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# STUDIES ON WINDOW FLOW CONTROL MECHANISM IN COMPUTER NETWORKS

ICHIRO AKIYOSHI

to my parents, Miwako and Taro Akiyoshi

# STUDIES ON WINDOW FLOW CONTROL MECHANISM IN COMPUTER NETWORKS

(コンピュータ・ネットワークにおける ) ウィンドウ・フロー制御方式に関する研究 )

ICHIRO AKIYOSHI

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## **ABSTRACT**

The objective of this thesis is to investigate the effect of window flow control mechanism on delay system models of computer networks.

Chapter 1 gives fundamental aspects of computer network and flow control procedure in computer network. Furthermore, the problem, which will be discussed in this thesis, is provided.

Chapter 2 describes a review of the previous researches on the window mechanism and also clarifies the difference between them and this research, by introducing performance measures for the evaluation of flow control, namely power in loss system models and total delay in delay system models respectively.

Chapter 3 investigates elementary effect of the window mechanism on delay performance in a simple delay system model under simplified and artificial assumptions. And from simulation results, we find the phenomenon that in a properly window flow controlled case the total delay, which consists of admission delay and network delay, can be minimized and can be lower than that in no controlled case. Additionally, we observe that packet admission interval can be regularized by the window mechanism.

Chapter 4 investigates effect of the window mechanism on delay performance in extended network models under various constraints. There consists of investigations in some network topologies, comparison between End-to-End flow controlled network and Link-by-Link flow controlled network, and affect of acknowledgement delay. For each case we find the similar phenomenon as Chapter 3.

Chapter 5 examines the reason of the phenomenon. We analyze Input-Sequencing of the window mechanism, which means rearrangement of packets on inputting to the network. And from numerical results, it is certified that the phenomenon should be due to Input-Sequencing.

Chapter 6 summarizes the overall conclusions obtained in this thesis.

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## CHAPTER 1

## INTRODUCTION

Both computer technology and communication technology have been playing an extremely important role in human society. The integration of these technologies has yielded a concept of computer network. The main purpose of computer network is to exploit and share resources (i.e. hardware, software and data base), by connecting geographical distributed computer systems.

In the recent past, we have witnessed an explosion in the analysis, design and implementation of computer network, however there exist several open problems. This dessertation studies window mechanism for flow control which is one of the most important problems in computer network.

#### 1.1 Computer Network

Fig. 1.1 shows a simplified computer network. It consists of local networks and a communication sub-network. A local network consists of one or more HOST computers (HOSTs) connected to communication sub-network via a switching center. Users' terminals are also connected to HOSTs. The communication sub-network is a network of facilities (hardware and software) to deliver messages from one HOST to another. It consists of switching centers which are connected to each other via communication media. There are two types of communication media; broadcast and point-to-point.

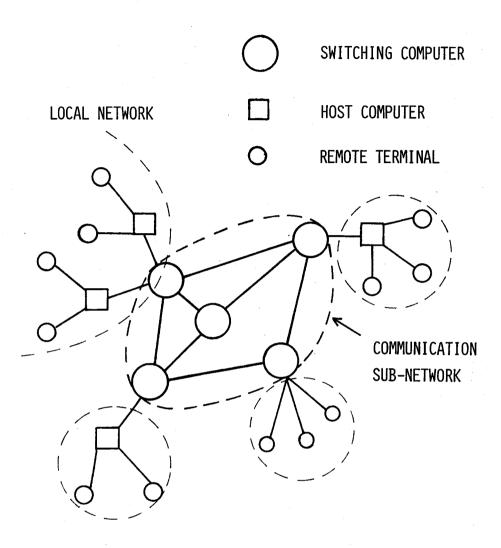


Fig. 1.1 Computer Network

In this thesis, we are concerned with point-to-point media and refer to them as channels, links or lines. Also we refer to a switching center simply as a node and the communication sub-network as a communication net.

The technique incorporated in a communication net to transmit messages from one HOST to another is called switching technique. Three different switching techniques are usually used in computer network, as follows:

#### (1) Circuit (or Line) Switching

In this technique, a complete path of communication must be set up by a call between source to destination, before communication begins. Once a path is set up, it is tied up from the time the first message begins to be sent until the last message is transmitted. Circuit switching is a common technique for telephone switching and it is suitable to transmit long messages as file messages.

#### (2) Message Switching

In this technique, a message is defined to be the logical unit of information to which address information (usually called header) is attached. First a message travels from its source to the next node. When this transmission is finished, this message selects channel towards the destination; if the selected channel is busy, the message waits in queue, and when the channel becomes available, transmission starts, namely store-and-forward fashion [4], [6].

#### (3) Packet Switching

This technique is basically the same as message switching, except that message is subdivided into packets with

address information. Many packets of the same message may be transmitted simultaneously. Higher channel utilization and lower network delay can be achieved by this technique, due to more complete resource sharing [2], [3], [6]. The ARPANET [7]-[9], the British NPL [10], the French CYCLADES [39] and the TELENET are examples of packet switching networks.

One thing is clear, if messages are very long as file messages, circuit switching is a good choice. On the other hand, if messages are relatively short, store-and-forward switching, especially packet switching is a good choice [11], [12]. There is wide range of message lengths in practice, the concept of hybrid switching incorporated both store-and-forward and circuit switching capability has been proposed [13], [14], [44]-[47].

# 1.2 Routing and Flow Control in Computer Network

In the previous section we described the concept of computer network and switching technique in computer network. Packet switching offers attractive advantages over circuit switching due to flexibility in setting up user connections, more effective use of resources by multi-nodal traffic and so on.

However, in designing a packet switching network we encounter many problems, as follows:

- (1) topological design
- (2) capacity allocations

- (3) routing procedures
- (4) flow control procedures

In this research, it is out of scope to study topological design and capacity allocation problems. Several formulations have been proposed in [4], [6], [15], [16] etc..

Advantages of packet switching do not come without any control. If demands are allowed to exceed the system capacity, unpleasant congestion may occur, which rapidly neutralize the delay and efficiency advantages. Internal network congestion may be relieved by rerouting a portion of traffic from heavily loaded channels to underutilized channels. There are some approaches to establish route for packets in packet switching network; namely, centrally or locally, fixed or adaptive, deterministic or stochastic. Several classifications have been devised to categorize routing policy, and various routing procedures have been proposed [17]-[21], [43]. Routing is also out of scope in this research.

Even with well defined routing procedure, network can not afford to accept all the offered traffic without control. This control is commonly called flow control procedure. The basic principle of flow control is to keep the excess load out of the network. In order to keep the network traffic within a limit, flow control procedures must be equipped with throttling mechanism. Existing flow control in packet switching network can be categorized into local control and global control. Local control is not sufficient by itself to prevent congestion, so global control is necessary to supervise the admission of packets to the network.

Examples of global control procedures are isarithmic flow control studied for the NPL network [22]-[24] and end-to-end flow control used in the ARPANET [25], [26]. End-to-end flow control mechanism is usually referred to as window mechanism, where the total number of packets simultaneously outstanding between a source and a destination is limitted to window size.

Several authors have discussed the mechanism for flow control, however due to complex environment of flow control, quantitative and analytical studies have been lacking [16],[27]-[31]. Critical problem is the optimal selection of system parameters, especially the window size. The recent researches on this issue are extremely welcome [32]-[38], [41]. However, they suffer from the limitation that they only focused on loss system models, hence there was no account of the admission delay to the network.

Computer network is basically a mixed structure which consists of loss system and delay system, therefore it should be necessary to study for the latter. On this point of view, in this research we investigate effect of the window mechanism on delay system models of computer networks [48]-[53].

### CHAPTER 2

# WINDOW MECHANISM AND PERFORMANCE MEASURE

#### 2.1 Introduction

It is essential to study for the optimal selection of system parameters in designing computer network with window mechanism for flow control of packet level, based on C.C.I.T.T. Recommendation X.25 [28]. Critical parameters in the mechanism are the window size and retransmission time-out interval if error and loss recovery will be provided. Some investigators have discussed on this problem and yielded some guidelines regarding optimal selection. However, the previous studies suffer from the limitation that only loss system models have been investigated. In real life, however, computer network is basically a mixed structure consisting of loss and delay systems, hence it should be welcome to study for the latter.

On this point of view, in this chapter we investigate elementary effect of the window mechanism on a simple delay system model.

#### 2.2 Review of the Previous Researches for Window Mechanism

End-to-end flow control is exercised on a logical channel (a virtual circuit) which is provided between source and destination nodes (devices). The main objective of end-to-end control is to protect the destination node from congestion, due to the fact that remote sources are sending traffic at a higher rate than

can be accepted by terminals, fed by the distination node. And important byproduct is the prevention of internal congestion.

Most end-to-end control mechanisms use a variant of the credit throttling technique, usually called window, which permits up to W (window size) packets to be outstanding on a logical channel.

We show an outline of the mechanism in Fig. 2.1.

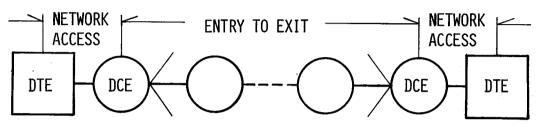


Fig.2.1 Network Structure

The logical channel is implemented as the concatenation of three protocol segments:

A packet level X.25 protocol from the source device (i.e., data terminating equipment or DTE) to source node (i.e., data communication equipment or DCE), an internal protocol from source DCE to destination DCE, and a packet level X.25 protocol from destination DCE to destination DTE.

Each one of three protocol segments is flow controlled by window mechanism. For example, let us assume that all windows are of size W=3, and that the window between source and destination DCE is full (i.e. there are three outstanding packets). The following packet arriving from the source DTE to DCE will be accepted, but will not be immediately acknowledged; rather the packet will be withheld until an ACK (acknowledgement) from the destination DCE

is received, thus a window opens. Therefore, window mechanism serves the function of promptly reflecting a congestion situation in network back to the source by withholding packets.

A variant of this mechanism was originally used in the French Cyclade Network [39] and the ARPANET VDH (Very Distant Host) connections [40]. The mechanism is categorized into two strategies, namely End-to-End window flow control and Link-by-Link window flow control [16]. The restriction is imposed on a logical channel for the former, and on each link belonging to a logical channel for the latter, respectively. We mainly concerned with the former, and for simplicity, when there is no ambiguity, we will use "window control" to indicate End-to-End window flow control, in this thesis.

It is essential to investigate the effect of the mechanism and study for the optimal selection of system parameters in designing computer network with the mechanism. Critical parameters in the window mechanism are the window size and retransmission time-out interval if error and loss recovery will be provided. Several analytic and simulation studies have been reported to investigate the effect of these parameters on throughput and delay performance. We now review some of the most significant contributions in this area.

Kleinrock and Kermani develop an analytic model in which a single source-to-destination traffic is flow controlled by window mechanism [36]. In this model, network delay is simplified as an M/M/1 queue delay, and round trip delay therefore follows an Erlang-2 distribution. The destination node is assumed to have finite storage and deliver packets to the destination host on

a finite capacity channel, therefore the destination node occasionally drops packets. To provide for transmission integrity, the source node must retransmit an unacknowledged packet a time-out interval. The simplified window model is solved analytically, yielding some guidelines regarding the optimal selection of system parameters, namely window size and time-out interval.

In a subsequent paper [37], an adaptive policy for the dynamic adjustment of window size to time- varying traffic rate, is proposed by the same authors. Numerical results shows that the delay versus throughput performance of adaptively controlled scheme is somewhat superior to that of non- adaptively controlled scheme. Network being regarded as a single queue in these models, therefore insight into the independence of window size on the number of intermediate hops has not offered.

In order to remove this limitation, the same authors also develop a simple multinode model, where a single source-to-destination traffic flowing through the network via a k-hop path is window controlled [33], [35]. In this model, infinite buffer storage and negligible error rate are assumed. And they find that "power", which is a compact measure of combined throughput and delay performance, is optimized by setting window size W=k.

The two previous models assuming a single source-to-destination traffic, interference at a node by other traffic traversing it is out of consideration. Pennotti and Schwartz take the effect of interference into the tandem network model in an approximate fashion [16], [27]. They analyze performance of the network congestion versus probability of lost packet at the source node.

in End-to-End and Link-by-Link flow controlled case, and show that the latter exhibits similar performance as the former. Some insight of multiuser flow control is obtained from this study, however the model suffer from the limitation that only one logical channel can be flow controlled at a time, the remaining traffic component being constant.

To remove this limitation, a number of multiple source, multiple destination models need to be developed, but the exact analysis for multinode network with individually controlled node pair, will be impractical. Reiser recently proposes an approximate solution technique applicable to large networks [38].

However, all the previous researches mainly focus on man-to-machine communication system, so these models developed in them are viewed as loss system models. This implies that once a request for service (i.e. packet) finding no available window at the entry to the network is dropped from the system and is lost, hence admission delay incurred by packets is out of consideration. In real life, however, computer network is basically a mixed structure consisting of man-to-machine and machine-to-machine communication system. So it should be welcome to study for the latter. On this point of view, in this thesis we will investigate effect of the window mechanism on delay system models, where packet finding no available window waits for a window to be free.

#### 2.3 Performance Measures

We wish to define a quantitative measure of flow control performance for various reasons.

First, we wish to establish a well defined performance criterion in order to tune the parameters of a given flow control. Second, we wish to carefully weigh performance benefit against overhead introduced by flow control. Third, we are interested in comparing the performance of alternative flow control schemes in quantitative term.

In this section, we clarify the difference between loss system and delay system by introducing performance measure suitable to each system. Before proceeding, we start with the following assumptions which are accepted throughout the analysis in this thesis:

- 1. External Poisson arrival
- 2. Exponential distributed message lengths
- 3. Infinite nodal capacity
- 4. Fixed routing
- 5. Independence Assumption [4]
- 6. Negligible propagation delay
- 7. Messages consisting of single packets

#### 2.3.1 Power in Loss System Model

We begin by introducing one of the previous reseaches by Kleinrock and Kermani [33],[35]. They developed a simple loss system model of k-hop tandem network with a certain input traffic, with each hop modelled by an M/M/1 queue and with an instantaneous end-to-end acknowledgement, as shown in Fig.2.2.

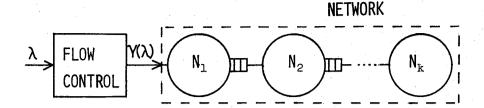


Fig. 2.2 Loss system Model

In loss system, if input traffic can not find a free window, it is lost by input reglation. Therefore, network accepts a portion of this traffic and delivers it to its destination.

The following notations are used:

 $\lambda$ : Input rate of messages applied to the network (msg/sec)

 $\gamma(\lambda)$ : Traffic carried by the network (= network throughput)

(msg/sec)

 $T(\gamma(\lambda))$ : Total average network delay (sec)

: Network capacity ( the maximum traffic that can be handled by the network) (msg/sec)

P : Power ( throughput/ network delay=  $\gamma(\lambda)/T(\gamma(\lambda))$  )

 $(msg/sec^2)$ 

As a single input traffic is flow controlled in this model, throughput and network delay increase for larger window size. What is the best choice for the window size is the question now. It should be first defined measure of "goodness". in order to answer this question. If high throughput is favored, a large window size is the best choice, but this results in large network

delay. On the other hand, if low network delay is favored, a window size W=0 is the best. In order to combine both throughput and network delay, it is need to define a proper performance measure for the evaluation which reflects the opposite effect.

They use the ratio of throughput and network delay as their performance measure, which is first defined in [31] and is referred to as "power", P:

$$P \triangleq \frac{\gamma(\lambda)}{T(\gamma(\lambda))}$$

(2.1)

The name "power" is originated the power (energy/time) used in physics.

Assuming that each hop may be modelled by an M/M/1 queue, each hop adds an amount  $1/(\mu-\gamma(\lambda))$  seconds to the average delay, so the total average network delay  $T(\gamma(\lambda))$  is given by

$$T(\gamma(\lambda)) = \frac{k}{\mu - \gamma(\lambda)}$$
(2.2)

Using Eq.(2.1) and (2.2) gives

$$P = \frac{\gamma(\lambda)(\mu - \gamma(\lambda))}{k}$$

(2.3)

In Fig.2.3 a sketch of power is shown as a function of throughput. The condition for the maximum value of power is that  $\lambda$  should be flow controlled such that throughput  $\gamma(\lambda)=\mu/2$  ( utilization  $\rho=0.5$ ), and also this means  $T(\gamma(\lambda))=2k/\mu$  ( the average number of messages  $\overline{N}=k$  ). That is, throughput and network delay

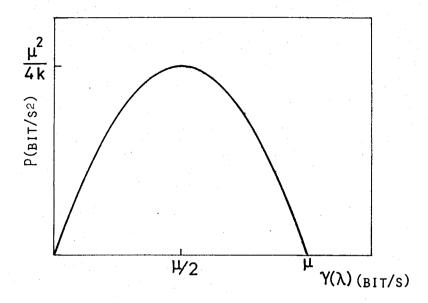


Fig. 2.3 Power vs. Throughput

should be balanced such that half the maximum throughput and twice the minimum delay.

Additionally, they considered flow control procedure using window mechanism, where the average number of messages in the network  $(\overline{N})$  is restricted to a maximum number W (window size). From well known Little's result,

or 
$$\overline{N} = W = \gamma(\lambda) T(\gamma(\lambda))$$

$$W = \frac{k\gamma(\lambda)}{u-\gamma(\lambda)}$$
(2.5)

which gives

$$\gamma(\lambda) = \frac{W}{k + W} \mu \qquad (2.6)$$

(2.5)

With Eq.(2.6) they described that:

For optimality, we know that  $\gamma(\lambda)=\mu/2$  which implies W=k. This implies that the network should contain k messages on average to give maximum power. That is, we should just keep the pipe full (i.e., one message per hop).

