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Elastoplastic Description for Friction Behavior of Interaction of Solids†

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Abstract

The subloading surface model is capable of rigorously describing the friction phenomenon between solids. In addition, it provides high efficiency in numerical calculation since the stress is attracted automatically to the yield surface. The model possesses the high capability of describing uniformly and rigorously the elastoplastic deformation and interaction of solids. This fact is analyzed and deliberated in this article.

KEY WORDS: (elastoplasticity), (constitutive equation), (cyclic plasticity), (friction theory), (rate-dependence), (subloading surface)

1. Introduction

A subloading surface model in the framework of the hypoelastic-based plastic constitutive equation is shown, which is based on the natural postulate that the plastic strain rate develops as the stress approaches the yield surface, describing pertinently the plastic strain rate induced by the rate of stress inside the yield surface. In addition to that, the pertinence and the generality for descriptions of elastoplastic deformation/sliding behavior of solids and the adaptability to the numerical calculation are materialized in the subloading surface model. Therefore, the subloading surface model would be regarded as providing the basic structure of interaction and has been studied briefly in this article.

2. Subloading-friction model

It is historically widely known that a friction resistance exhibits first a high friction, called the *static friction*, and thereafter tends to a low friction, called the *kinetic friction*. Further, it is also recognized that the static friction recovers if sliding commences again after the sliding ceases for a while.

The above-mentioned fundamental phenomena in friction can be described pertinently by extending the concept of the subloading surface which has been applied to the elastoplastic constitutive equation.

2.1 Decomposition of sliding velocity

The sliding velocity $\bar{\mathbf{v}}$ is defined as the relative velocity of the counter body to the main body and is additively decomposed into the normal part $\bar{\mathbf{v}}_n$ and the tangential part $\bar{\mathbf{v}}_t$ to the contact surface as follows :

$$\bar{\mathbf{v}} = \bar{\mathbf{v}}_n + \bar{\mathbf{v}}_t = -\bar{v}_n\mathbf{n} + \bar{v}_t\mathbf{t}_v \tag{1}$$

where

$$\left.\begin{aligned} \bar{\mathbf{v}}_n &= (\bar{\mathbf{v}}\bullet\mathbf{n})\mathbf{n} = (\mathbf{n}\otimes\mathbf{n})\bar{\mathbf{v}} = -\bar{v}_n\mathbf{n} \\ \bar{\mathbf{v}}_t &= \bar{\mathbf{v}} - \bar{\mathbf{v}}_n = (\mathbf{I}-\mathbf{n}\otimes\mathbf{n})\bar{\mathbf{v}} = \bar{v}_t\mathbf{t}_v \end{aligned}\right\} \tag{2}$$

whilst $\mathbf{n}$ is the unit outward-normal vector of the main body and

$$\left.\begin{aligned} &\bar{v}_n \equiv -\mathbf{n}\bullet\bar{\mathbf{v}}_n = -\mathbf{n}\bullet\bar{\mathbf{v}} \\ &\bar{v}_t = \|\bar{\mathbf{v}}_t\|,\ \ \mathbf{t}_v \equiv \frac{\bar{\mathbf{v}}_t}{\|\bar{\mathbf{v}}_t\|}\ (\mathbf{n}\bullet\mathbf{t}_v = 0,\ \ \|\mathbf{t}_v\| = 1) \end{aligned}\right\} \tag{3}$$

The minus sign is added for $\bar{v}_n$ to be positive when the counter body approaches the main body.

Here, it is assumed that $\bar{\mathbf{v}}$ is additively decomposed into elastic sliding velocity $\bar{\mathbf{v}}^e$ and plastic sliding velocity $\bar{\mathbf{v}}^p$, i.e.

