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Effect of Reflected Wave from the Wall of a Vessel and End Effect of the Plate in the Oscillating-plate Method

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The effect of reflected wave and the end effect of a new oscillating-plate viscometer, constructed by Sasahara *et al.*, have been investigated experimentally over a wide range of viscosity. The results obtained are as follows:

(1) The resonant frequencies of the plate oscillations in liquid samples decrease with increasing $\rho\mu$ values. However, the viscosity dependence of the resonant frequency is negligibly small when an adequate leaf-spring is used for chinning the plate attached to stem.

(2) The effect of reflected wave from the wall of a vessel is negligible when the distance between the plate and the wall is kept over one wavelength of the wave produced by the plate oscillations.

(3) The end effect of the oscillating-plate increases with increasing not only the thickness of the plates but also the viscosity of liquids.

(4) The working formula for viscosity determination by the oscillating-plate method should be corrected by considering the above two effects.

KEY WORDS: viscosity; oscillating-plate viscometer; correction factors of viscometer.

1. Introduction

The authors have constructed an oscillating-plate viscometer, and investigated its characteristics for the viscosity range below 1 Pa·s.[1,2] In addition, we have been participating in the project from Phase 1 to Phase 3 for the establishment of standard reference materials at high temperature viscosity measurements. This project is organized by Community Bureau of Reference in Europe.[3,4] Approximate ten laboratories in Europe and Japan have been participating in this project. However, only one of them, *i.e.*, Osaka University has been using the oscillating-plate method. Incidentally, Sumitomo Metal Industries, Ltd. and Kobe Steel, Ltd. will also participate in this project and use the oscillating-plate method in Phase 3. The designs of these equipments are based on the authors' original design. As such, detailed studies on the correction factors needed for this method are required.

On the basis of the authors' design, a new oscillating-plate viscometer has been constructed by Sasahara *et al.*[5] to measure the viscosities of molten slags and fluxes with high accuracy. This type of viscometer provides instantaneous and continuous data for the product of density and viscosity of the melts, even though the temperature of the melts changes continuously. Those data are recorded automatically by measuring the amplitudes of the plate oscillations. In this viscometer, a leaf-spring is equipped for chinning the plate attached to stem and a laser displacement transducer is used for determining precise amplitudes, as shown in **Fig. 1**.

The equation used to connect the measured amplitudes with the viscosity of the liquid in the oscillating-plate method has been derived on the basis of several assumptions as will be described later. The experimental conditions of the apparatus, however, do not satisfy all of those assumptions. The corrections of the viscometer, therefore, must sufficiently be weighted to obtain accurate viscosity values of molten slags and fluxes.

The purpose of the present work is to study experimentally the important correction factors, *i.e.*, the effect of reflected wave and the end effect of the oscillating-

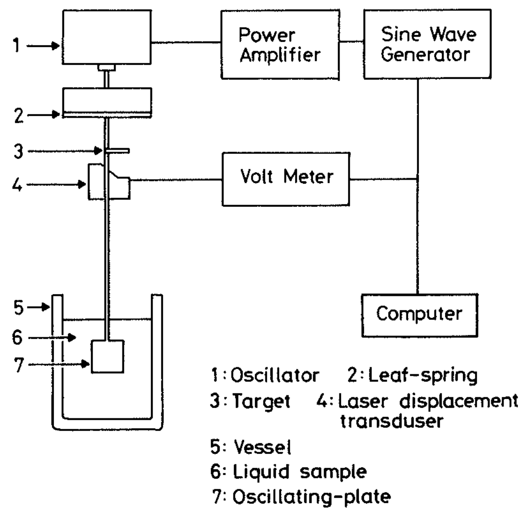

Fig. 1. Schematic diagram of the oscillating-plate viscometer.

plate viscometer over a wide range of viscosity, *i.e.* 1–100 Pa·s, which covers the viscosity range of molten slags and fluxes.

