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# Undressing the Kondo Effect near the Antiferromagnetic Quantum Critical Point

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The problem of a spin-1/2 magnetic impurity near an antiferromagnetic transition of the host lattice is shown to transform to a multichannel problem. A variety of fixed points is discovered asymptotically near the antiferromagnetic critical point. Among these is a new variety of stable fixed point of a multichannel Kondo problem which does not require channel isotropy. At this point Kondo screening disappears but coupling to spin fluctuations remains. In addition to its intrinsic interest, the problem is an essential ingredient in the problem of quantum critical points in heavy fermions.

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Theories of quantum critical phenomena involving fermions rely on extensions of the theory of classical critical phenomena to quantum problems by integrating out the fermions in favor of low-energy bosonic fluctuations [1,2]. Several experimental results on heavy fermion quantum critical points (QCP's) are in disagreement with such Gaussian fixed-point theories [3,4]. Some valiant efforts to address the problem are being made by using an extended dynamical mean field theory [5]. As an interesting problem in itself as well as to gain insight to the difficult problem of the lattice, we present here a systematic theory for a single impurity in a host with a diverging antiferromagnetic (AFM) correlation length. We show below that this necessarily leads to a multichannel problem with a variety of remarkable properties.

The  $S = 1/2$  local moment is coupled to the  $d$ -dimensional lattice, which is near an AFM instability due to electron-electron interactions, by the Hamiltonian  $(J/2)(2\pi)^{-2d} \iint d^d k d^d k' \psi_{\mathbf{k}}^\dagger \boldsymbol{\sigma} \psi_{\mathbf{k}'} \cdot \mathbf{S}$  where  $\psi_{\mathbf{k}}^\dagger$  ( $\psi_{\mathbf{k}'}$ ) is the creation (annihilation) operator for the host itinerant electron with momentum  $\mathbf{k}$  ( $\mathbf{k}'$ ), and  $\mathbf{S}$  is the localized spin. We assume the usual Gaussian spin fluctuations for the host electrons. Because of the interactions among the host electrons, the bare Kondo vertices  $J$  are renormalized. For a Fermi liquid such vertex corrections lead only to a numerical renormalization. However, qualitatively new effects can arise due to such renormalizations near a QCP in the host itinerant electrons. This problem has been solved for the case of a ferromagnetic (FM) instability of the itinerant electrons [6,7]. The problem of the AFM instability is both physically and technically quite different.

In order to perform a renormalization group (RG) procedure, we first derive the low-energy effective action in which the momenta of the host itinerant electrons are restricted within a narrow region near the Fermi surface,  $|\varepsilon_{\mathbf{k}}| \leq W$  ( $\varepsilon_{\mathbf{k}}$  is the energy of the host electrons relative to the Fermi level). Denote  $\psi_{\mathbf{k}}$  ( $\psi_{\mathbf{k}}^\dagger$ ) as  $\psi_{<}$  ( $\psi_{<}^\dagger$ ) for  $|\varepsilon_{\mathbf{k}}| \leq W$  otherwise  $\psi_{>}$  ( $\psi_{>}^\dagger$ ), then the Hamiltonian can be expressed as  $H[\psi, \psi^\dagger] = H[\psi_{<}, \psi_{<}^\dagger] + H[\psi_{>}, \psi_{>}^\dagger] +$

$H'[\psi_{<}, \psi_{<}^\dagger, \psi_{>}, \psi_{>}^\dagger]$ . Eliminating the modes for  $|\varepsilon_{\mathbf{k}}| > W$  in a path-integral formulation, we obtain the effective action as  $\mathcal{A}_{\text{eff}} = \mathcal{A}_{\text{eff}}^{(0)} + \mathcal{A}_{\text{eff}}^{(cc)} + \mathcal{A}_{\text{eff}}^{(cf)} + \dots$ ;  $\mathcal{A}_{\text{eff}}^{(0)}$  describes the free parts of the effective action,  $\mathcal{A}_{\text{eff}}^{(cc)}$  is the mutual-interaction term among the spins of the host itinerant electrons and  $\mathcal{A}_{\text{eff}}^{(cf)}$  corresponds to the effective interaction between the host electrons and the localized spin which is represented by Fig. 1:

Figure 1 presents Feynman diagrams for  $A_{\text{eff}}^{(cf)}$  in which solid and broken lines are associated with the host electron and the local moment, respectively. The exchange coupling in  $A_{\text{eff}}^{(cf)}$  has two parts: the first term  $j$  in Fig. 1 is irreducible with respect to the propagator  $\mathcal{D}_{\mathbf{k},\mathbf{k}'}(\omega)$  describing the AFM spin fluctuations of the host electrons, while the second part is reducible. The division into the two parts is such that  $g$  starts out as  $J$  while  $j$  starts out as  $O(J^2)$ , as explained through the first few terms of the series for  $j$ , and

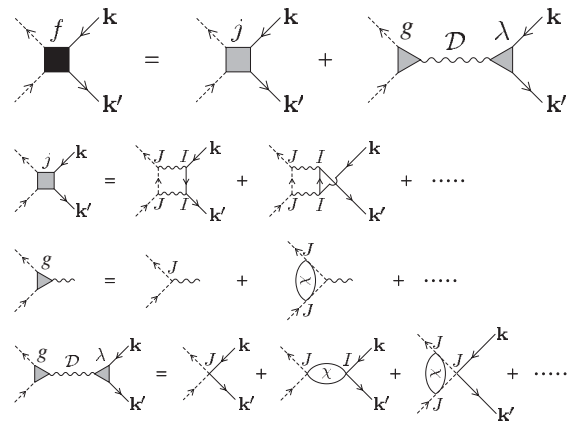


FIG. 1. Exchange interaction between the host electrons and the localized spin in the low-energy effective action. The first line gives the complete interaction; the subsequent lines show the first few terms of the series that is summed in each vertex in the first line.  $\chi$  represents the dynamical spin susceptibility of the host electrons.

$g$  in the figure.  $g$  is the coupling of the spin fluctuations to the local moments which are always coupled through electron-electron interaction vertices  $\lambda$  to the fermions. The vertices  $g$ , and  $\lambda$  are also irreducible with respect to the propagator  $\mathcal{D}_{\mathbf{k},\mathbf{k}'}$ .  $\lambda = I\Lambda^>(k, k')$  where  $I$  is the bare interaction among the spins of the host electrons and  $\Lambda^>(k, k')$  is the one-interaction irreducible vertex part in the spin channel for  $H[\psi_>, \psi_>^\dagger]$ .  $\mathcal{D}_{\mathbf{k},\mathbf{k}'}$  is related to the dynamical spin susceptibility  $\chi_{\mathbf{k},\mathbf{k}'}^>(\omega)$  of the host described by  $H[\psi_>, \psi_>^\dagger]$  as

$$\mathcal{D}_{\mathbf{k},\mathbf{k}'}$$

Note that even in the limit of  $W \rightarrow 0$  near the QCP,  $\Lambda^>$  is not singular (while  $\chi^>$  is). So, we can safely neglect the dependence of  $\lambda$  on the cutoff as well as the momenta, after  $\mathcal{D}_{\mathbf{k},\mathbf{k}'}$  is extracted.

In the limit that the energy cutoff  $W \rightarrow 0$ , the momenta  $\mathbf{k}$  and  $\mathbf{k}'$  are restricted to be on the Fermi surface  $S_F$ . If  $\mathbf{k}$  is represented by its projection onto  $S_F$  denoted by  $\mathbf{K}$  and the energy shell it belongs to  $\varepsilon = \varepsilon_{\mathbf{k}}$ , we can approximate  $\mathcal{D}_{\mathbf{k},\mathbf{k}'}$  as

$$\mathcal{D}(\mathbf{K}, \mathbf{K}') = \frac{AN_0}{\kappa(W)^2 + 2[d + \sum_{i=1}^d \cos(K_i - K'_i)]}, \quad (2)$$

where  $\kappa(W)$  is the inverse magnetic correlation length,  $N_0$  is the density of states, and  $A$  is a constant of the order of 1. The retardation of the interactions is properly included through a cutoff which appears through  $\kappa(W) \propto W^{1/z}$ , where  $z$  is the dynamical exponent. This procedure has been explicitly justified in Ref. [7] where it occurs in the same context.

