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Test of AdS/CFT correspondence by non-local operators

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Abstract

In this thesis, comparing the results from both side, we aim to find new evidence of the AdS/CFT correspondence. We introduce operators in the gauge theory with various dimensionality. Such operators have counterparts in string theory consisting of branes or fundamental strings. We find that considering these non-local objects is an effective method for testing the AdS/CFT correspondence. These operators introduce a new parameter in the theory. This parameter allows us to compare the results from both theories. An important non-local operator in this thesis is a 3-dimensional operator called an interface. This object is realized as a codimension-1 object in 4-dimensional conformal field theory. We considered the potential between this interface and a test particle. We find complete agreement between the gauge and gravity theory results in the classical level.

We also find a procedure to relate a brane configuration to a representation of a non-local operator called a 't Hooft operator. This representation is expressed as a Young diagram. This configuration can also be investigated in detail by virtue of the interface. Under the condition that our system preserves a quarter of the supersymmetry, we find the equations and boundary condition to determine the embedding of the D5-brane in bulk $AdS_5 \times S^5$ spacetime.

Contents

Chapter 1

Introduction

The AdS/CFT correspondence is a statement about equivalence between type IIB superstring theory and $\mathcal{N} = 4$ super Yang-Mills theory, conjectured by Juan Maldacena [1]. So far many affirmative responses are found but the proof is not found yet. Actually, this proof is not easy. What we can do immediately is to accumulate evidence of this correspondence. We show some examples of them in this thesis.

An important object in this thesis is a gauge theory operator which has non-trivial spacetime dimension. These are called "non-local operators." Non-local operators play a crucial role in the study of the AdS/CFT correspondence. These operators are extending in spacetime and are classified by their dimensionality. An operator which has one spacetime dimension is called a line operator or a loop operator if it is closed, an operator which has two spacetime dimensions is called a surface operator, and so forth. In particular, if an operator has no spacetime dimension like an instanton then it is a local operator. Their holographic duals have been studied, e.g. for surface operators [2] and for Wilson line operators [3]. In this thesis we mainly treat three-dimensional local operators which are called "interfaces."

The interface separates the whole 4-dimensional spacetime into two parts where different gauge groups can live. In addition to this we introduce some operators and calculate correlation functions between these operators and the interface. This calculation can be done both from the gauge theory side and the string theory side. Comparing these results gives new evidence of the AdS/CFT correspondence.

There is a big problem to confirm the AdS/CFT correspondence in this way. To confirm this correspondence one computes corresponding quantities in both theories and compares two results as said. However, the regions where perturbative computations are effective are different in two theories —the perturbative calculation is valid in string theory when the 't Hooft coupling $\lambda := g^2 N$ is large $\lambda \gg 1$, while in gauge theory when $\lambda \ll 1$.

One way to overcome this difficulty is to introduce another large parameter as in [4]. In [4] the R-charge J (the angular momentum in the gravity side) has been taken to be large and the effective expansion parameter has become λ/J^2. By virtue of this change of the effective coupling, the conformal dimensions of such operators have been successfully compared to the energy of the stringy excited states in the pp-wave geometry. Their result has given non-trivial evidence of the AdS/CFT correspondence. Other examples of similar phenomena are found in surface operators [5, 2] and also in the interface [3] as we see in this thesis.

This system enables us to compare these two theories, even if effective regions are different for the gauge side and the string side. In our case a new parameter k is introduced because of the existence of the interface. Physical quantities are expanded into power series of $\sqrt{\lambda}/k$. This combination, $\sqrt{\lambda}/k$, can be taken to be small by the tuning of the parameter $k \gg \sqrt{\lambda}$. In fact, later we compare the results of calculating a certain potential from the gauge theory side and the gravity theory side and then confirm these results agree in the leading term.

The gauge theory considered in this thesis is known as a defect CFT [6]. This theory has the interface between two gauge theories which have different gauge groups. Recently, 4-dimensional interface CFTs (or boundary CFTs) have been found to related to knot invariants [7, 8]. A system consisting of D3 and D5-branes realizes this theory through the AdS/CFT correspondence [6]. The corresponding supergravity description is called "Janus" and has been studied [9, 10, 11, 12, 13, 14].

The interface is introduced by considering a D3-D5 system [6]. In this scenario, multiple D3-branes form $AdS_5 \times S^5$ spacetime, while the D5-brane is treated as a probe brane and corresponds to the interface. In addition to this 3-dimensional operator, we consider adding several operators in this thesis — Wilson/'t Hooft line operators and chiral primary operators.

We treat two kinds of correlation functions. One is the correlation function between a local operator called a chiral primary operator and the interface. The other is the correlation function between the Wilson line operator and the interface.

First, we calculate the correlation function between the chiral primary operator and the interface. The correlation functions in the AdS/CFT correspondence are calculated by GKPW prescription [15, 16]. Due to GKPW there is one-to-one correspondence between local operators in the gauge theory and fields in the gravity theory. Using this relation we compare the result of the correlation function between two theories. In terms of the gauge theory, this one-point function usually vanishes since the background is a solution of the equation of motion and thus any variation of the action vanishes at this background. In our case this one-point function does not vanish in general because the interface is inserted. This insertion breaks the original symmetry of the system. In the gravity side, this one-point function does not vanish because we have, in addition to the supergravity, the probe D5-brane which gives non-vanishing contribution.

The other correlation function we want to calculate is the correlation function between a 1-dimensional non-local operator and the interface. We consider the Wilson loop as the 1-dimensional non-local operator. This correlation function is interpreted as the potential between the interface and a test particle. This system is an analogue of a dielectric substance and a charged particle in the electromagnetism.

In addition to these calculation of correlation functions, we try to relate a certain brane configuration with a representation of a 1-dimensional operator which is called a 't Hooft operator. This correspondence is derived by considering supersymmetry condition. This system enables us to study the detailed configuration of branes which cannot be seen only from the string description in terms of Young diagrams.

This paper is organized as follows: Chapter 2 and chapter 3 are devoted to the review of essential ingredients of our discussion. Chapter 4, chapter 5 and chapter 6 are devoted to show our own works.

The topic of each section is as follows. The main theme of 2 is gauge theory. Section 2.1 describes our set-up of gauge theory, $\mathcal{N} = 4$ supersymmetric Yang-Mills theory. After that, we introduce chiral primary operators in section 2.2 and non-local operators in section 2.3.

In chapter 3 we move to super string theory. First in section 3.1 we see type IIB string theory which is expected to be a dual theory of $\mathcal{N} = 4$ super Yang-Mills theory according to the AdS/CFT correspondence. This theory is introduced by the near horizon limit of the D3-brane system as viewed in section 3.2. After that in section 3.4 we consider a D3-D5 brane system to introduce a counterpart of a non-local operator called an interface in the gauge theory side.

Chapter 4 shows the first evidence of the AdS/CFT correspondence. The theme of this chapter is the correspondence of calculation of chiral primary operators in both theories. We calculate a physical quantity in gauge theory and later calculate the counterpart of this quantity in string theory. Then, the comparison of these quantities shows us a compatible result with the AdS/CFT scenario.

Chapter 5 shows the second evidence of the AdS/CFT correspondence. The theme of this chapter is the correspondence of a kind of non-local operators in both theories. As well as chapter 4 we did calculations and comparison.

In chapter 6 we go to an applications of the AdS/CFT correspondence. We try to find a certain brane configuration consisting of D3, D5 and D1-branes. This system introduces, in addition to the interface, the 't Hooft line operator embedded on it. Using non-local operators via AdS/CFT duality, it is expected to find a detailed configuration which cannot be seen directly only from the string side. Appendices show the basic tools for our analysis.

Chapter 2

Gauge theory side

In this section we first review $\mathcal{N}=4$ super Yang-Mills theory. And next we consider some examples of local/non-local operators. These operators are classified by dimensionality. First, in section 2.2 we introduce operators which have zero spacetime dimension — local operators. These are called chiral primary operators.

In the next section we introduce operators which have non-zero spacetime dimension — non-local operators. We treat 1-dimensional non-local operators in this thesis. These are Wilson operators and 't Hooft operators. The other non-local operator we consider in this thesis is 3-dimensional non-local operator called an "interface." What we want to do in this thesis is to calculate correlation functions between these local/non-local operators.

2.1 $\mathcal{N}=4$ super Yang-Mills theory

This section gives the setup and the action of $\mathcal{N}=4$ super Yang-Mills theory. This theory contains the fields, $A_\mu, \phi_i, \psi : \mu = 0, 1, \cdots, 3,\ i = 4, 5, \cdots, 9$. These are the gauge field, the real scalar fields and the 16 component spinor, respectively.

The action of this theory is derived from 10-dimensional super Yang-Mills theory by a trivial dimensional reduction. (See appendix A for the convention of 10-dimensional gamma matrices Γ_M.)

$$S = \frac{2}{g^2}\int d^4x\, \mathrm{tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}D_\mu\phi_i D^\mu\phi_i + \frac{i}{2}\bar{\psi}\Gamma^\mu D_\mu\psi + \frac{1}{2}\bar{\psi}\Gamma^i[\phi_i,\psi] + \frac{1}{4}[\phi_i,\phi_j][\phi_i,\phi_j]\right], \tag{2.1.1}$$

where the definition of the field strength and the covariant derivative are given by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu], \tag{2.1.2}$$

$$D_\mu\phi_i = \partial_\mu\phi_i - i[A_\mu, \phi_i], \tag{2.1.3}$$

$$D_\mu\psi = \partial_\mu\psi - i[A_\mu, \psi]. \tag{2.1.4}$$

This action possesses the following supersymmetry.

$$\delta A_\mu = i\bar{\epsilon}\Gamma_\mu\psi, \tag{2.1.5a}$$

$$\delta\phi_i = i\bar{\epsilon}\Gamma_i\psi, \tag{2.1.5b}$$

$$\delta\psi = \frac{1}{2}F_{\mu\nu}\Gamma^{\mu\nu}\epsilon + D_\mu\phi_i\Gamma^{\mu i}\epsilon - \frac{i}{2}[\phi_i,\phi_j]\Gamma^{ij}\epsilon, \tag{2.1.5c}$$

where ϵ, the 16 component spinor, is the parameter of the supersymmetry.

2.2 Chiral primary operators

Local operators in super Yang-Mills theory are classified into infinite dimensional families. These are irreducible representation of the 4-dimensional $\mathcal{N}=4$ superconformal algebra. Let us first look at these algebra. The superconformal algebra is expressed in appendix E.

Before introducing non-local operators, let us investigate local operators in $\mathcal{N}=4$ super Yang-Mills theory. In unitary field theories there is a lower bound on the dimension of fields — $\Delta=(d-2)/2$ for scalar fields. Therefore, each representation of the conformal group must have some operator of lowest dimension, which must then be annihilated by the conformal boost, K and the superconformal generator, S. Such operators are called 'primary operators," or PO for short. Among them there are special types of primary operators which are annihilated by some combination of the supercharges Q. Their representations are smaller than the generic representations. These are called "chiral primary operators." They have special proparties that their dimension is uniquely determined by their R-symmetry representations and does not receive any quantum corrections.

We introduce chiral primary operators in the $\mathcal{N}=4$ super Yang-Mills theory using scalar fields ϕ_i.

$$O^I_\Delta(x) := \mathcal{C}^I_{i_1\cdots i_\Delta}\mathrm{tr}(\phi_{i_1}(x)\cdots\phi_{i_\Delta}(x)), \tag{2.2.1}$$

where indices $i_1\cdots i_\Delta$ are $SO(6)$ vector indices. The trace in the formula above is over $SU(N)$ indices. $\mathcal{C}^I_{i_1\cdots i_\Delta}$ is a totally symmetric traceless rank Δ tensor of $SO(6)$. This tensor is totally symmetric in lower indices, $i_1, i_2, \cdots, i_\Delta$ and upper index, I, which distinguishes such operators. We can choose an orthonormal basis such that $\mathcal{C}^I_{i_1\cdots i_\Delta}\mathcal{C}^J_{i_1\cdots i_\Delta}=\delta^{IJ}$. The 2-point function of two CPOs is specified by tensors $\mathcal{C}^I_{i_1\cdots i_\Delta}$ and $\mathcal{C}^J_{j_1\cdots j_{\Delta'}}$. Consider

$$\left\langle \mathrm{tr}(\phi_{i_1}(x)\cdots\phi_{i_\Delta}(x))\mathrm{tr}(\phi_{j_1}(y)\cdots\phi_{j_{\Delta'}}(y))\right\rangle. \tag{2.2.2}$$

Because of the symmetry of the indices, $\mathcal{C}^I_{i_1 i_2\cdots i_\Delta}=\mathcal{C}^I_{i_2 i_1\cdots i_\Delta}$ and so on, this expectation value $\left\langle O^I_\Delta(x)O^J_{\Delta'}(y)\right\rangle$ becomes zero if it contain the factor like $\delta_{i_k i_l}$ from a pair between the same trace. So the surviving term is the contraction between ϕs in different traces in eq. (2.2.2). It is nonzero only if $\Delta=\Delta'$. So below we focus only on this case. Therefore, the 2pt function

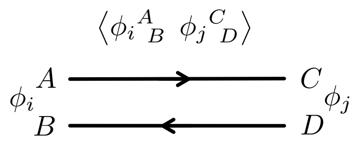

Figure 2.1: Propagator

can be expanded using "Wick's theorem":

$$\begin{aligned}
\left\langle \mathcal{O}^I_\Delta(x)\mathcal{O}^J_\Delta(y)\right\rangle &= \mathcal{C}^I_{i_1\cdots i_\Delta}\mathcal{C}^J_{j_\Delta\cdots j_1}\Big\langle \phi_{i_1}{}^{A_\Delta}{}_{A_1}(x)\ \phi_{i_2}{}^{A_1}{}_{A_2}(x)\ \phi_{i_3}{}^{A_2}{}_{A_3}(x)\ \cdots\phi_{i_\Delta}{}^{A_{\Delta-1}}{}_{A_\Delta}(x) \\
&\qquad \cdot\phi_{j_\Delta}{}^{B_\Delta}{}_{B_{\Delta-1}}(y)\ \phi_{j_{\Delta-1}}{}^{B_{\Delta-1}}{}_{B_{\Delta-2}}(y)\ \phi_{j_{\Delta-2}}{}^{B_{\Delta-2}}{}_{B_{\Delta-3}}(y)\ \cdots\phi_{j_1}{}^{B_1}{}_{B_\Delta}(y)\ \Big\rangle \\
&= \mathcal{C}^I_{i_1\cdots i_\Delta}\mathcal{C}^J_{j_1\cdots j_\Delta}\Bigg(\ \left\langle\phi_{i_1}{}^{A_\Delta}{}_{A_1}(x)\phi_{j_1}{}^{B_1}{}_{B_\Delta}(y)\right\rangle\left\langle\phi_{i_2}{}^{A_1}{}_{A_2}(x)\phi_{j_2}{}^{B_2}{}_{B_1}(y)\right\rangle \\
&\qquad \cdots\Big\langle\phi_{i_\Delta}{}^{A_{\Delta-1}}{}_{A_\Delta}(x)\phi_{j_\Delta}{}^{B_\Delta}{}_{B_{\Delta-1}}(y)\Big\rangle \\
&\qquad + \ \cdots\ \Bigg).
\end{aligned} \tag{2.2.3}$$

Here we write the $SU(N)$ indices explicitly. The summation in the parentheses includes all permutations in indices $j_1, j_2, \cdots, j_\Delta$. Each propagator in the expression (2.2.3) is calculated from the action (2.1.1) as follows:

$$\left\langle\phi_i{}^A{}_B(x)\ \phi_j{}^C{}_D(y)\right\rangle = \frac{g^2}{8\pi^2}\frac{\delta_{ij}\delta^A{}_D\delta^C{}_B}{|x-y|^2}, \tag{2.2.4}$$

being represented graphically as shown in figure 2.1.

