Human Capital and International Trade

Ph.D. Dissertation

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Preface

This monograph is the result of my research on human capital and international trade in Graduate School of Economics, Osaka University since 2000. After several years of the research, I left for Shobi University as a full time lecturer and have been keeping on studying the same subject.

The subject of human capital has been elaborated for many economic scientists for decades. Jacob Mincer first used the term in the modern neoclassical economic literature which is his pioneering article "Investment in Human Capital and Personal Income Distribution" in The Journal of Political Economy in 1958. A few years later, Mincer and Gary Becker of the Chicago school applied the idea of "human capital" in economics. In particular, Becker's book entitled Human Capital, published in 1964, is remarkable. The book has become a standard reference for many years especially in the field of economics.

The term human capital can be defined in many ways. Arthur Cecil Pigou first discussed that investment in human capital is just like investment in material capital in 1928. Human capital is also a means of production which is similar to factories and machines which are physical means of production. This is because one can invest in human capital via education and one's outpout depends partly on the rate of return on the human capital one owns. However, unlike the investment in material capital, after the investment in human capital one owns, he or she cannot transfer the additional endowment of human capital to other individuals.

The endowment of human capital can be increased through many channels such as education, training, experience, etc. In some way, human capital also accumulates, for instance, working experience, as many studies have examined. On the other hand, there are also quite many studies assume that human capital depends only on education period, such as Findlay-Kierzkowski (1983), which is the basic applied model in this monograph.

Many governments are trying to change their education policy. More human capital is reallocated into the education sector to increase the endowment of professional human capital. In the framework of Heckscher-Ohlin-Samuelson model, if human capital is used in an industry intensively, then the output of the good produced in the industry increases and tends to be exported. On the other hand, in the framework of Ricardo-Viner model, if human capital is used only in an industry, then the output of the good produced in the industry increases and tends to be exported. In either case, human capital is used intensively or solely in an industry, the industry tends to be a high-tech sector such as information technology (IT) sector.

Although there are some governments in Asian countries have succeeded in their policy, but there are also quite a few countries has failed. Instead of Heckscher-Ohlin-Samuelson model, Ricardo-Viner model will be applied to study not only on this issue but also on the issues of brain drain and technology change throughout this monograph. Besides human capital, there are still more than two kinds of factor appear in our study, namely, physical capital and unskilled labor. In the part II of this monograph, which deals with the issue of technology change and wage inequality, two kinds of human capital, namely, skilled labor in sector 1 and skilled labor in sector 2, and unskilled labor which is mobile between sectors will appear in our model.

In the context of international trade, human capital and brain drain relate to each other. This subject is often argued in the study of migration or mobility between nations in the international trade literature. Brain drain is some time regarded as human capital flight. Just like capital flight, in which the owner of the capital or financial capital invests in the other country rather than the country he or she lives. What is the difference between brain drain an capital flight? As I have argued before, one cannot transfer the human capital he or she owns to other individuals, hence the transfer of human capital from a country to another country involves the migration of the owner of human capital.

In many cases, when a country exports high-tech good, for example, Japan exports high-tech good to china or India, human capital flight will occur if mobility of human capital between nations is allowed. In fact, skilled labor who owns the human capital which is used to produce the high-tech good, tends to migrate to Japan from the countries which import the high-tech good. However, this cannot be explained in the traditional Ricardo-Viner model, since the traditional results in the standard model shows that a country with more human capital which is specific to a sector tends to export to a country with less human capital, while the factor price for the human capital is lower than the country with less human capital. If this is true, then skilled labor will not migrate to the country which exports the high-tech good because the reward for human capital is lower.

Human capital and technology change also relates to each other. The issue of human capital involves also the issue on wage inequality. Undoubtedly, the issue of the relationship between technology change and wage inequality is also important in the international trade context. The concept of formation of human capital in this monograph will also be applied to study on this issue.

Before I start talking about my study on the issues above in detailed, I

wish to mention how much human capital I have acquired from during the time I was preparing the manuscript. I discussed my ideas with numerous colleagues and friends. They have contributed to this study with their valuable remarks, suggestions, and critique. Three of them, I am in particularly indebted. The first one is my Ph.D. advisor, Professor Kenzo Abe, who stimulated my interest in issues of human capital and international trade relationships. His constant encouragement, valuable critique and appropriate suggestions led to the completion of my dissertation. I also have an exceptional debt to Professor Suezo Ishizawa and Yasuji Goto. They spared so much valuable time to guide me to go to graduate school when I was still an undergraduate student in Tezukayama University.

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In spite of all this help and advice, all errors, shortcomings, misconceptions and typos that have remained in the manuscript, are my own responsibility.

The research grant from Shobi University is much appreciated which enabled me to participate in many academic discourse on workshops, conferences and extramural seminars, and the publication of this monograph. Last but not least, I gratefully acknowledge the financial support of the Japanese Government (Monbukagakusho: MEXT) Scholarship which was a great help when I was a graduate student in Osaka University.

Finally I wish to mention the support from my family. Mio, my wife who allowed my Ph.D. program for several years inspite of our burden on household finance. Moi and Yuen Fong, my parents who are getting old, allowed my study in Japan for more than fifteen years, and have been desperately anxious for my return home. Most of the work on this monograph and Ph.D. program in Osaka University were borne by them. Without their patience, I could not have time and comtemplation to elaborate my ideas which is indispensable in my Ph.D. program and the completion of this monograph.

Contents

Chapter 1

Introduction

Over the last fifteen years, workplace and work force have changed dramatically. Since the world has changed to an information economy, people are the critical asset. Many outputs are intangible and human capital becomes a source of competitive advantage.

Human capital has become more critical to competitiveness. Many governments have taken notice of this and recognized the necessity to increase the endowment of human capital. Most governments are trying to change their education policy in order to enhance the competitiveness of the industry using human capital to produce high-tech good. This kind of policy brings about the issues of trade pattern, change in wages, brain drain and so on. Our study examines these issues in part I which includes chapter 2 and chapter 3.

In chapter 2, we examine the Ricardo-Viner (RV) trade model which incorporates an intermediate good under public service. We allow the government to use one of the sector-specific factors and the general factor to produce the sector-specific factor that has been used by the government. Hence from this chapter the intermediate good serves as input for both the public sector and the private sector. We will see some very similar results as those of the standard RV model. However, we will also show the effects of the public service on trade patterns and factor prices. The examination in this chapter is based on Chong (2002b).

Chapter 3 examines the relationship between trade patterns and brain drain with publicly provided education service which controls human capital formation. We apply RV model to show that when human capital mobility is allowed in a free trade world, brain drain does not occur necessarily in a country which exports the good using human capital.

The issue on relationship between technology change and wage inequality has been argued in many studies. In particular, United States has been experiencing the increase in income inequality due to the technical change. Many economists have argued that the rising income inequality is due to the skilled labor biased technical change. But this is not necessarily true.

In chapter 4, we construct a RV model with endogenous labor supplies and examine the effects of two types of technical change on relative wages. That is, (i) product-specific skilled labor augmentation and (ii) product-specific unskilled labor augmentation. We will clarify whether factor bias or sector bias matters for relative wages. Within this framework, there is also an additional indirect effect through the labor market compared to the traditional RV model. We will also show that the indirect effect is unambiguous in case (i) and ambiguous in case (ii). Both cases establish the validity of the earlier results in the traditional RV model, although it brings some additional results for individual wages in case (ii). The examination in this chapter is based on Chong (2002a).

A summary and some remarks are given in the last chapter.

Part I

Brain Drain and Trade Pattern

Chapter 2

The Ricardo-Viner Model with a Publicly Provided Intermediate Good

2.1 Introduction

There have been so many studies of examining the basic results of Ricardo-Viner (RV) model as well as Heckscher-Ohlin-Samuelson (HOS) model by reconstructing the conventional version of those models. For example, the assumption of constant returns to scale (CRS) is replaced by variable returns to scale (VRS) [e.g., Kemp and Negishi (1970), Helpman (1983,1984)]. On the other hand, the inelasticities of the factor supplies become elastic which can be done by incorporating intermediate goods into the production functions of the final goods. Most studies serve intermediate good as input of the final good solely. Some studies even deal with the presence of VRS in the production of intermediate goods [e.g., Ishikawa (1991) and Isikawa (1992)]. However, intermediate goods are not necessarily produced by the private sector, some studies show examples incorporating a government-financed public input which is simultaneously used by two industries [e.g., Tawada and Abe (1984), Ishizawa (1988)].

In this chapter, we allow the assumptions of CRS remain in all sectors, but serve intermediate good as input not only for a final good, but also for intermediate good itself. In the meantime, we concentrate on RV model rather than HOS model. Ishikawa (2000) constructs a RV model by replacing one of the sector-specific factors with a sector-specific intermediate good. In his study, there are two primary factors, two final goods, and one intermediate good. One of the factors is sector-specific and the other is general. The intermediate good is also specific to the sector where the specific factor is not used. Our model here is very similar to the model in Ishikawa (2000), however, the intermediate good in his model is produced with the general factor alone and served as private good. Our model will differ from his basic model in these two points.

It is possible to consider the general factor as labor (unskilled labor), the specific factor as land or plant, etc, and the intermediate good as human capital (skilled labor), in which case the model could be used to analyze an economy with a sector particularly using human capital such as software industry while the other sector particularly using land or machine such as food industry. In the sence of human capital, the public service can be considered as education and is the only channel through which human capital accumulates. We will construct an overlapping-generations model as what Findlay-Kierzkowski (1983) and Wong-Yip (1999) study, where the former consider T-genereation while the latter consider only 2-generation in each period.

2.2 The Model

2.2.1 Basic Ricardo-Viner Model with Intermediate Good

We consider a three-sector (2 private sectors and 1 public sector), two-primary-factor (unskilled labor and capital) framework. Private sectors produce final goods, good 1 and good 2, while public sector produces an intermediate good, say, human capital. Capital is specific to sector 2, while unskilled labor is general factor and freely moves across private sectors. The basic RV model in this chapter is expressed as

$$X_1 = X_1(H_p, L_1), \tag{2.1}$$

$$X_2 = X_2(K, L_2), \tag{2.2}$$

$$L_1 + L_2 = L, \tag{2.3a}$$

$$L = L(U_l), \tag{2.3b}$$

$$H_p = H - H_e, \tag{2.4a}$$

$$H = F(H_e, U_e), \tag{2.4b}$$

$$K = \bar{K}, \tag{2.5}$$

where U_l, U_e, X_i, L_i (i=1,2), L, K, $\bar{K}$, H, H_p, and H_e are, respectively, uneducated individual in the unskilled labor market, uneducated individual in the public sector, the output of sector i, the general factor (unskilled labor) hired by sector i, the general factor supply, factor(capital) specific to sector 1 which is used, its total endowment, gross output of intermediate good (human capital or skilled labor), net output of intermediate good which is specific to sector 2 and that serve as input in public sector. Equations (3.1), (3.1), (2.3a), (2.4a) and (3.5) are, respectively, the production functions of good 1 and good 2, full employment conditions of unskilled labor, human capital (skilled labor) and capital. Equation (2.3b) is the unskilled labor supply function while equation (2.4b) is the production function of the intermediate good (human capital) in public sector. Note that except the equations (2.3b) and (2.4b), the system above is very similar to that in the traditional RV model[1]. Good 2 is produced using unskilled labor and capital, while good 1 is produced using unskilled labor and the human capital. The human capital is produced using uneducated individual and human capital[2], hence it serves as an input in the public sector as well as in the sector 1. $X_i(\cdot)$ (i=1,2) and $F(\cdot)$ are increasing, strictly quasi-concave, positively linear homogeneous and twice continuously differentiable. We will explain in an explicit way for $L_i(\cdot)$ in the next subsection. Full employment and perfect competition are assumed.

2.2.2 Human Capital Formation

Let us consider an economy which N individuals are 'born' at each period and are uneducated in the beginning, each of them lives for 2 periods. The population of the economy at each period is $2N$.

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[1] Since we will treat H_e as an exogenous variable, it is exactly same as the traditional RV model. See also Jones (1971).

[2] When the uneducated individual and the human capital are used in public sector, they should be refered as 'student' and 'educator'

Government provide a public service for those who want to be educated under the public service for free but forgoing the opportunity to participate in the unskilled labor market during the education period (the 1st period of his lifetime). We can see from above, there are two kinds of decision maker among the new generation in each period. Let U_e denote those who decide to earn their income under wage rate (W_H) as a skilled labor after having enjoyed the public service. Those who decide not to enjoy the public service and participate in the unskilled labor market to earn their income under wage rate (W_L) is denoted by U_l. The population is expressed as

$$2N = 2(U_e + U_l),$$

which can be rewritten as

$$N = U_e + U_l, \tag{2.6}$$

Noting that the production function of the human capital is positively linear homogeneous, equation (2.4b) can be rewritten as

$$\frac{H}{U_e} = h = f(h_e),$$
$$f(0) > 0 \qquad f^{'} > 0 \qquad f_i^{''} < 0 \qquad h_e = \frac{H_e}{U_e},$$

where h and h_e are, respectively, human capital per capita acquired by U_e and educator-student ratio[3].

[3] Compare to Becker and Murphy (1992), which shows that the human capital acquired

For simplicity, we also assume that the domestic capital stock is owned by all the individuals and there is perfectly equality in distribution of the capital stock[4]. Then, in each period, each individual receives rk equally, where $k = K/2N$ and r is the factor price of the capital. An additional income as W_L will be earned in period 1 and period 2 if he of she becomes unskilled labor, while $W_H \cdot f(h_e)$ is earned only in period 2 if he or she decides to acquire the human capital through the public service. The lifetime income of U_l and U_e at present value are, respectively, can be expressed as

$$B_l = (1-\tau)[W_L + rk + (W_L + rk) \cdot \frac{1}{1+\rho}]$$
$$B_e = (1-\tau)\{rk + [W_H f(h_e) + rk] \cdot \frac{1}{1+\rho}\}.$$

ρ and τ are, respectively, fixed interest rate and income tax rate imposed by government to finance the public service.

Since $dB_e/dU_e < 0$, we know that U_e must be determined under the arbitrary condition as

$$B_l = B_e,$$

which yields

$$f(h_e) = \frac{W_L}{W_H} \cdot R, \tag{2.7}$$

by a student depends on the human capital of her teachers, and the number of teachers per student.

[4] Many studies assume this, for example, see Gupta (1994).

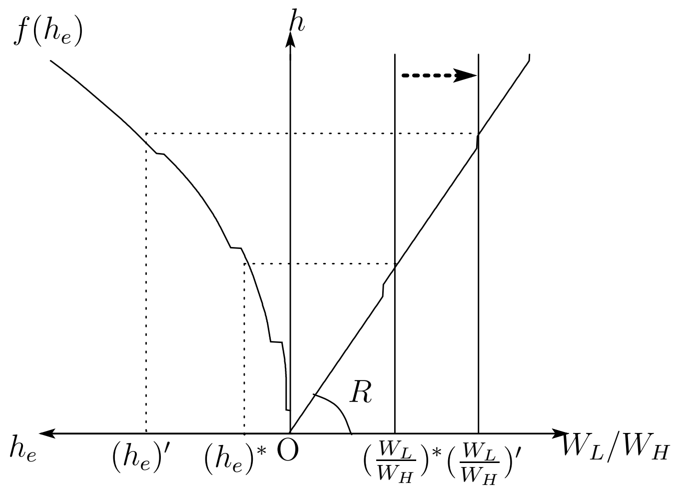

Figure 2.1: Determination of U_e given H_e, R and N as well as W_L/W_H, where h_e and h can be considered as the level of educator per student and the level of quality or human captital per student, respectively.

where $R \equiv 2 + \rho$ is assumed to be fixed, further , we define W_L/W_H as relative wage in this chapter. Given H_e and W_L/W_H as well as R and N, U_e is determined as shown in figure 2.1.

