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Theoretical Analysis of the Capacity Controlled Digital Mobile System in the Presence of Interference and Thermal Noise

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SUMMARY This paper analyzes the performance of the capacity controlled digital radio system, which controls the number of modulation levels according to the amount of traffic. These analyses are performed under thermal noise and co-channel interference. As a result, the throughput improvement is approximately 16 times comparing with the fixed capacity system which has the designed outage probability of 0.1%. Theoretical results are applied to the future mobile communication system which utilizes TDMA access method or burst co-dec, and it is found that the reuse distance can be improved to 1/5 times when the designed outage probability is 0.1%.
key words: *capacity controlled transmission, thermal noise, co-channel interference, multi-level modulation, mobile communication, reuse distance, fading channel*

1. Introduction

In the conventional mobile communication systems, the small number of modulation levels, such as QPSK, is used to prevent the quality degradations during the deep fading. In such systems, the number of radio subscribers in use are limited to small numbers, because the transmission capacity of the limited frequency bands cannot be increased due to small number of modulation levels.

On the other hand, it will be easy to increase the number of subscribers, using the large number of modulation levels such as 16QAM. However the modulated signal that has large number of modulation levels is easily impaired by the shallower fading compared with the QPSK case, and results in the lower transmission quality.

To compromise the higher quality with larger number of subscribers, we have proposed the capacity controlled system, which controls the transmission capacity by changing the number of modulation levels according to the amount of the subscribers(1). This system chooses the large number of modulation levels such as 16QAM, 32QAM or higher when the number of subscribers are increased, and also chooses the small number of modulation levels, such as QPSK or BPSK, vice versa. Using this large number of modulation levels in the capacity controlled system, the quality will not be degraded by the fading, because the conditional occurrence probability of the large amount of subscriber numbers and deep fading is small.

In this paper, we have analyzed the throughput improvement of the capacity controlled system, that indicates the increase of subscriber numbers, under the condition of co-channel interference.

Variable capacity system reported so far, controls the modulation speed to change the transmission capacity(2),(3), so the modulated signal bandwidth is changed when the transmission capacity is controlled. The system proposed here has the advantage of no band width expansion compared with the conventional systems.

As mentioned above, since the proposed system is effective to improve the peak traffic throughput, it has the improvement effect to the variable traffic loads that happens in the TDMA access mobile systems, multimedia communication systems that utilize the burst co-dec or packet radio communications.

The capacity controlled system is expected to be effective to enhance the throughput under the co-channel interference environment. And then it will reduce the reuse distance, because the allowable interference will become larger when it is applied to the cellular system with the same throughput as the fixed capacity system. Therefore this paper also analyzes the improvement of reuse distance when the capacity controlled system is used.

In this paper we first describe the system model of the capacity controlled system. Next, its throughput is derived theoretically. Finally throughput improvement and the effect on reuse distance are analyzed.

2. System Model

Figure 1 shows the system model for the capacity controlled system under the thermal noise and co-channel interference.

The traffic loss probability due to traffic fluctuation is improved by icreasing the channel capacity. In order to increase the channel capacity without widening bandwidth, we usually use a large number of modulation levels instead of binary ones. On the other hands, the channel outage probability becomes larger

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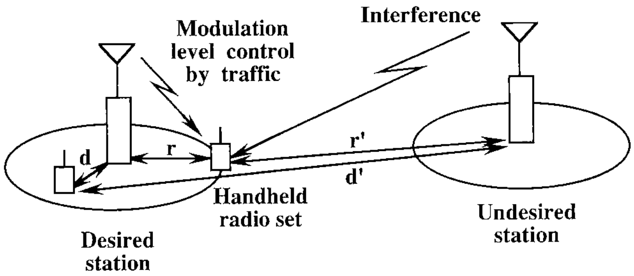


Fig. 1 System model.

because a large number of modulation levels requires a larger C/N. Thus there exists a trade-off between the traffic loss and the channel outage probability.

The capacity controlled system increases the number of modulation levels to withstand extraordinary traffic variation when the amount of traffic increases, and decreases it in accordance with the small traffic loads to endure the interference. By performing these control, the system has the endurance to multi-path fading in comparison with the fixed capacity system.

