Parity Violation in QCD process via Supersymmetry

(QCD過程における超対称性によるパリティの破れ)

Kunio Kaneta

Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan

January. 2013

Abstract

Supersymmetry (SUSY) is one of the most promising candidates beyond the Standard Model (SM), and has been searched at high energy colliders, for example, the Large Hadron Collider (LHC). The SUSY SM undergoes parity violation in QCD process through gauge interactions of quark-squark-gluino with non-degenerate spectrum between left-handed and right-handed sqaurks. In the SM QCD, of course there is no parity violation, however, the violation can appear via SUSY loop contributions in QCD process. First of all, we investigate parity violation in helicity dependent top quark pair production, where the violation can be observed as helicity asymmetry. Universal extra dimension (UED) model is also one of attractive ideas beyond the SM, and it is difficult to discriminate SUSY from UED by direct observation of new particles at the LHC. Since UED does not have parity violation in QCD process excepting renormalization group equations effects, we discuss a possibility to discriminate SUSY form UED by focusing on the parity violation. In order to estimate the helicity asymmetry, we utilize an effective operator analysis, and thus, we have also discussed an accurate calculation of effective operators. As our result, the helicity asymmetry induced by SUSY can be $\delta A_{LR} \simeq 0.05 \pm 0.01$ with specific parameters, and the sizable asymmetry can be observed with an integrated luminosity $\mathcal{L} \sim 100$ fb^{-1} at the LHC. Second of all, we study parity non-conserving heavy meson decay via SUSY, in particular, we evaluate the decay of η_c of $c\bar{c}$ meson, where a bound for the decay width has been experimentally obtained. We also develop a method to analyze a heavy meson system with SUSY loop effects, which is based on non-relativistic QCD (NRQCD) framework. Furthermore, we estimate parity violation from left-right non-degeneracy of $\tilde{u}$ and $\tilde{d}$ in nucleon interactions. Unfortunately, our results are below current experimental data. However, it is expected that our method is useful for research of an origin of parity violation.

Contents

Chapter 1

Introduction

The Standard Model (SM) is the most successful framework in particle physics. ATLAS/CMS collaborations at the Large Hadron Collider (LHC) has reported the discovery of Higgs boson [1, 2]. Thus all of the particles contained in the SM have been discovered. However, there are still mysteries in nature, for example, gauge hierarchy, absence of dark matter (DM) in the SM, and origin of neutrino mass, and therefore, the SM seems to be unsatisfactory.

Supersymmetry (SUSY) and extra dimension are most reliable candidates beyond the SM, which naturally contain stable DM particle [3, 4, 5, 6, 7, 8, 9]. In the SUSY with R-parity, the lightest SUSY particle is stable which can be a DM. For extra dimension theory, which might be related to string theory, we consider an universal extra dimension (UED) model as the simplest example. The UED model is a naive extension of the SM[10], where SM particles have extra dimensional modes, i.e., Kaluza-Klein (KK) particles, and their spins are the same as SM particles. The lightest KK particle is stable and can be a DM[6, 11, 12, 13, 14, 9]. Both SUSY and UED predict new heavy particles as superpartners and KK-particles, respectively.

For those models, it is important to experimentally distinguish one from the other, not only by the mass spectrum but also by the kinematic feature in the production. Actually, the production processes between SUSY and UED models are similar at hadron colliders, so that the study of the kinematic properly will become more important role to determine it. An example is shown in Ref.[15] ,which an event in those models are similar. In SUSY the cascade decay of squark $\tilde{g} \to q\tilde{\chi}_2^0 \to ql^{\pm}\tilde{l}_L^{\mp} \to ql^+l^-\tilde{\chi}_1^0$ is similar to the decay chain of the first KK quark $Q_1 \to qZ_1 \to ql^{\pm}l_1^{\mp} \to ql^+l^-\gamma_1$ in UED, where $Z_1, l_1^{\mp}$ and γ are the first

KK modes of $Z, l^{\mp}$, and γ, respectively. Final state in those case is one jet, two charged leptons with oposite charge, and missing energy from $\tilde{\chi}_1^0$ or γ_1. The spin information of new particles will be the direct probe to determine the model, although we meet a difficulty how to measure the spin in production at the LHC, in which the center of mass energy in each event is generally unknown at hadron colliders[16]. At e^+e^- collider, for example International Linear Collider (ILC), we can observe the spin information of produced particles, which process is, for example, $e^+e^- \to Z/\gamma \to \tilde{\mu}\tilde{\mu}$ or $\mu_1\mu_1$ [16]. In SUSY the angular distribution of $\tilde{\mu}^+\tilde{\mu}^-$ production is given by

$$\left.\frac{d\sigma}{d\cos\theta}\right|_{\rm SUSY} \sim 1 - \cos^2\theta, \tag{1.0.1}$$

and that of KK muon pair is given by

$$\left.\frac{d\sigma}{d\cos\theta}\right|_{\rm UED} \sim 1 + \cos^2\theta, \tag{1.0.2}$$

and thus, we can distinguish one from the other by the angular distribution.

One of the aim of this thesis is to study the discrimination of SUSY and UED at the LHC by focusing on the parity violation in QCD processes without discovering any new particles. * We investigate the determination of underlying theory through the parity violation in QCD, even if new particles are too heavy to be detected directly in the experiments. Of course the SM has no parity violation in QCD, and it can be happened through the effects of new physics. In SUSY, gluino-quark$_{L(R)}$-squark$_{L(R)}$ interactions can violate parity, since a mass of left-handed squark $\tilde{q}_L$ is different from that of right-handed squark $\tilde{q}_R$ in general. While the UED has no parity violating QCD interactions (at least in tree level). Therefore, we could distinguish SUSY from UED through the parity violation in QCD processes.

The $t\bar{t}$ helicity asymmetry is expected to be highly sensitive to new physics. In the SM, only top quark is enough heavy not to form a hadron, and thus, top quark maintains spin information until the decay. Therefore if parity is not conserved in $t\bar{t}$ pair production, which is induced by new physics, the observed top quark spin presents the parity violation. As we will discuss in this thesis, SUSY can arise sizable $t\bar{t}$ asymmetry through squark loop diagrams, because $\tilde{q}_L$ and $\tilde{q}_R$ have different mass spectrum in general, and $q_{L(R)}$-$\tilde{q}_{L(R)}$-$\tilde{g}$ is chiral interaction[17]. We will show that the sizable helicity asymmetry can be observed with specific parameters at the LHC.

*The discrimination of SUSY and UED at the LHC is also studied in [15].

For another QCD parity violating process, we will discuss parity non-conserving quarkonium decay. Quarkonium is heavy $q\bar{q}$ bound state, which is described by the framework of non-relativistic QCD (NRQCD). In order to investigate the parity violation in quarkonium decay, we formulate NRQCD by an improved new method[18], and discuss a case of charmonium decay. We also study light meson cases by use of an effective theory of nucleon interactions.

This thesis is organized as follows. In chapter 2, we explain a method to obtain effective field theory. We utilize effective operator analysis to study the parity violation in QCD process, therefore, we describe a strategy to obtain effective operators and give a simple example to calculate dimension six effective operators. Chapter 3 is devoted to study a discrimination of SUSY and UED by focusing on a helicity asymmetry in top pair production at the LHC. In order to estimate this helicity dependent production process, we calculate dimension six operators in SUSY and UED in section 3.1 and 3.2, respectively. In chapter 4 we discuss parity violation in quarkonium decay. In order to evaluate the violation in concrete decay mode, we develop a method in section 4.1. In section 4.2, we calculate parity violating potential induced by SUSY, and estimate a decay width of parity non-conserving charmonium decay in section 4.3. In section 4.4 we evaluate non-degeneracy bounds for light flavors, and, in section 4.5, we comment on other sfermions. Finally, we devote chapter 5 to summary and discussion.

Chapter 2

Effective field theory

In this chapter, we explain how we obtain higher dimensional operators in an effective theory. When new physics such as SUSY is characterized by larger mass scale than electroweak scale, it is reasonable to calculate matrix elements by use of effective operator analysis. For our analysis, we explain an effective field theory, and discuss how to obtain effective operators.

Suppose that a fundamental theory $\mathcal{L}_F$ has well-separated two different mass scales such as $E_{EW} \ll M_{NP}$, where E_{EW} and M_{NP} are electroweak scale and new physics scale, respectively. Of course $\mathcal{L}_F$ need not to be really "*fundamental*," and it is possible to consider that $\mathcal{L}_F$ itself is an effective theory, however, a required property is that $\mathcal{L}_F$ has the scale hierarchy $E_{EW} \ll M_{NP}$. An effective theory $\mathcal{L}_{\rm eff}$ can be obtained by integrating out all particles characterized by M_{NP}. Due to the scale hierarchy, $\mathcal{L}_{\rm eff}$ can be expanded by power of M_{NP},

$$\mathcal{L}_{\rm eff} = \mathcal{L}_0 + \frac{1}{M_{NP}}\mathcal{L}_1 + \frac{1}{M_{NP}^2}\mathcal{L}_2 + \cdots , \tag{2.0.1}$$

where $\mathcal{L}_0$ is the SM Lagrangian, and $\mathcal{L}_1$ ($\mathcal{L}_2$) contains dimension five (six) operators. Notice that all particles contained in $\mathcal{L}_i$ ($i = 0, 1, 2, \cdots$) are nothing but the SM particles, and their energy/momentum scale is up to E_{EW} in this expansion. In the SM, dimension five operator is only $(\tilde{H}^\dagger L)^T C(\tilde{H}^\dagger L)$, where $\tilde{H} \equiv i\sigma^2 H^*$, and C is the charge conjugation matrix [19]. This operator violates lepton number, and induces Majorana neutrino mass after electroweak symmetry breaking. $\mathcal{L}_2$ is represented as

$$\mathcal{L}_2 = \sum_i c_i \mathcal{O}_i, \tag{2.0.2}$$

where i is the label of all possible dimension six operators $\mathcal{O}_i$ allowed by the SM gauge symmetry, and c_i is called Wilson coefficient. Lorentz and gauge symmetry determine a complete set of dimension six operators, however, their coefficients can not be determined without a specific fundamental theory [20, 21, 22, 23, 24]. To evaluate an accurate physical quantity, we need to obtain exact Wilson coefficients of dimension six operators by integrating out high energy degrees of freedom. How can we calculate dimension six operators in the effective Lagrangian? One correct answer is to take a path integral of the fundamental theory as

$$Z = \int \mathcal{D}\phi_{SM}\mathcal{D}\phi_h e^{iS[\phi_{SM},\phi_h]}, \tag{2.0.3}$$

where ϕ_{SM} and ϕ_h represent the SM fields and heavy fields, respectively. By integrating out ϕ_h as

$$e^{iS_{\text{eff}}[\phi_{SM}]} = \int \mathcal{D}\phi_h e^{iS[\phi_{SM},\phi_h]}, \tag{2.0.4}$$

we can obtain an effective action,

$$S_{\text{eff}}[\phi_{SM}] = S_{SM} + S_1[\phi_{SM}] + S_2[\phi_{SM}] + \cdots, \tag{2.0.5}$$

where $S_1 = (1/M_{NP})\int \mathcal{L}_1$ and $S_2 = (1/M_{NP}^2)\int \mathcal{L}_2$. Coefficients of dimension six operators have been basically calculated in $S_2[\phi_{SM}]$.

As more concrete discussion, we consider a Lagrangian such as

$$\mathcal{L}[\phi_l,\phi_h] = \mathcal{L}[\phi_l] + F_i[\phi_l]\phi_{h,i} + \frac{1}{2}K_{ij}[\phi_l]\phi_{h,i}\phi_{h,j}, \tag{2.0.6}$$

where $\phi_l(\phi_h)$ represents light (heavy) scalar fields, and i denotes an index of scalar fields. The effective Lagrangian can be obtained using Gaussian integral of ϕ_h as

$$e^{iS_{\text{eff}}[\phi_l]} = \exp\left[-\frac{1}{2}\text{Tr}\log K[\phi_l] - \frac{i}{2}\int d^4xd^4yF_i[\phi_l](x)K_{ij}^{-1}[\phi_l](x,y)F_j[\phi_l](y)\right]. \tag{2.0.7}$$

Note that Eq.(2.0.7) is exact when the Lagrangian is up to $\mathcal{O}(\phi_h^2)$. Although the operators in Eq.(2.0.7) are non-local, they can be expanded by infinite number of local operators. Noting mass of $\phi_l(\phi_h)$ as $\mu(\Lambda)$, the non-local operators can be expanded in powers of μ/Λ. Then, $K[\phi_l]$ in Eq.(2.0.7) is divided into two parts as

$$K[\phi_l] = K^h + K^l[\phi_l], \tag{2.0.8}$$

where K^h is the kinetic term of ϕ_h, and $K^l[\phi_l]$ is ϕ_l dependent part of $K[\phi_l]$. Expanding momentum of $K^l[\phi_l]$ by Λ, and regarding $K^l[\phi_l]/K^h \sim \mathcal{O}(\mu^2/\Lambda^2) \ll 1$, $\text{Tr}\log K[\phi_l]$ and

$K[\phi_l]^{-1}$ are given by

$$
\begin{aligned}
\mathrm{Tr}\log K[\phi_l] &= \mathrm{Tr}\log(K^h)^{-1} + \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\mathrm{Tr}\left((K^h)^{-1}K^l[\phi_l]\right)^n, &(2.0.9)\\
K[\phi_l]^{-1} &= (K^h + K^l[\phi_l])^{-1} \sim (K^h)^{-1} - (K^h)^{-1}K^l[\phi_l](K^h)^{-1} + \cdots, &(2.0.10)
\end{aligned}
$$

where $(K^h)^{-1} = (K^h)^{-1}(x,y)$ is the propagator of ϕ_h except for imaginary factor i. Then, the action, $S_{\rm eff}$, can be written as

$$
\begin{aligned}
S_{\rm eff}[\phi_l] = & \int d^4x \Bigg[\mathcal{L}[\phi_l] + \frac{i}{2}\int d^4y\delta^4(y-x)(K^h)^{-1}(x,y)K^l[\phi_l](y)\\
&+\frac{i}{4}\int d^4y(K^h)^{-1}(x,y)K^l[\phi_l](y)(K^h)^{-1}(y,x)K^l[\phi_l](x) + \cdots\\
&-\frac{1}{2}\int d^4y F_i[\phi_l](x)\left\{(K^h)^{-1}(x,y)\right.\\
&\left.-\int d^4z(K^h)^{-1}(x,z)K^l[\phi_l](z)(K^h)^{-1}(z,y)K^l[\phi_l](y) + \cdots\right\}F_j[\phi_l](y)\Bigg].
\end{aligned}
\qquad (2.0.11)
$$

In this expression, we can obtain a effective theory with infinite number of local operators.

We comment that the perturbative expansion in Eqs.(2.0.9) and (2.0.10) do not depend on $F[\phi_l]$ in Eq.(2.0.7). We can also consider a theory with interactions of $\phi_h^3, \phi_h^4, \cdots$, where they can be treated perturbatively. For example, if there is a interaction, ϕ_h^3, with coupling G_{ijk}, the Lagrangian is given by

$$
\mathcal{L}'[\phi_l,\phi_h] = \mathcal{L}[\phi_l,\phi_h] + \frac{1}{3!}G_{ijk}[\phi_l]\phi_{h,i}\phi_{h,j}\phi_{h,k}. \qquad (2.0.12)
$$

By integrating out ϕ_h, the effective action, $S'_{\rm eff}[\phi_l]$, is given by

$$
\begin{aligned}
S'_{\rm eff}[\phi_l] = & S_{\rm eff}[\phi_l] + \int d^4x A_{ijk}(x)G_{ijk}[\phi_l](x)\\
&+\int\int d^4xd^4y B_{ijklmn}(x,y)G_{ijk}[\phi_l](x)G_{lmn}[\phi_l](y) + \cdots \qquad (2.0.13)
\end{aligned}
$$

with dimensionful couplings A_{ijk} and B_{ijklmn}. They are calculated as

$$
\begin{aligned}
A_{ijk}(x) &= \frac{\delta}{\delta G_{ijk}(x)}e^{iS'_{\rm eff}}\Big|_{G_{ijk}=0}, &(2.0.14)\\
B_{ijklmn}(x,y) &= \frac{\delta^2}{\delta G_{lmn}(y)\delta G_{ijk}(x)}e^{iS'_{\rm eff}}\Big|_{G_{ijk}=0}. &(2.0.15)
\end{aligned}
$$

Apparently, A_{ijk} is zero. B_{ijklmn} is $\langle(\phi_{h,i}\phi_{h,j}\phi_{h,k})_x(\phi_{h,l}\phi_{h,m}\phi_{h,n})_y\rangle_h$, where $\langle\mathcal{O}\rangle_h$ is defined by

$$
\langle\mathcal{O}\rangle_h \equiv \int \mathcal{D}\phi_h\ \mathcal{O}\ e^{i\int\mathcal{L}[\phi_l,\phi_h]}. \qquad (2.0.16)
$$

In this stage, $S'_{\rm eff}[\phi_l]$ is explicitly obtained.

Let us show a concrete calculation by using a toy model. We consider a Lagrangian,

$$\mathcal{L} = \overline{\psi}_l(i\partial\!\!\!/ - m)\psi_l + \overline{\psi}_h(i\partial\!\!\!/ - M)\psi_h + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}M^2\phi^2 - g\phi(\overline{\psi}_l\psi_h + \overline{\psi}_h\psi_l), \tag{2.0.17}$$

where ψ_l (ψ_h) denotes a light (heavy) Dirac fermion and ϕ is a heavy real scalar with $m \ll M$. We will obtain an effective action of ψ_l after integrating out heavy fields, where irrelevant operators must include traces of heavy particles and their interactions at high energy scale. Let us calculate the effective action by integrate out ψ_h, ϕ, and show dimension six operators by expanding $1/M^n$. The effective action should be given by

$$e^{iS_{\rm eff}[\psi_l]} = \int \mathcal{D}\phi\mathcal{D}\psi_h\mathcal{D}\overline{\psi}_h e^{iS[\psi_l,\psi_h,\phi]}, \tag{2.0.18}$$

and firstly, by integrating out the heavy fermion, it becomes

$$\begin{aligned} &= \int \mathcal{D}\phi\mathcal{D}\psi_h\mathcal{D}\overline{\psi}_h \exp i\left\{S_{\rm free}[\psi_l,\phi] + (\overline{\psi}_h - \overline{A}K_0^{-1})K_0(\psi_h - K_0^{-1}A) - \overline{A}K_0^{-1}A\right\}, \\ &= ({\rm Det}K_0)\exp i\left\{S_{\rm free}[\psi_l,\phi] - \overline{A}K_0^{-1}A\right\}, \end{aligned} \tag{2.0.19}$$

where

$$\begin{array}{l} K_0^{-1} = \int \frac{d^4k}{(2\pi)^4}\frac{1}{p\!\!\!/ - M}e^{-ip(x-y)} = -i\mathcal{D}^{(\psi_h)}(x-y), \\ A = g\phi\psi_l,\ \overline{A} = g\overline{\psi}_l\phi. \end{array} \tag{2.0.20}$$

The second term in Eq.(2.0.19) is written by

$$-\overline{A}K_0^{-1}A = -\frac{1}{2}\int d^4xd^4y\phi(x)\delta\tilde{K}(x,y)\phi(y) \equiv -\frac{1}{2}\phi\cdot\delta\tilde{K}\cdot\phi, \tag{2.0.21}$$

$$\delta\tilde{K}(x,y) \equiv -2ig^2\overline{\psi}_l(x)\mathcal{D}^{(\psi_h)}(x-y)\psi_l(y). \tag{2.0.22}$$

Next step is an integration of ϕ, which gives

$$\begin{aligned} e^{iS_{\rm eff}[\psi_l]} &= ({\rm Det}K_0)\int\mathcal{D}\phi\exp i\left\{S_{\rm free}[\psi_l] - \frac{1}{2}\phi\cdot(\tilde{K}_0 + \delta\tilde{K})\cdot\phi\right\}, \\ &= \left(\frac{{\rm Det}K_0}{{\rm Det}^{\frac{1}{2}}(\tilde{K}_0 + \delta\tilde{K})}\right)e^{iS_{\rm free}[\psi_l]}, \\ &= \left(\frac{{\rm Det}K_0}{{\rm Det}^{\frac{1}{2}}\tilde{K}_0}\right)\exp\left\{iS_{\rm free}[\psi_l] - \frac{1}{2}{\rm Tr}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}(\tilde{K}_0^{-1}\delta\tilde{K})^n\right\}, \end{aligned} \tag{2.0.23}$$

$$\tilde{K}_0^{-1}(x,y) \equiv \int\frac{d^4p}{(2\pi)^4}\frac{-1}{p^2 - M^2}e^{-ip(x-y)} = i\mathcal{D}^{(\phi)}(x-y). \tag{2.0.24}$$

Determinant of K_0 and $\tilde{K}_0$ are cancelled by normalization, so that we finally obtain the effective action of ψ_l as

$$S_{\text{eff}}[\psi_l] = S_{\text{free}}[\psi_l] + \frac{i}{2}\text{Tr}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}(\tilde{K}_0^{-1}\delta\tilde{K})^n. \tag{2.0.25}$$

Higher dimensional operators are included in the second term of Eq.(4.1.2), thus dimension six operators are calculated from the second order of $1/M^n$ expansion. A space integration of $\mathcal{O}(1/M^2)$ gives

$$\begin{aligned}
&-\frac{i}{4}\int d^4xd^4yd^4zd^4w\tilde{K}_0^{-1}(x,y)\delta\tilde{K}(y,z)\tilde{K}_0^{-1}(z,w)\delta\tilde{K}(w,x),\\
&= -ig^4\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\frac{d^4k_4}{(2\pi)^4}(2\pi)^4\delta^4(k_1-k_2+k_3-k_4),\\
&\times\frac{d^4p}{(2\pi)^4}\frac{1}{p^2-M^2}\overline{\psi}_l(k_1)\frac{1}{\not{p}+\not{k}_1-M}\psi_l(k_2)\frac{1}{(p+k_1-k_2)^2-M^2}\overline{\psi}_l(k_2)\frac{1}{\not{p}+\not{k}_4-M}\psi_l(k_4),
\end{aligned} \tag{2.0.26}$$

and integration of all momenta $p, k_i, (i = 1, \cdots, 4)$ with $k_i \ll M$ induces 4-Fermi operators,

$$\mathcal{O}_{4\text{F}}(x) = \frac{g^4}{192\pi^2}\frac{1}{M^2}\left[-(\overline{\psi}_l\gamma^\mu\psi_l)(\overline{\psi}_l\gamma_\mu\psi_l) + 2(\overline{\psi}_l\psi_l)(\overline{\psi}_l\psi_l)\right]. \tag{2.0.27}$$

These are the dimension six operators in this model. Notice that accurate coefficients are automatically obtained without care of symmetric factors. Other higher order operators can be calculated similarly.

Chapter 3

Discrimination of SUSY and UED

3.1 Minimal supersymmetric standard model

In this section, we calculate dimension six operators in QCD sector of SUSY SM.

SUSY is one of most promising candidates beyond the SM. Number of reviews of SUSY is shown in, for example, Refs. [25, 26, 27, 28, 29, 30, 31, 32, 33, 34]. One of motivations to consider SUSY is the fine tuning problem in Higgs mass. Gauge coupling unification also strongly supports SUSY. Although gauge couplings of $SU(3)_C \times SU(2)_L \times U(1)_Y$ are not unified in the SM, they are unified around 10^{16} GeV in SUSY SM. As other motivation, SUSY SM naturally contains DM candidate, which the lightest SUSY particle (LSP) can be DM. Due to a phenomenological reason, minimal SUSY SM (MSSM) is favored, and we consider MSSM in this thesis.

SUSY transformation exchanges bosonic states and fermionic states, and all particles in the SM have their partners whose spins differ by 1/2. Particle content of the MSSM is shown in Tab.3.1. Note that two Higgs doublets with oposite hypercharge are necessary to cancel gauge anomalies.

In general, supersymmetric Lagrangian is given by

$$\mathcal{L} = \left[\Phi^{*i}(e^{2ig_aV^aT^a})_i^j\Phi_j\right]_D + ([W(\Phi_i)]_F + c.c.) + \frac{1}{4}([\mathcal{W}^\alpha\mathcal{W}_\alpha]_F + c.c.), \qquad (3.1.1)$$

where Φ_i and V^a are chiral and vector supermultiplet, respectively. $\mathcal{W}_\alpha$ is a chiral supermultiplet defined by

$$\mathcal{W}_\alpha = -\frac{1}{4}D^\dagger D^\dagger(e^{-V}D_\alpha e^V), \qquad (3.1.2)$$

quantum number	spin 0	spin 1/2	supermultiplet
$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$	$\begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	Q
$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$	$\tilde{u}_R^*$	$u_R^\dagger$	$\bar{u}$
$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$	$\tilde{d}_R^*$	$d_R^\dagger$	$\bar{d}$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$\begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix}$	$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	L
$(\mathbf{1}, \mathbf{1}, 1)$	$\tilde{e}_R^*$	$e_R^\dagger$	$\bar{e}$
$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$\begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$	$\begin{pmatrix} \tilde{H}_u^+ \\ \tilde{H}_u^0 \end{pmatrix}$	H_u
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$\begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$	$\begin{pmatrix} \tilde{H}_d^0 \\ \tilde{H}_d^- \end{pmatrix}$	H_d

quantum number	spin 1/2	spin 1	names
$(\mathbf{8}, \mathbf{1}, 0)$	$\tilde{g}$	G	gluino, gluon
$(\mathbf{1}, \mathbf{3}, 0)$	$\tilde{W}^\pm$, $\tilde{W}^0$	$W^\pm$, W^0	winos, W boson
$(\mathbf{1}, \mathbf{1}, 0)$	$\tilde{B}^0$	B^0	bino, B boson

Table 3.1: Particle content of the MSSM.

where D_α is a chiral covariant derivative. Superpotential W is given by

$$W = \mu H_u \cdot H_d + y_u \bar{u} Q \cdot H_u + y_d \bar{d} Q \cdot H_d + y_e \bar{e} L \cdot H_d \tag{3.1.3}$$

in the MSSM, where "$\cdot$" represents $SU(2)$ product defined as $A \cdot B = A_1 B_2 - A_2 B_1$. In terms of MSSM fields, the first term in Eq.(3.1.1) gives gauge interaction of $SU(3)_C \times SU(2)_L \times U(1)_Y$, and kinetic terms of all scalar and fermion fields. The second term gives Yukawa interactions and F-term component of the scalar potential. The third term gives kinetic terms of gauginos and gauge fields, and D-term component of the potential.

Here we only interested in the QCD sector in the MSSM, and Lagrangian is given by

$$\begin{aligned}
\mathcal{L} =& \mathcal{L}_{SM} + \frac{1}{2}\bar{\tilde{g}}(i\not{\partial} - m_{\tilde{g}})\tilde{g} - \tilde{q}_R^\dagger(\partial^2 + m_{\tilde{q}_R}^2)\tilde{q}_R - \tilde{q}_L^\dagger(\partial^2 + m_{\tilde{q}_L}^2)\tilde{q}_L \\
&+ \frac{i}{2} g_s f^{abc} \overline{\tilde{g}^a} \gamma^\mu \tilde{g}^b G_\mu^c - i g_s \sum_q (\tilde{q}_L^\dagger \frac{\lambda^a}{2} \overleftrightarrow{\partial}^\mu \tilde{q}_L) G_\mu^a - i g_s \sum_q (\tilde{q}_R^\dagger \frac{\lambda^a}{2} \overleftrightarrow{\partial}^\mu \tilde{q}_R) G_\mu^a \\
&+ g_s^2 \sum_q \tilde{q}_L^\dagger \frac{\lambda^a}{2} \frac{\lambda^b}{2} \tilde{q}_L G_\mu^a G^{b\mu} + g_s^2 \sum_q \tilde{q}_R^\dagger \frac{\lambda^a}{2} \frac{\lambda^b}{2} \tilde{q}_R G_\mu^a G^{b\mu} \\
&- \sqrt{2} g_s \sum_q (\tilde{q}_L^\dagger \overline{\tilde{g}_R^a} \frac{\lambda^a}{2} q_L + \text{h.c.}) + \sqrt{2} g_s \sum_q (\tilde{q}_R^\dagger \overline{\tilde{g}_L^a} \frac{\lambda^a}{2} q_R + \text{h.c.}).
\end{aligned} \tag{3.1.4}$$

where $\sum_q$ represents a sum over all flavors. $m_{\tilde{g}}, m_{\tilde{q}_R}$ and $m_{\tilde{q}_L}$ are soft SUSY breaking parameters. The last line of Eq.3.1.4 is chiral interactions, which comes from the D-term in Eq.3.1.1, and plays an important role in this thesis.

3.1.1 Dimension six operators in MSSM

Let us calculate coefficients of dimension six operators in the QCD in the MSSM. In the SUSY with R-parity, SUSY particles propagate only inside of loop diagrams. Effective action should be obtained by integrating out $\tilde{q}_L, \tilde{q}_R$, and $\tilde{g}$ as

$$e^{iS_{\text{eff}}} = \int \mathcal{D}\tilde{g}\mathcal{D}\tilde{q}_L\mathcal{D}\overline{\tilde{q}_L}\mathcal{D}\tilde{q}_R\mathcal{D}\overline{\tilde{q}_R}\ e^{iS}. \tag{3.1.5}$$

S_{eff} includes all possible irrelevant operators. Calculating results are listed in Appendix B.1, where coefficients of dimension six operators, 4-Fermi $\mathcal{O}_{4F}$, quark-quark-gluon-gluon $\mathcal{O}_{qqGG}$, and quark-quark-gluon $\mathcal{O}_{qqG}$ are represented. They are operators up to $\mathcal{O}(g_s^4)$, which are useful to estimate phenomenology at the LHC[35] We overview explicit technique to calculate them in the following discussions.

The first step is integrating out $\tilde{q}_R$ as

$$\int \mathcal{D}\tilde{q}_R\mathcal{D}\overline{\tilde{q}_R}e^{iS} = \exp i\ [i\text{Tr}(\log K) + B^\dagger K^{-1}B], \tag{3.1.6}$$

$$\begin{aligned} &K = K_0 + \delta K, \\ &K_0 = (\partial^\mu\partial_\mu + m_R^2), \qquad \delta K = ig_s[2G_\mu\partial^\mu + (\partial^\mu G_\mu)] - g_s^2 G_\mu G^\mu, \\ &B = -\sqrt{2}g_s(\overline{\tilde{g}^a}\tfrac{\lambda}{2}^a P_R q), \quad B^\dagger = -\sqrt{2}g_s(\overline{q}P_L\tfrac{\lambda}{2}^a\tilde{g}^a), \end{aligned} \tag{3.1.7}$$

where $\text{Tr}(\log K)$ includes some loop diagrams which have external gluon lines (e.g Fig. 3.1). The second term, $B^\dagger K^{-1}B$, can be expanded by right-handed squark propagator as

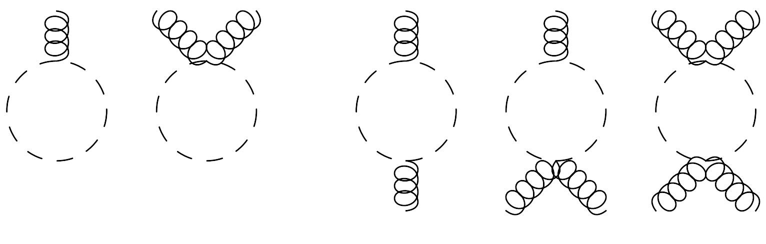

Figure 3.1: Diagrams of gluon external lines.

$$\begin{aligned} B^\dagger K^{-1}B &= B^\dagger(1 + K_0^{-1}\delta K)^{-1}K_0^{-1}B \\ &= B^\dagger K_0^{-1}B - B^\dagger K_0^{-1}\delta K K_0^{-1}B + B^\dagger K_0^{-1}\delta K K_0^{-1}\delta K K_0^{-1}B + \cdots. \end{aligned} \tag{3.1.8}$$

We take the expansion up to order of g_s^4 in S_{eff}. Similarly, $\tilde{q}_L$ integration can be performed, and after $\tilde{q}_R, \tilde{q}_L$ integrations, an "effective action" in this stage is given by

$$S^{(\tilde{g})} = \frac{1}{2}\int d^4x d^4y \overline{\tilde{g}}(x)_i^a \left[\tilde{K}_0 + \tilde{K}_I\right]_{xy}^{ab} \tilde{g}(y)_j^b, \tag{3.1.9}$$

where $\tilde{K}_0$ and $\tilde{K}_I$ are

$$\left[\tilde{K}_0\right]_{xy}^{ab} \equiv \delta^{ab}\delta^4(x-y)(i\partial\!\!\!/_y - m_{\tilde{g}})_{ij}, \tag{3.1.10}$$

$$\begin{aligned}
\left[\tilde{K}_I\right]_{xy}^{ab} \equiv\ & ig_s f^{abc}\delta^4(x-y) G\!\!\!/^c_{ij} - 4ig_s^2 \sum_{q=q_L,q_R} \overline{q}_j(y)\frac{\lambda}{2}^b \mathcal{D}^{(\tilde{q})}(y-x)\frac{\lambda}{2}^a q_i(x) \\
& - 4ig_s^3 \int d^4z \sum_{q=q_L,q_R} \overline{q}_j(y)\frac{\lambda}{2}^b \mathcal{D}^{(\tilde{q})}(y-z)\{2G_\mu\partial^\mu + (\partial^\mu G_\mu)\}_z \mathcal{D}^{(\tilde{q})}(z-x)\frac{\lambda}{2}^a q_i(x) \\
& - 4ig_s^4 \int d^4z d^4w \sum_{q=q_L,q_R} \overline{q}_j(y)\overline{q}_j(y)\frac{\lambda}{2}^b \mathcal{D}^{(\tilde{q})}(y-z)\{2G_\mu\partial^\mu + (\partial^\mu G_\mu)\}_z \mathcal{D}^{(\tilde{q})}(z-w) \\
& \cdot \{2G_\mu\partial^\mu + (\partial^\mu G_\mu)\}_w \mathcal{D}^{(\tilde{q})}(w-x)\frac{\lambda}{2}^a q_i(x) + \mathcal{O}(g_s^5),
\end{aligned} \tag{3.1.11}$$

and

$$\mathcal{D}^{(\tilde{q})}(x-y) = -iK_0^{-1} = \int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2 - m_{\tilde{q}}^2 + i\epsilon} \mathrm{e}^{-ik\cdot(x-y)}. \tag{3.1.12}$$

Here i, j denote spinor indexes. Next step is integrating out gluino, and the final effective action is obtained as

$$\begin{aligned}
S_{\text{eff}} =\ & \int d^4x d^4y \alpha(x,y)_{ij}^{ab} \tilde{K}_I(x,y)_{ij}^{ab} \\
& + \int d^4x d^4y d^4z d^4w \beta(x,y,z,w)_{ijkl}^{abcd} \tilde{K}_I(x,y)_{ij}^{ab} \tilde{K}_I(z.w)_{kl}^{cd} + \mathcal{O}(\tilde{K}_I^3),
\end{aligned} \tag{3.1.13}$$

where $\alpha(x,y)$, $\beta(x,y,z,w)$ consist of gluino propagator as

$$\alpha(x,y)_{ij}^{ab} = -\frac{1}{2}\delta^{ab}\mathcal{D}^{(\tilde{g})}(y-x)_{ji}, \tag{3.1.14}$$

$$\begin{aligned}
& \beta(x,y,z,w)_{ijkl}^{abcd} \\
& = \frac{i}{8}\delta^{ac}\delta^{bd}\left[C^\dagger \mathcal{D}^{(\tilde{g})}(x-z)\right]_{ik}\left[\mathcal{D}^{(\tilde{g})}(w-y)C^T\right]_{lj} - \frac{i}{8}\delta^{ad}\delta^{bc}\mathcal{D}^{(\tilde{g})}(w-x)_{li}\mathcal{D}^{(\tilde{g})}(y-z)_{lj}.
\end{aligned} \tag{3.1.15}$$

We can know these results by differentiating interacting parts of the effective action as

$$\left.\frac{\delta}{\delta \tilde{K}_I(x,y)^{ab}_{ij}} e^{iS_{\rm eff}}\right|_{\tilde{K}_I=0} = i\alpha(x,y)^{ab}_{ij}, \tag{3.1.16}$$

$$\left.\frac{\delta}{\delta \tilde{K}_I(z,w)^{cd}_{kl}} \frac{\delta}{\delta \tilde{K}_I(x,y)^{ab}_{ij}} e^{iS_{\rm eff}}\right|_{\tilde{K}_I=0} = -\alpha(x,y)^{ab}_{ij}\alpha(z,w)^{cd}_{kl} + i\beta(x,y,z.w)^{abcd}_{ijkl} + i\beta(z,w,x,y)^{cdab}_{klij}. \tag{3.1.17}$$

In this stage, all $\mathcal{O}_{4F}$ and $\mathcal{O}_{qqG}$ at 1-loop level are included in Eq.(3.1.13) up to the second order of $\tilde{K}_I$.