2. Principle of Viscosity Measurement by the Oscillating-plate Method

When a flat plate executing linear oscillations is immersed in a liquid, its motions are impeded by the retarding force exerted on the oscillating-plate by the viscous liquid. If the plate is now vibrating in the liquid with a constant driving force, the amplitude of motion of the plate is reduced to a degree dependent on the viscosity of the liquid. The oscillating-plate method is based on measurements of the amplitudes of plate oscillations in air and in a liquid sample. The relation between viscosity and amplitudes has been derived theoretically as follows:[1,2)]

$$\rho\mu=\frac{R_M^2}{\pi f A^2}\cdot\left(\frac{f_a\cdot E_a}{f\cdot E}-1\right)^2=K\cdot\theta_0 \quad \text{..................(1)}$$

$$K=\frac{R_M^2}{\pi f_a A^2} \quad \text{and} \quad \theta_0=\frac{f_a}{f}\cdot\left(\frac{f_a\cdot E_a}{f\cdot E}-1\right)^2$$

where ρ is the liquid's density, μ is the liquid's viscosity, E and E_a are the amplitudes in liquid and in air, respectively, f and f_a are the resonant frequencies in liquid and in air, respectively, K is a constant of the apparatus, R_M is the real component of the mechanical impedance, A is the area of the plate.

Equation (1) has been derived on the following assumptions[6)]:

(1) A liquid sample is a Newtonian fluid.

(2) The plate oscillation does not make turbulent flow.

(3) An oscillating-plate does not slip on the contact surface of a liquid sample.

(4) The size of an oscillating-plate, *e.g.* in the case of square oscillating-plate, the length of side, is larger than one wavelength of the wave, which is made by the oscillation. The wave must be a plane wave.

(5) The end effect of an oscillating-plate on the damping of the amplitude is negligible.

(6) The size of a vessel is large enough to neglect the effect of reflected wave from the wall of the vessel.

If the resonant frequency in a liquid f is assumed to be equal to that in air f_a, Eq. (1) can be written in the following form:

$$\rho\mu=K\left(\frac{E_a}{E}-1\right)^2=K\cdot\theta \quad \text{..................(2)}$$

where $K=R_M^2/\pi f_a A^2$ and $\theta=(E_a/E-1)^2$.

Hereafter, θ is called the damping-factor. The constant of the apparatus K is determined experimentally by the use of Standard Viscosity Samples at room temperature.

3. Oscillating-plate Viscometer

Figure 1 shows the schematic diagram of the oscillating-plate viscometer, which comprises an oscillator, an oscillating-plate, a displacement transduser and a computer. It would be expected that the highly accurate oscillation can be obtained by the use of the leaf-spring, which has been developed by Sasahara *et al.*[5)] The shape of the leaf-spring of bronze is shown in **Fig. 2**. The stem connecting the oscillating-plate is chinned from the center of the leaf-spring. The displacements of the target attached to the stem are measured by the laser displacement transduser for determining the amplitudes of the oscillations, which are instantaneously and continuously recorded in a computer. In these experiments, square oscillating-plates (length of side, 30 mm) are employed.

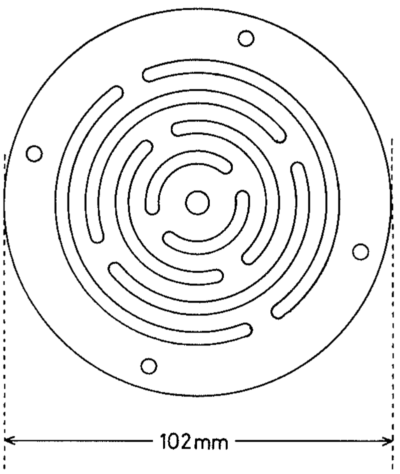

Fig. 2. Shape of a leaf-spring.