In this system model the following are assumed.

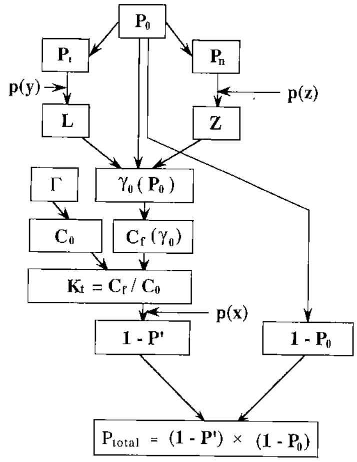
(a) control method
- The maximum modulation level is chosen according to the designed channel outage rate P_0, and a number of modulation levels is controlled according to the amount of traffic occurred at a transmitter.

(b) channel model
- The desired signal is Rayleigh distributed.
- The interference is also Rayleigh distributed, which is independent of the desired signal.
- The slow fading is assumed. The condition is satisfied when the fading speed is very slow as compared with the symbol rate. This condition is satisfied in case of hand held radio telephone set.
- The outage occurrence is caused by the desired signal power deterioration. The channel delay dispersion is neglected.
- The threshold bit error rate for the outage is 10^{-6}, because the future high quality multi-media communication is taken into account.

(c) traffic model
- Traffic is the exponentially distributed.
- Traffic and fading occurrence are statistically independent.

Details of the total throughput calculation are shown in the following section. For the easy understanding of the calculation process, Fig. 2 summarizes the whole calculation flow and parameters.

The total throughput at receiver is equal to the product of the following two as shown in the Sect. 3.3; the one is the traffic throughput, $1-P'$, considering traffic variation only, the other is the channel throughput, $1-P_0$, considering both thermal noise and co-channel interference occurrence. When the designed outage probability P_0 is small, the traffic blocking rate P' becomes large because the channel capacity becomes small for the sake of small number of modulation levels. So the total throughput $(1-P_0)(1-P')$ decreases. When P_0 is large, P' becomes small because the channel capacity becomes large. However the total throughput decreases because P_0 is large. It indicates that there is optimum point for P_0 because the total throughput is obtained from the product $1-P_0$ by $1-P'$. Therefore it is possible to transmit information with the maximum throughput by using the capacity controlled system at the optimum point.



P_i : Outage probability due to interference alone
P_0 : Outage probability due to interference and thermal noise
P_n : Outage probability due to thermal noise alone
L : Instantaneous interference to carrier power ratio
Z : Average thermal noise to instantaneous carrier power ratio
Γ : Average carrier to total noise power ratio
γ_0 : Instantaneous carrier to total noise power ratio
C_0 : Transmission capacity under the unfaded condition
C_f : Transmission capacity when maximum modulation level is used
p(x) : p.d.f. of the amount of traffic
p(y) : p.d.f. of the instantaneous interference to carrier power ratio
p(z) : p.d.f. of the Average thermal noise to instantaneous carrier power ratio

Fig. 2 Calculation procedure.

3. Throughput Analysis

3.1 Carrier to Total Noise Power Ratio

In this section, we determine the performance of the capacity controlled system in the presence of the interference and the thermal noise simultaneously. Generally the carrier power to the sum of the interference and thermal noise power ratio $C/(N+I)$ is given by as follows:

$$C/(N+I)=\frac{1}{(N/C)+(I/C)} \quad (1)$$

where N/C is the thermal noise to carrier power ratio and I/C is co-channel interference to carrier power ratio.

In the following analysis, we assume the slow fading condition. So the word "instantaneous" used in this paper is valid for the period, in which the desired signal and interference levels are quasi-stationary. And also "average" is valid for the same quasi-stationary period, where the thermal noise power is averaged. Therefore we call these instantaneous signal power, instantaneous interference power and averaged noise power in the following section.