For example, the 4-Fermi operators all $\mathcal{O}_{4F}$ are obtained by picking up $\mathcal{O}(g_s^2)$ order terms from each $\tilde{K}_I$ in $\beta\tilde{K}_I\tilde{K}_I$, which is given by

$$\begin{aligned}
&\int d^4x d^4y d^4z d^4w \beta(x,y,z,w)^{abcd}_{ijkl}\tilde{K}_I(x,y)^{ab}_{ij}\tilde{K}_I(z.w)^{cd}_{kl} \\
&\supset \int d^4x d^4y d^4z d^4w \left[\frac{i}{8}\delta^{ac}\delta^{bd}\left[C^\dagger \mathcal{D}^{(\tilde{g})}(x-z)\right]_{ik}\left[\mathcal{D}^{(\tilde{g})}(w-y)C^T\right]_{lj}\right. \\
&\left.-\frac{i}{8}\delta^{ad}\delta^{bc}\mathcal{D}^{(\tilde{g})}(w-x)_{li}\mathcal{D}^{(\tilde{g})}(y-z)_{lj}\right] \\
&\times\left[-4ig_s^2\overline{q}_j(y)\frac{\lambda}{2}^b\mathcal{D}^{(\tilde{q})}(y-x)\frac{\lambda}{2}^a q_i(x)\right]\cdot\left[-4ig_s^2\overline{q}'_l(w)\frac{\lambda}{2}^d\mathcal{D}^{(\tilde{q}')}(w-z)\frac{\lambda}{2}^c q'_k(z)\right].
\end{aligned} \tag{3.1.18}$$

The first term is given by

$$\begin{aligned}
&-2ig_s^4\int d^4x d^4y d^4z d^4w\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\frac{d^4k_4}{(2\pi)^4}\frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4}\frac{d^4p_3}{(2\pi)^4}\frac{d^4p_4}{(2\pi)^4} \\
&\times\left[\overline{q}_j(k_2)\frac{\lambda}{2}^b\frac{\lambda}{2}^a q_i(k_1)\right]\cdot\left[\overline{q}'_l(k_4)\frac{\lambda}{2}^d\frac{\lambda}{2}^c q'_k(k_3)\right]\frac{i[C^\dagger(\not{p}_1+m_{\tilde{g}})]_{ik}}{p_1^2-m_{\tilde{g}}^2}\frac{i[(\not{p}_2+m_{\tilde{g}})C^T]_{lj}}{p_2^2-m_{\tilde{g}}^2}\frac{i}{p_3^2-m_{\tilde{q}}^2}\frac{i}{p_4^2-m_{\tilde{q}'}^2} \\
&\times e^{ik_2y}e^{-ik_1x}e^{ik_4x}e^{-ik_3z}e^{-ip_1(x-z)}e^{ip_2(w-y)}e^{-ip_3(y-x)}e^{-ip_4(w-z)},
\end{aligned} \tag{3.1.19}$$

and by integrating out x,y,z,w,p_2,p_3,p_4, this term becomes

$$\begin{aligned}
&= -2ig_s^4\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\frac{d^4k_4}{(2\pi)^4}\delta^4(k_1-k_2+k_3-k_4) \\
&\times\left[\overline{q}_j(k_2)\frac{\lambda}{2}^b\frac{\lambda}{2}^a q_i(k_1)\right]\cdot\left[\overline{q}'_l(k_4)\frac{\lambda}{2}^d\frac{\lambda}{2}^c q'_k(k_3)\right]\left[A(C^\dagger\gamma^\mu)_{ik}(\gamma^\nu C^T)_{lj}+B(C^\dagger)_{ik}(C^T)_{lj}\right],
\end{aligned} \tag{3.1.20}$$

$$A = \int\frac{d^4p_1}{(2\pi)^4}\frac{p_{1\mu}(p_1+k_1-k_2)_\nu}{(p_1^2-m_{\tilde{g}}^2)[(p_1+k_1-k_2)^2-m_{\tilde{g}}^2][(p_1+k_1)^2-m_{\tilde{q}}^2][(p_1-k_1)^2-m_{\tilde{q}'}^2]}, \tag{3.1.21}$$

$$B = \int\frac{d^4p_1}{(2\pi)^4}\frac{m_{\tilde{g}}^2}{(p_1^2-m_{\tilde{g}}^2)[(p_1+k_1-k_2)^2-m_{\tilde{g}}^2][(p_1+k_1)^2-m_{\tilde{q}}^2][(p_1-k_1)^2-m_{\tilde{q}'}^2]}. \tag{3.1.22}$$

Here A, B are Feynman parameter integral, and they become

$$A \to -i\frac{6}{192\pi^2}f_1(m_{\tilde{q}}, m_{\tilde{g}}), \quad B \to i\frac{12}{192\pi^2}f_2(m_{\tilde{q}}, m_{\tilde{g}}), \tag{3.1.23}$$

when $k_i, (i = 1, \cdots, 4)$ are much smaller than masses of squarks and gluino. f_1, f_2 are shown in Appendix B.1, and the spinor can be rearranged by Fierz transformation of Eqs.(A.2.6) and (A.2.7) in Appendix A. Necessary Fierz transformations and color factors are shown in Appendix A.

We can summarize all 4-Fermi operators as separating color singlet $\mathcal{O}_{4F}^{(1)}$ or color octet $\mathcal{O}_{4F}^{(8)}$, which is shown in Appendix B.1.1. When their chiralities are $(LL)(LL)$ or $(RR)(RR)$ (L: left-handed, R: right-handed), the 4-Fermi operators are given by

$$\mathcal{O}_{4F}^{(1)} = \frac{12}{192\pi^2}g_s^4\left[\frac{2}{9}(f_1 + f_2)\right](\overline{q}\gamma^\mu q)(\overline{q}'\gamma^\mu q'), \tag{3.1.24}$$

$$\mathcal{O}_{4F}^{(8)} = \frac{12}{192\pi^2}g_s^4\left[-\frac{1}{3}f_1 - \frac{7}{6}f_2\right](\overline{q}\gamma^\mu \frac{\lambda^a}{2} q)(\overline{q}'\gamma^\mu \frac{\lambda^a}{2} q'). \tag{3.1.25}$$

On the other hand, when their chiralities are $(LL)(RR)$ or $(RR)(LL)$, the 4-Fermi operators are given by

$$\mathcal{O}_{4F}^{(1)} = \frac{12}{192\pi^2}g_s^4\left[\frac{2}{9}(-f_1 + f_2)\right](\overline{q}\gamma^\mu q)(\overline{q}'\gamma^\mu q'), \tag{3.1.26}$$

$$\mathcal{O}_{4F}^{(8)} = \frac{12}{192\pi^2}g_s^4\left[-\frac{7}{6}f_1 - \frac{1}{3}f_2\right](\overline{q}\gamma^\mu \frac{\lambda^a}{2} q)(\overline{q}'\gamma^\mu \frac{\lambda^a}{2} q'). \tag{3.1.27}$$

As for $\mathcal{O}_{qqG}$, vertex originates from two parts, $\alpha\tilde{K}_I$ and $\beta\tilde{K}_I\tilde{K}_I$, as

$$\begin{aligned}
&\int d^4x d^4y \alpha(x,y)_{ij}^{ab}\tilde{K}_I(x,y)_{ij}^{ab} \supset \int d^4x d^4y \left[-\frac{1}{2}\delta^{ab}\mathcal{D}^{(\tilde{g})}(y-x)_{ji}\right] \\
&\times \left[-4ig_s^3 \int d^4z \sum_{q=q_L,q_R} \overline{q}_j(y)\frac{\lambda^b}{2}\mathcal{D}^{(\tilde{q})}(y-z)\{2G_\mu\partial^\mu + (\partial^\mu G_\mu)\}_z \mathcal{D}^{(\tilde{q})}(z-x)\frac{\lambda^a}{2}q_i(x)\right],
\end{aligned} \tag{3.1.28}$$

$$\begin{aligned}
&\int d^4x d^4y d^4z d^4w \beta(x,y,z,w)_{ijkl}^{abcd}\tilde{K}_I(x,y)_{ij}^{ab}\tilde{K}_I(z.w)_{kl}^{cd} \\
&\supset 2\int d^4x d^4y d^4z d^4w \left[\frac{i}{8}\delta^{ac}\delta^{bd}\left[C^\dagger\mathcal{D}^{(\tilde{g})}(x-z)\right]_{ik}\left[\mathcal{D}^{(\tilde{g})}(w-y)C^T\right]_{lj}\right. \\
&\left.-\frac{i}{8}\delta^{ad}\delta^{bc}\mathcal{D}^{(\tilde{g})}(w-x)_{li}\mathcal{D}^{(\tilde{g})}(y-z)_{lj}\right] \\
&\times \left[ig_s f^{abc}\delta^4(x-y)\not{G}_{ij}^c\right] \cdot \left[-4ig_s^2 \sum_{q=q_L,q_R} \overline{q}_j(y)\frac{\lambda^b}{2}\mathcal{D}^{(\tilde{q})}(y-x)\frac{\lambda^a}{2}q_i(x)\right].
\end{aligned} \tag{3.1.29}$$

Equations (3.1.28) and (3.1.29) include not only dimension four operators but also all higher dimensional operators such as dimension six operator. Higher dimensional operators have been obtained by expanding the full operator by $k^2 \ll \Lambda^2$, where k^μ and Λ denote the momentum of the SM particles and SUSY particles, respectively. Anyhow, we can obtain all $\mathcal{O}_{qqG}$ in the similar calculations as 4-Fermi operators, which is shown Appendix B.1.2.

For $\mathcal{O}_{qqGG}$, they can be also obtained in the same manner. $\mathcal{O}_{qqGG}$ contains in Eq.(3.1.13), and there are two contributions in the first order of $\tilde{K}_I$ as

$$\begin{aligned} &\int d^4xd^4y\alpha(x,y)^{ab}_{ij}\tilde{K}_I(x,y)^{ab}_{ij} \\ &\supset \int d^4xd^4y\alpha(x,y)^{ab}_{ij}\cdot 4g_s^2\int d^4z\overline{q}_j(y)\frac{\lambda}{2}^a i\mathcal{D}^{(\tilde{q})}(y-z)[-g_s^2G_\mu G^\mu]_z i\mathcal{D}^{(\tilde{q})}(z-x)\frac{\lambda}{2}^a q_i(x), \end{aligned} \tag{3.1.30}$$

$$\begin{aligned} &\int d^4xd^4y\alpha(x,y)^{ab}_{ij}\tilde{K}_I(x,y)^{ab}_{ij} \\ &\supset \int d^4xd^4yd^4zd^4w\alpha(x,y)^{ab}_{ij}(-4g_s^2)\overline{q}_j(y)\frac{\lambda}{2}^a i\mathcal{D}^{(\tilde{q})}(y-z)ig_s[2G^\mu\partial_\mu+(\partial_\mu G^\mu)]_z \\ &\times i\mathcal{D}^{(\tilde{q})}(z-w)ig_s[2G^\mu\partial_\mu+(\partial_\mu G^\mu)]_w i\mathcal{D}^{(\tilde{q})}(w-x)\frac{\lambda}{2}^a q_i(x). \end{aligned} \tag{3.1.31}$$

Similarly, there is one contribution in the second order of $\tilde{K}_I$, which is shown as

$$\begin{aligned} &\int d^4xd^4yd^4zd^4w\beta(x,y,z,w)^{abcd}_{ijkl}\tilde{K}_I(x,y)^{ab}_{ij}\tilde{K}_I(z.w)^{cd}_{kl} \\ &\supset 2\int d^4xd^4yd^4zd^4w\beta(x,y,z,w)^{abcd}_{ijkl}[ig_sf^{abe}\not{G}^e_{ij}(x)\delta^4(x-y)] \\ &\times \int d^4z'4g_s^2\overline{q}_l(z)\frac{\lambda}{2}^d i\mathcal{D}^{(\tilde{q})}(z-z')\{ig_s(2G^\mu\partial_\mu+(\partial_\mu G^\mu)\}_{z'}i\mathcal{D}^{(\tilde{q})}(z'-w)\frac{\lambda}{2}^c q_k(w) \end{aligned} \tag{3.1.32}$$

There is one contribution in the third order in Eq.(3.1.13), $\gamma\tilde{K}_I\tilde{K}_I\tilde{K}_I$, where γ is given by

$$
\begin{aligned}
\gamma(x,y,z,w,u,v)^{abcdef}_{ijklmn} = \frac{1}{8}\{ & \delta^{ad}\delta^{be}\delta^{cf}\mathcal{D}^{(\tilde{g})}(w-x)_{li}\mathcal{D}^{(\tilde{g})}(y-u)_{jm}\mathcal{D}^{(\tilde{g})}(v-z)_{nk} \\
& + \delta^{af}\delta^{bc}\delta^{ed}\mathcal{D}^{(\tilde{g})}(v-x)_{ni}\mathcal{D}^{(\tilde{g})}(y-x)_{jk}\mathcal{D}^{(\tilde{g})}(w-u)_{lm} \\
& - \delta^{ac}\delta^{be}\delta^{df}[C^\dagger\mathcal{D}^{(\tilde{g})}(z-x)]_{ki}\mathcal{D}^{(\tilde{g})}(y-u)_{jm}[\mathcal{D}^{(\tilde{g})}(v-w)C^T]_{nl} \\
& - \delta^{ac}\delta^{bf}\delta^{de}[C^\dagger\mathcal{D}^{(\tilde{g})}(x-z)]_{ik}[\mathcal{D}^{(\tilde{g})}(v-y)C^T]_{nj}\mathcal{D}^{(\tilde{g})}(w-u)_{lm} \\
& - \delta^{ad}\delta^{bf}\delta^{ce}\mathcal{D}^{(\tilde{g})}(w-x)_{li}[\mathcal{D}^{(\tilde{g})}(y-v)C^T]_{jn}[C^\dagger\mathcal{D}^{(\tilde{g})}(u-z)]_{mk} \\
& - \delta^{ae}\delta^{bc}\delta^{fd}[C^\dagger\mathcal{D}^{(\tilde{g})}(u-x)]_{mi}\mathcal{D}^{(\tilde{g})}(y-z)_{jk}[\mathcal{D}^{(\tilde{g})}(w-v)C^T]_{ln} \\
& - \delta^{ae}\delta^{bd}\delta^{fc}[C^\dagger\mathcal{D}^{(\tilde{g})}(x-u)]_{im}[\mathcal{D}^{(\tilde{g})}(y-w)C^T]_{lj}\mathcal{D}^{(\tilde{g})}(v-z)_{nk} \\
& -\delta^{af}\delta^{bd}\delta^{ec}\mathcal{D}^{(\tilde{g})}(v-x)_{ni}[\mathcal{D}^{(\tilde{g})}(y-w)C^T]_{jl}[C^\dagger\mathcal{D}^{(\tilde{g})}(z-u)]_{km}\},
\end{aligned}
\tag{3.1.33}
$$

and the third order is shown as

$$
\begin{aligned}
& \int d^4x\cdots d^4v\gamma(x,y,z,w,u,v)^{abcdef}_{ijklmn}\tilde{K}_I(x,y)^{ab}_{ij}\tilde{K}_I(z,w)^{cd}_{kl}\tilde{K}_I(u,v)^{ef}_{mn} \\
& \supset \int d^4x\cdots d^4v\gamma(x,y,z,w,u,v)^{abcdef}_{ijklmn}\left[(-4g_s^2)\overline{q}_j(y)\frac{\lambda^b}{2}i\mathcal{D}^{(\tilde{q})}(y-x)\frac{\lambda^a}{2}q_i(x)\right] \\
& \times\left[ig_sf^{cdg}\not{G}^g(z)_{kl}\delta^4(z-w)\right]\left[ig_sf^{efh}\not{G}^h(u)_{mn}\delta^4(u-v)\right].
\end{aligned}
\tag{3.1.34}
$$

Although γ has eight terms in total, they are all the same in Eq.(3.1.34) since each term of γ corresponds to statistic factor in Feynman diagram. Notice again that we do not care about a statistic factor in each operator since it is automatically included in α, β, γ. We can obtain all $\mathcal{O}_{qqGG}$ induced from SUSY which is shown Appendix B.1.1.

3.2 Universal extra dimension model

This section aims to obtain the Lagrangian of UED model, and we calculate dimension six operators. Number of reviews and related topics of UED model can be found in, for example, Refs.[36, 37, 38, 39, 40, 41, 42, 43, 44, 45]. We consider five dimensional flat space-time which metric is given by

$$ds^2 = g_{MN}dx^Mdx^N = \eta_{\mu\nu}dx^\mu dx^\nu - dy^2, \tag{3.2.35}$$

$$g_{MN} = \text{diag}\ (+1,-1,-1,-1,-1), \tag{3.2.36}$$

$$\eta_{\mu\nu} = \text{diag}\ (+1,-1,-1,-1), \tag{3.2.37}$$

where $M, N, \cdots$ are indices of five dimensional space-time ($M, N = 0, 1, 2, 3, 5$), and $\mu, \nu, \cdots$ are indices of four dimensional space-time ($\mu, \nu = 0, 1, 2, 3$). We denote the fifth dimensional coordinate $x^5 \equiv y$, and consider the extra dimension is compactified on a circle S^1 with the radius R. In order to obtain chiral fermion, 5th direction should be projected on S^1/Z_2 orbifold where the region of y is given by $0 \leq y \leq \pi R$, which will be explained below. In UED model, all SM fields propagate in the bulk (y direction in our setup). If there are no extra fields other than the SM fields and their KK excited modes, the model is called minimal UED model, which we consider in this section.

Since the extra dimension is a compact space, all fields can be expanded by a complete set of y-dependent wave function $f(y)$,

$$\phi(x^\mu, y) = \sum_n \phi^{(n)}(x^\mu) f(y), \tag{3.2.38}$$

where $\phi^{(n)}(x^\mu)$ is nth KK mode. Before Z_2 orbifolding, theory has translational symmetry due to S^1 symmetry. Momentum in the fifth direction is now quantized, and KK modes carries a conserved quantum number, called KK number, under the symmetry. Dividing Z_2 symmetry, the KK number is no longer conserved, however, a discrete symmetry remains, which is called KK-parity defined by $(-1)^n$. The minimal UED assumes the conservation of KK-parity. KK-parity forbids a single production of KK particle, and guarantees that the lightest KK particle is stable. Due to S^1/Z_2 compactification, theory should be invariant under the orbifold projection $P_1 : y \to -y$. Thus there are two possibilities of the expansion of fields as follows,

$$\phi_+(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \phi_+^{(0)}(x^\mu) + \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} \phi_+^{(n)}(x^\mu) \cos \frac{ny}{R}, \tag{3.2.39}$$

$$\phi_-(x^\mu, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} \phi_-^{(n)}(x^\mu) \sin \frac{ny}{R}, \tag{3.2.40}$$

where ϕ_+ (ϕ_-) is P_1 even (odd) field. We comment on their normalization factors. For instance, when we write the expansion of ϕ_+ as

$$\phi_+(x^\mu, y) = N_0 \phi_+^{(0)}(x^\mu) + \sum_{n=1}^{\infty} N_n \phi_+^{(n)}(x^\mu) \cos \frac{ny}{R}, \tag{3.2.41}$$

4D quadratic term is given by

$$\int_0^{\pi R} |\phi_+(x^\mu, y)|^2 dy = \pi R |N_0|^2 |\phi_+^{(0)}(x^\mu)|^2 + \frac{\pi R}{2} \sum_{n=1}^{\infty} |N_n|^2 |\phi_+(x^\mu)|^2. \tag{3.2.42}$$

Suppose that N_n is the same for all $\phi_+^{(n)}$, the normalization factors are determined as $N_0 = \frac{1}{\sqrt{\pi R}}$ and $N_n = \sqrt{\frac{2}{\pi R}}$ in order to make the kinetic term into canonical.

On the boundaries of $y = 0$ and πR, ϕ_+ and ϕ_- satisfy the following boundary conditions,

$$\text{Neumann boundary conditions :} \quad \left.\frac{\partial \phi_+(x^\mu, y)}{\partial y}\right|_{y=0} = \left.\frac{\partial \phi_+(x^\mu, y)}{\partial y}\right|_{y=\pi R} = 0, \tag{3.2.43}$$

$$\text{Dirichlet boundary conditions :} \quad \phi_-(x^\mu, y = 0) = \phi_-(x^\mu, y = \pi R) = 0. \tag{3.2.44}$$

For a vector field $A_M(x^\mu, y)$, A_μ (A_5) is P_1 even (odd) so that a vector field transforms in the same way as the space-time coordinate. Therefore A_μ and A_5 respectively obey Neumann and Dirichlet boundary condition, and thus, they can be expanded as

$$A_\mu(x^\mu, y) = \frac{1}{\sqrt{\pi R}} A_\mu^{(0)}(x^\mu) + \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} A_\mu^{(n)}(x^\mu) \cos\frac{ny}{R}, \tag{3.2.45}$$

$$A_5(x^\mu, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} A_\mu^{(n)}(x^\mu) \sin\frac{ny}{R}. \tag{3.2.46}$$

All SM scalar and vector fields have zero mode, and even charge of KK-parity.

The expansion of a fermion is rather interesting. Clifford algebra in five dimensions is constructed as follows,

$$\{\Gamma^M, \Gamma^N\} = 2g^{MN}, \text{ where } \quad \Gamma^\mu = \gamma^\mu, \ \Gamma^5 = i\gamma^5. \tag{3.2.47}$$

Since the kinetic term of a fermion

$$\int d^5x \ \bar{\Psi} i \Gamma^M \partial_M \Psi \tag{3.2.48}$$

contains γ^5, theory can not be constructed in chiral, and a fermion needs a chiral partner and becomes a vector-like particle in five dimensions. Thus, a fermion can be expanded as follows,

$$\Psi(x^\mu, y) = \begin{pmatrix} \frac{1}{\sqrt{\pi R}} \psi_R^{(0)}(x^\mu) + \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \psi_R^{(n)}(x^\mu) \cos\frac{ny}{R} \\ \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \psi_L^{(n)}(x^\mu) \sin\frac{ny}{R} \end{pmatrix} \tag{3.2.49}$$

with Ψ_R (Ψ_L) obeys Neumann (Dirichlet) boundary condition, or

$$\Psi(x^\mu, y) = \begin{pmatrix} \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \psi_R^{(n)}(x^\mu) \sin\frac{ny}{R} \\ \frac{1}{\sqrt{\pi R}} \psi_L^{(0)}(x^\mu) + \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \psi_L^{(n)}(x^\mu) \cos\frac{ny}{R} \end{pmatrix} \tag{3.2.50}$$

with Ψ_L (Ψ_R) obeys Neumann (Dirichlet) boundary condition, where L and R denote the chiral projection in four dimensions. For the SM fermions, $\psi_L^{(0)}$ is $SU(2)_L$ doublet, and $\psi_R^{(0)}$ is $SU(2)_L$ singlet.

Using above expressions, the UED Lagrangian of QCD sector is obtained as follows,

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_{SM} + \mathcal{L}_{q^{(n)}} + \mathcal{L}_{G^{(n)}} + \mathcal{L}_{G_5^{(n)}}, \\
\mathcal{L}_{q^{(n)}} =& \sum_n \Big[\overline{q_L}^{(n)} i(\partial\!\!\!/ + i g_s G\!\!\!/ - m_L^{(n)}) q_L^{(n)} + \overline{q_R}^{(n)} i(\partial\!\!\!/ + i g_s G\!\!\!/ + m_R^{(n)}) q_R^{(n)} \\
& -g_s(\overline{q} G\!\!\!/^{(n)} P_L q^{(n)} + \overline{q} i\gamma^5 G_5^{(n)} P_R q^{(n)}) - (L \longleftrightarrow R) \Big] + \cdots , \\
\mathcal{L}_{G^{(n)}} =& \sum_n \Big[-\frac{1}{4}(\partial_\mu G_\nu^{(n)a} - \partial_\nu G_\mu^{(n)a})^2 + \frac{1}{2} m_g^{(n)2} G_\mu^{(n)a} G^{(n)a\mu} \\
& -\tfrac{1}{2} g_s f^{abc}(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a) G^{(n)b\mu} G^{(n)c\nu} - \tfrac{1}{2} g_s f^{abc}(\partial_\mu G_\nu^{(n)a} - \partial_\nu G_\mu^{(n)a}) G^{(n)b\mu} G^{c\nu} \\
& -\tfrac{1}{2} g_s f^{abc} \partial_\mu G_\nu^{(n)a} - \partial_\nu G_\mu^{(n)a}) G^{b\mu} G^{(n)c\nu} - \tfrac{1}{4} f^{abc} f^{ade} \Big\{ 2 G_\mu^{(0)b} G_\nu^{(0)c} G^{(n)d\mu} G^{(n)e\mu} \\
& +(G_\mu^{(0)b} G_\nu^{(n)c} + G_\mu^{(n)b} G_\nu^{(0)c})(G^{(0)d\mu} G^{(n)e\mu} + G^{(n)d\mu} G^{(0)e\mu}) \Big\} \Big] \cdots , \\
\mathcal{L}_{G_5^{(n)}} =& \sum_n \Big[\frac{1}{2} \partial_\mu G_5^{(n)a} \partial^\mu G_5^{(n)a} - \frac{1}{2} m_5^2 G_5^{(n)a} G_5^{(n)a} \\
& + g_s f^{abc}(m_g G_\nu^{(n)a} + \partial_\nu G_5^{(n)a}) G^{(n)b5} G^{c\nu} - \tfrac{1}{2} g_s^2 f^{abc} f^{ade} G_5^{(n)b} G_\nu^c G^{(n)d5} G^{e\nu} \Big] + \cdots ,
\end{aligned}
$$

where

$$
\begin{aligned}
m_L^{(n)} &= \frac{n}{R} + \delta m_L, \quad m_R^{(n)} = \frac{n}{R} + \delta m_R, \\
m_g^{(n)} &= \frac{n}{R} + \delta m_g, \quad m_5^{(n)} = \frac{n}{R} + \delta m_5.
\end{aligned}
$$

Here we take a 'tHooft-Feynman gauge fixing, and $m_L^{(n)}, m_R^{(n)}, m_g^{(n)}, m_5^{(n)}$ are $SU(2)$ doublet KK quark mass, $SU(2)$ singlet KK quark mass, KK gluon mass, and KK scalar (fifth dimensional component of KK gluon) mass with each radiative correction, respectively. At a tree level, these KK particles are degenerate in a minimal UED, but there is a slight difference between $m_L^{(n)}$ and $m_R^{(n)}$ when we consider radiative corrections.

3.2.1 Dimension six operators in minimal UED model

Next, we estimate QCD dimension six operators induced from UED. The UED has KK-parity so that KK particles can propagate only inside loop processes. As the SUSY case, we can calculate dimension six operators by integrating out KK particles.

The effective operators of $S_{\rm eff}$ in UED can be calculated by the similar technique of the previous subsection, and the results are shown in Appendix B.2.

Here we overview this calculation. By integrating out KK quarks, KK scalars, and KK gluons, the effective action becomes

$$\begin{aligned}S_{\rm eff} = \tilde{S} &+ \int d^4x d^4y \alpha(x,y)^{ab}_{\mu\nu} K_I(x,y)^{ab}_{\mu\nu} \\ &+ \int d^4x d^4y d^4z d^4w \beta(x,y,z,w)^{abcd}_{\mu\nu\rho\sigma} K_I(x,y)^{ab}_{\mu\nu} K_I(z,w)^{cd}_{\rho\sigma} + \cdots \ ,\end{aligned} \tag{3.2.51}$$

where $\tilde{S}$ does not include KK gluons that is given by

$$\begin{aligned}\tilde{S} = &-g_s^3 \int d^4x d^4y d^4z \mathcal{D}^{(s)}(x-y) \left[\overline{q}_L(x) \frac{\lambda}{2}^a \mathcal{D}^{(L)}(x-z) \not{G}(z) \mathcal{D}^{(L)}(z-y) \frac{\lambda}{2}^a q_L(y) + (L \leftrightarrow R) \right] \\ &+ i \int d^4x d^4y d^4z_1 d^4z_2 d^4z_3 \delta^4(x-y) \\ &\times \left[g_s f^{bcf} G^{f\mu}(z_1) \partial_{z_1\mu} \delta^4(z_1 - z_2) + i g_s^2 \left\{ \overline{q}_L(z_1) \frac{\lambda}{2}^b \mathcal{D}^{(L)}(z_1 - z_2) \frac{\lambda}{2}^c q_L(z) + (L \leftrightarrow R) \right\} \right] \\ &\times \left[\mathcal{D}^{(s)}(x - z_1) \delta^{ab} \right] \\ &\times \left[g_s f^{deg} G^{g\nu}(z_3) \partial_{z_3\nu} \delta^4(z_3 - y) + i g_s^2 \left\{ \overline{q}_L(z_3) \frac{\lambda}{2}^b \mathcal{D}^{(L)}(z_3 - y) \frac{\lambda}{2}^c q_L(y) + (L \leftrightarrow R) \right\} \right] \\ &\times \left[\mathcal{D}^{(s)}(z_2 - z_3) \delta^{cd} \right] .\end{aligned} \tag{3.2.52}$$

Here $\mathcal{D}^{(L)}$, $\mathcal{D}^{(R)}$, and $\mathcal{D}^{(s)}$ are propagators of KK quarks and KK scalars as

$$\mathcal{D}^{(L)} = \int \frac{d^4p}{(2\pi)^4} \frac{i}{\not{p} - m_L^{(n)}} e^{-ip(x-y)}, \tag{3.2.53}$$

$$\mathcal{D}^{(R)} = \int \frac{d^4p}{(2\pi)^4} \frac{i}{\not{p} + m_R^{(n)}} e^{-ip(x-y)}, \tag{3.2.54}$$

$$\delta^{ab}\mathcal{D}^{(s)} = \delta^{ab} \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m_5^{(n)2}} e^{-ip(x-y)}, \tag{3.2.55}$$

respectively. And α, β, K_I are given by

$$\alpha(x,y)^{ab}_{\mu\nu} = \frac{1}{2}\mathcal{D}^{(g)}(x-y)\delta^{ab}g_{\mu\nu} \equiv \frac{1}{2}\delta^{ab}g_{\mu\nu}\int\frac{d^4p}{(2\pi)^4}\frac{-i}{p^2-m_g^2}e^{-i(x-y)}, \tag{3.2.56}$$

$$\begin{aligned}&\beta(x,y,z,w)^{abcd}_{\mu\nu\rho\sigma}\\ &= \frac{i}{8}\left[\mathcal{D}^{(g)}(x-z)\mathcal{D}^{(g)}(y-w)\delta^{ac}\delta^{bd}g_{\mu\rho}g_{\nu\sigma} + \mathcal{D}^{(g)}(x-w)\mathcal{D}^{(g)}(y-z)\delta^{ad}\delta^{bc}g_{\mu\sigma}g_{\nu\rho}\right],\end{aligned} \tag{3.2.57}$$

$$\begin{aligned}&K_I(x,y)^{ab}_{\mu\nu} = -2g_s f^{abc}\left[\partial^\mu G^{c\nu}(x) + G^{c\mu}(x)\partial^\nu_x - g^{\mu\nu}G^{c\sigma}(x)\partial_{x\sigma}\right]\delta^4(x-y)\\
&+ 2ig_s^2\left[\overline{q}_L(x)\gamma^\mu\frac{\lambda^a}{2}\mathcal{D}^{(L)}(x-y)\gamma^\nu\frac{\lambda^b}{2}q_L(y) + (L\leftrightarrow R)\right]\\
&+ 2g_s^3\int d^4z\left[\overline{q}_L(x)\gamma^\mu\frac{\lambda^a}{2}\mathcal{D}^{(L)}(x-z)\not{G}(z)\mathcal{D}^{(L)}(z-y)\gamma^\nu\frac{\lambda^b}{2}q_L(y) + (L\leftrightarrow R)\right]\\
&- ig_s^4\int d^4z d^4w\\
&\times\left[\left(\overline{q}_L(x)\gamma^\mu\frac{\lambda^a}{2}\mathcal{D}^{(L)}(x-z)\frac{\lambda^c}{2}q_L(z)\right)\mathcal{D}^{(s)}(z-w)\left(\overline{q}_L(y)\gamma^\nu\frac{\lambda^b}{2}\mathcal{D}^{(L)}(y-w)\frac{\lambda^c}{2}q_L(w)\right)\right.\\
&+\left(\overline{q}_L(z)\frac{\lambda^c}{2}\mathcal{D}^{(L)}(z-x)\gamma^\mu\frac{\lambda^a}{2}q_L(x)\right)\mathcal{D}^{(s)}(z-w)\left(\overline{q}_L(w)\frac{\lambda^c}{2}\mathcal{D}^{(L)}(w-y)\gamma^\nu\frac{\lambda^b}{2}q_L(y)\right)\\
&-2\left(\overline{q}_L(x)\gamma^\mu\frac{\lambda^a}{2}\mathcal{D}^{(L)}(x-z)\frac{\lambda^c}{2}q_L(z)\right)\mathcal{D}^{(s)}(z-w)\left(\overline{q}_L(w)\frac{\lambda^c}{2}\mathcal{D}^{(L)}(w-y)\gamma^\nu\frac{\lambda^b}{2}q_L(y)\right)\\
&-\left(\overline{q}_L(x)\gamma^\mu\frac{\lambda^a}{2}\mathcal{D}^{(L)}(x-z)\frac{\lambda^c}{2}q_L(z)\right)\mathcal{D}^{(s)}(z-w)\left(\overline{q}_R(y)\gamma^\nu\frac{\lambda^b}{2}\mathcal{D}^{(R)}(y-w)\frac{\lambda^c}{2}q_R(w)\right)\\
&-\left(\overline{q}_L(z)\frac{\lambda^c}{2}\mathcal{D}^{(L)}(z-x)\gamma^\mu\frac{\lambda^a}{2}q_L(x)\right)\mathcal{D}^{(s)}(z-w)\left(\overline{q}_R(w)\frac{\lambda^c}{2}\mathcal{D}^{(R)}(w-y)\gamma^\nu\frac{\lambda^b}{2}q_R(y)\right)\\
&+\left(\overline{q}_L(x)\gamma^\mu\frac{\lambda^a}{2}\mathcal{D}^{(L)}(x-z)\frac{\lambda^c}{2}q_L(z)\right)\mathcal{D}^{(s)}(z-w)\left(\overline{q}_R(w)\frac{\lambda^c}{2}\mathcal{D}^{(R)}(w-y)\gamma^\nu\frac{\lambda^b}{2}q_R(y)\right)\\
&+\left(\overline{q}_L(w)\frac{\lambda^c}{2}\mathcal{D}^{(L)}(w-y)\gamma^\nu\frac{\lambda^b}{2}q_L(y)\right)\mathcal{D}^{(s)}(z-w)\left(\overline{q}_R(x)\gamma^\mu\frac{\lambda^a}{2}\mathcal{D}^{(R)}(x-z)\frac{\lambda^c}{2}q_R(z)\right)\\
&\left.+(L\leftrightarrow R)\right],\end{aligned} \tag{3.2.58}$$

respectively. Here $\partial^\nu_x, \partial_{x\sigma}$ in the first line of Eq.(3.2.58) means derivatives of KK gluons.

Dimension six operators induced from UED can be also obtained as taking KK particles are enough heavy than SM particles. In UED case, we also analyze up to $\mathcal{O}(g_s^4)$. We can show that $(K_I)^n$ with $n = 1, 2, 3$ once contribute $\mathcal{O}_{4F}$, respectively, and shown in Appendix B.1.2. Similarly, $(K_I)^n$ with $n = 1, 3$ once contribute $\mathcal{O}_{qqG}$, respectively, and $(K_I)^2$ twice contributes $\mathcal{O}_{qqG}$ and shown in Appendix B.1.2. As for $\mathcal{O}_{qqGG}$, K_I four times, $(K_I)^2$ six times, $(K_I)^3$ five times, and $(K_I)^4$ once contribute, respectively, and shown in Appendix B.1.2.

3.3 Helicity asymmetry in top pair production at the LHC

The helicity of top pair can be measured, since top quark immediately decays before hadronization differently from other quarks. The observed property in the decay products assessed the helicity information of the t and $\bar{t}$ [46, 47, 48, 49, 50]. Then a measurement of the cross section depending on helicities of $t\bar{t}$ is possible. We denote the helicity dependent cross section as $\sigma_{\lambda_t \lambda_{\bar{t}}}$, where $\lambda_t(\lambda_{\bar{t}}) = \pm 1$ refers to the $t(\bar{t})$ spin projection onto the $t(\bar{t})$ flight direction. In this basis, we introduce the helicity asymmetry as

$$\delta A_{LR} = \frac{\frac{d\sigma_{+-}}{dm_{t\bar{t}}} + \frac{d\sigma_{++}}{dm_{t\bar{t}}} - \frac{d\sigma_{-+}}{dm_{t\bar{t}}} - \frac{d\sigma_{--}}{dm_{t\bar{t}}}}{\frac{d\sigma_{+-}}{dm_{t\bar{t}}} + \frac{d\sigma_{++}}{dm_{t\bar{t}}} + \frac{d\sigma_{-+}}{dm_{t\bar{t}}} + \frac{d\sigma_{--}}{dm_{t\bar{t}}}}. \tag{3.3.59}$$

When we neglect a chirality flip via a top Yukawa coupling, which is $\mathcal{O}(\alpha_s^2 y_t)$ contribution, σ_{-+} and σ_{+-} are redundant, and it is useful to define δA_{LR} as

$$\delta A_{LR} = \frac{\frac{d\sigma_{+-}}{dm_{t\bar{t}}} - \frac{d\sigma_{-+}}{dm_{t\bar{t}}}}{\frac{d\sigma_{+-}}{dm_{t\bar{t}}} + \frac{d\sigma_{-+}}{dm_{t\bar{t}}}}. \tag{3.3.60}$$

This is leading order contribution (up to $\mathcal{O}(\alpha_s^2)$), and we use this expression in this paper. For experimental analyses, we consider how the helicity information is extracted in a final state. Top quark almost decay into a W boson and a b quark, which branching ratio is $\Gamma(t \to Wb)/\Gamma(t \to Wq(q = b, s, d)) \simeq 0.99$ [51]. The W boson from the top quark decay has three polarization states of longitudinal, left-handed, and right-handed. Right-handed W bosons are strongly suppressed because the top quark decay is $V - A$ interaction. Helicities of t and $\bar{t}$ can be observed in $2l + 2\nu$ and 2 b-jets event, for example. In this process, t decays into bW^+, and W^+ decays into $l^+\nu$, where ν is left-handed. In the W^+ rest frame, right-handed l^+ is emitted transversely to the direction of W^+ polarization due to angular momentum conservation. That is, W^+ polarization is observed as the angular distribution of the charged lepton. Then, the helicity information of t and $\bar{t}$ propagates into the final state of charged leptons.

