Chao, X.
Osaka J. Math.
50 (2013), 715–723

COMPLETE SPACELIKE HYPERSURFACES IN THE DE SITTER SPACE

XIAOLI CHAO

(Received August 8, 2011, revised December 7, 2011)

Abstract

In this paper, by modifying Cheng–Yau's technique to complete spacelike hypersurfaces in the de Sitter $(n+1)$-space $S_1^{n+1}(1)$, we prove a rigidity under the hypothesis of the mean curvature and the normalized scalar curvature being linearly related. As a corollary, we have the Theorem 1.1 of [3].

1. Introduction

Let L^{n+2} be the $(n+2)$-dimensional Lorentz–Minkowski space, that is, the real vector space R^{n+2} endowed with the Lorentzian metric

$$\langle v, w\rangle = -v_0 w_0 + \sum_{i=1}^{n+1} v_i w_i \tag{1.1}$$

for $v, w \in R^{n+2}$. Then, the $(n+1)$-dimensional de Sitter space $S_1^{n+1}(1)$ can be defined as the following hyperquadric of L^{n+2}

$$S_1^{n+1}(1) = \{x \in L^{n+2}\colon \langle x, x\rangle = 1\}. \tag{1.2}$$

A smooth immersion $\varphi\colon M^n \to S_1^{n+1}(1) \subset L^{n+2}$ of an n dimensional connected manifold M^n is said to be a *spacelike* hypersurface if the induced metric via φ is a Riemannian metric on M^n. As is uaual, the spacelike hypersurface is said to be complete if the Riemannian induced metric is a complete metric on M^n. By endowing M^n with the induced metric we can suppose M^n to be Riemannian and φ to be an isometric spacelike immersion.

The interest in the study of spacelike hypersurfaces immersed in the de Sitter space is motivated by their nice Bernstein-type properties. It was proved by E. Calabi [2] (for $n \leq 4$) and by S.Y. Cheng and S.T. Yau [7] (for all n) that a complete maximal spacelike hypersurface in L^{n+2} is totally geodesic. In [13], S. Nishikawa obtained similar results for others Lorentzian manifolds. In particular, he proved that a complete maximal spacelike hypersurface in $S_1^{n+1}(1)$ is totally geodesic.

2000 Mathematics Subject Classification. 53C40, 53C42.
Research supported by grant No. 10971029 of NSFC.

Goddard [9] conjectured that complete spacelike hypersurface with constant mean curvature in the de Sitter $S_1^{n+1}(1)$ should be umbilical. Although the conjecture turned out to be false in its original statement, it motivated a great deal of work of several authors trying to find a positive answer to the conjecture under appropriate additional hypotheses. For instance, in 1987 Akutagawa [1] proved the Goddard conjecture when $H^2 < 1$ if $n = 2$ (see also Ramanathan [16]) and $H^2 < 4(n-1)/n^2$ if $n > 2$. He and Ramanathan also showed that when $n = 2$, for any constant $H^2 > c^2$, there exists a non-umbilical surface of mean curvature H in the de Sitter space $S_1^3(c)$ of constant curvature $c > 0$. One year later, S. Montiel [11] (and Akutagawa [1], Ramanathan [16] when $n = 2$) solved Goddard's problem in the compact case in $S_1^{n+1}(1)$ without restriction over the range of H. He also gave examples of non-umbilical complete spacelike hypersurfaces in $S_1^{n+1}(1)$ with constant H satisying $H^2 \geq 4(n-1)/n^2$ if $n > 2$, including the so-called hyperbolic cylinders. In [12], Montiel proved that the only complete spacelike hypersurface in $S_1^{n+1}(1)$ with constant $H = 2\sqrt{n-1}/n$ with more than one topological end is a hyperbolic cylinder. At the same time, the complete hypersurfaces in the de Sitter space have been characterized by Cheng [5] under the hypothesis of the mean curvature and the scalar curvature being linearly related.

