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Relations between Pion Photoproduction and πN Scattering

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The pion photoproduction due to the magnetic moment of nucleon is shown to possess a property similiar to πN scattering. The magnetic multipole amplitudes $M_{l\pm}$ are found to be related to the corresponding πN scattering amplitudes $f_{l\pm}$ as

$$M_{l\pm}=\frac{e\mu_v}{g}\frac{1}{l}\left(\frac{k}{q}\right)^l f_{l\pm}.$$

The proportionality relation is obtained also for the case of electric multipole radiation. The full amplitudes for pion photoproduction are expressed in terms of the amplitudes of the processes $\pi N\to\pi N$ and $\pi N\to\eta N$ without any freely adjustable parameter. Numerical results for differential cross sections and polarizations agree fairly well with the experimental data in the wide energy range 180～1000 MeV.

§ 1. Introduction

The amplitudes of the π meson produced by a low energy photon have been calculated by Chew, Goldberger, Low and Nambu[1] by using the dispersion relation. They derived the relation (CGLN relation) that the amplitudes M_{1+} of the photoproduction is proportional to that of the P_{33} wave for πN elastic scattering. But in a higher energy range including the second and third resonances, the pion photoproduction have hardly*) been theoretically investigated, because four or five resonances are expected to contribute to the production amplitudes and the inelastic channels give rise to complicated effects. Recently Chau et al.[3] calculated the amplitude of pion photoproduction by the phenomenological method for the energy range 535～850 MeV. They showed that the photoproduction amplitudes can be expressed by the sum of two terms with many adjustable parameters; the one is a background term independent of the energy while the other is proportional to the πN elastic amplitudes. Furthermore, according to their results, it seems that the pion photoproduction is closely related with πN elastic scattering in those energy range and that the idea of the CGLN relation for P_{33} wave may be applied to the other partial waves.

In the present paper it will be shown that the proportionality between πN

*) There are a few approaches by the use of the isobar model,[2] but such an approach needs many free parameters to fit the results to the experimental data.

scattering and γ-π production amplitudes is valid for any wave with arbitrary angular momentum and for the wide energy range such as those from the π-production threshold to 1 GeV.

The CGLN relation was derived from the fact that the dispersion relation of the pion photoproduction has the same form as that of πN elastic scattering. However, the solution of the dispersion relation is not unique and consequently the photoproduction amplitude is not necessarily proportional to the πN elastic amplitude. In spite of these situations, the CGLN relation agrees well with the experimental data in the low energy range. The reason for this agreement is that the interaction of pion photoproduction is very similiar to that of πN elastic scattering. In particular, a magnetic multipole radiation has the same parity as that of a π meson with the same angular momentum. Accordingly, when a nucleon absorbs the magnetic multipole radiation, the nucleon receives the same angular momentum and the same parity as those in the case of the corresponding πN elastic scattering. In addition, the final state interaction of the pion photoproduction is exactly the same as that of πN elastic scattering. Thus, it may be conjectured that the partial wave amplitudes of the magnetic transition should be proportional to πN elastic amplitudes for all partial waves.

We shall examine these situations in detail, and derive a set of relations between the photoproduction amplitudes and πN scattering. It will also be shown that the proportionality relation is valid not only for the cases of magnetic multipole radiation but also the cases of electric radiation. According to our results, the amplitudes of the pion photoproduction can be expressed in terms of the amplitudes of the reactions $\pi N \rightarrow \pi N$ and $\pi N \rightarrow \eta N$ without any free parameter (see § 4). Angular distributions and polarizations are calculated in the photon energy range 180∼1000 MeV in the laboratory system. The results agree fairly well with the experimental data in spite of no freely adjustable parameters and a wide energy range (see § 5).

§ 2. Relations between magnetic multipole transition and πN scattering

In dealing with the various kinds of hadrons, there are, roughly speaking, two different viewpoints.

(I) The so-called democracy of elementary particles. Namely every hadron, regardless of its stability for strong interaction, should be treated as equally fundamental. According to this viewpoint, any scattering amplitude should include even contributions from Feynman diagrams where some resonant particles occur, in addition to the contributions from stable particles in intermediate states. In such a consideration, coupling constants of each hadron should be treated as unknown parameters which should be determined from the experimental data.

(II) Each resonance is regarded as a compound state of some (stable) elementary particles of a limited number of kinds. According to this point of view any physical amplitude should correspond to Feynman diagrams where all the internal lines represent only elementary particles of the kinds stated above. It is, of course, an open question that what kind of particles is eligible for being elementary.

In the present paper the second viewpoint will be adopted because the first one will necessarily needs a number of parameters (for example many unknown coupling constants mentioned above), and there seems no theoretical basis for the determination of these arbitrary constants. Thus the only way to determine these constants is to choose them in such a way as to get a best fit of the calculated results to experimental data. Almost all the articles[2] so far published on the photo-pion-production may be said to belong more or less to this kind of approach.

Therefore in what follows the second viewpoint will exclusively be adopted together with the definition that all the stable particles (when weak and electromagnetic interactions are neglected) are elementary particles. Thus the contributions from pions and nucleons in the intermediate states are enough for the investigation of the γ-π processes in the energy range 180～1000 MeV, since the strange particles may give a negligibly small contribution.

Let us compare the Feynman diagram of πN scattering with that of the γ-π process where the photon is absorbed by the anomalous magnetic moment*) of a nucleon. Any diagram representing the γ-π process can be derived from the corresponding diagram of πN scattering by replacing the π-N interaction with the electromagnetic interaction due to the anomalous magnetic moment (see Fig. 1). The interaction due to the anomalous magnetic moment is represented

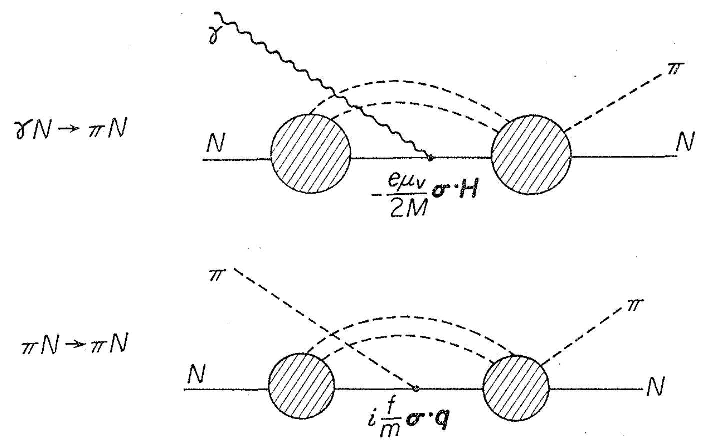

Fig. 1. Examples for the diagram of πN scattering and its corresponding diagram of $\gamma\pi$ production.

*) In this section since we are only concerned with the transition due to the magnetic moment of a nucleon, the absorption of a photon by a charged pion may be excluded. The latter effect will be taken into account in § 4 by a different approach.

by the following Hamiltonian:

$$H_{\text{anom}} = -\frac{e}{4M}\bar{\psi}(\mu_v\tau_3+\mu_s)\sigma_{\mu\nu}\cdot F_{\mu\nu}\psi\,, \tag{2·1}$$

where $F_{\mu\nu}$ denotes a electromagnetic field tensor, M is a nucleon mass, $e^2/4\pi=1/137$, $\mu_v=1.853$, $\mu_s=-0.06$ and τ_3 is an isospin matrix. On the other hand, the strong interaction Hamiltonian is given by

$$H_s = i\frac{f}{m}\bar{\psi}\gamma_5\gamma_\mu\boldsymbol{\tau}\cdot\partial_\mu\boldsymbol{\pi}\psi\,, \tag{2·2}$$

where $f^2/4\pi=0.077$ and m is a pion mass. In the static limit $(p/(E_p+M)\ll 1)$ the expectation value of H_{anom} is very similiar to that of H_s:

$$\langle\psi_b|H_{\text{anom}}|\psi_a\rangle \approx -e\mu_v/2M\cdot\chi_b^+\tau_3\boldsymbol{\sigma}\cdot\boldsymbol{H}\chi_a\,, \tag{2·3}$$

$$\langle\psi_b|H_s|\psi_a\rangle \approx if/m\cdot\chi_b^+\boldsymbol{\tau}\cdot\boldsymbol{\pi}\boldsymbol{\sigma}\cdot\boldsymbol{q}\chi_a\,, \tag{2·4}$$

where χ_a, χ_b denote the spin wave functions of nucleon, $\boldsymbol{H}$ is a magnetic field and $\boldsymbol{q}$ is a c.m. momentum of pion.

Now recalling the perfect analogy between the diagrams of πN scattering and pion photoproduction mentioned just above, and taking account of the expressions (2·3) and (2·4), we are led to the following expressions*) for each reaction amplitude:

$$T_{\text{anom}} = -\frac{e\mu_v}{2M}\boldsymbol{H}\cdot\langle\pi N|\boldsymbol{V}|N'\rangle\,, \tag{2·5}$$

$$T_{\pi N} = i\frac{f}{m}\boldsymbol{q}\cdot\langle\pi N|\boldsymbol{V}|N\rangle\,, \tag{2·6}$$

which are exactly valid provided the graphical consideration is approved (c.f. Appendix 1). Here $\langle\pi N|\boldsymbol{V}|N\rangle$ is a vector representing a sum of the contributions from all the diagrams of πN scattering (or γ-π production) in which the incident π-line (or incident γ-line) is removed. Thus the matrix elements $\langle\pi N|\boldsymbol{V}|N\rangle$ and $\langle\pi N|\boldsymbol{V}|N'\rangle$ have the same form except for the little difference of the incident momentum of nucleon. Therefore, by decomposing it into partial waves this reaction amplitude is rewritten as a sum of the reduced matrix elements of $\boldsymbol{V}$ (the detail of this laborious calculation is given in Appendix 2.):

*) In deriving the above expressions we have assumed that the internal nucleon line which absorbs the incident photon or pion can be considered as a real line (in the sense of the old perturbation) satisfying approximately the conservation of energy. The reason for this is that the energy of the incident particle (which have the c. m. momentum below 500 MeV/c) justifies the validity of the above assumption ($p/E_p+M\ll 1$) and it is easily understood that the contribution from the virtual pair of nucleons can be neglected for such a range of energy.

$$M_{l\pm}^{\rm anom}=-i\frac{e\mu_v}{2M}\frac{\sqrt{6(2l+1)}}{\sqrt{l(l+1)}}\begin{Bmatrix} l & 1 & l-1 \\ 1 & l & 1 \end{Bmatrix}(-1)^{J+l-1/2}\Bigg[\begin{Bmatrix} l & 1 & l-1 \\ 1 & l & 0 \end{Bmatrix}$$

$$\begin{Bmatrix} 1/2 & 0 & 1/2 \\ l & J & l \end{Bmatrix}\sqrt{6}(l\|V_a\|l-1)-\begin{Bmatrix} l & 1 & l-1 \\ 1 & l & 1 \end{Bmatrix}\begin{Bmatrix} 1/2 & 1 & 1/2 \\ l & J & l \end{Bmatrix}$$

$$\times 3\sqrt{2}(l\|V_b\|l-1)\Bigg](l-1\|k\|l), \tag{2·7}$$

$$f_{l\pm}=i\frac{f}{m}(-1)^{J+l-1/2}\Bigg[\begin{Bmatrix} l & 1 & l-1 \\ 1 & l & 0 \end{Bmatrix}\begin{Bmatrix} 1/2 & 0 & 1/2 \\ l & J & l \end{Bmatrix}\sqrt{6}(l\|V_a\|l-1)$$

$$-\begin{Bmatrix} l & 1 & l-1 \\ 1 & l & 1 \end{Bmatrix}\begin{Bmatrix} 1/2 & 1 & 1/2 \\ l & J & l \end{Bmatrix}3\sqrt{2}(l\|V_b\|l-1)\Bigg](l-1\|q\|l), \tag{2·8}$$

where $M_{l\pm}^{\rm anom}$ and $f_{l\pm}$ are the amplitudes of magnetic multipole transition and πN scattering respectively, $l\pm$ denotes $J=l\pm 1/2$, $\{\ \}$ is a 6-j symbol and $(\|\|)$ is a reduced matrix element. Here the vector $\boldsymbol{V}$ is devided into spin non-flip and flip parts:

$$\boldsymbol{V}=\boldsymbol{V}_a+i/\sqrt{2}\,\boldsymbol{V}_b\times\boldsymbol{\sigma}\,. \tag{2·9}$$

From Eqs. (2·7) and (2·8), it is seen that the transition amplitude due to the magnetic multipole is related with the corresponding partial wave amplitude of πN scattering in the following way:

$$M_{l\pm}=\frac{e\mu_v}{g}\frac{1}{l}f_{l\pm}\,, \tag{2·10}$$

where $g=2Mf/m$ $(g^2/4\pi=14.5)$. This equation is obtained by using the approximation*) that the c.m. momentum k of the photon is put equal to the c.m. momentum q of the pion (pion mass being neglected). In the vicinity of the threshold energy of the incident particle, the pion mass cannot be neglected and consequently the above relation (2·10) should be modified by a factor $(k/q)^l$ (which is approximately unity for the moderately high incident energy) as

$$M_{l\pm}=\frac{e\mu_v}{g}\frac{1}{l}\left(\frac{k}{q}\right)^l f_{l\pm}\,. \tag{2·11}$$

The line of discussion stated above may be applied to the isoscalar part of magnetic moment. In this case, the photoproduction amplitudes must be compared with those of the reaction $\eta+N\to\pi+N$, where the strong interaction Hamiltonian for η meson has the same form as the isoscalar part of the electromagnetic interaction:

$$H_s^{\eta}=if_{NN\eta}/m_\eta\bar{\psi}\gamma_5\gamma_\mu\psi\partial_\mu\eta\,. \tag{2·12}$$

*) The difference of k and q is 16% at 300 MeV and 5% at 600 MeV.

