State Feedback Control of Discrete Event Systems

（離散事象システムの状態フィードバック制御）

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Shigemasa Takai

Abstract

The conventional control theory deals with systems which can be modeled by difference or differential equations. Modern technology, however, has created an important class of systems, called discrete event systems (DESs), which are not suitable for modeling by difference or differential equations. A DES is a dynamic system that evolves according to the spontaneous and asynchronous occurrence of events.

This thesis studies state feedback control of DESs. State feedback control was initiated by Ramadge and Wonham, and is a useful control technique for a class of logical control problems where control specifications are given in terms of predicates on the state set. Several typical control problems such as deadlock avoidance problems and mutual exclusion problems can be solved in this framework.

First, we address state feedback control problems for automata based models. We present necessary and sufficient conditions for the existence of a state feedback controller under partial as well as complete observations. We then extend these results to decentralized state feedback control where, instead of a global controller, a collection of local controllers controls the system so that the global behavior satisfies the global specification.

Next, we study state feedback control of concurrent DESs modeled by controlled Petri nets. The effect of concurrency is an important problem in real-time control of DESs. Petri nets can represent concurrency explicitly. We derive a necessary and sufficient condition under which there exists the unique maximally permissive controller.

Finally, we study blocking in state feedback control in the context of stability of the system. We present an algorithm to compute the minimally restrictive nonblocking controller. But a nonblocking controller may be restrictive because it disables all behaviors which may lead to blocking. In this sense, blocking controllers can be practically more efficient than nonblocking ones if blocking in the closed-loop system is resolved easily

by some external intervention such as rollback mechanism. To achieve this purpose we present an optimization technique to improve some logical performance measures of a blocking controller.

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Contents

Chapter 1

Introduction

This thesis studies state feedback control of *discrete event systems* (DESs). State feedback control was initiated by Ramadge and Wonham [68]. This theory provides systematic methods of synthesizing controllers for a class of logical control problems on DESs.

1.1 Discrete Event Systems

The conventional control theory deals with systems which can be modeled by difference or differential equations. Modern technology, however, has created an important class of systems, called discrete event systems (DESs) [10, 21, 102, 69, 3, 109], which are not suitable for modeling by difference or differential equations. A DES is a dynamic system that evolves according to the spontaneous and asynchronous occurrence of events. For example, sending a packet in a communication network, machine breakdown in a flexible manufacturing system and arrival of a customer in a queueing system can be regarded as events. States of a DES have logical or symbolic values, and state trajectories are piecewise constant and event-driven as shown in Figure 1.1, where x^1, x^2, ..., are states and α, β and λ are events [69]. Examples of DESs include flexible manufacturing systems, communication networks, computer operating systems, traffic systems and database management systems.

Several models for DESs have been proposed. The simplest models, called untimed models, ignore the timing of occurrence of events, and deal with only the order of event sequences. These are suitable models for dealing with logical control problems such as deadlock avoidance problems [5, 27], concurrency control problems [1, 40] and mutual exclusion problems [66, 19]. Untimed models are classified into logical models such as automata [23] and Petri nets [65] and algebraic models such as communicating sequential processes (CSP) [22], a calculus for communicating systems (CCS) [56] and finitely recursive processes (FRP) [31].

On the other hand, the timing of occurrence of events is important and must be taken into account in order to analyze some quantitative properties such as average throughput, waiting time and so on. This leads to timed models. These can be also classified into nonstochastic and stochastic models according to whether the timing of occurrence of events is deterministic or stochastic. Logical models including the timing information such as temporal logic [51] and timed Petri nets [70] are said to be deterministic logical models. Process models with the timing information such as the min-max algebra [15] are called deterministic algebraic models.

Stochastic models such as queueing networks [21] and stochastic Petri nets [55] have been used to study both quantitative and qualitative properties [69]. These models, however, have a shortcoming that it is generally difficult to analyze them.

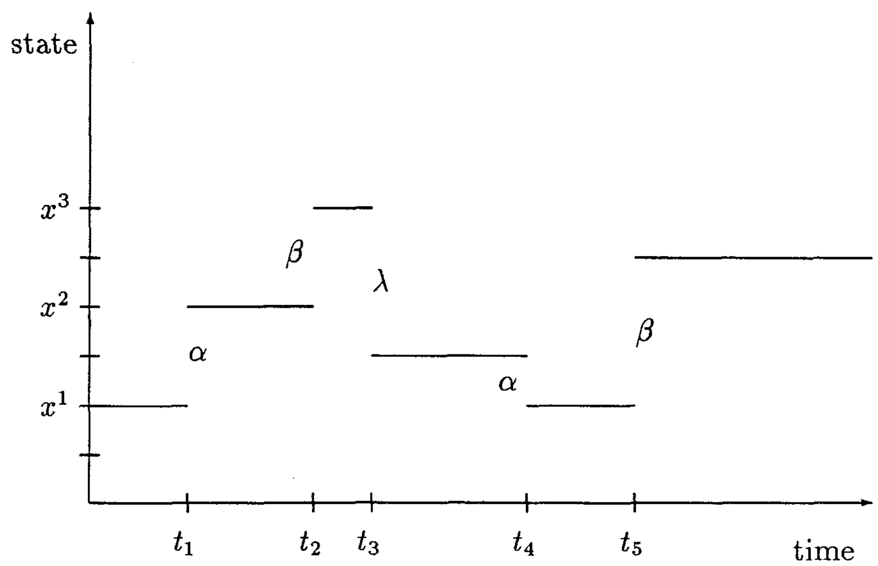

Figure 1.1: A state trajectory of a DES.

1.2 Control of Discrete Event Systems

Typical control problems of DESs include concurrency control problems, mutual exclusion problems and so on. For each of these problems, several control techniques have been investigated based upon the specified feature of each problem. However, as the complexity of control specifications for DESs has been increasing, it is getting important to discover general principles of control of DESs. This leads the need of systematic methods of synthesizing controllers which do not depend on the specified feature of the problem. For example, recent manufacturing systems are quite flexible and must adapt to changes of complex specifications efficiently. In this connection, we need a multitasking control system which supports the fast development of reliable control software [4]. A systematic control theory of DESs can provide a formal design method of such a system as shown in [4].

Ramadge and Wonham have proposed two control techniques for logical control problems: *supervisory control* [67] and *state feedback control* [68] in order to establish a systematic control theory of DESs. In their framework, the set of events are decomposed into two subsets of controllable and uncontrollable events. Each controllable event can be enabled or disabled by external control while any uncontrollable event cannot be disabled. A controller disables some of the controllable events based upon observations of events or states so that the system satisfies the given control specification [105, 106].

When a control specification is given in terms of a formal language, supervisory control is useful. A controller, called a *supervisor*, disables some controllable events according to the event string executed by the system. Ramadge and Wonham proposed the notion of controllability of languages, and showed that there exists a supervisor for a given language if and only if it is controllable [67]. Then this result has been extended to modular control [107], control under partial observations [52, 14], decentralized control [14, 53, 103, 72, 32], hierarchical control [108], on-line control [13] and real-time control [45, 59, 6, 2].

On the other hand, state feedback control is suitable for control specifications given by predicates on the set of states. A state feedback controller disables some controllable events based upon the current state of the system. Ramadge and Wonham considered a control problem that the given predicate remains true at all reachable states in the closed-loop system invariantly. In other words, any forbidden states are nerver reached in the closed-loop system, which is one of the fundamental requirements in control of DESs. It has been showed that there exists such a state feedback controller if and only if the predicate is Σ_u-invariant [68]. Mutual exclusion problems and deadlock avoidance problems can be formulated as the problem.

Li and Wonham addressed a control problem where desirable behavior of the system is specified by the set of all reachable states. Intuitively, the problem requires that all desirable states are reachable without visiting any forbidden states, which means that some tasks can be accomplished successfully. It has been shown that there exists a controller such that the set of all reachable states in the closed-loop system is equal to the given predicate if and only if the predicate is controllable [46, 38]. These results have been extended to modular control [68, 47], control under partial observations [49, 38], control under strict concurrency [48, 100] and real-time control [60, 75].

Moreover, these theories have been applied to several application areas such as manufacturing systems [35, 7], database systems [40, 33, 74], communication systems [71, 99, 61] and rapid thermal multiprocessors [4].

1.3 Contributions and Organization of the Thesis

This section provides an outline of the main contributions and the organization of the thesis.

Chapter 2 gives basic notations on controlled DESs and state feedback controllers. We first introduce two kinds of controlled DES models, automata based models [67, 20] and controlled Petri nets [29, 34], and give the definitions of state feedback controllers for each of these models [68, 34]. Next, we define several predicates on the set of states which will be needed in the subsequent chapters. Moreover, we review basic results on the state feedback control theory initiated by Ramadge and Wonham.

In Chapter 3, we address the state feedback control problem (SFCP) formulated in [46] and [49] under complete observations. The SFCP requires that the set of all reachable states in the closed-loop system is equal to the given predicate, namely, a control specification. Li and Wonham studied the problem in the Ramadge-Wonham model [67, 68] where each controllable event is assigned to be enabled or disabled by external control independently [49]. They gave a necessary and sufficient condition for the existence of a solution to the problem. The condition is called the controllability condition. However, a given predicate is not always controllable. In this case, we have to synthesize a controller for its controllable subpredicate. Li and Wonham also presented a closed form expression of the supremal controllable subpredicate of a given predicate [49]. A simple method for synthesizing a controller for the subpredicate is given by Kumar et al. [38].

Golaszewski and Ramadge considered controlled DESs with arbitrary control patterns, which is called the Golaszewski-Ramadge model [20]. The Golaszewski-Ramadge model is a generalization of the Ramadge-Wonham model. They showed that $(L(G),\Gamma)$-controllability introduced in [20] is a necessary and sufficient condition for the existence of a supervisor, where control specifications are given in terms of formal languages. It was also shown that the supremal $(L(G),\Gamma)$-controllable sublanguage of any language exists if the set Γ of control patterns is closed under union.

We study SFCPs in the Golaszewski-Ramadge model under the assumption that the set Γ of control patterns is closed under union [93, 92]. First, we derive a necessary and sufficient condition for the existence of a state feedback controller. We will call the condition the Γ-controllability condition. However, the given predicate is not necessarily Γ-controllable. So we derive a closed form expression of the supremal Γ-controllable subpredicate of the given predicate. These results are a generalization of those obtained in [49, 38].

Chapter 4 studies state feedback control under partial observations. In some real situations, states of the system are not completely observed. Such situations can be represented by introducing a "mask" which is a mapping from the state space to the observation space [38, 37]. In this framework, a state feedback controller must take the control action based upon the observation under the mask. In the Ramadge-Wonham model, Li and Wonham first studied this problem and defined observability of predicates under a restrictive assumption on the mask [46]. The assumption holds for vector DESs [44, 49] but it does not in general. Kumar et al. proposed another definition of observability without imposing any restriction on the mask [38]. They then showed that controllability and observability defined in [38] are necessary and sufficient conditions for the existence of a *dynamic* controller which uses the entire history of state observations and control actions. But it is practically impossible to check their observability condition if a DES generates an infinite number of observation sequences because the condition requires that the set of all possible observation sequences satisfies some property.

We study (static) state feedback control under partial observations without any assumptions on the mask [84, 85, 91, 87]. First, we consider a balanced state feedback controller [49] in the Golaszewski-Ramadge model. At a reachable state of the closed-loop system, a balanced controller enables every event whose occurrence keeps the control specification true. We give a necessary and sufficient condition for the existence of a balanced controller.

Next, we consider the Ramadge-Wonham model. In the case of complete observations, any state feedback controller can be replaced by a balanced controller without changing the set of reachable states [49]. However, we show that this property does not hold under partial observations. So we present a necessary and sufficient condition for the existence of a (not necessarily balanced) state feedback controller. We will call the condition the M-controllability condition. Kumar et al. gave necessary and sufficient conditions for the existence of a dynamic controller [38]. However, they did not discuss the existence of a state feedback controller. Obviously, a state feedback controller is a special case of a dynamic one. But a state feedback controller is easier to implement than a dynamic one. Moreover, our condition has computational advantage in contrast to those obtained by Kumar et al. because the computational complexity to check our condition is polynomial if the system is modeled by a finite automaton. So our condition is useful from the practical point of view. It is also noted that our condition is a generalization of the result obtained in [49].

Moreover, we study modular control synthesis in the Ramadge-Wonham model. In the case where a predicate is decomposed into conjunction of component predicates, modular control synthesis [68] is very effective. Li and Wonham studied modular state feedback control under complete observations [47, 49]. We consider modular (not necessarily balanced) state feedback control under partial observations. We show that M-controllability of component predicates implies M-controllability of their conjunction under a certain condition. We then present a necessary and sufficient condition under which a state feedback controller can be constructed in a modular fashion.

In Chapter 5, we consider decentralized state feedback control [86, 85, 81, 83, 94]. For distributed systems such as communication systems, a decentralized controller is more suitable than a centralized one. In the context of supervisory control [69] based upon formal languages, several decentralized control problems have been studied in [14, 53, 103, 72, 32]. However, the decentralized state feedback control problem based upon predicates has not been discussed.

First, we consider the decentralized state feedback control problem (DSFCP), which requires that the set of reachable states in the closed-loop system is equal to the specified predicate. We introduce the notion of n-observability of predicates, and prove that the controllability and n-observability are necessary and sufficient conditions for the existence of a solution to the DSFCP.

Next, we consider the decentralized state feedback control problem with tolerance (DSFCPT), which requires that the set of reachable states in the closed-loop system is in the given admissible range. We show that the infimal controllable and n-observable superpredicate of a given predicate plays an important role in solving the DSFCPT. We then prove that there exists the infimal controllable and n-observable superpredicate of a given predicate under a certain condition, and derive its closed form expression.

Chapter 6 studies maximally permissive controllers (MPCs) [35] for controlled Petri nets (CPNs). In real DESs, there are two possible control schemes:

(S1) Control is done by event assignment i.e., the corresponding events are assigned to be enabled concurrently.

(S2) Control is done by resource allocation i.e., the corresponding events have to share the resources.

For example, consider a situation in which two users share one resource, and the control specification is to insure that more than one user cannot occupy the resource simultaneously. Resource allocation control is done by assigning the number of users who can occupy the resource, and it is not specified beforehand which user is permitted to occupy it. On the other hand, event assignment control has to decide who occupies it. Because two users may start to use the resource simultaneously if both are permitted to occupy it, resource allocation control is more effective than event assignment control in real-time systems. Also the effect of concurrency is a very important problem in the design of real-time control of DESs [48, 100]. If we restrict to serial DESs, both event assignment and resource allocation can be modeled by automata based models, whereas, in concurrent DESs, resource allocation cannot be modeled because such models cannot represent the multiplicity of available resources.

In this connection, we note that a Petri net can represent the number of resources by the corresponding number of tokens as well as simultaneous occurrence of events by a bag over the set of transitions [65]. Especially, a CPN is an adequate model for controlled DESs with concurrency. A CPN can model both event assignment and resource allocation with suitable firing rules. Resource allocation can be modeled by assigning patterns of input tokens to the external input places. In order to model event assignment, we use a permission arc connected to each specified transition, and the assignment is represented by putting one token in the corresponding external input place.

A given predicate Q is said to be control-invariant if there exists a state feedback controller f such that all reachable markings in the CPN with f satisfy Q whenever the initial marking satisfies it [68, 34]. Such a controller f is called a permissive controller. In general, there are more than one permissive controller for a control-invariant predicate. A maximal element in the set of permissive controllers is called a maximally permissive controller (MPC). The MPC is not necessarily unique in CPNs because, when we assign patterns of input tokens to the external input places, it does not mean that each transition is assigned to be enabled or disabled independently [95]. Krogh has proposed an algorithm for computing all MPCs [34]. Then, however, if there are more than one MPC, we have to select one MPC among them. The unique MPC is optimal in the sense that it allows the largest set of transitions to fire at each marking. Also, from the theoretical point of view, it is interesting to study the uniqueness of the MPC. Ushio has given a necessary and sufficient condition for the unique existence of the MPC [95, 98]. However, we have to construct the set of all permissive controllers in order to check the condition.

We first consider CPNs without concurrency. We derive the necessary and sufficient conditions for the unique existence of the MPC under partial as well as complete observations, which can be checked without constructing the set of all permissive controllers [78, 90]. Next, we extend the results to CPNs with concurrency controlled by either event assignment or resource allocation [88, 79]. We then show that the unique existence of the MPC in resource allocation control implies that the same is true in event assignment control.

In Chapter 7, we study blocking in state feedback control in the context of stability of the system [89, 82, 80]. Control problems in DESs such as manufacturing systems are often specified by both admissible states and target states. Admissible states represent a set of states in which state trajectories of a system should reside. Target states, which is a subset of admissible states, represent the completion of some tasks. Desirable behaviors of a system are represented by trajectories of admissible states which reach target states in a finite number of transitions. Such trajectories mean that the system has completed some task successfully.

If control specifications are given not only by admissible states but also by target states, the notion of the stabilization of DESs [64, 63, 8, 9, 104, 39, 76, 77] plays an important role to design state feedback controllers. Intuitively, a DES is called stable if all possible state trajectories visit target states in a finite number of transitions. We define blocking [67, 12] in the context of state feedback control as follows: a system is said to be blocking if some trajectories of admissible states cannot reach the target states. In other words, a system cannot complete the execution of the task. So the notion of stabilization is useful to study blocking. To the best of our knowledge, blocking has never been studied in the context of state feedback control. Note that in contrast to [12] where behavior over finite transitions of the system are considered, we can also deal with behavior over infinite transitions in the context of state feedback control. Therefore blocking in state feedback control includes not only deadlock but also livelock.

In the context of supervisory control, blocking can be dealt with by marked traces of events, and both nonblocking and blocking supervisors have been studied [68, 12, 42]. The optimization techniques of a blocking supervisor in terms of two logical performance measures, a satisficing measure and a blocking measure, have been proposed [12]. We define two similar performance measures, called a prestabilizing measure and a blocking measure, to analyze blocking in state feedback control. The former measure is described by the predicate indicating states such that all admissible trajectories starting from them can be extended to target states, while the latter by the predicate indicating states which may lead to blocking. Note that the meaning of the blocking measure in state feedback control is different from that in supervisory control. In supervisory control based upon formal languages, blocking means that there is a state such that no state trajectory from it can reach target states [54]. But state feedback control can treat a wider class of blocking, that is, the following case is also regarded as blocking: there is a state such that some (*infinite*) state trajectory from it cannot reach target states. Note that such a state may transit to a target state by an adequate firing sequence of events.

We first present an algorithm to compute the minimally restrictive nonblocking solution [41, 12]. But a nonblocking controller may be restrictive because it prevents all behaviors which may lead to blocking. In this sense, blocking controllers can be practically more efficient than nonblocking ones if blocking in the closed-loop system is resolved easily by some external intervention such as rollback mechanism. A manufacturing system is a typical example of such a system. Then by similar techniques to [12], we perform the optimization of a given blocking controller in terms of the two performance measures.

Chapter 8 concludes the thesis with listing possible directions of future research.

Chapter 2

Preliminaries

This chapter gives basic notations on controlled DESs and state feedback controllers, and reviews the state feedback control theory initiated by Ramadge and Wonham [68].

2.1 Controlled Discrete Event Systems and State Feedback Controllers

This section introduces two kinds of controlled DES models, automata based models and controlled Petri nets, and gives the definitions of state feedback controllers for each of these models.

2.1.1 Automata Based Model

Let G be a DES modeled by an automaton [23].

$$G = (X, \Sigma, \delta, x^0), \tag{2.1}$$

where X is the (possibly infinite) set of states, Σ is the finite set of events, a partial function $\delta : \Sigma \times X \to X$ is the transition function, and $x^0 \in X$ is the initial state. Let m and n be the numbers of elements of Σ and X, respectively. Let Σ^* be the set of all finite strings of elements in Σ, including the empty string ϵ. The function δ can be generalized to $\delta : \Sigma^* \times X \to X$ as follows [23]: for any $x \in X$ and any $s \in \Sigma^*$,

$$\delta(\epsilon, x) = x,$$

$$\delta(s\sigma, x) = \delta(\sigma, \delta(s, x)),$$

whenever $\delta(s,x)$ and $\delta(\sigma,\delta(s,x))$ are defined. We shall write $\delta(s,x)!$ for any $s \in \Sigma^*$ and any $x \in X$ if $\delta(s,x)$ is defined.

Ramadge and Wonham incorporated a control mechanism into G as follows [67, 68]. The set Σ is decomposed into two subsets Σ_c and Σ_u of controllable and uncontrollable events, respectively, where $\Sigma_c \cup \Sigma_u = \Sigma$ and $\Sigma_c \cap \Sigma_u = \emptyset$. Each controllable event can be enabled or disabled independently while all uncontrollable events cannot be prevented from occurring by external controllers.

A state feedback controller proposed by Ramadge and Wonham [68] disables some controllable events based upon the current state of G so that the behaviors of the closed-loop system satisfy the given control specification. A subset γ of Σ including Σ_u is called a control pattern, which indicates the events enabled by a controller. The condition $\Sigma_u \subseteq \gamma$ implies that all uncontrollable events cannot be disabled by a controller. Let Γ be a set of all control patterns.

$$\Gamma = \{\gamma \in 2^{\Sigma};\ \Sigma_u \subseteq \gamma \subseteq \Sigma\}, \tag{2.2}$$

where 2^{Σ} is the power set of Σ. A controlled DES G with Γ given by Eq. (2.2) is called the Ramadge-Wonham model.

Golaszewski and Ramadge introduced a controlled DES which allows arbitrary control patterns [20]. In their model, Γ is given by a subset of 2^{Σ} satisfying the following condition (C2-1) [20].

(C2-1) There exists a control pattern $\gamma \in \Gamma$ with $\sigma \in \gamma$ for any $\sigma \in \Sigma$.

In general, Γ is a poset with respect to the set inclusion "$\subseteq$". Such a controlled DES is called the Golaszewski-Ramadge model [20]. The Golaszewski-Ramadge model is a generalization of the Ramadge-Wonham model.

A state feedback controller $f \in \Gamma^X$ for G is formally defined by a mapping from X to Γ [68], where Γ^X is the set of all mappings from X to Γ. $f \in \Gamma^X$ is static in the sense that it selects a control pattern based upon only the current state of G. A closed-loop system $G \mid f$ for Eq. (2.1) with a controller f is described as follows [68]:

$$G \mid f = (X, \Sigma, \delta_{cf}, x^0), \tag{2.3}$$

where a partial function $\delta_{cf} : \Sigma \times X \to X$ is defined by

$$\delta_{cf}(\sigma,x) = \begin{cases} \delta(\sigma,x) & \text{if } \sigma \in f(x), \\ \text{undefined} & \text{otherwise.} \end{cases}$$

That is, each event $\sigma \in \Sigma$ with $\sigma \notin f(x)$ is disabled at the state $x \in X$. A block diagram of $G \mid f$ is shown in Figure 2.1.

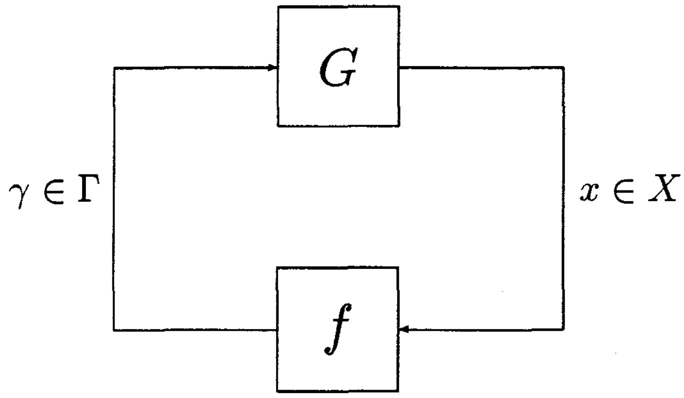

Figure 2.1: A block diagram of the closed-loop system $G \mid f$.

2.1.2 Controlled Petri Nets without Concurrency

Let $\mathcal{N}$ be the set of all nonnegative integers. A Petri net is described by a 5-tuple [65, 58]

$$N = (P, T, I, O, M_0), \tag{2.4}$$

where P is the finite set of places, T is the finite set of transitions, $M_0 : P \to \mathcal{N}$ is the initial marking, and $I : T \times P \to \mathcal{N}$ (respectively, $O : T \times P \to \mathcal{N}$) is the number of arcs from a place to a transition (respectively, from a transition to a place). It is assumed that $P \cap T = \emptyset$. Let $M : P \to \mathcal{N}$ be a marking of N, that is, $M(p)$ denotes the number of tokens in a place $p \in P$.

We review execution rules for a Petri net N briefly [65, 58]. We assume that no two transitions can fire simultaneously in N. A transition $t \in T$ is said to be enabled at a marking M, denoted by $M[t >$, if and only if the following condition holds.

$$I(t,p) \leq M(p) \quad \forall p \in P.$$

After the firing of t, M changes to M', defined as follows, and we shall write $M[t > M'$. For each $p \in P$,

$$M'(p) = M(p) + (O(t,p) - I(t,p)).$$

A (finite) sequence $w = t_1t_2...t_j$ is called a firing sequence from M to M', denoted by $M[w > M'$, if and only if there exist markings $M_1, M_2, ...,$ and M_{j-1} such that $M[t_1 > M_1, M_1[t_2 > M_2, ...,$ and $M_{j-1}[t_j > M'$. A marking M is called a reachable marking if and only if there exists a firing sequence from the initial marking M_0 to M. Let $R(N)$ be the reachable set for N.

We introduce a controlled Petri net (CPN) N_c for Eq. (2.4) as follows [30, 29]:

$$N_c = (P \cup P_c, T, I, I_c, O, M_{c0}), \tag{2.5}$$

where P_c is the finite set of external input places, $I_c : T \times P_c \rightarrow \{0, 1\}$ is the number of arcs from an external input place to a transition, and $M_{c0} : (P \cup P_c) \rightarrow \mathcal{N}$ is the initial marking. It is assumed that $P \cap P_c = T \cap P_c = \emptyset$. Let $M_c : (P \cup P_c) \rightarrow \mathcal{N}$ be a marking of N_c . For each $p \in P$, $M_{c0}(p) = M_0(p)$, while $M_{c0}(p_c)$ for each $p_c \in P_c$ is assigned by a state feedback controller defined later. Let T_c and $T_u \subseteq T$ be the set of transitions whose firings can and cannot be controlled by the external input places, respectively [34, 95].

$$T_c = \{t \in T;\ I_c(t, p_c) = 1 \quad \text{for some } p_c \in P_c\}.$$

$$T_u = T - T_c := \{t \in T;\ t \notin T_c\}.$$

For each $t \in T$, let ${}^c t \subseteq P_c$ be the set of external input places for t.

$${}^c t = \{p_c \in P_c;\ I_c(t, p_c) = 1\}.$$

For a CPN N_c, a mapping $\gamma : P_c \rightarrow \{0, 1\}$ is called a control pattern, and $\gamma(p_c)$ is the number of tokens assigned to the external input place $p_c \in P_c$ [95]. Let $\Gamma = \{0, 1\}^{P_c}$ be the set of all control patterns. For each γ_a , $\gamma_b \in \Gamma$, the sum $\gamma_a + \gamma_b$ is defined as follows: for each $p_c \in P_c$,

$$(\gamma_a + \gamma_b)(p_c) = \max\{\gamma_a(p_c), \gamma_b(p_c)\}. \tag{2.6}$$

A controlled DES is modeled by a pair $\mathcal{G} = (N_c, \Gamma)$. A transition $t \in T$ is enabled at M_c, denoted by $M_c[t >$, if and only if the following conditions hold [95].

$$I(t, p) \leq M_c(p) \quad \forall p \in P,$$

$$I_c(t, p_c) \leq M_c(p_c) \quad \forall p_c \in P_c.$$

After the firing of t, M_c changes to M'_c, defined as follows, and we shall write $M_c[t > M'_c$. For each $p \in P$

$$M'_c(p) = M_c(p) + (O(t,p) - I(t,p)),$$

and, for each $p_c \in P_c$, $M'_c(p_c)$ is determined by a state feedback controller defined later.

A state feedback controller $f \in \Gamma^{R(N)}$ is defined by a mapping from a reachable marking of N to a control pattern. Let $\mathcal{G} \mid f$ denote the closed-loop system with a controller f. Letting M be a marking of N, the corresponding marking M_c of the closed-loop system $\mathcal{G} \mid f$ is described by

$$M_c(p) = \begin{cases} M(p) & \text{if } p \in P, \\ f(M)(p) & \text{if } p \in P_c. \end{cases}$$

It is obvious that, for each reachable marking M_c of $\mathcal{G} \mid f$, there exists a reachable marking $M \in R(N)$ such that $M(p) = M_c(p)$ for any $p \in P$ [95]. For each f_1 and $f_2 \in \Gamma^{R(N)}$, we define the sum $f_1 + f_2$ as follows: for any $M \in R(N)$,

$$(f_1 + f_2)(M) = f_1(M) + f_2(M). \tag{2.7}$$

2.1.3 Controlled Petri Nets with Concurrency

Let $N = (P, T, I, O, M_0)$ be a Petri net defined by Eq. (2.4). We extend N in order to include concurrency, that is, simultaneous firing of transitions. Let

$$N^{con} = (P, T, I, O, M_0) \tag{2.8}$$

be the extended model. In N^{con}, we permit the case where a transition can fire n-times ($n \geq 1$) at the same time. A bag over T will be called a transition bag, or a b-transition for short. (Reader are referred to [65] for definitions and notations of bags.) Let T^ω be the set of all b-transitions. T can be regarded as a subset of T^ω by identifying $\{t\} \in T^\omega$ with $t \in T$. A b-transition t_b of N^{con} is said to be (concurrently) enabled at a marking M, denoted by $M[t_b >$, if and only if the following condition holds.

$$\sum_{t \in t_b} I(t,p) \leq M(p) \quad \forall p \in P.$$

After the firing of t_b, M changes to M', defined as follows, and we shall write $M[t_b > M'$. For each $p \in P$,

$$M'(p) = M(p) + \sum_{t \in t_b} (O(t,p) - I(t,p)).$$

It is obvious that the reachable set for N^{con} is equal to $R(N)$. So we also denote the reachable set for N^{con} by $R(N)$. Let $T_b \subseteq T^{\omega}$ be the set of b-transitions which can be enabled at some reachable marking $M \in R(N)$.

$$T_b = \{t_b \in T^{\omega};\ M[t_b > \quad \text{for some } M \in R(N)\}.$$

Let $N_c^{con} = (P \cup P_c, T, I, I_c, O, M_{c0})$ be the CPN for Eq. (2.8). We consider two control schemes for N_c^{con}:

(S1) Control is done by event assignment i.e., the corresponding events are assigned to be enabled concurrently.

(S2) Control is done by resource allocation i.e., the corresponding events have to share the resources.

Let T_c^{ω} and T_u^{ω} be the set of all bags over T_c and T_u, respectively. For each $t_b \in T_b$, ${}^c t_b \subseteq P_c$ will be defined by

$${}^c t_b = \bigcup_{t \in t_b} {}^c t.$$

In the control scheme (S1) (respectively, (S2)) for N_c^{con}, a mapping $\gamma : P_c \rightarrow \{0,1\}$ (respectively, $P_c \rightarrow \mathcal{N}$) is called a control pattern, and $\gamma(p_c) \in \{0,1\}$ (respectively, $\in \mathcal{N}$) is the number of tokens assigned to the external input place $p_c \in P_c$ [95]. Let $\Gamma_1 = \{0,1\}^{P_c}$ (respectively, $\Gamma_2 = \mathcal{N}^{P_c}$) be the set of control patterns for (S1) (respectively, (S2)). For each γ_a , $\gamma_b \in \Gamma_1$ or Γ_2, the sum $\gamma_a + \gamma_b$ is defined by Eq. (2.6).

Controlled DESs with the control schemes (S1) and (S2) are modeled by pairs $\mathcal{G}_1 = (N_c^{con}, \Gamma_1)$ and $\mathcal{G}_2 = (N_c^{con}, \Gamma_2)$, respectively. In $\mathcal{G}_1$, a b-transition t_b is enabled at M_c, if and only if the following conditions hold [24].

$$\sum_{t \in t_b} I(t,p) \le M_c(p) \quad \forall p \in P,$$

$$I_c(t,p_c) \le M_c(p_c) \quad \forall t \in t_b \text{ and } \forall p_c \in P_c.$$

The second inequality is different from the usual firing rules in Petri nets, and an arc from an external input place to a transition will be called a permission arc. In $\mathcal{G}_2$, a b-transition t_b is enabled at M_c if and only if the following conditions hold [95].

$$\sum_{t \in t_b} I(t,p) \le M_c(p) \quad \forall p \in P,$$

$$\sum_{t \in t_b} I_c(t, p_c) \leq M_c(p_c) \quad \forall p_c \in P_c.$$

Under the marking M_c shown in Figure 2.2, for example, $\{t_1, t_2\}$ is enabled in $\mathcal{G}_1$ while it is not enabled in $\mathcal{G}_2$. Note that, in both $\mathcal{G}_1$ and $\mathcal{G}_2$, $M_c[t'_b >$ for every $t'_b \subseteq t_b$ if $M_c[t_b >$. After the firing of t_b, M_c changes to M'_c, defined as follows: for each $p \in P$,

$$M'_c(p) = M_c(p) + \sum_{t \in t_b}(O(t,p) - I(t,p)),$$

and $M'_c(p_c)$ is determined by a state feedback controller defined later.

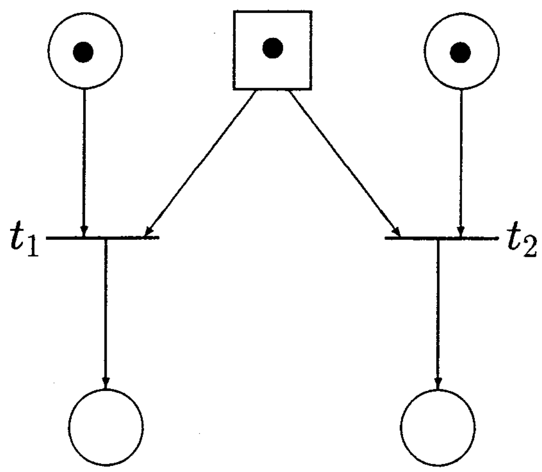

Figure 2.2: An example of CPNs where a place, an external input place and a transition are represented by a circle, a box and a bar, respectively.

In a controlled DES $\mathcal{G}_1$ (respectively, $\mathcal{G}_2$), a state feedback controller $f \in \Gamma_1^{R(N)}$ (respectively, $\in \Gamma_2^{R(N)}$) is defined by a mapping from $R(N)$ to Γ_1 (respectively, Γ_2). Let $\mathcal{G}_1 \mid f$ (respectively, $\mathcal{G}_2 \mid f$) denote a closed-loop system with a controller f. Letting M be a marking of N^{con}, the corresponding marking M_c of the closed-loop system $\mathcal{G}_1 \mid f$ (respectively, $\mathcal{G}_2 \mid f$) is described by

$$M_c(p) = \begin{cases} M(p) & \text{if } p \in P, \\ f(M)(p) & \text{if } p \in P_c. \end{cases}$$

For each f_1 and $f_2 \in \Gamma_1^{R(N)}$ or $\in \Gamma_2^{R(N)}$, the sum $f_1 + f_2$ is defined by Eq. (2.7).

2.2 Predicates

In this thesis, we consider the case that control specifications are given in terms of predicates on the set X of states (respectively, the set $R(N)$ of reachable markings) in automata based models (respectively, in CPNs). Let $\mathbf{Q}$ be the set of all predicates on X (respectively, $R(N)$). In this section, we only consider the case that the system is modeled by an automaton G. Note that all predicates defined in this section can be applied to CPNs directly.

We say that a predicate $Q \in \mathbf{Q}$ is true (respectively, false) at $x \in X$ if $Q(x) = 1$ (respectively, $= 0$). For any predicates Q_1 and $Q_2 \in \mathbf{Q}$, negation $\sim Q_1$, conjunction $Q_1 \wedge Q_2$ and disjunction $Q_1 \vee Q_2$ on $\mathbf{Q}$ are defined as follows: for any $x \in X$,

$$\begin{aligned} \sim Q_1(x) = 1 &\Leftrightarrow Q_1(x) = 0, \\ (Q_1 \wedge Q_2)(x) = 1 &\Leftrightarrow Q_1(x) = 1 \text{ and } Q_2(x) = 1, \end{aligned}$$

$$Q_1 \vee Q_2 = \sim ((\sim Q_1) \wedge (\sim Q_2)).$$

We define a partial order on $\mathbf{Q}$ as follows: $Q_1 \leq Q_2$ if $Q_1(x) \leq Q_2(x)$ for every $x \in X$. Also we shall write $Q_1 < Q_2$ if $Q_1 \leq Q_2$ and $Q_1(x) < Q_2(x)$ for at least one $x \in X$. We say that Q_1 is a subpredicate of Q_2 or Q_2 is a superpredicate of Q_1 if $Q_1 \leq Q_2$. The predicates $\mathbf{0}$ and $\mathbf{1} \in \mathbf{Q}$ are defined by $\mathbf{0}(x) = 0$ and $\mathbf{1}(x) = 1$ for each $x \in X$, respectively.

For each $\sigma \in \Sigma$, the predicate $D_\sigma \in \mathbf{Q}$ and the transformations wp_σ, wlp_σ and sp_σ on $\mathbf{Q}$ are defined as follows [16, 17, 68]:

$$D_\sigma(x) = \begin{cases} 1 & \text{if } \delta(\sigma, x)!, \\ 0 & \text{otherwise.} \end{cases}$$

$$wp_\sigma(Q)(x) = \begin{cases} 1 & \text{if } \delta(\sigma, x)! \text{ and } Q(\delta(\sigma, x)) = 1, \\ 0 & \text{otherwise.} \end{cases}$$

$$wlp_\sigma(Q) = wp_\sigma(Q) \vee \sim D_\sigma.$$

$$sp_\sigma(Q)(x) = \begin{cases} 1 & \text{if } \exists x' \in X \text{ with } Q(x') = 1 \text{ and } \delta(\sigma, x') = x, \\ 0 & \text{otherwise.} \end{cases}$$

(Readers are referred to [16] and [17] for detail explanations of these predicates.) The predicate $wlp_\sigma(Q)$ holds on the states where the occurrence of σ leads them to states satisfying Q, or σ is not defined. The predicate $sp_\sigma(Q)$ holds on the states which are reachable from states satisfying Q by the occurrence of σ. The transformations wp_σ, wlp_σ and sp_σ are called the weakest precondition, the weakest liberal precondition and the strongest postcondition, respectively [16, 17].

Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$. The predicate $Re(G, Q) \in \mathbf{Q}$ is defined inductively as follows [49]:

(C2-2) $Re(G, Q)(x^0) = 1$;

(C2-3) If $Re(G, Q)(x) = 1$ and $wp_\sigma(Q)(x) = 1$ for some $\sigma \in \Sigma$, then $Re(G, Q)(\delta(\sigma, x)) = 1$;

(C2-4) Every state which satisfies $Re(G, Q)$ is obtained as in (C2-2) and (C2-3).

Any $x \in X$ with $Re(G, Q)(x) = 1$ is reachable from x^0 via states satisfying Q. That is, for any $x \in X$ with $Re(G, Q)(x) = 1$, there exist $x^1, x^2, ..., x^m \in X$ and $\sigma^0, \sigma^1, ..., \sigma^{m-1} \in \Sigma$ satisfying the following conditions (C2-5)–(C2-7) [49]:

(C2-5) $\delta(\sigma^j, x^j) = x^{j+1}$ for $j = 0, 1, ..., m-1$;

(C2-6) $Q(x^j) = 1$ for $j = 0, 1, ..., m$;

(C2-7) $x^m = x$.

For any state feedback controller $f \in \Gamma^X$, the predicate $Re(G \mid f, \mathbf{1})$ represents the set of reachable states in $G \mid f$. For convenience denote $Re(G \mid f, \mathbf{1})$ by $Re(G \mid f)$. We call $Re(G \mid f)$ the closed-loop predicate for f. For any $x \in X$ with $Re(G \mid f)(x) = 1$, there exist $x^1, x^2, ..., x^m \in X$ and $\sigma^0, \sigma^1, ..., \sigma^{m-1} \in \Sigma$ satisfying the following conditions (C2-8)–(C2-10) [49]:

(C2-8) $\delta(\sigma^j, x^j) = x^{j+1}$ for $j = 0, 1, ..., m-1$;

(C2-9) $\sigma^j \in f(x^j)$ for $j = 0, 1, ..., m-1$;

(C2-10) $x^m = x$.

2.3 Review of the State Feedback Control Theory

This section reviews the state feedback control theory initiated by Ramadge and Wonham in [68].