Loss system means that once a request for service (in this case message or packet) finds no available window, it is dropped from the system and is lost. So power, which represents a trade-off between two major criteria: throughput and network delay, should be regarded as a proper measure in loss system. On the other hand, packet in delay system must wait outside the system and is not lost, so constant throughput can be obtained with properly flow controlled case. Therefore, in delay system there is no difference between power and delay as a measure for evaluating.

In the following section, we focus on delay system and introduce "total delay" as a measure.

#### 2.3.2 Total Delay in Delay System Model

In this section we consider effect of window mechanism on delay system model, comparing with the loss system model developed in the previous section. The difference between two models is as follows:

In loss system, if input traffic (packet) finds no available window, it is dropped from the system and is lost, hence admission delay is out of consideration. On the other hand, in delay system, if input traffic can not find a free window, it must wait at IQ (Input Queue) for its window to become open (rather than

being lost). This implies that traffic applied to network is garanteed delivery to its destination and constant throughput can be obtained in delay system. So we had better evaluate "total delay", which consists of admission delay incurred in IQ and network delay incurred in network, as a measure, rather than "power" raised in the last section.

We develop a simple delay system model of 2-hop tandem network with a certain input traffic (LC<sub>13</sub> packet), as shown in Fig. 2.4. This model is similar to the model in the last section, except that the former has IQ (Input Queue). Furthermore, based on the result obtained from the last section, constraints are accepted that window size  $W_{13}=2$  and network utilization  $\rho=0.5$ .

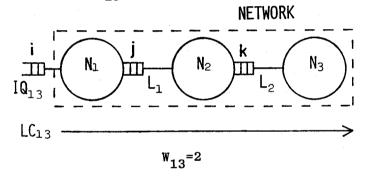


Fig. 2.4 Delay System Model

We define the probability of the state finding  $IQ_{13}$  with i packets,  $L_1(line\ 1)$  with j packets and  $L_2(line\ 2)$  with k packets, as P(i,j,k). The state-transition-rate diagram is given in Fig. 2.5, where input rate  $\lambda$  and line capacity  $\mu$ . We also require the conservation relation:

$$\sum_{j=0}^{\infty} \sum_{j=0}^{2} \sum_{k=0}^{2} P(i,j,k) = 1$$
i=0j=0k=0 (2.7)

(i,j,k): state (i,j,k)

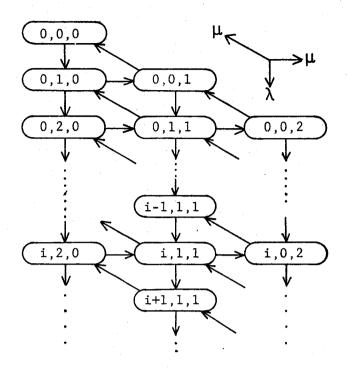


Fig. 2.5 State-Transition-Rate Diagram

From Fig.2.5, we can immediately write down the following equilibrium equations:

$$i = 0$$

$$\lambda P(0,0,0) = \mu P(0,0,1)$$
 (2.8)

$$(\lambda + \mu)P(0,0,1) = \mu P(0,1,0) + \mu P(0,0,2)$$
 (2.9)

$$(\lambda + \mu)P(0,0,2) = \mu P(0,1,1)$$
 (2.10)

$$(\lambda + \mu)P(0,1,0) = \lambda P(0,0,0) + \mu P(0,1,1)$$
 (2.11)

$$(\lambda + \mu)P(0,2,0) = \lambda P(0,1,0) + \mu P(1,1,1)$$
 (2.12)

$$(\lambda+2\mu)P(0,1,1) = \lambda P(0,0,1) + \mu P(0,2,0) + \mu P(1,0,2)$$
 (2.13)

$$(\lambda + \mu)P(i,2,0) = \lambda P(i-1,2,0) + \mu P(i+1,1,1)$$
 (2.14)

$$(\lambda+2\mu)P(i,l,l) = \lambda P(i-l,l,l) + \mu P(i,2,0) + \mu P(i+l,0,2)$$
 (2.15)

$$(\lambda + \mu)P(i,0,2) = \lambda P(i-1,0,2) + \mu P(i,1,1)$$
 (2.16)

We choose to use the method of z-transform for solving this set of equations. The z-transform of P(i,j,k) is defined as Eq.(2.17).

$$G(z_1, z_2, z_3) \triangleq \sum_{i=0, j=0}^{\infty} \sum_{k=0}^{2} P(i, j, k) z_i^i z_i^j z_3^k$$

= 
$$P(0,0,0) + P(0,1,0)z_2 + P(0,0,1)z_3$$

$$+\sum_{i=0}^{\infty} P(i,2,0)z_{1}^{1}z_{2}^{2} + \sum_{i=0}^{\infty} P(i,1,1)z_{1}^{1}z_{2}z_{3} + \sum_{i=0}^{\infty} P(i,0,2)z_{1}^{1}z_{3}^{2}$$

(2.17)

By applying Eqs. (2.8)-(2.16) to Eq. (2.17), we can obtain

$$G(z_1,z_2,z_3) = P(0,0,0) \times \left[1 + \frac{\lambda(\lambda^2 + 2\lambda\mu + \mu^2)}{2\mu^2(\lambda + \mu)} z_2 + \frac{\lambda}{\mu} z_3\right]$$

$$+ \frac{\lambda^2(\lambda+2\mu-\lambda z_1)\{(\lambda+\mu-\lambda z_1)z_2z_3+\mu z^2\}}{\mu\{\lambda z_1^2-\lambda(\lambda+3\mu)z_1+2\mu^2\}(\lambda+\mu-\lambda z_1)} + \frac{\lambda^2 z_2^2}{2\mu^2(\lambda+\mu)} \times$$

$$\left\{\frac{\lambda^{2}(\lambda^{2}+2\lambda\mu+2\mu^{2})z_{1}^{2}-2\lambda(\lambda^{3}+4\lambda^{2}\mu+5\lambda\mu^{2}+3\mu^{3})z_{1}+\lambda^{4}+6\lambda^{3}\mu+11\lambda^{2}\mu^{2}+8\lambda\mu^{3}+4\mu^{4}}{\{\lambda^{2}z_{1}^{2}-\lambda(\lambda+3\mu)z_{1}+2\mu^{2}\}(\lambda+\mu-\lambda z_{1})}\right\}$$

(2.18)

We may evaluate the constant P(0,0,0) by recognizing G(1,1,1)=1.

 $<sup>^{\</sup>dagger}$  Derivation of Eq.(2.18) is given in Appendix A.

$$P(0,0,0) = \frac{2\mu - 3\lambda}{2\mu + \lambda}$$
 (2.19)

Substituting this into Eq.(2.18), we find

$$G(z_1, z_2, z_3) = \frac{2\mu - 3\lambda}{2\mu + \lambda} \times \left[ 1 + \frac{\lambda(\lambda^2 + 2\lambda\mu + \mu^2)}{2\mu^2(\lambda + \mu)} z_2 + \frac{\lambda}{\mu} z_3 \right]$$

$$+ \frac{\lambda^{2}(\lambda + 2\mu - \lambda z_{1})\{(\lambda + \mu - \lambda z_{1})z_{2}z_{3} + \mu z^{2}\}}{\mu\{\lambda z_{1}^{2} - \lambda(\lambda + 3\mu)z_{1} + 2\mu^{2}\}(\lambda + \mu - \lambda z_{1})} + \frac{\lambda^{2}z_{2}^{2}}{2\mu^{2}(\lambda + \mu)} \times$$

$$\left\{ \frac{\lambda^{2}(\lambda^{2}+2\lambda\mu+2\mu^{2})z_{1}^{2}-2\lambda(\lambda^{3}+4\lambda^{2}\mu+5\lambda\mu^{2}+3\mu^{3})z_{1}+\lambda^{4}+6\lambda^{3}\mu+11\lambda^{2}\mu^{2}+8\lambda\mu^{3}+4\mu^{4}}{\{\lambda^{2}z_{1}^{2}-\lambda(\lambda+3\mu)z_{1}+2\mu^{2}\}(\lambda+\mu-\lambda z_{1})} \right\}$$
(2.20)

Various derivations of z-transform evaluated for z=1 gives the various moments of the random variable. Differentiating Eq.(2.20) with respect to z ,setting z=1 and moreover recalling Little's result, we find average admission delay  $\overline{\mathbf{T}}_1$ , average network delay  $\overline{\mathbf{T}}_2$  and average total delay  $\overline{\mathbf{T}}(\mathbf{W}_{13}=2)$ , in window flow controlled network with  $\mathbf{W}_{1,2}=2$ .

$$\overline{T}_{1} = \frac{1}{\lambda} \left\{ \frac{d}{dz_{1}} G(z_{1}, 1, 1) \Big|_{z_{1}=1} \right\} = \frac{\rho^{2}(9\rho^{2} + 16\rho + 32)}{2\mu(\rho + 2)(\rho + 1)(2 - 3\rho)}$$
(2.21)

$$\overline{T}_{2} = \frac{1}{\lambda} \left\{ \frac{d}{dz_{2}} G(1,z_{2},1) \Big|_{z_{2}=1} + \frac{d}{dz_{3}} G(1,1,z_{3}) \Big|_{z_{3}=1} \right\} = \frac{3\rho^{3} + 10\rho^{2} + 20\rho + 8}{2\mu(\rho+2)(\rho+1)}$$
(2.22)

$$\overline{T}$$
 (  $W_{13} = 2$ ) =  $\overline{T}_1 + \overline{T}_2 = \frac{4(\rho^2 - 2)}{\mu(\rho + 2)(3\rho - 2)}$  (2.23)

where  $\rho$  is line utilization factor  $(=\lambda/\mu)$ .

On the other hand, in no flow controlled case we can regard each hop as an M/M/1 queue independently, by accepting independence assumption [4]. So average total delay  $\overline{T}(W_{13}^{=\infty})$  in no flow controlled network is given by

$$\overline{T}$$
 (  $W_{13} = \infty$ ) =  $\frac{2}{\mu(1-\rho)}$  (2.24)

We show an example of delay performance obtained from above discussion in Table 2.1, where we set  $1/\mu$  =0.04(sec)<sup>+</sup> (average service time of a packet) and  $\rho$  =0.5.

Table 2.1 An Example of Delay Performance

|                         | ٠,                          | window control(W13=2) | no control |
|-------------------------|-----------------------------|-----------------------|------------|
| average admission delay | $\overline{\mathbf{T}}_{1}$ | 0.1128                | 0.0        |
| average network delay   | $\overline{\mathbf{T}}_{2}$ | 0.1112                | 0.16       |
| average total delay     | T                           | 0.2240                | 0.16       |

(sec)

From this table, it is recognized that in window controlled case network delay can be reduced, however total delay turns out to increase due to admission delay, comparing with no flow controlled case.

Up to this point we only consider the case for  $W_{13}=2$ . Even on a simple model as Fig.2.4, it is difficult to analyze the case for  $W_{13} \geq 3$ , because the number of states increases as a combinatorial function of  $W_{13}$ . So we will use simulation method for the latter case.

<sup>&</sup>lt;sup>+</sup> This corresponds average packet size equals to 2K (bit) and capacity of line equals to 50K (bit/sec).

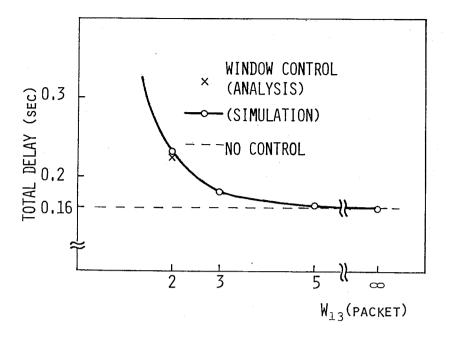


Fig. 2.6 Total Delay Characteristics

In Fig.2.6 the total delay characteristics for  $\rho$  =0.5 is shown as a function of W<sub>13</sub>. When W<sub>13</sub> is setted too small, the total delay increases unbounded due to the increase of the admission delay. Especially, packet may insignificantly incurr the admission delay at the state (i,0,W<sub>13</sub>), which implies all the windows are used on line 2. Because packet can not enter the network in spite of finding line 1 free at this state. We will call the situation which can not be neglected the probability of the state, "over control" in our future discussion. As W<sub>13</sub> increases, the total delay gradually decreases to be equivalent to that in no flow controlled case (dashed line), which is independent on W<sub>13</sub>.

From this figure, it is recognized that the optimal window size, which minimizes the total delay, is infinite, hence the price for window flow control procedure is a increased total delay in

delay system model as shown in Fig. 2.4.

### 2.4 Conclusion

In this chapter, we have investigated elementary effect of the window mechanism for flow control on a simple delay system model. First, we have given a fundamental aspect of the window mechanism. Then, we have described a review of the previous researches on the window mechanism and also clarified the difference between viewpoints of them and this reseach. Finally, we have considered the window mechanism having on total delay performance in a simple delay system model, where a single input traffic is flow controlled.

We summarize the significant findings in this chapter as follows

- (1) Power, which represents a tradeoff between throughput and network delay, should be regarded as a proper measure in loss system. On the other hand, since constant throughput can be obtained in delay system, we had better evaluate total delay, which consists of admission delay and network delay, as a measure
- (2) The price for window flow control procedure is an increased total delay in a simple delay system model with a single source-to-destination traffic.

# CHAPTER 3

# EFFECT OF THE WINDOW MECHANISM IN A SIMPLE DELAY SYSTEM MODEL

# 3.1 <u>Introduction</u>

In the last chapter, we have evaluated total delay performance in a simple delay system model with the window mechanism, and found that the price for window flow control procedure is an increased total delay. However, the model in the last chapter suffers from the limitation that only a single source-to-destination traffic is flow controlled; therefore, insight into model with simultaneously flow controlled multi-logical channels, has not yet offerd.

It is generally difficult to analyze the latter model. In this chapter, we simulate a simple delay system model of 3-node tandem network with individually window flow controlled logical channels, under simplified and artificial assumptions.

# 3.2 The Model and Assumptions

In this section, we develop a simulation model for a network with the window mechanism and elaborate on the assumptions involved in our simulation. We now describe a simple delay system model as shown in Fig. 3.1 for which simulation results can be obtained in Section 3.3.

# 3.2.1 Network Structure

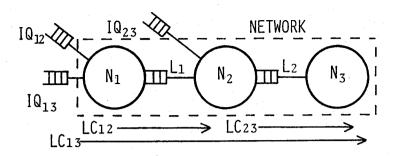


Fig. 3.1 Simulation Model (3-node tandem)

Assuming the routing strategy for the network fixed, messages are transmitted along the fixed path through the network. This path consists of series tandem nodes  $(N_i)$  connected by communication lines  $(L_k)$ , each modeled in the usual way by a queue. We will call this path between source node  $N_i$  and destination node  $N_j$ , logical channel  $LC_{ij}$ . And we will describe an amount of traffic on each  $LC_{ij}$  by using a traffic matrix  $[\gamma_{ij}]$  as Eq.(3.1). The total number of messages on each  $LC_{ij}$  is restricted to a maximum value  $W_{ij}$  (i.e. window size) for end-to-end window flow controlled network. We will describe this by using a window matrix [ W ] as Eq.(3.2).

Outside the network, each  $LC_{ij}$  has its input queue  $IQ_{ij}$  which is connected to the source node i  $(N_i)$ . Note that in end-to-end window flow controlled network, messages generated by users of  $LC_{ij}$ , when the number of messages outstanding on  $LC_{ij}$  is below  $W_{ij}$  (namely, window is open), can enter the network. On the other hand, when  $W_{ij}$  messages are already outstanding on  $LC_{ij}$  (namely, all the windows are busy), they are blocked from entry to the network and stored in  $IQ_{ij}$ ; until an acknowledgement is received by the source node  $N_i$ , indicating that a message has successfully reached its destination node  $N_j$ . So some messages can incur admission delay at  $IQ_{ij}$ .

On the other hand, in no controlled network, all the windows being assigned infinite, messages enter the network without any input regulation; hence, they do not incur admission delay at IQ.

### 3.2.2 Assumptions

Measurements on existing network, in particular on ARPANET [42], show that the average number of packets per message is very close to one; hence we make the simplifying assumption that messages consist of single packets and we do not differentiate between packet- and message- switching.

In our simulation we will accept the following assumptions:

- (1) Fixed message (packet) length (with 2.0 K bit).
- (2) Poisson arrival of messages to input queue (IQ)

- (3) Capacity of communication line between two nodes is 50K (bit/sec).
- (4) Negligible nodal processing time.
- (5) No failure in the system.
- (6) Size of input queue (IQ) is unlimited; hence no packet is lost at IQ.
- (7) Negligible transmission error rate.
- (8) Negligible transmission time of acknowledgement (ACK).
- (9) Parameter is the window size of logical channel (LC), which has maximum number of hops, and the others are fixed.

#### Remarks:

(1) As for assumption (4)

The network delay consists of the waiting time for nodal processing and the waiting time for transmission on line. Which affects influentially, the former or the latter, depends on the ratio the nodal processing rate to the line transmission rate. From assumption (3), it is natural that the nodal processing rate should be faster than the line transmission rate. And we will study characteristics under "heavy traffic "conditions, there almost always exists long queues on communication lines. In these conditions, the waiting time for nodal processing time is completely masked by the waiting time for transmission on line.