By the same procedure as in the case of the isovector part, we obtain the final result*)

$$M_{l\pm}^{(0)} = \frac{e\mu_s}{g_{NN\eta}} \frac{1}{l} \left(\frac{k}{q_\eta}\right)^l f_{l\pm}^\eta / \sqrt{3}\,, \tag{2·13}$$

where $M_{l\pm}^{(0)}$ is an isoscalar magnetic multipole amplitude, $f_{l\pm}^\eta$ is a physical amplitudes of $\eta + N \to \pi + N$ and $g_{NN\eta}^2/4\pi$ is equal to 4.1 from the result of the analysis of $\pi + N \to \eta + N$ in reference 4). Here the values $f_{l\pm}^\eta$ are equal to the amplitudes of the reaction $\pi + N \to \eta + N$ on the basis of the time reversability, which have been given in reference 5).

The transition amplitudes by the normal magnetic moment can be calculated by the same method. Adding this contribution, we obtain the following final result which is a generalization of the CGLN relation:

$$M_{l\pm}^{(1)} = \frac{e(\mu_v + 1/2)}{g} \frac{1}{l} \left(\frac{k}{q}\right)^l f_{l\pm}^{(I)}\,, \tag{2·14}$$

$$M_{l\pm}^{(0)} = \frac{e(\mu_s + 1/2)}{g_{NN\eta}} \frac{1}{l} \left(\frac{k}{q_\eta}\right)^l f_{l\pm}^\eta / \sqrt{3}\,, \tag{2·15}$$

where I denotes the total isospin in πN system.

§ 3. Electric multipole transition induced by anomalous magnetic moment

Since the electric l-th pole radiation have the natural parity $(-1)^l$, the same line of argument as those of § 2 cannot be applied without any modification to the present electric radiation. In spite of this different parity between the electric and magnetic radiations, there is still no difference with respect to the final state interactions between both cases, i.e. the electric type γ-π production and π-N scattering. Of course the electric transition is more difficult to be dealt with as compared with the magnetic case. The more detailed investigation**) shows that this difficulty seems to be caused by a mixing of a couple of scattering amplitudes with different parities. However, if one makes use of some approximation, namely, if one takes account of contributions from only diagrams of some particular type (see Fig. 2), then the above-mentioned fact concerning the final state interaction allows us to express the electric multipole amplitudes due to the interaction of the anomalous magnetic moment in terms of the π-N scattering amplitudes.

In the case of πN scattering, the diagrams of Fig. 2 produce the P_{33} resonance

*) The factor $1/\sqrt{3}$ in Eq. (2·13) comes from the difference of normalizations of $f_{l\pm}^\eta$ and $M_{l\pm}^{(0)}$ with respect to isotopic spin states.

**) An investigation of this problem following the line of reasoning of § 2 is now being carried on by the present author, the result of which will be published in the near future.

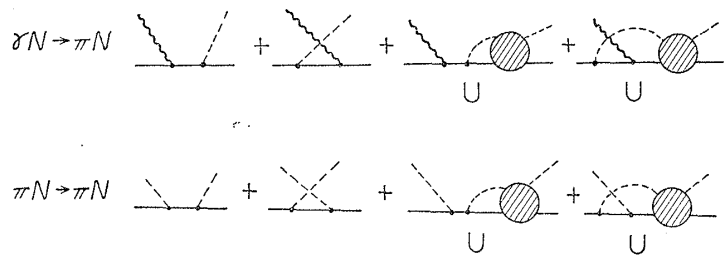

Fig. 2.

as was shown in the paper of Chew[6] where the crossed diagram and its repetition are evaluated. This fact shows that the present approximation, namely, taking a sum of contributions from all the diagrams in Fig. 2 gives a fairly close approximation to the true physical amplitudes. The transition matrices corresponding to these diagrams satisfy the following relations both of which depend upon a matrix U corresponding to πN scattering:*)

$$T_{\text{anom}}=B_{\text{anom}}+\int d^4pd^4qB_{\text{anom}}\frac{1}{q^2+m^2-i\epsilon}\frac{-i\gamma\cdot p+M}{p^2+M^2-i\epsilon}U\delta^4(p+q-p_1'-k_1), \tag{3·1}$$

$$T_{\pi N}=B_{\pi N}+\int d^4pd^4qB_{\pi N}\frac{1}{q^2+m^2-i\epsilon}\frac{-i\gamma\cdot p+M}{p^2+M^2-i\epsilon}U\delta^4(p+q-p_1-q_1), \tag{3·2}$$

where B denotes a Born term (on and off shell), p_1' and k_1 are incident four-momenta of nucleon and photon in the γN system, p_1 and q_1 are momenta of nucleon and pion in the πN system respectively. Equations (3·1) and (3·2) are rewritten by using the virtual masses $M_x=\sqrt{p_0^2-\boldsymbol{p}^2}$ and $m_x=\sqrt{q_0^2-\boldsymbol{q}^2}$ as follows:**)

$$T_{\text{anom}}=B_{\text{anom}}+\int dM_x\left(\frac{M_x}{E_p}\right)dm_x\left(\frac{m_x}{\omega_p}\right)\frac{E_p\omega_p}{pW}p^2d\Omega$$
$$\times B_{\text{anom}}\frac{1}{m_x^2-m^2+i\epsilon}\frac{-i\gamma\cdot p+M}{M_x^2-M^2+i\epsilon}U(W,M_x,m_x,\Theta,\Phi), \tag{3·3}$$

$$T_{\pi N}=B_{\pi N}+\int dM_x\left(\frac{M_x}{E_p}\right)dm_x\left(\frac{m_x}{\omega_p}\right)\frac{E_p\omega_p}{pW}p^2d\Omega$$
$$\times B_{\pi N}\frac{1}{m_x^2-m^2+i\epsilon}\frac{-i\gamma\cdot p+M}{M_x^2-M^2+i\epsilon}U(W,M_x,m_x,\Theta,\Phi), \tag{3·4}$$

*) Here the matrix element of U is a function of the initial and final momenta (both off and on shell states should be considered). The difference of $T_{\pi N}$ and U is that the former is an approximation to the latter with on-shell-momenta while U is an exact matrix element of πN elastic scattering (on-shell or off-shell).

**) The small letter x attached to any quantity means that this quantity represents a quantity in a virtual state.

where W is a total energy and $E_p=\sqrt{\boldsymbol{p}^2+M_x^2}$, $\omega_p=\sqrt{\boldsymbol{p}^2+m_x^2}$, while $\boldsymbol{p}$ is a c.m. momentum of the intermediate state for $W=E_p+\omega_p$ and Θ, Φ are the scattering angles between the intermediate and the final pion. If we introduce the four-spinor $u_x^{(s)}$ (s represents the helicity) of the virtual nucleon state which has a mass M_x, the projection operator for a positive energy state is given by

$$\sum_s u_x^{(s)}\bar{u}_x^{(s)}=-i\gamma\cdot p+M_x\,. \tag{3·5}$$

Neglecting the contribution of antinucleon in the intermediate state (it means that we put the expression (3·5) in place of the numerator of the nucleon propagator in (3·3) and (3·4)), we can decompose the transition matrices into a diagonal sum with respect to the total isopin I and the total angular momentum $J=l\pm 1/2$:

$$E_{l\pm}^{\text{anom}(I)}=\int E_{l\pm}^{B,\text{anom}(I)}(W,M_x,m_x)\Big[\delta(M_x-M)\cdot\delta(m_x-m)$$
$$+\frac{pm_xM_x}{W}\frac{1}{m_x^2-m^2+i\epsilon}\frac{1}{M_x^2-M^2+i\epsilon}U_{l\pm}^{(I)}(W,M_x,m_x)\Big]dm_xdM_x\,, \tag{3·6}$$

$$f_{l\pm}^{(I)}=\int f_{l\pm}^{B,(I)}(W,M_x,m_x)\Big[\delta(M_x-M)\cdot\delta(m_x-m)$$
$$+\frac{pm_xM_x}{W}\frac{1}{m_x^2-m^2+i\epsilon}\frac{1}{M_x^2-M^2+i\epsilon}U_{l\pm}^{(I)}(W,M_x,m_x)\Big]dm_xdM_x\,. \tag{3·7}$$

Here $E_{l\pm}^{B,\text{anom}}$ and $f_{l\pm}^{B}$ are Born amplitudes of electric multipole and πN elastic scattering with the final virtual masses m_x and M_x respectively. The Born amplitudes will be calculated exactly in Appendix 3. From the results of Appendix 3, it is seen that the ratios $E_{l\pm}^{B,\text{anom}(I)}/f_{l\pm}^{B,(I)}$ are nearly constant for the variation of M_x and m_x as shown in Table I.

Since the integrand of (3·6) and (3·7) become very largely near the physical mass, the above-mentioned independence of E^B/f^B from the mass variables allows us to rewrite (3·6) by means of (3·7) as follows:

$$E_{l\pm}^{\text{anom}(I)}(W)=C_{l\pm}^{(I)}(W)f_{l\pm}^{(I)}\,, \tag{3·8}$$

where $C_{l\pm}^{(I)}(W)=E_{l\pm}^{B,\text{anom}(I)}(W)/f_{l\pm}^{B,(I)}(W)$.

Using the same procedure, we obtain the following relation for the magnetic multipole amplitudes $M_{l\pm}^{\text{anom}(I)}$:

$$M_{l\pm}^{\text{anom}(I)}(W)=D_{l\pm}^{(I)}(W)f_{l\pm}^{(I)}(W), \tag{3·9}$$

where $D_{l\pm}^{(I)}(W)=M_{l\pm}^{B,\text{anom}(I)}(W)/f_{l\pm}^{B,(I)}(W)$. In the static limit, the value $D_{l\pm}^{(I)}(W)$ becomes (see Table I)

$$D_{l\pm}^{(I)}(W)=\frac{e\mu_v}{g}\frac{1}{l}\left(\frac{k}{q}\right)^l. \tag{3·10}$$

Table I. Ratios of Born amplitudes at $E_{\mathrm{Lab}}=500\,\mathrm{MeV}$. The values C' and D' shown in this table are related to the ratios C and D as

$$C_{l\pm}^{(I)}=\frac{e\mu_v}{g}\left(\frac{k_1}{q_1}\right)^l C_{l\pm}'^{(I)}, \qquad D_{l\pm}^{(I)}=\frac{e\mu_v}{g}\left(\frac{k_1}{q_1}\right)^l D_{l\pm}'^{(I)}.$$

$m_x(M_x=M)$	$C_{0+}'^{(1/2)}$	$C_{0+}'^{(3/2)}$	$C_{1+}'^{(1/2)(3/2)}$	$C_{2-}'^{(1/2)(3/2)}$	$C_{2+}'^{(1/2)(3/2)}$
37.3	−0.266	0.768	−0.006	2.522	−0.003
87.3	−0.273	0.776	−0.006	2.521	−0.003
137.3	−0.287	0.791	−0.006	2.520	−0.003
187.3	−0.306	0.814	−0.005	2.519	−0.002
237.3	−0.335	0.852	−0.004	2.516	−0.002

$m_x(M_x=M)$	$D_{1-}'^{(1/2)}$	$D_{1-}'^{(3/2)}$	$D_{1+}'^{(1/2)(3/2)}$	$D_{2-}'^{(1/2)(3/2)}$	$D_{2+}'^{(1/2)(3/2)}$
37.3	0.970	1.266	1.012	0.510	0.501
87.3	0.970	1.264	1.012	0.510	0.500
137.3	0.971	1.261	1.009	0.509	0.500
187.3	0.971	1.257	1.007	0.508	0.498
237.3	0.972	1.251	1.003	0.507	0.497

Therefore, Eq. (3·9) becomes

$$M_{l\pm}^{\mathrm{anom}(I)}(W)=\frac{e\mu_v}{g}\frac{1}{l}\left(\frac{k}{q}\right)^l f_{l\pm}^{(I)}(W), \tag{3·11}$$

which is in accordance with the result of § 2.