Since the LHC is pp collider, the cross section is given by

$$\sigma(pp \to t\bar{t}) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 \hat{\sigma}(ab \to t\bar{t}; \hat{s}, x_1, x_2, \mu_R) D_{a/p}(x_1, M_Z) D_{b/p}(x_2, M_Z), \tag{3.3.61}$$

where $D_{a/p}(x, \mu_F)$ is a parton distribution function (PDF) with a factorization scale of μ_F, which is chosen for Z boson mass, for simplicity. The a and b stand for gluon and quark flavor in the proton. The $\hat{\sigma}$ is a parton level cross section with an invariant mass of a and b as $\hat{s} = (p_a + p_b)^2$ and scaling parameter x. The μ_R is a renormalization scale which we take M_Z. We demonstrate the PDFs of all flavors excepting top quark in Fig. 3.2 and 3.3. The

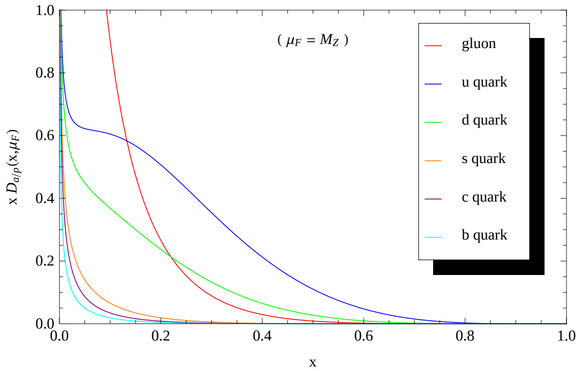

Figure 3.2: PDFs of gluon and all quarks excepting top quark. The factorization scale is set by $\mu_F = M_Z$.

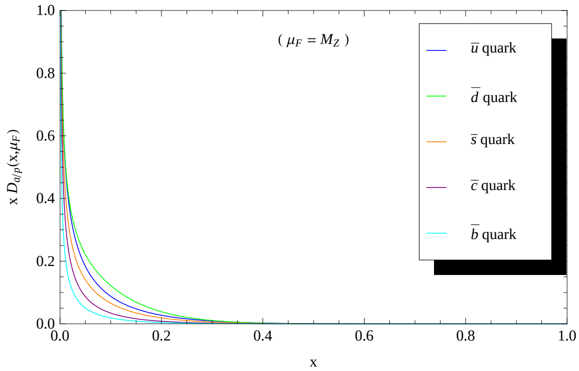

Figure 3.3: PDFs of all anti-quarks excepting anti-top quark. The factorization scale is set by $\mu_F = M_Z$.

PDFs satisfy a momentum sum rule such as

$$\sum_{a=\text{all flavors}} \int_0^1 dx\, x D_{a/p}(x, M_Z) = 1. \tag{3.3.62}$$

When we calculate the momentum sum for only gluon, we obtain

$$\int_0^1 dx\, x D_{g/p}(x, M_Z) \simeq 0.47, \tag{3.3.63}$$

therefore, the LHC, which is proton-proton collider, is almost gluon-gluon (or quark-gluon) collider.

At first, we represent parity violating dimension six operators, and, next, we investigate the QCD parity violation in SUSY and UED. Let us try to discriminate SUSY from UED through the parity violation even when masses of sparticles or KK-particles are too heavy to be detected at direct searches. We also estimate effects of weak parity violation in the SM and Little Higgs (LH) model.

3.3.1 Dimension six operators

We use an effective theory where particles of new physics, such as, sparticles and KK-particles, are integrated out. The QCD parity violation is represented by the SM field contents with dimension six operators as a leading order. These irrelevant operators in QCD are shown by $\mathcal{O}_{4F}^{(1)}, \mathcal{O}_{4F}^{(8)}, \mathcal{O}_{qqG}$, and $\mathcal{O}_{qqGG}$, which represent color-singlet 4-Fermi, color-octet 4-Fermi, quark-quark-gluon, and quark-quark-gluon-gluon operators, respectively. They are listed in Ref.[35], and given by

$$\begin{aligned}\mathcal{O}_{4F}^{(1)} =& \frac{12 g_s^4}{192\pi^2} \sum_{i,j=L,R} \int \frac{d^4k_1 d^4k_2 d^4k_3 d^4k_4}{(2\pi)^4(2\pi)^4(2\pi)^4(2\pi)^4} (2\pi)^4 \delta^4(-k_1+k_2-k_3+k_4) \\ & \times C_{ij} \left[\bar{q}(k_1)\gamma^\mu P_i q(k_2)\right] \left[\bar{q}'(k_3)\gamma_\mu P_j q'(k_4)\right], \qquad (3.3.64)\end{aligned}$$

$$\begin{aligned}\mathcal{O}_{4F}^{(8)} =& \frac{12 g_s^4}{192\pi^2} \sum_{i,j=L,R} \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\frac{d^4k_4}{(2\pi)^4} (2\pi)^4 \delta^4(-k_1+k_2-k_3+k_4) \\ & \times D_{ij} \left[\bar{q}(k_1)T^a\gamma^\mu P_i q(k_2)\right] \left[\bar{q}'(k_3)T^a\gamma_\mu P_j q'(k_4)\right], \qquad (3.3.65)\end{aligned}$$

$$\mathcal{O}_{qqG} = \frac{g_s^3}{96\pi^2} \sum_{i=L,R} \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4} (2\pi)^4 \delta^4(-k_1+k_2+k_3) \bar{q}(k_1) T^a E_i^\mu G_\mu^a(k_3) P_i q(k_2), \qquad (3.3.66)$$

$$\begin{aligned}\mathcal{O}_{qqGG} =& \frac{g_s^4}{192\pi^2} \sum_{i=L,R} \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\frac{d^4k_4}{(2\pi)^4} (2\pi)^4 \delta^4(-k_1+k_2+k_3+k_4) \\ & \times \bar{q}(k_1) \left[F_i^{\mu\nu}\delta^{ab} + H_i^{\mu\nu} T^a T^b\right] G_\mu^a(k_2) G_\nu^b(k_3) P_i q(k_4), \qquad (3.3.67)\end{aligned}$$

where P_i $(i = L,\ R)$ is the chirality projection, $P_L = \frac{1-\gamma^5}{2}$ $\left(P_R = \frac{1+\gamma^5}{2}\right)$, and $E_i^\mu,\ F_i^{\mu\nu},\ H_i^{\mu\nu}$ are defined as

$$\begin{aligned}E_i^\mu =& \{e_{1i}\not{k}_1 + e_{2i}\not{k}_2\}k_1^\mu + \{e_{3i}\not{k}_2 + e_{4i}\not{k}_1\}k_2^\mu \\ & + \{e_{5i}k_1^2 + e_{6i}k_2^2 - e_{7i}k_1\cdot k_2\}\gamma^\mu \\ & - e_{8i} i\epsilon^{\alpha\beta\mu\nu}\gamma_5\gamma_\nu k_{1\alpha}k_{2\beta}, \qquad (3.3.68)\\ F_i^{\mu\nu} =& f_{1i\alpha} i\epsilon^{\alpha\mu\nu\beta}\gamma_5\gamma_\beta + f_{2i\alpha}g^{\mu\nu}\gamma^\alpha + f_{3i\alpha}g^{\alpha\mu}\gamma^\nu \\ & + f_{4i\alpha}g^{\alpha\nu}\gamma^\mu, \qquad (3.3.69)\end{aligned}$$

$$\begin{aligned}H_i^{\mu\nu} =& h_{1i\alpha} i\epsilon^{\alpha\mu\nu\beta}\gamma_5\gamma_\beta + h_{2i\alpha}g^{\mu\nu}\gamma^\alpha + h_{3i\alpha}g^{\alpha\mu}\gamma^\nu \\ & + h_{4i\alpha}g^{\alpha\nu}\gamma^\mu. \qquad (3.3.70)\end{aligned}$$

In Eqs.(3.3.65) and (3.3.68), $C_{ij}, D_{ij}, e_{1i}, \cdots, e_{8i}$ are Wilson coefficients which have mass dimension M_{NP}^{-2}, where M_{NP} stands for a scale of new physics characterized by new particles' masses. In Eqs.(3.3.80) and (3.3.70), coefficients $f_{1i\alpha}, \cdots, h_{1i\alpha}, \cdots$ are some combination of the quark and the gluon momentum, e.g.

$$f_{1L\alpha} = f_{1L}^{(k_1)} k_{1\alpha} + f_{1L}^{(k_3)} k_{3\alpha} + f_{1L}^{(k_4)} k_{4\alpha} \tag{3.3.71}$$

for left-handed quarks, and $f_{1i}^{(k_1)}, \cdots, f_{4i}^{(k_4)}$, $h_{1i}^{(k_1)}, \cdots, h_{4i}^{(k_4)}$ are Wilson coefficients with mass dimension M_{NP}^{-2}. The QCD interactions in Eqs.(3.3.64)-(3.3.67) become chiral in the SUSY SM, since their coefficients are different between left- and right-handed quarks. We can see explicit coefficients of these operators in the SUSY SM and UED model in Appendix B and Ref.[35].

3.3.2 SUSY

In the SUSY SM with R-parity, SUSY particles can propagate only inside loop diagrams, and the maximal contributions of the parity violation come from 1-loop induced dimension six operators. There were some estimations previously, where SUSY particles have masses of $\mathcal{O}(100)$ GeV [52, 53, 54]. In particular, in Ref.[53], the asymmetry was estimated as $|\delta A_{LR}(m_{t\bar{t}})| \simeq 2.0\%$ with $\mathcal{O}(100)$ GeV sparticles. Here we show a similar estimation by use of dimension six operators by integrating out heavy SUSY particles. We neglect the left-right mixing in the stop mass matrix, which corresponds to neglect top Yukawa in the loop level.

We should take mass bounds of gluino and squarks constrained by LHC experiment[55]. Cross sections from SUSY dimension six operators at a center-of-mass energy $E_{\text{CM}} = 7$ TeV are listed in Table 3.2. Where we take some sample points of sparticle masses as $(m_{\tilde{g}}, m_{\tilde{t}_L}, m_{\tilde{t}_R}) = (2000, 2100, 1000)$, $(2000, 1200, 1000)$, and $(400, 1200, 410)$ GeV. Cases of (i) and (ii) show heavy sparticles consistent with LHC data[55]. In case of (iii), parameter set shows gluino and one of stop are degenerate within 30 GeV, which is not excluded experimentally, too. It is because there are experimental cut for p_Ts of multi-jets with missing transverse momentum in SUSY search at the LHC (Tevatron), where an event selection for jets is $p_T > 40$ GeV[55] ($p_T > 30$ GeV[56]), and p_T of jets are roughly estimated as the mass difference of gluino and squarks. For calculating a cross section, we should pay attention to the cut of collision energy at parton level. In the effective operator approach, operators are expanded by sparticle masses. This means the parton level invariant mass $\sqrt{\hat{s}}$ can not be larger than sparticle mass, $\sqrt{\hat{s}} < M_{\text{SUSY}}$. Here, we estimate the cross section under the limit of $\sqrt{\hat{s}} \leq 0.95 \times M_{\text{SUSY}}$, where M_{SUSY} stands for Min.$[m_{\tilde{g}}, m_{\tilde{t}_L}, m_{\tilde{t}_R}]$. Note that top and anti-top are mainly produced in QCD process in collider experiment and the experimental data shows $\sigma^{\rm exp}(pp \to t\bar{t}) = 179$ pb[57, 58], where the magnitude of $\sigma^{\rm exp}$ is almost same as that induced from the SM QCD processes, $\sigma^{\rm SM}$. Table 3.2 shows cross sections of (i) and (ii) are (roughly) 10^{-6} smaller than $\sigma^{\rm SM}$. Even if gluino mass is $\mathcal{O}(100)$ GeV as the case of (iii), the cross section does not drastically increase. We show the numerical calculation of the asymmetry δA_{LR} for the case of (i) together with UED result in section 3.3.5.

Table 3.3 shows cross sections at $E_{\rm CM} = 7$ TeV with various magnitudes of $m_{\tilde{t}_L}$ fixing $m_{\tilde{g}}$ and $m_{\tilde{t}_R}$. In Table 3.3, $\Delta\sigma$ is defined as $\Delta\sigma \equiv \sigma^{\rm SM+SUSY} - \sigma^{\rm SM}$, where $\sigma^{\rm SM+SUSY}$ stands for a cross section including QCD, SM electroweak (SMEW), and SUSY contributions. The SUSY contribution is small, i.e., cross section is $\mathcal{O}(10^{-4})$ pb and $|\Delta\sigma| \simeq \mathcal{O}(10^{-2})$ pb. The values of δA_{LR} depending on the masses are shown in Fig. 3.4. Apparently, the larger the mass difference between $\tilde{q}_L$ and $\tilde{q}_R$ becomes, the larger magnitude of δA_{LR} becomes.

$(m_{\tilde{g}}, m_{\tilde{t}_L}, m_{\tilde{t}_R})$ [GeV]	$\sigma^{\rm SUSY}(pp \to t\bar{t})$ [$\times 10^{-4}$pb]
(i), (2000, 2100, 1000)	1.5
(ii), (2000, 1200, 1000)	2.2
(iii), (400, 1200, 410)	8.8

Table 3.2: Sample points of sparticle masses and their cross sections at $E_{\rm CM} = 7$ TeV

$(m_{\tilde{g}}, m_{\tilde{t}_L}, m_{\tilde{t}_R})$ [GeV]	$\sigma^{\rm SUSY}$ [10^{-4}pb]	$\Delta\sigma$ [10^{-2}pb]
(2000, 1010, 1000)	2.5	-5.2
(2000, 1100, 1000)	2.3	-5.0
(2000, 1200, 1000)	2.2	-4.9
(2000, 1300, 1000)	2.3	-4.7
(2000, 1400, 1000)	2.0	-4.6
(2000, 1500, 1000)	1.9	-4.5
(2000, 1600, 1000)	1.8	-4.3
(2000, 1700, 1000)	1.8	-4.1
(2000, 1800, 1000)	1.7	-4.0
(2000, 1900, 1000)	1.6	-3.8
(2000, 2100, 1000)	1.5	-3.6
(2000, 2200, 1000)	1.5	-3.6
(2000, 2300, 1000)	1.5	-3.5

Table 3.3: Sparticles masses and corresponding cross sections are listed. $\sigma^{\rm SUSY} \equiv \sigma^{\rm SUSY}(pp \to t\bar{t})$ and $\Delta\sigma \equiv \sigma^{\rm SM+SUSY} - \sigma^{\rm SM}$.

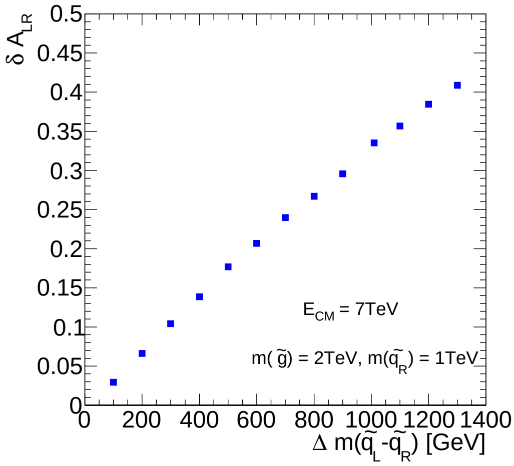

Figure 3.4: The relation between δA_{LR} and the mass differences in Table 3.3. The horizontal axis denotes $\Delta m(\tilde{q}_L - \tilde{q}_R) \equiv m_{\tilde{q}_L} - m_{\tilde{q}_R}$.

3.3.3 UED

In the UED with KK-parity, KK particles can propagate only inside loop diagrams, and the maximal contributions come from 1-loop induced dimension six operators. The QCD parity violation is induced only through non-degeneracy of KK masses induced from radiative corrections. The dimension six operators are listed in Appendix B.1 and Ref.[35], which contribute both $q\bar{q}$ annihilation processes and gluon fusion subprocesses. In UED we take the cut of collision energy to $\sqrt{\hat{s}} \leq 1/R$, where R is the compactification scale. At tree level, KK particles are degenerate, but there appears a slight difference between the left-handed KK quark mass, $m_L^{(n)}$, and right-handed one, $m_R^{(n)}$, through the renormalization effects [59] as

$$\delta m_{t_L^{(n)}} = \left(\frac{n}{R}\right)\left(3\frac{g_s^2}{16\pi^2} + \frac{27}{16}\frac{g^2}{16\pi^2} + \frac{1}{16}\frac{g'^2}{16\pi^2}\right)\log\frac{\Lambda^2}{\mu^2}, \tag{3.3.72}$$

$$\delta m_{t_R^{(n)}} = \left(\frac{n}{R}\right)\left(3\frac{g_s^2}{16\pi^2} + \frac{g'^2}{16\pi^2}\right)\log\frac{\Lambda^2}{\mu^2}, \tag{3.3.73}$$

where Λ and μ are the cutoff scale and the renormalization scale, respectively. Accurately speaking, these effects are beyond the order of α_s^2, however, here we take them into account, since it might be informative. We take $\mu = \sqrt{\hat{s}} \geq 2m_t$, and helicity asymmetry is plotted in a sample point as $(R^{-1}, \Lambda) = (2\ \text{TeV},\ 20\ \text{TeV})$. Fixing the ΛR means that KK-mode appears up to $n = 20$ below the cutoff scale. Here we take the sum of KK-modes up to infinity, for simplicity, because a difference of coefficients between the sum of n up to 20 and infinity is less than 3%. A numerical result of the magnitude of δA_{LR} in the UED model will be shown in Fig.3.7 and 3.8 in section 3.3.5.

3.3.4 SM electroweak background and Little Higgs model

Here we estimate δA_{LR} induced from not QCD but weak interactions, which is the SMEW background. The SMEW background is not negligible, although it was not estimated in Ref.[53]. The asymmetry from the SMEW, $\delta A_{LR}^{\rm SMEW}$, is induced by electroweak interactions at a tree level, and the cross section of the SMEW is estimated as $\sigma^{\rm SMEW}(pp \to t\bar{t}) \simeq 3.4 \times 10^{-1}$pb.* The SMEW contribution is larger than the SUSY contribution comparing to Table 3.2. It is worth noting that a magnitude of $\sigma^{\rm QCD+SMEW}$($\simeq$125 pb) is smaller than that of only QCD contribution $\sigma^{\rm QCD}$($\simeq$ 138 pb). The SMEW contributions in $pp \to t\bar{t}$ process is studied in Ref.[60, 61, 62].

We alos consider the Little Higgs (LH) model[63, 64, 65], and there is no QCD parity violation as in the SM. In the LH model, Higgs doublet is identified as Nambu-Goldstone boson (NGB), for example, associated with $SU(3)/SU(2)$, and quadratic divergence in Higgs mass correction does not appear. A minimal setup of the LH model is to take two global $SU(3)$ symmetries, and they are spontaneously broken into two $SU(2)$ symmetries by VEV of LHs such as

$$\langle \Sigma_1 \rangle = \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}, \qquad \langle \Sigma_2 \rangle = \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}, \tag{3.3.74}$$

where 10 NGBs appear. Here we introduce a $SU(3)$ gauge interaction with coupling g_3,

$$\mathcal{L} \;\sim\; |g_3 A_\mu \phi_1|^2 + |g_3 A_\mu \phi_2|^2, \tag{3.3.75}$$

where ϕ_1 and ϕ_2 are NGBs, and the $SU(3)$ gauge symmetry is *"diagonal."* The reason is as follows. When ϕ_1 and ϕ_2 transform under $SU(3)$ as

$$\phi_1 \to U_1 \phi_1, \qquad \phi_2 \to U_2 \phi_2, \tag{3.3.76}$$

gauge field A_μ should transform as

$$A_\mu \to U_1 A_\mu U_1^\dagger, \qquad A_\mu \to U_2 A_\mu U_2^\dagger, \tag{3.3.77}$$

*Input parameters are $(g_s, g, g') = (1.3, 0.65, 0.31)$, $M_Z = 91$ GeV, and $m_t = 173$ GeV.

and thus, we obtain $U_1 = U_2$. After symmetry breaking, 5 of NGBs are eaten by the gauge boson, and a part of remained NGBs can be identified as Higgs doublet. In this setup, the Higgs mass and the Higgs quartic coupling is induced by quantum corrections. A finite Higgs mass is obtained through a 1-loop diagram shown in Fig.3.5. Self-energy diagrams such as

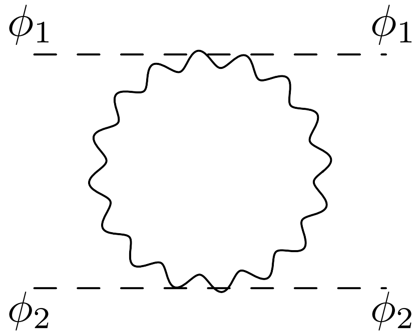

Figure 3.5: This 1-loop diagram induces a finite Higgs mass.

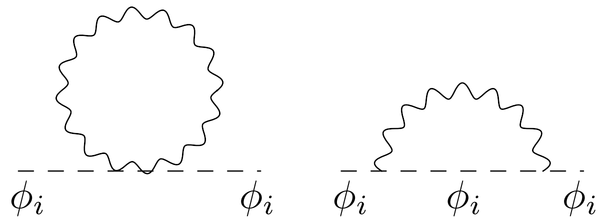

Figure 3.6: This 1-loop diagram does not contribute Higgs mass.

Fig.3.6 does not contribute the Higgs mass. We can check the contributions of the diagrams in Figs.3.5 and 3.6. When we write ϕ_1 and ϕ_2 as

$$\phi_1 \sim \exp\left[\frac{i}{f}\begin{pmatrix} 0 & h \\ h^\dagger & 0 \end{pmatrix}\right]\langle\Sigma_1\rangle, \qquad \phi_2 \sim \exp\left[-\frac{i}{f}\begin{pmatrix} 0 & h \\ h^\dagger & 0 \end{pmatrix}\right]\langle\Sigma_2\rangle, \tag{3.3.78}$$

where h is Higgs doublet, we obtain

$$(\text{Fig.3.5}) \sim \frac{g_3^2}{(4\pi)^2}\log\left(\frac{\Lambda^2}{\mu^2}\right)|\phi_1^\dagger\phi_2|^2 \sim f^2\frac{g_3^2}{(4\pi)^2}\log\left(\frac{\Lambda^2}{\mu^2}\right)h^\dagger h, \tag{3.3.79}$$

$$(\text{Fig.3.6}) \sim \frac{g_3^2}{(4\pi)^2}\Lambda^2(|\phi_1|^2 + |\phi_2|^2) \sim \frac{g_3^2}{(4\pi)^2}\Lambda^2(f^2 + f^2). \tag{3.3.80}$$

Thus a Higgs quadratic contribution does not appear in the Eq.(3.3.80). These results are understood as follows. When the gauge coupling of ϕ_2 is switched off, two $SU(3)$ sectors are independent, and ϕ_2 remains as exact NGB. Similarly, when the gauge coupling of ϕ_1 is switched off, ϕ_1 becomes exact NGB. Thus, the self-energy diagrams (Fig.3.6) does not give the Higgs mass. Only when the gauge couplings for both ϕ_1 and ϕ_2 are switched on, Higgs boson receives the mass induced by mixing of ϕ_1 and ϕ_2 through gauge interaction, where does not appear quadratic divergence. In matter sector, $SU(2)$ doublets are also enlarged into $SU(3)$ triplet as

$$\Psi = \begin{pmatrix} t \\ b \\ T \end{pmatrix} \equiv \begin{pmatrix} Q \\ T \end{pmatrix} \tag{3.3.81}$$

for the third generation. By introducing two right-handed singlets t_1^c and t_2^c, Yukawa couplings are written by

$$\begin{aligned} \mathcal{L}_Y &= \lambda_1 \phi_1^\dagger \Psi t_1^c + \lambda_2 \phi_2^\dagger \Psi t_2^c \\ &\sim h^\dagger Q(i\lambda_2 t_2^c - i\lambda_1 t_1^c) + T\left(f - \frac{h^\dagger h}{2f}\right)(\lambda_2 t_2^c + \lambda_1 t_1^c) + \cdots \\ &\sim \lambda_t h^\dagger Q t^c + f\lambda_t T\left(1 - \frac{h^\dagger h}{2f^2}\right) T^c + \cdots, \end{aligned} \tag{3.3.82}$$

where we assume $\lambda_1 = \lambda_2 = \lambda_t$ in the last line, and define $t^c = \frac{i}{\sqrt{2}}(t_2^c - t_1^c)$ as the SM top and $T^c = \frac{1}{\sqrt{2}}(t_2^c + t_1^c)$ for heavy top partner.

This minimal setup makes clear an important advantage of LH models, however, this setup suffere from precision electroweak constraints [66, 67, 68, 69, 70]. For a realistic model, LH model with T-parity is studied in Refs [71, 66, 68]. For example, in a miniml LH model with T-parity[72, 73, 71], the SM fields and a part of additional top partners are assigned to T-parity even, and other heavy fields are T-parity odd. In the model, top quark mixes the partner, and weak interactions are slightly modified. Ref.[74] discuss this mixing in single-top event at the Tevatron and the LHC.

For our concern, the effects of the LH is summarized as deviated weak interactions from the SM. The Lagrangian is given by

$$\mathcal{L}_{\text{int}} = -g_s \bar{t} \not{G} t - \frac{2}{3} e\bar{t} \not{A} t - \frac{g}{\sqrt{2}} \cos\beta (\bar{b} \not{W} P_L t + \text{h.c.}) - \frac{g}{\cos\theta_W} \bar{t} \not{Z} \left(-\frac{2}{3}\sin^2\theta_W + \frac{1}{2}\cos^2\beta P_L\right) t, \tag{3.3.83}$$

where β and m_t are $\tan^{-1}\frac{\lambda_1^2}{\lambda_1^2+\lambda_2^2}\frac{v}{f}$ and $\frac{v\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}$, respectively. The f denotes the VEV of the LH. When we take $\lambda_1 \simeq \lambda_2 \simeq 1$, we can estimate $\cos\beta \simeq 1 - \frac{v^2}{2f^2}$. Integrating out new heavy particles in the LH, the effective 4 Fermi operators are induced as

$$\begin{aligned} \mathcal{O}^{\text{LH}} &= \frac{g^2}{2}\cos^2\beta \frac{1}{k_W^2 - M_W^2}(\bar{t}\gamma^\mu P_L b)(\bar{b}\gamma^\mu P_L t) \\ &+ \sum_q^{\text{flavor}} \frac{g^2}{3}\tan^2\theta_W \cos^2\beta \frac{1}{k_Z^2 - M_Z^2}(\bar{q}\gamma^\mu P_L q)(\bar{t}\gamma^\mu P_L t), \end{aligned} \tag{3.3.84}$$

where k_W and k_Z stand for momenta of W and Z bosons, respectively. When the LH takes VEV as $f = 2$ TeV, the angle β becomes $\cos\beta = 0.992$. We will estimate the helicity asymmetry in the LH in Figs. 3.7 and 3.8. Note that the cross section of the LH model is the same order of that of the SMEW processes as $\sigma^{\rm LH}(pp \to t\bar{t}) = 2.4 \times 10^{-1}$ pb.

3.3.5 Discriminate SUSY from UED

Figures 3.7 $\sim$ 3.10 show results of numerical analyses of the magnitude of δA_{LR} in the SUSY, UED, SMEW, and LH depending on $m_{t\bar{t}}$ and p_T, respectively. For example, δA_{LR} in SUSY, denoted by $\delta A_{LR}^{\rm SUSY}$, is defined by $\delta A_{LR}^{\rm SUSY} = \dfrac{d\sigma_{+-}^{\rm SUSY}/dm_{t\bar{t}} - d\sigma_{-+}^{\rm SUSY}/dm_{t\bar{t}}}{d\sigma_{+-}^{\rm SUSY}/dm_{t\bar{t}} + d\sigma_{-+}^{\rm SUSY}/dm_{t\bar{t}}}$, while an observable magnitude of δA_{LR} in the experiments is given by $\delta A_{LR}^{\rm exp} = \dfrac{d\sigma_{+-}^{\rm exp}/dm_{t\bar{t}} - d\sigma_{-+}^{\rm exp}/dm_{t\bar{t}}}{d\sigma_{+-}^{\rm exp}/dm_{t\bar{t}} + d\sigma_{-+}^{\rm exp}/dm_{t\bar{t}}}$, where $\sigma^{\rm exp}$ is the total cross section.

Figures 3.7 and 3.8 show each contributions of the SM, SUSY, UED, and LH, and catch a trend whether the asymmetry is enhanced or not. The magnitude of $\delta A_{LR}^{\rm SUSY}$ in Figs. 3.7 and 3.8 does not include an interference of SM and SUSY. δA_{LR} of UED, SMEW, and LH in Figs. 3.7 and 3.8 are defined in the same way. We input SUSY mass parameters as ($m_{\tilde{g}}$, $m_{\tilde{q}_L}$, $m_{\tilde{q}_R}$) = (2TeV, 2.1TeV, 1TeV), and (2TeV, 1TeV, 2.1TeV). Parameters of UED model are taken as $(R^{-1}, \Lambda) = (2{\rm TeV}, 20{\rm TeV})$. Apparently, the helicity asymmetry in the SUSY SM can be larger than that in UED model, which is of course due to the squark mass splitting. For example, when $m_{\tilde{t}_L} \gg m_{\tilde{t}_R}$, left-handed top pair production should be suppressed, and then the sign of $\delta A_{LR}^{\rm SUSY}$ becomes positive because σ_{+-} is larger than σ_{-+}. The opposite case is similarly understood.

However, we should notice that the SUSY cross section is much smaller than that of the SM QCD, and unfortunately, once the SM interferes the SUSY contribution, the asymmetry could not seen by the large SM QCD contribution.

Figures 3.9 and 3.10 show δA_{LR} including the interference of SM and the new physics (SUSY, UED and LH), where $\delta A_{LR}^{\rm SM+SUSY}$ is around 3×10^{-3}, and a deviation of $\delta A_{LR}^{\rm SM+SUSY}$ from δA_{LR}^{SM} is roughly estimated as 1×10^{-3}. Here SM means QCD + SMEW. As for LH, there is a large contribution as $\delta A_{LR}^{\rm LH} \sim 5 \times 10^{-3}$. Since the helicity asymmetry is measured by a spin correlation, $\delta A_{LR}^{\rm exp}$ should be larger than an error of the spin correlation for observation. Thus, $\delta A_{LR}^{\rm SM+SUSY} (\sim 3 \times 10^{-3})$ is difficult to be observed. For example, in Ref.[75, 76], a correlation coefficient in a helicity basis is represented, and the statistic and the systematic

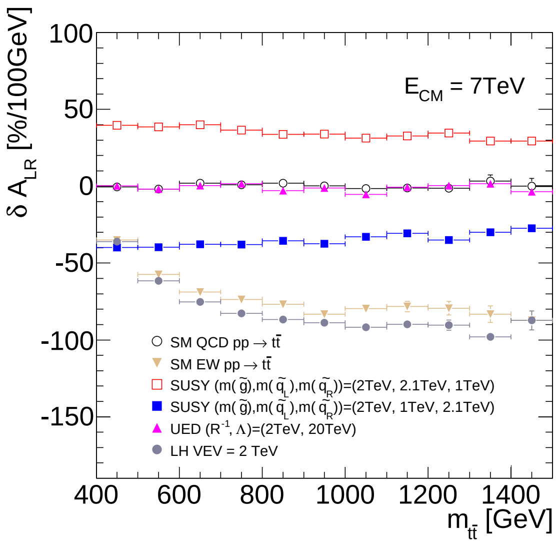

Figure 3.7: δA_{LR} of $m_{t\bar{t}}$ distribution.

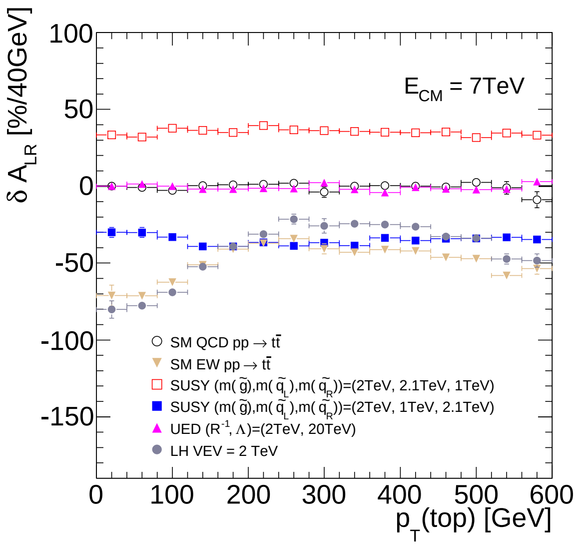

Figure 3.8: Dependence of δA_{LR} on p_T.

errors are of order 0.1. However, we could expect that the statistic error reduces about 1/3 by 10 times events in the future LHC experiments and the systematic error reduce about 1/10. In the next section, we discuss a potential to observe δA_{LR}. As for the LH model, $\delta A_{LR}^{\rm LH}$ is the same order as $\delta A_{LR}^{\rm SMEW}$ as shown in Figs. 3.7 and 3.8. Their asymmetries could be observable when the SM QCD cross section is suppressed, where the LH might be also discriminated from the SMEW in a specific value of f.

3.3.6 Discussions

The cross section of the SM QCD decreases comparing to that of the SUSY SM in high $m_{t\bar{t}}$ and p_T region, since gluino and squark contribution become dominant. So the SUSY signal could be significant, if we select the phase space (final states) as well as take cut to focus on high $m_{t\bar{t}}$ and p_T region, where the SM QCD contribution should be suppressed. Tables 3.4 and 3.5 show fractions of $(\lambda_t, \lambda_{\bar{t}}) = (+,+)/(-,-)$, $(-,+)$, and $(+,-)$ for SM QCD, SMEW, SUSY-L, and SUSY-R at $E_{\rm cm} = 7$TeV. Where SUSY-L and SUSY-R stand for $(m_{\tilde{g}}, m_{\tilde{q}_L}, m_{\tilde{q}_R}) = (2\text{TeV}, 2.1\text{TeV}, 1\text{TeV})$ and $(2\text{TeV}, 1\text{TeV}, 2.1\text{TeV})$, respectively. Notice that SM QCD contribution of $(-,+)$ and $(+,-)$ are suppressed in $gg \to t\bar{t}$ process. Moreover, when top Yukawa coupling is large, the parity violation could be enhanced since 1-loop diagrams with a charged Higgs(ino) inside the loop has the top Yukawa couplings, which have t_R and $\bar{t}_R$ in the external lines (while bottom Yukawa couplings have t_L and

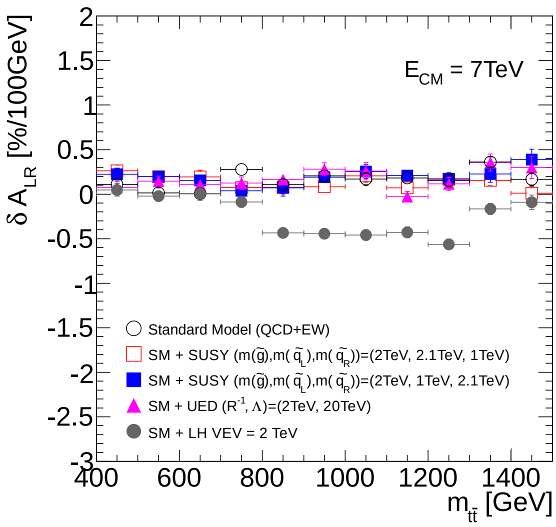

Figure 3.9: Dependence of δA_{LR} on $m_{t\bar{t}}$ with SM interference.

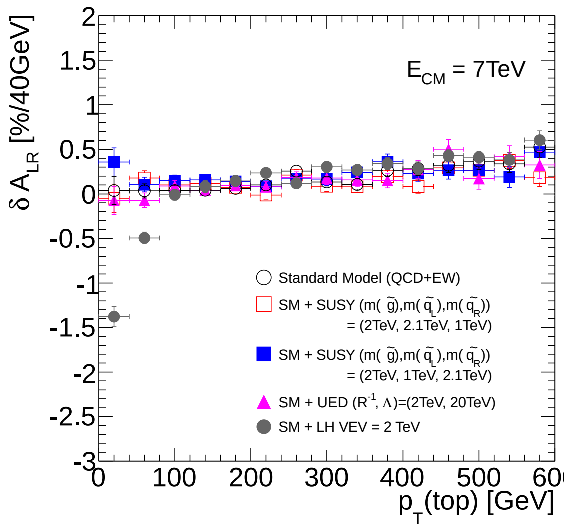

Figure 3.10: Dependence of δA_{LR} on p_T with SM interference.