Substituting it and taking the large N limit where only planar diagrams (ex. figure 2.2) contribute. Each loop in figure 2.2 gives the factor $\delta^A_A = N$. There are Δ such diagrams. Therefore,

$$\begin{aligned}
\left\langle O^I_\Delta(x)O^J_\Delta(y)\right\rangle &= \left(\frac{g^2}{8\pi^2}\frac{1}{|x-y|^2}\right)^\Delta N^\Delta\Delta\delta^{IJ} \\
&= \left(\frac{\lambda}{8\pi^2}\right)^\Delta \Delta\,\frac{\delta^{IJ}}{|x-y|^{2\Delta}},
\end{aligned} \tag{2.2.5}$$

where $\lambda := g^2N$ is the 't Hooft coupling. We rescale the CPOs such that they have normalized 2-point functions

$$\left\langle\mathcal{O}^I_\Delta(x)\mathcal{O}^J_\Delta(y)\right\rangle = \frac{\delta^{IJ}}{|x-y|^{2\Delta}}, \tag{2.2.6}$$

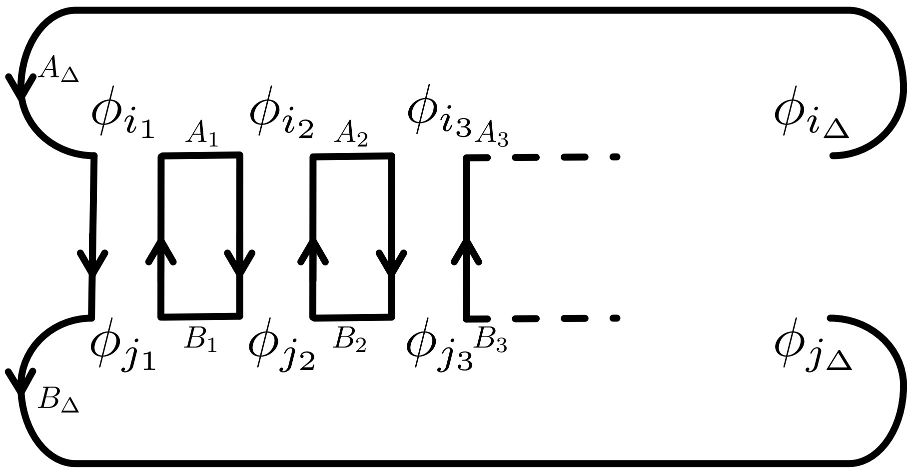

Figure 2.2: An example of a planar diagram

namely, $\mathcal{O}^I(x) := \left(\frac{8\pi^2}{\lambda}\right)^{\Delta/2} \frac{1}{\sqrt{\Delta}}\ O^I(x)$.

From the above, we obtain CPOs we use in this thesis:

$$\mathcal{O}^I_\Delta(x) := \frac{(8\pi^2)^{\Delta/2}}{\lambda^{\Delta/2}\sqrt{\Delta}}\mathcal{C}^I_{i_1 i_2\cdots i_\Delta}\mathrm{tr}(\phi_{i_1}(x)\phi_{i_2}(x)\cdots\phi_{i_\Delta}(x)). \tag{2.2.7}$$

2.3 Non-local operators

In the previous section 2.2 we reviewed local operators called chiral primary operators. In addition to these the gauge theory has other types of operators which are not defined locally. These are called non-local operators.

Non-local operators are classified by their dimensionality in spacetime. In this section we introduce some examples of these operators — Wilson/'t Hooft operators and an interface. Wilson/'t Hooft operators have one spacetime dimension and extend along the time axis in our case. Then, these are 1-dimensional non-local operators. The interface, on the other hand, extends in 3 dimensions in spacetime. This is a 3-dimensional non-local operator. Their correlation functions are calculated in this thesis. These results give evidence of the AdS/CFT correspondence.

2.3.1 Wilson operator and 't Hooft operator

Wilson loop operators are introduced to solve the problem of confinement of quarks in [17]. This operator is expressed for a gauge field $A_\mu(x)$ as

$$W(C) := \mathrm{tr}P\exp\left(i\oint_C A_\mu dx^\mu\right), \tag{2.3.1}$$

where C is a closed path in spacetime and P means the path-ordering products.

Next let us generalize the Wilson line operator by introducing a scalar field [18, 19]. The multiplets in the $\mathcal{N}=4$ super Yang-Mills are now (A_μ, ϕ_i, ψ), the gauge field, the scalar field and the fermion field, where μ is a vector index of $SO(1,3)$ and i is a vector index of the $SO(6)$ R-symmetry. In the case of the bosonic Wilson operator, the path is parametrized by $(x^\mu(s), y^i(s))$ [20].

$$W_R(C) = \mathrm{tr}_R P\mathrm{exp} \int_C ds \left(i(A_\mu(x)\dot{x}^\mu + \phi_i \dot{y}^i) \right) , \tag{2.3.2}$$

where the loop C is parametrized by $x^\mu(s)$ and $y^i(s)$. For gauge invariance the curve $x^\mu(x)$ must be closed while $y^i(s)$ can be taken arbitrary curve. The curve C is identified with the worldline of the inserted particle propagating in $\mathcal{N}=4$ superspace while the representation R corresponds to the charge carried by the inserted particle. In a case of supersymmetric Wilson loop the path C must be a straight line spanned by $x^0 = t$ and $\dot{y}^i = n^i$, a unit vector in 6-dimensional space (See appendix of [21]). Therefore, the Wilson loop becomes

$$W_R = \mathrm{tr}_R P \exp \left(i \int dt (A_0 + \phi) \right) , \tag{2.3.3}$$

where $\phi := \phi_i n^i$. This half Wilson loop is parametrized by only its representation R.

As an example, let us consider the simplest case of the representation — the trivial Young diagram $\Box$. It corresponds to a fundamental string propagating in the bulk and ending at the boundary of AdS along the curve C. In chapter 5 we consider the potential energy between a test particle and the interface. In higher representations of the gauge group, the holographic description of a half-BPS Wilson loop operator, that is an operator which preserves half of supersymmetry is described by D3 or D5-branes as briefly explained in appendix D or see Ref. [21]

A 't Hooft operator, on the other hand, corresponds to inserting a magnetic monopole of magnetic charge m, with the worldline C'. Wilson operators and 't Hooft operators are related under the S-duality transformation [22].

A representation of the 't Hooft operator is classified by the Young diagram in the same way as the Wilson operator. In chapter 6 we try to interpret this representation from string theory description.

2.3.2 Interface

An interface is a codimension one defect which connects two different theories. Here we consider an interface connecting two $\mathcal{N}=4$ super Yang-Mills theories with gauge groups SU(N) and SU($N-k$).

The counterpart of the interface in string theory is a probe D5-brane. The bulk spacetime $AdS_5 \times S^5$ is formed by multiple D3-branes. The back-reaction of the D5-brane can be neglected. So we consider the D5-brane in $AdS_5 \times S^5$ spacetime. It can be realized in the string theory [6] as the D3- and D5-brane configuration shown in the table 2.1.

Due to the presence of this interface, the fields have a nontrivial classical vacuum solution.

	0	1	2	3	4	5	6	7	8	9
D3	◯	◯	◯	◯	×	×	×	×	×	×
D5	◯	◯	◯	×	◯	◯	◯	×	×	×

Table 2.1: D3-D5 system. "◯" means the direction the brane is extended, while "×" means the normal direction.

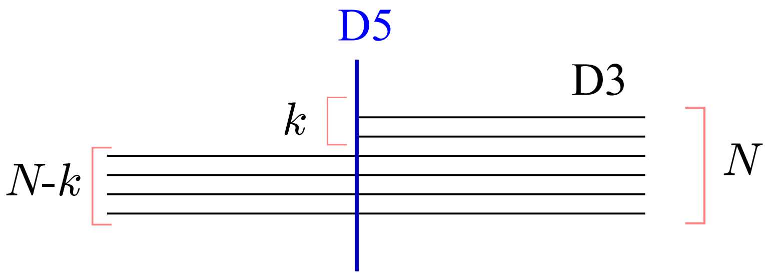

Figure 2.3: D3-D5 system. k semi-infinite D3-branes end on a D5-brane.

We analyze the supersymmetry of this classical solution in the gauge theory with the ansatz:

$$A_\mu = 0, \qquad \phi_i = \phi_i(x_3),\ (i = 4, 5, 6), \qquad \phi_i = 0,\ (i = 7, 8, 9).$$

We obtain the fermion condition from (2.1.5c)

$$0 = \delta\psi = \partial_3\phi_i\Gamma^{3i}\epsilon - \frac{i}{2}[\phi_i, \phi_j]\Gamma^{ij}\epsilon, \tag{2.3.4}$$

which is rewritten as Nahm's equations:

$$\partial_3\phi_i = -\frac{i}{2}\epsilon_{ijk}[\phi_j, \phi_k]. \tag{2.3.5}$$

The parameters of the remaining supersymmetries satisfy

$$(1 - \Gamma^{3456})\epsilon = 0. \tag{2.3.6}$$

Nahm's equations (2.3.5) have a fuzzy funnel solution [23]:

$$\phi_i = -\frac{1}{x_3}t_i \oplus 0_{(N-k)\times(N-k)}, \qquad (x_3 > 0) \tag{2.3.7}$$

where $t_i,\ i = 4, 5, 6$ are generators of a representation of SU(2). Namely, t_i are $k \times k$ matrices satisfying the commutation relations.

$$[t_i, t_j] = i\epsilon_{ijk}t_k, \quad i, j, k = 4, 5, 6,$$

ϵ_{ijk}: totally anti-symmetric tensor and $\epsilon_{456} = +1$.

In the rest of this thesis we only consider t_i of the k-dimensional irreducible representation.

Chapter 3

String theory side

In this section we propose the gravity counterparts corresponding to the gauge theory objects considered in chapter 2.

3.1 Type IIB superstring

The theory we consider in this thesis is type IIB superstring theory which is a holographic dual to $\mathcal{N} = 4$ super Yang-Mills theory according to the AdS/CFT correspondence.

Supertstrings with $\mathcal{N} = 2$ supersymmetry is classified by the chirality — The IIA theory is non-chiral while the IIB theory is chiral. Dp-brane is stable when p is even for type IIA superstring while p is odd for type IIB superstring. Our system consists of D3 and D5 branes. The D3-branes form bulk $AdS_5 \times S^5$ spacetime while the D5-brane is treated as a probe brane. This probe D5-brane breaks half of the supersymmetry and also carries the Ramond-Ramond (RR) charges [24].

The NS-NS sector fields of type II strings consist of the dilation, the metric and antisymmetric tensor fields:

$$\Phi,\ G_{\mu\nu},\ B_{\mu\nu}. \tag{3.1.1}$$

In addition to these type IIB string theory contains the following fields in R-R sector:

$$C_{(0)}, C_{(2)}, C_{(4)}, C_{(6)}, C_{(8)}. \tag{3.1.2}$$

Among of them, $C_{(6)}, C_{(8)}$ are Hodge duals of $C_{(0)}, C_{(2)}$, respectively and $C_{(4)}$ is a self-dual field. The coupling of these fields to the Dp-brane is

$$\mu_p \int_{\mathcal{M}_{p+1}} C_{(p+1)}, \tag{3.1.3}$$

where μ_p is the charge of the Dp-brane and $\mathcal{M}_{(p+1)}$ is the worldvolume of the Dp-brane.

3.2 D3-brane system

The classical solution describing multiple D3-branes is given by

$$ds^2 = H(r)^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + H(r)^{1/2}(dr^2 + r^2 d\Omega_5^2), \;\; H(r) = 1 + \frac{r_0^4}{r^4}, \tag{3.2.1}$$

where x^μ, $\mu = 0, 1, 2, 3$, are the coordinates in directions parallel to the brane's world-volume and perpendicular directions to this world-volume are parametrized by r in the radial direction and Ω_5 in the axial directions on S^5. The constant r_0 determines the scale of the solution and is related to the number of the D3-branes as follows:

$$r_0 = (4\pi N g_{\mathrm{s}})^{1/4}\sqrt{\alpha'}, \tag{3.2.2}$$

where g_{s} is the string coupling.

Let us focus on the near horizon region $r \ll r_0$. The function is approximately $H(r) = \frac{r_0^4}{r^4}$. The metric is decomposed into two spaces.

$$ds^2 = \left[\left(\frac{r_0}{r}\right)^2 dr^2 + \left(\frac{r}{r_0}\right)^2 dx^2\right] + r_0^2 d\Omega_5^2. \tag{3.2.3}$$

Using the coordinate transformation $y = r_0^2/r$, we obtain the $AdS_5 \times S^5$ metric in the following well-known form:

$$ds^2 = \frac{r_0^2}{y^2}(dy^2 + dx^2) + r_0^2 d\Omega_5^2. \tag{3.2.4}$$

We can certainly see that the near horizon of this D3-brane solution is $AdS_5 \times S^5$.

We add a D5-brane to this system. We note that supersymmetry of this system is preserved after adding this D5-brane.

3.3 Gubser-Klebanov-Polyakov-Witten relation

The correlation functions in the AdS/CFT correspondence are calculated by GKPW prescription [15, 16]. Due to GKPW there is one-to-one correspondence between local operators in the gauge theory and fields in the gravity theory. Let $\mathcal{O}$ be a scalar operator in the gauge theory, and s be the scalar field in the gravity theory which corresponds to $\mathcal{O}$. GKPW claims that the relation

$$\left\langle \mathrm{e}^{\int d^4x s_0(x)\mathcal{O}(x)} \right\rangle_{\mathrm{CFT}} = \mathrm{e}^{-S_{\mathrm{cl}}(s_0)} \tag{3.3.1}$$

is satisfied in the classical gravity limit. In this equation s_0 is a boundary condition of s up to a certain factor, $S_{\mathrm{cl}}(s_0)$ is the action evaluated by the classical solution with the boundary condition given by s_0.

Using this relation the one-point function is calculated as follows.

$$\langle \mathcal{O}(x) \rangle = \left.\frac{-\delta S_{\mathrm{cl}}(s_0)}{\delta s_0(x)}\right|_{s_0=0}. \tag{3.3.2}$$

We employ the normalization $\langle 1 \rangle = 1$.

