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Gender Specialization of Skill Acquisition

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Gender Specialization of Skill Acquisition*

Junichiro Ishida and Hiromi Nosaka

Abstract

This paper presents a model that can account for the gender specialization of skill acquisition in the presence of competitive matching. In particular we show that when the comparative advantage in nonmarket domestic activities belongs to women, an incentive arises for them to intentionally degrade the market value of acquired skills in order to secure gains from the marriage market. We then show that this incentive can be excessively strong and gives rise to the emergence of an inefficient asymmetric equilibrium where women concentrate excessively on acquiring skills that do not lead to higher wages in the labor market. The analysis reveals why policy interventions such as affirmative action programs or equal employment opportunity laws that directly subsidize the acquisition of skills for women would not be effective in closing the gender earnings gap in the long run, and instead suggests that extensive family policies are generally more effective in this regard.

KEYWORDS: gender specialization, human capital investment, marriage, intrahousehold bargaining, affirmative action

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1 Introduction

Personal attributes often dictate the way people acquire skills. Among them, gender in particular seems to be a crucial determinant of the pattern of skill acquisition. Even in countries where the gender differences in educational attainment no longer exist, in general the pattern of skill acquisition has been highly segregated by gender. A pattern consistently observed is that women tend to invest in skills that, on average, do not lead to high wages in the labor market whereas men acquire skills that are necessary for high-income occupations. In the US, for instance, women have been underrepresented in high-income college majors such as engineering and business, at least until recently: they made up only 9% of all business majors and 0.8% of all engineering majors in 1971, yet 74% of all education and foreign language literature majors (US Census Bureau, 2004–2005, No.285). As the salaries of engineering and business graduates were 43% and 13% higher, respectively, than those of humanities graduates in 1975 (US Census Bureau, 1980, No.283), the differences in college majors account for a substantial part of gender wage gaps (Eide and Grogger, 1995; Brown and Corcoran, 1997; Altonji and Blank, 1999). Apparently, this asymmetric pattern of skill acquisition is not an isolated phenomenon in the US. In Japan, besides the gender difference in college major choices, a substantial portion of women attend two-year junior colleges that place a clear emphasis on the acquisition of domestic skills such as home economics or domestic science.

To examine this gender specialization of skill acquisition (hereafter referred to simply as the gender specialization), our paper presents a theoretical framework that can account for this pattern and examines its welfare and policy implications. To this end, the first part of the paper is concerned with efficiency properties, asking whether the gender specialization ever undermines social welfare, and, if so, in what ways. The answer to this question is not necessarily straightforward as the gender specialization could be a result of the efficient division of labor within households. As Becker (1991) most notably suggested, the two parties in a household generally do not need to

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1 In the US, for instance, the gender differences in educational attainment had almost disappeared by the 1970s (Corcoran and Duncan, 1979).
2 The figures are the initial monthly salaries offered to new graduates.
3 In particular, Brown and Corcoran, (1997) show that 0.08 to 0.09 of a 0.2 gender wage gap is caused by differences in college majors.
4 The gender difference in the college enrollment rate had disappeared in Japan by the late 1980s. However, of those women attending college, about 60% attended junior colleges, whereas the corresponding figure for men was only 5% (Ministry of Education, Culture, Sports, Science and Technology of Japan, 2004).
acquire the same set of skills: if men invest in skills designed for market activities, it is often more beneficial for women to acquire skills designed for domestic activities.\(^5\) In order to reap the benefit of role specialization, the pattern of skill acquisition could be, and, perhaps to some extent, should be segregated by gender. Although Becker analyzed a situation where the investments take place after marriage, subsequent studies such as Echevarria and Merlo (1999), Hadfield (1999), Engineer and Welling (1999) and Ishida (2003) confirmed that Becker’s original insight still holds in various settings even if the investments take place before marriage.

Although these studies are certainly suggestive, there is an important caveat to their results: their models assume either that agents are homogeneous before investment decisions or that the matching is random.\(^6\) These assumptions may trivialize a critical aspect of marriage formation because the matching pattern in the marriage market is often positively assortative. The main purpose of the paper is to incorporate this competitive aspect of marriage formation and explore its consequences and implications. This addition proves to be critical, as it introduces a new dimension to the problem, i.e., the trade-off between the marriage and labor markets. More importantly, we argue that this trade-off distorts incentives, especially on the part of women, and ultimately leads to inefficient asymmetric equilibria where women concentrate excessively on the acquisition of nonmarketable domestic skills.

The intuition behind this inefficiency result is as follows. With marriage arises the benefit of role specialization. As full-time jobs normally require full-time effort, it is often optimal for at least one member of the household to focus exclusively on market activities. In many cases, it is women who expend more resources on domestic activities as they often possess a comparative advantage in them (Lazear and Rosen, 1990; Echevarria and Merlo, 1999). Provided that the productivity in domestic activities does not depend strongly on the level of accumulated marketable skills, this implies that at least some fraction of women’s skills must be wasted when they are married.

\(^5\) Throughout the paper, the term “domestic activities” is used to broadly refer to various nonmarket activities such as housekeeping, childbearing and child rearing.

\(^6\) The random matching assumption implies that marriages are formed based on exogenous (noneconomic) factors: Engineer and Welling (1999) referred to this as the “true-love” criteria. In this paper, we take the opposite stance, that marriages are formed on the basis of purely endogenous (mostly economic) factors. These views clearly represent the two extreme points of the spectrum, and reality must lie somewhere in between. It should be noted that our main results hold even when exogenous factors play some role in marriage formation, as long as endogenous factors are sufficiently important.
This fact gives rise to subtle strategic interactions when the total surplus is divided through intrahousehold (Nash) bargaining, as is often assumed in the recent literature. Because women with higher earnings potentially have stronger bargaining power (more precisely, higher threat points) but do not increase the total surplus of the household sufficiently, they tend to be perceived as less desirable as marital partners from the viewpoint of men. As a consequence, a serious trade-off arises for women: while they can raise their wages by acquiring more marketable skills, this could actually work to their disadvantage in the marriage market because their bargaining power becomes excessively strong. This “fear of success” seriously distorts women’s incentives and leads them to invest inefficiently in an attempt to reduce their threat points.

Given that gender specialization can be inefficient from the social point of view, the second part of this paper focuses on policies to fix this problem. To this end, it is important to note that whether this inefficient asymmetric equilibrium arises depends crucially on the magnitude of the cost of the domestic activities required to sustain a household. Women with more marketable skills are less preferred by men when the cost of domestic activities is large and women need to devote a significant fraction of resources to domestic activities once they are married. As the burden of domestic activities becomes less significant and the opportunity cost of marriage decreases, there may be a symmetric equilibrium where the gender difference in the pattern of skill acquisition disappears. Thus, the present analysis reveals that the source of the inefficiency lies in the asymmetric cost structure of domestic activities.

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7In many economic analyses, households are considered as the minimum decision-making unit: it is typically assumed that each household acts as a single decision-making unit and that each member of a household earns the same level of utility. Recent evidence seems to indicate that this pooled income approach is not consistent empirically (Thomas, 1990; Browning et al., 1994; Chiappori et al., 2002).

8Thus, our focus is on endogenously determined threat points in the bargaining process. Another strand of literature focuses on endogenously determined bargaining shares. See Basu (2006) and Iyigun and Walsh (2007a) for this approach.

9The term “fear of success” was suggested by a referee. We thank him/her for this intuitive expression.

10We argue that this is roughly consistent with the recent trend in the US. As stated, in the US in 1971, women made up only 9% of all business majors and 0.8 % of all engineering majors. In 2002, these figures had risen to 50% and 19%, respectively. The same trends can be observed for the gender differences in occupational choices. Black and Juhn (2000, Table 1) showed that the fraction of women in high-wage occupations, defined as the top 20% of jobs in terms of male wages, increased from 8% in 1967 to 24% by 1997. Changes in the pattern of skill acquisition have narrowed the gender earnings gap (Blau and Kahn, 1992; O’Neill and Polachek, 1993).
and the consequent earnings gap between single and married women.

Subsidizing women to acquire skills is not effective because such a policy benefits both single and married women equally and hence has no impact on the pattern of investment. An income transfer program that compensates married women for the opportunity cost of marriage (or alternatively the lost market income) is more effective as it can effectively reduce the earnings gap between single and married women. As a consequence, we argue that policy interventions such as paid maternity leave, childcare benefits or subsidies to nursery schools are much more effective in closing the gender gap in the long run than affirmative action programs or equal employment opportunity laws that directly subsidize the acquisition of skills for all women.