Helicities	SM QCD	SMEW	SUSY-L	SUSY-R
$(+,+)/(-,-)$	0.222	0.181	0.022	0.023
$(+,-)$	0.385	0.178	0.570	0.402
$(-,+)$	0.393	0.641	0.408	0.575

Table 3.4: Helicity fractions in $q\bar{q} \to t\bar{t}$ process

$\bar{t}_L$ in the external lines). Figure 3.11 shows Higgs(ino) contributions with top and bottom Yukawa couplings, and thus, $t_R\bar{t}_R$ production can be enhanced due to $y_t^2 \gg y_b^2$. This effect is order of $\alpha_s y_t^2$, which could be the same order as α_s^2 in small $\tan\beta$ region. Then, if the SUSY cross section is enhanced as $\sim 10^{-2}$ pb at the high $m_{t\bar{t}}$ and p_T region with the specific phase space (where the SM QCD cross section could be suppressed as $\sim$ 1 pb), $\delta A_{LR}^{\rm SUSY}$ could be large enough as 0.05. In this case, δA_{LR} can be observed when the statistic error is of oder 0.01, where we need 10^4 events of top pair production for this statistic error.

Helicities	SM QCD	SMEW	SUSY-L	SUSY-R
$(+,+)/(-,-)$	0.747	—	0.000	0.000
$(+,-)$	0.125	—	0.715	0.283
$(-,+)$	0.127	—	0.285	0.717

Table 3.5: Helicity fractions in $gg \to t\bar{t}$ process

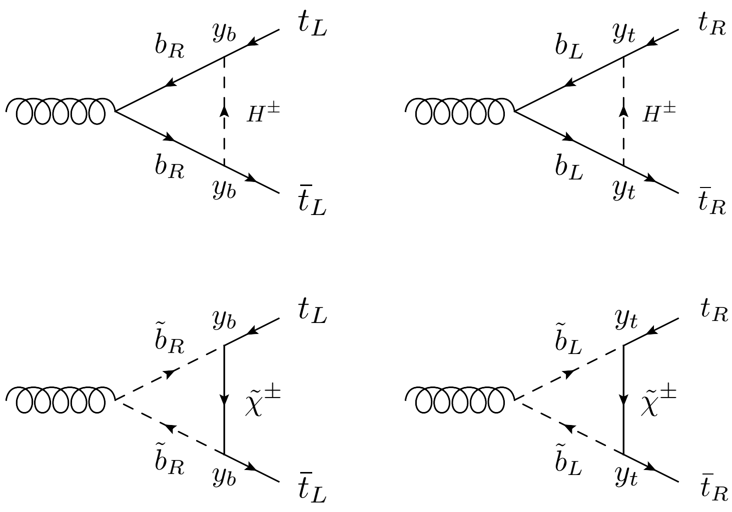

Figure 3.11: Higgs(ino) contributions with top and bottom Yukawa couplings. These diagrams enhance the parity violation.

Then, $\delta A_{LR} \simeq 0.05 \pm 0.01$ could be observed with an integrated luminosity 10 fb^{-1} which is obtained from the number of total events divided by the SM cross section (10 fb^{-1} $\sim 10^4/1$ pb). When we consider an acceptance of a detector, the luminosity should be about 10 times larger since we can not detect all of events. Thus the luminosity is roughly estimated as 100 fb^{-1}. Therefore, we should take reanalyses of δA_{LR} up to $\mathcal{O}(\alpha_s y_t^2)$ with no use of dimension six operators[77], and we need more detailed studies for the discrimination between the SUSY SM and UED model.

Chapter 4

Parity non-conserving quarkonium decay

In this chapter, we discuss SUSY induced parity violation in quarkonium decay. Heavy quakonium can be described by NRQCD[78], however, original NRQCD only explains heavy meson decay by on-shell asymptotic state of quarkonium. As we see in section 4.2, parity violating operators affect a mixing of wave functions with different parity eigenstate, and the original NRQCD is not suitable to estimate the mixing. Therefore, in section 4.1, we establish a method to analyze the parity violation, which framework is based on NRQCD criteria. In section 4.3, we analyze a concrete example of parity non-conserving decay of charmonium, where we estimate a decay width of $\eta_c \to \pi\pi$. Next, we estimate bounds for $\tilde{u}$ and $\tilde{d}$ by use of a similar technique in Ref.[79]. And finally, we comment on bounds for other sfermions.

4.1 Non-relativistic QCD

Let us consider a quarkonium of $q\bar{q}$ bound state in the NRQCD framework by introducing a bilocal field. It is applicable for heavy quarks, and a related work has been shown in, for example, Refs. [80, 81, 82, 7, 83, 84, 85].

In NRQCD, heavy quarkonium is characterized by quark velocity v and quark mass m. Quark field q can be expanded by its mass as

$$q(x) = \begin{pmatrix} \varphi e^{-imt} + i\frac{\vec{\nabla}\cdot\vec{\sigma}}{2m}\chi e^{imt} \\ \chi e^{imt} - i\frac{\vec{\nabla}\cdot\vec{\sigma}}{2m}\varphi e^{-imt} \end{pmatrix} \tag{4.1.1}$$

when m is enough large. φ and χ denote particle and anti-particle components, respectively, and this expansion is so-called Foldy-Wouthuysen-Tani transformation[86, 87]. Thus a two-body effective action in NRQCD is given by

$$S_{\text{eff}} = \int_X \int_{\vec{r}} \phi_X^{\mu\dagger}(\vec{r}) \left[i\partial_X^0 - \frac{\nabla_X^2}{4m} + H(r) \right] \phi_{\mu X}(\vec{r}), \tag{4.1.2}$$

where $H(r)$ is defined as $H(r) \equiv -\nabla_r^2/m - V(r)$. A detail derivation of Eq. (4.1.2) is given in Appendix C. Now we estimate the spectra of bound states $\phi_X^\mu(\vec{r})$. $\phi_X^\mu(\vec{r})$ can be expanded by a complete set of $\psi_n(\vec{r})$ as

$$\phi_X^\mu(\vec{r}) = \sum_n a_n^\mu(X)\psi_n(\vec{r}) = \sum_n \int \frac{d^3P}{(2\pi)^3} a_n^\mu(\vec{P})\psi_n(\vec{r})e^{-iP\cdot X}, \tag{4.1.3}$$

where $a_n^\mu(X)$ is a plane wave, and $\psi_n(\vec{r})$ is a possible bound state which this system can take. An eigenstate of $H(\vec{r})$, which satisfies

$$\hat{H}(\hat{\vec{r}})\psi_n(\vec{r}) = E_n\psi_n(\vec{r}), \tag{4.1.4}$$

is a quarkonium, and E_n denotes a binding energy of it. Orthogonality and completeness suggest

$$\int d^3r \psi_n^\dagger(\vec{r})\psi_m(\vec{r}) = \delta_{nm}, \quad \sum_n \psi_n(\vec{r})\psi_n^\dagger(\vec{s}) = \delta(\vec{r} - \vec{s}). \tag{4.1.5}$$

A hadron wave function is factorized by $a_n^\mu(X)$, which only depends on center of mass coordinate. Here μ represents spin singlet (triplet) state of meson when $\mu = 0$ ($\mu = i$). Note that a hadron labeled by n is created by $a_n^{\mu\dagger}(X)$ as $a_n^{\mu\dagger}(X)|0\rangle = |n\rangle$.

Here let us apply this formalism to a charmonium, for example. We denote $n = \eta_c, h_c, J/\psi, \chi_c, \cdots$, then a spin singlet state $\phi_X^0(\vec{r})$ and a spin triplet state $\phi_X^i(\vec{r})$ are represented by

$$\phi_X^0(\vec{r}) = a_{\eta_c}^0(X)\psi_{\eta_c}(\vec{r}) + a_{h_c}^0(X)\psi_{h_c}(\vec{r}) + \cdots, \tag{4.1.6}$$

$$\phi_X^i(\vec{r}) = a_{J/\psi}^i(X)\psi_{J/\psi}(\vec{r}) + a_{\chi_{cJ}}^i(X)\psi_{\chi_{cJ}}(\vec{r}) + \cdots, \tag{4.1.7}$$

respectively. We now obtain the effective action of charmonium in the SM QCD, where parity is conserved.

4.2 Parity violating potential induced by SUSY

4.2.1 Direct parity violation

In the SUSY SM, parity can be violated in quarkonium through the non-degeneracy of left-right squark masses. As we have shown in Appendix A, there are three parity-violating operators, $\mathcal{O}_{4F}^{(1)}$, $\mathcal{O}_{4F}^{(8)}$, and $\mathcal{O}_{qqG}$, where $\mathcal{O}_{qqGG}$ is next to leading order which we do not consider. At a direct decay vertex of quarkonium, $\mathcal{O}_{4F}^{(1)}$ gives the leading order of parity violation, and we call this process "direct parity violation". The explicit form of the direct parity violating operator is given by

$$\mathcal{O}_{4F}^{\mathrm{p.v}} = (A_{uc} + B_{cu})\delta^4(x-y)[\overline{u}(x)\gamma_\mu u(y)][\overline{c}(x)\gamma^\mu\gamma^5 c(y)], \tag{4.2.8}$$

where A_{uc} and B_{cu} are

$$A_{uc} \equiv \frac{12g_s^4}{192\pi^2}\frac{1}{4}(-C_{LL}^{(\tilde{u},\tilde{c})} + C_{RR}^{(\tilde{u},\tilde{c})} + C_{LR}^{(\tilde{u},\tilde{c})} - C_{RL}^{(\tilde{u},\tilde{c})}), \tag{4.2.9}$$

$$B_{cu} \equiv \frac{12g_s^4}{192\pi^2}\frac{1}{4}(-C_{LL}^{(\tilde{c},\tilde{u})} + C_{RR}^{(\tilde{c},\tilde{u})} - C_{LR}^{(\tilde{c},\tilde{u})} + C_{RL}^{(\tilde{c},\tilde{u})}), \tag{4.2.10}$$

respectively. We estimate u-quark contribution at first, and later include d-quark contribution. Note that squark flavor is labeled by $C_{ij}^{(\tilde{q},\tilde{q}')}$ $(i,j = L,R)$, and has squark mass dependence through $f_1(m_{\tilde{q}}, m_{\tilde{q}'})$ and $f_2(m_{\tilde{q}}, m_{\tilde{q}'})$. For example, $C_{LL}^{(\tilde{u},\tilde{c})}$ is denoted as

$$C_{LL}^{(\tilde{u},\tilde{c})} = \frac{2}{9}[f_1(m_{\tilde{u}_L}, m_{\tilde{c}_L}) + f_2(m_{\tilde{u}_L}, m_{\tilde{c}_L})], \tag{4.2.11}$$

and other C-factors are similarly obtained by using Eqs.(B.1.9)$\sim$(B.1.12).

As for $\mathcal{O}_{qqG}$ and $\mathcal{O}_{4F}^{(8)}$, they do not induce the leading order contributions in direct parity violation process, because they must emit a gluon in the decay vertex. We can neglect gluon exchange between in-going and out-going states at the decay instant in the NRQCD, since non-relativistic bound states are hadronized by space-like gluon exchanges. Therefore, we can neglect the contributions from $\mathcal{O}_{qqG}$ and $\mathcal{O}_{4F}^{(8)}$, and factorize this decay process by a vacuum insertion as in Fig. 1.

We focus on a charmonium, η_c, which is 0^{-+} under J^{PC}, and has mass of 2980 MeV. Notice that $\mathcal{O}_{4F}^{\mathrm{p.v}}$ is a contact interaction, where the decay constant is a value of wave function at an origin due to δ-function and a decay through the contact interaction is only possible with the S-state (angular momentum $L = 0$). Thus, reminding π is 0^{-+}, η_c can not decay to $\pi\pi$ until it pick up parity violation, since $\pi(p), \pi(-p)$ system* of S-state is 0^{++}. Note that there exits weak interaction, however, it also breaks C. Anyhow, as in Fig. 1, the direct parity violation through the SUSY effects, i.e., a two-body decay process, $\eta_c \to \pi\pi$, should be factorized as $\langle\pi\pi|\mathcal{O}_{4F}^{\mathrm{p.v}}|\eta_c\rangle \sim \langle\pi\pi|\overline{q}\gamma^\mu q|0\rangle\langle 0|\overline{q}\gamma_\mu\gamma^5 q|\eta_c\rangle$. Here $\langle\pi\pi|\overline{q}\gamma^\mu q|0\rangle$ is a pion

*It has $P = (-1)^L$ and $C = (-1)^{S+L}$.

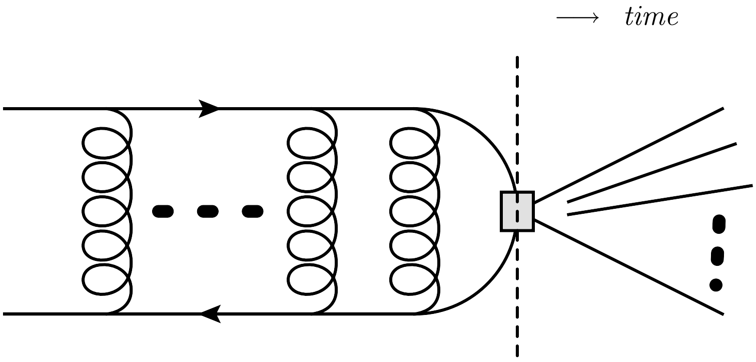

Figure 4.1: Factorization by a vacuum insertion in a direct parity violation process. A box stands for $\mathcal{O}_{4F}^{\rm p.v}$ in Eq.(4.2.8).

form factor, and we can estimate $\langle 0|\overline{q}\gamma_\mu\gamma^5 q|\eta_c\rangle$ by use of NRQCD. Actually, by regarding $\overline{q}\gamma^0\gamma^5 q \sim -\frac{1}{2}\chi^\dagger\varphi + \text{h.c.}$ in a non-relativistic picture, the S-matrix element of $\eta_c \to \pi\pi$ is given by

$$\langle \pi\pi|\mathcal{O}_{4F}^{\rm p.v}|\eta_c\rangle \sim -\frac{1}{2}(A_{uc}+B_{cu})\delta^4(x-y)\langle\pi\pi|u^\dagger(x)u(y)|0\rangle\langle 0|\chi^\dagger(x)\varphi(y)|\eta_c\rangle. \tag{4.2.12}$$

Here $\langle\pi\pi|u^\dagger(x)u(y)|0\rangle \sim F^s(k)$ is a scalar form factor of pion, which has non-trivial energy dependence.

In general, when a bound state $|n\rangle(\equiv a_n^{\nu\dagger}(P)|0\rangle)$ decays through a bilocal operator $\mathcal{O}_X^{\nu\lambda\cdots}(\vec{r}) = \phi_X^\mu(\vec{r})\phi_X^\lambda(\vec{r})\cdots$, its matrix element is given by

$$\langle 0|T[\mathcal{O}_X^{\nu\lambda\cdots}(\vec{r})]|n\rangle = i\int d^4Y\int d^3s F_P^n(Y;s)\left(i\partial_{Y^0} - \frac{\nabla_Y^2}{4m} - \hat{H}(s)\right)\langle 0|T[\mathcal{O}_X^{\nu\lambda\cdots}(\vec{r})\phi_Y^{\mu\dagger}(s)]|0\rangle,$$

where $F_P^n(Y;s) \equiv \psi_n(\vec{s})e^{-iP\cdot Y}$, and it satisfies $\int d^3Xd^3r\phi_X^{\mu\dagger}(r)F_P^n(X;r) = a_n^\dagger(\vec{P})$ from orthogonality and completeness. Thus, the transition amplitude in Eq.(4.2.13) is given by $\psi_n(\vec{r})e^{-iPX}$ with $\mathcal{O}_X^{\nu\lambda\cdots}(\vec{r}) = \phi_X^\nu(\vec{r})$.

Let us go back to a charmonium, and take q as c-quark in Eq.(4.1.1). Since a heavy quark is non-relativistically expanded as Eq.(4.1.1), the 4-Fermi operator can be also expanded similarly. In the leading order, components of $\chi^\dagger\varphi$ and $\varphi^\dagger\chi$ in the bilocal field, are only creating and annihilating operators of charmonium. Thus, $\phi_X^n(\vec{r})$ corresponds to $\chi^\dagger(x)\varphi(y)$, and we name a label $n=0$ η_c for the charmonium, which suggests

$$\langle 0|\phi_X^0(\vec{r})|\eta_c\rangle = \psi_{\eta_c}(\vec{r})e^{-iP\cdot X}. \tag{4.2.13}$$

Remind that $\mathcal{O}_{4F}^{\rm p.v}$ is a contact interaction, and we can use m_{η_c} for an energy of the pion form factor due to a momentum conservation. Then, we obtain

$$\langle \pi\pi | \mathcal{O}_{4F}^{\rm p.v} | \eta_c \rangle \sim -\frac{1}{2}(A_{uc} + B_{cu}) F^s(m_{\eta_c}) \psi_{\eta_c}(0). \qquad (4.2.14)$$

There is a d-quark contribution as well as u-quark ones, so that the effective 4-Fermi operator $\mathcal{O}_{4F}^{\rm p.v}$ becomes a linear combination of u and d. Therefore, $\Gamma(\eta_c \to \pi\pi)$ is estimated as

$$\Gamma(\eta_c \to \pi\pi) \sim \mid A_{uc} + A_{dc} + B_{cu} + B_{cd} \mid^2 \frac{|F^s(m_{\eta_c})|^2 |\psi_{\eta_c}(0)|^2}{16 m_{\eta_c}^2}. \qquad (4.2.15)$$

Since η_c is an S-state, the decay width depends only on the wave function at the origin . This is a characteristic feature in the direct parity violating process in the SUSY SM.

4.2.2 Indirect parity violation

The QCD dimension six operators from the SUSY SM can have the parity violating effects, and actually, they can also contribute organization of quarkoniums themselves. We call this effect "indirect parity violation", and we investigate it in this section. For this indirect parity violation, all $\mathcal{O}_{qqG}, \mathcal{O}_{4F}^{(1)}$, and $\mathcal{O}_{4F}^{(8)}$ contribute as in Fig. 2.

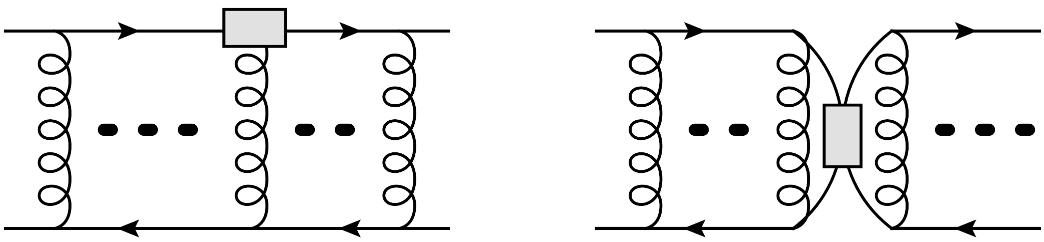

Figure 4.2: Diagrams which contribute indirect parity violation through dimension six operators (which are shown a box at a vertex). (Left): a contribution from $\mathcal{O}_{qqG}$, (Right): a contribution from $\mathcal{O}_{4F}^{(1)}$ or $\mathcal{O}_{4F}^{(8)}$.

The indirect parity violation induces a mixing between an even-parity state and an odd-parity state as well as a S-state and a P-state in a quarkonium. As the parity violating term is written by $\delta V^{\mu\nu}(r)$ in the potential, the effective action in Eq.(C.0.21) includes indirect parity violation by rewriting $V(r) g^{\mu\nu} \to V(r) g^{\mu\nu} + \delta V^{\mu\nu}(r)$. Here $\delta V^{\mu\nu}(r)$ is a matrix in a basis of S- and P-states, which has off-diagonal elements of hadron state labeled by n (and μ). Now let us calculate the mixing between asymptotic states in the SUSY SM by using the basis of the SM QCD. Since the potential only depends on relative coordinate, the wave function can be expanded by $\Psi_n(\vec{r})$ in the SUSY SM as

$$\phi_X^\mu(\vec{r}) = \sum_n A_n^\mu(X)\Psi_n(\vec{r}), \tag{4.2.16}$$

where $\Psi_n(\vec{r})$ satisfies eigenvalue equations,

$$[H^{\rm QCD}(\vec{r}) + \delta V(\vec{r})]\Psi_n(\vec{r}) = E_n^{\rm full}\Psi_n(\vec{r}), \tag{4.2.17}$$

for $E_n^{\rm full} \neq E_n$. Note that n is the label of the hadron, which contains an information of spin ($\mu = 0$: singlet, $\mu = i$: triplet). This $\Psi_n(\vec{r})$ must be $\Psi_n(\vec{r}) \to \psi_n(\vec{r})$ as $\delta V \to 0$, so that it is given by

$$\Psi_n(\vec{r}) = \psi_n(\vec{r}) + \sum_{k\neq n} \frac{V_{nk}}{E_n - E_k}\psi_k(\vec{r}), \tag{4.2.18}$$

up to the first order of perturbation. Note that $\Psi_n(\vec{r})$ must satisfy

$$\int d^3r \Psi_n^\dagger(\vec{r})\Psi_m(\vec{r}) = \delta_{mn} \tag{4.2.19}$$

for the zeroth order of perturbation. V_{nk} is defined by

$$V_{nk} \equiv \int d^3s \psi_k^\dagger(\vec{s})[\delta V(\vec{s})]\psi_n(\vec{s}). \tag{4.2.20}$$

The classical complete set $\Psi_n(\vec{r})$ should be written by the QCD complete set $\psi_n(\vec{r})$, while a hadron creation operator is given by $A_n^{\mu\dagger}(X)$, so that $A_n^\mu(X)$ corresponds to $a_n^\mu(X)$. $\Psi_n(\vec{r})$ and $\psi_n(\vec{r})$ are different complete bases as

$$\phi_X^\mu(\vec{r}) = \sum_n a_n^\mu(X)\psi_n(\vec{r}) = \sum_n A_n^\mu(X)\Psi_n(\vec{r}). \tag{4.2.21}$$

Thus, by use of orthogonalization of $\Psi_n(\vec{r})$, we obtain

$$A_n^{\mu\dagger}(X) = a_n^{\mu\dagger}(X) + \sum_{k\neq n} \frac{(V_{nk}a_k^\dagger(X))^\mu}{E_n - E_k} \ . \tag{4.2.22}$$

Let us consider a charmonium system. Equation (4.2.22) means an observed η_c is almost represented by a mixing state of η_c and χ_{c0} as

$$|\eta_c\rangle_{\rm obs.} = |\eta_c\rangle + \frac{V_{\eta_c,\chi_{c0}}}{E_{\eta_c} - E_{\chi_{c0}}}|\chi_{c0}\rangle. \tag{4.2.23}$$

χ_{c0} is 0^{++} with mass of 3415 MeV, and a decay of χ_{c0} to $\pi\pi$ is possible (see, Eq.(4.3.37)) when π-π system has angular momentum, $L = 1$. We estimate parity violating potential induced from the SUSY SM. As for $\mathcal{O}_{4F}^{(1)}$ in Fig. 2, its coefficient only depends on $m_{\tilde{c}_L}$ and $m_{\tilde{c}_R}$, since the bound state is charmonium. The parity violating terms in $\mathcal{O}_{4F}^{(1)}$ are given by

$$\mathcal{O}_{4F}^{(1)} \supset \frac{12g_s^4}{192\pi^2}\frac{1}{2}(-C_{LL}^{(\tilde{c},\tilde{c})} + C_{RR}^{(\tilde{c},\tilde{c})})\delta^4(x-y)[\bar{c}(x)\gamma^\mu c(x)][\bar{c}(y)\gamma_\mu\gamma^5 c(y)], \tag{4.2.24}$$

where we use spin relation, $\delta_{\alpha\beta}\delta_{\gamma\lambda} = \frac{1}{2}\delta_{\alpha\lambda}\delta_{\gamma\beta} + \frac{1}{2}\sigma^a_{\alpha\lambda}\sigma^a_{\gamma\beta}$, and $\sigma^a\sigma^b = \delta^{ab} + i\epsilon^{abc}\sigma^c$. A color factor is rewritten as $\frac{1}{2}\delta_{ij}\delta_{kl} = \frac{1}{2N_C}\delta_{il}\delta_{kj} + T^A_{il}T^A_{kj}$ for an exchange of spin. We must be careful for exchanges of spin and coordinate, where only spin-singlet changes its sign (Table.4.1). After careful calculations, $\mathcal{O}_{4F}^{(1)}$ is given by

	exchange of spin ($\varphi \leftrightarrow \chi$)	exchange of coordinate ($x \leftrightarrow y$)
spin singlet $\phi^0(x,y)$	asym.	sym.
spin triplet $\phi^i(x,y)$	sym.	asym.

Table 4.1: exchanges of spin or coordinate

$$\begin{aligned}\mathcal{O}_{4F}^{(1)} \to & \frac{12g_s^4}{192\pi^2}\frac{1}{2}(-C_{LL}^{(\tilde{c},\tilde{c})} + C_{RR}^{(\tilde{c},\tilde{c})})\left(\frac{i}{4m_c N_C}\right) \\ & \times \begin{pmatrix}\phi^0 \\ \phi^i\end{pmatrix}^\dagger_{x,y} \begin{pmatrix} 0 & 4\mathcal{V}(r)\partial_r^j \\ 4_r^i\mathcal{V}(r) & 4i\epsilon^{ijk}{}_r^k\mathcal{V}(r)\end{pmatrix}\begin{pmatrix}\phi^0 \\ \phi^j\end{pmatrix}_{x,y},\end{aligned} \tag{4.2.25}$$

where $\mathcal{V} \equiv \delta^4(x-y)$ and $\phi_r^{ii} \equiv -\partial_r^i\phi^i$. As for $\mathcal{O}_{4F}^{(8)}$, we can use the calculation result of $\mathcal{O}_{4F}^{(1)}$, since spin structure is the same. The different point is just color factor, and by using $T^A_{ij}T^A_{kl} = \frac{C_F}{2N_C}\delta_{il}\delta_{kj} - \frac{1}{N_C}T^A_{il}T^A_{kj}$, we show color octet part is C_F $(= (N_C^2-1)/(2N_C))$ times larger than $\mathcal{O}_{4F}^{(1)}$. Then, non-relativistic potential from $\mathcal{O}_{4F}^{(1)}$ and $\mathcal{O}_{4F}^{(8)}$ with parity violation is totally given by

$$\begin{aligned}&\delta V_{\mu\nu}^{4F}(r) \\ &= \frac{12g_s^4}{192\pi^2}\frac{i}{8m_cN_C}\left[(-C_{LL}^{(\tilde{c},\tilde{c})} + C_{RR}^{(\tilde{c},\tilde{c})}) + C_F(-D_{LL}^{(\tilde{c},\tilde{c})} + D_{RR}^{(\tilde{c},\tilde{c})})\right]\begin{pmatrix} 0 & 4\mathcal{V}(r)\partial_r^j \\ 4_r^i\mathcal{V}(r) & 4i\epsilon^{ijk}{}_r^k\mathcal{V}(r)\end{pmatrix}.\end{aligned} \tag{4.2.26}$$

For a non-relativistic potential from $\mathcal{O}_{qqG}$, we estimate leading part. Since $\mathcal{O}_{qqG}$ is not the contact interaction as $\mathcal{O}_{4F}^{(1)}$, its parity violation effects should be added to the gluon potential. The bilocal operator after integrating out gluon is given by

$$\mathcal{L} \sim \frac{g_s^2}{96\pi^2 C_F}\left[\bar{q}(x)T^A E^0_{L,R}P_{L,R}q(x)\right]V(r)\left[\bar{q}(y)T^A\gamma^0 q(y)\right], \tag{4.2.27}$$

where $E^0_{L,R}$ has eight terms in total, which are categorized as

$$\text{(i)} \quad \pm\, e_1(m_{\tilde{q}}) \frac{g_s^2}{96\pi^2 C_F} \left[(\overline{q}_x T^A \gamma_\mu \partial^\mu \partial^0 q_x) + (\partial^\mu \partial^0 \overline{q}_x T^A \gamma_\mu q_x) \right] V(r) \left[\overline{q}(y) T^A \gamma^0 q(y) \right], \tag{4.2.28}$$

$$\text{(ii)} \quad \pm\, e_2(m_{\tilde{q}}) \frac{g_s^2}{96\pi^2 C_F} \left[(\partial^\mu \overline{q}_x T^A \gamma_\mu \partial^0 q_x) + (\partial^0 \overline{q}_x T^A \gamma_\mu \partial^\mu q_x) \right] V(r) \left[\overline{q}(y) T^A \gamma^0 q(y) \right], \tag{4.2.29}$$

$$\begin{aligned} \text{(iii)} \quad \pm\, & \frac{g_s^2}{96\pi^2 C_F} \left[e_3(m_{\tilde{q}}) \left\{ (\overline{q}_x T^A \gamma^0 \partial^2 q_x) + (\partial^2 \overline{q}_x T^A \gamma^0 q_x) \right\} + e_4(m_{\tilde{q}}) (\partial^\mu \overline{q}_x T^A \gamma^0 \partial_\mu q_x) \right] \\ & \times V(r) \left[\overline{q}(y) T^A \gamma^0 q(y) \right], \end{aligned} \tag{4.2.30}$$

$$\text{(iv)} \quad \pm\, (-e_5(m_{\tilde{q}})) \frac{g_s^2}{96\pi^2 C_F} i\epsilon^{\alpha\beta 0 \nu} \left[\partial_\beta \overline{q}_x T^A \gamma_\nu \partial_\alpha q_x \right] V(r) \left[\overline{q}(y) T^A \gamma^0 q(y) \right]. \tag{4.2.31}$$

Here, sign $+$ $(-)$ means that quark chirality is R (L). In the non-relativistic limit, (i) and (ii) vanish, since components of $\mu = 0$ and $\mu = i$ are cancelled with each other. For this calculation, we have used a NRQCD result, $\partial^0 q \sim \mathcal{O}(m_c (m_c v)^{3/2})$ (v: c-quark velocity, m_c: c-quark mass) which counting rules are shown in Appendix C. Actually, (iii) induces the leading effects for the potential. By taking leading order of v, a power counting shows

$$\begin{aligned} \delta V^{qqG}_{\mu\nu}(r) = & \left[(e_4(m_{\tilde{q}_R}) - e_4(m_{\tilde{q}_L})) - 2(e_3(m_{\tilde{q}_R}) - e_3(m_{\tilde{q}_L})) \right] \\ & \times \frac{g_s^2}{96\pi^2} \left(\frac{-i m_q}{8 N_C} \right) \begin{pmatrix} 0 & V(r)\partial_r^j +_r^j V(r) \\ V(r)\partial_r^i +_r^i V(r) & i\epsilon^{ijk} [V(r)\partial_r^k +_r^k V(r)] \end{pmatrix}, \end{aligned} \tag{4.2.32}$$

where we use color factor $(C_F/(2N_C))$ from Fierz transformation. As for (iv), α, β must be *space*-index, so that the second derivative of *space*-index appears, which corresponds to D-state (or higher angular momentum states), so that it does not contribute the mixing between S- and P-states. The (iv) does not contribute the mixing between S- and P-states, too. Thus, the leading order of parity violating potential, which triggers the mixing between S- and P-states, is given by

$$\delta V^{\text{SUSY}}_{\mu\nu}(r) = \delta V^{4F}_{\mu\nu}(r) + \delta V^{qqG}_{\mu\nu}(r). \tag{4.2.33}$$

Then, we can calculate $V_{\eta_c, \chi_{c0}}$ in a charmonium, and a formula of decay width is given by

$$\Gamma(\eta_c \to \pi\pi) \sim \left| \frac{V_{\eta_c, \chi_{c0}}}{E_{\eta_c} - E_{\chi_{c0}}} \right|^2 \Gamma(\chi_{c0} \to \pi\pi). \tag{4.2.34}$$

A wave function of charmonium is given by $\psi(\vec{r}) = R_n(r)Y_{lm}(\theta,\phi)$, where $R_n(r)$ satisfies the Schrödinger equation (4.1.4) with Coulomb plus linear potential (Cornell potential),

$$V(r) \;\; = \;\; -\frac{\kappa}{r} + \frac{r}{a^2}. \tag{4.2.35}$$

We take $\kappa = 0.52$ and $a = 2.34$ GeV^{-1} for charmonium system [88]. Through the Schrödinger equation with this potential, we can obtain charmonium wave function numerically.

4.3 Parity non-conserving charmonium decay

Let us investigate the left-right non-degeneracy bound for the masses of $\tilde{c}_L$ and $\tilde{c}_R$ by use of the calculation method shown above. For a charmonium, we focus on η_c, whose decay has upper bounds of P and CP violations as [51]

$$Br(\eta_c \to \pi^+\pi^-) < 1.1 \times 10^{-4}, \quad Br(\eta_c \to \pi^0\pi^0) < 3.5 \times 10^{-5}. \tag{4.3.36}$$

Note again that η_c can not decay to $\pi\pi$ until it picks up parity violation. On the other hand, a branching ratio of $\chi_{c0} \to \pi\pi$ is

$$Br(\chi_{c0} \to \pi\pi) = (8.4 \pm 0.4) \times 10^{-3}. \tag{4.3.37}$$

A branching ratio of $\eta_c \to \pi\pi$ in a direct parity violation from Eq.(4.2.34) is given by

$$Br(\eta_c \to \pi\pi)_{\text{dir.}} = \mid A_{uc} + A_{dc} + B_{cu} + B_{cd} \mid^2 \frac{|F^s(m_{\eta_c})|^2 |\psi_{\eta_c}(0)|^2}{16 m_{\eta_c}^2 \Gamma_{\eta_c}}, \tag{4.3.38}$$

where Γ_{η_c} is the total decay width of η_c. Here we take a scalar form factor of pion F^s by an input parameter as $F^s(m_{\eta_c}^2) = 1,\ 0.1,\ 0.001$, since its theoretical estimation is difficult above 1 GeV. On the other hand, the indirect parity violation in $\eta_c \to \pi\pi$ suggests

$$Br(\eta_c \to \pi\pi)_{\text{indir.}} \sim \left| \frac{V^{\text{SUSY}}_{\eta_c,\chi_{c0}} + V^{\text{EW}}_{\eta_c,\chi_{c0}}}{E_{\eta_c} - E_{\chi_{c0}}} \right|^2 Br(\chi_{c0} \to \pi\pi), \tag{4.3.39}$$

where $V^{\text{EW}}_{\eta_c,\chi_{c0}}$ is the SM background induced from a Z-boson exchange. It gives an additional effect $\mathcal{V}(r) \equiv (\alpha/r)\exp(-m_Z r))$ in Eq.(4.2.26), which is shown as

$$\delta V^{\text{EW}}_{\mu\nu}(r) = \frac{g^2}{\cos^2\theta_W}\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)^2 \frac{iC_F}{8 m_c N} \begin{pmatrix} 0 & 4\mathcal{V}(r)\partial^j_r \\ 4^i_r \mathcal{V}(r) & 4i\epsilon^{ijk}{}^k_r \mathcal{V}(r) \end{pmatrix} \tag{4.3.40}$$

in a basis of (S-state, P-state) with $N = 2$ and $C_F = 3/2$. Then, we can evaluate $V^{\rm EW}_{\eta_c,\chi_{c0}}$ with Eq.(4.2.20), and the branching ratio is given by $Br(\eta_c \to \pi\pi)_{\rm SM} \sim \left|\frac{V^{\rm EW}_{\eta_c,\chi_{c0}}}{E_{\eta_c}-E_{\chi_{c0}}}\right|^2 Br(\chi_{c0} \to \pi\pi) \simeq 7.0 \times 10^{-22}$.

In Figs. 3 and 4, the branching ratios of $\eta_c \to \pi\pi$ from direct and indirect parity violation effects are plotted, respectively, where horizontal axis is a magnitude of $(m^2_{\tilde{c}_L} - m^2_{\tilde{c}_R})/m^2_{\tilde{g}}$. Note that the branching ratio from indirect parity violation is larger than that from direct parity violation. Unfortunately, we can show that the SUSY parity violating effect is smaller than the experimental bound of Eq.(4.3.36) in the parameter region, and it is difficult to obtain the non-degeneracy bound between $m_{\tilde{c}_L}$ and $m_{\tilde{c}_R}$. Figures 5 and 6 show a case that $\tilde{g}$ and $\tilde{c}_R$ are degenerate around 850 GeV in mass. The magnitude of the horizontal axis is varied from $(m^2_{\tilde{c}_L} - m^2_{\tilde{c}_R})/m^2_{\tilde{g}} = 4.5$, which is taken to be consistent with LHC data. Notice that the branching ratio becomes larger than that in Figs. 3 and 4, however, the experimental bound is also much higher, and we can not obtain the bounds.

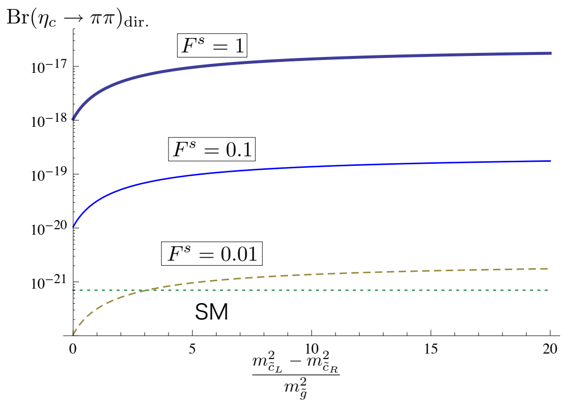

Figure 4.3: Branching ratios of $\eta \to \pi\pi$ from direct parity violation with $m_{\tilde{g}} = 1400$ GeV, $m_{\tilde{u}_R} = 2000$ GeV, $m_{\tilde{u}_L} = 2500$ GeV, $m_{\tilde{d}_R} = 2100$GeV, $m_{\tilde{d}_L} = 2600$ GeV, and $m_{\tilde{c}_R} = 2200$ GeV.