If no interface or other defects are inserted, this one-point function vanishes due to the conformal invariance. In terms of the gravity theory, this one-point function vanishes since the background is a solution of the equation of motion and thus any variation of the action vanishes at this background. In our case this one-point function does not vanish in general because the interface is inserted as we have seen in the previous section. In the gravity side, this one-point function does not vanish because we have, in addition to the supergravity, a probe D5-brane which gives non-vanishing contribution.

3.4 Addition of D5-brane

In order to introduce an interface we add a probe D5-brane to the previous D3-brane system. This D5-brane is located in $AdS_5 \times S^5$ spacetime formed by multiple D3-branes [6]. After addition of the D5-brane, our brane configuration becomes as showed in table 2.1. Physical quantities we calculate in this thesis are correlation functions between this 3-dimensional non-local operator and several test operators.

In section 4 we calculate correlation functions between the interface and local operators called chiral primary operators.

In section 5 we calculate correlation functions between the interface and a test particle. This corresponds to addition of a 1-dimensional non-local operator called a Wilson loop.

3.4.1 D3-brane background

Through the study of the AdS/CFT correspondence, it is widely known that the near horizon geometry of D3-branes, as the solution of the 10-dimensional type IIB supergravity, equivalently describe the world volume gauge theory on N D3-branes. We prepare this gravity background. The metric takes following $\mathrm{AdS}_5 \times S^5$ form, using the coordinates $y, x^\mu,\ \mu = 0, 1, 2, 3$: [1]

$$ds^2 = \frac{1}{y^2}(dy^2 + dx^\mu dx^\nu \eta_{\mu\nu}) + d\Omega_5^2, \tag{3.4.1}$$

with RR 4-form

$$G_4 = -\frac{1}{y^4}dx^0 dx^1 dx^2 dx^3 + 4\alpha_4, \tag{3.4.2}$$

where $\eta_{\mu\nu}$ and $d\Omega_5^2$ denote 4-dimensional Lorentzian metric $\eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1)$ and the unit S^5 metric, respectively. Here we also use 4-form α_4 in S^5 which satisfy $d\alpha_4 =$(volume form of S^5). In this thesis we employ the unit in which the radii of AdS_5 and S^5 are 1. In this unit the slope parameter α' can be written as $\alpha' = 1/\sqrt{\lambda} := 1/\sqrt{4\pi g_s N}$, where g_s is the string coupling constant and λ corresponds to the 't Hooft coupling in the gauge theory side.

[1] The boundary of the AdS_5 is at $y = 0$.

3.4.2 Probe D5-brane

For analyzing the gravity dual to the interface gauge theory, we put a single probe D5-brane, whose backreaction can be neglected, on the D3-brane background as realization of the interface. It is appropriate that we arrange the probe D5-brane on the AdS_4 in the AdS_5 and S^2 on the equator of the S^5. The action of a single D5-brane is given by

$$S_{D5} = -T_5 \int \sqrt{-\det(G+\mathcal{F})} + T_5 \int \mathcal{F} G_4, \tag{3.4.3}$$

which consists of two terms; the first term is the Dirac-Born-Infeld action and the second term is the Wess-Zumino term. Here we set the pull-back of metric as G and world volume gauge flux as $\mathcal{F}$. And the Dp-brane tension is defined as

$$T_p = \frac{1}{(2\pi)^p \alpha'^{(p+1)/2} g_s}. \tag{3.4.4}$$

Consider the solution of probe D5-brane in the background (3.4.1) and (3.4.2) under the following ansatz

$$y = y(x_3), \qquad \mathcal{F} = -\kappa \, \mathrm{vol}[S^2], \tag{3.4.5}$$

with a constant κ and the S^2 volume form $\mathrm{vol}[S^2]$. By substituting the ansatz, we can rewrite the action

$$S_{D5} = -4\pi T_5 V \int dx_3 \frac{1}{y^4} \left(\sqrt{\left((\partial_3 y)^2 + 1\right)(1+\kappa^2)} - \kappa \right), \tag{3.4.6}$$

where V means volume of the 3-dimensional subspace along (x^0, x^1, x^2) directions in the probe D5-brane and we use ∂_3 instead of $\partial/\partial x_3$. We solve the equation of motion

$$\partial_3 \left(\frac{\partial_3 y}{y^4} \sqrt{\frac{1+\kappa^2}{(\partial_3 y)^2 + 1}} \right) + \frac{4}{y^5} \left(\sqrt{\left((\partial_3 y)^2 + 1\right)(1+\kappa^2)} - \kappa \right) = 0, \tag{3.4.7}$$

and obtain the solution of probe D5-brane

$$x_3 = \kappa y, \tag{3.4.8}$$

which fixes the position of probe D5-brane located on the AdS_5. In addition, charges of D3-branes appear as magnetic flux in the D5-brane world volume, because the D5-brane are linked to D3-branes through the fuzzy funnel solution (2.3.7) in the world volume theory. Namely we can associate k with κ

$$k = -\frac{T_5}{T_3} \int \mathcal{F} = \frac{\kappa}{\pi \alpha'} = \kappa \frac{\sqrt{\lambda}}{\pi}. \tag{3.4.9}$$

Chapter 4

Correspondence I: Chiral primary operator

In this section we consider the one-point functions of chiral primary operators. The chiral primary operators are defined as

$$\mathcal{O}^I_\Delta(x) := \frac{(8\pi^2)^{\Delta/2}}{\lambda^{\Delta/2}\sqrt{\Delta}} C^I_{i_1 i_2 \cdots i_\Delta} \mathrm{tr}(\phi_{i_1}(x)\phi_{i_2}(x)\cdots\phi_{i_\Delta}(x)), \tag{4.0.1}$$

where Δ denotes the conformal dimension and $C^{i_1 i_2 \cdots i_\Delta}$ is a traceless symmetric tensor normalized as $C^{i_1 i_2 \cdots i_\Delta} C^{i_1 i_2 \cdots i_\Delta} = 1$. The normalization of the operator is determined so that the two point function without interface becomes

$$\langle \mathcal{O}^I_\Delta(x)\mathcal{O}^J_\Delta(y)\rangle = \frac{\delta^{IJ}}{|x-y|^{2\Delta}}, \tag{4.0.2}$$

as we defined in section 2.2. We followed the definition by [25] where they calculated the 3-point function.

In this section, on the other hand, we calculate the 1-point function of this operator $\left\langle \mathcal{O}^I_\Delta(x)\right\rangle$. Usually this 1-pt function becomes zero, $\left\langle \mathcal{O}^I_\Delta\right\rangle = 0$ if $\Delta \neq 0$ due to the conformal symmetry. But now we introduce the interface then this symmetry breaks partically. Therefore the 1-pt function may not be zero in the presence of the interface. It is a good example to test the AdS/CFT correspondence. First we calculate this 1-pt function at the classical level in section 4.1. Next we calculate the same quantity according to the prescription of AdS/CFT in section 4.2. After these, we compare the two results in section 4.3.

4.1 1-pt function from gauge theory

We would like to calculate the 1-point function of this operator. Let us insert this operator at a point $x_3 = \xi$ and consider the expectation value $\langle \mathcal{O}_\Delta(\xi)\rangle$. For calculating the classical expectation value of this operator we substitute the fuzzy funnel solution introduced in section 2.3.2. Since our fuzzy funnel solution preserves SO(3)×SO(3) symmetry which are rotations in

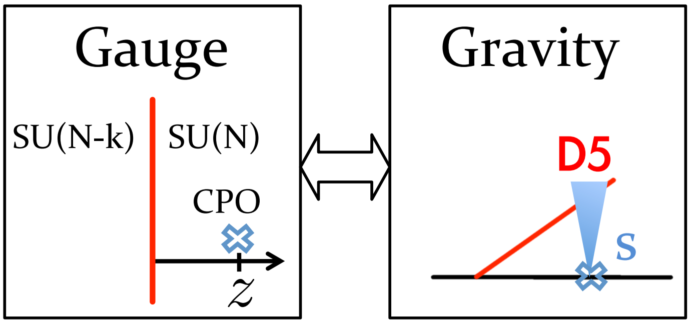

Figure 4.1: Interface and chiral primary operator

$4, 5, 6$ and $7, 8, 9$ spaces, only SO(3)×SO(3) invariant chiral primary operators can have non-vanishing expectation values. As shown in appendix C, Δ must be even and is denoted as $\Delta = 2\ell$. Moreover there is only one such chiral primary operator for each $\Delta = 2\ell,\ \ell = 0, 1, 2, 3, \cdots$.

The traceless symmetric tensors $\mathcal{C}_{i_1 \cdots i_\Delta}$ are related to the spherical harmonics (see appendix C).

$$\mathcal{C}_{i_1 i_2 \cdots i_\Delta} x_{i_1} \cdots x_{i_\Delta} = Y_\ell(\psi), \qquad \sum_{i=4}^{6} x_i^2 = \sin^2\psi, \qquad \sum_{j=7}^{9} x_j^2 = \cos^2\psi. \tag{4.1.1}$$

Spherical harmonics is expressed as eq. (C.2.6)

$$Y_\ell(\psi) = C_\ell F(-\ell, \ell+2, \frac{3}{2}; \cos^2\psi) = C_\ell(1 + \cos^2\psi P(\cos^2\psi)), \tag{4.1.2}$$

where $P(\cos^2\psi)$ is an inhomogeneous polynomial of $\cos^2\psi$. The normalization C_ℓ is determined so that $\mathcal{C}^{i_1 i_2 \cdots i_\Delta}\mathcal{C}^{i_1 i_2 \cdots i_\Delta} = 1$ is satisfied, or equivalently eq. (C.2.7). We can express this spherical harmonics by a homogeneous polynomial of $\sin^2\psi$ and $\cos^2\psi$. This is because if we have a inhomogeneous term, we can replace 1 by some power of $\sin^2\psi + \cos^2\psi$. In particular we can replace the first term 1 in the paren in eq. (4.1.2) by $(\sin^2\psi + \cos^2\psi)^\ell$ and get homogeneous expression

$$Y_\ell = C_\ell(\sin^{2\ell}\psi + \cos^2\psi\, Q(\sin^2\psi, \cos^2\psi)), \tag{4.1.3}$$

where $Q(\sin^2\psi, \cos^2\psi)$ is a homogeneous polynomial of $\sin^2\psi$ and $\cos^2\psi$. Then replacing $\sin^2\psi$ by $\sum_{i=4}^{6}\phi_i^2$ and $\cos^2\psi$ by $\sum_{j=7}^{9}\phi_j^2$, we obtain the relation[1]

$$\mathcal{C}_{i_1 \cdots i_\Delta}\phi_{i_1} \cdots \phi_{i_\Delta} = C_\ell \left\{ \left(\sum_{i=4}^{6}\phi_i^2\right)^\ell + \left(\sum_{j=7}^{9}\phi_j^2\right) Q\left(\sum_{i=4}^{6}\phi_i^2, \sum_{j=7}^{9}\phi_j^2\right) \right\}. \tag{4.1.4}$$

[1]Precisely speaking the right hand side is symmetrized product.

Substituted the solution (2.3.7), all terms except the first one vanish since $\phi_7 = \phi_8 = \phi_9 = 0$. Using the relations (4.1.4) we obtain the following result.

$$\begin{aligned} \langle \mathcal{O}_{2\ell}(\xi) \rangle_{\text{classical}} &= \frac{(8\pi^2)^{\Delta/2}}{\lambda^{\Delta/2}\sqrt{\Delta}} C_\ell \text{tr}\left[\left(\frac{1}{4\xi^2}(k^2-1) \right)^\ell \mathbf{1}_{k\times k} \right] \\ &= C_\ell \frac{(2\pi^2)^\ell}{\sqrt{2\ell}\lambda^\ell}(k^2-1)^\ell k \frac{1}{\xi^{2\ell}}. \end{aligned} \tag{4.1.5}$$

The behavior $1/\xi^{2\ell}$ is determined by the conformal symmetry and does not change by the quantum correction. The non-trivial part is the coefficient, which will change by the quantum correction. We compare this result with the gravity side calculation.

4.2 1-pt function from gravity theory

In this section we calculate the expectation values of the chiral primary operators in the gravity side. The AdS/CFT correspondence is a duality between $\mathcal{N}=4$ super Yang-Mills theory we discussed in the previous section and type IIB superstring theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$. Let us first review background of this theory and later we add a probe D5-brane which corresponds to the interface.

4.2.1 Background

We consider here type IIB superstring theory as the gravity theory. The near horizon limit of the supergravity solution of N coincident D3-branes is $\mathrm{AdS}_5 \times \mathrm{S}^5$. The coordinates of AdS_5 are denoted by $y, x^\mu, \mu = 0,1,2,3$. The metric on this space is given by

$$ds^2_{\mathrm{AdS}_5\times\mathrm{S}^5} = \frac{1}{y^2}(dy^2 + \eta_{\mu\nu}dx^\mu dx^\nu) + ds^2_{\mathrm{S}^5}. \tag{4.2.1}$$

In this thesis we choose the unit in which the radius of AdS_5 is 1. Thus the string coupling constant g_s and the slope parameter α' are related as

$$\lambda := 4\pi g_s N = \alpha'^{-2}. \tag{4.2.2}$$

Furthermore the RR 4-form [24, 26] is also excited

$$G_4 = -\frac{1}{y^4}dx^0 dx^1 dx^2 dx^3 + \cdots . \tag{4.2.3}$$

In addition to the D3-brane configuration discussed earlier, we introduce a D5-brane in order to study the corresponding theory of the interface CFT. The D5-brane action is the usual DBI+WZ action.

$$S = T_5 \int d^6\zeta \sqrt{\det(G+\mathcal{F})} + iT_5 \int \mathcal{F} \wedge G_4, \tag{4.2.4}$$

where $T_5 = (2\pi)^{-5}\alpha'^{-2}g_s^{-1}$ is the tension of the D5-brane, ζ's are the world-volume coordinates, G and $\mathcal{F}$ denote the induced metric and the field strength of the world-volume gauge field respectively.

The $\mathrm{AdS}_4\times\mathrm{S}^2$ solution is obtained by [6]. We use the convention of [3]. AdS_4 part is embedded in AdS_5 and expressed by the equation

$$x_3 = \kappa y \tag{4.2.5}$$

with a constant parameter κ. S^2 is embedded in S^5 as a great sphere. We denote world-volume coordinates of the D5-brane by $(y, x_0, x_1, x_2, \theta, \phi)$; (y, x_0, x_1, x_2) are coordinates of AdS_4 and (θ, ϕ) are ones of S^2. Substituting this solution into (4.2.1), we obtain the induced metric of the D5-brane:

$$ds^2_{\mathrm{D5}} = \frac{1}{y^2}(-dt^2 + dr^2 + r^2 d\psi^2 + (\kappa^2+1)dy^2) + d\theta^2 + \sin^2\theta d\phi^2. \tag{4.2.6}$$

The induced metric and the gauge field are summarized by a matrix $H = G + \mathcal{F}$. H takes the following form in this solution.

$$H = \left(\begin{array}{cccc|cc} (1+\kappa^2)y^{-2} & & & & & \\ & y^{-2} & & & & \\ & & y^{-2} & & & \\ & & & y^{-2} & & \\ \hline & & & & 1 & -\kappa\sin\theta \\ & & & & \kappa\sin\theta & \sin^2\theta \end{array}\right), \tag{4.2.7}$$

where the diagonal components come from G and the off-diagonal components come from $\mathcal{F}$. Actually the parameter κ is related with k as $\kappa = \frac{\pi}{\sqrt{\lambda}}k$ (3.4.9).

