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# Energy Gaps in Fractional Quantum Hall States

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**Abstract.** Energy values versus filling factors  $\nu$  are examined for fractional quantum Hall states (FQHS). First, the classical Coulomb energy of nearest electron pairs in FQHS is shown to be linearly dependent upon  $1/\nu$ . The residual Coulomb interaction produces quantum transitions. Examination of second order transitions indicates a discrepancy between the second order perturbation energy at  $\nu=2/3$  and the limiting energy value at  $\nu=(2s+1)/(3s+1)$  for an infinitely large value of  $s$ . Accordingly, an energy gap appears at  $\nu=2/3$ . On the other hand, the second order perturbation energy  $\varepsilon_2(\nu=3/4)$  is equal to the limiting energy value at  $\nu=(3s+1)/(4s+1)$  for an infinitely large  $s$ . Therefore, the energy spectrum is continuous near  $\nu=3/4$ , meaning that it is gapless at  $\nu=3/4$ . The same mechanisms appear in higher order perturbation calculations because the number of forbidden transitions in the higher order calculation is equal to that in the second order calculation. That is to say, the gap and gapless mechanisms can be extended to higher order calculations. In fractional filling factors other than  $\nu=2/3$  and  $3/4$ , either a gap mechanism or a gapless mechanism appears for each filling factor. Consequently, our results can theoretically explain the precise confinement of Hall resistance at fractional filling factors with a gap mechanism.

## 1. Introduction

Many local minima of diagonal resistivity  $\rho_{xx}$  have been identified in ultra-high-mobility samples at filling factors  $\nu=3/8, 3/10, 4/11, 4/13, 5/13, 5/17, 6/17$ , and so on [1, 2]. These states cannot be understood using the traditional theories [3–7]. Accordingly, many physicists have proposed their extended models [8–10]. Far fewer calculations have been made of binding energies or energy spectra for fractional quantum Hall states. Therefore, details of the energy spectrum are unknown. We calculate the binding energies using an improved method of Tao and Thouless [11]. According to those results, a gap mechanism or a gapless mechanism appears in the energy spectrum of FQHS.

The electron system in a quantum Hall device obeys the Hamiltonian  $H_0$ , as

$$H_0 = (\mathbf{p} + e\mathbf{A})^2 / (2m) + U(y) + W(z), \quad \mathbf{A} = (-yB, 0, 0), \quad (1)$$

when the Coulomb interactions among electrons are neglected. Therein,  $y$  and  $z$  respectively denote the directions of Hall voltage and magnetic field;  $U(y)$  and  $W(z)$  are confining potentials. The ground states of  $H_0$  are described with  $\psi_k$  as

$$H_0\psi_k = E_0\psi_k, \quad \psi_k \equiv \psi_k(x, y, z) = ue^{ikx}e^{-\alpha(y-c)^2}\phi(z), \quad \left[-\left(\hbar^2/(2m)\right)\partial^2/\partial z^2 + W(z)\right]\phi(z) = \lambda\phi(z), \quad (2)$$

$$E_0(k) = \lambda + U(c) + \hbar eB/(2m) = b + \hbar eB/(2m), \quad c \approx k\hbar/(eB), \quad (3)$$

where  $c$  indicates the centre position of the electron in the  $y$  direction. We consider the Coulomb interaction between many electrons, and then obtain the total Hamiltonian  $H_T$  of many electrons and the first-order wave function  $\Psi(k_1, \dots, k_N)$  as the following.

$$H_T = \sum_{i=1}^N H_0(x_i, y_i, z_i) + H_C, \quad H_C = \sum_{i=1}^{N-1} \sum_{j>i}^N e^2 / \left( 4\pi\epsilon \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2} \right) \quad (4)$$

$$\Psi(k_1, \dots, k_N) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \psi_{k_1}(x_1, y_1, z_1) & \dots & \psi_{k_1}(x_N, y_N, z_N) \\ \vdots & & \vdots \\ \psi_{k_N}(x_1, y_1, z_1) & \dots & \psi_{k_N}(x_N, y_N, z_N) \end{vmatrix} \quad (5)$$

The sum of the single electron energies and the classical Coulomb energies is

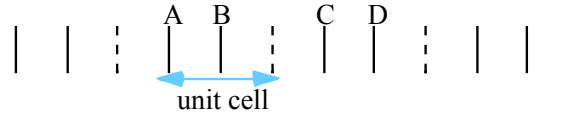
$$W(k_1, \dots, k_N) = \sum_{i=1}^N E_0(k_i) + C(k_1, \dots, k_N), \quad (6)$$

where  $C(k_1, \dots, k_N) = \int \dots \int \Psi(k_1, \dots, k_N)^* H_C \Psi(k_1, \dots, k_N) dx_1 dy_1 dz_1 \dots dx_N dy_N dz_N$ .