U_l is determined in equation (2.6). It follows that equation (2.3b) determines L, and equations (2.4a) and (2.4b) determine H_p. Since each individual lives for 2 periods, we can rewrite the equation (2.3b) into an explicit form as

$$L = 2U_l. \tag{2.8}$$

Equation (2.4a) and (2.4b) can also be rewritten as

$$H_p = f(h_e) \cdot U_e - H_e. \tag{2.9}$$

We also assume that the cost of the public service is financed by the government by equal income tax rate among all individuals[5], then the government budget constraint is expressed as

$$W_H H_e = \tau[W_H(H_p + H_e) + W_L L]. \tag{2.10}$$

Equation (2.10) will be used to solve for τ alone.

In general, a whole system has to be solved simultaneously including the determination of W_L/W_H. Let us use the dual unit cost functions to rewrite the basic RV model. The unit cost functions are expressed as

$$C^1(W_L, W_H) = P_1, \tag{2.11a}$$

$$C^2(W_L, r) = 1, \tag{2.11b}$$

where good 2 is numeraire good. The full employment conditions are expressed as

$$C^1_{W_H}(W_L, W_H)X_1 = H_p, \tag{2.12a}$$

$$C^2_r(W_L, r)X_2 = K, \tag{2.12b}$$

$$C^1_{W_L}(W_L, W_H)X_1 + C^2_{W_L}(W_L, r)X_2 = L, \tag{2.12c}$$

[5]See Abe (1990).

where the subscripts on each function represent the partial derivative of the function with respect to the corresponding variable; that is $C^1_{W_H} \equiv \partial C^1/\partial W_H$[6].

Thus, we have 10 equations from (2.6) to (2.12c) and 10 unknowns ($U_e, U_l, H_p, L, W_L, W_H, r, X_1, X_2, \tau$) with given variables, P, H_e, N and ρ. We call the economy above as public intermediate good economy.

2.3 Preliminaries

Before we go for the comparative static analysis, we can rearrange the equations above into the terms of rates of changes.

$$\hat{U}_l = -\gamma \hat{U}_e, \tag{2.13}$$

$$\hat{H}_e - \hat{U}_e = (1+\beta)(\hat{W}_L - \hat{W}_H), \tag{2.14}$$

$$\hat{L} = \hat{U}_l, \tag{2.15}$$

$$\hat{H}_p = \frac{1}{\lambda_H}[\frac{1}{1+\beta}(\hat{H}_e - \hat{U}_e) + \hat{U}_e - (1-\lambda_H)\hat{H}_e], \tag{2.16}$$

$$\theta_{L1}\hat{W}_L + \theta_H \hat{W}_H = \hat{P}_1, \tag{2.17a}$$

$$\theta_{L2}\hat{W}_L + \theta_r \hat{r} = 0, , \tag{2.17b}$$

[6]By Shepard's lemma, we should also further note that, for example, $C^1_{W_H}$ is the skilled labor input coefficient of sector 1.

$$\hat{C}^1_{W_L}(W_L, W_H) = -\theta_H \sigma_1(\hat{W}_L - \hat{W}_H), \tag{2.17c}$$

$$\hat{C}^2_{W_L}(W_L, r) = -\theta_r \sigma_2(\hat{W}_L - \hat{r}), \tag{2.17d}$$

$$\hat{C}^1_{W_H}(W_L, W_H) = \theta_{L1} \sigma_1(\hat{W}_L - \hat{W}_H), \tag{2.17e}$$

$$\hat{C}^2_r(W_L, r) = \theta_{L2} \sigma_2(\hat{W}_L - \hat{r}), \tag{2.17f}$$

$$\sum_{i=1}^{2} \lambda_{Li} \hat{X}_i = \hat{L} - \sum_{i=1}^{2} \lambda_{Li} \hat{C}^i_{W_L}(\cdot), \tag{2.18a}$$

$$\hat{X}_1 = \hat{H}_p - \hat{C}^1_{W_H}(W_L, W_H), \tag{2.18b}$$

$$\hat{X}_2 = \hat{K} - \hat{C}^2_r(W_L, r), \tag{2.18c}$$

where

$$\gamma \equiv \frac{U_e}{U_l},$$

$$\lambda_H \equiv \frac{H_p}{H_p + H_e}, \qquad 0 < \lambda_H < 1,$$

$$\beta \equiv \frac{1 - \sigma^q}{\sigma^q} > 0,$$

$$\sigma^q \equiv \frac{f'(h_e)}{f(h_e)} \cdot h_e, \qquad 0 < \sigma^q < 1,$$

$$\theta_{Li} \equiv \frac{W_L C^i_{W_L}(\cdot)}{P_i}, \qquad for\ i = 1, 2,$$

$$\theta_H \equiv \frac{W_H C^1_{W_H}(\cdot)}{P_1}, \qquad \theta_{L1} + \theta_H = 1,$$

$$\theta_r \equiv \frac{r C^2_r(\cdot)}{P_2}, \qquad \theta_{L2} + \theta_r = 1,$$

$$\sigma_1 \equiv \frac{\hat{C}^1_{W_H}(\cdot) - \hat{C}^1_{W_L}(\cdot)}{\hat{W}_L - \hat{W}_H} > 0,$$

$$\sigma_2 \equiv \frac{\hat{C}^2_r(\cdot) - \hat{C}^2_{W_L}(\cdot)}{\hat{W}_L - \hat{r}} > 0,$$

$$\lambda_{Li} \equiv \frac{C^i_{W_L}(\cdot) X_i}{L}, \qquad for\ i = 1, 2, \qquad \lambda_{L1} + \lambda_{L2} = 1.$$

$(\hat{\ })$ denotes a proportionate change, for example, $\hat{H}_p = dH_p/H_p$. σ^q is positive and smaller than 1 since we have assumed $f''(\cdot) < 0$. The equations (2.13)-(2.18c) can be obtained by differentiating equations (2.6)-(2.9) and equations (2.11a)-(2.12c). That is, equations (2.13)-(2.16) are from equations (2.6)-(2.9), equations (2.17a)-(2.17f) are from equations (2.11a) and (2.11b), and equations (2.18a)-(2.18c) are from equations (2.12a)-(2.12c). σ^q represents the change in percentage of the human capital acquired per capita due to the change of one percent in educator-student ratio. θ_{Li}, θ_H and θ_r, are the familiar income shares in HOS model or RV model. σ_i and λ_{Li} are, respectively, the substitution elasticity between factors and fraction of unskilled labor in ith sector.

Equations (2.17a)-(2.18c) represent exactly the equations those in the standard RV model. The traditional solutions for $\hat{W}_L$ and $\hat{X}_i$ (i=1,2) are expressed as

$$\hat{W}_L = \frac{1}{\Lambda^e}(\Lambda_1^e \hat{P}_1 + \lambda_{L1}\hat{H}_p + \lambda_{L2}\hat{K} - \hat{L}) \tag{2.19}$$

$$\begin{aligned} \hat{X}_1 &= \theta_{L1}e_{L1} \cdot \frac{\Lambda_2^e}{\Lambda^e} \cdot \hat{P}_1 + \hat{H}_p \\ &\quad + \frac{\theta_{L1}e_{L1}}{\Lambda^e}(\hat{L} - \lambda_{L1}\hat{H}_p - \lambda_{L2}\hat{K}) \qquad (2.20a) \\ \hat{X}_2 &= -\theta_{L2}e_{L2} \cdot \frac{\Lambda_1^e}{\Lambda^e} \cdot \hat{P}_1 + \hat{K} \\ &\quad + \frac{\theta_{L2}e_{L2}}{\Lambda^e}(\hat{L} - \lambda_{L1}\hat{H}_p - \lambda_{L2}\hat{K}). \qquad (2.20b) \end{aligned}$$

where

$$\Lambda_i^e \equiv \lambda_{Li} e_{Li}, \qquad for\ i = 1, 2, \qquad \Lambda_1^e + \Lambda_2^e = \Lambda^e,$$

$$e_{L1} \equiv \frac{\sigma_1}{\theta_H}, \qquad e_{L2} \equiv \frac{\sigma_2}{\theta_r}.$$

e_{Li} and Λ^e are, respectively, the elasticity of demand for unskilled labor in the ith sector and the aggregate general-equilibrium elasticity of demand for unskilled labor in the private sectors.

2.4 Comparative Statics

In the standard RV model, $\hat{H}_p$ and $\hat{L}$ are treated as exogenous variables, but in this chapter, since they are treated as endogenous variables, more equations are necessary to complete our story.

Rewrite equations (2.17a) and (2.17b) as

$$\hat{W}_L - \hat{W}_H = \frac{\hat{W}_L - \hat{P}_1}{\theta_H}, \tag{2.17a'}$$

$$\hat{W}_L - \hat{r} = \frac{\hat{W}_L}{\theta_r}, \tag{2.17b'}$$

and substitute equation $(2.17a')$ into equation (2.14), we have

$$\hat{H}_e - \hat{U}_e = (1 + \beta)(\frac{\hat{W}_L - \hat{P}_1}{\theta_H}). \tag{2.14$'$}$$

Substitute equation (2.13) into equation (2.15), we have

$$\hat{L} = -\gamma \hat{U}_e. \tag{2.15'}$$

Solving equations (2.14′), (2.15′), (2.16) and (2.19) simultaneously, we obtain $\hat{W}_L$, $\hat{H}_p$, $\hat{L}$ and $\hat{U}_e$ as

$$\begin{aligned}
\hat{W}_L &= \frac{1}{\Lambda^e + \tilde{\beta}}[(\Lambda_1^e + \tilde{\beta})\hat{P}_1 + (\lambda_{L1} + \gamma)\hat{H}_e + \lambda_{L2}\hat{K}], && (2.21)\\
\hat{H}_p &= \frac{1}{\tilde{\Lambda}}\{[\lambda_H \theta_H \Lambda^e + \lambda_H \gamma - (1-\lambda_H)\gamma\beta]\hat{H}_e \\
&\quad + \Lambda_2^e \beta \hat{P}_1 - \lambda_{L2}\beta\hat{K}\}, && (2.22)\\
\hat{U}_e &= \frac{1}{\tilde{\Lambda}}\{[\lambda_H \theta_H \Lambda^e + (1-\lambda_H)\lambda_{L1}\beta - \lambda_H \lambda_{L1}]\hat{H}_e \\
&\quad + \lambda_H(1+\beta)\Lambda_2^e \hat{P}_1 - \lambda_H(1+\beta)\lambda_{L2}\hat{K}\}, && (2.23)\\
\hat{L} &= \frac{\gamma}{\tilde{\Lambda}}\{[\lambda_H \lambda_{L1} - \lambda_H \theta_H \Lambda^e - (1-\lambda_H)\lambda_{L1}\beta]\hat{H}_e \\
&\quad - \lambda_H(1+\beta)\Lambda_2^e \hat{P}_1 + \lambda_H(1+\beta)\lambda_{L2}\hat{K}\}, && (2.24)
\end{aligned}$$

where

$$\tilde{\beta} \equiv \frac{(\lambda_{L1} + \lambda_H \gamma)\beta + \lambda_H \gamma}{\lambda_H \theta_H} > 0,$$

$$\tilde{\Lambda} \equiv \lambda_H \theta_H (\Lambda^e + \tilde{\beta}) > 0.$$

2.4.1 Commodity Prices and Factor Prices

From equation (2.21), we can see that the effects of P_1 and K on W_L is similar to that in the standard RV model, particularly the result where $0 <$

$\hat{W}_L/\hat{P}_1 < 1$[7]. To see this, we can compare equations (2.19) and (2.21). Let us define

$$\hat{S} \equiv \hat{L} - (\lambda_{L1}\hat{H}_p + \lambda_{L2}\hat{K}).$$

We call $\hat{S}$ as the proportionate change in the supply of unskilled labor relative to an "aggregate" of the specific factors[8]. Substitute equations (2.22) and (2.24) into $\hat{S}$, we obtain

$$\hat{S} = -\frac{\Lambda_2^e\tilde{\beta}}{\Lambda^e + \tilde{\beta}} \cdot \hat{P}_1 - \frac{\Lambda^e(\lambda_{L1} + \gamma)}{\Lambda^e + \tilde{\beta}} \cdot \hat{H}_e - \frac{\Lambda^e\lambda_{L2}}{\Lambda^e + \tilde{\beta}} \cdot \hat{K}. \tag{2.25}$$

Substitute equation (4.1) into equation (2.19), we have

$$\hat{W}_L = \frac{1}{\Lambda^e}(\Lambda_1^e\hat{P}_1 - \hat{S}),$$

or in an explicit form as

$$\hat{W}_L = \frac{1}{\Lambda^e}(\Lambda_1^e\hat{P}_1 + \frac{\Lambda_2^e\tilde{\beta}}{\Lambda^e + \tilde{\beta}} \cdot \hat{P}_1 + \frac{\Lambda^e(\lambda_{L1} + \gamma)}{\Lambda^e + \tilde{\beta}} \cdot \hat{H}_e + \frac{\Lambda^e\lambda_{L2}}{\Lambda^e + \tilde{\beta}} \cdot \hat{K}). \tag{2.19$'$}$$

Holding $\hat{H}_e$ and $\hat{K}$ being fixed, we have decomposed the effect of P_1 on W_L in more concrete way. In the right hand side, the first term in the bracket relates to the conventional direct effect as in the standard RV model, while

[7]Note further that, this eventually implies also that $\hat{W}_H < \hat{P}_1 < \hat{W}_L < \hat{P}_2 = 0 < \hat{r}$ which is the familiar one in the standard RV model.

[8]See Bhagwati and Jagdish (c1983)

the second term relates to the indirect effect through the change in factor supply (H_p and L) due to the change in relative wage (W_L/W_H) which is caused by the change in relative price of final goods (P_1). Further, because it is true that

$$\frac{1}{\Lambda^e}(\Lambda_1^e + \frac{\Lambda_2^e\tilde{\beta}}{\Lambda^e + \tilde{\beta}}) = \frac{\Lambda_1^e + \tilde{\beta}}{\Lambda^e + \tilde{\beta}} < 1,$$

since $\Lambda_1^e < \Lambda^e$. Hence we have

$$\hat{W}_L < \hat{P}_1.$$

From equations $(2.17a')$ and $(2.17b')$, it is easy to see that

$$\hat{W}_L - \hat{W}_H < 0, \qquad \hat{W}_L - \hat{r} > 0.$$

Let us make a clear distinction between $\hat{W}_L$ and $\hat{W'}_L$ where

$$\hat{W'}_L = \frac{\Lambda_1^e}{\Lambda^e} \cdot \hat{P}_1.$$

$(')$ represents the conventional direct effects in the standard RV model. Similarly, we can also define $\hat{W'}_H$ and $\hat{r}'$ in the same sense. Hence we have,

$$\hat{W}_L > \hat{W'}_L,$$

and from equations $(2.17a')$ and $(2.17b')$, we also have

$$\hat{W'}_H > \hat{W}_H, \qquad \hat{r}' > \hat{r},$$

which yields

$$\hat{W}'_H > \hat{W}_H > \hat{P}_1 > \hat{W}_L > \hat{W}'_L > \hat{P}_2 = 0 > \hat{r}' > \hat{r}.$$

Let us conclude the argument as below.

