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Determination of the Coupling Constant $g_{NN\eta}$

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At present, available information on η-N interactions can be obtained only*) from the process $\pi^-+p\to\eta+n$. In this note we determine the coupling constant $g_{NN\eta}$ using the dispersion relation of the amplitudes for the η-production.

The dispersion relation includes an integral over a wide range of energy from the low energy region below the η-N threshold to the high energy region.**) In order to avoid the unknown parts involved in this integral, we consider the dispersion relation for partial wave amplitudes. Among various partial waves, we exclusively consider a particular one for which the contribution from the integral becomes negligibly small as compared with that from the Born term. The P_{13} wave is appropriate for this purpose in the reaction $\pi^-+p\to\eta+n$ according to the results of a preceding paper.[1]

The following dispersion relation for the P_{13} wave is obtained in the same way as in the paper of C.G.L.N.[2] which is valid near the ηN threshold:

$$\frac{f_{P_{13}}}{q_1q_2}=\frac{f_{P_{13}}^{\rm Born}}{q_1q_2}+\frac{2}{3}\frac{1}{4\pi}\frac{\sqrt{(E_1+M)(E_2+M)}}{2W}\frac{1}{\pi}$$
$$\times\Big[\int_{(M+m_\pi)^2}^{\infty}[\mathrm{Im}\{A^{(1)}(s',t_0)+(W-M)B^{(1)}(s',t_0)\}/(s'-s-i\epsilon)]ds'$$
$$+\int_{(M+m_\pi)^2}^{\infty}[\mathrm{Im}\{A^{(1)}(s',t_0)+(W-M)B^{(1)}(s',t_0)\}/(s'-u)]ds'$$
$$-\int_{(M+m_\pi)^2}^{\infty}[\mathrm{Im}\{A(s',t_0)+(W-M)B(s',t_0)\}/(s'-u)^2]ds'\Big],\qquad(1)$$

*) There are experiments for the process $\gamma+p\to\eta+p$. However, the experimental data were obtained for only one particular angle with very large errors.

**) Experiments for the reaction $\pi^-+p\to\eta+n$ were performed only for the energy range from the threshold to 1.3 BeV.

where q_1, q_2 are the initial and final momenta, and E_1, E_2 are the nucleon energies. A and B are the invariant amplitudes for the reaction $\pi+N\to\eta+N$ as defined in reference 2) and $A^{(1)}$, $B^{(1)}$ are the derivatives of A, B with respect to the square of momentum transfer. W, M, s, t_0 and u are respectively the total energy, nucleon mass, W^2, square of momentum transfer at the threshold and $u=-s-t_0+2M^2+m_\pi^2+m_\eta^2$. $f_{P_{13}}^{\mathrm{Born}}$ is the Born amplitude and has the following form in the static limit:

$$f_{P_{13}}^{\mathrm{Born}}=\frac{-1}{\sqrt{3}}\frac{g_{NN\pi}g_{NN\eta}}{4\pi}\frac{\sqrt{(E_1+M)(E_2+M)}}{2W} \times(W-M)\frac{2q_1q_2}{(2E_1\omega_\eta-m_\eta^2)^2}\,. \tag{2}$$

In reference 1), it was shown that the η-production amplitudes have two types (Cases I and II) of the solutions. First, we discuss Case I. In this case the S_{11} and P_{11} waves have large imaginary parts. However, these waves contribute only to the third integral in Eq. (1) in which the denominator is equal to $(s'-u)^2$. Since the denominator has a large value, the contribution of this integral is negligibly small as compared with the l.h.s. of Eq. (1). In fact, the numerical value for the energy range 561∼1300 MeV is equal to -0.004*) (see Table I). Below the ηN threshold, the η-production amplitudes have no enhancement except for the P_{11} (1400 MeV) resonance. This contribution, however, is at most about a few per cent of the magnitude of l.h.s. of Eq. (1), when the integral is estimated by means of the extrapolation of physical amplitudes.

Table I. Numerical value of f_{P13}/q_1q_2 and the dispersion integrals in Eq. (1). (unit: η meson mass)

$f_{P13}/q_1q_2=-0.75^{-0.15}_{+0.07}$

Kinetic energy (MeV)	First integral	Sum of second and third integrals
0∼561	0∼0.01	−0.005∼−0.002
561∼1300		−0.004
above 1300	|absolute value|<0.005	|absolute value|<0.001

*) This value is estimated by using the partial wave amplitudes of the solution 3 in reference 1).

Consequently, the coupling constant $g_{NN\eta}$ can be determined from the value $f_{P_{13}}/q_1q_2$ for which we have, fortunately, a reliable value obtained by the analysis of reference 1). These considerations lead to the final result:

$$g_{NN\eta}^2/4\pi=4.1^{+1.8}_{-0.7}. \tag{3}$$

Secondly, $g_{NN\eta}^2$ for Case II in reference 1) is roughly estimated in the same method as for Case I, and we obtain $g_{NN\eta}^2/4\pi=0.4$. This value is too small to reproduce the enhancement of the η-production cross section near threshold. We, therefore, abandon this case and adopt Eq. (3) as the correct value $g_{NN\eta}^2$.

From $SU(3)$ symmetry,

$$\frac{g_{NN\eta}^2}{4\pi}=\frac{1}{3}(1-4f)^2\frac{g_{NN\pi}^2}{4\pi}\,. \tag{4}$$

By using $f=0.41\pm0.07$[3] and $g_{NN\pi}^2/4\pi=14.5\pm0.4$, the calculated value of $g_{NN\eta}^2/4\pi=2.0$. This value is a half of experimental value given by Eq. (3). Thus pure octet model does not agree with the experimental result, suggesting the necessity of η-χ mixing. The physical mesons η and χ are expressed by the mixing of singlet meson η_0 and octet meson η_8 as follows:

$$\begin{aligned}\eta&=\eta_8\cos\theta+\eta_0\sin\theta\,,\\ \chi&=-\eta_8\sin\theta+\eta_0\cos\theta\,.\end{aligned} \tag{5}$$

The coupling constants for these η_0 and η_8 mesons are determined by nonet model, where the strong interactions between baryons and mesons are derived from the following interaction Hamiltonian (the γ_5

matrix is abbreviated):

$$H_I = \sqrt{2} g_{NN\pi} \mathrm{Tr}[\{(f+d)\bar{B}B + (d-f)B\bar{B}\}$$
$$\times (M + (1/\sqrt{3})\eta_0 E)]. \qquad (6)$$

B, M are matrices of baryon octet and meson octet respectively, and E is a unit matrix. From these Eqs. (5) and (6), the value of $g_{NN\eta}$ is given by

$$g_{NN\eta} = (g_{NN\eta}/\sqrt{3})[(4f-1)\cos\theta$$
$$+2\sqrt{2}(1-f)\sin\theta]. \qquad (7)$$

Using $\theta = 10.4°$ determined[4] by the mass formula, we have

$$g^2_{NN\eta}/4\pi = 4.2, \qquad (8)$$

which agrees with the experimental result. The nonet model seems to be consistent with the value of $g_{NN\eta}$ given by Eq. (3) within the accuracy of the present experimental data.

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1) S. Sasaki, J. Takahashi and K. Ozaki, Prog. Theor. Phys. **38** (1967), 1326.
2) G. F. Chew, M. L. Goldberger, F. E. Low and Y. Nambu, Phys. Rev. **106** (1957), 1337.
3) J. K. Kim, Phys. Rev. Letters **19** (1967), 1079.
4) H. Rosenfeld et al., UCRL 8030 (Rev.) September 1967.