So mention above justifies assumption (4).

### (2) As for assumption (8)

This assumption implies that the acknowledgements generated by the destination node are not returned via the network, but are returned directly to the source node. Although the model becomes rather simplified and possibly artificial by this assumption, it should provide a first approach to a designer interested in implementing flow control procedure in networks.

Furthermore, in the following chapter we will focus on more realistic model in which acknowledgement is considered according to the RECOMMENDATION X.25.

With these assumptions, we will be mainly concerned with total delay, which consists of admission delay and network delay. All simulations run were made for the same fixed length of simulated time with the same traffic pattern offerd to the network.

### 3.3 Simulation Results and Considerations

In this section we describe some simulation results done on a simple delay system model with individually window flow controlled logical channels, and evaluate the effect of the window mechanism having on delay performance which contains admission delay. Except for Fig.3.8, we use the window matrix [W] as shown in Eq. (3.3), where W<sub>13</sub> (window size of 2-hop LC<sub>13</sub>) is a parameter and

the others are fixed 8.

$$[W] = \begin{bmatrix} 0 & 8 & W_{13} \\ 0 & 0 & 8 \\ 0 & 0 & 0 \end{bmatrix}$$
 (3.3)

# 3.3.1 Delay Characteristics

Fig.3.2 shows the congestion control behavior under heavy traffic condition by the window mechanism, where the average number of busy windows on each logical channel is shown as a function of  $W_{13}$ . Plots for  $LC_{23}$  (1-hop logical channel), which is similar to these for  $LC_{12}$ , will be omitted in our future figures. As  $W_{13}$  increases, the degree of input regulation for  $LC_{13}$  becomes lower,

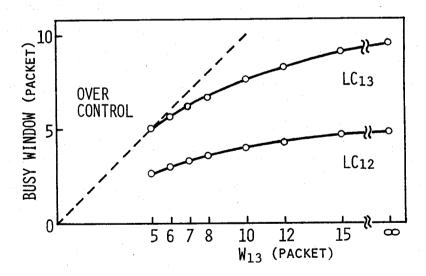


Fig. 3.2 Number of Busy Window Characteristics(p=0.964)

hence, the number of busy windows gradually grows to be a constant value; on the other hand, for  $LC_{12}$  the number of busy windows also grows due to the increase in the number of packets outstanding on  $LC_{13}$ . It can be seen that, when  $W_{13}$  is assigned to be less than or equal to 5, the number of busy windows turns out to be equal to  $W_{13}$ .

The reason of this is the following:

When  $W_{13}$  is too small, the number of  $LC_{13}$  packets generated is greater than the number admitted to enter the network; hence, all the windows for  $LC_{13}$  are always busy and there always exists packets in  $IQ_{13}$ . This situation is called over control as mentioned in the last chapter. As pointed out in the last chapter, in over control region packet will insignificantly incur admission delay in spite of finding line 1 ( $L_1$ ) is free, therefore network will perform ineffectively. Hence, we will exclude the situation of over control in our future discussion, and the offered traffic to the network is held fixed in our future figures.

Fig.3.3 shows the admission delay characteristics, where the average admission delay for each LC is shown as a function of  $\mathbf{W}_{13}$ . As  $\mathbf{W}_{13}$  increases, the admission delay for  $\mathbf{LC}_{13}$  rises beyond bound, due to over control.

Fig. 3.4 shows the network delay characteristics, where the average network delay for each LC is shown as a function of  $W_{13}$ , and also characteristics of no controlled cases, which are independent on  $W_{13}$ , are shown. It is recognized that this figure is equivalent to Fig.3.2, by invoking Little's result to relate the network delay to the number of busy windows. The network delay

in window controlled case is lower than that in no controlled case, because the former case has the effect on limiting the number of packets outstanding in the network at any time.

Fig. 3.5 shows the distribution characteristics of the network delay, where an Erlangian distribution of a degree K for each LC is shown as a function of  $W_{13}$ . As  $W_{13}$  decreases, K for  $LC_{13}$  grows, and when  $W_{13}$  is too small, it becomes beyond bound. On the other hand, K for  $LC_{12}$  is always about 3, independ on  $W_{13}$ . In addition, in window controlled network, K is always greater than that in no controlled network.

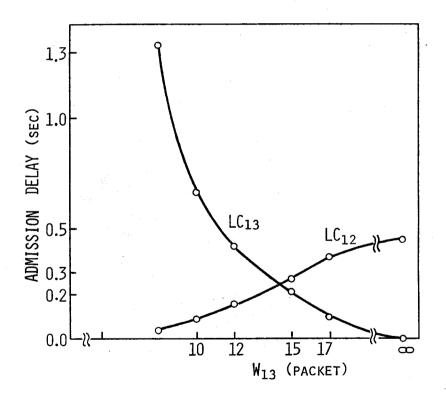


Fig. 3.3 Admission Delay Characteristics (ρ=0.964)

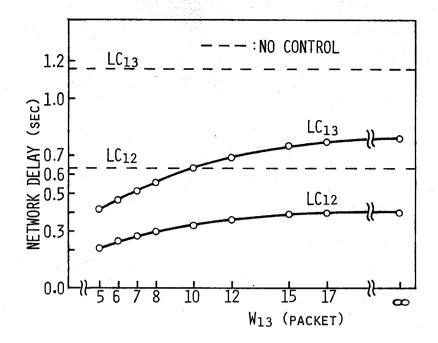


Fig. 3.4 Network Delay Characteristics (ρ=0.964)

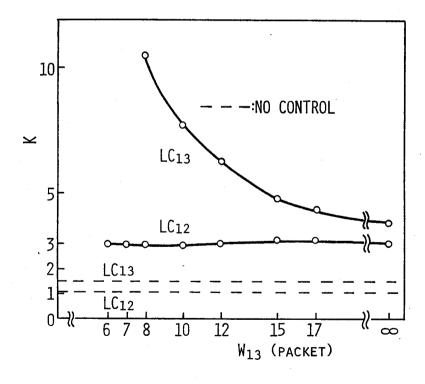


Fig. 3.5 Distribution Characteristics of Network Delay by Erlangian Degree (  $\rho$ =0.964)

From Fig. 3.4 and Fig. 3.5, it is recognized that we can settle the network delay lower and more stable than that in no controlled network.

Fig. 3.6 shows the total delay characteristics, where the average total delay for each LC, which consists the admission delay plus the network delay, is shown as a function of  $W_{13}$ .

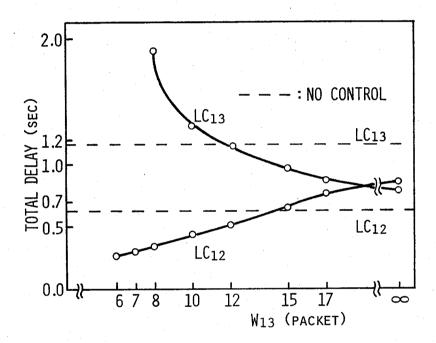


Fig. 3.6 Total Delay Characteristics for Each Logical Channel ( $\rho$ =0.964)

Fig.3.7 is the core of this thesis. The effect of the window mechanism on total delay is shown in this figure, where the average total delay, which is taken averaged on all the logical channels, is shown as a function of  $W_{13}$ , for various line utilization  $\rho$ 's.

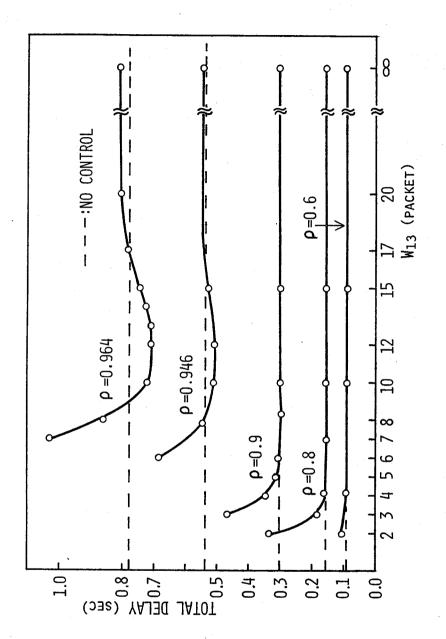


Fig. 3.7 Total Delay Characteristics

When  $\mathrm{W}_{13}$  is too small, the total delay rises unbounded due to the increase of the admission delay of  $\mathrm{LC}_{13}$  packets by over control as pointed out previously; As  $\mathrm{W}_{13}$  increases, the total delay first reaches a minimum and then gradually grows to be equal to or a little greater than that in no controlled case.

Here we encounter the unexpected phenomenon :

The window flow controlled total delay can be lower than the no flow controlled one under a heavy traffic condition  $\rho \geq 0.954$ ; though no packet is lost and the offered traffic is held fixed in delay system , hence some packets incur the admission delay in IQ due to input regulation. There exists the optimal window size which optimizes ( minimizes ) the total delay under a heavy traffic condition. ( Note that for  $\rho=0.964$  the optimal value of  $W_{13}$  is 12 or 13.) Similar phenomena appear for  $\rho \leq 0.9$  as well as  $\rho \geq 0.954$ . The lower is  $\rho$ , the less remarkable becomes the effect; that is, we can observe in this figure that the total delay curve for  $\rho \leq 0.9$ , after an initial decrease, remains flat to be equivalent to that in no controlled case. ( Note that for  $\rho \leq 0.9$ , the smaller is assigned the size of all windows, the more remarkable becomes the effect; which we have not drown.)

Overall, this figure gives us the following information:

For  $\rho \le 0.964$ , total delay can be improved or equivalent when the window size  $W_{13}$  is assigned about 13, compared with no flow controlled total delay.

The optimal assignment of window size for each LC, which we do not study analytically, should be almost equivalent to the average number of packets outstanding on each LC in no flow

controlled case  $(\overline{N}_{ij})$  , in our experience; for example,  $\overline{N}_{12}$ =. 7.59,  $\overline{N}_{23}$ =. 6.50 and  $\overline{N}_{13}$ =. 13.97 for  $\rho$  =0.964 .

Up to this point, we have fixed window sizes of 1-hop logical channels (  $W_{12}=W_{23}=8$  ). Similar phenomena can be observed in the additional figure 3.8, where  $W_{12}=W_{23}=7$ , and =11.

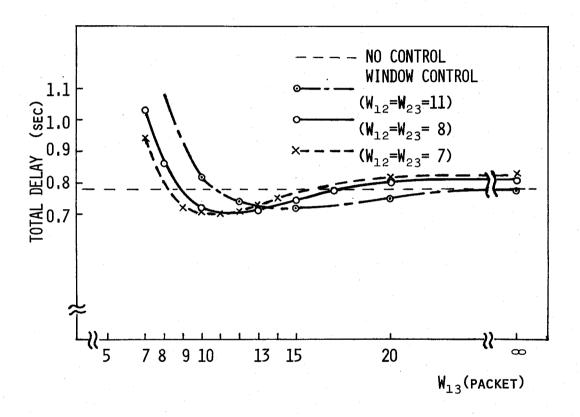


Fig. 3.8 Total Delay Characteristics (  $\rho$ =0.964)

# 3.3.2 Distribution Characteristics of Packet Admission Interval

Continuing our discussion about the effect of the window mechanism on the total delay, we now investigate the distribution characteristics of packet admission interval to the network.

In Fig.3.9 the number of sampled LC<sub>13</sub> packets is shown as a function of the admission interval of LC<sub>13</sub> packet, in no controlled case and in window controlled case, respectively. It can be seen that for no controlled case the admission intervals are exponentially distributed, because packets generated by user can immediately enter the network without any input regulation. On the other hand, for window controlled case, partially they are also exponentially distributed, except that some peaks appear at the multiple of 0.04 (sec) of interval. So, we can appreciate that packets forming peaks have been regulated on entering the network. We will call the first three peaks whose intervals are 0.04, 0.08, 0.12 (sec), peak A, B, C, respectively in our future discussion. Similar results are obtained for other logical channels (we have not shown these.).

Now let us mark packets which belong to peak. As pointed out above, the tagged packets have been regulated on entering the network, therefore the admission interval between the tagged packets depends on the arrival interval between acknowledgements which indicate windows open, not on its arrival interval at IQ (assumed Poisson arrival, as assumption (2)). Assuming that the acknowledgement delay is negligible (assumption (8)), the admission interval between the tagged packets is regarded as the interval at which previous packets reached the destination.

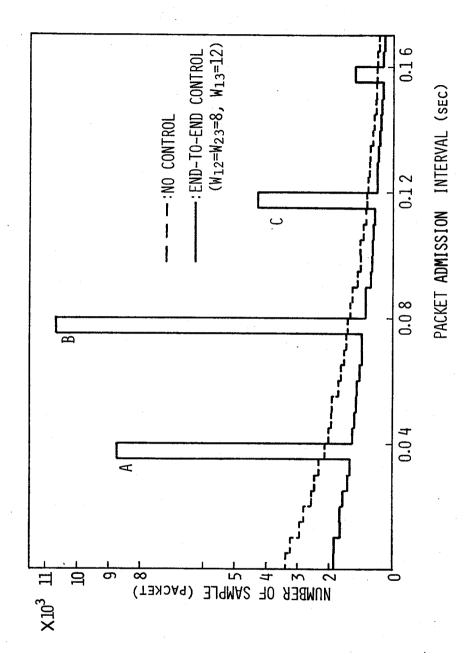


Fig. 3.9 Histgram of Packet Admission Interval (  $\rho\!=\!0.964)$ 

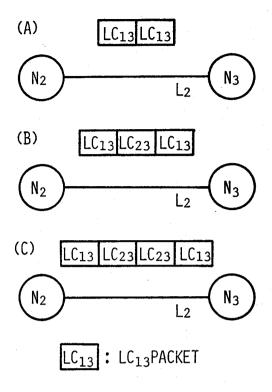


Fig. 3.10 Packet Transmission Behavior on Line 2

In order to explain the physical meaning of the peak, we show the transmission behavior of  $LC_{13}$  packet on line 2, in Fig. 3.10. In case (A), two of  $LC_{13}$  packets are transmitted successively on line and the interval at which they reaches their destination is 0.04 (sec) (1-hop transmission time of a packet is assumed 0.04 (sec), by accepting assumptions (1) and (3)); hence, this case is regarded to be correspond to the peak A in Fig. 3.9. In case (B),  $LC_{13}$  and  $LC_{23}$  packets are successively and alternatively transmitted on line 2. In case (C), two of  $LC_{23}$  packets, which are inserted between two of  $LC_{13}$  packets, are

successively transmitted on line 2. Case (B), (C) is corresponded to the peak B, C respectively, as well.

From these two figures, we can observe that:

In no controlled network, as packets are inputted in the order of their arrival ( in a first-in first-out fashion ), packets belonging to the same logical channel can be successively transmitted on line, if they arrive at their source node in a cluster. On the contrary, in window controlled network, packets can be replaced on inputting to the network, so that the nature of a cluster can be released and the sequence of the same logical channel packets on line can be more regular than that in no controlled network.

In no controlled network, the following case (of unfortunate  $LC_{13}$  packet (2-hop)) often can be seen.

 ${
m LC}_{12}$  packets arrive in a cluster at their source node  ${
m N}_1$  when line 2 is not so busy. The tagged  ${
m LC}_{13}$  packet arriving at  ${
m N}_1$  just a few seconds after their arrivals, must wait on line 1, until their transmission is finished. Still worse, the tagged  ${
m LC}_{13}$  packet, which arrives at node 2 ( ${
m N}_2$ ) when  ${
m LC}_{23}$  packets have already arrived in a cluster at  ${
m N}_2$ , must unfortunately wait on line 2. In this case, in order to decrease insignificant delay of  ${
m LC}_{13}$  packet, it should be rapidly transmitted on line 1 prior to  ${
m LC}_{12}$  packets until line 2 becomes so busy.

In window flow controlled network, we expect window will play an important role in decreasing this insignificant delay incurred by  ${
m LC}_{13}$  packets, for the sequence of the same logical channel packets on line can become more regular than that in no

controlled network. We call this effect of the window mechanism
"Input-Sequencing effect "in our future discussion." InputSequencing "implies rearrangement of packets on inputting adaptively to the network condition. The precise discussion for
"Input-Sequencing" being out of scope in this chapter, no longer we continue to discuss here. And we will analytically study it in Chapter 5.

### 3.4 Conclusion

In this chapter, we have simulated a simple delay system model of 3-node tandem network with individually window flow controlled logical channel, and have investigated elementary effect of the window mechanism on delay performance, under simplified and artificial assumptions.

We summarize the significant results in this chapter as follows:

- (1) The phenomenon can be observed that in a properly window flow controlled network, the total delay, which consists of admission delay and network delay, can be minimized and can be lower than that in no flow controlled network.
- (2) Network performs most effectively when each window size is assigned according to the average number of packets outstanding on each logical channel in no flow

controlled network.

(3) Packet admission interval can be regularized by the window mechanism.

# CHAPTER 4

# IN EXTENDED NETWORK MODELS

# 4.1 Introduction

The last chapter has investigated elementary effect of the window mechanism on delay performance in a simple delay system model. And from simulation results, we have found the phenomenon that in a properly window flow controlled network the total delay can be optimized and lower than that in no flow controlled network. However, the model in the last chapter suffers from the limitation that the model accepts some simplified and artificial assumptions, therefore insight into more realistic models has not yet offered.