Proposition 2.1

In the public intermediate good economy, the theory of standard RV model remains valid, that is $\hat{W}_H > \hat{P}_1 > \hat{W}_L > \hat{P}_2 = 0 > \hat{r}$. However, the mobile factor gains more, while the immobile factors gain less (lose more) than that in the standard RV model.

This is not surprising. In the standard RV model, $\hat{S}/\hat{P}_1 = 0$, but in this chapter, from equation (2.25), we know that $\hat{S}/\hat{P}_1 < 0$. This can be seen in more obvious way in equations (2.23) and (2.24), that is, an increase in P_1 raises U_e and reduces L, causing the supply of unskilled labor relative to an "aggregate" of the specific factors decrease, hence there is an additional positive indirect effect for the mobile factor and negative indirect effects for the specific factors.

2.4.2 Change in Human Capital

In this subsection, we examine the change in human capital or skilled labor. This examination is essential for clarifying the effect on final good output as shown in equations (2.20a) and (2.20b). From the equation (2.22), we know that an increase in P_1 raises H_p. On the other hand, an increase in K reduces H_p. This can be predicted easily from the proposition 2.1, that is, an increase in P_1 (K) reduces (raises) relative wage, it follows that from equations (2.14) and (2.16), we know that H_p increases (decreases) due to the increase (decrease) in U_e. However, the effect of H_e is a bit complicated here. From equation (2.21), we can see that the sign depends mainly on β which can be 0 or infinitively large if σ^q is 1 or 0. Let us rewrite the equation (2.16) as

$$\hat{H}_p = \hat{H}_A + \hat{H}_B + \hat{H}_C, \tag{2.16$'$}$$

where

$$\begin{aligned}
\hat{H}_A &\equiv \frac{1}{\lambda_H(1+\beta)}(\hat{H}_e - \hat{U}_e),\\
\hat{H}_B &\equiv \frac{1}{\lambda_H} \cdot \hat{U}_e,\\
\hat{H}_C &\equiv -\frac{1-\lambda_H}{\lambda_H} \cdot \hat{H}_e < 0.
\end{aligned}$$

$\hat{H}_A$, $\hat{H}_B$ and $\hat{H}_C$ are, respectively, the effect which positively depends educator-student ratio, the effect which positively depends on number of students and the negative input effect. We call $\hat{H}_A$, $\hat{H}_B$ and $\hat{H}_C$ as, respectively, quality effect, quantity effect and input effect. Let us first conclude from equation (2.16$'$) as below.

Lemma 2.1 *In the public intermediate good economy, the effect on the net output of human capital is decomposed into three parts which are called quality effect, quantity effect and input effect.*

There are many studies show that quality effect depends on positively on educator-student ratio[9]. From equation (2.23), we know that an increase in H_e may increase or decrease U_e hence the quantity effect is ambiguous. One may argue that in the case that U_e increases, an increase in H_e may even reduce the educator-student ratio, then the quality effect should also be ambiguous. But this is not right, since we have examined that $(\hat{W}_L - \hat{W}_H)/\hat{H}_e$ is positive[10], from equation (2.14) we know that an increase in H_e must raise the educator-student ratio. To see this in an explicit way, substitute equation (2.21) into equation (2.14′) and into equation (2.16′), we have

$$\begin{aligned}
\hat{H}_A &= \frac{1}{\tilde{\Lambda}}[(\lambda_{L1} + \gamma)\hat{H}_e - \Lambda_2^e \hat{P}_1 + \lambda_{L2}\hat{K}], \\
\hat{H}_B &= \frac{1}{\tilde{\Lambda}}\{[\theta_H(\Lambda^e + \tilde{\beta}) - (1+\beta)(\lambda_{L1} + \gamma)]\hat{H}_e \\
&\quad +(1+\beta)\Lambda_2^e \hat{P}_1 - (1+\beta)\lambda_{L2}\hat{K}\}.
\end{aligned}$$

[9]Becker and Murphy (1992) also note that some good empirical studies like Card and Krueger (1990) and Finn and Achilles (1990) found some evidence to the assumption above.

[10]See equations (2.21) and (2.17a').

At present moment, let $\hat{P}_1$ and $\hat{K}$ be 0 and sum up the quality effect and the quantity effect, we obtain

$$\hat{H}_A + \hat{H}_B = \frac{1}{\tilde{\Lambda}}\{\theta_H \Lambda^e + \gamma + \frac{1}{\lambda_H}[(1-\lambda_H)\lambda_{L1}\beta]\} > 0.$$

Further, since $\hat{H}_C < 0$, the total effect must be ambiguous. Recall the equation (2.22), its sign can be expressed as

$$\frac{\hat{H}_p}{\hat{H}_e} \overset{>}{\underset{<}{=}} 0 \qquad if \qquad \lambda_H \overset{>}{\underset{<}{=}} \eta_H.$$

where $\eta_H \equiv \dfrac{\gamma\beta}{\gamma\beta + \gamma + \theta_H \Lambda^e}$. Note that η_H and λ_H are smaller than 1. Let us conclude the argument above as

Lemma 2.2 *(a). $\lambda_H > \eta_H$ if σ^q or/and Λ^e is/are significantly large.*

(b). $\lambda_H < \eta_H$ if both σ^q and Λ^e are significantly small.

Further, we can establish the proposition as below.

Proposition 2.2

In the public intermediate good economy, the supply of human capital increases (decreases), if there is

(a). an increase (decrease) in relative price of commodity where human capital is used to produce that commodity, or/and

(b). an decrease (increase) in capital, or/and

(c). an increase in provision of public service if $\lambda_H > (<) \eta_H$.

The effects of P_1 and K are obvious since an increase in P_1 and decrease in K reduce W_L/W_H, this makes more individual become students, that is, U_e increases hence the quantity effect is positive. On the other hand, the quality effect becomes negative, but this will be dominated by the positive quantity effect as shown in equation (2.16′). The effect of an increase in provision says that there is an maximum point for the supply of human capital, this implies that if H_e is so large, hence $\lambda_H < \eta_H$, an decrease in the provision of public service can increase the supply of human capital.

2.4.3 Change in Outputs of Final Goods

In this subsection we will examine the effect on outputs of final goods before we examine the trade pattern in the next section. Substitute equations (2.22) and (4.1) into equation (2.20a), and equation (4.1) into equation (2.20b), we obtain

$$\hat{X}_1 = \frac{1}{\tilde{\Lambda}}\{\Lambda_2^e \ddot{\beta}\hat{P}_1 + [\lambda_H(\gamma\beta + \gamma + \tilde{\Lambda}_e) - \gamma\beta]\hat{H}_e - \lambda_{L2}\ddot{\beta}\hat{K}\}, \qquad (2.20a')$$

$$\hat{X}_2 = -\frac{\theta_{L2}e_{L2}}{\Lambda^e + \tilde{\beta}}[(\Lambda_1^e + \tilde{\beta})\hat{P}_1 + (\lambda_{L1} + \gamma)\hat{H}_e] + \frac{\ddot{\Lambda}^e + \tilde{\beta}}{\Lambda^e + \tilde{\beta}} \cdot \hat{K}, \qquad (2.20b')$$

where

$$\ddot{\beta} \equiv \lambda_H \theta_H \theta_{L1} e_{L1} + \beta > 0,$$

$$\tilde{\Lambda}^e \equiv \theta_H \{[(1 - \theta_{L1})\lambda_{L1} - \theta_{L1}\gamma] e_{L1} + \Lambda_2^e\},$$

$$\ddot{\Lambda}^e \equiv \Lambda_1^e + (1 - \theta_{L2})\Lambda_2^e + \tilde{\beta} < \Lambda^e.$$

Define $X \equiv X_1/X_2$ as relative output of commodity 1. From equation $(2.20a')$ and $(2.20b')$ we have

Lemma 2.3 *In the public intermediate good economy, an increase (decrease) in P_1 (K) raises X_1 but reduces X_2, hence we have $\hat{X}/\hat{P}_1 > 0$ and $\hat{X}/\hat{K} < 0$. An increase in H_e reduces X_2, whereas the effect on X_1 is ambiguous hence $\hat{X}/\hat{H}_e$ remains ambiguous as well.*

Let P_1 and K be fixed, we have

$$\frac{\hat{X}}{\hat{H}_e} = \frac{\lambda_H(\gamma\beta + \gamma + \bar{\Lambda}^e) - \gamma\beta}{\tilde{\Lambda}}, \tag{2.26}$$

where $\bar{\Lambda}^e \equiv \theta_H[\Lambda^e + (\lambda_{L1} + \gamma)(\theta_{L2}e_{L2} - \theta_{L1}e_{L1})]$.

Note that $\tilde{\Lambda}^e$ and $\bar{\Lambda}^e$ may be negative if θ_{L1} and e_{L1} are so large, for simplicity, let us assume that

Assumption 2.1

θ_{L1} is sufficiently small so that $\tilde{\Lambda}^e$ and $\bar{\Lambda}^e$ are positive.

From equations (2.20a') and (2.26) we know that

$$\frac{\hat{X}_1}{\hat{H}_e} \underset{<}{\overset{>}{=}} 0 \qquad if \qquad \lambda_H \underset{<}{\overset{>}{=}} \eta_{X_1},$$

$$\frac{\hat{X}}{\hat{H}_e} \underset{<}{\overset{>}{=}} 0 \qquad if \qquad \lambda_H \underset{<}{\overset{>}{=}} \eta_X,$$

where

$$\eta_{X_1} \equiv \frac{\gamma\beta}{\gamma\beta + \gamma + \tilde{\Lambda}^e}, \qquad \eta_X \equiv \frac{\gamma\beta}{\gamma\beta + \gamma + \bar{\Lambda}^e}.$$

This is very similar to the argument in the lemma 2.2. Thus we can conclude that

Lemma 2.4 *Let assumption 2.1 be satisfied, then*

(a). $\lambda_H > \eta_{X_1}$ and $\lambda_H > \eta_X$ are true if σ^q or/and Λ^e is significantly large.

(b). $\lambda_H < \eta_{X_1}$ and $\lambda_H < \eta_X$ are true if both σ^q and Λ^e are significantly small[11].

Then we can establish the proposition as below.

[11] Since assumption 2.1 is satisfied, the change in Λ^e corresponds to the change in both $\tilde{\Lambda}^e$ and $\bar{\Lambda}^e$.

Proposition 2.3

Let assumption 2.1 be satisfied, then, in the public intermediate good economy, an increase in provision of public service raises (reduces) the relative supply of commodity 1 as well as its output if σ^q or/and (and) Λ^e is (are) significantly large (small).

This relates to the proposition 2.2, once we know the effect on H_p, we can predict what would happen according to the results in standard RV model.

2.5 Trade Pattern

In this section, we examine the trade pattern by assuming two countries in the world.

To see how the relative price of commodities changes, the demand side of commodities has to be stated explicitly. We can express the relative demand D as a function of relative price of the commodities P_1 on the demand side, if we assume homothetic preferences , then the domestic market equilibrium is expressed as

$$X = D(P_1).$$

Differentiating the equation above and consider the sign of $\hat{X}/\hat{P}_1$ and $\hat{X}/\hat{H}_e$

which have been examined, we have[12]

$$\frac{\hat{P}_1}{\hat{H}_e} = -(\frac{\hat{X}}{\hat{P}_1} + \sigma_D)^{-1} \cdot \frac{\hat{X}}{\hat{H}_e}, \tag{2.27}$$

where $\sigma_D \equiv -D'(P_1)P_1/D(P_1) > 0$ is the price elasticity of demand. Since $\hat{X}/\hat{P}_1 > 0$, the sign of $\hat{P}_1/\hat{H}_e$ depends solely on $\hat{X}/\hat{H}_e$. Hence we will have the same argument again as in the proposition 2.3 as

Proposition 2.4

Suppose that there are two countries with the public intermediate good economy, where

(a). preferences, technology and population are identical,

(b). the share of the unskilled labor in the cost of producing commodity 1 is sufficiently small, that is θ_{L1} is sufficiently close to 0.

Then, the country allocate more (less) skilled labor into the public sector tends to be skilled labor abundant country, hence exports commodity 1 and imports commodity 2 if σ^q or/and (and) Λ^e is (are) significantly large (small).

The proposition implies that, for example, consider a government intends to enhance the competitiveness of a selective sector which uses human capital through the public service to develop and support the formation of human capital. If the effect on the quality per student is so small under the public

[12] See appendix in this chapter.

service, the fraction of the skilled labor allocated into the public sector may be too large hence the government's policy may bring the result which is opposite to the target[13].

2.6 Concluding Remarks

This chapter has examined how the difference in allocation of human capital and effect of the provision of the public service on the formation of human capital determine the change in labor supply. In particular, we have examined the formation of the skilled labor which can be decomposed into quality effect, quantity effect and input effect. The results then immediately tell us whether the private sectors expand or not as in the conventional results shown in the standard RV model. One of the most important point is that whether the effect on the quality per student is so small under the public service, this effect will affect the entire effectiveness of the skilled labor supply in the sector. If it is possible to have such case, all the private sectors in the public intermediate good economy will shrink, and the GNP in the country may fall as well although we have ignored the definition and the examination of welfare throughout this chapter. Another viewpoint of this chapter is, if brain drain occurs in the selective private sector, the fraction of the total

[13]Compare to the proposition 1 in Abe (1990).

skilled labor for the public sector will become larger than before. As a result, if the government reallocates additional skilled labor from the private sector into the public sector may aggravate the economy of the country, otherwise continue to increase the public service will enhance the competitiveness of the selective industry but gradually reach to the maximum point and turn to deteriorate. We should also point out that an endogenous provision of the public service model may be more appropriate. Further, the reason for government to enhance a selective sector should be incorporated into the model. What we have presented here are the basic framework for the future research.

A Appendix

In this appendix, we will show how we obtain the equation (2.27). Since we know X is a function of P_1 and H_e, while D is a function of P_1, and the domestic market equilibrium which can be expressed as

$$X(P_1, H_e) = D(P_1). \tag{A.1}$$

P_1 is determined in equation (A.1) as a function of H_e, which can be expressed as

$$P_1 = P_1^*(H_e). \tag{A.2}$$

Equation (A.1) becomes an identity if we substitute equation (A.2) into equation (A.1), which can be expressed as

$$X(P_1^*(H_e), H_e) = D(P_1^*(H_e)). \tag{A.3}$$

Differentiating equation (A.3), we obtain

$$\frac{\partial P_1^*}{\partial H_e} = -(\frac{\partial X}{\partial P_1} - \frac{\partial D}{\partial P_1})^{-1} \cdot \frac{\partial X}{\partial H_e}, \tag{A.4}$$

which can be rewritten as equation (2.27).

Notice also that the first term and the second term in the bracket of RHS are positive, hence we know that the sign of $\partial P_1^*/\partial H_e$ is negative if $\partial X/\partial H_e$ is positive. However, the sign of $\partial X/\partial H_e$ is ambiguous in this model.

Chapter 3

Trade Patterns and Brain Drain with Public Human Capital Formation

3.1 Introduction

It is widely known that skilled workers tend to migrate from developing countries to advanced industrial nations[1]. Developed countries usually have the comparative advantage in the production of high-tech good using skilled workers, in the meantime, skilled workers also get higher wage compared to

[1] See, for example, a 1984 report (July 20) by the United Nations Conference on Trade and Development (UNCTAD).

developing countries. This also implies that skilled workers prefer to migrate to developed countries as long as they prefer higher wage. However, this violates the basic propositions in the frame work of RV model, i.e., a country with larger supply of skilled workers which are specific to hich-tech sector, has the comparative advantage in the production of the good, but lower factor price for skilled workers.

