Reheating Mechanism in the Inflationary Universe Scenario Based on Grand Unified Theory of Particles

by

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Abstract

The inflationary Universe scenario is regarded as a possible model of cosmology which can solve the fundamental problems of the standard big bang model: The flatness, horizon and primordial monopole problems. After reviewing the inflationary cosmology, we focus our attention on a particularly promising model: The new inflationary universe scenario based on the Coleman - Weinberg potential among many versions. In the main part of this thesis we shall clarify the reheating mechanism of the new inflationary Universe scenario, which undergoes a gigantic order of supercooling by the exponential expansion of the Universe at the time $\sim 10^{-34}$ sec. Precisely speaking we shall investigate the time development of the phase transition at the inflationary stage, deriving the evolution equation of the order parameter and evaluating the value of the friction term which arises in the obtained equation. This friction force eventually heats up the supercooled Universe. The above approach to the inflationary phase transition is demonstrated in the minimal SU(5) grand unified model. It is shown that the friction force in this model is unfortunately too weak so that the order parameter travels many times around the minima of the SU(5) Coleman - Weinberg potential. This would cause unwanted large scale inhomogeneities of the Universe after the phase transition. Our estimation of friction force may give an important criterion for the successful inflationary scenario in future. Finally we also discuss the possible thermal effects on the friction.

Contents

§1. Introduction

The big bang Universe model [1,2,3], the so called standard model of cosmology, has been successful to describe the evolution of the Universe from about 10^{-2} sec after "the bang" until today. Especially this model gives nice explanations of the observed 2.7K cosmic microwave background radiation and the abundances of the light elements through nucleosynthesis [1,2,3,4].

Extending our understanding further back, to earlier time and higher temperature, requires knowledge about the fundamental particles and their interactions at very high energy. The interplay of the cosmology and the elementary particle physics, i.e. grand unified theories (GUTs) [5], has led to an attractive picture which can explain the baryon excess in the present Universe [6]. In the standard baryogenesis scenario the baryon number non-conserving process together with the time reversal non-invariance in GUTs generates the cosmological baryon asymmetry. Thus we can say that the interface between elementary particle physics and cosmology is now a very active area of research and it will stimulate progress of both fields.

The standard model, however, still has fundamental problems to be solved, i.e. the flatness [7] and horizon problems [1,8,9], which cannot be naturally explained within the framework of this model and appears as unreasonable initial conditions in the early Universe. The first problem is that we have to make an extremely unnatural assumption that the initial value of the Hubble constant must be fine-tuned to an extraordinary accuracy to produce a flat Universe of which mass density is even now near the critical one (the flatness problem). The second is the horizon problem that the early Universe is assumed to be highly homogeneous, in spite of the fact that separated regions were causally disconnected.

Moreover a GUT phase transition which occurs at the time $t \sim 10^{-34}$ sec, when the temperature of the Universe $T \sim 10^{14}$ GeV, causes another difficulty of the overproduction of t'Hooft-Polyakov monopole [10] (primordial monopole problem) [11,12]. This would lead to a catastrophic cosmological consequence that the Universe would have collapsed a long time ago due to the large mass density caused by the monopoles.

The interplay of elementary particle physics and cosmology again shed light upon these fundamental problems of the big bang cosmology. Guth [13] and Sato [14] independently proposed a very attractive idea called the inflationary Universe scenario which overcomes these problems. In the standard scenario we assume the adiabatic expansion of the Universe which is the cause of the difficulties. In the inflationary Universe scenario, on the contrary, the time evolution of the Universe becomes drastic. Namely during the GUT phase transition, the Universe experiences the large magnitude of exponential expansion. In this stage the energy density is dominated by the vacuum energy which works as the cosmological constant. The cosmological term brings about the exponential expansion. In the final stage of the phase transition the release of latent heat generates a huge amount of entropy. These huge expansion and entropy production make it possible to obtain the simultaneous solution of the previously mentioned fundamental problems of the standard scenario.

However it turned out that the original version of the inflationary Universe scenario, in which the GUT phase transition is assumed to be strongly first-order, cannot complete the phase transition and causes undesirable inhomogeneities of the Universe. That is, the necessary bubble collisions for the completion do not efficiently occur due to the rapid expansion of the Universe and the latent heat will never be thermalized [15,16]. (More detailed explanation is given in subsection 3-1.)

A new version of the inflationary Universe scenario was promptly proposed by Linde [17,18] and Albrecht et al. [19] which was free of the main difficulties of the original scenario. The new scenario is based on the Coleman-Weinberg (C-W) type symmetry breaking [20] in the SU(5) GUT. While the order parameter of the phase transition, i.e. 24-dimensional adjoint Higgs field, very slowly rolls down on the almost flat C-W potential (See Fig.4 in subsection 3-2), the scale of the Universe grows exponentially. The phase transition seems to be terminated by particle productions [21] due to a damped oscillation of the Higgs field around the bottom of the C-W potential.

Furthermore the new scenario is the first theory which can predict the initial spectrum of the density perturbation in the early Universe [22,23]. This brings about the possibility to solve the one of the outstanding problems of the standard cosmology : The explanation of rich structure of the Universe at present (stars, galaxies and clusters of the galaxies). However it was shown that the new inflationary Universe scenario based on the SU(5) C-W potential provides too large magnitude of the density fluctuation [22,23], which is a main difficulty of this scenario.

Moreover there is another problem in the new senario [24,25] which we shall be concerned with in the main part of this thesis. As we shall see in §4, e.g. see Fig.6, the local SU(4) × U(1) minima of the C-W potential are more accessible for the order parameter than the global and physical SU(3) × SU(2) × U(1) minima. Thus it is likely that the order parameter settles down at the undesirable SU(4) × U(1) minima, then subsequent strongly first-order phase transition to the global SU(3) × SU(2) × U(1) minima causes the same difficulty as the original inflationary Universe scenario: incompletion of the phase transition and resultant inhomogeneities of the Universe.

In order to overcome these difficulties, model building efforts has been continued, and newer versions of the inflationary Universe scenario have been proposed [18]: The supersymmetric new inflation [26], primordial inflation [27,28] and chaotic inflation [29,30], etc., which are devised to reduce the value of the density perturbation to the desirable amplitude by a suitable choice of parameters. However each of them suffers from its specific difficulties and cannot yet be regarded as a realistic model of inflationary cosmology. It seems that further investigations are necessary for the improvement of these models.

In this thesis we shall consider thermalization mechanism of latent heat in the new inflationary Universe Scenario, and especially investigate the second problem of this scenario which is closely related to the reheating of the Universe. In order to comprehend the process of the inflationary phase transition, we shall derive the evolution equation of the order parameter of the phase transition and evaluate the value of the friction term which arises in the obtained equation and characterizes dissipative processes, i.e. thermalization of Universe [31,33].

The above approach to the inflationary phase transition was initiated by Albrecht et al. [34]. However they introduced the friction term by hand. The present author and Hosoya [31], at first, formulated a systematic method which makes it possible to calculate the strength of the friction force from first principles at finite temperature. Furthermore this method was developed to the zero temperature case by Sasaki and Morikawa [32] (in the de Sitter stage the temperature of the Universe is effectively zero). They took account of the particle productions due to the time dependence of the Higgs field, which ref.31 ignores.

We shall evaluate the value of the friction term and investigate the second problem of the new inflationary Universe scenario, specifically taking the minimal SU(5) model as a prototype. We know that this model leads to a too large magnitude of density perturbation. Nevertheless we think that this investigation is worthwhile from the following reasons. First, there is an unreliable point in the estimation of the density perturbation (see subsection 3-3), so the new scenario should not entirely be abandoned and therefore its elaboration seems to make sense. Second, the minimal SU(5) model is simplest and can serve as a good testing ground for the application of our method to more sophisticated models of inflationary Universe.

As we shall see later, the magnitude of the friction term which arises in the evolution equation of the order parameter has an important role to determine whether the new inflationary Universe scenario works successfully or not [35]. If its value is too large, this scenario suffers from the same problem as the original one. On the other hand, if it is too small, the order parameter travels around many minima of C-W potential before it settles down at some minimum, and this causes the inhomogeneities of the Universe. If and only if its value lies in an adequate range, the order parameter can properly terminate at the physically favorable SU(3) × SU(2) × U(1) minima and this scenario can be free from the above difficulty.

Thus it is essential to evaluate the amplitude of the friction term and understand the aspects of the time evolution of the Higgs field, which will be performed in this thesis.

Finally we find that the value of the friction term, which we calculate in the SU(5) model at $T = 0$, is unfortunately too small so that the Universe becomes inhomogeneous [33]. However it is likely that we cannot neglect finite temperature effects to the friction in the final stage of the phase transition. We will suggest the possibility of saving the new inflationary Universe scenario based on the minimal SU(5) GUT from the above difficulty by means of the thermal friction.

This thesis is organized as follows. In §2 we briefly review the standard scenario of cosmology and discuss its fundamental problems. The history of the inflationary cosmology is surveyed in §3. In §4 we investigate the reheating mechanism of the new inflationary Universe scenario based on the C-W potential. We evaluate the magnitude of the friction term and numerically solve the evolution equation of the order parameter in the minimal SU(5) model. Section 5 is devoted to summary and discussion.

§2. Standard Hot Universe Scenario

2-1 A Brief Review of the Big Bang Cosmology

The big bang model is based on the "Cosmological Principle" which is the hypothesis that the Universe is spatially homogeneous and isotropic. The above hypothesis is supported by the 2.7K cosmic microwave background radiation and the universal (Hubble) expansion of the Universe. On the large scale (>> 100 Mpc), the Universe can accurately be described by the Robertson-Walker metric

$$ds^2 = dt^2 - R(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right) \quad , \tag{2.1}$$

where k = +1, -1 or 0 for a closed, open or flat Universe, respectively and R(t) is the cosmic scale factor. For k = +1 the spatial Universe is finite in extent and can be regarded as the surface of a sphere of radius R(t) in four-dimensional Euclidean space. So R(t) can be justly called the "radius of the Universe". For k = -1 and k = 0 it is infinite in extent and no such interpretation is possible, but R(t) still sets the scale of the geometry of space.

The evolution of the cosmic scale factor R(t) is determined by the Friedmann equations

$$\ddot{R} = -\frac{4\pi}{3} G(\rho + 3p)R \quad , \tag{2.2}$$

$$H^2 + \frac{k}{R^2} \equiv \left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = \frac{8\pi}{3} G\rho \quad , \tag{2.3}$$

where ρ in the energy density, p is the pressure and the dot denotes the derivative with respect to t. The Hubble parameter $H = \dot{R}/R$ characterizes the time evolution of R(t) and G is the gravitational constant, $G = M_p^{-2}$ (where $M_p = 1.22 \times 10^{19}$ GeV is the Planck mass). In the derivation of the Friedmann equation (2.2) and (2.3), we have chosen that the cosmological constant $\Lambda = 0$ and employed the energy-momentum tensor for ideal fluid in the Einstein equation. In addition we have the equation of energy conservation:

$$\frac{d}{dt}(\rho R^3) = - p \frac{d}{dt} R^3 \quad . \tag{2.4}$$

or equivalently

$$\dot{\rho} + 3H(\rho + p) = 0 \quad , \tag{2.5}$$

Here we note that the equation of energy conservation can be derived from the Friedmann equation. In the standard scenario it is usually assumed also that the expansion is adiabatic, in which case

$$\frac{d}{dt}(sR^3) = 0 \quad , \tag{2.6}$$

where s is the entropy density. This assumption is equivalent to adopting the energy-momentum tensor for ideal fluid.

According to the big bang model, the Universe has been expanding and gradually cooling from a state with very high temperature and density. The 2.7K background radiation is a relic of the above hot and dense radiation dominant stage. Furthermore that picture of the evolution of the Universe successfully accounts for the abundances of the light elements, D and ^{4}He, etc., through primordial nucleosynthesis [1,2,3,4]. In order to determine the evolution of the Universe, we must specify an equation of state for the matter. In the asymptotically free theories, one may neglect interactions among particles in the lowest order, when the Universe is at high temperature and high density. It follows that the equation of state is to a good approximation that of an ideal gas of massless particles. Provided that the temperature T is not near any mass thresholds, the values of ρ, p and s are given by

$$\rho = 3p = \frac{\pi^2}{30} N(T)T^4 \quad , \tag{2.7}$$

$$s = \frac{2\pi^2}{45} N(T)T^3 \quad , \tag{2.8}$$

where N(T) is the effective number of particle species

$$N(T) = N_b(T) + \frac{7}{8} N_f(T) \quad . \tag{2.9}$$

Here $N_f(N_b)$ denotes the number of fermionic (bosonic) spin degrees of freedom which are effectively massless at temperature T (e.g., the photon contributes two units to N_b and electrons and positron four units for N_f). From eqs. (2.6) and (2.8), we obtain

$$RT = \text{const.} \quad , \tag{2.10}$$

where the constant is determined by the total entropy in a volume specified by the scale factor R(t).

It can be shown that, in the very early stage of evolution of the Universe in the standard scenario, the Universe was very flat and one may neglect the term k/R^2 in eq.(2.3). Thus, in that case, from eqs. (2.3) and (2.10) we find that the scale factor evolves as

$$R(T) \propto t^{1/2} \quad , \tag{2.11}$$

and the age of the Universe is given by

$$t = \frac{1}{4\pi}\left(\frac{45}{\pi N}\right)^{1/2} \frac{M_p}{T^2} \quad . \tag{2.12}$$

This result is reliable only at $T \leq T_p \sim M_p/\sqrt{N} \sim 10^{18}$ GeV, and the density $\rho \leq \rho_p \sim M_p^4/N \sim 10^{92}\ \mathrm{gcm}^{-3}$. When $T > T_p$ or $\rho > \rho_p$, effects of quantum gravity become important. Furthermore it should be noted that the thermal equilibrium in the expanding Universe is established only at $T \leq 10^{16}$ GeV [36].

The hot big bang model seems to provide an accurate accounting of the Universe from about 10^{-4} sec after 'the bang' when the temperature was about 10 MeV, until today. In order to extend our understanding further back to earlier times and higher temperatures, we need to know about the fundamental particles (e.g. quarks and leptons) and their interactions at very high energies (e.g. GUT). It seems that the most successful result of the interplay between elementary particle physics and cosmology is comprehension of a mechanism of baryogenesis [6]. The B(baryon number), C(charge conjugation) and CP(time reversal) violating interaction in GUTs provides a dynamical explanation for the predominance of matter over antimatter.

Thermal history of the Universe based on GUTs is shown in Fig.1. At $t \sim 10^{-35}$ sec after the big bang, when the temperature drops down to $T \sim 10^{14} - 10^{15}$ GeV, the first phase transition occurs in GUTs. For example, in the SU(5) theory, this may be a transition $SU(5) \to SU(3) \times SU(2) \times U(1)$. After the phase transition decay processes of the superheavy X,Y bosons and superheavy Higgs bosons lead to the baryon asymmetry generation. The phase transition which separates weak and electromagnetic interactions ($SU(3) \times SU(2) \times SU(1) \to SU(3) \times SU(1)$) takes place when the temperature drops down to $T \sim 10^2$ GeV. It corresponds to $t \sim 10^{-10}$ sec after the big bang. At $T \sim 10^2$ MeV two phase transitions occur successively. One is a confining-deconfining transition of QCD. The other is a phase transition with breaking of chiral symmetry. However we do not know for sure which phase transition takes place earlier. The subsequent evolution of the Universe is nicely described by the standard scenario of cosmology.

2-2 Problems of the Standard Scenario

Although the hot big bang scenario has succeeded in describing the evolution of the Universe especially from about 10^{-2} sec after the big bang until today, there are fundamental problems which cannot be solved within the framework of the standard scenario. Here, among them, we will briefly present the problems closely related to the motivation of the inflationary Universe scenario, which we shall be concerned with in the subsequent sections.

1. Flatness Problem

Since the standard scenario assumes the adiabatic expansion of the Universe eq. (2.6), we have the relation (2.10), RT = const. From the fact that the present 'radius' of the Universe $R(t_0)$ exceeds 10^{28} cm, we can estimate the radius of the Universe at the Planck time t_p

$$R(t_p) \simeq \frac{T_0}{T_p} R(t_0) > 10^{29}\,\ell_p \quad , \tag{2.13}$$

where ℓ_p is the Planck length. A naive discussion from dimensional ground would suggest that $R(t_p) \sim \ell_p$. Here from eq. (2.13) we can say that our Universe is actually 10^{29} times as flat as the one which would be expected from the naive discussion. In terms of entropy $S_{ob} \sim (RT)^3$ this means that $S_{ob} > 10^{87}$, where S_{ob} is total entropy of present observable part of the Universe. In the standard scenario it is impossible to account for the flatness and the enormous amount of entropy of the observable Universe.

For the energy density of the Universe, we know only that $0.01 < \rho/\rho_{cr} < 10$ from the observation at present, where $\rho_{cr} = 3H^2/8\pi G$ which is a critical density corresponding to the flat case for a given H. We cannot determine whether the

Universe is open or close at present. From eqs. (2.3) and (2.8) it follows that

$$\left|\frac{\rho - \rho_{cr}}{\rho_{cr}}\right| = 0.21\ N(T)^{-1/3} S^{-2/3} \left(\frac{M_p}{T}\right)^2 \quad . \tag{2.14}$$

Equation (2,14) means that in the very early Universe the value of $|(\rho-\rho_{cr})/\rho_{cr}|$ was extremely small due to the huge amount of entropy S. At the Planck time, we obtain

$$\left|\frac{\rho - \rho_{cr}}{\rho_{cr}}\right| \leqq 10^{-59} \quad . \tag{2.15}$$

We see that the flatness of the present Universe can be guaranteed only if the energy density at Planck time is fine-tuned to 10^{-59}. If it was slightly greater than ρ_{cr} , say $\rho \geqq \rho_{cr}(1+10^{-59})$, the Universe would have collapsed millions years ago. On the other hand, if $\rho \leqq \rho_{cr}(1-10^{-59})$ at the Planck time, the present Universe would be curvature dominated, i.e., energy density at present would be negligibly small. That is another expression of the flatness problem.

2. Horizon Problem

The observable Universe ($d \simeq 10^{28}$ cm $\simeq$ 3000 Mpc) is to a high degree of precision isotropic and homogeneous on the largest scales (>> 100 Mpc). The best evidence for this is provided by the uniformity of the cosmic background radiation. Since $d \simeq 10^{28}$ cm is the maximum size of the currently observed region of the Universe, let us call it the present size of the Universe. The size of the Universe in the past or future is proportional to the cosmic scale factor. This means that the size of the

Universe varies as $t^{1/2}$ for a radiation dominated era, $t^{2/3}$ for a matter dominated era.