To show its empirical relevance, we would like to note that this policy implication is roughly consistent with what has happened in Nordic countries: those countries have long been known for their extensive and progressive family policies and the gender earnings gap in Nordic countries is one of the smallest among industrialized countries (Blau and Kahn, 1992; Harkness and Waldfogel, 2003). This fact illuminates a potentially important role for family policies in reducing, and possibly eliminating, the gender gap that exists in the labor market. Although there may be other contributing factors,\footnote{For instance, Milgrom et al. (2001) argued that the gender earnings gap in Sweden narrowed almost a decade before the passage of family legislation, indicating that there are other crucial factors at work. Blau and Kahn (2003) argued that the collective bargaining convention and the minimum wage laws significantly influence the international differences in the gender earnings gap.} we argue that extensive family policies are quite an effective way to reduce the gender earnings gap, and our model provides a plausible framework to understand this important connection.

While our primary focus is on welfare and policy implications, the present paper also raises several theoretical issues that set it apart from the existing literature. In our model, the interactions of three contributing factors—assortative matching, intrahousehold bargaining and the asymmetric cost structure—are crucial in giving rise to the distorted system of incentives. Competition in the matching market (assortative matching) typically has a positive incentive effect because agents have stronger incentives to invest, in general, when they have concerns about their future matching partners. The presence of intrahousehold bargaining by itself provides an additional incentive, compared to the case where the total income of a household is pooled and equally shared among its members, because agents can raise their threat points by acquiring skills.\footnote{The models of Echevarria and Merlo (1999), Konrad and Lommerud (2000), Lundberg} However, when these two factors
are combined with the fact that women possess a comparative advantage in domestic activities, they are turned into negative incentives that lead women to invest inefficiently in order to reduce their threat points and make themselves more attractive in the marriage market. In this respect, our result stands in strong contrast to the previous literature on *ex ante* investments with intrahousehold bargaining where agents are homogeneous and hence assortative matching is not an issue.

Several models incorporate competition for matching partners where investment decisions explicitly affect the type of matching partner (Cole et al., 2001a, 2001b; Peters and Siow, 2002; Burdett and Coles, 2001; Chiappori et al., 2006; Iyigun and Walsh, 2007b). In particular, Peters and Siow (2002) revealed that competition in the marriage market is instrumental in resolving the holdup problem, as investment is accompanied by the upgrading of marriage partners and this provides an additional incentive to invest. They argued that this matching concern leads to an efficient allocation under certain conditions.\(^\text{13}\) The basic structure of our model builds on that of Peters and Siow, but we show that competition in the marriage market can be the source of a different type of inefficiency, as discussed above.\(^\text{14}\) Burdett and Coles (2001), on the other hand, studied a holdup problem that includes search frictions in finding matching partners. When an individual can find another partner in a short time, he/she is more inclined to reject an undesirable current partner. This matching concern provides too much incentive to invest and may lead to inefficient overinvestment. Chiappori et al. (2006) analyzed the problem of premarital investments with a different focus, illustrating why women now attain higher education levels than men. Their model is based on the presumption that binding agreements can be made on the division of the marriage surplus and that this feature ensures that the consequent equilibrium allocations are efficient. In contrast, we focus more on the fact that marriage contracts are naturally incomplete and that this incompleteness distorts women’s behavior, thereby leading to inefficient allocations under some conditions.

A number of theoretical models have shown that individuals may acquire

\(^\text{13}\)It has also been pointed out that in the context of the competitive marriage market where agents compete for marital partners, standard models are no longer capable of replicating the strongly asymmetric pattern of skill acquisition between men and women (Rios-Rull and Sanchez-Marcos, 2002).

\(^\text{14}\)Nosaka (2007) showed another source of inefficiency that arises when the utility is submodular. In this case, the negatively assortative matching is more efficient, but the competition effect prevents this matching formation.

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unproductive attributes.\textsuperscript{15} Of these studies, Cole et al. (1992, 1998) and Mailath and Postlewaite (2006) come closest in spirit to ours. They presented models in which individuals prefer to marry partners with a specific social status that is unrelated to productivity because the acquisition of such a social status improves the matching partners of their offspring. However, the underlying mechanisms of those models differ totally from ours: the matching concern provides an incentive, distorted through the process of intrahousehold bargaining, to deliberately degrade the quality of investment in our model, whereas this effect is absent in theirs. This difference proves to be crucial, ultimately leading to different welfare and policy implications, which are at the core of this paper.

The rest of the paper is organized as follows. Section 2 briefly outlines the model. Section 3 illustrates the driving force of our model, the trade-off between the marriage and labor markets. Section 4 characterizes equilibria and shows that two distinct types of equilibrium arise, asymmetric and symmetric, depending on the relative cost of domestic activities. Section 5 discusses key properties of the model, given the results obtained in the previous section. In particular, we argue that the investment pattern of women tends to be inefficient in the asymmetric equilibrium and we offer a potential remedy for it. Finally, Section 6 makes some concluding remarks.

2 The model

2.1 Environment

There is a continuum of agents who belong to either one of the two gender subsets, each denoted by \( j \in \{f, m\} \) (\( j = f \) for female and \( j = m \) for male), and the population size of each gender subset is \( n_j \). There are three time periods. Agents acquire skills in the first period and search for marital partners (among the opposite sex) in the second. If two agents decide to marry, they form a household and bargain over the total surplus in the third period.\textsuperscript{16} Marriage contracts are incomplete and the married parties must

\textsuperscript{15}Some examples include signaling models where education may not improve workers’ skills (Spence, 1973), and overlapping generations models where money or other attributes are virtually unproductive (Samuelson, 1958; Cozzi, 1998). However, in these models, the matching concern is absent.

\textsuperscript{16}Note that the marriage market closes after the second period, which rules out the possibility of remarriage. In other words, we consider a situation where remarriage is prohibitively costly, perhaps because of search costs, so that it is not in anyone’s best interest to do so. Note that this assumption is made strictly to simplify the analysis, as
bargain over the total surplus \textit{ex post}.

Each agent is completely characterized by the ability type \( x \in X = [x, \bar{x}] \) and gender \( j \in \{f, m\} \). The distribution of the ability type is denoted by \( F(x_j) \), which is independent of gender.

### 2.2 Skill acquisition

Agents can accumulate skills in two dimensions, the level \( e_j(x) \in R_+ \) and the market value \( q_j(x) \in \{H, L\} \). We assume that agents choose between two market values, high \((q_j = H)\) and low \((q_j = L)\), where \( H > L \geq 0 \). Let \( h_j(x) = (q_j(x), e_j(x)) \in \{H, L\} \times R_+ \) denote the investment choice. For notational simplicity, we sometimes write this as \( h_j = (q_j, e_j) \) or simply \( h = (q, e) \), wherever it is not confusing. In what follows, we refer to the skill with high (low) market value as being marketable (nonmarketable).

The market value simply reflects differences in the nature of skills and is totally independent of how difficult or costly it is to acquire these skills. That is, given some ability type \( x \), the cost of skill acquisition depends on the investment level \( e \) but not on the market value \( q \), and is denoted by \( C(e, x) \).\(^{17}\)

### 2.3 Marriage and production

In the second period, each agent decides whether to enter the marriage market, which is assumed to be competitive and stable: no matched pair has an incentive to unilaterally dissolve the marriage in search for another partner. The resulting matching pattern must be positively assortative according to the attractiveness of agents to the potential marriage partners, which consists of the level and market value of the skills.

If an agent decides to remain single, then the agent concentrates on market activities. The total utility when an agent remains single is equal to the market productivity, which is the product of the two elements, \( qe \), regardless of gender.

If an agent enters the marriage market and finds a partner, the two matched agents form a household in the second period. The gains from our main results can be obtained even in the presence of a remarriage market as long as the probability of remarriage diminishes over time (after each divorce).