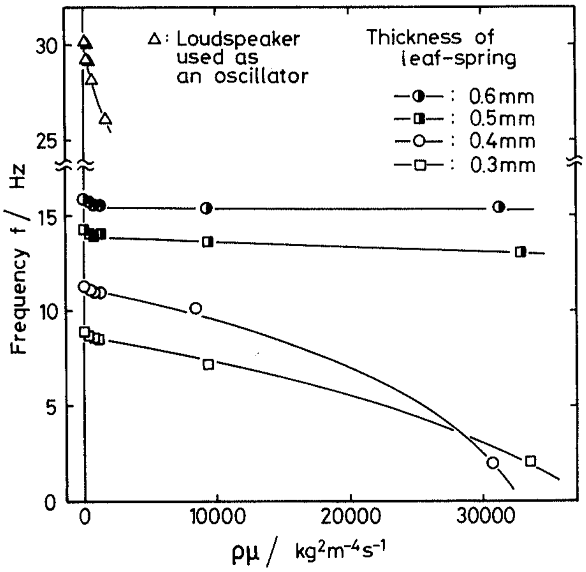

Fig. 3. Dependence of the resonant frequency on viscosity ($\rho\mu$) value. Thickness of the oscillating-plate: 0.5 mm.

4. Characteristics of the Oscillating-plate Viscometer

The authors have studied experimentally the characteristics of the oscillating-plate viscometer over a wide range of viscosity. In particular, the following were investigated: 1) dependence of the resonant frequency on viscosity ($\rho\mu$) of a liquid. 2) the effect of reflected wave from the wall of a vessel. 3) the end effect, *i.e.*, the effect

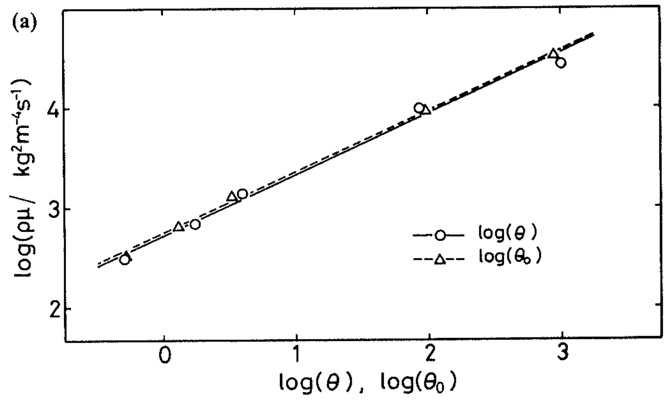

Fig. 4(a). Relations between $\log(\rho\mu)$ and $\log(\theta)$, and between $\log(\rho\mu)$ and $\log(\theta_0)$ for a leaf-spring of 0.5 mm thickness. Thickness of the oscillating-plate: 0.5 mm.

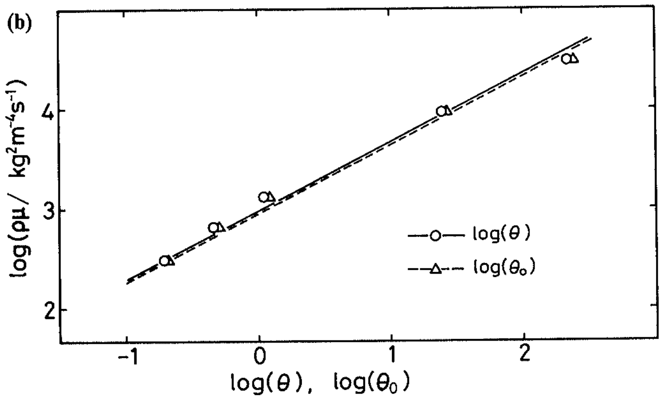

Fig. 4(b). Relations between $\log(\rho\mu)$ and $\log(\theta)$, and between $\log(\rho\mu)$ and $\log(\theta_0)$ for a leaf-spring of 0.6 mm thickness. Thickness of the oscillating-plate: 0.5 mm.

of the thickness of the oscillating-plate.

