1)
$\sigma_{15}\delta$	(-11 2 1 1 1 1)
$\sigma_{16}\delta$	(-11 1 1 1 1 1)

β_1	(1 0 0 0 0 0)
β_2	(1 0 1 0 0 0)
β_3	(1 0 1 1 0 0)
β_4	(1 0 1 1 1 0)
β_5	(1 1 1 1 0 0)
β_6	(1 0 1 1 1 1)
β_7	(1 1 1 1 1 0)
β_8	(1 1 1 1 1 1)
β_9	(1 1 1 2 1 0)
β_{10}	(1 1 1 2 1 1)
β_{11}	(1 1 2 2 1 0)
β_{12}	(1 1 2 2 1 1)
β_{13}	(1 1 1 2 2 1)
β_{14}	(1 1 2 2 2 1)
β_{15}	(1 1 2 3 2 1)
β_{16}	(1 2 2 3 2 1)

Table 2

p	q = 0	$1 \leqq q \leqq 15$, (a,b) shows q = a and k = b	q = 16
0	$k > -1$		$k < -11$
1	$k > 1$	(1,0)	$k < -11$
2	$k > 2$	(2,0), (14,-9)	$k < -11$
3	$k > 3$	(3,0), (15,-10)	$k < -11$
4	$k > 4$	(4,0), (12,-6), (15,-9), (15,-10)	$k < -11$
5	$k > 5$	(5,0), (3,2), (15,-8), (15,-9), (15,-10)	$k < -11$
6	$k > 6$	(6,0), (3,3), (2,4), (10,-3), (14,-7), (15,-8) (15,-9)	$k < -10$
7	$k > 7$	(7,0), (1,6), (2,5), (14,-6), (15,-8)	$k < -9$
8	$k > 8$	(8,0), (1,7), (2,5), (2,6), (14,-5), (14,-6) (14,-7)	$k < -8$
9	$k > 9$	(9,0), (1,8), (2,6), (14,-5), (15,-6)	$k < -7$
10	$k > 10$	(10,0), (1,8), (1,9), (2,7), (6,3), (13,-3) (14,-4)	$k < -6$
11	$k > 11$	(11,0), (1,8), (1,9), (1,10), (13,-2)	$k < -5$
12	$k > 11$	(12,0), (1,9), (1,10), (4,6)	$k < -4$
13	$k > 11$	(13,0), (1,10)	$k < -3$
14	$k > 11$	(14,0), (2,9)	$k < -2$
15	$k > 11$	(15,0)	$k < -1$
16	$k > 11$		$k < 1$

2.3. The case M is of type E VII. The Dinkin diagram of Π is:



,where α_7 ◎ shows that $\alpha_j = \alpha_7$ in this case. We have $\#\Delta(n^+) = 27$ and $\# W^1 = 56$. We express β of $\Delta(n^+)$ and $\sigma\delta$ for $\sigma \in W^1$ in a similar way as in 2.2. Then the values $(\sigma\delta, \frac{2\beta}{(\beta,\beta)})$ for $\sigma \in W^1$ and $\beta \in \Delta(n^+)$ are as in Table 3. From Table 3, Theorems 1 and 2, we obtain the following.

Theorem 5. Let M be of type E VII. Then the group $H^q(M, \Omega^p(E_{-k\omega_j})) = 0$ except for (p, q, k) listed in Table 4.

Table 3

Values $\left(\sigma\delta, \frac{2\beta}{(\beta,\beta)} \right)$ for $\sigma \in W^1$ and $\beta \in \Delta(n^+)$ (type E VII)

$\sigma\delta$ \ β	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}	β_{11}	β_{12}	β_{13}	β_{14}	β_{15}	β_{16}
$\sigma_0\delta$	1	2	3	4	5	5	6	6	7	7	8	8	9	9	9	10
$\sigma_1\delta$	-1	1	2	3	4	4	5	5	6	6	7	7	9	9	8	10
$\sigma_2\delta$	-2	-1	1	2	3	3	4	4	5	5	7	6	8	8	7	9
$\sigma_3\delta$	-3	-2	-1	1	2	2	3	3	5	4	7	6	8	7	7	8
$\sigma_4\delta$	-4	-3	-2	-1	1	1	3	2	3	4	4	5	5	6	7	7
$\sigma_5\delta$	-5	-4	-3	-2	1	-1	2	1	2	4	3	5	4	6	6	7
$\sigma_5'\delta$	-5	-4	-3	-2	-1	1	2	2	3	3	4	4	5	5	7	6
$\sigma_6\delta$	-6	-5	-4	-3	-1	-1	1	1	2	3	3	4	4	5	6	6
$\sigma_6'\delta$	-6	-5	-4	-3	1	-2	2	-1	3	3	4	4	5	5	5	6
$\sigma_7\delta$	-7	-6	-5	-4	-1	-2	1	-1	2	2	3	3	4	4	5	5
$\sigma_7'\delta$	-7	-6	-5	-3	-2	-2	-1	1	1	2	2	4	3	5	5	6
$\sigma_8\delta$	-8	-7	-6	-4	-2	-3	-1	-1	1	1	2	3	3	4	4	5
$\sigma_8'\delta$	-8	-7	-5	-4	-3	-3	-2	1	-1	2	1	3	2	5	4	6
$\sigma_9\delta$	-9	-8	-7	-4	-3	-3	-2	-2	1	-1	2	2	3	3	3	4
$\sigma_9'\delta$	-9	-8	-6	-5	-3	-4	-2	-1	-1	1	1	2	2	4	3	5
$\sigma_9''\delta$	-9	-7	-6	-5	-4	-4	-3	1	-2	2	-1	3	1	4	4	6
$\sigma_{10}\delta$	-10	-9	-7	-5	-4	-4	-3	-2	-1	-1	1	1	2	3	2	4
$\sigma_{10}'\delta$	-10	-8	-7	-6	-4	-5	-3	-1	-2	1	-1	2	1	3	3	5
$\sigma_{10}''\delta$	-9	-8	-7	-6	-5	-5	-4	1	-3	2	-2	3	-1	4	4	5
$\sigma_{11}\delta$	-11	-10	-7	-6	-5	-4	-3	-3	2	-2	11	-1	2	2	1	3
$\sigma_{11}'\delta$	-11	-9	-8	-6	-5	-5	-4	-2	-2	-1	-1	1	1	2	2	4
$\sigma_{11}''\delta$	-10	-9	-8	-7	-5	-6	-4	-1	-3	1	-2	2	-1	3	3	4
$\sigma_{12}\delta$	-12	-11	-7	-6	-5	-5	-4	-4	-3	-3	1	-2	2	2	-1	3
$\sigma_{12}'\delta$	-12	-10	-8	-7	-6	-5	-4	-3	-3	-2	-1	-1	1	1	1	3
$\sigma_{12}''\delta$	-11	-10	-9	-7	-6	-6	-5	-2	-3	-1	-2	1	-1	2	2	3
$\sigma_{13}\delta$	-13	-11	-8	-7	-6	-6	-5	-4	-4	-3	-1	-2	1	1	-1	3
$\sigma_{13}'\delta$	-13	-10	-9	-8	-7	-5	-4	-4	-3	-3	-2	-2	1	-1	1	2
$\sigma_{13}''\delta$	-12	-11	-9	-8	-7	-6	-5	-3	-4	-2	-2	-1	-1	1	1	2
$\sigma_{14}\delta$	-14	-11	-9	-8	-7	-6	-5	-5	-4	-4	-2	-3	1	-1	-1	2
$\sigma_{14}'\delta$	-13	-12	-9	-8	-7	-7	-6	-4	-5	-3	-2	-2	-1	1	-1	1
$\sigma_{14}''\delta$	-13	-11	-10	-9	-8	-6	-5	-4	-4	-3	-3	-2	-1	-1	1	1
$\sigma_{15}\delta$	-13	-12	-11	-10	-9	-6	-5	-5	-4	-4	-3	-3	-2	-2	1	-1
$\sigma_{15}'\delta$	-14	-12	-10	-9	-8	-7	-6	-5	-5	-4	-3	-3	-1	-1	-1	1
$\sigma_{15}''\delta$	-15	-11	-10	-8	-7	-7	-6	-6	-4	-5	-3	-3	1	-2	-2	2
$\sigma_{16}\delta$	-14	-13	-11	-10	-9	-7	-6	-6	-5	-5	-3	-4	-2	-2	-1	-1
$\sigma_{16}'\delta$	-15	-12	-11	-9	-8	-8	-7	-6	-5	-5	-4	-3	-1	-2	-2	1
$\sigma_{16}''\delta$	-16	-11	-10	-9	-7	-8	-6	-7	-5	-5	-4	-4	1	-3	-3	2
$\sigma_{17}\delta$	-15	-13	-12	-10	-9	-8	-7	-7	-5	-6	-4	-4	-2	-3	-2	-1
$\sigma_{17}'\delta$	-16	-12	-11	-10	-8	-9	-7	-7	-6	-5	-5	-4	-1	-3	-3	1
$\sigma_{17}''\delta$	-17	-11	-10	-9	-8	-8	-7	-7	-6	-6	-5	-5	1	-4	-4	2
$\sigma_{18}\delta$	-15	-14	-13	-10	-9	-9	-8	-8	-5	-7	-4	-4	-3	-3	-3	-2
$\sigma_{18}'\delta$	-16	-13	-12	-11	-9	-9	-7	-8	-6	-6	-5	-5	-2	-4	-3	-1
$\sigma_{18}''\delta$	-17	-12	-11	-10	-9	-9	-8	-7	-7	-6	-6	-5	-1	-4	-4	1
$\sigma_{19}\delta$	-16	-14	-13	-11	-9	-10	-8	-9	-6	-7	-5	-5	-3	-4	-4	-2
$\sigma_{19}'\delta$	-17	-13	-12	-11	-10	-9	-8	-8	-7	-7	-6	-6	-2	-5	-4	-1
$\sigma_{20}\delta$	-16	-15	-13	-12	-9	-11	-8	-10	-7	-7	-5	-6	-4	-4	-5	-3
$\sigma_{20}'\delta$	-17	-14	-13	-11	-10	-10	-9	-9	-7	-8	-6	-6	-3	-5	-5	-2
$\sigma_{21}\delta$	-16	-15	-14	-13	-9	-12	-8	-11	-7	-7	-6	-6	-5	-5	-5	-4
$\sigma_{21}'\delta$	-17	-15	-13	-12	-10	-11	-9	-10	-8	-8	-6	-7	-4	-5	-6	-3
$\sigma_{22}\delta$	-17	-15	-14	-13	-10	-12	-9	-11	-8	-8	-7	-7	5	-6	-6	-4
$\sigma_{22}'\delta$	-17	-16	-13	-12	-11	-11	-10	-10	-9	-9	-6	-8	-5	-5	-7	-4
$\sigma_{23}\delta$	-17	-16	-14	-13	-11	-12	-10	-11	-9	-9	-7	-8	-6	-6	-7	-5
$\sigma_{24}\delta$	-17	-16	-15	-13	-12	-12	-11	-11	-9	-10	-8	-8	-7	-7	-7	-6
$\sigma_{25}\delta$	-17	-16	-15	-14	-13	-12	-11	-11	-10	-10	-9	-9	-8	-8	-7	-7
$\sigma_{26}\delta$	-17	-16	-15	-14	-13	-13	-12	-11	-11	-10	-10	-9	-9	-8	-8	-7
$\sigma_{27}\delta$	-17	-16	-15	-14	-13	-13	-12	-12	-11	-11	-10	-10	-9	-9	-8	-8

