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Performance Analysis of the Capacity Controlled System with Adaptive Equalizer

Hee-Jin LEE†, Member, Takashi OMAE†‡, Nonmember, Shozo KOMAKI† and Norihiko MORINAGA†, Members

SUMMARY This paper analyzes the performance of the capacity controlled radio system under a frequency selective fading environment. This system controls the number of modulation levels according to the number of active subscribers. In the analysis, we consider the capacity controlled system either with or without adaptive equalizer. As the result of analysis, it is clarified that the system is superior to the conventional fixed capacity system, and can be considered as a good countermeasure for multi-pulse fading. And it is found that there exists a synergistic effect due to capacity controlling and adaptive equalizing.

key words: capacity controlled system, frequency selective fading, mobile communications, synergistic effect, multi-level modulation

1. Introduction

In radio communications, it is important to increase the number of subscribers in limited bandwidth. In order to achieve higher capacity system, there are two methods: the one is to increase frequency reuse by shortening the distance between cells, and the other is to use multi-level modulation such as 16QAM to improve spectrum utilization efficiency. When an M-ary signal with multiple modulation levels is used, the communication quality will be degraded by fading. To prevent the quality degradation the several types of counter-measures such as diversity and/or adaptive equalizer have been introduced.

On the other hand, we can use the small number of modulation levels when the number of active subscribers is small, to make a modulated signal robust to fading. Thus if the transmission capacity is controlled by changing the number of modulation levels according to the amount of active subscribers, the effect of fading will be mitigated. Such a capacity controlled system with or without diversity is considered to be effective against that fading and is also useful to mobile communication using TDMA burst co-dec, multi-media radio communication and so on, because of the large amount of traffic change.

The communication quality in a fading channel is degraded not only by the degradation of signal strength due to a flat fading but also by the waveform distortion due to a frequency selective fading. In order to compensate frequency selective fading, an adaptive equalizer or a diversity reception has been used. And it is well known that the outage improvement is remarkable when both diversity reception and adaptive equalizer are used simultaneously, compared to the improvement effects of solitary use of them. This phenomena is called as a synergistic effect between them.

This paper is devoted to the theoretical performance analysis of capacity controlled system with or without adaptive equalizer on a frequency selective fading channel. Through a comparison with the conventional fixed capacity system, a synergistic effect due to both capacity controlling and equalizing is clarified.

2. System Model

Figure 1 illustrates the system model for the capacity controlled system under a frequency selective fading.
channel. The capacity controlled system controls the number of modulation levels according to the amount of active subscribers, i.e., it uses the large number of modulation levels when the amount of active subscribers increases and conversely it uses the small number of modulation levels when the amount of active subscribers decreases. In this paper, it is supposed that the outage occurrence is caused only by the waveform distortion due to the multi-path fading for simplicity of the analysis. The outage occurrence caused by thermal noise and interference is not taken into account. In the following section, we will use the term “the amount of traffic” instead of the amount of active subscribers. The probability density function (p.d.f.) of the amount of traffic, \( p(x) \) is approximated by the exponential distribution, closely related to the Poisson Distribution:

\[
p(x) = \lambda \exp(-\lambda x)
\]

where traffic \( x \) is normalized by \( C_{nx} \), and \( 1/\lambda \) is the mean value of the amount of traffic. When the amount of traffic exceeds a transmittable normalized capacity \( C_t \), related to the modulation levels, the traffic loss \( P' \) occurs. The traffic throughput is given by

\[
1 - P' = \int_0^{C_t} p(x) \, dx = 1 - \exp(-\lambda C_t).
\]

There exists another outage which occurs due to the frequency selective fading caused by a multipath propagation, that is, the outage due to the waveform distortion. Denoting the outage due to the waveform distortion arising from the frequency selective fading by \( P_b \) and assuming that the fading and traffic occurrences are mutually independent, total throughput is represented by

\[
P_{\text{total}} = (1 - P') (1 - P_b)
\]

\[
P'o\text{total} = (1 - P_b) (1 - P') = (1 - P_b) (1 - \exp(-\lambda C_t)).
\]

In this paper, 2\textsuperscript{nd}QAM is chosen for the multi-level modulation in order to maximize the spectrum utilization efficiency under the same carrier to noise power ratio. It is assumed that selective fading channel is two-ray model and delay difference between two rays is not so large comparing with a symbol interval.