On the other hand, for the study of spacelike hypersurfaces with constant scalar curvature in de Sitter spaces, Y. Zheng [18] proved that a compact spacelike hypersurface in $S_1^{n+1}(1)$ with constant normalized scalar curvature r, $r < 1$ and non-negative sectional curvatures is totally umbilical. Later, Q.M. Cheng and S. Ishikawa [6] showed that Zheng's result in [18] is also true without additional assumptions on the sectional curvatures of the hypersurface. In [10], H. Li proposed the following problem: Let M^n be a complete spacelike hypersurface in $S_1^{n+1}(1)$, $n \geq 3$, with constant normalized scalar curvature r satisfying $(n-2)/n \leq r \leq 1$. Is M^n totally umbilical? A. Caminha [4] answered that question affirmatively under the additional condition that the supremum of H is attained on M^n. Recently, Camargo–Chaves–Sousa [3] showed that Li's question is also true if the mean curvature is bounded.

In this paper, by modifying Cheng–Yau's technique to complete spacelike hypersurfaces in $S_1^{n+1}(1)$, we prove a rigidity theorem under the hypothesis of the mean curvature and the normalized scalar curvature being linearly related. More precisely, we have

Theorem 1.1. *Let M^n be a complete spacelike hypersurface of $S_1^{n+1}(1)$ with bounded mean curvature. If $r = aH + b$, $a, b \in \mathbb{R}$, $a \geq 0$, $b \geq (n-2)/n$, $(n-1)a^2 + 4n(1-b) \geq 0$, then M^n is totally umbilical.*

If we choose $a = 0$ and $(n-2)/n \leq b \leq 1$ in Theorem 1.1, we obtain the Theorem 1.1 of [3]:

Corollary 1.2. *Let M^n be a complete spacelike hypersurface of $S_1^{n+1}(1)$ with constant normalized scalar curvature r satisfying $(n-2)/n \leq r \leq 1$. If M^n has bounded* *mean curvature, then M^n is totally umbilical.*

2. Preliminaries

Let M be an n-dimensional complete spacelike hypersurface of de Sitter space $S_1^{n+1}(1)$. For any $p \in M$, we choose a local orthonormal frame $e_1, \ldots, e_n, e_{n+1}$ in $S_1^{n+1}(1)$ around p such that $e_1, \ldots, e_n$ are tangent to M. Take the corresponding dual coframe $\omega_1, \ldots, \omega_n, \omega_{n+1}$, We use the following standard convention for indices:

$$1 \leq A, B, C, D, \ldots \leq n+1, \quad 1 \leq i, j, k, l, \ldots \leq n.$$

Let $\varepsilon_i = 1$, $\varepsilon_{n+1} = -1$, then the structure equations of $S_1^{n+1}(1)$ are given by

$$d\omega_A = \sum_B \varepsilon_B \omega_{AB} \wedge \omega_B, \quad \omega_{AB} + \omega_{BA} = 0, \tag{2.1}$$

$$d\omega_{AB} = \sum_C \varepsilon_C \omega_{AC} \wedge \omega_{CB} - \frac{1}{2} \sum_{C,D=1}^{n+1} R_{ABCD} \omega_C \wedge \omega_D, \tag{2.2}$$

$$R_{ABCD} = \varepsilon_A \varepsilon_B (\delta_{AC} \delta_{BD} - \delta_{AD} \delta_{BC}). \tag{2.3}$$

Restricting those forms to M, we get the structure equations of M

$$d\omega_i = \sum_{j=1}^n \omega_{ij} \wedge \omega_j, \quad \omega_{ij} + \omega_{ji} = 0, \tag{2.4}$$

$$d\omega_{ij} = \sum_{k=1}^n \omega_{ik} \wedge \omega_{kj} - \frac{1}{2} \sum_{k,l=1}^n R_{ijkl} \omega_k \wedge \omega_l. \tag{2.5}$$

The Gauss equations are

$$R_{ijkl} = (\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}) - (h_{ik}h_{jl} - h_{il}h_{jk}), \tag{2.6}$$

$$n(n-1)r = n(n-1) - n^2 H^2 + |B|^2, \tag{2.7}$$

where $r = (1/(n(n-1))) \sum_{i,j} R_{ijij}$ is the normalized scalar curvature of M and the norm square of the second fundamental form is

$$|B|^2 = \sum_{i,j} h_{ij}^2. \tag{2.8}$$

The Codazzi equations are

$$h_{ijk} = h_{ikj} = h_{jik}, \tag{2.9}$$

where the covariant derivative of h_{ij} is defined by

$$\sum_k h_{ijk}\omega_k = dh_{ij} + \sum_k h_{kj}\omega_{ki} + \sum_k h_{ik}\omega_{kj}. \tag{2.10}$$