Table 3 ——continued

$\sigma\delta$ \ β	β_{17}	β_{18}	β_{19}	β_{20}	β_{21}	β_{22}	β_{23}	β_{24}	β_{25}	β_{26}	β_{27}	the number which does not appear in the sequence
$\sigma_0\delta$	10	11	11	12	12	13	13	14	15	16	17	
$\sigma_1\delta$	9	11	10	12	11	11	13	14	15	16	17	0
$\sigma_2\delta$	9	10	10	11	11	13	12	14	15	16	17	0
$\sigma_3\delta$	8	9	10	11	11	12	12	13	15	16	17	0, 14
$\sigma_4\delta$	8	9	9	10	11	11	12	13	14	16	17	0, 15
$\sigma_5\delta$	7	8	8	9	10	11	12	13	14	15	17	0, 16
$\sigma_5'\delta$	8	9	9	10	10	11	11	12	13	16	17	0, 14, 15
$\sigma_6\delta$	7	8	8	9	10	10	11	12	13	15	17	-2, 0, 14, 16
$\sigma_6'\delta$	6	7	7	8	11	9	12	13	14	15	16	0, 10
$\sigma_7\delta$	6	7	7	8	10	9	11	12	13	15	16	-3, 0, 14
$\sigma_7'\delta$	6	7	8	9	9	10	10	11	13	14	17	-4, 0, 12, 15, 16
$\sigma_8\delta$	5	6	7	8	9	9	10	11	13	14	16	-5, 0, 12, 15
$\sigma_8'\delta$	6	7	7	8	8	10	9	11	12	13	17	-6, 0, 14, 15, 16
$\sigma_9\delta$	4	5	7	8	8	9	9	10	13	14	15	-5, -6, 0, 6, 11, 12
$\sigma_9'\delta$	5	6	6	7	8	9	9	11	12	13	16	-7, 0, 10, 14, 15
$\sigma_9''\delta$	5	7	6	8	7	9	9	10	11	12	17	-8, 0, 13, 14, 15, 16
$\sigma_{10}\delta$	4	5	6	7	7	9	8	10	12	13	15	-8, -6, 0, 11, 14
$\sigma_{10}'\delta$	4	6	5	7	7	8	9	10	11	12	16	-9, 0, 13, 14, 15
$\sigma_{10}''\delta$	5	6	6	7	7	8	8	9	10	11	17	0, 12, 13, 14, 15, 16
$\sigma_{11}\delta$	4	5	5	6	6	9	7	10	11	13	14	-8, -9, o, 8, 12
$\sigma_{11}'\delta$	3	5	5	7	6	8	8	9	11	12	15	-10, -7, -3, 0, 10, 13,14
$\sigma_{11}''\delta$	4	5	5	6	7	7	8	9	10	11	16	0, 12, 13, 14, 15
$\sigma_{12}\delta$	3	4	4	5	5	9	6	10	11	12	13	-10, -9, -8, 0, 7, 8
$\sigma_{12}'\delta$	3	5	4	6	5	8	7	9	10	12	14	-11, -9, 0, 2, 11, 13
$\sigma_{12}''\delta$	3	4	5	6	6	7	7	8	10	11	15	-8, -4, 0, 9, 12, 13, 14
$\sigma_{13}\delta$	2	4	3	5	4	8	6	9	10	11	13	-12, -10, -9, 0, 7, 12
$\sigma_{13}'\delta$	2	5	3	6	4	7	7	8	9	12	13	-12, -11, -6, 0, 10, 11
$\sigma_{13}''\delta$	3	4	4	5	5	7	6	8	9	11	14	-10, 0, 10, 12, 13
$\sigma_{14}\delta$	1	4	2	5	3	7	6	8	9	11	12	-13, -12, -10, 0, 10
$\sigma_{14}'\delta$	2	3	3	4	4	7	5	8	9	10	13	-11, -10, 0, 6, 11, 12
$\sigma_{14}''\delta$	2	4	3	5	4	6	6	7	8	11	13	-12, -7, 0, 9, 10, 12
$\sigma_{15}\delta$	2	3	3	4	4	5	5	6	7	11	12	-8, -7, 0, 8, 9, 10
$\sigma_{15}'\delta$	1	3	2	4	3	6	5	7	8	10	12	-13, -11, -2, 0, 9, 11
$\sigma_{15}''\delta$	-1	3	1	5	2	6	6	7	9	10	11	-14,-13,-12,-9, 0, 4, 9
$\sigma_{16}\delta$	1	2	2	3	3	5	4	6	7	10	11	-12, -8, 0, 8, 9
$\sigma_{16}'\delta$	-1	2	1	4	2	5	5	6	8	9	11	-14,-13,-10, 0, 3, 7, 10
$\sigma_{16}''\delta$	-2	3	-1	4	1	5	6	7	8	9	10	-15, -14, -13, -12, 0
$\sigma_{17}\delta$	-1	1	1	3	2	4	4	5	7	9	10	-14, -11 0, 6, 8
$\sigma_{17}'\delta$	-2	2	-1	3	1	4	5	6	7	8	10	-15, -14, -13, 0, 9
$\sigma_{17}''\delta$	-3	3	-2	4	-1	5	5	6	7	8	9	-16, -15, -14, -13, -12,0
$\sigma_{18}\delta$	-2	-1	-1	2	2	3	3	4	7	8	9	-12, -11, -6, 0, 5, 6
$\sigma_{18}'\delta$	-2	1	-1	2	1	3	4	5	6	8	9	-15, -14, -10, 0, 7
$\sigma_{18}''\delta$	-3	2	-2	3	-1	4	4	5	6	7	9	-16, -15, -14, -13, 0, 8
$\sigma_{19}\delta$	-3	-1	-1	1	1	2	3	4	6	7	8	-15, -12, 0, 5
$\sigma_{19}'\delta$	-3	1	-2	2	-1	3	3	4	5	7	8	-16, -15, -14, 0, 6
$\sigma_{20}\delta$	-3	-2	-2	-1	1	1	2	4	5	6	7	-14, 0, 3
$\sigma_{20}'\delta$	-4	-1	-2	1	-1	2	2	3	5	6	7	-16,-15, -12, 0, 4
$\sigma_{21}\delta$	-4	-3	-3	-2	1	-1	2	3	4	5	6	-10, 0
$\sigma_{21}'\delta$	-4	-2	-3	-1	-1	1	1	3	4	5	6	-16, -14, 0, 2
$\sigma_{22}\delta$	-5	-3	-4	-2	-1	-1	1	2	3	4	5	-16,0
$\sigma_{22}'\delta$	-4	-3	-3	-2	-2	1	-1	2	3	4	5	-15, -14, 0
$\sigma_{23}\delta$	-5	-4	-4	-3	-2	-1	-1	1	2	3	4	-15, 0
$\sigma_{24}\delta$	-6	-5	-4	-3	-3	-2	-2	-1	1	2	3	-14, 0
$\sigma_{25}\delta$	-6	-5	-5	-4	-4	-3	-3	-2	-1	1	2	0
$\sigma_{26}\delta$	-7	-6	-6	-5	-5	-4	-4	-3	-2	-1	1	0
$\sigma_{27}\delta$	-8	-7	-7	-6	-6	-5	-5	-4	-3	-2	-1	

, where $\sigma\delta$, $\sigma \in W^1$, and $\beta \in \Delta(n^+)$ are expressed as follows:

$\sigma_0\delta$	(1 1 1 1 1 1 1)
$\sigma_1\delta$	(1 1 1 1 1 2 -1)
$\sigma_2\delta$	(1 1 1 1 2 1 -2)
$\sigma_3\delta$	(1 1 1 2 1 1 -3)
$\sigma_4\delta$	(1 2 2 1 1 1 -4)
$\sigma_5\delta$	(2 3 1 1 1 1 -5)
$\sigma_5'\delta$	(1 1 3 1 1 1 -5)
$\sigma_6\delta$	(2 2 2 1 1 1 -6)
$\sigma_6'\delta$	(1 4 1 1 1 1 -6)
$\sigma_7\delta$	(1 3 2 1 1 1 -7)
$\sigma_7'\delta$	(3 1 1 2 1 1 -7)
$\sigma_8\delta$	(2 2 1 2 1 1 -8)
$\sigma_8'\delta$	(4 1 1 1 2 1 -8)
$\sigma_9\delta$	(1 1 1 3 1 1 -9)
$\sigma_9'\delta$	(3 2 1 1 2 1 -9)
$\sigma_9''\delta$	(5 1 1 1 1 2 -9)
$\sigma_{10}\delta$	(2 1 1 2 2 1 -10)
$\sigma_{10}'\delta$	(4 2 1 1 1 2 -10)
$\sigma_{10}''\delta$	(6 1 1 1 1 1 -9)
$\sigma_{11}\delta$	(1 1 2 1 3 1 -11)
$\sigma_{11}'\delta$	(3 1 1 2 1 2 -11)
$\sigma_{11}''\delta$	(5 2 1 1 1 1 -10)
$\sigma_{12}\delta$	(1 1 1 1 4 1 -12)
$\sigma_{12}'\delta$	(2 1 2 1 2 2 -12)
$\sigma_{12}''\delta$	(4 1 1 2 1 1 -11)
$\sigma_{13}\delta$	(2 1 1 1 3 2 -13)
$\sigma_{13}'\delta$	(1 1 3 1 1 3 -13)
$\sigma_{13}''\delta$	(3 1 2 1 2 1 -12)
$\sigma_{14}\delta$	(1 1 2 1 2 3 -14)
$\sigma_{14}'\delta$	(3 1 1 1 3 1 -13)
$\sigma_{14}''\delta$	(2 1 3 1 1 2 -13)
$\sigma_{15}\delta$	(1 1 4 1 1 1 -13)
$\sigma_{15}'\delta$	(2 1 2 1 2 2 -14)
$\sigma_{15}''\delta$	(1 1 1 2 1 4 -15)
$\sigma_{16}\delta$	(1 1 3 1 2 1 -14)
$\sigma_{16}'\delta$	(2 1 1 2 1 3 -15)
$\sigma_{16}''\delta$	(1 2 1 1 1 5 -16)
$\sigma_{17}\delta$	(2 1 2 2 1 2 -15)
$\sigma_{17}'\delta$	(2 2 1 1 1 4 -16)
$\sigma_{17}''\delta$	(1 1 1 1 1 6 -17)
$\sigma_{18}\delta$	(1 1 1 3 1 1 -15)
$\sigma_{18}'\delta$	(1 2 2 1 1 3 -16)
$\sigma_{18}''\delta$	(2 1 1 1 1 5 -17)
$\sigma_{19}\delta$	(1 2 1 2 1 2 -16)
$\sigma_{19}'\delta$	(1 1 2 1 1 4 -17)
$\sigma_{20}\delta$	(1 3 1 1 2 1 -16)
$\sigma_{20}'\delta$	(1 1 1 2 1 3 -17)
$\sigma_{21}\delta$	(1 4 1 1 1 1 -16)
$\sigma_{21}'\delta$	(1 2 1 1 2 2 -17)
$\sigma_{22}\delta$	(1 3 1 1 1 2 -17)
$\sigma_{22}'\delta$	(1 1 1 1 3 1 -17)
$\sigma_{23}\delta$	(1 2 1 1 2 1 -17)
$\sigma_{24}\delta$	(1 1 1 2 1 1 -17)
$\sigma_{25}\delta$	(1 1 2 1 1 1 -17)
$\sigma_{26}\delta$	(2 1 1 1 1 1 -17)
$\sigma_{27}\delta$	(1 1 1 1 1 1 -17)

β_1	(0 0 0 0 0 0 1)
β_2	(0 0 0 0 0 111)
β_3	(0 0 0 0 1 1 1)
β_4	(0 0 0 1 1 1 1)
β_5	(0 1 0 1 1 1 1)
β_6	(0 0 1 1 1 1 1)
β_7	(0 1 1 1 1 1 1)
β_8	(1 0 1 1 1 1 1)
β_9	(0 1 1 2 1 1 1)
β_{10}	(1 1 1 1 1 1 1)
β_{11}	(0 1 1 2 2 1 1)
β_{12}	(1 1 1 2 1 1 1)
β_{13}	(0 1 1 2 2 2 1)
β_{14}	(1 1 1 2 2 1 1)
β_{15}	(1 1 2 2 1 1 1)
β_{16}	(1 1 1 2 2 2 1)
β_{17}	(1 1 2 2 2 1 1)
β_{18}	(1 1 2 2 2 2 1)
β_{19}	(1 1 2 3 2 1 1)
β_{20}	(1 1 2 3 2 2 1)
β_{21}	(1 2 2 3 2 1 1)
β_{22}	(1 1 2 3 3 2 1)
β_{23}	(1 2 2 3 2 2 1)
β_{24}	(1 2 2 3 3 2 1)
β_{25}	(1 2 2 4 3 2 1)
β_{26}	(1 2 3 4 3 2 1)
β_{27}	(2 2 3 4 3 2 1)