2.3.1 Control-Invariance and Permissive Controllers

In this subsection, we consider a DES G given by Eq. (2.1). Let $Q \in \mathbf{Q}$ be a predicate. We interprete Q as the control specification. For a state feedback controller $f \in \Gamma^X$ and an event $\sigma \in \Sigma$, the predicate $f_\sigma \in \mathbf{Q}$ is defined by

$$f_\sigma(x) = \begin{cases} 1 & \text{if } \sigma \in f(x), \\ 0 & \text{otherwise.} \end{cases}$$

Ramadge and Wonham considered a control problem which requires that Q is true at all reachable states in the closed-loop system from each $x \in X$ with $Q(x) = 1$ [68]. Control-invariance of predicates was introduced in order to solve the problem. A state feedback controller $f \in \Gamma^X$ is said to be a *permissive controller* of Q if, for any $\sigma \in \Sigma$, the following equation holds [68].

$$Q \leq wlp_\sigma(Q) \vee \sim f_\sigma. \tag{2.9}$$

Q is said to be *control-invariant* if there exists a permissive controller of Q [68]. It has been proved in [68] that there exists a solution to the problem if and only if Q is control-invariant. In the Ramadge-Wonham model, Q is control-invariant if and only if it is Σ_u-invariant, that is, $Q \leq wlp_\sigma(Q)$ for any $\sigma \in \Sigma_u$ [68]. Let $Per(Q) \subseteq \Gamma^X$ be the set of all permissive controllers. A partial order "$\leq$" on Γ^X is defined as follows: $f_1 \leq f_2$ if $f_1(x) \subseteq f_2(x)$ for all $x \in X$. A controller $f_{max} \in Per(Q)$ is said to be a *maximally permissive controller* (MPC) for Q if no controller $f(\neq f_{max}) \in Per(Q)$ satisfies that $f_{max} \leq f$ [34]. In the Ramadge-Wonham model, for any control-invariant predicate Q, the MPC exists uniquely [96], and is given by

$$f(x) = \begin{cases} \Sigma - \{\sigma \in \Sigma_c;\ wlp_\sigma(Q)(x) = 0\} & \text{if } Q(x) = 1, \\ \Sigma & \text{otherwise.} \end{cases} \tag{2.10}$$

However, a given predicate may not be control-invariant. We define the two subsets $CI_<(Q)$ and $CI_>(Q)$ of $\mathbf{Q}$ as follows:

$$CI_<(Q) = \{Q' \in \mathbf{Q};\ Q' \leq Q \text{ and } Q' \text{ is control-invariant}\},$$

$$CI_>(Q) = \{Q' \in \mathbf{Q};\ Q' \geq Q \text{ and } Q' \text{ is control-invariant}\}.$$

In the Ramadge-Wonham model, both the supremal element of $CI_<(Q)$ and the infimal element of $CI_>(Q)$ under "$\leq$" always exist and are denoted by $Q^\uparrow$ and $Q^\downarrow$, respectively

[68]. $Q^{\uparrow}$ and $Q^{\downarrow}$ can be regarded the two optimal approximations of Q. Consider the two sequences $\{Q_j\}$ and $\{Q'_j\}$ of predicates defined by

$$Q_0 := Q, \quad Q_{j+1} := Q \wedge \left(\bigwedge_{\sigma \in \Sigma_u} wlp_\sigma(Q_j) \right) \quad \text{for } j = 0, 1, ...$$

and

$$Q'_0 := Q, \quad Q'_{j+1} := Q \vee \left(\bigvee_{\sigma \in \Sigma_u} sp_\sigma(Q'_j) \right) \quad \text{for } j = 0, 1, ...,$$

respectively. Then $Q^{\uparrow}$ and $Q^{\downarrow}$ are given by

$$Q^{\uparrow} = \bigwedge_{j \in \mathcal{N}} Q_j \text{ and } Q^{\downarrow} = \bigvee_{j \in \mathcal{N}} Q'_j,$$

respectively [68], where $\mathcal{N}$ is the set of all nonnegative integers.

Ushio studied MPCs in the Golaszewski-Ramadge model [96]. In general, there exist more than one MPC in the Golaszewski-Ramadge model. In [96], the notion of *weakly interaction* was introduced, and it was proved that there exists the unique MPC for a control-invariant predicate Q if and only if Q is weakly interactive.

2.3.2 Controllability and Observability

In this subsection, we consider a DES G modeled by the Ramadge-Wonham model. Let $Q \in \mathbf{Q}$ be a predicate. Control-invariance of Q ensures that there exists a state feedback controller such that all reachable states satisfy Q in the closed-loop system. However, control-invariance of Q does not characterize the set of all reachable states, that is, there may exist $x \in X$ with $Q(x) = 1$ which is not reachable from the initial state in the closed-loop system [46]. So Li and Wonham studied the following problem [49]:
State Feedback Control Problem (SFCP): For a predicate $Q \in \mathbf{Q}$ with $Q(x^0) = 1$, synthesize a state feedback controller $f \in \Gamma^X$ such that $Re(G \mid f) = Q$.

Li and Wonham introduced the notion of controllability of predicates in order to solve the SFCP.

Definition 2.1 *[49] Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$. Q is said to be controllable (with respect to G) if Q is Σ_u-invariant and the following equation holds.*

$$Q \leq Re(G, Q). \tag{2.11}$$

The following proposition has been proved in [49].

Proposition 2.1 *[49] Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$. Then there exists a state feedback controller $f \in \Gamma^X$ such that $Re(G \mid f) = Q$, that is, f is a solution to the SFCP if and only if Q is controllable.*

When Q is controllable, a solution to the SFCP is given by Eq. (2.10).

However, a given predicate is not always controllable. In this case, we have to synthesize a controller for its controllable subpredicate. Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$. We denote the set of all controllable subpredicates of Q by $\underline{C}(Q) \subseteq \mathbf{Q}$.

$$\underline{C}(Q) = \{Q' \in \mathbf{Q} \; ; \; Q'(x^0) = 1, \; Q' \leq Q \text{ and } Q' \text{ is controllable}\}.$$

The predicate $[Q] \in \mathbf{Q}$ is defined as follows [49]:

$$[Q](x) = \begin{cases} 1 & \text{if } x \text{ satisfies the following condition (C2-11),} \\ 0 & \text{otherwise.} \end{cases}$$

(C2-11) For any $s \in \Sigma_u^*$,

$$\delta(s, x)! \Rightarrow Q(\delta(s, x)) = 1,$$

where Σ_u^* is the set of all finite strings of elements in Σ_u, including ϵ.

Assume that $[Q](x^0) = 1$. Then it has been proved in [49] that the supremal element of $\underline{C}(Q)$ under "$\leq$", denoted by $\sup \underline{C}(Q)$, exists, and is given by

$$\sup \underline{C}(Q) = Re(G, [Q]).$$

$\sup \underline{C}(Q)$ is said to be the supremal controllable subpredicate of Q [49]. A simple method for synthesizing a controller for the subpredicate is given by Kumar et al. [38]. In vector DESs, Li and Wonhan showed that computation of $\sup \underline{C}(Q)$ can be reduced to a linear integer programming under certain conditions [50]. Ushio discussed relationship between controllability of predicates and that of formal languages [97].

In some real situations, however, states of the system are not completely observed. In order to represent such situations, we introduce a mask $M : X \to Y$ defined by a mapping from the state space X to the observation space Y, that is, $M(x) \in Y$ is observed when the current state of G is $x \in X$ [37, 38]. Note that the mask M is not necessarily injective. In this framework, a state feedback controller $f \in \Gamma^X$ selects a control pattern $f(x)$ based upon $M(x)$. That is, f satisfies the following condition (C2-12) [49, 38]:

(C2-12) For any x, $x' \in X$,

$$M(x) = M(x') \Rightarrow f(x) = f(x').$$

Let F_o be the set of all controllers which satisfy the condition (C2-12).

A state feedback controller $f \in \Gamma^X$ is said to be balanced if f satisfies the following condition (C2-13) [49]:

(C2-13) For any $\sigma \in \Sigma$ and any x, $x' \in X$,

$$Re(G \mid f)(x) = Re(G \mid f)(x') = 1 \text{ and } \delta(\sigma, x) = x' \ \Rightarrow \ \sigma \in f(x).$$

Let F_b be the set of all balanced controllers which satisfy the condition (C2-12).

Li and Wonham also studied the problem formulated as follows [49]:

Balanced State Feedback Control and Observation Problem (BSFCOP): For a predicate $Q \in \mathbf{Q}$ with $Q(x^0) = 1$, synthesize a balanced state feedback controller $f \in F_b$ such that $Re(G \mid f) = Q$.

For a predicate Q on X, the predicate $M(Q)$ on Y is defined as follows [49]:

$$M(Q)(y) = \begin{cases} 1 & \text{if } M(x) = y \text{ for some } x \in X \text{ with } Q(x) = 1, \\ 0 & \text{otherwise.} \end{cases}$$

For a predicate Q' on Y, we will define the predicate $M^{-1}(Q')$ on X as follows [49]:

$$M^{-1}(Q')(x) = \begin{cases} 1 & \text{if } Q'(M(x)) = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Li and Wonham imposed the following assumption (H) on the mask M in order to solve the BSFCOP [49]:

(H) For any $\sigma \in \Sigma$ and $x, x' \in X$ such that $\delta(\sigma, x)!$ and $\delta(\sigma, x')!$,

$$M(x) = M(x') \ \Leftrightarrow \ M(\delta(\sigma, x)) = M(\delta(\sigma, x')).$$

Indeed, the assumption (H) holds for vector DESs [44, 49] but it does not in general.

Definition 2.2 *[49] Let $Q \in \mathbf{Q}$ be a predicate. Q is said to be observable (with respect to G) if, for any $\sigma \in \Sigma_c$, the following equation holds.*

$$M^{-1}(M(sp_\sigma(Q) \wedge Q)) \wedge sp_\sigma(Q) \leq Q.$$

Proposition 2.2 *[49] Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$. Assume that the assumption (H) holds. Then there exists a balanced state feedback controller $f \in F_b$ such that $Re(G \mid f) = Q$, that is, f is a solution to the BSFCOP if and only if Q is controllable and observable.*

When Q is controllable and observable, a controller $f \in \Gamma^X$ given by the following equation is a solution to the BSFCOP.

$$f(x) = \begin{cases} \Sigma_u \cup \Sigma_c(Q, x) & \text{if } M(Q)(M(x)) = 1, \\ \Sigma & \text{otherwise,} \end{cases} \tag{2.12}$$

where

$$\Sigma_c(Q, x) = \{\sigma \in \Sigma_c;\ \text{there exists } x' \in M^{-1}(M(x)) \text{ with } (wp_\sigma(Q) \wedge Q)(x') = 1\}.$$

Kumar et al. proposed another definition of observability without imposing any restriction on the mask M [38]. They then showed that controllability and observability defined in [38] are necessary and sufficient conditions for the existence of a *dynamic* controller which uses the entire history of state observations and control actions.

2.3.3 Modular State Feedback Control

We consider a DES G given by Eq. (2.1) under complete observations. In this subsection, we deal with control specifications which are given in terms of conjunction and/or disjunction of component predicates.

Modular state feedback control was firstly studied by Ramadge and Wonham [68]. In the Ramadge-Wonham model, it has been proved in [68] that control-invariance of component predicates implies control-invariance of the total predicates, that is, for any control-invariant predicates $Q_1, Q_2 \in \mathbf{Q}$, $Q_1 \wedge Q_2$ and $Q_1 \vee Q_2$ are also control-invariant.

Li and Wonham considered the case that a control specification is constructed from conjunction of component predicates in the Ramadge-Wonham model [47, 49]. For state feedback controllers f and $g \in \Gamma^X$, we shall define the conjunction $f \wedge g \in \Gamma^X$ as follows:

$$f \wedge g(x) = f(x) \cap g(x) \quad \text{for any } x \in X.$$

Let Q_1 and $Q_2 \in \mathbf{Q}$ be predicates with $Q_1(x^0) = Q_2(x^0) = 1$. Assume that $[Q_1](x^0) = [Q_2](x^0) = [Q_1 \wedge Q_2](x^0) = 1$. Then there exist balanced state feedback controllers f_1 and

$f_2 \in \Gamma^X$ with $Re(G \mid f_1) = \sup \underline{C}(Q_1)$ and $Re(G \mid f_2) = \sup \underline{C}(Q_2)$, respectively [49]. It has been proved in [47] that $Re(G \mid f_1 \wedge f_2) = \sup \underline{C}(Q_1 \wedge Q_2)$. However, the following equation does not always hold.

$$\sup \underline{C}(Q_1) \wedge \sup \underline{C}(Q_2) = \sup \underline{C}(Q_1 \wedge Q_2). \tag{2.13}$$

Predicates Q and $Q' \in \mathbf{Q}$ with $Q(x^0) = Q'(x^0) = 1$ are said to be nonconflicting (with respect to G) if the following equation holds [47].

$$Re(G, Q) \wedge Re(G, Q') = Re(G, Q \wedge Q'). \tag{2.14}$$

It has been also proved that Eq. (2.13) holds if and only if $\sup \underline{C}(Q_1)$ and $\sup \underline{C}(Q_2)$ are nonconflicting [47].

However, the above results cannot be applied to the Golaszewski-Ramadge model directly. Ushio presented conditions under which the unique MPC can be constructed in a modular fashion for the Golaszewski-Ramadge model [96].

2.3.4 Concurrently Well-Posedness

Let $G = (X, \Sigma, \delta, x^0)$ be a DES modeled by the Ramadge-Wonham model. We extend G in order to include concurrency [48, 100]. The extended concurrent DES G^{con} is described by

$$G^{con} = (X, 2^{\Sigma}, \delta^{con}, x^0). \tag{2.15}$$

An event $e = \{\sigma_1, \sigma_2, ..., \sigma_m\} \in 2^{\Sigma}$ indicates the simultaneous occurrence of $\sigma_1, \sigma_2, ..., \sigma_m$. Let $\nu(e)$ be the set of all concatenations of all events belonging to e. We impose the assumption that $\delta(s_1, x) = \delta(s_2, x)$ for any s_1, $s_2 \in \nu(e)$ and any $x \in X$ if $\delta(s_1, x)!$. The assumption always holds for vector DESs [44, 49]. The transition function $\delta^{con} : 2^{\Sigma} \times X \to X$ is defined as follows: for any $e \in 2^{\Sigma}$ and any $x \in X$, $\delta^{con}(e, x)!$ if and only if $\delta(s, x)!$ for some $s \in \nu(e)$ and $\delta(\sigma, x)!$ for any $\sigma \in e$. If $\delta^{con}(e, x)!$, then $\delta^{con}(e, x) = \delta(s, x)$ for some $s \in \nu(e)$. δ^{con} is well-defined by the above assumption.

Let $Q \in \mathbf{Q}$ be a predicate, and $Per(Q)$ and $Per^{con}(Q)$ be the sets of all permissive controllers in G and G^{con}, respectively. Note that $Per(Q)$ is nonempty if and only if $Per^{con}(Q)$ is nonempty [100]. It is obvious that $Per^{con}(Q) \subseteq Per(Q)$. Ushio et. al. presented a necessary and sufficient condition for concurrency to have no effect on control, that is, $Per^{con}(Q) = Per(Q)$. A predicate $Q \in \mathbf{Q}$ is said to be *concurrently well-posed* (CWP) if the following condition (C2-14) holds [48]:

(C2-14) For any $\{\alpha, \beta\} \in 2^{\Sigma}$ and any $x \in X$ with $Q(x) = 1$,

$$\delta^{con}(\{\alpha,\beta\},x)!,\ Q(\delta(\alpha,x)) = 1 \text{ and } Q(\delta(\beta,x)) = 1 \ \Rightarrow Q(\delta^{con}(\{\alpha,\beta\},x)) = 1.$$

Let $Q \in \mathbf{Q}$ be a control-invariant predicate. Then it has been proved that $Per^{con}(Q) = Per(Q)$ if and only if Q is CWP [100].

2.3.5 Maximally Permissive Controllers for Petri Nets

Let $N_c^{con} = (P \cup P_c, T, I, I_c, O, M_{c0})$ be a CPN with concurrency. For each $t_b \in T_b$, the predicate $D_{t_b} \in \mathbf{Q}$ and transformations wp_{t_b} and wlp_{t_b} on $\mathbf{Q}$ are defined as follows [34, 95]:

$$D_{t_b}(M) = \begin{cases} 1 & \text{if } M[t_b>, \\ 0 & \text{otherwise.} \end{cases}$$

$$wp_{t_b}(Q)(M) = \begin{cases} 1 & \text{if } M[t_b > M' \text{ and } Q(M') = 1, \\ 0 & \text{otherwise.} \end{cases}$$

$$wlp_{t_b}(Q) = wp_{t_b}(Q) \vee \sim D_{t_b}.$$

In the controlled DES $\mathcal{G}_1$, for a state feedback controller $f \in \Gamma_1^{R(N)}$ and a b-transition $t_b \in T_b$, the predicate $f_{t_b} \in \mathbf{Q}$ is defined by

$$f_{t_b}(M) = \begin{cases} 1 & \text{if } I_c(t, p_c) \le f(M)(p_c) \quad \forall t \in t_b \text{ and } \forall p_c \in P_c, \\ 0 & \text{otherwise.} \end{cases}$$

Also, in $\mathcal{G}_2$, for $f \in \Gamma_2^{R(N)}$ and $t_b \in T_b$, $f_{t_b} \in \mathbf{Q}$ is defined by

$$f_{t_b}(M) = \begin{cases} 1 & \text{if } \sum_{t \in t_b} I_c(t, p_c) \le f(M)(p_c) \quad \forall p_c \in P_c, \\ 0 & \text{otherwise.} \end{cases}$$

Let $Q \in \mathbf{Q}$ be a predicate, and $Per_1(Q)$ and $Per_2(Q)$ be the sets of all permissive controllers of Q in $\mathcal{G}_1$ and $\mathcal{G}_2$, respectively. Note that $Per_1(Q)$ is nonempty if and only if $Per_2(Q)$ is nonempty. It has been proved that $Per_1(Q) \neq \emptyset$ ($Per_1(Q) \neq \emptyset$) if and only if Q is T_u^*-invariant, that is, $Q \le wlp_{t_b}(Q)$ for any $t_b \in T_u^* \cap T_b$ [68, 34]. In general, there are more than one permissive controller for a T_u^*-invariant predicate Q [95].

A partial order "$\le$" on $\Gamma_1^{R(N)}$ and $\Gamma_2^{R(N)}$ is defined as follows: $f_1 \le f_2$ if and only if $f_1(M)(p_c) \le f_2(M)(p_c)$ for any $M \in R(N)$ and any $p_c \in P_c$.

In this thesis, the following condition (C2-15) will be imposed on $f \in \Gamma_2^{R(N)}$, which is called the minimum token condition, or the MTC for short [95].

(C2-15) For any $M \in R(N)$ and any $p_c \in P_c$, $f(M)(p_c) = 0$ or there exists a b-transition t_b such that $M[t_b >$ and $f(M)(p_c) = \sum_{t \in t_b} I_c(t, p_c)$.

It is obvious that, for every state feedback controller $f \in \Gamma_2^{R(N)}$, there exists a controller $f^* \in \Gamma_2^{R(N)}$ with the MTC such that if $(f_{t_b} \wedge D_{t_b})(M) = 1$ then $f^*_{t_b}(M) = 1$ for any $M \in R(N)$ and any $t_b \in T_b$ [95].

Let $\Omega_2(Q) \subseteq Per_2(Q)$ be the set of permissive controllers with the MTC for Q. A controller $f_{max} \in Per_1(Q)$ (respectively, $\Omega_2(Q)$) is said to be *maximally permissive controller* (MPC) for Q in $\mathcal{G}_1$ (respectively, $\mathcal{G}_2$) if no controller $f(\neq f_{max}) \in Per_1(Q)$ (respectively, $\Omega_2(Q)$) satisfies that $f_{max} \leq f$ [34, 95]. In general, there exist more than one MPC in either of $\mathcal{G}_1$ and $\mathcal{G}_2$. Krogh has proposed an algorithm for computing all MPCs [34]. Holloway and Krogh presented a computationally efficient method to compute MPCs in controlled marked graphs [24]. It has been proved that the computational complexity of the method is polynomial [36]. They also derived sufficient conditions for liveness in the closed-loop system with a MPC [25]. The method proposed in [24] has been applied to other subclasses of CPNs: controlled state machines [101] and controlled complementary-places Petri nets [11]. Holloway and Guan defined a subclass of CPNs which includes both controlled marked graphs and controlled state machines, and extended the result in [24] to the subclass [26]. In [19], Giua et al. presented a synthesis method of Petri net supervisors for the problem same as [24].

However, if there are more than one MPC, we have to select one MPC among them. The unique MPC is optimal in the sense that it allows the largest set of transitions to fire at each marking. A necessary and sufficient condition for the unique existence of the MPC has been shown in [95].

Definition 2.3 *[95] Let $Q \in \mathbf{Q}$ be a T_u^*-invariant predicate. Q is said to be weakly interactive, (WI) in $\mathcal{G}_1$ (respectively, $\mathcal{G}_2$) if, for any f, $g \in Per_1(Q)$ (respectively, $\Omega_2(Q)$) and any $t_b \in T_b$, the following equation holds.*

$$Q \leq wlp_{t_b}(Q) \vee f_{t_b} \vee g_{t_b} \vee \sim (f+g)_{t_b}. \tag{2.16}$$

Proposition 2.3 *[95] Let $Q \in \mathbf{Q}$ be a T_u^*-invariant predicate. Then the MPC for Q exists uniquely in $\mathcal{G}_1$ (respectively, $\mathcal{G}_2$) if and only if Q is WI in G_1 (respectively, G_2).*

Chapter 3

State Feedback Control under Complete Observations

3.1 Introduction

In this chapter, we study the state feedback control problem (SFCP) formulated in **2.3.2** in the Golaszewski-Ramadge model. The SFCP requires that the set of all reachable states in the closed-loop system is equal to the given predicate. First, we derive a necessary and sufficient condition for the existence of a state feedback controller under the assumption that the set Γ of control patterns is closed under union. We will call the condition the Γ-controllability condition. However, a given predicate is not necessarily Γ-controllable. So we next derive a closed form expression of the supremal Γ-controllable subpredicate of a given predicate. Finally, we demonstrate synthesis of a controller for a simple manufacturing system.

3.2 Γ-Controllability and State Feedback Controllers

We consider a DES modeled by the Golaszewski-Ramadge model. Let $G = (X, \Sigma, \delta, x^0)$ be an automaton defined by Eq. (2.1). In this chapter, we assume the following condition (C3-1).

(C3-1) Γ is closed under union, that is, for any γ_1 and $\gamma_2 \in \Gamma$, $\gamma_1 \cup \gamma_2 \in \Gamma$.

Let $Q \in \mathbf{Q}$ be a given predicate with $Q(x^0) = 1$. For $Q \in \mathbf{Q}$ and $x \in X$, we define the subset $\Gamma(Q, x) \subseteq \Gamma$ as follows:

$$\Gamma(Q,x) = \{\gamma \in \Gamma;\ \bigwedge_{\sigma \in \gamma} wlp_\sigma(Q)(x) = 1\}.$$

$\Gamma(Q, x)$ consists of all control patterns disabling all events whose occurrence leads to a state where Q is false. We define the transformation Θ on $\mathbf{Q}$ as follows:

$$\Theta(Q)(x) = \begin{cases} 1 & \text{if } \Gamma(Q,x) \neq \emptyset, \\ 0 & \text{otherwise.} \end{cases}$$

Proposition 3.1 *For any predicate $Q \in \mathbf{Q}$ and any state $x \in X$ with $\Theta(Q)(x) = 1$, there exists the supremal element of $\Gamma(Q, x)$ under "$\subseteq$", denoted by $\sup \Gamma(Q, x)$.*

Proof: It is sufficient to prove that, for any γ_1, $\gamma_2 \in \Gamma(Q, x)$, $\gamma_1 \cup \gamma_2 \in \Gamma(Q, x)$. Since

$$\begin{aligned} \bigwedge_{\sigma \in \gamma_1 \cup \gamma_2} wlp_\sigma(Q)(x) &= \left(\bigwedge_{\sigma \in \gamma_1} wlp_\sigma(Q)\right) \wedge \left(\bigwedge_{\sigma \in \gamma_2} wlp_\sigma(Q)\right)(x) \\ &= 1, \end{aligned}$$

we have $\gamma_1 \cup \gamma_2 \in \Gamma(Q, x)$. □

The predicate $\Upsilon_D(Q) \in \mathbf{Q}$ is defined inductively as follows:

(C3-2) $\Upsilon_D(Q)(x^0) = 1$;

(C3-3) If $\Upsilon_D(Q)(x) = 1$ and $D_\sigma(x) = 1$ for some $\sigma \in \sup \Gamma(Q, x)$, then $\Upsilon_D(Q)(\delta(\sigma, x)) = 1$;

(C3-4) Every state satisfying $\Upsilon_D(Q)$ is obtained as in (C3-2) and (C3-3).

From the above definition, for any $x \in X$ with $\Upsilon_D(Q)(x) = 1$, there exist $x^1, x^2, ..., x^m \in X$ and $\sigma^0, \sigma^1, ..., \sigma^{m-1} \in \Sigma$ which satisfy the following conditions (C3-5)–(C3-8):

(C3-5) $\delta(\sigma^j, x^j) = x^{j+1}$ for $j = 0, 1, ..., m-1$,

(C3-6) $Q(x^j) = 1$ for $j = 0, 1, ..., m$,

(C3-7) $\sigma^j \in \sup \Gamma(Q, x^j)$ for $j = 0, 1, ..., m-1$,

(C3-8) $x^m = x$.

We introduce the notion of Γ-controllability of predicates in order to solve the SFCP.

Definition 3.1 *A predicate $Q \in \mathbf{Q}$ with $Q(x^0) = 1$ is said to be Γ-controllable (with respect to G) if the following equation holds.*

$$Q \leq \Upsilon_D(Q) \wedge \Theta(Q). \tag{3.1}$$

We now give a necessary and sufficient condition under which a solution to the SFCP exists.

Theorem 3.1 *Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$. Then there exists a state feedback controller $f \in \Gamma^X$ such that $Re(G \mid f) = Q$, that is, f is a solution to the SFCP if and only if Q is Γ-controllable.*

We need the following lemma in order to prove Theorem 3.1.

Lemma 3.1 *Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$. Then, for any state feedback controller $f \in \Gamma^X$ such that $Re(G \mid f) = Q$, the following equation holds.*

$$f(x) \in \Gamma(Q, x) \quad \forall x \in X \text{ with } Q(x) = 1. \tag{3.2}$$

Proof: Consider $x \in X$ with $Q(x) = 1$. Since $Re(G \mid f) = Q$, we have $Re(G \mid f)(x) = 1$. For any $\sigma \in f(x)$, if $D_\sigma(x) = 0$ then $wlp_\sigma(Q)(x) = 1$. If $D_\sigma(x) = 1$ then $Q(\delta(\sigma, x)) = Re(G \mid f)(\delta(\sigma, x)) = 1$, which implies that $wlp_\sigma(Q)(x) = 1$. Therefore, we have $f(x) \in \Gamma(Q, x)$. □

Proof of Theorem 3.1: ($\Leftarrow$) By Γ-controllability of Q, $\Theta(Q)(x) = 1$ for any $x \in X$ with $Q(x) = 1$. So we can construct a controller $f_s \in \Gamma^X$ as follows:

$$f_s(x) = \begin{cases} \sup \Gamma(Q, x) & \text{if } Q(x) = 1, \\ \Sigma & \text{otherwise.} \end{cases} \tag{3.3}$$

We shall show that $Re(G \mid f_s) = Q$. We shall first prove that $Re(G \mid f_s) \leq Q$. For any $x \in X$ with $Re(G \mid f_s)(x) = 1$, there exist $x^1, x^2, ..., x^m \in X$ and $\sigma^0, \sigma^1, ..., \sigma^{m-1} \in \Sigma$ satisfying the conditions (C2-8)–(C2-10) for $f = f_s$. We shall show by induction that $Q(x) = Q(x^m) = 1$. By the assumption, we have $Q(x^0) = 1$. For the induction step, suppose that $Q(x^k) = 1$. By the condition (C2-9) and Eq. (3.3), $\sigma^k \in \sup \Gamma(Q, x^k)$. So we have $wlp_{\sigma^k}(Q)(x^k) = 1$, which implies that $Q(x^{k+1}) = Q(\delta(\sigma^k, x^k)) = 1$. This completes the induction.

Next, we shall prove that $Q \leq Re(G \mid f_s)$. Since $Q \leq \Upsilon_D(Q)$, for any $x \in X$ with $Q(x) = 1$, there exist $x^1, x^2, ..., x^m \in X$ and $\sigma^0, \sigma^1, ..., \sigma^{m-1} \in \Sigma$ satisfying the conditions (C3-5)–(C3-8). In order to show that $Re(G \mid f_s)(x) = 1$, it is sufficient to prove that $\sigma^j \in f_s(x^j)$ $(j = 0, 1, ..., m-1)$. By the conditions (C3-6), (C3-7) and Eq. (3.3), we have $\sigma^j \in f_s(x^j)$.

($\Rightarrow$) Suppose that there exists $f \in \Gamma^X$ such that $Re(G \mid f) = Q$. Consider $x \in X$ with $Q(x) = 1$. By Lemma 3.1, we have $f(x) \in \Gamma(Q, x) \neq \emptyset$, which implies that $\Theta(Q)(x) = 1$.

We next prove that $\Upsilon_D(Q)(x) = 1$. Since $Re(G \mid f) = Q$, there exist $x^1, x^2, ..., x^m \in X$ and $\sigma^0, \sigma^1, ..., \sigma^{m-1} \in \Sigma$ which satisfy the conditions (C2-8)–(C2-10). We shall show by induction that $\Upsilon_D(Q)(x) = \Upsilon_D(Q)(x^m) = 1$. By the definition of $\Upsilon_D(Q)$, we have $\Upsilon_D(Q)(x^0) = 1$. For the induction step, suppose that $\Upsilon_D(Q)(x^k) = 1$. It follows that $Q(x^k) = 1$. Then by the condition (C2-9) and Lemma 3.1, $\sigma^k \in f(x^k) \subseteq \sup \Gamma(Q, x^k)$. Therefore, we have $\Upsilon_D(Q)(x^{k+1}) = 1$. This completes the induction. □

When Q is Γ-controllable, the controller $f_s \in \Gamma^X$ given by Eq. (3.3) is a solution to the SFCP. Note that Theorem 3.1 is a generalization of the results for the Ramadge-Wonham model obtained in [49, 38]. In the Ramadge-Wonham model, the condition of Γ-controllability is reduced to that of controllability.

Example 3.1 We consider a DES G shown in Figure 3.1, where $\Sigma = \{\sigma^1, \sigma^2, \sigma^3\}$, $X = \{x^0, x^1, x^2, x^3\}$ and x^0 is the initial state. Let $\Gamma = \{\{\sigma^1, \sigma^2\}, \Sigma\}$. Obviously, Γ satisfies the condition (C3-1). Consider a predicate $Q \in \mathbf{Q}$ given by

$$Q(x) = \begin{cases} 1 & \text{if } x \in \{x^0, x^1, x^2\}, \\ 0 & \text{otherwise,} \end{cases}$$

Then we have

$$\Gamma(Q, x^0) = \{\{\sigma^1, \sigma^2\}\} \text{ and } \Gamma(Q, x^1) = \Gamma(Q, x^2) = \Gamma,$$

which implies that $\Theta(Q)(x^0) = \Theta(Q)(x^1) = \Theta(Q)(x^2) = 1$. By the definition, $\Upsilon_D(Q)(x^0) = 1$. Since $D_{\sigma^1}(x^0) = 1$ and $\sigma^1 \in \sup \Gamma(Q, x^0)$ (respectively, $D_{\sigma^2}(x^0) = 1$ and $\sigma^2 \in \sup \Gamma(Q, x^0)$), we have $\Upsilon_D(Q)(x^1) = 1$ (respectively, $\Upsilon_D(Q)(x^2) = 1$). So Q is Γ-controllable, which implies together with Theorem 3.1 that a solution f to the SFCP exists, and is given by

$$f(x) = \begin{cases} \{\sigma^1, \sigma^2\} & \text{if } x = x^0, \\ \Sigma & \text{otherwise.} \end{cases}$$

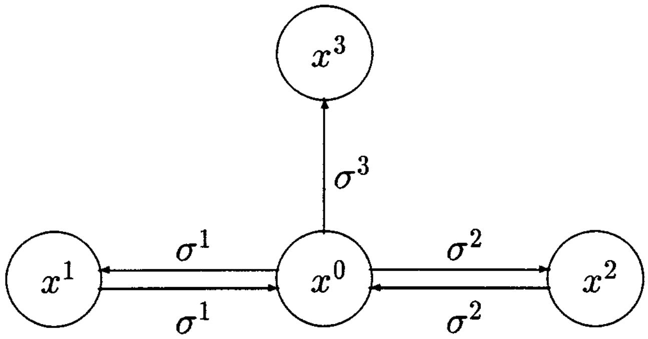

Figure 3.1: The DES of Example 3.1.

3.3 The Supremal Γ-Controllable Subpredicate

Theorem 3.1 shows that if $Q \in \mathbf{Q}$ with $Q(x^0) = 1$ is Γ-controllable, then there exists a state feedback controller $f \in \Gamma^X$ such that $Re(G \mid f) = Q$. However, a given predicate Q is not always Γ-controllable. In such a case, we have to synthesize a controller for its Γ-controllable subpredicate.

Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$. We define the subset $\Gamma C(Q) \subseteq \mathbf{Q}$ as follows:

$$\Gamma C(Q) = \{Q' \in \mathbf{Q};\ Q'(x^0) = 1,\ Q' \le Q \text{ and } Q' \text{ is } \Gamma\text{-controllable}\}.$$

The following proposition shows that there exists the supremal element of $\Gamma C(Q)$ under "$\le$", denoted by $\sup \Gamma C(Q)$, if $\Gamma C(Q) \neq \emptyset$.

Proposition 3.2 *For any $Q \in \mathbf{Q}$ with $Q(x^0) = 1$, there exists $\sup \Gamma C(Q)$ if $\Gamma C(Q) \neq \emptyset$.*
Proof: Let I be any index set, and $Q_\alpha \in \Gamma C(Q)$ for each $\alpha \in I$. Letting

$$\hat{Q} = \bigvee_{\alpha \in I} Q_\alpha,$$

it is obvious that $\hat{Q}(x^0) = 1$ and $\hat{Q} \le Q$. We shall show that $\hat{Q}$ is Γ-controllable. For any $x \in X$ with $\hat{Q}(x) = 1$, there exists $\beta \in I$ such that $Q_\beta(x) = 1$. By Γ-controllability of Q_β, we have $\Upsilon_D(Q_\beta)(x) = 1$ and $\Theta(Q_\beta)(x) = 1$. Since $Q_\beta \leq \hat{Q}$, it follows that $\Upsilon_D(\hat{Q})(x) = 1$ and $\Theta(\hat{Q})(x) = 1$. So $\hat{Q}$ is Γ-controllable. Therefore, $\Gamma C(Q)$ is closed under $\vee$, which implies that there exists $\sup \Gamma C(Q)$. □

Remark 3.1 For $Q \in \mathbf{Q}$ with $Q(x^0) = 1$, $\Gamma C(Q)$ is not always nonempty. We give a counter-example as follows. We consider a DES G shown in Figure 3.2, where $\Sigma = \{\sigma^1, \sigma^2\}$, $X = \{x^0, x^1, x^2\}$ and x^0 is the initial state. Let $\Gamma = \{\{\sigma^2\}, \{\sigma^1, \sigma^2\}\}$. Obviously, Γ satisfies the condition (C3-1). Consider a predicate $Q \in \mathbf{Q}$ given by

$$Q(x) = \begin{cases} 1 & \text{if } x \in \{x^0, x^1\}, \\ 0 & \text{otherwise,} \end{cases}$$

Then it is easily shown that $\Gamma(Q', x^0) = \emptyset$ for any subpredicate Q' of Q with $Q'(x^0) = 1$, which implies that $\Gamma C(Q) = \emptyset$.

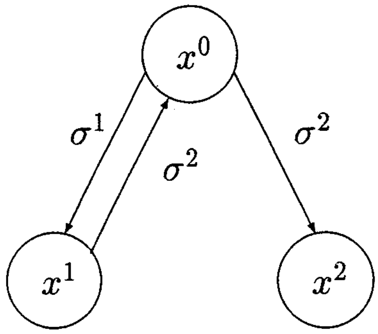

Figure 3.2: The DES of Remark 3.1.

We call $\sup \Gamma C(Q)$ the supremal Γ-controllable subpredicate of Q. In the following, we derive a closed form expression of $\sup \Gamma C(Q)$. First, we define Γ-invariance of predicates.

Definition 3.2 *A predicate $Q \in \mathbf{Q}$ is said to be Γ-invariant (with respect to G) if the following equation holds.*

$$Q \leq \Theta(Q). \tag{3.4}$$

In the Ramadge-Wonham model, the condition of Γ-invariance is reduced to that of Σ_u-invariance [68]. Let $Q \in \mathbf{Q}$ be a predicate. We define the subset $\Gamma I(Q) \subseteq \mathbf{Q}$ as follows:

$$\Gamma I(Q) = \{Q' \in \mathbf{Q};\ Q' \leq Q \text{ and } Q' \text{ is Γ-invariant}\}.$$

$\Gamma I(Q)$ is nonempty because $\mathbf{0} \in \Gamma I(Q)$. We denote the supremal element of $\Gamma I(Q)$ under "$\leq$" by $\sup \Gamma I(Q)$. We call $\sup \Gamma I(Q)$ the supremal Γ-invariant subpredicate of Q. The following corollary can be proved in the same way as Proposition 3.2.

Corollary 3.1 *For any $Q \in \mathbf{Q}$, there exists* $\sup \Gamma I(Q)$.

For $Q \in \mathbf{Q}$, we consider a sequence $\{Q_j\}$ of predicates defined by

$$Q_0 := Q, \quad Q_{j+1} := Q \wedge \Theta(Q_j) \quad \text{for } j = 0, 1, \ldots \tag{3.5}$$

Proposition 3.3 *Let $Q \in \mathbf{Q}$ be a predicate and $\{Q_j\}$ be the sequence of predicates defined by Eq. (3.5). Assume that there exists $m \in \mathcal{N}$ (the set of all nonnegative integers) such that $Q_m = Q_{m+1}$. Then*

$$\sup \Gamma I(Q) = Q_m. \tag{3.6}$$

We need the following lemma in order to prove Proposition 3.3.

Lemma 3.2 *Let $Q \in \mathbf{Q}$ be a predicate and $\{Q_j\}$ be the sequence of predicates defined by Eq. (3.5). Then, for any $Q' \in \Gamma I(Q)$,*

$$Q' \leq Q_j \qquad \forall j \in \mathcal{N}. \tag{3.7}$$

Proof: We shall prove Eq. (3.7) by induction. Obviously, $Q' \leq Q_0$. For the induction step, suppose that $Q' \leq Q_k$. Then by Γ-invariance of Q', the following equation holds.

$$\begin{aligned} Q' &= Q \wedge Q' \\ &\leq Q \wedge \Theta(Q') \\ &\leq Q \wedge \Theta(Q_k) \\ &= Q_{k+1}. \end{aligned}$$

This completes the induction.

Proof of Proposition 3.3: Since $Q_m = Q_{m+1} \leq \Theta(Q_m)$, we have $Q_m \in \Gamma I(Q)$. We also have by Lemma 3.2 that $Q' \leq Q_m$ for any $Q' \in \Gamma I(Q)$. □

The following corollary shows that $\sup \Gamma I(Q) = \bigwedge_{j \in \mathcal{N}} Q_j$ under a certain condition on Γ.

Corollary 3.2 *Let $Q \in \mathbf{Q}$ be a predicate and $\{Q_j\}$ be the sequence of predicates defined by Eq. (3.5). Assume that Γ is closed under intersection, that is, for any γ_1 and $\gamma_2 \in \Gamma$, $\gamma_1 \cap \gamma_2 \in \Gamma$. Then*

$$\sup \Gamma I(Q) = Q_\infty := \bigwedge_{j \in \mathcal{N}} Q_j. \tag{3.8}$$

We need the following lemma in order to prove Corollary 3.2.

Lemma 3.3 *Assume that Γ is closed under intersection. Then the transformation Θ on $\mathbf{Q}$ is conjunctive, that is, for any index set I on $\mathbf{Q}$,*

$$\Theta\left(\bigwedge_{\alpha \in I} Q_\alpha\right) = \bigwedge_{\alpha \in I} \Theta(Q_\alpha). \tag{3.9}$$

Proof: It is obvious that, for each $\alpha \in I$,

$$\Theta\left(\bigwedge_{\alpha \in I} Q_\alpha\right) \leq \Theta(Q_\alpha),$$

which implies that

$$\Theta\left(\bigwedge_{\alpha \in I} Q_\alpha\right) \leq \bigwedge_{\alpha \in I} \Theta(Q_\alpha).$$

We shall prove the reverse inequality. Consider $x \in X$ with $\bigwedge_{\alpha \in I} \Theta(Q_\alpha)(x) = 1$. For each $\alpha \in I$, there exists $\gamma_\alpha \in \Gamma$ with $\gamma_\alpha \in \Gamma(Q_\alpha, x)$. Let $\hat{\gamma} := \bigcap_{\alpha \in I} \gamma_\alpha$. Since Γ is closed under intersection, we have $\hat{\gamma} \in \Gamma$. For any $\sigma \in \hat{\gamma}$, if $D_\sigma(x) = 0$ then $wlp_\sigma\left(\bigwedge_{\alpha \in I} Q_\alpha\right)(x) = 1$. We consider the case that $D_\sigma(x) = 1$. Since $\sigma \in \hat{\gamma} \subseteq \gamma_\alpha \in \Gamma(Q_\alpha, x)$ for each $\alpha \in I$, we have

$$\begin{aligned} wlp_\sigma\left(\bigwedge_{\alpha \in I} Q_\alpha\right)(x) &= \bigwedge_{\alpha \in I} wlp_\sigma(Q_\alpha)(x) \\ &= 1. \end{aligned}$$

So we have $\hat{\gamma} \in \Gamma\left(\bigwedge_{\alpha \in I} Q_\alpha, x\right) \neq \emptyset$, which implies that $\Theta\left(\bigwedge_{\alpha \in I} Q_\alpha\right)(x) = 1$. □

Proof of Corollary 3.2: First, we shall prove that $Q_\infty \in \Gamma I(Q)$. By Lemma 3.3, we have

$$\begin{aligned} Q_\infty &= \bigwedge_{j \in \mathcal{N}} Q_j \\ &= Q \wedge \left(\bigwedge_{j \in \mathcal{N}} (Q \wedge \Theta(Q_j))\right) \end{aligned}$$

$$\begin{aligned} &\leq \bigwedge_{j\in\mathcal{N}} \Theta(Q_j) \\ &= \Theta\left(\bigwedge_{j\in\mathcal{N}} Q_j\right), \end{aligned}$$

which implies that $Q_\infty \in \Gamma I(Q)$. For any $Q' \in \Gamma I(Q)$, we have by Lemma 3.2 that

$$Q_\infty = \bigwedge_{j\in\mathcal{N}} Q_j \geq Q',$$

which implies that $\sup \Gamma I(Q) = Q_\infty$. □

Remark 3.2 When the set X of states is finite, it is easily proved that Eq. (3.5) always converges to $\sup \Gamma I(Q)$ after a finite number of iterations. Note that Eq. (3.5) may converge after a finite number of iterations even if X is infinite.