Above mentioned analysis procedure for this model is summarized in Fig. 2.

3. Performance Analysis

3.1 Total Outage Probability without Equalization

The inband amplitude dispersion (IAD) is defined as the received signal power ratio \( l = X_1/X_2 \) where \( X_1 \) and \( X_2 \) denote the received signal levels at both edges of the pass-band \( \Delta f \). Both \( X_1 \) and \( X_2 \) are the Rayleigh-distributed time variables with a mutual correlation coefficient \( \rho_{AF} \), because the received signal is Rayleigh-distributed on any frequency. \( \rho_{AF} \) is usually called a frequency correlation coefficient and the channel dispersion becomes large as \( \rho_{AF} \) decreases. A flat fading is denoted by \( \rho_{AF} = 1 \). The outage due to the frequency selective fading occurs when the inband amplitude dispersion \( l \) exceeds its threshold value \( b_l \). Thus the outage probability \( P_o \) is obtained by

\[
P_o = 1 + \frac{1 - b_l}{(1 + b_l) - 4 \rho_{AF} b_l}.
\]

From Eq. (4), the IAD threshold \( b_l \) is represented by

\[
b_l = \frac{1 + (1 - P_b)^{2}(1 - 2\rho_{AF})}{P_o (2 - P_b)} + \sqrt{(1 + (1 - P_b)^{2}(1 - 2\rho_{AF})^2 + P_o^2 (2 - P_b)^2
\]

\[
P_o (2 - P_b)}
\]

(5)

as a function of \( P_o \).

In the appendix, the capacity decrease factor \( C_t \) defined as the transmission capacity normalized by that of a conventional fixed capacity system is derived:

\[
C_t = \frac{\log_2 \left( 1 + f(b_l) \right)}{1.95}.
\]

The denominator of Eq. (6) is the transmission capacity of the fixed capacity system and

\[
f(b_l) = \frac{1 - \cos(2\pi T / T)}{(b_l^2 - 1) (\tau / T)}
\]
Fig. 3 Total outage probability \(1 - P_{\text{total}}(\tau/T = 0.2, \rho_M = 0.99, P^0 = 0.1\%)\).

\[
1 - P_{\text{total}} = \frac{\sqrt{(1 - \cos(2\pi T/T))^2 + 2(k^2 - 1)(1 - \cos(2\pi \tau/T))}}{(k^2 - 1)(\tau/T)} \quad (0 < \tau/T < 1). \tag{7}
\]

where \(\tau\) is delay difference between direct and reflected rays and \(T\) is a symbol interval. Substituting Eq. (6) into Eq. (3), the total throughput probability \(P_{\text{total}}\) is obtained as a function of \(P_0\):

\[
P_{\text{total}} = (1 - P_0) \left[ 1 - \exp(-\lambda \cdot \frac{\log(1 + f(k))}{1.95}) \right]. \tag{8}
\]

Figure 3 illustrates the total outage probability \(1 - P_{\text{total}}\) for the designed outage probability \(P_0\) and shows the existence of the optimum points of \(P_0\) to minimize the total outage probability. If the outage probability \(P_0\) is designed at a certain large value, i.e., corresponding IAD threshold \(k\) is to be small, the total outage probability becomes large since even a little bit inband amplitude dispersion may exceed the threshold \(k\). On the other hand, if the designed outage probability \(P_0\) is small, the transmissible capacity decreases because small number of modulation level must be used to prevent the system outage even in the large dispersion, then the total outage probability also becomes large. Thus as a result of the trade-off between system outages due to the frequency selective fading and the amount of traffic, there exists the optimum point for the designed outage probability \(P_0\). For example, when \(\lambda = 1\), a minimum total outage probability of \(1.5 \times 10^{-2}\) is obtained for an optimum designed \(P_0\) of \(10^{-3}\).

3.2 Total Outage Probability with Adaptive Equalizer

An adaptive equalizer has been used as a counter-measure of waveform distortion under a frequency selective fading environment. When the adaptive equalizer is equipped, the tolerable IAD increases since the outage occurrence due to the waveform distortion is suppressed by the adaptive equalizer, then the amount of transmission capacity is improved.