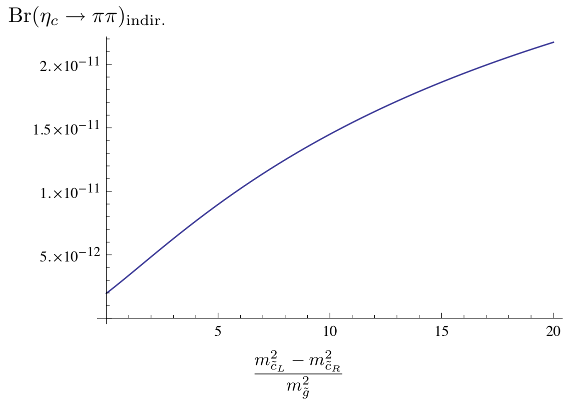

Figure 4.4: Branching ratios of $\eta \to \pi\pi$ from indirect parity violation with $m_{\tilde{g}} = 1400$ GeV, and $m_{\tilde{c}_R} = 2200$ GeV.

4.4 Estimation of bounds for $\tilde{u}$ and $\tilde{d}$

The left-right non-degeneracy bounds for $\tilde{u}$ and $\tilde{d}$ was studied by use of nuclear parity violation in Ref.[79]. Where they compared coefficients of (quark level) meson-nucleon couplings in the SM with those in the SUSY. However, studied parameter region was $m^2_{\tilde{q}} \ll m^2_{\tilde{g}} < \mathcal{O}(G_F^{-1})$, which is already experimentally excluded, so that we investigate

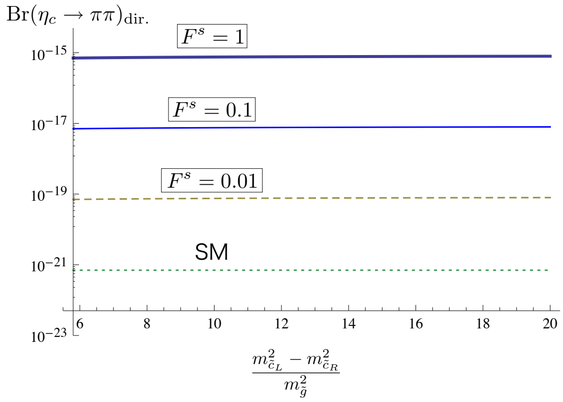

Figure 4.5: Branching ratios of $\eta \to \pi\pi$ from indirect parity violation with $m_{\tilde{g}} = 850$ GeV, $m_{\tilde{u}_R} = 860$ GeV, $m_{\tilde{u}_L} = 2500$ GeV, $m_{\tilde{d}_R} = 870$GeV, $m_{\tilde{d}_L} = 2600$ GeV, and $m_{\tilde{c}_R} = 880$ GeV.

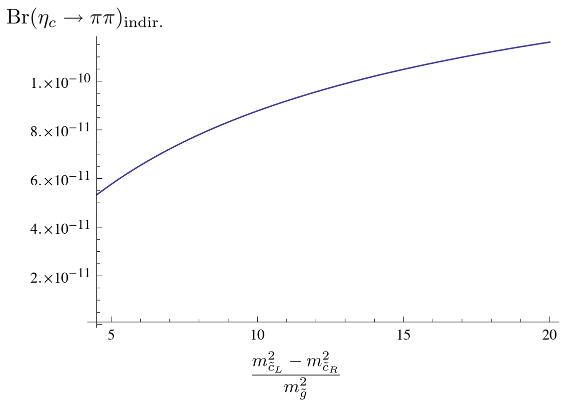

Figure 4.6: Branching ratios of $\eta \to \pi\pi$ from indirect parity violation with $m_{\tilde{g}} = 850$ GeV, and $m_{\tilde{c}_R} = 880$ GeV.

the left-right non-degeneracy bound in a wider parameter region, besides, without approximations used in Ref.[79].

We use π, ω, ρ and nucleon couplings for the meson-nucleon coupling. The notation of our dimension six operators corresponds to

$$\frac{G_2(m_{\tilde{q}}, m_{\tilde{q}'})}{3m^2_{\tilde{g}}} = f_1(m_{\tilde{q}}, m_{\tilde{q}'}), \quad \frac{G_1(m_{\tilde{q}}, m_{\tilde{q}'})}{3m^2_{\tilde{g}}} = f_2(m_{\tilde{q}}, m_{\tilde{q}'}), \tag{4.4.41}$$

in Ref.[79], where we neglect flavor mixings and squark left-right mixings (A-terms). On the other hand, coefficient of q-q-G vertex is written by

$$\frac{C(m^2_{\tilde{q}}/m^2_{\tilde{g}})}{m^2_{\tilde{q}}} = \frac{43m^6_{\tilde{g}} - 144m^4_{\tilde{g}}m^2_{\tilde{q}} + 153m^2_{\tilde{g}}m^4_{\tilde{q}} - 6\left(2m^6_{\tilde{g}} - 9m^2_{\tilde{g}}m^4_{\tilde{q}} + 6m^6_{\tilde{q}}\right)\log\left(\frac{m^2_{\tilde{g}}}{m^2_{\tilde{q}}}\right) - 52m^6_{\tilde{q}}}{54\left(m^2_{\tilde{g}} - m^2_{\tilde{q}}\right)^4} \tag{4.4.42}$$

in a massless approximation of u- and d-quarks. By using above equations, we calculate bounds from the SM as

$$\begin{aligned} &\text{(i)} \quad |C^p(\pi) + C^b_-(\pi)| < |C^{\mathrm{SEW}}(\pi)|, \\ &\text{(ii)} \quad |C^p(\omega) + C^b_+(\omega)| < |C^{\mathrm{SEW}}(\omega)|, \\ &\text{(iii)} \quad |C^p(\rho) + C^b_+(\rho)| < |C^{\mathrm{SEW}}(\rho)|, \end{aligned}$$

which are shown in Figs. 4.7, 4.8, and 4.9. $C(\pi)$, $C(\omega)$, and $C(\rho)$ are parity violating effects (coupling) from π-, ω-, and ρ-nucleon interactions, respectively. Indices p and b stand for penguin and box diagram contributions, respectively. Index SEW means an effect from the SM electroweak interactions [79] as $|C^{\rm SEW}(\pi)| = 8.5\times10^{-7}, |C^{\rm SEW}(\omega)| = 4.5\times10^{-6}$, and $|C^{\rm SEW}(\rho)| = 6.2\times10^{-7}$. The factor $c(m_{\tilde q})$ is defined by $c(m_{\tilde q}) \equiv C(m_{\tilde q}^2/m_{\tilde g}^2)/m_{\tilde q}^2$, and then

$$C^p(\pi) = \frac{4}{3}\frac{\alpha_s^2}{12}\rho\left[c(m_{\tilde u_R}) - c(m_{\tilde u_L}) - c(m_{\tilde d_R}) + c(m_{\tilde d_L})\right], \tag{4.4.43}$$

$$C^p(\omega) = \frac{1}{3}\frac{\alpha_s^2}{24}\rho\left[c(m_{\tilde u_R}) - c(m_{\tilde u_L}) + c(m_{\tilde d_R}) - c(m_{\tilde d_L})\right], \tag{4.4.44}$$

$$C^p(\rho) = \frac{2}{3}\frac{\alpha_s^2}{24}\rho\left[c(m_{\tilde u_R}) - c(m_{\tilde u_L}) + c(m_{\tilde d_R}) - c(m_{\tilde d_L})\right], \tag{4.4.45}$$

$$C^b_-(\pi) = -\frac{\alpha_s^2}{27}\rho\left[f_1(m_{\tilde u_L}, m_{\tilde d_R}) - f_1(m_{\tilde u_R}, m_{\tilde d_L}) - f_2(m_{\tilde u_L}, m_{\tilde d_R}) + f_2(m_{\tilde u_R}, m_{\tilde d_L})\right], \tag{4.4.46}$$

$$\begin{aligned} C^b_+(\omega) = &-\frac{3\alpha_s^2}{48}\left(\frac{2}{9}+\frac{8}{27}\right)\rho\left[2f_1(m_{\tilde u_L}, m_{\tilde d_L}) - 2f_1(m_{\tilde u_R}, m_{\tilde d_R}) - f_2(m_{\tilde u_L}, m_{\tilde d_L}) + 2f_2(m_{\tilde u_R}, m_{\tilde d_R})\right. \\ &- f_1(m_{\tilde d_L}, m_{\tilde d_L}) - f_1(m_{\tilde d_R}, m_{\tilde d_R}) - f_1(m_{\tilde u_L}, m_{\tilde u_L}) - f_1(m_{\tilde u_R}, m_{\tilde u_R}) \\ &\left.+f_2(m_{\tilde d_L}, m_{\tilde d_L}) + f_2(m_{\tilde d_R}, m_{\tilde d_R}) + f_2(m_{\tilde u_L}, m_{\tilde u_L}) + f_2(m_{\tilde u_R}, m_{\tilde u_R})\right], \end{aligned} \tag{4.4.47}$$

$$\begin{aligned} C^b_+(\rho) = &-\frac{\alpha_s^2}{48}\frac{32}{27}\rho\left[2f_1(m_{\tilde u_L}, m_{\tilde d_L}) - 2f_1(m_{\tilde u_R}, m_{\tilde d_R}) - f_2(m_{\tilde u_L}, m_{\tilde d_L}) + 2f_2(m_{\tilde u_R}, m_{\tilde d_R})\right. \\ &- f_1(m_{\tilde d_L}, m_{\tilde d_L}) - f_1(m_{\tilde d_R}, m_{\tilde d_R}) - f_1(m_{\tilde u_L}, m_{\tilde u_L}) - f_1(m_{\tilde u_R}, m_{\tilde u_R}) \\ &\left.+f_2(m_{\tilde d_L}, m_{\tilde d_L}) + f_2(m_{\tilde d_R}, m_{\tilde d_R}) + f_2(m_{\tilde u_L}, m_{\tilde u_L}) + f_2(m_{\tilde u_R}, m_{\tilde u_R})\right], \end{aligned} \tag{4.4.48}$$

where we take $\rho \sim \sqrt{10}$.

In Figs. 4.7, 4.8, and 4.9, we take sample points which are not excluded by experiment[55]. Under $m_{\tilde g} = 1400$ GeV, $m_{\tilde u_R} = 2000$ GeV, $m_{\tilde u_L} = 2500$ GeV, and $m_{\tilde d_R} = 2100$ GeV, we change a value of $(m_{\tilde d_L}^2 - m_{\tilde d_R}^2)/m_{\tilde g}^2$ from 1.2 for the consistent with the experimental data. Unfortunately, in this parameter space, $\tilde u$ and $\tilde d$ are too heavy to obtain bounds for degeneracies between $m_{\tilde u_L}$ and $m_{\tilde u_R}$, or, $m_{\tilde d_L}$ and $m_{\tilde d_R}$. On the other hand, when gluino and squarks degenerate within 30 GeV, π-, ω-, and ρ-nucleon couplings are shown in 4.10, 4.11, and 4.12, respectively. The magnitude of $(m_{\tilde d_L}^2 - m_{\tilde d_R}^2)/m_{\tilde g}^2$ is varied from 7.1 for the consistency with the LHC data. In this parameter space, $\tilde u$ and $\tilde d$ are again too heavy to obtain the bounds. The branching ratio is small because SUSY effects always have a loop factor, and it is the reason why there are the asymptotic values in Figs. 7∼12.

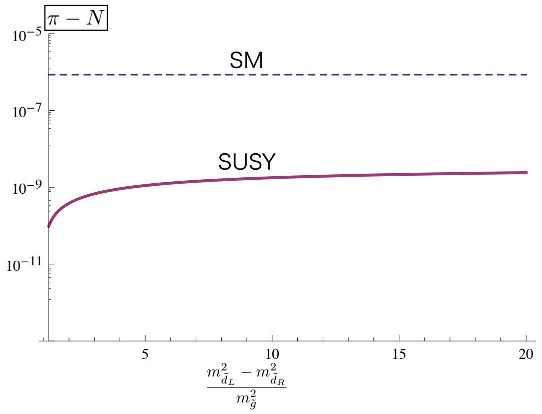

Figure 4.7: $\pi - N$ coupling ($|C^p(\pi) + C^b_-(\pi)| < |C^{\mathrm{SEW}}(\pi)|$) with $m_{\tilde{g}} = 1400$ GeV, $m_{\tilde{u}_R} = 2000$ GeV, $m_{\tilde{d}_R} = 2100$ GeV, and $m_{\tilde{u}_L} = 2500$ GeV. The magnitude closes in 4.0×10^{-9} as $(m^2_{\tilde{d}_L} - m^2_{\tilde{d}_R})/m^2_{\tilde{g}} \to \infty$.

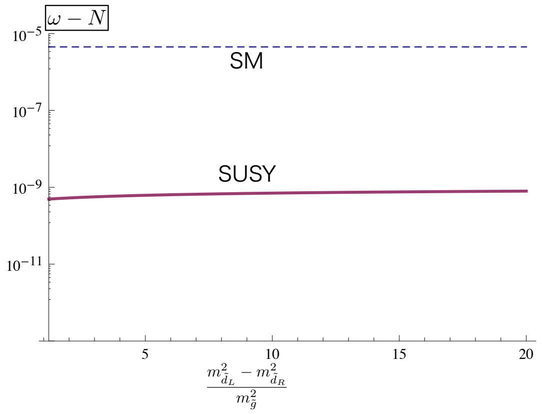

Figure 4.8: $\omega - N$ coupling ($|C^p(\omega) + C^b_+(\omega)| < |C^{\mathrm{SEW}}(\omega)|$) with $m_{\tilde{g}} = 1400$ GeV, $m_{\tilde{u}_R} = 2000$ GeV, $m_{\tilde{d}_R} = 2100$ GeV, and $m_{\tilde{u}_L} = 2500$ GeV. The magnitude closes in 1.1×10^{-9} as $(m^2_{\tilde{d}_L} - m^2_{\tilde{d}_R})/m^2_{\tilde{g}} \to \infty$.

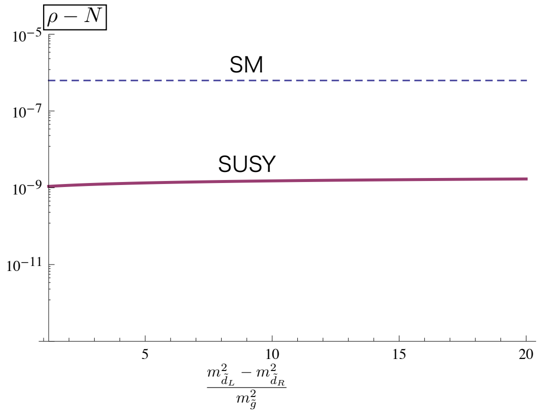

Figure 4.9: $\rho - N$ coupling ($|C^p(\rho) + C^b_+(\rho)| < |C^{\mathrm{SEW}}(\rho)|$) with $m_{\tilde{g}} = 1400$ GeV, $m_{\tilde{u}_R} = 2000$ GeV, $m_{\tilde{d}_R} = 2100$ GeV, and $m_{\tilde{u}_L} = 2500$ GeV. The magnitude closes in 2.3×10^{-9} as $(m^2_{\tilde{d}_L} - m^2_{\tilde{d}_R})/m^2_{\tilde{g}} \to \infty$.

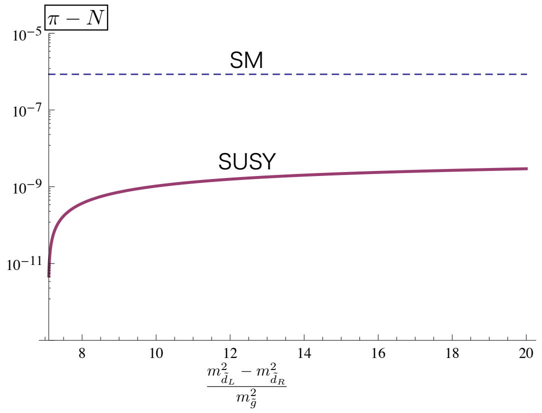

Figure 4.10: $\pi - N$ coupling ($|C^p(\pi) + C^b_-(\pi)| < |C^{\mathrm{SEW}}(\pi)|$) with $m_{\tilde{g}} = 850$ GeV, $m_{\tilde{u}_R} = 860$ GeV, $m_{\tilde{d}_R} = 870$ GeV, and $m_{\tilde{u}_L} = 2500$ GeV. The magnitude closes in 7.4×10^{-9} as $(m^2_{\tilde{d}_L} - m^2_{\tilde{d}_R})/m^2_{\tilde{g}} \to \infty$.

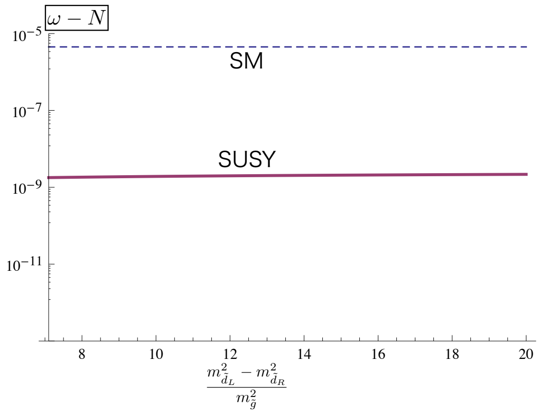

Figure 4.11: $\omega - N$ coupling ($|C^p(\omega) + C^b_+(\omega)| < |C^{\rm SEW}(\omega)|$) with $m_{\tilde{g}} = 850$ GeV, $m_{\tilde{u}_R} = 860$ GeV, $m_{\tilde{d}_R} = 870$ GeV, and $m_{\tilde{u}_L} = 2500$ GeV. The magnitude closes in 2.9×10^{-9} as $(m^2_{\tilde{d}_L} - m^2_{\tilde{d}_R})/m^2_{\tilde{g}} \to \infty$.

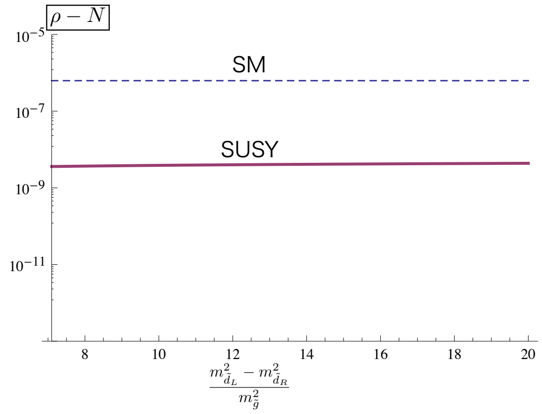

Figure 4.12: $\rho - N$ coupling ($|C^p(\rho) + C^b_+(\rho)| < |C^{\rm SEW}(\rho)|$) with $m_{\tilde{g}} = 850$ GeV, $m_{\tilde{u}_R} = 860$ GeV, $m_{\tilde{d}_R} = 870$ GeV, and $m_{\tilde{u}_L} = 2500$ GeV. The magnitude closes in 2.3×10^{-9} as $(m^2_{\tilde{d}_L} - m^2_{\tilde{d}_R})/m^2_{\tilde{g}} \to \infty$.

4.5 Comments on bounds for other sfermions

Let us comment on the bounds for left-right non-degeneracies of other sfermions. As for $\tilde{b}$, a total decay width of each bound state of $b\,\bar{b}$-meson has not experimentally measured yet. If we can know the width, the $b\,\bar{b}$-meson system can be analyzed, and a bound for a non-degeneracy between $m_{\tilde{b}_L}$ and $m_{\tilde{b}_R}$ can be calculated just as the bound between $m_{\tilde{c}_L}$ and $m_{\tilde{c}_R}$ was calculated from the charmonium. We will calculate the bounds by just replacing $\eta_c \to \eta_b$ (η_b: 0^{-+}) and $\chi_{c0} \to \chi_{b0}$ (χ_{b0}: 0^{++}). We hope our method is useful to give a bound between $m_{\tilde{b}_L}$ and $m_{\tilde{b}_R}$ from a future experiments of B-physics.

As for $\tilde{s}$, it is difficult to estimate the bound from the same method in section 6.1. The reason is as follows. If we include a mixing between d- and s-quarks through the Cabbibo angle, this effect is too small to induce the bound between $m_{\tilde{s}_L}$ and $m_{\tilde{s}_R}$ because Figs. 4.7, 4.8, and 4.9 can not give bounds for $\tilde{u}, \tilde{d}$, too. On the other hand, if we take s-quark as a heavy quark and calculate a quarkonium in NRQCD as c-quark, we might have bounds of $\tilde{s}$ for left-right non-degeneracy from parity violating decay mode of $\eta(548)$. Here, $\eta(548)$ is 0^{-+} which might have a mixing with $f_0(600)$ (0^{++}), if parity violation exists. The decay mode of $f_0(600)$ is dominated by 2π. Thus, the parity violation induces $\eta(548) \to \pi\pi$, whose experimental bounds are given by

$$\mathrm{Br}(\eta \to \pi^+\pi^-) < 1.3\times10^{-5}, \quad \mathrm{Br}(\eta \to 2\pi^0) < 3.5\times10^{-4}. \qquad (4.5.49)$$

However, these state are not composed only by s-quarks but also u-, d-quarks, so that a valid estimation is difficult. Also we should remind that mass of s-quark is about ten times smaller than that of c-quark which is too light to be treated in the NRQCD.

Finally, we comment on *sleptons*. Lepton flavor violation (LFV) experiments require stringent bounds of non-degeneracy among slepton flavors (generations). However, the LFV is suppressed when slepton masses are heavy enough even if their left- and right-handed slepton masses are not degenerate. That is, the left-right degeneracy is not required when sleptons are heavy enough. This situation is the same for squark sector as above (and also shown in $K^0 - \bar{K}^0$ system, where left-right degeneracy is not required with enough heavy squarks).

Chapter 5

Summary and discussion

The SUSY SM undergoes parity violation in QCD through chiral quark-squark-gluino interactions with non-degenerate masses between left-handed and right-handed squarks. In this thesis, we have studied parity violation in QCD process via SUSY.

In chapter 3, we have investigated parity violation in QCD process by focusing on helicity dependent top quark pair productions at the LHC experiment. Though no violation can be found in the SM, new physics beyond the SM predicts the violation in general. In order to evaluate the violation, we have utilized an effective operator analysis in a case that new particles predicted by the new physics are too heavy to be directly detected. By this method, we have tried to discriminate SUSY SM model from UED model via helicity asymmetry measurement of the top quark pair production. As our results, parity violation originated from SUSY induces larger helicity asymmetry than UED case. We have found that there is a possibility to discriminate SUSY from UED, and we have also estimated the asymmetries from the SMEW background and the LH model, where they are the same order and could be observable in the specific phase space. However, the SM QCD background is large, and the signal of the helicity asymmetry becomes tiny in our search (with specific parameters). In spite of the tiny signals of $t\bar{t}$ asymmetry in the SUSY and UED, there are still possibilities of the discrimination to succeed, i.e., we take the analyses of order $\alpha_s y_t^2$ in the small $\tan\beta$ region, and investigate without use of the effective operators in the specific phase space. For example, top Yukawa contribution is coming from t_R-$\tilde{b}_L$-$\tilde{\chi}^{\pm}$ interaction. On the other hand, t_L couples to $\tilde{b}_R$ and $\tilde{\chi}^{\pm}$ with bottom Yukawa coupling. Thus, in the small $\tan\beta$ region, the parity violation is expected to be large due to large top Yukawa contribution. In this case, if the SUSY cross section is enhanced as $\sim 10^{-2}$ pb at the high $m_{t\bar{t}}$ and p_T region with the specific phase space, $\delta A_{LR}^{\rm SUSY}$ could be large enough as 0.05. Then, δA_{LR} can be observed when the statistic error is of oder 0.01. We need 10^4 events of top pair production for this statistic error, and an integrated luminosity is roughly estimated as 10^2 fb^{-1}. Therefore, the sizable asymmetry can be observed with specific parameters at the LHC.

In chapter 4, we have studied parity non-conserving quarkonium decay. First of all, we have established the methods of analysis to estimate this bound for each squark. Second, we have investigated the non-degeneracy bound between $m_{\tilde{c}_L}$ and $m_{\tilde{c}_R}$ from experimental data of charmonium decay by use of NRQCD. Third, we evaluated the non-degeneracy bounds for $\tilde{u}$ and $\tilde{d}$ from nucleon-meson scattering data, and commented on other squarks. Unfortunately, our results are below current experimental data, and can not obtain the left-right degeneracy bounds for squark masses. However, it is expected that our method is useful for obtaining bounds from future experimental data and research of an origin of parity violation.

Acknowledgement

I am very grateful to my supervisor Prof. N. Haba for his great advises, encouragements, stimulating discussions and collaborations. I am also very grateful to Prof. Y. Hosotani for various supports and encouragements. I would like to thank T. Onogi for his instructive advise, fruitful discussions and collaboration, which study is a part of this thesis. I would like to thank Y. Mimura for his useful advises, exciting discussions and collaborations. I also thank S. Matsumoto, S. Tsuno, and T. Nabeshima, who are collaborators in main part of this thesis, for fruitful discussions and collaborations. I would like to express special thanks to R. Takahashi for his various helpful advices, discussions and collaborations. Finally, I thank all members of particle physics theory group at Osaka University and theoretical particle physics group at Hokkaido University.

Appendix A

Notations and helicity amplitude

A.1 Notations

The notation of this thesis is shown for convenience.

- Pauli matrix:

$$\sigma^1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} & -i \\ i & \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}. \tag{A.1.1}$$

- γ matrix (chiral rep.):

$$\gamma^0 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} & -\sigma^i \\ \sigma^i & \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \tag{A.1.2}$$

- γ matrix (standard rep.):

$$\gamma^0 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} & \sigma^i \\ -\sigma^i & \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, \tag{A.1.3}$$

where $\gamma^\mu_{\text{chiral}} = U\gamma^\mu_{\text{standard}}U^\dagger$, and U is defined by

$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{A.1.4}$$

Chiral projection is given by

$$P_R = \frac{1+\gamma^5}{2}, \quad P_L = \frac{1-\gamma^5}{2}. \tag{A.1.5}$$

A.2 Fierz transformation and color factor

Here let us summarize some formulas which are useful for calculations. Fierz transformation of γ-matrix in Eq.(3.1.20) shows

$$(C^\dagger\gamma^\mu)_{ik}(\gamma^\nu C^T)_{lj} = -\frac{1}{2}(\gamma^\mu)_{ji}(\gamma_\mu)_{lk} - \frac{1}{2}(\gamma^\mu\gamma^5)_{ji}(\gamma_\mu\gamma^5)_{lk}, \tag{A.2.6}$$

$$(C^\dagger)_{ik}(C^T)_{lj} = \frac{1}{4}(\gamma^\mu)_{ji}(\gamma_\mu)_{lk} - \frac{1}{4}(\gamma^\mu\gamma^5)_{ji}(\gamma_\mu\gamma^5)_{lk}. \tag{A.2.7}$$

We should notice that scalar nor pseudo-scalar components do not appear by the Fierz transformation, since spinor components, (i,j,k,l), always have the same chirality in each set of (i,j) and (k,l).

The color factor becomes

$$\left(\frac{\lambda^b}{2}\right)_{jn}\left(\frac{\lambda^a}{2}\right)_{ni}\left(\frac{\lambda^b}{2}\right)_{lm}\left(\frac{\lambda^a}{2}\right)_{mk} = \frac{2}{9}\delta_{lk}\delta_{jk} - \frac{1}{3}\left(\frac{\lambda^a}{2}\right)_{lk}\left(\frac{\lambda^a}{2}\right)_{jk}, \tag{A.2.8}$$

from Fierz transformation of spinors. Next formulas are useful for the second term of Eq.(3.1.18), whose spinor and color factor are different from those of the first term. The spinor is given by

$$(\gamma^\mu)_{li}(\gamma_\mu)_{jk} = -\frac{1}{2}(\gamma^\mu)_{ji}(\gamma_\mu)_{lk} + \frac{1}{2}(\gamma^\mu\gamma^5)_{ji}(\gamma_\mu\gamma^5)_{lk}, \tag{A.2.9}$$

$$\delta_{li}\delta_{jk} = \frac{1}{4}(\gamma^\mu)_{ji}(\gamma_\mu)_{lk} + \frac{1}{4}(\gamma^\mu\gamma^5)_{ji}(\gamma_\mu\gamma^5)_{lk}, \tag{A.2.10}$$

and the color factor becomes

$$\left(\frac{\lambda^b}{2}\right)_{jn}\left(\frac{\lambda^a}{2}\right)_{ni}\left(\frac{\lambda^a}{2}\right)_{lm}\left(\frac{\lambda^b}{2}\right)_{mk} = \frac{2}{9}\delta_{lk}\delta_{jk} + \frac{7}{6}\left(\frac{\lambda^a}{2}\right)_{lk}\left(\frac{\lambda^a}{2}\right)_{jk}, \tag{A.2.11}$$

while A, B are the same as the first term.

A.3 Helicity eigenstate

Plane wave solutions of Dirac Eq. satisfy

$$(\not{p} - m)u(\vec{p}, s) = 0, \quad (\not{p} + m)v(\vec{p}, s) = 0, \tag{A.3.12}$$

where $u(v)$ has positive (negative) energy, and s is eigenvalue of helicity operator $\sigma\cdot\vec{p}/|\vec{p}| \equiv \sigma\cdot n$. The solution u can be decomposed to 2-component spinors χ and ϕ such as

$$u(\vec{p}, s) \;=\; N\begin{pmatrix}\chi(\vec{p}, s)\\ \phi(\vec{p}, s)\end{pmatrix}, \tag{A.3.13}$$

where N is normalization factor. In the standard representation, we obtain following relations,

$$\chi = \frac{|\vec{p}|}{E-m}(\vec{\sigma}\cdot\vec{n})\ \phi, \quad \phi = \frac{|\vec{p}|}{E+m}(\vec{\sigma}\cdot\vec{n})\ \chi, \tag{A.3.14}$$

and N is determined by the normalization of $\bar{u}(\vec{p},s)u(\vec{p},s') = 2m\delta_{ss'}$ as $N = \sqrt{E+m}$. Thus the solution u can be written by

$$u(\vec{p},s) \ = \ \left(\begin{array}{c} \sqrt{E+m}\ \chi(\vec{p},s) \\ \sqrt{E-m}\ (\sigma\cdot\vec{n})\ \chi(\vec{p},s) \end{array} \right). \tag{A.3.15}$$

When we take $\chi(\vec{p},s)$ as helicity eigenstate, it is given by

$$\chi(\vec{p},s) = \frac{1}{2}\left(\begin{array}{c} (1+s)\cos\frac{\theta}{2} - (1-s)e^{is\phi}\sin\frac{\theta}{2} \\ (1-s)\cos\frac{\theta}{2} + (1+s)e^{is\phi}\sin\frac{\theta}{2} \end{array} \right) \equiv \chi^{(s)}, \qquad (\vec{\sigma}\cdot\vec{n})\ \chi^{(s)} = s\ \chi^{(s)}, \tag{A.3.16}$$

where $s = \pm 1$, and θ and ϕ are polar and azimuthal angles of $\vec{p}$. Negative energy solution is given by $v = C\bar{u}^T$, where $C = i\gamma^2\gamma^0$. Then we obtain

$$v(\vec{p},s) \ = \ \left(\begin{array}{c} -s\sqrt{E-m}\ \chi^{(s)'} \\ \sqrt{E+m}\ \chi^{(s)'} \end{array} \right), \tag{A.3.17}$$

where $\chi^{(s)'} \equiv -i\sigma^2\chi^{(s)*}$ which satisfies $(\vec{\sigma}\cdot\vec{n})\ \chi^{(s)'} = -s\ \chi^{(s)'}$. Helicity amplitude can be calculated by using (A.3.15), (A.3.16), and (A.3.17).

In the rest of this section, we show the relation between helicity and chirality. The solution in the standard representation, $u_{\mathrm{st.}}$, is related to that in the chiral representation, $u_{\mathrm{ch.}}$, by the unitary matrix U. Thus we obtain

$$\begin{aligned} u_{\mathrm{ch.}} \ &= \ \left(\begin{array}{c} u_R \\ u_L \end{array} \right) \\ &= \ U\ u_{\mathrm{st.}} \sim \frac{E}{\sqrt{2}} \left(\begin{array}{c} \left[(1+s) + \frac{m}{2E}(1-s)\right]\chi^{(s)} \\ \\ \left[(1-s) + \frac{m}{2E}(1+s)\right]\chi^{(s)} \end{array} \right) \end{aligned} \tag{A.3.18}$$

in high energy limit, $E \gg m$, and we find that u_R (u_L) almost comes from $\chi^{(s=+1)}$ ($\chi^{(s=-1)}$). The same relation for v is obtained by

$$\left(\begin{array}{c} v_R \\ v_L \end{array} \right) \ \sim \ \frac{E}{\sqrt{2}} \left(\begin{array}{c} \left[(1-s) + \frac{m}{2E}(1+s)\right]\chi^{(s)'} \\ \\ -\left[(1+s) + \frac{m}{2E}(1-s)\right]\chi^{(s)'} \end{array} \right), \tag{A.3.19}$$

and we find that v_R (v_L) almost comes from $\chi^{(s=-1)'}$ ($\chi^{(s=+1)'}$), which correspondence is opposite from the case of u. Note that the relation between helicity and chirality of $\bar{u}$ ($\bar{v}$) is the same as that of u (v). Thus an amplitude for each helicities, for example, s-channel gluon exchange

$$i\mathcal{M}_{s_1 s_2 s_3 s_4} = -g_s^2[\bar{u}(\vec{p}_1, s_1)\gamma^\mu T^a v(\vec{p}_2, s_2)]\frac{-ig_{\mu\nu}}{s}[\bar{v}(\vec{p}_3, s_3)\gamma^\nu T^a u(\vec{p}_4, s_4)] \quad \text{(A.3.20)}$$

can be obtained by the helicity eigenstates. Various techniques are shown in Refs. [89, 90, 91, 92, 93, 94].