The following theorem presents a necessary and sufficient condition under which $\Gamma C(Q)$ is nonempty.

Theorem 3.2 *Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$. Then $\Gamma C(Q) \neq \emptyset$ if and only if* $\sup \Gamma I(Q)(x^0) = 1$.

We need the following lemmas in order to prove Theorem 3.2.

Lemma 3.4 *Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$. For any $x \in X$ with $(\Theta(Q) \wedge \Upsilon_D(Q))(x) = 1$, the following equation holds.*

$$\sup \Gamma(Q, x) = \sup \Gamma(\Upsilon_D(Q), x). \tag{3.10}$$

Proof: Since $\Theta(Q)(x) = 1$, $\sup \Gamma(Q,x)$ exists. First, we shall show that $\sup \Gamma(Q,x) \subseteq \sup \Gamma(\Upsilon_D(Q),x)$. It is sufficient to prove that $\sup \Gamma(Q,x) \in \Gamma(\Upsilon_D(Q),x) \neq \emptyset$. Consider $\sigma \in \sup \Gamma(Q,x)$. If $D_\sigma(x) = 0$, then $wlp_\sigma(\Upsilon_D(Q))(x) = 1$. We consider the case that $D_\sigma(x) = 1$. Since $\Upsilon_D(Q)(x) = 1$, we have $\Upsilon_D(Q)(\delta(\sigma,x)) = 1$, which implies that $wlp_\sigma(\Upsilon_D(Q))(x) = 1$. So we have $\sup \Gamma(Q,x) \in \Gamma(\Upsilon_D(Q),x)$.

Conversely, since $\Upsilon_D(Q) \leq Q$, we have $\sup \Gamma(\Upsilon_D(Q),x) \subseteq \sup \Gamma(Q,x)$. □

Lemma 3.5 *The transformation Υ on $\mathbf{Q}$ is idempotent, that is,*

$$\Upsilon_D(\Upsilon_D(Q)) = \Upsilon_D(Q) \quad \forall Q \in \mathbf{Q} \text{ with } Q(x^0) = 1. \tag{3.11}$$

Proof: Since $\Upsilon_D(Q) \leq Q$, it follows that $\Upsilon_D(\Upsilon_D(Q)) \leq \Upsilon_D(Q)$. We shall prove that $\Upsilon_D(Q) \leq \Upsilon_D(\Upsilon_D(Q))$. Consider $x \in X$ with $\Upsilon_D(Q)(x) = 1$. Then there exist $x^1, x^2, ..., x^m \in X$ and $\sigma^0, \sigma^1, ..., \sigma^{m-1} \in \Sigma$ satisfying the conditions (C3-5)–(C3-8). We shall prove by induction that $\Upsilon_D(\Upsilon_D(Q))(x) = \Upsilon_D(\Upsilon_D(Q))(x^m) = 1$. By the definition of $\Upsilon_D(\Upsilon_D(Q))$, we have $\Upsilon_D(\Upsilon_D(Q))(x^0) = 1$. For the induction step, suppose that $\Upsilon_D(\Upsilon_D(Q))(x^k) = 1$. Then it is obvious that $(\Theta(Q) \wedge \Upsilon_D(Q))(x^k) = 1$. By the conditions (C3-5), (C3-7) and Lemma 3.4, we have $D_{\sigma^k}(x^k) = 1$ and $\sigma^k \in \sup\Gamma(Q, x^k) = \sup\Gamma(\Upsilon_D(Q), x^k)$, which implies that $\Upsilon_D(\Upsilon_D(Q))(x^{k+1}) = 1$. This completes the induction. □

Proof of Theorem 3.2: (⇐) We shall show that $\Upsilon_D(\sup\Gamma I(Q)) \in \Gamma C(Q)$. First, we shall prove that $\Upsilon_D(\sup\Gamma I(Q))$ is Γ-controllable, that is, the following equation holds.

$$\Upsilon_D(\sup\Gamma I(Q)) \leq \Upsilon_D(\Upsilon_D(\sup\Gamma I(Q))) \wedge \Theta(\Upsilon_D(\sup\Gamma I(Q))). \tag{3.12}$$

Consider $x \in X$ with $\Upsilon_D(\sup\Gamma I(Q))(x) = 1$. By Lemma 3.5, $\Upsilon_D(\Upsilon_D(\sup\Gamma I(Q)))(x) = 1$. We shall show that $\Theta(\Upsilon_D(\sup\Gamma I(Q)))(x) = 1$. Since $\Upsilon_D(\sup\Gamma I(Q))(x) = 1$, we have $\sup\Gamma I(Q)(x) = 1$, which implies that $\Theta(\sup\Gamma I(Q))(x) = 1$. So we have by Lemma 3.4 that

$$\sup\Gamma(\sup\Gamma I(Q), x) = \sup\Gamma(\Upsilon_D(\sup\Gamma I(Q)), x) \in \Gamma(\Upsilon_D(\sup\Gamma I(Q)), x) \neq \emptyset,$$

which implies that $\Theta(\Upsilon_D(\sup\Gamma I(Q)))(x) = 1$. Thus $\Upsilon_D(\sup\Gamma I(Q))$ is Γ-controllable. Since $\Upsilon_D(\sup\Gamma I(Q)) \leq \sup\Gamma I(Q) \leq Q$, we have $\Upsilon_D(\sup\Gamma I(Q)) \in \Gamma C(Q)$.

(⇒) By the definition of $\Gamma C(Q)$, $Q'(x^0) = 1$ for any $Q' \in \Gamma C(Q)$. Since $Q' \in \Gamma I(Q)$, we have $Q' \leq \sup\Gamma I(Q)$, which implies together with $Q'(x^0) = 1$ that $\sup\Gamma I(Q)(x^0) = 1$.
□

Example 3.2 We consider the same system as Remark 3.1. Since X is finite, we have by Proposition 3.3 that $\sup\Gamma I(Q) = \mathbf{0}$, which implies together with Theorem 3.2 that $\Gamma C(Q)$ is empty.

The following theorem presents a closed form expression of $\sup\Gamma C(Q)$.

Theorem 3.3 *Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$. Assume that $\sup\Gamma I(Q)(x^0) = 1$. Then $\sup\Gamma C(Q)$ is given by the following equation.*

$$\sup\Gamma C(Q) = \Upsilon_D(\sup\Gamma I(Q)). \tag{3.13}$$

Proof: From the proof of Theorem 3.2, we have $\Upsilon_D(\sup \Gamma I(Q)) \in \Gamma C(Q)$. Obviously, $Q' \leq \sup \Gamma I(Q)$ for any $Q' \in \Gamma C(Q)$. So we have by Γ-controllability of Q' that

$$Q' \leq \Upsilon_D(Q') \leq \Upsilon_D(\sup \Gamma I(Q)),$$

which implies that Eq. (3.13) holds. □

3.4 Example

We consider a simple manufacturing system consisting of two groups of two machines and a buffer shown in Figure 3.3. The machines in Group 1 take materials and pass products to the buffer B after processings are completed. Then the machines in Group 2 take products from B in order to carry out further processing. Each machine M_{ij} $(i, j = 1, 2)$ is modeled by an automaton whose state transition diagram is shown in Figure 3.4, where states I_{ij} and W_{ij} are "idle" and "working", respectively. The buffer B is also modeled by an automaton with the state set $\mathcal{N}$ (the set of nonnegative integers). Its state transitions are as follows:

$$\beta_{1j}\ (j = 1, 2) \quad : n \rightarrow n + 1,$$
$$\alpha_{2j}\ (j = 1, 2) \quad : n \rightarrow n - 1.$$

The sets Σ and X of events and states are given by

$$\Sigma = \{\alpha_{ij}, \beta_{ij}\ ;\ i, j = 1, 2\},$$

$$X = \{(x_{11}, x_{12}, x_{21}, x_{22}, x_b)\ ;\ x_{ij} \in \{I_{ij}, W_{ij}\}\ (i, j = 1, 2) \text{ and } x_b \in \mathcal{N}\},$$

where x_{ij} $(i, j = 1, 2)$ is the state of the machine M_{ij} and x_b is the state of the buffer B. Let $x^0 = (I_{11}, I_{12}, I_{21}, I_{22}, 0)$ be the initial state and Γ be the set of control patterns given by

$$\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\},$$

where

$$\begin{aligned}
\gamma_1 &= \Sigma, \\
\gamma_2 &= \{\alpha_{2j}, \beta_{ij}\ ;\ i, j = 1, 2\}, \\
\gamma_3 &= \{\alpha_{1j}, \beta_{ij}\ ;\ i, j = 1, 2\}, \\
\gamma_4 &= \{\beta_{ij}\ ;\ i, j = 1, 2\}.
\end{aligned}$$

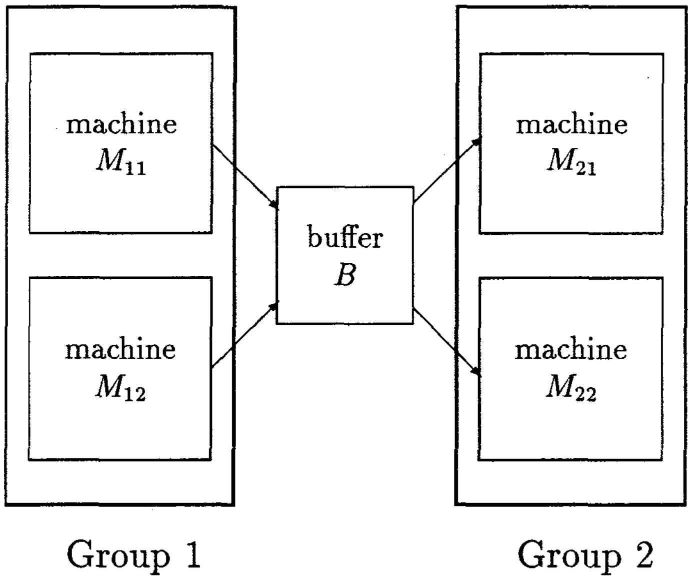

Figure 3.3: A simple manufacturing system.

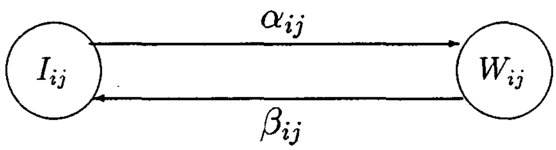

Figure 3.4: A state transition diagram of M_{ij}.

It is obvious that Γ satisfies the condition (C3-1).

Now we consider a control specification that the buffer content x_b is always at most e (≥ 1), which is given by the following predicate $Q \in \mathbf{Q}$:

$$Q((x_{11}, x_{12}, x_{21}, x_{22}, x_b)) = \begin{cases} 1 & \text{if } x_b \leq e, \\ 0 & \text{otherwise.} \end{cases}$$

For simplicity let $e = 1$. It is easily shown that Q is not Γ-controllable, and $\sup \Gamma C(Q)$ is given by

$$\sup \Gamma C(Q)((x_{11}, x_{12}, x_{21}, x_{22}, x_b)) = \begin{cases} 1 & \text{if } \sharp(x_{11}, x_{12}) + x_b \leq 1, \\ 0 & \text{otherwise,} \end{cases}$$

where

$$\sharp(x_{11}, x_{12}) = \begin{cases} 2 & \text{if } x_{11} = W_{11} \text{ and } x_{12} = W_{12}, \\ 1 & \text{if } (x_{11} = W_{11} \text{ and } x_{12} = I_{12}) \text{ or } (x_{11} = I_{11} \text{ and } x_{12} = W_{12}), \\ 0 & \text{otherwise.} \end{cases}$$

A state feedback controller $f \in \Gamma^X$ such that $Re(G \mid f) = \sup \Gamma C(Q)$ is given by

$$f((x_{11}, x_{12}, x_{21}, x_{22}, x_b)) = \begin{cases} \gamma_2 & \text{if } \sharp(x_{11}, x_{12}) + x_b = 1, \\ \gamma_1 & \text{otherwise.} \end{cases}$$

3.5 Concluding Remarks

We studied the SFCP in the Golaszewski-Ramadge model. First, we defined Γ-controllability of predicates, and showed that Γ-controllability is a necessary and sufficient condition for the existence of a state feedback controller under the assumption that the set Γ of control patterns is closed under union. We then derived a closed form expression of the supremal Γ-controllable subpredicate of a given predicate. These results can be applied to the case where there is no assumption on Γ by adapting nondeterministic state feedback controllers [48] instead of deterministic ones [73, 93].

Chapter 4

State Feedback Control under Partial Observations

4.1 Introduction

In this chapter, we study state feedback control under partial observations without any assumptions on the mask. First, we consider the balanced state feedback control and observation problem (BSFCOP) formulated in **2.3.2** in the Golaszewski-Ramadge model. We show a necessary and sufficient condition for the existence of a balanced state feedback controller.

Next, we consider the Ramadge-Wonham model. We present a necessary and sufficient condition for the existence of a (not necessarily balanced) state feedback controller. We will call the condition the M-controllability condition. Kumar et al. showed necessary and sufficient conditions for the existence of a dynamic controller [38]. However, they did not discuss the existence of a state feedback controller. Obviously, a state feedback controller is a special case of a dynamic one. But a state feedback controller is easier to implement than a dynamic one. Moreover, our condition has computational advantage in contrast to those obtained by Kumar et al. because the computational complexity to check our condition is polynomial if the system is modeled by a finite automaton. So our condition is useful in the practical point of view.

Finally, we study modular control synthesis in the Ramadge-Wonham model. We show that M-controllability of component predicates implies that of their conjunction under a certain condition. We then present a necessary and sufficient condition under which a controller can be constructed in a modular fashion.

4.2 Balanced State Feedback Control of the Golaszewski-Ramadge Model

In this section, we study the BSFCOP in the Golaszewski-Ramadge model. Let $G = (X, \Sigma, \delta, x^0)$ be an automaton defined by Eq. (2.1), and $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$.

We present a necessary and sufficient condition for the existence of a solution to the BSFCOP.

Theorem 4.1 *Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$. Then there exists a balanced state feedback controller $f \in F_b$ such that $Re(G \mid f) = Q$, that is, f is a solution to the BSFCOP if and only if the following conditions (C4-1)–(C4-3) hold.*

(C4-1) $Q = Re(G, Q)$, *equivalently* $Q \leq Re(G, Q)$.

(C4-2) *For any $y \in Y$ with $M(Q)(y) = 1$, there exists a control pattern $\gamma_y \in \Gamma$ satisfying the following equation.*

$$\Sigma(y) \cap \gamma_y = \Sigma_Q(y), \tag{4.1}$$

where

$$\Sigma(y) = \{\sigma \in \Sigma \ ; \ D_\sigma(x) = 1 \text{ for some } x \in M^{-1}(y) \text{ with } Q(x) = 1\}$$

and

$$\Sigma_Q(y) = \{\sigma \in \Sigma \ ; \ wp_\sigma(Q)(x) = 1 \text{ for some } x \in M^{-1}(y) \text{ with } Q(x) = 1\}.$$

(C4-3) *For any $\sigma \in \Sigma$, the following equation holds.*

$$Q \wedge M^{-1}(M(wp_\sigma(Q) \wedge Q)) \leq wlp_\sigma(Q). \tag{4.2}$$

Note that computational complexity to verify the conditions in Theorem 4.1 is $O(mn^2)$ if X is finite.

We need the following results in order to prove Theorem 4.1. The following lemma gives an interpretation of the condition (C4-3).

Lemma 4.1 Let $Q \in \mathbf{Q}$ be a predicate. Then the condition (C4-3) holds if and only if, for any $\sigma \in \Sigma$ and any $x, x' \in X$ with $Q(x) = Q(x') = 1$, the following condition (C4-4) holds.

(C4-4) $\quad M(x) = M(x'),\ D_\sigma(x) = D_\sigma(x') = 1$ and $Q(\delta(\sigma, x')) = 1 \ \Rightarrow\ Q(\delta(\sigma, x)) = 1.$

Proof: ($\Leftarrow$) Consider $\sigma \in \Sigma$ and $x \in X$ satisfying

$$(Q \wedge M^{-1}(M(wp_\sigma(Q) \wedge Q)))(x) = 1. \tag{4.3}$$

Obviously, $wlp_\sigma(Q)(x) = 1$ if $D_\sigma(x) = 0$. Suppose that $D_\sigma(x) = 1$. Eq. (4.3) implies that there exists $x' \in X$ such that $M(x) = M(x')$, $wp_\sigma(Q)(x') = 1$ and $Q(x') = 1$. It follows that $D_\sigma(x') = 1$ and $Q(\delta(\sigma, x')) = 1$. So we have by condition (C4-4) that $Q(\delta(\sigma, x)) = 1$, which implies that $wlp_\sigma(Q)(x) = 1$. Therefore, for any $\sigma \in \Sigma$, Eq. (4.2) holds.

($\Rightarrow$) Consider $\sigma \in \Sigma$ and $x, x' \in X$ such that $Q(x) = Q(x') = 1$, $M(x) = M(x')$, $D_\sigma(x) = D_\sigma(x') = 1$ and $Q(\delta(\sigma, x')) = 1$. Then we have $wp_\sigma(Q)(x') = 1$, and Eq. (4.3) holds by the above assumption. So by Eq. (4.2), we have $wlp_\sigma(Q)(x) = 1$, that is, $Q(\delta(\sigma, x)) = 1$. □

The following proposition gives a role of the conditions (C4-2) and (C4-3).

Proposition 4.1 *Let $Q \in \mathbf{Q}$ be a predicate. Then there exists a state feedback controller $f \in F_o$ which satisfies the following condition (C4-5) if and only if the conditions (C4-2) and (C4-3) hold.*

(C4-5) For any $\sigma \in \Sigma$ and any $x \in X$ with $Q(x) = 1$,

$$D_\sigma(x) = 1 \text{ and } \sigma \in f(x) \ \Leftrightarrow\ Q(\delta(\sigma, x)) = 1.$$

Proof: ($\Leftarrow$) Suppose that the conditions (C4-2) and (C4-3) hold. Then consider $f \in \Gamma^X$ given by

$$f(x) = \begin{cases} \gamma_{M(x)} & \text{if } M(Q)(M(x)) = 1, \\ \text{any} & \text{otherwise,} \end{cases} \tag{4.4}$$

where $\gamma_{M(x)}$ is a control pattern satisfying Eq. (4.1) with $y = M(x)$. It is obvious that $f \in F_o$. We shall show that f satisfies the condition (C4-5). Consider $x \in X$ with $Q(x) = 1$, and let $y = M(x)$. Then $M(Q)(y) = 1$. For any $\sigma \in \Sigma$ such that $D_\sigma(x) = 1$

and $\sigma \in f(x)$, we have $\sigma \in \Sigma(y) \cap f(x) = \Sigma_Q(y)$, which implies that there exists $x' \in X$ such that $M(x') = y$, $Q(x') = 1$, $D_\sigma(x') = 1$ and $Q(\delta(\sigma, x')) = 1$. So by Lemma 4.1, we have $Q(\delta(\sigma, x)) = 1$. Conversely, consider $\sigma \in \Sigma$ such that $Q(\delta(\sigma, x)) = 1$. Then $\sigma \in \Sigma_Q(y)$, and by the condition (C4-2) and Eq. (4.4), we have $\sigma \in f(x)$. Therefore, f satisfies the condition (C4-5).

($\Rightarrow$) Suppose that there exists $f \in F_o$ which satisfies the condition (C4-5). First, we shall prove the condition (C4-2). Consider $y \in Y$ with $M(Q)(y) = 1$. Let $\gamma_y = f(x)$ for $x \in M^{-1}(y)$. Note that γ_y is uniquely defined since f satisfies the condition (C2-12). We shall show that

$$\Sigma(y) \cap \gamma_y \subseteq \Sigma_Q(y).$$

For any $\sigma \in \Sigma(y) \cap \gamma_y$, there exists $x' \in X$ such that $M(x') = y$, $Q(x') = 1$, $D_\sigma(x') = 1$ and $\sigma \in f(x')$. Since f satisfies the condition (C4-5), we have $Q(\delta(\sigma, x')) = 1$, which implies that $\sigma \in \Sigma_Q(y)$. Conversely, we shall show that

$$\Sigma(y) \cap \gamma_y \supseteq \Sigma_Q(y).$$

For any $\sigma \in \Sigma_Q(y)$, there exists $x' \in X$ such that $M(x') = y$, $Q(x') = 1$, $D_\sigma(x') = 1$ and $Q(\delta(\sigma, x')) = 1$. Then it is obvious that $\sigma \in \Sigma(y)$. Since f satisfies the condition (C4-5), we have $\sigma \in f(x')$, which implies together with $M(x') = y$ that $\sigma \in f(x') = \gamma_y$. Therefore, γ_y satisfies Eq. (4.1), and the condition (C4-2) holds.

It remains to prove the condition (C4-3). Suppose that there exist $\sigma \in \Sigma$ and $x, x' \in X$ such that $Q(x) = Q(x') = 1$, $M(x) = M(x')$, $D_\sigma(x) = D_\sigma(x') = 1$ and $Q(\delta(\sigma, x')) = 1$. By Lemma 4.1, it is sufficient to show that $Q(\delta(\sigma, x)) = 1$. By the condition (C4-5), we have $\sigma \in f(x')$, which implies together with $M(x) = M(x')$ that $\sigma \in f(x)$. Since $D_\sigma(x) = 1$ and $\sigma \in f(x)$, the condition (C4-5) asserts that $Q(\delta(\sigma, x)) = 1$. □

By the proof of Proposition 4.1, it is shown that, for a predicate $Q \in \mathbf{Q}$, a controller $f \in F_o$ given by Eq. (4.4) satisfies the condition (C4-5) if the conditions (C4-2) and (C4-3) hold. The following lemma shows that f is a solution to the BSFCOP if the conditions (C4-1)–(C4-3) hold.

Lemma 4.2 Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$. Then if the conditions (C4-1)–(C4-3) hold, then $Re(G \mid f) = Q$ for a state feedback controller $f \in F_o$ given by Eq. (4.4), and f is balanced.

Proof: First, we shall prove that $Re(G \mid f) \leq Q$. For any $x \in X$ with $Re(G \mid f)(x) = 1$, there exist $x^1, x^2, ..., x^m \in X$ and $\sigma^0, \sigma^1, ..., \sigma^{m-1} \in \Sigma$ satisfying the conditions (C2-8)–(C2-10). We shall prove by induction that $Q(x) = Q(x^m) = 1$. By the assumption, we know that $Q(x^0) = 1$. For the induction step, suppose that $Q(x^k) = 1$. By the conditions (C2-8) and (C2-9), it follows that $D_{\sigma^k}(x^k) = 1$ and $\sigma^k \in f(x^k)$. The proof of Proposition 4.1 shows that f satisfies the condition (C4-5). Therefore, we have $Q(x^{k+1}) = Q(\delta(\sigma^k, x^k)) = 1$. This completes the induction.

Next, we shall prove that $Q \leq Re(G \mid f)$. The condition (C4-1) implies that, for any $x \in X$ with $Q(x) = 1$, there exist $x^1, x^2, ..., x^m \in X$ and $\sigma^0, \sigma^1, ..., \sigma^{m-1} \in \Sigma$ satisfying the conditions (C2-5)–(C2-7). In order to show that $Re(G \mid f)(x) = 1$, it is sufficient to prove that $\sigma^j \in f(x^j)$ ($j = 0, 1, ..., m-1$). By the conditions (C2-5) and (C2-6), $Q(x^j) = 1$ and $Q(\delta(\sigma^j, x^j)) = 1$ for each j. Recall f satisfies the condition (C4-5). So, we have $\sigma^j \in f(x^j)$.

Finally, we shall show that f is balanced. Consider $\sigma \in \Sigma$ and $x, x' \in X$ such that $Re(G \mid f)(x) = Re(G \mid f)(x') = 1$ and $\delta(\sigma, x) = x'$. Since $Re(G \mid f) = Q$, we have $Q(x) = 1$ and $Q(\delta(\sigma, x)) = 1$, which imply that $\sigma \in f(x)$ since f satisfies the condition (C4-5). So f is balanced. □

Proof of Theorem 4.1: ($\Leftarrow$) By Lemma 4.2.

($\Rightarrow$) In order to derive the condition (C4-1), it is sufficient to prove that $Q \leq Re(G, Q)$. Consider $f \in F_b$ such that $Re(G \mid f) = Q$. Then, for any $x \in X$ with $Q(x) = 1$, there exist $x^1, x^2, ..., x^m \in X$ and $\sigma^0, \sigma^1, ..., \sigma^{m-1} \in \Sigma$ satisfying the conditions (C2-8)–(C2-10). We shall show by induction that $Re(G, Q)(x^j) = 1$ ($j = 0, 1, .., m$). From the definition of $Re(G, Q)$, we have $Re(G, Q)(x^0) = 1$. For the induction step, suppose that $Re(G, Q)(x^k) = 1$. Then it is obvious that $Q(x^k) = 1$, which implies that $Re(G \mid f)(x^k) = 1$. By the conditions (C2-8) and (C2-9), we have $\delta(\sigma^k, x^k) = x^{k+1}$ and $\sigma^k \in f(x^k)$, which imply that $Re(G \mid f)(x^{k+1}) = 1$, that is, $Q(x^{k+1}) = 1$. So we have $Re(G, Q)(x^{k+1}) = 1$. This completes the induction, and we have $Re(G, Q)(x) = 1$.

It remains to prove the conditions (C4-2) and (C4-3). Since $f \in F_o$, Proposition 4.1 implies that it is sufficient to show that f satisfies the condition (C4-5). Consider $x \in X$ with $Q(x) = 1$. Then $Re(G \mid f)(x) = 1$. For any $\sigma \in \Sigma$ such that $D_\sigma(x) = 1$ and

$\sigma \in f(x)$, we have $Re(G \mid f)(\delta(\sigma, x)) = 1$, that is, $Q(\delta(\sigma, x)) = 1$. Conversely, for any $\sigma \in \Sigma$ such that $Q(\delta(\sigma, x)) = 1$, it follows that $D_\sigma(x) = 1$ and $Re(G \mid f)(\delta(\sigma, x)) = 1$. Since f is balanced, we have $\sigma \in f(x)$. Therefore, f satisfies the condition (C4-5). □

Example 4.1 We consider the DES shown in Figure 3.1. Let $\Gamma = \{\{\sigma^1\}, \Sigma\}$. Assume that the mask $M : X \to Y$ is given by

$$M(x) = y \quad \forall x \in X.$$

Consider a predicate $Q \in \mathbf{Q}$ given by

$$Q(x) = \begin{cases} 1 & \text{if } x \in \{x^0, x^1\}, \\ 0 & \text{otherwise.} \end{cases}$$

By the definition, we have $Re(G, Q)(x^0) = 1$, which implies together with $wp_{\sigma^1}(Q)(x^0) = 1$ that $Re(G, Q)(x^1) = 1$. So the condition (C4-1) holds. It is obvious that $M(Q)(y) = 1$ and $\{\sigma^1\} \in \Gamma$ satisfies Eq. (4.1), which implies that the condition (C4-2) holds. Additionally, since $wlp_{\sigma^1}(Q)(x^0) = wlp_{\sigma^1}(Q)(x^1) = 1$, we can easily show that the condition (C4-3) holds. Thus, by Theorem 4.1, a solution f to the BSFCOP exists, and is given by

$$f(x) = \{\sigma^1\} \quad \forall x \in X.$$

Example 4.2 We consider the same manufacturing system as **3.4**. Let $M : X \to \{0, 1, 2\} \times Z^+$ be the mask as follows:

$$M((x_{11}, x_{12}, x_{21}, x_{22}, x_b)) = (\sharp(x_{11}, x_{12}), x_b),$$

where

$$\sharp(x_{11}, x_{12}) = \begin{cases} 2 & \text{if } x_{11} = W_{11} \text{ and } x_{12} = W_{12}, \\ 1 & \text{if } (x_{11} = W_{11} \text{ and } x_{12} = I_{12}) \text{ or } (x_{11} = I_{11} \text{ and } x_{12} = W_{12}), \\ 0 & \text{otherwise.} \end{cases}$$

That is, we can observe only the number of working machines in Group 1 and the buffer content.

It is easily shown that Q does not satisfy the conditions in Theorem 4.1. We consider a subpredicate Q' of Q given by

$$Q'((x_{11}, x_{12}, x_{21}, x_{22}, x_b)) = \begin{cases} 1 & \text{if } \sharp(x_{11}, x_{12}) + x_b \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Then Q' satisfies the conditions in Theorem 4.1, and there exists a balanced controller $f \in F_b$ such that $Re(G \mid f) = Q'$. In fact, f is given by

$$f((x_{11}, x_{12}, x_{21}, x_{22}, x_b)) = \begin{cases} \gamma_2 & \text{if } \sharp(x_{11}, x_{12}) + x_b = 1, \\ \gamma_1 & \text{otherwise.} \end{cases}$$

4.3 State Feedback Control of the Ramadge-Wonham Model

In this section, we assume that a DES G is modeled by the Ramadge-Wonham model [67, 68], where Σ is decomposed into two subsets Σ_c and Σ_u of controllable and uncontrollable events, respectively, and the set Γ of control patterns is given by Eq. (2.2), that is,

$$\Gamma = \{\gamma \in 2^{\Sigma}; \Sigma_u \subseteq \gamma \subseteq \Sigma\}.$$

Li and Wonham imposed the assumption (H) shown in **2.3.2** in order to solve the BSFCOP. Indeed, it holds for vector DESs [44, 49] but it does not in general.

We propose a definition of observability.

Definition 4.1 *A predicate $Q \in \mathbf{Q}$ is said to be observable (with respect to G) if, for any $\sigma \in \Sigma_c$, Eq. (4.2) holds.*

The following corollary shows that in the Ramadge-Wonham model, the conditions obtained in Theorem 4.1 are reduced to those of controllability and observability.

Corollary 4.1 *For a predicate $Q \in \mathbf{Q}$ with $Q(x^0) = 1$, there exists a balanced state feedback controller $f \in F_b$ such that $Re(G \mid f) = Q$ if and only if Q is controllable and observable (in the sense of Definition 4.1).*

Li and Wonham has proposed a concept of observability, and obtained necessary and sufficient conditions for the solvability of the BSFCOP under the assumption (H) [49]. Their observability is equivalent to that defined above under the assumption (H). So Corollary 4.1 is a generalization of the Li and Wonham's result. Note that the computational complexity to verify the conditions in Corollary 4.1 is also $O(mn^2)$ if X is finite.

In the case of complete observations, for any $f' \in \Gamma^X$, there exists a balanced controller $f \in \Gamma^X$ such that $Re(G \mid f) = Re(G \mid f')$ [44, 49]. But this property does not always hold under partial observations. We shall give a counter-example as follows.

Example 4.3 We consider a DES G shown in Figure 3.2. Let $\Sigma_c = \{\sigma^2\}$ and $\Sigma_u = \{\sigma^1\}$. Assume that the mask $M : X \to Y$ is given by

$$M(x) = y \quad \forall x \in X.$$

Consider a predicate $Q \in \mathbf{Q}$ given by

$$Q(x) = \begin{cases} 1 & \text{if } x \in \{x^0, x^1\}, \\ 0 & \text{otherwise,} \end{cases}$$

and a controller $f' \in \Gamma^X$ given by

$$f'(x) = \Sigma_u \quad \forall x \in X. \tag{4.5}$$

Then $f' \in F_o$, and $Re(G \mid f') = Q$. But f' is not balanced because $\delta(\sigma^2, x^1)$ is disabled by f'. Moreover, it is easily shown that there does not exist a balanced controller $f \in F_b$ such that $Re(G \mid f) = Re(G \mid f')$.

The above example implies that even if $Q \in \mathbf{Q}$ does not satisfy the conditions in Corollary 4.1, there may exist a controller f (which is not balanced) such that $Re(G \mid f) = Q$. The following proposition can be easily proved by Corollary 4.1.

Proposition 4.2 *For a state feedback controller $f' \in F_o$, there exists a balanced state feedback controller $f \in F_b$ such that $Re(G \mid f) = Re(G \mid f')$ if and only if $Re(G \mid f')$ is observable (in the sense of Theorem 4.1).*

Next, we consider a problem formulated as follows:

State Feedback Control and Observation Problem (SFCOP): For a predicate $Q \in \mathbf{Q}$ with $Q(x^0) = 1$, synthesize a state feedback controller $f \in F_o$ such that $Re(G \mid f) = Q$.

Note that the SFCOP does not require a balanced controller.

For $Q \in \mathbf{Q}$ and $y \in Y$, we shall define the subset $A(Q, y) \subseteq \Sigma_c$ as follows:

$$A(Q, y) = \{\sigma \in \Sigma_c \text{ ; there exists } x \in M^{-1}(y) \text{ such that } (Q \wedge \sim wlp_\sigma(Q))(x) = 1\}. \tag{4.6}$$

The predicate $Re^*(G, Q) \in \mathbf{Q}$ is defined inductively as follows:

(C4-6) $Re^*(G, Q)(x^0) = 1$;

(C4-7) If $Re^*(G,Q)(x) = 1$ and $wp_\sigma(Q)(x) = 1$ for some $\sigma \in \Sigma - A(Q, M(x))$, then $Re^*(G,Q)(\delta(\sigma,x)) = 1$, where $\Sigma - A(Q,y) = \{\sigma \in \Sigma \ ; \ \sigma \notin A(Q,y)\}$;

(C4-8) Every state satisfying $Re^*(G,Q)$ is obtained as in (C4-6) and (C4-7).

From the above definition, for any $x \in X$ with $Re^*(G,Q)(x) = 1$, there exist $x^1, x^2, ..., x^m \in X$ and $\sigma^0, \sigma^1, ..., \sigma^{m-1} \in \Sigma$ satisfying the following conditions (C4-9)–(C4-12):

(C4-9) $\delta(\sigma^j, x^j) = x^{j+1}$ for $j = 0, 1, ..., m-1$;

(C4-10) $Q(x^j) = 1$ for $j = 0, 1, ..., m$;

(C4-11) $\sigma^j \in \Sigma - A(Q, M(x^j))$ for $j = 0, 1, ..., m-1$;

(C4-12) $x^m = x$.

Note that the definition of $Re(G,Q)$ does not require the condition (C4-11) (see the conditions (C2-5)–(C2-7)). Obviously, $Re^*(G,Q) \leq Re(G,Q)$.

We introduce the notion of M-controllability of predicates.

Definition 4.2 *A predicate $Q \in \mathbf{Q}$ with $Q(x^0) = 1$ is said to be M-controllable (with respect to G) if Q is Σ_u-invariant and the following equation holds.*

$$Q \leq Re^*(G,Q). \tag{4.7}$$

We give a necessary and sufficient condition under which a solution to the SFCOP exists.

Theorem 4.2 *Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$. Then there exists a state feedback controller $f \in F_o$ such that $Re(G \mid f) = Q$, that is, f is a solution to the SFCOP if and only if Q is M-controllable.*

Proof: ($\Leftarrow$) Consider $f \in \Gamma^X$ given by

$$f(x) = \begin{cases} \Sigma - A(Q, M(x)) & \text{if } M(Q)(M(x)) = 1, \\ \Sigma & \text{otherwise.} \end{cases} \tag{4.8}$$

It is obvious that $f \in F_o$. We shall show that $Re(G \mid f) = Q$. We shall first prove that $Re(G \mid f) \leq Q$. For any $x \in X$ with $Re(G \mid f)(x) = 1$, there exist $x^1, x^2, ..., x^m \in X$ and $\sigma^0, \sigma^1, ..., \sigma^{m-1} \in \Sigma$ satisfying the conditions (C2-8)–(C2-10). We shall show by induction that $Q(x) = Q(x^m) = 1$. By the assumption, we have $Q(x^0) = 1$. For the induction step, suppose that $Q(x^k) = 1$. If $\sigma^k \in \Sigma_u$ then we have $Q(x^{k+1}) = Q(\delta(\sigma^k, x^k)) = 1$ since $Q \leq wlp_{\sigma^k}(Q)$. Suppose that $\sigma^k \in \Sigma_c$. Letting $y = M(x^k)$, $M(Q)(y) = 1$. By the condition (C2-9) and Eq. (4.8), we have $\sigma^k \in f(x^k) = \Sigma - A(Q, y)$. Since $\sigma^k \notin A(Q, y)$, we have $Q(x^{k+1}) = Q(\delta(\sigma^k, x^k)) = 1$. This completes the induction.

Next, we shall prove that $Q \leq Re(G \mid f)$. Since $Q \leq Re^*(G, Q)$, for any $x \in X$ with $Q(x) = 1$, there exist $x^1, x^2, ..., x^m \in X$ and $\sigma^0, \sigma^1, ..., \sigma^{m-1} \in \Sigma$ satisfying the conditions (C4-9)–(C4-12). In order to show that $Re(G \mid f)(x) = 1$, it is sufficient to prove that $\sigma^j \in f(x^j)$ $(j = 0, 1, ..., m-1)$. Obviously, $M(Q)(M(x^j)) = 1$. By the condition (C4-11) and Eq. (4.8), we have $\sigma^j \in \Sigma - A(Q, M(x^j)) = f(x^j)$.

($\Rightarrow$) Suppose that there exists $f \in F_o$ such that $Re(G \mid f) = Q$. It is obvious that, for any $\sigma \in \Sigma_u$, $Q \leq wlp_\sigma(Q)$. We shall prove that $Q \leq Re^*(G, Q)$. Since $Re(G \mid f) = Q$, for any $x \in X$ with $Q(x) = 1$, there exist $x^1, x^2, ..., x^m \in X$ and $\sigma^0, \sigma^1, ..., \sigma^{m-1} \in \Sigma$ which satisfy the conditions (C2-8)–(C2-10). We shall show by induction that $Re^*(G, Q)(x) = Re^*(G, Q)(x^m) = 1$. By the definition of $Re^*(G, Q)$, we have $Re^*(G, Q)(x^0) = 1$. For the induction step, suppose that $Re^*(G, Q)(x^k) = 1$. It is proved that $Q(x^{k+1}) = 1$ in the same way as the proof of Theorem 4.1.

It remains to show that $\sigma^k \in \Sigma - A(Q, M(x^k))$. Suppose that $\sigma^k \in A(Q, M(x^k))$. Then there exists $x' \in X$ such that $M(x') = M(x^k)$ and $(Q \wedge \sim wlp_{\sigma^k}(Q))(x') = 1$. So we have $Q(\delta(\sigma^k, x')) = 0$. Since $Re(G \mid f) = Q$, we have $Re(G \mid f)(x') = 1$. We also have $\sigma^k \in f(x^k) = f(x')$ since $M(x') = M(x^k)$. It follows that $Re(G \mid f)(\delta(\sigma^k, x')) = 1$. This contradicts the hypothesis that $Re(G \mid f) = Q$. Therefore, we have by the definition of $Re^*(G, Q)$ that $Re^*(G, Q)(x^{k+1}) = 1$. This completes the induction. □

When Q is M-controllable, the controller $f \in \Gamma^X$ given by Eq. (4.8) is a solution to the SFCOP. Note that the computational complexity to verify the condition in Theorem 4.2 is also $O(mn^2)$ if X is finite.

Obviously, a state feedback controller is a special case of a dynamic one studied in [38]. So the condition in Theorem 4.2 satisfies those for the existence of a dynamic controller obtained by Kumar et al. [38]. However, they did not discuss a necessary and sufficient condition for the existence of a static controller. A static controller is easier to implement than dynamic one. So from the practical point of view, Theorem 4.2 is useful. Moreover, the condition in Theorem 4.2 has computational advantage in contrast to those obtained in [38].

We shall show that, for any M-controllable predicate Q, there exists the unique maximally permissive controller [34, 96]. Let $F_o(Q)$ be the set of all state feedback controllers $f \in F_o$ such that $Re(G \mid f) = Q$. The following proposition shows that there always exists the supremal element of $F_o(Q)$ under "$\leq$", denoted by $\sup F_o(Q)$.

Proposition 4.3 *For any M-controllable predicate $Q \in \mathbf{Q}$ with $Q(x^0) = 1$, $\sup F_o(Q)$ exists and is given by Eq. (4.8).*

Proof: Consider $f \in \Gamma^X$ given by Eq. (4.8). Then, from the proof of Theorem 4.2, we have $f \in F_o(Q)$. Suppose that there exists $g \in F_o(Q)$ which does not satisfy that $g \leq f$. Then there exist $\sigma \in \Sigma$ and $x \in X$ such that $\sigma \in g(x)$ and $\sigma \notin f(x)$. It is obvious that $\sigma \in \Sigma_c$ and $M(Q)(M(x)) = 1$. Since $\sigma \notin f(x)$, we have $\sigma \in A(Q, M(x))$ which implies that there exists $x' \in X$ such that $M(x') = M(x)$ and $(Q \wedge \sim wlp_\sigma(Q))(x') = 1$. It is easily proved that $Q(\delta(\sigma, x')) = 0$ and $Re(G \mid g)(\delta(\sigma, x')) = 1$, which contradict the fact that $g \in F_o(Q)$. □

Example 4.4 We consider the same system as Example 4.3. We have $Re^*(G, Q)(x^0) = 1$ by the definition of $Re^*(G, Q)$. It can be easily shown that $A(Q, y) = \{\sigma^2\}$, which implies together with $wp_{\sigma^1}(Q) = 1$ that $Re^*(G, Q)(x^1) = 1$. Thus, Q is M-controllable, which implies together with Theorem 4.2 that a solution to the SFCOP exists, and given by Eq. (4.5).