\(^{17}\)We can easily extend the model to the case where it is less costly to acquire the nonmarketable skill. This modification does not alter our main result because it enhances the possibility of the equilibrium occurring in which female agents obtain the nonmarketable skill. We make this assumption to emphasize that female agents may select the nonmarketable skill even when there is no cost advantage in acquiring such a skill. The assumption is also instrumental in making our welfare implications more transparent.
marriage arise from the investment choices made in the first period. Consider a household where the investment choice for the female agent is $h_f$ and that for the male agent is $h_m$. Then, we specify that the joint outcome for this household is given by:

$$y(h_f, h_m) = (1 + \alpha)(q_m e_m + (1 - \theta)q_f e_f + \theta \delta e_f), \quad \theta \in (0, 1).$$

The first term represents the contribution of the male agent who devotes his time entirely to market activities. The second and third terms indicate the contribution of the female agent who devotes a fraction $\theta$ of her time to domestic activities in order to produce various household public goods, such as children and a clean and tidy household. The parameter $\delta$ measures the productivity of domestic activities and it does not depend on the market value of skills $q_j$. This captures the fact that in order to enrich married lives, many different types of knowledge and skills are valuable, including those that are less productive in the labor market. The market value of skills $q_j$ is no longer appropriate to measure the impact of skills. We sometimes refer to $\delta$ as the intrinsic value of skills, partly to contrast with the endogenously chosen market value $q_j(x)$. Finally, the total market income is multiplied by $1 + \alpha$, where $\alpha > 0$, to represent additional benefits of marriage. This term arises as a result of the creation of public goods within a household because some consumed goods may be non-rivalry.\(^8\)

We let $\theta \in (0, 1)$. In other words, we assume that women tend to possess a comparative advantage in domestic activities.\(^9\) As a consequence, their market productivity is lower by assumption (Echevarria and Merlo, 1999).\(^{20}\) Although this is debatable, there are several ways to justify this. First and foremost, only women can bear children. This gender difference lowers their productivity in market activities in many ways. For instance, Corcoran and Duncan (1979), Mincer and Ofek (1982), Cox (1984) and Lazear and Rosen (1990) emphasized the connection between career interruptions and earnings growth for women. Moreover, Echevarria and Merlo (1999) constructed a public goods framework that extends to the family environment.\(^{18}\)

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\(^{18}\) Evidently, household public goods such as children and a clean and tidy household are inherently non-rivalry and their benefits can be shared at little cost. Examples of private goods that become (at least partially) non-rivalry within a household are consumer durable goods such as refrigerators, TVs, telephones and so on.

\(^{19}\) If each household endogenously decides which partner is to specialize in domestic activities, the one whose skill has a lower market value would specialize in them. Then, there arises the possibility of a gender role reversal in which some female agents invest in the marketable skills substantially more than do male agents in order to work full time in the labor market. We exclude this possibility for the reasons suggested in the main text.

\(^{20}\) A similar assumption, that women bear the cost of child rearing more than do men, is employed by Iyigun and Walsh (2007a).
dynamic household bargaining model and showed that when the cost associated with childbearing is positive, then, in equilibrium, women also bear the entire cost associated with child rearing. In addition, there are many works that provide mechanisms through which gender differences arise endogenously from *ex ante* identical agents. See, among others, Francois (1998), Bjerk and Han (2006) and Lommerud and Vagstad (2000). In any event, it is natural that women devote more resources to domestic activities. Note that $\theta$ represents the opportunity cost of marriage for female agents, which leads to the earnings gap with respect to the marital status.

### 2.4 Intrahousehold bargaining

Because the private good is transferable, the agents in a household negotiate over how to divide the total surplus in the third period. In the absence of complete marriage contracts, the outcome of the negotiation is characterized by the Nash bargaining solution. The threat point for each agent is the total utility when the agent remains single, which is simply given by the skill level in the market (note that there is no household public good in this case). The formula for the Nash solution that produces the bargaining outcome $V_j(h_f, h_m)$ for each agent as a function of the investment choices is:

$$V_j(h_f, h_m) = \frac{1}{2} \left( y(h_f, h_m) - q_f e_f - q_m e_m \right) + q_f e_j,$$

$$= \frac{\alpha}{2} q_m e_m + \Delta(q_f) e_f + q_f e_j,$$

where,

$$\Delta(q_f) \equiv \frac{1}{2} \left[ (\alpha - (1 + \alpha)\theta) q_f + (1 + \alpha)\theta \delta \right].$$

The marginal gain from $e_f$ is influenced by $\Delta(q_f)$, which in turn depends on the market value of the acquired skill.

The use of Nash bargaining that explicitly introduces the agents’ threat points into intrahousehold bargaining is critical. Besides the fact that it is relatively common in applied works, we would like to point out two desirable properties. First, empirical studies indicate that bargaining power clearly matters in the final allocation of household resources (Thomas, 1990; Browning et al., 1994; Chiappori et al., 2002). In this sense, Nash bargaining is more consistent than other nonstrategic sharing rules that preclude the

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21 In other words, we view the ultimate threat point of intrahousehold bargaining as a divorce. An alternative approach is to view it as a noncooperative marriage. See Lundberg and Pollak (1993, 1994) for this approach.
presence of threat points. Second, Nash bargaining leads to \textit{ex post} efficient allocations and it is reasonable for a couple to use it as a way to divide the surplus.

3 \hspace{3pt} \textbf{The trade-off}

This section illustrates the trade-off between the marriage and labor markets. We modify (2) as follows:

\begin{align*}
V_f(h_f, h_m) &= a_m(h_m) + \left(q_f + \Delta(q_f)\right)e_f, \\
V_m(h_f, h_m) &= a_f(h_f) + (1 + \frac{\alpha}{2})q_m e_m,
\end{align*}

(3)

(4)

where $a_j$ is what we call “attractiveness,” defined as:

\begin{align*}
a_f &= \Delta(q_f)e_f, \\
 a_m &= \frac{\alpha}{2}q_m e_m.
\end{align*}

(5)

(6)

Attractiveness represents the gross benefit that an agent can provide to the marital partner. Thus, the preferences for marital partners are entirely summarized by this scalar $a_j$. More attractive agents are more desirable as marital partners.

There are two critical implications of the preferences for marital partners. First, female agents always prefer male agents with the marketable skill. While agents with the marketable skill have stronger bargaining powers, this negative effect is totally dominated by their higher income because they can devote all of their resources to market activities. As a consequence, male agents always choose to acquire the marketable skill in equilibrium, and we can set $q_m = H$.

An increase in the market value of skills, on the other hand, may or may not increase the attractiveness of female agents. The deciding factor is the fraction of time required to sustain a household relative to $\alpha$. Under some conditions, a serious trade-off arises for female agents where they actually reduce their attractiveness by acquiring the marketable skill.

\textbf{Proposition 1} \hspace{3pt} Female agents with the marketable skill are less preferred by male agents for any given investment level $e_f$, i.e., $\Delta(L) > \Delta(H)$, if

\[
\frac{\alpha}{1 + \alpha} < \theta.
\]
Proof. One simply needs to derive a condition for $\Delta(L) > \Delta(H)$, which can be written as:

$$\left(\alpha - (1 + \alpha)\theta\right)L + (1 + \alpha)\theta\delta > \left(\alpha - (1 + \alpha)\theta\right)H + (1 + \alpha)\theta\delta.$$  (7)

Rearranging this yields the result. Q.E.D.

When this condition holds, female agents are not sufficiently productive in the labor market because they need to spend a significant fraction of time on domestic activities once they are married. As they cannot fully utilize their acquired skill, their bargaining power is perceived to be excessively strong from the viewpoint of male agents. This fact gives rise to a serious trade-off for female agents: while they can raise their wages in the labor market by acquiring the marketable skill, it could actually work to their disadvantage in the marriage market because their threat points are now simply too high. Because of this trade-off, an incentive may arise for them to intentionally degrade the marketable skill and acquire the nonmarketable skill in order to secure the benefit of marriage.

4 Equilibrium

Given the trade-off between the two markets of our interest, it is intuitively clear that some sort of “gender asymmetry” could arise in a qualitative sense. However, the exact form of the equilibrium depends crucially on the minute details of the model structure and we need to add more assumptions to the basic model to fully characterize the equilibria. As it is more of a technical problem to characterize how agents behave in equilibrium, which by itself does not yield much economic insight, we focus on cases that are relatively tractable to illustrate the gist of the model. Obviously, similar conclusions can be obtained in other specifications as long as the trade-off described in the previous section is present, although it is simply not tractable in many cases.

The basic model needs to be more tightly specified to precisely characterize the equilibria. First, we make the following assumption concerning the ability distribution and the population size:

Assumption 1 $F(x_j)$ is twice continuously differentiable and strictly increasing in $x_j \in X$. In addition, the female population is smaller than the male population, i.e., $n_f < n_m$. 