3.1.1 Average Thermal Noise to Instantaneous Carrier Power Ratio Z

Here is devoted to analyzing the average thermal noise to instantaneous carrier power ratio, called Z. During the fading periods, the p. d. f. of the ratio of the average power of thermal noise to instantaneous carrier power, $p(z)$, is approximated as Pearson's Distribution[4]. That is

$$p(z)=\frac{\eta^{\eta}}{n_0\Gamma(\eta)}\left(\frac{z}{n_0}\right)^{-(\eta+1)}\exp\left(-\frac{\eta n_0}{z}\right) \quad (2)$$

where $\Gamma(\cdot)$ is the gamma function and η is a parameter. n_0 is the average power ratio of thermal noise to carrier for total observed periods. For the received signal with Rayleigh-distribution, $\eta=1$. In this case, the p. d. f. of the ratio of the average thermal noise power to instantaneous carrier power is rewritten by

$$p(z)=\frac{n_0}{z^2}e^{-n_0/z} \quad (\eta=1). \quad (3)$$

Outage due to thermal noise occurs when the thermal noise z exceeds the threshold value Z. Consequently, the outage probability P_n due to the thermal noise alone is given by

$$\begin{aligned}P_n &\equiv P[z>Z]=\int_Z^{\infty}p(z)\,dz \\ &=1-\exp\left(-\frac{n_0}{Z}\right) \\ &\approx\frac{n_0}{Z} \quad (Z\gg n_0). \quad (4)\end{aligned}$$

From Eq.(4), Z is obtained by

$$Z=\frac{n_0}{P_n}=\frac{1}{\Gamma_n P_n} \quad (5)$$

where Γ_n is defined by the inverse of n_0 and denotes the average carrier to thermal noise power ratio for total observed periods.

3.1.2 Instantaneous Interference to Carrier Power Ratio L

Here is devoted to analyzing the instantaneous interference to carrier power ratio, called L. When signal and interference are statistically independent and Rayleigh distributed, the p. d. f. of instantaneous interference to carrier power ratio, y, is given by[5]

$$p(y)=\frac{i_0}{(y+i_0)^2}. \quad (6)$$

Similarly as the above, system outage due to interference occurs when the y exceeds the threshold value, so the outage probability P_i due to interference alone is obtained by

$$P_i\equiv P[y>L]=\int_L^{\infty}p(y)\,dy=\frac{i_0}{L+i_0} \quad (7)$$

where i_0 is the average interference to carrier power ratio for the total observation period. From Eq. (7), L can be written by

$$L=i_0\left(\frac{1}{P_i}-1\right)=\frac{1}{\Gamma_i}\left(\frac{1}{P_i}-1\right) \quad (8)$$

where Γ_i is defined by the $1/i_0$ and denotes the average carrier to interference power ratio for the total observation period.

3.1.3 Instantaneous Carrier to Total Noise Power Ratio γ_0

From the concept of the composite fade margin[6], the total outage probability P_0 can be shown by the sum of P_i due to interference and P_n due to thermal noise. So, the followings are derived if we introduce the design parameter a, which denotes the ratio of P_i and P_0.

$$P_0=P_i+P_n \quad (9a)$$

$$P_i=aP_0 \quad (9b)$$

$$P_n=(1-a)P_0 \quad (9c)$$

Similarly, we will obtain the following relations in normal propagation condition:

$$\Gamma_i=\frac{\Gamma}{b} \quad (10a)$$

$$\Gamma_n=\frac{\Gamma}{1-b} \quad (10b)$$

$$\Gamma=\frac{1}{(1/\Gamma_i)+(1/\Gamma_n)}=\frac{\Gamma_i\Gamma_n}{\Gamma_i+\Gamma_n} \quad (10c)$$

where b denotes the ratio of Γ and Γ_i.