$$\bar{\mathbf{v}} = \bar{\mathbf{v}}^e + \bar{\mathbf{v}}^p \tag{4}$$

Then, $\bar{\mathbf{v}}_n$ and $\bar{\mathbf{v}}_t$ are expressed by the elastic and the plastic parts as follows:

$$\bar{\mathbf{v}}_n = \bar{\mathbf{v}}_n^e + \bar{\mathbf{v}}_n^p,\quad \bar{\mathbf{v}}_t = \bar{\mathbf{v}}_t^e + \bar{\mathbf{v}}_t^p \tag{5}$$

and thus

$$\bar{\mathbf{v}}^e = \bar{\mathbf{v}}_n^e + \bar{\mathbf{v}}_t^e = -\bar{v}_n^e\mathbf{n} + \bar{v}_t\mathbf{t}_v^e \tag{6}$$

$$\bar{\mathbf{v}}^p = \bar{\mathbf{v}}_n^p + \bar{\mathbf{v}}_t^p = -\bar{v}_n^p\mathbf{n} + \bar{v}_t\mathbf{t}_v^p \tag{7}$$

where

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$$\left.\begin{aligned}\bar{\mathbf{v}}_n^e &= (\bar{\mathbf{v}}^e \bullet \mathbf{n})\mathbf{n} = (\mathbf{n}\otimes\mathbf{n})\bar{\mathbf{v}}^e = -\bar{v}_n^e\mathbf{n}\\ \bar{\mathbf{v}}_t^e &= \bar{\mathbf{v}}^e - \bar{\mathbf{v}}_n^e = (\mathbf{I}-\mathbf{n}\otimes\mathbf{n})\bar{\mathbf{v}}^e = \bar{v}_t^e\mathbf{t}_v^e\end{aligned}\right\} \tag{8}$$

$$\left.\begin{aligned}\bar{\mathbf{v}}_n^p &= (\bar{\mathbf{v}}^p \bullet \mathbf{n})\mathbf{n} = (\mathbf{n}\otimes\mathbf{n})\bar{\mathbf{v}}^p = -\bar{v}_n^p\mathbf{n}\\ \bar{\mathbf{v}}_t^p &= \bar{\mathbf{v}}^p - \bar{\mathbf{v}}_n^p = (\mathbf{I}-\mathbf{n}\otimes\mathbf{n})\bar{\mathbf{v}}^p = \bar{v}_t^p\mathbf{t}_v^p\end{aligned}\right\} \tag{9}$$

setting

$$\left.\begin{aligned}&\bar{v}_n^e \equiv -\mathbf{n}\bullet\bar{\mathbf{v}}_n^e = -\mathbf{n}\bullet\bar{\mathbf{v}}^e\\ &\bar{v}_t^e = \|\bar{\mathbf{v}}_t^e\|,\ \mathbf{t}_v^e \equiv \frac{\bar{\mathbf{v}}_t^e}{\|\bar{\mathbf{v}}_t^e\|}\ (\mathbf{n}\bullet\mathbf{t}_v^e = 0,\ \|\mathbf{t}_v^e\| = 1)\end{aligned}\right\} \tag{10}$$

$$\left.\begin{aligned}&\bar{v}_n^p \equiv -\mathbf{n}\bullet\bar{\mathbf{v}}_n^p = -\mathbf{n}\bullet\bar{\mathbf{v}}^p\\ &\bar{v}_t^p = \|\bar{\mathbf{v}}_t^p\|,\ \mathbf{t}_v^p \equiv \frac{\bar{\mathbf{v}}_t^p}{\|\bar{\mathbf{v}}_t^p\|}\ (\mathbf{n}\bullet\mathbf{t}_v^p = 0,\ \|\mathbf{t}_v^p\| = 1)\end{aligned}\right\} \tag{11}$$

The contact traction vector $\mathbf{f}$ acting on the body is expressed by the normal traction vector $\mathbf{f}_n$ and the tangential traction vector $\mathbf{f}_t$ as follows:

$$\mathbf{f} = \mathbf{f}_n + \mathbf{f}_t = -f_n\mathbf{n} + f_t\mathbf{t}_f \tag{12}$$

where

$$\left.\begin{aligned}\mathbf{f}_n &\equiv (\mathbf{n}\bullet\mathbf{f})\mathbf{n} = (\mathbf{n}\otimes\mathbf{n})\mathbf{f} = -f_n\mathbf{n}\\ \mathbf{f}_t &\equiv \mathbf{f} - \mathbf{f}_n = (\mathbf{I}-\mathbf{n}\otimes\mathbf{n})\mathbf{f} = f_t\mathbf{t}_f\end{aligned}\right\} \tag{13}$$

$$\left.\begin{aligned}&f_n \equiv -\mathbf{n}\bullet\mathbf{f}\\ &f_t = \|\mathbf{f}_t\|,\ \mathbf{t}_f \equiv \frac{\mathbf{f}_t}{\|\mathbf{f}_t\|}\ (\mathbf{n}\bullet\mathbf{t}_f = 0,\ \|\mathbf{t}_f\| = 1)\end{aligned}\right\} \tag{14}$$

The minus sign is added for f_n to be positive when the compression is applied to the main body by the counter body.