In this chapter, we focus on extended network models. We consider

- (1) Investigations in some network topologies
- (2) Comparison between End-to-End window flow controlled network and Link-by-Link window flow controlled network
- (3) Affect of acknowledgement delay

### 4.2 Some Network Topologies

In this section we investigate effect of the window mechanism in some network topologies. We focus on tandem networks, 3-node loop network and 6-node ladder network. Similar assumptions are accepted in this section as in the last chapter.

### 4.2.1 Tandem Networks

We simulate 4-node and 5-node tandem networks as shown in Table 4.1. (the next page) Each network is assumed to be assigned its own window matrix.

Optimal window size is not offered in the table, however, in any case we can recognize that window controlled total delay can be lower than no controlled one.

### 4.2.2 Loop Network

We simulate 3-node loop network as shown in Fig.4.1, where the uniform loading [  $\gamma_a$  ] ( case (A)) and the non-uniform loading [  $\gamma_b$  ] ( case (B)) to the network are assumed. For each case, we set the window matrix [ Wa ],[ Wb ] as shown in Eq. (4.3), (4.4) respectively, where window size for 2-hop logical channel (W<sub>2</sub>) is a parameter. And also assuming the routing strategy fixed as shown in the figure.

Table 4.1 Total Delay Characteristics in 4- and 5-Node Tandem Networks

|   | •                                       |  |  |   |
|---|---|--|--|---|
|   |   | 4-node tandem  | 5-node tandem 1.   | 5-node tandem 2.  |
|   | Network Topology                        | 1 2 3 4  |  | 4 5   |
|   | Traffic Matrix                          | $ \begin{pmatrix} 0.0 & 1.0 & 0.0 & 1.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{pmatrix} $ | (0.0 1.0 0.0 0.0 1.0)<br>0.0 0.0 1.0 0.0 0.0<br>0.0 0.0 0.0 1.0 0.0<br>0.0 0.0 0.0 0.0 1.0<br>(0.0 0.0 0.0 0.0 0.0 | (0.0 0.5 0.5 0.0 1.0)<br>0.0 0.0 0.5 0.0 0.0<br>0.0 0.0 0.0 0.5 0.5<br>0.0 0.0 0.0 0.0 0.5<br>0.0 0.0 0.0 0.0 0.0 |
|   | Window Matrix                           | 0 8 0 20<br>0 0 8 0<br>0 0 8 0<br>0 0 0 8  | 0 8 0 0 23<br>0 0 8 0 0<br>0 0 0 8 0<br>0 0 0 0 8  | 0 4 8 0 26<br>0 0 4 0 0<br>0 0 0 4 8<br>0 0 0 0 4<br>0 0 0 0 0  |
| H | Line Utilization $\rho$                 | 0.962  | 096.0  | 0.963   |
|   | Average Total Delay<br>(Window Control) | 0.786 (sec)  | 0.729 (sec)  | 1.007 (sec)   |
|   | Average Total Delay<br>(No Control)     | 0.810 (sec)  | 0.761 (sec)  | 1.046 (sec)   |

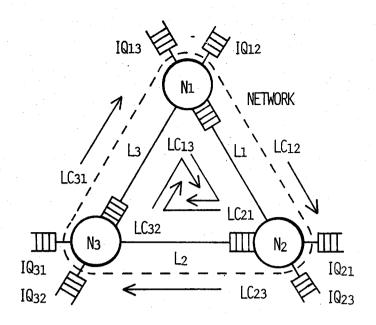


Fig. 4.1 Simulation Model (3-node loop)

Fig. 4.2 shows total delay characteristics, where average total delay for each case is shown as a function of  $W_2$ . Similar phenomena as Fig. 3.7, that lower total delay can be achieved in window flow controlled network than in no controlled one, are obtained for both cases. Additionally, although we do not study analytically, we recognize in our experience that we should assign window size according to the average number of packets outstanding on each logical channel in no controlled network; for example, the average number on 1-hop, on 2-hop logical channel in case (A) is nearly equal to 3.60, 7.17 respectively.

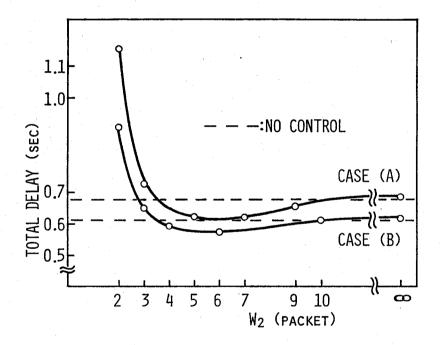


Fig. 4.2 Total Delay Characteristics (  $\rho$ =0.96)

# 4.2.3 Ladder Network

We simulate 6-node ladder network as shown in Fig.4.3, where the uniform loading to the network  $[\gamma_{i,j}]$  is assumed.

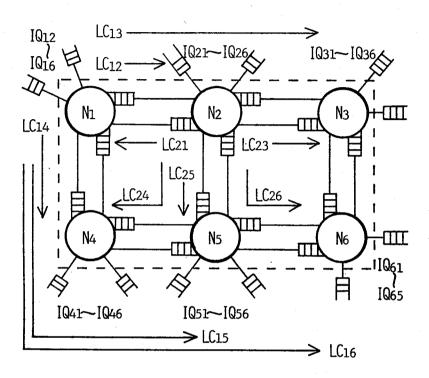


Fig. 4.3 Simulation Model (6-node ladder)

$$[ Y_{ij} ] = Y \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$(4.5)$$

We set the window matrix [ W ],

$$[W] = \begin{bmatrix} 0 & 3 & 6 & 1 & 3 & W_3 \\ 3 & 0 & 3 & 3 & 1 & 3 \\ 6 & 3 & 0 & W_3 & 3 & 1 \\ 1 & 3 & W_3 & 0 & 3 & 6 \\ 3 & 1 & 3 & 3 & 0 & 3 \\ W_3 & 3 & 1 & 6 & 3 & 0 \end{bmatrix}$$

$$(4.6)$$

where window size for 3-hop logical channel  $(W_3)$  is a parameter. Also we assume the routing strategy fixed as shown in the figure.

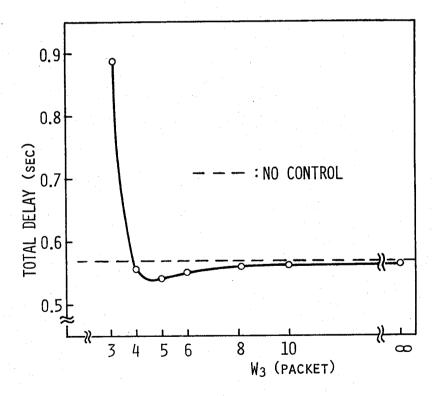


Fig. 4.4 Total Delay Characteristics ( $\rho=0.96$ )

Fig. 4.4 shows total delay characteristics, where average total delay is shown as a function of  $\mathbb{W}_3$ . Similar phenomenon can be observed in this figure as well as the previous figures.

# 4.3 Comparison between End-to-End Flow Controlled Network and Link-by-Link Flow Controlled Network

Up to this point in this thesis, we have evaluated effect of End-to-End flow control procedure. In this section, we compare End-to-End flow control procedure with Link-by-Link flow control procedure.

Fig. 4.5 shows simulation model of Link-by-Link window flow controlled network.

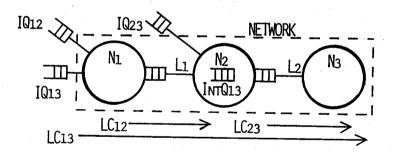


Fig.4.5 Simulation Model (Link-by-Link flow control)

Total number of packets on a logical channel is restricted by End-to-End flow control procedure, on the contrary, total number on each link of a logical channel is restricted by Link-by-Link flow control procedure; hence, for the former we use the window matrix [ W ] as follows

$$[W] = \begin{bmatrix} 0 & 5 & W_{13} \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$
 (4.7)

But for the latter, the k-th hop window matrix [  $W^k$  ] is used as Eq.(4.8) and (4.9),

$$[W^{1}] = \begin{bmatrix} 0 & 5 & W_{13}^{1} \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$
(4.8)

$$[ W^{2} ] = \begin{bmatrix} 0 & 0 & W_{13}^{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
(4.9)

where  $W_{ij}^k$  is the window size of the k-th link on  $LC_{ij}$ . Additionally, in the latter there is intermediate queue INT  $Q_{13}$ , in which  $LC_{13}$  packet finding no available window at node  $2(N_2)$  is stored until window  $(W_{13}^2)$  becomes open.

Fig.4.6 shows total delay characteristics, where average total delay in End-to-End, Link-by-Link window flow controlled network, is shown as a function of  $W_{13}$ ,  $(W_{13}^1, W_{13}^2)$ , respectively.

Similar curve can be seen in the latter as well. Also it is recognized that the former performs more effectively than the latter.

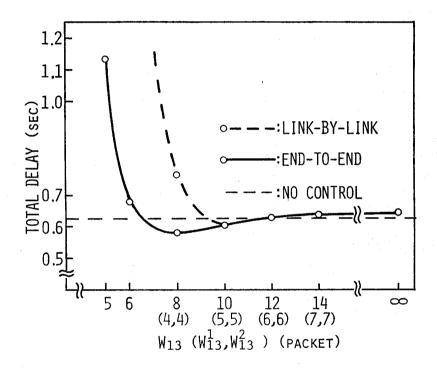


Fig. 4.6 Total Delay Characteristics (ρ=0.96)

Fig. 4.7 shows histgram of buffer occupancy time (line 2).

Upper bound of queue is limitted 14 for the former, 9 for the latter. This implies that more adaptive control according to the network condition can be achieved by the former than by the latter. latter.

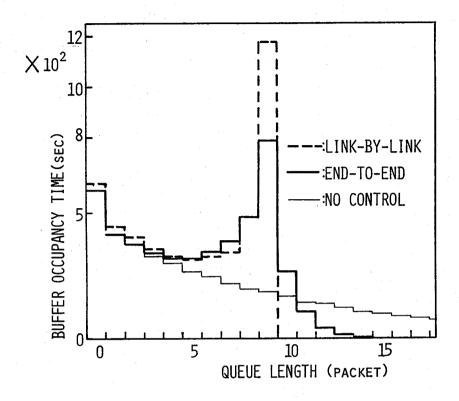


Fig.4.7 Histgram of Buffer Occupancy Time (  $\rho$ =0.96) (End-to-End:  $W_{13}$ =10, Link-by-Link:( $W_{13}^1$ ,  $W_{13}^2$ )=(5,5))

# 4.4 Affect of Acknowledgement Delay

Up to this point of this thesis, the models were rather simplified and possibly artificial, by the assumption of negligible acknowledgement (ACK) delay. In this section we examine affect of ACK delay. We start with basic affect, using a simple model shown in Fig. 3.1, where constant ACK delay (0.02(sec)×( number of hops)) is assumed.

 $<sup>^{+}</sup>$  0.02 (sec) corresponds to 1/2 packet of service time.

Fig. 4.8 shows total delay characteristics in both models with ACK delay and without ACK delay. Similar curves are observed, except that the former rises unbounded at larger window size than the latter. This implies that the price for ACK delay is substantially a reducing window size.

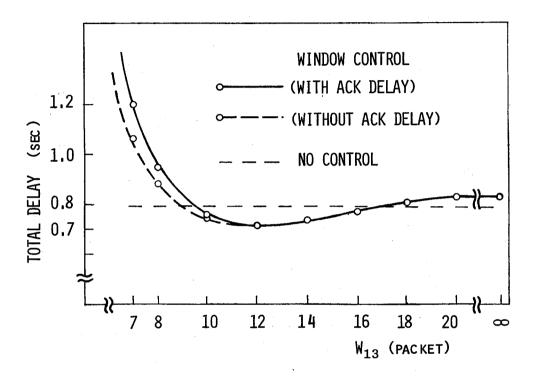


Fig. 4.8 Total Delay Characteristics in Both Models with and without ACK Delay (  $\rho$ =0.96)

Then, we develop more realistic model, where end-to-end ack-nowledgement is considered according to the RECOMMENDATION X.25, as follows:

Acknowledgement, which indicates that packet has successfully reached its destination, is piggy backed onto return packet and sent back to its source via a network. If none is available

or the reverse window is closed, an explicit acknowledgement RR (Receive Ready packet) is generated.

Fig. 4.9 shows simulation model,

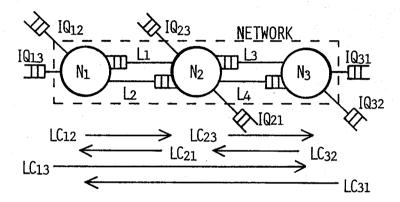


Fig. 4.9 Simulation Model (3-node tandem, both directional traffic)

where uniform loading to the network  $[\gamma_{ij}]$  (including both directional traffic) is assumed.

$$[\gamma_{1,j}] = \gamma \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$
 (4.10)

We use window matrix [ W ] as

$$[W] = \begin{bmatrix} 0 & 7 & W_2 \\ 7 & 0 & 7 \\ W_2 & 7 & 0 \end{bmatrix}$$
 (4.11)

where window size of 2-hop logical channel ( $W_2$ ) is a parameter. Additionally, RR is assumed to be fixed with 100 bits of lengths. (cf. X.25 recommends 72 bits)

Fig. 4.10 shows total delay characteristics, where average total delay is shown as a function of  $\mathbf{W}_2$ . Comparison between models with and without ACK delay can not be offered, due to the increase of RR traffic for the former, however, also similar phenomenon can be observed in this figure as well as the previous figures.

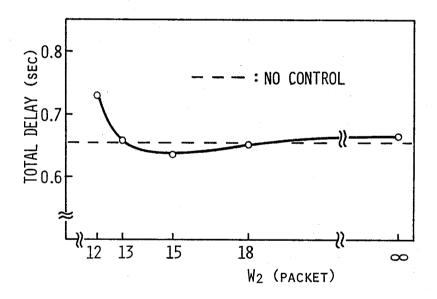


Fig. 4.10 Total Delay Characteristics in the Model with ACK Delay (  $\rho$ =0.96)

### 4.5 Conclusion

In this chapter, we have investigated effect of window mechanism on delay performance in extended network models.

First, we have examined effect in some network topologies. Then, we have compared End-to-End window flow control procedure with Link-by-Link window flow control procedure. Finally, we have examined affect of acknowledgement delay.

We summarrize the significant results as follows:

- (1) In any case, we can observe the phenomenon that, in a properly window flow controlled network, the total delay can be lower than that in no flow controlled network.
- (2) Network performs more effectively by End-to-End window flow control procedure than by Link-by-Link window flow control procedure, on a viewpoint of total delay.
- (3) The price for acknowledgement delay is substantially a reducing window size.

# CHAPTER 5

# INPUT-SEQUENCING EFFECT OF THE WINDOW MECHANISM

## 5.1 Introduction

The last two chapters were mainly concerned with the delay performance by window flow control mechanism, on delay system of computer communication network. From simulation results, we observed the phenomenon that in a properly flow controlled network the performance of the total delay can be superior to that in no controlled network. And we suggested this phenomenon should be due to "Input-Sequencing" of the window mechanism, which implies rearrangement of packets on inputting to the network.

In this chapter we analyze "Input-Sequencing Effect "and certify that the phenomenon should be due to the effect.

### 5.2 Outline of Input-Sequencing

In this section we describe an outline of Input-Sequencing by using the End-to-End window flow controlled network model shown in Fig.5.1, from which we will obtain numerical and simulation results in Section 5.4. This model is the same that we used in Chapter 3. And also in this chapter, we accept similar assumptions as well as in Chapter 3, except for the distribution of message lengths. The exponential distribution is assumed in this chapter.

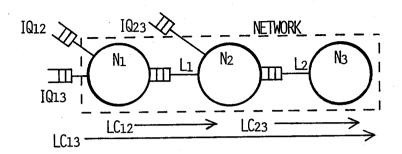


Fig. 5.1 3-Node Tandem Network

We assume the uniform loading [  $\gamma_{\mbox{\scriptsize ij}}$  ]  $\,$  and the window matrix [ W ] as

$$[ \gamma_{ij} ] = \gamma \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[ W ] = \begin{bmatrix} 0 & 4 & W_{13} \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(5.1)$$

Now we begin by considering the transmission behavior of  $LC_{13}$  packets on line 1 ( $L_1$ ). Fig. 5.2 shows how the tagged two  $LC_{13}$  packets possibly arrive at their intermediate node 2 ( $N_2$ ). There are (N+2) cases of conditions on transmitting. In our future discussion, we will call each of them, case 0, case 1, case 2, ...., case (N+1) in order.

(5.2)

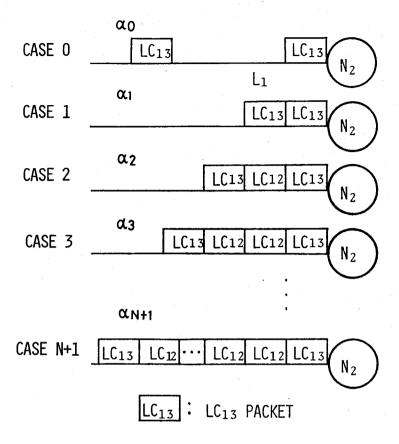


Fig. 5.2 Packet Transmission Behavior on Line 1

Each case is defined as follows:

## case 0;

the case that  $L_1$  is temporally free, since the interarrival time between the tagged two packets happens to be very long.

# case i ( $1 \le i \le N+1$ );

the case that a mass of packets are successively transmitted, where (i-1) of  ${\rm LC}_{12}$  packets are inserted between the tagged two  ${\rm LC}_{13}$  packets.