Appendix B

Dimension six operators and Wilson coefficients

B.1 Explicit Coefficients of dimension six operators

We define the following functions to write coefficients in terms of linear combinations of them,

$$\begin{aligned}
\alpha\left(a,b,m,n\right)\equiv& \int_0^1 dx \frac{x^n}{\left(x\left(a^2-b^2\right)+b^2\right)^m},\\
\beta\left(a,b,c,m,n,l\right)\equiv& \int_0^1 dx \int_x^1 dy \frac{x^n y^l}{\left(x\left(c^2-b^2\right)+y\left(b^2-a^2\right)+a^2\right)^m},\\
\gamma\left(a,b,c,m,n,l,r\right)\equiv& \int_0^1 dx \int_x^1 dy \int_y^1 dz \frac{x^n y^l z^r}{\left(x\left(c^2-a^2\right)+y\left(a^2-c^2\right)+z\left(c^2-b^2\right)+b^2\right)^m},\\
\delta\left(a,b,c,m,n,l,r\right)\equiv& \int_0^1 dx \int_x^1 dy \int_y^1 dz \frac{x^n y^l z^r}{\left(x\left(c^2-a^2\right)+z\left(a^2-b^2\right)+b^2\right)^m}.
\end{aligned} \tag{B.1.1}$$

B.1.1 SUSY

In SUSY SM, dimension six operators are written as follows,

$$\begin{aligned}
\int d^4x \mathcal{O}_{4F}^{(1)}(x) =& \frac{12g_s^4}{192\pi^2}\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\frac{d^4k_4}{(2\pi)^4}(2\pi)^4\delta^4(-k_1+k_2-k_3+k_4)\\
&\sum_{i,j=L,R} C_{ij}\left(\bar{q}(k_1)\gamma^\mu P_i q(k_2)\right)\left(\bar{q}'(k_3)\gamma_\mu P_j q'(k_4)\right),
\end{aligned} \tag{B.1.2}$$

$$\int d^4x\mathcal{O}_{4F}^{(8)}(x) = \frac{12g_s^4}{192\pi^2}\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\frac{d^4k_4}{(2\pi)^4}(2\pi)^4\delta^4(-k_1+k_2-k_3+k_4)$$
$$\sum_{i,j=L,R} D_{ij}\,(\bar{q}T^a\gamma^\mu P_i q)\,(\bar{q}'T^a\gamma_\mu P_j q')\,, \tag{B.1.3}$$
$$\int d^4x\mathcal{O}_{qqG}(x) = \frac{g_s^3}{96\pi^2}\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}(2\pi)^4\delta^4(-k_1+k_2+k_3)\bar{q}(k_2)T^aE^\mu_{L,R}G^a_\mu(k_3)P_{L,R}q(k_1), \tag{B.1.4}$$
$$\int d^4x\mathcal{O}_{qqGG}(x) = \frac{g_s^4}{192\pi^2}\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\frac{d^4k_4}{(2\pi)^4}(2\pi)^4\delta^4(-k_1+k_2+k_3+k_4)$$
$$\bar{q}(k_1)\left[F^{\mu\nu}_{L,R}\delta^{ab}+H^{\mu\nu}_{L,R}T^aT^b\right]G^a_\mu(k_2)G^b_\nu(k_3)P_{L,R}q(k_4), \tag{B.1.5}$$

where $E_i^\mu,\ F_i^{\mu\nu},\ H_i^{\mu\nu},\ (i=L,\ R)$ are

$$\begin{aligned} E_i^\mu &= \{e_{1i}\not{k}_1+e_{2i}\not{k}_2\}k_1^\mu+\{e_{1i}\not{k}_2+e_{2i}\not{k}_1\}k_2^\mu \\ &\quad +\{e_{3i}(k_1^2+k_2^2)-e_{4i}k_1\cdot k_2\}\gamma^\mu - e_{5i}i\epsilon^{\alpha\beta\mu\nu}\gamma_5\gamma_\nu k_{1\alpha}k_{2\beta}, \end{aligned} \tag{B.1.6}$$
$$F_i^{\mu\nu} = f_{1i\alpha}i\epsilon^{\alpha\mu\nu\beta}\gamma_5\gamma_\beta + f_{2i\alpha}g^{\mu\nu}\gamma^\alpha + f_{3i\alpha}g^{\alpha\mu}\gamma^\nu + f_{4i\alpha}g^{\alpha\nu}\gamma^\mu, \tag{B.1.7}$$
$$H_i^{\mu\nu} = h_{1i\alpha}i\epsilon^{\alpha\mu\nu\beta}\gamma_5\gamma_\beta + h_{2i\alpha}g^{\mu\nu}\gamma^\alpha + h_{3i\alpha}g^{\alpha\mu}\gamma^\nu + h_{4i\alpha}g^{\alpha\nu}\gamma^\mu. \tag{B.1.8}$$

Coefficients in $\mathcal{O}_{4F}$

The coefficients of 4-Fermi operator are given as

$$C_{LL}^{\rm SUSY} = \frac{2}{9}[f_1(m_{\tilde{q}_L},m_{\tilde{q}'_L})+f_2(m_{\tilde{q}_L},m_{\tilde{q}'_L})], \tag{B.1.9}$$
$$C_{RR}^{\rm SUSY} = \frac{2}{9}[f_1(m_{\tilde{q}_R},m_{\tilde{q}'_R})+f_2(m_{\tilde{q}_R},m_{\tilde{q}'_R})], \tag{B.1.10}$$
$$C_{LR}^{\rm SUSY} = -\frac{2}{9}[f_1(m_{\tilde{q}_R},m_{\tilde{q}'_L})-f_2(m_{\tilde{q}_L},m_{\tilde{q}'_R})], \tag{B.1.11}$$
$$C_{RL}^{\rm SUSY} = -\frac{2}{9}[f_1(m_{\tilde{q}_L},m_{\tilde{q}'_R})+f_2(m_{\tilde{q}_L},m_{\tilde{q}'_R})], \tag{B.1.12}$$
$$D_{LL}^{\rm SUSY} = -\frac{1}{3}f_1(m_{\tilde{q}_L},m_{\tilde{q}'_L})-\frac{7}{6}f_2(m_{\tilde{q}_L},m_{\tilde{q}'_L}), \tag{B.1.13}$$
$$D_{RR}^{\rm SUSY} = -\frac{1}{3}f_1(m_{\tilde{q}_R},m_{\tilde{q}'_R})-\frac{7}{6}f_2(m_{\tilde{q}_R},m_{\tilde{q}'_R}), \tag{B.1.14}$$
$$D_{LR}^{\rm SUSY} = -\frac{7}{6}f_1(m_{\tilde{q}_L},m_{\tilde{q}'_R})-\frac{1}{3}f_2(m_{\tilde{q}_L},m_{\tilde{q}'_R}), \tag{B.1.15}$$
$$D_{RL}^{\rm SUSY} = -\frac{7}{6}f_2(m_{\tilde{q}_R},m_{\tilde{q}'_L})-\frac{1}{3}f_1(m_{\tilde{q}_R},m_{\tilde{q}'_L}), \tag{B.1.16}$$

where f_1 and f_2 are given by

$$f_1(m_{\tilde{q}}, m_{\tilde{q}'}) = \beta\,(m_{\tilde{q}'}, m_{\tilde{q}}, m_{\tilde{g}}, 1, 1, 0), \tag{B.1.17}$$

$$f_2(m_{\tilde{q}}, m_{\tilde{q}'}) = m_{\tilde{g}}^2 \beta\,(m_{\tilde{q}'}, m_{\tilde{q}}, m_{\tilde{g}}, 2, 1, 0). \tag{B.1.18}$$

Coefficients in $\mathcal{O}_{qqG}$

The coefficients of q-q-G operator are given as

$$E_L^\mu \equiv E^\mu(m_{\tilde{q}} = m_{\tilde{q}_L}), \tag{B.1.19}$$

$$= \{e_1(m_{\tilde{q}_L})\not{k}_1 + e_2(m_{\tilde{q}_L})\not{k}_2\}k_1^\mu + \{e_1(m_{\tilde{q}_L})\not{k}_2 + e_2(m_{\tilde{q}_L})\not{k}_1\}k_2^\mu$$
$$+ \{e_3(m_{\tilde{q}_L})(k_1^2 + k_2^2) - e_4(m_{\tilde{q}_L})k_1 \cdot k_2\}\gamma^\mu - e_5(m_{\tilde{q}_L}) i\epsilon^{\alpha\beta\mu\nu}\gamma_5\gamma_\nu k_{1\alpha}k_{2\beta}, \tag{B.1.20}$$

$$E_R^\mu = E^\mu(m_{\tilde{q}_R}), \tag{B.1.21}$$

$$e_1(m_{\tilde{q}})$$
$$= \frac{107m_{\tilde{g}}^6 - 495m_{\tilde{g}}^4 m_{\tilde{q}}^2 + 477m_{\tilde{g}}^2 m_{\tilde{q}}^4 - 89m_{\tilde{q}}^6 - 6(m_{\tilde{g}}^6 + 3m_{\tilde{g}}^4 m_{\tilde{q}}^2 - 54m_{\tilde{g}}^2 m_{\tilde{q}}^4 + 18m_{\tilde{q}}^6)\log(m_{\tilde{g}}^2/m_{\tilde{q}}^2)}{18(m_{\tilde{g}}^2 - m_{\tilde{q}}^2)^4}, \tag{B.1.22}$$

$$e_2(m_{\tilde{q}})$$
$$= \frac{-203m_{\tilde{g}}^6 + 351m_{\tilde{g}}^4 m_{\tilde{q}}^2 - 189m_{\tilde{g}}^2 m_{\tilde{q}}^4 + 41m_{\tilde{q}}^6 + 6(m_{\tilde{g}}^6 + 51m_{\tilde{g}}^4 m_{\tilde{q}}^2 - 54m_{\tilde{g}}^2 m_{\tilde{q}}^4 + 18m_{\tilde{q}}^6)\log(m_{\tilde{g}}^2/m_{\tilde{q}}^2)}{18(m_{\tilde{g}}^2 - m_{\tilde{q}}^2)^4}, \tag{B.1.23}$$

$$e_3(m_{\tilde{q}}) = e_2(m_{\tilde{q}}), \tag{B.1.24}$$

$$e_4(m_{\tilde{q}})$$
$$= \frac{-155m_{\tilde{g}}^6 + 423m_{\tilde{g}}^4 m_{\tilde{q}}^2 - 333m_{\tilde{g}}^2 m_{\tilde{q}}^4 + 65m_{\tilde{q}}^6 + 6(m_{\tilde{g}}^6 + 27m_{\tilde{g}}^4 m_{\tilde{q}}^2 - 54m_{\tilde{g}}^2 m_{\tilde{q}}^4 + 18m_{\tilde{q}}^6)\log(m_{\tilde{g}}^2/m_{\tilde{q}}^2)}{9(m_{\tilde{g}}^2 - m_{\tilde{q}}^2)^4}, \tag{B.1.25}$$

$$e_5(m_{\tilde{q}}) = \frac{9(m_{\tilde{g}}^4 - m_{\tilde{q}}^4 - 2m_{\tilde{g}}^2 m_{\tilde{q}}^2 \log(m_{\tilde{g}}^2/m_{\tilde{q}}^2))}{(m_{\tilde{g}}^2 - m_{\tilde{q}}^2)^3}. \tag{B.1.26}$$

Coefficients in $\mathcal{O}_{qqGG}$

The coefficients of q-q-G-G operator are given as

$$F^{\mu\nu} = f_{1\alpha} i\epsilon^{\alpha\mu\nu\beta}\gamma_5\gamma_\beta + f_{2\alpha} g^{\mu\nu}\gamma^\alpha + f_{3\alpha} g^{\alpha\mu}\gamma^\nu + f_{4\alpha} g^{\alpha\nu}\gamma^\mu, \tag{B.1.27}$$

$$H^{\mu\nu} = h_{1\alpha} i\epsilon^{\alpha\mu\nu\beta}\gamma_5\gamma_\beta + h_{2\alpha} g^{\mu\nu}\gamma^\alpha + h_{3\alpha} g^{\alpha\mu}\gamma^\nu + h_{4\alpha} g^{\alpha\nu}\gamma^\mu, \tag{B.1.28}$$

$$f_{1\alpha} = \left[\frac{1}{2}S_{0\alpha} - 2(P_0 + Q_0)_\alpha\right](m_a = m_{\tilde{g}}, m_b = m_{\tilde{q}_i}), \tag{B.1.29}$$

$$f_{2\alpha} = \left[-3K_\alpha + \frac{1}{2}R_{3\alpha} + \frac{1}{2}S_{1\alpha} + 2(P_1 + Q_1)_\alpha\right](m_a = m_{\tilde{g}}, m_b = m_{\tilde{q}_i}), \tag{B.1.30}$$

$$f_{3\alpha} = \left[\frac{1}{2}R_{2\alpha} + \frac{1}{2}S_{2\alpha} + 2(P_3 + Q_3)_\alpha\right](m_a = m_{\tilde{g}}, m_b = m_{\tilde{q}_i}), \tag{B.1.31}$$

$$f_{4\alpha} = \left[\frac{1}{2}R_{1\alpha} + \frac{1}{2}S_{3\alpha} + 2(P_2 + Q_2)_\alpha\right](m_a = m_{\tilde{g}}, m_b = m_{\tilde{q}_i}), \tag{B.1.32}$$

$$h_{1\alpha} = [12(P_0 + Q_0)_\alpha](m_a = m_{\tilde{g}}, m_b = m_{\tilde{q}_i}), \tag{B.1.33}$$

$$h_{2\alpha} = \left[2K_\alpha - \frac{1}{3}R_{3\alpha} + 12(P_1 + Q_1)_\alpha\right](m_a = m_{\tilde{g}}, m_b = m_{\tilde{q}_i}), \tag{B.1.34}$$

$$h_{3\alpha} = \left[-\frac{1}{3}R_{2\alpha} + 12(P_3 + Q_3)_\alpha\right](m_a = m_{\tilde{g}}, m_b = m_{\tilde{q}_i}), \tag{B.1.35}$$

$$h_{4\alpha} = \left[-\frac{1}{3}R_{1\alpha} + 12(P_2 + Q_2)_\alpha\right](m_a = m_{\tilde{g}}, m_b = m_{\tilde{q}_i}).,\tag{B.1.36}$$

where $K(m_a, m_b), P(m_a, m_b), Q(m_a, m_b), R(m_a, m_b), S(m_a, m_b)$ are written in terms of a linear combination of the momentums;

$$K_\alpha = a_1 k_{1\alpha} + a_2 k_{4\alpha}, \tag{B.1.37}$$

$$P_{0\alpha} = (i_{11} + i_{12} + i_{13})k_{1\alpha} + (i_{21} + i_{22} + i_{23})k_{2\alpha} + (i_{31} + i_{32} + i_{33})k_{4\alpha}, \tag{B.1.38}$$

$$P_{1\alpha} = (i_{11} + i_{12} - i_{13})k_{1\alpha} + (i_{21} + i_{22} - i_{23})k_{2\alpha} + (i_{31} + i_{32} - i_{33})k_{4\alpha}, \tag{B.1.39}$$

$$P_{2\alpha} = (i_{11} - i_{12} + i_{13})k_{1\alpha} + (i_{21} - i_{22} + i_{23})k_{2\alpha} + (i_{31} - i_{32} + i_{33})k_{4\alpha}, \tag{B.1.40}$$

$$P_{3\alpha} = (-i_{11} + i_{12} + i_{13})k_{1\alpha} + (-i_{21} + i_{22} + i_{23})k_{2\alpha} + (-i_{31} + i_{32} + i_{33})k_{4\alpha}, \tag{B.1.41}$$

$$Q_{0\alpha} = (j_{11} - j_{12} + j_{13})k_{1\alpha} + (j_{21} - j_{22} + j_{23})k_{2\alpha} + (j_{31} - j_{32} + j_{33})k_{4\alpha}, \tag{B.1.42}$$

$$Q_{1\alpha} = Q_{0\alpha}, \tag{B.1.43}$$

$$Q_{2\alpha} = (-j_{11} + j_{12} + j_{13})k_{1\alpha} + (-j_{21} + j_{22} + j_{23})k_{2\alpha} + (-j_{31} + j_{32} + j_{33})k_{4\alpha}, \tag{B.1.44}$$

$$Q_{3\alpha} = (j_{11} + j_{12} - j_{13})k_{1\alpha} + (j_{21} + j_{22} - j_{23})k_{2\alpha} + (j_{31} + j_{32} - j_{33})k_{4\alpha}, \tag{B.1.45}$$

$$R_{1\alpha} = b_{11}k_{1\alpha} + b_{21}k_{2\alpha} + b_{31}k_{4\alpha}, \tag{B.1.46}$$

$$R_{2\alpha} = b_{12}k_{1\alpha} + b_{22}k_{2\alpha} + b_{32}k_{4\alpha}, \tag{B.1.47}$$

$$R_{3\alpha} = b_{13}k_{1\alpha} + b_{23}k_{2\alpha} + b_{33}k_{4\alpha}, \tag{B.1.48}$$

$$S_{0\alpha} = (-f_{12} + f_{13})k_{1\alpha} + (-f_{22} + f_{23})k_{2\alpha} + (-f_{32} + f_{33})k_{4\alpha}, \tag{B.1.49}$$

$$S_{1\alpha} = (f_{12} + f_{13})k_{1\alpha} + (f_{22} + f_{23})k_{2\alpha} + (f_{32} + f_{33})k_{4\alpha}, \tag{B.1.50}$$

$$S_{2\alpha} = S_{1\alpha}, \tag{B.1.51}$$

$$\begin{aligned} S_{3\alpha} = &(h_1 - 2f_{11} - f_{12} - f_{13})k_{1\alpha} + (h_2 - 2f_{21} - f_{22} - f_{23})k_{2\alpha} \\ &+ (h_3 - 2f_{31} - f_{32} - f_{33})k_{4\alpha}. \end{aligned} \tag{B.1.52}$$

These coefficients, $a_1, a_2, \cdots$, are given in terms of Feynman parameter integral as

$$\begin{aligned} a_1(m_a, m_b) &= a_1(m_a, m_b) = a_2(m_a, m_b) \\ &= -\alpha\,(m_a, m_b, 1, 2) + 2\alpha\,(m_a, m_b, 1, 1) - \alpha\,(m_a, m_b, 1, 0), \end{aligned} \tag{B.1.53}$$

$$\begin{aligned} i_{11}(m_a, m_b) &= -2i_{21}(m_a, m_b) = -2i_{22}(m_a, m_b) = 2i_{32}(m_a, m_b) = 2i_{33}(m_a, m_b) \\ &= -2\alpha(m_a, m_b, 1, 3), \end{aligned} \tag{B.1.54}$$

$$\begin{aligned} i_{12}(m_a, m_b) &= i_{13}(m_a, m_b) \\ &= -2\alpha(m_a, m_b, 1, 3) + 3\alpha(m_a, m_b, 1, 2), \end{aligned} \tag{B.1.55}$$

$$\begin{aligned} i_{23}(m_a, m_b) &= -i_{31}(m_a, m_b) \\ &= \alpha(m_a, m_b, 1, 3) - 3\alpha(m_a, m_b, 1, 2), \end{aligned} \tag{B.1.56}$$

$$\begin{aligned} j_{11}(m_a, m_b) &= j_{12}(m_a, m_b) \\ &= -2m_a^2\alpha(m_a, m_b, 2, 3) + 3m_a^2\alpha(m_a, m_b, 2, 2) \end{aligned} \tag{B.1.57}$$

$$\begin{aligned} j_{13}(m_a, m_b) &= 2j_{21}(m_a, m_b) = 2j_{23}(m_a, m_b) = -2j_{31}(m_a, m_b) = -2j_{32}(m_a, m_b) \\ &= 2m_a^2\alpha(m_a, m_b, 2, 3), \end{aligned} \tag{B.1.58}$$

$$\begin{aligned} j_{22}(m_a, m_b) &= -j_{33}(m_a, m_b) \\ &= m_a^2\alpha(m_a, m_b, 2, 3) - 3m_a^2\alpha(m_a, m_b, 2, 2), \end{aligned} \tag{B.1.59}$$

$$\begin{aligned} b_{11}(m_a, m_b) &= 2b_{12}(m_a, m_b) = b_{13}(m_a, m_b) = -2b_{21}(m_a, m_b) = -2b_{22}(m_a, m_b) = -2b_{23}(m_a, m_b) \\ &= -2b_{31}(m_a, m_b) = 2b_{32}(m_a, m_b) = 2b_{33}(m_a, m_b) \\ &= 8\alpha(m_a, m_b, 1, 3) - 9\alpha(m_a, m_b, 1, 2) + 6\alpha(m_a, m_b, 1, 1) - \alpha(m_a, m_b, 1, 0), \end{aligned} \tag{B.1.60}$$

$$f_{11}(m_a,m_b) = -6\alpha\,(m_a,m_b,1,3) + 6\alpha\,(m_a,m_b,1,2), \tag{B.1.61}$$

$$\begin{aligned} f_{12}(m_a,m_b) &= f_{13}(m_a,m_b) = f_{22}(m_a,m_b) \\ &= 12\alpha\,(m_a,m_b,1,1) - 12\alpha\,(m_a,m_b,1,0), \end{aligned} \tag{B.1.62}$$

$$\begin{aligned} f_{23}(m_a,m_b) &= 12\alpha\,(m_a,m_b,1,3) + 6\alpha\,(m_a,m_b,1,2) - 6\alpha\,(m_a,m_b,1,1), \\ f_{31}(m_a,m_b) &= -12\alpha\,(m_a,m_b,1,3) + 6\alpha\,(m_a,m_b,1,2) + 6\alpha\,(m_a,m_b,1,1), \end{aligned} \tag{B.1.63}$$

$$\begin{aligned} f_{32}(m_a,m_b) &= f_{33}(m_a,m_b) \\ &= 3\alpha\,(m_a,m_b,1,4) + 4\alpha\,(m_a,m_b,1,3) - \alpha\,(m_a,m_b,1,0), \end{aligned} \tag{B.1.64}$$

$$h_1(m_a,m_b) - m_a^2\alpha\,(m_a,m_b,2,4) - 2m_a^2\alpha\,(m_a,m_b,2,3) + 4m_a^2\alpha\,(m_a,m_b,2,1) - m_a^2\alpha\,(m_a,m_b,2,0), \tag{B.1.65}$$

$$h_3(m_a,m_b) = -12m_a^2\alpha(m_a,m_b,2,3) + 6m_a^2\alpha(m_a,m_b,2,2) + 6m_a^2\alpha(m_a,m_b,2,1), \tag{B.1.66}$$

and other coefficients are zero.

B.1.2 UED

Let us show the dimension six operators of QCD in UED.

Coefficients in $\mathcal{O}_{4F}$

When $(\overline{q}q)(\overline{q}'q')$ chirality is (LL)(LL) or (RR)(RR), 4-Fermi operator is

$$\mathcal{O}_{4F}(x) = \frac{g_s^4}{192\pi^2}\left[f_1(\overline{q}\gamma^\mu q)(\overline{q}'\gamma_\mu q') + f_2\left(\overline{q}\gamma^\mu \frac{\lambda^a}{2} q\right)\left(\overline{q}'\gamma^\mu \frac{\lambda^a}{2} q'\right)\right], \tag{B.1.67}$$

and the coefficients f_1, f_2 are

$$f_{1L} = \frac{4\left(m_g^4 + 4m_g^2 m_L^2 \log\left(\frac{m_L}{m_g}\right) - m_L^4\right)}{\left(m_g^2 - m_L^2\right)^3}, \tag{B.1.68}$$

$$f_{1R} = \frac{4\left(m_g^4 + 4m_g^2 m_R^2 \log\left(\frac{m_R}{m_g}\right) - m_R^4\right)}{\left(m_g^2 - m_R^2\right)^3}, \tag{B.1.69}$$

$$f_{2L} = \frac{1}{\left(m_g^2 - m_5^2\right)(m_5 - m_L)^2(m_5 + m_L)^2(m_g - m_L)^2(m_g + m_L)^2}$$
$$\Bigg[18m_L^2\left(m_g^2\left(m_5^4\log\left(\frac{m_L^2}{m_g^2}\right) - m_L^4\log\left(\frac{m_g^2}{m_L^2}\right)\right) + m_5^2\left(m_g^4 + m_L^4\right)\log\left(\frac{m_5^2}{m_L^2}\right)\right.$$
$$+ 2m_5^2 m_g^2 m_L^2 \log\left(\frac{m_g^2}{m_5^2}\right)$$
$$\left. + (m_5^2 - m_g^2)(m_5^2 - m_L^2)(m_g^2 - m_L^2)\right) + \frac{7\left(-m_5^4 + 2m_5^2 m_L^2\log\left(\frac{m_5^2}{m_L^2}\right) + m_L^4\right)}{2\left(m_5^2 - m_L^2\right)^3}$$
$$+ \frac{30\left(-m_g^4 + 2m_g^2 m_L^2 \log\left(\frac{m_g^2}{m_L^2}\right) + m_L^4\right)}{\left(m_g^2 - m_L^2\right)^3}\Bigg], \tag{B.1.70}$$

$$f_{2R} = \frac{1}{\left(m_g^2 - m_5^2\right)(m_5 - m_R)^2(m_5 + m_R)^2(m_g - m_R)^2(m_g + m_R)^2}$$
$$\Bigg[18m_R^2\left(m_g^2\left(m_5^4\log\left(\frac{m_R^2}{m_g^2}\right) - m_R^4\log\left(\frac{m_g^2}{m_R^2}\right)\right) + m_5^2\left(m_g^4 + m_R^4\right)\log\left(\frac{m_5^2}{m_R^2}\right)\right.$$
$$+ 2m_5^2 m_g^2 m_R^2 \log\left(\frac{m_g^2}{m_5^2}\right)$$
$$\left. + (m_5^2 - m_g^2)(m_5^2 - m_R^2)(m_g^2 - m_R^2)\right) + \frac{7\left(-m_5^4 + 2m_5^2 m_R^2\log\left(\frac{m_5^2}{m_R^2}\right) + m_R^4\right)}{2\left(m_5^2 - m_R^2\right)^3}$$
$$+ \frac{30\left(-m_g^4 + 2m_g^2 m_R^2 \log\left(\frac{m_g^2}{m_R^2}\right) + m_R^4\right)}{\left(m_g^2 - m_R^2\right)^3}\Bigg]. \tag{B.1.71}$$

When $(\overline{u}u)(\bar{t}t)$ chirality is (LL)(RR) or (RR)(LL), 4-Fermi operator is

$$\mathcal{O}_{4F}(x) = \frac{g_s^4}{192\pi^2}\left[f_3(\overline{q}\gamma^\mu q)(\overline{q}'\gamma_\mu q') + f_4\left(\overline{q}\gamma^\mu \frac{\lambda}{2}^a q\right)\left(\overline{q}'\gamma^\mu\frac{\lambda}{2}^a q'\right)\right], \tag{B.1.72}$$

and the coefficients f_3, f_4 are

$$f_3(m_q, m_{q'}) = -\frac{1}{\left(m_g^2 - m_q^2\right)^2\left(m_g^2 - m_{q'}^2\right)^2\left(m_q^2 - m_{q'}^2\right)}$$
$$\Big[8\Big(2m_g^2 m_{q'}^4\left(m_g^2 - 2m_q^2\right)\log(m_g) - 2m_{q'}^4\left(m_g^2 - m_q^2\right)^2\log(m_{q'})$$
$$+ m_g^2(m_g^2 - m_q^2)(m_g^2 - m_{q'}^2)(m_q^2 - m_{q'}^2)$$
$$+2m_q^4\left(m_g^4\log\left(\frac{m_q}{m_g}\right) + 2m_g^2 m_{q'}^2\log\left(\frac{m_g}{m_q}\right) + m_{q'}^4\log(m_q)\right)\Big)\Big], \tag{B.1.73}$$

$$
\begin{aligned}
f_4(m_q, m_{q'}) = &\frac{9}{2\left(m_5^2 - m_q^2\right)\left(m_5^2 - m_{q'}^2\right)\left(m_q^2 - m_{q'}^2\right)} \\
&\Bigg(\frac{\left(m_5^2 - m_q^2\right)\left(-m_g^4 + 4m_g^4\log(m_g) + m_{q'}^4 - 4m_{q'}^4\log(m_{q'})\right)}{m_g^2 - m_{q'}^2} \\
&- \frac{\left(m_5^2 - m_{q'}^2\right)\left(-m_g^4 + 4m_g^4\log(m_g) + m_q^4 - 4m_q^4\log(m_q)\right)}{m_g^2 - m_q^2} \\
&+ \frac{\left(m_q^2 - m_{q'}^2\right)\left(-m_5^4 + 4m_5^4\log(m_5) + m_g^4 - 4m_g^4\log(m_g)\right)}{m_5^2 - m_g^2}\Bigg) \\
&+ \frac{9}{2\left(m_5^2 - m_q^2\right)^2\left(m_5^2 - m_{q'}^2\right)^2\left(m_{q'}^2 - m_q^2\right)} \\
&\Big[2m_5^2 m_{q'}^4\left(m_5^2 - 2m_q^2\right)\log(m_5) - 2m_{q'}^4\left(m_5^2 - m_q^2\right)^2\log(m_{q'}) \\
&+ m_5^2(m_5^2 - m_q^2)(m_5^2 - m_{q'}^2)(m_q^2 - m_{q'}^2) \\
&+ 2m_q^4\left(m_5^4\log\left(\frac{m_q}{m_5}\right) + 2m_5^2 m_{q'}^2\log\left(\frac{m_5}{m_q}\right) + m_{q'}^4\log(m_q)\right)\Big] \\
&- \frac{30}{\left(m_g^2 - m_q^2\right)^2\left(m_g^2 - m_{q'}^2\right)^2\left(m_q^2 - m_{q'}^2\right)} \\
&\Big[2m_g^2 m_{q'}^4\left(m_g^2 - 2m_q^2\right)\log(m_g) - 2m_{q'}^4\left(m_g^2 - m_q^2\right)^2\log(m_{q'}) \\
&+ m_g^2(m_g^2 - m_q^2)(m_g^2 - m_{q'}^2)(m_q^2 - m_{q'}^2) \\
&+ 2m_q^4\left(m_g^4\log\left(\frac{m_q}{m_g}\right) + 2m_g^2 m_{q'}^2\log\left(\frac{m_g}{m_q}\right) + m_{q'}^4\log(m_q)\right)\Big],
\end{aligned}
\tag{B.1.74}
$$

where $m_q, m_{q'}$ are m_L or m_R $(m_q, \neq m_{q'})$.

Coefficients in $\mathcal{O}_{qqG}$

q-q-G operator in UED takes the form as

$$
\int d^4x \mathcal{O}_{qqG}(x) = -\frac{g_s^3}{192\pi^2}\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}(2\pi)^4\delta^4(-k_1 + k_2 + k_3)\overline{q}(k_1)C^\mu G_\mu(k_3)q(k_2),
\tag{B.1.75}
$$

$$
C^\mu \equiv c_1 i\epsilon^{\alpha\beta\mu\nu}\gamma^5\gamma_\nu + c_2 g^{\mu\alpha}\gamma^\beta + c_3 g^{\mu\beta}\gamma^\alpha + c_4 g^{\alpha\beta}\gamma^\mu,
\tag{B.1.76}
$$

$$
c_1 \equiv c_{11}k_{1\alpha}k_{2\beta},
\tag{B.1.77}
$$

$$
c_2 \equiv c_{21}k_{1\alpha}k_{1\beta} + c_{22}k_{1\alpha}k_{2\beta} + c_{23}k_{2\alpha}k_{1\beta} + c_{24}k_{2\alpha}k_{2\beta},
\tag{B.1.78}
$$

$$
c_3 \equiv c_{31}k_{1\alpha}k_{1\beta} + c_{32}k_{1\alpha}k_{2\beta} + c_{33}k_{2\alpha}k_{1\beta} + c_{34}k_{2\alpha}k_{2\beta},
\tag{B.1.79}
$$

$$
c_4 \equiv c_{41}k_{1\alpha}k_{1\beta} + c_{42}k_{1\alpha}k_{2\beta} + c_{43}k_{2\alpha}k_{2\beta}.
\tag{B.1.80}
$$

The coefficients are given as

$$c_{11} = 6\frac{m_g^4 - 4m_g^2 m_q^2 \log\left(\frac{m_g}{m_q}\right) - m_q^4}{\left(m_g^2 - m_q^2\right)^3}, \tag{B.1.81}$$

$$c_{21} = 6\frac{5m_g^6 - 27m_g^4 m_q^2 + 27m_g^2 m_q^4 + 12m_g^4\left(m_g^2 - 3m_q^2\right)\log\left(\frac{m_q}{m_g}\right) - 5m_q^6}{18\left(m_g^2 - m_q^2\right)^4}, \tag{B.1.82}$$

$$c_{22} = 6\frac{7m_g^6 - 27m_g^2 m_q^4 + 12\left(m_g^6 - 6m_g^4 m_q^2 + 6m_g^2 m_q^4\right)\log\left(\frac{m_g}{m_q}\right) + 20m_q^6}{18\left(m_g^2 - m_q^2\right)^4}, \tag{B.1.83}$$

$$c_{23} = 6\frac{12m_g^6\log\left(\frac{m_g}{m_q}\right) - 11m_g^6 + 18m_g^4 m_q^2 - 9m_g^2 m_q^4 + 2m_q^6}{18\left(m_g^2 - m_q^2\right)^4}, \tag{B.1.84}$$

$$c_{24} = 6\frac{12m_g^6\log\left(\frac{m_q}{m_g}\right) + 11m_g^6 - 18m_g^4 m_q^2 + 9m_g^2 m_q^4 - 2m_q^6}{18\left(m_g^2 - m_q^2\right)^4}, \tag{B.1.85}$$

$$c_{31} = \frac{1}{3}\left(\frac{5m_g^6 - 27m_g^4 m_q^2 + 27m_g^2 m_q^4 + 12\left(m_g^6 - 3m_g^4 m_q^2\right)\log\left(\frac{m_q}{m_g}\right) - 5m_q^6}{\left(m_g^2 - m_q^2\right)^4}\right.$$
$$\left. - \frac{6\left(2m_5^6 - 9m_5^4 m_q^2 + 18m_5^2 m_q^4 + 12m_q^6\log\left(\frac{m_q}{m_5}\right) - 11m_q^6\right)}{\left(m_5^2 - m_q^2\right)^4}\right), \tag{B.1.86}$$

$$c_{32} = \frac{1}{3}\left(\frac{3\left(7m_5^6 - 36m_5^4 m_q^2 + 45m_5^2 m_q^4 + 12\left(3m_5^2 m_q^4 - 2m_q^6\right)\log\left(\frac{m_5}{m_q}\right) - 16m_q^6\right)}{\left(m_5^2 - m_q^2\right)^4}\right.$$
$$\left. + \frac{7m_g^6 - 27m_g^2 m_q^4 + 12\left(m_g^6 - 6m_g^4 m_q^2 + 6m_g^2 m_q^4\right)\log\left(\frac{m_g}{m_q}\right) + 20m_q^6}{\left(m_g^2 - m_q^2\right)^4}\right), \tag{B.1.87}$$

$$c_{33} = \frac{1}{3}\left(\frac{12m_g^6\log\left(\frac{m_g}{m_q}\right) - 11m_g^6 + 18m_g^4 m_q^2 - 9m_g^2 m_q^4 + 2m_q^6}{\left(m_g^2 - m_q^2\right)^4}\right.$$
$$\left. - \frac{3\left(2m_5^6 - 9m_5^4 m_q^2 + 18m_5^2 m_q^4 + 12m_q^6\log\left(\frac{m_q}{m_5}\right) - 11m_q^6\right)}{\left(m_5^2 - m_q^2\right)^4}\right), \tag{B.1.88}$$

$$c_{34} = \frac{1}{3}\left(\frac{12m_g^6\log\left(\frac{m_q}{m_g}\right) + 11m_g^6 - 18m_g^4 m_q^2 + 9m_g^2 m_q^4 - 2m_q^6}{\left(m_g^2 - m_q^2\right)^4}\right.$$

$$-\frac{3\left(-5m_5^6+27m_5^4m_q^2-27m_5^2m_q^4+12m_q^4\left(m_q^2-3m_5^2\right)\log\left(\frac{m_5}{m_q}\right)+5m_q^6\right)}{\left(m_5^2-m_q^2\right)^4}\Bigg), \tag{B.1.89}$$

$$c_{41}=\frac{18m_gm_q}{\left(m_g^2-m_5^2\right)^3(m_5-m_q)(m_5+m_q)(m_g-m_q)^3(m_g+m_q)^3}$$
$$\Big[4m_g^2m_q^2\left(m_5^2-m_g^2\right)^3\log(m_q)+4m_5^2m_g^2\log(m_5)\left(m_g^2-m_q^2\right)^3$$
$$+\left(m_5^2-m_q^2\right)\left((m_5^2-m_g^2)(m_g^2-m_q^2)\left(m_5^2\left(m_g^2+m_q^2\right)-3m_g^4+m_g^2m_q^2\right)\right.$$
$$\left.-4m_g^2\log(m_g)\left(-3m_5^2m_g^2m_q^2+m_5^2m_q^2\left(m_5^2+m_q^2\right)+m_g^6\right)\right)\Big]$$
$$+\frac{5m_5^6-27m_5^4m_q^2+27m_5^2m_q^4-12m_q^4\left(m_q^2-3m_5^2\right)\log\left(\frac{m_5}{m_q}\right)-5m_q^6}{2\left(m_5^2-m_q^2\right)^4}$$
$$+\frac{5\left(-5m_g^6+27m_g^4m_q^2-27m_g^2m_q^4+12\left(m_g^6-3m_g^4m_q^2\right)\log\left(\frac{m_g}{m_q}\right)+5m_q^6\right)}{3\left(m_g^2-m_q^2\right)^4}$$
$$-\frac{5m_g^6-27m_g^4m_q^2+27m_g^2m_q^4+12\left(m_g^6-3m_g^4m_q^2\right)\log\left(\frac{m_q}{m_g}\right)-5m_q^6}{3\left(m_g^2-m_q^2\right)^4}, \tag{B.1.90}$$

$$c_{42}=\frac{1}{3}\Bigg(-\frac{108m_gm_q}{\left(m_g^2-m_5^2\right)^3(m_5-m_q)^2(m_5+m_q)^2(m_g-m_q)^2(m_g+m_q)^2}$$
$$\Big[-2m_q^4\left(m_5^2-m_g^2\right)^3\log(m_q)+\left(m_q^2-m_5^2\right)\left((m_5^2-m_g^2)(m_g^2-m_q^2)\left(2m_5^2m_g^2-m_q^2\left(m_5^2+m_g^2\right)\right)\right.$$
$$\left.+2m_g^2(m_5-m_q)(m_5+m_q)\log(m_g)\left(m_5^2\left(m_g^2-2m_q^2\right)+m_g^4\right)\right)$$
$$+2m_5^2\log(m_5)\left(m_g^2-m_q^2\right)^2\left(m_5^4+m_5^2m_g^2-2m_g^2m_q^2\right)\Big]$$
$$+\frac{3\left(-2m_5^6+9m_5^4m_q^2-18m_5^2m_q^4+12m_q^6\log\left(\frac{m_5}{m_q}\right)+11m_q^6\right)}{\left(m_5^2-m_q^2\right)^4}$$
$$+\frac{-12m_g^6\log\left(\frac{m_g}{m_q}\right)+11m_g^6-18m_g^4m_q^2+9m_g^2m_q^4-2m_q^6}{\left(m_g^2-m_q^2\right)^4}$$
$$-\frac{7m_g^6-27m_g^2m_q^4+12\left(m_g^6-6m_g^4m_q^2+6m_g^2m_q^4\right)\log\left(\frac{m_g}{m_q}\right)+20m_q^6}{\left(m_g^2-m_q^2\right)^4}$$
$$+\frac{10\left(12m_g^6\log\left(\frac{m_q}{m_g}\right)+11m_g^6-18m_g^4m_q^2+9m_g^2m_q^4-2m_q^6\right)}{\left(m_g^2-m_q^2\right)^4}\Bigg), \tag{B.1.91}$$

$$c_{43} = -\frac{18m_g m_q}{\left(m_g^2 - m_5^2\right)^3 (m_5 - m_q)^3 (m_5 + m_q)^3 (m_g - m_q)(m_g + m_q)}$$
$$\Big(4m_5^2 \Big(m_q^2 \left(m_g^2 - m_5^2\right)^3 \log(m_q) + m_g^2 \left(m_5^2 - m_q^2\right)^3 \log(m_g)$$
$$+ \log(m_5)(m_q^2 - m_g^2) \left(m_5^6 - 3m_5^2 m_g^2 m_q^2 + m_g^2 m_q^2 \left(m_g^2 + m_q^2\right)\right)\Big)$$
$$+(m_5^2 - m_g^2)(m_5^2 - m_q^2)(m_g^2 - m_q^2) \left(3m_5^4 - m_5^2 \left(m_g^2 + m_q^2\right) - m_g^2 m_q^2\right)\Big)$$

$$+ \frac{5m_5^6 - 27m_5^4 m_q^2 + 27m_5^2 m_q^4 - 12m_q^4 \left(m_q^2 - 3m_5^2\right) \log\left(\frac{m_5}{m_q}\right) - 5m_q^6}{2\left(m_5^2 - m_q^2\right)^4}$$
$$+ \frac{5\left(-5m_g^6 + 27m_g^4 m_q^2 - 27m_g^2 m_q^4 + 12\left(m_g^6 - 3m_g^4 m_q^2\right) \log\left(\frac{m_g}{m_q}\right) + 5m_q^6\right)}{3\left(m_g^2 - m_q^2\right)^4}$$
$$+ \frac{-12m_g^6 \log\left(\frac{m_q}{m_g}\right) - 11m_g^6 + 18m_g^4 m_q^2 - 9m_g^2 m_q^4 + 2m_q^6}{3\left(m_g^2 - m_q^2\right)^4}, \tag{B.1.92}$$