In the standard scenario, we know the existence of particle horizon which is the size of a region causally related at time t. This is equivalent to the distance that a light signal could have propagated since the bang,

$$d_H(t) = R(t) \int_0^t dt' \, 1/R(t')$$

$$= t/(1 - n) \quad , \quad \text{for } R(t) \propto t^n, \; n < 1 \; . \qquad (2.16)$$

When we go back to the past, the particle horizon shrinks more rapidly than the size of the Universe. This means that the observable Universe which is homogeneous and isotropic was not causally connected in the past. At $t = 10^{12}$ sec when the radiation was decoupled, the distribution of cosmic background radiation should already be uniform over the scale of 10 times that of the horizon.

3. Galaxy Formation Problem

The Universe is fairly homogeneous and isotropic on the large scale. However, on the small scale (< 100 Mpc), it has rich structures as stars, galaxies and clusters of galaxies. According to the investigation of Zel'dovich for gravitational instability and galaxy formation [37], small density perturbations $\delta\rho$ with an almost scale-independent spectrum $\delta\rho/\rho \sim 10^{-4}$ in the very early Universe is required to account for the small-scale inhomogeneities which is observed today. However it to date remains unanswered to explain the origin of the fluctuations with such a specific spectrum.

4. Primordial Monopole Problem

Although the interplay between GUTs and the standard cosmology gives an attractive explanation for predominance of matter over antimatter, it also causes a difficulty of the overproduction of t'Hooft-Polyakov monopole during the phase transitions in GUTs at $T \sim 10^{14} - 10^{15}$ GeV [11,12]. When spontaneous symmetry breaking occurs, the Higgs field can only be smoothly oriented on scales smaller than some characteristic correlation length ξ. On scales larger than ξ the Higgs field must be uncorrelated, and thus we expect of order 1 monopole per correlation volume ($\simeq \xi^3$) to be produced as a topological defect when the Higgs field freezes out [11]. Since the particle horizon sets an absolute upper bound on ξ, we can estimate the lower bound of the value of monopole-to-baryon number

$$\frac{n_M}{n_b} \geqq 10^{12}\left(\frac{T_c}{M_p}\right)^2 \quad . \tag{2.17}$$

As shown by Preskill [12], monopole-antimonopole annihilation does not effectively work to reduce the monopole abundance due to GUTs phase transition, unless $n_M/n_B > 1$. It follows that at present number density of monopole in the Universe is of the same order as that of protons. Since the density of matter in our Universe would be approximately 10^{15} times greater than the critical density due to monopoles, if that was the case, the Universe would have collapsed a long time ago.

In this subsection we have briefly explained several fundamental difficulties in the big bang cosmology which are especially related to the inflationary Universe scenario. However we know that there are other fundamental problems in the standard cosmology. Here we shall enumerate those problems.

1) The Robertson-Walker metric which describes the Universe on the large scale is singular at $t = 0$. One might think that this singularity is caused by the symmetric properties of the metric to a high degree. However Hawking and Penrose have proven that the existence of such an initial singularity is unavoidable in more general cases. This is the initial singularity problem [38].

2) At present Universe the cosmological constant in the Einstein equation is apparently negligibly small. With the possible exception of supersymmetry and supergravity theories, the absolute scale of the effective potential $V(\phi)$ in not determined. Since, at low temperature, $V(\phi)$ is equivalent to a cosmological terms, we should fine-tune the scale of $V(\phi)$ in order that the value of $V(\phi)$ at the true vacuum is zero to 10^{46} GeV^4. 3) The scenario of baryogenesis can nicely account for the mechanism to give baryon asymmetry. However it has not yet succeeded to explain the value of the baryon-to-photon ratio quantitatively.

4) It is likely that $N = 1$ supergravity coupled to matter can solve the gauge hierarchy problem [39]. One of the important feature of the proposed solution of the gauge hierarchy problem is the existence of the gravitino with mass $m_{3/2} \sim m_W$ 10^2 GeV [40]. However it gives undesirable influences for the baryon-to-photon ratio or the mass density of the present Universe [41,42].

§3. Inflationary Universe Scenario

3-1 Original Inflationary Universe Scenario

In this subsection we will describe the inflationary Universe scenario in its original form [13,14] which was intended to solve some fundamental problems of the standard cosmology that were explained in the previous section. Then some difficulties of the original scenario will be also discussed.

The original version of the inflationary Universe scenario requires that the GUT phase transition is strongly first-order [43]. The effective potential for a Higgs field of an adjoint representation Φ has a form shown schematically in Fig.2. At zero temperature, a symmetric minimum ($\Phi = 0$) is a local minimum (a false vacuum) of the effective potential and a global minimum (a true vacuum) lies at $\Phi = \Phi_0 \sim 10^{15}$ GeV. A phase transition from the false vacuum to the true vacuum proceeds through the nucleation of bubbles[44].

We note that the energy density $\rho(T)$ in the standard scenario (2.7) must now be modified. When a temperature of the Universe is very high compared to a critical temperature $T_C \sim 10^{14}$ GeV, the symmetric phase $\Phi = 0$ is stable. As we have assumed that the phase transition is strongly first-order, a rate of the phase transition to the true vacuum through the nucleation of bubbles of the new phase is small in comparison with an expansion rate of the Universe. So the Universe supercools below T_C and the energy density is dominated by ρ_0 ,which is an energy density of the false vacuum, at that time. Therefore the energy density $\rho(T)$ is approximately given by

$$\rho(T) = \frac{\pi^2}{30} N(T)T^4 + \rho_0 \quad . \tag{3.1}$$

Since the false vacuum is completely Lorentz invariant, its energy-momentum tensor must have the form

$$T_{\mu\nu} = \rho_0 g_{\mu\nu} \quad . \tag{3.2}$$

The energy of the false vacuum behaves as a cosmological constant $\Lambda = 8\pi G\rho_0$ in the Einstein equation at the supercooling stage. It leads to a rapid exponential expansion of the Universe. It should be noted that we have set the value of the effective potential at the true vacuum to vanish, $V_{eff}(\Phi_{true}) = 0$, since we know empirically that Λ of the present Universe is very small ($|\Lambda| < 10^{-46}$ GeV^4).

In the very hot Universe, it is likely that there are local regions which are sufficiently homogeneous and isotropic to be approximated by the Robertson-Walker metric (2.1). When these regions supercool below T_c, they start to expand exponentially as

$$R(t) \propto \exp(Ht) \quad , \tag{3.3}$$

where

$$H = \left(\frac{8\pi}{3} G\rho_0\right)^{1/2} \sim 10^{10}\ \text{GeV} \tag{3.4}$$

for $\rho_0 \sim (10^{14}\ \text{GeV})^4$. The scale factor $R(t)$ quickly becomes so large that the Robertson-Walker metric (2.1) can be approximated by its $k = 0$ form. The resulting space is called the de Sitter space [45].

Let us now make a working assumption that the exponential expansion continues for a time interval Δt, and that the phase transition through bubble formation then occurs with rapid thermalization of the latent heat. (It turns out, however, that the assumption of rapid thermalization is very unrealistic ; we will later discuss what would actually happen.) During the exponential expansion, the value of $R(t)$ increases by a factor

$$Z = \exp(H\Delta t) \quad . \tag{3.5}$$

In order to solve the cosmological problems, it is necessary that $Z > 10^{29}$. It has been shown that reasonable parameters can lead to values of Z as large as $10^{10^{10}}$ in the SU(5) model [46].

When the phase transition takes place, the energy density ρ_0 of the false vacuum will be released as latent heat. If this thermalization of the Universe is sufficiently rapid, the temperature will again rise to the order of 10^{14} GeV, which is comparable in order of magnitude to its value before the exponential expansion. From here on, the inflationary Universe scenario rejoins the standard scenario[#1,#2]. Since $R(t)$ increases by a factor of Z, it follows that the entropy $S \equiv R^3 s$ increases by a factor of order Z^3. Thus if S were initially of order of unity, it would be greater than 10^{87} after the phase transition. These gigantic order of the exponential expansion and entropy generation solve the flatness problem.

The horizon problem also disappears owing to the huge magnitude of the exponential expansion. After the reheating, the region which evolves to become our observable Universe has a size of order of 10 cm. Before inflation its size was less than 10^{-28} cm, and this is of order of 10^4 times smaller than the horizon length at that time.

#1) It is essential that baryogenesis occurs after the exponential expansion and themalization of the Universe, so that the baryon-to-entropy ratio is not diluted.

#2) In the case of the Weinberg-Salam phase transition, if inflation takes place, the baryon-to-entropy ratio is reduced. thus, in the context of current understanding of baryogenesis, it is necessary that the phase transition proceeds so rapidly that the magnitude of inflation does not exceed 10^2. This constraint gives a lower bound of the mass of Higgs doublet as $m_H > 9$ GeV [47].

As for the monopole problem, the mechanism which leads to the overproduction of monopoles in the standard scenario dose not obviously work in the inflationary scenario. However it is not clear how many monopoles are produced in the phase transition. In order to find an answer to this question, it is needed to investigate the detailed mechanism of nucleation of bubbles.

So far we have assumed that the phase transition occures with rapid thermalization of the latent heat. However unfortunately that is not the case. Here we will discuss what actually happens when a slow first order phase transitions takes place in an exponentially expanding space.

The process of bubble nucleation and growth in a false vacuum has been analyzed by Coleman et al. [44]. The nucleation rate λ (the expected number of nucleation per physical volume per time) is given by

$$\lambda = A \exp(-B) \quad , \tag{3.6}$$

where B is the classical action of a bubble solution and A is in order of the fourth power of the mass scale which characterizes the phase transition: $A \sim (10^{14}\ \mathrm{GeV})^4$. The bubble wall expands at a speed which rapidly approaches that of light. Since the energy of the false vacuum is first transferred to the kinetic energy of the bubble wall, the thermalization of the Universe proceeds by collisions of the bubbles as illustrated in Fig.3.

We can calculate the probability P(t) that an arbitrary point remains in the old phase at time t in the exponentially expanding space [48],

$$P(t) = \exp\left[-\frac{4\pi}{3}\varepsilon H(t-t_b)\right] \quad , \tag{3.7}$$

where

$$\varepsilon = \lambda/H^4 \tag{3.8}$$

and t_b is a time at which bubble formation begins. In order to have an exponential expansion by a factor $Z \sim 10^{29}$ before the phase transition is completed, i.e., $P(t) << 1$, one needs $\varepsilon \leq 10^{-2}$. According to the numerical calculation of the probability P(t) [15], it is easy to obtain the value $\varepsilon \leq 10^{-2}$ which gives sufficient amounts of expansion ,and reasonable parameters lead to values like $\varepsilon = 10^{-100}$ or even smaller.

However, in the case of a slow phase transitiion ($\varepsilon << 1$), it is shown that, due to the rapid expansion of outside of bubbles, it is difficult to terminate the phase transition through thermalization of the latent heat by means of bubble collision and coalescence [15]. It can be proven rigorously that if $\varepsilon < 10^{-6}$, then the bubble will form finite size clusters only, no matter how long one waits, even though $P(t) \to 0$. This means that the bubbles will never merge to form an infinite connected region; i.e., the system does not percolate.

Furthermore, Hawking, Moss and Stewart [16] have shown that, with $\varepsilon \sim 1/200$, about 1/3 of all bubbles will grow to a size which will evolve to galactic proportions in the present Universe, before undergoing their first collisions. The energy released by these wall collisions could not thermalize nor distribute itself more or less uniformly over a region until light had time to cross the region. In standard model, it corresponds to $t \sim 10^8$ sec, when $T \sim 10^6$ K. Thus one cannot have the usual explanations of the baryon and helium abundances. Moreover, thermalization would not be completed at this late epoch and so the microwave background radiation would be distorted. The scenario is unworkable, unless one finds a more graceful end of the phase transition.

3-2 New Inflationary Universe Scenario

Although, as was shown in the previous subsection, the original scenario of the inflationary Universe leads to some unacceptable cosmological consequences, its basic idea is very attractive. Fortunately, a new version of the inflationary Universe scenario was suggested [17,18,19], which kept the virtue of the original one of solving some fundamental problems of the standard cosmology, and which could overcome the main difficulties of the original scenario.

The new scenario is based on the Coleman-Weinberg type (C-W type) symmetry breaking [20]. For simplicity, we shall consider here a particular pattern of symmetry breaking $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ due to the appearance of the classical Higgs field Φ of the adjoint representation,#)

$$\Phi = \sqrt{15/2}\,\varphi\,\mathrm{diag}(1,1,1,-3/2,-3/2) \quad . \tag{3.9}$$

The effective potential in the SU(5) C-W theory at $T \gg M_X$, m_φ has a form of

$$V_{eff}(\varphi,T) = CT^4 + \frac{1}{2}m^2(T)\varphi^2 + \frac{25g^4}{256\pi^2}\varphi^4(\ln(\varphi^2/\varphi_0^2) - 1/2) + \frac{9M_X^4}{32\pi^2} \quad , \tag{3.10}$$

where $M_X^2 = (25/8)g^2\sigma^2$, $\varphi_0 = \sqrt{2/15}\sigma$, $g^2 \sim 0.3$, $\sigma \sim 4.5 \times 10^{14}$ GeV, C is some constant, C = O(10), and $m^2(T)$ is given by

$$m^2(T) = \left.\frac{d^2Veff}{d\varphi^2}\right|_{\varphi=0} = \frac{5}{4}g^2T^2 \quad . \tag{3.11}$$

At high temperature $T \gg \sigma$, the C-W potential has only minimum at $\varphi = 0$. This means that, at $T \gg \sigma$, the symmetry is restored. Even for the temperature $T \ll \sigma$, the equation (3.10) is reliable near the origin $\varphi = 0$, so that the point $\varphi = 0$ remains a local minimum of V_{eff} for any $T \neq 0$. The shape of the C-W potential $V_{eff}(\varphi,T)$ for $T \ll \sigma$ is illustrated in Fig.4.

#) Obviously the degrees of freedom of the Higgs field is more than unity. We will discuss the C-W potential for the multi-component Higgs field rather than the one restricted in the SU(3) × SU(2) × U(1) direction in the subsections 3-3 and 4-1.

We note that the effective potential (3.10) contains only the 1-loop correction due to the vector boson. The neglect of contribution of the Higgs field can be a consistent approximation of the C-W potential at $T, \varphi \sim \sigma$ [20]. However it is likely that the scalar field contribution becomes large and important at $T, \varphi \ll \sigma$ [49]. We will forget about this effect for a while for simplicity, but will discuss this point later.

The phase transition from the local minimum of the C-W potential at $\varphi = 0$ to the global minimum at $\varphi = \varphi_0$ proceeds by the formation and subsequent expansion of the bubbles of φ. A detailed study of the bubble formation [50] has shown that at the moment of the bubble formation its size is of order T_c^{-1}, and the maximal value of the field φ inside the bubble is approximately

$$\varphi \leq 3\varphi_1 = \frac{12\pi T_c}{g} \left(\frac{1}{5\times\ln(M_X/T_c)}\right)^{1/2} \ll \sigma \quad , \qquad (3.12)$$

where the point φ_1 is defined by the condition $V_{eff}(0,T_c) = V_{eff}(\varphi_1,T_c)$ (see Fig.4). Here T_c is a critical temperature of the phase transition through the nucleation of the bubbles at which this process proceeds very quickly. The value of T_c seems to be of order of $10^6 - 10^8$ GeV#).

#) As for T_c, Albrecht and Steinhardt [19] chose the value $T_c \sim 10^8$ GeV, at which a steepest descent approximation for calculation of the rate of the bubble formation breaks down, while Linde [17] used the value of $T_c \sim 10^6$ GeV, as it was claimed in Ref.[51], at which the SU(5) gauge coupling constant becomes of order of unity.

After the nucleation of the bubble, the value of the field φ is almost homogeneous inside the bubble and it obeys the equation of motion,

$$\ddot{\varphi} + 3H\dot{\varphi} + m^2 \quad = 0 \quad , \tag{3.13}$$

where the second term of eq. (3.13) appears due to the expansioin of the Universe. At the moment of the nucleation of the bubble, the value of φ inside the bubble is very small in comparison with σ and the (negative) mass squared of the field φ is

$$|m^2| = \left|\frac{d^2 Veff}{d \quad \varphi^2}\right| \lesssim 75g^2 T_c^2 \sim 25T^2{}_c \quad , \tag{3.14}$$

so that φ grows very slowly and stays in the region $\varphi \lesssim H$ during the time interval Δt,

$$\Delta t \sim 3H/|m|^2 \quad , \tag{3.15}$$

which is the most part of time spent by the travel of φ from the origin to the bottom of the C-W potential. In this interval the vacuum energy $V_{eff}(\varphi)$ remains almost equal to $V_{eff}(0)$, so that the part of the Universe inside the bubble expands exponentially just as it expanded before the bubble creation. This is the main difference between this new scenario and the original scenario, in which it was assumed that the exponential expansion finished immediately after bubble formation.

The amount of expansion which the Universe undergoes in the inflationary stage is given by

$$Z = \exp(H\Delta t) \sim \exp(3H^2/|m|^2)$$

$$\geq \exp(1200) \sim 10^{500} \qquad , \qquad (3.16)$$

where we have used the values, $H \sim 10^{10}$ GeV and $T_c \sim 10^8$ GeV. Since a typical size of the bubble at the moment of its formation is $O(T_c^{-1}) \sim 10^{-22}$ cm, after the period of exponential expansion this bubble grows into the one whose size is of order of 10^{500} cm. Therefore the whole observable part of the Universe is contained inside one bubble.

When the field grows sufficiently large, $\varphi > H$, it begins to evolve very rapidly and rolls down to the minimum of the C-W potential. The thermalization of the vacuum energy in the form of $V_{eff}(0)$ proceeds through a production of Higgs bosons and X, Y bosons caused by rapid oscillation of the field φ around the bottom of V_{eff} [21]. The Universe is reheated up to the temperature $T_R \sim V_{eff}^{1/4}(0) \sim 10^{14}$ GeV. In the new inflationary Universe the vacuum energy can be converted into the radiation energy inside the bubble without bubble wall collisions which led some difficulties in the original scenario. This means that in new scenario the GUT phase transition can be completed and does not cause inhomogeneities of the Universe which conflicts with present observation#).