4.2.2 Probe D5-brane

How this gravity side is modified when the interface is inserted? The object which corresponds to our interface is a probe D5-brane with k units of magnetic flux [6]. This gravity dual is obtained by the following way. We consider a D5-brane where k D3-branes end. Then $\mathrm{SU}(N)$ gauge theory is realized in the side where there are N D3 branes and $\mathrm{SU}(N-k)$ gauge theory is realized in the other side as low energy effective theories. This D5-brane is pulled by k D3-branes which end on it and becomes funnel-shaped with k units of magnetic flux. If we consider the supergravity solution of D3-branes and take the near horizon limit, we obtain the gravity dual mentioned above.

Here we make a remark on the value k. Although we take k large, it is still much smaller than N in order not to modify the supergravity background.

Now let us turn to the calculation of the one point function. The scalar fields which correspond to the chiral primary operators are identified in [27, 25]. These scalar fields come from the fluctuation of the metric and the RR 4-form as

$$h^{\text{AdS}}_{\mu\nu} = -\frac{2\Delta(\Delta-1)}{\Delta+1} s g_{\mu\nu} + \frac{4}{\Delta+1}\nabla_\mu\nabla_\nu s, \tag{4.2.8}$$

$$h^{\text{S}}_{\alpha\beta} = 2\Delta s g_{\alpha\beta}, \tag{4.2.9}$$

$$a^{\text{AdS}}_{\mu\nu\rho\sigma} = 4i\sqrt{g^{\text{AdS}}}\epsilon_{\mu\nu\rho\sigma\eta}\nabla^\eta s, \tag{4.2.10}$$

where $h^{\text{AdS}}_{\mu\nu}$, $h^{\text{S}}_{\alpha\beta}$ and $a^{\text{AdS}}_{\mu\nu\rho\sigma}$ are the fluctuation of AdS_5 part of the metric, S^5 part of the metric and AdS_5 part of the RR 4-form, respectively. $\Delta = 2\ell$ corresponds to the conformal dimension of the operator in the gauge theory.

The classical solution of s with the boundary condition can be written as

$$\begin{aligned} s(y, x, \theta, \phi, \psi, \cdots) &= \int d^4x' c_\Delta \frac{y^\Delta}{K(y, x, x')^\Delta} s_0(x') Y_{\Delta/2}(\psi), \\ K(y, x, x') &:= |x - x'|^2 + y^2, \\ c_\Delta &= \frac{\Delta+1}{2^{2-\Delta/2}N\sqrt{\Delta}}. \end{aligned} \tag{4.2.11}$$

where $Y_{\Delta/2}$ is the spherical harmonics obtained in appendix C. The normalization factor c_Δ is the correct one obtained in [28, 25]. It is determined so that the coefficient of the two point function is unity.

The first order fluctuation of the action is

$$S^{(1)} = \frac{T_5}{2}\int d^6\zeta\sqrt{\det H}(H^{-1}_{\text{sym}})^{ab}\partial_a X^M \partial_b X^N h_{MN} + iT_5\int \mathcal{F}\wedge a_4, \tag{4.2.12}$$

where $h_{\mu\nu}$ and a_4 are the fluctuation of the metric and the RR 4-form given in eqs. (4.2.8)-(4.2.10). H^{-1}_{sym} denotes the symmetric part of the inverse matrix of H.

The one-point function can be calculated by using eq. (3.3.2). The classical action S_{cl} in eq. (3.3.2) can be replaced by $S^{(1)}$ in eq. (4.2.12)

$$\langle \mathcal{O}(x)\rangle = -\frac{\delta S^{(1)}(s_0)}{\delta s_0(x)}. \tag{4.2.13}$$

The detailed derivation of the fluctuation $S^{(1)}$ is shown in section 4.2.3 and section 4.2.4. The final result of gravity side is given by eq. (4.2.36)

$$-\frac{\delta S_{\text{cl}}}{\delta s_0(\xi)} = C_\ell \frac{\sqrt{\lambda}2^\ell\Gamma(2\ell+1/2)}{\pi^{3/2}\sqrt{2\ell}\Gamma(2\ell)}\frac{1}{\xi^{2\ell}}\int_0^\infty du \frac{u^{2\ell-2}}{\left[(1-\kappa u)^2 + u^2\right]^{2\ell+1/2}}. \tag{4.2.14}$$

Here ξ is the distance between the interface and the point where the chiral primary operator is inserted.

In eq. (4.2.14), the dependence of ξ is $1/\xi^{2\ell}$ and this is determined by the conformal symmetry. We will compare the coefficient with the gauge theory side in the next section.

4.2.3 Fluctuations h and a

In this subsection we show the detailed calculations of fluctuations h and a defined by the scalar field $s(x)$ as (4.2.8), (4.2.9) and (4.2.10). Actually it is enough to calculate them when s_0 is a delta function as

$$s_0(x) = \delta^4(x - x'). \tag{4.2.15}$$

In this case the classical solution (4.2.11) becomes

$$s(y, x, \theta, \phi, \psi) = c_\Delta \frac{y^\Delta}{K(y, x, x)^\Delta} Y_{\Delta/2}(\psi). \tag{4.2.16}$$

We use the convention for the covariant derivative and totally anti-symmetric tensor

$$\nabla_i T_{j_1 \cdots j_n} := \partial_i T_{j_1 \cdots j_n} - \sum_{l=1}^{n} \Gamma^k_{ij_l} T_{j_1 \cdots j_{l-1} k j_{l+1} \cdots j_n}, \tag{4.2.17}$$

$$\epsilon_{y0123} = 1, \tag{4.2.18}$$

where Christoffel symbols are $\Gamma^i_{jk} := \frac{1}{2} g^{il}(\partial_j g_{lk} + \partial_k g_{lj} - \partial_l g_{jk})$.

The first derivatives and the second derivatives of s are

$$\frac{\partial_y s}{s} = \Delta(\frac{1}{y} - \frac{2y}{K}), \tag{4.2.19}$$

$$\frac{\partial_i s}{s} = -\Delta \frac{2(x - x')_i}{K}, \tag{4.2.20}$$

$$\frac{\nabla_y \nabla_y s}{s} = \frac{\Delta^2}{y^2} + 4\Delta(\Delta + 1)\left(-\frac{1}{K} + \frac{y^2}{K^2}\right), \tag{4.2.21}$$

$$\frac{\nabla_y \nabla_i s}{s} = \Delta(\Delta + 1)\left(+4y\frac{(x - x')_i}{K^2} - 2\frac{(x - x')_i}{yK}\right), \tag{4.2.22}$$

$$\frac{\nabla_i \nabla_j s}{s} = -\Delta\frac{\delta_{ij}}{y^2} + 4\Delta(\Delta + 1)\frac{(x - x')_i (x - x')_j}{K^2}. \tag{4.2.23}$$

Using these results and the definition of h in AdS the expression of fluctuations are

$$\frac{h^{\mathrm{AdS}}_{yy}}{\Delta s} = \frac{2}{y^2} - \frac{16}{K} + \frac{16}{K^2}, \tag{4.2.24}$$

$$\frac{h^{\mathrm{AdS}}_{yi}}{\Delta s} = 16y\frac{(x - x')_i}{K} - 8\frac{(x - x')_i}{yK}, \tag{4.2.25}$$

$$\frac{h^{\mathrm{AdS}}_{ij}}{\Delta s} = -2\frac{\delta_{ij}}{y^2} + \frac{16(x - x')_i (x - x')_j}{K^2}, \tag{4.2.26}$$

and in 2-sphere

$$\frac{h^{\mathrm{S}}_{\theta\theta}}{\Delta s} = 2, \quad \frac{h^{\mathrm{S}}_{\phi\phi}}{\Delta s} = 2\sin^2\theta. \tag{4.2.27}$$

4.2.4 D5-brane action

When we give fluctuation to the metric and the RR 4-form, the D5-brane action is deformed as follows in the first order. We use the notation $v_i = x_i - x'_i$ and p, q run $0, 1, 2$. The first order fluctuation is calculated as follows.

$$\begin{aligned} S^{(1)} &= \frac{T_5}{2}\int d^6\zeta\sqrt{\det H}(H^{-1}_{\mathrm{sym}})^{ab}\partial_a X^M \partial_b X^N h_{MN} + iT_5\int \mathcal{F}\wedge a_4 \\ &= T_5\int d^6\zeta(\mathcal{L}^{(1)}_{\mathrm{DBI}} + \mathcal{L}^{(1)}_{\mathrm{WZ}}). \end{aligned} \tag{4.2.28}$$

In this equation we need the explicit form of the symmetric part of H^{-1}.

$$H^{-1}_{\mathrm{sym}} = \begin{pmatrix} (1+\kappa^2)^{-1}y^2 & & & & & \\ & y^2 & & & & \\ & & y^2 & & & \\ & & & y^2 & & \\ & & & & (1+\kappa^2)^{-1} & \\ & & & & & [\sin^2\theta(1+\kappa^2)]^{-1} \end{pmatrix}. \tag{4.2.29}$$

Eq. (4.2.28) is calculated as follows.

$$\begin{aligned} \mathcal{L}^{(1)}_{\mathrm{DBI}} :=& \frac{1}{2}\sqrt{\det H}(H^{-1}_{\mathrm{sym}})^{ab}\partial_a X^M \partial_b X^N h_{MN} \\ =& \frac{(1+\kappa^2)\sin^2\theta}{2y^4}\{H^{yy}\partial_y X^M\partial_y X^N h^{\mathrm{AdS}}_{MN} + H^{ij}\partial_i X^M \partial_j X^N h^{\mathrm{AdS}}_{MN} \\ & + H^{\theta\theta}\partial_\theta X^M \partial_\theta X^N h^{\mathrm{S}}_{MN} + H^{\phi\phi}\partial_\phi X^M\partial_\phi X^N h^{\mathrm{S}}_{MN}\} \\ =& \frac{\Delta s\sin\theta}{y^4K^2}\{-8y^2v_3^2 + \kappa(16y^3v_3 - 8yv_3K) + \kappa^2(8y^2(v_pv_p + v_3^2) - 4K^2)\}. \end{aligned} \tag{4.2.30}$$

$$\begin{aligned} \mathcal{L}^{(1)}_{\mathrm{WZ}} :=& i\mathcal{F}_{\theta\phi}\frac{1}{4!}\epsilon^{abcd}(Pa)_{abcd} \\ =& i2\kappa\sin\theta(a_{y012} + \kappa a_{3012}) \\ =& i2\kappa\sin\theta\left\{\Delta 4s\frac{1}{y^3}\frac{2v_3}{K} + \kappa\Delta 4s\frac{1}{y^3}\left(\frac{1}{y} - \frac{2y}{K}\right)\right\}. \end{aligned} \tag{4.2.31}$$

$S^{(1)}$ is the sum of these two terms

$$\begin{aligned} S^{(1)} &= T_5\int d^6\zeta\left(\mathcal{L}^{(1)}_{\mathrm{DBI}} + \mathcal{L}^{(1)}_{\mathrm{WZ}}\right) \\ &= -8T_5\int d^6\zeta\frac{\sin\theta\cdot\Delta s}{y^2K^2}(v_3 - \kappa y)^2 \\ &= -8T_5\int d^6\zeta\frac{\sin\theta\cdot\Delta s}{y^2K^2}x'^2_3. \end{aligned} \tag{4.2.32}$$

This formula with the classical solution (4.2.15) $s_0(x) = \delta^4(x-x')$ is the functional derivative $\delta S^{(1)}/\delta s_0(x')$. This functional derivative evaluated at $x'_3 = \xi$ is the quantity we want. Notice that the D5-brane sits at $\psi = \pi/2$, thus the spherical harmonics should be evaluated at this surface. This value is given by (see eq. (C.2.6))

$$Y_\ell(\psi = \pi/2) = C_\ell. \tag{4.2.33}$$

Putting all these things together, we obtain

$$\begin{aligned}-\frac{\delta S^{(1)}}{\delta s_0(\xi)} =& 32T_5\pi\Delta c_\Delta C_\ell \int_0^\infty dy \int dx^0 dx^1 dx^2 \frac{y^{\Delta-2}\xi^2}{((\kappa y-\xi)^2 + x^p x^p + y^2)^{\Delta+2}} \\ =& 32T_5\pi^{5/2}\Delta c_\Delta C_\ell \frac{\Gamma(\Delta+1/2)}{\Gamma(\Delta+2)}\xi^2 \int_0^\infty dy \frac{y^{\Delta-2}}{((\kappa y-\xi)^2 + y^2)^{\Delta+1/2}}.\end{aligned} \tag{4.2.34}$$

In the above calculation we used the formula.

$$\int d^D x \frac{1}{(x^2+A)^\alpha} = \frac{\Gamma(-D/2+\alpha)}{\Gamma(\alpha)} \frac{\pi^{D/2}}{A^{-D/2+\alpha}}. \tag{4.2.35}$$

In our unit (4.2.2) the D5-brane tension is written as $T_5 = \frac{2N\sqrt{\lambda}}{(2\pi)^4}$. Finally by substituting T_5, c_Δ and $\Delta = 2\ell$ to eq. (4.2.34), and the change of valuable as $y = \xi u$, we obtain

$$-\frac{\delta S_{\mathrm{cl}}}{\delta s_0(\xi)} = C_\ell \frac{\sqrt{\lambda}2^\ell \Gamma(2\ell+1/2)}{\pi^{3/2}\sqrt{2\ell}\Gamma(2\ell)} \frac{1}{\xi^{2\ell}} \int_0^\infty du \frac{u^{2\ell-2}}{\left[(1-\kappa u)^2 + u^2\right]^{2\ell+1/2}}. \tag{4.2.36}$$

4.3 Comparison of CPOs from both theories

In the previous sections, 4.1 and 4.2, we calculated the one-point function in the gauge theory side and the gravity side. Our goal is to confirm the correspondence between the gauge theory and the gravity theory. Let us compare these results in this section. We consider the limits $k \gg 1$ and $\lambda/k^2 \ll 1$, and compare the leading terms.