This chapter incorporates public human capital formation into the basic model of Findlay and Kierzkowski (1983) to provide an explanation of the issue above. They construct a model with two kinds of individual with equal lifetime incomes in terms of present value which is based on the standard HOS Model [2]. They show the additional effects of the change in prices compared to the conventional model. In their model, publicly provided education service does not exit and the education cost is fully financed by the students [3].

[2]Mayer (1982), shows factor quality considerations into Heckscher-Ohlin framework and examines the importance of factors skills in determining a country's production pattern and income distribution, while Mayer (1991) shows also the impacts of world price, capital endowment on labor supply, output and national income.

[3]Although there is a trend that many universities start charging tuition to the students in many countries, but the role of publicly provided education service is still significant nowadays. In order to make our results more clearly, we focus only on the role of publicly provided education and assume that privately provided education does not exit, which is crucially different from Findlay and Kierzkowski (1983).

In this chapter, there are three kinds of factors, that is, capital, unskilled workers and skilled workers which are refered as human capital. However, the human capital is assumed to be produced by government through public service in our model. Government can reallocate more human capital into the public sector by extracting human capital [4] from the private sector.

This chapter shows that the supply of human capital does not necessarily increase even if government employs more educators for public sector. On the issue between international trade and brain drain[5] , this chapter also shows that even if a country exports a good using human capital which is specific factor, the factor price for the human capital in the country can still be higher than that in the foreign. As a result, the human capital flows from the foreign into the country. This result is opposite from the traditional RV model, which does not help to explain the relationship between trade patterns and factor mobility in most cases for many countries.

Miyagiwa (1991) and Wong and Yip (1999), emphasize the role of increasing returns to scale in education and overlapping-generations model of endogenous growth, respectively. Compared to their studies, this chapter presents only a very simple model following the basic assumptions such as

[4] In this case, educators in universities are referred as human capital.

[5] Some recent studies on brain drain are remarkable, e.g., see Mullan (2005) and Horvat (2005).

constant returns to scale in education and perfect competition in private sectors, but still provide some explanations for the issue of human capital mobility between advanced industrial countries and developing countries. Other than that, this chapter also examines whether a government can enhance the competitiveness of high-tech sector by hiring more educators.

The model is presented in the next section. The effects of public service are examined in section 3. Our propostion about the trade patterns is obtained in section 4. Section 5 discuss the issue of brain drain. Some remarks on our conclusion appear in the final section.

3.2 The Model

We introduce a country with public human capital formation. There are two private and one public sectors in the country, where one of the private sectors produces high-tech final good using human capital and unskilled workers, while the other private sector produces low-tech final good using physical capital and unskilled workers[6]. Unskilled workers is mobile between private sectors while human capital and physical capital are factor specific to high-tech sector and low-tech sector, respectively. Public sector provides education service to the students for free. We assume that only educators

[6]At the present moment, we implicitly assume a small open country model.

are required for the education service. Therefore, the public sector produces human capital using only educators and students[7]. For the time being, let us show the standard RV model here. The production functions are expressed as[8]

$$X_1 = L_1^{\alpha} H_p^{1-\alpha}, \qquad 0 < \alpha < 1,$$

$$X_2 = L_2^{\beta} K^{1-\beta}, \qquad 0 < \beta < 1,$$

where X_1, X_2, L_1, L_2, H_p and K are high-tech final good produced in high-tech sector (i.e., sector 1), low-tech final good produced in low-tech sector (i.e., sector 2), unskilled workers employed in high-tech sector and low-tech sector, human capital specific to high-tech sector and physical capital specific to low-tech sector, respectively. Let W_L, W_H and r denote the factor prices of unskilled workers, human capital and physical capital, respectively. Using the unit cost functions[9], the final goods market equilibrium conditions will

[7]Educators are also regarded as human capital. On the other hand, students themselves also become the human capital after graduation.

[8]Cobb-Douglas functions will make our analysis become simpler. Moreover, we can obtain sharper results easily compared to those general functions which will not make significant difference.

[9]The unit cost functions are defined as

$\min_{L_1, H_p} \{W_L L_1 + W_H H_p | L_1^{\alpha} H_p^{1-\alpha} \geq 1\}$, and

$\min_{L_2, K} \{W_L L_2 + rK | L_2^{\beta} K^{1-\beta} \geq 1\}$.

be given by

$$\left(\frac{W_L}{\alpha}\right)^{\alpha}\left(\frac{W_H}{1-\alpha}\right)^{1-\alpha} = P, \tag{3.1}$$

$$\left(\frac{W_L}{\beta}\right)^{\beta}\left(\frac{r}{1-\beta}\right)^{1-\beta} = 1, \tag{3.2}$$

where low-tech good serves as the numeraire, and P is the relative price of high-tech good in terms of the numeraire. Full employment conditions are expressed as

$$\left(\frac{\alpha}{1-\alpha}\cdot\frac{W_H}{W_L}\right)^{1-\alpha}X_1 + \left(\frac{\beta}{1-\beta}\cdot\frac{r}{W_L}\right)^{1-\beta}X_2 = L, \tag{3.3}$$

$$\left(\frac{1-\alpha}{\alpha}\cdot\frac{W_L}{W_H}\right)^{\alpha}X_1 = H_p, \tag{3.4}$$

$$\left(\frac{1-\beta}{\beta}\cdot\frac{W_L}{r}\right)^{\beta}X_2 = K. \tag{3.5}$$

Given P, K, L, H_p, α and β, we can solve for W_L, W_H, r, X_1 and X_2 from equations (3.1) to (3.5). This is only the familiar basic RV model[10] which is much simpler than what we are going to extend[11].

At the present model, we only consider a small open country without any international factor mobility. Human capital can be allocated into either private sector (i.e., high-tech sector) or public sector which can be expressed as

$$H = H_p + H_e, \tag{3.6}$$

[10]See Jones (1971).

[11]L and H_p will be endogenously determined.

where H and H_e denote the total supply of domestic human capital and the supply of educators, respectively.

As in the traditional RV model, we assume the conditions of full employment and perfect competition are always satisfied in the country. However, unskilled workers and human capital are treated as endogenous variables in this chapter. We follow the basic concept of Findlay and Kierzkowski (1983)[12]. N individuals are born and N individuals die in each period in the economy, all live for T periods. This means that the population will always be NT in the steady state[13]

We assume education service is publicly provided by government for individuals free of charge in the country, rather than privately provided as assumed in Findlay and Kierzkowski (1983). Either individuals can be "unskilled workers" and immediately start earning W_L for their whole life, or they can become "students," acquire an "education" that last for a fixed length of time θ, and become "skilled workers," earning W_H for the fixed length of time $(T - \theta)$. Thus, for each generation,

$$N = U_l + U_e \tag{3.7}$$

[12]The pioneering contribution of Kemp and Jones (1962) and elaboration by Frenkel and Razin (1975), Martin (1976) and Martin and Neary (1980) in the literature on variable labor supply are also remarkable.

[13]There are T generations and each generation has N individuals.

must be satisfied, where U_l and U_e donote the individuals who choose to become unskilled workers and students respectively.

Government employs skilled workers as "educators" from high-tech sector into the public sector. The term of "human capital" in our model includes both skilled workers and educators. We assume that human capital is mobile between high-tech sector and public sector. This means that the government will only pay to the educators with the same going wage for skilled workers. Therefore, education cost is expressed as

$$W_H H_e.$$

The education cost is financed by the income tax[14], then the government budget constraint is expressed as

$$W_H H_e = \tau(W_L L + W_H H + rK),$$

where τ is the income tax rate[15].

We assume domestic human capital can be produced with Cobb-Douglas production function[16] in the public sector which can be expressed as[17]

$$H = f(\theta) H_e^{\gamma} U_e^{1-\gamma}, \qquad 0 < \gamma < 1, \tag{3.8}$$

[14]See Abe (1990)

[15]τ can be solved with this equation, but we do not focus on the its effects in this chapter.

[16]See footnote 8

[17]Since θ is assumed to be fixed throughout this chapter, differentiation of $f(\theta)$ could be omitted.

where γ can be interpreted as effect of educator-student ratio (H_e/U_e) on human capital quality, or in other words, on human capital per student (H/U_e) can be acquired by individuals who choose to be educated since it can be expressed as

$$\gamma = \frac{\partial(H/U_e)}{\partial(H_e/U_e)} \cdot \frac{H_e/U_e}{H/U_e}.$$

The government acts like a producer who produces 'human capital'[18] at each period of t, using students and human capital itself as inputs[19].

Now, we need to describe how U_e and U_l make their decisions. For simplicity, we also assume that the domestic physical capital stock is owned by all the individuals and there is perfectly equality in distribution of the capital stock[20]. Then, in each period of t, each individual receives rk equally, where $k = K/NT$. The lifetime income after tax for an unskilled worker and skilled worker would therefore result in

[18] Compared to Ishikawa (2000) who shows a RV model with an intermediate good. However, in his model, intermediate good is served as input for final good but not for itself.

[19] Compared to Becker and Murphy (1992), which show that human capital acquired by a student depends on the human capital of her teachers, and the number of teachers per student. They also note that some good empirical studies like Card and Krueger (1990) and Finn and Achilles (1990) found some evidence to the assumption above.

[20] Many studies assume this, for example, see Gupta (1994).

$$(1-\tau)\int_0^T (rk+W_L)\cdot e^{-\rho t}dt$$
$$= (1-\tau)\cdot\frac{1}{\rho}\Big[(rk+W_L)(1-e^{-\rho T})\Big],$$
$$(1-\tau)\Big[\int_0^T rk\cdot e^{-\rho t}dt+\int_\theta^T W_H\cdot\frac{H}{U_e}\cdot e^{-\rho t}dt\Big]$$
$$= (1-\tau)\cdot\frac{1}{\rho}\Big[rk(1-e^{-\rho T})+W_H\cdot\frac{H}{U_e}(e^{-\rho\theta}-e^{-\rho T})\Big],$$

respectively, where ρ is fixed interest rate. As Findlay and Kierzkowski (1983) points out, the lifetime income after tax for every individuals must be equal in the long run equilibrium, which implies that

$$\frac{H}{U_e}=\frac{W_L}{W_H}\cdot R, \tag{3.9}$$

where $R\equiv(1-e^{-\rho T})/(e{-}\rho\theta-e^{-\rho T})$[21]

H and U_e can be solved with equations (3.8) and (3.9) simultaneously, given H_e , W_L and W_H. Substituting H into equation (3.6) we can solve for H_p.

U_l can be solved with equation (3.7). Since there are only 2 generations at each period of t, unskilled workers supply is given by

$$L=U_lT. \tag{3.10}$$

[21]Note that $dB_H/dU_e<0$, where $B_H\equiv(1-\tau)\cdot\frac{1}{\rho}\Big[rk(1-e^{-\rho T})+W_H\cdot\frac{H}{U_e}(e^{-\rho\theta}-e^{-\rho T})\Big]$ thus we know that U_e must be determined under the arbitrary condition.

The model can be solved from equations (3.1) to (3.10) to solve for 10 variables, that is, W_L, W_H, r, X_1, X_2, H_p, L, U_e, U_l and H, given P, H_e, as well as α, γ, β, K, N and i which are assumed to be fixed throughout this chapter[22].

3.3 Preliminaries

Let N, K and i be fixed throughout this chapter. Differentiating equations from (3.1) to (3.10), the equations can be reduced as

$$\begin{bmatrix} \Lambda & -\lambda_{L1} & -\dfrac{1-\lambda_{Ul}}{\lambda_{Ul}} \\ 0 & 1 & -\dfrac{(1-\gamma)}{\delta_p} \\ \dfrac{1}{\gamma(1-\alpha)} & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{W}_L \\ \\ \hat{H}_p \\ \\ \hat{U}_e \end{bmatrix}$$

[22]In particular, H_p and L can be solved as functions of $(W_L, W_H, r; \cdot)$ while W_L, W_H and r can be solved as functions of $(P, H_p, L; \cdot)$, where $(\cdot)$ represents other variables treated exogenously. Since W_H and r can also be solved as functions of (P, W_L) which is familiar in RV model. In the end, the system above can be reduced to 3 equations which must be solved simultaneously for W_L, H_p and L.

$$= \begin{bmatrix} \Lambda_1 \\ \\ 0 \\ \\ \dfrac{1}{\gamma(1-\alpha)} \end{bmatrix} \hat{P} + \begin{bmatrix} 0 \\ \\ \dfrac{\gamma - (1-\delta_p)}{\delta_p} \\ \\ 1 \end{bmatrix} \hat{H}_e \tag{3.1}$$

where

$$\lambda_{Li} \equiv \frac{L_i}{L}, \qquad 0 < \lambda_{Li} < 1, \qquad for\ i = 1, 2, \qquad \lambda_{L1} + \lambda_{L2} = 1,$$
$$\delta_p \equiv \frac{H_p}{H_p + H_e}, \qquad 0 < \delta_p < 1,$$
$$\lambda_{Ul} \equiv \frac{U_l}{U_l + U_e}, \qquad 0 < \lambda_{Ul} < 1,$$
$$\Lambda_1 \equiv \frac{\lambda_{L1}}{1-\alpha}, \qquad \Lambda_2 \equiv \frac{\lambda_{L2}}{1-\beta}, \qquad \Lambda \equiv \Lambda_1 + \Lambda_2.$$

($\hat{\ }$) denotes a proportionate change, for example, $\hat{W}_L = dW_L/W_L$. In particular, notice that δ_p represents the allocative share of domestic human capital in private sector. In addition, we also have

$$\hat{X}_1 = \hat{H}_p - \frac{\alpha}{1-\alpha}(\hat{W}_L - \hat{P}), \tag{3.2}$$
$$\hat{X}_2 = -\frac{\beta}{1-\beta} \cdot \hat{W}_L, \tag{3.3}$$

Equations (3.2) and (3.3) are so familiar where the first term in the RHS of equation (3.2) represents the direct effect of human capital on the high-tech good while keeping the factor prices hypothetically constant. We call this effect as the direct effect. The second terms in the RHS of equation (3.2)

	$\hat{P}$	$\hat{H}_e$
$\hat{W}_L$	$+$	$+$
$\hat{H}_p$	$+$	?
$\hat{X}_1$	$+$	?
$\hat{X}_2$	$-$	$-$
$\hat{X}_1 - \hat{X}_2$	$+$	?

Table 3.1: The table shows the effects of P, H_e on W_L, H_p, X_1, X_2 and X_1/X_2, respectively. For example, the effect of P on W_L is shown as '+', and so on. '?', '+' and '−' refer to indefinite effect, positive effect and negative effect, respectively.

and RHS of equation (3.3) represent effects on high-tech good and low-tech good, respectively, due to the change in the factor prices which originated from the disturbance in the factor markets. We call this as the indirect effect.

Using Cramel's rule to solve equation (3.1), then substitute $\hat{W}_L$ and $\hat{H}_p$ into equations (3.2) and (3.3), we obtain the direct effects, indirect effects and total effects of P, H_e on X_1, X_2 and X_1/X_2 which are shown in table 3.1.