The new inflationary Universe scenario leads to a sufficient magnitude of expansion of the Universe and homogeneous radiation of the latent heat, so that it can solve the flatness and horizon problems in the standard cosmology. Furthermore, since the primordial monopoles in GUTs are created only in the points, in which bubbles with different types of Higgs field φ collide, in this new scenario no monopole are produced in the observable part of the Universe, which solves the primordial monopole problem.

The process of the baryogenesis in the new scenario is also considerably different from the usual one [52]. In this scenario the baryon asymmetry is generated by the decay of the superheavy particles, X, Y-bosons or φ-bosons which are created by the oscillation of the classical φ field. It was shown that the value of the baryon-to-entropy ratio is about two order of magnitude larger than that obtained in the standard scenario [6]. It is also important that any initial baryon asymmetry of the Universe vanishes after the inflation even in the theory which conserves B-L.

#) The thermalization processes can be expressed by <u>a friction term</u> which appears in the evolution equation of the field φ, and which , at first, has been introduced by Albrecht, et al. [34] by hand. We will discuss how that term arises in the equation in the § 4.

So far we have neglected the one-loop contribution of Higgs fields to the effective potential, since Higgs coupling constants are assumed to be of order of g^4 in the C-W theory [20]. Although that is the case at T or $\varphi \sim \sigma$, we should take into account the scalar field contribution at T, $\varphi \ll \sigma$, which changes the curvature of $V_{eff}(\varphi,T)$ near $\varphi = 0$ [49]:

$$m^2(T) = \frac{d^2 V_{eff}}{d\varphi^2}\Big| = \left(\frac{5}{4} g^2 + \frac{65a+47b}{30}\right) T^2 , \tag{3.17}$$

where a and b is the Higgs coupling constant in the interaction Lagrangian of the Higgs field ϕ in the SU(5) theory,

$$L_{int} = -\frac{1}{4} a(\mathrm{Tr}\phi^2)^2 - \frac{1}{2} b\, \mathrm{Tr}\phi^4 . \tag{3.18}$$

The analysis of the renormalization group equation for the effective coupling constants a(T), b(T) at $\varphi = 0$ suggests that at smaller values of T the coupling constants a(T) and b(T) become negative and rapidly grows in magnitude, so that $m^2(T)$ becomes negative at some critical temperature T_c [49]. This critical temperature is crucially model-dependent. Since in the theory $T_c > H$ the sufficient inflation does not occur, we will consider only those theories in which $T_c \ll H$. However, in such theories all high temperature effects are irrelevant for the phase transition and effects connected with the non-vanishing space-time curvature and the exponential expansion of the Universe are important.

Hawking and Moss showed that gravitational effect plays a very important role in the early stage of the phase transition [53]. They claimed that in the case of $\frac{d^2 V_{eff}}{d\varphi^2}\Big|_{\varphi=0} < 2H^2$, the tunneling proceeds from the minimum of V_{eff} at $\varphi = 0$ to the nearby maximum of V_{eff} at $\varphi = \varphi_1$, with the probability

$$P \sim \left(\frac{d^2 V_{eff}}{d\varphi^2}\Big|_{\varphi=0}\right)^2 \exp(-B) \qquad (3.19)$$

per unit four-volume. Here the exponent B is given by

$$B = \frac{1}{8} M_p^4 \left(\frac{1}{V_{eff}(0)} - \frac{1}{V_{eff}(\varphi_1)}\right) , \qquad (3.20)$$

which is the difference between the combined gravitational and scalar field actions of the $\varphi = \varphi_1$, and the $\varphi = 0$ solutions. This result should be interpreted as the probability of tunnelling which is not absolutely homogeneous but looks homogeneous in the domains of the Universe of size $\ell \geq H^{-1}$ [54].

On the contrary for $\frac{d^2 V_{eff}}{d\varphi^2}\Big|_{\varphi=0} > 2H^2$, the effects of gravity and the curvature of the Universe does not seriously change the aspect of the phase transition. Thus, in this case, the phase transition proceeds as the scenario proposed by Linde, or Albrecht and Steinhardt.

3-3 Difficulties of the New Inflationary Universe Scenario

As we have seen in the previous subsection the new inflationary Universe scenario has a graceful termination of the period of inflation, preserving the virtue of the original one which solves the flatness, horizon and primordial monopole problems. However the new scenario turned out not to be free from difficulties. In this subsection we shall critically discuss two main problems of the new inflationary Universe scenario based on the SU(5) C-W type symmetry breaking.

The first one is concerned with the galaxy formation [22,23]. During the evolution of the energy-density perturbation the Hubble radius H^{-1} (= particle horizon in the standard cosmology) represents an important scale [22]. Consider a density perturbation which is described by its wavelength λ or its wavenumber k ($= 2\pi/\lambda$), and its amplitude. When the perturbation is inside the horizon, $\lambda \leq H^{-1}$, microphysical processes can affect the amplitude of the perturbation. However when $\lambda > H^{-1}$ the perturbation is just a wrinkle in the space-time which are evolving kinematically, since microphysics can only operate coherently on proper length scales less than H^{-1}. This means that the development of $\delta\rho/\rho$ is determined solely by gravitational effects#).

Since the evolution of perturbations is crucially determined by the relative sizes of its wavelength λ and the horizon H^{-1}, it is convenient to specify the amplitude of density perturbations by the value $(\delta\rho/\rho)_H$ at the epoch when it crosses the horizon, i.e. $\lambda = H^{-1}$.

#). Since the quantity $\delta\rho/\rho$ is not gauge invariant, its evolution depend on the choice of gauge conditions. Fortunately Bardeen [55] has developed an elegant gauge invariant formalism to handle density perturbations in a gauge invariant way.

In order to explain the structure of the Universe at present, it is known that one must assume that the density perturbations is such that they have an amplitude of $O(10^{-4})$ on a scale which corresponds to a size of a galaxy. The perturbation may be smoothly continued with the same amplitude to larger and smaller scales:

$$(\delta\rho/\rho)_H = O(10^{-4}) \quad , \tag{3.21}$$

which is so-called Zel'dovich spectrum [37] and has an attractive feature that all scales cross the horizon with the same amplitude. Such a spectrum is not necessarily required by the observations. However many people believe that it leads to an acceptable picture of galaxy formation.

In the standard cosmology the effective horizon H^{-1} grows monotonically, $H^{-1} \propto t$. So the size of the perturbed region crosses the horizon only once (see Fig.5). Since the microphysics only operates on scales $\leq H^{-1}$, adiabatic perturbations were either there ab initio or they are not present in the big bang cosmology.

On the contrary, in the new inflationary Universe scenario the features of the evolution of the density perturbation change dramatically. Since in the exponentially expanding stage the horizon is constant, a perturbation can cross the horizon ($\lambda = H^{-1}$) twice. The evolution of two scales (λ_G = galaxy and λ_H = presently observable Universe) is shown in Fig.5. In earlier times that tı microphysics can affect the evolution of perturbations on the scale of a galaxy. Thus in the new inflationary Universe scenario microscopic processes are possible to cause the initial density perturbations in the epoch when $\lambda_G < H^{-1}$, which eventually show up as galaxies. When $t = t_1$ microphysics freezes out on this scale; the density perturbation which exists on this scale then evolves kinematically until it reenters the horizon at $t = t_H$.

The theory of generation of the density perturbations after inflation has been developed by many authors [22,23]. The spectrum of the density perturbation at the time when one scale reenters the horizon has been derived as

$$(\delta\rho/\rho)_H = 4H\delta\tau \quad , \tag{3.22}$$

where $\delta\tau$ is a position dependent time delay of the rolling down motion of the order parameter,

$$\delta\tau \sim \delta\varphi/\dot{\varphi} \quad , \tag{3.23}$$

which can be caused by the fluctuation of the field φ. The amplitude of the fluctuation $\delta\varphi$ has been estimated by means of the <u>quantum fluctuation</u> of massless scalar fields in the de Sitter space [56,57],

$$(\delta\varphi)^2 \sim H^2/16\pi^3 \quad , \tag{3.24}$$

although this evaluation seems quite questionable and is under the criticism.

In the new inflationary Universe scenario based on the SU(5) C-W potential (3.10) which can be approximated for $\varphi \leq H$ as

$$V_{eff}(\varphi) = V_{eff}(0) - \frac{\lambda}{2}\varphi^4 \tag{3.25}$$

with $\lambda \sim 1/2$. We refer the method of the evaluation of the time delay to references [22,23] which is too involved to present here. We finally obtain the almost scale-independent spectrum of the density perturbation

$$(\delta\rho/\rho)_H = (4\lambda/3\pi^3)^{1/2}\ \ln^{3/2}(Hk^{-1}) \quad , \tag{3.26}$$

where k is the wavelength of the perturbation. Although the spectrum is roughly scale invariant as desired, its magnitude is unfortunately of the order of 50 for a galactic scale, which is about 10^5 times too large to explain the structure of the observable Universe at present.

In order to overcome this problem, several scenarios are proposed, which realize flatter potential with smaller value of λ than the new inflationary Universe scenario based on the SU(5) C-W theory. We will discuss these scenarios in the next subsection.

The problem of the galaxy formation looks quite serious for the new inflationary Universe scenario. However the estimation of the fluctuation of the classical Higgs field $\delta\varphi$ does not seem completely justified, which assumed that quantum fluctuations can be directly realized as classical fluctuations. Thus this first difficulty may not be fatal and may be overcome somehow. Therefore in the author's opinion the new inflationary Universe scenario still seems to survive as one of the possible scenario which can solve the fundamental difficulties of the standard cosmology.

Here we discuss another problem of the new inflationary Universe scenario [24,25]. When this scenario was explained in the previous subsection, the evolution of the adjoint Higgs field Φ was restricted only in the $SU(3) \times SU(2) \times U(1)$ direction. However, the order parameter Φ obviously has the degrees of freedom of more than one. Thus we must investigate the evolution of Φ in the full 24-dimensional space of the adjoint representation of SU(5).

Taking into account of the many degrees of freedom of Φ, it turns out that the SU(5) C-W potential possesses additional local $SU(4) \times U(1)$ minima besides the global $SU(3) \times SU(2) \times U(1)$ minima. Furthermore these $SU(4) \times U(1)$ minima are more accessible for the order parameter Φ rather than the $SU(3) \times SU(2) \times U(1)$ minima. Thus the Higgs field probably settles down to the $SU(4) \times U(1)$ minima first and a succeeding strongly first-order phase transition to the global $SU(3) \times SU(2) \times U(1)$ minima causes the same difficulties as the original scenario.

In the § 4 we will investigate the second problem of the new inflationary Universe scenario in detail.

3-4 Other Versions of the Inflationary Universe Scenario

Now many authors who concern themselves with the inflationary cosmology are looking for new models which can overcome the difficulties of the new inflationary Universe scenario explained in the previous subsection. Here we will briefly report the status of some of the models which have recently received major attention in a critical way.

As we have seen in the previous subsection it might be a fatal defect of the new inflationary Universe based on the SU(5) C-W type symmetry breaking to give too large density perturbation, $(\delta\rho/\rho)_H \sim 50$, although the estimation of $(\delta\rho/\rho)_H$ has a questionable point. In order to reduce the magnitude of the density perturbation, it is the simplest way to adopt supersymmetric versions of the C-W potential. In supersymmetric theories (partial) cancellation of the contributions between bosons and fermions leads to a very small value of the parameter λ in eq.(3.26) which describes a slope of an effective potential near an origin as eq.(3.25). Thus we can obtain much smaller value of the density perturbation in those theories. This possibility was explicitly demonstrated by Albrecht et al. [26] in the context of the Witten-Dimopoulos-Raby inverted hierarchy model [58]. However it was shown that we cannot obtain efficient reheating and baryon asymmetry after inflation in this model [26,59,60]. So the first attempt of supercosmology failed.

These problems of reheating and baryon asymmetry seems to be solved if the inflation scale is moved up towards the Planck mass, $M_p \sim 10^{19}$ GeV (primordial inflation). This possibility was first suggested in the context of a globally supersymmetric theory by Ellis et al. [27]. Later Nanopoulos et al. considered primordial inflation in N = 1 supergravity coupled to matter [28]. In this model of the primordial inflation, the effective potential $V(z,z^*)$ of a scalar component of a chiral superfield Σ coupled to N = 1 supergravity is given by [61]

$$V(z,z^*) = \exp(zz^*/2)\left(2\left|\frac{dg}{dz} - \frac{1}{2}z^*g\right|^2 - 3|g|^2\right) , \tag{3.27}$$

where g(z) is some arbitrary function called superpotential,

$$g(z) = \mu^3 f(z) . \tag{3.28}$$

Here μ is some mass parameter and $f(z)$ is some dimensionless function of the field z. In this subsection we use the system of units in which $M_p/\sqrt{8\pi} = 1$ [61]. We write the function $f(z)$ as

$$f(z) = \sum_{n=1}^{m} \frac{\lambda_n}{n} z^n , \tag{3.29}$$

and assume $\lambda_0 \geqq 0$, $\lambda_1 \sim 0$. Then the effective potential of φ which is a real part of the field z is expressed as

$$V(\varphi) = \mu^6(\alpha + \beta\varphi + \gamma\varphi^2 + \delta\varphi^3 + \ldots) \tag{3.30}$$

where α, β, γ and δ are some functions of λ_n. This is the basic form for the potential we consider for inflation.

Similarly to the original and new inflationary Universe scenario, it is expected that due to high temperature effects, the field φ is initially zero, and the Universe undergoes the inflationary stage, while φ proceeds from the origin to the absolute minimum of $V(\varphi)$ at $\varphi = \varphi_0$ [28,62]. Thus we assume that the effective potential has an absolute minimum at $\varphi = \varphi_0 = 1$ of a vanishing value of the potential, $V(\varphi_0) = 0$, since the cosmological constant is apparently negligibly small at present. Furthermore it is also demanded that $g(\varphi_0) = 0$. The reason that we require the above condition is as follows. In the theory under consideration the mass of the gravitino is proportional to $g(\varphi_0)$ and is usually assumed to be small, $m_{3/2} \sim 10^2$GeV [40]. This is necessary to obtain the proposed solution of the gauge hierarchy problem in the context of supergravity [39].

However it can be proved that there is a deeper minimum $V(\tilde{\varphi})$ at some point $\tilde{\varphi}$ between $\varphi = 0$ and $\varphi = \varphi_0$, if we demand that $g(\varphi_0) = 0$ [30,60]. Therefore the above usual picture of the inflationary phase transition is impossible to realize if we take the gauge hierarchy problem in particle physics seriously.

There is a possibility, however, that the gauge hierarchy problem should be solved in other unknown mechanism in particle physics rather than supergravity. Let us consider this possibility for a moment and disconnect supergravity from the hierarchy problem. Anyway, from the astrophysical point of view, the gravitino with mass $m_{3/2} \sim 10^2$ GeV may cause the serious difficulty of the cosmology based on supergravity as was explained in subsection 2-2 (gravitino problem) [41,42].

If we did not assume that $g(\varphi_0) = 0$, it was shown that we could construct a model which realizes the primordial inflation in the context of supergravity [63].

We consider the simplest effective potential which can be obtained from eq.(3.27) by a proper choice of g(Z) [63],

$$V(\varphi) = 3\mu^6(1 - \alpha^2\varphi^2 + \frac{\alpha^4}{4}\varphi^4) \quad . \tag{3.31}$$

The minimum of $V(\varphi)$ lies at $\varphi = \varphi_0 = \sqrt{2}/\alpha$. In this model we can show that the density perturbation at the galactic scale has the magnitude

$$\delta\rho/\rho \sim \mu^3 \exp(10^2\alpha^2)/20\alpha \quad , \tag{3.32}$$

and the duration of the inflationary stage is given by

$$\Delta t \sim 6\mu^{-3}\alpha^{-2} \quad . \tag{3.33}$$

Thus a reasonable choice of parameters, e.g. $\alpha \sim 10^{-1}$, $\mu^3 \sim 10^{-4}$, gives the desirable value of the density perturbation $\delta\rho/\rho \sim 10^{-4}$ and the sufficient amplitude of inflation, $\Delta t \sim 600\ H^{-1} \sim (3 \times 10^{11}\ \mathrm{GeV})^{-1}$. Therefore we can say that the second attempt of primordial inflation based on supergravity can solve the problem of density perturbation, sacrificing the solution to the gauge hierarchy problem.

Now let us consider the primordial monopole problem in this scenario. In the first papers on primordial inflation [27,28] it was assumed that the monopole production due to the SU(5) symmetry breaking occurs after primordial inflation, and therefore one should find some other method to solve the primordial monopole problem in this scenario. However we note that the temperature of the Universe during the primordial inflation is much smaller than the critical temperature of the SU(5) phase transition, and a typical time $\tau \sim (10^{15}\ \text{GeV})^{-1}$, which is necessary for the SU(5) symmetry breaking to occur, is much shorter than the time of inflation $\Delta t \sim (10^{11}\ \text{GeV})^{-1}$. Therefore the SU(5) phase transition with monopole production takes place not after inflation, but long before the end of inflation [63]. Furthermore, since the field φ is very weakly coupled to each other and to all other matter fields, the reheating temperature T_R in this scenario is typically of the order of 10^{11} GeV [62,63], which is much smaller than the critical temperature of the SU(5) phase transition. Thus we can also solve the primordial monopole problem in this scenario [63].

So far we have just assumed that the Universe was initially in the symmetric state corresponding to a local minimum of the effective potential $V(\varphi,T)$. However it was pointed out that the above assumption is not valid in the primordial inflationary scenario in the context of supergravity [29].

A typical curvature of the effective potential in the high temperature limit is given by

$$m^2(T) = \frac{d^2V}{d\varphi^2} = CT^4 \tag{3.34}$$

where C is some combination of coupling constants: $C = \mu^6\alpha^2$ for the model (3.31). The characteristic time which is necessary for the field φ to roll down to the minimum of $V(\varphi)$ exceeds the time

$\tau \sim m^{-1}(T) \sim C^{-1/2}T^{-1}$. On the other hand the age of the Universe is given by eq.(2.12) for the case that $N \sim 200$

$$t \leq \frac{M_p}{50T^2} \quad . \tag{3.35}$$

By comparison of τ and t it follows that the field φ can be influenced by high temperature effects only at

$$T \leq T^* \sim 10^{-2}C^{1/2}M_p \quad . \tag{3.36}$$

Thus we conclude that the usual picture of the inflationary phase transition which assumes that the field φ initially lies in the symmetric state due to the high temperature effects, breaks down in the primordial inflationary scenario based on supergravity. Therefore the second attempt of supercosmology finally failed.