For the interaction due to electric current, the ratio of Born terms of γ-π process and πN scattering varies very largely for the variation of m_x. Therefore the method of this section cannot be applied to the current-type interaction which will be dealt with in the next section.

§ 4. Full amplitudes of pion photoproduction

The photoproduction amplitudes produced by the electric charge have a contribution from the pion exchange term. This kind of contribution is, in general, important and especially for a low energy photon, it becomes the main part of the photoproduction amplitudes. From the discussion of the previous section, since this part cannot be given by the generalized CGLN relation, it is necessary to find a method of evaluation of this part.

Fortunately as the pion mass is small, this interaction is very peripheral. Therefore, the effect of the strong interaction for this case is perhaps dominated by the final state interaction. In fact, the pion photoproduction near the threshold is well reproduced by the Born amplitude due to the electric charge. Thus we adopt the Born amplitude with a modification by the final state interaction as follows:

$$E_{l\pm}^{\text{charge}(I)} = E_{l\pm}^{B,\text{charge}(I)}[1+iqf_{l\pm}^{(I)}], \tag{4·1}$$

$$M_{l\pm}^{\text{charge}(I)} = M_{l\pm}^{B,\text{charge}(I)}[1+iqf_{l\pm}^{(I)}], \tag{4·2}$$

$$E_{l\pm}^{\text{charge}(0)} = E_{l\pm}^{B,\text{charge}(0)}[1+iqf_{l\pm}^{(1/2)}], \tag{4·3}$$

$$M_{l\pm}^{\text{charge}(0)} = M_{l\pm}^{B,\text{charge}(0)}[1+iqf_{l\pm}^{(1/2)}]. \tag{4·4}$$

Equations (4·1) and (4·2) are the isovector part, Eqs. (4·3) and (4·4) are the isoscalar part and $E_{l\pm}^{B,\text{charge}}$, $M_{l\pm}^{B,\text{charge}}$ are the Born amplitudes due to the electric charge.

Summing up the transition amplitudes caused by the electric charge and magnetic moment, we obtain the full amplitudes for the pion photoproduction as follows:

$$E_{l\pm}^{(I)} = E_{l\pm}^{B,\text{charge}(I)}(1+iqf_{l\pm}^{(I)}) + C_{l\pm}^{(I)}f_{l\pm}^{(I)}, \tag{4·5}$$

$$M_{l\pm}^{(I)} = \left[M_{l\pm}^{B,\text{charge}(I)} - \frac{1}{2\mu_v}M_{l\pm}^{B,\text{anom}(I)}\right](1+iqf_{l\pm}^{(I)}) + \frac{e(\mu_v+1/2)}{g}\frac{1}{l}\left(\frac{k}{q}\right)^l f_{l\pm}^{(I)}, \tag{4·6}$$

$$E_{l\pm}^{(0)} = E_{l\pm}^{B,\text{charge}(0)}(1+iqf_{l\pm}^{(1/2)}), \tag{4·7}$$

$$M_{l\pm}^{(0)} = \left[M_{l\pm}^{B,\text{charge}(0)} - \frac{1}{2\mu_s}M_{l\pm}^{B,\text{anom}(0)}\right](1+iqf_{l\pm}^{(1/2)}) + \frac{e(\mu_s+1/2)}{g_{NN\eta}}\frac{1}{l}\left(\frac{k}{q}\right)^l f_{l\pm}^{\eta}/\sqrt{3}, \tag{4·8}$$

where the Born amplitudes of magnetic multipole transition due to the normal magnetic moment should be excluded from the charge part, since these are already included in (2·14) and (2·15). In order to calculate Eq. (4·8), it is necessary to know the values of $f_{l\pm}^{\eta}$ below the ηN threshold. We, therefore, have to use the analytic continuation from the physical amplitudes which are obtained in reference 5) by the analysis of the reaction $\pi+N\to\eta+N$ in the energy range $T_\pi^{\text{Lab}}=561-1300$ MeV. This continuation can be performed by simply substituting the c.m. momentum q for q_η in the amplitudes of $\pi N\to\eta N$.

Consequently, we can express all the amplitudes of the pion photoproduction without any free parameter.

§ 5. Cross sections and polarizations

Using the results given in Eqs. (4·5)∼(4·8), let us obtain the helicity amplitudes. First, we define the invariant amplitudes F as follows:[1)]

$$F_1^{(I)} = \sum_{l=0}^{\infty}[lM_{l+}^{(I)}+E_{l+}^{(I)}]P'_{l+1}(\cos\theta) + [(l+1)M_{l-}^{(I)}+E_{l-}^{(I)}]P'_{l-1}(\cos\theta), \tag{5·1}$$

$$F_2^{(I)} = \sum_{l=1}^{\infty}[(l+1)M_{l+}^{(I)}+lM_{l-}^{(I)}]P_l'(\cos\theta), \tag{5·2}$$

$$F_3^{(I)} = \sum_{l=1}^{\infty}[E_{l+}^{(I)}-M_{l+}^{(I)}]P''_{l+1}(\cos\theta) + [E_{l-}^{(I)}+M_{l-}^{(I)}]P''_{l-1}(\cos\theta), \tag{5·3}$$

$$F_4^{(I)} = \sum_{l=2}^{\infty} [M_{l+}^{(I)} - E_{l+}^{(I)} - M_{l-}^{(I)} - E_{l-}^{(I)}] P_l''(\cos\theta), \tag{5·4}$$

where the sufix I takes the value 1/2, 3/2 or 0. The helicity amplitudes are expressed by the following equations: helicity-nonflip amplitudes are

$$H^{\pm}(s, t) = \langle \boldsymbol{q}, \nu_2 = 1/2 | T | \boldsymbol{k}, \lambda = \pm 1, \nu_1 = 1/2 \rangle,$$

helicity-flip amplitudes are

$$\Phi^{\pm}(s, t) = \langle \boldsymbol{q}, \nu_2 = 1/2 | T | \boldsymbol{k}, \lambda = \pm 1, \nu_1 = -1/2 \rangle,$$

where λ is a helicity of photon and ν_1, ν_2 are helicities of the initial and final nucleons respectively. These helicity amplitudes are written in terms of the amplitudes F given in Eqs. (5·1)~(5·4) as follows:

$$H^- = -\frac{1}{\sqrt{2}} \sin\theta \cos\frac{\theta}{2} (F_3 + F_4), \tag{5·5}$$

$$H^+ = -\sqrt{2} \sin\frac{\theta}{2} (F_1 + F_2) + H^-, \tag{5·6}$$

$$\Phi^+ = \frac{1}{\sqrt{2}} \sin\theta \sin\frac{\theta}{2} (F_3 - F_4), \tag{5·7}$$

$$\Phi^- = -\sqrt{2} \cos\frac{\theta}{2} (F_1 - F_2) + \Phi^+. \tag{5·8}$$

The cross section for an unpolarized photon is given by

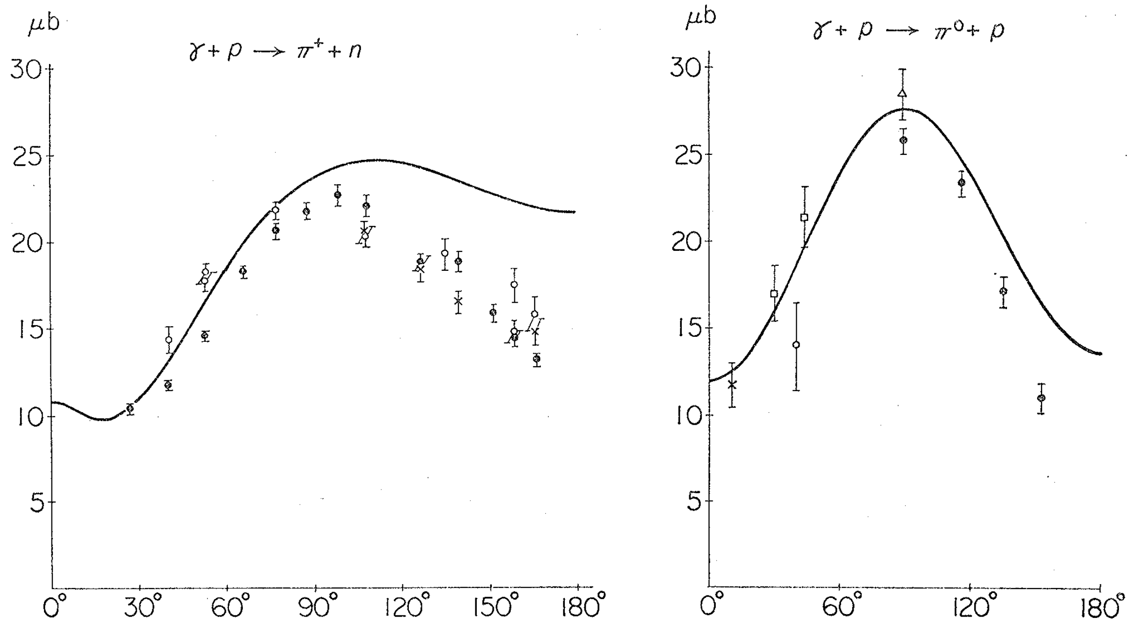

Fig. 3. The differential cross sections for pion photoproduction at $E_{\mathrm{Lab}} = 320$ MeV. The experimental data are cited from references 8) and 9).

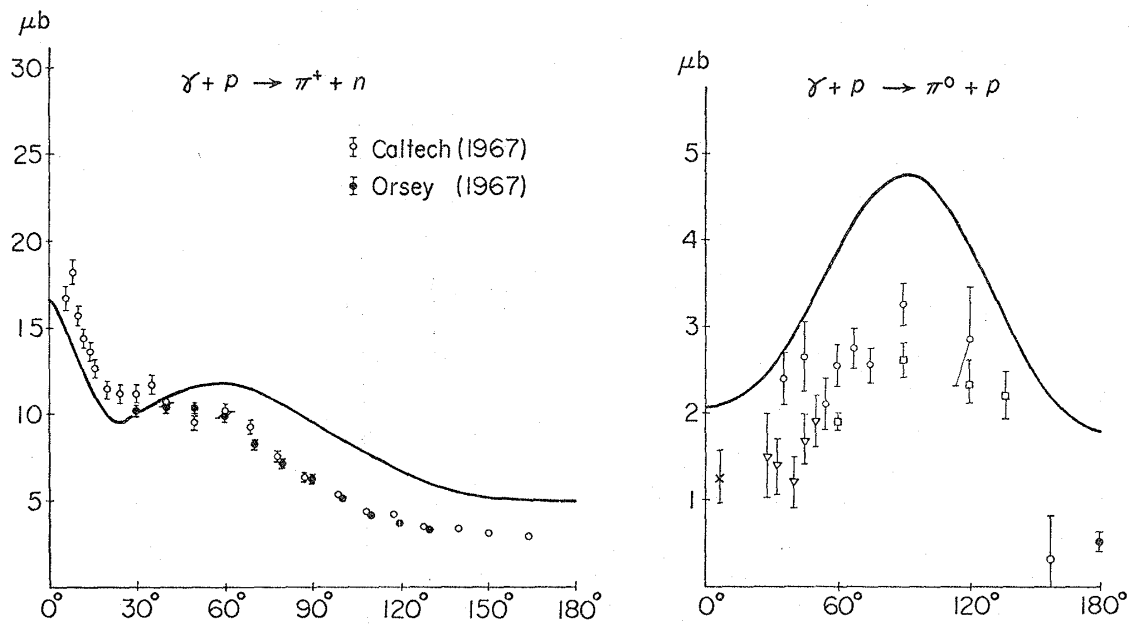

Fig. 4. The differential cross sections for pion photoproduction at E_{Lab}=600 MeV. The experimental data are cited from references 8) and 9).