The effects of H_e on H_p, X_1 and X_1/X_2 are ambiguous. Since the ambiguous effects mainly originated from the change in H_p, in particular, we show that the effects of H_e on H_p can be decomposed into three parts which can be expressed as

$$\hat{H}_p = \frac{1}{\delta_p}\Big[\gamma(\hat{H}_e - \hat{U}_e) + \hat{U}_e - (1-\delta_p)\hat{H}_e\Big], \tag{3.4}$$

or alternatively,

$$\frac{\hat{H}_p}{\hat{H}_e} = \frac{1}{|A|\delta_p}\left[\Lambda\delta_p + \frac{(1-\lambda_{Ul})(\gamma+\delta_p-1)}{\lambda_{Ul}\gamma(1-\alpha)}\right], \tag{3.5}$$

where

$$|A| \equiv \Lambda + \frac{1}{\gamma(1-\alpha)}\left[\frac{\Lambda_1(1-\gamma)}{\delta_p} + \frac{1-\lambda_{Ul}}{\lambda_{Ul}}\right] > 0,$$

In general, the sign of $\hat{H}_p/\hat{H}_e$ is not determined, however, we know that the sign is positive (negative) if and only if

$$\delta_p > (<) \frac{(1-\lambda_{Ul})(1-\gamma)}{1-\lambda_{Ul}+\gamma\lambda_{Ul}\Big(\lambda_{L1}+\dfrac{1-\alpha}{1-\beta}\cdot\lambda_{L2}\Big)}, \tag{3.6}$$

where RHS is obviously between 0 and 1 since the numerator is smaller than the denominator and both of them are positive as well. In particular, we can see that the condition is satisfied easier with larger β and γ, but smaller α.

The first term in the brace of equation (3.4) represents the effects on educator-student ratio. Since higher quality of human capital can be acquired as the ratio is higher, we call this as quality effect. The second term in the brace of equation (3.4) represents the effects on number of individuals who decide to be educated, we call this as quantity effect. The last term represents the input effect which is negative, we call this as crowding out effect. The total effect particularly depends on δ_p, that is, allocative share of domestic human capital in private sector. In particular, we can conclude as

Lemma 3.1 *In the country with public human capital formation, an increase in P always increases the human capital in private sector. On the other hand, an increase in H_e increases (decreases) human capital in private sector, if γ and β are sufficiently large (small) while α is sufficiently small (large).*

Recall that γ represents the effect of educator-student ratio on human capital quality. Larger effect means larger human capital per capita that students can acquire, hence higher productivity and income they can get. This also makes more individuals are willing to choose being educated. The problem is whether the number of students will increase significantly hence overcome the negative crowding out effect. This depends on the elasticities of demand for unskilled workers in high-tech sector and low-tech sector, which are donoted by α and β, respectively. Recall the familiar traditional RV model, if β is large and α is small, then elasticity of demand for unskilled workers is large in high-tech sector and small in low-tech sector, it follows that higher W_H/W_L can be realized hence more individuals are willing to choose being educated

.

The indeterminacy of effect of H_e on H_p also brings ambiguous effects on X_1 and X_1/X_2. The signs of $\hat{X}_1/\hat{H}_e$ and $(\hat{X}_1-\hat{X}_2)/\hat{H}_e$ are positive (negative) if and only if

$$\delta_p > (<) \frac{A}{B}, \tag{3.7}$$

$$\delta_p > (<) \frac{C}{D}, \tag{3.8}$$

respectively, where

$$
\begin{aligned}
A &\equiv (1-\lambda_{Ul})(1-\gamma) \\
B &\equiv (1-\lambda_{Ul})(1-\alpha\gamma) + \gamma(1-\alpha)\lambda_{Ul}\Big(\lambda_{L1} + \frac{\lambda_{L2}}{1-\beta}\Big) \\
C &\equiv (1-\lambda_{Ul})(1-\gamma) \\
D &\equiv (1-\lambda_{Ul})(1-\alpha\gamma) + \frac{\gamma(1-\alpha)}{1-\beta}\Big[\lambda_{Ul} + (1-\lambda_{Ul})\beta\Big]
\end{aligned}
$$

Since $A > 0$, $B > 0$, $C > 0$, $D > 0$, and $A < B$, $C < D$, the RHS of equations (3.7) and (3.8) are between 0 and 1. It follows that we can conclude the results above as

Lemma 3.2 *In the country with public human capital formation,*

(a). an increase in P always increases X_1 but decreases X_2, hence increases X_1/X_2, and

(b). an increase in H_e always decreases X_2.

(c). On the other hand, if γ and β are sufficiently large (small) while α is sufficiently small (large), an increase in H_e increases (decreases) X_1/X_2 as well as X_1,

Lemma 3.2(a) says that X_1/X_2 is an increasing function of P while lemma 3.2(c) says that relative supply curve does not necessarily shift to the right due to an increase in H_e.

3.4 Trade Patterns

To see how the relative price of final goods change, the demand side of the final goods has to be stated explicitly. We can express the relative demand which is denoted by D as a function of P on the demand side, if we assume homothetic preferences [23]. Then the domestic market equilibrium is expressed as

$$X = D(P),$$

where $X \equiv X_1/X_2$. Differentiating the equation above and using the results in table 3.1, we obtain

$$\frac{\hat{P}}{\hat{H}_e} = -(\frac{\hat{X}}{\hat{P}} + \sigma_D)^{-1} \cdot \frac{\hat{X}}{\hat{H}_e}, \tag{3.1}$$

[23] Although there are two kinds of individual in this model, identical preferences assumption is unnecessary as long as their lifetime income are all the same as well as their fixed rate of time preferences which are equal to the market rate of interest.

where $\sigma_D \equiv -D'(P)P/D(P)$ is the price elasticity of demand.

In the lemma 3.1 and 3.2, we have examined the effect of P on X which is positive, whereas the effect of H_e on X is ambiguous, hence the total effect is ambiguous as well. Let us define that

Definition 3.1

A country with more (less) human capital employed in private sector is called human capital abundant (scarce) country.

Hence we can establish the following proposition.

Proposition 3.1

Suppose that there are two countries with public human capital formation where preferences, technology, capital endowment and population are identical. If the effect of educator-student ratio on human capital quality and income share of unskilled workers in low-tech sector are sufficiently large (small), then the country that allocates more domestic human capital into the public sector tends to be human capital abundant (scarce) country, hence exports (imports) high-tech final good and imports (exports) low-tech final good.

Again, the argument in the proposition 3.1 can easily be predicted from the lemma 3.1 and 3.2. If an increase in H_e decreases the human capital supply in private sector instead, then the government will fail to enhance the competitiveness of high-tech sector.

Notice also that since an increase in H_e decreases H_p but increases H when γ is small[24], if 'human capital abundant country' is defined as a country with larger H instead of H_p, then we can conclude as 'human capital abundant country imports high-tech final good and exports low-tech final good', which is a paradox[25].

3.5 Foreign Human Capital Mobility

We examine the effect of H_e on factor mobility among countries in this section, let us focus only on the human capital mobility rather than capital mobility, the effect of H_e on $\hat{W}_L - \hat{W}_H$ can be obtained as

$$\hat{W}_L - \hat{W}_H = \frac{\hat{W}_L - \hat{P}}{1-\alpha}, \tag{3.1}$$

From the table 3.1, we know that

$$\frac{\hat{W}_L}{\hat{H}_e} > 0. \tag{3.2}$$

[24]See equations (3.6) and (3.5.)

[25]See Leontief (1956) and Ishizawa (1988), where Ishizawa (1988) shows the Leontief paradox through the public sectors which depends on assumptions of the factor intensities and the size of the economy. Furthermore, the definition of 'abundant' may have played a great role to the paradox.

From equation (3.1) and (3.2) it is easy to show that

$$\frac{\hat{W}_H}{\hat{H}_e} < 0. \tag{3.3}$$

Equation (3.2) says an increase in H_e always decreases W_H. From equation (3.9) we can also easily see that W_H is an decreasing function of H instead of H_p which is different from the traditional RV model.

Recall the lemma 3.1 and consider the case of a country where the effect of educator-student ratio on human capital quality and income share of unskilled workers in low-tech sector are sufficiently small, then we have

$$\frac{\hat{H}_p}{\hat{H}_e} < 0. \tag{3.4}$$

Equation (3.4) says that when the country allocates less domestic human capital into the public sector, the human capital employed in private sector increases. In the meantime, equation (3.3) shows the factor price for human capital in the country rises, which is opposite compared to the traditional effect. Suppose that equation (3.4) is satisfied. Consider the case in which there are only country A and country B exit in the world. If country A allocates less domestic human capital into the public sector, then country A has the comparative advantage in the production of high-tech good but higher wage for skilled workers compared to country B, which is totally opposite compared to that in the traditional RV model.

Suppose country A is the advanced industrial country while country B is the developing country in this case, human capital moves from developing country to advanced industrial country. Recall the words of World Development Report: "Can something be done to stop the exodus of trained workers from poorer countries?" (World Bank, 1995, p. 64)[26]. Government can reduce its public service by decreasing the number of educator but still can enhance its high-tech sector and improve the brain drain problem.

Hence we can conclude as

Proposition 3.2

Suppose that there are two countries with public human capital formation where preferences, technology, capital endowment and population are identical. If the effect of educator-student ratio on human capital quality and income share of unskilled workers in low-tech sector are sufficiently small, then the country that allocates less educators into the public sector tends to export high-tech final good and have higher factor price for human capital.

Proposition 3.2 implies that if human capital mobility is allowed between the two countries in a free trade world, human capital moves from the country which exports high-tech final good into the country which imports it. This is the crucial result in the present chapter. A government can reduce its educators but still can enhance the high-tech sector. More surprisingly,

[26] See Stark-Helmenstein-Prskawetz 1998.

despite the country becomes human capital abundant country and exports high-tech final good, the wage for human capital rises and creats an incentive for foreign human capital inflow. As a result, there is an additional positive effect on the output of high-tech final good, instead of crowding out effect brought by brain drain, as long as foreign human capital inflow is allowed. Notice that the total human capital supply decreases as a whole, which has caused the rise in factor price for human capital.

3.6 Concluding Remarks

In this chapter, we have examined the relationships between trade patterns and human capital mobility. We have found that the RV model still can be applied to explain why skilled workers tend to move from developing countries to developed countries. One of the most important characters is that we use only very simple model to capture the human capital formation and derive some different results compared to many studies.

Our results can best be concluded in proposition 3.2. Some other policy implications can also be discussed. For example, consider the case of foreign human capital inflow. If the effect of educator-student ratio on human capital quality and income share of unskilled workers in low-tech sector are sufficiently large (small), a government should hire foreign human capital to work as educators in education sector (skilled workers in private high-tech sector) to enhance the high-tech sector in a more effective way.

Notice that we have only compared the cases in an equilibrium. The analysis of welfare can also be done in this chapter. Other than that, the education can also be financed by the students. However, we need to obtain some more tractable results for the future research. The analysis of effect of brain drain on welfare is important for policy implications, but we just leave this to the future research.

B Appendix

B.1 Calculation

Differentiating equations from (3.1) to (3.10), we have[27]

$$\alpha\hat{W}_L + (1-\alpha)\hat{W}_H = \hat{P}, \tag{B.1}$$

$$\beta\hat{W}_L + (1-\beta)\hat{r} = 1, \tag{B.2}$$

$$\begin{aligned}\lambda_{L1}\hat{X}_1 + \lambda_{L2}\hat{X}_2 = \hat{L} - \lambda_{L1}(1-\alpha)(\hat{W}_H - \hat{W}_L) \\ -\lambda_{L2}(1-\beta)(\hat{r} - \hat{W}_L),\end{aligned} \tag{B.3}$$

$$\hat{X}_1 = \hat{H}_p - \alpha(\hat{W}_L - \hat{W}_H), \tag{B.4}$$

[27] Notice that i is fixed.

$$\hat{X}_2 = \hat{K} - \beta(\hat{W}_L - \hat{r}), \tag{B.5}$$

$$\hat{H} = \delta_p \hat{H}_p + (1 - \delta_p)\hat{H}_e, \tag{B.6}$$

$$\hat{N} = \lambda_{Ul}\hat{U}_l + (1 - \lambda_{Ul})\hat{U}_e, \tag{B.7}$$

$$\hat{H} = \gamma \hat{H}_e + (1 - \gamma)\hat{U}_e, \tag{B.8}$$

$$\hat{H} - \hat{U}_e = \hat{W}_L - \hat{W}_H, \tag{B.9}$$

$$\hat{L} = \hat{U}_l, \tag{B.10}$$

Since N and K are assumed to be fixed throughout this paper, $\hat{N} = \hat{K} = 0$ hold. Equations (B.1) to (B.5) are the familiar basic equations of the RV model. Considering equations (B.1) and (B.2) can also be rewritten as

$$\hat{W}_L - \hat{W}_H = \frac{\hat{W}_L - \hat{P}}{1 - \alpha}, \tag{B.11}$$

$$\hat{W}_L - \hat{r} = \frac{\hat{W}_L}{1 - \beta}, \tag{B.12}$$

we can solve $\hat{W}_L$ easily given $\hat{P}$, $\hat{H}_p$ and $\hat{L}$ as

$$\hat{W}_L = \frac{1}{\Lambda}\Big(\Lambda_1 \hat{P} + \lambda_{L1}\hat{H}_p - \hat{L}\Big), \tag{B.13}$$

which is familiar. Substitute equations (B.11) and (B.12) into equations (B.4) and (B.5), we obtain equations (3.2) and (3.3).

Using equations (B.6) and (B.8) to solve for $\hat{H}_p$, we obtain

$$\hat{H}_p = \frac{1}{\delta_p}\Big[(\gamma + \delta_p - 1)\hat{H}_e + (1 - \gamma)\hat{U}_e\Big]. \tag{B.14}$$

Substitute equation (B.8) into equation (B.9) to eliminate $\hat{H}$ and rewrite it by using equation (B.11), we have

$$\hat{H}_e - \hat{U}_e = \frac{1}{\gamma} \cdot \frac{\hat{W}_L - \hat{P}}{1-\alpha}. \tag{B.15}$$

Substitute equation (B.7) into equation (B.10) to eliminate U_l, we obtain

$$\hat{L} = -\frac{(1-\lambda_{Ul})\hat{U}_e}{\lambda_{Ul}}. \tag{B.16}$$

We can also substitute equation (B.15) into equations (B.14) and (B.16) to eliminate $\hat{U}_e$, but since we are more interested in the effects on U_e instead, we substitute equation (B.16) into equation (B.13) to eliminate $\hat{L}$, hence we have

$$\hat{W}_L = \frac{1}{\Lambda}\Big(\Lambda_1 \hat{P} + \lambda_{L1}\hat{H}_p + \frac{(1-\lambda_{Ul})\hat{U}_e}{\lambda_{Ul}}\Big). \tag{B.17}$$

From equations (B.14), (B.15) and (B.17), we obtain equation (3.1) as shown in context.