Recently Linde suggested a new idea of the inflationary Universe, supposing quite different picture of the phase transition from the usual one (chaotic inflation) [29]. At the Planck time $t_p \sim M_p^{-1}$ the value of the effective potential seems to be defined only with an accuracy of $O(M_p^4)$ due to the uncertainty principle. Therefore one may expect that in the hot Universe at $t \sim t_p$ any field configuration $\varphi(x)$ such that $V(\varphi) \leq M_p^4$, $(\partial_\mu \varphi)^2 \leq M_p^4$ can appear in any point x with an almost -independent probability.

Let consider a model which has a very flat effective potential, e.g. $V(\varphi) = \lambda\varphi^4/4$, $\lambda << 1$. We assume that there were domains in which the field φ was initially sufficiently homogeneous at a scale of $\ell \geq H^{-1}$ and sufficiently large $\varphi \geq M_p$. Since the field φ inside the domains varies very slowly and the temperature rapidly falls, the space-time inside the domains enters into the de Sitter stage with the Hubble constant

$$H = \left(\frac{2\pi\lambda}{3}\right)^{1/2} \frac{\varphi^2}{M_p} \quad . \tag{3.37}$$

Furthermore we can show that these parts of the Universe expand more than exp(70) times if $\varphi \geq 5M_p$. In this case each domain becomes a mini-Universe in which life may exist. This is the chaotic inflationary Universe scenario.era

Using the conception of the chaotic inflation, we can construct a model [30] which realizes the inflationary Universe scenario in the context of supergravity and, furthermore, satisfies the condition $g(\varphi_0) = 0$, which is necessary to solve the gauge hierarchy problem. In this model, although we do not explain here, we also obtain the desirable magnitude of the density perturbation for a reasonable choice of parameters. However in order to solve the gravitino problem, it is necessary to have a reheating temperature T_R as small as 10^9 GeV [64], which can suppress the gravitino production after inflation. This seems a rather stringent constraint on the model. Although it is still possible to generate the baryon asymmetry of the Universe at $T_R \sim 10^9$ GeV, it seems not to be very easy and it would be much better to make the gravitino heavy and harmless, $m_{3/2} \geq 10^4$ GeV [42].

Furthermore the large average value of the kinetic energy $(\partial_\mu\varphi)^2 \leq M_p^4$ suggests that the sufficiently homogeneous regions which can inflate are in fact extremely rare. Thus it is possible that these regions are sufficiently suppressed and would not be significant even after inflation.

In this subsection we have briefly surveyed the current status of model building efforts in the inflationary cosmology. Although the supersymmetric new inflatinary scenario and primordial inflationary scenario could lead to desirable magnitude of the density perturbation for the galaxy formation, it was pointed out that each of them suffered from specific serious difficulties. In the context of the chaotic inflation in supergravity we might be able to construct a realistic scenario of the inflationary cosmology. However this model still has unconvincing points to be clarified. Further investigation is needed to elaborate this scenario.

§4. Fate of the Order Parameter in the New Inflationary Universe Scenario

4-1 Difficulty in the SU(5) Phase Transition

So far we have surveyed the current status of the inflationary cosmology, which is expected to overcome some fundamental problems of the standard big bang cosmology. In this section, which is the main part of this thesis, we shall clarify the reheating mechanism of the new inflationary Universe scenario, taking account of a dissipative process, i.e. friction. Specifically we shall apply our method for inflationary phase transition to the new inflationary Universe scenario based on the SU(5) C-W potential in order to investigate the second difficulty of the model, which is concerned to many degrees of freedom of 24-dimensional adjoint Higgs field (see subsection 3-3).

One might consider that the new inflationary Universe scenario is no longer a candidate of realistic models for the inflationary cosmology, since this scenario gives the magnitude of the density perturbation which is about 10^5 times too large for galaxy formation. However, as was mentioned in the subsection 3-3, the evaluation of the amplitude of the density perturbation does not seem to be reliable as to the source of the perturbation in the early Universe, i.e. quantum fluctuation in the de Sitter space. Further investigations must be performed on this problem. Therefore the new inflationary Universe scenario based on the minimal SU(5) C-W potential is still alive as the simplest model which may realize the inflationary cosmology. Thus it is meaningful to elaborate this model by solving its another difficulty.

At the first stage of the investigation of the new inflationary Universe Scenario, we assumed that the evolution of the order parameter Φ goes along only in the $SU(3) \times SU(2) \times U(1)$ direction. However the order parameter Φ can actually take values in the full 24-dimensional space of the adjoint representation of SU(5). Taking account of the above degrees of freedom, we encounter one of the serious difficulties of the new inflationary Universe scenario based on the SU(5) C-W type symmetry breaking due to the existence of additional local minima in the $SU(4) \times U(1)$ direction in the C-W potential [24,25]. Since the $SU(4) \times U(1)$ minima are more accessible for the order parameter Φ than the global $SU(3) \times SU(2) \times U(1)$ minima, it seems likely that the order parameter Φ settles down at the $SU(4) \times U(1)$ minimum and the bubble succeedingly goes to the $SU(3) \times SU(2) \times U(1)$ minimum by a strongly first-order phase transition.

We do not have to consider the evolution of the full 24-dimensional Higgs field [24]. It is sufficient to consider the diagonal elements at the 24-dimensional Higgs field. The reason is as follows. At the instant that the bubble of nonzero Φ forms we assume that $\dot{\Phi} = 0$ and the initial value of Φ can be any arbitrary 5×5 Hermitian traceless matrix. It is possible to diagonalize Φ by means of a global gauge transformation which preserves $\dot{\Phi} = 0$. The evolution equation for the full matrix Φ does not change the diagonal form of Φ. The C-W potential for the adjoint Higgs field is then given by

$$V_{eff}(\Phi) = \frac{3g^4}{256\pi^2}\left[C\{\sum_{i=1}^{5}\Phi_i^4 - \frac{7}{30}(\sum_{i=1}^{5}\Phi_i^2)^2\}\right.$$

$$\left. + \sum_{i,j=1}^{5}(\Phi_i-\Phi_j)^4(\ln(\Phi_i-\Phi_j/\mu)^2 - 1/2)\right]$$

$$+ \frac{5625g^4}{1024\pi^2}\frac{\sigma^4}{2} \quad , \tag{4.1}$$

where

$$\Phi = \mathrm{diag}(\Phi_1,\Phi_2,\Phi_3,\Phi_4,\Phi_5) \tag{4.2}$$

with a constraint $\sum_{i=1}^{5}\Phi_i = 0$, C is a dimensionless arbitrary parameter, g is the gauge coupling constant and $\mu = 5\sigma/2$, ($\sigma \sim 4.5 \times 10^{14}$ GeV). We need the parameter C to be 0(1) in order that the C-W potential be a consistent approximation [20].

For the convenience of numerical computation which we shall carry out later, we introduce a linearly independent combination of Φ [35],

$$\tilde{\Phi}_i = \Phi_i + \frac{1}{1+\sqrt{5}}\sum_{j=1}^{4}\Phi_j \quad , \quad i=1\ldots4 \quad . \tag{4.3}$$

A contour map of the C-W potential on a plane ($\tilde{\Phi}_1 = \tilde{\Phi}_2 = \tilde{\Phi}_3 = X/\sqrt{3}$, $\tilde{\Phi}_4 = Y$) is shown in Fig.6 for the case that C = 1. The physically favorable SU(3) × SU(2) × U(1) minima lie on this plane. From simple analysis of the C-W potential (4.1) it turns out that lines from origin to the SU(3) × SU(2) × U(1) minima are ridges of the potential. Furthermore local minima appear in the SU(4) × U(1) direction for C < 15 and these become global minima for $C < -15\ln 1.5$. Since the slope of the potential is steepest in the SU(4) × U(1) direction, the SU(4) × U(1) minima are more accessible for order parameter Φ than the physical SU(3) × SU(2) × U(1) minima. Thus the bubble first rolls down in the SU(4) × U(1) direction. If the bubble settles down to one of the local SU(4) × U(1) minima for a while, a succeeding first-order phase transition to the true SU(3) × SU(2) × U(1) minimum causes similar inhomogeneities of the Universe as happened in the old scenario [15,16].

The time evolution of the order parameter has been numerically analysed by Sato and Kodama [35] in the multi-component case. The equation of motion for the order parameter $\tilde{\Phi}$ is given by

$$\ddot{\tilde{\Phi}}_i + 3H\dot{\tilde{\Phi}}_i + V^{(1)}_{\mathrm{eff},i} + \eta|\tilde{\Phi}_i|\dot{\tilde{\Phi}}_i = 0 \quad , \tag{4.4}$$

where $V^{(1)}_{\mathrm{eff},i}$ is the first derivative of the C-W potential (4.1) with respect to $\tilde{\Phi}_i$ and a friction term $\eta|\tilde{\Phi}_i|\dot{\tilde{\Phi}}_i$ is introduced in order to convert the energy of the order parameter into radiation energy. The second term appears due to the expansion of the Universe and the Hubble parameter H is given by

$$H = \left[\frac{8\pi G}{3}(\rho_r + \rho_\Phi)\right]^{1/2} \quad , \tag{4.5}$$

where we have assumed that the Universe is spatially flat, which is adequate in the early Universe even if it is not flat exactly. The radiation energy density ρ_r and the energy density of the

Higgs field ρ_{Φ} change with time according to equations

$$\dot{\rho}_r + 4H\rho_r = \sum_{i=1}^{4} \eta |\tilde{\Phi}_i| \dot{\tilde{\Phi}}_i^2 \tag{4.6}$$

and

$$\rho_{\Phi} = \frac{1}{2} \sum_{i=1}^{4} \dot{\tilde{\Phi}}_i^2 + V_{eff}(\Phi) \tag{4.7}$$

As was mentioned before, the eq.(4.6) means that the vacuum energy which is stored as the energy of the order parameter $\tilde{\Phi}_i$ is liberated to the radiation energy through the friction term which characterizes dissipative effects. An unsatisfactory point in Eq.(4.6) is that the dissipative term has been introduced by hand. We shall see later how it comes arise from the first principle.

The initial conditions for numerical computation are as follows. At the first stage of GUT phase transition, the bubble in which the Higgs field takes a value $\tilde{\Phi}_i$ forms by means of a tunneling. We parametrize the Higgs field $\tilde{\Phi}_i$ using four quantities: one is the norm of the Higgs field $|\tilde{\Phi}| = \sqrt{\sum \tilde{\Phi}_i^2} = \sqrt{\sum \Phi_i^2}$ and the others are angles α, θ, φ of the vector $(\tilde{\Phi}_1, \tilde{\Phi}_2, \tilde{\Phi}_3, \tilde{\Phi}_4)$ in the four dimensional $\tilde{\Phi}_i$ space. Here α represents the deviation angle from the SU(4) $\times$ U(1) direction $\tilde{\Phi}_1 = \tilde{\Phi}_2 = \tilde{\Phi}_3 = \tilde{\Phi}_4 > 0$ on the plane $\tilde{\Phi}_1 = \tilde{\Phi}_2 = \tilde{\Phi}_3$ on which the physically favorable SU(3) $\times$ SU(2) $\times$ U(1) minimum lies, and θ and φ represent the deviation angles from this plane. We assume that the initial magnitude of the order parameter $|\tilde{\Phi}|_0 = 0.2H = 1.5 \times 10^9$ GeV, $\dot{\tilde{\Phi}}_i = 0$ and restrict θ and φ in the very narrow range $|\theta| < 10^{-4}$ and $|\varphi| < 10^{-4}$ for a convenience of the numerical computation.

Some typical results of the numerical analysis are shown in Fig.7 for the case that $C = 1$. For any choice of the initial angle α $(-0.29\pi < \alpha < 0.21\pi)$ and the magnitude of the friction coefficient, the order parameter rolls down to the $SU(4) \times U(1)$ minimum at first. The evolution of the order parameter after the arrival to that minimum is crucially determined by the magnitude of the friction η. In the case that the friction coefficient is too large, according to Sato and Kodama $\eta > 10^{-2}$, the order parameter settles down to the $SU(4) \times U(1)$ minimum as illustrated in Fig.7(a). Thus the new inflationary Universe scenario has the same serious difficulty as the old one due to the first-order phase transition from the $SU(4) \times U(1)$ minimum to the $SU(3) \times SU(2) \times U(1)$ minimum. On the other hand if the friction is too small, $\eta < 2 \times 10^{-3}$, the order parameter travels around the minima of C-W potential many times. For a very small deviation of the initial angle α, θ and φ, the order parameter will finally get a different minimum. A typical result of numerical computation is shown in Fig.7(b). This result means that a coherent region, which is formed by nucleation of bubbles or spinodal decomposition in the early stage $|\tilde{\Phi}| < H$, is fragmented into many $SU(3) \times SU(2) \times U(1)$ and $SU(4) \times U(1)$ minima by the fluctuation of the order parameter associated with the initial state. It follows that the large scale inhomogeneities of the Universe appear. In order to avoid these difficulties the magnitude of the friction coefficient must lie in a suitable range, $2 \times 10^{-3} < \eta < 10^{-2}$. In this case, as shown in Fig.7(c), the order parameter can goes through the SU(4) × U(1) minimum and can terminate at the SU(3) × SU(2) × U(1) minimum without a tunnelling for some range of initial angles.

Thus the evaluation of the magnitude of the friction coefficient is essential in order to decide weather the new inflationary Universe scenario does work or not.

4-2 Evaluation of the Friction Term : A Simple Model

As explained in the previous subsection, the magnitude of the friction η, which appears in the equation of motion for the Higgs field $\tilde{\Phi}_i$, is crucial for the fate of the Higgs field $\tilde{\Phi}_i$. The friction was first phenomenologically introduced by Albrecht et al. [34] in order to realize the thermalization of the latent heat and show that the GUT phase transition can complete itself in the new inflationary Universe scenario. Then some authors used it to analyse the time evolution of the Higgs field [24,35]. Since, however, they have introduced it by hand, they could not tell the strength of the friction from the grand unified theory. Although Abbot. et al. [21] have discussed the particle production and reheating of the Universe due to the damped oscillation of the order parameter around a minimum of C-W potential, unfortunately their method could not predict the magnitude of the friction.

It is a work of the present author and Hosoya [31] that has dealt with the derivation of the evolution equation for the order parameter including the friction term at finite temperature for the first time. They considered a small deviation from the thermal equilibrium caused by the rolling down motion of the Higgs field and evaluated the friction coefficient by means of linear response theories [65,66]. However that formalism cannot be straightforwardly applied to the early stage of the thermalization process, since at that time, the Universe was nearly at zero temperature due to the de Sitter expansion so that the thermal friction vanishes out. Even if we take account of the Hawking radiation which is intrinsic to the de Sitter space [57,67,68], the Hawking temperature is fairly low compared to the GUT mass scale so that its effect is negligible. Thus we should consider the zero temperature situation.

Recently Morikawa and Sasaki [32] have estimated the value of the friction coefficient for the $\lambda\phi^4$ theory at zero temperature. We explain their method here. Consider a simple model of scalar field ϕ with Lagrangian

$$L = \frac{1}{2} (\partial_\mu \phi)^2 - V(\phi) \quad , \tag{4.8}$$

where the potential $V(\phi)$ is supposed to be convex everywhere. Although we discuss only the simplest case here, the extension to other models, e.q. SU(5) GUT, is straightforward and will be considered in the next subsection. Furthermore we consider the case in flat spacetime, since gravitational effect does not qualitatively change the feature of dissipative processes. We split the scalar field ϕ as

$$\phi = \Phi + \varphi \quad , \tag{4.9}$$

where φ is a quantum fluctuation around the time dependent c-number part Φ which walks about in the C-W potential. Performing a canonical transformation to change a dynamical variable from ϕ to φ, we obtain transformed Hamiltonian [31],

$$H = H_\Phi + H_1 + H_\varphi + J\varphi \quad , \tag{4.10}$$

where

$$H_\Phi = \frac{1}{2}\dot{\Phi}^2 + \frac{1}{2}(\partial_k\Phi)^2 + V(\Phi) \quad , \tag{4.11a}$$

$$H_1 = \varphi(\partial^2\Phi + V^{(1)}(\Phi)) \quad , \tag{4.11b}$$

$$H_\varphi = \frac{1}{2}\pi^2 + \frac{1}{2}(\partial_k\varphi)^2 + \frac{1}{2}V^{(2)}(\Phi)\varphi^2$$

$$+ \frac{1}{3!}V^{(3)}(\Phi)\varphi^3 + \frac{1}{4!}V^{(4)}(\Phi)\varphi^4 \quad . \tag{4.11c}$$

Here H_Φ and H_φ are the c-number part and the quantumpart of the Hamiltonian and $J\varphi$ is a source term. We keep the linear term (4.11b) in the fluctuation field, since Φ is chosen as the order parameter rather than a classical solution. The term $H_1 + J\varphi \equiv F\varphi$ cancels the tadpole contribution as shown in Fig.8. So we can uniquely determine the source term F (or J).

Dissipative effect, as we shall see, arises from the explicit time dependence of mass and coupling of quanta φ due to the time development of the Higgs field Φ. The order parameter Φ is defined as an expectation value of ϕ,

$$\Phi = \langle\phi\rangle \quad . \tag{4.12}$$

We note that the bracket <...> contains not only usual quantum effects but also effect caused by explicit time dependence of Φ. Taking the expectation value of the field equation for ϕ,

$$\partial^2\phi + V^{(1)}(\phi) = 0 \quad , \tag{4.13}$$

we obtain the equation for the order parameter

$$\ddot{\Phi} + V^{(1)}(\Phi) + \frac{1}{2} V^{(3)}(\Phi)\langle\varphi^2\rangle + \frac{1}{3!} V^{(4)}(\Phi)\langle\varphi^3\rangle = 0 \quad , \tag{4.14}$$

where the potential term has been expanded around Φ. Here we evaluate the second term $\langle\varphi^2\rangle$ in eq.(4.14), since this term gives a dominant contribution in the perturbative expansion. That is, this is the lowest order term in the coupling constant which is contained in the potential $V(\phi)$.