Example 4.5 We consider the system consisting of three processes and two resources shown in Figure 4.1. The processes 1 and 3 use the resources 1 and 2, respectively, while the process 2 uses either the resource 1 or 2. Each process is modeled by an automaton shown in Figure 4.2, where the state I_i and W_{ij} are "idle" and "using the resource j", respectively. The sets Σ and X of events and states are given by

$$\Sigma = \{\alpha_1, \alpha_{21}, \alpha_{22}, \alpha_3, \beta_i \ (i = 1, 2, 3)\},$$

$$X = \{(x_1, x_2, x_3) \ ; \ x_1 \in \{I_1, W_{11}\}, \ x_2 \in \{I_2, W_{21}, W_{22}\}, \ x_3 \in \{I_3, W_{32}\}\},$$

where x_i $(i = 1, 2, 3)$ is the state of the process i. Let $x^0 = (I_1, I_2, I_3)$ be the initial state. Assume that $\Sigma_c = \{\alpha_1, \alpha_{21}, \alpha_{22}, \alpha_3\}$ and $\Sigma_u = \{\beta_i \ (i = 1, 2, 3)\}$.

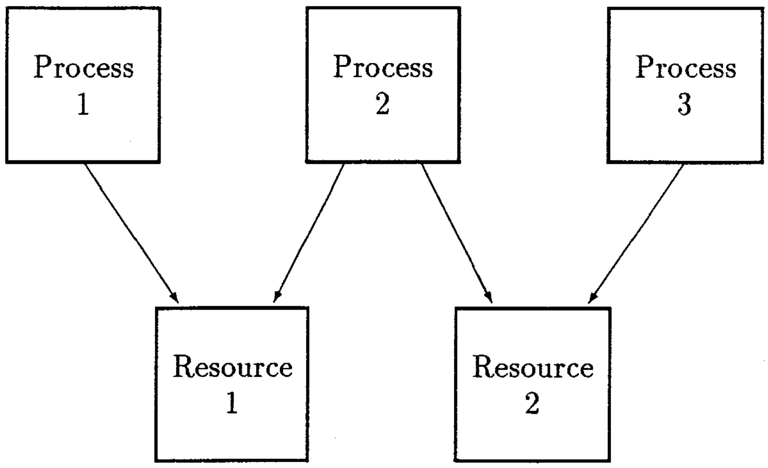

Figure 4.1: The system consisting of processes and resources.

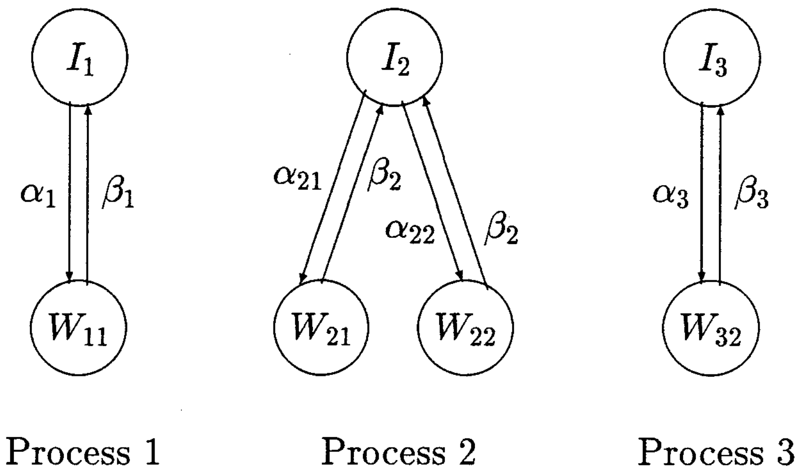

Figure 4.2: State transition diagrams of processes.

Now we consider a control specification that at most one process can use a resource simultaneously, which is given by the following predicate $Q \in \mathbf{Q}$:

$$Q((x_1,x_2,x_3)) = \begin{cases} 0 & \text{if } (x_1 = W_{11} \text{ and } x_2 = W_{21}) \text{ or } (x_2 = W_{22} \text{ and } x_3 = W_{32}), \\ 1 & \text{otherwise.} \end{cases}$$

We consider the case that we cannot know which resource a process is using while we can observe whether a process is idle or using a resource. So the mask M is given by

$$M(x_1,x_2,x_3) = (y_1,y_2,y_3),$$

where

$$y_i = \begin{cases} I_i & \text{if } x_i = I_i, \\ W_i & \text{otherwise.} \end{cases}$$

It is easily shown that Q is not observable but M-controllable. By Theorem 4.2, the SFCOP is solvable and $\hat{f} := \sup F_o(Q)$ is given by

$$\hat{f}((x_1,x_2,x_3)) = \begin{cases} \Sigma - \{\alpha_{21}\} & \text{if } x_1 = W_{11},\ x_2 = I_2 \text{ and } x_3 = I_3, \\ \Sigma - \{\alpha_{22}\} & \text{if } x_1 = I_1,\ x_2 = I_2 \text{ and } x_3 = W_{32}, \\ \Sigma - \{\alpha_{21},\alpha_{22}\} & \text{if } x_1 = W_{11},\ x_2 = I_2 \text{ and } x_3 = W_{32}, \\ \Sigma - \{\alpha_1\} & \text{if } x_1 = I_1,\ x_2 \in \{W_{21},W_{22}\} \text{ and } x_3 = W_{32}, \\ \Sigma - \{\alpha_3\} & \text{if } x_1 = W_{11},\ x_2 \in \{W_{21},W_{22}\} \text{ and } x_3 = I_3, \\ \Sigma - \{\alpha_1,\alpha_3\} & \text{if } x_1 = I_1,\ x_2 \in \{W_{21},W_{22}\} \text{ and } x_3 = I_3, \\ \Sigma & \text{otherwise.} \end{cases}$$

4.4 Modular State Feedback Control

4.4.1 Modular Specification and M-controllability

In this subsection, we consider the case that a control specification is constructed from a finite number of conjunctions of component predicates. Let Q_1 and $Q_2 \in \mathbf{Q}$ be M-controllable predicates with $Q_1(x^0) = Q_2(x^0) = 1$. Then $Q_1 \wedge Q_2$ may not be M-controllable. We shall give a counter-example as follows.

Example 4.6 We consider a DES G shown in Figure 4.3, where $\Sigma = \Sigma_c = \{\sigma^1,\sigma^2,\sigma^3\}$ and $X = \{x^0,x^1,x^2,x^3,x^4\}$. Let x^0 be the initial state. Assume that the mask $M : X \to Y$ is given by

$$M(x) = y \quad \forall x \in X.$$

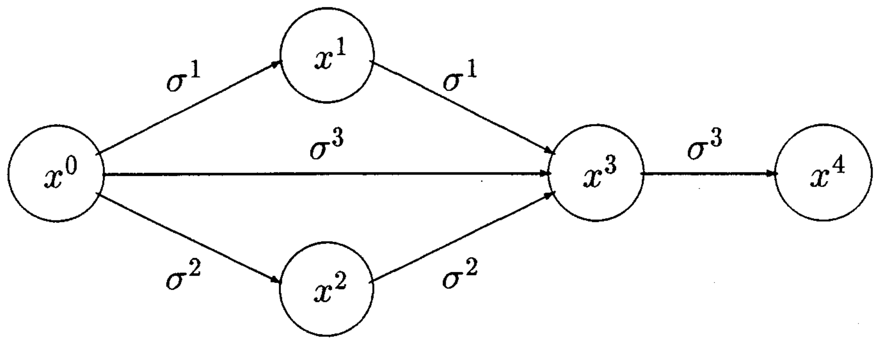

Figure 4.3: The DES of Example 4.6.

Consider M-controllable predicates Q_1 and $Q_2 \in \mathbf{Q}$ given by

$$Q_1(x) = \begin{cases} 1 & \text{if } x \in \{x^0, x^1, x^3\}, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad Q_2(x) = \begin{cases} 1 & \text{if } x \in \{x^0, x^2, x^3\}, \\ 0 & \text{otherwise,} \end{cases}$$

respectively. Then $Q_1 \wedge Q_2$ is given by

$$Q_1 \wedge Q_2(x) = \begin{cases} 1 & \text{if } x \in \{x^0, x^3\}, \\ 0 & \text{otherwise.} \end{cases}$$

It is easily shown that $Q_1 \wedge Q_2$ is not M-controllable.

Theorem 4.3 *Let Q_1 and $Q_2 \in \mathbf{Q}$ be M-controllable predicates with $Q_1(x^0) = Q_2(x^0) = 1$. Then $Q_1 \wedge Q_2$ is M-controllable if and only if the following equation holds.*

$$Re^*(G, Q_1) \wedge Re^*(G, Q_2) \leq Re^*(G, Q_1 \wedge Q_2). \tag{4.9}$$

Proof: ($\Leftarrow$) Since Q_1 and Q_2 are M-controllable, the following equations hold for any $\sigma \in \Sigma_u$.

$$Q_1 \leq Re^*(G, Q_1) \wedge wlp_\sigma(Q_1),$$
$$Q_2 \leq Re^*(G, Q_2) \wedge wlp_\sigma(Q_2).$$

From Eq. (4.9), we have

$$\begin{aligned} Q_1 \wedge Q_2 &\leq Re^*(G, Q_1) \wedge wlp_\sigma(Q_1) \wedge Re^*(G, Q_2) \wedge wlp_\sigma(Q_2) \\ &\leq Re^*(G, Q_1 \wedge Q_2) \wedge wlp_\sigma(Q_1 \wedge Q_2), \end{aligned}$$

which implies that $Q_1 \wedge Q_2$ is M-controllable.

($\Rightarrow$) It is obvious that $Re^*(G, Q_1) \leq Q_1$ and $Re^*(G, Q_2) \leq Q_2$. So we have

$$Re^*(G, Q_1) \wedge Re^*(G, Q_2) \leq Q_1 \wedge Q_2,$$

which implies together with M-controllability of $Q_1 \wedge Q_2$ that Eq. (4.9) holds. □

4.4.2 Modular Feedback Synthesis

This subsection discusses the modular feedback synthesis in the Ramadge-Wonham model. It is obvious that if f and $g \in F_o$, then $f \wedge g \in F_o$.

We consider the problem formulated as follows:

Modular State Feedback Control and Observation Problem (MSFCOP): For M-controllable predicates Q_1 and $Q_2 \in \mathbf{Q}$ with $Q_1(x^0) = Q_2(x^0) = 1$, synthesize state feedback controllers $f \in F_o(Q_1)$ and $g \in F_o(Q_2)$ such that $Re(G \mid f \wedge g) = Q_1 \wedge Q_2$.

The MSFCOP requires that a controller for $Q_1 \wedge Q_2$ is constructed by conjunction of $f \in F_o(Q_1)$ and $g \in F_o(Q_2)$.

Proposition 4.4 *Let Q_1 and $Q_2 \in \mathbf{Q}$ be M-controllable predicates with $Q_1(x^0) = Q_2(x^0) = 1$. Then there exist state feedback controllers $f \in F_o(Q_1)$ and $g \in F_o(Q_2)$ such that $Re(G \mid f \wedge g) = Q_1 \wedge Q_2$ if and only if $Re(G \mid f_1 \wedge f_2) = Q_1 \wedge Q_2$, where $f_1 := \sup F_o(Q_1)$ and $f_2 := \sup F_o(Q_2)$, respectively.*

Proof: ($\Leftarrow$) Obvious.

($\Rightarrow$) Since $f_1 \in F_o(Q_1)$ and $f_2 \in F_o(Q_2)$, we have

$$Re(G \mid f_1 \wedge f_2) \leq Re(G \mid f_1) = Q_1$$

and

$$Re(G \mid f_1 \wedge f_2) \leq Re(G \mid f_2) = Q_2.$$

Moreover, $f \leq f_1$ and $g \leq f_2$. Therefore,

$$\begin{aligned} Q_1 \wedge Q_2 &= Re(G \mid f \wedge g) \\ &\leq Re(G \mid f_1 \wedge f_2) \\ &\leq Q_1 \wedge Q_2, \end{aligned}$$

which implies that $Re(G \mid f_1 \wedge f_2) = Q_1 \wedge Q_2$. □

From the above proposition, the MSFCOP can be reduced to check whether $Re(G \mid f_1 \wedge f_2) = Q_1 \wedge Q_2$. In general, even if $Q_1 \wedge Q_2$ is M-controllable, $f_1 \wedge f_2 \notin F_o(Q_1 \wedge Q_2)$. We shall give a counter-example as follows.

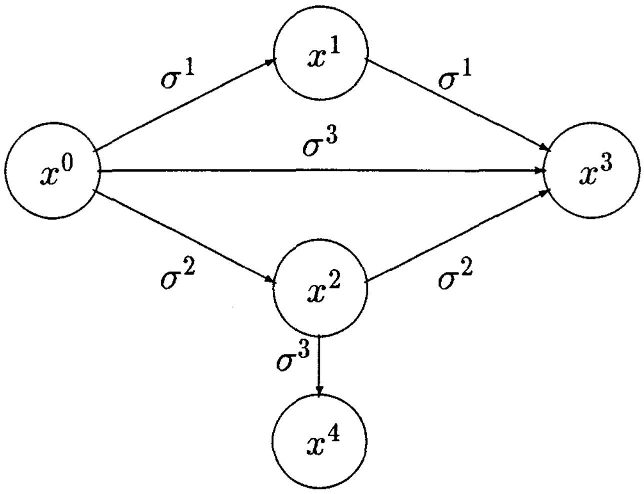

Figure 4.4: The DES of Example 4.7.

Example 4.7 We consider a DES G shown in Figure 4.4, where $\Sigma = \Sigma_c = \{\sigma^1, \sigma^2, \sigma^3\}$ and $X = \{x^0, x^1, x^2, x^3, x^4\}$. Let x^0 be the initial state. Assume that the mask $M : X \to Y$ is given by

$$M(x) = y \quad \forall x \in X.$$

Consider M-controllable predicates Q_1 and $Q_2 \in \mathbf{Q}$ given by

$$Q_1(x) = \begin{cases} 1 & \text{if } x \in \{x^0, x^1, x^3\}, \\ 0 & \text{otherwise,} \end{cases} \text{ and } Q_2(x) = \begin{cases} 1 & \text{if } x \in \{x^0, x^2, x^3\}, \\ 0 & \text{otherwise,} \end{cases}$$

respectively. Then $Q_1 \wedge Q_2$ is given by

$$Q_1 \wedge Q_2(x) = \begin{cases} 1 & \text{if } x \in \{x^0, x^3\}, \\ 0 & \text{otherwise.} \end{cases}$$

It is easily shown that $Q_1 \wedge Q_2$ is also M-controllable, and f_1 and f_2 are given by

$$f_1(x) = \{\sigma^1, \sigma^3\} \text{ and } f_2(x) = \{\sigma^2\} \quad \forall x \in X,$$

respectively. Then $f_1 \wedge f_2$ is given by

$$f_1 \wedge f_2(x) = \emptyset \quad \forall x \in X.$$

So we have

$$Re(G \mid f_1 \wedge f_2)(x) = \begin{cases} 1 & \text{if } x = x^0, \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, $Re(G \mid f_1 \wedge f_2) \neq Q_1 \wedge Q_2$.

We define the predicate $Re^*(G,Q_1)\Diamond Re^*(G,Q_2) \in \mathbf{Q}$ inductively as follows:

(C4-13) $Re^*(G,Q_1)\Diamond Re^*(G,Q_2)(x^0) = 1$;

(C4-14) If $Re^*(G,Q_1)\Diamond Re^*(G,Q_2)(x) = 1$ and $wp_\sigma(Q_1 \wedge Q_2)(x) = 1$ for some $\sigma \in \Sigma - (A(Q_1, M(x)) \cup A(Q_2, M(x)))$, then $Re^*(G,Q_1)\Diamond Re^*(G,Q_2)(\delta(\sigma,x)) = 1$, ;

(C4-15) Every state satisfying $Re^*(G,Q_1)\Diamond Re^*(G,Q_2)$ is obtained as in (C4-13) and (C4-14).

From the above definition, for any $x \in X$ with $Re^*(G,Q_1)\Diamond Re^*(G,Q_2)(x) = 1$, there exist $x^1, x^2, ..., x^m \in X$ and $\sigma^0, \sigma^1, ..., \sigma^{m-1} \in \Sigma$ satisfying the following conditions (C4-16)–(C4-19):

(C4-16) $\delta(\sigma^j, x^j) = x^{j+1}$ for $j = 0, 1, ..., m-1$;

(C4-17) $Q_1 \wedge Q_2(x^j) = 1$ for $j = 0, 1, ..., m$;

(C4-18) $\sigma^j \in \Sigma - (A(Q_1, M(x^j)) \cup A(Q_2, M(x^j)))$ for $j = 0, 1, ..., m-1$;

(C4-19) $x^m = x$.

We now give a necessary and sufficient condition under which a solution to the MSF-COP exists.

Theorem 4.4 *Let Q_1 and $Q_2 \in \mathbf{Q}$ be M-controllable predicates with $Q_1(x^0) = Q_2(x^0) = 1$. Then $Re(G \mid f_1 \wedge f_2) = Q_1 \wedge Q_2$ if and only if the following equation holds.*

$$Re^*(G,Q_1) \wedge Re^*(G,Q_2) \leq Re^*(G,Q_1)\Diamond Re^*(G,Q_2). \tag{4.10}$$

Proof: ($\Leftarrow$) From the proof of Proposition 4.4, it is sufficient to prove that $Q_1 \wedge Q_2 \leq Re(G \mid f_1 \wedge f_2)$. M-controllability of Q_1 and Q_2 implies that $Q_1 \leq Re^*(G,Q_1)$ and $Q_2 \leq Re^*(G,Q_2)$. By Eq. (4.10), for any $x \in X$ with $Q_1 \wedge Q_2(x) = 1$, $Re^*(G,Q_1)\Diamond Re^*(G,Q_2)(x) = 1$, which implies that there exist $x^1, x^2, ..., x^m \in X$ and $\sigma^0, \sigma^1, ..., \sigma^{m-1} \in \Sigma$ satisfying the conditions (C4-16)–(C4-19). Using these conditions and Proposition 4.3, it is proved that $Re(G \mid f_1 \wedge f_2)(x) = 1$ in the same way as the proof of Theorem 4.2.

($\Rightarrow$) Since

$$Re^*(G,Q_1) \wedge Re^*(G,Q_2) \leq Q_1 \wedge Q_2 = Re(G \mid f_1 \wedge f_2),$$

it is sufficient to prove that

$$Re(G \mid f_1 \wedge f_2) \leq Re^*(G, Q_1) \Diamond Re^*(G, Q_2).$$

It is obvious that, for any $x \in X$ with $Re(G \mid f_1 \wedge f_2)(x) = 1$, there exist $x^1, x^2, ..., x^m \in X$ and $\sigma^0, \sigma^1, ..., \sigma^{m-1} \in \Sigma$ satisfying the following conditions (C4-20)–(C4-22):

(C4-20) $\delta(\sigma^j, x^j) = x^{j+1}$ for $j = 0, 1, ..., m-1$;

(C4-21) $\sigma^j \in f_1(x^j) \cap f_2(x^j)$ for $j = 0, 1, ..., m-1$;

(C4-22) $x^m = x$.

It is proved by induction that $Re^*(G, Q_1) \Diamond Re^*(G, Q_2)(x) = 1$ in the same way as the proof of Theorem 4.2. □.

For f, $g \in \Gamma^X$ and $Q \in \mathbf{Q}$, we shall write $f = g$ (rel Q) if $f(x) = g(x)$ for all $x \in X$ with $Q(x) = 1$. If Eq. (4.10) holds, then $Q_1 \wedge Q_2$ is M-controllable. Then letting $f_s := \sup F_o(Q_1 \wedge Q_2)$, the following equation does not always hold.

$$f_1 \wedge f_2 = f_s \text{ (rel } Q_1 \wedge Q_2). \tag{4.11}$$

So $f_1 \wedge f_2$ does not always act in the same way as f_s at each state satisfying $Q_1 \wedge Q_2$. We shall give a counter-example as follows.

Example 4.8 We consider a DES G shown in Figure 4.5, where $\Sigma = \Sigma_c = \{\sigma^1, \sigma^2, \sigma^3, \sigma^4\}$, $X = \{x^0, x^1, x^2, x^3, x^4\}$ and x^0 is the initial state. Assume that the mask $M : X \to Y$ is given by

$$M(x) = y \quad \forall x \in X.$$

Consider M-controllable predicates Q_1 and $Q_2 \in \mathbf{Q}$ given by

$$Q_1(x) = \begin{cases} 1 & \text{if } x \in \{x^0, x^1, x^3\}, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad Q_2(x) = \begin{cases} 1 & \text{if } x \in \{x^0, x^2, x^3\}, \\ 0 & \text{otherwise,} \end{cases}$$

respectively. Then f_1 and f_2 are given by

$$f_1(x) = \{\sigma^1, \sigma^3\} \text{ and } f_2(x) = \{\sigma^2, \sigma^3, \sigma^4\} \quad \forall x \in X,$$

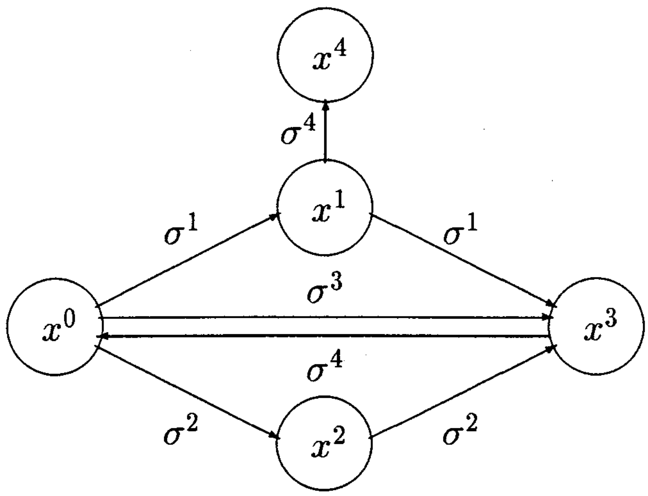

Figure 4.5: The DES of Example 4.8.

respectively. Then $f_1 \wedge f_2$ is given by

$$f_1 \wedge f_2(x) = \{\sigma^3\} \quad \forall x \in X.$$

It is easily shown that $Re(G \mid f_1 \wedge f_2) = Q_1 \wedge Q_2$. However, f_s is given by

$$f_s(x) = \{\sigma^3, \sigma^4\} \quad \forall x \in X.$$

Therefore, Eq. (4.11) does not hold.

Theorem 4.5 *Let Q_1 and $Q_2 \in \mathbf{Q}$ be M-controllable predicates with $Q_1(x^0) = Q_2(x^0) = 1$. Assume that the condition in Theorem 4.4 holds. Then $f_1 \wedge f_2 = f_s$ (rel $Q_1 \wedge Q_2$) if and only if, for any $y \in Y$ with $M(Q_1 \wedge Q_2)(y) = 1$, the following equation holds.*

$$A(Q_1, y) \cup A(Q_2, y) \subseteq A(Q_1 \wedge Q_2, y). \tag{4.12}$$

Proof: ($\Leftarrow$) Since $f_1 \wedge f_2 \in F_o(Q_1 \wedge Q_2)$ and $f_s = \sup F_o(Q_1 \wedge Q_2)$, we have $f_1 \wedge f_2 \leq f_s$. We shall show that

$$f_s \leq f_1 \wedge f_2 \quad (\text{rel } Q_1 \wedge Q_2). \tag{4.13}$$

Consider $x \in X$ with $Q_1 \wedge Q_2(x) = 1$. Letting $y = M(x)$, $M(Q_1 \wedge Q_2)(y) = 1$. By Proposition 4.3, we have

$$f_s(x) \quad = \quad \Sigma - A(Q_1 \wedge Q_2, y)$$

$$
\begin{aligned}
&\subseteq \Sigma - (A(Q_1,y) \cup A(Q_2,y)) \\
&= (\Sigma - A(Q_1,y)) \cap (\Sigma - A(Q_2,y)) \\
&= f_1(x) \cap f_2(x) \\
&= f_1 \wedge f_2(x).
\end{aligned}
$$

Therefore, Eq. (4.13) holds.

($\Rightarrow$) For any $y \in Y$ with $M(Q_1 \wedge Q_2)(y) = 1$, there exists $x \in X$ such that $M(x) = y$ and $Q_1 \wedge Q_2(x) = 1$. Since $f_1 \wedge f_2(x) = f_s(x)$, we have

$$\Sigma - (A(Q_1,y) \cup A(Q_2,y)) = \Sigma - A(Q_1 \wedge Q_2, y),$$

that is,

$$A(Q_1,y) \cup A(Q_2,y) = A(Q_1 \wedge Q_2, y).$$

□

Finally, we consider the case that both Q_1 and $Q_2 \in \mathbf{Q}$ are controllable and observable predicates with $Q_1(x^0) = Q_2(x^0) = 1$. Obviously, both Q_1 and Q_2 are M-controllable. Then $f_i := \sup F_o(Q_i)$ $(i = 1,2)$ always exists and is balanced. The following proposition shows that $f_1 \wedge f_2$ is also balanced.

Proposition 4.5 *Let Q_1 and $Q_2 \in \mathbf{Q}$ be controllable and observable (in the sense of Definition 4.1) predicates with $Q_1(x^0) = Q_2(x^0) = 1$. Then $f_1 \wedge f_2 \in F_b$.*

Proof: By Corollary 4.1, it is obvious that f_1 and $f_2 \in F_b$. Also, clearly, $f_1 \wedge f_2$ satisfies the condition (C2-12). We shall show that $f_1 \wedge f_2$ is balanced. Consider $\sigma \in \Sigma$ and $x, x' \in X$ such that $Re(G \mid f_1 \wedge f_2)(x) = Re(G \mid f_1 \wedge f_2)(x') = 1$ and $\delta(\sigma, x) = x'$. Since $Re(G \mid f_1 \wedge f_2) \leq Re(G \mid f_i)$ $(i = 1,2)$, we have $Re(G \mid f_i)(x) = Re(G \mid f_i)(x') = 1$. Also since f_1 and f_2 are balanced, we have $\sigma \in f_1(x) \cap f_2(x) = f_1 \wedge f_2(x)$. So $f_1 \wedge f_2$ is balanced. □

Theorem 4.6 *Let Q_1 and $Q_2 \in \mathbf{Q}$ be controllable and observable (in the sense of Definition 4.1) predicates with $Q_1(x^0) = Q_2(x^0) = 1$. Then $Re(G \mid f_1 \wedge f_2) = Q_1 \wedge Q_2$ if and only if Q_1 and Q_2 are nonconflicting.*

We need the following results in order to prove Theorem 4.6.

Lemma 4.3 *Let $Q \in \mathbf{Q}$ be a controllable and observable (in the sense of Definition 4.1) predicate with $Q(x^0) = 1$. Then*

$$Re(G, Q) = Re^*(G, Q) \tag{4.14}$$

Proof: Obviously, $Re(G, Q) \geq Re^*(G, Q)$. We prove the reverse inequality. Consider $x \in X$ with $Re(G, Q)(x) = 1$. Then there exist $x^1, x^2, ..., x^m \in X$ and $\sigma^0, \sigma^1, ..., \sigma^{m-1} \in \Sigma$ satisfying the conditions (C2-5)–(C2-7). We shall prove by induction that $Re^*(G, Q)(x) = Re^*(G, Q)(x^m) = 1$. From the definition of $Re^*(G, Q)$, $Re^*(G, Q)(x^0) = 1$. For the induction step, suppose that $Re^*(G, Q)(x^k) = 1$. In order to show that $Re^*(G, Q)(x^{k+1}) = 1$, it is sufficient to prove that $\sigma^k \notin A(Q, M(x^k))$. If $\sigma^k \in \Sigma_u$, then $\sigma^k \notin A(Q, M(x^k))$. Suppose that $\sigma^k \in \Sigma_c$. By the condition (C2-6), we have $Q(x^k) = Q(x^{k+1}) = 1$. So observability of Q implies with Lemma 4.1 that $Q(\delta(\sigma^k, x')) = 1$ for any $x' \in X$ such that $Q(x') = 1$, $M(x') = M(x^k)$ and $D_{\sigma^k}(x') = 1$. So $\sigma^k \notin A(Q, M(x^k))$. This completes the induction. □

The following corollary can be proved in the same way as Lemma 4.3.

Corollary 4.2 *Let Q_1 and $Q_2 \in \mathbf{Q}$ be controllable and observable (in the sense of Definition 4.1) predicates with $Q_1(x^0) = Q_2(x^0) = 1$. Then*

$$Re(G, Q_1 \wedge Q_2) = Re^*(G, Q_1) \Diamond Re^*(G, Q_2). \tag{4.15}$$

□

Proof of Theorem 4.6: ($\Leftarrow$) By Theorem 4.4, it is sufficient to prove Eq. (4.10). We have by Lemma 4.3 and Corollary 4.2 that

$$\begin{aligned} Re^*(G, Q_1) \wedge Re^*(G, Q_2) &= Re(G, Q_1) \wedge Re(G, Q_2) \\ &= Re(G, Q_1 \wedge Q_2) \\ &= Re^*(G, Q_1) \Diamond Re^*(G, Q_2). \end{aligned}$$

($\Rightarrow$) Obviously, $Re(G, Q_1) \wedge Re(G, Q_2) \geq Re(G, Q_1 \wedge Q_2)$. We shall prove the reverse inequality. By Proposition 4.5 and Corollary 4.1, $Q_1 \wedge Q_2$ is controllable and observable. So we have

$$Re(G, Q_1) \wedge Re(G, Q_2) \leq Q_1 \wedge Q_2$$

$$\leq Re(G, Q_1 \wedge Q_2).$$

□

By Theorem 4.6, the following corollary can be easily proved.

Corollary 4.3 *Let Q_1 and $Q_2 \in \mathbf{Q}$ be controllable and observable (in the sense of Definition 4.1) predicates with $Q_1(x^0) = Q_2(x^0) = 1$. Then the following statements are equivalent.*

(a) $Q_1 \wedge Q_2$ *is controllable and observable (in the sense of Definition 4.1).*

(b) $Re(G \mid f_1 \wedge f_2) = Q_1 \wedge Q_2$. □

Note that there exist controllable and observable predicates Q_1 and Q_2 such that Eq. (4.11) does not hold. However, $f_1 \wedge f_2$ practically acts in the same way as f_s at each state satisfying $Q_1 \wedge Q_2$, as shown in the following proposition.

Proposition 4.6 *Let Q_1 and $Q_2 \in \mathbf{Q}$ be controllable and observable (in the sense of Definition 4.1) predicates with $Q_1(x^0) = Q_2(x^0) = 1$. Assume that $Q_1 \wedge Q_2$ is controllable and observable (in the sense of Definition 4.1). Then for any $x \in X$ with $Q_1 \wedge Q_2(x) = 1$ and any $\sigma \in f_s(x) - f_1 \wedge f_2(x)$, $D_\sigma(x) = 0$.*

Proof: Assume that $\delta(\sigma, x)!$. Since $\sigma \in f_s(x)$, we have $Q_1 \wedge Q_2(\delta(\sigma, x)) = 1$. So observability of Q_i (i=1,2) implies together with Lemma 4.1 and Proposition 4.3 that $\sigma \in f_i(x)$, which contradicts the hypothesis that $\sigma \notin f_1 \wedge f_2(x)$. □

4.5 Concluding Remarks

In this chapter, we studied state feedback control under partial observations. We first presented a necessary and sufficient condition for the existence of a balanced state feedback controller in the Golaszewski-Ramadge model. Next, in the Ramadge-Wonham model, we showed a necessary and sufficient condition for the existence of a state feedback controller which is not necessarily balanced. Moreover, we discussed modular state feedback control under partial observations, where a control specification is given in terms of conjunction of component predicates. We showed that M-controllability of component predicates implies that of their conjunction under a certain condition. We then presented a necessary and sufficient condition under which a state feedback controller can be constructed in a modular fashion.

Chapter 5

Decentralized State Feedback Control

5.1 Introduction

For distributed systems such as communication systems, a decentralized controller is more suitable than a centralized one. In the context of supervisory control [67] based upon formal languages, two types of decentralized control problems have been studied [14, 53, 103, 72, 32]. One is the synthesis problem without tolerance [72] which requires that the behavior of the closed-loop system equals the given legal language. The other is the synthesis problem with tolerance [72] which requires that the behavior of the closed-loop system lies in the given admissible range. However, the decentralized state feedback control problem based upon predicates has not been discussed.

In this chapter, we study decentralized state feedback control based upon predicates. First, we consider the decentralized state feedback control problem (DSFCP), which requires that the set of reachable states in the closed-loop system is equal to the specified predicate. We introduce the notion of n-observability of predicates, which is a natural extension of observability defined in Definition 4.1, and prove that controllability and n-observability are necessary and sufficient conditions for the existence of a solution to the DSFCP.

Next, we consider the decentralized state feedback control problem with tolerance (DSFCPT), which requires that the set of reachable states in the closed-loop system is in the given admissible range. We show that the infimal controllable and n-observable superpredicate of a given predicate plays an important role in solving the DSFCPT. So we prove that there exists the infimal controllable and n-observable superpredicate of a given predicate under a certain condition, and derive its closed form expression.

5.2 Decentralized State Feedback Controllers

In this section, we define a decentralized state feedback controller. Let $G = (X, \Sigma, \delta, x^0)$ be an automaton defined by Eq. (2.1). A control mechanism for G is as follows. We assume that the controlled DES is modeled by the Ramadge-Wonham model. Given subsets $\Sigma_{1c}, \Sigma_{2c}, ..., \Sigma_{nc}$ (not necessarily disjoint) with $\Sigma_c = \bigcup_{i=1}^{n} \Sigma_{ic}$, for each i, the set Γ_i of control patterns is given by

$$\Gamma_i = \{\gamma_i \in 2^{\Sigma}; \Sigma - \Sigma_{ic} \subseteq \gamma_i \subseteq \Sigma\},$$

where $\Sigma - \Sigma_{ic} = \{\sigma \in \Sigma \ ; \ \sigma \notin \Sigma_{ic}\}$. Every local state feedback controller $f_i : X \to \Gamma_i$ $(i = 1, 2, ..., n)$ is defined by a mapping from X to Γ_i. That is, each f_i controls only the events which belong to Σ_{ic}. Also f_i must take the control action according to the corresponding local information. Let $M_i : X \to Y_i$ $(i = 1, 2, ..., n)$ be n masks defined by a mapping from the state space X to the corresponding local observation space Y_i, where $M_i(x) \in Y_i$ is observed by a local controller f_i when the current state of G is $x \in X$. Every local controller f_i selects a control pattern $f_i(x)$ based upon $M_i(x)$. That is, f_i satisfies the following condition (C5-1):

(C5-1) For any x, $x' \in X$,

$$M_i(x) = M_i(x') \Rightarrow f_i(x) = f_i(x').$$

A collection $\{f_i\}_{i=1}^{n}$ of n local state feedback controllers f_i $(i = 1, 2, ..., n)$ satisfying the condition (C5-1) is called a decentralized state feedback controller. The closed-loop system $G \mid \{f_i\}_{i=1}^{n}$ with a decentralized state feedback controller $\{f_i\}_{i=1}^{n}$ is as follows:

$$G \mid \{f_i\}_{i=1}^{n} = (X, \Sigma, \delta_{cf}, x^0), \tag{5.1}$$

where a partial function $\delta_{cf} : \Sigma \times X \to X$ is defined by

$$\delta_{cf}(\sigma, x) = \begin{cases} \delta(\sigma, x) & \text{if } \forall i \in \{1, 2, ..., n\},\ \sigma \in f_i(x), \\ \text{undefined} & \text{otherwise.} \end{cases}$$

That is, an event σ is enabled by $\{f_i\}_{i=1}^n$ if and only if σ is enabled by each f_i. A block diagram of $G \mid \{f_i\}_{i=1}^n$ is shown in Figure 5.1. Let $Re(G \mid \{f_i\}_{i=1}^n)$ be the closed-loop predicate for $\{f_i\}_{i=1}^n$, which is true at all and only reachable states in $G \mid \{f_i\}_{i=1}^n$. For any $x \in X$ with $Re(G \mid \{f_i\}_{i=1}^n)(x) = 1$, there exist $x^1, x^2, ..., x^m \in X$ and $\sigma^0, \sigma^1, ..., \sigma^{m-1} \in \Sigma$ satisfying the following conditions (C5-2)–(C5-4).

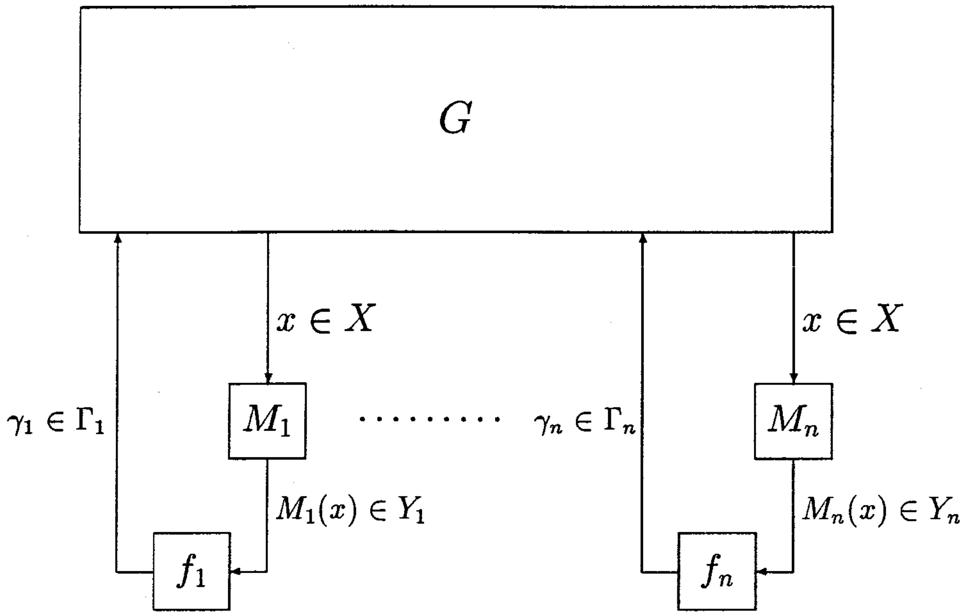

Figure 5.1: A block diagram of the closed-loop system $G \mid \{f_i\}_{i=1}^n$.

(C5-2) $\delta(\sigma^j, x^j) = x^{j+1}$ for $j = 0, 1, ..., m-1$;

(C5-3) $\sigma^j \in f_i(x^j)$ for $i = 1, 2, ..., n$ and $j = 0, 1, ..., m-1$;

(C5-4) $x^m = x$.

A decentralized controller $\{f_i\}_{i=1}^n$ is said to be balanced if $\{f_i\}_{i=1}^n$ satisfies the following condition (C5-5).

(C5-5) For any $\sigma \in \Sigma$ and any x, $x' \in X$,

$$Re(G \mid \{f_i\}_{i=1}^n)(x) = 1,\ Re(G \mid \{f_i\}_{i=1}^n)(x') = 1 \text{ and } \delta(\sigma, x) = x'$$
$$\Rightarrow \quad \forall i \in \{1, 2, ..., n\},\ \sigma \in f_i(x).$$

Given a control specification $Q \in \mathbf{Q}$, a balanced decentralized controller $\{f_i\}_{i=1}^n$ such that $Re(G \mid \{f_i\}_{i=1}^n) = Q$ permits the enablement of every event whose occurrence leads to the states satisfying Q at every $x \in X$ with $Q(x) = 1$. Let F_d be the set of all balanced decentralized state feedback controllers $\{f_i\}_{i=1}^n$.

5.3 N-Observability and Decentralized Controller

Let $Q \in \mathbf{Q}$ be a given predicate with $Q(x^0) = 1$. We interpret Q as the (global) control specification for the system G. We formulate the decentralized state feedback control problem as follows:

Decentralized State Feedback Control Problem (DSFCP): For a predicate $Q \in \mathbf{Q}$ with $Q(x^0) = 1$, synthesize a balanced decentralized state feedback controller $\{f_i\}_{i=1}^n \in F_d$ such that $Re(G \mid \{f_i\}_{i=1}^n) = Q$.

The DSFCP requires that the set of reachable states in the closed-loop system with a balanced decentralized controller is equal to the set of states satisfying Q. Note that if $n = 1$, then the DSFCP is reduced to the state feedback control and observation problem discussed in [49, 38] and Chapter 4.

For a predicate Q on X, the predicate $M_i(Q)$ on Y_i ($i = 1, 2, ..., n$) is defined as follows [49]:

$$M_i(Q)(y_i) = \begin{cases} 1 & \text{if } M_i(x) = y_i \text{ for some } x \in X \text{ with } Q(x) = 1, \\ 0 & \text{otherwise.} \end{cases}$$

For a predicate Q_i on Y_i ($i = 1, 2, ..., n$), we will define the predicate $M_i^{-1}(Q_i)$ on X as follows [49]:

$$M_i^{-1}(Q_i)(x) = \begin{cases} 1 & \text{if } Q_i(M_i(x)) = 1, \\ 0 & \text{otherwise.} \end{cases}$$

For each $\sigma \in \Sigma_c$, $In(\sigma)$ is defined by $In(\sigma) = \{i \,;\, \sigma \in \Sigma_{ic}\}$. We shall define the notion of n-observability which plays an important role in the DSFCP.

Definition 5.1 *A predicate $Q \in \mathbf{Q}$ is said to be n-observable (with respect to G) if, for any $\sigma \in \Sigma_c$, the following equation holds.*

$$Q \wedge \left(\bigwedge_{i \in In(\sigma)} M_i^{-1}(M_i(wp_\sigma(Q) \wedge Q)) \right) \leq wlp_\sigma(Q). \tag{5.2}$$

Note that the computational complexity to check n-observability is $O(mn^2)$ if X is finite. If $n = 1$, then the condition of n-observability is reduced to that of the observability defined in Definition 4.1. Note also that the n-observability condition requires that, for any $\sigma \in \Sigma_c$ and any $x \in X$ with $Q(x) = 1$, the following condition (C5-6) holds.

(C5-6) If there exists $x_i \in X$ such that $M_i(x) = M_i(x_i)$, $Q(x_i) = 1$ and $Q(\delta(\sigma, x_i)) = 1$ for each $i \in In(\sigma)$, then $Q(\delta(\sigma, x)) = 1$ or $\delta(\sigma, x)$ is undefined.

The notion of n-observability corresponds to those of $(\{M_i\}, \{\Sigma_{i,c}\}, L(G))$-controllability [14], coobservability [72] and decomposability [32] in supervisory controls based upon formal languages.