Now, let the elastic sliding velocity be given by the following isotropic hypo-elastic relation.

$$\bar{\mathbf{v}}^e = \mathbf{C}^{e-1}\overset{\circ}{\mathbf{f}},\quad \overset{\circ}{\mathbf{f}} = \mathbf{C}^e\bar{\mathbf{v}}^e \tag{15}$$

where the second-order tensor $\mathbf{C}^e$ is the contact elastic modulus tensor given by

$$\left.\begin{aligned}\mathbf{C}^e &= \alpha_n\mathbf{n}\otimes\mathbf{n} + \alpha_t(\mathbf{I}-\mathbf{n}\otimes\mathbf{n})\\ \mathbf{C}^{e-1} &= \frac{1}{\alpha_n}\mathbf{n}\otimes\mathbf{n} + \frac{1}{\alpha_t}(\mathbf{I}-\mathbf{n}\otimes\mathbf{n})\end{aligned}\right\} \tag{16}$$

α_n and α_t are the contact elastic moduli in the normal and the tangential directions to the contact surface.

$$\bar{\mathbf{v}}^e = \frac{1}{\alpha_n}\overset{\circ}{\mathbf{f}}_n + \frac{1}{\alpha_t}\overset{\circ}{\mathbf{f}}_t,\quad \overset{\circ}{\mathbf{f}} = \alpha_n\bar{\mathbf{v}}_n^e + \alpha_t\bar{\mathbf{v}}_t^e \tag{17}$$

2.2 Normal sliding-yield and sliding-subloading surfaces

Assume the following Coulomb-type sliding-yield surface with the isotropic hardening/softening, which describes the sliding-yield condition.

$$f(\mathbf{f}) = \mu \tag{18}$$

μ is the isotropic hardening/softening function denoting the variation of the size of the sliding-yield surface.

In what follows, we assume that the interior of the sliding-yield surface is not a purely elastic domain but that the plastic sliding velocity is induced by the rate of traction inside that surface. Therefore, let the sliding-yield surface be renamed as the normal-sliding surface.

Then, based on the concept of the subloading surface described in the preceding sections, we introduce the subloading-sliding surface, which always passes through the current contact traction point $\mathbf{f}$ and retains a similar shape and orientation to the normal sliding-yield surface with respect to the origin of contact traction space, i.e. $\mathbf{f} = 0$. Let the ratio of the size of the sliding-subloading surface to that of the normal sliding-yield surface be called the normal-sliding ratio, denoted by $r\ (0 \le r \le 1)$. Then, the sliding-subloading surface is described by

$$f(\mathbf{f}) = r\mu \tag{19}$$

The material-time derivative of Eq. (19) leads to

$$\frac{\partial f(\mathbf{f})}{\partial\mathbf{f}}\bullet\overset{\circ}{\mathbf{f}} = r\dot{\mu} + \dot{r}\mu \tag{20}$$

2.3 Evolution rules of sliding-hardening function and normal-sliding ratio

Evolution rules of the isotropic hardening function and the normal sliding-yield ratio are formulated so as to reflect experimental facts.

2.3.1 Evolution rule of sliding-hardening function

The followings might be stated from the results of experiments.

i) The friction coefficient first reaches the maximal value of static-friction and then decreases to the minimum stationary value of kinetic-friction. Physically, this phenomenon might be interpreted to result from separations of the adhesions of surface asperities between contact bodies because of the sliding. Then, let it be assumed that the plastic sliding causes the contraction of the normal sliding-yield surface, i.e., the plastic softening.

ii) The friction coefficient recovers gradually with the elapse of time and the identical behavior as the initial sliding behavior exhibiting the static friction is reproduced if sufficient time has elapsed after the sliding ceases. Physically, this phenomenon might be interpreted to result from the reconstructions of the adhesions of surface asperities during the elapsed time under a quite high contact pressure between edges of surface asperities. Then, let it be assumed that the recovery results from the viscoplastic hardening.