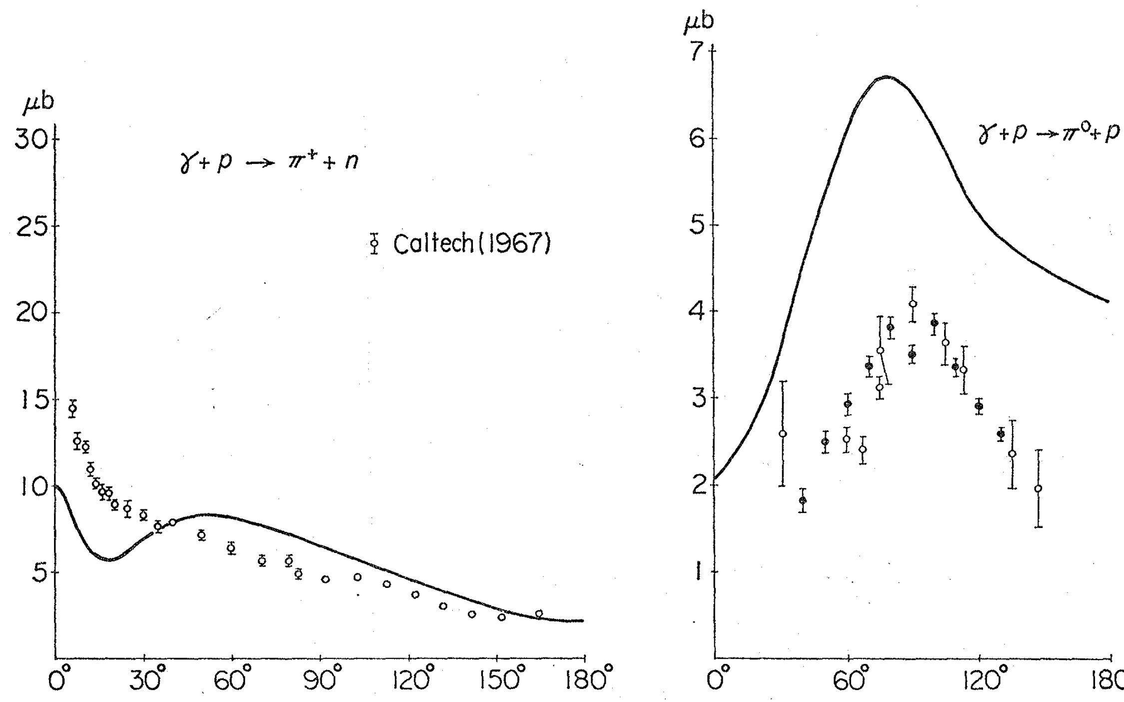

Fig. 5. The differential cross sections for pion photoproduction at E_{Lab}=800 MeV. The experimental data are cited from references 8) and 9).

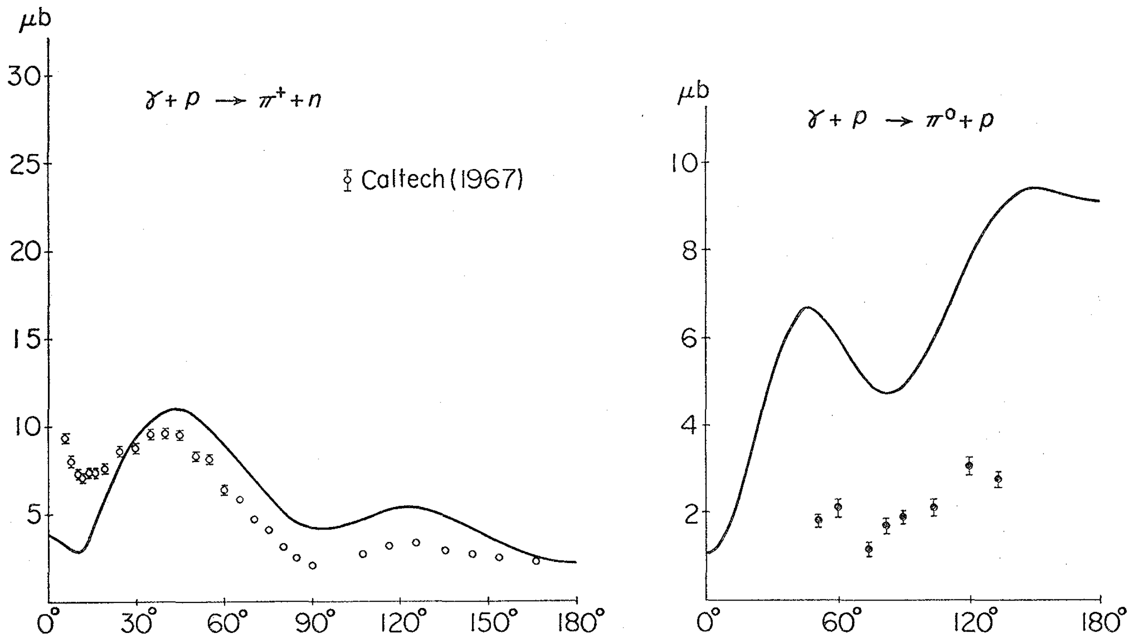

Fig. 6. The differential cross sections for pion photoproduction at $E_{\text{Lab}}=1000$ MeV. The experimental data are cited from references 8) and 9).

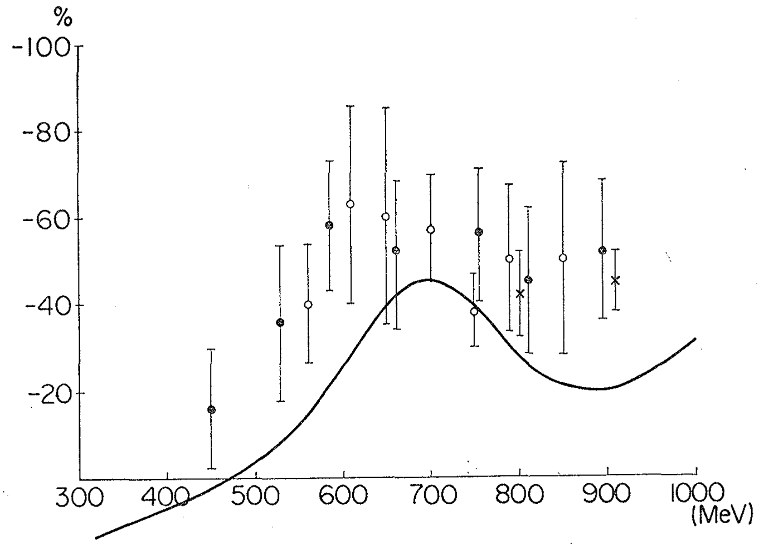

Fig. 7. The polarizations of recoil nucleon at $\theta_{\text{c.m.}}=90°$ for the reaction $\gamma+p\to\pi^0+p$. The experimental data are cited from references 10).

$$\frac{d\sigma}{d\Omega}=\frac{1}{2}\frac{q}{k}(|H^+|^2+|H^-|^2+|\Phi^+|^2+|\Phi^-|^2). \tag{5·9}$$

The polarization $P(\theta)$ of the final nucleon is

$$P(\theta)=\frac{2\,\mathrm{Im}(\phi^+H^{-*}+\phi^-H^{+*})}{|H^+|^2+|H^-|^2+|\Phi^+|^2+|\Phi^-|^2}. \tag{5·10}$$

For the energy range $E^{\gamma}_{\text{Lab}}=180\sim1000$ MeV, the angular distributions and polarizations have been evaluated, the results of which are shown in Figs. 3~7. Here we have made use of the partial wave amplitudes of πN elastic scattering given in references 7). These results agree with the experimental data fairly well in spite of the fact that there is no arbitrarily adjustable parameters.

§ 6. Discussion

From the experimental data the backward cross section for γ-π production is small and uniform for the variation of the incident energy. It probably means that the sum of the electric and magnetic multipole amplitudes for resonant wave vanishes at $\theta_{\text{c.m.}}=180°$. Therefore the following relation may hold for each partial wave:

$$E_{l-}/M_{l-}=(l+1)/(l-1). \tag{6·1}$$

From the results of this paper, the ratios E_{l-}/M_{l-} for the resonant waves are

$$\begin{aligned} &E_{2-}^{(1/2)}/M_{2-}^{(1/2)}\approx4.0 \quad \text{at } E_{\text{lab}}=700\text{ MeV},\\ &E_{3-}^{(1/2)}/M_{3-}^{(1/2)}\approx2.6 \quad \text{at } E_{\text{lab}}=1000\text{ MeV}. \end{aligned} \tag{6·2}$$

These numerical values (6·2) agree with the relation (6·1) within 30%. However, the small differences between (6·1) and (6·2) give large effects for the calculated angular distributions. The inagreement shows that the calculation of the electric multipole amplitudes is insufficient. In order to improve this inagreement, it may be considered that the coefficients $C_{l\pm}^{(I)}$ of the proportionality relations are varied as free parameters.

According to the results of this paper, we can predict the angular distributions for photon-neutron scattering. In our treatment, the isoscalar part is expressed by the sum of Born term and the ηN production amplitude. The amplitudes of $\pi N\to\eta N$ have no resonances in the D_{13}, D_{15} and F_{15} waves (see solution 3 in reference 5)). Therefore, the angular distributions $\gamma+n\to\pi^0+n$ (or $\gamma+n\to\pi^-+p$) will perhaps be very similar to those of $\gamma+p\to\pi^0+p$ (or $\gamma+p\to\pi^++n$) in the energy range 800~1200 MeV. In order to examine this prediction, we hope that the experiments on γ-n scattering will be performed.

Acknowledgements

The author would like to thank Professor R. Utiyama for valuable discussions and advice. We also wish to thank Professor M. Kawaguchi for helpful discussions and encouragements.

Appendix 1

In § 2, we derived Eqs. (2·5) and (2·6) on the basis of the perfect analogy between the diagrams of πN scattering and pion photoproduction. But these relations may be obtained more easily and in a convincing way by means of the perturbation approach. Of course, we are very sceptical of the applicability of the perturbation method to the strong interaction, but if we are concerned only with the correspondence between any mathematical expression derived by the perturbation method and a corresponding diagram, and if the problem of the convergence is not the subject of our discussion, we can say that the graphical consideration is essentially equivalent to the method of perturbation. The graphical approach is, from the heuristic point of view, a very powerful tool for the derivation of equations in an intuitive way and it is said that the recent theory of the strong interaction is based on this approach. Therefore it may be granted to apply the perturbation method to the derivation of the graphical analogy of γ-π process with πN scattering.

The transition matrix elements are given by the perturbation theory:

$$\langle\pi N|T|\gamma N\rangle=\frac{1}{2ik}\sum_{n=1}^{\infty}\sum_{k=0}^{n}\frac{(-i)^{n+1}}{(n+1)!}\int\cdots\int d^4x_1\cdots d^4x_n d^4y \\ \times\langle\pi N|P[H_s(x_1)\cdots H_s(x_k)H_{\text{anom}}(y)H_s(x_{k+1})\cdots H_s(x_n)]|\gamma N\rangle, \tag{A1·1}$$

$$\langle\pi N|T|\pi N\rangle=\frac{1}{2iq}\sum_{n=1}^{\infty}\sum_{k=0}^{n}\frac{(-i)^{n+1}}{(n+1)!}\int\cdots\int d^4x_1\cdots d^4x_n d^4y \\ \times\langle\pi N|P[H_s(x_1)\cdots H_s(x_k)H_s(y)H_s(x_{k+1})\cdots H_s(x_n)]|\pi N\rangle, \tag{A1·2}$$

where P denotes the time-ordered product and y is the coordinate of the point where the incident photon (or pion) is absorbed by a nucleon-line (c.f. the footnote on page 602). Using the explicit form of the interaction Hamiltonian (2·1) and (2·2), we obtain

$$\langle\pi N|T|\gamma N\rangle=\frac{1}{2ik}\sum_{n=1}^{\infty}\frac{(-i)^{n+1}}{n!}\int\cdots\int d^4x_1\cdots d^4x_n d^4y \\ \times\langle\pi N|P\Big[H_s(x_1)\cdots H_s(x_n)\frac{-e}{4M}\bar{\psi}(y)(\mu_v\tau_3 \\ +\mu_s)\sigma_{\mu\nu}\psi(y)\Big]i(k_\mu\epsilon_\nu-k_\nu\epsilon_\mu)\frac{e^{iky}}{\sqrt{2(2\pi)^3\omega_k}}|N\rangle, \tag{A1·3}$$