Now, consider the unitary transformation which diagonalizes  $\mathcal{D}(\mathbf{K}, \mathbf{K}')$  on the Fermi surface  $S_F$ . Assuming that the impurity sits in a site with the full point group symmetry of the lattice, the symmetry operations  $R$  of a point group  $G$  (say,  $C_{4v}$  for the 2D square symmetry) may be used such that it is enough to find eigenvalues of  $\mathcal{D}(\mathbf{K}, \mathbf{K}')$  with  $\mathbf{K}$  restricted to an irreducible portion of the Brillouin zone,  $\Omega$  (a triangle determined by the vertices  $(0, 0)$ ,  $(\pi, 0)$ , and  $(\pi, \pi)$  for  $C_{4v}$ ). We obtain

$$\int_{\mathbf{K}' \in \Omega} \sum_{m'=1}^{d_\alpha} \mathcal{D}_\alpha(\mathbf{K}, \mathbf{K}')_{m,m'} u_l(\mathbf{K}')_{m'} = \mathcal{D}_l(\kappa) u_l(\mathbf{K})_m. \quad (3)$$

Here  $\int_{\mathbf{K}}$  stands for the average over  $S_F$ ,  $N_0^{-1}(2\pi)^{-d} \times \int_{\mathbf{k}} \delta(\varepsilon_{\mathbf{k}})$ ;  $\mathcal{D}_\alpha(\mathbf{K}, \mathbf{K}')_{m,m'} = \sum_{R \in G} R \mathcal{D}(\mathbf{K}, \mathbf{K}') \Gamma_\alpha(R)^*_{m,m'}$ .  $\Gamma_\alpha(R)$  is the unitary matrix for the irreducible representation  $\alpha$ , the dimension of which is  $d_\alpha$ . Therefore,  $l$  can be represented by a set of  $\alpha$  and  $i$  in which  $i$  tells apart eigenvalues in the space of  $\{1, 2, \dots, d_\alpha\} \otimes \{\mathbf{K} | \mathbf{K} \in \Omega\}$  for each  $\alpha$ . In the whole space of the Fermi surface, the number of degeneracies  $d_l$  is equal to  $d_\alpha$ . This general result is always true but more interesting is the generic case in which the AFM vectors  $\mathbf{Q}$ , (assumed commensurate [8]) connect points on the Fermi surface (“hot spots” in two

dimensions but “hot lines” in three dimensions). In that case the problem acquires larger degeneracies.

Expanding  $\psi_{\mathbf{k}}$  as  $\psi_{\mathbf{k}} = \psi_{\mathbf{K},\varepsilon} = \sum_{l,m} u_l(\mathbf{K})_m \psi_{l,m,\varepsilon}$ , the equation represented by Fig. 1 leads to the effective interaction of the local moment:

$$\frac{1}{2} \sum_{l,m} [j_l + g\lambda \mathcal{D}_l(\kappa)] a_{l,m}^\dagger \boldsymbol{\sigma} a_{l,m} \cdot \mathbf{S}, \quad (4)$$

where  $a_{l,m} \equiv \int \psi_{l,m,\varepsilon} d\varepsilon$ . At the initial cutoff  $W_0$ ,  $g = O(J)$  and  $j_l = O(J^2)$  for all  $l$  so that  $g \gg j_l$  for weak couplings  $J$ . Equation (4) has the form of a multichannel Kondo Hamiltonian with the number of channel  $d_l$  for each  $l$ , but the interactions depend *explicitly* on  $W$  through  $\kappa$ .

The RG equations for Eq. (4) are now derived for any given  $\mathcal{D}_l(\kappa)$ . Figure 2 presents perturbative corrections up to the third order of  $j$  and  $g$  in the successive elimination of modes for  $W' < \varepsilon_{\mathbf{k}} \leq W$ . We define a crossover parameter  $W_1 \sim rW_0$ ;  $r$  is the distance from the QCP. For the present case of  $z = 2$ ,  $\kappa/\kappa_0 = \sqrt{W/W_0}$  for  $W > W_1$ , i.e., the “quasiclassical” regime while  $\kappa/\kappa_0 = \sqrt{r}$  for  $W < W_1$ , i.e., the quantum regime.  $\kappa_0$  is a constant of the order of 1. In this Letter we present results only for the “quasiclassical” regime. It is also useful to introduce  $\epsilon \equiv (d/dt) \ln[\lambda^2 \sum_l d_l \mathcal{D}_l^2]$  with  $t = \ln(W_0/W)$ . Since the imaginary part of the local spin susceptibility for the host electrons scales as  $\text{Im} \chi_{\text{loc}}(\omega) \sim \lambda^2 \sum_l d_l \mathcal{D}_l^2 \omega \sim W^{-\epsilon} \omega$ ,  $\epsilon$  determines the power of  $\chi_{\text{loc}}(\tau)$  for the long-time limit  $\tau \rightarrow \infty$  as  $\chi_{\text{loc}}(\tau) \sim 1/\tau^{(2-\epsilon)}$ . Note that  $\epsilon = (4-d)/2$  for  $W > W_1$ , (while  $\epsilon = 0$  for  $W < W_1$  as for Fermi liquids). In principle, the dependence of  $\kappa$  on  $W$  can be derived from another RG equation of the mutual coupling among the host electrons in the present scheme and/or better assumptions than in the Gaussian picture for the pure system employed.