Gauge theory

Since we consider the limit $k \gg 1$ the gauge theory result (4.1.5) becomes

$$\begin{aligned}\langle \mathcal{O}_{2\ell} \rangle_{\mathrm{classical}} =& C_\ell \frac{(2\pi^2)^\ell}{\sqrt{2\ell}\lambda^\ell}(k^2-1)^\ell k \frac{1}{\xi^{2\ell}} \\ \approx& C_\ell \frac{(2\pi^2)^\ell}{\sqrt{2\ell}\lambda^\ell} k^{2\ell+1} \frac{1}{\xi^{2\ell}}.\end{aligned} \tag{4.3.1}$$

This result is compared with the gravity side.

Gravity theory

We consider the behavior of the gravity side result in the limit $\epsilon := \frac{1}{\kappa^2+1} \to 0$, $\kappa = \frac{\pi}{\sqrt{\lambda}} k \gg 1$. The following expression of the Dirac delta function is convenient[2].

$$\delta(x) = \lim_{\epsilon \to 0} \frac{1}{\sqrt{\pi}} \frac{\Gamma(n)}{\Gamma(n-1/2)} \frac{\epsilon^{2n-1}}{(x^2+\epsilon^2)^n}. \tag{4.3.2}$$

Using this formula the integrand of the equation (4.2.14) can be approximated by the Dirac delta function.

$$\frac{1}{\left((1-\kappa u)^2+u^2\right)^{2\ell+1/2}} \longrightarrow \frac{1}{\epsilon^{4\ell}} \frac{\Gamma(2\ell)\Gamma(\frac{1}{2})}{\Gamma(2\ell+\frac{1}{2})} \delta(u-\kappa\epsilon). \tag{4.3.3}$$

After integration we obtain the result

$$-\frac{\delta S^{(1)}}{\delta s_0(\xi)} = C_\ell \frac{(2\pi^2)^\ell}{\lambda^\ell \sqrt{2\ell}} k^{2\ell+1} \frac{1}{\xi^{2\ell}}. \tag{4.3.4}$$

Comparing (4.3.1) and (4.3.4), we can conclude that these two quantities completely agree in the leading order of λ/k^2 series.

We can go to next-to-leading order in the gravity side. Actually the integral in eq. (4.2.14) can be rewritten as

$$\begin{aligned} I :&= \int_0^\infty du \frac{u^{2\ell-2}}{[(1-\kappa u)^2+u^2]^{2\ell+1/2}} \\ &= \kappa^{2\ell+1}\left(1+\frac{1}{\kappa^2}\right)^{3/2} \int_{-\arctan\kappa}^{\pi/2} d\theta (\cos\theta)^{4\ell-1}\left(1+\frac{1}{\kappa}\tan\theta\right)^{2\ell-2}, \end{aligned} \tag{4.3.5}$$

by the change of variable as $\tan\theta = (1+\kappa^2)u - \kappa$. This function can be expanded around $\kappa \to \infty$ as [3]

$$I = \kappa^{2\ell+1} \frac{\Gamma(2\ell)\Gamma(1/2)}{\Gamma(2\ell+1/2)} \left(1+\frac{1}{\kappa^2} I_1 + O(\frac{1}{\kappa^4})\right), \tag{4.3.6}$$

$$I_1 = \frac{3}{2} + \frac{(2\ell-2)(2\ell-3)}{4(2\ell-1)}. \tag{4.3.7}$$

Using this I_1 the gravity result up to next-to-leading order is

$$-\frac{\delta S^{(1)}}{\delta s_0(x)} = C_\ell \frac{(2\pi^2)^\ell}{\lambda^\ell \sqrt{2\ell}} k^{2\ell+1} \frac{1}{\xi^{2\ell}} \left(1+\frac{\lambda}{\pi^2 k^2} I_1 + \cdots\right). \tag{4.3.8}$$

These corrections are formally a positive power series of λ/k^2. The expansion eq. (4.3.8) indicates the reason why we can compare the gravity side and the gauge theory side. In the gravity side λ/k^2 can be small even though λ is large because k^2 can be larger. Thus one can suppress the sub-leading terms by sending $\lambda/k^2 \to 0$ which has superficially the same effects as $\lambda \to 0$. A heuristic arguments of λ/k^2 scaling in the gauge theory side is given in the discussion section 8.

An interesting future work is to compare the prediction of the 1-loop correction in eq. (4.3.8) from the gravity side to the 1-loop calculation in the gauge theory side.

[2]The case $n = 1$ is well known.

[3]This expansion is correct for $\ell \geq 2$

Chapter 5

Correspondence II: Test particle

We consider two $\mathcal{N} = 4$ supersymmetric gauge theories connected by an interface and the gravity dual of this system. This interface is expressed by a fuzzy funnel solution of Nahm's equation in the gauge theory side. The gravity dual is a probe D5-brane in $\mathrm{AdS}_5 \times S^5$. The potential energy between this interface and a test particle is calculated in both the gauge theory side and the gravity side by the expectation value of a Wilson loop. In the gauge theory it is evaluated by just substituting the classical solution to the Wilson loop. On the other hand it is done by the on-shell action of the fundamental string stretched between the AdS boundary and the D5-brane in the gravity. We show the gauge theory result and the gravity one agree with each other.

5.1 Test particle potential from gauge theory

In this section we would like to discuss the potential energy between our interface and a test particle. In order to calculate this potential energy, we adopt the idea of Wilson loop operators W_C, eq. (2.3.2), inserted at the distance z from the interface. We take a loop to be parallel to the time axis, the parameter to be $s = t$ and the unit six-vector as $\theta^4 = -1, \theta^i = 0, (i \neq 4)$. It is known that the expectation value of the Wilson loop operator [17] is related to the potential energy as

$$\langle W(z) \rangle \cong \exp(-TV(z)). \tag{5.1.1}$$

T denotes the time interval which is taken to be infinity.

Here we introduce the Wilson loop operator and evaluate its expectation value classically. Let us consider the Wilson loop in Euclidean space.

$$W(z) = \mathrm{tr} P \exp \int_{x_3=z} dt (iA_0 - \phi_4), \tag{5.1.2}$$

where "tr" is the trace in the fundamental representation and "P" means a path-ordered product. The expectation value of this operator is evaluated classically by substituting the

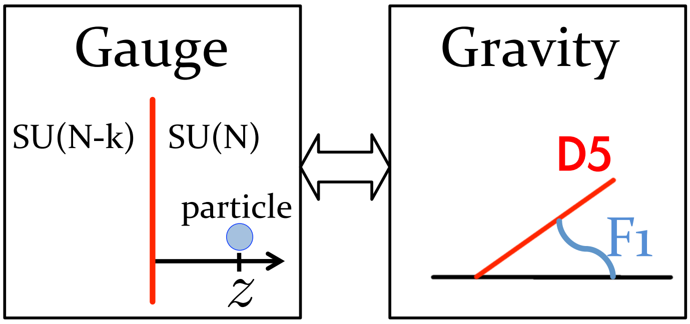

Figure 5.1: Interface and test particle

classical solution (2.3.7) to eq. (5.1.2).

$$\begin{aligned}
\langle W(z) \rangle &= \mathrm{tr} P \exp \int dt \left(\frac{1}{z} t_4 \right) \\
&= \sum_{\ell\text{: eigen values of } t_4} \exp \left(T \frac{1}{z} \ell \right) \\
&\cong \exp \left(T \frac{1}{z} \ell_{\max} \right) \qquad (T \to \infty) \\
&= \exp \left(T \frac{k-1}{2z} \right) .
\end{aligned} \tag{5.1.3}$$

In the last line we used the expression of the maximal eigen value of t_4 [1], $\ell_{\max} = \frac{k-1}{2}$. By using the relation (5.1.1) the potential energy in this configuration is

$$V(z) = -\frac{k-1}{2z}. \tag{5.1.4}$$

We will compare this result with the gravity dual calculation in the next section.

5.2 Test particle potential from gravity theory

In this section we try to calculate the potential energy between the interface and a test particle distance z away from the interface.

[1]This is the highest weight of the representation.

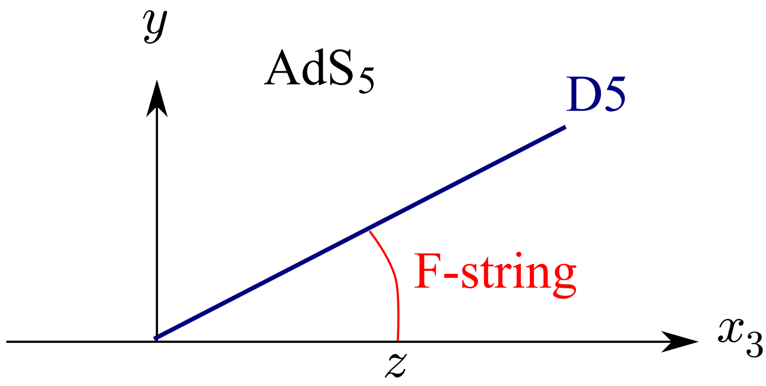

Figure 5.2: The probe D5-brane and the fundamental string in the AdS$_5$ expressed by the solution (3.4.8).

5.2.1 String and potential

Now let us focus a string ending on the probe D5-brane from the infinite distance corresponds to the Wilson loop (5.1.2) in the gauge theory with interface. Therefore we can identify the interface-particle potential from the on-shell string action.

In the conformal gauge, the Polyakov action and the Virasoro constraints are

$$S = \frac{1}{4\pi\alpha'}\int d\tau d\sigma (\dot{X}^M \dot{X}_M + X'^M X'_M), \tag{5.2.1}$$

$$\dot{X}^M \dot{X}_M - X'^M X'_M = 0, \qquad \dot{X}^M X'_M = 0, \tag{5.2.2}$$

where τ, σ are string world sheet coordinates and differentials with respect to them are denoted by " $\cdot$ " and " $'$ " respectively. We assume the region of σ as $0 \leq \sigma \leq \sigma_1$. The string ends on the AdS boundary at $\sigma = 0$ and is attached to the D5-brane at $\sigma = \sigma_1$. We can set following ansatz for the string to be static:

$$t = t(\tau), \quad y = y(\sigma), \quad x_3 = x_3(\sigma). \tag{5.2.3}$$

Then the action and the constraints are translated into

$$S = \frac{1}{4\pi\alpha'}\int d\tau d\sigma \frac{1}{y^2}(\dot{t}^2 + y'^2 + x_3'^2), \tag{5.2.4}$$

$$\dot{t}^2 = y'^2 + x_3'^2. \tag{5.2.5}$$

The equations of motion are given by

$$\ddot{t} = 0, \tag{5.2.6}$$

$$\left(\frac{x_3'}{y^2}\right)' = 0, \tag{5.2.7}$$

$$-\frac{2}{y^3}(\dot{t}^2 + y'^2 + x_3'^2) - \left(\frac{2y'}{y^2}\right)' = 0. \tag{5.2.8}$$

Note that we impose boundary conditions

$$\kappa x_3'(\sigma_1) + y'(\sigma_1) = 0, \tag{5.2.9}$$

$$-x_3(\sigma_1) + \kappa y(\sigma_1) = 0. \tag{5.2.10}$$

These boundary conditions denote the string is attached to the probe D5-brane.

In particular the first line is Neumann boundary condition along the probe D5-brane and the second line is Dirichlet boundary condition transverse to the probe D5-brane.

Next we solve the equations of motion with above boundary conditions under the gauge $t = \tau$. Eq. (5.2.7) gives

$$\frac{x_3'}{y^2} = -c, \qquad (c: \text{ constant}). \tag{5.2.11}$$

And the Virasoro constraint becomes

$$-1 + y'^2 + c^2 y^4 = 0, \tag{5.2.12}$$

which takes the form

$$y' = \sqrt{1 - c^2 y^4}. \tag{5.2.13}$$

Taking the boundary condition $x_3(0) = z$ into account, eq. (5.2.11) is solved as

$$\begin{aligned} \int_z^{x_3} dx_3 &= -c \int_0^{\sigma} d\sigma y^2 = -c \int_0^y dy \frac{y^2}{y'} = -c \int_0^y dy \frac{y^2}{\sqrt{1 - c^2 y^4}} \\ x_3 - z &= -\frac{1}{\sqrt{c}} (E(\varphi, i) - F(\varphi, i)), \end{aligned} \tag{5.2.14}$$

where we have introduced the elliptic integrals $E(\varphi, i)$ and $F(\varphi, i)$ for convenience (see appendix B for detail). The boundary condition (5.2.9) indicate $y_1 = y(\sigma_1)$ by using (5.2.11) and (5.2.13),

$$\sqrt{c} y_1 = (1 + \kappa^2)^{-1/4}. \tag{5.2.15}$$

On the other hand, we can solve the boundary condition (5.2.10) and determine the constant c

$$\sqrt{c} = \frac{1}{z} [E(\varphi_1, i) - F(\varphi_1, i) + \kappa (1 + \kappa^2)^{-1/4}], \qquad \left(\sin \varphi_1 := \sqrt{c} y_1\right). \tag{5.2.16}$$

With the use of the formula

$$\begin{aligned} &\frac{1}{u^2 \sqrt{(1 - u^2)(1 - h^2 u^2)}} \\ &= \frac{d}{du} \left[-\frac{1}{u} \sqrt{(1 - u^2)(1 - h^2 u^2)} \right] - \sqrt{\frac{1 - h^2 u^2}{1 - u^2}} + \frac{1}{\sqrt{(1 - u^2)(1 - h^2 u^2)}}, \end{aligned} \tag{5.2.17}$$

we can rewrite the action

$$
\begin{aligned}
S &= \frac{1}{4\pi\alpha'}\int d\tau d\sigma \frac{1}{y^2}\left(\dot{t}^2 + y'^2 + x_3'^2\right) \\
&= \frac{T}{4\pi\alpha'}\int_{\epsilon}^{\sigma_1} d\sigma \frac{2}{y^2} \\
&= \frac{T}{2\pi\alpha'}\int_{\epsilon}^{y_1} dy \frac{1}{y^2\sqrt{1-c^2y^4}} \\
&= \frac{T}{2\pi\alpha'}\sqrt{c}\left(\frac{1}{\sqrt{c}\epsilon} + O(\epsilon) - \frac{\sqrt{1-c^2y_1^4}}{\sqrt{c}y_1} - E(\varphi_1, i) + F(\varphi_1, i)\right), \qquad (5.2.18)
\end{aligned}
$$

where we chose the integral region $\epsilon \to y_1$, due to decompose the divergence originate with the string self-energy. The potential piece, to compare with the gauge theory, is extracted by removing the divergence from (5.2.18) as in [18, 19, 29]. The potential is read off from (5.2.18) as

$$
\begin{aligned}
V(z) =& \frac{1}{2\pi\alpha'}\sqrt{c}\left(-\frac{\sqrt{1-c^2y_1^4}}{\sqrt{c}y_1} - E(\varphi_1, i) + F(\varphi_1, i)\right) \\
=& -\frac{1}{2\pi\alpha' z}\left(\frac{\kappa}{(1+\kappa^2)^{1/4}} + E(\varphi_1, i) - F(\varphi_1, i)\right)^2, \qquad (5.2.19)
\end{aligned}
$$

where φ_1 is defined as $\sin\varphi_1 = (1+\kappa^2)^{-1/4}$.