Table 4

p	q = 0	1 ≦ q ≦ 26, (a,b) shows q = a and k = b	q = 27
0	k > -1		k < -17
1	k > 1	(1,0)	k < -17
2	k > 2	(2,0)	k < -17
3	k > 3	(3,0), (24, -14)	k < -17
4	k > 4	(4,0), (25,-15)	k < -17
5	k > 5	(5,0), (25,-14∿-15)	k < -17
6	k > 6	(6,0), (4,2), (21,-10), (25,-14), (26,-16)	k < -17
7	k > 7	(7,0), (3,4), (4,3), (24,-12), (25,-14), (26,-15∿-16)	k < -17
8	k > 8	(8,0), (2,6), (3,5), (24,-12), (26,-14∿-16)	k < -17
9	k > 9 >	(9,0), (1,8), (2,7), (3,5 6), (18,-6), (23,-10), (24,-11∿-12), (26,-13∿-16)	k < -17
10	k > 10	(10,0), (1,9), (2,8), (3,6), (24,-11), (26,-12∿-16)	k < -17
11	k > 11	(11,0), (1,10), (2,8∿9), (3,7), (7,3), (22,-8), (24,-10), (25,-12), (26,-12∿-15)	k < -16
12	k > 12	(12,0), (1,11), (2,8∿10), (3,8), (7,4), (15,-2), (22,-7∿-8), (24,-9), (25,-11), (26,-12∿-14)	k < -15
13	k > 13	(13,0), (1,11∿12), (2,9∿10), (5,6), (22,-7), (25,-10∿-11), (26,-12∿-13)	k < -14
14	k > 14	(14,0), (1,12∿13), (2,10∿11), (5,7), (22,-6), (25,-9∿-10), (26,-11∿-12)	k < -13
15	k > 15	(15,0), (1,12∿14), (2,11), (5,7∿8), (3,9), (12,2), (20,-4), (24,-8), (25,-8∿-10), (26,-11)	k < -12
16	k > 16	(16,0), (1,12∿15), (2,12), (3,10), (5,8), (20,-3), (24,-7), (25,-8∿-9), (26,-10)	k < -11
17	k > 17	(17,0), (1,12∿16), (3,11), (24,-6), (25,-8), (26,-9)	k < -10
18	k > 17	(18,0), (1,13∿16), (3,11∿12), (4,10), (9,6),	k < -9
19	k > 17	(19,0), (1,14∿16), (3,12), (24,-5), (25,-6)	k < -8
20	k > 17	(20,0), (1,15∿16), (2,14), (3,12), (23,-3), (24,-4)	k < -7
21	k > 17	(21,0), (1,16), (2,14) (6,10), (23,-2)	k < -6
22	k > 17	(22,0), (1,16), (2,14∿15)	k < -5
23	k > 17	(23,0), (2,15)	k < -4
24	k > 17	(24,0), (3,14)	k < -3
25	k > 17	(25,0)	k < -2
26	k > 17	(26,0)	k < -1
27	k > 17		k < 1

2.4. Other cases. If M is of type A III, D III or C I, it is not known completely when the groups $H^q(M, \Omega^p(E))$ vanish. in this section we consider for the case when p is equal to 0 or 1.

We denote by K_N the canonical line bundle of a complex manifold N. There exists an integer λ such that $K_N = E_{\lambda\omega_j}$. Further we know

$$\lambda = 2\left(\sum_{\beta\in\Delta(n^+)} \beta,\ \alpha_j\right)/(\alpha_j,\ \alpha_j)$$

(Borel-Hirzebruch [2]). Applying this formula, we may calculate λ for each type and get the following table.

A III	$SU(m+n)/S(U(m)\times U(n))$,	$\lambda = m + n$,
D III	$SO(2n)/U(n)$,	$\lambda = 2n - 2$,
C I	$Sp(n)/U(n)$,	$\lambda = n + 1$,
BD I	$SO(n+2)/SO(2)\times SO(n)$,	$\lambda = n$,
E III	$E_6/Spin(10)\times T^1$,	$\lambda = 12$,
E VII	$E_7/E_6\times T^1$	$\lambda = 18$.

Theorem 6. Let M be an n-dimensional irreducible Hermitian symmetric space of compact type. Then the group $H^q(M, \Omega E_{-k\omega_j}) = 0$ except for the following cases: (i) $q = 1$ and $k \geqq 0$, (ii) $q = n$ and $k \leqq -\lambda$.

Proof. By the theorem of Bott, we get

(2.1) $$H^0(M, \Omega E_{-k\omega_j}) \neq 0 \qquad \text{if } k \geqq 0,$$

(2.2) $$H^j(M, \Omega E_{-k\omega_j}) = 0 \quad \text{for } j > 0, \text{ if } k \geqq 0,$$

(2.3) $$H^0(M, \Omega E_{-k\omega_j}) = 0 \qquad \text{if } k < 0.$$

By serre's duality theorem, we have

$$\dim H^q(M, \Omega E_{-k\omega_j}) = \dim H^{n-q}(M, \Omega (K_M \otimes E_{k\omega_j})).$$

Hence we obtain, from (2.1) and (2.2)

(2.4) $\quad H^n(M, \Omega E_{-k\omega_j}) \neq 0 \qquad$ if $k \leqq -\lambda$,

(2.5) $\quad H^j(M, \Omega E_{-k\omega_j}) = 0 \qquad$ for $j < n$, if $k \leq -\lambda$.

We note $E_{-k\omega_j}$ is positive if $k > 0$. Then by Kodaira's vanishing theorem, we see

(2.6) $\quad H^j(M, \Omega E_{-k\omega_j}) = 0 \qquad$ for $j > 0$, if $k > -\lambda$.

The conclusion follows from (2.1), (2.3), (2.4), (2.5) and (2.6).

Remark. If M is a Kahler C-space whose 2nd Betti number is 1, we get the same conclusion in the same way as above.

Theorem 7. Let M be an irreducible Hermitian symmetric space of compact type. Assume that M is not $P_n(C)$, $Sp(2)/U(2)$, $SO(6)/U(3)$ or $SO(8)/U(4)$. Then the group $H^q(M, \Omega^1(E_{-k\omega_j})) = 0$ except for the following cases: (i) $q = 0$ and $k > 1$, (ii) $q = 1$ and $k = 0$, (iii) $q = n$ and $k < -\lambda + 1$.

Proof. We may assume that M is of type A III, C I or D III by Theorems 3, 4 and 5.

It is known

$$n(\sigma) = \min \{ k;\ \sigma = \tau_{\alpha_{i_1}} \cdots \tau_{\alpha_{i_k}},\ \alpha_{i_j} \in \Pi \} \qquad \text{for } \sigma \in W$$

, where τ_α denotes the symmetry with respect to $\alpha \in \Delta$. Therefore by the definition of $W^1(1)$, we have

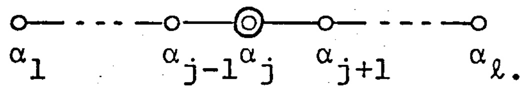
$$W^1(1) = \{ \tau_{\alpha_j} \}.$$

Since $\tau_{\alpha_j}\delta = \delta - \alpha_j$, we have by Theorem 2

(2.7) $\dim H^q(\, M,\ \Omega^1(E_{-k\omega_j})\,) = \dim H^q(\, M,\ \Omega(E_{-(k\omega_j-\alpha_j)})\,)$

for $q = 0, 1, \cdots$.

1. The case $M = SU(\ell+1)/S(U(j)\times U(\ell+1-j))$ for $1 < j < \ell$. The Dinkin diagram of Π is:



We may assume that h_0 is the set of points $(x_i) \in R^{\ell+1}$ such that $\sum_{i=1}^{\ell+1} x_i = 0$. Let $\{\varepsilon_i\}_{i=1}^{+1}$ be the natural basis of $R^{\ell+1}$. Then

$$\alpha_j = \varepsilon_j - \varepsilon_{j+1},$$

$$\delta = \ell\varepsilon_1 + (\ell-1)\varepsilon_2 + \cdots + 2\varepsilon_{\ell-1} + \varepsilon_\ell,$$

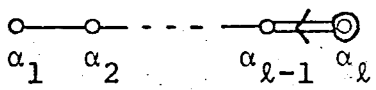
$$\Delta(n^+) = \{\ \varepsilon_s - \varepsilon_t;\ 1 \leqq s \leqq j < t \leqq \ell+1\ \}.$$

It follows that

$$\{\ (\delta-\alpha_j, \tfrac{2\beta}{(\beta,\beta)});\ \beta \in \Delta(n^+)\ \} = \{\ -1,\ 1,\ 2,\ \cdots,\ \ell\}.$$

Further if $\beta \in \Delta(n^+)$ satisfies $(\delta-\alpha_j, \frac{2\beta}{(\beta,\beta)}) = -1$, then $\beta = \alpha_j$. Therefore the conclusion follows from Theorem 1.

2. The case $M = Sp(\ell)/U(\ell)$ for $\ell \geqq 3$. The Dinkin diagram of Π is:



,where α_ℓ ◎ means $\alpha_j = \alpha_\ell$. Let $\{\varepsilon_i\}_{i=1}$ be the basis of h_0 which satisfies $(\varepsilon_i, \varepsilon_j) = \delta_{ij}$. Then

$\alpha_\ell = 2\varepsilon_\ell$,

$\omega_\ell = \varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_\ell$,

$\delta = \ell\varepsilon_1 + (\ell-1)\varepsilon_2 + \cdots + 2\varepsilon_{\ell-1} + \varepsilon_\ell$,

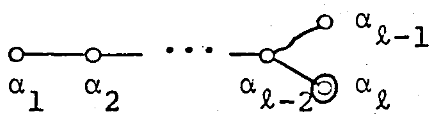
$\Delta(n^+) = \{ \varepsilon_i + \varepsilon_j;\ 1 \leqq i < j \leqq \ell,\ 2\varepsilon_i;\ 1 \leqq i \leqq \ell\}$.

Hence we have

$$\{ (\delta-\alpha_\ell,\beta);\ \beta \in \Delta(n^+) \} = \begin{cases} \{-2,1,2,\cdots,2\ -1,2\} & \text{if } \ell > 3, \\ \{-2,\ 1,\ 2,\ 4,\ 5,\ 6\} & \text{if } \ell = 3. \end{cases}$$

Further if $\beta \in \Delta(n^+)$ satisfies $(\delta-\alpha_\ell,\beta) = -2$, then $\beta = \alpha_\ell$. Since $(k\omega_\ell,\beta) = 2k$, $\beta \in \Delta(n^+)$, the conclusion follows then from Theorem 1.

3. $M = SO(2\ell)/U(\ell)$ for $\ell \geqq 5$. The Dinkin diagram of is:



, where α_ℓ◎ means $\alpha_j = \alpha_\ell$. Let $\{\varepsilon_i\}_{i=1}$ be a basis of h_0 such that $(\varepsilon_i,\varepsilon_j) = \delta_{ij}$. Then

$\alpha_\ell = \varepsilon_{\ell-1} + \varepsilon_\ell$,

$\omega_\ell = 1/2(\varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_\ell)$,

$\delta = (\ell-1)\varepsilon_1 + (\ell-2)\varepsilon_2 + \cdots + \varepsilon_{\ell-1}$,

$\Delta(n^+) = \{ \varepsilon_i + \varepsilon_j;\ 1 \leqq i < j \leqq \ell\}$.

Hence we have

$\{(\delta-\alpha_\ell,\beta);\ \beta \in \Delta(n^+) \} = \{ -1,\ 1,\ 2,\ \cdots,\ 2\ell-3\}$,

$(k\omega_\ell,\beta) = k$ for any $\beta \in \Delta(n^+)$.

Further, if $\beta \in \Delta(n^+)$ satisfies $(\delta-\alpha_\ell,\beta) = -1$, then $\beta = \alpha_\ell$. The conclusion follows then from Theorem 1.

q.e.d.

Remark. Assume that M is one of the following Hermitian symmetric spacesof compact type. Then the group $H^q(M, \Omega^p(E_{-k\omega_j}))$ $= 0$ except for the following cases:

Sp(2)/U(2) (i) $q = 0$ and $k > 1$, (ii) $q = 1$ and $k = 0$, (iii) $q = 2$ and $k = -1$, (iv) $q = 3$ and $k < -2$,

SO(6)/U(3) (i) $q = 0$ and $k > 1$, (ii) $q = 1$ and $k = 0$, (iii) $q = 3$ and $k < -2$,

SO(8)/U(4) (i) $q = 0$ and $k > 1$, (ii) $q = 1$ and $k = 0$, (iii) $q = 5$ and $k = -4$, (iv) $q = 6$ and $k < -5$.

