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Nonlinear resonance interaction of ultrasonic waves under applied stress

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The stress effect on the nonlinear resonance interaction $T + T \rightarrow L$ is experimentally investigated using an Al 5083 specimen. Compressive stress applied at the localized region where two noncollinear transverse waves intersect each other is shown to vary, up to $\pm 8.5\%$, of the amplitude of the longitudinal wave generated through the resonance interaction. The amplitude variation with stress is explained by relating the stress-induced change of elastic wave velocities (acoustoelastic effect) to the change in the resonance angle, and then to the directional characteristics of the two incoming transverse beams. The weak anisotropy of the material is taken into account.

Nonlinear resonance interaction of noncollinear ultrasonic waves in solids has been studied in the last two decades.[1–6] The study set forth with the purpose of determining the values of the third-order elastic constants, which characterize the nonlinear elasticity of continua or the anharmonicity of lattice vibrations. In general, the interaction occurs and as the result a third wave appears, when two ultrasonic waves intersect each other at an angle appropriate to their wave types, frequencies, and propagation velocities. Knowing that the propagation velocities depend on the stress in the medium (acoustoelastic effect[7–10]) and moreover the resonance condition includes the velocities as the governing parameters, one could expect that application of stress should induce some influence on the nonlinear resonance. Our preliminary experiment is devoted to a verification of this intuitive expectation.

The theoretical aspect of the resonance interaction belongs to the first-order approximation theory of nonlinear elasticity.[1,2,4] It is known that the conservation law of energy and momentum,

$$\omega_1 \pm \omega_2 = \omega_3 \quad \text{and} \quad \mathbf{k}_1 \pm \mathbf{k}_2 = \mathbf{k}_3, \tag{1}$$

is satisfied by the three waves involved; ω_i is the angular frequency and $\mathbf{k}_i$ the wave vector ($i = 1, 2, 3$). In the following experiments, we observe the resonance interaction $T(\omega_1) + T(\omega_2) \rightarrow L(\omega_1 + \omega_2)$ in stressed polycrystalline material. Microstructural properties of the material result in a degree of acoustic anisotropy and the velocity varies depending on the directions of propagation and polarization. For this, we use the notation C^i_{tj} to designate the velocity of transverse wave of the wave vector $\mathbf{k}_j$ ($j = 1, 2$); the superscript i is called N when the direction of polarization is normal to the surfaces of plate specimen and it is called P when parallel to them. Moreover, the stress influences the velocities C^i_{tj} and C_l. According to the first-order theory of acoustoelasticity the elastic wave velocities are linear functions of stress σ; so we can write

$$C^i_{tj}(\sigma) = C^i_{tj}(0) + \alpha^i_{tj}\sigma,$$

and

$$C_l(\sigma) = C_l(0) + \alpha_l\sigma, \tag{2}$$

for the case of uniaxial loading. The velocities at zero stress and the constants α's will be experimentally determined. The condition for the resonance interaction $T(\omega_1) + T(\omega_2) \rightarrow L(\omega_1 + \omega_2)$ can then be obtained from Eq. (1) as

$$\cos\theta^i_r(\sigma) = \frac{C^i_{t1}(\sigma)C^i_{t2}(\sigma)}{2\omega_1\omega_2} \times \left\{\left[\frac{\omega_1 + \omega_2}{C_l(\sigma)}\right]^2 - \frac{\omega_1^2}{C^i_{t1}(\sigma)^2} - \frac{\omega_2^2}{C^i_{t2}(\sigma)^2}\right\}. \tag{3}$$

Here we have emphasized the stress dependence of the resonance angle θ_r, i.e., the intersecting angle at which the resonance interaction occurs. As the velocities change linearly with the stress, θ_r also changes linearly with the stress in the first-order approximation.