In order to represent the effect due to the adaptive equalizer, we will use a waveform factor that is mentioned in Appendix and Ref. (8). This analysis is applicable to transversal equalizer because its performance is represented easily by the waveform factor. To evaluate the increase in the transmission capacity due to adaptive equalizer, we define a ratio of waveform factors, \(k\), as follows:

\[
k = \frac{\sum |k(i)|}{\sum \sum \frac{|k(i)|}{k(i)}} \tag{9}
\]

where \(N\) is the number of taps of the adaptive equalizer, and \(\sum |k(i)|\) is the sum of the absolute of differential coefficient \(k(i)\) of the Nyquist pulse \(S(t)\) in BPSK system when the impulse response crosses zero, given by

\[
k(i) = \left( \frac{dS(t)}{dt} \right)_{t=iT} \tag{10}
\]

where \(T\) is a symbol interval.

In Eq. (9), the numerator is the waveform factor of BPSK signal and the denominator is that in the case with adaptive equalizer under no intersymbol interference at \(t = iT\), \(i = 1, 2, \cdots, (N - 1)/2\). Equation (9), therefore, means that the waveform factor in case of using adaptive equalizer is \(1/k\) times of that without equalizer, namely the equalizer effect can be denoted by the increase in \(k\). The relation between \(k\) and the number \(N\) of equalizer taps is shown in Fig. 4.

Thus the transmissible capacity of the capacity controlled system with adaptive equalizer is equivalent to that of no equalizer system with waveform factor reduced by \(1/k\). When the adaptive equalizer is used in the capacity controlled system, the transmissible capacity \(C_t\) is represented by
\[ C_t = \frac{\log_2(1 + \frac{k f(b) }{1.95})}{\log_2(1 + f(b(P_t^a)) \frac{1}{1.95})} \]  

(11)

from Eqs. (A·9) and (9).

Figure 5 illustrates the relation between the number of modulation levels \(2^n\) and allowable IAD \(b\) with the number of taps as parameter. It is obvious that the permissible IAD increases when the number of taps increases and the number of modulation levels decreases.

Therefore, the total throughput with equalizer is obtained by substituting Eq. (11) into Eq. (3) as

\[ P_{total} = (1 - P_b) \left\{ 1 - \exp \left( -\lambda \cdot \frac{\log_2(1 + \frac{k f(b) }{1.95})}{\log_2(1 + f(b(P_t^a)) \frac{1}{1.95})} \right) \right\} \]  

(12)

Figure 6 illustrates the total outage probability \(1 - P_{total}\) of the system with adaptive equalizer, derived from Eqs. (9) and (12), where \(k = 1\) in Eq. (9) means no equalizer. In this figure, the total outage probabilities for both the capacity controlled and the fixed capacity systems are show by a solid line and a broken line, respectively. We see that both systems are improved as the number of taps increases and the capacity controlled system is superior to the fixed capacity system.

4. Improvement Effect of the Capacity Controlled System

4.1 Fading Countermeasure Effect of the Capacity Controlled System

In order to evaluate improvement effect by the capacity controlled system on a frequency selective channel, we define an improvement factor \(I_{var}\) as follows;

\[ I_{var} = \frac{1 - P_{fix}}{1 - P_{var}} \]  

(13)

where \(1 - P_{var}\) is the total outage probability of capacity controlled system and \(1 - P_{fix}\) is that of conventional fixed capacity system where \(P_{fix}\) is derived from Eq. (8) as follows;

\[ P_{fix} = (1 - P_b) \left\{ 1 - \exp \left( -\lambda \cdot \frac{\log_2(1 + f(b(P_t^a)) \frac{1}{1.95})}{\log_2(1 + f(b(P_t^a)) \frac{1}{1.95})} \right) \right\} \]  

\[ = (1 - P_b) \{1 - \exp(-\lambda)\} \]  

(14)