Let us split the Hamiltonian (4.10) into two parts;

$$H(t) = H_0 + \bar{H}(t) \quad , \tag{4.15}$$

where

$$H_0 = H(t)\big|_{t=t_0} \quad . \tag{4.16}$$

The argument t denotes the explcit time dependence due to the time evolution of Φ. The former part H_0 is the Hamiltonian which we would obtain if the time development of Φ were frozen at $t = t_0$, and the latter part $\bar{H}$ is the rest of H. We can evaluate the effects of the explicit time dependence of $H(t)$ by means of the same method as the usual perturbative calculation in the interaction picture, regarding $\bar{H}(t)$ as the interaction Hamiltonian,

$$\langle\varphi^2(t)\rangle = \langle U^\dagger(t,t_0)\hat{\varphi}^2(t)U(t,t_0)\rangle_0 \quad , \tag{4.17}$$

where

$$U(t,t_0) = T\exp\left[-i\int_{t_0}^{t}dt'\int d^3x'\ \bar{H}(\hat{\varphi}(t'))\right] \quad . \tag{4.18}$$

Here $\hat{\varphi}$ is the field operator whose dynamics is determined by H_0 and brackets $\langle\ldots\rangle_0$ denote an expectation value by means of H_0. In the lowest order with respect to $\bar{H}$, i.e. a linear response approximation, we obtain

$$\begin{aligned}
\langle\varphi^2(t)\rangle &= \langle\hat{\varphi}^2(t)\rangle_0 \\
&\quad - i\int_{t_0}^{t}dt'\int d^3x'(\langle\hat{\varphi}^2(t)\hat{\varphi}^2(t')\rangle_0 - \langle\hat{\varphi}^2(t')\hat{\varphi}^2(t)\rangle_0) \\
&\qquad \times \frac{1}{2}\left[V^{(2)}(\Phi(t')) - V^{(2)}(\Phi(t_0))\right] \\
&= \langle\hat{\varphi}^2(t)\rangle_0 \\
&\quad + \left[V^{(2)}(t) - V^{(2)}(t_0)\right]\times\int_{t_0}^{t}dt'\int d^3x'\,\mathrm{Im}\langle T\hat{\varphi}^2(t)\hat{\varphi}^2(t')\rangle_0 \\
&\quad - \int_{t_0}^{t}dt'\int d^3x'\left[V^{(2)}(t) - V^{(2)}(t')\right]\times\mathrm{Im}\langle T\hat{\varphi}^2(t)\hat{\varphi}^2(t')\rangle_0 \quad .
\end{aligned} \tag{4.19}$$

The second and third terms do not appear in the usual perturbation theory and the second term is divergent. If we adopt the idea of time-dependent mass renormalization and assume the mass squared of quanta $\hat{\varphi}$ at time t is $V^{(2)}(\Phi(t))$ rather than $V^{(2)}(\Phi(t_0))$, the divergence of the first and second terms can be completely absorbed by appropriate counter terms. The net effect of these terms on the right-hand-side of eq.(4.19) is the quantum correction to the potential $V(\phi)$. It is the last term on the right-hand-side of eq.(4.19) that gives dissipative effects.

We assume that the order parameter varies slowly in comparison with the characteristic time scale of the correlation function. If and only if the above assumption is satisfied, the lowest order approximation with respect to the explicit time dependence of the Hamiltonian is valid. Furthermore we approximate a potential term in the last term of eq.(4.19) as

$$V^{(2)}(\Phi(t)) - V^{(2)}(\Phi(t')) \sim V^{(3)}(\Phi(t))\dot{\Phi}(t)(t-t') \quad ,$$

and replace the lower limit of the time integral by $-\infty$, since the quantum correlation vanishes sufficiently fast. Then the last term of eq.(4.19) is given by

$$\langle\hat{\varphi}^2\rangle_d = - V^{(3)}(\Phi(t))\dot{\Phi}(t)$$

$$\times \int_{-\infty}^{t} dt' \int d^3x'(t-t')\,\mathrm{Im}\langle T\hat{\varphi}^2(t)\hat{\varphi}^2(t')\rangle_0 \quad . \qquad (4.20)$$

Inserting eqs.(4.19) and (4.20) to eq.(4.14), we obtain the equation of motion for the order parameter Φ,

$$\ddot{\Phi} + V_{eff}^{(1)}(\Phi) + \eta(\Phi)\dot{\Phi} = 0 \quad , \tag{4.21}$$

where

$$\begin{aligned} V_{eff}^{(1)}(\Phi) &= V^{(1)}(\Phi) \\ &\quad + \frac{1}{2} V^{(3)}(\Phi)\Big[\langle\hat{\varphi}^2\rangle + (V^{(2)}(t) - V^{(2)}(t_0)) \\ &\qquad \times \int_{-\infty}^{t} dt' \int d^3x' \, \mathrm{Im}\langle T\hat{\varphi}^2(t)\hat{\varphi}^2(t')\rangle_0 \Big] \end{aligned} \tag{4.22}$$

and

$$\eta(\Phi) = -\frac{1}{2}(V^{(3)}(\Phi))^2 \int_{-\infty}^{t} dt' \int d^3x' (t-t') \, \mathrm{Im}\langle T\hat{\varphi}^2(t)\hat{\varphi}^2(t')\rangle_0 \; . \tag{4.23}$$

We do not evaluate the effective potential and the friction coefficient η. That will be performed in the next subsection in the case of SU(5) GUT.

4-3 Evaluation of the Friction Term: SU(5) GUT

In this subsection by applying the method explained in the previous subsection we shall derive the evolution equation for $\tilde{\Phi}_i$ which is the order parameter of the SU(5) GUT phase transition, and evaluate the friction term which appears in the obtained equation [33]. Let us consider the minimal SU(5) Lagrangian,

$$L = -\frac{1}{2}\,\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu} + \sum_{k=1}^{3}\left(i\overline{\psi}^k(10)\not{\nabla}\psi^k(10) + i\overline{\psi}^k(5)\not{\nabla}\psi^k(5)\right)$$

$$+ \frac{1}{2}\,\mathrm{Tr}\,\nabla_\mu\phi\nabla^\mu\phi + \nabla_\mu H^\dagger\nabla^\mu H - \overline{V}(\phi,H) + L_{\mathrm{Yukawa}} \quad , \qquad (4.24)$$

where ϕ and H are 24-dimensional and 5-dimensional Higgs fields, $\psi(10)$ and $\psi(5)$ are 10-dimensional and 5-dimensional fermions, k is a generation index and $\not{\nabla}$ is the covariant derivative. We choose the unitary gauge for convenience of calculation, in which the adjoint Higgs field becomes diagonal. Since dissipative effects arise due to the time dependence of mass of quanta, as explained in the previous subsection, massless gauge bosons which correspond to unbroken gauge symmetry do not contribute to the friction term. Thus these degrees of freedom need not be considered.

We split the diagonal part of the adjoint Higgs field into c-number and fluctuating parts,

$$\tilde{\phi}_i = \langle\tilde{\phi}_i\rangle + \tilde{\varphi}_i = \tilde{\Phi}_i + \tilde{\varphi}_i \quad , \quad i=1...4 \quad , \qquad (4.25)$$

where wavy line over the field indicates the linearly independent combination defined in eq.(4.3). The canonically transformed Hamiltonian is given by

$$H = H_{\mathrm{gauge}} + H_\Phi + H_1 + H_\varphi + H_H + H_{\mathrm{fermion}} + H_{\mathrm{Yukawa}} \quad , \qquad (4.26)$$

where

$$H_{gauge} = \frac{1}{2}\,\Omega^a(\Phi+\varphi)^{-1}\,(\partial_k P^{ak} + gC^{abc}\,P^{bk}\,A^{ck})^2$$

$$+\,(P^{ak})^2 + \frac{1}{4}\,(F^a_{kl})^2 + \frac{1}{2}\,\Omega^a(\Phi+\varphi)(A^{ak})^2 \quad , \qquad (4.27a)$$

$$H_\Phi = \frac{1}{2}\,(\dot{\tilde{\Phi}}_i) + \frac{1}{2}\,(\partial_k\tilde{\Phi}_i)^2 + V(\Phi) \quad , \qquad (4.26b)$$

$$H_1 = \tilde{\varphi}_i(\partial^2\tilde{\Phi}_i + \bar{V}(\Phi,H)) \quad , \qquad (4.27c)$$

$$H_\varphi = \frac{1}{2}\,(\tilde{\pi}^i_\varphi)^2 + \frac{1}{2}\,(\partial_k\tilde{\varphi}_i)^2 + \frac{1}{2}\,V^{(2)}_{,ij}(\Phi)\tilde{\varphi}_i\tilde{\varphi}_j$$

$$+\,\frac{1}{3!}\,V^{(3)}_{,ijk}(\Phi)\tilde{\varphi}_i\tilde{\varphi}_j\tilde{\varphi}_k + \frac{1}{4!}\,V^{(4)}_{,ijkl}(\Phi)\tilde{\varphi}_i\tilde{\varphi}_j\tilde{\varphi}_k\tilde{\varphi}_l \quad , \qquad (4.27d)$$

$$H_H = \pi^\dagger_H\pi_H + \nabla_k H^\dagger\nabla_k H + V(\Phi,H) + V(H) + \frac{1}{2}\,V^{(2)}_{,ij}(\Phi,H)\tilde{\varphi}_i\tilde{\varphi}_j \quad . \qquad (4.27e)$$

Here the Higgs potential has been divided into three parts.

$$\bar{V}(\phi,H) = V(\phi) + V(\phi,H) + V(H) \quad , \qquad (4.28)$$

and

$$\Omega^a(\phi) = \frac{g^2}{2}|\phi_i - \phi_j|^2 \quad , \quad a=(\alpha,i,j) \quad , \quad \alpha=1,2 \quad , \quad i,j=1\ldots4 \quad . \qquad (4.29)$$

The lower indices $i\ \ldots$ after comma represent the derivatives with respect to $\tilde{\Phi}_i\ \ldots$, a is a group index and g is the SU(5) gauge coupling constant.

Evolution equation for the order parameter $\tilde{\Phi}_i$ can be derived by taking the expectation value of equation of motion for $\tilde{\phi}_i$,

$$\partial^2\tilde{\Phi}_i - \frac{1}{2}(\mu^a)^2_{,i}(\Phi)\langle A^a_\mu A^{a\mu}\rangle - \frac{1}{2}(\mu^a)^2_{,ij}(\Phi)\langle\tilde{\varphi}_j A^a_\mu A^{a\mu}\rangle$$

$$+ \langle\overline{V}^{(1)}_{,i}(\Phi,H)\rangle + \frac{1}{2}V^{(3)}_{,ijk}(\Phi)\langle\tilde{\varphi}_j\tilde{\varphi}_k\rangle + \frac{1}{3!}V^{(4)}_{,ijkl}(\Phi)\langle\tilde{\varphi}_i\tilde{\varphi}_j\tilde{\varphi}_l\rangle = 0 \quad , \tag{4.30}$$

where $(\mu^a(\Phi))^2$ is a mass squared of gauge boson,

$$(\mu^a(\Phi))^2 = \Omega^a(\Phi) = \frac{g^2}{2}|\Phi_i - \Phi_j|^2 \quad ,$$

$$a=(\alpha,i,j) \quad , \quad \alpha=1,2 \quad , \quad i,j=1\ldots4 \quad . \tag{4.31}$$

In order to evaluate the expectation values of field operators in eq.(4.30), we split the Hamiltonian for the SU(5) GUT in unitary gauge (4.26) into two parts as already prescribed in eq.(4.15) and (4.16). Then we can calculate them which contain the effects due to the explicit time dependence of the Hamiltonian $\overline{H}(t)$ by means of the perturbative expansion, regarding H(t) as the interaction Hamiltonian. We obtain the evolution equation for the order parameter $\tilde{\Phi}_i$ in the lowest order with respect to $\overline{H}$,

$$\ddot{\tilde{\Phi}}_i + 3H\dot{\tilde{\Phi}}_i + V^{(1)}_{eff,i}(\Phi) + \eta_{ij}(\Phi)\dot{\tilde{\Phi}}_j = 0 \quad , \tag{4.32}$$

where $V^{(1)}_{eff}$ is the C-W potential for the adjoint Higgs field in the SU(5) GUT whose explicit form expressed in eq.(4.1), and

$\eta_{ij}(\Phi)$ is a friction matrix,

$$\eta_{ij}(\Phi)$$

$$= -\frac{1}{2}(\mu^a)^2_{,i}(\mu^b)^2_{,j}\int_{-\infty}^{t}dt'\int d^3x'(t-t')\mathrm{Im}\langle T(A^a_\mu(x,t))^2(A^b_\nu(x't'))^2\rangle_0$$

$$-\frac{1}{2}V^{(3)}_{,ilk}V^{(3)}_{,jmn}\int_{-\infty}^{t}dt'\int d^3x'(t-t')\mathrm{Im}\langle T\,\tilde\varphi_l\tilde\varphi_k(x,t)\tilde\varphi_m\tilde\varphi_n(x',t')\rangle_0$$

$$-2m^2_{k,i}\,m^2_{l,j}\int_{-\infty}^{t}dt'\int d^3x'(t-t')\mathrm{Im}\langle T\,\overline{H}_kH_k(x,t)\overline{H}_lH_l(x',t')\rangle_0 \quad . \tag{4.33}$$

Here m_k^2 is a mass squared of 5-dimensional Higgs field,

$$m_k^2\,\overline{H}_k\,H_k = V(\Phi,H) \quad , \tag{4.34}$$

and the second term in eq.(4.32) appears due to the expansion of the Universe. The first term in eq.(4.33) is the contribution of the massive gauge bosons, the second and third terms are that of 24-dimensional and 5-dimensional Higgs, respectively. Since the Higgs coupling constants are generally of the order of g^4 in order for the consistency of the C-W potential [20], the dominant contribution to the friction matrix (4.33) comes from the gauge bosons and we can neglect the second and third terms in eq.(4.33).

In order to evaluate the correlation function in eq.(4.33), we have to use the full propagator for the gauge boson [69] (Fig.9),

$$D^{ab}_{\mu\nu}(x-x') = -\delta^{ab} \int \frac{d^4k}{(2\pi)^4} \left[\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \frac{1}{k^2-(\mu^a)^2+i\pi^a} \right.$$

$$\left. + \left(1 - \frac{k^2}{(\mu^a)^2}\right) \frac{k_\mu k_\nu}{k^2} \frac{1}{k^2-(\mu^a)^2+i\varepsilon} \right] e^{-ik(x-x')} , \tag{4.35}$$

where Π^a is a vacuum polarization (Fig.10). Integration over k^0 gives

$$D^{ab}_{\mu\nu}(x-x') = i\delta^{ab} \int \frac{d^3k}{(2\pi)^3} \left[\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \Bigg|_{k_0=\omega-i\Gamma^a} \times \frac{e^{-i(\omega-i\Gamma^a)|t-t'|}}{2(\omega-i\Gamma^a)} \right.$$

$$\left. + \left(1 - \frac{k^2}{(\mu^a)^2}\right) \frac{k_\mu k_\nu}{k^2} \Bigg|_{k_0=\omega} \times \frac{e^{-i\omega|t-t'|}}{2(\omega-i\varepsilon)} \right] e^{i\vec{k}(\vec{x}-\vec{x}')} , \tag{4.36}$$

where $\omega = |\vec{k}|$ and width Γ^a is given by

$$\Gamma^a = \frac{1}{2} \,\mathrm{Im}\, \Pi^a = \frac{g^2}{4\pi} \frac{(\mu^a)^2}{\omega} . \tag{4.37}$$

The width Γ^a is an imaginary part of the vacuum polarization Π^a and the decay rate of heavy gauge boson to light fermion pairs. On the other hand, the inverse decay is forbidden due to the enormous temperature drop by the de Sitter expansion of the Universe. This time-irreversibility is essential for dissipative process [33]. As explained in the previous subsection, one of the necessary conditions for the dissipation is the heavy particle production due to the explicit time dependence of its mass. However that is not enough to cause dissipative effects. Decay of produced particles which is out of equilibrium is also necessary for appearance of the friction and generation of entropy at zero temperature situation#). Since decay processes are contained in the imaginary part of vacuum polarization Π, i.e. Γ, we must employ full propagators for evaluation of the friction coefficients in order to include such processes sufficiently. Furthermore we may neglect the finite part of the real part of Π.

Using the full propagator for the gauge boson (4.36), we finally obtain an expression for the friction matrix,

$$\eta_{ij}(\Phi) = \frac{3g^2}{2^9\pi^2} \frac{(\mu^a)^2_{,i}(\mu^a)^2_{,j}}{\mu^a} \quad , \quad i,j=1\ldots4 \quad , \tag{4.38}$$

where the group index a is summed up.

For example the value of η_{ij} at the SU(3) $\times$ SU(2) $\times$ U(1) minimum, $\Phi = \sigma \times \mathrm{diag}(1,1,1,-3/2,-3/2)$ is given by

$$\eta_{ij} = |\tilde{\Phi}| \times \begin{pmatrix} 4.4 & 1.4 & 1.4 & -0.6 \\ 1.4 & 4.4 & 1.4 & -0.6 \\ 1.4 & 1.4 & 4.4 & -0.6 \\ -0.6 & -0.6 & -0.6 & 5.0 \end{pmatrix} \times 10^{-4} \tag{4.39}$$

#) At finite temperature, scattering processes with quanta which are produced by thermal effects can work for that purpose in place of decay processes [31,69].

and four eigenvalues of the matrix is $(7.6,\ 4.5,\ 3.0,\ 3.0) \times |\tilde{\Phi}| \times 10^{-4}$, where $|\tilde{\Phi}| = \sum\tilde{\Phi}_i^2 = \sqrt{15/2}\ \sigma$. Here we have used the gauge coupling constant at GUT mass scale, $g^2 \simeq 0.3$, since each coupling comes from the gauge boson mass term or the decay rate of the gauge boson. We note that the magnitude of the components of the friction matrix η_{ij} does not change very much and all the eigenvalues are positive at any point of the four dimensional $\tilde{\Phi}_i$ space.

4-4 Time Evolution of the Order Parameter and Thermalization of the Universe

Since the form of the friction term which we have obtained is different from that assumed by Sato and Kodama [35], an appropriate range of the friction coefficient which they have found can not directly apply to our results. We numerically solve the differential equations [33]

$$\ddot{\tilde{\Phi}}_i + 3H\dot{\tilde{\Phi}}_i + V^{(1)}_{eff,i}(\Phi) + \eta_{ij}(\Phi)\dot{\tilde{\Phi}}_i = 0 \quad , \tag{4.40}$$

$$\dot{\rho}_r + 4H\rho_r = \eta_{ij}(\Phi)\dot{\tilde{\Phi}}_i\dot{\tilde{\Phi}}_j \quad , \tag{4.41}$$

where

$$H = \left(\frac{8\pi G}{3}\,(\rho_r + \rho_\Phi)\right)^{1/2} \quad , \tag{4.42}$$

$$\rho_\Phi = \frac{1}{2}\sum_{i=1}^{4}\dot{\tilde{\Phi}}_i^2 + V_{eff}(\Phi) \quad . \tag{4.43}$$

Here $\eta_{ij}(\Phi)$ is the friction matrix (4.38) which has been obtained in previous subsection, and the precise form of the C-W potential $V_{eff}(\Phi)$ is expressed in eq.(4.1).