$$\langle\pi N|T|\pi N\rangle=\frac{1}{2iq}\sum_{n=1}^{\infty}\frac{(-i)^{n+1}}{n!}\int\cdots\int d^4x_1\cdots d^4x_n d^4y \\ \times\langle\pi N|P\Big[H_s(x_1)\cdots H_s(x_n)\frac{if}{m}\bar{\psi}(y)\gamma_5\gamma_\mu\tau_\alpha\psi(y)\Big]iq_\mu\frac{e^{iqy}}{\sqrt{2(2\pi)^3\omega_q}}|N\rangle, \tag{A1·4}$$

where α is a sufix corresponding to the isospin state of incident pion. In the static limit $(p/E_p+M\ll 1)$, the above equations become

$$\langle \pi N|T|\gamma N\rangle = \frac{1}{2ik}\sum_{n=1}^{\infty}\frac{(-i)^{n+1}}{n!}\int\cdots\int d^4x_1\cdots d^4x_n d^4y$$
$$\langle \pi N|P[H_s(x_1)\cdots H_s(x_n)\bar{\psi}(y)\boldsymbol{\sigma}\tau_3\psi(y)]|N\rangle\left(\frac{-e\mu_v}{2M}\right)i(\boldsymbol{k}\times\boldsymbol{\epsilon})\frac{e^{iky}}{\sqrt{2(2\pi)^3\omega_k}}, \qquad (\text{A1}\cdot 5)$$

$$\langle \pi N|T|\pi N\rangle = \frac{1}{2iq}\sum_{n=1}^{\infty}\frac{(-i)^{n+1}}{n!}\int\cdots\int d^4x_1\cdots d^4x_n d^4y$$
$$\langle \pi N|P[H_s(x_1)\cdots H_s(x_n)\bar{\psi}(y)\boldsymbol{\sigma}\tau_\alpha\psi(y)]|N\rangle i\frac{f}{m}\boldsymbol{q}\frac{e^{iqy}}{\sqrt{2(2\pi)^3\omega_q}}. \qquad (\text{A1}\cdot 6)$$

Since the interaction Hamiltonians are invariant with respect to the space-rotation, the common factor between (A1·5) and (A1·6) transforms as a vector. When we represent this common factor by $\boldsymbol{V}$, Eq. (A1·5) and (A1·6) become,

$$T_{\text{anom}} = -\frac{e\mu_v}{2M}\boldsymbol{H}\cdot\langle \pi N|\boldsymbol{V}|N'\rangle, \qquad (\text{A1}\cdot 7)$$

$$T_{\pi N} = i\frac{f}{m}\boldsymbol{q}\cdot\langle \pi N|\boldsymbol{V}|N\rangle. \qquad (\text{A1}\cdot 8)$$

Thus we have arrived at the final results stated in § 2.

Appendix 2

(a) *Decomposition of the πN scattering amplitudes into partial waves*

In the system of two free particles π and N, the eigenfunction of the total angular momentum J and the orbital angular momentum l is given by the spherical harmonics Y_{lm} and spherical Bessel function j_l as follows:

$$\Psi_{q,JM}^{(1/2,l)}(r,\theta,\varphi) = \sum_{s,m}(lm, 1/2s|l1/2;JM)\,2qj_l(qr)\,Y_{lm}(\theta,\varphi)\,u_s, \qquad (\text{A2}\cdot 1)$$

with the normalization condition

$$\int \Psi_{q',J'M'}^{(1/2,l')*}\Psi_{q,JM}^{(1,2,l)}r^2 dr d\Omega = \delta(q-q')\delta_{ll'}\delta_{JJ'}\delta_{MM'}, \qquad (\text{A2}\cdot 2)$$

where u_s is the spin wave function. Let us calculate the expectation value of transition matrix (2·6) using the wave function (A2·1). The partial wave amplitudes $f_{l\pm}$ become

$$f_{l\pm} = i\frac{f}{m}\langle l1/2;JM|\boldsymbol{V}\cdot\boldsymbol{q}|l1/2;JM\rangle, \qquad (\text{A2}\cdot 3)$$

which can be rewritten by using the usual reduction formula[11] as follows:

$$\langle l1/2;JM|\boldsymbol{V}\cdot\boldsymbol{q}|l1/2;JM\rangle = \frac{1}{2J+1}\sum_{l'J'}(-1)^{J'-J}$$
$$\times(l1/2;J\|V\|l'1/2;J')(l'1/2;J'\|q\|l1/2;J). \qquad (\text{A2}\cdot 4)$$

Because of the kinematical independence of the momentum q and the spin, the final factor in Eq. (A2·4) can be further simplified,

$$\langle l1/2;JM|\boldsymbol{V}\cdot\boldsymbol{q}|l1/2;JM\rangle=\frac{1}{\sqrt{2J+1}}\sum_{l'J'}(-1)^{J'+l+1/2}\sqrt{2J'+1}\times\begin{Bmatrix} l & J & 1/2 \\ J' & l' & 1 \end{Bmatrix}(l1/2;J\|V\|l'1/2;J')\,(l'\|q\|l), \tag{A2·5}$$

where $\begin{Bmatrix} l & J\,1/2 \\ J' & l' & 1 \end{Bmatrix}$ denotes the 6-j symbol,[11] and l' takes only the value $l-1$ or $l+1$ for non-vanishing $(l'\|q\|l)$.

The operator V is divided into a spin non-flip part and a spin flip part as follows:

$$\boldsymbol{V}=\boldsymbol{V}_a+\frac{i}{\sqrt{2}}\boldsymbol{V}_b\times\boldsymbol{\sigma}, \tag{A2·6}$$

where $\boldsymbol{V}_a$ and $\boldsymbol{V}_b$ depend only on the orbital part. Then, the reduced matrix element of $\boldsymbol{V}$ becomes by using the usual reduction formula

$$(l1/2;J\|V\|l'1/2;J')=(-1)^{l+1/2+J'+1}\sqrt{(2J+1)(2J'+1)}\times(l\|V_a\|l')\begin{Bmatrix} l & J & 1/2 \\ J' & l' & 1 \end{Bmatrix}+\sqrt{3(2J+1)(2J'+1)}\times\begin{Bmatrix} l & 1/2 & J \\ l' & 1/2 & J' \\ 1 & 1 & 1 \end{Bmatrix}(l\|V_b\|l')\,(1/2\|\sigma\|1/2), \tag{A2·7}$$

where $\begin{Bmatrix} l & 1/2 & J \\ l' & 1/2 & J' \\ 1 & 1 & 1 \end{Bmatrix}$ is a 9-j symbol. Since $(1/2\|\sigma\|1/2)$ is equal to $\sqrt{6}$, Eq. (A2·7) is rewritten as

$$(l1/2;J\|V\|l'1/2;J')=\sqrt{(2J+1)(2J'+1)}\left[\begin{Bmatrix} J & l & 1/2 \\ J' & l' & 1/2 \\ 1 & 1 & 0 \end{Bmatrix}\sqrt{6}(l\|V_a\|l')+\begin{Bmatrix} 1 & 1 & 1 \\ l & 1/2 & J \\ l' & 1/2 & J' \end{Bmatrix}3\sqrt{2}(l\|V_b\|l')\right]. \tag{A2·8}$$

If we substitute (A2·8) into (A2·5) and sum with respect to J', we obtain

$$(l1/2;JM|\boldsymbol{V}\cdot\boldsymbol{q}|l1/2;JM)=\sum_{l'}(-1)^{J+l'+1/2}\left[\begin{Bmatrix} l & 1 & l' \\ 1 & l & 0 \end{Bmatrix}\begin{Bmatrix} 1/2 & 0 & 1/2 \\ l & J & l \end{Bmatrix}\times\sqrt{6}(l\|V_a\|l')-\begin{Bmatrix} l & 1 & l' \\ 1 & l & 1 \end{Bmatrix}\begin{Bmatrix} 1/2 & 1 & 1/2 \\ l & J & l \end{Bmatrix}3\sqrt{2}(l\|V_b\|l')\right](l'\|q\|l), \tag{A2·9}$$

where we have used the relation with 6-j and 9-j symbols (see reference 11)). The reduced matrix element of V_a is expressed in the form

$$(l\|V_a\|l')\propto\int j_l(qr)\,V_a(r)\,j_{l'}(qr)\,r^2dr\,. \tag{A2·10}$$

(See Eq. (A2·1).) According to the assumption of § 2 that the exchanging particle is the nucleon, this range is smaller than $1/M$. Therefore, the integral of (A2·10) involves mainly the contribution from the region $qr<q/M$. In the energy range discussed in this paper, as q/M is smaller than 0.6, the spherical Bessel function of order $l+1$ is negligibly small compared with that of order $l-1$ (see Table II). Thus, the scattering amplitude $f_{l\pm}$ becomes

Table II. Values of spherical Bessel functions $j_l(qr)$.

qr	j_0	j_1	j_2	j_3	j_4
0.5	0.958	0.162	0.0163	0.0011	0.00006
1.0	0.841	0.301	0.0620	0.0090	0.00101

$$f_{l\pm}=i\frac{f}{m}(-1)^{J+l-1/2}\left[\begin{Bmatrix} l & 1 & l-1\\ 1 & l & 0\end{Bmatrix}\begin{Bmatrix} 1/2 & 0 & 1/2\\ l & J & l\end{Bmatrix}\sqrt{6}\,(l\|V_a\|l-1)\right.$$
$$\left.-\begin{Bmatrix} l & 1 & l-1\\ 1 & l & 1\end{Bmatrix}\begin{Bmatrix} 1/2 & 1 & 1/2\\ l & J & l\end{Bmatrix}3\sqrt{2}\,(l\|V_b\|l-1)\right](l-1\|q\|l)\,. \tag{A2·11}$$

(b) *Decomposition of pion photoproduction amplitudes due to the magnetic moment*

The photon field is expanded into a sum of electric and magnetic multipole radiations each of which has a definite angular momentum and parity as follows:

$$\Psi^E_{l,m}=\sqrt{\frac{l+1}{2l+1}}\,i\sum_{\alpha,m'}(l-1,\,m',\,1,\,\alpha|l-1,\,1;\,lm)\,2kj_{l-1}Y_{l-1,m'}\epsilon_\alpha$$
$$+\sqrt{\frac{l}{2l+1}}\,i\sum_{\alpha,m'}(l+1,\,m',\,1,\,\alpha|l+1,\,1;\,lm)\,2kj_{l+1}Y_{l+1,m'}\epsilon_\alpha\,, \tag{A2·12}$$

$$\Psi^M_{lm}=i\sum_{\alpha m'}(lm',\,1\alpha|l1;\,lm)\,2kj_lY_{lm'}\epsilon_\alpha\,, \tag{A2·13}$$

where Ψ^E_{lm} and Ψ^M_{lm} are the electric and magnetic multipole radiations, k and ϵ_α are a c.m. momentum and a polarization of the photon respectively. The eigen-states of γN system with respect to the angular momentum are written in terms of the wave functions (A2·12), (A2·13) and the spin wave function u_s of nucleon:

$$|El,\,1/2;\,JM\rangle=\sum_{m,s}(lm,\,1/2s|l1/2;\,JM)\,\Psi^E_{lm}u_s\,, \tag{A2·14}$$

$$|Ml,\,1/2;\,JM\rangle=\sum_{m,s}(lm,\,1/2s|l1/2;\,JM)\,\Psi^M_{lm}u_s\,. \tag{A2·15}$$

When we represent the wave functions on the right-hand side of (A2·12)～(A2·15) by

$$|lm, 1/2s\rangle = 2kj_l Y_{lm} u_s\,, \tag{A2·16}$$

$$|1\alpha\rangle = \epsilon_\alpha\,, \tag{A2·17}$$

(A2·14) and (A2·15) become as follows:

$$|El, 1/2; JM\rangle = i\sum_{msl'm'\alpha} h_l(l')\,(l'm', 1\alpha|l'1; lm) \times (lm, 1/2s|l1/2; JM)\,|l'm', 1/2s\rangle|1\alpha\rangle\,, \tag{A2·18}$$

$$|Ml, 1/2; JM\rangle = i\sum_{msm'\alpha} (lm', 1\alpha|l1; lm)\,(lm, 1/2s|l1/2; JM)\,|lm', 1/2s\rangle|1\alpha\rangle\,, \tag{A2·19}$$

where

$$h_l(l-1) = \sqrt{\frac{l+1}{2l+1}}\,,$$

$$h_l(l+1) = \sqrt{\frac{l}{2l+1}}$$

and for other l' $h_l(l')=0$.