Suppose $a\approx b$, then from Eqs. (1), (5), (8), (9a) and (10c), when fading occurs the instantaneous carrier to total noise power ratio γ_0 is written by

$$\gamma_0=\frac{1}{L+Z}=\frac{P_0\Gamma}{2-aP_0}. \tag{11}$$

3. 2 Traffic Throughput

3. 2. 1 Normalized Transmission Capacity K_t

When the 2^{2n} QAM is selected for the multi-level modulation, the carrier to noise power ratio (CNR) ρ_{QAM} is given by[7]

$$\rho_{QAM}=\frac{1}{3}(2^{2n}-1)\,\rho_{QPSK} \tag{12}$$

where ρ_{QPSK} is the CNR of quaternary PSK. Assuming the bit interval is T, the information transmission rate C is written by

$$C\equiv\frac{2n}{T}=\frac{1}{T}\log_2\left(1+3\cdot\frac{\rho_{QAM}}{\rho_{QPSK}}\right). \tag{13}$$

Therefore, in fading channel the information transmission capacity C_0 for QAM signal can be represented by

$$C_0=\frac{1}{T}\log_2\left(1+3\cdot\frac{\Gamma}{\rho_{QPSK}}\right), \tag{14}$$

while for the capacity controlled QAM system the information transmission capacity C_f becomes

$$C_f=\frac{1}{T}\log_2\left(1+3\cdot\frac{\gamma_0}{\rho_{QPSK}}\right) \tag{15}$$

where γ_0 is the instantaneous carrier to total noise power ratio, as shown Eq. (11). Consequently, when the outage occurs, the transmission capacity normalized by C_0, which is denoted by K_t. In the following part, we call K_t as capacity decrease factor which is given as follows:

$$K_t\equiv\frac{C_f}{C_0}=\frac{\log_2\left(1+3\cdot\frac{\gamma_0}{\rho_{QPSK}}\right)}{\log_2\left(1+3\cdot\frac{\Gamma}{\rho_{QPSK}}\right)}. \tag{16}$$

3. 2. 2 Traffic Throughput $1-P'$

We assume that the p. d. f., $p(x)$, of the traffic volume x is given by the following exponential distribution with the average $1/\lambda$:

$$p(x)=\lambda\exp(-\lambda x). \tag{17}$$

The traffic throughput may be defined by the probability that the amount of traffic is below possible transmission capacity. Since K_t is the possible transmission capacity, the traffic throughput $1-P'$ is represented by

$$1-P'=\int_0^{K_t}p(x)\,dx=1-\exp(-\lambda K_t) \tag{18}$$

where P' denotes the channel blocking probability.

3. 3 Total Throughput

The total throughput is obtained from the following equation:

$$P_{total}=\int_0^{K_t}\int_{\gamma(x)}^{\infty}p(x,\gamma)\,dxd\gamma \tag{19}$$

where $p(x,\gamma)$ is the joint p. d. f. of traffic volume variable x and the CNR variable γ, K_t is the possible transmission capacity using the maximum modulation level number and $\gamma(x)$ is the CNR required when a modulation level number is chosen. If traffic and interference occurrences are statistically independent, Eq. (19) is replaced by

$$\begin{aligned}P_{total}&=\int_0^{K_t}p(x)\int_{\gamma(x)}^{\infty}p(\gamma)\,dxd\gamma\\&\geqq\int_0^{K_t}p(x)\,dx\int_{\gamma_0}^{\infty}p(\gamma)\,d\gamma\\&=(1-P')(1-P_0).\end{aligned} \tag{20}$$

When the maximum number of modulation level is used, the outage probability P_0 due to the total noise is the probability that the received signal power is below γ_0. Since the actual modulation levels are selected such that it is no larger than the maximum modulation level, the channel outage probability is no larger than P_0. In this paper, P_0 is used as a measure for the worst case channel outage probability.

Consequently, using Eqs. (11), (18) and (20), the total throughput is given by

$$P_{total}=(1-P_0)\cdot\left\{1-\exp\left(-\lambda\frac{\log_2\left(1+3\cdot\frac{P_0\Gamma}{(2-aP_0)\,\rho_{QPSK}}\right)}{\log_2\left(1+3\cdot\frac{\Gamma}{\rho_{QPSK}}\right)}\right)\right\}. \tag{21}$$

Figure 3 shows the results of the total throughput with Γ as a parameter.