Similarly, the components h_{ijkl} of the second derivative $\nabla^2 h$ are given by

$$\sum_l h_{ijkl}\omega_l = dh_{ijk} + \sum_l h_{ljk}\omega_{li} + \sum_l h_{ilk}\omega_{lj} + \sum_l h_{ijl}\omega_{lk}. \tag{2.11}$$

By exterior differentiation of (2.10), we can get the following *Ricci formula*

$$h_{ijkl} - h_{ijlk} = \sum_m h_{im}R_{mjkl} + \sum_m h_{jm}R_{mikl}. \tag{2.12}$$

The Laplacian $\triangle h_{ij}$ of h_{ij} is defined by $\triangle h_{ij} = \sum_k h_{ijkk}$, from the Codazzi equation and Ricci formula, we have

$$\triangle h_{ij} = \sum_k h_{kkij} + \sum_{m,k} h_{km}R_{mijk} + \sum_{m,k} h_{im}R_{mkjk}. \tag{2.13}$$

Set $\phi_{ij} = h_{ij} - H\delta_{ij}$. It is easy to check that ϕ is traceless and $|\phi|^2 = \sum_{i,j}\phi_{ij}^2 = |B|^2 - nH^2$. Following Cheng–Yau [8], for each $a \geq 0$, we introduce a modified operator $\square$ acting on any C^2-function f by

$$\square(f) = \sum_{i,j}\left(\left(nH + \frac{n-1}{2}a\right)\delta_{ij} - h_{ij}\right) f_{ij}, \tag{2.14}$$

where f_{ij} is given by the following

$$\sum_j f_{ij}\omega_j = df_i + f_j\omega_{ij}.$$

Lemma 2.1. *Let M^n be a complete spacelike hypersurface of $S_1^{n+1}(1)$ with $r = aH + b$, $a, b \in \mathbb{R}$ and $(n-1)a^2 + 4n - 4nb \geq 0$. Then we have*

$$|\nabla B|^2 \geq n^2|\nabla H|^2. \tag{2.15}$$

Proof. From Gauss equation, we have

$$|B|^2 = n^2H^2 + n(n-1)(r-1) = n^2H^2 + n(n-1)(aH + b - 1).$$

Taking the covariant derivative of the above equation, we have

$$2\sum_{i,j} h_{ij}h_{ijk} = 2n^2HH_k + n(n-1)aH_k.$$

Therefore,

$$4|B|^2|\nabla B|^2 \geq 4\sum_k \left(\sum_{i,j} h_{ij}h_{ijk}\right)^2 = [2n^2H + n(n-1)a]^2|\nabla H|^2.$$

Since we know

$$\begin{aligned}[2n^2H + n(n-1)a]^2 - 4n^2|B|^2 &= 4n^4H^2 + n^2(n-1)^2a^2 + 4n^3(n-1)aH \\ &\quad - 4n^2(n^2H^2 + n(n-1)(aH + b - 1)) \\ &= n^2(n-1)^2a^2 - 4n^3(n-1)(b-1) \\ &= n^2(n-1)[(n-1)a^2 + 4n - 4nb] \geq 0,\end{aligned}$$

it follows that

$$|\nabla B|^2 \geq n^2|\nabla H|^2. \qquad \square$$

Lemma 2.2. *Let M^n be a complete spacelike hypersurface of $S_1^{n+1}(1)$ with $r = aH + b$, $a, b \in \mathbb{R}$. Then we have*

$$\begin{aligned}\square(nH) \geq &\sum_{i,j,k} h_{ijk}^2 - n^2|\nabla H|^2 \\ &+ |\phi|^2\left(|\phi|^2 - \frac{n(n-2)}{\sqrt{n(n-1)}}|H|\,|\phi| + n(1-H^2)\right).\end{aligned} \tag{2.16}$$

Proof. First, from (2.6) and (2.12), we have

$$\begin{aligned}\frac{1}{2}\triangle|B|^2 = \frac{1}{2}\triangle\sum_{i,j} h_{ij}^2 &= \sum_{i,j} h_{ij}\triangle h_{ij} + \sum_{i,j,k} h_{ijk}^2 \\ &= \sum_{i,j,k} h_{ijk}^2 + n\sum_{i,j} h_{ij}H_{ij} + n(|B|^2 - nH^2) - nHf_3 + |B|^4,\end{aligned}$$

where $f_3 = \sum_{i,j,k} h_{ij}h_{jk}h_{ki}$. On the other side, from Gauss equation and $r = aH + b$, we have

$$\begin{aligned}\triangle|B|^2 &= \triangle(n^2H^2 + n(n-1)(r-1)) \\ &= \triangle(n^2H^2 + n(n-1)(aH + b - 1)) \\ &= \triangle(n^2H^2 + (n-1)anH) \\ &= \triangle(nH + \frac{1}{2}(n-1)a)^2.\end{aligned} \tag{2.17}$$