The electric and magnetic multipole amplitudes $E_{l\pm}$ and $M_{l\pm}$ are defined by the following normalization:

$$M_{l\pm} = \frac{1}{\sqrt{l(l+1)}}\langle l1/2; JM|T|Ml, 1/2; JM\rangle\,, \tag{A2·20}$$

$$E_{l+} = \frac{1}{\sqrt{(l+1)(l+2)}}\langle l1/2; JM|T|E(l+1), 1/2; JM\rangle, \tag{A2·21}$$

$$E_{l-} = \frac{-1}{\sqrt{l(l-1)}}\langle l1/2; JM|T|E(l-1), 1/2; JM\rangle\,, \tag{A2·22}$$

where T is the transition matrix of photoproduction. Equation (2·5) is expressed by the polarization $\boldsymbol{\epsilon}$ of photon as follows:

$$T_{\text{anom}} = -\frac{e\mu_v}{2M} i\boldsymbol{V}\cdot(\boldsymbol{k}+\boldsymbol{\epsilon})\,. \tag{A2·23}$$

Since ϵ_α corresponds to the anihilation of a photon, the factor of the right-hand side of (A2·23) is rewritten as

$$\boldsymbol{V}\cdot(\boldsymbol{k}\times\boldsymbol{\epsilon}) = -\sqrt{2}i\sum_\alpha [V^{(1)}\times k^{(1)}]_\alpha^{(1)}\langle 1\alpha|\,, \tag{A2·24}$$

where we have used the tensor product of rank 1. By a further use of (A2·19), (A2·20), the above expression becomes,

$$M_{l\pm}^{\text{anom}} = -\frac{e\mu_v}{2M} i \frac{1}{\sqrt{l(l+1)}} \sum_{m,s,m',\alpha} \sqrt{2}\langle l1/2; JM| \times [V^{(1)}\times k^{(1)}]_\alpha^{(1)}|lm', 1/2s\rangle (lm', 1\alpha|l1; lm)\,(lm, 1/2s|l1/2; JM)\,. \tag{A2·25}$$

Applying the relation[11] between 3-j symbols and 6-j symbol, we obtain

$$M_{l\pm}^{\text{anom}} = -\frac{e\mu_v}{2M} i \frac{1}{\sqrt{l(l+1)}} \sum_{J'l'} \frac{(-1)^{J'+l-1/2}}{\sqrt{2J+1}} \sqrt{2}\sqrt{(2J'+1)(2l+1)}$$
$$\times (l1/2; J\| (V^{(1)} \times k^{(1)})^{(1)} \| l'1/2; J') \begin{Bmatrix} l & J & 1/2 \\ J' & l' & 1 \end{Bmatrix}. \qquad \text{(A2·26)}$$

By decomposing the tensor product $(V\times k)^{(1)}$, (A2·26) becomes

$$M_{l\pm}^{\text{anom}} = -\frac{e\mu_v}{2M} i \frac{1}{\sqrt{l(l+1)}} \sqrt{6} \frac{\sqrt{2l+1}}{\sqrt{2J+1}} \sum_{l'J'} (-1)^{J'+l+1/2} \sqrt{2J'+1}$$
$$\times \begin{Bmatrix} l & J & 1/2 \\ J' & l' & 1 \end{Bmatrix} (l1/2; J\|V\| l'1/2; J') (l' \|k\| l) \begin{Bmatrix} l & 1 & l' \\ 1 & l & 1 \end{Bmatrix}. \qquad \text{(A2·27)}$$

This expression is the same as Eq. (A2·5) except the factor $\sqrt{6}\sqrt{2l+1}\begin{Bmatrix} l & 1 & l' \\ 1 & l & 1 \end{Bmatrix}$. By following the same line of argument as the case of πN scattering, V is divided into spin-flip and non-flip parts as follows:

$$M_{l\pm}^{\text{anom}} = -\frac{e\mu_v}{2M} i \frac{1}{\sqrt{l(l+1)}} \sqrt{6}\sqrt{2l+1} (-1)^{J+l-1/2} \begin{Bmatrix} l & 1 & l-1 \\ 1 & l & 1 \end{Bmatrix}$$
$$\times \left[\begin{Bmatrix} l & 1 & l-1 \\ 1 & l & 0 \end{Bmatrix} \begin{Bmatrix} 1/2 & 0 & 1/2 \\ l & J & l \end{Bmatrix} \sqrt{6} (l \| V_a \| l-1) \right. \qquad \text{(A2·28)}$$
$$\left. - \begin{Bmatrix} l & 1 & l-1 \\ 1 & l & 1 \end{Bmatrix} \begin{Bmatrix} 1/2 & 1 & 1/2 \\ l & J & l \end{Bmatrix} 3\sqrt{2} (l \| V_b \| l-1) \right] (l-1 \|k\| l).$$

Since the difference of the momenta q and k is negligibly small above 300 MeV of the laboratory photon energy, we obtain the relation between $M_{l\pm}^{\text{anom}}$ and $f_{l\pm}$ from (A2·11) and (A2·28),

$$M_{l\pm}^{\text{anom}} = -\frac{e\mu_v}{2M} \frac{m}{f} \frac{\sqrt{6}\sqrt{2l+1}}{\sqrt{l(l+1)}} \begin{Bmatrix} l & 1 & l-1 \\ 1 & l & 1 \end{Bmatrix} f_{l\pm}$$
$$= \frac{e\mu_v}{g} \frac{1}{l} f_{l\pm}, \qquad \text{(A2·29)}$$

where $g = 2Mf/m$.

Appendix 3

There is no essential difficulty in the partial wave decomposition of the Born amplitudes, but it may be quite useful for the practical calculation to give an explicit formula of this decomposition. Therefore in the present appendix we dare give many formula concerning this decomposition especially for the two cases (a) πN scattering (b) pion photoproduction.

(a) *Partial wave decomposition of the Born amplitude of πN elastic scattering*

As is well known,[12] the transition matrix of πN elastic scattering is expressed by the invariant amplitudes A and B:

$$T_{\pi N} = -A + i\gamma \cdot QB\,, \tag{A3·1}$$

where $Q = (q_i + q_f)/2$ and q_i, q_f denote the four-momenta of initial and final pions respectively. In order to calculate the Born amplitude stated in § 3, we take the off-shell masses m_x and M_x for the final pion and nucleon masses respectively (namely $q_f^2 = -m_x^2$ and $p_f^2 = -M_x^2$). Then, the Born amplitude corresponding

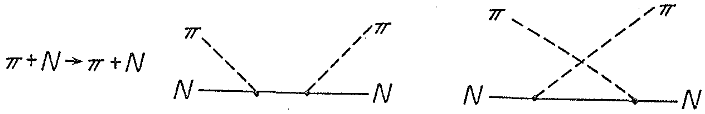

Fig. 8. Born diagrams for πN scattering.

to the diagrams shown in Fig. 8 becomes

$$A_{\beta\alpha} = \frac{f^2}{m^2}\Big[\Big(\frac{3}{2}M + \frac{1}{2}M_x\Big)(\tau_\beta\tau_\alpha + \tau_\alpha\tau_\beta) + \frac{M(M_x^2 - M^2)}{q_i^2 + 2p_i\cdot q_i}\tau_\beta\tau_\alpha + \frac{M(M_x^2 - M^2)}{q_f^2 - 2p_i\cdot q_f}\tau_\alpha\tau_\beta\Big], \tag{A3·2}$$

$$B_{\beta\alpha} = -\frac{f^2}{m^2}\Big[(\tau_\beta\tau_\alpha - \tau_\alpha\tau_\beta) - \frac{2M(M_x + M)}{q_i^2 + 2p_i\cdot q_i}\tau_\beta\tau_\alpha + \frac{2M(M_x + M)}{q_f^2 - 2p_i\cdot q_f}\tau_\alpha\tau_\beta\Big], \tag{A3·3}$$

where τ_α and τ_β are the isospin matrices, α and β are the isospin states of initial and final pions respectively. The amplitudes for the eigenstates of the total isotopic spin are represented by the following relations:

$$A^{(3/2)} = A^{(+)} - A^{(-)}, \quad A^{(1/2)} = A^{(+)} + 2A^{(-)}, \tag{A3·4}$$

$$A_{\beta\alpha} = \delta_{\beta\alpha}A^{(+)} + \tfrac{1}{2}[\tau_\beta, \tau_\alpha]A^{(-)}. \tag{A3·5}$$

By using the usual notations[11] f_1 and f_2, the partial wave amplitudes $f_{l\pm}$ are related as follows:

$$f_1 = \sum_{l=0}^{\infty} f_{l+}P'_{l+1}(z) - \sum_{l=2}^{\infty} f_{l-}P'_{l-1}(z), \quad f_2 = \sum_{l=1}^{\infty}(f_{l-} - f_{l+})P_l'(z), \tag{A3·6}$$

where

$$f_1 = \frac{1}{4\pi}\xi\Big[A + \Big(W - \frac{M + M_x}{2}\Big)B\Big], \quad f_2 = \frac{1}{4\pi}\zeta\Big[-A + \Big(W + \frac{M + M_x}{2}\Big)B\Big]. \tag{A3·7}$$

Here W is a total energy, $z=\cos\theta$ and

$$\xi=\frac{\sqrt{(E_i+M)(E_f+M_x)}}{2W}, \tag{A3·8}$$

$$\zeta=\frac{\sqrt{(E_i-M)(E_f-M_x)}}{2W} \tag{A3·9}$$

in the c.m. system, E_i and E_f are the energies of the initial and final nucleons in the c.m. system. Let us introduce the following amplitudes h_1 and h_2:

$$h_1=f_1+zf_2, \tag{A3·10}$$

$$h_2=f_2, \tag{A3·11}$$

which are written as

$$h_1=\sum_{l=0}^{\infty}[(l+1)f_{l+}+lf_{l-}]P_l(z), \tag{A3·12}$$

$$h_2=\sum_{l=1}^{\infty}[f_{l-}-f_{l+}]P_l'(z). \tag{A3·13}$$

From Eqs. (A3·2) and (A3·3), the amplitudes h_1 and h_2 for the Born diagrams become

$$h_1^{(\pm)}=\frac{-g^2}{4\pi}\left[\left\{\frac{W^2-2M_xW+M^2}{4M^2(W+M)}\xi+\frac{W^2+2M_xW+M^2}{4M^2(W-M)}\zeta z\right\}\right.$$
$$\left.\pm\left\{\frac{\xi(W-M_x)+z\zeta(W+M_x)}{\alpha z+\beta}\frac{M_x+M}{2M}-\frac{\xi(W+M)+z\zeta(W-M)}{4M^2}\right\}\right], \tag{A3·14}$$

$$h_2^{(\pm)}=\frac{-g^2}{4\pi}\left[\frac{W^2+2M_xW+M^2}{4M^2(W-M)}\zeta\pm\left\{\frac{\zeta(W+M_x)}{\alpha z+\beta}\frac{M_x+M}{2M}-\frac{\zeta(W-M)}{4M^2}\right\}\right], \tag{A3·15}$$

where $\alpha=2|\boldsymbol{q}_i|\cdot|\boldsymbol{q}_f|$ and $\beta=2E_i\sqrt{m_x^2+\boldsymbol{q}_f^2}-m_x^2$. By using the orthogonality of Legendre polynominals, the partial wave amplitudes are given by

$$(l+1)f_{l+}+lf_{l-}=\frac{2l+1}{2}\int_{-1}^{1}h_1P_l(z)\,dz, \tag{A3·16}$$

$$f_{l-}-f_{l+}=\frac{2l+1}{2l(l+1)}\int_{-1}^{1}h_2(1-z^2)P_l'(z)\,dz. \tag{A3·17}$$

Let us consider the following functions u_l and v_l:

$$u_l=\frac{2l+1}{2}\int_{-1}^{1}\frac{1}{\alpha z+\beta}P_l(z)\,dz, \tag{A3·18}$$

$$v_l=\frac{2l+1}{2l(l+1)}\int_{-1}^{1}\frac{1-z^2}{\alpha z+\beta}P_l'(z)\,dz. \tag{A3·19}$$

The explicit forms of u_l and v_l are shown in Table III.