3 Hypersurfaces of Hermitian symmetric spaces of compact type.

We retain the notations and assumptions introduced in the previous sections.

Let V be a hypersurface, that is, closed codimension 1 complex submanifold in a Kähler C-space M. Taking a sufficiently fine finite covering $\{U_j\}$ of V, V is defined in each U_j by a holomorphic equation $s_j = 0$. We associate with V the complex line bumdle $\{V\}$ over M determined by the system $\{s_{jk}\}$ of non-vanising holomorphic functions $s_{jk} = s_j/s_k$ on $U_j \cap U_k$. There is an integer d such that $\{V\} = E_{-d\omega_j}$. Since $\{V\}$ has a holomorphic section, $d > 0$. We call d the degree of V. If $M = P_n(C)$, this definition coinsides with the usual definition of degree. We denote by Θ (resp. Ω) the sheaf of germs of holomorphic vector fields (resp. holomorphic functions) on V. we spall compute the dimensions of $H^q(V, \Theta)$ and $H^q(V, \Omega)$.

By Serre's duality theorem, we have

$$\dim H^0(V, \Theta) = \dim H^n(V, \Omega^1(K_V)).$$

Denote by $E|_V$ the restriction to V of a holomorphic vector bundle over M. Since $K_V = (K_M \otimes \{V\})|_V$, we have

$$\dim H^0(V, \Theta) = \dim H^n(V, \Omega^1(E_{-(d-\lambda)\omega_j}|_V)). \tag{3.1}$$

Let us recall the following vanishing theorem of Akizuki-Nakano [1]. If L is a ~~positive~~ holomorphic line bundle over a compact complex manifold N. Then we have

$$H^q(N, \Omega^p(L)) = 0 \qquad \text{for} \quad p+q \geq n+1, \quad \text{if} \quad L \text{ is positive.} \tag{3.2}$$

Therefore we get

$$H^0(V, \Theta) = 0 \qquad \text{if} \quad d > \lambda,$$

by (3.1).

Theorem 8. Let M be an irreducible Hermitian symmetric space of compact type BD I, E III or E VII, and let V be a hypersurface of M whose degree is d. Then we have

$$H^0(V, \Theta) = 0 \qquad \text{if} \quad d \geq 2.$$

The following lemma follows from Theorems 3,4 and 5.

Lemma 3. Let M be an n-dimensional irreducible Hermitian symmetric space of compact type BD I, E III or E VII. Then we have

$$H^q(M, \Omega^p(E_{-k\omega_j})) = 0, \quad H^{q+1}(M, \Omega^p(E_{-(k-d)\omega_j})) = 0$$

for $p + q = n + 2$, $k = pd - \lambda$ if $2 \leqq p \leqq n$ and $d \geqq 2$.

Proof of Theorem 8. Recall the pair of exact sequences (Kodaira and Spencer [6])

$$\cdots \longrightarrow H^{q-1}(\ V,\ \Omega^{p}(E_{-k\omega_j}|_V)\ ) \longrightarrow H^{q}(\ M,\ \Omega''^{p}(E_{-k\omega_j})\ ) \longrightarrow$$

$$H^{q}(\ M,\ \Omega^{p}(E_{-k\omega_j})\ ) \longrightarrow \cdots,$$

$$\cdots \longrightarrow H^{q}(\ M,\ \Omega''^{p}(E_{-k\omega_j})\ ) \longrightarrow H^{q}(\ V,\ \Omega^{p-1}(E_{-(k-d)\omega_j}|_V)\ ) \longrightarrow$$

$$H^{q+1}(\ M,\ \Omega^{p}(E_{-(k-d)\omega_j})\ ) \longrightarrow \cdots$$

, where $\Omega''^{p}(L)$ is the kernel of the canonical map of $\Omega^{p}(L)$ onto $\Omega^{p}(L|_V)$ for a holomorphic line bundle L over M.

We see from the above pair of exact sequences and Lemma 3 that

$$H^{\overset{n}{\chi}-p-1}(\ V,\ \Omega^{p}(E_{-(pd-\lambda)\omega_j}|_V)\ ) \longrightarrow H^{\overset{n}{\chi}-p+2}(\ M,\ \Omega''^{p}(E_{-(pd-\lambda)\omega_j})) \longrightarrow 0,$$

$$H^{\overset{n}{\chi}-p+2}(\ M,\ \Omega''^{p}(E_{-(pd-\lambda)\omega_j})) \longrightarrow H^{\overset{n}{\chi}-p+2}(\ V,\ \Omega^{p-1}(E_{-((p-1)d-\lambda)\omega_j}|_V)) \longrightarrow$$

0.

Thus $H^{\overset{n}{\chi}-p+1}(\ V,\ \Omega^{p}(E_{-(pd-\lambda)\omega_j}|_V)\ ) = 0$ implies

$H^{\overset{n}{\chi}-p+2}(\ V,\ \Omega^{p-1}(E_{-((p-1)d-\lambda)\omega_j}|_V)\ ) = 0$, while we have

$H^{1}(\ V, \Omega^{n}(E_{-(nd-\lambda)\omega_j}|_V)\ )=0$ by (3.2). Hence we obtain

$H^{n}(\ V,\ \Omega^{1}(E_{-(d-\lambda)\omega_j}|_V)\ )=0.$

q.e.d.

Remark. The above proof is motivated by Kodaira and Spencer [5].

Let N be a complex manifold and let $W \longrightarrow N$ be a holomorphic vector bundle over N. Assume that V is a hypersurface of N.

We denote by $\hat{\Omega}(W|_V)$ the trivial extension of $\Omega(W|_V)$ to N. Then we have the following exact sequence (Kodoira and Spencer [6])

$$(3.3) \qquad 0 \longrightarrow \Omega(W\times\{V\}^{-1}) \longrightarrow \Omega(W) \longrightarrow \hat{\Omega}(W|_V) \longrightarrow 0 .$$

Assume that V is a hypersurface of M with degree d. It is easy to see that the normal bundle of V is equivaliant to $\{V\}|_V$. Hence, by Kimura [4], the nullity of V as a minimal submanifold of M is given as follows:

$$(3.4) \qquad n(V) = \dim_R H^0(\ V,\ \Omega(\{V\}|_V)\ ).$$

Denote by C the trivial line bundle over M. Then, by (3.3), we have the exact sequence:

$$0 \longrightarrow \Omega(C) \longrightarrow \Omega(\{V\}) \longrightarrow \hat{\Omega}(\{V\}|_V) \longrightarrow 0.$$

Since $H^1(\ M,\ \Omega(C)\ ) = 0$,

$$\dim H^0(\ V,\ \Omega(\{V\}|_V)\ ) = \dim H^0(\ M,\ \Omega(\{V\})\ ) - 1.$$

Since $\{V\} = E_{-d\omega_j}$, we get

$$\dim H^0(\ M,\ \Omega(\{V\})\ ) = \dim V_{-d\omega_j}$$

by the theorem of Bott. Therefore,

$$(3.5) \qquad \dim H^0(\ V, \Omega(\{V\}|_V)\ ) = \dim V_{-d\omega_j} - 1,$$

and by (3.4)

$$n(V) = 2(\dim V_{-d\omega_j} - 1).$$

We prove the following lemma.

Lemma 4. Let M be an irreducible Hermitian symmetric space of compact type of dimension > 3. Assume that M is not $P_n(C)$, $Sp(2)/U(2)$, $SO(6)/U(3)$ or $SO(8)/U(4)$. Then for a hypersurface V of M, we have

$$\dim H^0(V, \Omega(T(M)|_V)) = \dim H^0(M, \Omega T(M)),$$

$$H^1(V, \Omega(T(M)|_V)) = 0.$$

Proof. We have the exact sequence:

$$\cdots \longrightarrow H^q(M, \Omega(T(M)\otimes E_{d\omega_j})) \longrightarrow H^q(M, \Omega T(M))$$

$$\longrightarrow H^q(V, \Omega(T(M)|_V)) \longrightarrow \cdots$$

by (3.3). On the other hand, by Serre's duality theorem

$$\dim H^j(M, \Omega(T(M)\otimes E_{d\omega_j})) = \dim H^{n-j}(M, \Omega^1(E_{-(d-\lambda)\omega_j})).$$

Hence, since $H^j(M, \Omega T(M)) = 0$, $j = 1, 2$, the lemma follows from Theorem 7.

From this lemma we get the following.

Theorem 9. Let M be an irreducible Hermitian symmetric space of compact type of dimension > 3. Assume that M is not $P_n(C)$, $Sp(2)/U(2)$, $SO(6)/U(3)$ or $SO(8)/U(4)$. Then for a hypersurface V of M, we have

$$\dim H^1(V, \Theta) = \dim H^0(V, \{V\}|_V) + \dim H^0(V, \Theta)$$

$$- \dim H^0(M, \Omega T(M)).$$

By Theorems 8, 9 and (3.5), we obtain the following.

Theorem 10. Let M be an irreducible Hermitian symmetric space of compact type: BD I, E III or E VII, and let V be a hypersurface of M. Assume that $\dim M > 3$ and the degree of $V \geqq 2$. Then we have

$$\dim H^1(\, V,\, \Theta\,) = \dim V_{-d\omega_j} - \dim H^0(\, M,\, \Omega T(M)\,) - 1.$$

Finally the following theorem follows from Theorem 6.

Theorem 11. Let M be an n-dimensional irreducible Hermitian symmetric space of compact type, and let V be a hypersurface of M with degree d. Then the group $H^q(\, V,\, \Omega\,)$ vanishes except for the following cases:

$$q = 0 \text{ or } n-1 \qquad \text{if } d \geqq \lambda,$$
$$q = 0 \qquad \text{if } d < \lambda.$$

Proof. By Serre's duality theorem we have

$$(3.7) \quad \dim H^q(\, V,\, \Omega\,) = \dim H^{n-1-q}(\, V,\, \Omega(E_{-(d-\lambda)\omega_j}|_V)\,)$$

for $q = 0,\cdots,n-1$. On the other hand, by applying (3.3), we obtain the exact sequence:

$$(3.8) \quad \cdots \longrightarrow H^j(\, M,\, \Omega(E_{\lambda\omega_j})\,) \longrightarrow H^j(\, M,\, \Omega(E_{-(d-\lambda)\omega_j})\,) \longrightarrow H^j(\, V,\, \Omega(E_{-(d-\lambda)\omega_j}|_V)\,) \longrightarrow \cdots.$$

It follows from Theorem 6 that:

$$H^q(\, M,\, \Omega(E_{\lambda\omega_j})\,) = 0, \qquad \text{for } q = 0, 1, \cdots, n-1,$$
$$H^n(\, M,\, \Omega(E_{\lambda\omega_j})\,) \neq 0,$$

$$H^q(\ M,\ \Omega(E_{-(d-\lambda)\omega_j})\ ) = 0, \qquad \text{for any } q, \text{ if } d < \lambda,$$

$$H^q(\ M,\ \Omega(E_{-(d-\lambda)\omega_j})\ ) = 0, \qquad \text{for } q > 0, \text{ if } d \geqq \lambda,$$

$$H^0(\ M,\ \Omega(E_{-(d-\lambda)\omega_j})\ ) \neq 0, \qquad \text{if } d \geqq \lambda.$$

Hence the theorem is obtained by (3.7) and (3.8).

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ON THE HYPERSURFACES OF HERMITIAN SYMMETRIC SPACES OF COMPACT TYPE II

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1. Introduction.

Let M be an irreducible Hermitian symmetric space of compact type and let L be a holomorphic line bundle over M. Denote by $\Omega^p(L)$ the sheaf of germs of L-valued holomorphic p-forms on M. In the previous paper [1] we have studied the cohomology groups $H^q(M, \Omega^p(L))$ of M if M is of type BDI, EIII or EVII. This note is the continuation of [1], and we retain the notations introduced in [1]. In this note we study the cohomology groups $H^q(M, \Omega^p(L))$ of M of type AIII, CI or DIII and show the following theorem.