The latter part of the assumption, that the female population is smaller in size than the male population, is made strictly to ease computation and has no qualitative impact: note that \( n_f \) needs to be smaller than \( n_m \) only by an arbitrarily slight margin. We focus on this case in order to emphasize that female agents indeed choose to acquire the nonmarketable skill even when their bargaining power is inherently strong (because female agents are scarce). We can obtain similar results when the male population is smaller as long as the difference between \( n_f \) and \( n_m \) is small.\(^{22}\)

We assume the following fairly standard conditions on the cost function:

**Assumption 2** \( C \) is twice continuously differentiable. In addition, for \( x \in X \):

\[
C(0, x) = \frac{\partial}{\partial e} C(0, x) = 0, \quad \lim_{e \to +\infty} \frac{\partial}{\partial e} C(e, x) = \lim_{x \to 0} \frac{\partial}{\partial e} C(e, x) = +\infty.
\]

\[
\frac{\partial}{\partial e} C(e, x) > 0, \quad \frac{\partial}{\partial x} C(e, x) < 0, \quad \frac{\partial^2}{\partial e^2} C(e, x) > 0, \quad \frac{\partial^2}{\partial e \partial x} C(e, x) < 0,
\]

for \( e > 0 \).

Finally, we place some restrictions on the range of the parameter values in order to reduce the number of cases we need to consider:

**Assumption 3** \( L + \Delta(L) > (1 - \frac{\alpha}{2})H \).

This assumption says that the value of marriage is sufficiently large so that all agents actually have an incentive to marry.\(^{23}\)

### 4.1 Matching functions and equilibrium conditions

In what follows, we restrict our attention to the case where the equilibrium matching pattern is positively assortative in the sense that the male and female agents with the highest abilities are married to each other, the agents

\(^{22}\)A nontrivial difference arises for female agents who remain single in equilibrium when the male population is smaller in size. As they do not enter the marriage market, they have no incentive to acquire the nonmarketable skill to raise their attractiveness. This implies that single female agents always acquire the marketable skill, although married female agents acquire the nonmarketable skill in some equilibria.

\(^{23}\)The assumption ensures that marriage is preferable in equilibrium even if female agents acquire the nonmarketable skill. Note, on the other hand, that male agents always choose to marry whenever it is feasible to do so. As male agents always acquire the marketable skill in equilibrium, it follows from (4) that the marginal benefit of investment \((e_m)\) for male agents is at least \((1 + \alpha/2)H\) when they are married, whereas it is \(H\) when they remain single.
with the next highest abilities are married to each other, and so on. Thus, we introduce a function $\tau : X \rightarrow X$ that indicates the matching pattern under positive assortative mating:

$$x_f = \tau(x_m).$$

This function $\tau$ must satisfy:

$$n_f(1 - F(\tau(x_m))) = n_m(1 - F(x_m)) \quad \text{for} \quad x_m \in [x^c, x] \equiv X^m,$$

where $x^c$ is the least ability level among married male agents under the positive assortative mating, i.e., $n_f = n_m(1 - F(x^c))$. Male agents with abilities below this threshold $x^c$ remain single.

For the analysis, it is often useful to introduce the cost function of a matched pair. In terms of the function $\tau$, we redefine the cost functions as follows:

$$C_m(e, x) = C(e, x) \quad \text{for} \quad x \in X, \quad C_f(e, x) = C(e, \tau(x)) \quad \text{for} \quad x \in X^m,$$

where $x$ indicates the ability type of the male agent in the household. In the following analysis, we denote each household by the ability type of the male agent.

In addition, we redefine the utility function for each agent in terms of $a_j$, instead of $e_j$ as in (3) and (4). From the definitions of $a_j$ in (5) and (6) that implicitly determine $e_j = e_j(a_j, q_j)$, the utility functions can be represented as:

$$U_f(a_f, a_m, q_f, x) = V_f(e_f(a_f, q_f), q_f, e_m(a_m, q_m), q_m) - C_f(e_f(a_f, q_f), x),$$

$$U_m(a_f, a_m, q_m, x) = V_m(e_f(a_f, q_f), q_f, e_m(a_m, q_m), q_m) - C_m(e_m(a_m, q_m), x),$$

where we impose the assumption that $q_m = H$, as male agents always acquire the marketable skill in equilibrium.

Now, let $\phi : R \rightarrow R_+$ denote a matching function where $\phi(a_f)$ indicates the attractiveness of the male agent with which each female agent with attractiveness $a_f$ expects to match. The equilibrium matching pattern is completely described by this matching function.\footnote{Although the characteristics of agents are two dimensional ($q_j$ and $e_j$), we can still take this approach because the preferences of agents are summarized by the scalar variable $a_j$.} In order for the match to
be stable, the matching function \( \phi \) must be a strictly increasing function of \( a_f \).

The equilibrium allocation consists of the optimal investment choices \((a_j(x), q_j(x))\) for \( x \in X \) and a matching function \( \phi \). We are now ready to derive the conditions that need to be satisfied in any equilibrium. First, optimality implies that the following condition must be satisfied for married agents:

\[
(a^e_j(x), q^e_j(x)) = \arg\max_{a_j, q_j} U_j(a_f, a_m, q_j, x),
\]

\[\text{s.t. } a_m = \phi(a_f), \quad \text{for } x \in X^m.\]  

Let \( U^e_e(x) \) denote the equilibrium value of marriage (the expected utility when married).

Second, as the matching pattern is positively assortative, all male agents whose ability is lower than the threshold \( x_c \) are unable to marry in equilibrium. The problem for those agents is defined as:

\[
(e^s_j(x), q^s_j(x)) = \arg\max_{e_j, q_j} q_j e_j - C_j(e_j, x),
\]

\[\text{for male agents with } x \in X \setminus X^m.\]

Clearly, all male agents who remain single choose to acquire the marketable skill. We define \( S_j(x) \) for \( x \in X \) as the expected utility when single. Then, the male agent at the threshold \( x_c \) must be indifferent between marrying and staying single:

\[U^e_m(x_c) = S_m(x_c).\]  

In what follows, we refer to this agent at the threshold \( x_c \) as the boundary agent.

The above conditions along with a consistent matching function fully characterize the equilibrium allocation, but the nature of equilibrium differs markedly according to the market value of skills chosen by female agents \( q_f(x) \). While there are many possible equilibria such as (partially) pooling equilibria, we focus on two of the most illuminating separating equilibria in the next two subsections.

When the slope of the matching function is negative over some range, we can find a pair \((a_f^1, a_f^2)\) such that \( a_f^1 < a_f^2 \) and \( \phi(a_f^1) > \phi(a_f^2) \). Then, the female agent with \( a_f^2 \) and the male agent with \( \phi(a_f^1) \) would be made better off by changing their currently assigned partners and marrying each other. This violates the requirement of stable matching, which is that no pair has an incentive to unilaterally dissolve the match.

In general, we cannot rule out the possibility of partial pooling where a subset of agents choose the same investment levels. We do not pursue this possibility as it is clearly out of the scope of the paper.
4.2 Symmetric equilibrium

First, we consider a situation where all agents choose to acquire the marketable skill. This type of equilibrium could emerge when the condition in Proposition 1 fails to hold, i.e., \( \alpha/(1 + \alpha) > \theta \), and, hence, \( \Delta(H) > \Delta(L) > 0 \).\(^{27}\) In this case, attractiveness is strictly increasing in the market value of skills for both gender subsets, and, hence, there exists no trade-off between the marriage and labor markets. As the marketable skill is unambiguously the preferred choice for all agents, \( q_f(x) = q_m(x) = H \) for all \( x \in X \), and the problem is greatly simplified. As all agents in both gender subsets choose the identical market value, we call it the symmetric equilibrium, and its allocation is denoted by \((a_f^{\text{sym}}(x), a_m^{\text{sym}}(x))\).