Figure 7 illustrates outage improvement \(I_{var}\). The abscissa represents the designed inband amplitude dispersion (IAD) \(b(P_t^a)\) of the fixed capacity system, which depends on \(P_t^a\) of the fixed capacity system. From Fig. 7 it is shown that when the allowable IAD \(b\) is 30 dB, 5 times improvement effect is obtained by using the capacity controlled system. Figure 8 shows the ratio of throughput in capacity controlled system to that in the fixed capacity system, \(P_{var}/P_{fix}\), or the improvement effect of total throughput. When the
allowable IAD is 30 dB, 8 times improvement effect is obtained. From these results, the throughput and outage probability improvements increase as the designed IAD of the fixed capacity system becomes large, and are insensitive to the change of $\tau/T$. Therefore the capacity controlled system can be considered as a countermeasure for multi-path fading.

4.2 Synergistic Effect of Two Countermeasures

Figure 9 illustrates the improvement effects of the two countermeasures, i.e., capacity controlling and equalizing, on outage probabilities. A solid line shows the ratio of outage probability of the fixed capacity system without adaptive equalizer, $1 - P_{fix}$, to that of the capacity controlled system with equalizer, $1 - P_{var+eql}$, that is, the degree of improvement, $I_{var+eql}$, due to both capacity controlling and equalizing:

$$I_{var+eql} = \frac{1 - P_{fix}}{1 - P_{var+eql}}$$

(15)

where $P_{fix}$ is the total throughput of the fixed capacity system and $P_{var+eql}$ is that of the capacity controlled system with adaptive equalizer. A broken line shows the degree of improvement, $I_{fix+eql}$, due to only adaptive equalizing in the fixed capacity system, denoted by

$$I_{fix+eql} = \frac{1 - P_{fix}}{1 - P_{fix+eql}}$$

(16)

where $P_{fix+eql}$ is the total throughput of the fixed capacity system with adaptive equalizer and represents as follows;

$$P_{fix+eql} = (1 - P_0) \cdot \left\{ 1 - \exp\left( -\lambda \cdot \frac{\log_2 \left( 1 + \frac{k_f \left( b_0 \left( P_0^a \right) \right)}{1.95} \right)}{\log_2 \left( 1 + \frac{k_f \left( b_0 \left( P_0^a \right) \right)}{1.95} \right)} \right) \right\}$$

(17)

We can see from Fig. 9 that $I_{var+eql}$ is larger than that by $I_{fix+eql}$, and the degree of improvement becomes large as the number of taps increases. Furthermore, a dotted line shows the product of $I_{var}$, the degree of improvement due to only capacity controlling, by $I_{fix+eql}$. And if we write as

$$I_{var+eql} = \xi I_{var} \cdot I_{fix+eql}$$

(18)

we may consider that there exists a synergistic effect by using both capacity controlling and equalizing for the case of $\xi > 1$.

From Fig. 9 it is shown that there exists a synergistic effect $\xi > 1$, when the number of taps is smaller than 17. For more than 17 taps, there exists no synergistic effect, but the outage improvement by the capacity controlled system is still considerably large. Such a synergistic effect arising from both capacity controlling and equalizing is similar to that with using both diversity and adaptive equalizer.

Figure 10 shows $P_{var+eql}/P_{fix}$ and $P_{fix+eql}/P_{fix}$, that is, the degree of the improvement with the throughput. We see that the capacity controlled system with 5 taps is equivalent to the capacity fixed system with about 13 taps.
5. Conclusion

This paper analyzed the performance of the capacity controlled system with or without adaptive equalizer under the frequency selective fading environment. This system can increase the number of active subscribers by changing the modulation levels. It enforces the endurance of multi-path fading by using the small number of modulation levels when the number of active subscribers decreases.

As the results, the following had been found.
1. Both total throughput and outage probability improvement effect increase as the allowable IAD of capacity fixed system becomes large. For example, 5 times improvement effect of outage probability is obtained when the allowable IAD is 30 dB.
2. Thus it is clarified the capacity controlled system can be considered as a countermeasure for frequency selective fading.
3. The effect obtained by the capacity controlled system with adaptive equalizer is larger than that by the conventional system with it, and improvement ratio become large as number of taps increase. For example, 13 times improvement effect of outage probability is obtained in the capacity controlled system with equalizer of 5 taps, and 3.5 in the fixed capacity system with equalizer of the same taps.
4. Consequently, it is considered that there exists synergistic effect in the outage probability.
5. The capacity controlled system with 5 taps is equivalent to the capacity fixed system with about 13 taps.