5.3 Comparison of the particle-interface potential

We discuss in this section the behavior of the potential in large $\kappa = \pi k/\sqrt{\lambda}$ limit. Since we assume $N \gg \kappa$, large κ limit does not affect to the gravity background. Then the potential (5.2.19) is expanded as

$$
V = -\frac{k}{2z}\left(1 + \frac{1}{6\pi^2}\frac{\lambda}{k^2} + O(\frac{\lambda^2}{k^4})\right). \qquad (5.3.1)
$$

Even if λ is large, λ/k^2 can be small. Thus this expansion is formally positive power series of λ and could be compared with the gauge theory side. At the leading contribution, we confirmed the AdS/CFT correspondence of the interface-particle potential (5.1.4) in the gauge theory picture. The next to leading term is the prediction for the λ correction in the gauge theory side.

5.4 Generalization

In this section we consider a kind of generalization for the test particle while the interface is not changed. We compute the potential energy between the interface and this generalized test particle both in the gauge theory side and the gravity side. Those two results agree to each other in the leading order.

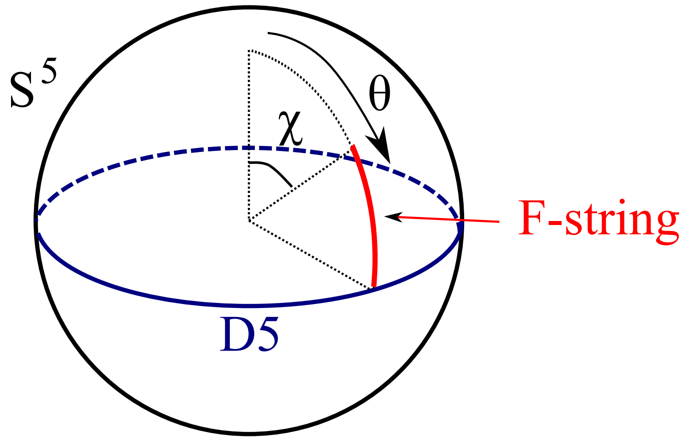

Figure 5.3: The D5-brane and F-string configuration on the S^5.

5.4.1 Gauge theory side

We consider a test particle parameterized by χ, $0 \le \chi \le \pi/2$ expressed by the Wilson loop

$$W(z,\chi) = \mathrm{tr} P \exp \int dt (iA_0 - \sin\chi\phi_4 - \cos\chi\phi_7). \tag{5.4.1}$$

When $\chi = \pi/2$ this particle is the same as the previous one, while this is mutually supersymmetric to the interface when $\chi = 0$.

The potential energy between the interface and this generalized test particle is evaluated by substituting the solution (2.3.7)[2] to the Wilson loop (5.4.1) as the same way as before. The result turns out to be

$$V(z) = -\frac{k-1}{2z}\sin\chi. \tag{5.4.2}$$

This is the same as eq. (5.1.4) when $\chi = \pi/2$. On the other hand the potential energy (5.4.2) vanishes when $\chi = 0$ as expected since the interface and the test particle are mutually supersymmetric in this case.

5.4.2 Gravity side: Result

Here in this section, we calculate the potential between the interface and the generalized test particle in the gravity side as the same way as section 5.2.1. The only difference is the boundary condition at $y = 0$. Let θ be the angle from the North pole of S^5 as shown in figure 5.3. We impose the boundary condition $\theta = \chi$ at $y = 0$ and $\theta = \pi/2$ at the other end of the string.

First we show the result. See next subsection 5.4.3 for the detail of the calculation.

[2] $\phi_7 = 0$ in this solution

Eq. (5.4.20), eq. (5.4.22) and eq. (5.4.24) give three equations for three unknowns y_1, m, c.

$$\frac{\pi}{2} - \chi = \frac{m}{\sqrt{A}} F(\varphi_1, h), \tag{5.4.3}$$

$$1 - m^2 y_1^2 - c^2(1+\kappa^2) y_1^4 = 0, \tag{5.4.4}$$

$$\kappa y_1 = z + \frac{c}{\sqrt{A}B}(E(\varphi_1, h) - F(\varphi_1, h)), \tag{5.4.5}$$

where we use the notation for short hand:

$$\begin{aligned} A &:= \frac{1}{2}(m^2 + \sqrt{m^4 + 4c^2}), \quad B := \frac{1}{2}(m^2 - \sqrt{m^4 + 4c^2}), \\ h^2 &:= \frac{B}{A}, \quad \sin\varphi_1 := \frac{y_1}{\sqrt{A}}. \end{aligned} \tag{5.4.6}$$

The potential is written as

$$V(z) = \frac{1}{2\pi\alpha'}\sqrt{A}\left[-\frac{\cos\varphi_1}{\sin\varphi_1}\sqrt{(1 - \frac{B}{A}\sin^2\varphi_1)} - E(\varphi_1, h) + F(\varphi_1, h)\right], \tag{5.4.7}$$

As in the previous case we can estimate this potential in the limit $\kappa \to \infty$ as

$$V(z) = -\frac{k\sin\chi}{2z}\left[1 + \frac{\sin\chi}{4\kappa^2\cos^3\chi}\left(\frac{\pi}{2} - \chi - \frac{1}{2}\sin 2\chi\right) + O(\kappa^{-4})\right]. \tag{5.4.8}$$

The leading term in this expansion agrees with the gauge theory side (5.4.2) (if k is large) and the second term gives the prediction for $\kappa^{-2} = \frac{\lambda}{\pi^2 k^2}$ correction.

5.4.3 Gravity side: Calculation

Here we show the detailed calculation of the potential discussed above (5.4.8).

Let us put the ansatz:

$$t = t(\tau), \quad y = y(\sigma), \quad x_3 = x_3(\sigma), \quad \theta = \theta(\sigma). \tag{5.4.9}$$

Then the action becomes

$$S = \frac{1}{4\pi\alpha'}\int d\tau d\sigma \left[\frac{1}{y^2}(\dot{t}^2 + y'^2 + x_3'^2) + \theta'^2\right]. \tag{5.4.10}$$

$t = \tau$ is a solution of the equation of motion for t. The equation of motion for θ is simply $\theta'' = 0$. This can be integrated as

$$\theta' = m = (\text{constant}). \tag{5.4.11}$$

x_3 is solved just the same way as (5.2.11) and thus the Virasoro constraint becomes

$$\frac{1}{y^2}(-1 + y'^2 + c^2 y^4) + m^2 = 0, \tag{5.4.12}$$

and the expression of y' as

$$y' = \sqrt{1 - m^2 y^2 - c^2 y^4}. \tag{5.4.13}$$

Integration of this equation gives the relation between σ_1(upper bound for σ) and $y_1 := y(\sigma_1)$ as

$$\int_0^{y_1} dy \frac{1}{\sqrt{1 - m^2 y^2 - c^2 y^4}} = \sigma_1. \tag{5.4.14}$$

It is convenient to introduce the number A, B:

$$A = \frac{1}{2}(m^2 + \sqrt{m^4 + 4c^2}), \tag{5.4.15}$$

$$B = \frac{1}{2}(m^2 - \sqrt{m^4 + 4c^2}), \tag{5.4.16}$$

since we can rewrite the inside the square root in eq. (5.4.13) as

$$1 - m^2 y^2 - c^2 y^4 = (1 - Ay^2)(1 - By^2). \tag{5.4.17}$$

Notice that $B < 0 < A$ is satisfied. Eq. (5.2.11) can be integrated and gives the value of x_3 at $\sigma = \sigma_1$.

$$x_3(\sigma_1) = z + cA^{-1/2}\frac{1}{B}(E(\varphi_1, h) - F(\varphi_1, h)), \tag{5.4.18}$$

where $\sin\varphi_1 = \sqrt{A}y_1$. θ is also solved as

$$\theta = m\sigma + \chi. \tag{5.4.19}$$

Since $\theta(0) = \chi$ and $\theta(\sigma_1) = \frac{\pi}{2}$ we obtain

$$\frac{\pi}{2} - \chi = m\sigma_1. \tag{5.4.20}$$

At $\sigma = \sigma_1$ we should impose the boundary conditions. One of them is

$$\kappa x_3'(\sigma_1) + y'(\sigma_1) = 0. \tag{5.4.21}$$

This equation can be rewritten as

$$1 - m^2 y_1^2 - c^2(1 + \kappa^2)y_1^4 = 0, \tag{5.4.22}$$

where we use eq. (5.4.13) and eq. (5.2.11). The other boundary condition at $\sigma = \sigma_1$:

$$-x_3(\sigma_1) + \kappa y_1 = 0. \tag{5.4.23}$$

Substituting $x_3(\sigma_1)$ by (5.4.18) we obtain

$$\kappa y_1 = z + \frac{c}{\sqrt{A}B}(E(\varphi_1, h) - F(\varphi_1, h)). \tag{5.4.24}$$

The action becomes

$$S = \frac{T}{2\pi\alpha'}\sqrt{A}\left[\frac{1}{\sqrt{A}\epsilon} + O(\epsilon) - \frac{\cos\varphi_1}{\sin\varphi_1}\sqrt{(1-\frac{B}{A}\sin^2\varphi_1)} - E(\varphi_1,h) + F(\varphi_1,h)\right]. \tag{5.4.25}$$

Thus the regularized action S_{reg} is obtained by subtracting the divergent part. We can then read off the potential from S_{reg} as

$$V(z) = \frac{1}{2\pi\alpha'}\sqrt{A}\left[-\frac{\cos\varphi_1}{\sin\varphi_1}\sqrt{(1-\frac{B}{A}\sin^2\varphi_1)} - E(\varphi_1,h) + F(\varphi_1,h)\right]. \tag{5.4.26}$$

Chapter 6

Bubbing D5-branes

In section 2.3.2 we have studied a 3-dimensional non-local operator, interface. Its counterpart in the string theory side is a probe D5-brane.

6.1 Configuration of the D3-D5-D1 system

The AdS/CFT correspondence with a probe D5-brane has been studied in [6]. Let us first briefly review this correspondence. This system consists of N D3-branes and a D5-brane. The D3-branes extend along the directions 0123 in 10-dimensional spacetime and the D5-brane extends 012456 (see table 6.1). The D5-brane does not extend in the direction 3, so D3-branes can end on the D5-brane in this direction. Let k D3-branes out of N end on this D5-branes, and suppose $k \ll N$. This system can be seen from two different points of view: the gravity side and the gauge theory side. These two theories are conjectured to be equivalent.

In the gravity side, these multiple D3-branes warp the spacetime and give rise to $AdS_5 \times S^5$ spacetime in the near horizon limit. Meanwhile, the backreaction of the D5-brane is negligible, and therefore the D5-brane is treated as a probe brane. Consequently, this system describes the superstring theory with the probe D5-brane in the $AdS_5 \times S^5$.

In the gauge theory side, the D5-brane is regarded as a wall between gauge theories with different gauge groups $SU(N)$ and $SU(N-k)$ where N is the total number of the D3-branes and k is the number of D3-branes which end on the D5-brane. This wall gives the boundary condition of each gauge theory and is called "an interface."

In this thesis we would like to insert a 't Hooft operator on the interface in the gauge theory. This corresponds to adding D1-branes ending on the D3-branes in string theory. The total system is then made of N D3-branes, a D5-brane and D1-branes as shown in table 6.1.

Similar to the previous case, the D3-branes forming the space-time give $AdS_5 \times S^5$ geometry, while the D5-brane and the D1-branes are treated as probes. The D1-branes are embedded as a worldvolume flux in the D5-brane and there is a symmetry $U(1) \times U(1) \times SO(3)$ related to the rotations in the directions 12, 56 and 789, respectively. This configuration preserves 1/4 of original supersymmetry in the near-horizon.

	0	1	2	3	4	5	6	7	8	9
D3	○	○	○	○						
D5	○	○	○		○	○	○			
D1	○				○					

Table 6.1: The brane system. In this table "○" denotes the directions along which branes extend.

6.2 Adding 't Hooft operator

A 't Hooft operator is a magnetic dual of the Wilson loop operator which is introduced in section 2.3.1, This operator is introduced in [30]. In string theory the 't Hooft operator corresponds to adding the D1-branes in the worldvolume of the multiple D3-branes. These D1-branes introduce the magnetic charge on the worldvolume of the D3-branes. So these D1-branes are magnetic charged point particle in the non-Abelian gauge theory realized by the D3-branes. Its charge is classified by Young diagrams.

6.3 Ansatz for D5-brane

We consider a bound state of a D5-brane and D1-branes in the $AdS_5 \times S^5$ spacetime. The D1-branes are realized as the worldvolume gauge flux on the D5-brane. Thus we consider a probe D5-brane with the worldvolume gauge flux. We define the worldvolume coordinates of the D5-brane as $(t, y, \psi, \phi, u_1, u_2)$ where the coordinates (t, y, ψ, ϕ) are identified with the coordinates of the bulk spacetime. According to the symmetry $U(1)^2 \times SO(3)$ we put the ansatz on the embedding as:

$$r = ys(u), \;\; x_3 = yz(u), \;\; \theta = \theta(u), \tag{6.3.1}$$

where $s(u)$, $z(u)$ and $\theta(u)$ are unknown functions of coordinates u^i, $i = 1, 2$. Since (u^1, u^2) are not fixed yet, there remains the general coordinate transformation symmetry of (u^1, u^2). Some of the D3-branes end on the D5-brane. Thus the ansatz for the worldvolume gauge flux is written as

$$\mathcal{F} = dP \wedge d\psi + dQ \wedge d\phi, \tag{6.3.2}$$

where potentials P and Q are functions of u. Then we have unknown functions of u

$$s(u), \;\; z(u), \;\; \theta(u), \;\; P(u), \;\; Q(u). \tag{6.3.3}$$

Our goal is to determine these functions.

6.4 Example of kappa symmetry projection

Let us calculate some examples of this symmetry. We show below the case of a D1-brane and a D5-brane. The results is used in our analysis of D5-D1 bound state case.