Taking account of these facts, let the evolution rule of the isotropic hardening/softening function μ be postulated as follows:

$$\dot{\mu} = -\kappa\left(\frac{\mu}{\mu_k} - 1\right)\|\bar{\mathbf{v}}^p\| + \xi\left(1 - \frac{\mu}{\mu_s}\right) \tag{21}$$

where μ_s and μ_k $(\mu_s \ge \mu \ge \mu_k)$ are the maximum and minimum values of μ for the static and kinetic frictions, respectively. κ is the material constant influencing the decreasing rate of μ under the plastic sliding, and ξ is the material constant influencing the recovering rate of μ by the elapse of time, whereas ξ is a function of absolute temperature in general.

2.3.2 Evolution rule of sliding-hardening function

Analogously to the evolution rule for the normal-yield ratio R in the subloading surface model, we assume the evolution rule of the normal sliding ratio r as follows:

$$\dot{r} = \bar{U}(r)\|\bar{\mathbf{v}}^p\| \quad \text{for } \bar{\mathbf{v}}^p \neq \mathbf{0} \tag{22}$$

where $\bar{U}(r)$ is a monotonically decreasing function of r fulfilling the following condition.

$$\left.\begin{array}{ll} \bar{U}(r) \to +\infty & \text{for } r = 0, \\ \bar{U}(r) = 0 & \text{for } r = 1, \\ (\bar{U}(r) < 0 & \text{for } r > 1). \end{array}\right\} \tag{23}$$

The function $\bar{U}(r)$ satisfying Eq. (23) can be simply given by

$$\bar{U}(r) = \tilde{u}\cot(\frac{\pi}{2}r) \tag{24}$$

2.4 Relation of contact traction rate and sliding velocity

The substitution of Eqs. (21) and (22) into Eq. (20) leads to

$$\frac{\partial f(\mathbf{f})}{\partial \mathbf{f}} \bullet \mathring{\mathbf{f}} = r\Big\{-\kappa\Big(\frac{\mu}{\mu_k}-1\Big)\|\bar{\mathbf{v}}^p\| + \xi\Big(1-\frac{\mu}{\mu_s}\Big)\Big\} + \bar{U}(r)\|\bar{\mathbf{v}}^p\|\mu \tag{25}$$

Assume that the direction of plastic sliding velocity is tangential to the contact plane and outward-normal to the curve generated by the intersection of the sliding-yield surface and the constant normal traction plane $\mathbf{f}_n = \text{const.}$, leading to the tangential associated flow rule

$$\bar{\mathbf{v}}^p = \dot{\lambda}\mathbf{t}_v^p \quad (\mathbf{n} \bullet \bar{\mathbf{v}}^p = 0) \tag{26}$$

by specializing $\mathbf{t}_v^p$ in Eq. (11) as

$$\mathbf{t}_v^p \equiv \frac{(\mathbf{I} - \mathbf{n}\otimes\mathbf{n})\dfrac{\partial f(\mathbf{f})}{\partial \mathbf{f}}}{\left\|(\mathbf{I} - \mathbf{n}\otimes\mathbf{n})\dfrac{\partial f(\mathbf{f})}{\partial \mathbf{f}}\right\|} \tag{27}$$

$\dot{\lambda}\ (>0)$ is a plastic multiplier describing the magnitude of plastic sliding velocity.