Using Cramel's rule to solve equation (3.1), then substitute $\hat{W}_L$ and $\hat{H}_p$ into equations (3.2) and (3.3), we obtain the results shown in table 3.2, where

$$|A| \equiv \Lambda + \frac{1}{\gamma(1-\alpha)}\left[\frac{\Lambda_1(1-\gamma)}{\delta_p} + \frac{1-\lambda_{Ul}}{\lambda_{Ul}}\right] > 0,$$

$$|A_{11}| \equiv \Lambda_1 + \frac{1}{\gamma(1-\alpha)}\left[\frac{\Lambda_1(1-\gamma)}{\delta_p} + \frac{1-\lambda_{Ul}}{\lambda_{Ul}}\right] > 0,$$

$$|A_{12}| \equiv \frac{1}{\lambda_{Ul}}\Big[(1-\lambda_{Ul}) + \lambda_{Ul}\lambda_{L1}\Big] > 0,$$

$$|A_{21}| \equiv \frac{\Lambda_2(1-\gamma)}{\gamma(1-\alpha)\delta_p} > 0,$$

	$\hat{P}$	$\hat{H}_e$
$\hat{W}_L$	$\frac{\lvert A_{11}\rvert}{\lvert A\rvert} > 0$	$\frac{\lvert A_{12}\rvert}{\lvert A\rvert} > 0$
$\hat{H}_p$	$\frac{\lvert A_{21}\rvert}{\lvert A\rvert} > 0$	$\frac{\lvert A_{22}\rvert}{\lvert A\rvert}$?
$\hat{U}_e$	$\frac{\lvert A_{31}\rvert}{\lvert A\rvert} > 0$	$\frac{\lvert A_{32}\rvert}{\lvert A\rvert} > 0$
$\hat{X}_1$	$\frac{\lvert A_{41}\rvert}{\lvert A\rvert} > 0$	$\frac{\lvert A_{42}\rvert}{\lvert A\rvert}$?
$\hat{X}_2$	$\frac{\lvert A_{51}\rvert}{\lvert A\rvert} < 0$	$\frac{\lvert A_{52}\rvert}{\lvert A\rvert} < 0$
$\hat{X}_1 - \hat{X}_2$	$\frac{\lvert A_{61}\rvert}{\lvert A\rvert} > 0$	$\frac{\lvert A_{62}\rvert}{\lvert A\rvert}$?

Table 3.2: The table shows the effects of P and H_e on W_L, H_P, U_e, X_1, X_2 and X_1/X_2, respectively. For example, the effect of P on W_L is shown as $|A_{11}|/|A| > 0$, and so on. '?' refers to indefinite effect.

$$
\begin{aligned}
|A_{22}| &\equiv \frac{1}{\delta_p}\left[\Lambda\delta_p + \frac{(1-\lambda_{Ul})(\gamma+\delta_p-1)}{\lambda_{Ul}\gamma(1-\alpha)}\right],\\
|A_{31}| &\equiv \frac{\Lambda_2}{\gamma(1-\alpha)} > 0,\\
|A_{32}| &\equiv \Lambda_2 + \Lambda_1\left[\frac{(1-\delta_p)(1-\gamma)}{\delta_p\gamma}\right] > 0,\\
|A_{41}| &\equiv |A_{21}| + \frac{\alpha}{1-\alpha}\Big(|A| - |A_{11}|\Big) > 0, \qquad \because \quad |A| > |A_{11}|,\\
|A_{42}| &\equiv \frac{(1-\lambda_{Ul})[\delta_p(1-\gamma\alpha) - (1-\gamma)]}{(1-\alpha)\lambda_{Ul}\delta_p\gamma} + \lambda_{L1} + \Lambda_2,\\
|A_{51}| &\equiv -\frac{\beta}{1-\beta}\cdot|A_{11}| < 0,\\
|A_{52}| &\equiv -\frac{\beta}{1-\beta}\cdot|A_{12}| > 0,\\
|A_{61}| &\equiv |A_{41}| - |A_{51}| > 0,
\end{aligned}
$$

$$|A_{62}| \equiv \frac{1}{\lambda_{Ul}}\left[\frac{\lambda_{Ul}}{1-\beta} + (1-\lambda_{Ul})\left(\frac{1-\gamma\alpha}{\gamma(1-\alpha)} + \frac{\beta}{1-\beta}\right) + \frac{(1-\lambda_{Ul})}{\delta_p(1-\alpha)}\left(1-\frac{1}{\gamma}\right)\right]$$

B.2 Decomposition of Effects on H_p

Let us explain why an increase in H_e does not necessarily increase H_p. Rewritting equation (B.14), we obtain equation (3.4) as shown in context.

Let us see the quality effect and the quantity effect of H_e as defined in context. Holding P fixed, from equations (B.8), (B.9) and (B.11), the quality effect can be expressed as

$$\gamma(\hat{H}_e - \hat{U}_e) = \frac{\hat{W}_L}{1-\alpha}. \tag{B.18}$$

Since $\hat{W}_L/\hat{H}_e = |A_{12}| > 0$ as shown in table 3.2, the quality effect of H_e is positive. On the other hand, the quantity effect of H_e is also positive which can be predicted from $\hat{U}_e/\hat{H}_e = |A_{32}|/|A|$ as shown in table 3.2. Thus the sum of the quality effect and quantity effect of H_e is positive which can be expressed as

$$\gamma(\hat{H}_e - \hat{U}_e) + \hat{U}_e = \left(\frac{|A_{12}|}{1-\alpha} + |A_{32}|\right)\hat{H}_e > 0. \tag{B.19}$$

Unfortunately, not only the crowding out effect is negative but also the total effect of H_e on H_p ends up an ambiguous effect which can be predicted from

$\hat{H}_p/\hat{H}_e = |A_{22}|/|A|$ as shown in table 3.2. The sign is positive (negative) if and only if

$$\delta_p > (<) \frac{(1-\lambda_{Ul})(1-\gamma)}{1-\lambda_{Ul}+\gamma\lambda_{Ul}\Big(\lambda_{L1}+\dfrac{1-\alpha}{1-\beta}\cdot\lambda_{L2}\Big)}, \tag{B.20}$$

where RHS is obviously between 0 and 1 since numerator is smaller than denominator and both of them are positive as well. In particular, we can see that the condition is satisfied easier with larger β and γ, and smaller α, hence we have lemma 3.1.

Part II

Technology Change and Wage Inequality

Chapter 4

Technology Change and Endogenous Labor Supply in a Small Open Country

4.1 Introduction

Technical change in Europe and Asia and Latin America could have contributed to the lower wage of unskilled labor and increased income inequality that the United States has been experiencing. Many economists have argued that skilled labor biased technical change has contributed to rising income equality in those countries. However, in the framework of traditional HOS model, it is possible that factor bias of technical progress does not affect relative factor prices.

Following the dispute between Leamer (1998) and Krugman (2000), we repeat some explanation of the terminology regarding sector bias and factor bias. When technical progress occurs in a sector definitely raises relative factor prices in the same way regardless of which factor is involved in the technical progress, we say sector bias does matter for relative factor prices but factor bias does not. On the other hand, when the effect of technical progress on relative factor prices depends on which factor is involved, we say factor bias does matter for relative prices but sector bias does not.

In fact, biased technical change may even lower the price of the factor reward in which it is biased if the biased technical change occurs in the sector where the factor is not used intensively. What would happen if technical progress occurs in a sector but only one of the factors is involved? As Krugman (2000) emphasizes, in the case of one-good economy where only skilled labor and unskilled labor are used in production, Hicks-neutral (HN) technical progress will not change relative wages, whereas skilled labor biased technical progress will raise skilled labor wage, hence factor bias rather than sector bias matters for relative wages. On the other hand, in the case of multi-good models, for example, as Leamer (1998) emphasizes that any improvement in the technology for producing a good will raise skilled labor wage if skilled labor is intensively used for the good, regardless of whether skilled labor biased or unskilled labor biased technical progress takes place, hence sector bias rather than factor bias matters for relative wages. It finally leads to a conclusion that the emphasis on factor bias suggested by the one-good model is all wrong when we are consider multi-good, that is, sector bias matters for relative wages but factor bias does not. Krugman (2000) shows an opposite view and argues that in the case of a small-economy model, HN technical progress in the skilled labor intensive sector does not necessarily raise skilled to unskilled wage, and indeed, lowers skilled to unskilled wage under certain assumptions, for example, in the case of Cobb-Douglas demand. He concludes that a two-sector model behaves just like the one-sector model under this assumption.

Xu (2001) investigates the effects of technical progress on relative wages in the 2x2x2 HOS model and sorts out the arguments between Leamer (1998) and Krugman (2000). He classifies technical progress according to factor-augmenting bias, factor-using bias and HN and clarifies the conditions for factor bias and sector bias to impact relative wages. On the other hand, as many studies insist, exploration in other conceptual framework, namely the RV[1] (RV) framework that has sectoral immobile unskilled labor is also important. We examine the effects of technical change on relative wages in RV model instead of HOS model, moreover we will also let labor supplies be

[1]See Jones 1971

elastic.

Unlike in the HOS model, RV model makes difference for the effects of technical progress on wages. Under the assumption of inelastic factor supply, elasticities of factor demand are not significant for the signs. Indeed, Rodrik (1997) explains important implications of more-elastic factor demands. The factor supplies we apply to this chapter, are all in terms of labor, that is, two kinds of sectoral-immobile skilled labor and one kind of common unskilled labor. Moreover, their supplies are all endogenously determined. We apply the basic idea of Findlay-Kierzkowski (1983)[2].

We examine two kinds of technical progress, that is, (1) Product-specific skilled labor augmenting and (2) Product-specific unskilled labor augmenting technical progress. Both of the technical progress take place at the same sector. As we have refered previously, if skilled to unskilled wage rises in both of the cases, then sector bias rather than factor bias matters for relative wages. On the contrary, if skilled to unskilled wage declines in the case of product-specific unskilled labor augmenting but rises in the case of product-specific skilled labor augmenting, then factor bias rather than sector bias matters for relative wages. We will show that whether factor bias or sector bias matters for relative wages in this framework, depends on the elasticities of substitu-

[2]Ishikawa (2000) is also remarkable, he constructs a RV model by replacing one of the sector-specific factors with a sector-specific intermediate good.

tion between factors (ESF) which comes to a conclusion as in the traditional RV model, although there are some additional effects on individual wages rather than relative wages.

4.2 The Model

4.2.1 Traditional Ricardo-Viner Model with Technical Change

Let us consider a small open country. There are two private sectors which produce final goods, X_1 and X_2, respectively. ith sector produces X_i using ith skilled labor and unskilled labor, $(i = 1, 2)$, which are denoted by L_i^p and L_3^p. L_i^p is sectoral-immobile and L_3^p is sectoral-mobile. The production function is expressed as $X_i = X_i(\beta_i L_{3_i}^p, \alpha_i L_i^p),\ (i = 1, 2)$, where $L_{3_i}^p$ represents the amount of unskilled labor used by ith sector. α_i and β_i are positive and initially equal to one, which are shift parameters of technical change of L_i^p and $L_{3_i}^p$ in ith sector, respectively. There are technical progress if their value are larger than one. For example, unskilled labor augmenting technical progress occurs in $1st$ sector if $\beta_1 > 1$. Define $\tilde{W}_i \equiv W_i/\alpha_i$ and $\tilde{W}_{3_i} \equiv W_3/\beta_i$, where W_i and W_3 are ith skilled labor wage and unskilled labor wage.

Following the traditional RV model, the profit conditions for the ith private sector is expressed as

$$C^i(\tilde{W}_{3_i}, \tilde{W}_i) = P_i, \qquad for\ i = 1, 2, \tag{4.1}$$

where C^i and P_i are the unit cost function of the ith private sector and the price of the ith good, respectively. $P_2 \equiv 1$ is chosen as a numeraire. The full employment conditions are expressed as

$$\sum_{i=1}^{2} C_3^i(\tilde{W}_{3_i}, \tilde{W}_i)X_i = L_3^p, \tag{4.2}$$

$$C_i^i(\tilde{W}_{3_i}, \tilde{W}_i)X_i = L_i^p, \qquad for\ i = 1, 2, \tag{4.3}$$

where the subscripts for the unit cost functions with a variable are the partial derivatives of the factor price related to the variables and represent the input coefficients of the factor in the ith private sector. For example, $C_3^i(\tilde{W}_{3_i}, \tilde{W}_i) \equiv \partial C^i(\tilde{W}_{3_i}, \tilde{W}_i)/\partial \tilde{W}_{3_i}$ is the input coefficient of the unskilled labor in the ith private sector. From the equations (4.1), (4.2) and (4.3), given L_3^p, L_1^p, L_2^p and P_1, we can solve for $\tilde{W}_{3_i}, \tilde{W}_1, \tilde{W}_2, X_1$ and X_2. W_i and W_3 can be solved given α_i and β_i.

Differentiate equations (4.1) to (4.3), setting $\hat{P}_i = 0$ and solve for $\hat{W}_3$ and $\hat{W}_i$, yields

$$\hat{W}_3 = \frac{\Lambda_1\hat{\beta}_1 + \Lambda_2\hat{\beta}_2}{\Lambda} - \frac{\hat{S}}{\Lambda}, \tag{4.4}$$

$$\hat{W}_1 = \frac{\Lambda_2\theta_{3_1}(\hat{\beta}_1 - \hat{\beta}_2) + \Lambda\theta_1\hat{\alpha}_1}{\Lambda\theta_1} + \frac{\theta_{3_1}\hat{S}}{\Lambda\theta_1}, \tag{4.5}$$

$$\hat{W}_2 = \frac{\Lambda_1\theta_{3_2}(\hat{\beta}_2 - \hat{\beta}_1) + \Lambda\theta_2\hat{\alpha}_2}{\Lambda\theta_2} + \frac{\theta_{3_2}\hat{S}}{\Lambda\theta_2}, \tag{4.6}$$

where

$$\hat{S} \equiv \hat{L}_3^p - (\lambda_{3_1}\hat{L}_1^p + \lambda_{3_2}\hat{L}_2^p),$$

$$\Lambda_i \equiv \lambda_{3_i} e_{3_i}, \qquad for\ i = 1, 2, \qquad \Lambda_1 + \Lambda_2 = \Lambda.$$

$$e_{3_i} \equiv \frac{\sigma_i}{\theta_i}, \qquad for\ i = 1, 2$$

$$\theta_{3_i} \equiv \frac{W_{3_i} C_3^i(\cdot)}{P_i}, \qquad for\ i = 1, 2,$$

$$\theta_i \equiv \frac{W_i C_i^i(\cdot)}{P_i}, \qquad \theta_{3_i} + \theta_i = 1, \qquad for\ i = 1, 2$$

$$\sigma_i \equiv \frac{\hat{C}_i^i(\cdot) - \hat{C}_3^i(\cdot)}{\hat{W}_3 - \hat{W}_i} > 0, \qquad for\ i = 1, 2$$

$$\lambda_{3_i} \equiv \frac{C_3^i(\cdot) X_i}{L_3^p}, \qquad for\ i = 1, 2, \qquad \lambda_{3_1} + \lambda_{3_2} = 1.$$

$(\hat{\ })$ denotes a proportionate change, for example, $\hat{W}_3 = dW_3/W_3$. θ_{3_i} and θ_i are the familiar income shares in HOS model or RV model. σ_i and λ_{3_i} are, respectively, the ESF and the fraction of unskilled labor in ith sector. e_{3_i} and Λ are, respectively, the elasticity of demand for unskilled labor in the ith sector and the aggregate general-equilibrium elasticity of demand for unskilled labor in the private sectors. We call $\hat{S}$ as the proportionate change in the supply of unskilled labor relative to an "aggregate" of skilled labor[3].

The first terms in the RHS of equations (4.4) to (4.6) represent the traditional effects while the second terms represent the additional effect due to the change in labor supplies through the labor market which may bring some

[3]Note that the definition of S is not $L_1^p + L_2^p$. The definition of $\hat{S}$ here follows Bhagwati and Srinivasan (c1983).

perverse results.