Then from (2.17) and Okumura's inequality [14], we get

$$\begin{aligned}\square(nH) &= \sum_{i,j}((nH + \frac{1}{2}(n-1)a)\delta_{ij} - h_{ij})(nH)_{ij}\\ &= (nH + \frac{1}{2}(n-1)a)\triangle(nH) - \sum_{i,j} h_{ij}(nH)_{ij}\\ &= (nH + \frac{1}{2}(n-1)a)\triangle(nH + \frac{1}{2}(n-1)a) - \sum_{i,j} h_{ij}(nH)_{ij}\\ &= \frac{1}{2}\triangle\left(nH + \frac{1}{2}(n-1)a\right)^2 - \left|\nabla(nH + \frac{1}{2}(n-1)a)\right|^2 - \sum_{i,j} h_{ij}(nH)_{ij}\\ &= \frac{1}{2}\triangle\left(nH + \frac{1}{2}(n-1)a\right)^2 - n^2|\nabla H|^2 - \sum_{i,j} h_{ij}(nH)_{ij}\\ &= \frac{1}{2}\triangle|B|^2 - n^2|\nabla H|^2 - \sum_{i,j} h_{ij}(nH)_{ij}\\ &= \sum_{i,j,k} h_{ijk}^2 - n^2|\nabla H|^2 + n(|B|^2 - nH^2) - nHf_3 + |B|^4\\ &\geq \sum_{i,j,k} h_{ijk}^2 - n^2|\nabla H|^2 + |\phi|^2\left(|\phi|^2 - \frac{n(n-2)}{\sqrt{n(n-1)}}|H|\,|\phi| + n(1-H^2)\right). \qquad \square\end{aligned}$$

Proposition 2.3. *Let M^n be a complete spacelike hypersurface of $S_1^{n+1}(1)$ with bounded mean curvature. If $r = aH + b$, $a, b \in \mathbb{R}$, $a \geq 0$, $(n-1)a^2 + 4n - 4nb \geq 0$, then there is sequence of points $\{p_k\} \in M^n$ such that*

$$\lim_{k\to\infty} nH(p_k) = n \sup H; \qquad \lim_{k\to\infty} |\nabla nH(p_k)| = 0; \qquad \lim_{k\to\infty} \sup(\square(nH)(p_k)) \leq 0.$$

Proof. Choose a local orthonormal frame field $e_1, \ldots, e_n$ at $p \in M^n$ such that $h_{ij} = \lambda_i\delta_{ij}$. Thus

$$\square(nH) = \sum_i \left[\left(nH + \frac{1}{2}(n-1)a\right) - \lambda_i\right](nH)_{ii}.$$

If $H \equiv 0$ the proposition is obvious. Let us suppose that H is not identically zero. By changing the orientation of M^n if necessary, we may assume $\sup H > 0$. From

$$\begin{aligned}\lambda_i^2 \leq |B|^2 &= n^2H^2 + n(n-1)(aH + b - 1)\\ &= (nH)^2 + (n-1)a(nH) + n(n-1)(b-1)\\ &= \left(nH + \frac{1}{2}(n-1)a\right)^2 - \frac{1}{4}(n-1)((n-1)a^2 + 4n - 4nb)\\ &\leq \left(nH + \frac{1}{2}(n-1)a\right)^2,\end{aligned}$$

we have

$$|\lambda_i| \leq \left|nH + \frac{1}{2}(n-1)a\right|. \tag{2.18}$$

Then, for i, j with $i \neq j$,

$$R_{ijij} = 1 - \lambda_i\lambda_j \geq 1 - \left(nH + \frac{1}{2}(n-1)a\right)^2. \tag{2.19}$$