4.1. Dependence of the Resonant Frequency on Viscosity ($\rho\mu$) of a Liquid

As described in the preceding sections, Eq. (2) can be obtained on the assumption that the resonant frequency f in liquid is equal to f_a in air. It is known, however, that f is dependent upon the viscosity of a liquid and then the assumption is not always satisfied.[6] The relation between f and $\rho\mu$ for the present oscillating-plate viscometer with the leaf-spring, of which thickness is 0.3, 0.4, 0.5 or 0.6 mm, has been investigated experimentally. The liquids used for the experiments are Standard Viscosity Samples of 5 kinds of viscosities. The separation distance between the oscillating-plate and the wall in the vessel is kept to be 40 mm. **Figure 3** shows the experimental results of the relation between the resonant frequencies f and $\rho\mu$ values. The relation obtained in the oscillating-plate viscometer with a loudspeaker as an oscillator[2] is also indicated in this figure. As shown in Fig. 3, the frequencies f decrease with increasing $\rho\mu$ values for each leaf-spring. For comparatively high viscosity range 30 000–35 000 $\mathrm{kg^2\,m^{-4}\,s^{-1}}$ (≒30–35 Pa·s), the values of $(f-f_a)/f_a \times 100$ are of the order of 2–3 % for the leaf-spring of 0.6 mm thickness, 8 % for 0.5 mm, 80 % for 0.3 mm, respectively. These results stem mainly from the difference in the mechanical properties of the leaf-springs.

Figures 4(a) and **4(b)** represent the experimental results of the relation between $\log(\rho\mu)$ and $\log(\theta)$, and between $\log(\rho\mu)$ and $\log(\theta_0)$. These results show that the discrepancy between $\log(\theta_0)$ and $\log(\theta)$ is negligibly small for the viscosity range of 250–35 000 $\mathrm{kg^2\,m^{-4}\,s^{-1}}$ (≒0.25–35 Pa·s) when the leaf-spring of 0.5 or 0.6 mm thickness is used in the present apparatus.

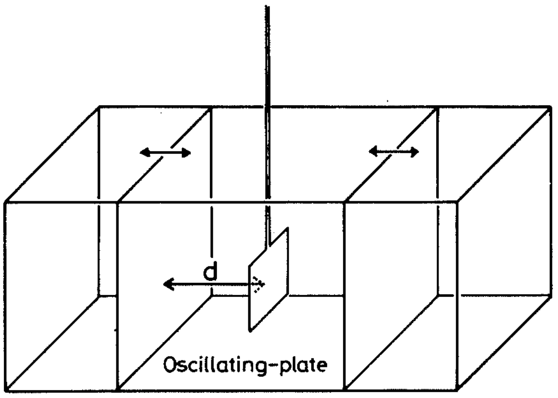

Fig. 5. Schematic diagram of the vessel. The distance between the walls of the vessel and the oscillating-plate is variable.

4.2. Effect of Reflected Wave from the Wall of a Vessel

The wave, which is produced by the plate oscillations, travels towards the direction perpendicular to the plate. Its amplitude is damped by the factor $\exp(-2\pi)$ as the wave advances by one wavelength, owing to the liquid's viscosity. If the distance between the plate and the wall is larger than one wavelength of the wave produced by the plate oscillations, the effect of reflected wave from the wall is negligibly small. This has been already found out for the liquids over a viscosity range of 25–749 $\mathrm{kg^2\,m^{-4}\,s^{-1}}$ (≒0.025–0.74 Pa·s).[2] In the present work, the effect of reflected wave from the wall has been studied for liquids over a viscosity range of 10^3–10^5 $\mathrm{kg^2\,m^{-4}\,s^{-1}}$ (≒1–100 Pa·s). The experiments are carried out as fol-

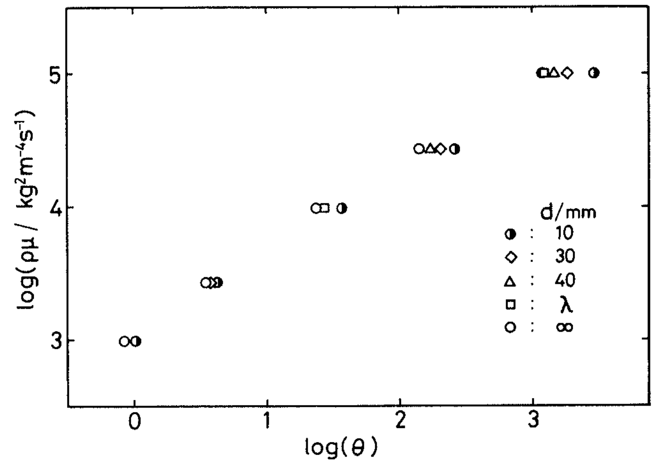

Fig. 6. Plots of log($\rho\mu$) against log(θ) for various distances between the oscillating-plate and the walls of the vessel. Thickness of the oscillating-plate: 0.5 mm.