The symmetric equilibrium is essentially a variant of standard models such as that of Peters and Siow (2002) and it is relatively straightforward to characterize. The intuition behind the symmetric equilibrium can be best seen graphically, as in Figure 1. The first equilibrium condition (10) dictates that an agent with \( x \) chooses a point \((a_f^{\text{sym}}(x), a_m^{\text{sym}}(x))\) that maximizes

\(^{27}\)It is clear that \( \Delta(q_f) > 0 \) when the condition in Proposition 1 does not hold.
the utility on the matching function \( \phi \). As a result, the indifference curve that provides an equilibrium utility \( U_e^j(x) \) is tangent to this matching function, and this provides the condition that the slope of the matching function must satisfy. The second equilibrium condition (12) imposes a restriction on the utility level of the boundary agent (the male agent at the threshold) and this determines his equilibrium level of attractiveness and consequently the investment level. As we now have the slope and the initial value of the matching function, we can solve its differential equation. Under the maintained assumptions, the following proposition can be obtained.

**Proposition 2 (Symmetric equilibrium)** Under Assumptions 1–3, there exists a symmetric equilibrium where all agents choose to acquire the marketable skill if \( \Delta(H) > \Delta(L) \).

**Proof.** See Appendix.

### 4.3 Asymmetric equilibrium

Now, we turn to a more interesting situation where the condition in Proposition 1 holds and thus \( \Delta(H) < \Delta(L) \). Then, female agents face a trade-off between the marriage and labor markets. This trade-off may lead to the emergence of an asymmetric equilibrium where female agents deliberately acquire the nonmarketable skill even though the cost of skill acquisition is totally independent of the market value. In particular, we seek a type of equilibrium where \( q_f(x) = L \) for \( x \in X^m \). We refer to this as the asymmetric equilibrium, and its allocation is denoted by \((a_f^{\text{asym}}(x), a_m^{\text{asym}}(x))\).

The asymmetric equilibrium is more complicated. Here, we focus our attention on a case where \( \Delta(L) > 0 > \Delta(H) \); we extend the analysis to the case where \( \Delta(L) > \Delta(H) > 0 \) in Section 5.3. Under this condition, we impose some additional conditions to characterize an asymmetric equilibrium in a tractable way:

**Assumption 4 (Cost function)** For all \((e, x) \in R_+ \times X\),

\[
\frac{\partial C(e, x)}{\partial e} \equiv \gamma(e, x) < 1 + \frac{-\Delta(H)}{H + \Delta(H)} \frac{\alpha}{2 + \alpha}
\]

Note that \( \gamma \) is the elasticity of the cost function with respect to the investment level \( e \) and that Assumption 4 places an upper bound upon it. As the elasticity measures how much the investment costs, the assumption means that the investment is not very costly. Under the maintained assumptions, we have:
Proposition 3 (Asymmetric equilibrium) Under Assumptions 1-4, there exists an asymmetric equilibrium where all female agents choose to acquire the nonmarketable skill if \( \Delta(L) > 0 > \Delta(H) \).

Proof. See Appendix.

The proposition establishes a sufficient condition for a type of asymmetric equilibrium where \( q_f(x) = L \) for \( x \in X^m \). Figure 2 graphically illustrates this situation. Note that, in the figure, male agents have an identical indifference curve to that in Figure 1, but the situation is different for female agents.

Given that \( \Delta(L) > \Delta(H) \), female agents face a trade-off; an increase in the market value leads to a decrease in attractiveness. Under this situation, there are generally two distinct options the female agents can take. One is to acquire the marketable skill, thereby increasing their market productivity, but at the expense of becoming less attractive. The indifference curve in this case is depicted as \( BB' \) in Figure 2, where \( a_f \) takes a negative value. This could happen if an increase in wages (earned in the labor market) is large enough to compensate for the loss in the marriage market. The other is to acquire the nonmarketable skill, thereby becoming more attractive, but at the
expense of a decrease in market productivity. In Figure 2, the corresponding indifference curve is depicted as $AA'$, where the domain of $a_f$ is positive. The asymmetric equilibrium arises when a positive value of attractiveness is chosen by female agents, as depicted in the figure. In this case, female agents choose the nonmarketable skill even if the acquisition of the marketable skill is a viable option. Provided that the cost of skill acquisition is independent of the market value, this incentive entails a welfare loss. This fact leaves room for some government interventions, which will be discussed later in Section 5.2.

The results obtained thus far suggest that the equilibrium matching pattern can change drastically as the parameters change. Consider the effect of an improvement in the productivity of the household public goods, which can be seen as a reduction in $\theta$. Apparently, a change in $\theta$ has a significant impact on the relative location of $\Delta(H)$ and $\Delta(L)$, and, hence, on the resulting equilibrium pattern. An improvement in the productivity of the household public goods allows female agents to shift their endowed resources more heavily toward market activities and thereby reduces the earnings differential with respect to the marital status. At some point, an investment in the market value of skills becomes sufficiently profitable for female agents and this leads to the emergence of the symmetric equilibrium. In the US, for instance, there has been a steady increase in the proportion of female college students choosing high-income majors such as engineering and business. The model’s prediction is largely consistent with this recent trend.

As a final point, it should be noted that Assumption 4 only presents a sufficient condition and that the asymmetric equilibrium may still exist even without it. However, when Assumption 4 fails to hold, another type of equilibrium where female agents choose the marketable skill may also emerge. Suppose that Assumption 4 does not hold and that investment is now more costly. In this case, the equilibrium investment level is lower and more compressed. That makes male agents more homogenous in terms of market productivity and consequently decreases the relative importance of the marriage market, with the matching function becoming flatter. As the loss in the marriage market can be compensated for easily by an increase in the market income, this provides female agents with a stronger incentive to deviate from the asymmetric equilibrium and acquire the marketable skill. Therefore, when Assumption 4 fails to hold, it is actually feasible to have an equilibrium where the matching pattern is negatively assortative.\footnote{To understand this, note first that stable matching requires that the slope of the matching function be positive. As female agents with higher ability invest more in any...}
we do not pursue this possibility any further as this type of equilibrium is not empirically prevalent, although it is certainly theoretically interesting.

5 Discussion

5.1 Welfare

The fact that female agents may choose to acquire the nonmarketable skill yields a serious welfare implication. In our model setup, social efficiency requires that \( q_j(x) = H \) for all \( x \) because the cost of skill acquisition is independent of the market value, whereas the intrinsic value \( \delta \) is identical across the two types of skills. In other words, our model depicts a situation where everyone should specialize in the marketable skill for efficiency. Needless to say, our point is not to insist that everyone should specialize in high-income majors, such as business and engineering, because the social value of skills cannot be measured solely by their market value: there are certain skills that cannot be sold at higher prices in the labor market, yet they benefit society in both tangible and intangible ways.\(^{29}\) Rather, our aim is simply to point out that there exists a structural factor that tends to distort incentives for women: as a result, the incentive to acquire the nonmarketable skill arises independently of its intrinsic value \( \delta \).

Now, we investigate the efficient allocation of the economy in order to derive some welfare implications. The social planner’s problem is fairly simple: as all terms are linear under the current setup, the efficient allocation is achieved if and only if each agent’s contribution to the social welfare is maximized. First, it can immediately be observed that the planner always chooses the marketable skill to maximize its market value. Given this observation, the planner’s problem is defined as finding \( e_{j}^{\text{opt}}(x), j = f, m \), such that:

\[
\begin{align*}
  e_{f}^{\text{opt}}(x) & = \arg\max_{e} (1 + \alpha)(1 - \theta)H + \theta \delta)e - C_{f}(e, x), \\
  e_{m}^{\text{opt}}(x) & = \arg\max_{e} (1 + \alpha)He - C_{m}(e, x),
\end{align*}
\]

separating equilibrium, their attractiveness \( a_f \) is lower, with \( \Delta(H) \) being negative. As a consequence, negatively assortative matching could emerge in some equilibria.

\(^{29}\)To pursue this point further, our model does not consider the social value of skills nor individual differences in tastes and aptitudes. The latter part is especially important because the relative cost between the two types of skills should vary in a horizontal way across individuals. We ignore this “horizontal individual heterogeneity” to emphatically make our conclusion that even female agents who possess comparative advantage in the marketable skill would acquire the nonmarketable skill.
for married agents with $x \geq x^c$. Under the maintained assumptions, the optimal investment levels are interior and, therefore, the first-order condition characterizes the efficient allocation of the investment levels, denoted by $e_j^{\text{opt}}(x)$. The next proposition characterizes the efficient allocation of the economy (proof omitted).

**Proposition 4 (Efficiency)** In the efficient allocation, male agents marry if $x \geq x^c$. All agents acquire the marketable skill in the efficient allocation and the investment levels of married agents are determined by $e_j^{\text{opt}}(x)$ for $x \in X^m$.