We adopt the same initial conditions as the ones which have been used in the numerical computation in subsection 4-1. The initial magnitude of the order parameter is taken to be $|\tilde{\Phi}|_0 = 0.2H = 1.5 \times 10^9$ GeV. In the first couse of numerical calculation, we take $\theta = \varphi = 0$ and $-0.29\pi < \alpha < 0.21\pi$ as the initial angle in order that the order parameter $\tilde{\Phi}_i$ keeps lying on the (X,Y)-plane on which $\tilde{\Phi}_1 = \tilde{\Phi}_2 = \tilde{\Phi}_3$ in the four dimensional $\tilde{\Phi}_i$ space.

One of the results of numerical computation is shown in Fig.11. The obtained magnitude of the friction coefficient is too small so that the order parameter $\tilde{\Phi}_i$ travels around the minima of the C-W potential many times. Thus the order parameter $\tilde{\Phi}_i$ settles down to a different minimum for a very small deviation of initial angle α. Furthermore if we lift the order parameter $\tilde{\Phi}_i$ slightly off the (X,Y)-plane, e.g. we take $|\theta| \sim 10^{-5}$ or $|\varphi| \sim 10^{-5}$ as initial angle, it gets out of the plane after visiting several minima of C-W potential and finally it terminates at a physically unfavorable minimum which does not lie on the (X,Y) plane.

Due to the chaotic behavior of the time evolution of the order parameter, a coherent region, which is formed by nucleation of bubbles or spinodal decomposition in the early stage of GUT phase transition, is fragmented into many minima by the fluctuation of the order parameter associated with the initial state. We conclude that the large scale inhomogeneities appear, which conflict with present observation.

Here we discuss the thermalization of the Universe during the time evolution of the order parameter $\tilde{\Phi}$ around bottoms of the C-W potential. The gauge bosons, which are produced by the explicit time dependence of the Hamiltonian, decay into light fermion pairs. If reaction rates among the light particles (i.e. massless gauge boson, Higgs doublet and fermions) are larger than the expansion rate of the Universe, these particles form a plasma of massless particles which is in (near) thermal equilibrium. Furthermore if interactions between heavy particles (i.e. massive gauge boson, φ , $H^{\pm 1/3}$) is sufficiently rapid, these heavy particles would come into the thermal equilibrium. The above processes would put forward the liberation of the vacuum energy originally in the form of the C-W potential energy into the radiation energy and also generate entropy.

Let us define the effective temperature T_{eff} by the fraction ε of the radiation energy density ρ_r to the vacuum energy V_0 at the beginning of the GUT phase transition:

$$\varepsilon = \frac{\rho_r}{V_0} = 1.5 \times 10^3 \left(\frac{T_{eff}}{M_X}\right)^2$$

or

$$T_{eff} = 0.16 \times \varepsilon^{1/4} M_X \quad . \tag{4.44}$$

We have said this temperature as an "effective" one, since the temperature has a meaning only if the radiation is in thermal equilibrium. It should be noted that the effective temperature increases to 0.16 M_x at most due to the smallness of the Higgs coupling constants. A typical value of the fractions ε obtained by our numerical computation is shown in Fig.11 at several representative points along the trajectory of $\tilde{\Phi}$. At the first stage for example, $|\tilde{\Phi}| \sim 10^{10}$ GeV, the fraction $\varepsilon \sim 10^{-19}$ and the effective temperature $T_{eff} \sim 3 \times 10^{-6}$ M_x. When $\tilde{\Phi}$ first gets to the SU(4) × U(1) minimum, $\varepsilon \sim 10^{-2}$ and $T_{eff} \sim 5 \times 10^{-2}$ M_x. After a few oscillations around the SU(4) × U(1) minimum, $\tilde{\Phi}$ goes to the neighboring SU(3) × SU(2) × U(1) minimum in most cases. At that time, $\varepsilon \sim 10^{-1}$ and $T_{eff} \sim 0.9 \times 10^{-1}$ M_x. In the last two cases in which the order parameter is near a minimum of the C-W potential, thermal effects may not be negligibly small.

So far we have investigated the time evolution of the GUT phase transition in the SU(5) new inflationary Universe. Unfortunately it has been shown that the value of the friction term is too small so that the time development of the order parameter becomes chaotic, and this leads to large scale inhomogeneities of the Universe. However it should be noted that the effective temperature of the Universe increases up to $T_{eff} \sim (5 \times 10^{-2} - 0.9 \times 10^{-1})$ M_x in the final stage of the inflationary phase transition. So finally we shall discuss possible thermal effects on the friction, which may lead to the successful termination of the GUT phase transition in the new inflationary Universe.

First let us consider the equilibrium condition in the final stage of the rolling-down phase transition. We assume that the (quasi) thermal equilibrium is realized, if a reaction rate γ is larger than the expansion rate of the Universe $H(= 0.8 \times 10^{10}$ GeV for the SU(5) inflationary scenario). In the case of light particles, i.e. massless gauge bosons, fermions, etc., the reaction rate is approximately given by

$$\gamma_{\ell\ell} \sim \sigma_{\ell\ell} n_\ell v \sim 10\alpha^2 T \tag{4.45}$$

on the dimensional ground. Here $\sigma_{\ell\ell}$ is a typical cross section for their collisions with n_ℓ and v being their number density and relative velocity, we take the coupling constant to be $\alpha = g^2/4\pi \sim 1/41$. Hence if $\gamma_{\ell\ell} > H$, i.e. if $T > 1.3 \times 10^{12}$ GeV for light particles, the system of light particles is in the thermal equilibrium. From eq. (4.44) we find that the equilibrium condition for light particles is $\varepsilon > \varepsilon_{eq} \equiv 1.3 \times 10^{-7}$. The numerical computation shows that this condition has already been met before the first arrival of the Higgs field at the SU(4) × U(1) minima.

Heavy particles, i.e. massive gauge bosons and 5-dimensional Higgs, are thermalized through the scattering on the light particles by the exchange of massless gauge bosons. Thus the equilibrium condition for the heavy particles seems to be the same as that for the light particles.

These facts mean that it makes sense to consider possible effects on the friction due to excited particles in (near) thermal equilibrium. We note that it is difficult to thermalize the -particle, which does not couple to the light particles and thus this does not participate in this game.

The friction coefficient at finite temperature $\eta(\Phi)$ was obtained by the present author and Hosoya [31] for a scalar particle:

$$\eta(\Phi) \sim \left(\frac{dm^2(\Phi)}{d\Phi}\right)^2 \beta \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega(p)^2\gamma} n_0(1+n_0) \quad , \qquad (4.46)$$

where

$$n_0 = \frac{1}{e^{\beta\omega(p)} - 1} \quad ,$$

$$\omega(p) = (\vec{p}^2 + m^2(\Phi))^{1/2} \quad .$$

Here β is inverse temperature, $\beta = 1/T$ and γ is a reaction rate of the scalar particle in the scattering to the light particles.

Equation (4.46) implies that the thermal friction coefficient is proportional to an inverse power of a coupling constant [31,69], since γ is essentially scattering amplitude of the scalar particle to the light particles. Therefore it follows that the magnitude of the thermal friction is much larger than that of the friction at $T = 0$. Here we should note the mass threshold effects on the thermal friction. Generally the magnitude of the thermal friction rapidly varies with the temperature near the mass of the particle which causes the friction. When $T \geq m$, its value is much larger than that of the friction at $T = 0$. However as the temperature goes down below the mass threshold, it rapidly decreases and becomes negligible. This temperature dependence of the friction seems very important for the time evolution of the order parameter in the inflationary phase transition.

In the minimal SU(5) inflationary Universe scenario, the massive gauge boson and 5-dimensional Higgs potentially cause the thermal friction. However since the reheating temperature is at most one order of magnitude smaller than M_X , contributions from the massive gauge boson is negligibly small. Thus we can concentrate our attention on the effects which come from 5-dimensional Higgs H.

Taking account of rapid change of the thermal friction near the mass threshold, we can suggest the possibility which might save the SU(5) new inflationary Universe scenario from the difficulty: When the order parameter $\tilde{\Phi}$ approaches to the $SU(3) \times SU(2) \times U(1)$ minimum after a few oscillations around the $SU(4) \times U(1)$ minimum, the strength of the thermal friction force rapidly increases and $\tilde{\Phi}$ settles down to the $SU(3) \times SU(2) \times U(1)$ minimum without traveling around the minima many times. It seems that the value of the 5-dimensional Higgs mass crucially determines whether the above scenario successfully works or not.

The proposed solution for the difficulty of the SU(5) new inflationary Universe scenario is still tentative. Further quantitative investigation should be carried out for the elaboration of this scenario by means of the proposed solution.

§5. Summary and Discussion

After reviewing the inflationary cosmology we have investigated the reheating problem at the final stage of the new inflationary Universe scenario.

We have derived the evolution equation of the order parameter $\tilde{\Phi}$ and evaluated the magnitude of the friction term which appears in the obtained equation for $\tilde{\Phi}$. Numerical calculation has shown that the value of the friction term is too small so that the order parameter $\tilde{\Phi}$ travels around many minima of the C-W potential as shown in Fig.11. This result means that the Universe would be fragmented into many $SU(3) \times SU(2) \times U(1)$ and $SU(4) \times U(1)$ symmetric worlds by the tiny fluctuation of the order parameter associated with the initial state. It follows that large scale inhomogeneities appear.

However thermal effects on the friction may be significant, since it is also shown by numerical calculation that the effective temperature of the Universe T_{eff} grows up to $T_{eff} \sim (5 \times 10^{-2} - 10^{-1})\, M_X$ in the final stage of the rolling down phase transition of the Universe. Therefore it may be possible that when the order parameter $\tilde{\Phi}$ approaches to the physical $SU(3) \times SU(2) \times U(1)$ minimum the strength of the friction force significantly increases and $\tilde{\Phi}$ settles down to this minimum without traveling around the minima many times.

It seems that dominant contributions to the thermal friction comes from the 5-dimensional Higgs field which is probably easy to be thermally excited. Therefore the value of its mass crucially determines whether the thermal friction can make the order parameter terminate at the SU(3) $\times$ SU(2) $\times$ U(1) minimum without traveling around the minima many times or not. Further quantitative investigation is needed to obtain a definite conclusion about the success or the failure of the SU(5) new inflationary model, though the analysis of the intermediate region from zero temperature to the finite temperature would be complicated.

We have considered the specific model of the new inflationary Universe scenario based on the minimal SU(5) C-W potential. However our method can also be applied to the reheating problems of other versions of the inflationary Universe. Especially it is important to estimate the reheating temperature T_R in the Chaotic inflationary Universe scenario in the context of N = 1 supergravity, since T_R should be sufficiently low so that the gravitino problem is solved by suppression of thermal gravitino productions [64].

One may wonder why we did not discuss anomalous behavior of quantum fluctuations of scalar field $\langle\phi^2\rangle$ in the de Sitter space which succeedingly emerges after the initial Friedmann Universe stage in the inflationary Universe scenario [54,68,70]. Several authors discussed its consequences to inflationary Universe scenario [71]. However their arguments do not seem to be convincing from the similar reason as the one which we previously mentioned in the case of the source of the density perturbation. Thus we have not touched upon this subject in order to avoid the possible confusion.

In this thesis we have discussed the effects of the friction only in cases which are concerned with the thermalization of Universe. However there is another processes in which the friction may play an important role, e.g. tunneling processes. It is shown that in ordinary case in flat space-time a tunneling amplitude is reduced by the friction [72]. Thus we might have to take account of this effects in the discussion of the bubble formation at the early stage of the inflationary phase transition. Furthermore the effects of the friction (or particle creation) seem to enhance a probability of a tunneling in gravitational cases [73]. Therefore these effect might become significant in Quantum cosmology [74].

As shown in this thesis, the inflationary cosmology is still far from a completion. However its basic idea is very attractive and seems to be approved as a probable scenario which solves the fundamental problems of the standard cosmology. The research efforts are now going on in order to elaborate the inflationary Universe scenarios. In the author's opinion it is sure that the methods which we have developed in this thesis will have an important role for that purpose.

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Figure Captions

Fig.1 Some important stages of evolution of the Universe. The temperature of the Universe is T and t denotes the age of the Universe after the bang.

Fig.2 A schematic description of the Higgs effective potential which leads to a strongly first-order phase transition.

Fig.3 An illustration of bubble collisions at the final stage of phase transition. The hatched regions represent trapped false vacuum regions.

Fig.4 (a) Schematic description of the C-W potential V_{eff} at $T \ll \sigma$, and (b) its shape near the origin. The arrow represents a tunnelling with the bubble formation.

Fig.5 The evolution of the horizon ($\simeq H^{-1}$) and the physical size of perturbations on the scale of a galaxy (λ_G) and on the scale of the present observable Universe (λ_H) in the inflationary Universe. The broken line shows the evolution of H^{-1} in the standard cosmology.

Fig.6 A contour map of the SU(5) C-W potential for the case of C = 1 on the (X,Y) plane. Four * and two □ represent SU(4) × U(1) and SU(3) × SU(2) × U(1) minima respectively. The angle α indicates deviation from the SU(4) × U(1) direction.

Fig.7 Time evolution of the order parameter is displayed for three typical cases; (a) for the case C = 1, η = 0.01 and α = 0.2π, (b) for the case C = 1, η = 0.001 and α = 0.2π, (c) for the case C = 1, η = 0.005 and α = -0.1π.

Fig.8 Cancellation of the tad pole contribution by the source term.

Fig.9 Skelton expansion for the full propagator $D_{\mu\nu}$.

Fig.10 Feynman diagram for the vacuum polarization $\Pi_{\mu\nu}$.

Fig.11 Typical case of time evolution of the order parameter is displayed on the (X,Y) plane. The values of the fraction ε is also shown for three stages; at $|\tilde{\Phi}| \sim 10^{10}$ GeV, at the SU(4) × U(1) minimum and at the SU(3) × SU(2) × U(1) minimum.

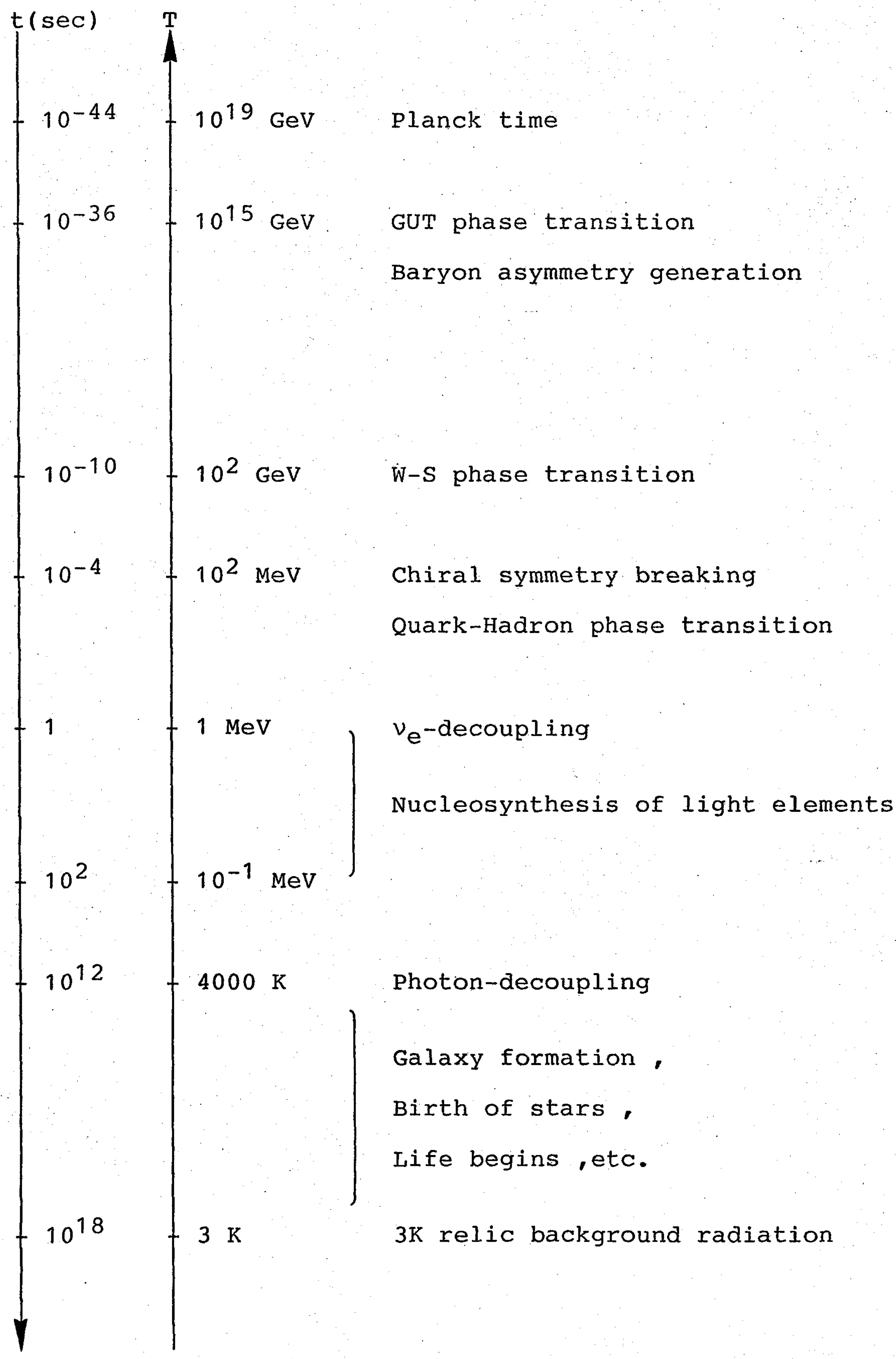
t(sec)
T
10^{-44}
10^{19} GeV
Planck time
10^{-36}
10^{15} GeV
GUT phase transition
Baryon asymmetry generation
10^{-10}
10^{2} GeV
W-S phase transition
10^{-4}
10^{2} MeV
Chiral symmetry breaking
Quark-Hadron phase transition
1
1 MeV
ν_e-decoupling
Nucleosynthesis of light elements
10^{2}
10^{-1} MeV
10^{12}
4000 K
Photon-decoupling
Galaxy formation ,
Birth of stars ,
Life begins ,etc.
10^{18}
3 K
3K relic background radiation