We now present necessary and sufficient conditions for the existence of a solution to the DSFCP.

Theorem 5.1 *Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$. Then there exists a balanced decentralized state feedback controller $\{f_i\}_{i=1}^n \in F_d$ such that $Re(G \mid \{f_i\}_{i=1}^n) = Q$, that is, $\{f_i\}_{i=1}^n$ is a solution to the DSFCP if and only if Q is controllable and n-observable.*

In order to prove Theorem 5.1, we shall construct local state feedback controller $f_i \in \Gamma_i^X$ $(i = 1, 2, ..., n)$ in the following manner. For each $y_i \in Y_i$ with $M_i(Q)(y_i) = 1$, let $\gamma_i(y_i) \in \Gamma_i$ be a control pattern given by

$$\gamma_i(y_i) = (\Sigma - \Sigma_{ic}) \cup \{\sigma \in \Sigma_{ic} \;;\; wp_\sigma(Q)(x) = 1 \text{ for some } x \in M_i^{-1}(y_i) \text{ with } Q(x) = 1\}.$$

Then f_i is given by

$$f_i(x) = \begin{cases} \gamma_i(M_i(x)) & \text{if } M_i(Q)(M_i(x)) = 1, \\ \Sigma & \text{otherwise.} \end{cases} \tag{5.3}$$

Note that the computational complexity to construct f_i is $O(mn^2)$ if X is finite. It is obvious that each f_i satisfies the condition (C5-1). So $\{f_i\}_{i=1}^n$ consisting of f_i $(i = 1, 2, ..., n)$ is a decentralized state feedback controller, and a candidate of the solutions to the DSFCP.

Lemma 5.1 *Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$. Then if Q is controllable and n-observable (with respect to G), then $Re(G \mid \{f_i\}_{i=1}^n) = Q$ holds for the decentralized* *state feedback controller* $\{f_i\}_{i=1}^n$ *consisting of* f_i *(*$i = 1, 2, ..., n$*) given by Eq. (5.3).*

Proof: First, we shall prove that $Re(G \mid \{f_i\}_{i=1}^n) \leq Q$. For any $x \in X$ with $Re(G \mid \{f_i\}_{i=1}^n)(x) = 1$, there exist $x^1, x^2, ..., x^m \in X$ and $\sigma^0, \sigma^1, ..., \sigma^{m-1} \in \Sigma$ satisfying the conditions (C5-2)–(C5-4). We shall show by induction that $Q(x) = Q(x^m) = 1$. By the assumption, we have $Q(x^0) = 1$. For the induction step, suppose that $Q(x^k) = 1$. If $\sigma^k \in \Sigma_u$, then controllability of Q implies that $Q(x^{k+1}) = Q(\delta(\sigma^k, x^k)) = 1$. Suppose that $\sigma^k \in \Sigma_c$. For each $i \in In(\sigma^k)$, let $y_i = M_i(x^k)$. Then $M_i(Q)(y_i) = 1$. By the condition (C5-3) and Eq. (5.3), we have $\sigma^k \in f_i(x^k) = \gamma_i(y_i)$, which implies that there exists $x_i \in X$ such that $M_i(x_i) = y_i$, $wp_{\sigma^k}(Q)(x_i) = 1$ and $Q(x_i) = 1$. So $M_i^{-1}(M_i(wp_{\sigma^k}(Q) \wedge Q))(x^k) = 1$. It follows that

$$\left(Q \wedge \left(\bigwedge_{i \in In(\sigma^k)} M_i^{-1}(M_i(wp_{\sigma^k}(Q) \wedge Q))\right)\right)(x^k) = 1.$$

Thus, by n-observability of Q, we have $wlp_{\sigma^k}(Q)(x^k) = 1$, which implies that $Q(x^{k+1}) = Q(\delta(\sigma^k, x^k)) = 1$. This completes the induction.

Next, we shall prove that $Q \leq Re(G \mid \{f_i\}_{i=1}^n)$. Controllability of Q shows that, for any $x \in X$ with $Q(x) = 1$, $Re(G, Q)(x) = 1$, which implies that there exist $x^1, x^2, ..., x^m \in X$ and $\sigma^0, \sigma^1, ..., \sigma^{m-1} \in \Sigma$ satisfying the conditions (C2-5)–(C2-7). It is sufficient to prove that $\sigma^j \in f_i(x^j)$ ($i = 1, 2, ..., n$ and $j = 0, 1, ..., m-1$). Suppose that $\sigma^k \notin f_l(x^k)$ for some l and k. Then it is obvious that $\sigma^k \in \Sigma_{lc}$. Let $y_l = M_l(x^k)$. Then $M_l(Q)(y_l) = 1$. By the conditions (C2-5) and (C2-6), it follows that $\delta(\sigma^k, x^k)!$ and $Q(x^{k+1}) = Q(\delta(\sigma^k, x^k)) = 1$. Therefore, we have by the definition of $\gamma_l(y_l)$ that $\sigma^k \in \gamma_l(y_l)$, which implies together with Eq. (5.3) that $\sigma^k \in f_l(x^k)$. This is a contradiction. □

Proof of Theorem 5.1: ($\Leftarrow$) We have by Lemma 5.1 that $Re(G \mid \{f_i\}_{i=1}^n) = Q$ for $\{f_i\}_{i=1}^n$ consisting of f_i ($i = 1, 2, ..., n$) given by Eq. (5.3). We shall show by contradiction that $\{f_i\}_{i=1}^n$ is balanced. Suppose that there exist $\sigma \in \Sigma$ and $x, x' \in X$ such that $Re(G \mid \{f_i\}_{i=1}^n)(x) = Re(G \mid \{f_i\}_{i=1}^n)(x') = 1$, $\delta(\sigma, x) = x'$ and $\sigma \notin f_l(x)$ for some l. Then it is obvious that $\sigma \in \Sigma_{lc}$. Since $Re(G \mid \{f_i\}_{i=1}^n) = Q$, it follows that $Q(x) = Q(x') = 1$. Let $y_l = M_l(x)$. Then $M_l(Q)(y_l) = 1$. We have by the definition of $\gamma_l(y_l)$ that $\sigma \in \gamma_l(y_l)$, which implies together with Eq. (5.3) that $\sigma \in f_l(x)$. This is a contradiction. Therefore, $\{f_i\}_{i=1}^n \in F_d$.

($\Rightarrow$) Suppose that there exists $\{f_i\}_{i=1}^n \in F_d$ such that $Re(G \mid \{f_i\}_{i=1}^n) = Q$. We have to prove the following three conditions.

(C5-7) $Q \leq Re(G, Q)$;

(C5-8) $Q \leq wlp_\sigma(Q)$ for any $\sigma \in \Sigma_u$;

(C5-9) Eq. (5.2) holds for any $\sigma \in \Sigma_c$.

First, we shall prove (C5-7). Since $Re(G \mid \{f_i\}_{i=1}^n) = Q$, for any $x \in X$ with $Q(x) = 1$, it follows that $Re(G \mid \{f_i\}_{i=1}^n)(x) = 1$, which implies that there exist $x^1, x^2, ..., x^m \in X$ and $\sigma^0, \sigma^1, ..., \sigma^{m-1} \in \Sigma$ satisfying the conditions (C5-2)–(C5-4). We shall show by induction that $Re(G, Q)(x) = Re(G, Q)(x^m) = 1$. By the definition of $Re(G, Q)$, we have $Re(G, Q)(x^0) = 1$. For the induction step, suppose that $Re(G, Q)(x^k) = 1$. Then it is obvious that $Q(x^k) = 1$, which implies that $Re(G \mid \{f_i\}_{i=1}^n)(x^k) = 1$. By the conditions (C5-2) and (C5-3), $\delta(\sigma^k, x^k) = x^{k+1}$ and $\sigma^k \in f_i(x^k)$ $(i = 1, 2, ..., n)$. So we have $Re(G \mid \{f_i\}_{i=1}^n)(x^{k+1}) = 1$, which implies that $Q(x^{k+1}) = 1$. Therefore, we have by the definition of $Re(G, Q)$ that $Re(G, Q)(x^{k+1}) = 1$. This completes the induction.

Next, we shall prove (C5-8) by contradiction. Suppose that there exist $\sigma \in \Sigma_u$ and $x \in X$ such that $Q(x) = 1$ and $wlp_\sigma(Q)(x) = 0$. Since $Re(G \mid \{f_i\}_{i=1}^n) = Q$, it follows that $Re(G \mid \{f_i\}_{i=1}^n)(x) = 1$. Also since $wlp_\sigma(Q)(x) = 0$ and $\sigma \in \Sigma_u$, it follows that $\delta(\sigma, x)!$ and $\sigma \in f_i(x)$ $(i = 1, 2, ..., n)$. So we have $Re(G \mid \{f_i\}_{i=1}^n)(\delta(\sigma, x)) = 1$, which implies that $Q(\delta(\sigma, x)) = 1$. This contradicts the hypothesis that $wlp_\sigma(Q)(x) = 0$.

Finally, we shall prove (C5-9) by contradiction. Suppose that there exist $\sigma \in \Sigma_c$ and $x \in X$ such that

$$Q(x) = 1, \left(\bigwedge_{i \in In(\sigma)} M_i^{-1}(M_i(wp_\sigma(Q) \wedge Q)) \right)(x) = 1 \text{ and } wlp_\sigma(Q)(x) = 0.$$

Since $Re(G \mid \{f_i\}_{i=1}^n) = Q$, it follows that $Re(G \mid \{f_i\}_{i=1}^n)(x) = 1$. For each $i \in In(\sigma)$, there exists $x_i \in X$ such that $M_i(x) = M_i(x_i)$, $Q(x_i) = 1$ and $wp_\sigma(Q)(x_i) = 1$ since $M_i^{-1}(M_i(wp_\sigma(Q) \wedge Q))(x) = 1$. It follows that $Re(G \mid \{f_i\}_{i=1}^n)(x_i) = Re(G \mid \{f_i\}_{i=1}^n)(\delta(\sigma, x_i)) = 1$. Since $\{f_i\}_{i=1}^n$ is balanced, we have $\sigma \in f_i(x_i)$. Additionally, since $M_i(x) = M_i(x_i)$, we have by the condition (C5-1) that $\sigma \in f_i(x)$. On the other hand, for any l with $l \notin In(\sigma)$, it is obvious that $\sigma \in f_l(x)$. Since $wlp_\sigma(Q)(x) = 0$, it follows that

$\delta(\sigma, x)!$. Therefore, we have $Re(G \mid \{f_i\}_{i=1}^n)(\delta(\sigma, x)) = 1$, that is, $Q(\delta(\sigma, x)) = 1$. This contradicts the hypothesis that $wlp_\sigma(Q)(x) = 0$. □

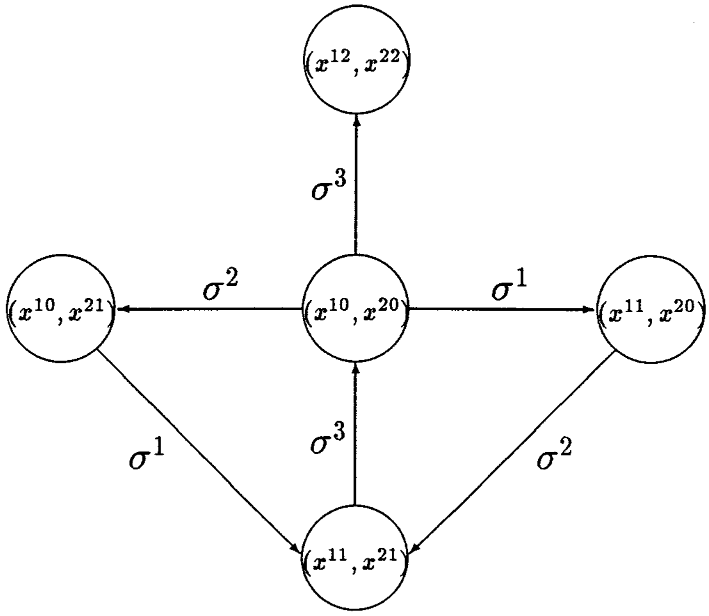

Figure 5.2: The DES of Example 5.1.

When Q is controllable and n-observable, the decentralized state feedback controller $\{f_i\}_{i=1}^n$ consisting of f_i $(i = 1, 2, ..., n)$ given by Eq. (5.3) is a solution to the DSFCP.

Example 5.1 We consider a DES G shown in Figure 5.2, where $\Sigma = \Sigma_c = \{\sigma^1, \sigma^2, \sigma^3\}$, $X = \{(x^{10}, x^{20}), (x^{10}, x^{21}), (x^{11}, x^{20}), (x^{11}, x^{21}), (x^{12}, x^{22})\}$ and (x^{10}, x^{20}) is the initial state. Let M_i $(i = 1, 2)$ be the mask given by

$$M_i(x_1, x_2) = x_i.$$

Assume that $\Sigma_{1c} = \{\sigma^1, \sigma^3\}$ and $\Sigma_{2c} = \{\sigma^2, \sigma^3\}$. We consider a predicate $Q \in \mathbf{Q}$ given by

$$Q(x) = \begin{cases} 0 & \text{if } x = (x^{12}, x^{22}), \\ 1 & \text{otherwise.} \end{cases}$$

Obviously, Q is controllable. Also we can easily show that Q is n-observable, that is, for any $\sigma \in \Sigma_c$ and $x \in X$ with $Q(x) = 1$, the condition (C5-6) holds. For example, consider $\sigma^1 \in \Sigma_c$ and $(x^{10}, x^{20}) \in X$. Then $In(\sigma^1) = \{1\}$, $M_1((x^{10}, x^{20})) = M_1((x^{10}, x^{21}))$, $Q((x^{10}, x^{21})) = Q(\delta(\sigma^1, (x^{10}, x^{21}))) = 1$. Additionally, $Q(\delta(\sigma^1, (x^{10}, x^{20}))) = 1$, which implies that (C5-6) holds. Thus, by Theorem 5.1, a solution to the DSFCP exists, and consists of f_i $(i = 1, 2)$ given by

$$f_i((x_1, x_2)) = \begin{cases} \{\sigma^1, \sigma^2\} & \text{if } x_i = x^{i0}, \\ \Sigma & \text{otherwise.} \end{cases}$$

Example 5.2 We consider a simple manufacturing system consisting of two machines G_1 and G_2 shown in Figure 5.3. We have three events as follows:

a: The machine G_1 starts working.

b: The machine G_1 completes working, and the machine G_2 starts working.

c: The machine G_2 completes working.

Each machine G_i $(i = 1, 2)$ is modeled by an automaton whose state transition diagram is shown in Figure 5.4, where I_i means that G_i is idle and W_{ij} $(j = 1, 2, \ldots)$ implies that G_i is processing the j parts simultaneously. The entire system G is also modeled by an automaton. Then the sets Σ and X of events and states are given as follows:

$$\Sigma = \{a, b, c\},$$

$$X = \{(x_1, x_2)\ ;\ x_i \in \{I_i, W_{ij}\ (j = 1, 2, \ldots)\}\ (i = 1, 2)\},$$

where x_i is the state of G_i. Let $x^0 = (I_1, I_2)$ be the initial state and $M_i : X \longrightarrow Y_i$ $(i = 1, 2)$ be the mask given by

$$M_i((x_1, x_2)) = x_i.$$

That is, a local state feedback f_i $(i = 1, 2)$ can only observe the current state of G_i. We assume that $\Sigma_{1c} = \{a\}$, $\Sigma_{2c} = \{b\}$ and $\Sigma_u = \{c\}$.

Now we consider a control specification that each machine G_i $(i = 1, 2)$ can process at most one part simultaneously, which is given by a predicate Q:

$$Q(x) = \begin{cases} 1 & \text{if } x \in \{(I_1, I_2), (I_1, W_{21}), (W_{11}, I_2), (W_{11}, W_{21})\}, \\ 0 & \text{otherwise.} \end{cases}$$

It is easily shown that Q is controllable and n-observable. Therefore, by Theorem 5.1, there exists a balanced decentralized controller $\{f_i\}_{i=1}^2$ such that $Re(G \mid \{f_i\}_{i=1}^2) = Q$, and f_1 and f_2 are given by

$$f_1(x) = \begin{cases} \{b, c\} & \text{if } M_1(x) = W_{11}, \\ \Sigma & \text{otherwise,} \end{cases}$$

and

$$f_2(x) = \begin{cases} \{a, c\} & \text{if } M_2(x) = W_{21}, \\ \Sigma & \text{otherwise,} \end{cases}$$

respectively.

Figure 5.3: A simple manufacturing system.

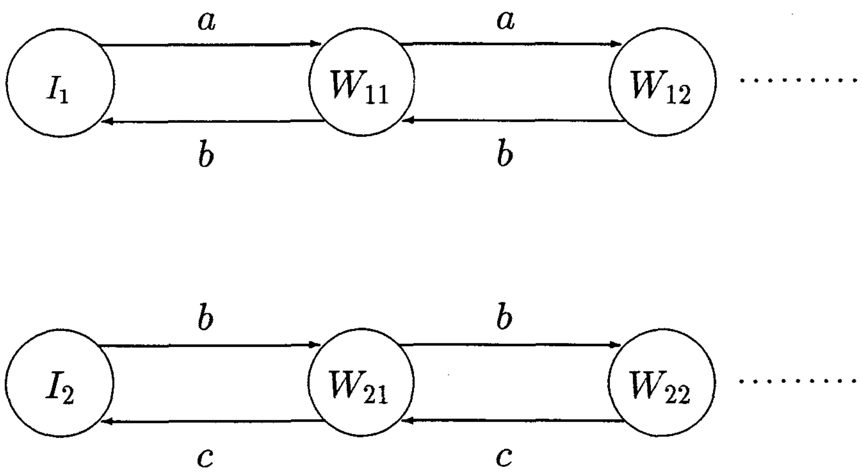

Figure 5.4: State transition diagrams of G_1 and G_2.

5.4 Decentralized Control with Tolerance

In the last section, we showed that controllability and n-observability are the necessary and sufficient conditions for the existence of a solution to the DSFCP. If the given predicate is not controllable and n-observable, then we have to synthesize a decentralized state feedback controller for its controllable and n-observable subpredicate. In this case, it is important to find a controllable and n-observable subpredicate which guarantees acceptable behaviors. In this section, we consider the decentralized state feedback control problem with tolerance (DSFCPT), which requires that the set of reachable states in the closed-loop system is in the given admissible range. Without loss of generality, we assume in this section that G is accessible [67], that is, all states are reachable from x^0.

5.4.1 The Infimal Controllable Superpredicate

Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$, and $\overline{C}(Q) \subseteq \mathbf{Q}$ be the set of all controllable superpredicates of Q.

$$\overline{C}(Q) = \{Q' \in \mathbf{Q}\ ;\ Q \leq Q' \text{ and } Q' \text{ is controllable}\}.$$

$\overline{C}(Q)$ is nonempty because $\mathbf{1} \in \overline{C}(Q)$. In general, the set $\overline{C}(Q)$ is not closed under "$\wedge$". We shall give a counter-example as follows.

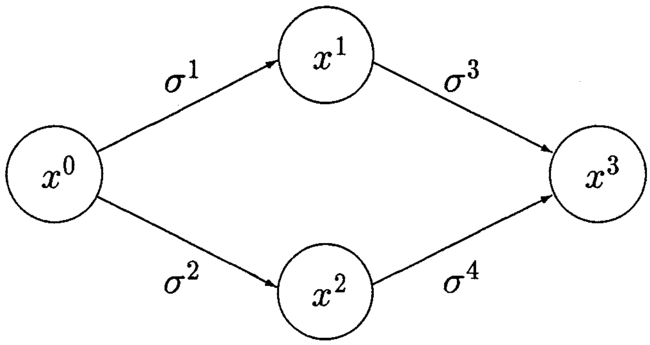

Figure 5.5: The DES of Example 5.3.

Example 5.3 We consider a DES G shown in Figure 5.5, where $\Sigma_c = \{\sigma^1, \sigma^2\}$, $\Sigma_u = \{\sigma^3, \sigma^4\}$ and $X = \{x^0, x^1, x^2, x^3\}$. Let x^0 be the initial state, and $Q \in \mathbf{Q}$ be a predicate given by

$$Q(x) = \begin{cases} 1 & \text{if } x = x^0, \\ 0 & \text{otherwise.} \end{cases}$$

Consider controllable predicates Q_1 and $Q_2 \in \overline{C}(Q)$ given by

$$Q_1(x) = \begin{cases} 1 & \text{if } x \in \{x^0, x^1, x^3\}, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad Q_2(x) = \begin{cases} 1 & \text{if } x \in \{x^0, x^2, x^3\}, \\ 0 & \text{otherwise,} \end{cases}$$

respectively. Then it is easily shown that $Q_1 \wedge Q_2 \notin \overline{C}(Q)$.

However, we shall show that the infimal element of $\overline{C}(Q)$ under "$\leq$", denoted by $\inf \overline{C}(Q)$, exists under a certain condition. We call $\inf \overline{C}(Q)$ the infimal controllable superpredicate of Q.

We define the transformation Ψ on $\mathbf{Q}$ as follows:

$$\Psi(Q)(x) = \begin{cases} 1 & \text{if } \exists s \in \Sigma_u^* \text{ and } x' \in X \text{ with } Q(x') = 1 \text{ and } \delta(s, x') = x, \\ 0 & \text{otherwise.} \end{cases}$$

Theorem 5.2 *Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$. Assume that $Q \leq Re(G, Q)$. Then $\inf \overline{C}(Q)$ exists, and is given by*

$$\inf \overline{C}(Q) = \Psi(Q). \tag{5.4}$$

Obviously, we have $Re(G, Q) \leq Q$ by the definition of $Re(G, Q)$, and the assumption of Theorem 5.2 is equivalent to $Q = Re(G, Q)$, which means that every state $x \in X$ with $Q(x) = 1$ is reachable from the initial state x^0 via states satisfying Q. So the assumption is reasonable.

We need the following lemma in order to prove Theorem 5.2.

Lemma 5.2 *Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$. If $Q \leq Re(G, Q)$, then $\Psi(Q) \in \overline{C}(Q)$.*

Proof: Since $\epsilon \in \Sigma_u^*$, it is obvious that $Q \leq \Psi(Q)$. We shall prove that $\Psi(Q)$ is controllable. First, we show that $\Psi(Q) \leq Re(G, \Psi(Q))$. For any $x \in X$ with $\Psi(Q)(x) = 1$, there exist $x_{(0)}, x_{(1)}, ..., x_{(m)} \in X$ and $\sigma_{(0)}, \sigma_{(1)}, ..., \sigma_{(m-1)} \in \Sigma_u$ satisfying the following conditions (C5-10)–(C5-12).

(C5-10) $\delta(\sigma_{(j)}, x_{(j)}) = x_{(j+1)}$ for $j = 0, 1, ..., m-1$;

(C5-11) $Q(x_{(0)}) = 1$;

(C5-12) $x_{(m)} = x$.

We shall prove by induction that $Re(G, \Psi(Q))(x) = Re(G, \Psi(Q))(x_{(m)}) = 1$. Since $Q \leq Re(G, Q)$ and $Q \leq \Psi(Q)$, we have $Q \leq Re(G, \Psi(Q))$. It follows that $Re(G, \Psi(Q))(x_{(0)}) = 1$. For the induction step, suppose that $Re(G, \Psi(Q))(x_{(k)}) = 1$, which implies that $\Psi(Q)(x_{(k)}) = 1$. Then obviously, $\Psi(Q)(x_{(k+1)}) = 1$. So we have $Re(G, \Psi(Q))(x_{(k+1)}) = 1$. This completes the induction.

It remains to show that $\Psi(Q) \leq wlp_\sigma(\Psi(Q))$ for any $\sigma \in \Sigma_u$. Consider $\sigma \in \Sigma_u$ and $x \in X$ with $\Psi(Q)(x) = 1$. If $D_\sigma(x) = 0$ then $wlp_\sigma(\Psi(Q))(x) = 1$. If $D_\sigma(x) = 1$ then, by the definition of $\Psi(Q)$, we have $\Psi(Q)(\delta(\sigma, x)) = 1$, which implies that $wlp_\sigma(\Psi(Q))(x) = 1$. □

Proof of Theorem 5.2: By Lemma 5.2, it is sufficient to show that, for any $Q' \in \overline{C}(Q)$, $\Psi(Q) \leq Q'$. Assume that there exists $x \in X$ such that $\Psi(Q)(x) = 1$ and $Q'(x) = 0$. Then there exist $x_{(0)}, x_{(1)}, ..., x_{(m)} \in X$ and $\sigma_{(0)}, \sigma_{(1)}, ..., \sigma_{(m-1)} \in \Sigma_u$ satisfying the conditions (C5-10)–(C5-12). Since $Q \leq Q'$, we have $Q'(x_{(0)}) = 1$. So there exists k $(1 \leq k \leq m)$ such that $Q'(x_{(k)}) = 0$ and $Q'(x_{(j)}) = 1$ for $j = 0, 1, ..., k-1$. Obviously, $wlp_{\sigma_{(k-1)}}(Q')(x_{(k-1)}) = 0$, which contradicts controllability of Q'. □

Note that even if the assumption in Theorem 5.2 holds, $\overline{C}(Q)$ is not necessarily closed under $\wedge$ (see Example 5.4).

5.4.2 The Infimal N-Observable Superpredicate

Let $Q \in \mathbf{Q}$ be a predicate, and let $\overline{O}(Q) \subseteq \mathbf{Q}$ be the set of all n-observable superpredicates of Q.

$$\overline{O}(Q) = \{Q' \in \mathbf{Q} \ ; \ Q \leq Q' \text{ and } Q' \text{ is } n\text{-observable}\}.$$

Since $\mathbf{1} \in \overline{O}(Q)$, $\overline{O}(Q)$ is nonempty. The following proposition shows that $\overline{O}(Q)$ is closed under "$\wedge$" in contrast to $\overline{C}(Q)$.

Proposition 5.1 *For any predicate $Q \in \mathbf{Q}$, $\overline{O}(Q)$ is closed under $\wedge$.*

In order to prove the above proposition, we need the following lemma, which can be easily proved.

Lemma 5.3 *For any index set I on $\mathbf{Q}$ and any mask M_i $(i = 1, 2, ..., n)$, the following equation holds.*

$$M_i^{-1}\left(M_i\left(\bigwedge_{\alpha \in I} Q_\alpha\right)\right) \leq \bigwedge_{\alpha \in I} M_i^{-1}(M_i(Q_\alpha)). \tag{5.5}$$

Proof of Proposition 5.1: Let I be any index set, and $Q_\alpha \in \overline{O}(Q)$ for each $\alpha \in I$. Then it is obvious that $Q \leq \bigwedge_{\alpha \in I} Q_\alpha$. We show that $\bigwedge_{\alpha \in I} Q_\alpha$ is n-observable. By Lemma

5.3 and n-observability of each Q_α, we have, for any $\sigma \in \Sigma_c$,

$$
\begin{aligned}
&\left(\bigwedge_{\alpha\in I} Q_\alpha\right) \wedge \left(\bigwedge_{i\in In(\sigma)} M_i^{-1}\left(M_i\left(wp_\sigma\left(\bigwedge_{\alpha\in I} Q_\alpha\right) \wedge \left(\bigwedge_{\alpha\in I} Q_\alpha\right)\right)\right)\right) \\
= &\left(\bigwedge_{\alpha\in I} Q_\alpha\right) \wedge \left(\bigwedge_{i\in In(\sigma)} M_i^{-1}\left(M_i\left(\bigwedge_{\alpha\in I}(wp_\sigma(Q_\alpha) \wedge Q_\alpha)\right)\right)\right) \\
\leq &\left(\bigwedge_{\alpha\in I} Q_\alpha\right) \wedge \left(\bigwedge_{i\in In(\sigma)} \bigwedge_{\alpha\in I} M_i^{-1}(M_i(wp_\sigma(Q_\alpha) \wedge Q_\alpha))\right) \\
= &\bigwedge_{\alpha\in I}\left(Q_\alpha \wedge \left(\bigwedge_{i\in In(\sigma)} M_i^{-1}(M_i(wp_\sigma(Q_\alpha) \wedge Q_\alpha))\right)\right) \\
\leq &\bigwedge_{\alpha\in I} wlp_\sigma(Q_\alpha) \\
= &\ wlp_\sigma\left(\bigwedge_{\alpha\in I} Q_\alpha\right)
\end{aligned}
$$

□

By Proposition 5.1, there always exists the infimal element of $\overline{O}(Q)$ under "$\leq$", denoted by $\inf \overline{O}(Q)$. We call $\inf \overline{O}(Q)$ the infimal n-observable superpredicate of Q.

We define the transformation Φ on $\mathbf{Q}$ as follows:

$$\Phi(Q) = \bigvee_{\sigma\in\Sigma_c} sp_\sigma\left(Q \wedge \left(\bigwedge_{i\in In(\sigma)} M_i^{-1}(M_i(wp_\sigma(Q) \wedge Q))\right)\right).$$

Then, for $Q \in \mathbf{Q}$, we consider a sequence $\{Q_j\}$ of predicates defined by

$$Q_0 := Q, \quad Q_{j+1} := Q_j \vee \Phi(Q_j) \quad \text{for } j = 0, 1, \ldots. \tag{5.6}$$

Theorem 5.3 *Let $Q \in \mathbf{Q}$ be a predicate and $\{Q_j\}$ be the sequence of predicates defined by Eq. (5.6). Then*

$$\inf \overline{O}(Q) = Q_\infty := \bigvee_{j\in\mathcal{N}} Q_j, \tag{5.7}$$

where $\mathcal{N}$ is the set of all nonnegative integers.

We need the following lemma in order to prove Theorem 5.3.

Lemma 5.4 *Let $Q \in \mathbf{Q}$ be a predicate and $\{Q_j\}$ be the sequence of predicates defined by Eq. (5.6). Then $Q_\infty := \bigvee_{j\in\mathcal{N}} Q_j$ is n-observable.*

Proof: Consider $\sigma \in \Sigma_c$ and $x \in X$ such that

$$\left(Q_\infty \wedge \left(\bigwedge_{i \in In(\sigma)} M_i^{-1}(M_i(wp_\sigma(Q_\infty) \wedge Q_\infty))\right)\right)(x) = 1.$$

If $D_\sigma(x) = 0$ then $wlp_\sigma(Q_\infty)(x) = 1$. We consider the case that $D_\sigma(x) = 1$. Since $Q_\infty(x) = 1$, we have $Q_k(x) = 1$ for some $k \in \mathcal{N}$. Also, for each $i \in In(\sigma)$, there exists $x_i \in X$ such that $(wp_\sigma(Q_\infty) \wedge Q_\infty)(x_i) = 1$ and $M_i(x) = M_i(x_i)$. So we have $Q_\infty(x_i) = Q_\infty(\delta(\sigma, x_i)) = 1$, which implies that $Q_{k_i}(x_i) = 1$ and $Q_{k_i'}(\delta(\sigma, x_i)) = 1$ for some k_i and $k_i' \in \mathcal{N}$. Consider $\bar{k} \in \mathcal{N}$ such that $k \leq \bar{k}$, $k_i \leq \bar{k}$ and $k_i' \leq \bar{k}$ for all $i \in In(\sigma)$. Since $\{Q_j\}$ is the monotonically increasing sequence, we have $Q_{\bar{k}}(x) = 1$ and $(wp_\sigma(Q_{\bar{k}}) \wedge Q_{\bar{k}})(x_i) = 1$ for all $i \in In(\sigma)$, which implies that

$$\left(Q_{\bar{k}} \wedge \left(\bigwedge_{i \in In(\sigma)} M_i^{-1}(M_i(wp_\sigma(Q_{\bar{k}}) \wedge Q_{\bar{k}}))\right)\right)(x) = 1.$$

It follows that $\Phi(Q_{\bar{k}})(\delta(\sigma, x)) = 1$, that is, $Q_{\bar{k}+1}(\delta(\sigma, x)) = 1$. Since $Q_{\bar{k}+1} \leq Q_\infty$, we have $Q_\infty(\delta(\sigma, x)) = 1$, which implies that $wlp_\sigma(Q_\infty)(x) = 1$. □

Proof of Theorem 5.3: Since $Q \leq Q_\infty$, we have by Lemma 5.3 that $Q_\infty \in \overline{O}(Q)$. It remains to show that, for any $Q' \in \overline{O}(Q)$,

$$Q_\infty \leq Q'.$$

We shall prove by induction that $Q_j \leq Q'$ for any $j \in \mathcal{N}$. Obviously, $Q_0 \leq Q'$. For the induction step, suppose that $Q_k \leq Q'$. Consider $x \in X$ with $Q_{k+1}(x) = 1$. If $Q_k(x) = 1$ then $Q'(x) = 1$. We consider the case that $\Phi(Q_k)(x) = 1$. Then there exist $\sigma \in \Sigma_c$ and $x' \in X$ such that

$$\delta(\sigma, x') = x \text{ and } \left(Q_k \wedge \left(\bigwedge_{i \in In(\sigma)} M_i^{-1}(M_i(wp_\sigma(Q_k) \wedge Q_k))\right)\right)(x') = 1.$$

Since $Q_k \leq Q'$, we have

$$\left(Q' \wedge \left(\bigwedge_{i \in In(\sigma)} M_i^{-1}(M_i(wp_\sigma(Q') \wedge Q'))\right)\right)(x') = 1.$$

So n-observability of Q' implies that $Q'(x) = 1$. This completes the induction. Therefore, we have $Q_\infty = \bigvee_{j \in \mathcal{N}} Q_j \leq Q'$. □

5.4.3 Decentralized State Feedback Control Problem with Tolerance

Let $Q \in \mathbf{Q}$ be a given predicate with $Q(x^0) = 1$. In this subsection, we consider the following problem:

Decentralized State Feedback Control Problem with Tolerance (DSFCPT): Let A and $Q \in \mathbf{Q}$ be predicates such that $A \leq Q$ and $A(x^0) = 1$. Then synthesize a balanced decentralized state feedback controller $\{f_i\}_{i=1}^n \in F_d$ such that $A \leq Re(G \mid \{f_i\}_{i=1}^n) \leq Q$.

We interpret A as the minimally acceptable behavior. The DSFCPT requires that the set of reachable states in the closed-loop system with a balanced decentralized controller lies between two sets of states satisfying A and Q, respectively.

Let $\overline{CO}(A) \subseteq \mathbf{Q}$ be the set of all controllable and n-observable superpredicates of A.

$$\overline{CO}(A) = \{Q' \in \mathbf{Q} \; ; \; A \leq Q' \text{ and } Q' \text{ is controllable and } n\text{-observable}\}.$$

$\overline{CO}(A)$ is nonempty because $\mathbf{1} \in \overline{CO}(A)$. We denote the infimal element of $\overline{CO}(A)$ under "$\leq$" by $\inf \overline{CO}(A)$. We call $\inf \overline{CO}(A)$ the infimal controllable and n-observable superpredicate of A. Assume that there exists $\inf \overline{CO}(A)$. Then it is easily proved that there exists a solution to the DSFCPT if and only if

$$\inf \overline{CO}(A) \leq Q.$$

When $\inf \overline{CO}(A) \leq Q$, the decentralized controller $\{f_i\}_{i=1}^n$ consisting of f_i $(i = 1, 2, ..., n)$ given by the following equation is a solution to the DSFCPT.

$$f_i(x) = \begin{cases} \gamma_i(M_i(x)) & \text{if } M_i(\inf \overline{CO}(A))(M_i(x)) = 1, \\ \Sigma & \text{otherwise.} \end{cases} \tag{5.8}$$

where

$$\begin{aligned} \gamma_i(M_i(x)) \;\; = \;\; & (\Sigma - \Sigma_{ic}) \cup \{\sigma \in \Sigma_{ic} \; ; \; wp_\sigma(\inf \overline{CO}(A))(x') = 1 \text{ for some } x' \in M_i^{-1}(M_i(x)) \\ & \text{with } \inf \overline{CO}(A)(x') = 1\}. \end{aligned}$$

Unfortunately, $\overline{CO}(A)$ is not always closed under "$\wedge$" because $\overline{C}(A)$ is not necessarily closed under $\wedge$. So $\inf \overline{CO}(A)$ does not always exist. However, we show that under a certain condition, there exists the infimal controllable and n-observable superpredicate of a given predicate.

Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$. We consider a sequence $\{\tilde{Q}_j\}$ of predicates defined by

$$\tilde{Q}_0 := Q, \quad \tilde{Q}_{j+1} := \inf \overline{O}(\Psi(\tilde{Q}_j)) \quad \text{for } j = 0, 1, \ldots. \tag{5.9}$$

Theorem 5.4 *Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$ and $\{\tilde{Q}_j\}$ be the sequence of predicates defined by Eq. (5.9). Assume that $Q \leq Re(G, Q)$. Then*

$$\inf \overline{CO}(Q) = \tilde{Q}_\infty := \bigvee_{j \in \mathcal{N}} \tilde{Q}_j. \tag{5.10}$$

We need the following lemmas in order to prove Theorem 5.4.

Lemma 5.5 *Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$. Assume that $Q \leq Re(G, Q)$. Then*

$$\inf \overline{O}(Q) \leq Re(G, \inf \overline{O}(Q)). \tag{5.11}$$

Proof: Let $\{Q_j\}$ be the sequence of predicates defined by Eq. (5.6). We shall prove by induction that

$$Q_j \leq Re(G, Q_j) \ \text{ for all } j \in \mathcal{N}.$$

By the assumption, we know that $Q_0 \leq Re(G, Q_0)$. For the induction step, suppose that $Q_k \leq Re(G, Q_k)$. Consider $x \in X$ with $Q_{k+1}(x) = 1$. Since

$$Q_k \leq Re(G, Q_k) \leq Re(G, Q_{k+1}),$$

if $Q_k(x) = 1$ then $Re(G, Q_{k+1})(x) = 1$. We consider the case that $\Phi(Q_k)(x) = 1$. Then there exist $\sigma \in \Sigma_c$ and $x' \in X$ such that $\delta(\sigma, x') = x$ and $Q_k(x') = 1$. Since $Re(G, Q_{k+1})(x') = 1$ and $Q_{k+1}(x) = 1$, we have $Re(G, Q_{k+1})(x) = 1$. This completes the induction. Therefore, we have

$$\inf \overline{O}(Q) = \bigvee_{j \in \mathcal{N}} Q_j \leq \bigvee_{j \in \mathcal{N}} Re(G, Q_j) \leq Re(G, \inf \overline{O}(Q)).$$

□

Lemma 5.6 *Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$ and $\{\tilde{Q}_j\}$ be the sequence of predicates defined by Eq. (5.9). Assume that $Q \leq Re(G, Q)$. Then*

$$\tilde{Q}_i \leq Re(G, \tilde{Q}_i) \ \text{ for all } i \in \mathcal{N}. \tag{5.12}$$

Proof: We shall prove Eq. (5.12) by induction. By the assumption, $\tilde{Q}_0 \leq Re(G, \tilde{Q}_0)$. For the induction step, suppose that $\tilde{Q}_k \leq Re(G, \tilde{Q}_k)$. By Theorem 5.2, $\Psi(\tilde{Q}_k)$ is controllable, which implies that $\Psi(\tilde{Q}_k) \leq Re(G, \Psi(\tilde{Q}_k))$. Then by Lemma 5.5, we have $\inf \overline{O}(\Psi(\tilde{Q}_k)) \leq Re(G, \inf \overline{O}(\Psi(\tilde{Q}_k)))$, that is, $\tilde{Q}_{k+1} \leq Re(G, \tilde{Q}_{k+1})$. This completes the induction. □

Lemma 5.7 *Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$ and $\{\tilde{Q}_j\}$ be the sequence of predicates defined by Eq. (5.9). Assume that $Q \leq Re(G, Q)$. Then $\tilde{Q}_\infty := \bigvee_{j \in \mathcal{N}} \tilde{Q}_j$ is controllable.*

Proof: By Lemma 5.6, we have

$$\tilde{Q}_\infty = \bigvee_{j \in \mathcal{N}} \tilde{Q}_j \leq \bigvee_{j \in \mathcal{N}} Re(G, \tilde{Q}_i) \leq Re(G, \tilde{Q}_\infty).$$

By Theorem 5.2, it is sufficient to show that $\tilde{Q}_\infty = \Psi(\tilde{Q}_\infty)$. Obviously, $\tilde{Q}_\infty \leq \Psi(\tilde{Q}_\infty)$. We shall prove that $\Psi(\tilde{Q}_\infty) \leq \tilde{Q}_\infty$. Consider $x \in X$ with $\Psi(\tilde{Q}_\infty)(x) = 1$. Then there exist $s \in \Sigma_u^*$ and $x' \in X$ with $\tilde{Q}_\infty(x') = 1$ and $\delta(s, x') = x$. So we have $\tilde{Q}_k(x') = 1$ and $\Psi(\tilde{Q}_k)(x) = 1$ for some $k \in \mathcal{N}$. Since $\Psi(\tilde{Q}_k) \leq \tilde{Q}_{k+1} \leq \tilde{Q}_\infty$, we have $\tilde{Q}_\infty(x) = 1$. □

Lemma 5.8 *Let $Q \in \mathbf{Q}$ be a predicate with $Q(x^0) = 1$ and $\{\tilde{Q}_j\}$ be the sequence of predicates defined by Eq. (5.9). Assume that $Q \leq Re(G, Q)$. Then $\tilde{Q}_\infty := \bigvee_{j \in \mathcal{N}} \tilde{Q}_j$ is n-observable.*

Proof: It is sufficient to show that $\tilde{Q}_\infty = \inf \overline{O}(\tilde{Q}_\infty)$. Obviously, $\tilde{Q}_\infty \leq \inf \overline{O}(\tilde{Q}_\infty)$. We shall prove that $\inf \overline{O}(\tilde{Q}_\infty) \leq \tilde{Q}_\infty$. We consider a sequence $\{Q'_j\}$ of predicates defined by

$$Q'_0 := \tilde{Q}_\infty, \quad Q'_{j+1} := Q'_j \vee \Phi(Q'_j) \quad \text{for } j = 0, 1, \ldots.$$

By Theorem 5.3, we have $\inf \overline{O}(\tilde{Q}_\infty) = \bigvee_{j \in \mathcal{N}} Q'_j$. So it is sufficient to show that $Q'_j \leq \tilde{Q}_\infty$ for all $j \in \mathcal{N}$. By the definition, $Q'_0 = \tilde{Q}_\infty$. For the induction step, suppose that $Q'_k \leq \tilde{Q}_\infty$. Consider $x \in X$ with $Q'_{k+1}(x) = 1$. If $Q'_k(x) = 1$ then $\tilde{Q}_\infty(x) = 1$. We consider the case that $\Phi(Q'_k)(x) = 1$. Then there exist $\sigma \in \Sigma_c$ and $x' \in X$ such that

$$\delta(\sigma, x') = x \text{ and } \left(Q'_k \wedge \left(\bigwedge_{i \in In(\sigma)} M_i^{-1}(M_i(wp_\sigma(Q'_k) \wedge Q'_k)) \right) \right)(x') = 1.$$

Since $Q'_k \leq \tilde{Q}_\infty$, we have

$$\left(\tilde{Q}_\infty \wedge \left(\bigwedge_{i \in In(\sigma)} M_i^{-1}(M_i(wp_\sigma(\tilde{Q}_\infty) \wedge \tilde{Q}_\infty)) \right) \right)(x') = 1.$$

Then we can prove in the same way as the proof of Lemma 5.4 that

$$\left(\tilde{Q}_{\bar{k}} \wedge \left(\bigwedge_{i\in In(\sigma)} M_i^{-1}(M_i(wp_\sigma(\tilde{Q}_{\bar{k}}) \wedge \tilde{Q}_{\bar{k}}))\right)\right)(x') = 1 \text{ for some } \bar{k} \in \mathcal{N}.$$

By n-observability of $\tilde{Q}_{\bar{k}}$, we have $\tilde{Q}_{\bar{k}}(x) = 1$, which implies that $\tilde{Q}_\infty(x) = 1$. This completes the induction. □

Proof of Theorem 5.4: By Lemmas 5.7 and 5.8, we have $\tilde{Q}_\infty \in \overline{CO}(Q)$. We show that, for any $Q' \in \overline{CO}(Q)$,

$$\tilde{Q}_\infty \leq Q'. \tag{5.13}$$

It is sufficient to prove that $\tilde{Q}_j \leq Q'$ for all $j \in \mathcal{N}$. Obviously, $\tilde{Q}_0 \leq Q'$. For the induction step, suppose that $\tilde{Q}_k \leq Q'$. Then it is obvious that $Q' \in \overline{C}(\tilde{Q}_k)$. By Lemma 5.6 and Theorem 5.2, we have $\Psi(\tilde{Q}_k) \leq Q'$. Moreover, since $Q' \in \overline{O}(\Psi(\tilde{Q}_k))$, we have $\tilde{Q}_{k+1} = \inf \overline{O}(\Psi(\tilde{Q}_k)) \leq Q'$. This completes the induction. □

Note that even if the assumption in Theorem 5.4 holds, $\overline{CO}(Q)$ is not necessarily closed under $\wedge$.