Note that our model includes premarital investments, and, therefore, has a holdup property. In the standard holdup problem, the investment level typically falls below its efficient level because concerned parties fail to take potential partners' benefits into consideration. However, this problem can be alleviated substantially when agents compete for spouses: in a competitive marriage market, there may arise an additional incentive to invest for attracting potential partners. Further, Peters and Siow (2002) showed that efficient allocation can be achieved even when the utility is nontransferable.

In our model with intrahousehold bargaining and transferable utility, this competition effect influences the equilibrium allocation in different ways, depending crucially on the relative magnitudes of $\Delta(H)$ and $\Delta(L)$. When $\Delta(L) > \Delta(H)$, along with other auxiliary conditions, there is an asymmetric equilibrium where female agents have an incentive to strategically degrade the market value of skills. The competition effect is actually the source of another type of inefficiency and fails to resolve the holdup problem.

### 5.2 Policy implications

The asymmetric equilibrium is inefficient in that female agents intentionally degrade the market value of skills, owing to their fear of success, even though the cost of skill acquisition is totally independent of the market value of the skill. Our analysis reveals that a direct subsidy to encourage the acquisition

---

30In the literature on the holdup problem, there is a debate over whether exclusive contracts foster relation-specific investment. For instance, De Meza and Selvaggi (2003) recently argued that some firms would deliberately expose themselves to holdup by signing exclusive contracts because these eliminate inefficient equilibria. However, the focus of this strand of literature is clearly different from ours because it is on the incentive for *ex post* investments. In contrast, agents in our model would reduce bargaining power through *ex ante* investments in order to find better trading partners.
of skills is not effective against this inefficiency. To see this, consider an alternative specification of the cost function for female agents:

$$\tilde{C}_f(e, x) = \beta^{-1}C_f(e, x),$$  \quad (15)

where $\beta$ is a parameter to measure the cost of skill acquisition. Within the current setup, a subsidy to encourage the acquisition of skills for female agents can be seen as an increase in $\beta$, as defined in (15), possibly financed by lump-sum taxes imposed on all agents. Apparently, an increase in $\beta$ could have some quantitative effects on the equilibrium investment levels, even when the asymmetric equilibrium prevails. However, it is clear that the nature of the equilibrium remains basically unchanged because the condition in Proposition 1 is totally independent of it. That is, if the asymmetric equilibrium prevails in the first place, the nonmarketable skill remains the equilibrium investment choice for female agents, regardless of $\beta$.\(^{31}\)

By the same logic, affirmative action programs or equal employment opportunity laws are equally ineffective unless they are specifically targeted at married women. To see the effects of such policies, we modify the model so that the market income is lower for female agents by design, possibly owing to labor market discrimination. Now, we denote the market income as $\lambda_iq_i e_i$, $i = m, f$, where $\lambda_m \geq \lambda_f$. Moreover, to simplify notation, let $\lambda_m = 1$ and $\lambda_f = \lambda$. The bargaining outcome for male agents (4) is modified to:

$$V_m = \frac{1}{2} \left( y(h_f, h_m) - \lambda q_f e_f - q_m e_m \right) + q_m e_m,$$

$$= a_f + (1 + \frac{\alpha}{2})q_m e_m, \quad (16)$$

where $a_f$ is now redefined as $a_f = \Delta(\lambda q_f)e_f$. The asymmetric equilibrium exists when $\Delta(\lambda L) > 0 > \Delta(\lambda H)$ under Assumptions 1–4, where $q_j$ is replaced by $\lambda q_j$. Next, we consider a policy intervention that raises $\lambda$ towards one, through, say, an affirmative action program. Even with this improvement, the preferences of male agents are qualitatively unchanged, as $\Delta(\lambda' L) > \Delta(\lambda' H)$ for any $\lambda' \in (\lambda, 1)$ when the condition in Proposition 1 holds. Moreover, when $\Delta(L) > 0$, we can apply the same analysis as in the change of $\beta$ because $\Delta(\lambda' L) > \Delta(L) > 0 > \Delta(\lambda H) > \Delta(\lambda' H)$, for any $\lambda' \in [\lambda, 1]$. Therefore, the nature of the equilibrium is identical to that in

\(^{31}\)This is, of course, when the increase in $\beta$ is within some reasonable range. When $\beta$ is extremely large, female agents would choose not to marry because their potential marriage partners are relatively less attractive to them.
Proposition 3, and the asymmetric equilibrium exists as long as Assumptions 1–4 hold. In conclusion, such policy interventions fail to resolve the inefficiency at its origin as these policies could benefit both single and married agents proportionally, and, hence, the policy interventions are not at all effective in narrowing the earnings gap.

The previous argument clearly indicates that the source of the inefficiency lies in the earnings differential between single and married women. Therefore, the key to the solution is to reduce $\theta$ by compensating married women for the market income that they would have earned if they had not engaged in domestic activities. In our model, a decrease in $\theta$ raises the value of the marketable skill for married female agents without affecting their market income when they remain single or, equivalently and more importantly, without affecting their threat point in the bargaining process. This could alter the equilibrium skill choice as the perverse incentive faced by women diminishes. There is indeed a wide array of possible compensation programs to achieve this goal: monetary transfers to compensate for the opportunity cost of childbearing and child rearing through providing paid maternity leave, childcare benefits (made proportional to beneficiaries’ potential earnings), or subsidies to nursery schools can be simple and effective policy measures in eliminating this inefficient outcome.

5.3 The intermediate case

Thus far, we have restricted our attention to two tractable cases, $\Delta(H) > \Delta(L)$ and $\Delta(L) > 0 > \Delta(H)$, in order to illustrate the gist of the model. A plausible situation that has not been covered is $\Delta(L) > \Delta(H) > 0$, which occurs when $\theta$ is in some intermediate range. Here, we briefly analyze this intermediate case.

When $\Delta(L) > \Delta(H) > 0$, female agents can raise their attractiveness by acquiring either type of skill, although the nonmarketable skill is more effective in this regard. This means that female agents still face the same trade-off: they can reap the benefit of marriage more effectively by acquiring the nonmarketable skill but at the expense of earning less in the labor market. As above, a deciding factor turns out to be the slope of the matching function,

32Clearly, this argument does not follow when $\Delta(\lambda L)$ is negative or Assumptions 1–4 with $q_j$ replaced by $\lambda'q_j$ do not hold for some $\lambda' > \lambda$.

33There is another possibility, that $0 > \Delta(L) > \Delta(H)$. As discussed at the end of the last section, the matching pattern is negatively assortative in terms of $x$ if a separating equilibrium ever exists, as $\Delta(q_f)$ is negative. As this situation is unrealistic, we do not pursue this case any further.
which roughly measures the relative importance of the marriage market. For instance, suppose that the slope is relatively steep, meaning that male agents are more diverse and a unit increase in $a_f$ results in a larger increase in the attractiveness of the spouse. In this case, the gains from the marriage market outweigh the gains from the labor market, and, hence, it is optimal for female agents to choose the nonmarketable skill. More precisely, a sufficient condition for the existence of the asymmetric (symmetric) equilibrium is:

$$\phi'(a_f) > \frac{2}{(1+\alpha)\theta - \alpha} - 1, \text{ for all } a_f \in R_+.$$  

(17)

See the Appendix for the derivation of this condition. By exactly the same logic, the symmetric equilibrium exists if the slope is sufficiently flat.

As well as the slope of the matching function, the nature of the equilibrium depends again on the value of $(1+\alpha)\theta - \alpha$. Note that $(1+\alpha)\theta > \alpha$ when $\Delta(L) > \Delta(H)$. If $(1+\alpha)\theta \approx \alpha$, the symmetric equilibrium emerges for any given matching function. As $\theta$ increases, the right-hand side of (17) decreases and the condition for the asymmetric equilibrium is more likely to hold. This argument confirms our intuition that the asymmetric (symmetric) equilibrium is more likely when $\theta$ is relatively large (small). Although one may need to specify the ability distribution and the cost function more tightly to obtain more complete conditions in terms of exogenous variables, we can say that the same basic mechanism is still at work.

6 Conclusion

This paper provides a model to account for the gender specialization of skill acquisition when the marriage market is competitive. The key ingredients of the model are the process of intrahousehold bargaining and the cost asymmetry of domestic activities. We show that when the cost of domestic activities is asymmetrically placed on women, their investment pattern may be distorted to gain advantages in the marriage market. This leads to the emergence of an inefficient asymmetric equilibrium where the majority of women intentionally degrade the market value of their acquired skills.