Table III. Functions u_l, v_l and w_l. $(\delta=\beta/\alpha)$

$$u_0(\delta)=\frac{1}{2\alpha}\log\frac{\delta+1}{\delta-1},$$

$$u_1(\delta)=\frac{3}{2\alpha}\left[2-\delta\log\frac{\delta+1}{\delta-1}\right],$$

$$u_2(\delta)=\frac{5}{2\alpha}\left[-\frac{6}{2}\delta+\left(\frac{3}{2}\delta^2-\frac{1}{2}\right)\log\frac{\delta+1}{\delta-1}\right],$$

$$u_3(\delta)=\frac{7}{2\alpha}\left[\frac{10}{2}\delta^2-\frac{4}{3}+\left(-\frac{5}{2}\delta^3+\frac{3}{2}\delta\right)\log\frac{\delta+1}{\delta-1}\right],$$

$$u_4(\delta)=\frac{9}{2\alpha}\left[-\frac{35}{4}\delta^3+\frac{55}{12}\delta+\left(\frac{35}{8}\delta^4-\frac{15}{4}\delta^2+\frac{3}{8}\right)\log\frac{\delta+1}{\delta-1}\right],$$

$$v_1(\delta)=\frac{3}{4\alpha}\left[2\delta+(-\delta^2+1)\log\frac{\delta+1}{\delta-1}\right],$$

$$v_2(\delta)=\frac{5}{4\alpha}\left[-2\delta^2+\frac{4}{3}+(\delta^3-\delta)\log\frac{\delta+1}{\delta-1}\right],$$

$$v_3(\delta)=\frac{7}{16\alpha}\left[10\delta^3-\frac{26}{3}\delta+(-5\delta^4+6\delta^2-1)\log\frac{\delta+1}{\delta-1}\right],$$

$$v_4(\delta)=\frac{9}{16\alpha}\left[-14\delta^4+\frac{46}{3}\delta^2-\frac{32}{15}+(7\delta^5-10\delta^3+3\delta)\log\frac{\delta+1}{\delta-1}\right],$$

$$w_2(\delta)=\frac{5}{16\alpha}\left[-2\delta^3+\frac{10}{3}\delta+(\delta^4-2\delta^2+1)\log\frac{\delta+1}{\delta-1}\right],$$

$$w_3(\delta)=\frac{7}{16\alpha}\left[2\delta^4-\frac{10}{3}\delta^2+\frac{16}{15}+(-\delta^5+2\delta^3-\delta)\log\frac{\delta+1}{\delta-1}\right],$$

$$w_4(\delta)=\frac{3}{32\alpha}\left[-14\delta^5+\frac{76}{3}\delta^3-\frac{54}{5}\delta+(7\delta^6-15\delta^4+9\delta^2-1)\log\frac{\delta+1}{\delta-1}\right].$$

Then we obtain the partial wave amplitudes as follows (for $l\geqq 2$):

$$f_{l+}^{(\pm)}=\mp\frac{g^2}{4\pi}\frac{1}{2l+1}\left[\xi(W-M_x)u_l-\frac{\zeta\beta}{\alpha}(W+M_x)\left(u_l+l\frac{\alpha}{\beta}v_l\right)\right]\frac{M_x+M}{2M},\tag{A3·20}$$

$$f_{l-}^{(\pm)}=\mp\frac{g^2}{4\pi}\frac{1}{2l+1}\Big[\xi(W-M_x)u_l$$
$$-\frac{\zeta\beta}{\alpha}(W+M_x)\left(u_l-(l+1)\frac{\alpha}{\beta}v_l\right)\Big]\frac{M_x+M}{2M},\tag{A3·21}$$

$$f_{1+}^{(\pm)}=\mp\frac{g^2}{4\pi}\frac{1}{3}\left[\xi(W-M_x)u_1-\frac{\zeta\beta}{\alpha}(W+M_x)\left(u_1+\frac{\alpha}{\beta}v_1\right)\right]\frac{M_x+M}{2M},\tag{A3·22}$$

$$f_{1-}^{(\pm)} = -\frac{g^2}{4\pi}\left[\frac{W^2+2M_\pi W+M^2}{4M^2(W-M)}\zeta \pm \left\{-\frac{W-M}{4M^2}\zeta + \frac{\xi(W-M_\pi)(M+M_\pi)}{6M}u_1 - \frac{\zeta\beta(W+M_\pi)(M+M_\pi)}{6\alpha M}\left(u_1 - 2\frac{\alpha}{\beta}v_1\right)\right\}\right], \tag{A3·23}$$

$$f_{0+}^{(\pm)} = -\frac{g^2}{4\pi}\left[\frac{W^2-2M_\pi W+M^2}{4M^2(W+M)}\xi \pm \left\{-\frac{W+M}{4M^2}\xi + \frac{\xi(W-M_\pi)(M+M_\pi)}{2M}u_0 - \frac{\zeta\beta(W+M_\pi)(M+M_\pi)}{2\alpha M}\left(u_0 - \frac{1}{\beta}\right)\right\}\right]. \tag{A3·24}$$

(b) *Born amplitude of pion photoproduction*

The transition amplitude is expressed by the four invariant amplitudes[1] A, B, C and D:

$$T = i\gamma_5\gamma\cdot\epsilon\gamma\cdot kA + 2i\gamma_5(P\cdot\epsilon q\cdot k - P\cdot kq\cdot\epsilon)B + \gamma_5(\gamma\cdot\epsilon q\cdot k - \gamma\cdot kq\cdot\epsilon)C + 2\gamma_5(\gamma\cdot\epsilon P\cdot k - \gamma\cdot kP\cdot\epsilon - iM\gamma\cdot\epsilon\gamma\cdot k)D, \tag{A3·25}$$

where $P=(p_i+p_f)/2$ and ϵ is a polarization of the photon. For the Born-type

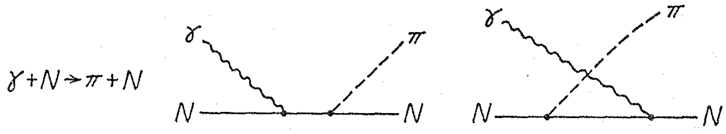

Fig. 9. Born diagrams for the pion photoproduction due to the magnetic moment of nucleon.

diagram shown in Fig. 9, the invariant amplitudes A, B, C and D become

$$A = \frac{ge\mu_v}{2M}\left[\frac{1}{2M}(\tau_\beta\tau_3+\tau_3\tau_\beta) + \frac{M-M_\pi}{q^2-2q\cdot p_i}\tau_3\tau_\beta\right], \tag{A3·26}$$

$$B = 0, \tag{A3·27}$$

$$C = \frac{ge\mu_v}{2M}\left[-\frac{1}{W^2-M^2}\tau_\beta\tau_3 - \frac{1}{q^2-2q\cdot p_i}\tau_3\tau_\beta\right], \tag{A3·28}$$

$$D = \frac{ge\mu_v}{2M}\left[-\frac{1}{W^2-M^2}\tau_\beta\tau_3 + \frac{1}{q^2-2q\cdot p_i}\tau_3\tau_\beta\right]. \tag{A3·29}$$

Let us introduce the amplitudes h_1, h_2, h_3 and h_4 which are related with the usual amplitudes F_1, F_2, F_3 and F_4 defined in reference 1), in the following way:

$$h_1 = 2F_1 - 2zF_2 + (1-z^2)F_4, \tag{A3·30}$$

$$h_2 = F_2, \tag{A3·31}$$

$$h_3 = F_3 + zF_4, \tag{A3·32}$$

$$h_4 = F_4. \tag{A3·33}$$

Let these quantities be decomposed into the sums of the electromagnetic multipole amplitudes:

$$h_1=\sum_l[l(l+1)M_{l+}+(l+1)(l+2)E_{l+}-l(l+1)M_{l-}+l(l-1)E_{l-}]P_l(z), \tag{A3·34}$$

$$h_2=\sum_l[(l+1)M_{l+}+lM_{l-}]P_l'(z), \tag{A3·35}$$

$$h_3=\sum_l[-(l+2)M_{l+}+(l+2)E_{l+}-(l-1)M_{l-}-(l-1)E_{l-}]P_l'(z), \tag{A3·36}$$

$$h_4=\sum_l[M_{l+}-E_{l+}-M_{l-}-E_{l-}]P_l''(z). \tag{A3·37}$$

In the Born approximation corresponding to the diagrams shown in Fig. 9, these h take the form

$$\begin{aligned}h_1^{(\substack{+0\\-})}=&-\frac{ge\mu}{4\pi}\frac{W-M}{2M}\Big[2\xi\Big(\frac{1}{W+M}-\frac{1}{2M}\Big)-2\zeta\frac{E_i+M}{|\boldsymbol{k}|}\Big(\frac{1}{W-M}+\frac{1}{2M}\Big)z\\&\pm\Big\{2\xi\Big(-\frac{1}{W-M}-\frac{1}{2M}+\eta_1\frac{1}{\alpha z+\beta}\Big)+2\zeta\frac{E_i+M}{|\boldsymbol{k}|}\Big(-\frac{1}{2M}z\\&-\frac{W+M-2|\boldsymbol{k}|}{2|\boldsymbol{k}|(W+M)}z-\frac{\eta_2-\varepsilon|\boldsymbol{q}|}{\alpha}+\frac{\beta\eta_2}{\alpha}\frac{1}{\alpha z+\beta}\Big)\Big\}\Big],\end{aligned} \tag{A3·38}$$

$$\begin{aligned}h_2^{(\substack{+0\\-})}=&-\frac{ge\mu}{4\pi}\frac{W-M}{2M}\zeta\frac{E_i+M}{|\boldsymbol{k}|}\Big[\frac{1}{W-M}+\frac{1}{2M}\\&\pm\Big(-\frac{1}{W+M}+\frac{1}{2M}+\frac{\eta_3}{\alpha z+\beta}\Big)\Big],\end{aligned} \tag{A3·39}$$

$$h_3^{(\substack{+0\\-})}=\mp\frac{ge\mu}{4\pi}\frac{W-M}{2M}\Big[2\xi\frac{\varepsilon\eta_4}{\alpha z+\beta}+2\zeta\frac{E_i+M}{|\boldsymbol{k}|}\eta_4\Big(\frac{\varepsilon}{\alpha}-\frac{1}{\alpha z+\beta}\Big)\Big], \tag{A3·40}$$

$$h_4^{(\substack{+0\\-})}=\mp\frac{ge\mu}{4\pi}\frac{W-M}{2M}\zeta\frac{E_i+M}{|\boldsymbol{k}|}2\eta_4\frac{\varepsilon}{\alpha z+\beta}, \tag{A3·41}$$

where we adopt the upper sign in the case of the isotopic superscript[1] $(+)$ or (0) and the lower sign in the case of superscript $(-)$. The value of the magnetic moment μ is equal to μ_v for the isotopic superscript $(+)$ or $(-)$ and μ_s for superscript (0). In addition,

$$\begin{aligned}&\alpha=2|\boldsymbol{k}||\boldsymbol{q}|, \qquad \beta=2E_i\omega_f-m_x^2,\\&\omega_f=\sqrt{\boldsymbol{q}^2+m_x^2}, \qquad \varepsilon=\alpha/\beta,\\&\xi=\sqrt{(E_i+M)(E_f+M_x)}/2W, \qquad \zeta=\sqrt{(E_i-M)(E_f-M_x)}/2W,\\&\eta_1=M_x-W+\frac{2W\omega_x-m_x^2}{W-M},\end{aligned} \tag{A3·42}$$

$$\eta_2=-W-M_x-\frac{\beta}{2|\boldsymbol{k}|}+\frac{2W\omega_x-m_x^2}{W+M}+\varepsilon^2\frac{\beta}{2|\boldsymbol{k}|}, \tag{A3·43}$$

$$\eta_3=-W-M_x+\frac{2W\omega_x-m_x^2}{W+M}, \tag{A3·44}$$

$$\eta_4=\frac{\beta}{2|\boldsymbol{k}|}. \tag{A3·45}$$

In addition to u_l and v_l, let us introduce the following function w_l:

$$w_l=\frac{2l+1}{2l(l-1)(l+1)(l+2)}\int_{-1}^{1}\frac{(1-z^2)^2}{\alpha z+\beta}P_l''(z)\,dz, \tag{A3·46}$$

which is explicitly shown in Table III. The multipole amplitudes of Born terms of γ-π production are given by (for $l\geqq 2$)

$$M_{l+}^{\binom{+0}{-}}=\mp\frac{ge\mu}{4\pi}\frac{W-M}{2M(l+1)(l+2)}\Big[\xi(\eta_1u_l-2\eta_4\varepsilon v_l)+\zeta\frac{E_i+M}{\varepsilon|\boldsymbol{k}|}\{\eta_2u_l+(l\eta_3+2\eta_4)\varepsilon v_l+(l-1)(l+2)\eta_4\varepsilon^2w_l\}\Big], \tag{A3·47}$$