Theorem. *Let M be an irreducible Hermitian symmetric space of compact type but not a complex projective space or a complex quadric of even dimension. Let V be a hypersurface of M whose degree ≥ 2. Then*

$$H^0(V, \Theta) = (0)$$

where Θ is the sheaf of germs of holomorphic vector fields on V.

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2. Proof of the theorem.

Theorem 8 and Lemma 3 in the previous paper [1] is incorrect. The followings are true.

Theorem 8. *Let M be an irreducible Hermitian symmetric space of type EIII, EVII or a complex quadoric of odd dimension (resp. a complex quadoric of even dimension), and let V be a hypersurface of M whose degree is d. Then*

$$H^0(V, \mathcal{O}) = (0) \quad \text{if} \quad d \geqq 2 \ (\text{resp. } 3).$$

Lemma 3. *Let M be an n-dimensional irreducible Hermitian symmetric space of compact type EIII, EVII or a complex quadoric of odd dimension (resp. a complex quadoric of even dimension). Then*

$$H^q(M, \Omega^p(E_{-k\omega_j})) = (0), \quad H^{q+1}(M, \Omega^p(E_{-(k-d)\omega_j})) = (0)$$

for $p+q = n+1$, $k = pd-\lambda$ *if* $2 \leqq p \leqq n-1$ *and* $d \geqq 2$ *(resp. 3).*

From the above theorem we may assume that M is of type AIII, CI or DIII but not a complex projective space or a complex quadoric. If we prove the following proposition, we get the above theorem in the same way as in the proof of Theorem 8 in [1].

Proposition 1. If $d \geqq 2$

$$H^q(M, \Omega^p(E_{-k\omega_j})) = (0), \quad H^{q+1}(M, \Omega^p(E_{-(k-d)\omega_j})) = (0),$$

for $p+q \geqq n+1$, $k = pd-\lambda$.

By Theorems 1 and 2 in [1], we get Proposition 1 if we prove the following inequalities:

$$\#\{\beta \in \Delta(\mathfrak{n}^+)\ ;\ (\sigma\delta + (dn(\sigma)-\lambda)\omega_j, \beta) < 0\} < n+1-n(\sigma),$$
$$\#\{\beta \in \Delta(\mathfrak{n}^+)\ ;\ (\sigma\delta + (dn(\sigma)-d-\lambda)\omega_j, \beta) < 0\} < n+2-n(\sigma),$$

for $\sigma \in W^1$ and $d \geqq 2$.

Since $(\omega_j, \beta) > 0$ for $\beta \in \Delta(\mathfrak{n}^+)$, we only have to prove the inequalities in the case of $d = 2$. Recall that $\#\Delta(\mathfrak{n}^+) = n$. We can restate the inequalities, in the case of d = 2, sa follows:

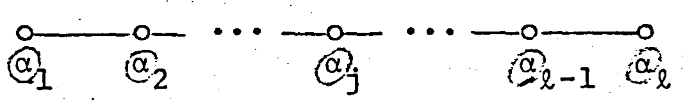
Proposition 2. For $\sigma \in W^1$

$$\#\{\beta \in \Delta(\mathfrak{n}^+)\ ;\ (\sigma\delta, \beta) \geqq ((\lambda - 2n(\sigma))\omega_j, \beta)\} > n(\sigma)-1,$$

$$\#\{\beta \in \Delta(\mathfrak{n}^+)\ ;\ (\sigma\delta, \beta) \geqq ((\lambda + 2 - 2n(\sigma))\omega_j, \beta)\} > n(\sigma)-2.$$

In the following we shall prove Proposition 2 in each case.

2.1. The case that M is of type AIII but not a complex projective space, that is $M = SU(\ell+1)/S(U(j)\times U(\ell+1-j))$, $\ell \geqq 3$ and $2 \leqq j \leqq \ell-1$. We immedietely see that $n = j(\ell+1-j)$ and $\lambda = \ell+1$. The Dynkin diagram of Π is as follows:



Let $\{\varepsilon_i\ ;\ 1 \leqq i \leqq \ell+1\}$ be a usual basis of $R^{\ell+1}$. Then we have:

$$\mathfrak{h}_0 = \{ \sum_{i=1}^{\ell+1} a_i \varepsilon_i \in R^{\ell+1};\ \sum_{i=1}^{\ell+1} a_i = 0 \},$$

$$\Delta = \{ \varepsilon_i - \varepsilon_k;\ 1 \leqq i,\ k \leqq \ell+1,\ i \neq k \},$$

$$\Pi = \{ \alpha_1 = \varepsilon_1 - \varepsilon_2,\ \alpha_2 = \varepsilon_2 - \varepsilon_3,\ \cdots,\ \alpha_\ell = \varepsilon_\ell - \varepsilon_{\ell+1} \},$$

$$\Delta(\mathfrak{n}^+) = \{ \varepsilon_i - \varepsilon_k;\ 1 \leqq i \leqq j < k \leqq \ell + 1 \},$$

$$2\delta = \ell\varepsilon_1 + (\ell - 2)\varepsilon_2 + (\ell - 4)\varepsilon_3 + \cdots - (\ell - 2)\varepsilon_\ell - \ell\varepsilon_{\ell+1},$$

$$\omega_j = \varepsilon_1 + \cdots + \varepsilon_j - \frac{j}{\ell+1} \sum_{i=1}^{\ell+1} \varepsilon_i .$$

An element $\sigma \in W$ acts on $R^{\ell+1}$ by $\sigma\varepsilon_i = \varepsilon_{\sigma(i)}$ for $1 \leqq i \leqq \ell + 1$, where σ in the index is a permutation of $\{ 1, 2, \cdots, \ell + 1 \}$. We represent σ by 次頁へ続く

$$\begin{pmatrix} 1 & 2 & \cdots & \ell+1 \\ \sigma(1) & \sigma(2) & \cdots & \sigma(\ell+1) \end{pmatrix}.$$

Then

$$W^1 = \left\{ \sigma \in W;\ \sigma^{-1} = \begin{pmatrix} 1 & \cdots & \ell+1 \\ \sigma^{-1}(1) & \cdots & \sigma^{-1}(\ell+1) \end{pmatrix},\ \begin{matrix} \sigma^{-1}(1) < \cdots < \sigma^{-1}(j) \\ \sigma^{-1}(j+1) < \cdots < \sigma^{-1}(\ell+1) \end{matrix} \right\}.$$

The index $n(\sigma)$ of $\sigma \in W^1$ is given by

$$n(\sigma) = \sum_{i=1}^{j} (\sigma^{-1}(i) - i)$$

(Takeuchi [2]). We see easily that

$$(\omega_j, \beta) = 1 \quad \text{for any } \beta \in \Delta(\mathfrak{n}^+),$$

$$(\sigma\delta, \varepsilon_i - \varepsilon_k) = \sigma^{-1}(k) - \sigma^{-1}(i) \quad \text{for } 1 \leqq i, k \leqq \ell+1.$$

Therefore we have to prove that the following two inequalities are true for any $\sigma \in W^1$

(1.1) $\#\{ (i,k);\ 1 \leqq i \leqq j < k \leqq \ell+1,\ \sigma^{-1}(k) - \sigma^{-1}(i) \geqq \ell+1-2n(\sigma) \} > n(\sigma)-1,$

(1.2) $\#\{ (i,k);\ 1 \leqq i \leqq j < k < \ell+1,\ \sigma^{-1}(k) - \sigma^{-1}(i) \geqq \ell+3-2n(\sigma) \} > n(\sigma)-2.$

First we prove the inequality (1.1).

Lemma 1.1. *Let $\sigma \in W^1$. If $n(\sigma) \geqq \ell+1$, the inequality (1.1) is true.*

Proof. Since $n(\sigma) \geqq \ell+1$, $\ell+1-2n(\sigma) \geqq -(\ell+1)$. There existe no pair (i, k), $i \neq k$, which satisfies

$$\sigma^{-1}(k) - \sigma^{-1}(i) < -(\ell+1).$$

Therefore

$$\#\{(i,k);\ 1 \leqq i \leqq j < k \leqq \ell+1,\ \sigma^{-1}(k)-\sigma^{-1}(i) \geqq \ell+1-2n(\sigma)\} = n.$$

From the definition of the index $n(\sigma) \leqq n$, it follows that $n > n(\sigma)-1$.

Q.E.D.

Lemma 1.2. Let $\sigma \in W^1$. Assume that $\sigma(1) \neq 1$ and $\sigma(\ell+1) \neq \ell+1$. Then $n(\sigma) \geqq \ell$.

Proof. By the assumption $\sigma^{-1}(j) = \ell+1$ and $\sigma^{-1}(i)-i \geqq 1$, $1 \leqq i \leqq j$. Therefore

$$\begin{aligned} n(\sigma) &= \sum_{i=1}^{j} (\sigma^{-1}(i)-i) \\ &= \sigma^{-1}(j) - j + \sum_{i=1}^{j-1} (\sigma^{-1}(i)-i) \\ &\geqq (\ell+1-j) + (j-1) \\ &= \ell. \end{aligned}$$

Q.E.D.

Lemma 1.3. Let $\sigma \in W^1$. Assume that $\sigma(1) \neq 1$ and $\sigma(\ell+1) \neq \ell+1$. Then the inequality (1.1) is true for σ.

Proof. By Lemmas 1.1 and 1.2 we assume that $n(\sigma) = \ell$. Then such an element σ is unique and given by

$$\sigma^{-1} = \begin{pmatrix} 1 & \cdots & j-1 & j & j+1 & j+2 & \cdots & \ell+1 \\ 2 & \cdots & j & \ell+1 & 1 & j+1 & \cdots & \ell \end{pmatrix}.$$

The pair (i, k), $1 \leqq i \leqq j < k \leqq \ell + 1$, which satisfies

$$\sigma^{-1}(k) - \sigma^{-1}(i) < \ell + 1 - 2n(\sigma) = 1 - \ell$$

is $(j, j+1)$. Hence

$\#\{(i,k);\ 1 \leqq i \leqq j < k \leqq \ell+1,\ \sigma^{-1}(k) - \sigma^{-1}(i) \geqq 1-\ell\} = n-1 > n(\sigma)-1.$

Q.E.D.

Lemma 1.4. If $j = 2$, the inequality (1.1) is true for any $\sigma \in W^1$.

Proof. From the definition of $n(\sigma)$

$$(1.3) \qquad n(\sigma) = \sigma^{-1}(1) + \sigma^{-1}(2) - 3.$$

If $n(\sigma) = 0$, the inequation (1.1) is clearly true. Let $n(\sigma) = 1$. Then $\sigma^{-1}(1) = 1$, $\sigma^{-1}(2) = 3$ and

$$\sigma^{-1}(\ell+1) - \sigma^{-1}(1) = \ell > \ell + 1 - 2n(\sigma).$$

It follows that the inequality (1.1) is true. Let $n(\sigma) = 2$. It is easy to see that the inequality (1.1) is true.

By Lemma 1.1 we have already seen that if $n(\sigma) \geqq \ell + 1$ the inequality is true. Hence we only have to show that (1.1) is true under the following condition:

$$(1.4) \qquad 5 < \sigma^{-1}(1) + \sigma^{-1}(2) < \ell + 4.$$

By (1.3)

$$\ell + 1 - 2n(\sigma) = \ell + 7 - 2(\sigma^{-1}(1) + \sigma^{-1}(2)).$$

Since $\sigma^{-1}(k) \geqq k - 2$ for $2 < k \leqq \ell + 1$,

$$\#\{k;\ 2<k\leqq\ell+1,\ \sigma^{-1}(k) - \sigma^{-1}(1) \geqq \ell+7-2(\sigma^{-1}(1)+\sigma^{-1}(2))\}$$

$$\geqq \min\{\sigma^{-1}(1) + 2\sigma^{-1}(2) - 7,\ \ell - 1\}.$$

Similarly

$$\#\{\ k;\ 2<k\leqq \ell+1,\ \sigma^{-1}(k)-\sigma^{-1}(2)\ \geqq\ \ell+7-2(\sigma^{-1}(1)+\sigma^{-1}(2))\}$$
$$\geqq \min\{\ 2\sigma^{-1}(1)\ +\sigma^{-1}(2)\ -\ 7,\ \ell\ -\ 1\ \}.$$

Therefore

$$\#\{(i,k);\ 1\leqq i\leqq 2<k\leqq \ell+1,\ \sigma^{-1}(k)-\sigma^{-1}(i)\ \geqq\ \ell+1-2n(\sigma)\}$$
$$\geqq \min\{\ 3(\sigma^{-1}(1)+\sigma^{-1}(2))-14,\ \ell+2\sigma^{-1}(1)+\sigma^{-1}(2)-8,\ 2\ell-2\ \}.$$

It is easy to see that $3(\sigma^{-1}(1)+\sigma^{-1}(2))-14$, $\ell+2\sigma^{-1}(1)+\sigma^{-1}(2)-8$ and $2\ell-2$ are both larger than $n(\sigma)-1=\sigma^{-1}(1)+\sigma^{-1}(2)-4$ under the condition (1.4).