Suppose that two plane monochromatic transverse waves of fixed frequencies are propagated in given directions making an angle θ through the stressed material. In this situation, the resonance interaction does take place and the longitudinal wave of the sum frequency appears, only when the stress changes the resonance angle $\theta_r(\sigma)$ to make it exactly equal to the intersecting angle θ. Otherwise, the interaction will not occur at all. Hence the appearance of the longitudinal wave distinctly indicates the relevant stress level at the intersecting zone. The above discussion is valid only in the ideal case of plane monochromatic wave incidence, but this is the basic idea of the stress detection technique proposed in this paper.

The stress effect on the noncollinear interaction $T + T \rightarrow L$ was measured using a hexagonal plate specimen, 18 mm thick and made of aluminum alloy 5083 (Fig. 1). The specimen angle ϕ was successively changed by machining the plate from 118.34° to 121.92°. The angle ϕ was deter-

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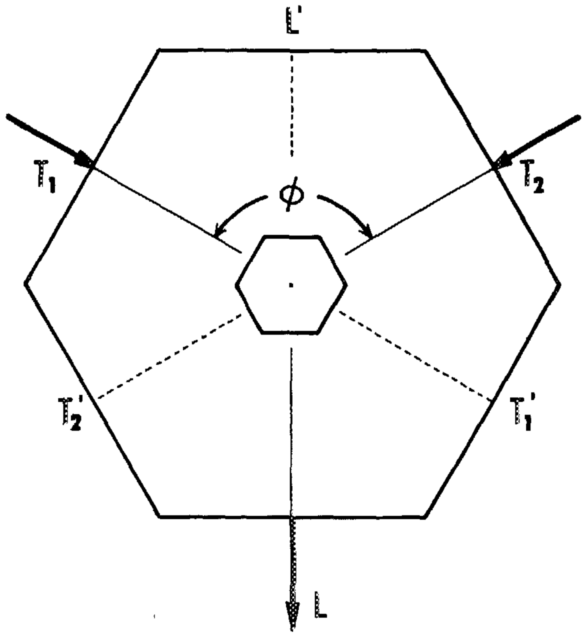

FIG. 1. Experimental configuration for the interaction $T + T \to L$. The distance between the opposite sides of hexagonal specimen is 80 mm and that of loading attachments is 16.8 mm. The specimen angle ϕ measures the angle made by two normals to the specimen sides where transducers T_1 and T_2 are bonded.

mined using a microscope with an accuracy of 10^{-2} deg. We assume that the measured voltage is proportional to the corresponding wave amplitude. Measurements of amplitudes A_1, A_2, and A_3 were carried out, applying compressive loads to a pair of attachments, made of steel in a hexagonal shape, which were mounted at the center of both surfaces of the plate specimen. The compressive stress σ was raised to a maximum value of 72 MPa, which corresponds to 61% of the elastic limit of Al 5083.

Electric pulses of 50 ns width and up to 1 kV amplitude, were fed simultaneously from a pulse generator to the air-backed PZT thickness-shear mode transducers T_1 and T_2, which were 7 mm square and bonded directly on the specimen. The nominal frequency was 5 MHz. The generated longitudinal wave was detected by a 3×10 mm thickness-expansion mode transducer L with a nominal frequency of 10 MHz. The transducer signals were processed with a signal averager which reduced the random noise by digitalizing and summing up a series of signals. The signals were then analyzed to study the spectral structure. The transducers T'_1, T'_2, and L' were used for monitoring the amplitudes A_1 and A_2 and for obtaining the necessary data for calibration, the wave velocities and their changes with the stress σ.

The application of stress can affect the resonance interaction in various ways besides that described earlier. The in-plane stress alters the intersecting angle and the distances from the transmitting transducers to the point of intersection as well as the wave velocities. The bending of the plate specimen may also shift the resonance condition. It should be added that the stress normal to the propagation paths leads to a delay of the arrival times at the intersecting point, which is due to the Poisson effect and the acoustoelastic effect. We have chosen the experimental configuration shown in Fig. 1. This enables us to eliminate all these influences and observe the stress effect with fixed interaction volume V and intersecting angle θ.