Remark 5.1 When the set X of states is finite, it is easily proved that Eqs. (5.6) and (5.9) always converge to $\inf \overline{O}(Q)$ and $\inf \overline{CO}(Q)$, respectively, after a finite number of iterations. Note that Eqs. (5.6) and (5.9) may converge after a finite number of iterations even if X is infinite (see Example 5.4).

Example 5.4 We consider a simple manufacturing system consisting of two machines G_1, G_2 and two buffers B_1, B_2 shown in Figure 5.6. This system processes two types of parts. Parts of both types are firstly processed by G_1, and completed parts of type 1 and 2 are passed to B_1 and B_2, respectively. Then they are taken by G_2 for further processing. Each machine G_j ($j = 1, 2$) is modeled by an automaton whose state transition diagram is shown in Figure 5.7, where I_j and W_{ji} ($i = 1, 2$) are representing "idle" and "processing a part of type i", respectively. Each buffer B_i ($i = 1, 2$) is also modeled by an automaton with the state set $\mathcal{N}$ (the set of nonnegative integers), and its state transitions are as follows:

$$\beta_{1i} : n \rightarrow n + 1,$$

$$\alpha_{2i} : n \rightarrow n - 1,$$

The sets Σ and X of events and states are given as follows:

$$\Sigma = \{\alpha_{ji}, \beta_{ji}\ (i,j=1,2)\},$$

$$X = \{(x_1, x_2, b_1, b_2)\ ; x_j \in \{I_j, W_{j1}, W_{j2}\} \text{ and } b_i \in \mathcal{N}\ (i,j=1,2)\},$$

where x_j $(j=1,2)$ is the state of the machine G_j and b_i $(i=1,2)$ is the state of the buffer B_i. Let $x_0 = (I_1, I_2, 0, 0)$ be the initial state and $M_i : X \to Y_i$ $(i=1,2)$ be the mask given by

$$M_i((x_1, x_2, b_1, b_2)) = b_i.$$

That is, a local state feedback controller f_i $(i=1,2)$ can observe the current state of B_i. We assume that $\Sigma_{1c} = \{\alpha_{11}\}$, $\Sigma_{2c} = \{\alpha_{12}\}$ and $\Sigma_u = \{\alpha_{2i}, \beta_{ji}\ (i,j=1,2)\}$. Now we consider a control specification that buffer contents b_1 and b_2 are always at most $e_1(\geq 1)$ and $e_2(\geq 1)$, respectively, which is given by a predicate $Q \in \mathbf{Q}$:

$$Q((x_1, x_2, b_1, b_2)) = \begin{cases} 1 & \text{if } b_i \leq e_i\ (i=1,2), \\ 0 & \text{otherwise.} \end{cases}$$

For simplicity let $e_i = 1$ $(i=1,2)$. Assume that the minimally acceptable behavior $A \in \mathbf{Q}$ is given by

$$A(x) = \begin{cases} 1 & \text{if } x \in \{(I_1, I_2, 0, 0), (W_{11}, I_2, 0, 0), (I_1, I_2, 1, 0), (I_1, W_{21}, 0, 0) \\ & \qquad (W_{12}, I_2, 0, 0), (I_1, I_2, 0, 1), (I_1, W_{22}, 0, 0)\}, \\ 0 & \text{otherwise.} \end{cases}$$

It is easily shown that Q is not controllable and n-observable, and the infimal controllable and n-observable superpredicate $\inf \overline{CO}(A)$ of A is given as follows:

$$\begin{aligned} &\inf \overline{CO}(A)(x_1, x_2, b_1, b_2) \\ &\quad = \begin{cases} 0 & \text{if } (b_1 \geq 2) \text{ or } (b_2 \geq 2) \text{ or } (x_1 = W_{11} \text{ and } b_1 = 1) \text{ or } (x_1 = W_{12} \text{ and } b_2 = 1), \\ 1 & \text{otherwise.} \end{cases} \end{aligned}$$

Since $\inf \overline{CO}(A) \leq Q$, The DSFCPT is solvable. Then a balanced decentralized state feedback controller $\{f_i\}_{i=1}^2$ such that $Re(G \mid \{f_i\}_{i=1}^2) = \inf \overline{CO}(A)$ is consisting of f_i $(i=1,2)$ given as follows:

$$f_i((x_1, x_2, b_1, b_2)) = \begin{cases} \Sigma - \{\alpha_{1i}\} & \text{if } b_i = 1, \\ \Sigma & \text{otherwise.} \end{cases}$$

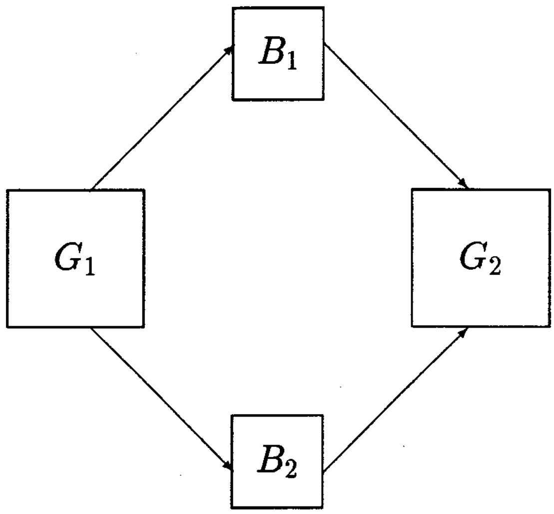

Figure 5.6: A simple manufacturing system.

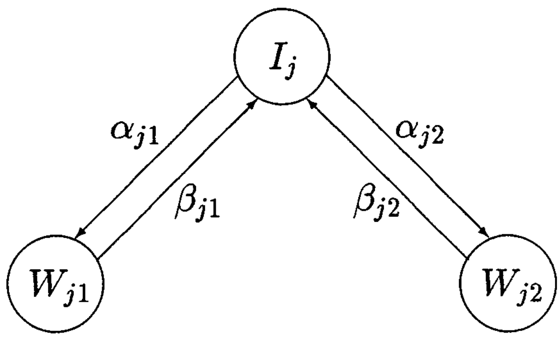

Figure 5.7: A state transition diagram of the machine G_j.

5.5 Concluding Remarks

In this chapter, we studied decentralized state feedback control based upon predicates. First, we addressed the decentralized state feedback control problem (DSFCP), which requires that the set of reachable states in the closed-loop system is equal to the specified predicate. We introduced the notion of n-observability of predicates and proved that controllability and n-observability are necessary and sufficient conditions for the existence of a solution to the DSFCP.

Next, we considered the decentralized state feedback control problem with tolerance (DSFCPT), which requires that the set of reachable states in the closed-loop system is in the given admissible range. We showed that the infimal controllable and n-observable superpredicate of a given predicate plays an important role in solving the DSFCPT. So we derived closed form expressions of the infimal controllable superpredicate, the infimal n-observable superpredicate and the infimal controllable and n-observable superpredicate, respectively, under a certain condition.

Chapter 6

State Feedback Control of Petri Nets

6.1 Introduction

This chapter studies maximally permissive controllers (MPCs) for controlled Petri nets (CPNs). Ushio has given a necessary and sufficient condition for the unique existence of the MPC as shown in **2.3.4** [95, 98]. However, we have to construct the set of all permissive controllers in order to check the condition.

In this chapter, we first consider CPNs without concurrency. We derive necessary and sufficient conditions for the unique existence of the MPC under partial as well as complete observations, which can be checked without constructing the set of all permissive controllers. Next, we extend the results to CPNs with concurrency controlled by either event assignment or resource allocation. We then show that the unique existence of the MPC in resource allocation control implies that the same is true in event assignment control.

6.2 Controllers for Petri Nets without Concurrency

We consider a serial controlled DES $\mathcal{G} = (N_c, \Gamma)$ where N_c is a CPN defined by Eq. (2.5) and $\Gamma = \{0,1\}^{P_c}$ is the set of all control patterns. This section presents necessary and sufficient conditions for the unique existence of the MPC under partial as well as complete observations.

6.2.1 Maximally Permissive Controllers under Complete Observations

In $\mathcal{G}$, for a state feedback controller $f \in \Gamma^{R(N)}$ and a transition $t \in T$, the predicate $f_t \in \mathbf{Q}$ is defined by

$$f_t(M) = \begin{cases} 1 & \text{if } I_c(t, p_c) \leq f(M)(p_c) \quad \forall p_c \in P_c, \\ 0 & \text{otherwise.} \end{cases}$$

Note that if $t \in T_u$ then $f_t(M) = 1$ for any $f \in \Gamma^{R(N)}$ and any $M \in R(N)$. There exists a permissive controller for a predicate $Q \in \mathbf{Q}$ if and only if Q is T_u-invariant, that is, $Q \leq wlp_t(Q)$ for any $t \in T_u$ [68, 34]. Ushio has shown in [98] that there exists the unique MPC for a T_u-invariant predicate Q if and only if Q is weakly interactive (WI) in $\mathcal{G}$, that is, for any permissive controllers f, $g \in Per(Q)$ (the set of all permissive controllers for Q) and any $t \in T$,

$$Q \leq wlp_t(Q) \vee f_t \vee g_t \vee \sim (f+g)_t. \tag{6.1}$$

We derive another form of a necessary and sufficient condition for the unique existence of the MPC. For each $p_c^* \in P_c$ and each $M^* \in R(N)$, the basis feedback controller $b(p_c^*, M^*) \in \Gamma^{R(N)}$ is defined by

$$b(p_c^*, M^*)(M)(p_c) = \begin{cases} 1 & \text{if } M = M^* \text{ and } p_c = p_c^*, \\ 0 & \text{otherwise.} \end{cases}$$

Let F_{bas} be the set of all basis feedback controllers.

Proposition 6.1 *[96] Assume that a predicate $Q \in \mathbf{Q}$ is T_u-invariant and that a state feedback controller $f \in \Gamma^{R(N)}$ is a permissive controller for Q. Then a state feedback controller $g \in \Gamma^{R(N)}$ is a permissive controller for Q whenever $g \leq f$.*

Lemma 6.1 *Assume that a predicate $Q \in \mathbf{Q}$ is T_u-invariant. Then there exists the unique MPC for Q if and only if, for any $M \in R(N)$ with $Q(M) = 1$, the following condition (C6-1) holds.*

(C6-1) For any $t \in T_c$ with $| {}^c t | \geq 2$,

$$b(p_{c1}, M),\ b(p_{c2}, M), \ldots,\ b(p_{cm}, M) \in Per(Q) \ \Rightarrow\ wlp_t(Q)(M) = 1,$$

where ${}^c t = \{p_{c1}, p_{c2}, \ldots, p_{cm}\}$ and $| \cdot |$ represents the number of elements of a set or bag.

Proof: ($\Leftarrow$) It is sufficient to prove that Q is WI in $\mathcal{G}$. Obviously, Eq. (6.1) holds for any f and $g \in Per(Q)$ if $t \in T_u$. Suppose that Eq. (6.1) does not hold for some $t \in T_c$. Then there exist f, $g \in Per(Q)$ and $M \in R(N)$ such that

$$Q(M) = 1 \text{ and } (wlp_t(Q) \vee f_t \vee g_t \vee \sim (f+g)_t)(M) = 0. \tag{6.2}$$

If $\mid {}^c t \mid = 1$ then, since $(f+g)_t(M) = 1$, we have $(f+g)(M)(p_c) = 1$ where ${}^c t = \{p_c\}$. So

$$f(M)(p_c) = 1 \text{ or } g(M)(p_c) = 1.$$

Hence

$$f_t \vee g_t(M) = 1,$$

which contradicts Eq. (6.2).

If $\mid {}^c t \mid \geq 2$ then we have, for any $p_c \in {}^c t$, $(f+g)(M)(p_c) = 1$ since $(f+g)_t(M) = 1$. Let ${}^c t = \{p_{c1}, p_{c2}, ..., p_{cm}\}$. Then, for each $p_{c_j} \in {}^c t$ $(j = 1, 2, ..., m)$,

$$f(M)(p_{c_j}) = 1 \text{ or } g(M)(p_{c_j}) = 1.$$

Hence

$$\flat(p_{c_j}, M) \leq f \text{ or } \flat(p_{c_j}, M) \leq g.$$

Since f, $g \in Per(Q)$, we have by Proposition 6.1 that $\flat(p_{c_j}, M) \in Per(Q)$ $(j = 1, 2, .., m)$, which implies together with the condition (C6-1) that $wlp_t(Q)(M) = 1$. This contradicts Eq. (6.2).

($\Rightarrow$) Assume that, for $M \in R(N)$ with $Q(M) = 1$, there exists $t \in T_c$ with $\mid {}^c t \mid \geq 2$ such that $\flat(p_{c1}, M)$, $\flat(p_{c2}, M)$,..., $\flat(p_{cm}, M) \in Per(Q)$ where ${}^c t = \{p_{c1}, p_{c2}, ..., p_{cm}\}$. Let $f := \sum_{p_{c_j} \in {}^c t} \flat(p_{c_j}, M)$. Since the unique MPC exists, $Per(Q)$ is closed under sum [100], and we have $f \in Per(Q)$. It is obvious that $f_t(M) = 1$, which implies together with $f \in Per(Q)$ that $wlp_t(Q)(M) = 1$. □

For each $p_c \in P_c$, the subset $T(p_c) \subseteq T_c$ is defined by

$$T(p_c) = \{t \in T_c;\ {}^c t = \{p_c\}\}.$$

We define the transformation wlp_{p_c} on $\mathbf{Q}$ for each $p_c \in P_c$ as follows:

$$wlp_{p_c}(Q)(M) = \begin{cases} 0 & \text{if } \exists t \in T(p_c) \text{ with } wlp_t(Q)(M) = 0, \\ 1 & \text{otherwise.} \end{cases}$$

That is, for $M \in R(N)$, if there exists $t \in T(p_c)$ such that Q is false at the marking after the firing of t then $wlp_{p_c}(Q)(M) = 0$, otherwise $wlp_{p_c}(Q)(M) = 1$.

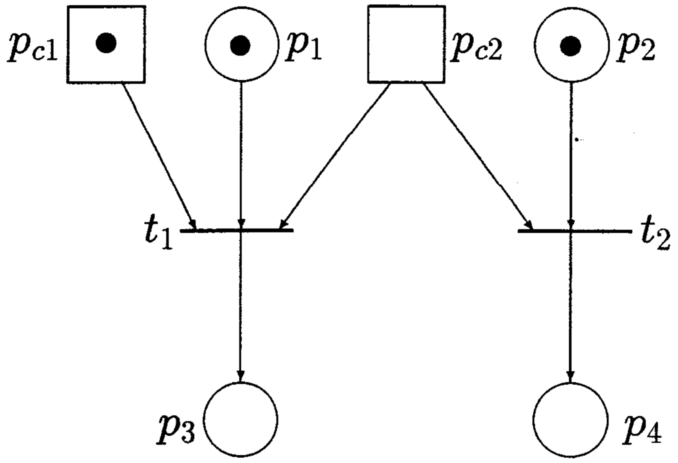

Figure 6.1: The CPN of Example 6.1.

Example 6.1 We consider a CPN shown in Figure 6.1, and a predicate Q such that $Q(M) = 1$ if $M(p_4) = 0$, and otherwise $Q(M) = 0$. For the marking M shown in Figure 6.1, $wlp_{t_2}(Q)(M) = 0$ and $t_2 \in T(p_{c2})$, which implies that $wlp_{p_{c2}}(Q)(M) = 0$. On the other hand, since $T(p_{c1}) = \emptyset$, we have $wlp_{p_{c1}}(Q)(M) = 1$.

Lemma 6.2 *Assume that a predicate $Q \in \mathbf{Q}$ is T_u-invariant. Then, for a marking $M^* \in R(N)$ with $Q(M^*) = 1$ and an external input place $p_c^* \in P_c$, $b(p_c^*, M^*) \in Per(Q)$ if and only if $wlp_{p_c^*}(Q)(M^*) = 1$.*

Proof: ($\Leftarrow$) T_u-invariance of Q implies that, for any $t \in T_u$,

$$Q \leq wlp_t(Q).$$

Consider $t \in T$ with $t \in T_c$. Then we have, for any $M \in R(N)$ such that $M \neq M^*$ and $Q(M) = 1$, $b(p_c^*, M^*)_t(M) = 0$. If ${}^c t \neq \{p_c^*\}$ then $b(p_c^*, M^*)_t(M^*) = 0$. Moreover, if ${}^c t = \{p_c^*\}$ then $wlp_t(Q)(M^*) = 1$ since $wlp_{p_c^*}(Q)(M^*) = 1$. Thus, for any $t \in T$,

$$Q \leq wlp_t(Q) \vee \sim b(p_c^*, M^*)_t, \tag{6.3}$$

which implies that $b(p_c^*, M^*) \in Per(Q)$.

($\Rightarrow$) Since $b(p_c^*, M^*) \in Per(Q)$ for $M^* \in R(N)$ with $Q(M^*) = 1$ and $p_c^* \in P_c$, Eq. (6.3) holds for any $t \in T$. Also since $Q(M^*) = 1$ and $b(p_c^*, M^*)_t(M^*) = 1$ for any $t \in T_c$ with ${}^c t = \{p_c^*\}$, Eq. (6.3) implies that $wlp_t(Q)(M^*) = 1$. Therefore, $wlp_{p_c^*}(Q)(M^*) = 1$. □

The following theorem can be proved by Lemmas 6.1 and 6.2.

Theorem 6.1 *Assume that a predicate $Q \in \mathbf{Q}$ is T_u-invariant. Then there exists the unique MPC for Q if and only if, for any $t \in T_c$ with $\mid {}^c t \mid \geq 2$, the following equation holds.*

$$Q \wedge \left(\bigwedge_{p_c \in {}^c t} wlp_{p_c}(Q) \right) \leq wlp_t(Q). \tag{6.4}$$

Eq. (6.4) implies that, for any $M \in R(N)$ with $Q(M) = 1$, if $b(p_c, M)$ is a permissive controller for all $p_c \in {}^c t$, then Q is true at the marking after the firing of t or t is not enabled. Using Theorem 6.1, we can check the uniqueness of the MPC without constructing $Per(Q)$.

We show a synthesis method for the unique MPC if it exists. Each controller $f \in \Gamma^{R(N)}$ can be decomposed into basis feedback controllers; $f = \sum_{i \in I} b_i$ with $b_i \in F_{bas}$ for each $i \in I$, where I is an index set. Hence, we can prove the following proposition.

Proposition 6.2 *If there exists the unique MPC $\hat{f} \in \Gamma^{R(N)}$ for a T_u-invariant predicate $Q \in \mathbf{Q}$, then $\hat{f}$ is given by*

$$\hat{f} = \sum_{f \in Per(Q) \cap F_{bas}} f. \tag{6.5}$$

Also, for any $t \in T_c$,

$$\hat{f}_t = \sim Q \vee \left(\bigwedge_{p_c \in {}^c t} wlp_{p_c}(Q) \right). \tag{6.6}$$

For any $p_c \in P_c$ and $t \in T_c$, if $\mid T(p_c) \mid = 1$ and $\mid {}^c t \mid = 1$ then

$$\bigwedge_{p_c \in {}^c t} wlp_{p_c}(Q) = wlp_t(Q),$$

and Eq. (6.6) is reduced as follows:

$$\hat{f}_t = \sim Q \vee wlp_t(Q),$$

which is equivalent to the unique MPC in the Ramadge-Wonham model [68]. Thus, Proposition 6.2 is a generalization of the result obtained in [68].

Example 6.2 We consider a simple manufacturing system consisting of two machines MA_1 and MA_2 and a buffer shown in Figure 6.2. p_i $(i = 1, 2, 3, 4, 5)$, p_{cj} $(j = 1, 2, 3)$ and t_k $(k = 1, 2, 3, 4)$ are assigned as follows:

p_1: MA_1 is idle.

p_2: MA_1 is working.

p_3: The buffer content.

p_4: MA_2 is idle.

p_5: MA_2 is working.

p_{c1}: Control of the start of MA_1.

p_{c2}: Control of the buffer.

p_{c3}: Control of the start of MA_2.

t_1: MA_1 completes working and passes the product to the buffer.

t_2: MA_2 fetches a product from the buffer and starts working.

t_3: MA_2 completes working.

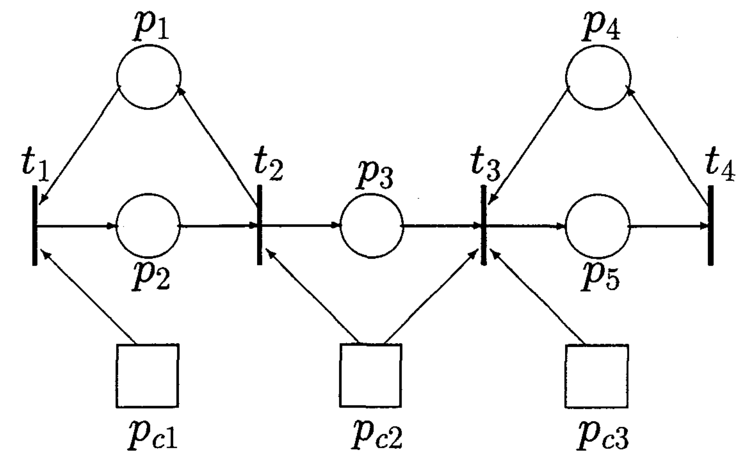

Figure 6.2: A Petri net model of a simple manufacturing system.

We write a marking M and a control pattern $f(M)$ as a 5-tuple

$$(M(p_1), M(p_2), M(p_3), M(p_4), M(p_5))$$

and a 3-tuple

$$(f(M)(p_{c1}), f(M)(p_{c2}), f(M)(p_{c3})),$$

respectively. Let $M_0 = (1,0,0,1,0)$. We consider a control specification $Q \in \mathbf{Q}$ given by

$$Q(M) = \begin{cases} 1 & \text{if } M(p_3) \leq 1, \\ 0 & \text{otherwise,} \end{cases}$$

that is, the buffer content is always at most one. Then reachable markings satisfying Q are as follows:

$m_1 = (1,0,0,1,0)$
$m_2 = (1,0,1,1,0)$
$m_3 = (1,0,0,0,1)$
$m_4 = (1,0,1,0,1)$
$m_5 = (0,1,0,1,0)$
$m_6 = (0,1,1,1,0)$
$m_7 = (0,1,0,0,1)$
$m_8 = (0,1,1,0,1)$

It is easily shown that Q is T_u-invariant. We shall show that there exists the unique MPC for Q using Theorem 6.1. In Figure 6.2, the transition t_3 is the only one such that $t \in T_c$ and $| {}^c t | \geq 2$, and ${}^c t_3 = \{p_{c2}, p_{c3}\}$. In order to prove the uniqueness of the MPC it is sufficient to show that, for m_j $(j = 1, 2, ..., 8)$, the following condition (C6-2) holds.

(C6-2) $\left(\bigwedge_{p_c \in {}^c t_3} wlp_{p_c}(Q)\right)(m_j) = 1 \;\Rightarrow\; wlp_{t_3}(Q)(m_j) = 1.$

We only consider the marking m_1. For m_j $(j = 2, 3, ..., 8)$, we can verify (C6-2) in the same way as m_1. For $p_{c2} \in {}^c t_3$, the transition t_2 is the only one such that $t \in T_c$ and ${}^c t = \{p_{c2}\}$. Since $wlp_{t_2}(Q)(m_1) = 1$, we have $wlp_{p_{c2}}(Q)(m_1) = 1$. Additionally, for $p_{c3} \in {}^c t_3$, there exists no transition t such that $t \in T_c$ and ${}^c t = \{p_{c3}\}$, which implies that $wlp_{p_{c3}}(Q)(m_1) = 1$. It follows that

$$\left(\bigwedge_{p_c \in {}^c t_3} wlp_{p_c}(Q)\right)(m_1) = 1.$$

Moreover, since t_3 is not enabled at m_1, we have $wlp_{t_3}(Q)(m_1) = 1$. Therefore, the condition (C6-2) holds. Thus, by Theorem 6.1, there exists the unique MPC $\hat{f}$ for Q, and

$\hat{f}$ is given by

$$\hat{f}(M) = \begin{cases} (1,0,1) & \text{if } M \in \{m_6, m_8\}, \\ (1,1,1) & \text{otherwise.} \end{cases}$$

6.2.2 Maximally Permissive Controllers under Partial Observations

In this subsection, we consider the case that only a marking of places belonging to the subset $P_o \subseteq P$ can be observed [28]. In this case, a state feedback controller $f \in \Gamma^{R(N)}$ selects a control pattern based upon a making of P_o. For M, $M' \in R(N)$, we shall write $M \equiv M'$ if $M(p) = M'(p)$ for all $p \in P_o$. Then "$\equiv$" is an equivalence relation on $R(N)$. We denote the equivalence class of $M \in R(N)$ mod $\equiv$ by $C(M)$, that is,

$$C(M) = \{M' \in R(N);\ M' \equiv M\}.$$

Let Z be the set of all equivalence classes. We define a mapping $\Lambda : R(N) \to Z$ as follows: for any $M \in R(N)$,

$$\Lambda(M) = C(M).$$

A state feedback controller $f \in \Gamma^{R(N)}$ under partial observations satisfies the following condition (C6-3).

(C6-3) For any M, $M' \in R(N)$

$$\Lambda(M) = \Lambda(M') \Rightarrow f(M) = f(M').$$

Let $Per_o(Q)$ be the set of all permissive controllers for Q satisfying the condition (C6-3). It can be easily proved that $Per_o(Q) \neq \emptyset$ if and only if Q is T_u-invariant. We call a maximal element of $Per_o(Q)$ under "$\leq$" a *maximally permissive controller under partial observations* (MPCPO). For a T_u-invariant predicate, there does not always exist the MPCPO uniquely.

For a predicate Q on $R(N)$, we define the predicate $\Lambda(Q)$ on Z as follows:

$$\Lambda(Q)(z) = \begin{cases} 1 & \text{if } \exists M \in R(N) \text{ with } \Lambda(M) = z \text{ and } Q(M) = 1, \\ 0 & \text{otherwise.} \end{cases}$$

For a predicate Q' on Z, we define the predicate $\Lambda^{-1}(Q')$ on $R(N)$ as follows:

$$\Lambda^{-1}(Q')(M) = \begin{cases} 1 & \text{if } Q'(\Lambda(M)) = 1, \\ 0 & \text{otherwise.} \end{cases}$$

We define the transformation $owlp_{p_c}$ on $\mathbf{Q}$ as follows:

$$owlp_{p_c}(Q) = \sim \Lambda^{-1}(\Lambda(Q \wedge \sim wlp_{p_c}(Q))).$$

For $M \in R(N)$, if there exists $M' \in R(N)$ such that $\Lambda(M') = \Lambda(M)$, $Q(M') = 1$ and $wlp_{p_c}(Q)(M') = 0$, then $owlp_{p_c}(Q)(M) = 0$, otherwise $owlp_{p_c}(Q)(M) = 1$.

For each $p_c^* \in P_c$ and each $M^* \in R(N)$, the basis feedback controller under partial observations $b_o(p_c^*, M^*) \in \Gamma^{R(N)}$ is defined by

$$b_o(p_c^*, M^*)(M)(p_c) = \begin{cases} 1 & \text{if } \Lambda(M) = \Lambda(M^*) \text{ and } p_c = p_c^* \\ 0 & \text{otherwise} \end{cases}$$

Obviously, $b_o(p_c^*, M^*)$ satisfies the condition (C6-3). Let F_{bas}^o be the set of all basis feedback controllers under partial observations.

Lemma 6.3 *Assume that a predicate $Q \in \mathbf{Q}$ is T_u-invariant. Then, for a marking $M^* \in R(N)$ and an external input place $p_c^* \in P_c$, $b_o(p_c^*, M^*) \in Per_o(Q)$ if and only if $owlp_{p_c^*}(Q)(M^*) = 1$.*

Proof: ($\Leftarrow$) T_u-invariance of Q implies that, for any $t \in T_u$,

$$Q \leq wlp_t(Q).$$

Consider $t \in T$ with $t \in T_c$. Then we have, for any $M \in R(N)$ such that $\Lambda(M) \neq \Lambda(M^*)$ and $Q(M) = 1$, $b_o(p_c^*, M^*)_t(M) = 0$. Also, for any $M \in R(N)$ such that $\Lambda(M) = \Lambda(M^*)$ and $Q(M) = 1$, if ${}^c t \neq \{p_c^*\}$ then $b(p_c^*, M^*)_t(M) = 0$. We consider the case that ${}^c t = \{p_c^*\}$. Since $owlp_{p_c^*}(Q)(M^*) = 1$, we have

$$\Lambda^{-1}(\Lambda(Q \wedge \sim wlp_{p_c^*}(Q)))(M^*) = 0,$$

which implies together with $\Lambda(M) = \Lambda(M^*)$ that

$$(Q \wedge \sim wlp_{p_c^*}(Q))(M) = 0.$$

So we have $wlp_{p_c^*}(Q)(M) = 1$, which implies together with ${}^c t = \{p_c^*\}$ that $wlp_t(Q)(M) = 1$. Therefore, the following equation holds for any $t \in T$.

$$Q \leq wlp_t(Q) \vee \sim b_o(p_c^*, M^*)_t, \tag{6.7}$$

which implies that $b_o(p_c^*, M^*) \in Per_o(Q)$.

($\Rightarrow$) Assume that $owlp_{p_c^*}(Q)(M^*) = 0$. Then there exists $M \in R(N)$ such that $\Phi(M) = \Phi(M^*)$ and $(Q\wedge \sim wlp_{p_c^*}(Q))(M) = 1$. Since $wlp_{p_c^*}(Q))(M) = 0$, there exists $t \in T_c$ such that ${}^c t = \{p_c^*\}$ and $wlp_t(Q)(M) = 0$. We also have $b_o(p_c^*, M^*)_t(M) = 1$. Since $b_o(p_c^*, M^*) \in Per_o(Q)$, Eq. (6.7) holds, which implies that $wlp_t(Q)(M) = 1$. This is a contradiction. □

Lemma 6.4 *Assume that a predicate $Q \in \mathbf{Q}$ is T_u-invariant. Then if there exists the unique MPCPO, then $Per_o(Q)$ is closed under sum.*

Proof: It can be proved in the same way as Theorem 2 of [100] that, for any f and $g \in Per_o(Q)$, $f + g$ is a permissive controller. Additionally, For any M, $M' \in R(N)$ with $\Lambda(M) = \Lambda(M')$, we have

$$\begin{aligned}(f+g)(M) &= f(M)+g(M)\\ &= f(M')+g(M')\\ &= (f+g)(M'),\end{aligned}$$

which implies that $f+g$ satisfies the condition (C6-3). Therefore, we have $f+g \in Per_o(Q)$. □

The following proposition can be proved in the same way as Theorem 1 of [98].

Proposition 6.3 *Assume that a predicate $Q \in \mathbf{Q}$ is T_u-invariant. Then there exists the unique MPCPO for Q if and only if the following condition (C6-4) holds for any f, $g \in Per_o(Q)$.*

(C6-4) For any $t \in T$, the following equation holds.

$$Q \leq wlp_t(Q) \vee f_t \vee g_t \vee \sim (f+g)_t \tag{6.8}$$

Using Lemmas 6.3, 6.4 and Proposition 6.3, we can prove the following theorem in the same way as Theorem 6.1.

Theorem 6.2 *Assume that a predicate $Q \in \mathbf{Q}$ is T_u-invariant. Then there exists the unique MPCPO for Q if and only if, for any $t \in T_c$ with $\mid {}^c t \mid \geq 2$, the following equation holds.*

$$Q \wedge \left(\bigwedge_{p_c \in {}^c t} owlp_{p_c}(Q) \right) \leq wlp_t(Q). \tag{6.9}$$

Eq. (6.9) implies that, for any $M \in R(N)$ with $Q(M) = 1$, if $b_o(p_c, M)$ is a permissive controller for all $p_c \in {}^c t$, then Q is true at the marking after the firing of t or t is not enabled. The following proposition presents a synthesis method of the unique MPCPO if it exists.

Proposition 6.4 *If there exists the unique MPCPO $\hat{f} \in \Gamma^{R(N)}$ for a T_u-invariant predicate $Q \in \mathbf{Q}$, then $\hat{f}$ is given by*

$$\hat{f} = \sum_{f \in Per_o(Q) \cap F^o_{bas}} f. \tag{6.10}$$

Also, for any $t \in T_c$,

$$\hat{f}_t = \bigwedge_{p_c \in {}^c t} owlp_{p_c}(Q). \tag{6.11}$$

Proof: First, we prove Eq. (6.10). By Lemma 6.4, $Per_o(Q)$ is closed under sum, which implies that

$$\sum_{f \in Per_o(Q) \cap F^o_{bas}} f \in Per_o(Q).$$

So we have

$$\sum_{f \in Per_o(Q) \cap F^o_{bas}} f \leq \hat{f}.$$

We prove the reverse inequality. Consider $M \in R(N)$ and $p_c \in P_c$ with $\hat{f}(M)(p_c) = 1$. Then $\hat{f}(M')(p_c) = 1$ for any $M' \in R(N)$ with $\Lambda(M) = \Lambda(M')$, which implies that $b_o(p_c, M) \leq \hat{f}$. Since $\hat{f} \in Per_o(Q)$, we have by Proposition 6.1 that $b_o(p_c, M) \in Per_o(Q) \cap F^o_{bas}$, which implies that

$$\left(\sum_{f \in Per_o(Q) \cap F^o_{bas}} f \right) (M)(p_c) = 1.$$

Therefore,

$$\hat{f} \leq \sum_{f \in Per_o(Q) \cap F^o_{bas}} f.$$

Next, we prove Eq. (6.11). We prove that, for any $t \in T_c$,

$$\hat{f}_t \leq \bigwedge_{p_c \in {}^c t} owlp_{p_c}(Q). \tag{6.12}$$

Consider $M \in R(N)$ with $\hat{f}_t(M) = 1$. Then $\hat{f}(M)(p_c) = 1$ for any $p_c \in^c t$. By Eq. (6.10), we have $b_o(p_c, M) \in Per_o(Q) \cap F^o_{bas}$, which implies together with Lemma 6.3 that $owlp_{p_c}(Q)(M) = 1$. Thus, Eq. (6.12) holds.

We prove the reverse inequality by contradiction. Suppose that there exist $t \in T_c$ and $M \in R(N)$ such that

$$\bigwedge_{p_c \in {}^c t} owlp_{p_c}(Q)(M) = 1 \text{ and } \hat{f}_t(M) = 0.$$

Since $\hat{f}_t(M) = 0$, there exists $p'_c \in {}^c t$ with $\hat{f}(M)(p'_c) = 0$, which implies together with Eq. (6.10) that $b_o(p'_c, M) \notin Per_o(Q) \cap F^o_{bas}$. However, since $\bigwedge_{p_c \in {}^c t} owlp_{p_c}(Q)(M) = 1$, we have by Lemma 6.3 that $b_o(p'_c, M) \in Per_o(Q) \cap F^o_{bas}$. This is a contradiction. □

Example 6.3 We consider the same manufacturing system as Example 6.2. We assume that $P_o = \{p_2, p_3\}$. By Theorem 6.2 and Proposition 6.4, we can show that the unique MPCPO for Q exists, and is given by

$$\hat{f}(M)(p_c) = \begin{cases} 0 & \text{if } M \in \{m_6, m_8\} \text{ and } p_c = p_{c2}, \\ 1 & \text{otherwise.} \end{cases}$$

6.3 Controllers for Petri Nets with Concurrency

In this section, we study MPCs for CPNs with concurrency controlled by either event assignment or resource allocation.

6.3.1 Event Assignment Control

We consider a controlled DES $\mathcal{G}_1 = (N_c^{con}, \Gamma_1)$, where N_c^{con} is a CPN with concurrency and $\Gamma_1 = \{0, 1\}^{P_c}$ is the set of all control patterns.

Let $T(p_c)^\omega$ be the set of all bags over $T(p_c)$. We define the transformation $cwlp_{p_c}$ on $\mathbf{Q}$ for each $p_c \in P_c$:

$$cwlp_{p_c}(Q)(M) = \begin{cases} 0 & \text{if } T(p_c) \neq \emptyset \text{ and } wlp_{t_b}(Q)(M) = 0 \text{ for some } t_b \in T(p_c)^\omega, \\ 1 & \text{otherwise.} \end{cases}$$

That is, for $M \in R(N)$, if there exists $t_b \in T(p_c)^\omega$ such that Q is false at the marking after the firing of t_b then $cwlp_{p_c}(Q)(M) = 0$, otherwise $cwlp_{p_c}(Q)(M) = 1$. Note that $cwlp_{p_c}(Q)$ is defined with respect to a b-transition $t_b \in T(p_c)^\omega$ while $wlp_{p_c}(Q)$ is defined with respect to a transition $t \in T(p_c)$.

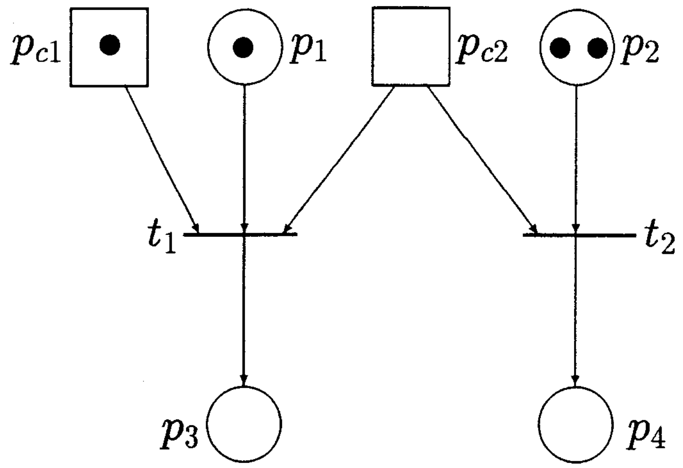

Figure 6.3: The CPN of Example 6.4.

Example 6.4 We consider a CPN shown in Figure 6.3, and a predicate Q such that $Q(M) = 1$ if $M(p_4) \leq 1$, and otherwise $Q(M) = 0$. Let $t_b = \{t_2, t_2\}$. For the marking M shown in Figure 6.3, $wlp_{t_b}(Q)(M) = 0$ and $t_b \in T(p_{c2})^\omega$, which implies that $cwlp_{p_{c2}}(Q)(M) = 0$. On the other hand, since $wlp_{t_2}(Q)(M) = 1$, we have $wlp_{p_{c2}}(Q)(M) = 1$.