4.2.2 Formation of Labor Supply

In this section, we apply the basic idea of Findlay-Kierzkowski (1983) to show how labor supplies are endogenously determined.

We assume that N individuals are born and N individuals die in each period in the economy, all live for 2 periods, this means that the population is in the steady state. Government provides two public services for those who want to be educated under the public service for free but forgoing the opportunity to participate in the unskilled labor market during the first education period. We can see from above, there are three kinds of decision maker (E_1, E_2, E_3) among the new generation(N) in each period of t, where E_i $(i = 1, 2)$denotes the ith students who decide to earn their income under wage rate (W_i) as a ith skilled labor after having enjoyed the ith public service, while E_3 denotes those who decide not to enjoy the public services and participate in the unskilled labor market to earn their income under wage rate (W_3). Now, we can describe how E_1, E_2 and E_3 make their decisions. We assume that the ith public service is provided by the government by employing the ith skilled labor from the ith private sector into the ith public sector. Let Q_i denote the ith 'skilled labor' or human capital that the government can 'produce' at each period of t, while L_i^g denotes the amount of the ith skilled labor or educators as public servants employed by the government in the ith public sector at each period of t. The government acts like a producer who produces 'skilled labor' at each period of t, using educators (i.e. L_i^g) and students (i.e. E_i) as inputs[4]. The production function of the ith public sector can be expressed as

$$Q_i = F_i(L_i^g, E_i), \qquad for\ i = 1, 2.$$

$F_i(\cdot)$ is increasing, strictly quasi-concave, positively linear homogeneous and twice continuously differentiable. Because of the linear homogenity, the equation above also can be expressed as

$$\frac{Q_i}{E_i} = q_i = f_i(l_i^g),$$
$$f_i(0) > 0 \qquad f_i^{'} > 0 \qquad f_i^{''} < 0 \qquad l_i^g = \frac{L_i^g}{E_i}, \qquad for\ i = 1, 2,$$

where l_i^g is the ith public service-beneficiary ratio, we can also put it in a more particular way such as ith educator-student ratio. L_i^g is valued at efficiency unit, and is the amount of ith skilled labor as public input in the ith public sector, which is treated as exogenous. q_i is the effectiveness or quality per ith student they can acquire as a member of ith skilled labor in the future[5] .

[4]Note that, however, the government does not act as a profit maximizer.

[5]Compare to Becker and Murphy (1992), which shows that the human capital acquired by a student depends on the human capital of her teachers, and the number of teachers per student.

The lifetime income of E_3 and E_i valued at present value are

$$B_3 = (1-\tau)W_3(1+\frac{1}{1+\rho}),$$
$$B_i = (1-\tau)W_i f_i(l_i^g)\cdot\frac{1}{1+\rho},$$

$for\ i = 1,2$, where ρ is the fixed interest rate, and $0<\tau<1$ is the income tax rate imposed by the government to finance the public services[6]. Because $dB_i/dE_i < 0$, we know that E_i must be determined under the arbitrary condition as

$$B_1 = B_2 = B_3,$$

which yields

$$f_i(l_i^g) = \frac{W_3}{W_i}\cdot R, \qquad for\ i = 1,2. \tag{4.7}$$

where $R \equiv 2+\rho$ is assumed to be fixed. From the equation (4.7), we know that E_i depends on f_i as well as W_3/W_1 and W_3/W_2. Given ρ, N, L_1^g, L_2^g, in addition to W_1, W_2 and, W_3, we can solve for E_1 and E_2, then, we get the value of E_3 with the new generation of the population constraint as

$$E_1 + E_2 + E_3 = N. \tag{4.8}$$

The total ith labor is expressed as

$$F_i(L_i^g, E_i) = f_i(l_i^g)E_i = L_i, \qquad for\ i = 1,2, \tag{4.9}$$

[6]We will show how this income tax rate is to be solved by the government budget constraint.

$$L_3 = 2E_3, \tag{4.10}$$

For the time being, the full employment conditions can be expressed as

$$L_i = L_i^g + L_i^p, \qquad for\ i = 1, 2 \tag{4.11}$$

$$L_3 = L_3^p, \tag{4.12}$$

The government budget constraint is expressed as

$$\sum_{i=1}^{2} W_i L_i^g = \tau[\sum_{i=1}^{2} W_i(L_i^p + L_i^g) + W_3 L_3]. \tag{4.13}$$

Let us call the economy above as endogenous labor supply economy and consider also equations (4.4) to (4.6), we have

Lemma 4.1 *In the endogenous labor supply economy, the effect of technical change on factor prices is decomposed into two parts which are called direct effect and indirect effect. The indirect effect is due to the change in labor supply which is caused by the change in factor prices through technical change.*

4.3 Indirect Effect

Differentiating equations from (4.7) to (4.10), holding other exogenous variables fixed, we can solve L_3^p and L_i^p in terms of $\hat{W}_3$ and $\hat{W}_i$, which eventually solve all the endogenous variables in the model. Since all factors in this model are endogenously determined, it is more important to see the change in the supply of unskilled labor relative to an "aggregate" of skilled labor rather than the change L_3^p or L_i^p individually when we want to see the effects on factor prices. Hence, it is convenient to solve L_3^p and L_i^p in terms of $\hat{S}$ as

$$\begin{aligned}\hat{S} \; = \; & \frac{[\Lambda_1\Lambda_2^\eta + \Lambda_1^\eta(\theta_1\Lambda_1 - \theta_{3_1}\Lambda_2)]\hat{\beta}_1 + [\Lambda_2\Lambda_1^\eta + \Lambda_2^\eta(\theta_2\Lambda_2 - \theta_{3_2}\Lambda_1)]\hat{\beta}_2}{\Lambda + \Lambda^\eta} \\ & - \frac{\Lambda\Lambda_1^\eta\theta_1\hat{\alpha}_1 + \Lambda\Lambda_2^\eta\theta_2\hat{\alpha}_2}{\Lambda + \Lambda^\eta}, \qquad (4.1)\end{aligned}$$

where

$$\begin{aligned}
&\Lambda_i^\eta \equiv \frac{1 + \Lambda_i^p\gamma_i}{\Lambda_i^p\theta_i} \cdot \eta_i > 0, \qquad for\ i = 1, 2, \qquad \Lambda^\eta = \Lambda_1^\eta + \Lambda_2^\eta, \\
&\sigma_i^q \equiv \frac{f'_i(l_i^g)l_i^g}{f_i(l_i^g)}, \qquad for\ i = 1, 2, \qquad 0 < \sigma_i^q < 1 \\
&\gamma_i \equiv \frac{E_i}{E_3} > 0, \qquad for\ i = 1, 2, \\
&\Lambda_i^p \equiv \frac{L_i^p}{L_i}, \qquad for\ i = 1, 2 \qquad 0 < \Lambda_i^p < 1 \\
&\eta_i \equiv \frac{1 - \sigma_i^q}{\sigma_i^q} > 0
\end{aligned}$$

Note also that if $\hat{\beta}_1 = \hat{\beta}_2 = \hat{\beta}$, from equation 4.1 we have

$$\frac{\hat{S}}{\hat{\beta}} = \frac{\Lambda(\theta_1\Lambda_1^\eta + \theta_2\Lambda_2^\eta)}{\Lambda + \Lambda^\eta} > 0,$$

Equation (4.1) can be used to examine the total effects on wages of various types of technical change. Before we go for the analysis in the next section, we can conclude here as

Lemma 4.2 *In the endogenous labor supply economy,* *(a). a product-specific skilled labor augmenting technical progress always decreases the supply of unskilled labor relative to an "aggregate" of skilled labor, hence it brings a negative indirect effect for W_1 and W_2 but positive indirect effect for W_3.*

(b). a product-specific unskilled labor augmenting technical progress in the 1st (2nd) sector increases the supply of unskilled labor relative to an "aggregate" of skilled labor if Λ_1^η (Λ_2^η), θ_{3_1} (θ_{3_2}) and Λ_2 (Λ_1) are sufficiently small, hence it brings a positive indirect effect for W_1 and W_2 but negative indirect effect for W_3.

(c). if product-specific unskilled labor augmenting technical progress in the 1st sector and the 2nd sector are the same, it always increases the supply of unskilled labor relative to an "aggregate" of skilled labor., hence it brings a positive indirect effect for W_1 and W_2 but negative indirect effect for W_3.

When there is a product-specific skilled labor augmenting technical progress, for instance, in the $1st$ sector, holding other fixed, we can see that from equations (4.4) to (4.6) W_3/W_1 will decline, hence increases E_1 while dereases E_3. Thus we have lemma 4.2(a). When there is a product-specific unskilled labor augmenting technical progress, for instance, in the $1st$ sector, from the equations (4.4) to (4.6), we know that W_3/W_2 will rise due to the direct effect through a rise in β_1. L_2 will decrease in only a smaller extent if there is smaller Λ_2^η, that is, smaller marginal quality which can be acquired in the $2nd$ education is given up. On the other hand, W_3/W_1 may rise or decline since both W_1 and W_3 rise through the direct effect due to the rise in β_1. If θ_{3_1} is sufficiently large or/and the elasticity of demand for uskilled labor in $2nd$ sector is sufficiently larger than that in $1st$ sector (i.e. $\Lambda_2 > \Lambda_1$), obviously W_3 will rise with a smaller extent than W_1. It follows that W_3/W_1 rises in this case hence we have a rise in E_1 and a decline in E_3. Again, L_1 will increase in only a larger extent if there is larger Λ_1^η since individuals (i.e. E_1) are getting larger marginal quality which can be acquired in the $1st$ education. After all, the sign of $\hat{S}$ depends on Λ_1^η, Λ_1, Λ_2 and θ_{3_1}. When Λ_1^η, Λ_1, Λ_2 and θ_{3_1} are sufficiently large, and Λ_2^η is sufficiently small, then there are sufficiently rise in L_1, small decline in L_2 and decline in L_3. Hence we have negative sign of $\hat{S}$ and lemma 4.2(b). Lemma 4.2(c) can be explained as in lemma 4.2(a) but just in the opposite way.

4.4 Technical Change

In this subsection, we are going to examine whether factor bias or sector bias matters for relative wages in one of the sectors. We examine two types of technical change in this section, which are specified as

1. product-specific skilled labor (sectoral-immobile) augmenting, where $\hat{\alpha_1} > 0 = \hat{\beta}_1 = \hat{\beta}_2 = \hat{\alpha_2}$

2. product-specific unskilled labor (sectoral-mobile) augmenting, where $\hat{\beta}_1 > 0 = \hat{\beta}_2 = \hat{\alpha_1} = \hat{\alpha_2}$

Substitute the classified parameter into equations (4.4) and (4.5), we can examine product-specific skilled labor augmenting and product-specific unskilled labor augmenting types of technical change[7].

We repeat what we are going to do here, that is, if skilled to unskilled wage rises in both of the cases, then we can conclude that sector bias rather than factor bias matters for relative wages. On the contrary, if skilled to unskilled wage rises in the case of product-specific skilled labor augmenting but declines in the case of product-specific unskilled labor augmenting, then factor bias rather than sector bias matters for relative wages.

4.4.1 Product-specific Skilled Labor Augmenting Technical Progress (in 1*st* sector)

We first examine product-specific skilled labor augmenting technical progress (in 1*st* sector) in this subsection. We are going to see whether product-specific skilled labor augmenting technical progress will raise skilled to un-

[7] Since W_1 and W_2 are symmetric, we focus only on W_1.

skilled wage or not. The final conclusion will not be established in this subsection, but it is important for us to conclude whether factor bias or sector bias matters for skilled to unskilled wage. Hence the result we obtain here is essential for the whole story.

The value of parameters correspond with this case are

$$\hat{\alpha_1} > 0 = \hat{\beta}_1 = \hat{\beta}_2 = \hat{\alpha_2}. \tag{4.1}$$

Substitute equation (4.1) into equation (4.1), we have

$$\frac{\hat{S}}{\hat{\alpha}_1} = -\frac{\theta_1 \Lambda \Lambda_1^{\eta}}{\Lambda + \Lambda^{\eta}} < 0, \tag{4.2}$$

The effect on $\hat{S}$ due to the change in the wages which is caused by the technical progress. This repeats what we have concluded in lemma 4.2(a), that is, product-specific skilled labor augmenting technical progress always decreases the supply of unskilled labor relative to an "aggregate" of skilled labor, which brings positive effect on W_3 but negative effects for W_1 and W_2.

The most important thing in this subsection is to see the effects on skilled to unskilled wage. Substitute equation (4.2) into equation (4.4) and (4.5), we obtain

$$\frac{\hat{W}_3}{\hat{\alpha}_1} = 0 + \frac{\theta_1 \Lambda_1^{\eta}}{\Lambda + \Lambda^{\eta}} > 0,$$

$$\frac{\hat{W}_1}{\hat{\alpha}_1} = 1 - \frac{\theta_{3_1} \Lambda_1^{\eta}}{\Lambda + \Lambda^{\eta}}$$

$$= \quad \frac{\Lambda + \theta_1 \Lambda_1^{\eta} + \Lambda_2^{\eta}}{\Lambda + \Lambda^{\eta}} > 0,$$

which yields

$$\frac{\hat{W}_1 - \hat{W}_3}{\hat{\alpha}_1} = 1 - \frac{\theta_{3_1} \Lambda_1^{\eta}}{\Lambda + \Lambda^{\eta}} - \frac{\theta_1 \Lambda_1^{\eta}}{\Lambda + \Lambda^{\eta}} = \frac{\Lambda + \Lambda_2^{\eta}}{\Lambda + \Lambda^{\eta}} > 0. \tag{4.3}$$

To see the effects of product-specific skilled labor augmenting on relative wage in $1st$ sector, let us focus on equation (4.3). The first term in the RHS of the first equality, that is, 1 represents the traditional direct effect of the product-specific skilled labor augmenting technical progress, which affects skilled to unskilled wage positively. This is because there is no effect on unskilled labor wage under product-specific skilled labor augmenting technical progress in the traditional RV model. The second term and the third term represent the indirect effects which affect skilled to unskilled wage negatively. Although the direct effect and the indirect effect have oppositive signs in this framework, the total effect remains positive. Hence in this framework, the effects of product-specific skilled labor augmenting technical progress on relative wages remains unchanged compared to the traditional RV model. However, the most important implication is whether skilled to unskilled wage rises or not under product-specific skilled labor augmenting technical progress. What we have obtained here is that skilled to unskilled wage must rise under product-specific skilled labor augmenting technical progress.

The next things is to clarify whether skilled to unskilled wage rises even in the case of product-specific unskilled labor augmenting technical progress. If it rises, then we can conclude that sector bias rather that factor bias matters for relative wages and vice versa.

4.4.2 Product-specific Unskilled Labor Augmenting Technical Progress (in 1st sector)

In this subsection, we will show the effect of product-specific unskilled labor augmenting technical progress on skilled to unskilled wage. As concluded in lemma 4.2(b), we will have some ambiguous indirect effect. However, the important thing is to see the total effect on skilled to unskilled wage and then conclude whether factor bias or sector bias matters for skilled to unskilled wage with the consideration of the result we have obtained in the previous subsection.

