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## LETTER

# An Application of Simulated Annealing to the Design of Block Coded Modulation

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**SUMMARY** This paper proposes a new block coded quadrature amplitude modulation (BC-QAM) scheme, which is designed by an optimization technique based on simulated annealing. Simulated annealing is an effective nonlinear optimization technique and can be applied to both the discrete and the continuous optimization problems. In this paper, the simulated annealing technique is used to design the optimum BC-QAM signal, which minimizes the upper bound on the bit error rate (BER) in a Rayleigh fading channel. The computer simulation shows that the proposed BC-QAM can improve the BER performance. This paper also proposes a simplified design method to reduce the number of variables to be optimized. The proposed simplified method optimizes the in-phase and quadrature components of the BC-QAM signal separately. The computer simulation also shows that the BC-QAM designed by the simplified method gives little degradation on the BER performance, although the simplified method can significantly reduce the number of optimization variables.

**key words:** coded modulation, fading, QAM, simulated annealing

## 1. Introduction

Coded modulation is an effective technique to improve the bit error rate (BER) performance of digital mobile radio communication systems, which suffers from severe performance degradation due to Rayleigh fading. In particular, the trellis coded modulation schemes which are based on convolutional codes, are widely studied and implemented in various radio systems [1], [2].

Block coded modulation (BCM) is an alternative technique to improve the BER performance. BCM schemes are more flexible than TCM ones from the code design point of view. Furthermore, since BCM schemes require no tail sequence, which is required for the decoding of the TCM signals, the overall bandwidth efficiency can be improved for shorter packet and the burst mode transmissions. In the previous works [3]–[6], we have proposed a block coded modulation (BCM) scheme, where the signal sets have been designed without restriction on both amplitude level and phase angle, using the quasi-Newton method [7], which is a nonlinear optimization technique to optimize continuous nonlinear functions. We have shown that the proposed BCM scheme can drastically improve the BER performance in a Rayleigh fading channel.

However, since the use of the signal sets without the restriction makes the BCM modulator complicated, it is necessary to design a BCM with symmetric signal sets, such as the conventional QAM (Quadrature Amplitude Modulation) signals having rectangular signal constellations. Therefore, although we have to solve a discrete nonlinear optimization problem to optimize a BCM with symmetric signal sets, the quasi-Newton method can not be applied to the discrete problem. In this paper, we propose a new BC-QAM (Block Coded Quadrature Amplitude Modulation) scheme with the conventional rectangular QAM signal sets, whose signals are designed by simulated annealing. Simulated annealing is a nonlinear optimization technique [8], which can solve not only continuous but discrete nonlinear optimization problems.

To improve the BER and the spectral efficiency of BCM, we have to increase the code length [6] and the number of the code words. However, when either of these parameters becomes large, the number of variables to be optimized increases exponentially, and the design becomes impossible. In this paper, we propose a simplified BCM design method, which designs the in-phase and quadrature phase components of the BC-QAM signals separately to reduce the number of optimization variables. Then, we show the BER performance of the proposed BC-QAM.

## 2. System Description

The block coded modulation system considered in this paper is shown in Fig. 1. A binary sequence of  $K$  bits

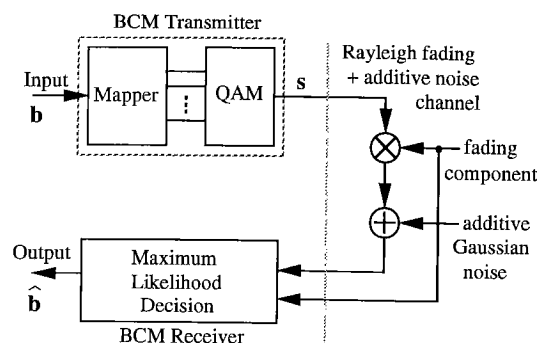


Fig. 1 System model.

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is fed into the block coded quadrature amplitude modulation (BC-QAM) transmitter, which consists of the binary block encoder and the  $2^L$ -ary QAM modulator. The block encoder maps the  $K$ -bit sequence into  $N$  binary blocks of  $L$  bits. The QAM modulator then maps each  $L$ -bit block onto a  $2^L$ -ary QAM symbol, and generates an  $N$ -symbol BC-QAM signal. There are  $2^K$  BC-QAM signals which is chosen within  $2^{LN}$  different possible QAM signals.