Lemma 6.5 *Assume that a predicate Q is T_u^ω-invariant. Then for a marking $M^* \in R(N)$ with $Q(M^*) = 1$ and an external input place $p_c^* \in P_c$, $b(p_c^*, M^*) \in Per_1(Q)$ (the set of all permissive controllers in $\mathcal{G}_1$) if and only if $cwlp_{p_c^*}(Q)(M^*) = 1$.*

Proof: ($\Leftarrow$) T_u^ω-invariance of Q implies that $Q \leq wlp_{t_b}(Q)$ for any $t_b \in T_u^\omega \cap T_b$. Let $t_b \in T_b$ be a b-transition such that $t_b \cap T_c \neq \emptyset$. Then we have $b(p_c^*, M^*)_{t_b}(M) = 0$ for any $M \in R(N)$ such that $M \neq M^*$ and $Q(M) = 1$. If $t_b \cap T_c \not\subseteq T(p_c^*)$, then $b(p_c^*, M^*)_{t_b}(M^*) = 0$. Moreover, if $t_b \cap T_c \subseteq T(p_c^*)$ and $D_{t_b}(M^*) = 0$ then $wlp_{t_b}(Q)(M^*) = 1$. Thus we consider the case that $t_b \cap T_c \subseteq T(p_c^*)$ and $D_{t_b}(M^*) = 1$. We partition t_b as

$$t_b = t_{bc} \cup t_{bu},$$

where $t_{bc} \in T_c^\omega$ and $t_{bu} \in T_u^\omega$. Since $cwlp_{p_c^*}(Q)(M^*) = 1$, we have $wlp_{t_{bc}}(Q)(M^*) = 1$. Let M' and M'' be markings such that $M^*[t_{bc} > M'$ and $M'[t_{bu} > M''$. Then we have

$Q(M') = 1$. Moreover, T_u^ω-invariance of Q implies that $Q(M'') = 1$. Since $M^*[t_b > M''$, we have $wlp_{t_b}(Q)(M^*) = 1$. Therefore, for any $t_b \in T_b$,

$$Q \leq wlp_{t_b}(Q) \vee \sim b(p_c^*, M^*)_{t_b},$$

which implies that $b(p_c^*, M^*) \in Per_1(Q)$.

($\Rightarrow$) Since $b(p_c^*, M^*) \in Per_1(Q)$ for $M^* \in R(N)$ with $Q(M^*) = 1$ and $p_c^* \in P_c$, the following equation holds for any $t_b \in T_b$.

$$Q \leq wlp_{t_b}(Q) \vee \sim b(p_c^*, M^*)_{t_b}. \tag{6.13}$$

Since $Q(M^*) = 1$ and $b(p_c^*, M^*)_{t_b}(M^*) = 1$ for any $t_b \in T(p_c^*)^\omega$, we have $wlp_{t_b}(Q)(M^*) = 1$ by Eq. (6.13). Therefore, $cwlp_{p_c^*}(Q)(M^*) = 1$. □

We show a necessary and sufficient condition for the unique existence of the MPC in $\mathcal{G}_1$, which can be checked without constructing $Per_1(Q)$.

Theorem 6.3 *Assume that a predicate Q is T_u^ω-invariant. Then there exists the unique MPC in $\mathcal{G}_1$ if and only if for any $t_b \in T_c^\omega \cap T_b$, the following equation holds.*

$$Q \wedge \bigwedge_{p_c \in {}^c t_b} cwlp_{p_c}(Q) \leq wlp_{t_b}(Q). \tag{6.14}$$

Proof: ($\Leftarrow$) By Proposition 2.3, it is sufficient to prove that Q is WI in $\mathcal{G}_1$. Obviously, Eq. (2.16) holds for any f and $g \in Per_1(Q)$ if $t_b \in T_u^\omega \cap T_b$. Suppose that Eq. (2.16) does not hold for some $t_b \in T_b$ such that $t_b \cap T_c \neq \emptyset$. Then there exist f, $g \in Per_1(Q)$ and $M \in R(N)$ such that

$$Q(M) = 1 \text{ and } (wlp_{t_b}(Q) \vee f_{t_b} \vee g_{t_b} \vee \sim (f+g)_{t_b})(M) = 0. \tag{6.15}$$

We partition t_b as

$$t_b = t_{bc} \cup t_{bu},$$

where $t_{bc} \in T_c^\omega$ and $t_{bu} \in T_u^\omega$. Since $(f+g)_{t_b}(M) = 1$, we have $(f+g)_{t_{bc}}(M) = 1$. Thus for any $p_c \in {}^c t_{bc}$, $(f+g)(M)(p_c) = 1$, that is,

$$f(M)(p_c) = 1 \text{ or } g(M)(p_c) = 1.$$

Hence

$$b(p_c, M) \leq f \text{ or } b(p_c, M) \leq g.$$

Since f, $g \in Per_1(Q)$, we have by Proposition 6.1 that $b(p_c, M) \in Per_1(Q)$, which implies together with Lemma 6.5 that $cwlp_{p_c}(Q)(M) = 1$. Then,

$$Q(M) = 1 \text{ and } \left(\bigwedge_{p_c \in {}^c t_{bc}} cwlp_{p_c}(Q) \right) (M) = 1.$$

So $wlp_{t_{bc}}(Q)(M) = 1$ by Eq. (6.14). Let M' and M'' be markings such that $M[t_{bc} > M'$ and $M'[t_{bu} > M''$. Then we have $Q(M') = 1$. Moreover, T_u^ω-invariance of Q implies that $Q(M'') = 1$. Since $M[t_b > M''$, we have $wlp_{t_b}(Q)(M) = 1$, which contradicts Eq. (6.15).

($\Rightarrow$) Assume that there exist $M \in R(N)$ and $t_b \in T_c^\omega \cap T_b$ such that

$$\left(Q \wedge \bigwedge_{p_c \in {}^c t_b} cwlp_{p_c}(Q) \right) (M) = 1.$$

Then by Lemma 6.5, $b(p_c, M) \in Per_1(Q)$ for any $p_c \in {}^c t_b$. Let $f = \sum_{p_c \in {}^c t_b} b(p_c, M)$. Since the unique MPC exists in $\mathcal{G}_1$, $Per_1(Q)$ is closed under sum [100], and we have $f \in Per_1(Q)$. It is obvious that $f_{t_b}(M) = 1$, which implies that $wlp_{t_b}(Q)(M) = 1$. Therefore, Eq. (6.14) holds for any $t_b \in T_c^\omega \cap T_b$. □

Eq. (6.14) implies that, for any $M \in R(N)$ with $Q(M) = 1$, if $b(p_c, M)$ is a permissive controller for all $p_c \in {}^c t_b$, then Q is true at the marking after the firing of t_b or t_b is not enabled.

6.3.2 Resource Allocation Control

We consider a controlled DES $\mathcal{G}_2$. For $M \in R(N)$ with $Q(M) = 1$ and $p_c \in P_c$, $X(M, p_c) \in \mathcal{N} \cup \{\infty\}$ is defined as follows:

$$X(M, p_c) = \begin{cases} 0 & \text{if } wlp_{p_c}(Q)(M) = 0, \\ k_{\max} & \text{if } (wlp_{p_c}(Q) \wedge \sim cwlp_{p_c}(Q))(M) = 1, \\ \infty & \text{otherwise,} \end{cases}$$

where

$$k_{\max} = \max\{k \in \mathcal{N};\ wlp_{t_b}(Q)(M) = 1 \text{ for any } t_b \in T(p_c)^\omega \text{ such that } \mid t_b \mid \leq k\},$$

the symbol ∞ satisfies that $k < \infty$ for any $k \in \mathcal{N}$. Intuitively, $X(M, p_c)$ denotes the maximum number of tokens we can put into p_c at $M \in R(N)$ under permissive controllers when $M(p'_c) = 0$ for all $p'_c (\neq p_c) \in P_c$.

Example 6.5 We consider the same system as Example 6.4. For the marking M shown in Figure 6.3, we know that $(wlp_{p_{c2}}(Q) \wedge \sim cwlp_{p_{c2}}(Q))(M) = 1$ by Example 6.4. Then it is easily shown that $X(M, p_{c2}) = k_{\max} = 1$. On the other hand, since $T(p_{c1}) = \emptyset$, we have $wlp_{p_{c1}}(Q)(M) = cwlp_{p_{c1}}(Q)(M) = 1$, which implies that $X(M, p_{c1}) = \infty$.

Lemma 6.6 *Assume that a predicate Q is T_u^ω-invariant. Then for any $f \in Per_2(Q)$, any $p_c \in P_c$ and any $M \in R(N)$ with $Q(M) = 1$, the following equation holds.*

$$f(M)(p_c) \leq X(M, p_c). \tag{6.16}$$

Proof: We shall prove the above lemma by contradiction. Suppose that there exist $f \in Per_2(Q)$, $p_c \in P_c$ and $M \in R(N)$ such that $Q(M) = 1$ and $X(M, p_c) < f(M)(p_c)$. Obviously, $X(M, p_c) \neq \infty$. If $wlp_{p_c}(Q)(M) = 0$, then $X(M, p_c) = 0$, which implies $f(M)(p_c) \geq 1$. Thus there exists $t \in T(p_c)$ such that $f_t(M) = 1$ and $wlp_t(Q)(M) = 0$, which contradicts the fact that $f \in Per_2(Q)$. If $wlp_{p_c}(Q)(M) = 1$, then there exists $t_b \in T(p_c)^\omega$ such that $| t_b | \leq f(M)(p_c)$ and $wlp_{t_b}(Q)(M) = 0$, and obviously, $f_{t_b}(M) = 1$, which also contradicts the fact that $f \in Per_2(Q)$. □

For $M^* \in R(N)$ with $Q(M^*) = 1$, $p_c^* \in P_c$ and $k \in \mathcal{N}$, we define $g(p_c^*, M^*, k) \in \Gamma_2^{R(N)}$ as follows:

$$g(p_c^*, M^*, k)(M)(p_c) = \begin{cases} k & \text{if } M = M^* \text{ and } p_c = p_c^*, \\ 0 & \text{otherwise.} \end{cases} \tag{6.17}$$

Lemma 6.7 *Assume that a predicate Q is T_u^ω-invariant. Then $g(p_c^*, M^*, k) \in \Gamma_2^{R(N)}$ defined by Eq. (6.17) is a permissive controller in $\mathcal{G}_2$ whenever $k \leq X(M^*, p_c^*)$.*

Proof: T_u^ω-invariance of Q implies that

$$Q \leq wlp_{t_b}(Q) \quad \text{for any } t_b \in T_u^\omega \cap T_b.$$

Let $t_b \in T_b$ be a b-transition such that $t_b \cap T_c \neq \emptyset$. It is sufficient to consider the case that $M = M^*$, $t_b \cap T_c \subseteq T(p_c^*)$ and $D_{t_b}(M^*) = 1$ because, in other cases, it is proved in the same way as Lemma 6.5. We partition t_b as

$$t_b = t_{bc} \cup t_{bu},$$

where $t_{bc} \in T_c^\omega$ and $t_{bu} \in T_u^\omega$. If $| t_{bc} | > k$, then $\sim g(p_c^*, M^*, k)_{t_{bc}}(M^*) = 1$. If $| t_{bc} | \leq k \leq X(M^*, p_c^*)$, then $wlp_{t_{bc}}(Q)(M^*) = 1$ by the definition of $X(M^*, p_c^*)$. Let M' and

M'' be markings such that $M^*[t_{bc} > M'$ and $M'[t_{bu} > M''$. Then we have $Q(M') = 1$. Moreover, T_u^ω-invariance of Q implies that $Q(M'') = 1$. Thus, we have $wlp_{t_b}(Q)(M^*) = 1$. Therefore, for any $t_b \in T_b$,

$$Q \leq wlp_{t_b}(Q) \vee \sim g(p_c^*, M^*, k)_{t_b},$$

which implies that $g(p_c^*, M^*, k) \in Per_2(Q)$. □

We define the predicate $B_{t_b} \in \mathbf{Q}$ for $t_b \in T^\omega$ as follows:

$$B_{t_b}(M) = \begin{cases} 1 & \text{if } \sum_{t \in t_b} I_c(t, p_c) \leq X(M, p_c) \quad \forall p_c \in P_c, \\ 0 & \text{otherwise.} \end{cases} \tag{6.18}$$

That is, for $M \in R(N)$, if the total number of arcs from p_c to some $t \in t_b$ is at most $X(M, p_c)$ for $p_c \in P_c$ then $B_{t_b}(M) = 1$, otherwise $B_{t_b}(M) = 0$.

The following theorem shows a necessary and sufficient condition for the unique existence of the MPC in $\mathcal{G}_2$, which does not require to construct $\Omega_2(Q)$.

Theorem 6.4 *Assume that a predicate Q is T_u^ω-invariant. Then there exists the unique MPC in $\mathcal{G}_2$ if and only if the following equation holds for any $t_b \in T_c^\omega \cap T_b$.*

$$Q \wedge B_{t_b} \leq wlp_{t_b}(Q). \tag{6.19}$$

Proof: ($\Rightarrow$) It is sufficient to prove that Q is WI in $\mathcal{G}_2$. It is obvious that Eq. (2.16) holds for any f and $g \in \Omega_2(Q)$ if $t_b \in T_u^\omega \cap T_b$. Suppose that Eq. (2.16) does not hold for some $t_b \in T_b$ with $t_b \cap T_c \neq \emptyset$. Then there exist f, $g \in \Omega_2(Q)$ and $M \in R(N)$ such that

$$Q(M) = 1 \text{ and } (wlp_{t_b}(Q) \vee f_{t_b} \vee g_{t_b} \vee \sim (f+g)_{t_b})(M) = 0. \tag{6.20}$$

We partition t_b as

$$t_b = t_{bc} \cup t_{bu},$$

where $t_{bc} \in T_c^\omega$ and $t_{bu} \in T_u^\omega$. Since $(f+g)_{t_b}(M) = 1$, we have $(f+g)_{t_{bc}}(M) = 1$. Thus for any $p_c \in P_c$,

$$\sum_{t \in t_{bc}} I_c(t, p_c) \leq f(M)(p_c) \text{ or } \sum_{t \in t_{bc}} I_c(t, p_c) \leq g(M)(p_c).$$

Also, since f and $g \in \Omega_2(Q)$, by Lemma 6.6,

$$f(M)(p_c) \leq X(M, p_c) \text{ and } g(M)(p_c) \leq X(M, p_c),$$

which implies that

$$\sum_{t \in t_{bc}} I_c(t, p_c) \le X(M, p_c).$$

Thus $B_{t_{bc}}(Q)(M) = 1$. Since $(Q \wedge B_{t_{bc}})(M) = 1$, Eq. (6.19) implies that $wlp_{t_{bc}}(Q) = 1$. Let M' and M'' be markings such that $M[t_{bc} > M'$ and $M'[t_{bu} > M''$. Then we have $Q(M') = 1$. Moreover, T_u^ω-invariance of Q implies that $Q(M'') = 1$. Therefore, we have $wlp_{t_b}(Q)(M) = 1$, which contradicts Eq. (6.20).

($\Leftarrow$) For $t_b \in T_c^\omega \cap T_b$, let M^* be a marking such that $(Q \wedge B_{t_b})(M^*) = 1$. Then the following equation holds.

$$\sum_{t \in t_b} I_c(t, p_c) \le X(M^*, p_c) \quad \forall p_c \in P_c.$$

Obviously, if $D_{t_b}(M^*) = 0$ then $wlp_{t_b}(Q)(M^*) = 1$. So we consider the case that $D_{t_b}(M^*) = 1$. We define $f(p_c^*, M^*) \in \Gamma_2^{R(N)}$ as follows:

$$f(p_c^*, M^*)(M)(p_c) = \begin{cases} \sum_{t \in t_b} I_c(t, p_c^*) & \text{if } M = M^* \text{ and } p_c = p_c^*, \\ 0 & \text{otherwise.} \end{cases}$$

By Lemma 6.7, $f(p_c^*, M^*) \in Per_2(Q)$, and it is obvious that $f(p_c^*, M^*)$ satisfies the MTC. Moreover, we define $g \in \Gamma_2^{R(N)}$ by

$$g = \sum_{p_c^* \in P_c} f(p_c^*, M^*).$$

Since the unique MPC exists in $\mathcal{G}_2$, $\Omega_2(Q)$ is closed under sum [100]. Therefore, we have $g \in \Omega_2(Q)$ and $g_{t_b}(M^*) = 1$, which implies that $wlp_{t_b}(Q)(M^*) = 1$. □

The above theorem shows that, for any $M \in R(M)$ and any $t_b \in T_c^\omega \cap T_b$ such that $(Q \wedge B_{t_b})(M) = 1$, if Q is true at the marking after the firing of t_b or t_b is not enabled, then the unique MPC exists in $\mathcal{G}_2$.

6.3.3 Comparison between $\mathcal{G}_1$ and $\mathcal{G}_2$

In this subsection, we discuss the relationship between the unique existence of the MPC in $\mathcal{G}_1$ and that in $\mathcal{G}_2$.

Theorem 6.5 *Assume that a predicate Q is T_u^ω-invariant. Then if the unique MPC exists in $\mathcal{G}_2$, then it also exists in $\mathcal{G}_1$.*

Proof: By the definition of $X(M, p_c)$, the conditions $cwlp_{p_c}(Q)(M) = 1$ and $X(M, p_c) = \infty$ are equivalent for any $M \in R(N)$ with $Q(M) = 1$ and any $p_c \in P_c$. For $M^* \in R(N)$ with $Q(M^*) = 1$, let $t_b \in T_c^\omega \cap T_b$ be a b-transition satisfying the following equation.

$$\bigwedge_{p_c \in {}^c t_b} cwlp_{p_c}(Q)(M^*) = 1.$$

Obviously, if $X(M^*, p_c) = \infty$ then $\sum_{t \in t_b} I_c(t, p_c) \leq X(M^*, p_c)$. If $X(M^*, p_c) < \infty$, we have $I_c(t, p_c) = 0$ for any $t \in t_b$. Thus the following equation holds.

$$\sum_{t \in t_b} I_c(t, p_c) \leq X(M^*, p_c) \quad \forall p_c \in P_c,$$

which implies that $B_{t_b}(M^*) = 1$. The unique existence of the MPC in $\mathcal{G}_2$ implies together with Theorem 6.4 that $wlp_{t_b}(Q)(M^*) = 1$. Then for any $t_b \in T_c^\omega \cap T_b$, Eq. (6.14) holds. Therefore, there exists the unique MPC in $\mathcal{G}_1$ by Theorem 6.3. □

Note that the reverse implication of Theorem 6.5 does not always hold. We shall give a counter-example as follows.

Example 6.6 We consider a CPN N_c shown in Figure 6.4. Let Q be the predicate such that $Q(M) = 1$ if $M(p_1) + M(p_2) \leq 1$, and otherwise 0. We will write a marking M and a control pattern $f(M)$ as a 3-tuple $(M(p_1), M(p_2), M(p_3))$ and a 2-tuple $(f(M)(p_{c1}), f(M)(p_{c2}))$, respectively. Let $M_0 = (0, 0, 2)$. It is easily shown that Q is T_u^ω-invariant, and that there exists the unique MPC $\hat{f}$ in $\mathcal{G}_1$ given by

$$\hat{f}(M) = \begin{cases} (0,0) & \text{if } M \in \{(0,0,2), (0,1,1), (1,0,1)\}, \\ (1,1) & \text{otherwise.} \end{cases}$$

On the other hand, the following two controllers f_1 and f_2 are permissive controllers with the MTC in $\mathcal{G}_2$.

(a) $f_1(M) = (0, 1)$ and $f_2(M) = (1, 0)$ if $M = (0, 0, 2)$.

(b) $f_1(M) = f_2(M) = (0, 0)$ otherwise.

The controller $f_1 + f_2$ is not, however, a permissive controller, that is, the MPC does not exist uniquely.

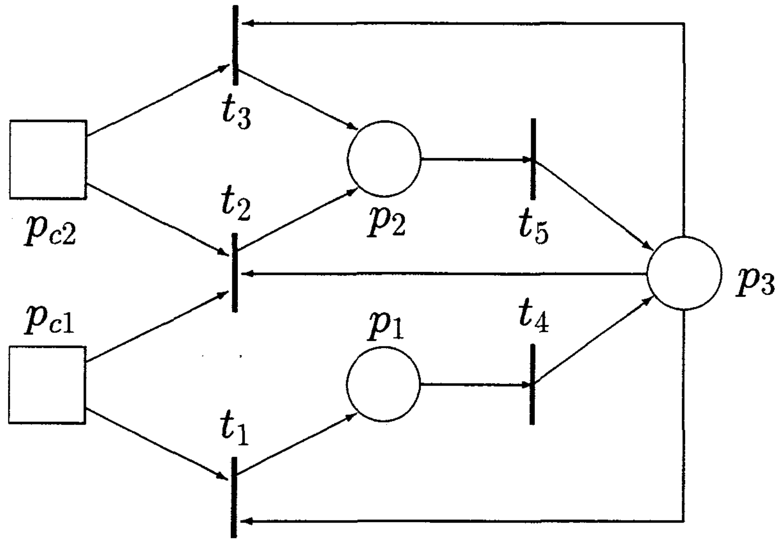

Figure 6.4: The CPN of Example 6.6.

6.4 Concluding Remarks

In this chapter, we studied the unique existence of the MPC in CPNs. First, we considered CPNs without concurrency. We presented necessary and sufficient conditions for the unique existence of the MPC under partial as well as complete observations, which can be checked without constructing the set of all permissive controllers. Next, we extended the results to CPNs with concurrency controlled by either event assignment or resource allocation. We then showed that the unique existence of the MPC in resource allocation control implies that the same is true in event assignment control.

Chapter 7

Stabilization and Blocking

7.1 Introduction

Control problems in DESs such as manufacturing systems are often specified by both admissible and target states. Admissible states represent a set of states in which state trajectories of a system should reside. Target states, which is a subset of admissible states, represent the completion of some tasks. It is shown in this chapter that if control specifications are given not only by admissible states but also by target states, the notion of stabilization of DESs [64, 63, 62, 8, 9, 103, 39, 76, 77] plays an important role to design state feedback controllers.

In the context of supervisory control [67], the optimization techniques of a blocking supervisor in terms of two logical performance measures, a satisficing measure and a blocking measure, have been proposed [12]. In the closed-loop system, the former measure indicates admissible marked traces enabled by the supervisor, and the latter indicates traces which lead to blocking. We define two similar performance measures called a prestabilizing measure and a blocking measure to analyze blocking in state feedback control. The former measure is described by the predicate indicating states such that all admissible trajectories starting from them can be extended to target states, while the latter by the predicate indicating states which may lead to blocking.

In this chapter, we first present an algorithm to compute the minimally restrictive nonblocking solution [41, 12]. But a nonblocking controller may be restrictive because it disables all behaviors which may lead to blocking. In this sense, blocking controllers can be practically more efficient than nonblocking ones if blocking in the closed-loop system is resolved easily by some external intervention such as rollback mechanism. A manufacturing system is a typical example of such a system. Then by similar techniques to [12], we perform the optimization of a given blocking controller in terms of the two performance measures.

7.2 Stability and Stabilizability

In this chapter, we consider the Ramadge-Wonham model. Let G be a finite state automaton $G = (X, \Sigma, \delta, x^0)$ defined by Eq. (2.1), where X is the finite set. We define notions of stability and stabilizability in terms of predicates. A (possibly infinite) sequence of states, $\mathbf{x} = x^0x^1x^2...$, is said to be a (state) trajectory from x^0 in G if there exist $\sigma^0, \sigma^1, ..., \in \Sigma$ such that $x^{j+1} = \delta(\sigma^j, x^j)$ for all $j = 0, 1, 2, ...,$. We say $x \in \mathbf{x}$ if $x_j = x$ for some j. Let $T(G, x)$ denote the set of all trajectories $\mathbf{x}$ from x in G such that $\mathbf{x}$ is infinite or ends with a state where no event can occur. A trajectory $\mathbf{x}'$ is said to be a prefix of $\mathbf{x}$ if there exists a sequence $\mathbf{s}$ of states such that $\mathbf{x} = \mathbf{x}'\mathbf{s}$. Let Q and E be predicates such that $E \leq Q$. A finite trajectory $\mathbf{x}$ is said to be (Q, E)-*attracting* if every state $x \in \mathbf{x}$ satisfies Q and $\mathbf{x}$ ends with a state satisfying E.

Definition 7.1 *Let Q, K and E be predicates such that $E \leq K \leq Q$. Then a subpredicate Q' of Q is (K, E)-prestable if, for all $x \in X$ with $Q'(x) = 1$, every trajectory $\mathbf{x} \in T(G, x)$ has a (K, E)-attracting prefix. Moreover Q' is (K, E)-stable if, for all $x \in X$ with $Q'(x) = 1$, every trajectory $\mathbf{x} \in T(G, x)$ satisfies either of the following two conditions.*

- $\mathbf{x}$ *is infinite and resides in K and visits states satisfying E infinitely often.*
- $\mathbf{x}$ *is finite and (K, E)-attracting.*

If $K = Q$ then we simply say that Q' is E-prestable or E-stable.

The above definitions are equivalent to the ones defined by Özveren et al.[64, 63, 62] except that our definitions require that every trajectory consists of states satisfying K.

Remark 7.1 Obviously, if a subpredicate Q' of Q is (K, E)-prestable ((K, E)-stable), then Q' is also a subpredicate of K.

Definition 7.2 *Let Q, K and E be predicates such that $E \leq K \leq Q$. Then a subpredicate Q' of Q is (K,E)-prestabilizable (respectively, (K,E)-stabilizable) if there exists a state feedback controller f such that Q' is (K,E)-prestable (respectively, (K,E)-stable) in the closed-loop system $G \mid f$. Such a controller is said to be a (K,E)-prestabilizing (respectively, (K,E)-stabilizing) controller of Q'. If $K = Q$ then we simply say that Q' is E-prestabilizable or E-stabilizable (f is E-prestabilizing or E-stabilizing controller).*

Remark 7.2 If K is control-invariant and a subpredicate Q' of Q is (K,E)-prestabilizable (respectively, (K,E)-stabilizable), then there exists a (K,E)-prestabilizing (respectively, (K,E)-stabilizing) controller of Q' which is also a permissive controller of K.

It can be shown in the same way as Corollary 3.3 (respectively, Corollary 3.17) in [64] that the supremal (K,E)-prestabilizable (respectively, (K,E)-stabilizable) subpredicate of Q always exists. Let $P(K,E)$ (respectively, $S(K,E)$) denote the supremal (K,E)-prestabilizable (respectively, (K,E)-stabilizable) subpredicate of Q.

Proposition 7.1 *Let Q, K, K' and E be predicates with $E \leq K' \leq K \leq Q$. Then if $P(K,E) \leq K'$ then the following equation holds.*

$$P(K,E) = P(K',E). \tag{7.1}$$

Proof: Since $K' \leq K$, it follows that $P(K',E) \leq P(K,E)$. We shall prove the reverse inequality. Let f be a (K,E)-prestabilizing controller of $P(K,E)$. It is sufficient to prove that, for any $x \in X$ with $P(K,E)(x) = 1$, every trajectory $\mathbf{x} \in T(G \mid f, x)$ has a (K',E)-attracting prefix. Consider $x \in X$ with $P(K,E)(x) = 1$. Then every trajectory $\mathbf{x} \in T(G \mid f, x)$ has a (K,E)-attracting prefix. Let $\mathbf{x}'$ be the shortest (K,E)-attracting prefix of $\mathbf{x}$. Suppose that there exists $x' \in \mathbf{x}'$ with $P(K,E)(x') = 0$. Then every trajectory $\mathbf{x}'' \in T(G \mid f, x')$ has a (K,E)-attracting prefix. We define the predicate $P'(K,E) \in \mathbf{Q}$ as follows:

$$P'(K,E)(x) = \begin{cases} 1 & \text{if } P(K,E)(x) = 1 \text{ or } x = x', \\ 0 & \text{otherwise.} \end{cases}$$

Then it is obvious that $P'(K,E) \leq Q$ and f is also a (K,E)-prestabilizing controller of $P'(K,E)$. This contradicts the fact that $P(K,E)$ is the supremal (K,E)-prestabilizable subpredicate of Q. Therefore, every trajectory $\mathbf{x} \in T(G \mid f, x)$ has a $(P(K,E),E)$-attracting prefix. Since $P(K,E) \leq K'$, $\mathbf{x}$ has a (K',E)-attracting prefix. Therefore

f is also a (K', E)-prestabilizing controller of $P(K, E)$, which implies that $P(K, E) \leq P(K', E)$. □

7.3 Blocking and E-stability

Let Q and E with $E \leq Q$ be predicates representing the set of all "admissible" and "target " states, respectively. Desirable behaviors of a system are represented by trajectories of admissible states which visit target states in a finite number of transitions. We define blocking in the context of state feedback control using the notion of E-stability.

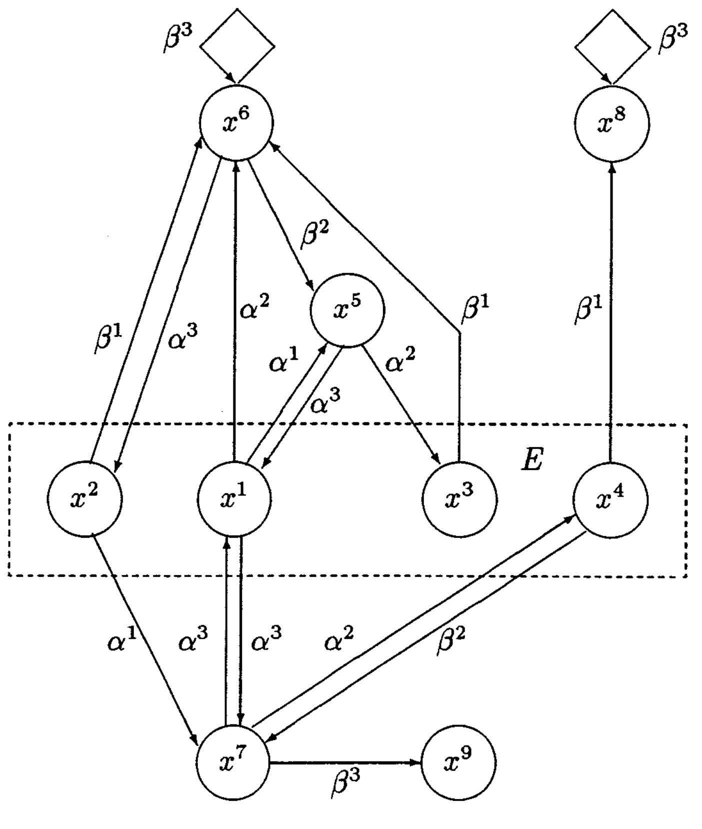

Figure 7.1: The DES of Example 7.1.

Example 7.1 We consider the DES shown in Figure 7.1, where it is assumed that $\Sigma_c =$

$\{\alpha^1, \alpha^2, \alpha^3\}$ and $\Sigma_u = \{\beta^1, \beta^2, \beta^3\}$. Let Q and E be predicates given by

$$Q(x) = 1 \quad \forall x \in X,$$

and

$$E(x) = \begin{cases} 1 & \text{if } x \in \{x_1, x_2, x_3, x_4\}, \\ 0 & \text{otherwise,} \end{cases}$$

respectively.

There are two types of blocking in state feedback control.

- There is a state such that no state trajectory from it can reach target states.
- There is a state such that some (infinite) state trajectory from it cannot reach target states. Note that such a state may transit to a target state by an adequate firing sequence of events.

Supervisory control based upon formal languages can treat only the former case [54]. Such blocking occurs at x_8 and x_9. On the other hand, the latter blocking occurs at x_6. In fact, state trajectories may drop into a infinite loop at x_6 and they never reach target states. However, this case is not regarded as blocking in supervisory control because it treats only finite sequences of events.

Definition 7.3 *Let Q' be a predicate with $Q' \le Q$. Then the system G is nonblocking for Q' if Q' is E-stable in G, otherwise blocking.*

Definition 7.4 *Let Q' be a control-invariant predicate with $Q' \le Q$. Then a permissive controller f of Q' is nonblocking for Q' if Q' is E-stable in the closed-loop system $G \mid f$, otherwise blocking.*

We shall show several properties of $P(Q, E)$ and $S(Q, E)$.

Proposition 7.2 *The following algorithm computes $P(Q, E)$.*
Algorithm 7.1: *Let $X_0 := E$ and iterate:*

$$\tilde{X}_{k+1} := Q \wedge \sim X_k \wedge \left(\bigwedge_{\sigma \in \Sigma_u} wlp_\sigma(X_k) \right) \wedge \left(\bigvee_{\sigma \in \Sigma} wp_\sigma(X_k) \right).$$

$$X_{k+1} := X_k \vee \tilde{X}_{k+1}.$$

Terminate when $X_{k+1} = X_k$. *Then* $P(Q,E) := X_k$.

If Algorithm 7.1 terminates when $X_{i+1} = X_i$, *then an* E*-prestabilizing controller* f *of* $P(Q,E)$ *is given by*

$$f(x) = \begin{cases} \Sigma_u \cup \{\sigma \in \Sigma_c; wlp_\sigma(X_{k-1})(x) = 1\} & \text{if } \tilde{X}_k(x) = 1 \ (k = 1, 2, ..., i), \\ \Sigma & \text{otherwise.} \end{cases} \tag{7.2}$$

Proof: Since X is the finite set and $\{X_k\}$ is a monotonically increasing sequence with $X_k \leq Q$, Algorithm 7.1 terminates in a finite number of steps. Suppose that Algorithm 7.1 terminates when $X_{i+1} = X_i$. First, we shall prove by induction that f defined by Eq. (7.2) is an E-prestabilizing controller of X_i, which implies that X_i is an E-prestabilizable subpredicate of Q. Clearly, X_0 is E-prestable in the closed-loop system $G \mid f$. Suppose that X_k is E-prestable in $G \mid f$. That is, for all $x \in X$ with $X_k(x) = 1$, every trajectory $\mathbf{x} \in T(G \mid f, x)$ has a (Q,E)-attracting prefix. Let $x^* \in X$ be a state such that $\tilde{X}_{k+1}(x^*) = 1$. Then

$$\bigwedge_{\sigma \in \Sigma_u} wlp_\sigma(X_k)(x^*) = 1,$$

which implies together with Eq. (7.2) that, for any $\sigma \in \Sigma$, $X_k(\delta_{cf}(\sigma, x^*)) = 1$ if $\delta_{cf}(\sigma, x^*)!$. Also since

$$\bigvee_{\sigma \in \Sigma} wp_\sigma(X_k)(x^*) = 1,$$

there exists at least one $\sigma \in \Sigma$ such that $\delta_{cf}(\sigma, x^*)!$. So every trajectory $\mathbf{x}^* \in T(G \mid f, x^*)$ is described by $\mathbf{x}^* = x^*\mathbf{x}$, where $\mathbf{x} \in T(G \mid f, x)$ for some $x \in X$ with $X_k(x) = 1$. Since $Q(x^*) = 1$, $\mathbf{x}^*$ has a (Q,E)-attracting prefix. Then X_{k+1} is E-prestable in $G \mid f$. This completes the induction. Thus, f is an E-prestabilizing controller of X_i and X_i is an E-prestabilizable subpredicate of Q. This implies that $X_i \leq P(Q,E)$.

Next, we shall prove that $P(Q,E) \leq X_i$ by contradiction. Suppose that there exists $x \in X$ such that $P(Q,E)(x) = 1$ and $X_i(x) = 0$. For all $x' \in X$ such that $X_i(x') = 0$ and $Q(x') = 1$, there exists $\sigma \in \Sigma_u$ such that $\delta(\sigma, x')!$ and $X_i(\delta(\sigma, x')) = 0$, otherwise there is no event $\sigma \in \Sigma$ such that $X_i(\delta(\sigma, x')) = 1$. Since X is the finite set, for any state feedback controller g, there exists at least one trajectory $\mathbf{x} \in T(G \mid g, x)$ which does not have a (Q, X_i)-attracting prefix. Since $E \leq X_i$, such a trajectory does not have a (Q,E)-attracting prefix, which contradicts that $P(Q,E)(x) = 1$.

Consequently, we have $P(Q,E) = X_i$ and f is an E-prestabilizing controller of $P(Q,E)$. □

Note that Algorithm 7.1 has complexity $O(mn^2)$.

Corollary 7.1 *For any $\sigma \in \Sigma_u$, the following equation holds.*

$$P(Q,E) \wedge \sim E \leq wlp_\sigma(P(Q,E)). \tag{7.3}$$

Proof: Let $x \in X$ be a state such that $(P(Q,E) \wedge \sim E)(x) = 1$. From Algorithm 7.1, we have $\tilde{X}_k(x) = 1$ for some k, which implies that

$$\bigwedge_{\sigma \in \Sigma_u} wlp_\sigma(X_{k-1})(x) = 1.$$

Clearly, $X_{k-1} \leq P(Q,E)$. Thus we have

$$\bigwedge_{\sigma \in \Sigma_u} wlp_\sigma(P(Q,E))(x) = 1.$$

□

Proposition 7.3 *$S(Q,E)$ is control-invariant, and any E-stabilizing controller of $S(Q,E)$ is also a permissive controller of $S(Q,E)$.*

Proof: Obviously, it is sufficient to prove that any E-stabilizing controller f of $S(Q,E)$ satisfies

$$S(Q,E) \leq wlp_\sigma(S(Q,E)) \vee \sim f_\sigma \quad \forall \sigma \in \Sigma. \tag{7.4}$$

Suppose that there exist $x \in X$ and $\sigma \in \Sigma$ such that

$$S(Q,E)(x) = 1 \tag{7.5}$$

and

$$(wlp_\sigma(S(Q,E)) \vee \sim f_\sigma)(x) = 0. \tag{7.6}$$

Eq. (7.6) shows that $\delta_{cf}(\sigma,x)!$ in $G \mid f$. Letting $x' = \delta_{cf}(\sigma,x)$, $S(Q,E)(x') = 0$ and $Q(x') = 1$. On the other hand, every trajectory $\mathbf{x} \in T(G \mid f, x')$ resides in Q and visits states satisfying E infinitely often, otherwise $\mathbf{x}$ is (Q,E)-attracting. We define the predicate $S'(Q,E) \in \mathbf{Q}$ as follows:

$$S'(Q,E)(x) = \begin{cases} 1 & \text{if } S(Q,E)(x) = 1 \text{ or } x = x', \\ 0 & \text{otherwise.} \end{cases}$$

Then it is obvious that $S'(Q,E) \leq Q$ and f is also an E-stabilizing controller of $S'(Q,E)$. This contradicts the fact that $S(Q,E)$ is the supremal E-stabilizable subpredicate of Q. Therefore, Eq. (7.4) holds, which implies that $S(Q,E)$ is control-invariant and f is a permissive controller of $S(Q,E)$. □

From Proposition 7.3, we can prove the following corollary.

Corollary 7.2 *Any E-stabilizing controller f of $S(Q,E)$ is nonblocking for $S(Q,E)$.*

Remark 7.3 In general, an E-prestabilizable subpredicate of Q is not control-invariant.

Example 7.2 We consider the same system as Example 7.1. We shall show that $P(Q,E)$ is not control-invariant. We apply Algorithm 7.1 to compute $P(Q,E)$. Then

$$P(Q,E)(x) = \begin{cases} 1 & \text{if } x \in \{x^1, x^2, x^3, x^4, x^5\}, \\ 0 & \text{otherwise.} \end{cases}$$

It is easily shown that for $x^2 \in X$ and $\beta^1 \in \Sigma_u$,

$$P(Q,E)(x^2) = 1 \text{ and } wlp_{\beta^1}(P(Q,E))(x^2) = 0,$$

which implies that $P(Q,E)$ is not control-invariant.

7.4 Logical Performance Measures

Let K be a control-invariant subpredicate of Q. In this section, we introduce the following two performance measures, a *prestabilizing measure* (PM) and a *blocking measure* (BM) to evaluate logical performances of prestability and blocking in state feedback control.

$$\begin{aligned} PM(K) &:= P(K, E \wedge K), \\ BM(K) &:= K \wedge \sim P(K, E \wedge K). \end{aligned}$$

There exists a permissive controller f of K such that, for all $x \in X$ with $PM(K)(x) = 1$, every trajectory $\mathbf{x} \in T(G \mid f, x)$ visits a state satisfying $E \wedge K$ in a finite number of transitions. Note that $PM(K)$ is $(K, E \wedge K)$-prestabilizable, not $(K, E \wedge K)$-stabilizable. On the other hand, for any permissive controller f of K, each state $x \in X$ with $BM(K)(x) = 1$ may lead to blocking in $G \mid f$. Note that BM indicates states which have the possibility of blocking, but a state $x \in X$ with $BM(K)(x) = 1$ may transit to a target state by an adequate firing sequence of events. So it is desirable that we select a control-invariant subpredicate K of Q such that $PM(K)$ is as large as possible and $BM(K)$ is as small as possible, which is a trade-off.

Remark 7.4 Chen and Lafortune have proposed two performance measures called a satisficing measure (SM) and a blocking measure (BM) in the context of supervisory control [12]. In the closed-loop system, SM indicates admissible marked traces, and BM indicates traces which lead to blocking. Note that the meaning of BM in state feedback control is different from that in supervisory control as shown in Example 7.1.

Lemma 7.1 *Assume that $K \in \mathbf{Q}$ is a control-invariant subpredicate of Q. Then the following equation holds for any control-invariant predicate $K' \in \mathbf{Q}$ with $PM(K) \leq K' \leq K$.*

$$PM(K') = PM(K). \tag{7.7}$$

Proof: We first show that

$$E \wedge K' = E \wedge K. \tag{7.8}$$

Since $PM(K) = P(K, E \wedge K) \leq K'$, we have $E \wedge K \leq K'$, that is, $E \wedge K \leq E \wedge K'$. Conversely, we have $E \wedge K' \leq E \wedge K$ since $K' \leq K$. Thus, Eq. (7.8) holds.

By Eq. (7.8) and Proposition 7.1, we have

$$\begin{aligned} P(K, E \wedge K) &= P(K, E \wedge K') \\ &= P(K', E \wedge K'), \end{aligned}$$

which implies that $PM(K') = PM(K)$. □

By the above lemma, we immediately show the following corollary.