6.4.1 D1-brane case

First, let us calculate the D1-brane case. The induced metric for the D1-brane with worldvolume coordinates (t, y) is

$$ds^2_{\mathrm{D1}} = \frac{1}{y^2}(-dt^2 + dr^2). \tag{6.4.1}$$

Since the dilaton Φ is zero and there is no flux, $\mathcal{F} = 0$,

$$d^2\xi \cdot \Gamma_{\mathrm{D1}} = -y^2 \cdot \chi|_{2-\mathrm{form}}. \tag{6.4.2}$$

Substituting

$$\chi|_{2-\mathrm{form}} = -dt\, dr K(-i)\frac{1}{y^2}\Gamma_{04}, \tag{6.4.3}$$

we obtain

$$\Gamma_{\mathrm{D1}} = \Gamma_{04}K(-i), \tag{6.4.4}$$

where K, charge conjugation, and i are expressed by matrices as

$$K = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad i = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \tag{6.4.5}$$

The necessary and sufficient condition for satisfying $\Gamma_{D1}\epsilon = \epsilon$ is

$$(iK\Gamma_{04} - 1)\xi = 0. \tag{6.4.6}$$

6.4.2 D5-brane case

Next, let us consider the D5-brane with ansatz [6, 31]

$$x_3 = \kappa y, \quad \mathcal{F} = f \sin\theta \, d\theta \wedge d\phi, \quad (\kappa, f : \text{constant}). \tag{6.4.7}$$

The induced metric of the D5-brane with coordinates $(t, r, \psi, y = \frac{1}{\kappa}x_3, \theta, \phi)$ is

$$ds^2_{\mathrm{D5}} = \frac{1}{y^2}(-dt^2 + dr^2 + r^2 d\psi^2 + (\kappa^2 + 1)dy^2) + d\theta^2 + \sin^2\theta d\phi^2. \tag{6.4.8}$$

We need to calculate the determinant of

$$G_{\mathrm{ind}} + \mathcal{F} = \begin{bmatrix} -1/y^2 & & & & & \\ & 1/y^2 & & & & \\ & & r^2/y^2 & & & \\ & & & (1+\frac{1}{\kappa^2})/y^2 & & \\ & & & & 1 & f\sin\theta \\ & & & & -f\sin\theta & \sin^2\theta \end{bmatrix}, \tag{6.4.9}$$

where all empty components denote zeros. The result is

$$\sqrt{-\det(G_{\text{ind}}+\mathcal{F})}=\frac{r\sin\theta}{y^4}\sqrt{1+1/\kappa^2}\sqrt{1+f^2}. \tag{6.4.10}$$

Since $e^{\mathcal{F}}=1+f\sin^2\theta d\theta\wedge d\phi$,

$$\begin{aligned}d^6\xi\cdot\Gamma_{\text{D5}}&=\left(-\frac{1}{\sqrt{1+1/\kappa^2}\sqrt{1+f^2}}\frac{y^4}{r\sin\theta}\Big(1+f\sin\theta d\theta\wedge d\phi\Big)\chi\right)\Big|_{6-\text{form}}\\&=-\frac{1}{\sqrt{1+1/\kappa^2}\sqrt{1+f^2}}\frac{y^4}{r\sin\theta}\Big(\chi\Big|_{6-\text{form}}+f\sin\theta d\theta\wedge d\phi\cdot\chi\Big|_{4-\text{form}}\Big).\end{aligned} \tag{6.4.11}$$

$\chi|_{6-\text{form}}$ and $\chi|_{4-\text{form}}$ are

$$\chi|_{6-\text{form}}=dt\;dr\;d\psi\;dy\;d\theta\;d\phi K i\frac{r\sin\theta}{y^4}\left(\Gamma_{012356}+\frac{1}{\kappa}\Gamma_{012456}\right), \tag{6.4.12}$$

$$\chi|_{4-\text{form}}=dt\;dr\;d\psi\;dy(-i)\frac{r}{y^4}\left(\Gamma_{0123}+\frac{1}{\kappa}\Gamma_{0124}\right). \tag{6.4.13}$$

We obtain the following result by putting together them.

$$\Gamma_{\text{D5}}=\frac{-1}{\sqrt{(\kappa^2+1)(f^2+1)}}\gamma(K\Gamma_{56}+f)(\Gamma_{34}+\kappa). \tag{6.4.14}$$

The necessary and sufficient condition for ϵ to satisfy $\Gamma_{\text{D5}}\epsilon=\epsilon$ is $\kappa=-f$ and

$$(K\Gamma_{3456}+\gamma)\xi=0. \tag{6.4.15}$$

Both the conditions (6.4.6) and (6.4.15) must be satisfied in our bound state of a D5-brane and D1-branes.

6.5 Derivation of the kappa symmetry projector Γ

We calculate Γ defined in (F.0.2) and (F.0.3) for a D5-brane with worldvolume coordinates (t,ψ,ϕ,y,u^1,u^2). There is a flux on the D5-brane,

$$\mathcal{F}=dP(u)\wedge d\psi+dQ(u)\wedge d\phi. \tag{6.5.1}$$

In our situation, the dilation is zero, and

$$e^{\mathcal{F}}=1+\partial_aPdu^a\wedge d\psi+\partial_aQdu^a\wedge d\phi-\partial_aP\partial_bQ\epsilon^{ab}d\psi\wedge d\phi\wedge du^1\wedge du^2, \tag{6.5.2}$$

$$\begin{aligned}(e^{\mathcal{F}}\chi)|_{6-\text{from}}&=\chi|_{6-\text{form}}+\chi|_{4-\text{form}}\cdot\partial_aPdu^a\wedge d\psi+\chi|_{4-\text{form}}\cdot\partial_aQdu^a\wedge d\phi\\&\quad+\chi|_{2-\text{form}}\cdot(-\partial_aP\,\partial_bQ\epsilon^{ab})d\psi\wedge d\phi\wedge du^1\wedge du^2.\end{aligned} \tag{6.5.3}$$

Here the first $\chi|_{4-\text{form}}$ in the expression (6.5.3) is proportional to $dt\wedge d\phi\wedge dy\wedge du^b,(b\neq a)$, while the second is proportional to $dt\wedge d\psi\wedge dy\wedge du^b,(b\neq a)$ and we use the notation

$$\{A,B\}:=\epsilon^{ab}\partial_aA\partial_bB=\frac{\partial A}{\partial u^1}\frac{\partial B}{\partial u^2}-\frac{\partial A}{\partial u^2}\frac{\partial B}{\partial u^1}, \tag{6.5.4}$$

in the following. Each term of eq.(6.5.3) is calculated as follows.

$$\chi|_{6-\text{form}} = d^6\xi \cdot \frac{s\sin\theta}{y^2}\Big(\{z,\theta\}\Gamma_{35} + \{s,\theta\}\Gamma_{15} - s^2\{\frac{z}{s},\theta\}\Gamma_{1345} + \{s,z\}\Gamma_{13}\Big)\Gamma_{04}\Gamma_{62}K(-i), \tag{6.5.5–i}$$

$$\begin{aligned}\chi|_{4-\text{form}} \cdot \partial_a P du^a d\psi = \frac{\sin\theta}{y^2}\Big(& s^2\{P,\frac{z}{s}\}\Gamma_{13} - \{P,s\}\Gamma_{14} - \{P,z\}\Gamma_{34} \\ &+ s\{P,\theta\}\Gamma_{15} + z\{P,\theta\}\Gamma_{35} + \{P,\theta\}\Gamma_{45}\Big)\Gamma_{60}(-i)d^6\xi,\end{aligned} \tag{6.5.5–ii}$$

$$\begin{aligned}\chi|_{4-\text{form}} \cdot \partial_a Q du^a d\psi = \frac{s}{y^2}\Big(& - s^2\{Q,\frac{z}{s}\}\Gamma_{13} + \{Q,s\}\Gamma_{14} + \{Q,z\}\Gamma_{34} \\ &- s\{Q,\theta\}\Gamma_{15} - z\{Q,\theta\}\Gamma_{35} - \{Q,\theta\}\Gamma_{45}\Big)\Gamma_{20}(-i)d^6\xi,\end{aligned} \tag{6.5.5–iii}$$

$$\chi|_{2-\text{form}} \cdot (-\partial_a P\partial_b Q\epsilon^{ab})d\psi\, d\phi\, dt\, dy = \frac{1}{y^2}(\epsilon^{ab}\partial_a P\partial_b Q)(s\Gamma_{14} + z\Gamma_{34} + 1)\Gamma_{04}K(-i) \cdot d^6\xi, \tag{6.5.5–iv}$$

where $d^6\xi = dt \wedge d\psi \wedge d\phi \wedge dy \wedge du^1 \wedge du^2$.

In the definition (F.0.2), $\mathcal{L}_{\text{DBI}}$ is

$$\mathcal{L}_{\text{DBI}} = \sqrt{-\det(G_{\text{ind}} + \mathcal{F})} =: \frac{W}{y^2}. \tag{6.5.6}$$

Under our ansatz, see eq.(6.3.1) in section 6.3, the induced metric G_{ind} is

$$ds^2_{\text{ind}} = -\frac{1}{y^2}dt^2 + s^2 d\psi^2 + \sin^2\theta d\phi^2 + \frac{\beta}{y^2}dy^2 + h_{ij}du^i du^j + \frac{\partial_a\beta}{y}du^a dy, \tag{6.5.7}$$

$$h_{ij} := \sum_{\lambda=s,z,\theta} \partial_i\lambda\partial_j\lambda. \tag{6.5.8}$$

We define a convenient variable $\beta := 1 + s^2 + z^2$. W is calculated as the following determinant.

$$W^2 = -y^4 \det \left[\begin{array}{c|cc|c|cc} -1/y^2 & & & & & \\ \hline & s^2 & & & -J_1 & -J_2 \\ & & \sin^2\theta & & -L_1 & -L_2 \\ \hline & & & \frac{\beta}{y^2} & \frac{1}{2y}\partial_1\beta & \frac{1}{2y}\partial_2\beta \\ \hline & J_1 & L_1 & \frac{1}{2y}\partial_1\beta & h_{11} & h_{12} \\ & J_2 & L_2 & \frac{1}{2y}\partial_2\beta & h_{21} & h_{22} \end{array}\right]$$

$$= \det \left[\begin{array}{cc|c|cc} s^2 & & & -J_1 & -J_2 \\ & \sin^2\theta & & -L_1 & -L_2 \\ \hline & & \beta & \frac{1}{2}\partial_1\beta & \frac{1}{2}\partial_2\beta \\ \hline J_1 & L_1 & \frac{1}{2}\partial_1\beta & h_{11} & h_{12} \\ J_2 & L_2 & \frac{1}{2}\partial_2\beta & h_{21} & h_{22} \end{array}\right], \tag{6.5.9}$$

where $J_a := \partial P/\partial u^a$ and $L_a := \partial Q/\partial u^a$. To calculate this determinant the following formula is convenient.

$$\det \begin{bmatrix} A & D \\ C & B \end{bmatrix} = \det A \cdot \det(B - CA^{-1}D). \tag{6.5.10}$$

One can check this decomposition by a straightforward calculation from the righthand side.

We use this formula for

$$A = \begin{bmatrix} s^2 & & \\ & \sin^2\theta & \\ & & \beta \end{bmatrix}, \quad D = \begin{bmatrix} -J_1 & -J_2 \\ -L_1 & -L_2 \\ \frac{1}{2}\partial_1\beta & \frac{1}{2}\partial_2\beta \end{bmatrix},$$

$$C = \begin{bmatrix} J_1 & L_1 & \frac{1}{2}\partial_1\beta \\ J_2 & L_2 & \frac{1}{2}\partial_2\beta \end{bmatrix}, \quad B = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}.$$

Then W is written explicitly as

$$\begin{aligned} W^2 =& s^2\sin^2\theta\{s,z\}^2 \\ &+ s^2\sin^2\theta((z^2+1)\{s,\theta\}^2 + (s^2+1)\{z,\theta\}^2 - 2sz\{s,\theta\}\{z,\theta\}) \\ &+ \sin^2\theta((z^2+1)\{s,P\}^2 + (s^2+1)\{z,P\}^2 - 2sz\{s,P\}\{z,P\}) \\ &+ s^2((z^2+1)\{s,Q\}^2 + (s^2+1)\{z,Q\}^2 - 2sz\{s,Q\}\{z,Q\}) \\ &+ \beta\{P,Q\}^2. \end{aligned} \tag{6.5.11}$$

Summarizing the above, the operator Γ is

$$\Gamma = \frac{1}{W}\Big\{s\sin\theta\,\mathcal{A}\Gamma_{62}K(-i)\Gamma_{04} + \sin\theta\,\mathcal{B}(-i)\Gamma_{60} - s\,\mathcal{C}(-i)\Gamma_{20} + \mathcal{D}K(-i)\Gamma_{04}\Big\}, \tag{6.5.12}$$

where

$$\mathcal{A} := -\{s,z\}\Gamma_{13} - \{s,\theta\}\Gamma_{15} - \{z,\theta\}\Gamma_{35} + s^2\{\frac{z}{s},\theta\}\Gamma_{1345}, \tag{6.5.13–i}$$

$$\mathcal{B} := -\{P,\frac{z}{s}\}\Gamma_{13} + \{P,s\}\Gamma_{14} + \{P,z\}\Gamma_{34} - s\{P,\theta\}\Gamma_{15} - z\{P,\theta\}\Gamma_{35} - \{P,\theta\}\Gamma_{45}, \tag{6.5.13–ii}$$

$$\mathcal{C} := -\{Q,\frac{z}{s}\}\Gamma_{13} + \{Q,s\}\Gamma_{14} + \{Q,z\}\Gamma_{34} - s\{Q,\theta\}\Gamma_{15} - z\{Q,\theta\}\Gamma_{35} - \{Q,\theta\}\Gamma_{45}, \tag{6.5.13–iii}$$

$$\mathcal{D} := -\{P,Q\}(1 + s\Gamma_{14} + z\Gamma_{34}), \tag{6.5.13–iv}$$

$\mathcal{C}$ is obtained by replacing all P's in $\mathcal{B}$ by Q's, and W is given by eq.(6.5.11).

6.6 Boundary behavior of probe D5-brane

First, let us consider the worldvolume of our D5-brane. Its boundary behavior is important in our investigation. The boundary of the u-plane (the base 2-dimensional space coordinated by (u^1, u^2)) is given by $s = 0$ or $\sin\theta = 0$. The boundary condition is not arbitrary and it contains the detailed information of the associated operators in the gauge theory as in [32, 33, 34, 35, 36]. We explain the relation between the boundary behavior of our system and Young diagrams which label the 't Hooft operators.

The structure of the D5-brane worldvolume is a fiber bundle over the u-plane with the fiber $S^1 \times S^1$ coordinated by ϕ and ψ. Each point of the boundary is distinguished by whether $s = 0$ or $\sin\theta = 0$ and the boundary is divided into segments as shown in figure 6.1. Let $I_i,\ i = 1, \ldots, \ell$ denote the i-th $s = 0$ segment and $J_j,\ j = 1, \ldots, \ell - 1$ denote the j-th $\sin\theta = 0$ segment. The pullback $dP|_{I_i}$ vanishes and P is a constant P_i on I_i for smoothness since $d\psi$ is singular at I_i and $dP \wedge d\psi$ must vanish. The pullback $dQ|_{J_j}$ also vanishes and Q is a constant Q_j on J_j in the same way. Thus the gauge flux reduces to $\mathcal{F} = dQ \wedge d\phi$ at I_i and $\mathcal{F} = dP \wedge d\psi$ at J_j. At each internal point on I_i the fiber reduces to S^1 coordinated by ϕ and at both end points of I_i the radius of this S^1 fiber vanishes. Therefore these S^1 fibers make a non-contractible S^2 cycle denoted by S_i^2. There is also a non-contractible S^2 cycle (denoted by $\widetilde{S}_j^2$) on J_j in the same way.