It is convenient to write the RG equations in terms of  $\bar{t} = \ln(\kappa_0/\kappa)^2$  and to rescale  $g$  and  $\mathcal{D}_l$  as  $\bar{g} = g\lambda\sqrt{\sum_l d_l \mathcal{D}_l^2}$  and  $\bar{\mathcal{D}}_l = \mathcal{D}_l/\sqrt{\sum_l d_l \mathcal{D}_l^2}$ . After some manipulations the RG equations are derived as

$$\frac{dj_l}{d\bar{t}} = N_0 f_l^2 - \frac{1}{2} N_0^2 j_l \sum_{l'} d_{l'} f_{l'}^2, \quad (5a)$$

$$\frac{d\bar{g}}{d\bar{t}} = \bar{g} \left[ \frac{\epsilon}{2} - \frac{1}{2} N_0^2 \sum_{l'} d_{l'} f_{l'}^2 \right], \quad (5b)$$

where  $f_l = j_l + \bar{g} \bar{\mathcal{D}}_l$ .

In order to solve Eqs. (5), we must first find  $\bar{\mathcal{D}}_l(\kappa)$  from Eq. (3). We have done this for two dimensions as well as

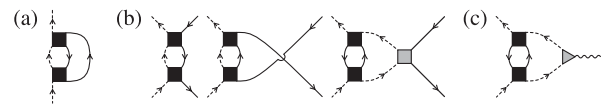


FIG. 2. Perturbative corrections to (a) the self-energy of the local moment represented in terms of pseudofermions, (b) the vertex  $j$ , and (c) the vertex  $g$  describing the coupling of the local moment to the magnetic fluctuations.

three dimensions analytically for  $\kappa \rightarrow 0$  and checked it numerically for a range of  $\kappa$ . Figure 3 shows the dependence of  $\bar{\mathcal{D}}_l$  on  $\kappa$  obtained from numerical solutions of Eq. (3) for a circular Fermi line with radius  $\sqrt{2\pi}$  in a 2D square lattice (i.e., at half filling); the Fermi line has 8 hot spots, which are 4 pairs of points connected by the AFM wave vector  $\mathbf{Q}$ . In the case of  $C_{4v}$ , there exist 5 irreducible representations in which  $d_\alpha = 1$  for  $\alpha = A_1, A_2, B_1,$  and  $B_2$  while  $d_\alpha = 2$  for  $\alpha = E$ . We find quite clearly that the absolute values of eight eigenvalues approach each other and  $A_1, B_2,$  and  $E$  ( $A_2, B_1,$  and  $E$ ) states bunch up to constitute fourfold degenerate states with a positive (negative) value of  $\bar{\mathcal{D}}_l$  in the limit of  $\kappa \rightarrow 0$ . From the inset,  $\sum_l d_l \bar{\mathcal{D}}_l = \text{const}$  while  $\sum_l d_l \mathcal{D}_l^2 \propto \kappa^{-2\epsilon}$  with  $\epsilon = 1$  so that  $\sum_l d_l \bar{\mathcal{D}}_l \rightarrow 0$  as  $\kappa \rightarrow 0$ .