Q.E.D.

We get the following lemma in the similar way as above.

<u>Lemma 1.5.</u> <u>If $j=\ell-1$, the inequality (1.1) is true for any $\sigma \in W^1$.</u>

We shall prove that the inequality (1.1) is true for any $\sigma \in W^1$ by using induction on ℓ. If $\ell = 3$ so that $j = 2$, it follows, by Lemma 1.4, our assertion is true.

Let $\ell = \ell_0 \geqq 4$. We can assume that $3 \leqq j = j_0 \leqq \ell_0 - 2$ and whether $\sigma(1) = 1$ or $\sigma(\ell_0+1) = \ell_0+1$ by Lemmas 1.3, 1.4 and 1.5.

Case 1: $\sigma(1) = 1$. Define the element τ of W^1, which is considered as an element of W^1 for $\ell = \ell_0 - 1$ and $j = j_0 - 1$, by

$$\tau^{-1} = \begin{pmatrix} 1 & 2 & \cdots & \ell_0 \\ \sigma^{-1}(2)-1 & \sigma^{-1}(3)-1 & \cdots & \sigma^{-1}(\ell_0+1)-1 \end{pmatrix}.$$

We immedietly see that $2 \leqq j \leqq \ell-2$ and $n(\tau) = n(\sigma)$. By the assumption of the induction,

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$$\#\{\ (i,k);\ 1\leqq i\leqq j_0-1<k\leqq \ell_0,\ \hat{\tau}^{-1}(k)-\hat{\tau}^{-1}(i)\ \geqq \ell_0-2n(\hat{\tau})\}\ >\ n(\hat{\tau})\ -\ 1.$$

Hence

$$(1.5)\quad \#\{(i,k);\ 2\leqq i\leqq j_0<k\leqq \ell_0+1,\ \hat{\sigma}^{-1}(k)-\hat{\sigma}^{-1}(i)\ \geqq\ \ell_0-2n(\hat{\sigma})\}\ >\ n(\hat{\sigma})\ -\ 1.$$

For any k, $j_0\leqq k\leqq \ell_0+1$, if there existes i, $2\leqq i\leqq j_0$, which satisfies the following:

$$\hat{\sigma}^{-1}(k)\ -\hat{\sigma}^{-1}(i)\ =\ \ell_0-\ 2n(\hat{\sigma}),$$

such an integer i is unique and

$$\hat{\sigma}^{-1}(k)\ -\ \hat{\sigma}^{-1}(1)\ \geqq\ \ell_0+\ 1\ -\ 2n(\hat{\sigma}).$$

Hence (1.5) leads to (1.1).

Case 2: $\hat{\sigma}(\ell_0+1)\ =\ \ell_0+1$. Define the element $\hat{\tau}\in W^1$, which is considered (as an element of W^1) for $\ell\ =\ \ell_0\ -\ 1$ and $j\ =\ j_0$, by

$$\hat{\tau}^{-1}\ =\ \begin{pmatrix} 1 & \cdots & \ell_0 \\ \hat{\sigma}^{-1}(1) & \cdots & \hat{\sigma}^{-1}(\ell_0) \end{pmatrix}.$$

Then $n(\hat{\tau})\ =\ n(\hat{\sigma})$. By the assumption of the induction, $3\leq j\leq \ell-1$ and

$$(1.6)\quad \#\{\ (i,k);\ 1\leqq i\leqq j_0<k\leqq \ell_0,\ \hat{\sigma}^{-1}(k)-\hat{\sigma}^{-1}(i)\ \geqq \ell_0-2n(\hat{\sigma})\}\ >\ n(\hat{\sigma})\ -\ 2.$$

For any i, $1\leqq i\leqq j_0$, if there existes k, $j_0<k\leqq \ell_0$, which satisfies the following:

$$\hat{\sigma}^{-1}(k)\ -\ \hat{\sigma}^{-1}(i)\ =\ \ell_0\ -\ 2n(\hat{\sigma}),$$

such an integer k is unique and

$$\hat{\sigma}^{-1}(\ell_0+1)\ -\ \hat{\sigma}^{-1}(i)\ \geqq\ \ell_0+\ 1\ -\ 2n(\hat{\sigma}).$$

Hence (1.5) leads to (1.1).

Thus we have proved that the inequality (1.1) is true for any $\sigma \in W^1$.

In the following we shall prove that the inequality (1.2) is true for any $\sigma \in W^1$.

Lemma 1.6. *Let $\sigma \in W^1$. If $n(\sigma) \geqq \ell+1$, the inequality (1.2) is true.*

Proof. Since $n(\sigma) \geqq \ell+1$

$$\ell + 3 - 2n(\sigma) \leqq 1 - \ell.$$

If there existe a pair (i, k), $i \neq k$, which satisfies

$$\sigma^{-1}(k) - \sigma^{-1}(i) < 1 - \ell,$$

such a pair is unique. Therefore

$$\#\{ (i,k);\ 1 \leqq i \leqq j < k \leqq \ell+1,\ \sigma^{-1}(k) - \sigma^{-1}(i) \geqq \ell+1-2n(\sigma) \} \geqq n-1 > n(\sigma)-2.$$

Q.E.D.

Lemma 1.7. *Let $\sigma \in W^1$. Assume that $\sigma(1) \neq 1$ and $\sigma(\ell+1) \neq \ell+1$. Then the inequality (1.2) is true.*

Proof. By Lemmas 1.2 and 1.6 we may assume that $n(\sigma) = \ell$. Such an element σ is unique and represented by

$$\sigma^{-1} = \begin{pmatrix} 1 & \cdots & j-1 & j & j+1 & j+2 & \cdots & \ell+1 \\ 2 & \cdots & j & \ell+1 & 1 & j+1 & \cdots & \ell \end{pmatrix}.$$

The pairs (i,k), $1 \leqq i \leqq j < k \leqq \ell+1$, which satisfy

$$\sigma^{-1}(k) - \sigma^{-1}(i) < \ell + 3 - 2n(\sigma) = 3 - \ell$$

are at most 2. Therefore

$$\#\{ (i,k);\ 1 \leqq i \leqq j < k \leqq \ell+1,\ \sigma^{-1}(k) - \sigma^{-1}(i) \geqq \ell+3-2n(\sigma) \} \geqq n - 2.$$

Since $n(\sigma) = \ell$, $n(\sigma) < n$. It follows that (1.2) is true.

Q.E.D.

Lemma 1.8. If $j = 2$, the inequality (1.2) is true for any $\sigma \in W^1$.

Proof. It is easy to see that (1.2) is true if $n(\sigma) \leqq 3$. By Lemma 1.6 and (1.3), we only have to show that (1.2) is true under the condition:

$$(1.7) \qquad 6 < \sigma^{-1}(1) + \sigma^{-1}(2) < \ell + 4.$$

We get the following inequality in the same way as in the proof of Lemma 1.4.

$$\#\{ (i,k);\ 1 \leqq i \leqq 2 < k \leqq \ell+1,\ \sigma^{-1}(k) - \sigma^{-1}(i) \leqq \ell+3-2n(\sigma) \}$$
$$\geqq \min\{ 3(\sigma^{-1}(1)+\sigma^{-1}(2))-18,\ \ell+2\sigma^{-1}(1)+\sigma^{-1}(2)-10,\ 2\ell-2 \}.$$

It is easy to see that $3(\sigma^{-1}(1)+\sigma^{-1}(2))-18$, $\ell+2\sigma^{-1}(1)+\sigma^{-1}(2)-10$ and $2\ell-2$ are both larger than $n(\sigma)-2 = \sigma^{-1}(1)+\sigma^{-1}(2)-4$ under the condition (1.7).

Q.E.D.

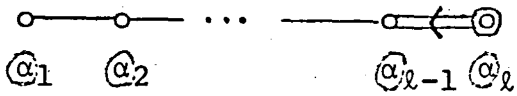
We get the following lemma in the similar way as above.

Lemma 1.9. If $j = \ell - 1$, the inequality (1.2) is true for any $\sigma \in W^1$.

From Lemmas 1.7, 1.8 and 1.9, we can prove that the inequality (1.2) is true for any $\sigma \in W^1$ in the same way as in the proof of the inequality (1.1).

2.2. The case that M is of type CI, that is $M = Sp(\ell)/U(\ell)$. If $\ell = 1$, $M = P_1(C)$. If $\ell = 2$, M is a complex quadoric of dimension 3. Hence we assume that $\ell \geqq 3$. 次頁へ続く.

In this case $n = \frac{1}{2}\ell(\ell+1)$ and $\lambda = \ell + 1$. The Dynkin diagram of Π is as follows:



where α_ℓ◎ shows ~~that~~ $\alpha_j = \alpha_\ell$. Let $\{ \varepsilon_i ;\ 1 \leqq i \leqq \ell \}$ be the basis of $\mathfrak{h}_0$ which satisfies ~~that~~ $(\varepsilon_i , \varepsilon_j) = \delta_{ij}$. Then we have:

$$\Delta = \{ \pm 2\varepsilon_i ;\ 1 \leqq i \leqq \ell,\ \pm \varepsilon_i \pm \varepsilon_j ;\ 1 \leqq i < j \leqq \ell \},$$

$$\Pi = \{ \alpha_1 = \varepsilon_1 - \varepsilon_2 , \cdots , \alpha_{\ell-1} = \varepsilon_{\ell-1} - \varepsilon_\ell , \alpha_\ell = 2\varepsilon_\ell \},$$

$$\Delta(\mathfrak{n}^+) = \{ 2\varepsilon_i ;\ 1 \leqq i \leqq \ell,\ \varepsilon_i + \varepsilon_j ;\ 1 \leqq i < j \leqq \ell \},$$

$$\delta = \ell\varepsilon_1 + (\ell-1)\varepsilon_2 + \cdots + \varepsilon_\ell ,$$

$$\omega_\ell = \varepsilon_1 + \cdots + \varepsilon_\ell .$$

An element $\sigma \in W$ acts on $\mathfrak{h}_0$ by $\sigma\varepsilon_i = \pm \varepsilon_{\bar{\sigma}(i)}$ for $1 \leqq i \leqq \ell$, where $\bar{\sigma}$ in the index is a permutation of $\{ 1, 2, \cdots , \ell \}$. We denote the element $\sigma \in W$ by the symbol

$$\begin{pmatrix} 1 & 2 & \cdots & \ell \\ \pm\bar{\sigma}(1) & \pm\bar{\sigma}(2) & \cdots & \pm\bar{\sigma}(\ell) \end{pmatrix}$$

Then

$$W^1 = \left\{ \sigma \in W ;\ \sigma^{-1} = \begin{pmatrix} 1 & \cdots & r & r+1 & \cdots & \ell \\ \bar{\sigma}^{-1}(1) & \cdots & \bar{\sigma}^{-1}(r) & -\bar{\sigma}^{-1}(r+1) & \cdots & -\bar{\sigma}^{-1}(\ell) \end{pmatrix} \right.$$

$$\left. \text{for } 0 \leqq r \leqq \ell,\ \bar{\sigma}^{-1}(1) < \cdots < \bar{\sigma}^{-1}(r),\ \bar{\sigma}^{-1}(r+1) > \cdots > \bar{\sigma}^{-1}(\ell) \right\}.$$

The index $n(\sigma)$ of $\sigma \in W^1$ is given by

$$n(\sigma) = \sum_{i=1}^{r} (\sigma^{-1}(i) - i) + {}_{\ell+1-r}C_2$$

(Takeuchi [2]). We see easily that

$$(\omega_\ell, \beta) = 2 \qquad \text{for any } \beta \in \Delta(\mathfrak{n}^+),$$

$$(\sigma\delta, \varepsilon_i) = \begin{cases} (\ell + 1 - \hat{\sigma}^{-1}(i)) & \text{if } 1 \leqq i \leqq r \\ -(\ell + 1 - \hat{\sigma}^{-1}(i)) & \text{if } r < i \leqq \ell. \end{cases}$$

Therefore we have to prove that the following inequalities are true for any $\sigma \in W^1$

(2.1) $\#\{ \beta \in \Delta(\mathfrak{n}^+) ; (\sigma\delta, \beta) \geqq 2(\ell+1) - 4n(\sigma) \} > n(\sigma) - 1,$

(2.2) $\#\{ \beta \in \Delta(\mathfrak{n}^+) ; (\sigma\delta, \beta) \geqq 2(\ell+3) - 4n(\sigma) \} > n(\sigma) - 2.$

Since $(\sigma\delta, \beta) \geqq -2\ell$, $\beta \in \Delta(\mathfrak{n}^+)$, we immedietely see that if $n(\sigma) \geqq \ell + 1$ (resp. $\ell + 2$), the inequality (2.1) (resp.(2.2)) is true for any $\sigma \in W^1$.