Coefficients in $\mathcal{O}_{qqGG}$

The q-q-G-G operator is written as

$$\int d^4x \mathcal{O}_{qqGG}(x) = \frac{g_s^4}{192\pi^2} \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \frac{d^4k_3}{(2\pi)^4} \frac{d^4k_4}{(2\pi)^4} (2\pi)^4 \delta^4(-k_1 + k_2 + k_3 + k_4)$$
$$\overline{q}(k_1) \left[F_{L,R}^{\mu\nu} \delta^{ab} + H_{L,R}^{\mu\nu} T^a T^b\right] G_\mu^a(k_2) G_\nu^b(k_3) P_{L,R} q(k_4), \tag{B.1.93}$$

where $E_i^\mu,\ F_i^{\mu\nu},\ H_i^{\mu\nu}$, $(i = L,\ R)$ are

$$F_i^{\mu\nu} = f_{1i\alpha} i\epsilon^{\alpha\mu\nu\beta}\gamma_5\gamma_\beta + f_{2i\alpha} g^{\mu\nu}\gamma^\alpha + f_{3i\alpha} g^{\alpha\mu}\gamma^\nu + f_{4i\alpha} g^{\alpha\nu}\gamma^\mu, \tag{B.1.94}$$
$$H_i^{\mu\nu} = h_{1i\alpha} i\epsilon^{\alpha\mu\nu\beta}\gamma_5\gamma_\beta + h_{2i\alpha} g^{\mu\nu}\gamma^\alpha + h_{3i\alpha} g^{\alpha\mu}\gamma^\nu + h_{4i\alpha} g^{\alpha\nu}\gamma^\mu. \tag{B.1.95}$$

The coefficients of color singlet part, $F_i^{\mu\nu}$, are given as

$$f_{1\alpha} = \Big\{3[g_1(m_g, m_q) + g_4(m_g, m_q) + g_7(m_g, m_q)] + h_1(m_g, m_q) - h_4(m_g, m_q) - h_7(m_g, m_q)$$
$$- \frac{1}{2}[g_1(m_5, m_q) + g_4(m_5, m_q) + g_7(m_5, m_q) - h_1(m_5, m_q) + h_4(m_5, m_q) - h_7(m_5, m_q)]$$
$$+ \frac{1}{4}[i_1(m_g, m_q) - 2j_1(m_g, m_q, m_5) + 2s_1(m_g, m_q)]$$
$$+\frac{1}{2}[-s_8(m_g, m_q, m_5) + s_{12}(m_g, m_q, m_5)]\Big\} k_{1\alpha}$$

$$
\begin{aligned}
&+\{3[g_2(m_g,m_q)+g_5(m_g,m_q)+g_8(m_g,m_q)]+h_2(m_g,m_q)-h_5(m_g,m_q)-h_8(m_g,m_q)\\
&-\frac{1}{2}[g_2(m_5,m_q)+g_5(m_5,m_q)+g_8(m_5,m_q)-h_2(m_5,m_q)+h_5(m_5,m_q)-h_8(m_5,m_q)]\\
&+\frac{1}{4}[i_2(m_g,m_q)+2j_2(m_g,m_q,m_5)-2s_1(m_g,m_q)+n_1(m_g,m_q)+2n_{11}(m_g,m_q)]\\
&-\frac{1}{2}[-s_8(m_g,m_q,m_5)+s_{12}(m_g,m_q,m_5)]\Big\}k_{3\alpha}\\
&+\{3[g_3(m_g,m_q)+g_6(m_g,m_q)+g_9(m_g,m_q)]+h_3(m_g,m_q)-h_6(m_g,m_q)-h_9(m_g,m_q)\\
&-\frac{1}{2}[g_3(m_5,m_q)+g_6(m_5,m_q)+g_9(m_5,m_q)-h_3(m_5,m_q)+h_6(m_5,m_q)-h_9(m_5,m_q)]\\
&+\frac{1}{4}[i_3(m_g,m_q)+2j_3(m_g,m_q,m_5)-2s_1(m_g,m_q)\\
&-\frac{1}{2}[-s_8(m_g,m_q,m_5)+s_{12}(m_g,m_q,m_5)]\Big\}k_{4\alpha},
\end{aligned}
\tag{B.1.96}
$$

$$
\begin{aligned}
f_{2\alpha}=&\{-3g_1(m_g,m_q)+3g_4(m_g,m_q)+2g_7(m_g,m_q)+h_1(m_g,m_q)-h_4(m_g,m_q)-h_7(m_g,m_q)\\
&-\frac{1}{2}[g_1(m_5,m_q)-g_4(m_5,m_q)+g_7(m_5,m_q)-h_1(m_5,m_q)+h_4(m_5,m_q)-h_7(m_5,m_q)]\\
&+\frac{1}{4}[i_4(m_g,m_q)-2j_1(m_g,m_q,m_5)-2j_1(m_5,m_q,m_g)]\\
&-3[e_1(m_g,m_q,m_5)-e_1(m_5,m_q,m_g)]\\
&+\frac{1}{4}[n_2(m_g,m_q)+2n_{12}(m_g,m_q)+l_1(m_q,m_5)-2s_2(m_g,m_q)]\\
&+\frac{1}{2}[r_1(m_g,m_q,m_5)+r_{10}(m_g,m_q,m_5)-s_8(m_g,m_q,m_5)+s_{12}(m_g,m_q,m_5)]\Big\}k_{1\alpha}\\
&+\{-3g_2(m_g,m_q)+3g_5(m_g,m_q)+2g_8(m_g,m_q)+h2(m_g,m_q)-h5(m_g,m_q)-h_8(m_g,m_q)\\
&-\frac{1}{2}[g_2(m_5,m_q)-g_5(m_5,m_q)+g_8(m_5,m_q)-h_2(m_5,m_q)+h_5(m_5,m_q)-h_8(m_5,m_q)]\\
&+\frac{1}{4}[i_5(m_5,m_q)-2j_2(m_g,m_q,m_5)-2j_2(m_5,m_q,m_g)\\
&+n_3(m_g,m_q)+2n_{13}(m_g,m_q)+l_2(m_q,m_5)-2s_3(m_g,m_q)]\\
&-\frac{1}{2}[-r_2(m_g,m_q,m_5)-r_{11}(m_g,m_q,m_5)-s_8(m_g,m_q,m_5)+s_{12}(m_g,m_q,m_5)]\Big\}k_{3\alpha}\\
&+\{-3g_3(m_g,m_q)+3g_6(m_g,m_q)+2g_9(m_g,m_q)+h3(m_g,m_q)-h6(m_g,m_q)-h_9(m_g,m_q)\\
&-\frac{1}{2}[g_3(m_5,m_q)-g_6(m_5,m_q)+g_9(m_5,m_q)-h_3(m_5,m_q)+h_6(m_5,m_q)-h_9(m_5,m_q)]\\
&+\frac{1}{4}[i_6(m_5,m_q)-2j_3(m_g,m_q,m_5)-2j_3(m_5,m_q,m_g)\\
&-3[e_2(m_g,m_q,m_5)-e_2(m_5,m_q,m_g)]\\
&+\frac{1}{4}[n_4(m_g,m_q)+2n_{14}(m_g,m_q)+l_3(m_q,m_5)-2s_4(m_g,m_q)]\\
&-\frac{1}{2}[-r_3(m_g,m_q,m_5)-r_{12}(m_g,m_q,m_5)-s_8(m_g,m_q,m_5)+s_{12}(m_g,m_q,m_5)]\Big\}k_{4\alpha},
\end{aligned}
\tag{B.1.97}
$$

$$
\begin{aligned}
f_{3\alpha} = \Bigg\{ & \frac{1}{4}[g_1(m_g,m_q) + 4g_4(m_g,m_q) - 8g_7(m_g,m_q) + 4(h_1(m_g,m_q) + h_4(m_g,m_q) + h_7(m_g,m_q))] \\
& - \frac{1}{2}[g_1(m_5,m_q) + g_4(m_5,m_q) - g_7(m_5,m_q) - h_1(m_5,m_q) - h_4(m_5,m_q) + h_7(m_5,m_q)] \\
& + \frac{1}{4}(i_7(m_5,m_q) - 2j_1(m_g,m_q,m_5)) - 3e_1(m_g,m_q,m_5) \\
& + \frac{1}{4}(n_5(m_g,m_q) + 2n_{12}(m_g,m_q)) + \frac{1}{4}[l_4(m_q,m_5) - 2s_2(m_g,m_q)] \\
& - \frac{1}{2}[-t_1(m_g,m_q,m_5) - r_4(m_g,m_q,m_5) - r_{13}(m_g,m_q,m_5) \\
& + s_9(m_g,m_q,m_5) - s_{13}(m_g,m_q,m_5)]\Bigg\}\, k_{1\alpha} \\
& + \Bigg\{\frac{1}{4}[g_2(m_g,m_q) + 4g_5(m_g,m_q) - 8g_8(m_g,m_q) + 4(h_2(m_g,m_q) + h_5(m_g,m_q) + h_8(m_g,m_q))] \\
& - \frac{1}{2}[g_2(m_5,m_q) + g_5(m_5,m_q) - g_8(m_5,m_q) - h_2(m_5,m_q) - h_5(m_5,m_q) + h_8(m_5,m_q)] \\
& + \frac{1}{4}[i_8(m_5,m_q) + 2j_2(m_g,m_q,m_5) + n_6(m_g,m_q) + 2n_{13}(m_g,m_q)] \\
& + \frac{1}{4}l_5(m_q,m_5) - \frac{1}{2}[-s_3(m_g,m_q) - t_2(m_g,m_q,m_5) - r_5(m_g,m_q,m_5) - r_{14}(m_g,m_q,m_5) \\
& + s_{10}(m_g,m_q,m_5) - s_{14}(m_g,m_q,m_5)]\Bigg\}\, k_{3\alpha} \\
& + \Bigg\{\frac{1}{4}[g_3(m_g,m_q) + 4g_6(m_g,m_q) - 8g_9(m_g,m_q) + 4(h_3(m_g,m_q) + h_6(m_g,m_q) + h_9(m_g,m_q))] \\
& - \frac{1}{2}(g_3(m_5,m_q) + g_6(m_5,m_q) - g_9(m_5,m_q) - h_3(m_5,m_q) - h_6(m_5,m_q) + h_9(m_5,m_q)) \\
& + \frac{1}{4}[i_9(m_5,m_q) + 2j_3(m_g,m_q,m_5)] - 3e_2(m_5,m_q,m_g) + \frac{1}{4}[n_7(m_g,m_q) + 2n_{14}(m_g,m_q)] \\
& + \frac{1}{4}l_6(m_q,m_5) - \frac{1}{2}[-s_4(m_g,m_q,m_5) - t_3(m_g,m_q,m_5) - r_6(m_g,m_q,m_5) - r_{15}(m_g,m_q,m_5) \\
& + s_{11}(m_g,m_q,m_5) - s_{15}(m_g,m_q,m_5)]\Bigg\}\, k_{4\alpha} \qquad \text{(B.1.98)}
\end{aligned}
$$

$$
\begin{aligned}
f_{4\alpha}, = \Bigg\{ & \frac{1}{4}[12g_1(m_g,m_q) - 12g_4(m_g,m_q) + g_7(m_g,m_q) \\
& + 4(-h_1(m_g,m_q) + h_4(m_g,m_q) + h_7(m_g,m_q))] \\
& - \frac{1}{2}(-g_1(m_5,m_q) + g_4(m_5,m_q) + g_7(m_5,m_q) + h_1(m_5,m_q) - h_4(m_5,m_q) - h_7(m_5,m_q)) \\
& + \frac{1}{4}[i_{10}(m_5,m_q) - 2j_1(m_g,m_q,m_5)] - 3e_1(m_5,m_q,m_g) + \frac{1}{4}[n_8(m_g,m_q) + l_7(m_q,m_5)] \\
& + \frac{1}{2}[s_5(m_g,m_q) + s_2(m_g,m_q)] - \frac{1}{2}[-t_4(m_g,m_q,m_5) - r_7(m_g,m_q,m_5) - r_{16}(m_g,m_q,m_5) \\
& - s_8(m_g,m_q,m_5) + s_{12}(m_g,m_q,m_5)]\Bigg\}\, k_{1\alpha}
\end{aligned}
$$

$$
\begin{aligned}
&+\left\{\frac{1}{4}[12g_2(m_g,m_q)-12g_5(m_g,m_q)+g_8(m_g,m_q)\right.\\
&+4(-h_2(m_g,m_q)+h_5(m_g,m_q)+h_8(m_g,m_q))]\\
&-\frac{1}{2}(-g_2(m_5,m_q)+g_5(m_5,m_q)+g_8(m_5,m_q)+h_2(m_5,m_q)-h_5(m_5,m_q)-h_8(m_5,m_q))\\
&+\frac{1}{4}[i_{11}(m_5,m_q)+2j_2(m_g,m_q,m_5)+n_9(m_g,m_q)+l_8(m_q,m_5)]\\
&+\frac{1}{2}[s_6(m_g,m_q)+s_3(m_g,m_q)]-\frac{1}{2}[-t_5(m_g,m_q,m_5)-r_8(m_g,m_q,m_5)-r_{17}(m_g,m_q,m_5)\\
&\left.+s_8(m_g,m_q,m_5)-s_{12}(m_g,m_q,m_5)]\right\}k_{3\alpha}\\
&+\left\{\frac{1}{4}[12g_3(m_g,m_q)-12g_6(m_g,m_q)+g_9(m_g,m_q)\right.\\
&+4(-h_3(m_g,m_q)+h_6(m_g,m_q)+h_9(m_g,m_q))]\\
&-\frac{1}{2}(-g_3(m_5,m_q)+g_6(m_5,m_q)+g_9(m_5,m_q)+h_3(m_5,m_q)-h_6(m_5,m_q)-h_9(m_5,m_q))\\
&+\frac{1}{4}[i_{12}(m_5,m_q)+2j_3(m_g,m_q,m_5)]-3e_2(m_5,m_q,m_g)+\frac{1}{4}[n_{10}(m_g,m_q)+l_9(m_q,m_5)]\\
&+\frac{1}{2}[s_7(m_g,m_q)-s_4(m_g,m_q)]-\frac{1}{2}[-t_6(m_g,m_q)-r_9(m_g,m_q,m_5)-r_{18}(m_g,m_q,m_5)\\
&\left.+s_8(m_g,m_q,m_5)-s_{12}(m_g,m_q,m_5)]\right\}k_{4\alpha}. \qquad \text{(B.1.99)}
\end{aligned}
$$

The coefficients of color octet part, $H_i^{\mu\nu}$, are given as

$$
\begin{aligned}
h_{1\alpha}=&\left\{-2(g_1(m_g,m_q)+g_4(m_g,m_q)+g_7(m_g,m_q))-\frac{2}{3}(h_1(m_g,m_q)-h_4(m_g,m_q)-h_7(m_g,m_q))\right.\\
&+\frac{1}{3}(g_1(m_5,m_q)+g_4(m_5,m_q)+g_7(m_5,m_q)-h_1(m_5,m_q)+h_4(m_5,m_q)-h_7(m_5,m_q))\\
&\left.+\frac{3}{2}[i_1(m_5,m_q)-2j_1(m_g,m_q,m_5)]-18e_1(m_5,m_q,m_g)\right\}k_{1\alpha}\\
&+\left\{-2(g_2(m_g,m_q)+g_5(m_g,m_q)+g_8(m_g,m_q))-\frac{2}{3}(h_2(m_g,m_q)-h_5(m_g,m_q)-h_8(m_g,m_q))\right.\\
&+\frac{1}{3}(g_2(m_5,m_q)+g_5(m_5,m_q)+g_8(m_5,m_q)-h_2(m_5,m_q)+h_5(m_5,m_q)-h_8(m_5,m_q))\\
&\left.+\frac{3}{2}[i_2(m_5,m_q)+2j_2(m_g,m_q,m_5)]\right\}k_{3\alpha}\\
&+\left\{-2(g_3(m_g,m_q)+g_6(m_g,m_q)+g_9(m_g,m_q))-\frac{2}{3}(h_3(m_g,m_q)-h_6(m_g,m_q)-h_9(m_g,m_q))\right.\\
&+\frac{1}{3}(g_3(m_5,m_q)+g_6(m_5,m_q)+g_9(m_5,m_q)-h_3(m_5,m_q)+h_6(m_5,m_q)-h_9(m_5,m_q))\\
&\left.+\frac{3}{2}[i_3(m_5,m_q)-2j_3(m_g,m_q,m_5)]-18e_2(m_5,m_q,m_g)\right\}k_{4\alpha}, \qquad \text{(B.1.100)}
\end{aligned}
$$

$$
\begin{aligned}
h_{2\alpha} = &-\Big\{2g_1(m_g,m_q) - 2g_4(m_g,m_q) - \frac{4}{3}g_7(m_g,m_q) - \frac{2}{3}(h_1(m_g,m_q) - h_4(m_g,m_q) - h_7(m_g,m_q)) \\
&+ \frac{1}{3}(g_1(m_5,m_q) - g_4(m_5,m_q) + g_7(m_5,m_q) - h_1(m_5,m_q) + h_4(m_5,m_q) - h_7(m_5,m_q)) \\
&+ \frac{3}{2}[i_4(m_5,m_q) - 2j_1(m_g,m_q,m_5) - 2j_1(m_5,m_q,m_g)] - 18[e_1(m_g,m_q,m_5) - e_1(m_5,m_q,m_g)] \\
&+ \frac{3}{2}[l_1(m_q,m_5) - 2r_1(m_g,m_q,m_5) - 2r_{10}(m_g,m_q,m_5)]\Big\}\, k_{1\alpha} \\
&- \Big\{2g_2(m_g,m_q) - 2g_5(m_g,m_q) - \frac{4}{3}g_8(m_g,m_q) - \frac{2}{3}(h_2(m_g,m_q) - h_5(m_g,m_q) - h_8(m_g,m_q)) \\
&+ \frac{1}{3}(g_2(m_5,m_q) - g_5(m_5,m_q) + g_8(m_5,m_q) - h_2(m_5,m_q) + h_5(m_5,m_q) - h_8(m_5,m_q)) \\
&+ \frac{3}{2}[i_5(m_5,m_q) - 2j_2(m_g,m_q,m_5) - 2j_2(m_5,m_q,m_g)] \\
&- 3[r_2(m_g,m_q,m_5) + r_{11}(m_g,m_q,m_5)]\}\, k_{3\alpha} \\
&- \Big\{2g_3(m_g,m_q) - 2g_6(m_g,m_q) - \frac{4}{3}g_9(m_g,m_q) - \frac{2}{3}(h_2(m_g,m_q) - h_5(m_g,m_q) - h_8(m_g,m_q)) \\
&+ \frac{1}{3}(g_3(m_5,m_q) - g_6(m_5,m_q) + g_9(m_5,m_q) - h_3(m_5,m_q) + h_6(m_5,m_q) - h_9(m_5,m_q)) \\
&+ \frac{3}{2}[i_6(m_5,m_q) - 2j_3(m_g,m_q,m_5) - 2j_3(m_5,m_q,m_g)] - 18[e_3(m_g,m_q,m_5) - e_3(m_5,m_q,m_g)] \\
&+ \frac{3}{2}[l_3(m_q,m_5) - 2r_3(m_g,m_q,m_5) - 2r_{12}(m_g,m_q,m_5)]\Big\}\, k_{4\alpha}, \qquad \text{(B.1.101)}
\end{aligned}
$$

$$
\begin{aligned}
h_{3\alpha} = &-\frac{1}{6}(g_1(m_g,m_q) + 4g_4(m_g,m_q) - 8g_7(m_g,m_q) + 4(h_1(m_g,m_q) + h_4(m_g,m_q) + h_7(m_g,m_q))) \\
&+ \Big\{\frac{1}{3}g_1(m_5,m_q) + g_4(m_5,m_q) - g_7(m_5,m_q) - h_1(m_5,m_q) - h_4(m_5,m_q) + h_7(m_5,m_q)) \\
&+ \frac{3}{2}[i_7(m_5,m_q) - 2j_1(m_g,m_q,m_5)] - 18e_1(m_g,m_q,m_5) + \frac{3}{2}l_4(m_q,m_5) \\
&-3(t_1(m_5,m_q) + r_4(m_g,m_q,m_5) + r_{13}(m_g,m_q,m_5))\}\, k_{1\alpha} \\
&- \frac{1}{6}(g_2(m_g,m_q) + 4g_5(m_g,m_q) - 8g_8(m_g,m_q) + 4(h_2(m_g,m_q) + h_5(m_g,m_q) + h_8(m_g,m_q))) \\
&+ \Big\{\frac{1}{3}g_2(m_5,m_q) + g_5(m_5,m_q) - g_8(m_5,m_q) - h_2(m_5,m_q) - h_5(m_5,m_q) + h_8(m_5,m_q)) \\
&+ \frac{3}{2}[i_8(m_5,m_q) + 2j_2(m_g,m_q,m_5)] + \frac{3}{2}l_5(m_q,m_5) \\
&-3(t_2(m_5,m_q) + r_5(m_g,m_q,m_5) + r_{14}(m_g,m_q,m_5))\}\, k_{3\alpha} \\
&- \frac{1}{6}(g_3(m_g,m_q) + 4g_6(m_g,m_q) - 8g_9(m_g,m_q) + 4(h_3(m_g,m_q) + h_6(m_g,m_q) + h_9(m_g,m_q))) \\
&+ \Big\{\frac{1}{3}g_3(m_5,m_q) + g_6(m_5,m_q) - g_9(m_5,m_q) - h_4(m_5,m_q) - h_6(m_5,m_q) + h_9(m_5,m_q)) \\
&+ \frac{3}{2}[i_9(m_5,m_q) - 2j_3(m_g,m_q,m_5)] - 18e_3(m_g,m_q,m_5) + \frac{3}{2}l_6(m_q,m_5) \\
&-3(t_3(m_5,m_q) + r_6(m_g,m_q,m_5) + r_{15}(m_g,m_q,m_5))\}\, k_{4\alpha}, \qquad \text{(B.1.102)}
\end{aligned}
$$

$$
\begin{aligned}
h_{4\alpha} = &-\bigg\{\frac{1}{6}(12g_1(m_g,m_q) - 12g_4(m_g,m_q) + g_7(m_g,m_q) \\
&+ 4(-h_1(m_g,m_q) + h_4(m_g,m_q) + h_7(m_g,m_q))) \\
&+ \frac{1}{3}(-g_1(m_5,m_q) + g_4(m_5,m_q) + g_7(m_5,m_q) + h_1(m_5,m_q) - h_4(m_5,m_q) - h_7(m_5,m_q)) \\
&+ \frac{3}{2}[i_{10}(m_5,m_q) - 2j_1(m_g,m_q,m_5)] - 18e_1(m_g,m_q,m_5) + \frac{3}{2}l_7(m_q,m_5) \\
&- 3\left(t_4(m_5,m_q) + r_7(m_g,m_q,m_5) + r_{16}(m_g,m_q,m_5)\right)\bigg\}\, k_{1\alpha} \\
&- \bigg\{\frac{1}{6}(12g_2(m_g,m_q) - 12g_5(m_g,m_q) + g_8(m_g,m_q) \\
&+ 4(-h_2(m_g,m_q) + h_5(m_g,m_q) + h_8(m_g,m_q))) \\
&+ \frac{1}{3}(-g_2(m_5,m_q) + g_5(m_5,m_q) + g_8(m_5,m_q) + h_2(m_5,m_q) - h_5(m_5,m_q) - h_8(m_5,m_q)) \\
&+ \frac{3}{2}[i_{11}(m_5,m_q) - 2j_2(m_g,m_q,m_5)] + \frac{3}{2}l_8(m_q,m_5) \\
&- 3\left(t_5(m_5,m_q) + r_8(m_g,m_q,m_5) + r_{17}(m_g,m_q,m_5)\right)\bigg\}\, k_{3\alpha} \\
&- \bigg\{\frac{1}{6}(12g_3(m_g,m_q) - 12g_6(m_g,m_q) + g_9(m_g,m_q) \\
&+ 4(-h_3(m_g,m_q) + h_6(m_g,m_q) + h_9(m_g,m_q))) \\
&+ \frac{1}{3}(-g_3(m_5,m_q) + g_6(m_5,m_q) + g_9(m_5,m_q) + h_3(m_5,m_q) - h_6(m_5,m_q) - h_9(m_5,m_q)) \\
&+ \frac{3}{2}[i_{12}(m_5,m_q) - 2j_3(m_g,m_q,m_5)] - 18e_3(m_g,m_q,m_5) + \frac{3}{2}l_9(m_q,m_5) \\
&- 3\left(t_6(m_5,m_q) + r_9(m_g,m_q,m_5) + r_{18}(m_g,m_q,m_5)\right)\bigg\}\, k_{4\alpha}, \qquad \text{(B.1.103)}
\end{aligned}
$$

These coefficients, $g(m_a,m_b), h(m_a,m_b), \cdots$, are written in terms of Feynman integral as follows;

$$g_1(m_a,m_b) = -\alpha(m_b,m_a,1,3) + 3\alpha(m_b,m_a,1,2), \qquad \text{(B.1.104)}$$

$$
\begin{aligned}
g_2(m_a,m_b) &= g_8(m_a,m_b) = \frac{1}{2}g_3(m_a,m_b) = g_4(m_a,m_b) = g_7(m_a,m_b)) \\
&= -\alpha(m_b,m_a,1,3), \qquad \text{(B.1.105)}
\end{aligned}
$$

$$
\begin{aligned}
g_5(m_a,m_b) &= g_6(m_a,m_b) = g_9(m_a,m_b) \\
&= -\alpha(m_b,m_a,1,3) + 3\alpha(m_b,m_a,1,2), \qquad \text{(B.1.106)}
\end{aligned}
$$

$$
\begin{aligned}
h_1(m_a,m_b) &= h_2(m_a,m_b) = h_8(m_a,m_b)) \\
&= -m_b^2\alpha(m_b,m_a,2,3) + 3m_b^2\alpha(m_b,m_a,2,2), \qquad \text{(B.1.107)}
\end{aligned}
$$

$$h_3(m_a,m_b) = -2m_b^2\alpha(m_b,m_a,2,3), \qquad \text{(B.1.108)}$$

$$\begin{aligned} h_4(m_a,m_b) &= h_7(m_a,m_b) \\ &= -m_b^2\alpha(m_b,m_a,2,3), \end{aligned} \tag{B.1.109}$$

$$\begin{aligned} h_5(m_a,m_b) &= h_6(m_a,m_b) = h_9(m_a,m_b) \\ &= -m_b^2\alpha(m_b,m_a,2,3) + 3m_b^2\alpha(m_b,m_a,2,2), \end{aligned} \tag{B.1.110}$$

$$\begin{aligned} i_1(m_a,m_b) &= i_4(m_a,m_b) \\ &= 6\alpha(m_a,m_b,1,3) + f9\alpha(m_a,m_b,1,2), \end{aligned} \tag{B.1.111}$$

$$\begin{aligned} i_2(m_a,m_b) &= i_5(m_a,m_b) \\ &= 6\alpha(m_a,m_b,1,3) - 24\alpha(m_a,m_b,1,2), \end{aligned} \tag{B.1.112}$$

$$\begin{aligned} i_3(m_a,m_b) &= i_6(m_a,m_b) \\ &= 12\alpha(m_a,m_b,1,3) - 12\alpha(m_a,m_b,1,2), \end{aligned} \tag{B.1.113}$$

$$i_7(m_a,m_b) = -12\alpha(m_a,m_b,1,3) + 3\alpha(m_a,m_b,1,2), \tag{B.1.114}$$

$$i_8(m_a,m_b) = 12\alpha(m_a,m_b,1,3) + 6\alpha(m_a,m_b,1,2), \tag{B.1.115}$$

$$i_9(m_a,m_b) = -24\alpha(m_a,m_b,1,3), \tag{B.1.116}$$

$$i_{10}(m_a,m_b) = -35\alpha(m_a,m_b,1,3), \tag{B.1.117}$$

$$i_{11}(m_a,m_b) = -26\alpha(m_a,m_b,1,3) + 42\alpha(m_a,m_b,1,2), \tag{B.1.118}$$

$$i_{12}(m_a,m_b) = -52\alpha(m_a,m_b,1,3) + 66\alpha(m_a,m_b,1,2), \tag{B.1.119}$$

$$j_1(m_a,m_b,m_c) = 3m_b^2\beta\,(m_a,m_c,m_b,2,1,3), \tag{B.1.120}$$

$$j_2(m_a,m_b,m_c) = 6m_b^2\beta\,(m_a,m_c,m_b,2,1,1) - 6m_b^2\beta\,(m_a,m_c,m_b,2,2,0), \tag{B.1.121}$$

$$j_3(m_a,m_b,m_c) = 6m_b^2\beta\,(m_a,m_c,m_b,2,0,3) - 3m_b^2\beta\,(m_a,m_c,m_b,2,1,3), \tag{B.1.122}$$

$$\begin{aligned} e_1(m_a,m_b) &= e_2(m_a,m_b) \\ &= \alpha\,(a,b,1,2), \end{aligned} \tag{B.1.123}$$

$$n_1(m_a,m_b) = 6\alpha\,(m_a,m_b,1,1) - 6\alpha\,(m_a,m_b,1,2), \tag{B.1.124}$$

$$\begin{aligned} n_2(m_a,m_b) &= n_8(m_a,m_b) = n_4(m_a,m_b) = \frac{1}{2}n_7(m_a,m_b) = n_{10}(m_a,m_b) \\ &= 12\alpha\,(m_a,m_b,1,3) - 24\alpha\,(m_a,m_b,1,2) + 12\alpha\,(m_a,m_b,1,1), \end{aligned} \tag{B.1.125}$$

$$n_5(m_a,m_b) = -24\alpha\,(m_a,m_b,1,2) - 24\alpha\,(m_a,m_b,1,1) + 48\alpha\,(m_a,m_b,1,0), \tag{B.1.126}$$

$$n_6(m_a,m_b) = -24\alpha\,(m_a,m_b,1,3) + 84\alpha\,(m_a,m_b,1,2) - 27\alpha\,(a,b,1,1), \tag{B.1.127}$$

$$n_{11}(m_a,m_b) = 6m_b^2\alpha\,(m_a,m_b,2,2) - 6m_b^2\alpha\,(m_a,m_b,2,1), \tag{B.1.128}$$

$$n_{12}(m_a,m_b) = 6m_b^2\alpha\,(m_a,m_b,2,3) - 12m_b^2\alpha\,(m_a,m_b,2,2) + 6m_b^2\alpha\,(m_a,m_b,2,1), \tag{B.1.129}$$

$$n_{14}(m_a,m_b) = 6m_b^2\alpha\,(m_a,m_b,2,1) - 12m_b^2\alpha\,(m_a,m_b,2,1) + 6m_b^2\alpha\,(m_a,m_b,2,1), \tag{B.1.130}$$

$$\begin{aligned} t_1(m_a,m_b,m_c) &= t_2(m_a,m_b,m_c) \\ &= -12m_am_b\gamma\,(m_a,m_b,m_c,2,1,0,0) + 6m_am_b\gamma\,(m_a,m_b,m_c,2,0,0,0), \end{aligned} \tag{B.1.131}$$

$$\begin{aligned} t_3(m_a,m_b,m_c) &= 12m_am_b\gamma\,(m_a,m_b,m_c,2,1,0,0) - 12m_am_b\gamma\,(m_a,m_b,m_c,2,0,0,1) \\ &\quad +6m_am_b\gamma\,(m_a,m_b,m_c,2,0,0,0), \end{aligned} \tag{B.1.132}$$

$$t_4(m_a,m_b,m_c) = 12m_am_b\gamma\,(m_a,m_b,m_c,2,1,0,0), \tag{B.1.133}$$

$$\begin{aligned} t_5(m_a,m_b,m_c) &= 12m_am_b\gamma\,(m_c,m_b,m_a,2,0,1,0) - 12m_am_b\gamma\,(m_c,m_b,m_a,2,0,1,0) \\ &\quad +6m_am_b\gamma\,(m_c,m_b,m_a,2,0,0,0), \end{aligned} \tag{B.1.134}$$

$$\begin{aligned} t_6(m_a,m_b,m_c) &= 12m_am_b\gamma\,(m_c,m_b,m_a,2,1,0,0) - 12m_am_b\gamma\,(m_c,m_b,m_a,2,0,0,1) \\ &\quad +12m_am_b\gamma\,(m_c,m_b,m_a,2,0,0,0), \end{aligned} \tag{B.1.135}$$

$$\begin{aligned} l_1(m_b,m_c) &= l_2(m_b,m_c) = \frac{1}{2}l_3(m_b,m_c) = l_7(m_b,m_c) \\ &= 4\alpha(m_c,m_b,1,3), \end{aligned} \tag{B.1.136}$$

$$\begin{aligned} l_4(m_b,m_c) &= l_5(m_b,m_c) = l_8(m_b,m_c) \\ &= 4\alpha(m_c,m_b,1,3) - 6\alpha(m_c,m_b,1,2), \end{aligned} \tag{B.1.137}$$

$$l_6(m_b,m_c) = 8\alpha(m_c,m_b,1,3) - 6\alpha(m_c,m_b,1,2), \tag{B.1.138}$$

$$l_9(m_b,m_c) = 8\alpha(m_c,m_b,1,3) - 12\alpha(m_c,m_b,1,2), \tag{B.1.139}$$

$$\begin{aligned} r_1(m_a,m_b,m_c) &= r_4(m_a,m_b,m_c) = -\frac{1}{2}r_7(m_a,m_b,m_c) \\ &= -6m_am_b\delta\,(m_a,m_b,m_c,2,1,0,0), \end{aligned} \tag{B.1.140}$$

$$\begin{aligned} r_2(m_a,m_b,m_c) &= 6m_am_b\delta\,(m_a,m_b,m_c,2,0,1,0) - 6m_am_b\delta\,(m_a,m_b,m_c,2,0,0,1) \\ &\quad +6m_am_b\delta\,(m_a,m_b,m_c,2,0,0,0), \end{aligned} \tag{B.1.141}$$

$$\begin{aligned} r_3(m_a,m_b,m_c) &= 6m_am_b\delta\,(m_a,m_b,m_c,2,1,0,0) - 6m_am_b\delta\,(m_a,m_b,m_c,2,0,0,1) \\ &\quad +6m_am_b\delta\,(m_a,m_b,m_c,2,0,0,0), \end{aligned} \tag{B.1.142}$$

$$r_5(m_a,m_b,m_c)= 6m_am_b\delta\,(m_a,m_b,m_c,2,0,1,0)-6m_am_b\delta\,(m_a,m_b,m_c,2,0,0,1), \tag{B.1.143}$$

$$r_6(m_a,m_b,m_c)= 6m_am_b\delta\,(m_a,m_b,m_c,2,1,0,0)-6m_am_b\delta\,(m_a,m_b,m_c,2,0,0,1) +6m_am_b\delta\,(m_a,m_b,m_c,2,0,0,0), \tag{B.1.144}$$

$$r_8(m_a,m_b,m_c)= 12m_am_b\delta\,(m_a,m_b,m_c,2,0,1,0)-12m_am_b\delta\,(m_a,m_b,m_c,2,0,0,1) -6m_am_b\delta\,(m_a,m_b,m_c,2,0,0,0), \tag{B.1.145}$$

$$r_9(m_a,m_b,m_c)= 12m_am_b\delta\,(m_a,m_b,m_c,2,1,0,0)-12m_am_b\delta\,(m_a,m_b,m_c,2,0,0,1) -12m_am_b\delta\,(m_a,m_b,m_c,2,0,0,0), \tag{B.1.146}$$

$$r_{10}(m_a,m_b,m_c)= 6m_am_b\delta\,(m_a,m_b,m_c,2,1,0,0), \tag{B.1.147}$$

$$r_{11}(m_a,m_b,m_c)= 6m_am_b\delta\,(m_a,m_b,m_c,2,0,1,0)-6m_am_b\delta\,(m_a,m_b,m_c,2,0,0,1) +6m_am_b\delta\,(m_a,m_b,m_c,2,0,0,0), \tag{B.1.148}$$

$$r_{12}(m_a,m_b,m_c)= 6m_am_b\gamma\,(m_a,m_b,m_c,2,1,0,0)-6m_am_b\gamma\,(m_a,m_b,m_c,2,0,0,1) -6m_am_b\gamma\,(m_a,m_b,m_c,2,0,0,0), \tag{B.1.149}$$

$$r_{13}(m_a,m_b,m_c)= -12m_am_b\gamma\,(m_a,m_b,m_c,2,1,0,0)+6m_am_b\gamma\,(m_a,m_b,m_c,2,0,0,0), \tag{B.1.150}$$

$$r_{14}(m_a,m_b,m_c)= 12m_am_b\gamma\,(m_a,m_b,m_c,2,0,1,0)-12m_am_b\gamma\,(m_a,m_b,m_c,2,0,0,1) +6m_am_b\gamma\,(m_a,m_b,m_c,2,0,0,0), \tag{B.1.151}$$