The value of parameters correspond with this case are

$$\hat{\beta}_1 > 0 = \hat{\beta}_2 = \hat{\alpha_1} = \hat{\alpha_2}. \tag{4.4}$$

Substitute equation (4.4) into equation (4.1), we have

$$\frac{\hat{S}}{\hat{\beta}_1} = \frac{\Lambda_1 \Lambda_2^\eta + \Lambda_1^\eta (\theta_1 \Lambda_1 - \theta_{3_1} \Lambda_2)}{\Lambda + \Lambda^\eta}. \tag{4.5}$$

As shown in lemma 4.2(b), product-specific unskilled labor augmenting technical progress in the 1st (2nd) sector increases the supply of unskilled labor relative to an "aggregate" of skilled labor if Λ_1^η (Λ_2^η), θ_{3_1} (θ_{3_2}) and Λ_2 (Λ_1) are sufficiently small, hence it brings a positive indirect effect for W_1 and W_2 but negative indirect effect for W_3.

To see effects on skilled to unskilled wage, we first substitute equation (4.5) into equation (4.4) and (4.5) and obtain

$$\begin{aligned}\frac{\hat{W}_3}{\hat{\beta}_1} &= \frac{\Lambda_1}{\Lambda} - \frac{\Lambda_1\Lambda_2^\eta + \Lambda_1^\eta(\theta_1\Lambda_1 - \theta_{3_1}\Lambda_2)}{\Lambda(\Lambda+\Lambda^\eta)} \\ &= \frac{\theta_{3_1}\Lambda_1^\eta + \Lambda_1}{\Lambda+\Lambda^\eta} > 0,\end{aligned}$$

$$\begin{aligned}\frac{\hat{W}_1}{\hat{\beta}_1} &= \frac{\theta_{3_1}\Lambda_2}{\theta_1\Lambda} + \frac{\theta_{3_1}[\Lambda_1\Lambda_2^\eta + \Lambda_1^\eta(\theta_1\Lambda_1 - \theta_{3_1}\Lambda_2)]}{\theta_1\Lambda(\Lambda+\Lambda^\eta)} \\ &= \frac{\theta_{3_1}(\theta_1\Lambda_1^\eta + \Lambda_2 + \Lambda_2^\eta)}{\theta_1(\Lambda+\Lambda^\eta)} > 0,\end{aligned}$$

which yields

$$\begin{aligned}\frac{\hat{W}_1 - \hat{W}_3}{\hat{\beta}_1} &= \frac{\theta_{3_1}\Lambda_2 - \theta_1\Lambda_1}{\theta_1\Lambda} + \frac{\Lambda_1\Lambda_2^\eta + \Lambda_1^\eta(\theta_1\Lambda_1 - \theta_{3_1}\Lambda_2)}{\theta_1\Lambda(\Lambda+\Lambda^\eta)} \\ &= \frac{\theta_{3_1}(\Lambda_2^\eta + \Lambda_2) - \theta_1\Lambda_1}{\theta_1(\Lambda+\Lambda^\eta)}. \end{aligned} \tag{4.6}$$

To see the effects of product-specific unskilled labor augmenting on the relative wage in 1*st* sector, let us focus on equation (4.6). The first term in the RHS of the first equality, shows the ambiguity of the effect on skilled to unskilled wage in the sector while the second term represents the indirect effect where its sign is determined. However, the total effect remains ambiguous which is shown in the second equality. Hence it establishes the validity in the case of product-specific unskilled labor augmenting compared to the traditonal RV model since it remains true whether skilled to unskilled wage rises or declines depends on the ESF, that is, if Λ_1 is sufficiently large, then the sign in equation (4.6) is negative hence skilled to unskilled wage declines and vice versa.

The determination of the sign in equation (4.6) is very important for our conclusion since we have to clarify whether unskilled labor augmentation in a sector results in the same way as skilled labor augmentation in the same sector, if so, any improvement of technology in a sector brings the same result for relative wages, hence sector bias rather than factor bias matters for relative wages as Leamer (1998) insists, otherwise we will have the opposite result as Krugman (2000) argues.

4.4.3 Sector Bias vs Factor Bias

From the subsections previous, we know that whether sector bias or factor bias matters for relative wages, depends on how skilled to unskilled wage reacts in the case of product-specific unskilled labor augmenting technical progress. If Λ_1 is sufficiently small, then skilled to unskilled wage rises. It follows that any improvement of technology in a sector will affect the relative wages in the same way. Hence we can conclude that sector bias rather than factor bias matters for relative wages. On the contrary, if Λ_1 is sufficiently large, then skilled to unskilled wage declines. Hence we can conclude that improvement of technology towards skilled labor (unskilled labor) in any sector will raise (reduce) skilled to unskilled wage in the sector. Hence we can conclude that factor bias rather than sector bias matters for relative wages.

Finally, we can conclude the results above as

Proposition 4.1

In the endogenous labor supply economy, sector bias (factor bias) matters for relative wages if ESF is sufficiently small (large) in the sector where technical progress occurs.

4.5 Concluding Remarks

This chapter has examined the effect of different types of technical progress on wages under the traditional RV model, but also with endogenous labor supplies. In this chapter, there is an additional indirect effects through labor market where elasticities of demand for labor and the endogenous labor supply matter. On the other hand, in the case of product-specific skilled labor augmenting, there are some additional results compared to the traditional RV model. That is, under product-specific skilled labor augmenting technical progress, unskilled labor wage and skilled labor wage in the other sector reacts as well as skilled labor wage in the sector where product-specific skilled labor augmenting technical progress occurs. In the context of whether sector bias or factor bias matters for relative wages, as Xu (2001) points out that, the argument between Leamer (1998) and Krugman (2000) seems to depend on the assumptions of small open country, large open country or an integrated world economy. However, in this framework, ESF which is related to elasticities of factor demand, determines whether sector bias or factor bias matters for relative wages. If ESF is sufficiently small in the $1st$ sector or in other words, the elasticity of demand for unskilled labor is sufficiently small, then skilled to unskilled wage rises in both product-specific skilled labor augmenting and product-specific unskilled labor augmenting technical progress. This means that the technical progress occurs in the sector does not matter whether it is skilled labor augmentation or unskilled labor augmentation but results in a rise in skilled to unskilled wage, or in other words, sector bias does matter. Hence Leamer (1998) is supported in this case in our framework. On the contrary, if ESF is sufficiently large in the $1st$ sector or elasticity of demand for unskilled labor is sufficiently large, then skilled to unskilled wage rises in product-specific skilled labor augmenting technical progress but declines in product-specific unskilled labor augmenting technical progress. This means that the change in skilled to unskilled wage depends on whether it is skilled labor augmentation or unskilled labor augmentation in the sector, or in other words, factor bias does matter. Hence Krugman (2000) is supported in this case in our framework.

Chapter 5

Summary

We have examined the issues of brain drain and international trade in part I. In particular, we have examined the formation of the skilled labor which can be decomposed into quality effect, quantity effect and input effect. Compared to the standard RV model, there is an additional effect of price on wages. In our model, when relative price rises, unskilled labor which is mobile between sectors gains more, while the immobile factors gain less or lose more compared to the standard one[1]. Remember that we have ambiguous effect of the educator on human capital employed in private sector. On the other hand, effect of educator on total human capital is unambiguously positive, which brings an negative effect on the wage of human capital. So it is possible

[1]Examination of the validity of the factor price equalization theorem in Ricardo-Viner model is also challenging, see Chong (2004b).

that if effect of the educator on human capital employed in private sector is negative, an increase in educator decreases the amount of human capital employed in private sector hence decreases the output of it. In the meantime, it also increases the amount of total human capital which decreases the wage of human capital. As a result, a country which allocates too much human capital in public sector tends to be inefficient, that is, causing the country to import high-tech good from the country where wage of human capital is higher, hence human capital in the country which imports high-tech good tends to migrate to the country which exports high-tech good. This is the crucial result of our study compared to the traditional RV model which cannot explain the brain drain issue.

We should also point out that an endogenous provision of the public service model may be more appropriate. The study of endogenous provision of public service model has been examined by Chong (2004a). The comparation between publicly provided education service and privately provided education service has also been examined in the paper. Further, the reason for government to enhance a selective sector should be incorporated into the model. What we have presented here are the basic framework for the future research.

In part II, we have examined the effect of different types of technical progress on wages under the traditional RV model, but also with endogenous labor supplies. The examination is quite simple, that is, to know whether sector bias or factor bias matters for relative wages, we just need to know whether both cases of product-specific skilled labor augmenting and product-specific skilled labor augmenting change the relative wage in the same direction. If both cases change the relative wage in the same direction, we can tell that factor bias technology change does not matter, and we can conclude that sector bias technology change does matter for the relative wages, and vice versa. However, the method of argument and examination in our study is quite different from Leamer (1998) and Krugman (2000) which depend on the assumptions of small open country, large open country or an integrated world economy. In our study, ESF which is related to elasticities of factor demand, determines whether sector bias or factor bias matters for relative wages. Hence Leamer (1998) is supported if ESF is sufficiently small in the $1st$ sector while Krugman (2000) is supported if ESF is sufficiently large in the $1st$ sector.

Using the idea of human capital formation, we have studied the relationship between brain drain and international trade as well as the relationship between technology change and wage inequality for several years. However, there are still quite many challenges lie ahead.

References

[1] Abe, Kenzo, 1990, "A Public Input as a Determinant of Trade." *Canadian Journal of Economics* 23(2), 400-407.

[2] Becker, Gary S.; Kevin M. Murphy, 1993, "The Division of Labor, Coordination Costs, and Knowledge." Becker, Gary S. *Human capital: a theoretical and empirical analysis, with special reference to education,* 3rd ed. Chicago; London: The University of Chicago Press, Chapter XI, 299-322.

[3] Bhagwati, Jagdish N.; T.N. Srinivasan, c1983, *Lectures on international trade,* Cambridge, Mass.: MIT Press.

[4] Chong, Fatt Seng, 2002a, "The Ricardo-Viner Model with a Publicly Provided Intermediate Good." *Osaka Economic Papers* 51(4), 166-181.

[5] Chong, Fatt Seng, 2002b, "Technology Change and Endogenous Labor Supply in a Small Open Country." *Osaka Economic Papers* 52(1), 183-

195.

[6] Chong, Fatt Seng, 2004a, "Trade Patterns and Human Capital under Publicly Provided Education and Privately Provided Education." *Bulletin of Policy and Management* 8, 39-47.

[7] Chong, Fatt Seng, 2004b, "Factor Price Equalization and Ricardo-Viner Model." *Shobi Journal of Policy Studies* 1, 89-93.

[8] Card, David; Alan B. Krueger, 1990, "Does School Quality Matter? Returns to Education and the Characteristic of Public School in the United States." *NBER Working Paper* May.

[9] Dixit, Avinash; Victor Norman, 1980, *a dual, general equilibrium approach,* Digswell Place, Welwyn [England] : James Nisbet. - [Cambridge, Cambridgeshire] : Cambridge University Press.

[10] Finn, Jerem D.; Charles M. Achilles, 1990, "Answer and Question About Class Size: A Statewide Experiment." *American Education Research Journal* 27, 557-577.

[11] Findlay, Ronald; Henryk Kierzkowski, 1983, "International Trade and Human Capital: A Simple General Equilibrium Model." *Journal of Political Economy* 91, 957-978.

[12] Frenkel, Jacob A.; Assaf Razin, 1975, "Variable Factor Supplies and the Production Possibility Frontier." *Southern-Economic-Journal* 41(3), 410-419.

[13] Gupta, Manash Ranjan, 1994, "Foreign capital, income inequality and welfare in a Harris-Todal model." *Journal of Development Economics* 45, 407-414.

[14] Helpman, E., 1983, "Variable returns to scale, and international trade, two generalization." *Economics Letters* 11, 167-174.

[15] Horvat, Vedran, 2004, "Brain Drain. Threat to Successful Transition in South East Europe?" *Southeast European Politics* 5(1), 76-93.

[16] Ishikawa, J., 1991, "The gains-from-trade theorem with variable returns to scale in the presence of intermediate goods." *Economics Letters* 36, 397-401.

[17] Ishikawa, J., 1992, "Trade patterns and gains from trade with an intermediate good produced under increasing returns to scale." *Journal of International Economics* 32, 57-81.

[18] Ishikawa, J., 2000, "The Ricardo-Viner Trade Model With An Intermediate Good." *Hitotsubashi Journal of Economics* 41, 65-75.

[19] Ishizawa, Suezo, 1988, "Increasing Returns, Public Inputs, and International Trade." *American Economic Review* 78(4), 794-795.

[20] Jones, Ronald W., 1971, "A Three-Factor Model in Theory, Trade, and History." *In Trade, Balance of Payments and Growth: Papers in International Economics in Honor of Charles P. Kindleberger,* edited by jagdish N. Bhagwati et al. Amsterdam: North-Holland.

[21] Kemp, M.C.; T. Negishi, 1970, "Variable returns to scale, commodity taxes, factor market distortions and trade theory," *Swedish Journal of Economics* 72, 1-11.

[22] Kemp, M.C. and R.W. Jones, 1962, "Variable labor supply and the theory of international trade." *Journal of Political Economy* 70, 30-36.

[23] Krugman, P.R., 2000, "Technology, trade, and factor prices." *Journal of International Economics* 50, 51-71.

[24] Leamer, E.E., 1998, "In search of Stolper-Samuelson linkages between international trade and lower wages. In: Collins, S.M. (Ed.), Imports, Exports, and the American Worker." *Brookings Institution, Washington, DC*, 141-203.

[25] Leontief, Wassily W., 1986, "Factor Proportions and the Structure of American Trade: Further Theoretical and Empirical Analysis." *Input-*

output economics. Second edition, New York and Oxford: Oxford University Press, 94-128. Previously published: [1956].

[26] Mincer, Jacob, 1958, "Investment in Human Capital and Personal Income Distribution." *The Journal of Political Economy* 66, 281-302.

[27] Martin, John P., 1976, "Variable Factor Supplies and the Heckscher-Ohlin-Samuelson Model." *Economic Journal* 86(344), 820-831.

[28] Martin, John P.; J. Peter Neary, 1980, "Variable Labour Supply and the Pure Theory of International Trade: An Empirical Note." *Journal of International Economics* 10(4), 549-559.

[29] Mullan, Fitzhugh, 2005, "The Metrics of the Physician Brain Drain." *New England Journal of Medicine* 353(17), 1810-1818.

[30] Mayer, Wolfgang, 1982, "Factor Quality, Factor Prices and Production Patterns." *Journal of International Economics* 12(1/2), 25-40.

[31] Mayer, Wolfgang, 1991, "Endogenous labor supply in international trade theory." *Journal of International Economics* 30, 105-120.

[32] Miyagiwa, Kaz, 1991, "Scale Economies in Education and the Brain Drain problem." *International Economic Review* 32, 743-759.

[33] Pigou, Arthur Cecil, 1928, “A Study in Public Finance.” *Macmillan, London*, page 29.

[34] Rodrik, D., 1997, “Has globalization gone too far?.” *Insitute for International Economics, Washington DC.*.

[35] Stark, O., Helmenstein, C., Prskawetz, A., 1998, “Human capital depletion, human capital formation, and migration: a blessing or a “curse”?” *Economics Letters* 60, 363-367.

[36] Tawada, Makoto; Kenzo Abe, 1984, “Production Possibilities, and International Trade.” *Canadian Journal of Economics* 17, 232-248.

[37] Xu, Bin, 2001, “Factor bias, sector bias, and the effects of technical progress on relative factor prices.” *Journal of International Economics* 54, 5-25.

[38] Wong, Kar-yiu; Chong Kee Yip, 1999, “Education, economic growth, and brain drain.” *Journal of Economic Dynamics Control* 23, 699-726.