The total Hamiltonian  $H_T$  is divisible into the diagonal component  $H_D$  and the off-diagonal component  $H_I$ , as in reference [12]:

$$H_D = \sum_{k_1, \dots, k_N} |\Psi(k_1, \dots, k_N)\rangle W(k_1, \dots, k_N) \langle \Psi(k_1, \dots, k_N)|, \text{ and } H_I = H_T - H_D. \quad (7)$$

Figure 1 shows the most uniform distribution at  $\nu = 2/3$ . The straight lines indicate Landau orbitals filled with electrons and the dotted lines indicate empty Landau orbitals. We describe the classical Coulomb energy as  $\xi$  for the nearest electron pair AB, and that of the second-nearest pair BC by  $\eta$ .



**Figure 1.** Most uniform distribution of electrons at  $\nu = 2/3$ .

For the more general case of  $\nu = p/q$  ( $\frac{1}{2} < \nu < 1$ ),  $p$  electrons exist and  $q - p$  orbitals are empty in a unit cell. Considering the condition  $\frac{1}{2} < \nu < 1$ , there are  $q - p$  second-nearest electron pairs for each unit cell. These pairs have Coulomb energy of  $\eta$  per pair. The number of the nearest electron pairs is  $2p - q$ . They have energy  $\xi$  per pair. Consequently,  $C(k_1, \dots, k_N)$  at  $\nu = p/q$  ( $\frac{1}{2} < \nu < 1$ ) is nearly equal to

$$C(\nu = p/q) \approx (2p - q)\xi N/p + (q - p)\eta N/p. \quad (8)$$

Therein, we neglect  $n$ -th order pairs ( $n \geq 3$ ) because the other electrons are sandwiched between the electron pair (these higher order energies are small because of the screening effect). According to Eqs. (3), (6) and (8), the energy of diagonal part  $H_D$  becomes

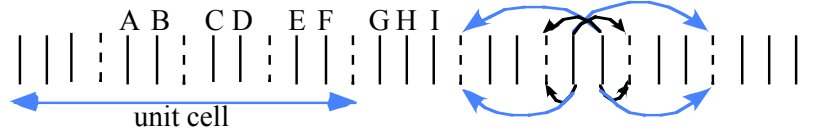
$$\begin{aligned} W(\nu = p/q) &\approx (b + \hbar eB/(2m))N + (2\xi - \eta)N - (\xi - \eta)Nq/p \\ W/N &\approx \beta + \hbar eB/(2m) - (\xi - \eta)/\nu, \text{ where } \beta = b + (2\xi - \eta). \end{aligned} \quad (9)$$

Consequently, the classical Coulomb energy per electron depends linearly on  $1/\nu$ .

## 2. Energy Gaps

The second order and higher order perturbation energies are examined in this section. Transitions via  $H_I$  should satisfy momentum conservation of the  $x$  direction. Therefore, the sum of two wave numbers  $k_1$  and  $k_2$  before the transition is equal to the sum of  $k'_1$  and  $k'_2$  after the transition, as  $k'_1 + k'_2 = k_1 + k_2$ . The centre positions for each electron are  $c_1$  and  $c_2$  before the transition. They

become  $c'_1$  and  $c'_2$  after the transition. When position  $c_1$  is transferred to the fourth orbital to the left, position  $c_2$  is transferred to the fourth orbital to the right because of momentum conservation and relation (3).



**Figure 2.** Most uniform distribution of electrons at  $\nu = 9/13$

We examine fractional quantum Hall states with the filling factor of  $(4s+1)/(6s+1)$ . The electron configuration of the ground state in  $H_D$  is drawn schematically in figure 2 for  $s=2$ , i.e.  $\nu = (4s+1)/(6s+1) = 9/13$ . The quantum transitions via  $H_1$  are shown using arrow pairs. Nine electrons are in each unit cell. The nearest electron pairs (AB), (CD), and (EF) can transfer to other empty orbitals, but nearest electron pairs (GH) and (HI) cannot transfer to other empty orbitals because of the Pauli exclusion principle. It is noteworthy here that all spins of electrons have the same direction under strong magnetic field strength. We introduce an integral value  $Z_2$  as

$$Z_2 = - \sum_{\Delta k \neq 0, 2\pi/\ell} \frac{\langle k_1, k_2 | H_1 | k'_1, k'_2 \rangle \langle k'_1, k'_2 | H_1 | k_1, k_2 \rangle}{W_G - W_{\text{excite}}(k_1 \rightarrow k'_1, k_2 \rightarrow k'_2)}. \quad (10)$$

Then the second order perturbation energy of the electron pair (CD) becomes  $-Z_2 \times (4/13)$ ; in addition, that of pair (AB) or (EF) becomes  $-Z_2 \times (2/13)$  because the interval value between transfer momenta is  $2\pi\hbar/\ell$ , which is extremely small. Pairs (GH) and (HI) have zero value for the second order perturbation energy because all the quantum transitions are forbidden. A new integral value,  $Z_3$ , is

$$Z_3 = - \sum_{\Delta k' \neq 0, 2\pi/\ell} \sum_{\Delta k'' \neq 0, 2\pi/\ell} \frac{\langle k_1, k_2 | H_1 | k'_1, k'_2 \rangle \langle k'_1, k'_2 | H_1 | k''_1, k''_2 \rangle \langle k''_1, k''_2 | H_1 | k_1, k_2 \rangle}{(W_G - W_{\text{excite}}(k_1 \rightarrow k'_1, k_2 \rightarrow k'_2)) (W_G - W_{\text{excite}}(k'_1 \rightarrow k''_1, k'_2 \rightarrow k''_2))}. \quad (11)$$