$$M_{l-}^{\binom{+0}{-}}=\pm\frac{ge\mu}{4\pi}\frac{W-M}{2Ml(2l+1)}\Big[\xi(\eta_1u_l-2\eta_4\varepsilon v_l)+\zeta\frac{E_i+M}{\varepsilon|\boldsymbol{k}|}\{\eta_2u_l-((l+1)\eta_3-2\eta_4)\varepsilon v_l+(l-1)(l+2)\eta_4\varepsilon^2w_l\}\Big], \tag{A3·48}$$

$$E_{l+}^{\binom{+0}{-}}=\mp\frac{ge\mu}{4\pi}\frac{W-M}{2M(l+1)(2l+1)}\Big[\xi(\eta_1u_l+2l\eta_4\varepsilon v_l)+\zeta\frac{E_i+M}{\varepsilon|\boldsymbol{k}|}\{\eta_2u_l+l(\eta_3-2\eta_4)\varepsilon v_l-l(l-1)\eta_4\varepsilon^2w_l\}\Big], \tag{A3·49}$$

$$E_{l-}^{\binom{+0}{-}}=\mp\frac{ge\mu}{4\pi}\frac{W-M}{2Ml(2l+1)}\Big[\xi(\eta_1u_l-2(l+1)\eta_4\varepsilon v_l)+\zeta\frac{E_i+M}{\varepsilon|\boldsymbol{k}|}\{\eta_2u_l-(l+1)(\eta_3-2\eta_4)\varepsilon v_l-(l+1)(l+2)\eta_4\varepsilon^2w_l\}\Big], \tag{A3·50}$$

$$E_{0+}^{\binom{+0}{-}}=-\frac{ge\mu}{4\pi}\frac{W-M}{2M}\Big[\xi\Big(\frac{1}{W+M}-\frac{1}{2M}\Big)\pm\Big\{\xi\Big(-\frac{1}{W-M}-\frac{1}{2M}+\eta_1u_0\Big)+\zeta\frac{E_i+M}{\varepsilon|\boldsymbol{k}|}\Big(\eta_2\Big(u_0-\frac{\varepsilon}{\alpha}\Big)+\eta_4\frac{\varepsilon^3}{\alpha}\Big)\Big\}\Big], \tag{A3·51}$$

$$M_{1-}^{\binom{+0}{-}}=\frac{ge\mu}{4\pi}\frac{W-M}{2M}\frac{1}{3}\Big[-3\zeta\frac{E_i+M}{\varepsilon|\boldsymbol{k}|}\Big(\frac{1}{W-M}+\frac{1}{2M}\Big)\varepsilon\pm\Big\{\xi(\eta_1u_1-2\eta_4\varepsilon v_1)+\zeta\frac{E_i+M}{\varepsilon|\boldsymbol{k}|}\Big(\frac{3\varepsilon}{W+M}-\frac{3\varepsilon}{2M} \tag{A3·52}$$

$$-\frac{3\varepsilon}{2|\boldsymbol{k}|}+\eta_2 u_1-2(\eta_3-\eta_4)\varepsilon v_1\Big)\Big\}\Big],$$

$$M_{1+}^{(\substack{+0\\-}) }=\mp\frac{ge\mu}{4\pi}\frac{W-M}{2M}\frac{1}{6}\Big[\xi(\eta_1 u_1-2\eta_4\varepsilon v_1)+\zeta\frac{E_i+M}{\varepsilon|\boldsymbol{k}|}\Big\{\eta_2 u_1+(\eta_3+2\eta_4)\varepsilon v_1-3\eta_4\frac{\varepsilon^2}{\alpha}\Big\}\Big], \tag{A3·53}$$

$$E_{1+}^{(\substack{+0\\-}) }=\mp\frac{ge\mu}{4\pi}\frac{W-M}{2M}\frac{1}{6}\Big[\xi(\eta_1 u_1+2\eta_4\varepsilon v_1)+\zeta\frac{E_i+M}{\varepsilon|\boldsymbol{k}|}\Big\{\eta_2 u_1+(\eta_3-2\eta_4)\varepsilon v_1+\eta_4\frac{\varepsilon^2}{\alpha}\Big\}\Big]. \tag{A3·54}$$

The Born amplitudes due to electric charge are calculated by a method similiar to the magnetic moment part. The final results are (for $l\geqq 2$)

$$M_{l+}^{(\substack{+0\\-}) }=\mp\frac{ge}{8\pi}\frac{W-M}{(l+1)(2l+1)}\Big[\xi(u_l-2\lambda_2\varepsilon v_l)+\zeta\frac{E_i+M}{\varepsilon|\boldsymbol{k}|}\{-u_l-l\varepsilon v_l+\lambda_1((1-\varepsilon^2)u_l-2\varepsilon v_l-(l-1)(l+2)\varepsilon^2 w_l)\}-(1\mp 1)\Big\{-2\xi\lambda_2\varepsilon v_l'+\zeta\frac{E_i+M}{|\boldsymbol{k}|}\lambda_1\Big(\frac{\gamma^2-\alpha^2}{\alpha^2}\varepsilon u_l'-2\frac{\gamma}{\alpha}\varepsilon v_l'-(l-1)(l+2)\varepsilon w_l'\Big)\Big\}\Big], \tag{A3·55}$$

$$M_{l-}^{(\substack{+0\\-}) }=\pm\frac{ge}{8\pi}\frac{W-M}{l(2l+1)}\Big[\xi(u_l-2\lambda_2\varepsilon v_l)+\zeta\frac{E_i+M}{\varepsilon|\boldsymbol{k}|}\{-u_l+(l+1)\varepsilon v_l+\lambda_1((1-\varepsilon^2)u_l-2\varepsilon v_l-(l-1)(l+2)\varepsilon^2 w_l)\}-(1\mp 1)\Big\{-2\xi\lambda_2\varepsilon v_l'+\zeta\frac{E_i+M}{|\boldsymbol{k}|}\lambda_1\Big(\frac{\gamma^2-\alpha^2}{\alpha^2}\varepsilon u_l'-2\frac{\gamma}{\alpha}\varepsilon v_l'-(l-1)(l+2)\varepsilon w_l'\Big)\Big\}\Big], \tag{A3·56}$$

$$E_{l+}^{(\substack{+0\\-}) }=\mp\frac{ge}{8\pi}\frac{W-M}{(l+1)(2l+1)}\Big[\xi(u_l+2l\lambda_2\varepsilon v_l)+\zeta\frac{E_i+M}{\varepsilon|\boldsymbol{k}|}\{-u_l-l\varepsilon v_l+\lambda_1((1-\varepsilon^2)u_l+2l\varepsilon v_l+l(l-1)\varepsilon^2 w_l)\}-(1\mp 1)\Big\{2\xi\lambda_2 l\varepsilon v_l'+\zeta\frac{E_i+M}{|\boldsymbol{k}|}\lambda_1\Big(\frac{\gamma^2-\alpha^2}{\alpha^2}\varepsilon u_l'+2\frac{\gamma}{\alpha}l\varepsilon v_l'+l(l-1)\varepsilon w_l'\Big)\Big\}\Big], \tag{A3·57}$$

$$E_{l-}^{(\stackrel{+0}{-})}=\mp\frac{ge}{8\pi}\frac{W-M}{l(2l+1)}\Big[\xi(u_l-2(l+1)\lambda_2\varepsilon v_l)+\zeta\frac{E_i+M}{\varepsilon|\boldsymbol{k}|}\Big\{-u_l$$
$$+(l+1)\varepsilon v_l+\lambda_1((1-\varepsilon^2)u_l-2(l+1)\varepsilon v_l+(l+1)(l+2)\varepsilon^2 w_l)\Big\} \quad (A3\cdot58)$$
$$-(1\mp1)\Big\{-2\xi\lambda_2(l+1)\varepsilon v_l'+\zeta\frac{E_i+M}{|\boldsymbol{k}|}\lambda_1\Big(\frac{\gamma^2-\alpha^2}{\alpha^2}\varepsilon u_l'$$
$$-2\frac{\gamma}{\alpha}(l+1)\varepsilon v_l'+(l+1)(l+2)\varepsilon w_l'\Big)\Big\}\Big],$$

$$E_{0+}^{(\stackrel{+0}{-})}=\frac{ge}{8\pi}(W-M)\Big[\xi\frac{1}{W^2-M^2}\mp\Big\{\xi u_0+\zeta\frac{E_i+M}{\varepsilon|\boldsymbol{k}|}\Big((\lambda_1-1)\Big(u_0-\frac{\varepsilon}{\alpha}\Big)$$
$$-\lambda_1\varepsilon^2u_0\Big)\Big\}-(1\mp1)\zeta\frac{E_i+M}{|\boldsymbol{k}|}\lambda_1\Big(\frac{\gamma^2-\alpha^2}{\alpha^2}\varepsilon u_0'-\frac{\gamma}{\alpha}\frac{\varepsilon}{\alpha}\Big)\Big], \quad (A3\cdot59)$$

$$M_{1-}^{(\stackrel{+0}{-})}=-\frac{ge}{8\pi}\frac{W-M}{3}\Big[3\zeta\frac{E_i+M}{|\boldsymbol{k}|}\frac{1}{W^2-M^2}\mp\Big\{\xi(u_1-2\lambda_2\varepsilon v_1)$$
$$+\zeta\frac{E_i+M}{\varepsilon|\boldsymbol{k}|}\Big(-u_1+2\varepsilon v_1+\lambda_1\Big((1-\varepsilon^2)u_1-2\varepsilon v_1+3\frac{\varepsilon^2}{\alpha}\Big)\Big)\Big\} \quad (A3\cdot60)$$
$$-(1\mp1)\Big\{-2\xi\lambda_2\varepsilon v_1'+\zeta\frac{E_i+M}{|\boldsymbol{k}|}\lambda_1\Big(\frac{\gamma^2-\alpha^2}{\alpha^2}\varepsilon u_1'-2\frac{\gamma}{\alpha}\varepsilon v_1'+3\frac{\varepsilon}{\alpha}\Big)\Big\}\Big],$$

$$M_{1+}^{(\stackrel{+0}{-})}=\mp\frac{ge}{8\pi}\frac{W-M}{6}\Big[\xi(u_1-2\lambda_2\varepsilon v_1)+\zeta\frac{E_i+M}{\varepsilon|\boldsymbol{k}|}\Big\{-u_1-\varepsilon v_1$$
$$+\lambda_1\Big((1-\varepsilon^2)u_1-2\varepsilon v_1+3\frac{\varepsilon^2}{\alpha}\Big)\Big\}-(1\mp1)\Big\{-2\xi\lambda_2\varepsilon v_1' \quad (A3\cdot61)$$
$$+\zeta\frac{E_i+M}{|\boldsymbol{k}|}\lambda_1\Big(\frac{\gamma^2-\alpha^2}{\alpha^2}\varepsilon u_1'-2\frac{\gamma}{\alpha}\varepsilon v_1'+3\frac{\varepsilon}{\alpha}\Big)\Big\}\Big],$$

$$E_{1+}^{(\stackrel{+0}{-})}=\mp\frac{ge}{8\pi}\frac{W-M}{6}\Big[\xi(u_1+2\lambda_2\varepsilon v_1)+\zeta\frac{E_i+M}{\varepsilon|\boldsymbol{k}|}\Big\{-u_1+2\varepsilon v_1$$
$$+\lambda_1\Big((1-\varepsilon^2)u_1-2\varepsilon v_1+3\frac{\varepsilon^2}{\alpha}\Big)\Big\}-(1\mp1)\Big\{2\xi\lambda_2\varepsilon v_1' \quad (A3\cdot62)$$
$$+\zeta\frac{E_i+M}{|\boldsymbol{k}|}\lambda_1\Big(\frac{\gamma^2-\alpha^2}{\alpha^2}\varepsilon u_1'+2\frac{\gamma}{\alpha}\varepsilon v_1'-\frac{\varepsilon}{\alpha}\Big)\Big\}\Big],$$

where

$$\lambda_1=\frac{\beta}{2|\boldsymbol{k}|}\frac{1}{W-M},\qquad \lambda_2=\frac{\beta}{2|\boldsymbol{k}|}\frac{1}{W+M},$$
$$\gamma=-2|\boldsymbol{k}|\omega_f$$

and the functions u_l', v_l', w_l' are given by substituting γ for the functional variable β in u_l, v_l, w_l respectively.

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