Corollary 7.3 *Assume that $K \in \mathbf{Q}$ is a control-invariant subpredicate of Q. Then the following equation holds.*

$$PM(PM(K)^{\downarrow}) = PM(K). \tag{7.9}$$

Lemma 7.2 *Assume that $K \in \mathbf{Q}$ is a subpredicate of Q such that $K = P(K, E \wedge K)^{\uparrow}$. Then K is an E-stabilizable subpredicate of Q.*

Proof: Since $K = P(K, E \wedge K)^{\uparrow}$, we have $K \leq P(K, E \wedge K)$. Conversely, it is obvious that $P(K, E \wedge K) \leq K$. So we have $K = P(K, E \wedge K)$, which implies that K is the supremal $(K, E \wedge K)$-prestabilizable subpredicate of Q. Since K is control-invariant and $E \wedge K \leq K$, the following equation holds for any $\sigma \in \Sigma_u$.

$$E \wedge K \leq wlp_{\sigma}(K).$$

By using a $(K, E \wedge K)$-prestabilizing controller f of K constructed by Eq. (7.2), we will define a state feedback controller g as follows:

$$g(x) = \begin{cases} \Sigma_u \cup \{\sigma \in \Sigma_c; wlp_\sigma(K)(x) = 1\} & \text{if } (E \wedge K)(x) = 1, \\ f(x) & \text{otherwise.} \end{cases}$$

For all $x \in X$ with $K(x) = 1$, every trajectory $\mathbf{x} \in T(G \mid g, x)$ has a $(K, E \wedge K)$-attracting prefix $\mathbf{x}'$. For any $x \in X$ with $(E \wedge K)(x) = 1$ and any $\sigma \in \Sigma$, $K(\delta_{cg}(\sigma, x)) = 1$ if $\delta_{cg}(\sigma, x)!$. Therefore, it is obvious that K is a $(K, E \wedge K)$-stabilizable subpredicate of Q. Since $K \leq Q$, K is also an E-stabilizable subpredicate of Q. □

By the above lemma, we immediately show the following corollary.

Corollary 7.4 *Assume that $K \in \mathbf{Q}$ is a control-invariant subpredicate of Q such that $BM(K) = \mathbf{0}$. Then there exists a nonblocking controller for K.*

Proposition 7.4

$$BM(S(Q, E)) = \mathbf{0}, \text{ equivalently, } S(Q, E) = P(S(Q, E), E \wedge S(Q, E)), \tag{7.10}$$

holds. Also for any control-invariant subpredicate K of Q such that $BM(K) = \mathbf{0}$,

$$K \leq S(Q, E). \tag{7.11}$$

Proof: First, we shall prove Eq. (7.10). From Proposition 7.3, every E-stabilizing controller f of $S(Q, E)$ is also a permissive controller of $S(Q, E)$. So, for any $x \in X$ with $S(Q, E)(x) = 1$, every trajectory $\mathbf{x} \in T(G \mid f, x)$ resides in $S(Q, E)$ and visits states satisfying $E \wedge S(Q, E)$ infinitely often, otherwise $\mathbf{x}$ is $(S(Q, E), E \wedge S(Q, E))$-attracting. Clearly, $S(Q, E)$ is the supremal $(S(Q, E), E \wedge S(Q, E))$-prestabilizable subpredicate of Q. Therefore, Eq. (7.10) holds.

Let $K \in \mathbf{Q}$ be a control-invariant subpredicate of Q such that $BM(K) = \mathbf{0}$. Since $BM(K) = \mathbf{0}$, we have $K = P(K, E \wedge K)$. Also control-invariance of K implies that $K = P(K, E \wedge K)^\dagger$. Thus, K is an E-stabilizable subpredicate of Q by Lemma 7.2. Therefore, $K \leq S(Q, E)$. □

From Proposition 7.4, $S(Q, E)$ is the largest control-invariant subpredicate of Q such that $BM(\cdot) = \mathbf{0}$. $S(Q, E)$ is said to be the *minimally restrictive nonblocking solution* (MRNBS) [41, 12].

On the other hand, if a control-invariant subpredicate K of Q satisfies that $PM(K) = P(Q,E)$, we call K the *completely prestabilizing solution* (CPS), which is similar to the completely satisficing solution considered in [41] and [12]. But the CPS does not always exist. We show a necessary and sufficient condition for the existence of the CPS.

Proposition 7.5 *There exists the CPS if and only if* $P(Q,E)^{\downarrow} \leq Q^{\uparrow}$.
Proof: ($\Leftarrow$) Since $P(Q,E)^{\downarrow} \leq Q^{\uparrow} \leq Q$, we have

$$\begin{aligned} PM(P(Q,E)^{\downarrow}) &= P(P(Q,E)^{\downarrow}, E \wedge P(Q,E)^{\downarrow}) \\ &= P(P(Q,E)^{\downarrow}, E) \\ &\leq P(Q,E). \end{aligned}$$

Conversely, since $P(Q,E) \leq P(Q,E)^{\downarrow}$, we have

$$\begin{aligned} PM(P(Q,E)^{\downarrow}) &= P(P(Q,E)^{\downarrow}, E) \\ &\geq P(P(Q,E), E) \\ &= P(Q,E). \end{aligned}$$

Therefore, $PM(P(Q,E)^{\downarrow}) = P(Q,E)$. Since $P(Q,E)^{\downarrow}$ is control-invariant subpredicate of Q, $P(Q,E)^{\downarrow}$ is the CPS.

($\Rightarrow$) Let a predicate K be the CPS. Since $PM(K) = P(Q,E)$, $P(Q,E) \leq K$. By control-invariance of K, we have

$$\begin{aligned} P(Q,E) &\leq P(Q,E)^{\downarrow} \\ &\leq K. \end{aligned}$$

Moreover, $K \leq Q^{\uparrow}$ since $K \leq Q$. Thus, we have $P(Q,E)^{\downarrow} \leq Q^{\uparrow}$. □

If $P(Q,E)^{\downarrow} \leq Q^{\uparrow}$, then it is obvious that $P(Q,E)^{\downarrow}$ is the CPS. $P(Q,E)^{\downarrow}$ can be computed by using Proposition 8.2 in [68] and Algorithm 7.1.

7.5 Nonblocking State Feedback Control

In the last section, we showed that $S(Q,E)$ is the MRNBS. This section presents an algorithm to compute $S(Q,E)$.

Theorem 7.1 *The following algorithm computes* $S(Q,E)$.

Algorithm 7.2: *Let* $X_0 := Q$ *and iterate:*

$$X_{k+1} := P(X_k, E \wedge X_k)^{\dagger}.$$

Terminate when $X_{k+1} = X_k$. *Then* $S(Q,E) := X_k$.

Proof: Since $P(X_k, E \wedge X_k)^{\dagger} \leq P(X_k, E \wedge X_k) \leq X_k$, $\{X_k\}$ is a monotonically decreasing sequence and X is finite, which implies that Algorithm 7.2 terminates in a finite number of steps. Suppose that Algorithm 7.2 terminates when $X_i = P(X_i, E \wedge X_i)^{\dagger}$. From Lemma 7.2, X_i is an E-stabilizable subpredicate of Q. Thus, we have $X_i \leq S(Q,E)$.

Next, we shall prove that $S(Q,E) \leq X_i$ by induction on the number of steps in Algorithm 7.2. Obviously, $S(Q,E) \leq X_0 = Q$. Suppose that $S(Q,E) \leq X_k$. Then from Propositions 7.3 and 7.4, it follows that

$$\begin{aligned} S(Q,E) &= S(Q,E)^{\dagger} \\ &= P(S(Q,E), E \wedge S(Q,E))^{\dagger}, \end{aligned}$$

which implies together with $S(Q,E) \leq X_k$ that

$$\begin{aligned} S(Q,E) &\leq P(X_k, E \wedge X_k)^{\dagger} \\ &= X_{k+1}. \end{aligned}$$

This completes the induction. Therefore, we have $S(Q,E) \leq X_i$. □

Note that Algorithm 7.2 has complexity $O(mn^3)$.

Remark 7.5 If $Q = \mathbf{1}$, then Algorithms 7.1 and 7.2 are reduced to algorithms for computing the supremal E-prestabilizable and E-stabilizable predicates proposed in [64], respectively.

Let f be a $(S(Q,E), E \wedge S(Q,E))$-prestabilizing controller of $S(Q,E)$ constructed by Eq. (7.2). Thus an E-stabilizing controller g of $S(Q,E)$ is given by

$$g(x) = \begin{cases} \Sigma_u \cup \{\sigma \in \Sigma_c; wlp_\sigma(S(Q,E))(x) = 1\} & \text{if } (E \wedge S(Q,E))(x) = 1, \\ f(x) & \text{otherwise.} \end{cases}$$

Example 7.3 We consider the same system as Example 7.1. We apply Algorithm 7.2 to compute $S(Q,E)$. Then

$$S(Q,E)(x) = \begin{cases} 1 & \text{if } x \in \{x^1, x^5\}, \\ 0 & \text{otherwise.} \end{cases}$$

Then an E-stabilizing controller f of $S(Q,E)$ is given by

$$f(x) = \begin{cases} \Sigma_u \cup \{\alpha^1\} & \text{if } x = x^1, \\ \Sigma_u \cup \{\alpha^3\} & \text{if } x = x^5, \\ \Sigma & \text{otherwise.} \end{cases}$$

7.6 Improvement of Logical Performances

In **7.4**, we defined two extreme solutions called the MRNBS and the CPS. The MRNBS is optimal with respect to BM, but the behaviors of the corresponding closed-loop system may be very restrictive. On the other hand, the CPS is optimal with respect to PM, but blocking may occur very often. Thus, both the MRNBS and the CPS may not be adequate solutions from the practical point of view.

It is obvious that, for any control-invariant subpredicate K of Q, $PM(K) \le PM(Q^\uparrow)$. But, in general, $BM(Q^\uparrow) \ne \mathbf{0}$. Let $\tilde{K} = PM(Q^\uparrow)^\downarrow = P(Q^\uparrow, E \wedge Q^\uparrow)^\downarrow$. From Corollary 7.3, we have $PM(\tilde{K}) = PM(Q^\uparrow)$. Since $\tilde{K}$ is the smallest control-invariant superpredicate of $PM(Q^\uparrow)$, we have $BM(\tilde{K}) \le BM(K)$ for any control-invariant predicate K with $PM(K) = PM(Q^\uparrow)$. Therefore, we consider the set of admissible solutions [12] as follows:

$$Q_{AS} = \{K \in \mathbf{Q}; S(Q,E) \le K \le P(Q^\uparrow, E \wedge Q^\uparrow)^\downarrow \text{ and } K \text{ is control-invariant}\}. \quad (7.12)$$

If an arbitrary admissible solution $K \in Q_{AS}$ is given, we may be able to improve PM and/or BM. In the following subsections, we first present the operation to reduce the blocking measure (respectively, enlarge the prestabilizing measure) without degrading the prestabilizing measure (respectively, the blocking measure). Using the operations, we then perform the optimization of a given blocking controller in terms of the two performance measures by similar techniques to [12].

7.6.1 Improving BM

Let $K_{IS} \in Q_{AS}$ be the "initial" solution in designing a state feedback controller. In this subsection, we show a design method to improve $BM(K_{IS})$ without changing $PM(K_{IS})$, that is, find $K_{BM} \in Q_{AS}$ such that $BM(K_{BM})$ is the smallest subject to $PM(K_{BM}) = PM(K_{IS})$.

Proposition 7.6 $PM(K_{IS})^{\downarrow} \in Q_{AS}$.

Proof: Since $K_{IS} \in Q_{AS}$, $S(Q,E) \leq K_{IS}$. So we have by Proposition 7.4 that

$$\begin{aligned} S(Q,E) &= P(S(Q,E), E \wedge S(Q,E)) \\ &\leq P(K_{IS}, E \wedge K_{IS}) \\ &\leq P(K_{IS}, E \wedge K_{IS})^{\downarrow} \\ &= PM(K_{IS})^{\downarrow}. \end{aligned}$$

Also $P(K_{IS}, E \wedge K_{IS})^{\downarrow} \leq K_{IS} \leq P(Q^{\uparrow}, E \wedge Q^{\uparrow})^{\downarrow}$ since $K_{IS} \in Q_{AS}$. Therefore, we have

$$\begin{aligned} PM(K_{IS})^{\downarrow} &= P(K_{IS}, E \wedge K_{IS})^{\downarrow} \\ &\leq P(Q^{\uparrow}, E \wedge Q^{\uparrow})^{\downarrow}. \end{aligned}$$

Since $PM(K_{IS})^{\downarrow}$ is control-invariant, we have $PM(K_{IS})^{\downarrow} \in Q_{AS}$. □

We can prove the following theorem which presents a method for improving $BM(K_{IS})$.

Theorem 7.2 *If $PM(K_{IS})^{\downarrow} < K_{IS}$, then the following equations hold.*

$$BM(PM(K_{IS})^{\downarrow}) < BM(K_{IS}), \tag{7.13}$$

$$PM(PM(K_{IS})^{\downarrow}) = PM(K_{IS}). \tag{7.14}$$

Proof: Eq. (7.14) is shown by Corollary 7.3.

We shall prove Eq. (7.13). Let $K_{BM} = PM(K_{IS})^{\downarrow}$. By Eq. (7.14) and the assumption, we have

$$\begin{aligned} BM(K_{BM}) &= K_{BM} \wedge \sim P(K_{BM}, E \wedge K_{BM}) \\ &= K_{BM} \wedge \sim P(K_{IS}, E \wedge K_{IS}) \\ &\leq K_{IS} \wedge \sim P(K_{IS}, E \wedge K_{IS}) \\ &= BM(K_{IS}). \end{aligned}$$

Moreover, there exists at least one $x \in X$ such that $K_{BM}(x) = 0$ and $K_{IS}(x) = 1$. Then it is obvious that $P(K_{IS}, E \wedge K_{IS})(x) = 0$. So

$$\begin{aligned} BM(K_{BM})(x) &= (K_{BM} \wedge \sim P(K_{IS}, E \wedge K_{IS}))(x) \\ &= 0, \end{aligned}$$

and

$$BM(K_{IS})(x) = 1,$$

which asserts Eq. (7.13). □

We define the BM improvement operation $A_{BM} : Q_{AS} \to Q_{AS}$ as follows: for each $K \in Q_{AS}$,

$$A_{BM}(K) := PM(K)^{\downarrow}. \tag{7.15}$$

Corollary 7.5

$$A_{BM}(A_{BM}(K_{IS})) = A_{BM}(K_{IS}). \tag{7.16}$$

Proof: Since K_{IS} is a control-invariant subpredicate of Q, we have by Corollary 7.3 that

$$PM(PM(K_{IS})^{\downarrow}) = PM(K_{IS}).$$

Therefore,

$$PM(PM(K_{IS})^{\downarrow})^{\downarrow} = PM(K_{IS})^{\downarrow},$$

which asserts (7.16). □

Corollary 7.5 means that the improvement of BM applying A_{BM} can be done by one step.

Example 7.4 We consider the same system as Example 7.1. Let $K_{IS} \in Q_{AS}$ be given by

$$K_{IS}(x) = \begin{cases} 1 & \text{if } x \in \{x^1, x^3, x^5, x^6, x^7, x^9\}, \\ 0 & \text{otherwise.} \end{cases}$$

Then

$$PM(K_{IS})(x) = \begin{cases} 1 & \text{if } x \in \{x^1, x^3, x^5\}, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$BM(K_{IS})(x) = \begin{cases} 1 & \text{if } x \in \{x^6, x^7, x^9\}, \\ 0 & \text{otherwise.} \end{cases}$$

$A_{BM}(K_{IS})$ is computed as follows:

$$A_{BM}(K_{IS})(x) = \begin{cases} 1 & \text{if } x \in \{x^1, x^3, x^5, x^6\}, \\ 0 & \text{otherwise.} \end{cases}$$

Since

$$PM(A_{BM}(K_{IS}))(x) = \begin{cases} 1 & \text{if } x \in \{x^1, x^3, x^5\}, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$BM(A_{BM}(K_{IS}))(x) = \begin{cases} 1 & \text{if } x \in \{x^6\}, \\ 0 & \text{otherwise,} \end{cases}$$

we have $BM(A_{BM}(K_{IS})) < BM(K_{IS})$ and $PM(A_{BM}(K_{IS})) = PM(K_{IS})$, which shows that the blocking measure is improved by applying the operator A_{BM} without changing the prestabilizing measure. Also it is easily shown that $A_{BM}(A_{BM}(K_{IS})) = A_{BM}(K_{IS})$.

7.6.2 Improving PM

Let $K_{IS} \in Q_{AS}$. In this subsection, a design method to improve $PM(K_{IS})$ without degrading $BM(K_{IS})$ is shown, that is, find K_{PM} such that $PM(K_{PM})$ is the largest subject to $BM(K_{PM}) \leq BM(K_{IS})$.

We now introduce an algorithm for improving $PM(K_{IS})$ for $K_{IS} \in Q_{AS}$.

Algorithm 7.3: Let $K_0 := K_{IS}$ and iterate:

$$E_k^* := E \wedge (E \vee K_k)^\dagger,$$

$$K_{k+1} := P(Q, E_k^*) \vee K_k.$$

Terminate when $K_{k+1} = K_k$. Then $K_{PM} := K_k$.

Note that the computation of $P(Q, E_k^*)$ in the above algorithm can be done by applying Algorithm 7.1. Since $\{K_k\}$ is a monotonically increasing sequence with $K_k \leq Q$, Algorithm 7.3 always terminates in a finite number of steps, and has complexity $O(mn^3)$.

Lemma 7.3 *In Algorithm 7.3, K_k is control-invariant and $K_k \leq Q^\dagger$ for all k.*

Proof: We shall prove the above lemma by induction. Obviously, $K_0 = K_{IS}$ is control-invariant and $K_0 \leq Q^\dagger$. Suppose that K_i is control-invariant and $K_i \leq Q^\dagger$. Since $K_{i+1} = P(Q, E_i^*) \vee K_i$, we have

$$\begin{aligned} K_i &\leq \bigwedge_{\sigma \in \Sigma_u} wlp_\sigma(K_i) \\ &\leq \bigwedge_{\sigma \in \Sigma_u} wlp_\sigma(K_{i+1}). \end{aligned}$$

Also by Corollary 7.1,

$$
\begin{aligned}
P(Q, E_i^*) \wedge \sim E_i^* &\leq \bigwedge_{\sigma \in \Sigma_u} wlp_\sigma(P(Q, E_i^*)) \\
&\leq \bigwedge_{\sigma \in \Sigma_u} wlp_\sigma(K_{i+1}).
\end{aligned}
$$

Let $x \in X$ be a state such that $(P(Q, E_i^*) \wedge E_i^*)(x) = 1$. For any $\sigma \in \Sigma_u$, if $D_\sigma(x) = 0$, then $wlp_\sigma(K_{i+1})(x) = 1$. We consider the case that $D_\sigma(x) = 1$. Let $x' = \delta(\sigma, x)$. Since $E_i^*(x) = (E \wedge (E \vee K_i)^\uparrow)(x) = 1$, control-invariance of $(E \vee K_i)^\uparrow$ implies that $(E \vee K_i)^\uparrow(x') = 1$. If $E(x') = 1$, then $E_i^*(x') = 1$, and $wlp_\sigma(E_i^*)(x) = 1$. Since $E_i^* \leq K_{i+1}$, we have $wlp_\sigma(K_{i+1})(x) = 1$. Moreover, if $E(x') = 0$, then $K_i(x') = 1$, and

$$
wlp_\sigma(K_i)(x) = 1,
$$

which implies that $wlp_\sigma(K_{i+1})(x) = 1$. Therefore, K_{i+1} is control-invariant.

Next, we shall prove that $K_{i+1} \leq Q^\uparrow$. Obviously, $P(Q, E_i^*) \leq Q$. So we have

$$
\begin{aligned}
K_{i+1} &= P(Q, E_i^*) \vee K_i \\
&\leq Q \vee Q^\uparrow \\
&= Q.
\end{aligned}
$$

Since K_{i+1} is control-invariant, we have $K_{i+1} \leq Q^\uparrow$. □

By Lemma 7.3, we immediately have the following corollary.

Corollary 7.6 *K_{PM} is control-invariant and $K_{PM} \leq Q^\uparrow$.*

Proposition 7.7 *$K_{PM} \in Q_{AS}$.*

Proof: K_{PM} is control-invariant by Corollary 7.6. We have $S(Q, E) \leq K_{PM}$ since $K_{IS} \in Q_{AS}$ and $K_{IS} \leq K_{PM}$. We shall prove that $K_{PM} \leq P(Q^\uparrow, E \wedge Q^\uparrow)^\downarrow$ by induction on the number of steps in Algorithm 7.3. We know that $K_0 = K_{IS} \leq P(Q^\uparrow, E \wedge Q^\uparrow)^\downarrow$. Suppose that $K_k \leq P(Q^\uparrow, E \wedge Q^\uparrow)^\downarrow$. Since $K_{k+1} = P(Q, E_k^*) \vee K_k$, it is sufficient to prove that $P(Q, E_k^*) \leq P(Q^\uparrow, E \wedge Q^\uparrow)^\downarrow$. We have

$$
\begin{aligned}
E \wedge K_{k+1} &\geq E \wedge P(Q, E_k^*) \\
&\geq E \wedge E_k^* \\
&= E_k^*.
\end{aligned}
$$

Since $P(Q, E_k^*) \leq K_{k+1} \leq Q$, we have by Proposition 7.1 and Lemma 7.3 that

$$\begin{aligned} P(Q, E_k^*) &= P(K_{k+1}, E_k^*) \\ &\leq P(K_{k+1}, E \wedge K_{k+1}) \\ &\leq P(Q^\uparrow, E \wedge Q^\uparrow) \\ &\leq P(Q^\uparrow, E \wedge Q^\uparrow)^\downarrow. \end{aligned}$$

Therefore, $K_{PM} \in Q_{AS}$. □

Lemma 7.4 $E \wedge K_{PM} = E \wedge (E \vee K_{PM})^\uparrow$.

Proof: Let $E^* = E \wedge (E \vee K_{PM})^\uparrow$. Then, from Algorithm 7.3, we have

$$K_{PM} = P(Q, E^*) \vee K_{PM}. \tag{7.17}$$

First, we shall show that $E^* \geq E \wedge K_{PM}$. We have by Corollary 7.6 that

$$\begin{aligned} E^* &= E \wedge (E \vee K_{PM})^\uparrow \\ &\geq E \wedge K_{PM}^\uparrow \\ &= E \wedge K_{PM}. \end{aligned}$$

Next, we shall show that $E^* \leq E \wedge K_{PM}$. From Eq. (7.17),

$$\begin{aligned} E \wedge K_{PM} &= E \wedge (P(Q, E^*) \vee K_{PM}) \\ &= (E \wedge P(Q, E^*)) \vee (E \wedge K_{PM}) \\ &\geq E \wedge P(Q, E^*) \\ &\geq E \wedge E^* \\ &= E^*. \end{aligned}$$

Thus, $E^* = E \wedge K_{PM}$. □

Lemma 7.5

$$K_{PM} = PM(K_{PM}) \vee K_{IS}. \tag{7.18}$$

$$PM(K_{PM}) = P(Q, E \wedge K_{PM}). \tag{7.19}$$

Proof: We shall first prove Eq. (7.19). From Lemma 7.4 and Eq. (7.17), we have

$$K_{PM} = P(Q, E \wedge K_{PM}) \vee K_{PM}.$$

Hence,

$$\begin{aligned}
PM(K_{PM}) &= P(K_{PM}, E \wedge K_{PM}) \\
&= P(P(Q, E \wedge K_{PM}) \vee K_{PM}, E \wedge K_{PM}) \\
&\geq P(P(Q, E \wedge K_{PM}), E \wedge K_{PM}) \\
&= P(Q, E \wedge K_{PM}).
\end{aligned}$$

Conversely, by Corollary 7.6, we have $K_{PM} \leq Q$, which implies that $P(K_{PM}, E \wedge K_{PM}) \leq P(Q, E \wedge K_{PM})$. Therefore, Eq. (7.19) holds.

Next, we shall prove Eq. (7.18). From Eq. (7.19) and the fact that $K_{IS} \leq K_{PM}$,

$$\begin{aligned}
K_{PM} &= P(Q, E \wedge K_{PM}) \vee K_{PM} \\
&= PM(K_{PM}) \vee K_{PM} \\
&\geq PM(K_{PM}) \vee K_{IS}.
\end{aligned} \tag{7.20}$$

We shall prove the reverse inequality by induction on the number of steps in Algorithm 7.3. Obviously, $K_0 = K_{IS} \leq PM(K_{PM}) \vee K_{IS}$. Suppose that $K_i \leq PM(K_{PM}) \vee K_{IS}$. Let $E^* = E \wedge (E \vee K_{PM})^{\dagger}$. Clearly, $E_i^* \leq E^*$. So we have by Eq. (7.19) and Lemma 7.4 that

$$\begin{aligned}
K_{i+1} &= P(Q, E_i^*) \vee K_i \\
&\leq P(Q, E^*) \vee K_i \\
&= PM(K_{PM}) \vee K_i \\
&\leq PM(K_{PM}) \vee K_{IS}.
\end{aligned}$$

This completes the induction. Therefore, we have $K_{PM} \leq PM(K_{PM}) \vee K_{IS}$, which implies together with Eq. (7.20) that Eq. (7.18) holds. □

Using Lemma 7.5, we can prove the following theorem which presents a method for improving $PM(K_{IS})$.

Theorem 7.3 *Let K_{PM} be the predicate computed by Algorithm 7.3 with the initial predicate K_{IS}. Then if $K_{IS} < K_{PM}$ for K_{PM}, then the following equations hold.*

$$PM(K_{IS}) < PM(K_{PM}), \tag{7.21}$$

$$BM(K_{PM}) \leq BM(K_{IS}). \tag{7.22}$$

Proof: First, we shall prove Eq. (7.21). By Eq. (7.18), we have

$$K_{PM} \wedge \sim K_{IS} = PM(K_{PM}) \wedge \sim K_{IS},$$

which implies together with $PM(K_{IS}) \leq K_{IS} < K_{PM}$ that

$$\begin{aligned} PM(K_{PM}) \wedge \sim PM(K_{IS}) &\geq PM(K_{PM}) \wedge \sim K_{IS} \\ &= K_{PM} \wedge \sim K_{IS} \\ &\neq \mathbf{0}. \end{aligned} \tag{7.23}$$

Also since $K_{IS} < K_{PM}$,

$$PM(K_{IS}) \leq PM(K_{PM}). \tag{7.24}$$

From Eqs. (7.23) and (7.24), Eq. (7.21) holds.

Next, we shall prove Eq. (7.22). From Eq. (7.18), it follows that

$$\begin{aligned} BM(K_{PM}) &= K_{PM} \wedge \sim PM(K_{PM}) \\ &= K_{IS} \wedge \sim PM(K_{PM}). \end{aligned}$$

Therefore, since $PM(K_{IS}) < PM(K_{PM})$, we have

$$\begin{aligned} BM(K_{PM}) &\leq K_{IS} \wedge \sim PM(K_{IS}) \\ &= BM(K_{IS}). \end{aligned}$$

□

We define the PM improvement operation $A_{PM} : Q_{AS} \to Q_{AS}$ as follows:

$$A_{PM}(K_{IS}) := K_{PM}. \tag{7.25}$$

The following corollary can be easily proved.

Corollary 7.7 $A_{PM}(A_{PM}(K_{IS})) = A_{PM}(K_{IS})$.

Corollary 7.7 shows that the improvement of PM applying A_{PM} can be also done by one step.

Proposition 7.8 *Let K' and $K'' \in Q_{AS}$ be predicates such that $K' \leq K''$. Then the following equation holds.*

$$P(Q,E') \vee K' \leq P(Q,E'') \vee K'', \tag{7.26}$$

where $E' = E \wedge (E \vee K')^{\uparrow}$ and $E'' = E \wedge (E \vee K'')^{\uparrow}$.

Proof: Since $K' \leq K''$, we have $E' \leq E''$, which implies that

$$P(Q,E') \leq P(Q,E'').$$

Therefore, Eq. (7.26) holds. □

By applying the above proposition to each step in Algorithm 7.3, the following corollary can be obtained.

Corollary 7.8 *Let K' and $K'' \in Q_{AS}$ be predicates such that $K' \leq K''$. Then the following equation holds.*

$$A_{PM}(K') \leq A_{PM}(K''). \tag{7.27}$$

Example 7.5 We consider the same system as Example 7.1. Let $K_{IS} \in Q_{AS}$ be the same as Example 7.4. Then $A_{PM}(K_{IS})$ is computed as follows:

$$A_{PM}(K_{IS})(x) = \begin{cases} 1 & \text{if } x \in \{x^1, x^2, x^3, x^5, x^6, x^7, x^9\}, \\ 0 & \text{otherwise.} \end{cases}$$

Since

$$PM(A_{PM}(K_{IS}))(x) = \begin{cases} 1 & \text{if } x \in \{x^1, x^2, x^3, x^5\}, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$BM(A_{PM}(K_{IS}))(x) = \begin{cases} 1 & \text{if } x \in \{x^6, x^7, x^9\}, \\ 0 & \text{otherwise,} \end{cases}$$

we have $PM(A_{PM}(K_{IS})) > PM(K_{IS})$ and $BM(A_{PM}(K_{IS})) = BM(K_{IS})$, which shows that the prestabilizing measure is improved by applying the operator A_{PM} without degrading the blocking measure. Also it is easily shown that $A_{PM}(A_{PM}(K_{IS})) = A_{PM}(K_{IS})$.

7.6.3 Successive Improvements of BM and PM

In the last two subsections, we presented design methods to improve BM and PM applying operators A_{BM} and A_{PM}, respectively. In this subsection, we consider the case that both operators are applied successively to optimize a given initial solution in terms of both BM and PM.

For $K_{IS} \in Q_{AS}$, we shall define four predicates as follows:

$$\begin{aligned} K_a &:= A_{BM}(K_{IS}). \\ K_b &:= A_{PM}(K_a). \\ K_c &:= A_{PM}(K_{IS}). \\ K_d &:= A_{BM}(K_c). \end{aligned}$$

Lemma 7.6 $A_{BM}(K_b) = K_b$.

Proof: First, we shall prove that $K_b \le A_{BM}(K_b)$. Since $K_a = P(K_{IS}, E \wedge K_{IS})^{\downarrow}$, we have by Lemma 7.5 that

$$K_b = P(Q, E \wedge K_b) \vee P(K_{IS}, E \wedge K_{IS})^{\downarrow}. \tag{7.28}$$

Also $E \wedge K_{IS} = E \wedge K_a$ by Eq. (7.8). Moreover, since $E \wedge K_{IS} = E \wedge K_a \le E \wedge K_b$ and $K_{IS} \le Q$, it follows from Eq. (7.28) that

$$K_b \le P(Q, E \wedge K_b)^{\downarrow}.$$

Thus, we have by Eq. (7.19) that

$$K_b \le P(K_b, E \wedge K_b)^{\downarrow}$$

On the other hand, note that $P(K_b, E \wedge K_b) \le K_b$. Then by control-invariance of K_b, we have

$$P(K_b, E \wedge K_b)^{\downarrow} \le K_b.$$

Therefore, we have $K_b = P(K_b, E \wedge K_b)^{\downarrow} = A_{BM}(K_b)$. □

Lemma 7.7 $A_{PM}(K_d) = K_d$.

Proof: Let $K_0 := K_d$ and we apply Algorithm 7.3. Then, since $K_d \le E \vee K_d$ and K_d is control-invariant,

$$\begin{aligned} E_1^* &= E \wedge (E \vee K_d)^{\uparrow} \\ &\geq E \wedge K_d. \end{aligned}$$

On the other hand, by the fact that $K_d \leq K_c$ and Lemma 7.4,

$$\begin{aligned} E_1^* &= E \wedge (E \vee K_d)^{\uparrow} \\ &\leq E \wedge (E \vee K_c)^{\uparrow} \\ &= E \wedge K_c. \end{aligned}$$

Note that $PM(K_c) \leq K_d \leq K_c$. Then we have by Eq. (7.8) that $E \wedge K_d = E \wedge K_c$. Therefore, $E_1^* = E \wedge K_c$. We have by Eq. (7.19) that

$$\begin{aligned} K_1 &= P(Q, E_1^*) \vee K_d \\ &= P(Q, E \wedge K_c) \vee K_d \\ &= PM(K_c) \vee K_d \\ &\leq PM(K_c)^{\downarrow} \vee K_d \\ &= K_d. \end{aligned}$$

Conversely, it is obvious that $K_d \leq K_1$. Therefore, we have $K_d = K_1 = K_0$, which implies that $A_{PM}(K_d) = K_d$. □

Lemma 7.8 $K_b \leq K_d$.

Proof: From Lemma 7.5,

$$K_b = P(K_b, E \wedge K_b) \vee P(K_{IS}, E \wedge K_{IS})^{\downarrow}.$$

On the other hand, we know that $K_d = P(K_c, E \wedge K_c)^{\downarrow}$. Since $K_{IS} \leq A_{PM}(K_{IS}) = K_c$, we have

$$\begin{aligned} P(K_{IS}, E \wedge K_{IS})^{\downarrow} &\leq P(K_c, E \wedge K_c)^{\downarrow} \\ &= K_d. \end{aligned}$$

It remains to show that $P(K_b, E \wedge K_b) \leq P(K_c, E \wedge K_c)^{\downarrow}$. In order to show that, it is sufficient to prove that $K_b \leq K_c$. Since $K_a \leq K_{IS}$, we have by Corollary 7.8 that $A_{PM}(K_a) \leq A_{PM}(K_{IS})$, that is, $K_b \leq K_c$. Therefore, $K_b \leq K_d$. □

The following theorem can be shown by Corollaries 7.5 and 7.7 and Lemmas 7.6, 7.7 and 7.8.

Theorem 7.4 *The result of successive improvements of* $K_{IS} \in Q_{AS}$ *by means of* $A_{BM}(\cdot)$ *and* $A_{PM}(\cdot)$ *is presented in Figure 7.2 (where* $\rightarrow$ *denotes an application of the operator labeling the arc). The partial order on the predicates in Figure 7.2 is shown in Figure 7.3 (where* $\rightarrow$ *implies* $\leq$*).*

The above theorem shows that the task of optimization of a blocking controller can be done by two steps. Figures 7.2 and 7.3 are graphically identical to Figures 8 and 9 in [12] by replacing PM by SM, respectively. It is very interesting that formal representations of results on optimizing operations are the same structures as those in [12] while meanings of measures used in this chapter are completely different from those in [12].

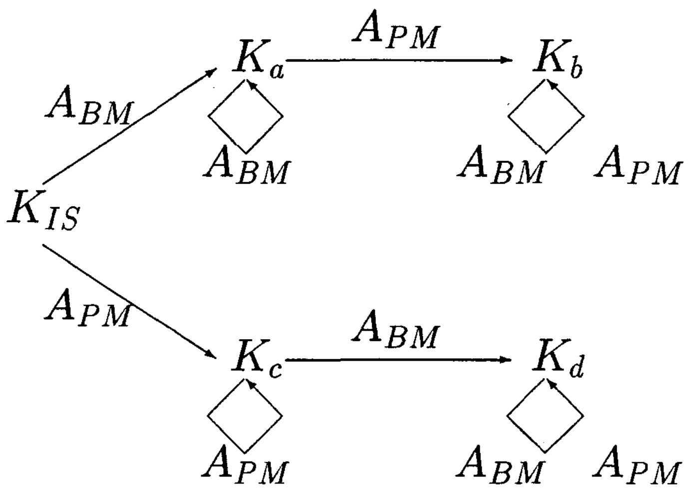

Figure 7.2: Successive improvements of K_{IS}.

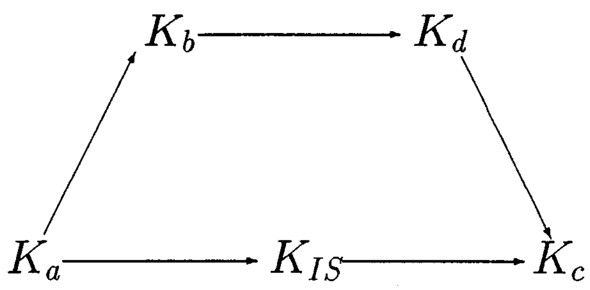

Figure 7.3: Relationship among predicates shown in Figure 7.2.

Example 7.6 We consider the same system as Example 7.1. Let $K_{IS} \in Q_{AS}$ be the same as Example 7.4. Then predicates K_b and K_d defined above are computed as follows:

$$K_b(x) = \begin{cases} 1 & \text{if } x \in \{x^1, x^2, x^3, x^5, x^6\}, \\ 0 & \text{otherwise.} \end{cases}$$

$$K_d(x) = \begin{cases} 1 & \text{if } x \in \{x^1, x^2, x^3, x^5, x^6\}, \\ 0 & \text{otherwise.} \end{cases}$$

Since

$$PM(K_b)(x) = PM(K_d)(x) = \begin{cases} 1 & \text{if } x \in \{x^1, x^2, x^3, x^5\}, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$BM(K_b)(x) = BM(K_d)(x) = \begin{cases} 1 & \text{if } x \in \{x^6\}, \\ 0 & \text{otherwise,} \end{cases}$$

we have $PM(K_b) = PM(K_d) > PM(K_{IS})$ and $BM(K_b) = BM(K_d) < BM(K_{IS})$. This shows that K_b and K_d are improved in terms of both BM and PM.

7.7 Concluding Remarks

In this chapter, we studied blocking in state feedback control in the context of stability of the system. We introduced two performance measures, called a prestabilizing measure and a blocking measure, for control-invariant predicates. First, we presented an algorithm to compute the minimally restrictive nonblocking solution. Next, we presented techniques for improving each of the two performance measures. Note that the complexities of these techniques are polynomial. Moreover, we showed that the task of optimization of a blocking controller in terms of the performance measures can be done by two steps.

Chapter 8

Conclusions

This thesis has studied state feedback control of DESs where control specifications are given in terms of predicates on the set of states. A state feedback controller disables some controllable events based upon the current state of the system so that the system satisfies the given control specification. The state feedback control theory based upon predicate can provide an efficient method of synthesizing controllers for a complex DES possibly with an infinite state space [38, 18]. We summarize our contributions as follows.

Chapter 3 has addressed the state feedback control problem in the Golaszewski-Ramadge model. First, we proposed the notion of Γ-controllability of predicates, and showed that Γ-controllability is a necessary and sufficient condition for the existence of a state feedback controller under the assumption that the set of control patterns is closed under union. We then derived a closed form expression of the supremal Γ-controllable subpredicate of the given predicate. These results are a generalization of those obtained in [49, 38].

In Chapter 4, we have studied state feedback control under partial observations. First, we showed a necessary and sufficient condition for the existence of a balanced controller in the Golaszewski-Ramadge model. This problem is also a generalization of that addressed in [49, 38], where the system is modeled by the Ramadge-Wonham model.

Next, we considered the Ramadge-Wonham model. We defined M-controllability of predicates, and proved that M-controllability is a necessary and sufficient condition for the existence of a state feedback controller which is not necessarily balanced. Kumar et al. have given necessary and sufficient conditions for the existence of a dynamic controller [38]. Obviously, a state feedback controller is a special case of a dynamic one. But a state feedback controller is easier to implement than a dynamic one. Moreover, our condition has computational advantage in contrast to those obtained by Kumar et al. because the computational complexity to check our condition is polynomial if the system is modeled by a finite automaton. So our condition is useful from the practical point of view. It is also noted that our condition is a generalization of the result obtained in [49].

Finally, we have dealt with modular control synthesis in the Ramadge-Wonham model. In the case where a predicate is decomposed into conjunction of component predicates, modular control synthesis [68] is very effective. We showed that M-controllability of component predicates implies M-controllability of their conjunction under a certain condition. We then presented a necessary and sufficient condition under which a state feedback controller can be constructed in a modular fashion.

Chapter 5 has studied decentralized state feedback control. For distributed systems such as communication systems, a decentralized controller is more suitable than a centralized one.

First, we addressed the decentralized state feedback control problem (DSFCP), which requires that the set of reachable states in the closed-loop system is equal to the specified predicate. We introduced the notion of n-observability of predicates and proved that the controllability and n-observability are necessary and sufficient conditions for the existence of a solution to the DSFCP.

Next, we considered the decentralized state feedback control problem with tolerance (DSFCPT), which requires that the set of reachable states in the closed-loop system is in the given admissible range. We showed that the infimal controllable and n-observable superpredicate of a given predicate plays an important role in solving the DSFCPT. So we derived closed form expressions of the infimal controllable superpredicate, the infimal n-observable superpredicate and the infimal controllable and n-observable superpredicate, respectively, under a certain condition.

In Chapter 6, we studied the unique existence of maximally permissive controllers (MFC) in controlled Petri nets (CPNs). Ushio has given a necessary and sufficient condition for the unique existence of the MPC [95, 98]. However, we have to construct the set of all permissive controllers in order to check the condition.

First, we considered CPNs without concurrency. We presented the necessary and sufficient conditions for the unique existence of the MPC under partial as well as complete observations, which can be checked without constructing the set of all permissive controllers. Next, we extended the results to CPNs with concurrency controlled by either event assignment or resource allocation. We then showed that the unique existence of the MPC in resource allocation control implies that the same is true in event assignment control.

Chapter 7 has considered the case that control specifications are given not only by admissible states but also by target states. In this case, the notion of the stabilization of DESs plays an important role to design state feedback controllers. In particular, we studied blocking in the context of stability of the system.

We first presented an algorithm to compute the minimally restrictive nonblocking solution. But a nonblocking controller may be restrictive because it disables all behaviors which may lead to blocking. In this sense, blocking controllers can be practically more efficient than nonblocking ones if blocking in the closed-loop system is resolved easily by some external intervention such as rollback mechanism. We defined two performance measures, called a prestabilizing measure and a blocking measure, for control-invariant predicates. Then we presented techniques for improving each of the two performance measures. Note that the complexities of these techniques are polynomial. Moreover, we showed that the task of optimization of a blocking controller in terms of the performance measures can be done by two steps.

The state feedback control theory leaves possible directions of future research listed as follows:

- It is well-known that hierarchical structure is suitable for designing, analyzing and controlling complex DESs. In this connection, hierarchical state feedback control needs to be addressed.

- Real-time state feedback control initiated by O'Young [60] needs to be extended to control under partial observations, decentralized control and so on.

- The supervisory control theory based upon formal languages has been extended to stochastic DESs [43, 57]. The state feedback control theory for stochastic DESs should be established.

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