Each case i is assumed to occur with the probability  $\alpha_{i}$ .

The distribution characteristics of packet arrival interval (in the M/D/1 queueing system) shown in Fig.3.9 , suggests that the value of  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$  might be larger in window flow controlled network than in no controlled network. This suggestion leads us to the following fact :

In no controlled network, since packets are inputted in the order of thier arrival without any input regulation (so, in first-in first-out fashion), packets belonging to the same logical channel will be successively transmitted on line, if they enter the network in a cluster.

On the other hand, in a properly window flow controlled network, packets are sometimes replaced on entering the network by the window mechanism as the case may be, so that the nature of a cluster might be released and the sequence of the same logical channel packets on line might be more regular than in no controlled network.

We will call this "Input-Sequencing" and call the effect of this "Input-Sequencing effect" simply, in our future discussion.

Now we describe an example, where Input-Sequencing becomes effective. In no controlled network, sometimes the following situation can be seen in the model of Fig.5.1.  $LC_{12}$  packets arrive in a cluster at their source node ( $N_1$ ) when the line 2 ( $L_2$ ) is not so busy.  $LC_{13}$  packet arriving at  $N_1$  just after their arrivals, must wait on the line 1 ( $L_1$ ) until their transmission

is finished. Still worse, the tagged  $LC_{13}$  packet, which arrives at the node 2  $(N_2)$  when a mass of  $LC_{23}$  packets have already arrived at their source node  $N_2$ , must unfortunately wait on  $L_2$ . In this circumstance, the tagged  $LC_{13}$  packet should be rapidly transmitted on  $L_1$  prior to  $LC_{12}$  packets until  $L_2$  becomes busy, so that  $L_2$  might be more effectively utilized. Input-Sequencing is expected to play an important role in decreasing in significant delay of  $LC_{13}$  packet, as mentioned above.

We continue our discussion about Input-Sequencing effect on delay performance. Average total delay  $\overline{T}$ , which consists of admission delay and network delay, may be given by

$$\overline{T} = \frac{(N_{12} + N_{13}) \overline{T}_1 + (N_{13} + N_{23}) \overline{T}_2}{N_{12} + N_{13} + N_{23}}$$
(5.3)

where  $\overline{T}_1$  is the average delay incurred on  $L_1$  (including  $IQ_{12}$  and  $IQ_{13}$ ) by  $LC_{12}$  and  $LC_{13}$  packets,  $\overline{T}_2$  is the average delay incurred on  $L_2$  (including  $IQ_{23}$ ) by  $LC_{13}$  and  $LC_{23}$  packets, and  $N_{ij}$  is the number of sampled  $LC_{ij}$  packets.

Our presumption for Input-Sequencing effect on delay performance is as follows:

Input-Sequencing will not bring in a decreasing delay on  $L_1$ , since  $\overline{T}_1$  will be invariable whether in a properly flow controlled network or not, even if  $LC_{12}$  and  $LC_{13}$  packets are inputted to the

network in any sequence. However, in a properly window flow controlled network, due to Input-Sequencing ,  $\rm LC_{13}$  packets being able to arrive at  $\rm N_2$  more regularly than in no controlled one, their intermediate line ( $\rm L_2$ ) can be more effectively utilized ; hence, we presume that  $\rm \overline{T}_2$  will possibly decrease due to Input-Sequencing.

Later in this chapter , we will analyze Input-Sequencing effect on the total delay performance, where we will apply M/M/1 to  $\overline{T}_1$  , and G/M/1 to  $\overline{T}_2$  in our future calculation.

## 5.3 Analysis of Input-Sequencing

In this section, we develop an analytic model for  $LC_{13}$ ,  $LC_{23}$  packet's interarrival process and find the Laplace transform for the pdf of the compound interarrival time  $LC_{13}$ ,  $LC_{23}$  packets feeding the line 2 ( $L_2$ ).

#### 5.3.1 The Model and Assumptions

Fig. 5.3 shows the model for interarrival process of  $LC_{13}$ ,  $LC_{23}$  packets, where that of  $LC_{13}$  packets is based on Fig. 5.2 (transmission behavior of them on the line 1).

In our analysis, we accept the following assumptions:

(1) Poisson arrivals of  $LC_{ij}$  packets to their input queue  $IQ_{ij}$  (with mean  $1/\lambda_{ij}$  of interarrival time).

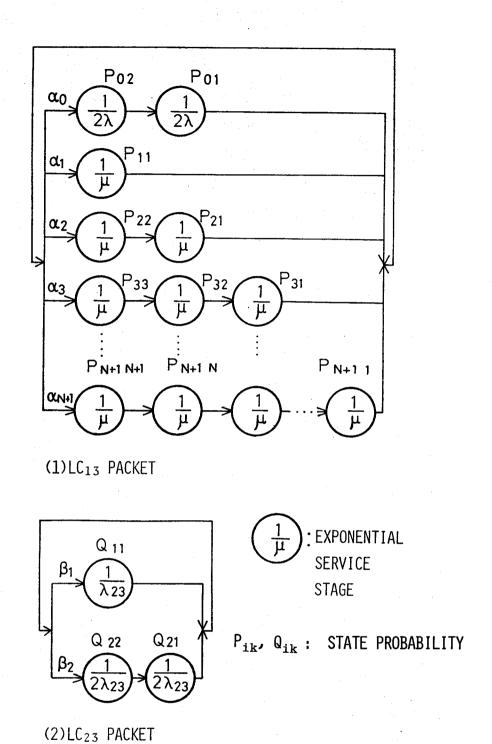


Fig. 5.3 Model for Packet's Interarrival Process

- (2) Service times of packets are exponentially distributed with mean  $1/\mu$  .
- (3) In a window flow controlled network, arrival process of  $LC_{23}$  packet is assumed to be modelled as shown in Fig.5.3 (2).

This diagram shows two parallel " stages ", upon entry into the arrival facility,  $LC_{23}$  packet will proceed to an exponential stage (with parameter  $\lambda_{23}$ ) with the probability  $\beta_1$ , or will proceed to two exponential stages (with parameter  $2\lambda_{23}$ ) with probability  $\beta_2$ . ^+ Additionally, in no flow controlled network, since  $LC_{23}$  packets are inputted without any input regulation, we assume  $\beta_1 = 1$  and  $\beta_2 = 0$ . The conservation relation is required as

$$\beta_1 + \beta_2 = 1 \tag{5.4}$$

 $<sup>^+</sup>$  We will make a comparison between numerical result and simulation result in the section 5.4,where we will calculate the former by using values of parameters ( $\alpha_i$ ,  $\beta_i$  etc.) obtained from the latter. Assumption (3) comes from the fact that we can observe more regular packet arrival in a window controlled network than in no controlled one. A verification of this modelling is offered in Appendix B .

The state probability of finding  $LC_{23}$  packet on the (i-k+1)-th stage in the i-th branch, which is denoted by  $Q_{ik}$  in the figure, is given by

$$\begin{cases} Q_{11} = \beta_1 \\ Q_{22} = Q_{21} = \beta_2/2 \end{cases}$$
 (5.5)

(4) Arrival process of  $LC_{13}$  packet is modelled as shown in Fig.5.3 (1), due to the transmission behavior of  $LC_{13}$  packets on  $L_1$  (Fig.5.2) .

This diagram shows (N+2) parallel branches, which are categorized into the first branch and others. We will call the former "  $\alpha_0$  branch ", and the latter "  $\alpha_i$  branch ", in our future discussion. The  $\alpha_0$  branch, which corresponds to case 0 in Fig. 5.2, is assumed to consist of m-stage series system. On the other hand, the  $\alpha_i$  branch, which corresponds to case i in Fig.5.2, consists of i-stage series system ( i= 1, 2, ..., N+1 ).

Each branch being with the probability  $\boldsymbol{\alpha}_{\dot{\mathbf{I}}}$  , we require the conservation relation :

<sup>\*</sup> In assumption (4), we assume that interarrival times of  $LC_{13}$  packets on the  $\alpha_0$  branch are Erlangian distributed with degree m. And we will let m be 2 on calculation ; the reason of this is discussed in Appendix C .

$$\sum_{i=0}^{N+1} \alpha_i = 1$$
(5.6)

Each stage in the  $\alpha_0$  branch, the  $\alpha_i$  branch, is an exponential arrival facility with parameter maa,  $\mu$  respectively.

The state probability of finding  $LC_{13}$  packet on the (m-k+1)-th stage in the  $\alpha_0$  branch, on the (i-k+1)-th stage in the  $\alpha_i$  branch, is denoted by  $P_{0k}$ ,  $P_{ik}$  respectively as follows:

$$\begin{cases} P_{ik} = \alpha_{i} / \mu \Lambda & (l \leq k \leq i \leq N+1) \\ P_{0k} = \alpha_{0} / m \lambda \Lambda & (k=1,2,...,m) \end{cases}$$
 (5.7)

where  $\Lambda$  is the normalization constant, given by

$$\Lambda = \frac{1}{\lambda} \alpha_0 + \frac{1}{\mu} \sum_{i=1}^{N+1} \sum_{k=1}^{i} \alpha_i \quad (= \frac{1}{\lambda_{13}})$$
 (5.8)

## 5.3.2 Analysis

In this section, we find the Laplace transform for the pdf of the compound interarrival time, at which  $LC_{13}$ ,  $LC_{23}$  packets feed the line 2 ( $L_2$ ). Derivations of the equations in this section are given in Appendix D

Let the Laplace transform be denoted by A\*(s). In order to calculate the unconditional transform A\*(s), we must devide it

into two conditional transforms denoted by B\*(s) and C\*(s), where B\*(s); on the condition that  $LC_{13}$  packet has just arrived (departed from its arrival process), on the other hand, C\*(s); on the condition that  $LC_{23}$  packet has just arrived. Because it is necessary for us to take it into consideration which stage one packet exists on its arrival process, when another has just arrived. We can write down the unconditional transform as

$$A^{*}(s) = \frac{\lambda 13}{\lambda 13 + \lambda 23} B^{*}(s) + \frac{\lambda 23}{\lambda 13 + \lambda 23} C^{*}(s)$$
 (5.9)

Let us now calculate the first conditional transform B\*(s). The basic question is to solve for  $LC_{13}$  packet's interarrival time distribution feeding  $L_2$ . Let the PDF be denoted by  $B_{LC_{13}}$  (t), the pdf;  $b_{LC_{13}}$  (t) and its Laplace transform;  $B_{LC_{13}}^*$  (s). We can obtain them from Fig.5.3 (1), as follows:

$$B_{LC_{13}}^{N+1} (s) = \sum_{i=1}^{N+1} \alpha_{i} (\frac{\mu}{s+\mu})^{i} + \alpha_{0} (\frac{m\lambda}{s+m\lambda})^{m}$$
 (5.10)

$$b_{LC_{13}}^{(t)} = \sum_{i=1}^{N+1} \alpha_i \frac{\mu(\mu t)^{i-1}}{(i-1)!} e^{-\mu t} + \alpha_0 \frac{m\lambda(m\lambda t)}{(m-1)!}^{m-1} e^{-m\lambda t}$$
(5.11)

$$B_{LC_{13}}(t) = 1 - e^{-\mu t} \sum_{i=1}^{N+1} \alpha_{i}^{i} \frac{(\mu t)^{i-k}}{(i-k)!} + \alpha_{0} e^{-m\lambda t} \sum_{k=1}^{m} \frac{(m\lambda t)^{m-k}}{(m-k)!}$$
(5.12)

To calculate the PDF of the compound (LC $_{13}$  and LC $_{23}$  packet) interarrival time, we need the individual PDF's. For LC $_{13}$  packet, Eq.(5.12) is available as it is, however, for LC $_{23}$  packet we must consider the position at which it exists on its arrival facility, as mentioned above; hence, we categorize the PDF of LC $_{23}$  packet's interarrival time B $_{\rm LC}_{23}$  (t) into the following two cases:

LC<sub>23</sub> packet exists on

(I): the first stage in  $\beta_1$  branch

$$B_{LC_{23}}(t) = 1 - e^{-\lambda_{23} t}$$
 (5.13)

(II): the (3-1)th stage in  $\beta_2$  branch

$$B_{LC_{23}}(t) = 1 - e^{-\lambda_{23} t} \sum_{j=1}^{l-1} \frac{(2\lambda_{23} t)^{j}}{j!}$$
 (5.14)

For (I), let the PDF of the compound interarrival time be denoted by  $B_{11}(t)$ , the pdf;  $b_{11}(t)$  and the Laplace transform for the pdf;  $B_{11}^*(s)$ .

For (II), let the PDF be denoted by  $B_{2l}(t)$ , the pdf;  $b_{2l}(t)$  and the Laplace transform;  $B_{2l}^*(s)$ .

 $B_{11}(t)$  is obtained from Eqs.(5.12)and (5.13),  $B_{21}(t)$  from Eqs.(5.12) and (5.14), respectively. So we have Equations (5.15)-(5.20).

(I)

$$B_{11}(t) = 1 - \beta_1 e^{-\lambda_{23} t} \left\{ e^{-\mu t} \sum_{i=1}^{N+1} \frac{i}{\alpha_{i}} \frac{(\mu t)^{i-k}}{(i-k)!} + \alpha_0 e^{-m\lambda t} \sum_{k=1}^{m} \frac{(m\lambda t)^{m-k}}{(m-k)!} \right\}$$
(5.15)

$$b_{11}(t) = \beta_{1} \sum_{i=1}^{N+1} \frac{(\mu t)^{i-1}}{(i-1)!} (\mu \alpha_{i} + \lambda_{23} \sum_{k=i}^{N+1} \alpha_{k}) e^{-(\mu + \lambda_{23})t}$$

$$+ \alpha_{0} \left\{ \lambda_{23} \sum_{k=1}^{m} \frac{(m\lambda t)^{k-1}}{(k-1)!} + \frac{m\lambda(m\lambda t)}{(m-1)!} \right\} e^{-(m\lambda + \lambda_{23})t}$$
(5.16)

$$B_{11}^{*}(s) = \beta_{1} \left[ \sum_{i=1}^{N+1} \frac{1 - 1(\mu \alpha_{1} + \lambda_{23} \sum_{k=1}^{n-1} \alpha_{k})}{(s + \mu_{1} + \lambda_{23})^{1}} + \alpha_{0} \left\{ \lambda_{23} \sum_{i=1}^{m} \frac{(m\lambda)^{1-1}}{(s + m\lambda + \lambda_{23})^{1}} + \frac{(m\lambda)^{m}}{(s + m\lambda + \lambda_{23})^{m}} \right\} \right]$$

(5.17)

We derivate a component of Eq.(5.16) from a component of Eq.(5.15), in Appendix D.1.

(II)

$$B_{21}(t) = 1 - \frac{\beta_2}{2} \sum_{j=0}^{l-1} \frac{(2\lambda_{23} t)^j}{j!} e^{-2\lambda_{23} t} \left\{ e^{-\mu t} \sum_{i=1}^{N+1} \frac{i}{k-1(i-k)!} + \alpha_0 e^{-m\lambda t} \sum_{k=1(m-k)!}^{m} \frac{(m\lambda t)^{m-k}}{(m-k)!} \right\}$$

(5.18)

$$= \frac{\beta_2}{2} \left[ e^{-(\mu+2\lambda_{23})t} \begin{cases} \sum_{i=1(i-1)!}^{N+1} \sum_{j=1}^{\alpha_i \mu^i} \sum_{(j-1)!}^{l(2\lambda_{23})^{j-1}t^{j+1-2}} + \frac{(2\lambda_{23})}{(l-1)!} \sum_{i=1}^{N+1} \sum_{k=1}^{i} \frac{\mu^{k-1}t^{k+l-2}}{(k-1)!} \right]$$

$$+ \alpha_{0} e^{-(m\lambda+2\lambda_{23})t} \left\{ \frac{(m\lambda)^{2}}{(m-1)!} \sum_{j=1}^{l} \frac{(2\lambda_{23})^{j-1}t^{j}}{(j-1)!} + \frac{(2\lambda_{23})^{l}}{(l-1)!} \sum_{k=1}^{m} \frac{(m\lambda)^{k-1}t^{k+l-2}}{(k-1)!} \right\}$$
(5.19)

B\*<sub>7</sub>(s)

$$=\frac{\beta_{2}}{2} \left[ \sum_{i=1(i-1)!}^{N+1} \frac{\mu_{i}\alpha^{i}}{\sum_{j=1(i-1)!}^{2} \frac{(2\lambda_{23})^{j-1}(j+i-2)!}{(j-1)!(s+\mu+2\lambda_{23})^{j+i-1}} + \frac{(2\lambda_{23})^{2}N+1}{(\ell-1)!} \sum_{i=1}^{N+1} \alpha_{i} \frac{\frac{1}{2}\mu^{k-1}(k+\ell-2)!}{k=1(k-1)!(s+\mu+2\lambda_{23})^{k+\ell-1}} \right]$$

$$+ \alpha_{0} \left\{ \frac{(m\lambda)^{m}}{(m-1)!} \sum_{j=1}^{l} \frac{(2\lambda_{23})^{j-1}(j+m-2)!}{(j-1)!(s+m\lambda+2\lambda_{23})^{j+1}} + \frac{(2\lambda_{23})^{l}}{(l-1)!} \sum_{k=1(k-1)!(s+m\lambda+2\lambda_{23})^{k+l-1}}^{m} \frac{(m\lambda)^{k-1}(k+l-2)!}{(l-1)!(s+m\lambda+2\lambda_{23})^{k+l-1}} \right\}$$

(5.20)

We derivate a component of Eq.(5.19) from a component of Eq.(5.18), in Appendix D.2.