The model indicates that the inefficient asymmetric equilibrium arises when the cost asymmetry of domestic activities is more significant. This result offers a critical policy implication: the fundamental source of the gender specialization of skill acquisition lies in the earnings gap between single and married women, rather than that between men and women. Once married, women devote more resources to domestic activities in order to reap
the benefit of role specialization. This lowers the returns to marketable skills when they are married, compared to if they had remained single. The model indicates that an effective remedy for this is to correct the cost structure of domestic activities so that \( \Delta(H) > \Delta(L) \) holds. A policy intervention to redistribute the cost of domestic activities, which is initially concentrated more heavily on married women, to all members of the economy, can be much more effective than an intervention such as affirmative action, which directly subsidizes the acquisition of marketable skills for women.

Appendix

**Proof of Proposition 2.**

As noted in the main body of the text, there is no incentive to acquire the nonmarketable skill in equilibrium when:

\[
\Delta(H) > \Delta(L) \iff \frac{\alpha}{1 + \alpha} > \theta, \tag{A.1}
\]

and we can impose \( q_j(x) = H \) for \( x \in X \). Then, following Peters and Siow (2002), we can characterize the equilibrium conditions (10), (11) and (12).

The first condition (10) provides a pair of first-order conditions for each \( x \in [x^c, \bar{x}] \):

\[
0 = \frac{d}{da_f} U_f(a_f, \phi(a_f), H, x),
\]

\[
= \phi'(a_f) + \left[ (H + \Delta(H)) - \frac{\partial}{\partial e_f} C_f(e_f(a_f, H), x) \right] \frac{1}{\Delta(H)}, \tag{A.2}
\]

\[
0 = \frac{d}{da_m} U_m(\phi^{-1}(a_m), a_m, H, x),
\]

\[
= \frac{1}{\phi'(a_f)} + \left[ (1 + \frac{\alpha}{2})H - \frac{\partial}{\partial e_m} C_m(e_m(a_m, H), x) \right] \frac{2}{\alpha H}. \tag{A.3}
\]

By substituting out \( x \) from the above equations, we have a unique differential equation of the form: \( \phi'(a_f) = g(a_f, \phi(a_f)) \) for some \( g \), as \( (a_f, a_m) \) uniquely determines \( \phi'(a_f) > 0 \) under our assumptions about the cost function.

The third condition (12) yields the utility level of the boundary male agent at the threshold \( x^c \). From this condition and the above tangency condition,

\[\text{We note that the value of marriage is always larger than the value of staying single for all agents in a symmetric equilibrium. In addition, the first-order conditions and an increasing matching function are sufficient for optimality because of the single crossing property of the utility function } U_j, \text{ i.e., } \partial^2 U_j(a_f, a_m, q_j, x)/(\partial x \partial a_j) > 0.\]
the investment levels are determined by the following maximization problem for the boundary agent:

\[
(a_f(x^c), a_m(x^c)) = \arg \max_{a_f, a_m} U_f(a_f, a_m, H, x^c) \\
\text{s.t. } U_m(a_f, a_m, H, x^c) \geq S_m(x^c), \quad a_j \geq 0.
\]

(A.4)

The assumption about the cost function ensures the unique interior solution. This provides the boundary condition of the differential equation: \(a_m(x^c) = \phi(a_f(x^c))\). With this boundary condition and the differential equation, we can obtain the matching function \(\phi\) and the equilibrium choice of \(a_j(x)\) for \(x \in [x^c, \bar{x}]\).

Male agents below the threshold \(x^c\) stay single in an equilibrium for which the investment level is obtained by (11). The matching function \(\phi\) below \(a_f(x^c)\) should be determined so that it is optimal for these male agents to stay single, as depicted in Figure 1, which completes the characterization of the symmetric equilibrium. Q.E.D.

**Proof of Proposition 3.**

As we do not know which skill type is preferred by female agents, we follow two steps in the proof of the proposition. First, we construct an asymmetric equilibrium as if only the nonmarketable skill is available for female agents. Second, we examine whether this choice is globally optimal even if the marketable skill becomes an available option.

In the first step, we suppose that all female agents choose the nonmarketable skill, i.e., \(q_f(x) = L\). Then, we can follow the same procedure as in the proof of Proposition 1. The first-order conditions and the boundary condition are unchanged with the exception that \(q_f = L\) in the expression.\(^{35}\) Male agents below the threshold \(x^c\) remain single in equilibrium and the investment choice is identical to the case in the symmetric equilibrium. This completes the construction of the equilibrium investment choice \(e_j(x)\) in \(x \in X\) and the matching function \(\phi(a_f)\) such that \(a_f \geq a_f(x^c)\).

In addition, it is necessary to show that all agents actually prefer to marry. This fact is clear for male agents, as shown in footnote 23. In order to investigate the case of female agents, we consider two cases depending on whether \(e_f^{\text{asym}}(x) \leq e_f^s(s)\), in which \(e_f^{\text{asym}}(x)\) is the investment level in the asymmetric equilibrium. First, when \(e_f^{\text{asym}}(x) \leq e_f^s(x)\), the optimality

\(^{35}\)Specifically, \(U_f(a_f, \phi(a_f), H, x)\) is replaced with \(U_f(a_f, \phi(a_f), L, x)\) in (A.2) and (A.4). As in the case of the symmetric equilibrium, the first-order conditions are sufficient for optimality.
requires that female agents are not better off when they choose $e_f^*(x)$ and marry. Letting $U_f^{\text{asym}}(x)$ be the equilibrium utility level when a female agent chooses to marry, we have:

$$U_f^{\text{asym}}(x) \geq \phi \left( \Delta(L)e_f^*(x) \right) + (L + \Delta(L))e_f^*(x) - C_f(e_f^*(x), x),$$

$$> \frac{\alpha}{2}He_f^s(x) + (L + \Delta(L))e_f^*(x) - C_f(e_f^*(x), x) > S_f(x),$$

where we use the fact that $\phi(\Delta(L)e_f^s(x)) \geq \phi(\Delta(L)e_f^{\text{asym}}(x)) = a_m^{\text{asym}}(x) > (\alpha/2)He_f^s(x) > (\alpha/2)He_f^s(x)^{36}$ in the second inequality, and Assumption 3 in the last inequality. Second, when $e_f^{\text{asym}}(x) > e_f^*(x)$, the direct application of the envelope theorem reveals the following:

$$\frac{\partial}{\partial x}(U_f^{\text{asym}}(x) - S_f(x)) = -\frac{\partial C_f(e_f^{\text{asym}}(x), x)}{\partial x} + \frac{\partial C_f(e_f^s(x), x)}{\partial x},$$

$$= -\int_{e_f^*(x)}^{e_f^{\text{asym}}(x)} \frac{\partial^2 C_f(e_f, x)}{\partial e \partial x} de_f > 0. \quad (A.5)$$

That implies that $U_f^{\text{asym}}(x') - S_f(x') > U_f^{\text{asym}}(x) - S_f(x)$ for $x' > x$ if $e_f^{\text{asym}}(\bar{x}) > e_f^*(\bar{x})$ for all $\bar{x} \in [x, x']$. When this inequality does not hold for some low value of $\bar{x}$, then we can apply the result of the first case and show that $U_f^{\text{asym}}(x) - S_f(x) > U_f^{\text{asym}}(\bar{x}) - S_f(\bar{x}) > 0$ for all $x \geq \bar{x}$. Otherwise, $U_f^{\text{asym}}(x) - S_f(x) > U_f^{\text{asym}}(x^c) - S_f(x^c)$ where $x^c$ is the ability level of the boundary agent. As the boundary female agent is clearly better off when married,\(^{37}\) the last expression is positive.

In the second step, we examine whether this potential equilibrium is indeed optimal even if female agents can choose the marketable skill. When the marketable skill is available, female agents may choose a negative value of $a_f$ (by acquiring the marketable skill), and, thus, we need to show that it is not a profitable choice for them for this range of $a_f$. As illustrated in Figure 2, the conditions for “no profitable deviation” require that the matching function $\phi$ should pass between the two indifference curves that yield the equilibrium utility levels. To see this, we first examine the indifference curves of male agents and then those of female agents.

**The case of male agents:** There are two types of male agents, depending on whether the ability type is above or below the threshold $x^c$. We show that it is enough to restrict attention to that of the boundary agent.