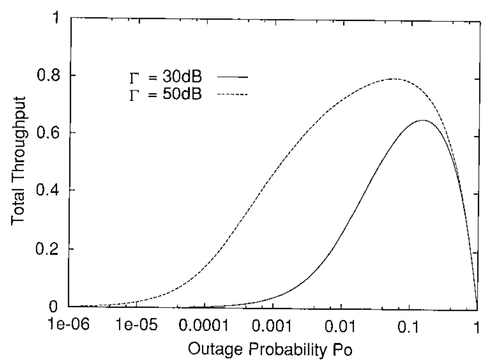


Fig. 3 Total throughput (a=0.5).

4. Effect on Throughput and Reuse Distance

4.1 Improvement Throughput Comparing to Fixed Capacity System

The throughput improvement of the capacity controlled system is shown in the Fig. 4. The capacity controlled system is performed with the maximum throughput in Fig. 3, while the fixed capacity system is performed with throughput at a designed outage probability P_0. Therefore, in this figure the improvement factor is defined by the ratio of the throughput of the capacity controlled system, denoted by the maximum throughput in Fig. 3, and the fixed capacity system, denoted by the throughput for a designed outage probability. These results indicate the throughput improvement exceeding 16 times is obtained when the designed outage probability is 0.1%. The improvement becomes smaller as the outage probability become larger, and for instance, there exists no improvement for 10% outage probability. Considering the improvement of the communication quality in the future mobile system, outage rates less than 10% may usually be achieved. For this reason, in this paper, the designed outage probability of the fixed capacity system in the range of 0.1∼10% is taken into account.

4.2 Improvement Effect on Reuse Distance

Since the capacity controlled system is effective in enhancing the throughput in the presence of interference, we can consider that the system has an ability to allow the stronger interference comparing with the constant transmission system for the same throughput.

The radio zones must be mutually separated geographically so that users do not suffer unacceptable co-channel interferences. Denoting the distances from a handheld radio set to a desired base station and to an interfering base station by d and d' respectively, as in Fig. 1, since the received power is inversely proportional to 4th power of the distance, d or d', d, d' and Γ_i must be satisfied by the following relation:

$$\left(\frac{d'}{d}\right)^4 \geqq \Gamma_i. \tag{22}$$

In other words, the distance that can be reused between co-channel zones is determined by the carrier to interference power ratio, Γ_i which the system allows. If the mobile station is on the nearest cell edge to the interference base station, the co-channel interference is the greatest. In this case, the reuse distance of a radio channel, D, is defined as the distance $r+r'$ between desired and undesired base stations normalized by the zone radius r, as in Fig. 1[(8)], i.e.,

$$D=\frac{r+r'}{r}. \tag{23}$$

From the relation as shown Eq. (22), the Eq. (23) can be written by

$$D=1+\Gamma_i^{1/4}. \tag{24}$$

Let us examine the permissible amount of interference when the capacity controlled transmission is used. For this purpose, we define Γ_{iv} as allowable carrier to interference power ratio Γ_i need to achieve a throughput P_{th} with the capacity controlled system, and define Γ_{if} as Γ_i need to achieve the same throughput P_{th} with the fixed capacity system, as in Fig. 5. In addition P_{ov} is defined as the designed outage probability of the capacity controlled system and P_{of} as that of the fixed capacity system.

The required Γ_{if} as a function of ratio a is plotted in Fig. 6 for $\Gamma=50$ dB. It indicates that Γ_{if} becomes small as the outage caused by interference occurs frequently.

On the other hand, Γ_{if}/Γ_{iv} shows the effective property of the capacity controlled system for interference. Figure 7 gives a relation between Γ_{if}/Γ_{iv} and P_{of} when $\Gamma=50$ dB. It is obvious from this figure that the capacity controlled system is effective on interference and therefore the reuse distance may decrease as Γ_{if}/Γ_{iv} becomes larger and the designed outage of the fixed capacity system, P_{of}, becomes smaller. And it also

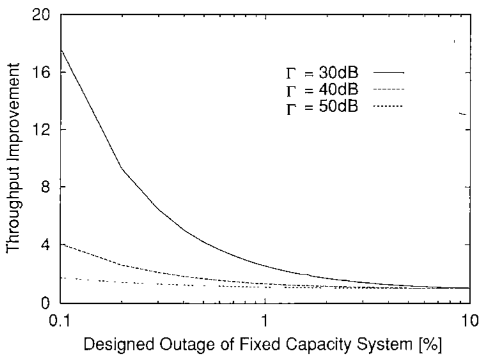


Fig. 4 Throughput improvement by the capacity controlled system ($a=0.5$).