The above realization of symmetry higher than that of the underlying lattice near the QCP can be understood from a general point of view; the 2D example is explained here. Consider a Fermi line with  $N_h = 2n_h$  equivalent hot spots, divided into  $n_h$  pairs with one member of the pair connected to the other by  $\mathbf{Q}$ . Let  $\mathbf{K}_h^1$  and  $\mathbf{K}_h^2$  be the vectors of two hot spots of one such pair, i.e.,  $\mathbf{K}_h^1 = \mathbf{K}_h^2 + \mathbf{Q}$ , and  $\mathbf{e}_h^1$  and  $\mathbf{e}_h^2$  are the unit vectors tangent to the Fermi line at these two hot spots. If we write  $\mathbf{K} = \mathbf{K}_h^\xi + p\mathbf{e}_h^\xi$  and  $\mathbf{K}' = \mathbf{K}_h^\eta + p'\mathbf{e}_h^\eta$  in which  $\xi = 1$  and  $\eta = 2$  or vice versa near these hot spots, then the singular parts of  $\mathcal{D}(\mathbf{K}, \mathbf{K}')$  can be approximated by

$$\mathcal{D}(\mathbf{K}, \mathbf{K}') \simeq \frac{N_0}{\kappa^2 + p^2 + p'^2 - 2pp' \cos\theta_h}, \quad (6)$$

where  $\cos\theta_h$  is given by  $\mathbf{e}_h^1 \cdot \mathbf{e}_h^2$ . Substituting Eq. (6) into Eq. (3), and dividing it by  $\sqrt{\sum_l d_l \mathcal{D}_l^2} \propto \kappa^{-1}$ , the eigenvalues of  $\bar{\mathcal{D}}_l(\kappa)$  in the limit of  $\kappa \rightarrow 0$  can be obtained by solving the following equation:

$$\int_{-\infty}^{\infty} dy \sum_{\eta=1}^2 \bar{\mathcal{D}}(x, y)_\eta^\xi u_l(y)_m^\eta = \bar{\mathcal{D}}_l(0) u_l(x)_m^\xi, \quad (7)$$

where  $u_l(x)_m^\xi = u_l(\mathbf{K}_h^\xi + \kappa x \mathbf{e}_h^\xi)_m$ ;  $\bar{\mathcal{D}}(x, y)_\eta^\xi$  is given by

$$\bar{\mathcal{D}}(x, y)_\eta^\xi = \frac{\sin\theta_h}{\sqrt{N_h \pi}} \frac{1 - \delta_\eta^\xi}{1 + x^2 + y^2 - 2xy \cos\theta_h}, \quad (8)$$

where  $\delta_\eta^\xi$  is 1 for  $\xi = \eta$  otherwise 0. If we diagonalize  $\bar{\mathcal{D}}(x, y)_\eta^\xi$  with respect to  $\xi$  and  $\eta$ , we necessarily find two eigenvalues of equal magnitude and opposite sign. For  $N_h = 8$  as in Fig. 3, the hot spots are connected by the operations of  $C_{4v}$ , i.e., the symmetry of the underlying lattice. The number of degeneracies is then half of the number of the symmetry operations in the group of the hot spots.

Equations (7) and (8) suggest that for  $\theta_h \rightarrow 0$ , all  $\bar{\mathcal{D}}_l(\kappa) \rightarrow 0$  as  $\kappa \rightarrow 0$  in two dimensions. We have explicitly found that the numerical results are consistent with this conjecture. In this case, as we explain below, the problem eventually acquires infinite degeneracy.

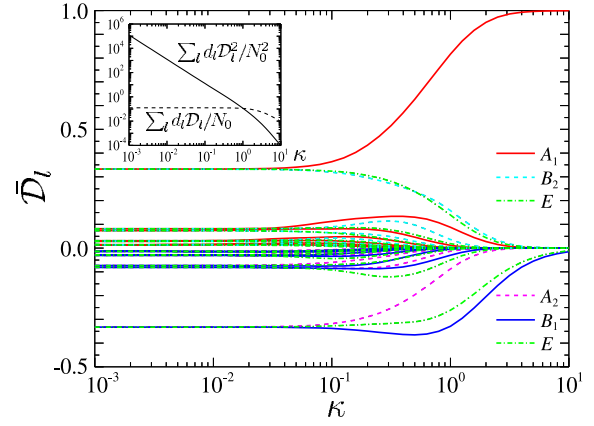


FIG. 3 (color online).  $\bar{\mathcal{D}}_l = \mathcal{D}_l / \sqrt{\sum_l d_l \mathcal{D}_l^2}$  versus  $\kappa$  for a circular Fermi line at half filling in a 2D square lattice. The largest  $7d_l$  eigenvalues for each symmetry are plotted. The inset shows dependences of  $\sum_l d_l \mathcal{D}_l^2 / N_0^2$  and  $\sum_l d_l \mathcal{D}_l / N_0$  on  $\kappa$ .