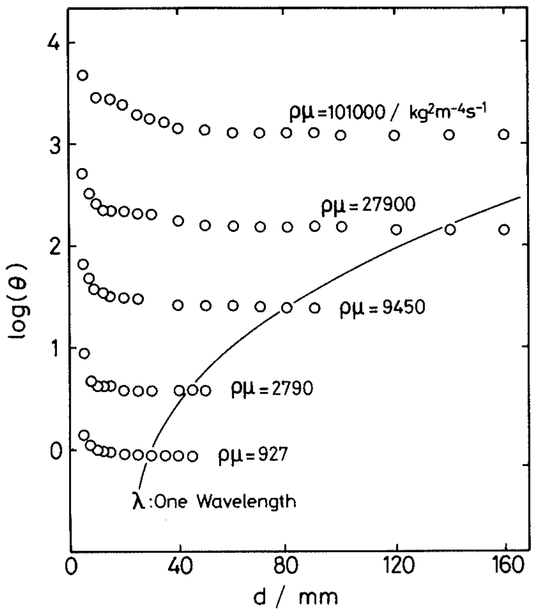

Fig. 7. Plots of log(θ) against the distance between the oscillating-plate and the walls of the vessel for various $\rho\mu$ values.

lows: The damping factor θ has been measured against various distances between the plate and the wall in the rectangular vessel as shown in **Fig. 5**. Five silicone-oils of different viscosity are employed as the liquid samples. The leaf-spring of 0.6 mm thickness is used for the experiments.

Figure 6 shows plots of log($\rho\mu$) *vs.* log(θ) for various distances, *i.e.*, $d=10$, 30, 40, λ and ∞ (extraporated value), between the plate and the walls. In **Fig. 7**, plots of log(θ) against the distance between the plate and the walls are shown for various $\rho\mu$ values; where the solid curve indicates one wavelength λ of the wave produced by the plate oscillations for each $\rho\mu$ value. As shown in these figures, the effect of reflected wave increases with decreasing the distance between the plate and the walls. In addition, the relations between log($\rho\mu$) and log(θ) for the distance of one wavelength λ are almost the same as those at infinity. In other words, the effect of reflected wave can be neglected for a viscosity range of 10^3–10^5 kg^{2}m^{-4}s^{-1} ($\fallingdotseq$1–100 Pa·s) when the distance is kept over one wavelength.

4.3. End Effect of the Oscillating-plate

Equations (1) and (2) have been derived on the assumption that the thickness of the plate is infinitely thin. Since the thickness of the plate is actually finite in any viscometer, the end effect of the oscillating-plate has been investigated experimentally in the present work. If we now assume that the effect of the thickness of the plate corresponds to the increase $\varDelta A_E$ in the area of the plate, the following equation is obtained from Eq. (2):

$$\frac{\theta}{\rho\mu}=\frac{1}{K}=\frac{\pi f_a}{R_M^2}(A+\varDelta A_E)^2 \quad \text{..................(3)}$$

Since $\varDelta A_E$ is considered to be much smaller than the area of the plate A, the term $(\varDelta A_E)^2$ can be neglected. In addition, if we assume that $\varDelta A_E$ is proportional to the thickness of the plate T, *i.e.*,

$$\varDelta A_E=C_E\cdot T \quad \text{.............................(4)}$$

Equation (3) can be rewritten in the form

$$\begin{aligned}\frac{\theta}{\rho\mu}&=\frac{\pi f_a}{R_M^2}(A^2+2A\varDelta A_E)\\&=\frac{\pi f_a A^2}{R_M^2}+\frac{2\pi f_a A C_E}{R_M^2}T\\&=a+b\cdot T \quad \text{......................................(5)}\end{aligned}$$

where $a=\pi f_a A^2/R_M^2$ and $b=2\pi f_a A C_E/R_M^2$.