The substitution of Eqs. (22) and (26) into Eq. (20) reads:

$$\frac{\partial f(\mathbf{f})}{\partial \mathbf{f}} \bullet \mathring{\mathbf{f}} = r\Big\{-\kappa\Big(\frac{\mu}{\mu_k}-1\Big)\dot{\lambda} + \xi\Big(1-\frac{\mu}{\mu_s}\Big)\Big\} + \bar{U}(r)\dot{\lambda}\mu \tag{28}$$

i.e.

$$\frac{\partial f(\mathbf{f})}{\partial \mathbf{f}} \bullet \mathring{\mathbf{f}} = \dot{\lambda}\ m^p + m^c \tag{29}$$

where

$$m^p \equiv -\kappa\Big(\frac{\mu}{\mu_k}-1\Big)r + \bar{U}(r)\mu \tag{30}$$

$$m^c \equiv \xi(1-\frac{\mu}{\mu_s})r \ (\geq 0) \tag{31}$$

It is obtained from Eqs. (26) and (29) that

$$\dot{\lambda} = \frac{\dfrac{\partial f(\mathbf{f})}{\partial \mathbf{f}} \bullet \mathring{\mathbf{f}} - m^c}{m^p}, \quad \bar{\mathbf{v}}^p = \frac{\dfrac{\partial f(\mathbf{f})}{\partial \mathbf{f}} \bullet \mathring{\mathbf{f}} - m^c}{m^p}\mathbf{t}_v^p \tag{32}$$

Substituting Eqs. $(15)_1$ and (32) into Eq. (4), the sliding velocity is given by

$$\bar{\mathbf{v}} = \mathbf{C}^{e-1}\mathring{\mathbf{f}} + \frac{\dfrac{\partial f(\mathbf{f})}{\partial \mathbf{f}} \bullet \mathring{\mathbf{f}} - m^c}{m^p}\mathbf{t}_v^p \tag{33}$$

The plastic multiplier in terms of the sliding velocity, denoted by the symbol $\dot{\Lambda}$, is given from Eqs. (33) as

$$\dot{\lambda} = \frac{\dfrac{\partial f(\mathbf{f})}{\partial \mathbf{f}} \bullet \mathbf{C}^e\bar{\mathbf{v}} - m^c}{m^p + \dfrac{\partial f(\mathbf{f})}{\partial \mathbf{f}} \bullet \mathbf{C}^e\mathbf{t}_v^p} \tag{34}$$

The rate of contact stress vector is derived from Eqs. (4), $(15)_1$, and (34) as follows:

$$\mathring{\mathbf{f}} = \mathbf{C}^e\left(\bar{\mathbf{v}} - \left\langle \frac{\dfrac{\partial f(\mathbf{f})}{\partial \mathbf{f}} \bullet \mathbf{C}^e\bar{\mathbf{v}} - m^c}{m^p + \dfrac{\partial f(\mathbf{f})}{\partial \mathbf{f}} \bullet \mathbf{C}^e\mathbf{t}_v^p} \right\rangle \mathbf{t}_v^p\right) \tag{35}$$

Therefore, the loading criterion is given as follows:

$$\left.\begin{array}{l} \bar{\mathbf{v}}^p \neq \mathbf{0}: \ \dfrac{\partial f(\mathbf{f})}{\partial \mathbf{f}} \bullet \mathbf{C}^e\bar{\mathbf{v}} - m^c > 0 \\ \bar{\mathbf{v}}^p = \mathbf{0}: \ \text{otherwise} \end{array}\right\} \tag{36}$$

3. Concluding remarks

The constitutive equations based on the subloading surface concept are formulated within the framework of the hypoelastic-based plasticity and the applications to the descriptions of sliding phenomena of solids are shown. The salient features of the concept and the constitutive equations based on this concept are summarized as follows:

1) It is capable of describing rigorously and concisely the friction phenomena between solids, describing the smooth reduction from the static to the kinetic friction by the sliding and the recovery of friction by the stop of sliding.

2) It possesses the distinctive advantage that the stress is automatically attracted to the yield surface. Therefore, it enables us to adopt rather large incremental steps in the forward Euler numerical calculation without the incorporation of particular algorithms to pull back the stress to the yield surface. Further, the plastic strain is automatically attracted to the isotropic hardening stagnation surface for metals, for which it is difficult for the return-mapping projection to be exploited. These advantages would be activated in large scale finite element analyses solving a big global stiffness matrix.

Needless to say, infinitesimal increments must be input for the deformation analyses in the curved loading process and under a material rotation in numerical calculations not only by the forward-Euler method but also by the return-mapping scheme.

Consequently, the physical and the mathematical pertinences and the numerical convenience are materialized in the subloading surface model. It is capable of describing the finite deformation and rotation under an infinitesimal elastic deformation.

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