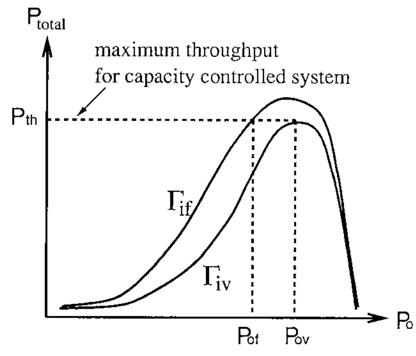


Fig. 5 Calculation of the allowable *CIR* improvement.

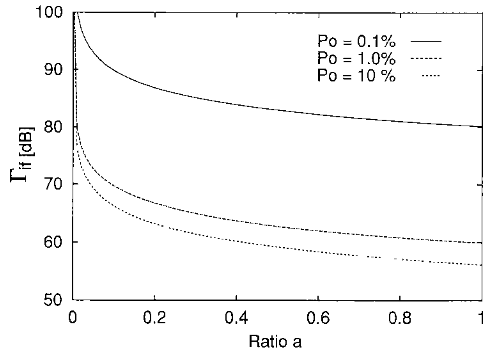


Fig. 6 Required carrier-to-interference power ratio of fixed capacity system that has the same throughput as the capacity controlled system with Γ (Γ=50 dB).

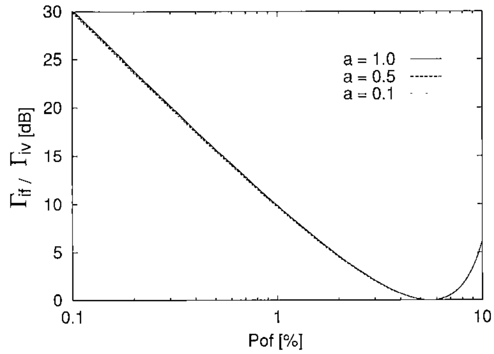


Fig. 7 Improvement of carrier-to-interference power ratio obtained by the capacity controlled system.

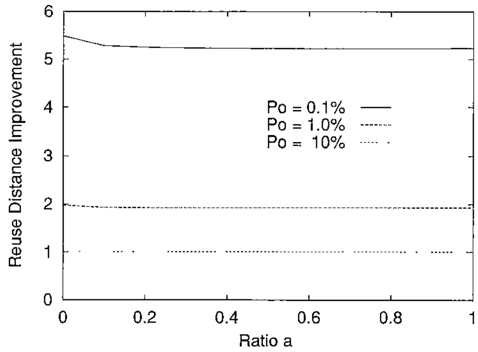


Fig. 8 Relation between the improvement of reuse distance and ratio a (Γ=40 dB).

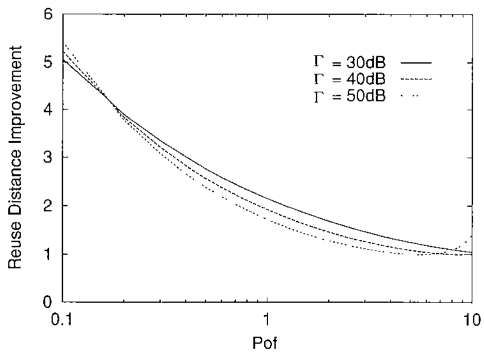


Fig. 9 Relation between the improvement of reuse distance and the outage probability P_{of} (a=0.5).

shows that the interference is the same as thermal noise with a same power since the differences among various ratios a are very small.

In order to compare the reuse distance, we define the improvement factor M as

$$M=\frac{D'}{D}=\frac{1+\Gamma_{if}^{1/4}}{1+\Gamma_{iv}^{1/4}} \tag{25}$$

where D' obtained by the fixed capacity system with D obtained by the capacity controlled system. Figure 8 illustrates the calculated results of Eq. (25) the designed outage probability of capacity fixed system P_0 as parameter. The results show that the advantage of reuse distance by using the capacity controlled system is approximately 5 times when the designed outage of the fixed capacity system is 0.1%.