Then, the third order perturbation energy of electron pair (CD) becomes  $-Z_3(4/13)^2$ , and that of the pair of (AB) or (EF) is  $-Z_3(2/13)^2$ . The higher order perturbation energies are calculable; their sums are

$$E_{\text{pairCD}} = - \sum_{n=2}^{\infty} Z_n \times (4/13)^{n-1}, \quad E_{\text{pairAB}} = - \sum_{n=2}^{\infty} Z_n \times (2/13)^{n-1}, \quad E_{\text{pairGH}} = 0. \quad (12)$$

Next, we examine perturbation energies for the other filling factors, i.e.  $\nu = (4s+1)/(6s+1)$  with a large value of  $s$ . In this case, the second order perturbation energy takes various values for various electron pairs within a unit cell as follows:

$$-Z_2 \times (2/(6s+1)), -Z_2 \times (4/(6s+1)), -Z_2 \times (6/(6s+1)), \dots, -Z_2 \times (2s/(6s+1)).$$

There are two pairs for  $-Z_2 \times (2/(6s+1))$ , ..., but one pair for  $-Z_2 \times (2s/(6s+1))$ . Consequently, the sum of the energies is  $-2Z_2 \times (2/(6s+1)) - 2Z_2 \times (4/(6s+1)) - \dots - Z_2 \times (2s/(6s+1)) = -Z_2 \times (2s^2/(6s+1))$ .

Because the number of electrons is  $(4s+1)$  in a unit cell, the energy per electron is

$$\varepsilon_2(\nu = (4s+1)/(6s+1)) = -Z_2 \times 2s^2 / ((6s+1)(4s+1)) \xrightarrow{s \rightarrow \infty} -Z_2/12. \quad (13)$$

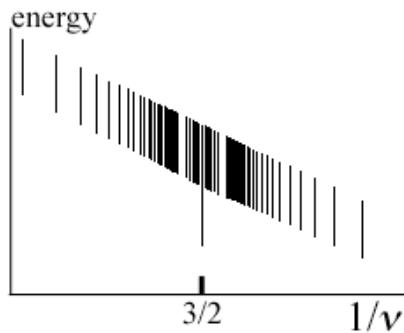
We can also obtain the perturbation energy per electron at the filling factor of  $2/3$  as

$$\varepsilon_2(\nu = 2/3) = -Z_2/6. \quad (14)$$

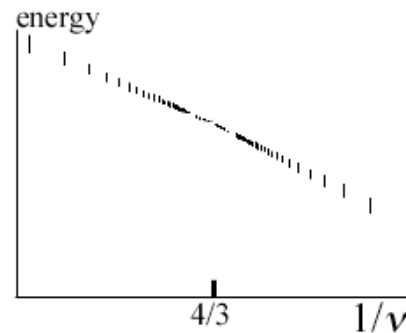
Considering the energy  $W/N$  in Eq. (9), the energy per electron  $\varepsilon(\nu)$  is described as

$$\varepsilon(\nu) = \beta + \hbar e B / (2m) - (\xi - \eta) / \nu + \varepsilon_2(\nu) + \varepsilon_3(\nu) + \dots. \quad (15)$$

This function  $\varepsilon(\nu)$  is discontinuous at  $\nu = 2/3$ ; therefore, an energy gap appears. Similar gaps appear at  $\nu = 4/5, 3/5, 2/5, 1/5$ , and so on. On the other hand, the function  $\varepsilon(\nu)$  is continuous at  $\nu = 3/4, 1/4, 5/6, 1/6$ , and so on, where the energy gap disappears. We draw the function near  $\nu = 2/3$  and  $\nu = 3/4$  in figures 3a) and 3b), respectively, where we have abbreviated many lines of binding energy near the centre of each figure.



**Figure 3a.** Gap mechanism at  $\nu = 2/3$ .



**Figure 3b.** Gapless mechanism at  $\nu = 3/4$ .

### 3. Conclusion

Hitherto, few calculations of energy spectrum for FQHS have been made. Halperin's result shows many cusps in the energy spectrum curve [5]. We examined the energy spectrum for this study and obtained a gap mechanism for  $\nu = 2/3, 1/3, 4/5, 3/5, 2/5, 1/5 \dots$  and a gapless mechanism for  $\nu = 3/4, 1/4, 5/6, 1/6, 7/8, 1/8 \dots$ . The gap mechanism produces precise confinement of Hall resistance because FQHS with only one filling factor is stable; the other states disappear because of the energy gap. Moreover, the gapless mechanism produces Hall resistance that is almost proportional to the magnetic field strength near the filling factor. The results described herein show good agreement with experimental data.

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