We found the material Al 5083 to be anisotropic to a considerable extent. The specimen configuration of Fig. 1 with the sing-around method allowed *in situ* measurements of velocities and their rates of change to the stress defined in Eq. (2); they are

$$\begin{aligned} C_{t1}^{P} &= 3.1303 - 1.213\times10^{-5}\sigma,\\ C_{t2}^{P} &= 3.1875 - 4.106\times10^{-5}\sigma,\\ C_{t1}^{N} &= 3.2076 + 6.800\times10^{-5}\sigma,\\ C_{t2}^{N} &= 3.1575 + 9.641\times10^{-5}\sigma,\\ C_{l} &= 6.3111 + 2.576\times10^{-6}\sigma. \end{aligned} \tag{4}$$

The velocities are in unit of km/s and the stress σ in MPa; the plus sign is used for compressive stress. These data were obtained with $\phi = 119.98°$, but will be used for calibration at all specimen angles.

An FFT analysis showed that the frequency spectra of the input transverse beams possessed rather narrow sidebands and their center frequencies were equally located around 8.3 MHz. Based on this observation, we make an assumption that $\omega_1 = \omega_2$ and neglect their sidebands, which corresponds to dealing with the monochromatic incident waves.

Using Eq. (3) with the assumption $\omega_1 = \omega_2$ and the velocity data given in Eqs. (4), we have the theoretical values of resonance angle θ_r,

$$\begin{aligned} \theta_r^P(\sigma) &= 119.94 + 5.70\times10^{-4}\sigma \text{ (deg)},\\ \theta_r^N(\sigma) &= 119.44 - 1.74\times10^{-3}\sigma \text{ (deg)}, \end{aligned} \tag{5}$$

where the superscripts P and N indicate the polarizations of the input transverse waves. It is seen that θ_r^N is three times more sensitive to the stress change than θ_r^P. For this reason, we mainly observed the stress effect using transverse waves polarized normally to the surfaces of plate specimen.

Open circles in Fig. 2 represent the amplitude A_3 of the generated longitudinal wave normalized with the product A_1A_2 of the pumping amplitudes in the stress-free state. The normalized amplitude does not respond sharply to the change of the specimen angle ϕ. This result is attributed to the divergence of the incident beams, which we attempted to lessen, but is still finite in reality. The incident wave vectors, $\mathbf{k}_1$ and $\mathbf{k}_2$, are not unidirectional but are spread within a small angular range; the amplitude is maximum at the center and decreases to the edges of the range. Each of the incident beams can be regarded as a group of plane waves whose propagation directions are continuously distributed over the angle of divergence. Two component waves, belonging to the different groups, intersect at an angle θ. If $\theta = \theta_r$, then this pair interacts where they overlap and generates a longitudinal wave with an amplitude proportional to the product of their amplitudes. The signal observed with transducer L corresponds to the aggregate of longitudinal waves generated by such interacting pairs. Thus, the resonance can occur to the extent that the beam divergence allows, even if the specimen

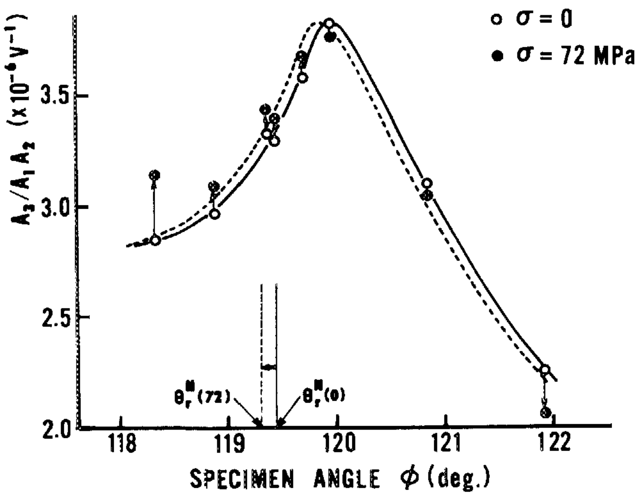

FIG. 2. Amplitude variation responding to the stress change from $\sigma = 0$ to $\sigma = 72$ MPa.