The generated BC-QAM signal is transmitted over a Rayleigh fading channel as shown in Fig. 1. The received signal is disturbed by the Rayleigh fading and the additive Gaussian noise. At the receiver, the received signal and the complex-valued fading component are fed into the decision block. The receiver estimates the transmitted binary sequence based on the maximum likelihood decoding. We suppose that the complex-valued fading components are known at the receiver.

Let  $\mathbf{b}^{(i)} = (b_1^{(i)}, \dots, b_K^{(i)})$  be the  $i$ -th input binary sequence within  $2^K$  possible binary sequences of  $K$  bits, and let  $\mathbf{s}^{(i)} = (s_1^{(i)}, \dots, s_N^{(i)})$  be the output QAM signal corresponding to  $\mathbf{b}^{(i)}$ . Each symbol in the sequence,  $s_k^{(i)}$ , which is chosen from one of the  $2^L$ -ary QAM signal points, is transmitted at discrete time  $k$ .

The upper bound on BER in a Rayleigh fading channel with ideal interleaving is expressed as [1], [6],

$$P_b = \sum_{i=0}^{2^K-1} P_M(\mathbf{b}^{(i)}) \sum_{\substack{i'=0 \\ i' \neq i}}^{2^K-1} a(\mathbf{b}^{(i)}, \mathbf{b}^{(i')}) p(\mathbf{s}^{(i)} \rightarrow \mathbf{s}^{(i')}) \quad (1)$$

where  $P_M(\mathbf{b}^{(i)})$  is the probability that  $\mathbf{b}^{(i)}$  is transmitted,  $a(\mathbf{b}^{(i)}, \mathbf{b}^{(i')})$  is the bit error rate when  $\mathbf{b}^{(i)}$  is transmitted and  $\mathbf{b}^{(i')}$  is received, and

$$p(\mathbf{s}^{(i)} \rightarrow \mathbf{s}^{(i')}) = \prod_{l=1}^N \left( 1 + \frac{E_s}{4N_0} |s_l^{(i)} - s_l^{(i')}|^2 \right)^{-1} \quad (2)$$

is the pairwise error probability that  $\mathbf{s}^{(i')}$  is received when  $\mathbf{s}^{(i)}$  is transmitted. In this equation,  $E_s$  is the average signal energy per symbol and  $N_0$  is the power spectral density of the noise.

### 3. Signal Design Formula

Now, we propose a BCM design method for the BCM which minimizes (1). In the proposed method, we use simulated annealing. Simulated annealing is an algorithm which has been proposed for finding a globally optimum point to minimize an evaluation function. Let  $S = (s^{(0)}, \dots, s^{(2^K-1)})$  be a set of the BC-QAM signals,

and let

$$f(S) = \log \left( \sum_{i=0}^{2^K-1} P_M(\mathbf{b}^{(i)}) \times \sum_{\substack{i'=0 \\ i' \neq i}}^{2^K-1} a(\mathbf{b}^{(i)}, \mathbf{b}^{(i')}) p(\mathbf{s}^{(i)} \rightarrow \mathbf{s}^{(i')}) \right) \quad (3)$$

be the evaluation function based on the upper bound shown in (1). Since  $\log x$  is the monotonically increasing function of  $x$ ,  $S$  which minimizes  $f(S)$  also minimizes the upper bound. The signal design algorithm based on the simulated annealing is described as follows:

1. Initialize a temperature  $T$  and a set of the BC-QAM signal  $S$ .
2. Compute:  $d = f(S)$
3. Choose an alternative set of BC-QAM signals

$$S' = (s'^{(0)}, \dots, s'^{(2^K-1)}), \quad (4)$$

where

$$s'^{(i)} = (s'_1^{(i)}, \dots, s'_N^{(i)}). \quad (5)$$

Each QAM symbol,  $s'_l^{(i)}$  is randomly chosen from  $s_l^{(i)}$  and its neighbor symbols.

4. Recompute the estimation function:  $d' = f(S')$
5. (a) If  $d' < d$ , replace  $S$  by  $S'$ .  
(b) If  $d' \geq d$ , replace  $S$  by  $S'$  with the probability:

$$P = \exp \left( -\frac{d' - d}{T} \right) \quad (6)$$

6. Reduce the temperature  $T \leftarrow \alpha T$ , where  $0 < \alpha < 1$  is a reduction factor.
7. Repeat step 2 to 6 until  $T < \epsilon$ , where  $\epsilon$  a small positive number.