\(^{36}\)Note that $x_m > \tau(x_m)$ owing to the fact that $n_f < n_m$. This implies that the ability of the female agent is lower than that of the male agent for any married pair, and, consequently, that $e_m^*(x) > e_f^*(x)$.

\(^{37}\)We can confirm this fact by substituting $(e_f, e_m) = (e_f^*(x^c), e_m^*(x^c))$ and $(q_f, q_m) = (L, H)$ into the maximization problem of (A.4). Then, Assumption 3 ensures the result.
First, consider male agents whose ability is above the threshold $x^c$. When female agents choose a negative value of $a_f$, the level of the attractiveness necessary for male agents to marry these female agents, $\phi(a_m)$, is lower than any equilibrium levels of $a_m$. It can be shown that male agents never choose such a low level of $a_m$ when the boundary agent does not prefer it. In order to confirm this, we consider the two choices of $a_m$ and $a_m(x^c)$, where $a_m < a_m(x^c)$. Then, we have the following relation because of the supermodularity of $U_m(\phi^{-1}(a_m), a_m, H, x)$ with respect to $(a_m, x)$:

$$U_m^{\text{asym}}(x) - U_m(\phi^{-1}(a_m), a_m, H, x) 
\geq U_m(\phi^{-1}(a_m(x^c)), a_m(x^c), H, x) - U_m(\phi^{-1}(a_m), a_m, H, x),$$

$$> U_m^{\text{asym}}(x^c) - U_m(\phi^{-1}(a_m), a_m, H, x^c), \quad \text{for } x > x^c.$$ 

Hence, male agents whose ability is above $x^c$ never choose this low level of $a_m$ when the boundary male agent does not prefer it. As a consequence, it is enough to focus on the boundary agent in order to investigate if male agents whose ability is above $x^c$ never prefer a negative value of $a_f$.

Now, consider male agents whose ability is below the threshold $x^c$. It is necessary to establish that they do not prefer marriage given the matching function of $\phi(a_f)$. First, it is noted that $U_m^{\text{asym}}(x) - S_m(x)$ is an increasing function of $x$, following the same logic as in (A.5), because $e_m^{\text{asym}}(x) > e_m^s(x)$. This implies that $U_m^{\text{asym}}(x) < S_m(x)$ below $x^c$ because the boundary male agent is indifferent. Therefore, it is sufficient to focus on the boundary agent in order to check the consistency of the matching function.

**The case of female agents:** Now, we consider female agents. As we have already seen that it is sufficient to consider the boundary agent in the case of male agents, we simply need to derive the condition for the existence of a matching function that passes between the indifference curve of any given female agent and that of the boundary male agent for $a_f < 0$.

In order to pin down a consistent matching function, the indifference curves of female agents and the boundary male agent must move away from each other. A sufficient condition is that the slope of the indifference curve of the female agent when $a_f$ is zero (one example is given at $B'$ in Figure 2) is flatter than that of the boundary male agent $(C)$. We will derive this condition explicitly.

The indifference curves in Figure 2 are characterized by a set of $(a_f, a_m)$ that keeps the utility levels identical to the equilibrium ones:

$$S_m(x^c) = U_m(a_f, a_m, H, x^c), \quad \text{for boundary male agents},$$

$$U_f^{\text{asym}}(x) = U_f(a_f, a_m, H, x), \quad \text{for female agents who acquire marketable skills}.$$
The slopes of the indifference curves when \( a_f = 0 \), denoted by \( \text{Slope}_j \) \((j = f, m)\), are obtained from the total differentiation of the above equations. From the definitions of utilities, (8) and (9), we have:

\[
\text{Slope}_f = \frac{H + \Delta(H)}{-\Delta(H)} > 0, \tag{A.7}
\]

\[
(\text{Slope}_m)^{-1} = \frac{2}{\alpha H} \left( -(1 + \frac{\alpha}{2})H + \frac{\partial}{\partial e_m} C_m \left( e_m(a_m^0, H), \bar{z}^c \right) \right),
\]

where \( a_m^0 \) satisfies the condition that \( S_m^0(\bar{z}^c) = U_m(0, a_m^0, H, \bar{z}^c) \). To derive a simple condition, we introduce \( a_m^1 > 0 \) such that \( 0 = U_m(0, a_m^1, H, \bar{z}^c) \). Then, it is clear that \( a_m^1 > a_m^0 \) because of the strict concavity of \( U_m \) with respect to \( a_m \). This specific value provides the lower bound for the slope of the indifference curve:\[38\]

\[
(\text{Slope}_m)^{-1} < \frac{2}{\alpha H} \left( -(1 + \frac{\alpha}{2})H + \frac{\partial}{\partial e_m} C_m \left( e_m(a_m^1, H), \bar{z}^c \right) \right),
\]

\[
= \left( \frac{2}{\alpha} + 1 \right) \left( \gamma(e_m(a_m^1, H), \bar{z}^c) - 1 \right). \tag{A.8}
\]

By comparing (A.7) and (A.8), we can conclude that the slope of the indifference curve is indeed steeper for the boundary male agent than for the boundary female agent under Assumption 4. Therefore, the indifference curves of male and female agents tend to move away from each other as the absolute value of \( a_f \) rises, and we can draw a matching function that passes between them for \( a_f < 0 \), as indicated by the bold curve in the figure. This completes the proof that there exists an asymmetric equilibrium under the maintained assumptions. Q.E.D.

**The derivation of (17).**

In this proof, we suppose that female agents choose to acquire the nonmarketable skill and then check when there is no incentive to deviate from the asymmetric equilibrium. The condition to support the symmetric equilibrium can be obtained by exactly the same logic.

Given that female agents acquire the nonmarketable skill, we end up with some matching function \( \phi(a_f) \). Suppose that female agents are now allowed to deviate by acquiring the marketable skill. It is not optimal to deviate this way if their indifference curve (A.6) is always above the matching function,

\[38\]From the definition of \( a_m^1, \partial C_m(e_m, \bar{z}^c)/\partial e_m = (1 + \alpha/2)H \gamma(e_m, \bar{z}^c) \) when \( e_m = e_m(a_m^1, H) \).
a set of \((a_f, a_m)\) available to female agents. In order to show that we can effectively ignore the choice of the marketable skill for all female agents, we construct the envelope of the indifference curves. The necessary conditions for the envelope curve provide the following equations in which \(x\) works as a shift parameter:

\[
a_f = \frac{\Delta(H)}{\Delta(L)} a_f^{\text{asym}}(x),
\]

\[
a_m = U_f(a_f^{\text{asym}}(x), a_m^{\text{asym}}(x), L, x) - \left( H + \Delta(H) \right) e_f(a_f, H) + C_f(e_f(a_f, H), x),
\]

\[
= a_m^{\text{asym}}(x) + (L + \Delta(L) - H - \Delta(H))e_f(a_f, H).
\]

When we substitute \(a_f^{\text{asym}}(x)\) out from the above conditions, we have the following envelope, denoted as \(a_m = \xi(a_f)\):

\[
a_m = \phi\left(\frac{\Delta(L)}{\Delta(H)}a_f\right) - (H + \Delta(H) - L - \Delta(L))\frac{a_f}{\Delta(H)} \equiv \xi(a_f).
\]

As noted, a sufficient condition is that this envelope \(\xi(a_f)\) is located above the matching function \(\phi(a_f)\) for all \(a_f \in \mathbb{R}_+\) (i.e., \(\xi(a_f) > \phi(a_f)\) for all \(a_f\)). This leads to the next condition:

\[
\phi\left(\frac{\Delta(L)}{\Delta(H)}a_f\right) - \phi(a_f) > \frac{H + \Delta(H) - L - \Delta(L)}{\Delta(L) - \Delta(H)} = \frac{2}{(1 + \alpha)\theta - \alpha - 1}.
\]

This inequality clearly holds when the slope of the matching function \(\phi'(a_f)\) is steeper than the value on the right-hand side for all \(a_f \in \mathbb{R}_+\). Q.E.D.

**References**


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39The envelope curve is obtained by the next two conditions: \(\frac{\partial}{\partial x}(U_f^{\text{asym}}(x) - U_f(a_f, a_m, H, x)) = 0, U_f^{\text{asym}}(x) - U_f(a_f, a_m, H, x) = 0\). By applying the envelope theorem, we obtain the two conditions.