The situation is actually simpler in three dimensions. It follows from Eq. (3) that  $|\mathcal{D}_l(\kappa)|$  are less singular than  $|\ln\kappa|$  for any  $l$  while  $\sum_l d_l \mathcal{D}_l^2 \propto \kappa^{-2\epsilon}$  with  $\epsilon = 1/2$ , so that  $\bar{\mathcal{D}}_l(\kappa) \rightarrow 0$  for all  $l$  as  $\kappa \rightarrow 0$ .

With this knowledge of  $\bar{\mathcal{D}}_l(\kappa)$ , we return to the RG equations (5) to study the fixed points. It is straightforward to prove that  $\bar{\mathcal{D}}_l(\kappa)$  approach constants as  $\kappa \rightarrow 0$  ( $d\bar{\mathcal{D}}_l/d\bar{t} \rightarrow 0$  as  $\bar{t} \rightarrow \infty$ ), as may also be seen in Fig. 3. Therefore, it is sufficient to analyze with  $\bar{\mathcal{D}}_l(0)$ . There are two possibilities for the fixed points of Eq. (5b). (i)  $\bar{g} \equiv \bar{g}^* = 0$ . Then, it follows from Eqs. (5) that the usual multichannel problem is realized. Because of the small channel anisotropy this is always unstable towards the single channel Fermi-liquid fixed point. On the other hand, a new class of singular or non-Fermi-liquid (NFL) fixed points is obtained for (ii)  $\sum_l d_l N_0^2 f_l^{*2} = \epsilon$ . Then the fixed-point values of  $\bar{g}^*, f_l^*$  are solutions of the following equations;

$$8/\epsilon = \sum_l d_l \left( 1 + \sigma_l \sqrt{1 - 8\bar{\mathcal{D}}_l(0) N_0 \bar{g}^* / \epsilon} \right), \quad (9a)$$

$$N_0 f_l^* = \frac{\epsilon}{4} \left( 1 + \sigma_l \sqrt{1 - 8\bar{\mathcal{D}}_l(0) N_0 \bar{g}^* / \epsilon} \right). \quad (9b)$$

The fixed-point values of  $j_l$  are given by  $j_l^* = f_l^* - \bar{g}^* \bar{\mathcal{D}}_l(0)$ . In Eqs. (9),  $\sigma_l$  is either of  $\pm 1$  for each  $l$ .

Of these NFL fixed points, one in which all of  $\sigma_l$  are  $-1$  can be shown to be linearly stable. For  $d = 2$ , this stable fixed point exists only when the Fermi line is almost tangent at the hot spots to the boundary of the magnetic Brillouin zone, i.e.,  $\theta_h \rightarrow 0$  [otherwise there is no solution of Eqs. (9) with  $\sigma_l = -1$  for all  $l$ ]. For  $d = 3$ , this fixed-point solution is always found.

When  $\theta_h \rightarrow 0$  for  $d = 2$ , and  $d = 3$ , a study of the solution of Eqs. (5) reveals that the RG flow of  $\bar{g}$  and  $j_l$  is toward the single channel Fermi-liquid fixed point (i) only for large initial coupling  $N_0 J \gg 1$ , i.e., for initial



$g \ll j_l$ ; the Kondo temperature is so high that AFM correlations do not determine the fixed point. For the interesting weak-coupling case  $N_0 J \ll 1$ , i.e., for initial  $g \gg j_l$ , the RG flow is sucked into the stable one of the new class of NFL fixed points (ii), i.e., the fixed point of the degenerate multichannel Kondo problem with a finite  $\bar{g}^*$ , as described below, is realized at the QCP:

In two dimensions, for  $0 < \theta_h \lesssim 0.07$  with 8 hot spots, the fixed point is a multiple degenerate four-channel fixed point, e.g., for  $\theta_h = 0.05$ ,  $|N_0 \bar{g}^*| = 0.92$ , and  $N_0 j_l^* = 0.067, 0.039, 0.029, \dots$ , in order of size with exactly 4 degenerate states at each  $N_0 j_l^*$  for both signs of coupling to the localized spin. In the limit of  $\theta_h \rightarrow 0$ , where the Fermi surface is tangent to the magnetic Brillouin zone, expansion of Eqs. (9) with respect to  $\bar{D}_l(0) \rightarrow 0$  with  $\sum_l d_l \bar{D}_l(0)^2 = 1$  leads us to

$$N_0^2 \bar{g}^{*2} = \epsilon, \quad N_0 j_l^* = 0 \quad \text{for all } l. \quad (10)$$

So  $N_0 f_l^* = \sqrt{\epsilon} \bar{D}_l(0) \rightarrow 0$  with  $\sum_l d_l N_0^2 f_l^{*2} = \epsilon$ . In three dimensions,  $\bar{D}_l(\kappa) \rightarrow 0$  as  $\kappa \rightarrow 0$  with  $\sum_l d_l \bar{D}_l(\kappa)^2 = 1$  so that the stable fixed point is also given by Eq. (10).

It is important to note that channel anisotropy is irrelevant at these stable fixed points for  $d = 2$  as well as  $d = 3$ . This can be proved by noting that  $\sum_l d_l N_0^2 f_l^{*2} = \epsilon$  and  $d \bar{D}_l / d\bar{t} \rightarrow 0$  as  $\bar{t} \rightarrow \infty$ . Then Eqs. (5) lead to  $d|f_l - f_{l'}| / d\bar{t} = |f_l - f_{l'}| (2N_0 f_l^* - \epsilon/2) < 0$  as  $\bar{t} \rightarrow \infty$  when  $\bar{D}_l(0) = \bar{D}_{l'}(0)$  for  $l \neq l'$ .

The other fixed points given by Eqs. (9) are unstable. In the case of all  $\bar{D}_l(0) \rightarrow 0$ ,  $N_0^2 \bar{g}^{*2} = \epsilon(1 - \epsilon n_+ / 4)$ ,  $N_0 j_l^* = \epsilon/2$  for  $\sigma_l = 1$ ,  $N_0 j_l^* = 0$  for  $\sigma_l = -1$ , where  $n_+$  is the number of channels for which  $\sigma_l = 1$ . Since  $(1 - \epsilon n_+ / 4)$  must be positive or zero, there exist four (eight) unstable fixed points in two (three) dimensions where  $\epsilon = 1$  ( $\epsilon = 1/2$ ). If  $\epsilon$  is assumed to be small, these may be related to the unstable fixed points of a multichannel version of the Bose-Fermi Kondo model in the  $\epsilon$  expansion [9].

The difference of the results from those for the Bose-Fermi Kondo model [5,9] is instructive. In the present work, the bosons or spin fluctuations enter the theory only as intermediate states and not in external vertices, see Figs. 1 and 2. This is the consistent formulation of the problem because the fluctuations arise in the first place due to the electron-electron interaction vertex  $\lambda$ . In Refs. [5,9],  $\lambda$  is implicitly included in part of the problem in defining the fluctuations but neglected in the other part, the conversion of the fluctuations back to fermions.

The correlation functions near the fixed point for  $W > W_1$  as well as the detailed properties when approaching the fixed point from  $W < W_1$  will be presented in a longer paper. At this point we can only say that Eq. (10)  $f_l^* \rightarrow 0$  suggests a decoupling of the local moment from the conduction electrons, while a finite  $\bar{g}^*$  at the fixed point suggests that the moment responds to the AFM correlations of the host. This is what may be expected if the Kondo effect is deconstructed such that the local moment is at

least partially recovered and the recovered moment participates in the AFM correlations [10]. The infinite degeneracy at the fixed point suggests that the ground state has finite entropy. This degeneracy may also be understood as the prelude to the participation of the moment in the infinite range spin-wave correlations below the AFM transition. This infinite channel fixed point may be thought of as the analog for staggered magnetization correlations of what happens for growing FM correlations [6], where a droplet of size  $\kappa^{-1}$  around the impurity leads to a number of channels  $\propto \kappa^{-(d-1)}$  [11]. However this analogy is only suggestive both because of the special condition of hot spots or hot lines required to get such a fixed point as well as the very special second property, on  $f_l^{*2}$ , noted after Eq. (10) at the fixed point.

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