In this algorithm, the temperature  $T$  decides the probability  $P$  that  $S$  is replaced by  $S'$  which makes BER worse. If the temperature is very low,  $S$  is subject to change towards the direction which the BER decreases and there is little chance to replace  $S$  by  $S'$  which makes the BER worse. However, in low temperature,  $S$  may be trapped by a local minimum, and never fall into a global optimum point. On the other hand, when the temperature becomes high, although  $S$  is not trapped by the local minima,  $S$  changes almost randomly and does not converge. In the simulated annealing algorithm, very

high temperature is chosen as an initial temperature to avoid trapping the local minima. Then the temperature is gradually reduced. When the temperature becomes low enough,  $S$  will converge to the optimum point.

In the proposed method, since the set of the BC-QAM signal,  $S$ , has  $2^K N$  complex variables, we shall have to optimize  $2^K N$  complex variables, or  $2^{K+1} N$  real variables. When  $K$  becomes large, the number of the variables increases significantly and it is impossible to optimize the BC-QAM, even if the simulated annealing method is used. To reduce the number of the variables, we propose a simplified design method, which designs the in-phase and the quadrature phase components of the BC-QAM signal independently. We suppose a  $2^{L/2}$ -ary ASK (amplitude shift keying) signal instead of a  $2^L$ -ary QAM signal. And a  $K/2$ -bit binary sequence is supposed to be mapped into a  $N$ -symbol  $2^{L/2}$ -ary ASK signal. In this assumption, the number of real variables, which we shall have to optimize, is reduced to  $2^{K/2} N$ .

#### 4. Numerical Results

In this section, we design BC-QAM using both design methods mentioned in Sect. 3, and analyze the BER performances in a Rayleigh fading channel. We suppose that the 256-ary QAM with rectangular signal sets is applied to the BC-QAM. We also suppose  $K = 8$  and  $N = 4$ , that is, the spectral efficiency of the BC-QAM is 2 (bits/symbol), or the spectral efficiency is all the same as that of the un-coded QPSK (Quadrature Phase Shift Keying). The optimization of this BC-QAM requires 2048 real variables. On the other hand, using the simplified design method, we can reduce the number of the variables to 64. We design the BC-QAM with using both the original and simplified optimization methods.  $E_s/N_0$  in (3) is set to 15 (dB). In the following discussion, we denote the BC-QAMs designed by the original and the simplified optimization methods as BC-QAM (I) and BC-QAM (II), respectively. In the optimization, we suppose that the initial temperature is 1000, the reduction factor  $\alpha = 0.95$ , and  $\epsilon = 10^{-9}$ .

Figure 2 shows computer simulation results for the BER performance of the BC-QAMs in a Rayleigh fading channel with the ideal interleaving. The dashed line illustrates the theoretical BER performance of the un-coded QPSK in a Rayleigh fading channel. We see that the proposed BC-QAMs can improve the BER performance, e.g., the proposed schemes give a coding gain of 6 dB at BER =  $10^{-3}$  against the un-coded QPSK. This figure also shows that there is no significant difference in BER performance between BC-QAM (I) and BC-QAM (II), although the number of the optimization variables are drastically reduced.

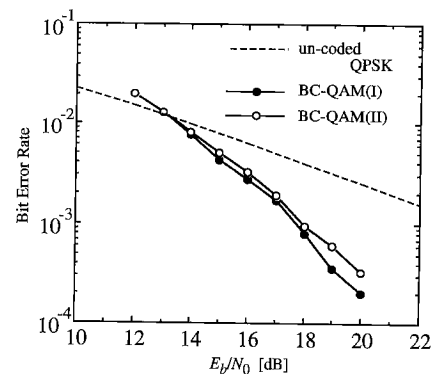


Fig. 2 The bit error rate performance of the proposed BC-256QAM schemes in a Rayleigh fading channel.

#### 5. Conclusions

In this paper, we have proposed a new BC-QAM scheme designed by simulated annealing. Furthermore, a simplified design method, which optimizes the in-phase and the quadrature components separately, is also proposed. Computer simulation has shown that the BC-QAM designed by the proposed method improves the BER performance significantly, and there is no significant difference in BER performance between BC-QAM designed by the original design method and that of simplified one. This indicates that the simplified design method is also a powerful technique to design the BC-QAM. In this study, it is supposed that the perfect estimation of the complex-valued fading components are performed at the receiver. However, in real systems, it is difficult to establish the perfect estimation of the fading parameters, and the accuracy of the estimation will affect the BER performance. Therefore, the overall performance evaluation of the proposed scheme with the imperfect estimation must be required in the further study.

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