$$r_{15}(m_a,m_b,m_c)= 12m_am_b\gamma\,(m_a,m_b,m_c,2,1,0,0)-12m_am_b\gamma\,(m_a,m_b,m_c,2,0,0,1) +6m_am_b\gamma\,(m_a,m_b,m_c,2,0,0,0), \tag{B.1.152}$$

$$r_{16}(m_a,m_b,m_c)= 6m_am_b\gamma\,(m_a,m_b,m_c,2,1,0,0)-6m_am_b\gamma\,(m_a,m_b,m_c,2,0,0,0), \tag{B.1.153}$$

$$r_{17}(m_a,m_b,m_c)= 6m_am_b\gamma\,(m_a,m_b,m_c,2,0,1,0)-6m_am_b\gamma\,(m_a,m_b,m_c,2,0,0,1), \tag{B.1.154}$$

$$r_{18}(m_a,m_b,m_c)= 6m_am_b\gamma\,(m_a,m_b,m_c,2,1,0,0)-6m_am_b\gamma\,(m_a,m_b,m_c,2,0,0,1), \tag{B.1.155}$$

$$s_1(m_b,m_c)= 6\alpha\,(m_c,m_b,1,1)+6\alpha\,(m_c,m_b,1,2), \tag{B.1.156}$$

$$s_2(m_b,m_c)= 18\alpha\,(m_c,m_b,1,3)-30\alpha\,(m_c,m_b,1,2)+6\alpha\,(m_c,m_b,1,1), \tag{B.1.157}$$

$$s_3(m_b,m_c)= -3\alpha\,(m_c,m_b,1,4)+2\alpha\,(m_c,m_b,1,3)+6\alpha\,(m_c,m_b,1,1)-5\alpha\,(m_c,m_b,1,0), \tag{B.1.158}$$

$$s_4(m_b, m_c) = 6\alpha\,(m_c, m_b, 1, 3) + 18\alpha\,(m_c, m_b, 1, 2) - 12\alpha\,(m_c, m_b, 1, 1), \tag{B.1.159}$$

$$\begin{aligned} s_5(m_b, m_c) =& \; 18m_b^2\alpha\,(m_c, m_b, 2, 3) - 36m_b^2\alpha\,(m_c, m_b, 2, 2) + 18m_b^2\alpha\,(m_c, m_b, 2, 1) \\ &+18m\alpha\,(m_c, m_b, 1, 3) - 36\alpha\,(m_c, m_b, 1, 2) + 18\alpha\,(m_c, m_b, 1, 1), \end{aligned} \tag{B.1.160}$$

$$\begin{aligned} s_6(m_b, m_c) =& -3m_b^2\alpha\,(m_c, m_b, 2, 4) + 2m_b^2\alpha\,(m_c, m_b, 2, 3) + 6m_b^2\alpha\,(m_c, m_b, 2, 1) \\ &-5m_b^2\alpha\,(m_c, m_b, 2, 0) - 3\alpha\,(m_c, m_b, 1, 4) + 2\alpha\,(m_c, m_b, 1, 3) \\ &+6\alpha\,(m_c, m_b, 1, 1) - 5\alpha\,(m_c, m_b, 1, 0), \end{aligned} \tag{B.1.161}$$

$$\begin{aligned} s_7(m_b, m_c) =& -6m_b^2\alpha\,(m_c, m_b, 2, 2) + 12m_b^2\alpha\,(m_c, m_b, 2, 1) - 6m_b^2\alpha\,(m_c, m_b, 2, 0) \\ &-6\alpha\,(m_c, m_b, 1, 2) + 12\alpha\,(m_c, m_b, 1, 1) - 6\alpha\,(m_c, m_b, 1, 0), \end{aligned} \tag{B.1.162}$$

$$s_8(m_a, m_b, m_c) = 6m_a m_b\beta\,(m_b, m_c, m_a, 2, 0, 0), \tag{B.1.163}$$

$$s_9(m_a, m_b, m_c) = -3m_a m_b\beta\,(m_b, m_c, m_a, 2, 0, 2) + 3m_a m_b\beta\,(m_b, m_c, m_a, 2, 0, 0), \tag{B.1.164}$$

$$\begin{aligned} s_{10}(m_a, m_b, m_c) =& -12m_a m_b\beta\,(m_b, m_c, m_a, 2, 0, 2) + 6m_a m_b\beta\,(m_b, m_c, m_a, 2, 0, 1) \\ &+12m_a m_b\beta\,(m_b, m_c, m_a, 2, 1, 1) - 12m_a m_b\beta\,(m_b, m_c, m_a, 2, 1, 0) \\ &+6m_a m_b\beta\,(m_b, m_c, m_a, 2, 0, 0), \end{aligned} \tag{B.1.165}$$

$$s_{11}(m_a, m_b, m_c) - 6m_a m_b\beta\,(m_b, m_c, m_a, 2, 0, 2) + 6m_a m_b\beta\,(m_b, m_c, m_a, 2, 0, 1), \tag{B.1.166}$$

$$s_{12}(m_a, m_b, m_c) = -6m_a m_b\beta\,(m_b, m_a, m_c, 2, 0, 1) + 6m_a m_b\beta\,(m_b, m_a, m_c, 2, 0, 0), \tag{B.1.167}$$

$$s_{13}(m_a, m_b, m_c) = -3m_a m_b\beta\,(m_b, m_a, m_c, 2, 0, 2) + 3m_a m_b\beta\,(m_b, m_a, m_c, 2, 0, 0), \tag{B.1.168}$$

$$\begin{aligned} s_{14}(m_a, m_b, m_c) =& -12m_a m_b\beta\,(m_b, m_a, m_c, 2, 0, 2) + 6m_a m_b\beta\,(m_b, m_a, m_c, 2, 0, 1) \\ &+12m_a m_b\beta\,(m_b, m_a, m_c, 2, 1, 1) - 12m_a m_b\beta\,(m_b, m_a, m_c, 2, 1, 0) \\ &+6m_a m_b\beta\,(m_b, m_a, m_c, 2, 0, 0), \end{aligned} \tag{B.1.169}$$

$$s_{15}(m_a, m_b, m_c) = -6m_a m_b\beta\,(m_b, m_a, m_c, 2, 0, 2) + 5m_a m_b\beta\,(m_b, m_a, m_c, 2, 0, 1), \tag{B.1.170}$$

and other coefficients are zero.

Appendix C

Two-body state effective action

Here we derive the effective action of heavy $q\bar{q}$-system in NRQCD, Eq.(4.1.2). At leading order in perturbation theory, we can write an effective QCD Lagrangian as

$$S = \int_x [\bar{q}(i\partial\!\!\!/ - m)q] + (-i)\int_x \int_y j^\dagger_\mu(x)\mathcal{D}^{\mu\nu}(x-y)j_\nu(y), \tag{C.0.1}$$

where $\mathcal{D}^{\mu\nu}(x-y)$ is gluon propagator. In non-relativistic limit, the gluon propagator induces a (gluon) potential as

$$\begin{aligned}\mathcal{D}^{\mu\nu}(x-y) &= \int \frac{d^4p}{(2\pi)^4}\frac{-ig_s^2 g^{\mu\nu}}{p_0^2 - |\vec{p}|^2}e^{-ip\cdot(x-y)} \\ &\simeq \delta(x^0-y^0)\frac{ig_s^2 g^{00}}{4\pi|\vec{x}-\vec{y}|} \equiv i\delta(x^0-y^0)\frac{V(r)}{C_F},\end{aligned} \tag{C.0.2}$$

where $r = |\vec{x}-\vec{y}|$ and $V(r) \equiv C_F g_s^2/(4\pi r)$. This is the "Coulomb" potential when energy level of $q\bar{q}$-system is low (for example, S-state in $c\bar{c}$-system, η_c). For high energy levels (for example, P-state in $c\bar{c}$-system, χ_c) the potential of the heavy $q\bar{q}$-system $V(r)$ should be well approximated by phenomenological potential such a "Coulomb" plus linear as Eq.(4.2.35). This is because, at longer distance, higher-order perturbation such as gluon self interaction gets more important. In fact, Refs.[95, 96, 97] show that the perturbatively calculated QCD potential agrees with lattice calculations or phenomenologically suggested potential. When q is a heavy quark, it is expanded by its mass as

$$q(x) = \begin{pmatrix} \varphi e^{-imt} + i\frac{\vec{\nabla}\cdot\vec{\sigma}}{2m}\chi e^{imt} \\ \chi e^{imt} - i\frac{\vec{\nabla}\cdot\vec{\sigma}}{2m}\varphi e^{-imt} \end{pmatrix}. \tag{C.0.3}$$

φ and χ denote particle and anti-particle components, respectively, and this expansion is so-called Foldy-Wouthuysen-Tani transformation[86, 87]. Taking a color singlet part in the second term of Eq.(C.0.1) (color octet part is the next leading order [78]), we can obtain a NRQCD action,

$$S_{\rm NRQCD} = \int_x \left[\varphi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m}\right)\varphi + \chi^\dagger \left(i\partial_0 - \frac{\nabla^2}{2m}\right)\chi\right]$$
$$+ \frac{1}{2N_C}\int_x\int_y \delta(x^0-y^0)\left[\varphi^\dagger(x)\chi(y)V(r)\chi^\dagger(y)\varphi(x) + \varphi^\dagger(x)\sigma^i\chi(y)V(r)\chi^\dagger(y)\sigma_i\varphi(x)\right], \tag{C.0.4}$$

where color factor comes from $T^A_{ij}T^A_{kl} = \frac{C_F}{2N_C}\delta_{il}\delta_{kj} - \frac{1}{N_C}T^A_{il}T^A_{kj}$ through Fierz transformation. Hereafter, we note N_C as a color number, which is, of course, $N_C = 3$. Next, by inserting the following identities,

$$1 = \int \prod_{\mu,\nu} \mathcal{D}s^\mu \mathcal{D}\phi^{\nu\dagger}\ \exp i \int_x\int_y \phi^\dagger_\mu(x,y)(s^\mu(x,y) - \varphi^\dagger(x)\sigma^\mu\chi(y)), \tag{C.0.5}$$

$$1 = \int \prod_{\mu,\nu} \mathcal{D}s^{\mu\dagger} \mathcal{D}\phi^{\nu}\ \exp i \int_x\int_y \phi_\mu(x,y)(s^{\mu\dagger}(x,y) - \chi^\dagger(x)\sigma^\mu\varphi(y)), \tag{C.0.6}$$

into Eq.(C.0.4), the QCD action becomes

$$S_{\rm NRQCD} = \int_x\int_y \Big[\varphi^\dagger(x)K_{\varphi\varphi}\varphi(y) + \chi^\dagger(x)K_{\chi\chi}\chi(y) - \phi^{\mu\dagger}(x,y)\varphi^\dagger(x)\sigma_\mu\chi(y) - \chi^\dagger(x)\sigma_\mu\varphi(y)\phi^\mu(x,y)$$
$$+\frac{1}{2N_C}\delta(x^0-y^0)s^{\mu\dagger}(x,y)V(r)s_\mu(x,y) + \phi^{\mu\dagger}(x,y)s_\mu(x,y) + s^\dagger_\mu(x,y)\phi^\mu(x,y)\Big], \tag{C.0.7}$$

where the kinetic terms denote

$$\varphi^\dagger(x)\delta^4(x-y)\left(i\partial_0 + \frac{\nabla^2}{2m}\right)\varphi(y) \equiv \varphi^\dagger(x)K_{\varphi\varphi}\varphi(y), \tag{C.0.8}$$

$$\chi^\dagger(x)\delta^4(x-y)\left(i\partial_0 - \frac{\nabla^2}{2m}\right)\chi(y) \equiv \chi^\dagger(x)K_{\chi\chi}\chi(y). \tag{C.0.9}$$

An effective action of the bilocal auxiliary field $\phi^\mu(x,y)$ will be obtained by integrating out s^μ, φ, and χ. This auxiliary field technique gives the same result obtained by inserting Gaussian configuration, which is explained in Appendix C.1, and we utilize the technique just for a convenience. A potential term is induced by integrating out s^μ as

$$-\int_x\int_y \phi^{\mu\dagger}(x,y)\left[2N_C\delta(x^0-y^0)V^{-1}(r)g_{\mu\nu}\right]\phi^\nu(x,y). \tag{C.0.10}$$

On the other hand, φ- and χ-integrations will derive a kinetic term of ϕ^μ as shown below. We can rewrite the first four terms in Eq.(C.0.7) as

$$\int_x\int_y \begin{pmatrix}\varphi(x)\\ \chi(x)\end{pmatrix}^\dagger \begin{pmatrix}K_{\varphi\varphi} & K_{\varphi\chi}\\ K_{\chi\varphi} & K_{\chi\chi}\end{pmatrix}\begin{pmatrix}\varphi(y)\\ \chi(y)\end{pmatrix}, \tag{C.0.11}$$

where $K_{\varphi\chi}$ and $K_{\varphi\chi}$ are denoted as $K_{\varphi\chi} = -\phi^{\mu\dagger}(x,y)\sigma_\mu$ and $K_{\chi\varphi} = -\sigma_\mu\phi^\mu(x,y)$. Then,by integrating out φ and χ in Eq.(C.0.7), we can obtain the term

$$i\mathrm{Tr}\log\begin{pmatrix} K_{\varphi\varphi} & K_{\varphi\chi} \\ K_{\chi\varphi} & K_{\chi\chi}\end{pmatrix} \simeq i\mathrm{Tr}\log\begin{pmatrix} K_{\varphi\varphi} & 0 \\ 0 & K_{\chi\chi}\end{pmatrix} + i\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\mathrm{Tr}\begin{pmatrix} 0 & K_{\varphi\varphi}^{-1}K_{\varphi\chi} \\ K_{\chi\chi}^{-1}K_{\chi\varphi} & 0\end{pmatrix}^n. \tag{C.0.12}$$

The Tr log is expanded in Eq.(C.0.12), where $n = 1$ is vanished by a trace, and the leading term is coming from $n = 2$. After taking traces of spinor, color, and coordinate indices, the leading term in Eq.(C.0.12) becomes

$$\begin{aligned}&\frac{-i}{2}\mathrm{Tr}\begin{pmatrix} K_{\varphi\varphi}^{-1}K_{\varphi\chi}K_{\chi\chi}^{-1}K_{\chi\varphi} & 0 \\ 0 & K_{\chi\chi}^{-1}K_{\chi\varphi}K_{\varphi\varphi}^{-1}K_{\varphi\chi}\end{pmatrix} \\ &= -iN_C\int_x\int_y\int_z\int_w \mathrm{Tr}_{\mathrm{spin}}K_{\varphi\varphi}^{-1}(x,y)K_{\varphi\chi}(y,z)K_{\chi\chi}^{-1}(z,w)K_{\chi\varphi}(w,x),\end{aligned} \tag{C.0.13}$$

where propagators are given by

$$\begin{aligned}K_{\varphi\varphi}^{-1}(x,y) &= \int\frac{d^4p}{(2\pi)^4}\frac{1}{p^0 - \frac{\vec{p}^2}{2m} + i\epsilon}e^{-ip(x-y)}\delta^{\alpha\beta}, \\ K_{\chi\chi}^{-1}(x,y) &= -\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^0 - \frac{\vec{q}^2}{2m} + i\epsilon}e^{iq(x-y)}\delta^{\alpha\beta}.\end{aligned} \tag{C.0.14}$$

We use a center of mass coordinate X^μ and relative coordinate $(0,\vec{r})^\mu$ as $x^\mu = X^\mu + \frac{1}{2}(0,\vec{r})^\mu$ and $y^\mu = X^\mu - \frac{1}{2}(0,\vec{r})^\mu$. The relative coordinate does not have *time*-component, since $\phi^\mu(x,y)$ is a coincident bilocal field for x and y. Then, $\phi^\mu(x,y)$ is represented by

$$\phi^\mu(x,y) \equiv \phi_X^\mu(\vec{r}) = \int_k\int_l \phi_k^\mu(\vec{l})e^{-ikX}e^{-il_\mu(0,\vec{r})^\mu} = \int_k\int_{l^0}\int_{\vec{l}}\phi_k^\mu(\vec{l})e^{-ikX}e^{i\vec{l}\cdot\vec{r}}, \tag{C.0.15}$$

with their momentums as $p^\mu = (\frac{k^0}{2} + l^0, \frac{\vec{k}}{2} + \vec{l})$ and $q^\mu = (\frac{k^0}{2} - l^0, \frac{\vec{k}}{2} - \vec{l})$. In this frame, Eq.(C.0.13) is written as

$$-2N_C\int_k\int_l\int_{\vec{r}}\int_{\vec{s}}\frac{1}{\left[\frac{k^0}{2} + l^0 - \frac{(\vec{k}/2+\vec{l})^2}{2m} + i\epsilon\right]\left[\frac{k^0}{2} - l^0 - \frac{(\vec{k}/2-\vec{l})^2}{2m} + i\epsilon\right]}\phi_k^{\mu\dagger}(\vec{r})\phi_{\mu k}(\vec{s})e^{-i\vec{l}\cdot(\vec{r}+\vec{s})}, \tag{C.0.16}$$

and we obtain

$$-2iN_C\int_k\int_{\vec{l}}\int_{\vec{r}}\int_{\vec{s}}\frac{1}{k^0 - \frac{\vec{k}^2}{4m} - \frac{\vec{l}^2}{m}}\phi_k^{\mu\dagger}(\vec{r})\phi_{\mu k}(\vec{s})e^{-i\vec{l}\cdot(\vec{r}+\vec{s})} \tag{C.0.17}$$

by integrating l^0. Then, the effective action of ϕ^μ is given by

$$S_{\text{eff}} = \int_X \int_{\vec{r}} \phi_X^{\mu\dagger}(\vec{r}) \left[\frac{1}{V(r)} - \frac{1}{K_X(r)} \right] \phi_{\mu X}(\vec{r}), \tag{C.0.18}$$

where $K_X(r) \equiv i\partial_X^0 - \frac{\nabla_X^2}{4m} - \frac{\nabla_r^2}{m}$. We omit overall factor $2N_C$ by use of normalization of the field. Note that a Green function $\langle \phi_X^\mu(\vec{r})\phi_Y^{\nu\dagger}(\vec{s})\rangle$ is given by

$$\begin{aligned}\langle \phi_X^\mu(\vec{r})\phi_Y^{\nu\dagger}(\vec{s})\rangle &\equiv \left[V^{-1} - K^{-1}\right]^{-1}_{\mu\nu}(X,\vec{r};Y,\vec{s}) \\ &= V(r)g_{\mu\nu}\delta^4(X-Y)\delta^3(\vec{r}-\vec{s}) + \left[V(K-V)^{-1}V\right]_{\mu\nu}(X,\vec{r};Y,\vec{s}). \end{aligned} \tag{C.0.19}$$

In asymptotic states, $X \neq Y$, the first term vanishes. The second term is what we want, and V is rotated out by field redefinition, then Eq.(C.0.19) becomes

$$\langle \phi_X^\mu(\vec{r})\phi_Y^{\nu\dagger}(\vec{s})\rangle = \left[(K-V)^{-1}\right]_{\mu\nu}(X,\vec{r};Y,\vec{s}). \tag{C.0.20}$$

This means that the effective action in Eq.(C.0.18) can be rewritten as

$$S_{\text{eff}} = \int_X \int_{\vec{r}} \phi_X^{\mu\dagger}(\vec{r}) \left[K_X(r) - V(r)\right] \phi_{\mu X}(\vec{r}). \tag{C.0.21}$$

This is the effective action of Eq.(4.1.2).We should notice that this form is correct when asymptotic states exist[98] and $\phi_X^\mu(\vec{r})$ is an on-shell state.

C.1 Auxiliary field technique

In this section, we exhibit that two types of auxiliary field technique is equivalent, where we consider Nambu–Jora-Nasinio model as an example. In Nambu–Jora-Nasinio model, a bi-linear operator can be replaced by a bi-local auxiliary field. The action of is given by

$$S = \int d^4x \Big(\bar{\psi}(x) i\partial\!\!\!/ \psi(x) + G\left[(\bar{\psi}(x)\psi(x))^2 + (\bar{\psi}(x)i\gamma^5\psi(x))^2 \right] \Big). \tag{C.1.22}$$

Theory does not change if we insert Gaussian configuration,

$$1 = \int \mathcal{D}\sigma\mathcal{D}\pi \exp i \left[-\frac{1}{4G}\int d^4x(\sigma(x)^2 + \pi(x)^2) \right]. \tag{C.1.23}$$

When σ and π are replaced by $\sigma' = \sigma + 2G\bar{\psi}\psi$ and $\pi' = \pi + 2G\bar{\psi}i\gamma^5\psi$, respectively, Eq.(C.1.23) does not change. Then, the action becomes

$$S = \int d^4x \Big(\bar{\psi} i\partial\!\!\!/ \psi - \frac{1}{4G}(\sigma^2 + \pi^2) - \bar{\psi}(\sigma + i\gamma^5\pi)\psi \Big). \tag{C.1.24}$$

On the other hand, we use different technique as follows. In stead of inserting Gaussian configuration, we use identities

$$1 = \int \mathcal{D}s\mathcal{D}\sigma \exp i\frac{1}{2G}\int d^4x\left[\sigma(s - 2G\bar{\psi}\psi)\right], \tag{C.1.25}$$

$$1 = \int \mathcal{D}t\mathcal{D}\pi \exp i\frac{1}{2G}\int d^4x\left[\pi(t - 2G\bar{\psi}i\gamma^5\psi)\right], \tag{C.1.26}$$

where s and σ are real. Then the action becomes

$$S = \int d^4x\Big(\bar{\psi}i\partial\!\!\!/\psi + \frac{1}{4G}(s^2+t^2) + \frac{1}{2G}\sigma(s - 2G\bar{\psi}\psi) + \frac{1}{2G}\pi(t - 2G\bar{\psi}i\gamma^5\psi)\Big) \tag{C.1.27}$$

$$= \int d^4x\Big(\bar{\psi}i\partial\!\!\!/\psi + \frac{1}{4G}(s+\sigma)^2 + \frac{1}{4G}(t+\pi)^2 - \frac{1}{4G}(\sigma^2+\pi^2) - \bar{\psi}(\sigma + i\gamma^5\pi)\psi\Big). \tag{C.1.28}$$

By integrating out s and t, we obtain

$$S = \int d^4x\Big(\bar{\psi}i\partial\!\!\!/\psi - \frac{1}{4G}(\sigma^2+\pi^2) - \bar{\psi}(\sigma + i\gamma^5\pi)\psi\Big). \tag{C.1.29}$$

Then this technique gives the same action which is obtained by the previous technique.

C.2 Power counting rules for NRQCD

We consider the power counting of non-relativistic fields as follows[99]. At first, probability of quark wave function is of course

$$\int d^3x\bar{q}q = 1. \tag{C.2.30}$$

Since quarks consisting quarkonium are localized within $\Delta x \sim 1/p$, the power of space integration is approximately $\int d^3x \sim \frac{1}{p}$, and thus, the power of q is $p^{3/2}$, where $p \sim mv$. Noting that the relative velocity can be estimated by the relation $p \sim 1/(\text{hadron size}) \sim mv$. Similarly, the kinetic term of φ, which is

$$\int d^3x\varphi^\dagger\left(D_0 + \frac{\vec{D}^2}{2m}\right)\varphi, \tag{C.2.31}$$

has an expectation value of kinetic energy of order $K \sim mv^2$. The covariant derivative in Eq. (C.2.31) is defined as

$$D_0\varphi = (\partial_0 + ig_s\phi)\varphi, \tag{C.2.32}$$

$$\vec{D}\varphi = (\vec{\nabla} - ig_s\vec{A})\varphi, \tag{C.2.33}$$

where ϕ and $\vec{A}$ are scalar and vector potential, respectively. We can find that the power of D_0 and $\vec{D}$ is

$$D_0 \sim \frac{\vec{D}^2}{2m} \sim K. \tag{C.2.34}$$

As for the gauge coupling, when we consider a potential in a meson is Coulomb potential $V = \alpha_s/r$, the potential energy and the kinetic energy should be balanced, and therefore, we obtain $\alpha_s \sim g_s^2 \sim K/p \sim v$. Next we consider the power of the gauge field in the Coulomb gauge. Equation of motion (EOM) for the scalar potential is given by

$$\nabla^2 g_s \phi \;=\; -g_s^2 \varphi^\dagger \varphi + \cdots , \tag{C.2.35}$$

where we consistently neglect the vector potential (we see below), and thus, φ is expected that $g_s\varphi \sim g_s^2 p \sim K$. For the vector potential, EOM is given by

$$(\partial_t^2 - \nabla^2) g_s \vec{A} \;=\; \frac{g_s^2}{m}\varphi^\dagger \nabla \varphi + g_s^2 \phi \nabla \phi + \cdots , \tag{C.2.36}$$

and we see that

$$g\vec{A} \sim \frac{1}{p^2}\left(\frac{g_s^2}{m}p^4 + pK^2\right) \sim vK, \tag{C.2.37}$$

which is smaller than the scalar potential. The power counting rule in NRQCD is summarized in Tab.C.1.

	field	power
quark filed	$\varphi(x)$	$(mv)^{3/2}$
anti-quark field	$\chi(x)$	$(mv)^{3/2}$
covariant time derivative	D_0	mv^2
covariant space derivative	$\vec{D}$	mv
scalar potential	$g_s\phi(x)$	mv^2
vector potential	$g_s\vec{A}(x)$	mv^3

Table C.1: The power counting rule in NRQCD with the quark mass m and typical velocity v.

Bibliography

[1] ATLAS Collaboration, G. Aad *et al.*, Phys.Lett. **B716**, 1 (2012), 1207.7214.

[2] CMS Collaboration, S. Chatrchyan *et al.*, Phys.Lett. **B716**, 30 (2012), 1207.7235.

[3] G. Jungman, M. Kamionkowski, and K. Griest, Phys.Rept. **267**, 195 (1996), hep-ph/9506380.

[4] L. Bergstrom, Rept.Prog.Phys. **63**, 793 (2000), hep-ph/0002126.

[5] D. Tucker-Smith and N. Weiner, Phys.Rev. **D64**, 043502 (2001), hep-ph/0101138.

[6] C. Munoz, Int.J.Mod.Phys. **A19**, 3093 (2004), hep-ph/0309346.

[7] J. Hisano, S. Matsumoto, M. M. Nojiri, and O. Saito, Phys.Rev. **D71**, 063528 (2005), hep-ph/0412403.

[8] E. A. Baltz, M. Battaglia, M. E. Peskin, and T. Wizansky, Phys.Rev. **D74**, 103521 (2006), hep-ph/0602187.

[9] G. Bertone, D. Hooper, and J. Silk, Phys.Rept. **405**, 279 (2005), hep-ph/0404175.

[10] T. Appelquist, H.-C. Cheng, and B. A. Dobrescu, Phys.Rev. **D64**, 035002 (2001), hep-ph/0012100.

[11] D. Hooper and K. M. Zurek, Phys.Rev. **D79**, 103529 (2009), 0902.0593.

[12] S. C. Park and J. Shu, Phys.Rev. **D79**, 091702 (2009), 0901.0720.

[13] Y. Bai and Z. Han, Phys.Rev. **D79**, 095023 (2009), 0811.0387.

[14] S. Arrenberg, L. Baudis, K. Kong, K. T. Matchev, and J. Yoo, Phys.Rev. **D78**, 056002 (2008), 0805.4210.

[15] A. Datta, K. Kong, and K. T. Matchev, Phys.Rev. **D72**, 096006 (2005), hep-ph/0509246.

[16] M. Battaglia, A. Datta, A. De Roeck, K. Kong, and K. T. Matchev, JHEP **0507**, 033 (2005), hep-ph/0502041.

[17] N. Haba, K. Kaneta, S. Matsumoto, T. Nabeshima, and S. Tsuno, Phys.Rev. **D85**, 014007 (2012), 1109.5082.

[18] N. Haba, K. Kaneta, and T. Onogi, (2011), 1109.5442.

[19] S. Weinberg, Phys.Rev.Lett. **43**, 1566 (1979).

[20] W. Buchmuller and D. Wyler, Nucl.Phys. **B268**, 621 (1986).

[21] C. Arzt, M. Einhorn, and J. Wudka, Nucl.Phys. **B433**, 41 (1995), hep-ph/9405214.

[22] J. Aguilar-Saavedra, Nucl.Phys. **B812**, 181 (2009), 0811.3842.

[23] J. Aguilar-Saavedra, Nucl.Phys. **B843**, 638 (2011), 1008.3562.

[24] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, JHEP **1010**, 085 (2010), 1008.4884.

[25] H. P. Nilles, Phys.Rept. **110**, 1 (1984).

[26] H. E. Haber and G. L. Kane, Phys.Rept. **117**, 75 (1985).

[27] S. P. Martin, (1997), hep-ph/9709356.

[28] M. Sohnius, Phys.Rept. **128**, 39 (1985).

[29] MSSM Working Group, A. Djouadi *et al.*, (1998), hep-ph/9901246.

[30] J. L. Lopez, Rept.Prog.Phys. **59**, 819 (1996), hep-ph/9601208.

[31] D. Chung *et al.*, Phys.Rept. **407**, 1 (2005), hep-ph/0312378.

[32] M. S. Carena and H. E. Haber, Prog.Part.Nucl.Phys. **50**, 63 (2003), hep-ph/0208209.

[33] P. Fayet and S. Ferrara, Phys.Rept. **32**, 249 (1977).

[34] M. Kuroda, (1999), hep-ph/9902340.

[35] N. Haba, K. Kaneta, S. Matsumoto, and T. Nabeshima, Acta Phys.Polon. **B43**, 405 (2012), 1106.6106.

[36] T. G. Rizzo, eConf **C040802**, L013 (2004), hep-ph/0409309.

[37] A. Datta, K. Kong, and K. T. Matchev, New J.Phys. **12**, 075017 (2010), 1002.4624.

[38] K. Kong, K. Matchev, and G. Servant, (2010), 1001.4801.

[39] C. Csaki, J. Heinonen, J. Hubisz, S. C. Park, and J. Shu, JHEP **1101**, 089 (2011), 1007.0025.

[40] G. Bhattacharyya, P. Dey, A. Kundu, and A. Raychaudhuri, Phys.Lett. **B628**, 141 (2005), hep-ph/0502031.

[41] T. Flacke, A. Menon, and D. J. Phalen, Phys.Rev. **D79**, 056009 (2009), 0811.1598.

[42] K. Nishiwaki, K.-y. Oda, N. Okuda, and R. Watanabe, Phys.Lett. **B707**, 506 (2012), 1108.1764.

[43] S. Matsumoto, J. Sato, M. Senami, and M. Yamanaka, Phys.Rev. **D80**, 056006 (2009), 0903.3255.

[44] H. Murayama, M. M. Nojiri, and K. Tobioka, Phys.Rev. **D84**, 094015 (2011), 1107.3369.

[45] H. Dohi and K.-y. Oda, Phys.Lett. **B692**, 114 (2010), 1004.3722.

[46] T. Stelzer and S. Willenbrock, Phys.Lett. **B374**, 169 (1996), hep-ph/9512292.

[47] M. Beneke *et al.*, (2000), hep-ph/0003033.

[48] W. Bernreuther, A. Brandenburg, Z. Si, and P. Uwer, Phys.Rev.Lett. **87**, 242002 (2001), hep-ph/0107086.

[49] W. Bernreuther, A. Brandenburg, Z. Si, and P. Uwer, Nucl.Phys. **B690**, 81 (2004), hep-ph/0403035.

[50] W. Bernreuther, M. Fuecker, and Z.-G. Si, Phys.Rev. **D74**, 113005 (2006), hep-ph/0610334.

[51] Particle Data Group, J. Beringer *et al.*, Phys.Rev. **D86**, 010001 (2012).

[52] W. Hollik, W. Mosle, C. Kao, and D. Wackeroth, (1997), hep-ph/9711419.

[53] S. Berge, W. Hollik, W. M. Mosle, and D. Wackeroth, Phys.Rev. **D76**, 034016 (2007), hep-ph/0703016.

[54] Z. Sullivan, Phys.Rev. **D56**, 451 (1997), hep-ph/9611302.

[55] Atlas Collaboration, G. Aad *et al.*, Phys.Lett. **B701**, 186 (2011), 1102.5290.

[56] D0 Collaboration, V. Abazov *et al.*, Phys.Lett. **B660**, 449 (2008), 0712.3805.

[57] ATLAS Collaboration, G. Aad *et al.*, Phys.Lett. **B707**, 459 (2012), 1108.3699.

[58] CMS Collaboration, S. Chatrchyan *et al.*, Phys.Rev. **D84**, 092004 (2011), 1108.3773.

[59] H.-C. Cheng, K. T. Matchev, and M. Schmaltz, Phys.Rev. **D66**, 036005 (2002), hep-ph/0204342.

[60] J. H. Kuhn, A. Scharf, and P. Uwer, Eur.Phys.J. **C45**, 139 (2006), hep-ph/0508092.

[61] J. H. Kuhn, A. Scharf, and P. Uwer, Eur.Phys.J. **C51**, 37 (2007), hep-ph/0610335.

[62] S. Moretti, M. Nolten, and D. Ross, Nucl.Phys. **B759**, 50 (2006), hep-ph/0606201.

[63] M. Schmaltz and D. Tucker-Smith, Ann.Rev.Nucl.Part.Sci. **55**, 229 (2005), hep-ph/0502182.

[64] M. Perelstein, Prog.Part.Nucl.Phys. **58**, 247 (2007), hep-ph/0512128.

[65] M. Schmaltz, JHEP **0408**, 056 (2004), hep-ph/0407143.

[66] C. Csaki, J. Hubisz, G. D. Kribs, P. Meade, and J. Terning, Phys.Rev. **D67**, 115002 (2003), hep-ph/0211124.

[67] J. L. Hewett, F. J. Petriello, and T. G. Rizzo, JHEP **0310**, 062 (2003), hep-ph/0211218.

[68] C. Csaki, J. Hubisz, G. D. Kribs, P. Meade, and J. Terning, Phys.Rev. **D68**, 035009 (2003), hep-ph/0303236.

[69] M.-C. Chen and S. Dawson, Phys.Rev. **D70**, 015003 (2004), hep-ph/0311032.

[70] W. Kilian and J. Reuter, Phys.Rev. **D70**, 015004 (2004), hep-ph/0311095.

[71] A. Belyaev, C.-R. Chen, K. Tobe, and C.-P. Yuan, Phys.Rev. **D74**, 115020 (2006), hep-ph/0609179.

[72] I. Low, JHEP **0410**, 067 (2004), hep-ph/0409025.

[73] J. Hubisz and P. Meade, Phys.Rev. **D71**, 035016 (2005), hep-ph/0411264.

[74] Q.-H. Cao, C. S. Li, and C.-P. Yuan, Phys.Lett. **B668**, 24 (2008), hep-ph/0612243.

[75] ATLAS Collaboration, (2011), ATLAS-CONF-2011-117, ATLAS-COM-CONF-2011-147.

[76] CMS Collaboration, (2011), CMS-PAS-TOP-11-014.

[77] N. Haba, K. Kaneta, Y. Takayasu, and S. Tsuno, to appear.

[78] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys.Rev. **D51**, 1125 (1995), hep-ph/9407339.

[79] M. J. Duncan, Nucl.Phys. **B214**, 21 (1983).

[80] H. Kleinert, Phys.Lett. **B62**, 429 (1976).

[81] T. Kugo, Phys.Lett. **B76**, 625 (1978).

[82] T. Morozumi and H. So, (1986).

[83] Quarkonium Working Group, N. Brambilla *et al.*, (2004), hep-ph/0412158.

[84] N. Brambilla, A. Pineda, J. Soto, and A. Vairo, Rev.Mod.Phys. **77**, 1423 (2005), hep-ph/0410047.

[85] N. Brambilla *et al.*, Eur.Phys.J. **C71**, 1534 (2011), 1010.5827.

[86] L. L. Foldy and S. A. Wouthuysen, Phys.Rev. **78**, 29 (1950).

[87] S. Tani, Progress of Theoretical Physics **6**, 267 (1951).

[88] E. Eichten, K. Gottfried, T. Kinoshita, K. Lane, and T.-M. Yan, Phys.Rev. **D17**, 3090 (1978).

[89] L. J. Dixon, (1996), hep-ph/9601359.

[90] Z. Bern, L. J. Dixon, and D. A. Kosower, Nucl.Phys. **B513**, 3 (1998), hep-ph/9708239.

[91] K. Hagiwara and D. Zeppenfeld, Nucl.Phys. **B274**, 1 (1986).

[92] G. Mahlon and S. J. Parke, Phys.Rev. **D53**, 4886 (1996), hep-ph/9512264.

[93] A. Ballestrero and E. Maina, Phys.Lett. **B350**, 225 (1995), hep-ph/9403244.

[94] L. Lukaszuk, D. Siemienczuk, and L. Szymanowski, Phys.Rev. **D35**, 326 (1987).

[95] S. Necco and R. Sommer, Phys.Lett. **B523**, 135 (2001), hep-ph/0109093.

[96] S. Recksiegel and Y. Sumino, Phys.Rev. **D65**, 054018 (2002), hep-ph/0109122.

[97] S. Recksiegel and Y. Sumino, Eur.Phys.J. **C31**, 187 (2003), hep-ph/0212389.

[98] H. D. Politzer, Nucl.Phys. **B172**, 349 (1980).

[99] G. P. Lepage, L. Magnea, C. Nakhleh, U. Magnea, and K. Hornbostel, Phys.Rev. **D46**, 4052 (1992), hep-lat/9205007.