Because H is bounded, it follows from (2.19) that the sectional curvatures are bounded from below. Therefore we may apply generalized maximun principle [15] [17] to nH, obtaining a sequence of points $\{p_k\} \in M^n$ such that

$$\lim_{k\to\infty} nH(p_k) = n \sup H; \qquad \lim_{k\to\infty} |\nabla nH(p_k)| = 0; \qquad \lim_{k\to\infty} \sup((nH)_{ii}(p_k)) \leq 0. \tag{2.20}$$

Since H is bounded, taking subsequences if necessary, we can arrive to a sequence $\{p_k\} \in M^n$ which satisfies (2.20) and such that $H(p_k) \geq 0$. Thus from (2.18) we get

$$\begin{aligned} 0 \leq nH(p_k) + \frac{1}{2}(n-1)a - |\lambda_i(p_k)| &\leq nH(p_k) + \frac{1}{2}(n-1)a - \lambda_i(p_k) \\ &\leq nH(p_k) + \frac{1}{2}(n-1)a + |\lambda_i(p_k)| \\ &\leq 2nH(p_k) + (n-1)a. \end{aligned} \tag{2.21}$$

Using once more the fact that H is bounded, from (2.21) we infer that $nH(p_k) + (1/2)(n-1)a - \lambda_i(p_k)$ is non-negative and bounded. By applying $\Box(nH)$ at p_k, taking the limit and using (2.20) and (2.21) we have

$$\begin{aligned} \lim_{k\to\infty} \sup(\Box(nH)(p_k)) &\leq \sum_i \lim_{k\to\infty} \sup \left[\left(nH + \frac{1}{2}(n-1)a\right) - \lambda_i\right](p_k)(nH)_{ii}(p_k) \\ &\leq 0. \end{aligned}$$

□

3. Proof of results

Proof of Theorem 1.1. If M^n is maximal, i.e., if $H \equiv 0$, due to Nishikawa's result [13], we know that M^n is totally geodesic. Let us suppose that H is not identically zero. In this case, by Proposition 2.3, it is possible to obtain a sequence of points $\{p_k\} \in M^n$ such that

$$\lim_{k\to\infty} \sup(\Box(nH)(p_k)) \leq 0, \qquad \lim_{k\to\infty} H(p_k) = \sup H > 0. \tag{3.1}$$

Moreover, using the Gauss equation, we have that

$$|\phi|^2 = |B|^2 - nH^2 = n(n-1)(H^2 + aH + b - 1). \tag{3.2}$$

In view of $\lim_{k\to\infty} H(p_k) = \sup H$ and $a \geq 0$, (3.2) implies that $\lim_{k\to\infty}|\phi|^2(p_k) = \sup|\phi|^2$. Now we consider the following polynomial given by

$$P_{\sup H}(x) = x^2 - \frac{n(n-2)}{\sqrt{n(n-1)}} \sup H x + n(1 - \sup H^2). \tag{3.3}$$

If $\sup H^2 < 4(n-1)/n^2$, then the discriminant of $P_{\sup H}(x)$ is negative. Hence, $P_{\sup H}(\sup|\phi|) > 0$. If $\sup H^2 \geq 4(n-1)/n^2$, let μ the biggest root of $P_{\sup H}(x) = 0$, which is positive. It's easy to check that $\sup|\phi|^2 - \mu^2 > 0$ provided $a \geq 0$ and $b \geq (n-2)/n$. In fact,

$$\begin{aligned}(\sup|\phi|)^2 = \sup|\phi|^2 &= n(n-1)(\sup H^2 + a \sup H + b - 1)\\ &\geq n(n-1)(\sup H^2 + b - 1),\end{aligned} \tag{3.4}$$

it is straightforward to verify that

$$\sup|\phi|^2 - \mu^2 \geq \frac{n-2}{2(n-1)}\Big(n^2 \sup H^2 - n \sup H \sqrt{n^2 \sup H^2 - 4(n-1)} + c\Big). \tag{3.5}$$

where $c = (2n(n-1)/(n-2))((n-1)b-(n-2))$. It can be easily seen that $\sup|\phi|^2 - \mu^2 > 0$ if and only if

$$n^2 \sup H^2 - n \sup H \sqrt{n^2 \sup H^2 - 4(n-1)} + c > 0.$$

So, if $b > (n-2)/n$, the last inequality is true. In fact, it's true if and only if

$$(n^2 \sup H^2 + c)^2 > n^2 \sup H^2(n^2 \sup H^2 - 4(n-1)), \tag{3.6}$$

i.e.

$$n^2 \sup H^2(2c + 4(n-1)) + c^2 > 0. \tag{3.7}$$

If $b = (n-2)/n$, then $2c + 4(n-1) = 0$. Hence

$$n^2 \sup H^2(2c + 4(n-1)) + c^2 > 0.$$

Then we deduce that $P_{\sup H}(\sup|\phi|) > 0$.