Then B\*(s) is given by

$$B^*(s) = B^*_{11}(s) + \sum_{l=1}^{2} B^*_{2l}(s)$$
 (5.21)

Substituting Eqs.(5.17) and (5.20) into Eq.(5.21), we arrive at Eq.(5.22).

$$B^{*}(s) = \beta_{1} \left[ \sum_{i=1}^{N+1} \frac{\mu^{i-1}(\mu\alpha_{1}^{i} + \lambda_{23} \sum_{k=1}^{N+1} \alpha_{k})}{(s+\mu+\lambda_{23})^{i}} + \alpha_{0} \left\{ \lambda_{23} \sum_{i=1}^{m} \frac{(m\lambda)^{i-1}}{(s+m\lambda+\lambda_{23})^{i}} + \frac{(m\lambda)^{m}}{(s+m\lambda+\lambda_{23})^{m}} \right\} \right]$$

$$+ \beta_{2} \left[ \sum_{i=1}^{N+1} \alpha_{1} \mu^{i} \left\{ \frac{1}{(s+\mu+2\lambda_{23})^{i}} + \frac{\lambda_{23} i}{(s+\mu+2\lambda_{23})^{i+1}} \right\} \right]$$

$$+ \lambda_{23} \sum_{i=1}^{N+1} \mu^{i-1} \left\{ \frac{1}{(s+\mu+2\lambda_{23})^{i}} + \frac{2\lambda_{23} i}{(s+\mu+2\lambda_{23})^{i+1}} \right\} \sum_{k=1}^{N+1} \alpha_{k}$$

$$+ \beta_{2} \alpha_{0} \left[ \lambda_{23} \sum_{i=1}^{m} (m\lambda)^{i-1} \left\{ \frac{1}{(s+m\lambda+2\lambda_{23})^{i}} + \frac{2\lambda_{23} i}{(s+m\lambda+2\lambda_{23})^{i+1}} \right\} \right]$$

$$+ (m\lambda)^{m} \left\{ \frac{1}{(s+m\lambda+2\lambda_{23})^{m}} + \frac{m\lambda_{23}}{(s+m\lambda+2\lambda_{23})^{m+1}} \right\}$$

$$+ (m\lambda)^{m} \left\{ \frac{1}{(s+m\lambda+2\lambda_{23})^{m}} + \frac{m\lambda_{23}}{(s+m\lambda+2\lambda_{23})^{m+1}} \right\}$$

$$(5.22)$$

In Appendix D.3, we derivate the first half of the second term in Eq.(5.22) from the second half of the first term in Eq.(5.20). We omit derivations of the other terms, since they can be easily derivated with the same process as Appendix D.3.

Next, let us calculate the second conditional transform C\*(s). The basic problem is to solve for  $LC_{23}$  packet's interarrival time distribution feeding  $L_2$ . Let the PDF be denoted by  $C_{LC_{23}}$  (t), the pdf;  $b_{LC_{23}}$  (t) and its Laplace transform;  $C_{LC_{23}}^*$  (s) . We can obtain them from Fig.5.3 (2), as follows:

$$C_{LC_{23}}^*(s) = \beta_1 \frac{\lambda_{23}}{s + \lambda_{23}} + \beta_2 (\frac{2\lambda_{23}}{s + \lambda_{23}})^2$$
 (5.23)

$$c_{LC_{23}}(t) = \beta_1 \lambda_{23} e^{-\lambda_{23} t} + \beta_2 (2\lambda_{23})^2 t e^{-2\lambda_{23} t}$$
 (5.24)

$$C_{LC_{23}}(t) = 1 - \{\beta_1 e^{-\lambda_{23} t} + \beta_2 (1 + 2\lambda_{23} t) e^{-2\lambda_{23} t}\}$$
 (5.25)

To calculate the PDF of the compound (LC $_{13}$  and LC $_{23}$  packet) interarrival time, we need the individual PDF's. For LC $_{23}$  packet, Eq.(5.25) is available as it is, however, for LC $_{13}$  packet we must consider the position at which it exists on its arrival facility, as mentioned above; hence, we categorize the PDF of LC $_{13}$  packet's interarrival time  $\rm C_{LC_{13}}$  (t) into the following two cases:

$$C_{LC_{13}}(t) = 1 - \sum_{j=0}^{k-1} \frac{(\mu t)^j}{j!} e^{-\mu t}$$
 (5.26)

(IV): the (m-k+1)th stage in  $\alpha_0$  branch ( k=1,2,...,m )

$$C_{LC_{13}}(t) = 1 - \sum_{j=0}^{k-1} \frac{(m\lambda t)^{j}}{j!} e^{-m\lambda t}$$
 (5.27)

For (III), let the PDF of the compound interarrival time be denoted by  $C_{ik}(t)$ , the pdf;  $c_{ik}(t)$  and the Laplace transform for the pdf;  $C_{ik}^*(s)$ .

For (IV), let the PDF be denoted by  $C_{0k}(t)$ , the pdf;  $c_{0k}(t)$  and the Laplace transform;  $C_{0k}^{\star}(s)$ .

 $C_{ik}(t)$  is obtained from Eqs.(5.25) and (5.26),  $C_{0k}(t)$  from Eqs.(5.25) and (5.27), respectively.

So we have Equations (5.28)-(5.33).

(田)

$$C_{ik}(t) = 1 - \frac{\alpha_i}{\mu \Lambda} e^{-\mu t} \left\{ \beta_1 e^{-\lambda_{23} t} + \beta_2 (1 + 2\lambda_{23} t) e^{-2\lambda_{23} t} \sum_{j=0}^{k-1} \frac{(\mu t)^j}{j!} \right\}$$
(5.28)

$$c_{ik}(t) = \frac{\alpha_{i}}{\mu \Lambda} \left[ \beta_{1} e^{-(\mu + \lambda_{23})t} \left\{ \lambda_{23} \sum_{j=1}^{k} \frac{(\mu t)^{j-1}}{j-1} + \frac{\mu(\mu t)^{k-1}}{(k-1)!} \right\} + \beta_{2} e^{-(\mu + 2\lambda_{23})t} \left\{ (2\lambda_{23})^{2} t \sum_{j=1}^{k} \frac{(\mu t)^{j-1}}{(j-1)!} + \frac{\mu(1 + 2\lambda_{23} t)(\mu t)^{k-1}}{(k-1)!} \right\}$$

$$(5.29)$$

$$C_{1k}^{*}(s) = \frac{\alpha_{1}}{\mu \Lambda} \left\{ \beta_{1} \left\{ \lambda_{23} \sum_{j=1}^{k} \frac{\mu^{j-1}}{(s+\mu+\lambda_{23})^{j}} + \frac{\mu^{k}}{(s+\mu+\lambda_{23})^{k}} \right\} + \beta_{2} \left\{ 2\lambda_{23} \right\} \sum_{j=1}^{2k} \frac{\mu^{j-1}j}{(s+\mu+2\lambda_{23})^{j+1}} + \frac{2\lambda_{23} \mu^{k}k}{(s+\mu+2\lambda_{23})^{k+1}} + \frac{\mu^{k}}{(s+\mu+2\lambda_{23})^{k}} \right\}$$
where 
$$\Lambda = \frac{1}{\lambda_{23}} \quad \text{(from Eq.(5.8))}$$
 (5.30)

We derivate Eq.(5.29) from Eq.(5.28), in Appendix D.4.

(IV)

$$C_{0k}(t) = 1 - \frac{\alpha_0}{m\lambda \hbar} \left\{ \beta_1 e^{-\lambda_{23} t} + \beta_2 (1 + 2\lambda_{23} t) e^{-2\lambda_{23} t} \right\} e^{-m\lambda t} \int_{j=0}^{k-1} \frac{(m\lambda t)^j}{j!}$$
(5.31)

$$c_{0k}(t) = \frac{\alpha_0}{m\lambda\Lambda} \left[ \beta_1 e^{-(m\lambda + \lambda_{23})t} \begin{cases} \lambda_{23} \sum_{j=1(j-1)!}^{k} + \frac{m\lambda(m\lambda t)^{k-1}}{(k-1)!} \end{cases} \right]$$

$$+ \beta_{2}e^{-(m\lambda+2\lambda_{23})t} \left\{ (2\lambda_{23})^{2} t \sum_{j=1(j-1)!}^{k} \frac{m\lambda(1+2\lambda_{23}t)(m\lambda t)^{k-1}}{(k-1)!} \right\}$$
(5.32)

$$C_{0k}^{*}(s) = \frac{\alpha_{0}}{m\lambda\Lambda} \left[ \beta_{1} \left( \frac{k}{23} \sum_{j=1}^{k} \frac{(m\lambda)^{j-1}}{(s+m\lambda+\lambda_{23})^{j}} + \frac{(m\lambda)^{k}}{(s+m\lambda+\lambda_{23})^{k}} \right) + \beta_{2} \left\{ (2\lambda_{23})^{2} \sum_{j=1}^{k} \frac{(m\lambda)^{j-1}j}{(s+m\lambda+2\lambda_{23})^{j}} + \frac{2\lambda_{23}(m\lambda)^{k}k}{(s+m\lambda+2\lambda_{23})^{k+1}} + \frac{(m\lambda)^{k}}{(s+m\lambda+2\lambda_{23})^{k}} \right\}$$

$$(5.33)$$

Derivation of Eq.(5.32) is omitted, since it can be easily derivated with the same process as Appendix D.4.

Then, C\*(s) is given by

$$C^{*}(s) = \sum_{i=1}^{N+1} \sum_{k=1}^{i} C^{*}_{ik}(s) + \sum_{k=1}^{N+1} C^{*}_{ik}(s)$$
(5.34)

Substituting Eqs.(5.30) and (5.33) into Eq.(5.34), we arrive at Eq.(5.35)  $\cdot$ 

$$C^{*}(s) = \frac{\beta_{1}}{\Lambda} \left[ \sum_{i=1}^{N+1} \frac{\sum_{k=i}^{\mu-(k-i+1)\lambda_{23}} \alpha_{k}}{\sum_{i=1}^{\mu-(k-i+1)\lambda_{23}} \alpha_{k}} + \alpha_{0} \sum_{i=1}^{m} \frac{(m\lambda)^{i-2} \{m\lambda + (m-i+1)\lambda_{23}\}}{(s+m\lambda + \lambda_{23})^{i}} \right]$$

$$+ \frac{\beta_{2}}{\Lambda} \left[ 2\lambda_{23}^{N+1} \sum_{i=1}^{\mu^{1-2}} \sum_{k=i}^{N+1} (s+\mu+2\lambda_{23})^{i+1} + \sum_{i=1}^{N+1} (s+\mu+2\lambda_{23})^{i} + \sum_{k=i}^{N+1} (s+\mu+2\lambda_{23})^{i} \right]$$

$$+ \alpha_{0} \left\{ 2\lambda_{23} \sum_{i=1}^{m} \frac{(m\lambda)^{i-2} i \{m\lambda + (m-i+1)2\lambda_{23}\}}{(s+m\lambda + 2\lambda_{23})^{i+1}} + \sum_{i=1}^{m} \frac{(m\lambda)^{i-1}}{(s+m\lambda + 2\lambda_{23})^{i}} \right\}$$
(5.35)

where

$$\frac{1}{\Lambda} = \lambda_{13}$$

In Appendix D.5, we derivate the first half of the second term in Eq.(5.35), from the second half of Eq.(5.30). We omit derivations of the other terms, since they can be easily derivated with the same process as Appendix D.5.

Substituting Eqs. (5.22) and (5.35) into Eq. (5.9), we can finally find the Laplace transform A\*(s) for the pdf of the compound interarrival time, as shown in Eq. (5.36).

$$\begin{split} A^{*}(s) &= \frac{\lambda_{13}}{\lambda_{13} + \lambda_{23}} \left[ \beta_{1} \left[ \sum_{j=1}^{N+1} \mu^{j-1} (\mu \alpha_{1} + \lambda_{23} \sum_{k=1}^{j} \alpha_{k}) + \alpha_{k} \lambda_{23} \sum_{j=1}^{m} (m\lambda)^{j-1} + \frac{(m\lambda)^{m}}{(s+m\lambda + \lambda_{23})^{m}} \right] \right] \\ &+ \beta_{2} \left[ \sum_{j=1}^{N+1} \mu^{j} \left\{ \frac{1}{(s+\mu + 2\lambda_{23})^{j}} + \frac{\lambda_{23} 1}{(s+\mu + 2\lambda_{23})^{j+1}} \right\} \right] \\ &+ \lambda_{23}^{N+1} \sum_{j=1}^{j-1} \mu^{j-1} \left\{ \frac{1}{(s+\mu + 2\lambda_{23})^{j}} + \frac{2\lambda_{23} 1}{(s+\mu + 2\lambda_{23})^{j+1}} \right\} \sum_{k=1}^{N+1} \lambda_{k} \\ &+ \beta_{2} \alpha_{0} \left[ \lambda_{23}^{m} \sum_{j=1}^{m} (m\lambda)^{j-1} \left\{ \frac{1}{(s+m\lambda + 2\lambda_{23})^{j}} + \frac{2\lambda_{23} 1}{(s+m\lambda + 2\lambda_{23})^{j+1}} \right\} \right] \\ &+ (m\lambda)^{2} \left\{ \frac{1}{(s+m\lambda + 2\lambda_{23})^{m}} + \frac{2\lambda_{23} 1}{(s+m\lambda + 2\lambda_{23})^{m+1}} \right\} \right] \\ &+ \frac{\lambda_{13} \lambda_{23}}{\lambda_{13} + \lambda_{23}} \left[ \beta_{1} \left[ \sum_{j=1}^{N+1} \frac{\mu^{j-2} \sum_{k=1}^{j} \{\mu + \lambda_{23} (k-1+1) \} \alpha_{k}}{(s+\mu + \lambda_{23})^{j}} + \alpha_{0}^{m} \sum_{j=1}^{m} \frac{(m\lambda)^{j-2} \{m\lambda + \lambda_{23} (m-1+1) \}}{i=1} \right] \right] \\ &+ \beta_{2} \left[ 2\lambda_{23} \sum_{j=1}^{n} \frac{\mu^{j-2} \sum_{k=1}^{j} \{\mu + 2\lambda_{23} (k-1+1) \} \alpha_{k}}{(s+\mu + 2\lambda_{23})^{j+1}} + \frac{N+1}{i=1} \frac{\mu^{j-1} \sum_{k=1}^{n} \alpha_{k}}{k=1} \right] \\ &+ \beta_{2} \left[ 2\lambda_{23} \sum_{j=1}^{n} \frac{\mu^{j-2} \sum_{k=1}^{j} \{\mu + 2\lambda_{23} (k-1+1) \} \alpha_{k}}{(s+\mu + 2\lambda_{23})^{j+1}} + \frac{N+1}{i=1} \frac{\mu^{j-1} \sum_{k=1}^{n} \alpha_{k}}{k=1} \right] \\ &+ \beta_{2} \left[ 2\lambda_{23} \sum_{j=1}^{n} \frac{\mu^{j-2} \sum_{k=1}^{n} \{\mu + 2\lambda_{23} (k-1+1) \} \alpha_{k}}{(s+\mu + 2\lambda_{23})^{j+1}} \right] \\ &+ \beta_{2} \left[ 2\lambda_{23} \sum_{j=1}^{n} \frac{\mu^{j-2} \sum_{k=1}^{n} \{\mu + 2\lambda_{23} (k-1+1) \} \alpha_{k}}{(s+\mu + 2\lambda_{23})^{j+1}} \right] \\ &+ \beta_{2} \left[ 2\lambda_{23} \sum_{j=1}^{n} \frac{\mu^{j-2} \sum_{k=1}^{n} \{\mu + 2\lambda_{23} (k-1+1) \} \alpha_{k}}{(s+\mu + 2\lambda_{23})^{j+1}} \right] \\ &+ \beta_{2} \left[ 2\lambda_{23} \sum_{j=1}^{n} \frac{\mu^{j-2} \sum_{k=1}^{n} \{\mu + 2\lambda_{23} (k-1+1) \} \alpha_{k}}{(s+\mu + 2\lambda_{23})^{j+1}} \right] \\ &+ \beta_{2} \left[ 2\lambda_{23} \sum_{j=1}^{n} \frac{\mu^{j-2} \sum_{k=1}^{n} \{\mu + 2\lambda_{23} (k-1+1) \} \alpha_{k}}{(s+\mu + 2\lambda_{23})^{j+1}} \right] \\ &+ \beta_{2} \left[ 2\lambda_{23} \sum_{j=1}^{n} \frac{\mu^{j-2} \sum_{k=1}^{n} \{\mu + 2\lambda_{23} (k-1+1) \} \alpha_{k}}{(s+\mu + 2\lambda_{23})^{j+1}} \right] \\ &+ \beta_{2} \left[ 2\lambda_{23} \sum_{j=1}^{n} \frac{\mu^{j-2} \sum_{k=1}^{n} \{\mu + 2\lambda_{23} (k-1+1) \} \alpha_{k}}{(s+\mu + 2\lambda_{23})^{j+1}} \right] \\ &+ \beta_{2} \left[ 2\lambda_{23} \sum_{j=1}^{n} \frac{\mu^{j-2} \sum_{k=1}^{n} \{\mu + 2\lambda_{23} (k-1+1) \} \alpha_{k}}{($$

(5.36)

 $+ \alpha_{0} \left\{ 2\lambda_{23} \sum_{i=1}^{m} \frac{(m\lambda)^{i-2} i \{m\lambda + 2\lambda_{23} (m-i+1)\}}{(s+m\lambda + 2\lambda_{23})^{i+1}} + \sum_{i=1}^{m} \frac{(m\lambda)^{i-1}}{(s+m\lambda + 2\lambda_{23})^{i}} \right\}$ 

### 5.4 Numerical Results and Considerations

We are now ready to calculate the total delay analytically by using the result obtained in last section. In this section, the quantitative study for the total delay is shown by both simulation and calculation under the same conditions. As mentioned in Section 5.2, simulation is performed on the network shown in Fig.5.1, and similar assumptions as in Chapter 3 are accepted.