Fig.1

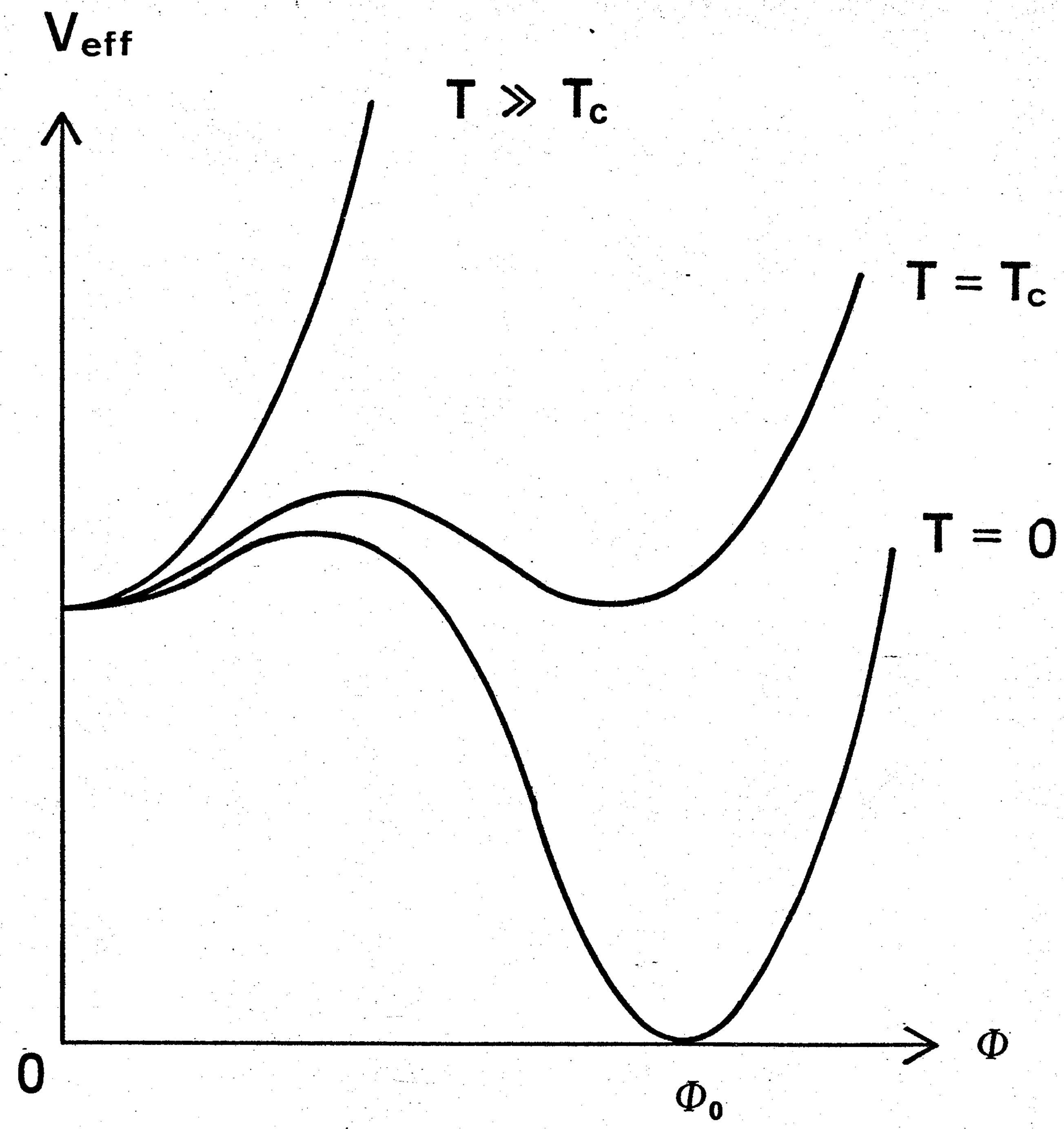

Fig.2

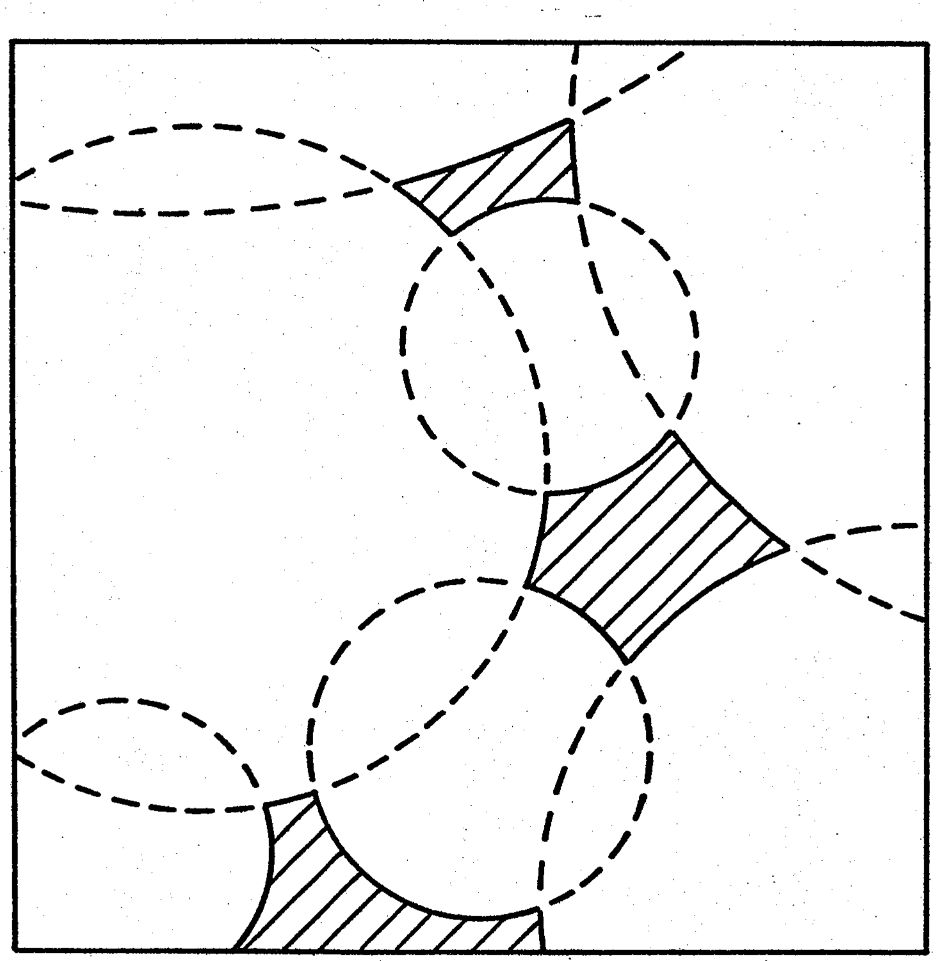

Fig.3

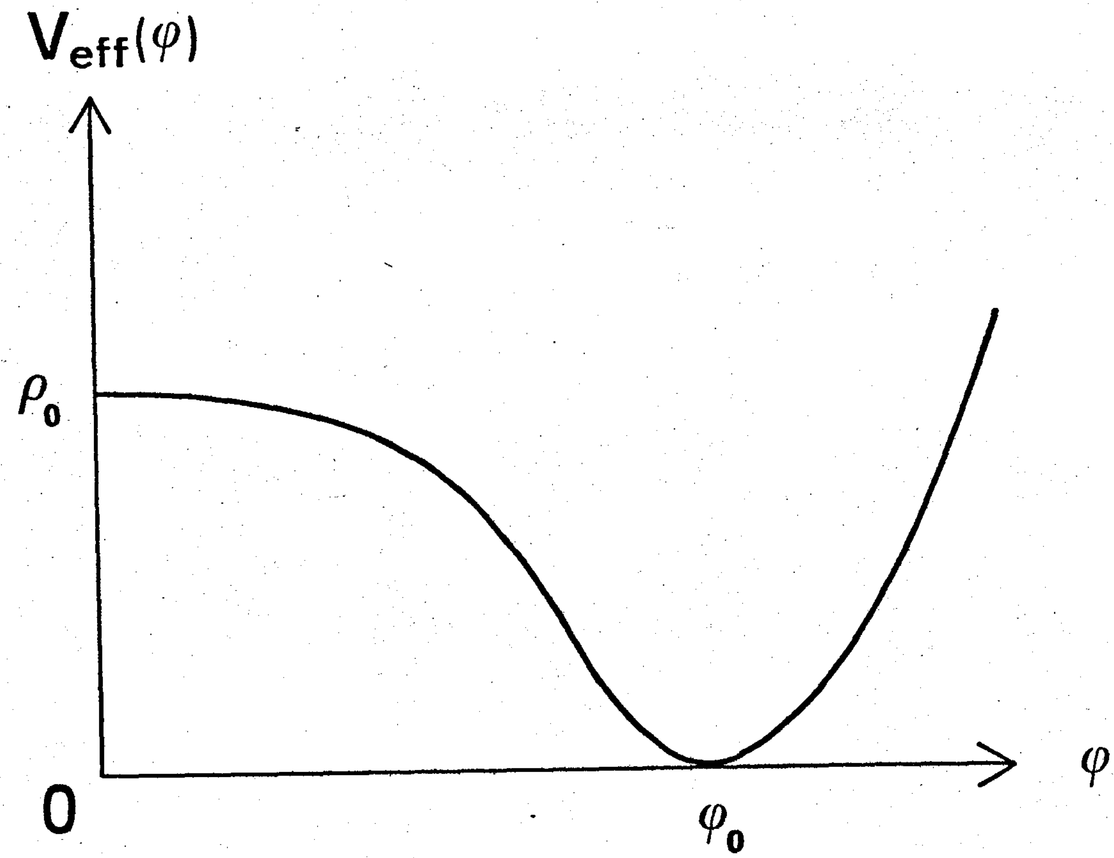

Fig.4(a)

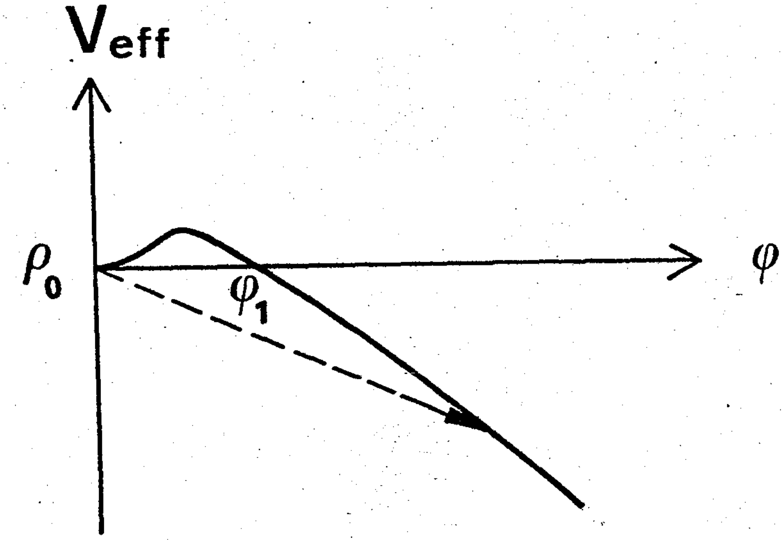

Fig.4(b)