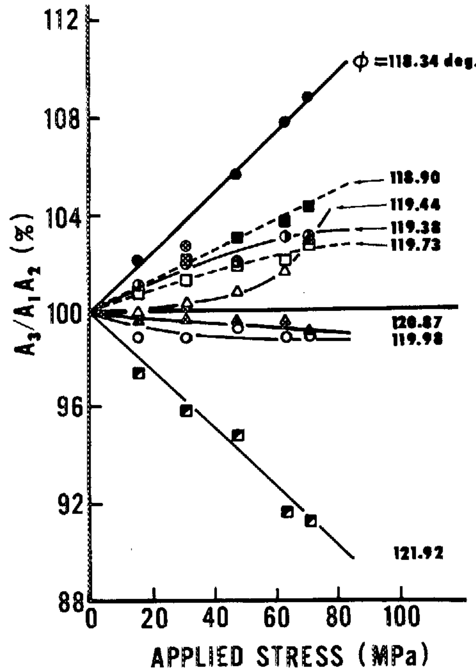

FIG. 3. Relative changes of amplitude with compressive stress σ at different specimen angles ϕ.

angle ϕ does not coincide with the resonance angle.

The peak of A_3/A_1A_2 missed the theoretical resonance angle $\theta_r^N(0) = 119.44°$ by about 0.5°. We ascribe this deviation to material inhomogeneity and the assumption that $\omega_1 = \omega_2$. The velocity data of Eqs. (4) at $\sigma = 0$ are not relevant to the small region of interaction; they are the average velocities across the specimen.

A marked response of A_3/A_1A_2 to the compressive stress σ is seen in Fig. 3. In Fig. 2, the final amplitudes (solid circles) are compared with those observed at zero stress (open circles). The stress contributes to vary the resonance angle θ_r owing to the acoustoelastic effect as stated in Eq. (3) and explicitly in Eq. (5). As a consequence, a pair of plane component waves that is involved in resonance at a stress level no longer interact with each other at another stress level. Instead, the component waves meet different partners so that the intersecting angles equal the resonance angle at new stress level. The amplitudes of the generated longitudinal waves are changed and accordingly transducer L receives a signal of different amplitude. Considering the beam divergence in connection with the change in resonance angle with stress, we find that the application of stress is equivalent to the alteration of the specimen angle ϕ by an amount appropriate to the stress. If this is true, the amplitudes A_3/A_1A_2 measured at $\sigma = 72$ MPa should be equal to those for $\sigma = 0$ shifted by an angle $-0.12°$ in Fig. 2. The correspondence, at least qualitatively, is satisfactory considering the simplifying assumptions used.

It was shown that the velocity change due to stress of the order of 10^{-4}–10^{-3} was effectively magnified to the order of 10^{-2}–10^{-1} of amplitude variation. The rate of magnification would be enlarged with the use of sharper directivity. The present results would indicate the possibility of an ultrasonic method for detecting the local stress. We should, however, notice several limitations.

(i) The nonlinear resonance interaction is a selective phenomenon and the angle available to give rise to the interaction is limited. But frequency scanning technique will cover some range of available angle together with the full use of five possible interaction cases.

(ii) The material inhomogeneity and anisotropy, which are the common impediments of acoustoelastic stress measurements, are more influential than in the ordinary acoustoelastic methods. To obtain the relevant data for calibration, *in situ* measurement is required.

(iii) So far as the plane waves are used, a pointwise measurement would be impossible.

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