The relation between the damping-factor θ and the thickness T has been determined experimentally. Square oscillating-plates of polyvinyl chloride (length of side, 30 mm; thickness, 2.15, 3.10, 3.90 and 5.00 mm) are used for the experiments. The leaf-spring of 0.6 mm thickness is equipped for chinning the plates. The value of 40 mm is employed for the distance between the plate and the walls of the vessel. The relations between $\theta/\rho\mu$ and T for various $\rho\mu$ values are shown in **Fig. 8**, where the linear relations are obtained as expected from Eq. (5). **Figure 9** shows the relations between log($\rho\mu$) and log(θ) for various thicknesses of the oscillating-plates. It is obvious from these figures that the end effect of the oscillating-plate increases with increasing not only the thickness of the plates but also the viscosity of the liquid samples.

5. Empirical Relation between the Damping Factor θ and $\rho\mu$ Values

As described in the preceding sections, Eq. (2) can be applied to the viscosity measurements of molten slags and fluxes if the resonant frequency is independent of the liquids' $\rho\mu$ values, and the value of the apparatus' constant K is determined, previous to viscosity measurements, by the use of Standard Viscosity Samples at room temperature. It is shown in Figs. 4(a) and 4(b) that the effect of the resonant frequency f, being dependent on $\rho\mu$ values, is negligible when the leaf-spring of 0.5 or 0.6 mm thickness is equipped in th apparatus.

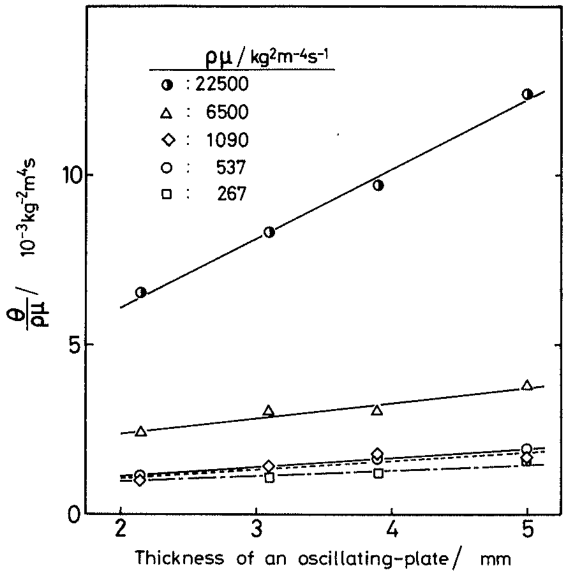

Fig. 8. Effect of the thickness of the oscillating-plate on $\rho\mu/\theta$ ($=1/K$) values.

Equation (2) can be written as

$$\log(\rho\mu)=\log K+\log(\theta) \quad \text{.....................(6)}$$

As can be seen from Fig. 9, the relation between $\log(\rho\mu)$ and $\log(\theta)$, *i.e.*, a linear relation with the slope of 45 degree given by Eq. (6), does not, to a good approximation, hold because of the end effect of the oscillating-plates and the effect of reflected wave from the walls of the vessel. Therefore, an empirical relation between $\log(\rho\mu)$ and $\log(\theta)$ should be determined so as to best fit the experimental results indicated in Fig. 9. For example, a relation between $\log(\rho\mu)$ and $\log(\theta)$ for the triangles plotted in Fig. 9 can be represented as a function of $\log(\theta)$ in the following empirical form:

$$\log(\rho\mu)=3.03+8.17\times10^{-1}\log(\theta)-7.53\times10^{-2}\{\log(\theta)\}^2 \quad \text{.........(7)}$$

Figure 10 shows the above relation. The above relation given by Eq. (7) can be used to determine accurate viscosities of molten slags and fluxes at high temperatures when the values of θ are measured by the oscillating-plate viscometer under the same experimental conditions for the two effects described above.