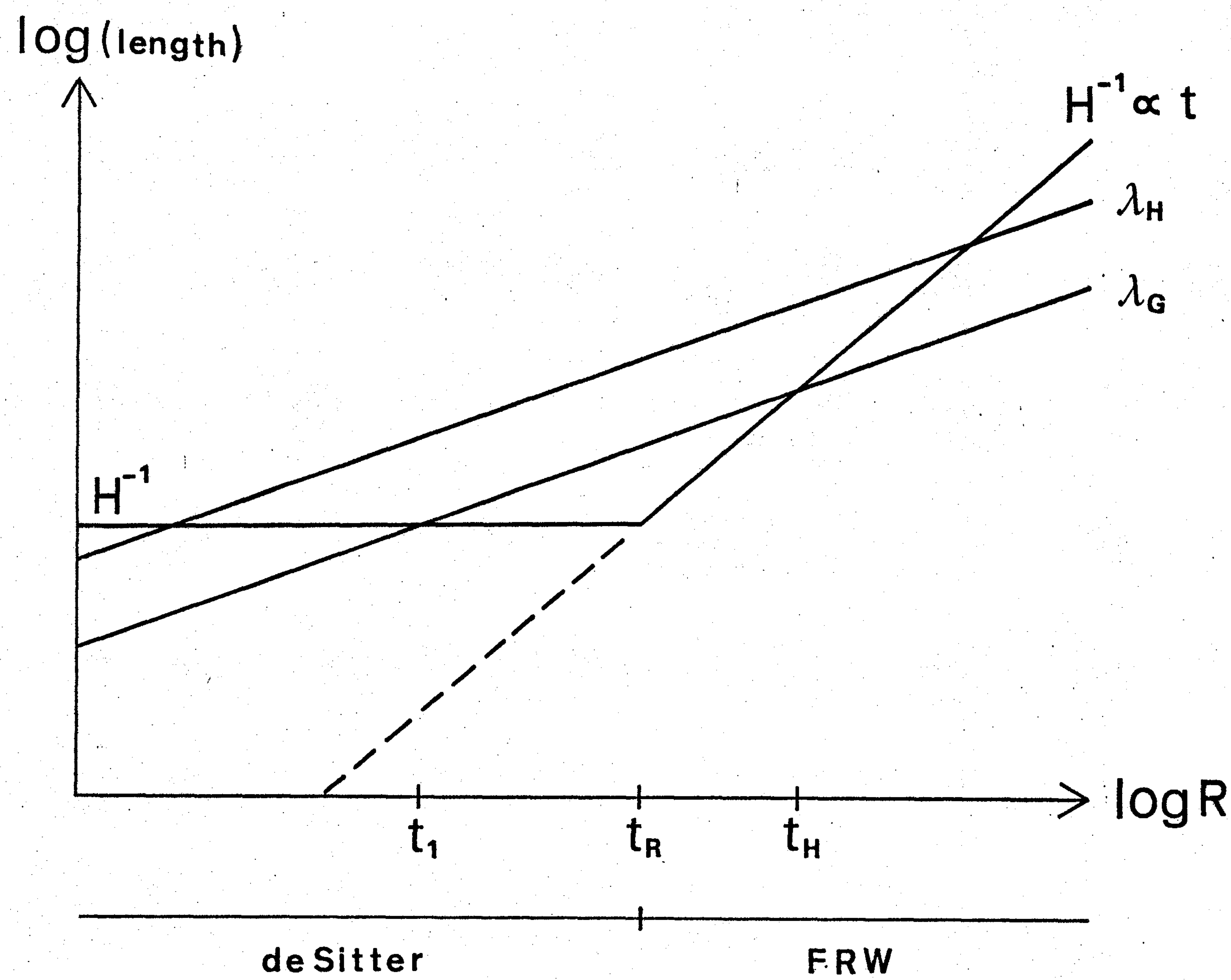

Fig.5

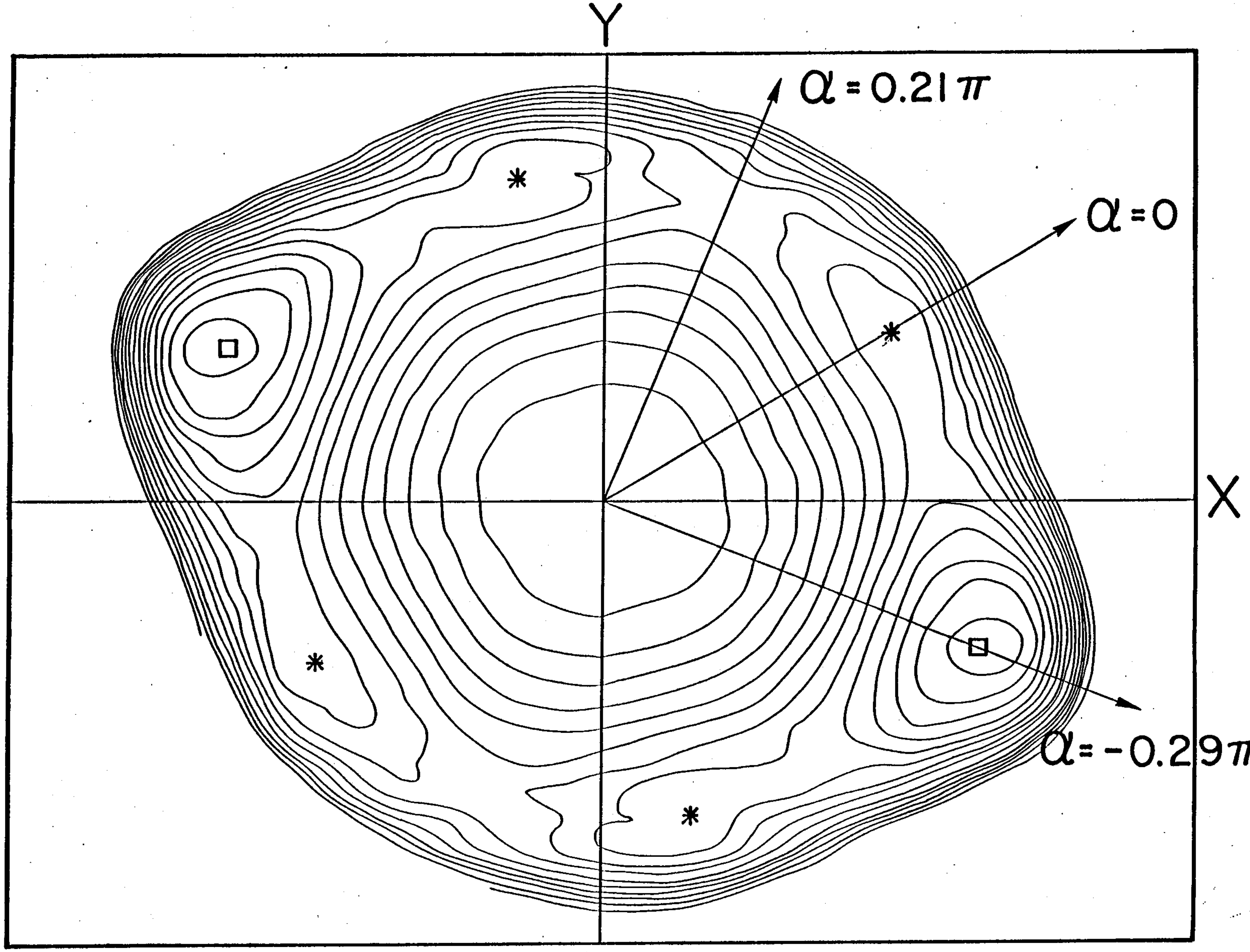

Fig.6

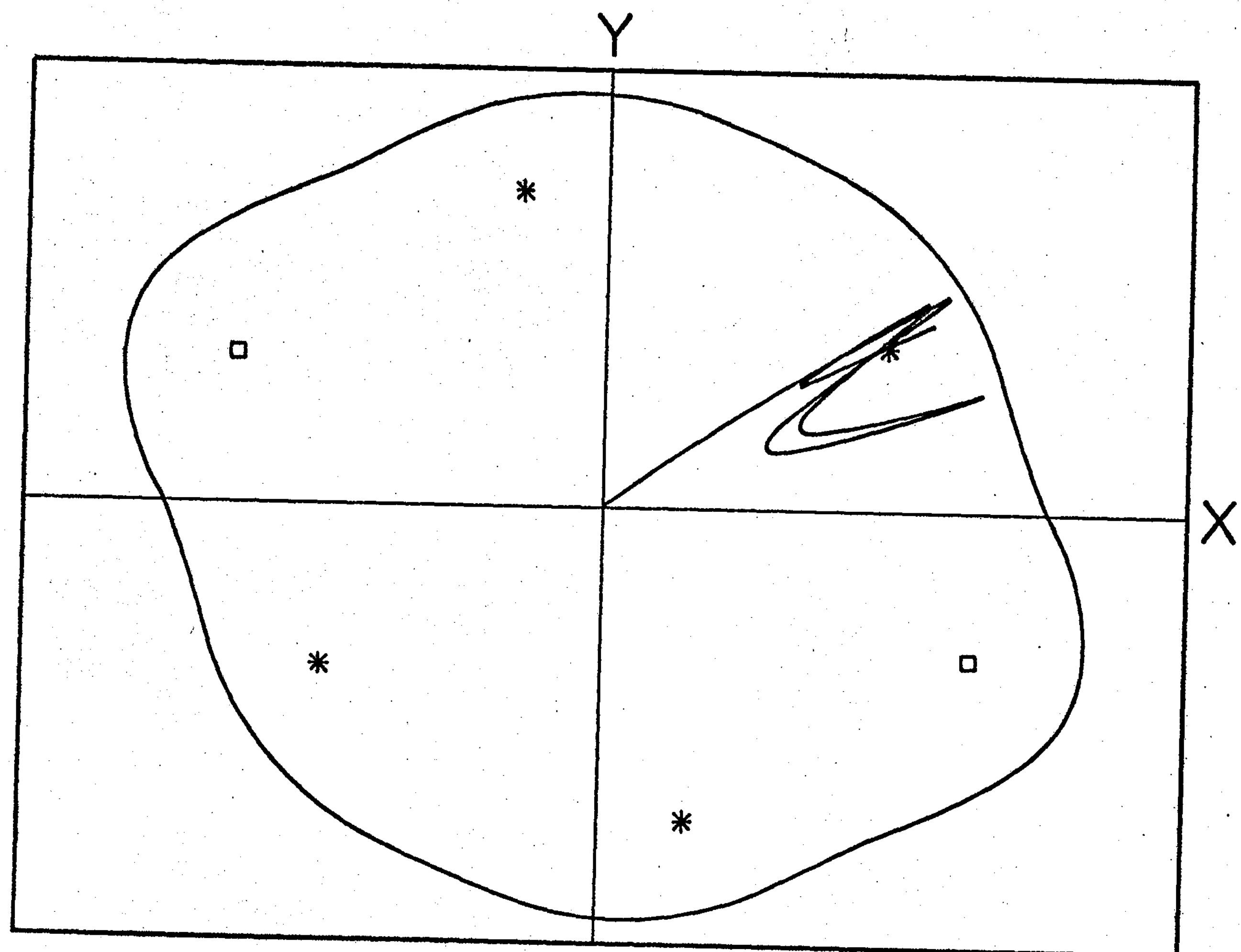

Fig.7(a)

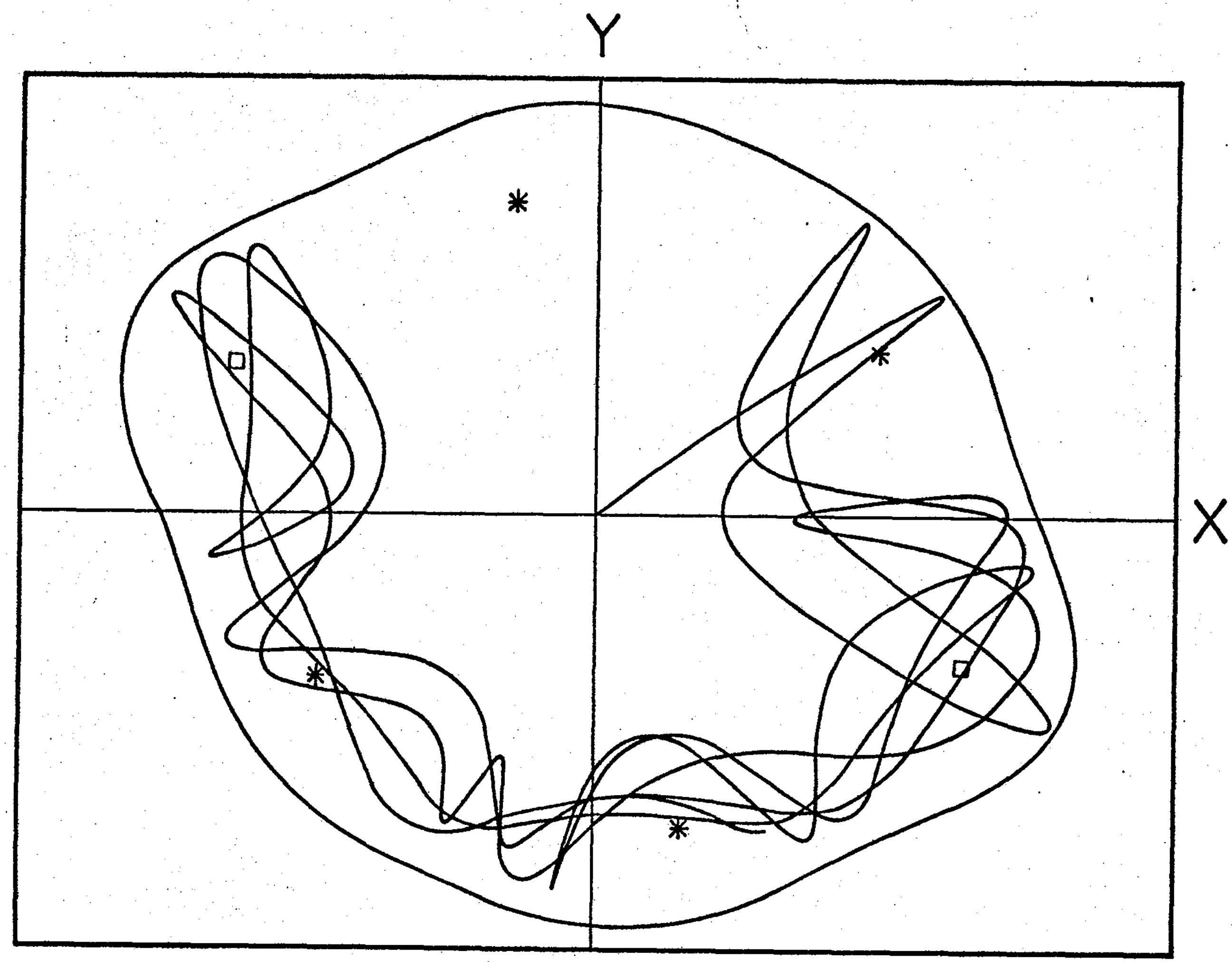

Fig. 7(b)

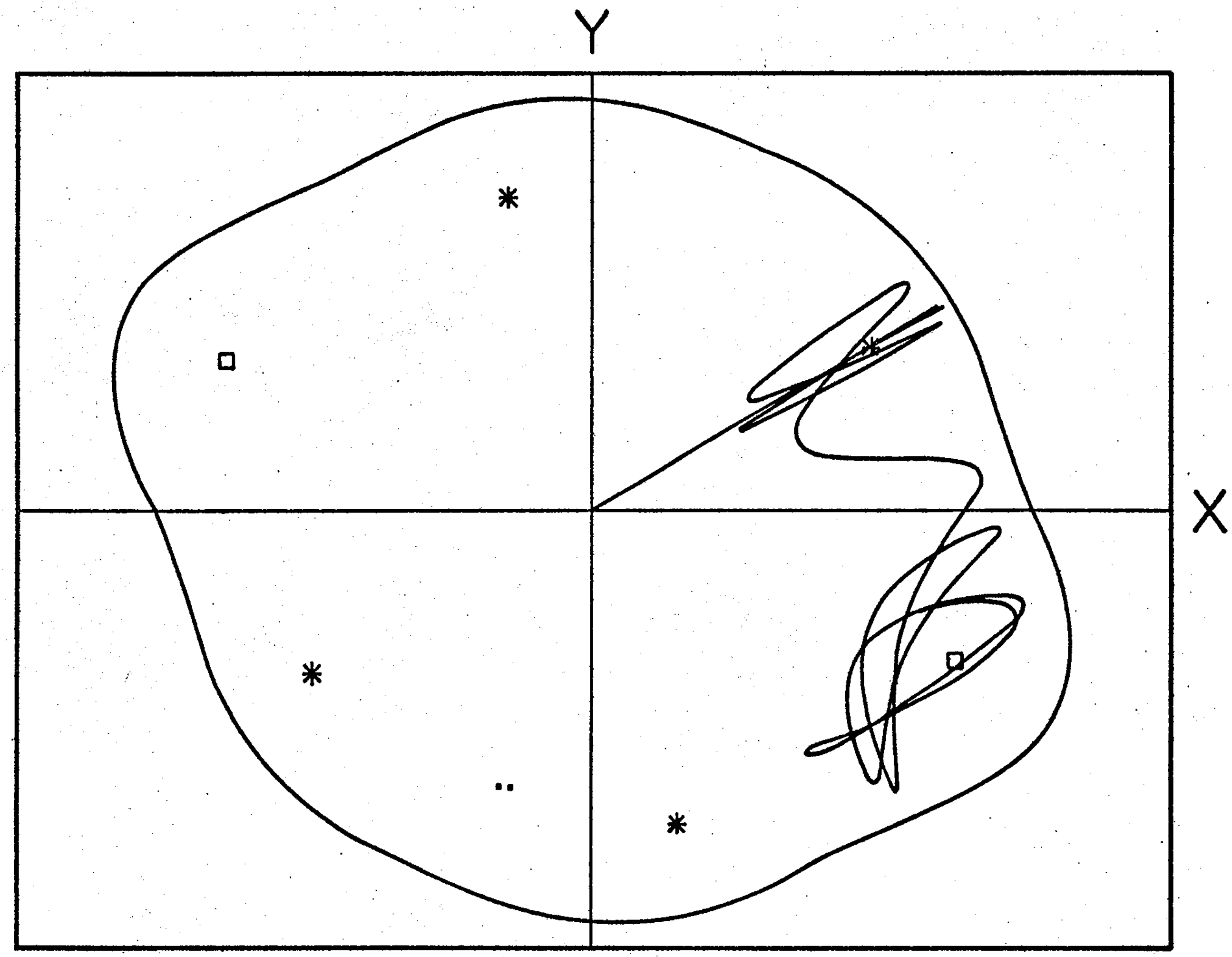

Fig.7(c)

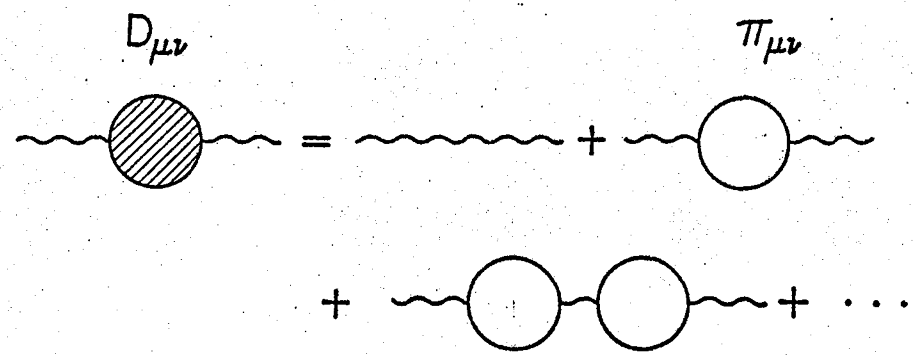

Fig.9

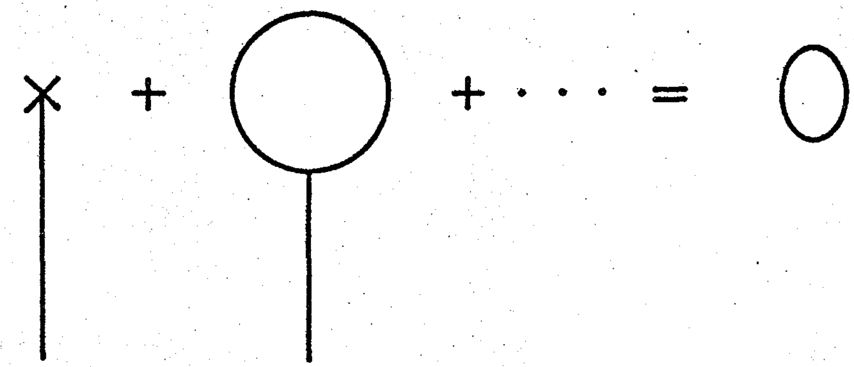

Fig.8

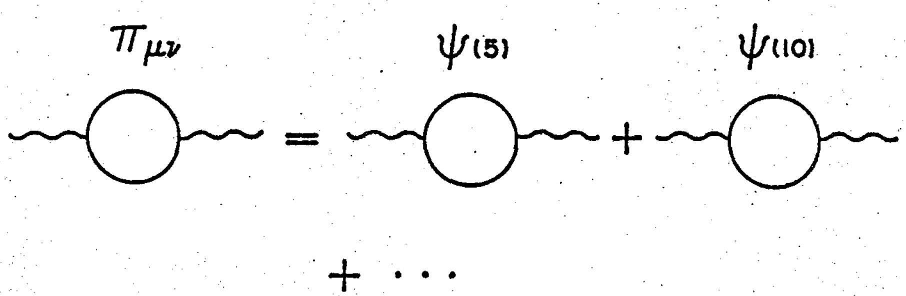

Fig.10

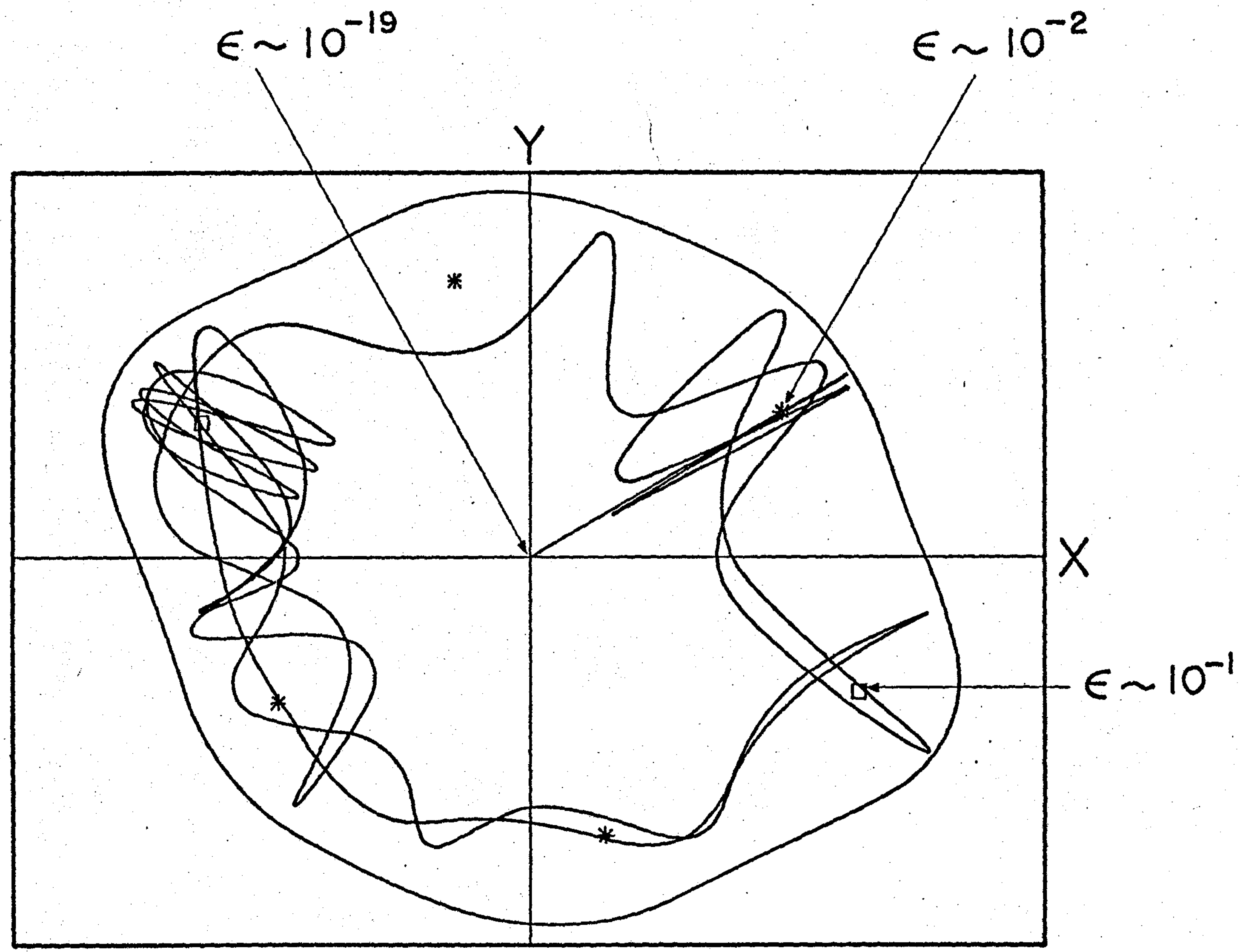

Fig.11