__Lemma 2.1.__ *Let $\sigma \in W^1$. If $n(\sigma) \geqq \ell$, the inequality (2.1) is true.*

__Proof.__ From the above notice we can assume that $n(\sigma) = \ell$. In this case

$$2(\ell+1) - 4n(\sigma) = 2 - 2\ell.$$

It is easy to see that

$$\#\{ \beta \in \Delta(\mathfrak{n}^+) ; (\sigma\delta, \beta) < 2 - 2\ell \} \leqq 2.$$

Hence

$$\#\{\beta \in \Delta(\mathfrak{n}^+)\,;\ (\sigma\delta, \beta) \geqq 2 - 2\ell\} \leqq {}_{\ell+1}C_2 - 2 > \ell - 1 = n(\sigma) - 1.$$

Q.E.D.

Lemma 2.2. Let $\sigma \in W^1$. If $n(\sigma) \geqq \ell$, the inequality (2.2) is true.

Proof. If $n(\sigma) \geqq \ell + 1$, the inequality is ture in the same way as above. Therefore we assume that $n(\sigma) = \ell$.

Case 1 : $\ell = 3$. If $r = 0$, $n(\sigma) = 6 \neq 3$. Hence $r > 0$, and σ is one of the following elements:

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & -3 & -2 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & -1 \end{pmatrix}.$$

In each case (2.2) is true.

Case 2 : $\ell = 4$. If $r \leqq 1$, $n(\sigma) \geqq 6 > 4$. Hence $r \geqq 2$

It follows that $(\sigma\delta, 2\varepsilon_1)$, $(\sigma\delta, \varepsilon_1 + \varepsilon_2)$ and $(\sigma\delta, 2\varepsilon_2)$ are larger than $2(\ell+3) - 4n(\sigma) = -2$.

On the other hand $n(\sigma) - 2 = 2$. Therefore (2.2) is true.

Case 3 : $\ell \geqq 5$. If $\beta \in \Delta(\mathfrak{n}^+)$ satisfies

$$(\sigma\delta, \beta) < 2(\ell+3) - 4n(\sigma) = 6 - 2\ell,$$

β is one of the following 12 elements:

$$2\varepsilon_\ell,\ \varepsilon_\ell + \varepsilon_{\ell-1},\ \varepsilon_\ell + \varepsilon_{\ell-2},\ \varepsilon_\ell + \varepsilon_{\ell-3},\ \varepsilon_\ell + \varepsilon_{\ell-4},\ \varepsilon_\ell + \varepsilon_{\ell-5},$$

$$2\varepsilon_{\ell-1},\ \varepsilon_{\ell-1} + \varepsilon_{\ell-2},\ \varepsilon_{\ell-1} + \varepsilon_{\ell-3},\ \varepsilon_{\ell-1} + \varepsilon_{\ell-4},\ 2\varepsilon_{\ell-2},\ \varepsilon_{\ell-2} + \varepsilon_{\ell-3}.$$

On the other hand

$$
\begin{aligned}
&{}_{\ell+1}C_2 - 12\ (\ell-2)\\
&= \tfrac{1}{2}\{\ell(\ell+1) - 20 - 2\ell\}\\
&= \tfrac{1}{2}(\ell^2 - \ell - 20)\\
&= \tfrac{1}{2}(\ell+4)(\ell-5) \geqq 0.
\end{aligned}
$$

The equality holds only in the case $\ell = 5$. But if $\ell = 5$, $\beta \neq \varepsilon_\ell + \varepsilon_{\ell-5}$ for $\beta \in \Delta(\mathfrak{n}^+)$. Therefore the inequality is true.

Q.E.D.

Lemma 2.3. Let $\sigma \in W$. If $\sigma(1) \neq 1$, $n(\sigma) \geqq \ell$.

Proof. By the assumption,

$$\sum_{i=1}^{r} (\sigma(i) - i) \geqq r.$$

Hence

$$
\begin{aligned}
&n(\sigma) - \ell\\
&\geqq r + {}_{\ell+1-r}C_2 - \ell\\
&= \tfrac{1}{2}(\ell-r-1)(\ell-r) \geqq 0.
\end{aligned}
$$

Q.E.D.

We shall prove that the inequality (2.1) is true for any $\sigma \in W^1$ by using induction on ℓ. Let $\ell = 3$. If $n(\sigma) \geqq 3$. the inequatlity is true by Lemma 2.1. If $n(\sigma) = 0$, the inequality is also true for $n(\sigma) - 1 < 0$. If $n(\sigma) = 1$ (resp.2), $\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & -3 \end{pmatrix}$ $\left(\text{resp. } \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & -2 \end{pmatrix}\right)$, and (2.1) is true.

Let $\ell = \ell_0 > 3$. By Lemmas 2.1 and 2.3, we may assume that $\sigma(1) = 1$. Define the element $\tau \in W^1$, which is considered for $\ell = \ell_0 - 1$, by

$$\tau^{-1} = \begin{pmatrix} 1 & \cdots & r-1 & r & \cdots & \ell_0-1 \\ \sigma^{-1}(2)-1 & \cdots & \sigma^{-1}(r)-1 & -(\sigma^{-1}(r+1)-1) & \cdots & -(\sigma^{-1}(\ell_0)-1) \end{pmatrix}.$$

We easily see that $n(\tau) = n(\sigma)$. By the assumption of the induction, $\#\{\varepsilon_i + \varepsilon_j;\ 1 \leqq i,j \leqq \ell_0 - 1,\ (\tau\delta', \varepsilon_i + \varepsilon_j) \geqq 2 - 4n(\tau)\} > n(\tau) - 1$, where $\delta' = (\ell_0-1)\varepsilon_1 + (\ell_0-2)\varepsilon_2 + \cdots + \varepsilon_{\ell_0-1}$. It follows, by the fact that $(\tau\delta', \varepsilon_{i-1}) = (\sigma\delta, \varepsilon_i)$ for $2 \leqq i \leqq \ell_0$, that

(2.3) $\#\{\varepsilon_i+\varepsilon_j;\ 2\leqq i,j\leqq \ell_0,\ (\sigma\delta, \varepsilon_i+\varepsilon_j) \geqq 2\ell - 4n(\sigma)\} > n(\sigma) - 1.$

<u>Lemma 2.4</u>. <u>Let</u>

$s = \#\{\varepsilon_i;\ 2 \leqq i \leqq \ell_0, \exists\varepsilon_j,\ 2 \leqq j \leqq \ell_0,\ j \neq i$, such that $(\sigma\delta, \varepsilon_i + \varepsilon_j) = 2\ell - 4n(\sigma)$ or $2\ell + 1 - 4n(\sigma)\}$.

<u>Then</u>

$\#\{\varepsilon_i + \varepsilon_j;\ 2 \leqq i < j \leqq \ell_0,\ (\sigma\delta, \varepsilon_i + \varepsilon_j) = 2\ell - 4n(\sigma)$ or $2\ell + 1 - 4n(\sigma)\} \leqq s - 1.$

<u>Proof.</u> Let ε_i, $2 \leqq i \leqq \ell_0$, satisfy the condition that there exists ε_j, $2 \leqq j \leqq \ell_0$, $j \neq i$, such that $(\sigma\delta, \varepsilon_i + \varepsilon_j) = 2\ell - n(\sigma)$ or $2\ell + 1 - n(\sigma)$. For the element ε_i,

(2.4) $\#\{\varepsilon_i + \varepsilon_j;\ 2 \leqq j \leqq \ell_0,\ j \neq i,\ (\sigma\delta, \varepsilon_i + \varepsilon_j) = 2\ell - 4n(\sigma)$ or $2\ell + 1 - 4n(\sigma)\} \leqq 2.$

In this way we find at most 2s ordered pairs (i, j), $2 \leqq i \leqq \ell_0$, $j \neq i$, which satisfies $(\sigma\delta, \varepsilon_i + \varepsilon_j) = 2\ell - 4n(\sigma)$ or $2\ell + 1 - 4n(\sigma)$. On the other hand the distinct pairs (i, j) and (j, i) induce the same element $\varepsilon_i + \varepsilon_j$. Therefore

(2.5) $\#\{ \varepsilon_i + \varepsilon_j ;\ 2 \leqq i < j \leqq \ell_0,\ (\sigma\delta, \varepsilon_i + \varepsilon_j) = 2\ell - 4n(\sigma)$ or $2\ell + 1 - 4n(\sigma) \} \leqq s$,

and the equality holds if and only if the equality in (2.4) holds for any ε_i, $2 \leqq i \leqq \ell_0$.

Define the integer i_0 (resp. i_m) by

$$\min \text{ (resp. } \max \text{) } \{ i;\ 2 \leqq i \leqq \ell_0,\ \exists j,\ 2 \leqq j \leqq \ell_0,\ j \neq i \text{ such that }$$
$$(\sigma\delta, \varepsilon_i + \varepsilon_j) = \ell - 2n(\sigma) \text{ or } \ell + 1 - 2n(\sigma) \},$$

If the equality in (2.4) holds for ε_{i_0} and ε_{i_m}, there exiset the integers i and j such that

$$(\sigma\delta, \varepsilon_{i_0} + \varepsilon_j) = \ell - 2n(\sigma) \quad \text{or} \quad \ell + 1 - 2n(\sigma),$$

$$(\sigma\delta, \varepsilon_i + \varepsilon_{i_m}) = \ell - 2n(\sigma) \quad \text{or} \quad \ell + 1 - 2n(\sigma),$$

$$i_0 < i \quad \text{and} \quad j < i_m.$$

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Hence

$$(\sigma\delta, \varepsilon_i + \varepsilon_{i_m}) \leqq (\sigma\delta, \varepsilon_{i_0} + \varepsilon_j) - 2.$$

This is impossible, and therefore the equality does not hold.

Q.E.D.

Let ε_i, $2 \leqq i \leqq \ell_0$, satisfy that there exists ε_j, $2 \leqq j \leqq \ell_0$, $j \neq i$, such that

$$(\sigma\delta, \varepsilon_i + \varepsilon_j) = 2\ell - 4n(\sigma) \text{ or } 2\ell + 1 - 4n(\sigma).$$

For this element ε_i,

$$(\sigma\delta, \varepsilon_i + \varepsilon_1) \geqq 2\ell + 2 - 4n(\sigma),$$

in all but the following case:

$$(\sigma\delta, \varepsilon_i + \varepsilon_2) = 2\ell - 4n(\sigma).$$

Therefore

$$\#\{\varepsilon_i + \varepsilon_j ;\ 1 \leqq i < j \leqq \ell_0,\ (\sigma\delta, \varepsilon_i + \varepsilon_j) \leqq 2\ell + 2 - 4n(\sigma)\}$$

$$\geqq \#\{\varepsilon_i + \varepsilon_j ;\ 2 \leqq i < j \leqq \ell_0,\ (\sigma\delta, \varepsilon_i + \varepsilon_j) \leqq 2\ell - 4n(\sigma)\}.$$

There exists at most one element ε_i, $2 \leqq i \leqq \ell_0$, such that $(\sigma\delta, 2\varepsilon_i) = 2\ell - 4n(\sigma)$ or $2\ell + 1 - 4n(\sigma)$.