Using Lemma 2.1 and evaluating (2.16) at the points p_k of the sequence, taking the limit and using (3.1), we obtain that

$$0 \geq \limsup_{k\to\infty}(\square(nH)(p_k)) \geq \sup|\phi|^2 P_{\sup H}(\sup|\phi|) \geq 0,$$

and so $\sup|\phi|^2 P_{\sup H}(\sup|\phi|) = 0$. Therefore, since $P_{\sup H}(\sup|\phi|) > 0$, we conclude that $\sup|\phi|^2 = 0$ which shows that M^n is totally umbilical. $\square$

ACKNOWLEDGEMENT. This work was done while the author was visiting University of Notre Dame. The author is grateful for the hospitality of the mathematical department of University of Notre Dame. We would like to express our gratitude to the referee for valuable suggestions.

References

[1] K. Akutagawa: *On spacelike hypersurfaces with constant mean curvature in the de Sitter space*, Math. Z. **196** (1987), 13–19.

[2] E. Calabi: *Examples of Bernstein problems for some nonlinear equations*, Math. Proc. Cambridge Phil. Soc. **82** (1977), 489–495.

[3] F.E.C. Camargo, R.M.B. Chaves and L.A.M. Sousa, Jr.: *Rigidity theorems for complete spacelike hypersurfaces with constant scalar curvature in de Sitter space*, Differential Geom. Appl. **26** (2008), 592–599.

[4] A. Caminha: *A rigidity theorem for complete CMC hypersurfaces in Lorentz manifolds*, Differential Geom. Appl. **24** (2006), 652–659.

[5] Q.M. Cheng: *Complete space-like hypersurfaces of a de Sitter space with $r = kH$*, Mem. Fac. Sci. Kyushu Univ. Ser. A **44** (1990), 67–77.

[6] Q.-M. Cheng and S. Ishikawa: *Spacelike hypersurfaces with constant scalar curvature*, Manuscripta Math. **95** (1998), 499–505.

[7] S.Y. Cheng and S.T. Yau: *Maximal space-like hypersurfaces in the Lorentz–Minkowski spaces*, Ann. of Math. (2) **104** (1976), 407–419.

[8] S.Y. Cheng and S.T. Yau: *Hypersurfaces with constant scalar curvature*, Math. Ann. **225** (1977), 195–204.

[9] A.J. Goddard: *Some remarks on the existence of spacelike hypersurfaces of constant mean curvature*, Math. Proc. Cambridge Philos. Soc. **82** (1977), 489–495.

[10] H. Li: *Global rigidity theorems of hypersurfaces*, Ark. Mat. **35** (1997), 327–351.

[11] S. Montiel: *An integral inequality for compact spacelike hypersurfaces in de Sitter space and applications to the case of constant mean curvature*, Indiana Univ. Math. J. **37** (1988), 909–917.

[12] S. Montiel: *A characterization of hyperbolic cylinders in the de Sitter space*, Tohoku Math. J. (2) **48** (1996), 23–31.

[13] S. Nishikawa: *On maximal spacelike hypersurfaces in a Lorentzian manifold*, Nagoya Math. J. **95** (1984), 117–124.

[14] M. Okumura: *Hypersurfaces and a pinching problem on the second fundamental tensor*, J. Math. Soc. Japan **19** (1967), 205–214.

[15] H. Omori: *Isometric immersions of Riemannian manifolds*, J. Math. Soc. Japan **19** (1967), 205–214.

[16] J. Ramanathan: *Complete spacelike hypersurfaces of constant mean curvature in de Sitter space*, Indiana Univ. Math. J. **36** (1987), 349–359.

[17] S.T. Yau: *Harmonic functions on complete Riemannian manifolds*, Comm. Pure Appl. Math. **28** (1975), 201–228.

[18] Y. Zheng: *Space-like hypersurfaces with constant scalar curvature in the de Sitter spaces*, Differential Geom. Appl. **6** (1996), 51–54.

Department of Mathematics
Southeast University, 210096 Nanjing
P.R. China
e-mail: xlchao@seu.edu.cn