The improvement effect on the designed outage probability P_0 is portrayed schematically in Fig. 9 with parameter Γ. Obviously the advantage of reuse distance increases as P_0 decreases. When the capacity controlled system is used, large improvement effect is obtained for small Γ in the range of P_0=0.2–10% and for large Γ in the vicinity of P_0= 0.1%.

5. Conclusion

This paper analyzed the performance of the capacity controlled digital mobile system in the presence of thermal noise and co-channel interference. The system controls the transmission capacity according to the amount of traffic by changing the modulation levels.

As a result, the following were found.

1. The throughput improvement of 16 times, which the designed outage probability of the fixed capacity system is 0.1%, is available by using the capacity controlled system under the condition of interference and thermal noise. Then the system can absorb the traffic peak more than 16 times, comparing to the conventional fixed capacity system.
2. It is possible the reuse distance is 1/5 times comparing with the fixed capacity system which the designed outage probability is 0.1%. The frequency utilization efficiency per unit area will be improved 25 times.
3. The improvement effect depends on the designed outage probability of the fixed capacity system, and its effect decreases as the designed outage probability increases.

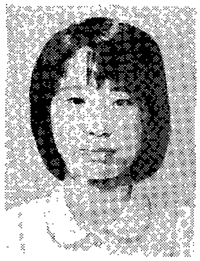
4. There is little dependency between interference and thermal noise power. So the total power of the interference and thermal noise is the important factor.

The analytical results for throughput improvement can be applicable not only to mobile system but also to the other systems, i.e. the fixed radio or satellite communication systems.

In this paper, we had assumed that the channel outage occurrence was caused by the Rayleigh-distributed desired signal level deterioration or interference level increase. In the future, it is necessary to analyze that the capacity controlled system in long term level deterioration by blocking or shadowing, and frequency-selective channel. Furthermore, although exponential distribution has been used as the traffic distribution for simplicity, the analysis for different type of traffic distribution will be necessary for multimedia communication. Also further study for capacity controlled modem and control protocols will be required to realize this system.

References

(1) Komaki S.: "Theoretical analyses of a capacity controlled digital microwave radio", Trans. IEICE, **J73-B-II**, 10, pp. 498-503 (Oct. 1990).
(2) Goodman D. J. and Sundberg C.-E.: "Combined source and channel coding for variable-bit-rate speech transmission", Bell Syst. Tech. J., **62**, pp. 2017-2036 (Sept. 1983).
(3) Katakol B. S. and Maskara S. L.: "Adaptive variable rate communication system for fading channels", J. INSTN. Electronics Telecom. Engrs., **32**, 3 (1986).
(4) Morita K.: "Estimation methods for distribution of thermal and interference noise power in microwave", Trans. IEICE, **J64-B**, 12 (Dec. 1981).
(5) Okui S., Morinaga N. and Namekawa T.: "Probability distribution for ratio of fading signal envelopes and their generalization", Trans. IECE Japan, **J64-B**, 11, pp. 1228-1235 (Nov. 1981).
(6) Rummler W. D.: "A comparison of calculated and observed performance of digital radio in the presence of interference", IEEE Trans. Commun., **COM-30**, pp. 1693-1700 (July 1982).
(7) Hancock J. C. and Lucky R. W.: "Performance of Combined Amplitude and Phase-Modulated Communication Systems", IRE Trans. Commun., pp. 232-237 (Dec. 1960).
(8) French R. C.: "The effect of fading and shadowing on channel reuse in mobile radio", IEEE Trans. Veh. Technol., **VT-28**, 3, pp. 171-181 (Aug. 1979).



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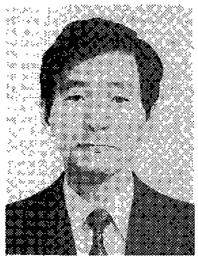

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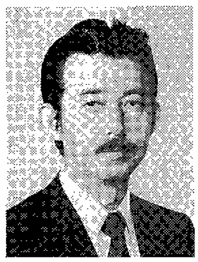

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