We calculate  $\overline{T}_1$  (the delay incurred on the line 1) and  $\overline{T}_2$  (the delay incurred on the line 2) which were shown in Eq. (5.3). The system M/M/1 is applicable to the former, and the system G/M/1 to the latter [5], as pointed out in Section 5.2; hence, they are given by

$$\overline{T}_1 = \frac{1}{\mu(1-\rho_1)}$$
 (5.37)

$$\overline{T}_2 = \frac{1}{\mu(1-\sigma)} \tag{5.38}$$

where  $1/\mu$  is the mean service time of packet,  $\rho_1$  is the utilization factor of the line 1. And  $\sigma$  is the unique root of

$$\sigma = A*(\mu+\mu\sigma)$$
 (5.39)  
(  $(\mu-s)/\mu = A*(s)$  )

in the range  $0 < \sigma < 1$ . Clearly, more regular arrival of packets feeding the line 2 permits  $\sigma$  to be smaller than the utilization of the line 2 ( $\rho_2$ ). The right-hand side of Eq.(5.39), namely A\*(s), being of the (N+2)th degree, Eq.(5.38) is the equation of

the (N+3)th degree. We apply Newton-Raphson method to obtain  $\sigma$  from Eq.(5.38).

The parameters, which we use on our calculation of the total delay  $\overline{T}$ , are the same that are obtained from simulation. We show an example of a set of the parameters in Table 5.1.

Table 5.1 An Example of Parameters Used on Calculation (  $\mathbb{W}_{13} = 9$  )

| μ               | 25.0   | βι | 0.82    | α7              | 0.00291 |
|-----------------|--------|----|---------|-----------------|---------|
| $\rho_1$        | 0.901  | β2 | 0.18    | α8              | 0.00184 |
| λ12             | 11.286 | ao | 0.13785 | α9              | 0.00096 |
| λ13             | 11.307 | α1 | 0.43316 | Q10             | 0.00054 |
| λ23             | 11.275 | α2 | 0.22423 | αη              | 0.00030 |
| N <sub>12</sub> | 108193 | α3 | 0.11012 | α12             | 0.00017 |
| N <sub>13</sub> | 108393 | α4 | 0.05288 | α <sub>13</sub> | 0.00011 |
| N <sub>23</sub> | 108080 | α5 | 0.02879 | 044             | 0,00006 |
|                 |        | α6 | 0.00591 | α <sub>15</sub> | 0.00004 |

Furthermore, we let m be 2 on our calculation. m is the number of the stages  $LC_{13}$  packet proceeds in the  $\alpha_0$  branch, as shown in Fig.5.3 (1). This reason is discussed in Appendix C.

Fig. 5.4 shows total delay caracteristics under a heavy traffic condition ( $\rho$  = 0.901), where the average total delay  $\overline{T}$  is shown as a function of the window size  $W_{13}$ . From simulation result, we can observe that :

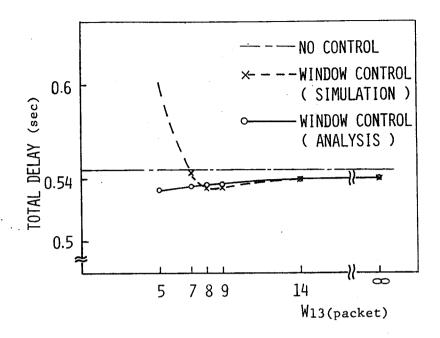


Fig. 5.4 Total Delay Characteristics (  $\rho=0.901$  )

When  $W_{13}$  is set too small,  $\overline{T}$  rises unbounded due to the increase of admission delay for  $LC_{13}$  packets by over control, as pointed out in the previous chapters. As  $W_{13}$  increases,  $\overline{T}$  first reaches a minimum and then gradually grows to be a constant value which is a little smaller than that in no controlled case. Similar result is obtained from an analytical work; ie., as  $W_{13}$  is set smaller,  $\overline{T}$  becomes lower. It is because  $\overline{T}_2$  (the delay incurred on the line 2 by  $LC_{13}$  and  $LC_{23}$  packets) decreases due to Input-Sequencing effect of the window flow control mechanism.

In our analytical study we neglect the increase of the admission delay due to over control, however, at least we clarify by computation that in a properly window flow controlled network

the total delay can be lower than that in no flow controlled network.

## 5.5 Conclusion

In this chapter, we have examined the reason of the phenomenon, that in a properly window flow controlled network the total delay can be lower than that in no flow controlled network.

First, we have describe an outline of Input-Sequencing which implies rearrangement of packet inputting to the network.

Then, we have developed an analytic model for packet arrival process due to Input-Sequencing, and found the Laplace transform for the pdf of the compound interarrival time. And finally we have calculated the total delay.

From numerical result, we have certified that the phenomenon should be due to Input-Sequencing.

## CHAPTER 6

# CONCLUSIONS AND SUGGESTION FOR FURTHER RESEARCH

This research investigated the effect of the window mechanism for flow control in delay system model of computer network.

The main conclusions of this research are as follows:

- (1) The total delay, which consists of the admission delay and the network delay, is a proper measure for performance evaluation in delay system, since the network eventually can pass the offered traffic and constant throughput can be obtained, if the network is properly flow controlled.
- on delay performance in simple delay system model was demonstrated. The phenomenon was found that the total delay in a properly window flow controlled network can be minimized and can
  be lower than that in no flow controlled network. Additionally,
  it was recognized that the network performed most effectively
  when each window size was assigned according to the average
  number of packets outstanding on their logical channel in no
  flow controlled case.
- (3) In Chapter 4, the effect of the window flow control on delay performance in extended network models was demonstrated. In any case, similar phenomenon was found as shown in

Chapter 3. And also it was shown that network performed more effectively by End-to-End flow control procedure than by Link-by-Link flow control procedure on a viewpoint of the total delay, and the price for acknowledgement delay was substantially a reducing window size.

in Chapter 5, the reason of the phenomenon observed in Chapters 3 and 4, was examined. Input-Sequencing of the window mechanism, which implies rearrangement of packets on inputting to the network, was analyzed, and it was certified that the phenomenon should be due to Input-Sequencing.

### Extensions for Further Research

The results obtained from this research will make us recognize the importance of the window mechanism, however, the analytic and quantitative studies for flow control are still in infancy.

#### For the former :

Delay system model of queueing network we focused on is a basic model for computer system, where a concept of wondow is only regarded as one of mechanism for flow control. Hence, if analytic study for the optimal selection of system parameters is given, approaches should appear to extend application of the window mechanism, not only to computer system but also to distributed processing computer system, in the future.

#### For the latter:

Flow control procedure is basically a defence technique against transitional overload, however, throughout this thesis we focused on models under steady traffic condition; hence, we will proceed quantitative study for transient traffic fluctuation, in no distant future.

We hope this research will be added to the understanding for further research.

# APPENDIX A: PROCESS FOR DERIVATION OF EQ.(2.18)

Now we define the following functions, which are the respective components of the 4th, the 5th and the 6th terms in Eq.(2.17).

$$f(z_1) \triangleq \sum_{i=0}^{\infty} P(i,2,0) z_1^{i}$$
 (A.1)

$$g(z_1) \triangleq \sum_{i=0}^{\infty} P(i,1,1)z_1^i$$
 (A.2)

$$h(z_1) \triangleq \sum_{i=0}^{\infty} P(i,0,2) z_1^{i}$$
 (A.3)

By using Eqs.(2.14)-(2.16), we can rewrite Eqs.(A.1)-(A.3) as follows:

$$f(z_1) = \frac{1}{\lambda + \mu - \lambda z_1} \left\{ \frac{\mu}{z_1} g(z_1) + \lambda P(0,1,0) - \frac{\mu}{z_1} P(0,1,1) \right\}$$
(A.4)

$$g(z_1) = \frac{1}{\lambda + 2\mu - \lambda z_1} \left\{ \mu f(z_1) + \frac{\mu}{z_1} (h(z_1) - P(0,0,2)) + \lambda P(0,0,0) \right\}$$
(A.5)

$$h(z_1) = \frac{\mu}{\lambda + \mu - \lambda z_1} g(z_1)$$
(A.6)

Well Eqs. (2.8)-(2.11) turn to be Eqs. (A.7)-(A.10).

$$P(0,0,1) = \frac{\lambda}{11} P(0,0,0)$$
 (A.7)

$$P(0,1,0) = \frac{\lambda(\lambda^2 + 2\lambda\mu + \mu^2)}{2\mu^2(\lambda + \mu)} P(0,0,0)$$
 (A.8)

$$P(0,1,1) = \frac{\lambda^2(\lambda + 2\mu)}{2\mu^3} P(0,0,0)$$
 (A.9)

$$P(0,0,2) = \frac{\lambda^2(\lambda+2\mu)}{2\mu^2(\lambda+\mu)} P(0,0,0)$$
 (A.10)

Solving the simultaneous equations (A.4)-(A.6) and using equations (A.7)-(A.10), we arrive at

$$f(z_1) = \frac{\lambda^2}{2u^2(\lambda+u)} P(0,0,0) \times$$

$$\left\{ \frac{\lambda^{2} (\lambda^{2} + 2\lambda\mu + 2\mu^{2}) z_{1}^{2} - 2\lambda(\lambda^{3} + 4\lambda^{2}\mu + 5\lambda\mu^{2} + 3\mu^{3}) z_{1} + \lambda^{4} + 6\lambda^{3}\mu + 11\lambda^{2}\mu^{2} + 8\lambda\mu^{3} + 4\mu^{4}}{\{\lambda^{2} z_{1}^{2} - \lambda(\lambda + 3\mu) z_{1} + 2\mu^{2}\}(\lambda + \mu - \lambda z_{1})} \right\}$$

(A.11)

$$g(z_1) = \frac{\lambda^2(\lambda + 2\mu - \lambda z_1)}{\mu \{\lambda z_1^2 - \lambda(\lambda + 3\mu) z_1 + 2\mu^2\}} P(0,0,0)$$
(A.12)

$$h(z_1) = \frac{\lambda^2(\lambda + 2\mu - \lambda z_1)}{\{\lambda z_1^2 - \lambda(\lambda + 3\mu) z_1 + 2\mu^2\}(\lambda + \mu - \lambda z_1)} P(0,0,0)$$
(A.13)

Substituting Eqs.(A.7), (A.8) and (A.11)-(A.13) into Eq.(2.17), we can obtain Eq.(2.18).

# APPENDIX B: DISCUSSION ABOUT ASSUMPTION (3) IN SECTION 5.3

Now we discuss the modelling for  $LC_{23}$  packet arrival process. In Fig. B we plot the pdfs of interarrival time  $LC_{23}$  packet feeding line 2 by simulation result and analytic result respectively. We obtain the latter from Eq. (5.24) due to the model shown in Fig.5.3 (2). And notice that parameters, which are shown in Table 5.1, are used.

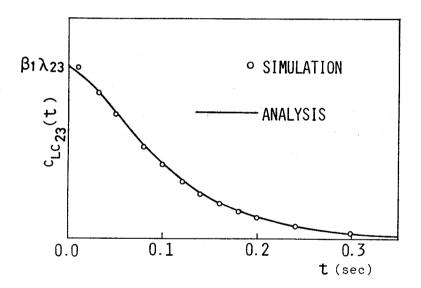


Fig. B The pdf of Interarrival Time  $LC_{23}$  Packet Feeding Line 2 ( $c_{LC_{23}}$ (t))

A comparison between analytic and simulation results shows that our modelling yields very accurate results. This justifies assumption (3) in Section 5.3.

# APPENDIX C: DISCUSSION ABOUT ASSUMPTION (4) IN SECTION 5.3

We consider the packet interarrival time distribution on the condition that line is free.

In Fig. C.1 we show unfinished work U(t) which exists in the system at time t.

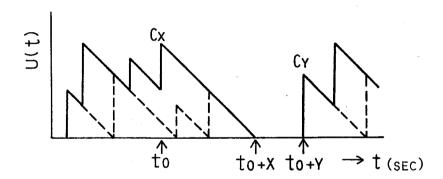


Fig. C.1 Unfinished Work

U(t) jumps up with the arrival of customer X ( $C_X$ ) at the instant  $t_0$ . As time progresses from  $t_0$ , U(t) decreases with slope equal to  $-\mu$  ( $\mu$  is the service rate of the system). And U(t) continues to decrease as the server works on the customers in the system until it reaches the instant ( $t_0^{+X}$ ), at which the server has successfully emptied the system of all customers. This terminates the busy period and initiates a new idle period. This idle period is

terminated with the arrival of customer Y  $(C_{\underline{Y}})$  at the instant  $(t_0^{+\underline{Y}})$ . It is recognized that the situation corresponds to case 0 in Fig. 5.2.

Assuming that the interarrival times and the service times of customers are exponentially distributed with mean  $1/\lambda$  and  $1/\mu$  respectively, we will find the conditional probability

$$P(X \le t \le Y \le t + dt \mid X \le Y)$$

The limiting probability  $\mathbf{p}_{\mathbf{k}}$ , which implies  $\mathbf{k}$  customers in the system, is given by

$$p_k = (1-\rho)\rho^k$$
 (C.1)

where  $\rho$  is the utilization of the system. So the probability  $q_{k,}$  which implies k customers (including  $C_{X}$ ) in the system at the instant  $t_0$  , is given by

$$q_{k} = \begin{cases} p_{k-1} (= (1-\rho) \rho^{k-1}) & (k=1,2,...) \\ 0 & (k=0) \end{cases}$$
 (C.2)

On the other hand, the probability density  $b_k(X)$ , the system of k customers is emptied in just X (sec), is given by

$$b_{k}(X) = \frac{\mu(\mu X)^{k-1}}{(k-1)!} e^{-\mu X}$$
 (C.3)

The pdf b(X) and the PDF  $P(X \le t)$ , for the interval X between

the instant  $t_0$  (the last customer arrives) and the instant ( $t_0+X$ ) (the busy period terminates), are obtained from Eqs.(C.2) and (C.3), as follows:

$$b(X) = \sum_{k=1}^{\infty} q_k b_k(X) = \sum_{k=1}^{\infty} (1-\rho) \rho^{k-1} \frac{\mu(\mu X)^{k-1}}{(k-1)!} e^{-\mu X} = (\mu - \lambda) e^{-(\mu - \lambda)X}$$
(C.4)

$$P(X \le t) = \int_0^t (\mu - \lambda) e^{-(\mu - \lambda)X} dX = 1 - e^{-(\mu - \lambda)t}$$
 (C.5)

On the other hand, the probability, with which the last customer entered the system at some arbitrary time  $t_0$  and the next one enters after t (sec), is given by

$$P(t < Y \leq t + dt) = \lambda e^{-\lambda t} dt$$
 (C.6)

So Eq.(C.7) is obtained from Eqs.(C.5) and (C.6).

$$P(X \le t < Y \le t + dt) = \lambda e^{-\lambda t} (1 - e^{-(\mu - \lambda)t}) dt$$
 (C.7)

Conditioning Eq.(C.7) with the event (X < Y), Eq.(C.7) turns out to be Eq.(C.8)

$$P(X \le t < Y \le t + dt \mid X < Y) = \frac{P(X \le t < Y \le t + dt)}{P(X < Y)}$$
 (C.8)

By the way, P(X < Y) being interpreted as the limiting probability that we observe the system is idle, that is  $(1-\rho)$ , we can arrive at Eq.(C.9).

$$P(X \le t < Y \le t + dt \mid X < Y) = \frac{\lambda}{1 - o} e^{-\lambda t} (1 - e^{-(\mu - \lambda)t}) dt \qquad (C.9)$$

We can obtain the pdf f(t) for the interarrival time on the condition that the system is idle, the mean  $\overline{t}$ , The variance  $\sigma^2$  and the Erlangian degree r , as follows :

$$f(t) = \frac{\mu \lambda}{u - \lambda} (e^{-\lambda t} - e^{-\mu t})$$
 (C.10)

$$\overline{t} = \frac{1}{u} + \frac{1}{\lambda}$$
 (C.11)

$$\sigma^2 = \frac{1}{\mu^2} + \frac{1}{\lambda^2}$$
 (C.12)

$$r = 1 + \frac{2\rho}{1+\rho^2}$$
 (C.13)

Fig. C.2 shows the distribution characteristics of packet interarrival time by Erlangian degree r, where the system is idle. It is obtained from Eq.(C.13). From this figure we can observe that r increases as a function of  $\rho$  ( r=2 when  $\rho=1$  ). This permits us to let m be 2, which is the number of stages on  $\alpha_0$  branch

in Fig. 5.3 (1); so far as we are concerned with heavy traffic condition.