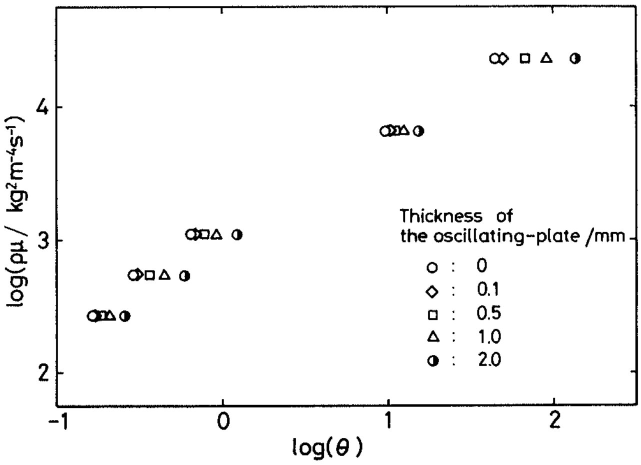

Fig. 9. Plots of $\log(\rho\mu)$ against $\log(\theta)$ for various thicknesses of oscillating-plates.

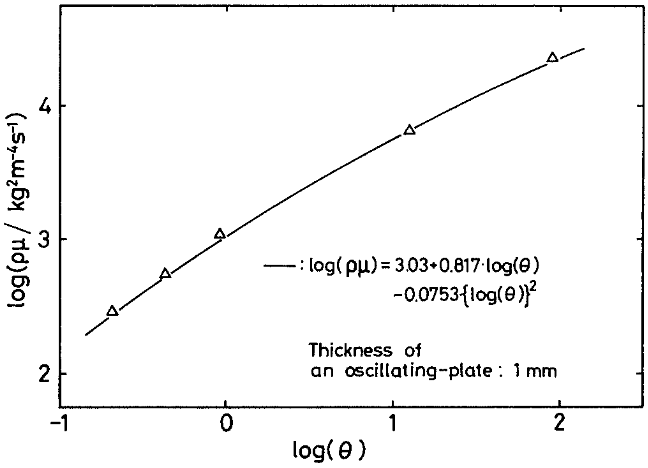

Fig. 10. Plots of $\log(\rho\mu)$ as a function of the second order regression with $\log(\theta)$.

6. Conclusions

The effect of reflected wave and the end effect of the oscillating-plate viscometer, constructed by Sasahara *et al.*,[5] have been investigated experimentally. The results obtained are as follows:

(1) The resonant frequencies of the plate oscillations in liquid samples decrease with increasing $\rho\mu$ values. When the leaf-spring of 0.3 mm or 0.4 mm thickness is used for chinning the oscillating-plate, the resonant frequencies decrease drastically with increasing $\rho\mu$ values. On the other hands, in the case of the leaf-spring of 0.5 mm or 0.6 mm thickness, the discrepancy between the resonant frequencies for the plate in liquid and in air for a viscosity range of approximate 35 Pa·s below is negligibly small.

(2) The effect of reflected wave can be neglected when the distance between the plate and the walls of a vessel is kept over one wavelength produced by the plate oscillations.

(3) The end effect of the oscillating-plate increases with increasing not only the thickness of the plates but also the viscosity of liquids.

(4) Owing to the above two effects, the working equation should be corrected experimentally for accurate viscosity determinations of molten slags and fluxes by the oscillating-plate method.

Acknowledgments

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REFERENCES

1) Z. Morita, T. Iida, M. Kawamoto and A. Mohri: *Tetsu-to-Hagané*, **70** (1984), 1242.
2) T. Iida, M. Kawamoto, S. Fujimoto and Z. Morita: *Tetsu-to-Hagané*, **71** (1985), 1490.
3) T. Iida: National Physical Laboratory Report DMM, (A), Vol. 29, ed. by K. C. Mills and N. Machingawuta, (1991), 1.
4) T. Iida: National Physical Laboratory Report DMM, Vol. 30, ed. by K. C. Mills, (1991), 1.
5) S. Sasahara, S. Inaba, M. Tomita, Y. Nimura and K. Ogawa: *KOBELCO Technol. Rev.*, **12** (1991), 56.
6) J. G. Woodward: *J. Acous. Soc. Amer.*, **25** (1953), 147.