If such ε_i exists,

$$(\sigma\delta, 2\varepsilon_1) \geqq 2\ell + 2 - 4n(\sigma).$$

Therefore the inequality (2.1) is true.

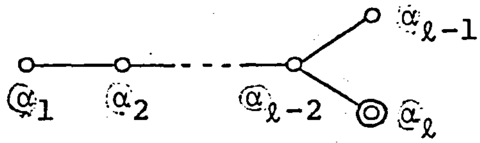
Thus we have proved that the inequality (2.1) is true for any $\sigma \in W^1$.

From Lemmas 2.2 and 2.3, we can prove that the inequality (2.2) is true for any $\sigma \in W^1$ in the same way as above.

If $\ell = 4$, M is a complex quadoric of dimension 6.

2.3. The case that M is of type DIII, that is $M = SO(2\ell)/U(\ell)$

If $\ell = 3$, $M = P_3(C)$. Hence we assume that $\ell \geqq 5$. In this case $n = \frac{1}{2}\ell(\ell-1)$ and $\lambda = 2\ell - 2$. The Dinkin diagram of Π is as follows:



where α_ℓ ◎ shows ~~that~~ $\alpha_j = \alpha_\ell$. Let $\{ \varepsilon_i ; 1 \leqq i \leqq \ell \}$ be the basis of h_0 which satisfies that $(\varepsilon_i, \varepsilon_j) = \delta_{ij}$. Then we have:

$$\Delta = \{ \pm\varepsilon_i \pm \varepsilon_j ; 1 \leqq i < j \leqq \ell \},$$

$$\Pi = \{ \alpha_1 = \varepsilon_1 - \varepsilon_2, \cdots, \alpha_{\ell-1} = \varepsilon_{\ell-1} - \varepsilon_\ell, \alpha_\ell = \varepsilon_{\ell-1} + \varepsilon_\ell \},$$

$$\Delta(n^+) = \{ \varepsilon_i + \varepsilon_j ; 1 \leqq i < j \leqq \ell \},$$

$$\delta = (\ell-1)\varepsilon_1 + (\ell-2)\varepsilon_2 + \cdots + \varepsilon_{\ell-1},$$

$$\omega = \tfrac{1}{2}(\varepsilon_1 + \cdots + \varepsilon_\ell).$$

An element $\sigma \in W$ acts on h by $\sigma\varepsilon_i = \pm\varepsilon_{\bar\sigma(i)}$ for $1 \leqq i \leqq \ell$, where $\bar\sigma$ ~~in the index~~ is a permutation of $\{ 1, 2, \cdots, \ell \}$. We denote the element $\sigma \in W$ by the symbol

$$\begin{pmatrix} 1 & 2 & \cdots & \ell \\ \pm\bar\sigma(1) & \pm\bar\sigma(2) & \cdots & \pm\bar\sigma(\ell) \end{pmatrix}.$$

Then

$$W^1 = \left\{ \sigma \in W ; \sigma^{-1} = \begin{pmatrix} 1 & \cdots & r & r+1 & \cdots & \ell \\ \bar\sigma^{-1}(1) & \cdots & \bar\sigma^{-1}(r) & -\bar\sigma^{-1}(r+1) & \cdots & -\bar\sigma^{-1}(\ell) \end{pmatrix}, \right.$$

$$\left. \ell - r \text{ is even}, \bar\sigma^{-1}(1) < \cdots < \bar\sigma^{-1}(r), \bar\sigma^{-1}(r+1) > \cdots > \bar\sigma^{-1}(\ell) \right\}.$$

The index $n(\sigma)$ of $\sigma \in W^1$ is given by

$$n(\sigma) = \sum_{i=1}^{r} (\bar{\sigma}^{-1}(i) - i) + {}_{\ell-r}C_2$$

(Takeuchi [2]). We see easily that

$$(\omega_\ell, \beta) = 1 \quad \text{for any} \quad \beta \in \Delta(n^+),$$

$$(\sigma\delta, \varepsilon_i) = \begin{cases} \ell - \bar{\sigma}^{-1}(i) & \text{if } 1 \leqq i \leqq r \\ -(\ell - \bar{\sigma}^{-1}(i)) & \text{if } r < i \leqq \ell. \end{cases}$$

Therefore we have to prove that the following inequalities are true for any $\sigma \in W^1$.

(3.1) $\#\{ \beta \in \Delta(n^+) ; (\sigma\delta, \beta) \geqq 2\ell-2-2n(\sigma) \} > n(\sigma)-1,$

(3.2) $\#\{ \beta \in \Delta(n^+) ; (\sigma\delta, \beta) \geqq 2\ell-2n(\sigma) \} > n(\sigma)-2.$

<u>Lemma 3.1.</u> *Let $\sigma \in W^1$. If $n(\sigma) \geqq 2\ell-3$, the inequality (3.1) is true.*

<u>Proof.</u> By the assumption $2\ell-2-2n(\sigma) \leqq 4-2\ell$. Let β be an element of $\Delta(n^+)$ which satisfies that

$$(\sigma\delta, \beta) < 4-2\ell,$$

then $\beta = \varepsilon_{\ell-1} + \varepsilon_\ell$. Therefore

$$\#\{ \beta \in \Delta(n^+) ; (\sigma\delta, \beta) \geqq 2\ell-2-2n(\sigma) \} \geqq n-1.$$

If the equality holds, $n(\tilde{\sigma}) = 2\ell-3$ and $n - n(\tilde{\sigma}) = \frac{1}{2}(\ell-2)(\ell-3) > 0.$

Q.E.D.

Lemma 3.2. Let $\sigma \in W^1$. If $n(\sigma) \geqq 2\ell-3$, the inequality (3.2) is true.

Proof. If $n(\sigma) \geqq 2\ell-2$, the inequality is true in the same way as above. Therefore we assume that $n(\sigma) = 2\ell-3$. The number of the elements $\beta \in \Delta(n^+)$ such that

$$(\sigma\delta, \beta) < 2\ell-2n(\sigma) = 6-2\ell$$

is at most 4. Since $\ell \geqq 5$,

$$(n - 4) - (n(\sigma) - 2) = \frac{1}{2}\ell(\ell - 1) - 4 - 2\ell - 1 = \frac{1}{2}\ell(\ell - 5) + 1 > 0.$$

Q.E.D.

Lemma 3.3. If $\sigma^{-1}(1) \geqq 3$, then $n(\sigma) \geqq 2\ell-3$.

Proof By the assumption

$$\sum_{i=1}^{r}(\sigma^{-1}(i) - i) \geqq 2r.$$

It follows that

$$\begin{aligned} & n(\sigma) - (2\ell - 3) \\ \geqq\ & 2r + {}_{\ell-r}C_2 - (2\ell - 3) \\ =\ & \frac{1}{2}(\ell - r - 2)(\ell - r - 3) \geqq 0. \end{aligned}$$

Q.E.D.

We prove that the inequality (3.1) is true for all $\sigma \in W^1$ by using induction on ℓ. If $\ell = 5$, we easily see that the inequation is true.

Let $\ell = \ell_0 > 5$. By Lemmas 3.1 and 3.3, we can assume that $\sigma^{-1}(1) = 1$ or 2.

Case 1 : $\sigma^{-1}(1) = 1$. Define the element $\tau \in W^1$, which is considered for $\ell = \ell_0 - 1$, by

$$\tau^{-1} = \begin{pmatrix} 1 & \cdots & r-1 & r & \cdots & \ell-1 \\ \sigma^{-1}(2)-1 & \cdots & \sigma^{-1}(r)-1 & -(\sigma^{-1}(r+1)-1) & \cdots & -(\sigma^{-1}(\ell)-1) \end{pmatrix}.$$

Then $n(\tau) = n(\sigma)$. By the assumption of the induction,

$$\#\{ \varepsilon_i + \varepsilon_j ;\ 2 \leqq i < j \leqq \ell_0,\ (\sigma\delta, \varepsilon_i + \varepsilon_j) \geqq 2\ell - 4 - 2n(\sigma) \}$$

$$> n(\sigma) - 1.$$

Let

$$s = \#\{\varepsilon_i ;\ 2 \leqq i \leqq \ell_0,\ \exists \varepsilon_j,\ 2 \leqq j \neq i \leqq \ell_0, \text{ such that } (\sigma\delta, \varepsilon_i + \varepsilon_j)$$

$$= 2\ell - 4 - 2n(\sigma) \text{ or } 2\ell - 3 - 2n(\sigma) \}.$$

Then, in the same way as in Lemma 2.4, we see that

$$\#\{\varepsilon_i + \varepsilon_j ;\ 2 \leqq i < j \leqq \ell_0,\ (\sigma\delta, \varepsilon_i + \varepsilon_j) = 2\ell - 4 - 2n(\sigma) \text{ or }$$

$$2\ell - 3 - 2n(\sigma) \} \leqq s - 1.$$

Let ε_i satisfy that there exists ε_j, $2 \leqq j \leqq \ell_0$, $j \neq i$, such that

$$(\sigma\delta, \varepsilon_i + \varepsilon_j) = 2\ell - 4 - 2n(\sigma) \text{ or } 2\ell - 3 - 2n(\sigma).$$

Then

$$(\bar{\sigma}\delta, \varepsilon_i + \varepsilon_1) \geqq 2\ell - 2 - 2n(\bar{\sigma}) \quad .$$

in all but the following case:

$$(\bar{\sigma}\delta, \varepsilon_i + \varepsilon_2) = 2\ell - 4 - 2n(\bar{\sigma}) \quad \text{and} \quad \bar{\sigma}^{-1}(2) = 2.$$

Therefore the inequality is true.

Case 2: $\bar{\sigma}^{-1}(1) = 2$. By the definition of W^1

$$\bar{\sigma}^{-1} = \begin{pmatrix} 1 & 2 & \cdots & r & r+1 & \cdots & \ell_0 \\ 2 & \bar{\sigma}^{-1}(2) & \cdots & \bar{\sigma}^{-1}(r) & -\bar{\sigma}^{-1}(r+1) & \cdots & -1 \end{pmatrix}.$$

Define the element $\bar{\sigma}' \in W^1$ by

$$(\bar{\sigma}')^{-1} = \begin{pmatrix} 1 & 2 & \cdots & r & r+1 & \cdots & \ell_0-1 & \ell_0 \\ 1 & \bar{\sigma}^{-1}(2) & \cdots & \bar{\sigma}^{-1}(r) & -\bar{\sigma}^{-1}(r+1) & \cdots & -\bar{\sigma}^{-1}(\ell_0-1) & -2 \end{pmatrix}.$$

Then $n(\bar{\sigma}') = n(\sigma) - 1$. Define another element $\tau \in W^1$, which is considered for $\ell = \ell_0 - 1$, by

$$\tau^{-1} = \begin{pmatrix} 1 & \cdots & r & r+1 & \cdots & \ell_0-1 \\ \bar{\sigma}^{-1}(2)-1 & \cdots & \bar{\sigma}^{-1}(r)-1 & -(\bar{\sigma}^{-1}(r+1)-1) & \cdots & -1 \end{pmatrix}.$$

Then $n(\tau) = n(\sigma')$.

Assume that the inequality (3.2) is true for τ. If we notice that $(\bar{\sigma}')^{-1}(2) > 2$, we get the following inequality in the same way as in case 1.

$$\#\{\beta \in \Delta(n^+);\ (\bar{\sigma}'\delta, \beta) \geqq 2\ell - 2 - 2n(\bar{\sigma}')\} > n(\bar{\sigma}').$$

Clearly

$$(\sigma\delta, \beta) \geq (\sigma'\delta, \beta\beta) - 2 \qquad \text{for any } \beta \in \Delta(n^+).$$

Hence if $\beta \in \Delta(n^+)$ satisfies that

$$(\sigma'\delta, \beta) \geqq 2\ell - 2 - 2n(\sigma'),$$

then

$$(\sigma\delta, \beta) \geqq 2\ell - 2 - 2n(\sigma).$$

Therefore

$$\#\{\beta \in \Delta(n^+);\ (\sigma\delta, \beta) \geqq 2\ell - 2 - 2n(\sigma)\} > n(\sigma) - 1.$$

Thus we have proved that the inequality (3.1) is true for any $\sigma \in W^1$. We can prove that the inequality (3.2) is true in the same way as above.

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