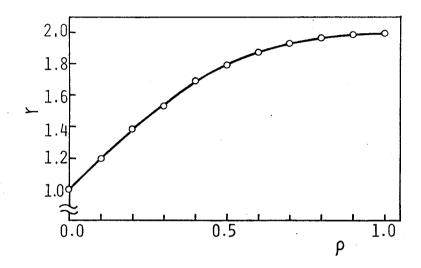


Fig. C.2 Distribution Characteristics of Packet
Interarrival Time by Erlangian Degree
( where system is idle )

# APPENDIX D: DERIVATIONS OF EQUATIONS IN CHAPTER 5

# D.1 <u>Derivation of Eq.(5.16)</u> from Eq.(5.15)

We define the following function, which is a component of  $B_{11}(t)$ .

$$F_1(t) \triangleq e^{-\lambda_{23} t} \left\{ e^{-\mu t} \sum_{i=1}^{N+1} \alpha_{i} \sum_{k=1}^{i} \frac{(\mu t)^{i-k}}{(i-k)!} \right\}$$
 (D.1)

We further define the function as:

$$f_1(t) \triangleq \frac{dF_1(t)}{dt}$$
 (D.2)

f<sub>1</sub>(t) can be obtained as follows:

$$f_{1}(t) = \begin{bmatrix} N+1 \\ \mu \sum_{i=1}^{n} \alpha_{i}^{1} \frac{(\mu t)^{i-k}}{(i-k)!} & \frac{i}{\sum_{k=1}^{n} \frac{(\mu t)^{i-k-1}}{(i-k-1)!}} + \lambda_{23} \sum_{i=1}^{n} \frac{(\mu t)^{i-k}}{(i-k)!} \end{bmatrix} e^{-(\mu + \lambda_{23})t}$$

$$= \begin{bmatrix} N+1 \\ \mu \sum_{i=1}^{n} \alpha_{i}^{1} \frac{(\mu t)^{i-1}}{(i-1)!} + \lambda_{23} \sum_{i=1}^{n} \alpha_{i}^{1} \frac{(\mu t)^{k-1}}{(k-1)!} \end{bmatrix} e^{-(\mu + \lambda_{23})t}$$

$$= \sum_{i=1}^{n+1} \frac{(\mu t)^{i-1}}{(i-1)!} (\mu \alpha_{i}^{1} + \lambda_{23} \sum_{k=1}^{n} \alpha_{k}^{1}) e^{-(\mu + \lambda_{23})t}$$

$$= \sum_{i=1}^{n+1} \frac{(\mu t)^{i-1}}{(i-1)!} (\mu \alpha_{i}^{1} + \lambda_{23} \sum_{k=1}^{n} \alpha_{k}^{1}) e^{-(\mu + \lambda_{23})t}$$

(D.3)

 $f_1(t)$  appears to be the first term of  $b_{11}(t)$  (Eq.(5.16)). Derivation of the other term is omitted, since it can be easily derivated with same process.

## D.2 Derivation of Eq.(5.19) from Eq.(5.18)

We define the following function, which is a component of  $B_{27}(t)$ .

$$F_{2}(t) \triangleq -\sum_{j=0}^{l-1} \frac{(2\lambda_{23} t)^{j}}{j!} e^{-2\lambda_{23} t} \begin{cases} e^{-\mu t} \sum_{k=1}^{N+1} \frac{1}{k!} \frac{(\mu t)^{j-k}}{(i-k)!} \end{cases}$$
 (D.4)

We further define the function as:

$$f_2(t) \triangleq \frac{dF_2(t)}{dt}$$
 (D.5)

f<sub>2</sub>(t) can be obtained as follows:

$$f_{2}(t) = \begin{bmatrix} N+1 \\ \sum_{i=1}^{\alpha} i \\ 1 \end{bmatrix} \left( \frac{1-1}{2\lambda_{23}} \sum_{j=0}^{2-1} \frac{(2\lambda_{23} t)^{j}}{j!} \frac{\sum_{i=1}^{j} \frac{(\mu t)^{i-k}}{(i-k)!}}{\sum_{j=0}^{2} \frac{(2\lambda_{23} t)^{j-1}}{j!} \frac{\sum_{i=1}^{2} \frac{(\mu t)^{i-k}}{(\mu t)!}} \right)$$

$$- \frac{1 - 1(2\lambda_{23} t)^{\frac{1}{3}}}{\sum_{j=0}^{3} \frac{1}{j!} \sum_{k=1(i-k-1)!}^{(\mu t)^{1-k-1}}} e^{-(\mu + 2\lambda_{23})t}$$

$$= \begin{bmatrix} N+1 & \sum_{i=1}^{l} \alpha_i & \sum_{j=0}^{l-1} \frac{(2\lambda_{23} t)^{j} (\mu t)^{i-1}}{j!} + 2\lambda_{23} \frac{(2\lambda_{23} t)^{l-1}}{(l-1)!} & \sum_{k=1(k-1)!}^{i} \frac{(\mu t)^{k-1}}{k!} \end{bmatrix} e^{-(\mu+2\lambda_{23})t}$$

$$= e^{-(\mu+2\lambda_{23})t} \begin{cases} \sum_{i=1}^{N+1} \alpha_i \mu^i & \sum_{j=1}^{l} \frac{(2\lambda_{23})^{j-1}t^{j+i-2}}{(j-1)!} + \frac{(2\lambda_{23})^{l}}{(l-1)!} & \sum_{i=1}^{l} \alpha_i \sum_{k=1}^{i} \frac{\mu^{k-1}t^{k+l-2}}{(k-1)!} \end{cases}$$

(D.6)

 $f_2(t)$  appears to be the first half of  $b_{2l}(t)$  (Eq.(5.19)). Derivation of the other is omitted.

## D.3 Derivation of Eq.(5.22) from Eq.(5.20)

We define the following function, which is a component of  $B_{21}^*(s)$ .

$$F_{3}(s) \triangleq \frac{(2\lambda_{23})^{l} \sum_{i=1}^{l} \alpha_{i} \sum_{k=1}^{i} \frac{\mu^{k-1}(k+l-2)!}{(k-1)!(s+\mu+2\lambda_{23})^{k+l-1}}$$
(D.7)

By summing  $F_3(s)$  with l,  $f_3(s)$  which is a component of B\*(s) appears to be as follows:

$$f_{3}(s) = \sum_{l=1}^{2} \frac{(2\lambda_{23})^{2}N+1}{(l-1)!} \sum_{i=1}^{1} \alpha_{i} \sum_{k=1}^{i} \frac{\mu^{k-1}(k+l-2)!}{(k-1)!(s+\mu+2\lambda_{23})^{k+l-1}}$$

$$= \sum_{i=1}^{N+1} \alpha_{i} \sum_{k=1}^{i} \frac{\mu^{k-1}}{(k-1)!(s+\mu+2\lambda_{23})^{k}} \sum_{l=1}^{2} \frac{(2\lambda_{23})^{l}(k+l-2)!}{(l-1)!(s+\mu+2\lambda_{23})^{l-1}}$$

$$= \sum_{i=1}^{N+1} \alpha_{i} \sum_{k=1}^{i} \frac{\mu^{k-1}}{(k-1)!(s+\mu+2\lambda_{23})^{k}} \left\{ 2\lambda_{23}(k-1)! + \frac{(2\lambda_{23})^{2}k!}{s+\mu+2\lambda_{23}} \right\}$$

$$= \sum_{i=1}^{N+1} \alpha_{i} \sum_{k=1}^{i} \frac{\mu^{k-1}}{(k-1)!(s+\mu+2\lambda_{23})^{k}} \left\{ 2\lambda_{23}(k-1)! + \frac{(2\lambda_{23})^{2}k!}{s+\mu+2\lambda_{23}} \right\}$$

$$= \sum_{i=1}^{N+1} \alpha_{i} \sum_{k=1}^{i} \frac{\mu^{k-1}}{(k-1)!(s+\mu+2\lambda_{23})^{k}} \left\{ 2\lambda_{23}(k-1)! + \frac{(2\lambda_{23})^{2}k!}{s+\mu+2\lambda_{23}} \right\}$$

$$= \sum_{i=1}^{N+1} \alpha_{i} \sum_{k=1}^{i} \frac{\mu^{k-1}}{(k-1)!(s+\mu+2\lambda_{23})^{k}} \left\{ 2\lambda_{23}(k-1)! + \frac{(2\lambda_{23})^{2}k!}{s+\mu+2\lambda_{23}} \right\}$$

$$= \sum_{i=1}^{N+1} \alpha_{i} \sum_{k=1}^{i} \frac{\mu^{k-1}}{(k-1)!(s+\mu+2\lambda_{23})^{k}} \left\{ 2\lambda_{23}(k-1)! + \frac{(2\lambda_{23})^{2}k!}{s+\mu+2\lambda_{23}} \right\}$$

$$= \sum_{i=1}^{N+1} \alpha_{i} \sum_{k=1}^{i} \frac{\mu^{k-1}}{(k-1)!(s+\mu+2\lambda_{23})^{k}} \left\{ 2\lambda_{23}(k-1)! + \frac{(2\lambda_{23})^{2}k!}{s+\mu+2\lambda_{23}} \right\}$$

$$= \sum_{i=1}^{N+1} \alpha_{i} \sum_{k=1}^{i} \frac{\mu^{k-1}}{(k-1)!(s+\mu+2\lambda_{23})^{k}} \left\{ 2\lambda_{23}(k-1)! + \frac{(2\lambda_{23})^{2}k!}{s+\mu+2\lambda_{23}} \right\}$$

$$= \sum_{i=1}^{N+1} \alpha_{i} \sum_{k=1}^{i} \frac{\mu^{k-1}}{(k-1)!(s+\mu+2\lambda_{23})^{k}} \left\{ 2\lambda_{23}(k-1)! + \frac{(2\lambda_{23})^{2}k!}{s+\mu+2\lambda_{23}} \right\}$$

Then, we may interchange the order of summation for the double sum such that.

$$\begin{array}{ccc}
N+1 & \mathbf{i} \\
\sum & \sum \\
\mathbf{i}=1 & \mathbf{k}=1
\end{array} = 
\begin{array}{c}
N+1 & N+1 \\
\sum & \sum \\
\mathbf{k}=1 & \mathbf{i}=\mathbf{k}
\end{array}$$

and exchange subscript i for k.

$$f_{3}(s) = \lambda_{23} \sum_{i=1}^{N+1} \mu^{i-1} \left\{ \frac{1}{(s+\mu+2\lambda_{23})^{i}} + \frac{2\lambda_{23} i}{(s+\mu+2\lambda_{23})^{i+1}} \right\}_{k=i}^{N+1} \alpha_{k}$$
(D.9)

 $f_3(s)$  turns out to be the first half of the second term in Eq. (5.22). We omit derivations of the other terms, since they can easily derivated with the same process as mentioned above.

# D.4 Derivation of Eq.(5.29) from Eq.(5.28)

$$C_{ik}(t) = 1 - \frac{\alpha_i}{\mu \Lambda} e^{-\mu t} \left\{ \beta_1 e^{-\lambda_{23} t} + \beta_2 (1 + 2\lambda_{23} t) e^{-2\lambda_{23} t} \sum_{j=0}^{k-1} \frac{(\mu t)^j}{j!} \right\}$$
(5.28)

Differentiating  $C_{ik}(t)$  (Eq.(5.28)) with respect to t, we find  $c_{ik}(t)$  (Eq.(5.29)), as follows:

$$c_{ik}(t) = \frac{\alpha_{i}}{\mu \hbar} \left[ \beta_{1} e^{-(\mu + \lambda_{23})t} \left\{ (\mu + \lambda_{23}) \sum_{j=1}^{k} \frac{(\mu t)^{j-1}}{(j-1)!} - \mu \sum_{j=1}^{k-1} \frac{(\mu t)^{j-1}}{(j-1)!} \right\} \right]$$

$$+ \beta_{2}e^{-(\mu+2\lambda_{23})t} \left[ \{ (\mu+2\lambda_{23})(1+2\lambda_{23}t)-2\lambda_{23} \} \underbrace{\sum_{j=1(j-1)!}^{k} (\mu t)^{j-1}}_{j=1(j-1)!} -\mu(1+2\lambda_{23}t) \underbrace{\sum_{j=1(j-1)!}^{k-1} (\mu t)^{j-1}}_{j=1(j-1)!} \right]$$

$$= \frac{\alpha_{i}}{\mu \Lambda} \left[ \beta_{1} e^{-(\mu + \lambda_{23})t} \left\{ \lambda_{23} \sum_{j=1(j-1)!}^{k} \frac{(\mu t)^{j-1}}{(k-1)!} + \frac{\mu(\mu t)^{k-1}}{(k-1)!} \right\} \right]$$

$$+ \beta_{2}e^{-(\mu+2\lambda_{23})t}\left\{(2\lambda_{23})^{2}t\sum_{j=1(j-1)!}^{k}(\mu t)^{j-1}+\frac{\mu(1+2\lambda_{23}t)(\mu t)^{k-1}}{(k-1)!}\right\}$$

(5.29)

## D.5 Derivation of Eq. (5.35) from Eq. (5.30)

We define the following function, which is a component of  $C^*_{ik}(s)$  (Eq.(5.30)).

$$\mathbf{F_{4}(s)} \triangleq \frac{\alpha_{1}}{\mu \Lambda} \beta_{2} \left\{ 2\lambda_{23} \right\}^{2} \sum_{j=1}^{k} \frac{\mu^{j-1} j}{(s+\mu+2\lambda_{23})^{j+1}} + \frac{2\lambda_{23} \mu^{k} k}{(s+\mu+2\lambda_{23})^{k+1}} + \frac{\mu^{k}}{(s+\mu+2\lambda_{23})^{k}} \right\}$$
(D.10)

By summing  $F_4(s)$  with respect to i and k, we can obtain  $f_4(s)$ , which is a component of C\*(s) (Eq.(5.35)), as follows:

$$f_4(s) =$$

$$\beta_{2}^{N+1} = \frac{1}{1} \frac{\alpha_{1}}{\mu \Lambda} \left\{ 2\lambda_{23} \right\}^{2} \frac{\sum_{j=1}^{k} \mu^{j-1} j}{(s+\mu+2\lambda_{23})^{j+1}} + \frac{2\lambda_{23} \mu^{k} k}{(s+\mu+2\lambda_{23})^{k+1}} + \frac{\mu^{k}}{(s+\mu+2\lambda_{23})^{k}} \right\}$$
(D.11)

By interchanging the order of summation for the double sum such that

we obtain

$$f_{4}(s) = \frac{\beta_{2} \sum_{\mu \Lambda}^{N+1} \sum_{i=k}^{N+1} \left\{ (2\lambda_{23})^{2} \sum_{j=1}^{k} \frac{\mu^{j-1} j}{(s+\mu+2\lambda_{23})^{j+1}} + \frac{2\lambda_{23} \mu^{k} k}{(s+\mu+2\lambda_{23})^{k+1}} + \frac{\mu^{k}}{(s+\mu+2\lambda_{23})^{k}} \right\}}$$
(D.12)

We find

$$f_4(s) =$$

$$\frac{\beta_{2}}{\mu \Lambda} \sum_{j=1}^{N+1} \left\{ \frac{(2\lambda_{23})^{2} \mu^{j-1} j}{(s+\mu+2\lambda_{23})^{j+1} k} \sum_{k=j}^{N+1} \sum_{i=k}^{N+1} + \frac{2\lambda_{23} \mu^{j-1} j}{(s+\mu+2\lambda_{23})^{j+1}} \sum_{i=j}^{N+1} + \frac{\mu^{j}}{(s+\mu+2\lambda_{23})^{j}} \sum_{i=j}^{N+1} k \sum_{j=j}^{N+1} \sum_{k=j}^{N+1} \frac{\lambda_{23} \mu^{j-1} j}{(s+\mu+2\lambda_{23})^{j}} \sum_{k=j}^{N+1} \frac{\lambda_{23} \mu^{j-1} j}{(s+\mu+2\lambda_{23})^{j}} \sum_{i=j}^{N+1} \frac{\lambda_{23} \mu^{j-1} j}{(s+\mu+2\lambda_{23})^{j}} \sum_{j=j}^{N+1} \frac{\lambda_{23} \mu^{j-1} j}{(s+\mu+2\lambda_{23})^{j}} \sum_{i=j}^{N+1} \frac$$

$$= \frac{1}{\Lambda} \left[ \beta_{2} 2 \lambda_{23} \sum_{j=1}^{N+1} \left\{ \frac{\mu^{j-2} 2 \lambda_{23} j}{(s+\mu+2\lambda_{23})^{j+1}} \sum_{i=j}^{N+1} (i-j+1) \alpha_{i} + \frac{\mu^{j-2} \mu j}{(s+\mu+2\lambda_{23})^{j+1}} \sum_{i=j}^{N+1} \alpha_{i} \right] + \beta_{2} \sum_{j=1}^{N+1} \frac{\mu^{j-1}}{(s+\mu+2\lambda_{23})^{j}} \sum_{i=j}^{N+1} \alpha_{i} \right]$$

$$(D.13)$$

Then, exchanging subscripts j→i and i→k, we arrive at

$$f_{4}(s) = \frac{\beta_{2}}{\Lambda} \left[ \begin{cases} N+1 & \mu^{i-2} i \sum_{k=i}^{N+1} \{\mu + (k-i+1)\lambda_{23}\} \alpha_{k} \\ 2\lambda_{23} \sum_{i=1}^{N+1} & k=i \end{cases} + \sum_{i=1}^{N+1} \frac{\mu^{i-1} \sum_{k=i}^{i+1} \alpha_{k}}{(s+\mu+2\lambda_{23})^{i+1}} \right]$$
(D.14)

 $f_4(s)$  appears to be the first half of the second term in Eq.(5.35). We omit derivations of the other terms, since they can be easily derivated with the same process.

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