References


Appendix A

The inband dispersion takes the maximum value when the fading notch passes through the inband edge frequency. Intersymbol interference increases in proportion to a waveform factor, which is susceptible to frequency selective fading. So we will introduce a waveform factor $K_{BPSK}$ defined by the sum of the absolute of differential coefficient of Nyquist pulse $S(t)$ at the point which $S(t)$ crosses the zero in BPSK system:

$$K_{BPSK} = \sum_{i=0}^{\infty} dS(t) \left|_{t=it} \right.$$

(A.1)

If an equalizer is adopted, $K_{BPSK}$ in Eq. (A.2) is modified as

$$K_{BPSK} = \sum_{i=0, \pm 1, \pm 2, \pm 3} dS(t) \left|_{t=it} \right.$$

(A.2)

In this case the relation between $\lambda$ and waveform factor $K$ is given by

$$\lambda = \left[ \frac{1}{1 + K_r} \right]^2 + \left[ \frac{\sin(2\pi T)}{1 + K_r} \right]^2$$

(A.3)

$$\lambda = \frac{K_r}{1 + K_r}$$

where $\tau$ is delay difference between direct and reflected rays. In Eq. (A.3), we assume that flat fading margin (FFM) is infinite, so threshold decision level is zero. Therefore, from Eqs. (5) and (A.3), following relation is satisfied:

$$KT = \frac{1 - \cos(2\pi T)}{(\lambda - 1)(\lambda/T)}$$

$$\frac{\sqrt{1 - \cos(2\pi T)^2 + 2(\lambda - 1)[1 - \cos(2\pi T)]}}{(\lambda - 1)(\lambda/T)}$$

$$\leq 0 < \tau/T < 1$$

(A.4)

where $T$ is symbol interval.

When $2^{2^n}$ QAM is used as multi-level modulation for good frequency utilization, waveform factor for $2^{2^n}$ QAM schemes are given by

$$KT = (2^n - 1) K_{BPSK}$$

(A.5)

where $K_{BPSK} T = 1.95$, which is the waveform factor of BPSK signal with roll off factor of 0.5.

Therefore, using the capacity controlled system,
information transmission capacity $C_{\text{bar}}$ is

$$C_{\text{bar}} = \frac{2n}{T} = \frac{2}{T} \log_2 \left( 1 + \frac{KT}{K_{\text{BPSK}}} \right)$$  \hspace{1cm} (A·6)

$$= \frac{2}{T} \log_2 \left( 1 + \frac{f(k)}{1.95} \right)$$  \hspace{1cm} (A·7)

where $f(k)$ is defined as $KT$.

On the other hand, if the designed outage probability of fixed capacity system is denoted by $P_0^*$, the transmission capacity is

$$C_{\text{fix}} = \frac{2}{T} \log_2 \left( 1 + \frac{f(k(P_0^*))}{1.95} \right).$$  \hspace{1cm} (A·8)

From Eqs. (A·7) and (A·8), the capacity decrease factor $C_t$ is defined as the transmission capacity normalized by that of conventional fixed capacity system, that is,

$$C_t = \frac{\log_2 \left( 1 + \frac{f(k)}{1.95} \right)}{\log_2 \left( 1 + \frac{f(k(P_0^*))}{1.95} \right)}.$$  \hspace{1cm} (A·9)

**Shozo Komaki** was born in Osaka, Japan in 1947. He received the B.E., M.E., and D.E. degrees in Electrical Communication Engineering from Osaka University, in 1970, 1972 and 1983 respectively. In 1972, he joined the NTT Radio Communication Labs., where he was engaged in repeater development for a 20-GHz digital radio system, 16-QAM and 256-QAM systems. From 1990, he has moved to Osaka University, Faculty of Engineering, and is currently a professor of Osaka University. Dr. Komaki is a member of IEEE, and the Institute of Television Engineers of Japan. He was awarded the Paper Award by IECE of Japan in 1977 and the NTT President Award in 1983.

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