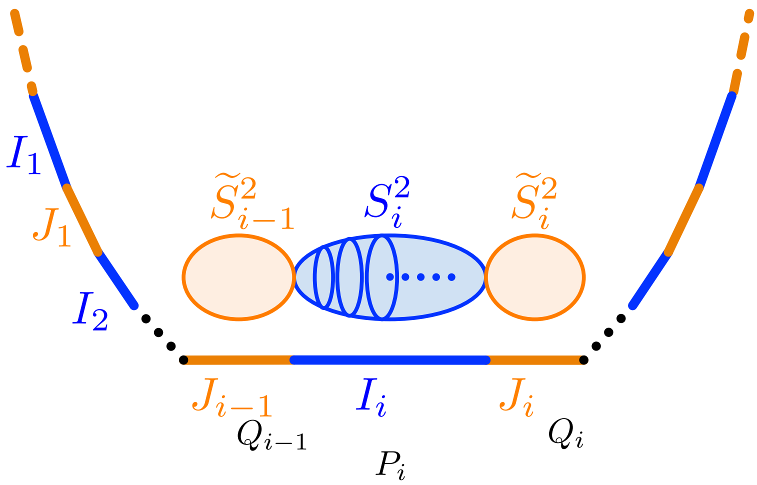

Figure 6.1: The boundary line and 2-spheres composed of ψ and ϕ-cycles.

The charge is defined as the integration of the flux on each non-contractible S^2 and we define these quantities as

$$n_i := \frac{\sqrt{\lambda}}{(2\pi)^2}\int_{S_i^2} dQ \wedge d\phi = \frac{\sqrt{\lambda}}{2\pi}\int_{I_i} dQ = \frac{\sqrt{\lambda}}{2\pi}(Q_i - Q_{i-1}), \tag{6.6.1}$$

$$m_j := \frac{\sqrt{\lambda}}{(2\pi)^2}\int_{\widetilde{S}_j^2} dP \wedge d\psi = \frac{\sqrt{\lambda}}{2\pi}\int_{J_j} dP = \frac{\sqrt{\lambda}}{2\pi}(P_{j+1} - P_j). \tag{6.6.2}$$

Here Q_0 is defined as the value of Q on the first $\theta = 0$ half line J_0. The normalization is determined so that n_i and m_j are integers as follows. In a general D5-brane with worldvolume flux the number of the D3-branes and the number of the D1-branes are calculated by the integration of the gauge flux as seen from the Wess-Zumino term of the D5-brane action.

$$(\text{number of D3-branes}) = \frac{T_5}{T_3}\int_{\mathcal{M}_2} \mathcal{F} = \frac{1}{(2\pi)^2\alpha'}\int_{\mathcal{M}_2} \mathcal{F}, \tag{6.6.3}$$

$$(\text{number of D1-branes}) = \frac{T_5}{T_1}\int_{\mathcal{M}_4} \frac{1}{2}\mathcal{F} \wedge \mathcal{F} = \frac{1}{32\pi^4\alpha'^2}\int_{\mathcal{M}_4} \mathcal{F} \wedge \mathcal{F}, \tag{6.6.4}$$

where the integral over $\mathcal{M}_2$ or $\mathcal{M}_4$ denotes the integral over the perpendicular directions to D3-branes or D1-branes on the D5-brane worldvolume. We also use the Dp-brane tension T_p

$$T_p = \frac{1}{(2\pi)^p \alpha'^{(p+1)/2} g_s}, \tag{6.6.5}$$

and $\alpha' = 1/\sqrt{\lambda}$ in our unit. Here g_s is the string coupling constant.

Since the quantities n_i and m_j are integers, these can be related to the number of boxes in the Young diagram as follows. First we deform the boundary as stepwise by bending it at the edges of each segment. After that deformation this boundary line can be interpreted as the right down edge of the Young diagram as shown in figure 6.2. The integers n_i and m_j correspond to each length of the edge of the Young diagram.

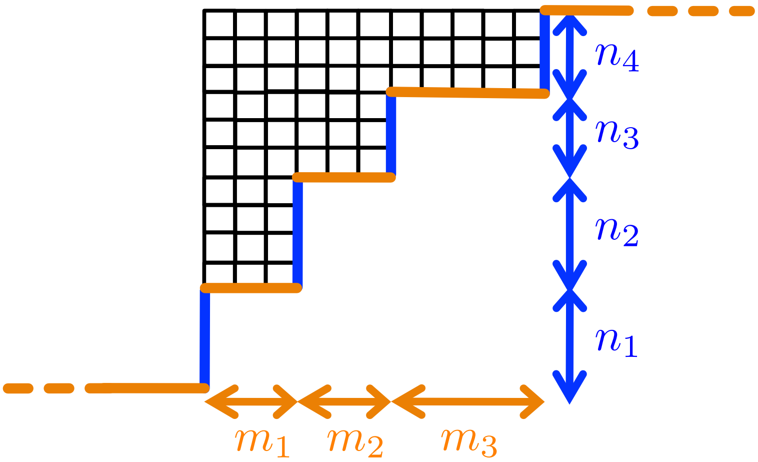

Figure 6.2: The relation between a deformed boundary line and the Young diagram.

Let us consider the relation between the number of branes and the Young diagram for a consistency check. The number of the D3-branes ending on the D5-brane, denoted by k, is related to the vertical length of the Young diagram as follows.

$$\begin{aligned} k &= \frac{\lambda}{4\pi^2} \int_{\mathcal{M}_2} \mathcal{F} \\ &= \frac{\lambda}{4\pi^2} \sum_i \int_{S_i^2} dQ \wedge d\phi \\ &= \frac{\lambda}{2\pi} \sum_i \int_{I_i} dQ \\ &= \sum_i n_i, \end{aligned} \tag{6.6.6}$$

where $\mathcal{M}_2$ is a 2-cycle shown in figure 6.3.

On the other hand, the number of the D1-branes k' can be interpreted as the total number of boxes in the Young diagram which characterize the boundary condition as expected. This

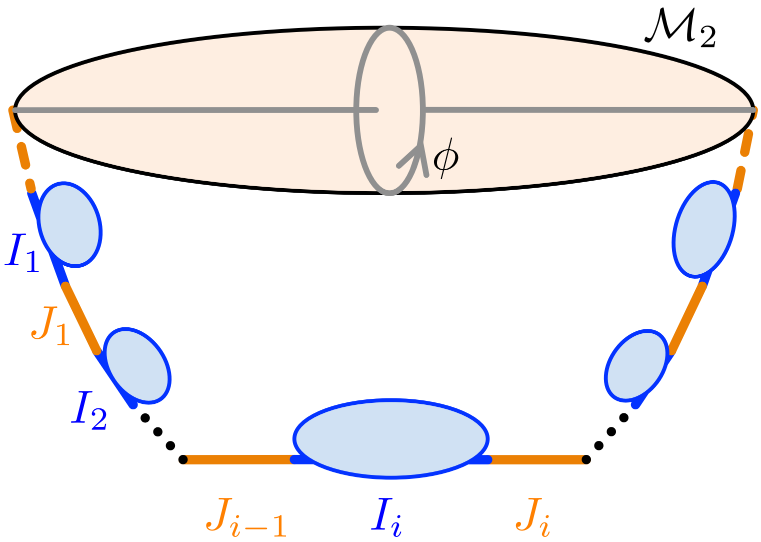

Figure 6.3: $\mathcal{M}_2$ is a 2-dimensional manifold located at sufficiently far. It can be deformed into 2-spheres located in the boundary without changing the value of the integral.

relation is derived as follows.

$$
\begin{aligned}
k' &= \frac{\lambda}{32\pi^4} \int_{\mathcal{M}_4} 2 \ dP \wedge d\psi \wedge dQ \wedge d\phi \\
&= -\frac{\lambda}{4\pi^2} \int_{u-\text{plane}} dP \wedge dQ \\
&= -\frac{\lambda}{4\pi^2} \int_{u-\text{plane}} d(P \wedge dQ) \\
&= -\frac{\lambda}{4\pi^2} \int_{\partial(u-\text{plane})} P \wedge dQ \\
&= -\frac{\lambda}{4\pi^2} \sum_i P_i \int_{I_i} dQ \\
&= -\frac{\lambda}{4\pi^2} \sum_{i\geq 2} \left(\sum_{j \leq i-1} \frac{2\pi}{\sqrt{\lambda}} m_j \right) \left(\frac{2\pi}{\sqrt{\lambda}} n_i \right) \\
&= -\sum_{i \geq 2} \left(\sum_{j \leq i-1} m_j \right) n_i \\
&= -\left(\sum_{j \leq 1} m_j \right) n_2 - \left(\sum_{j \leq 2} m_j \right) n_3 \cdots .
\end{aligned} \tag{6.6.7}
$$

Here $\mathcal{M}_4$ is a 4-cycle coordinated by u^1, u^2, ψ, ϕ. In the 5th line we used the fact that P is a constant at each I_i. Then in the next line the integral can be rewritten by (6.6.1) and the potential functions P_i can be translated by adding a constant to all P_i. Using this ambiguity we set $P_1 = 0$. The first term of the final expression (6.6.7) ($i = 2$) is equal to the number of the boxes in the lowest set of columns of the corresponding Young diagram. The second term is equal to the number of the boxes in the second lowest set of columns, and so forth (figure 6.2).

From the above calculations (6.6.6), (6.6.7), we see a correspondence between the brane configuration and the number of the boxes in the Young diagram. Namely, k, the number of the D3-branes ending on the D5-brane, corresponds to the vertical length of the Young diagram, and k', the number of the D1-branes embedded on the D5-brane, is the total number of the boxes in the Young diagram. These are consistent with our conjectured relation.

6.7 Supersymmetry in bulk space

In the next section 6.8 we investigate the embedding of the D5-brane through the BPS condition. Before that let us study the supersymmetry of the bulk spacetime, $AdS_5 \times S^5$.

We investigate supersymmetry in $AdS_5 \times S^5$ spacetime with metric

$$ds^2 = \frac{1}{y^2}(-dt^2 + dy^2 + dr^2 + r^2 d\psi^2 + dx_3^2) + d\theta^2 + \sin^2\theta d\phi^2. \tag{6.7.1}$$

In order to preserve supersymmetry, the gravitino transformation must give zero,

$$\nabla_M \epsilon + \frac{i}{2^4}\Gamma^{M_1 M_2 \cdots M_5} F^{(5)}_{M_1 M_2 \cdots M_5} \Gamma_M \epsilon = 0, \tag{6.7.2}$$

$$\nabla_M = \partial_M + \frac{1}{4}\Omega_M{}^{AB}\Gamma_{AB}, \tag{6.7.3}$$

where gamma matrices with indices $M = t, r, \psi, x_3, y, \theta, \phi$ are $\Gamma_t := E_t^A \Gamma_A = \frac{1}{y}\Gamma_0$ and so on. Γ_A, $A = 0, \ldots, 9$, are constant gamma matrices in 10-dimensional spacetime. They satisfy $\{\Gamma_A, \Gamma_B\} = 2\eta_{AB}$ where $\eta_{AB} = \mathrm{diag}(-1, +1, \ldots, +1)$. We use the notation for antisymmetrized products of gamma matrices as

$$\Gamma_{A_1 A_2 \ldots A_n} := \frac{1}{n!}\sum_{\sigma \in \mathfrak{S}_n} \mathrm{sign}(\sigma)\Gamma_{A_{\sigma(1)}}\Gamma_{A_{\sigma(2)}} \cdots \Gamma_{A_{\sigma(n)}}. \tag{6.7.4}$$

The SUSY parameter ϵ is a complex Weyl spinor which satisfies $\Gamma^{01\ldots 9}\epsilon = \epsilon$. In this thesis we choose vielbein as

$$\begin{aligned} &E^0 = \frac{dt}{y},\ E^1 = \frac{dr}{y},\ E^2 = \frac{r d\psi}{y},\ E^3 = \frac{dx_3}{y},\ E^4 = \frac{dy}{y}, \\ &E^5 = d\theta,\ E^6 = \sin\theta d\phi. \end{aligned} \tag{6.7.5}$$

The spin connections $\Omega^{AB} = \Omega_M{}^{AB} E^M$ are related to vielbein as $dE^A = -\Omega^A{}_B E^B$, and calculated using this relation as follows.

$$\begin{aligned} &\Omega^{04} = -\frac{dt}{y},\ \Omega^{12} = -d\psi,\ \Omega^{14} = -\frac{dr}{y}, \\ &\Omega^{24} = -\frac{r d\psi}{y},\ \Omega^{34} = -\frac{dx_3}{y},\ \Omega^{56} = -\cos\theta d\phi, \end{aligned} \tag{6.7.6}$$

and the other components are zero. The equations (6.7.2) for 7 components, $M = t, r, \psi, x_3, y, \theta, \phi$, are

$$\partial_t \epsilon - \frac{1+\gamma}{2y}\Gamma_{04}\epsilon = 0, \tag{6.7.7–i}$$

$$\partial_r \epsilon - \frac{1+\gamma}{2y}\Gamma_{14}\epsilon = 0, \tag{6.7.7–ii}$$

$$\partial_\psi \epsilon - \frac{1}{2}\Gamma_{12}\epsilon - \frac{1+\gamma}{2y}\Gamma_{24}\epsilon = 0, \tag{6.7.7–iii}$$

$$\partial_{x_3} \epsilon - \frac{1+\gamma}{2y}\Gamma_{34}\epsilon = 0, \tag{6.7.7–iv}$$

$$\partial_y \epsilon + \frac{1}{2y}\gamma\epsilon = 0, \tag{6.7.7–v}$$

$$\partial_\theta \epsilon + \frac{1}{2}\gamma\Gamma_{45}\epsilon = 0, \tag{6.7.7–vi}$$

$$\partial_\phi \epsilon - \frac{1}{2}e^{-\gamma\Gamma_{45}}\Gamma_{56}\epsilon = 0, \tag{6.7.7–vii}$$

where we have used the matrix $\gamma := -i\Gamma_{0123}$. Solving the equations (6.7.7–i)-(6.7.7–vii) we obtain the supersymmetry parameter in the bulk spacetime.

$$\epsilon = e^{-\frac{\theta}{2}\gamma\Gamma_{45}}e^{\frac{\phi}{2}\Gamma_{56}}e^{-\frac{1}{2}\ln y\cdot\gamma}e^{r\frac{1+\gamma}{2}\Gamma_{14}}e^{x_3\frac{1+\gamma}{2}\Gamma_{34}}e^{t\frac{1+\gamma}{2}\Gamma_{04}}e^{\frac{\psi}{2}\Gamma_{12}}\epsilon_0, \tag{6.7.8}$$

where ϵ_0 is an arbitrary constant complex Weyl spinor. For convenience, we define $\xi := e^{\frac{\phi}{2}\Gamma_{56}}e^{\frac{\psi}{2}\Gamma_{12}}\epsilon_0$. Then ϵ is rewritten as

$$\epsilon = e^{-\frac{\theta}{2}\gamma\Gamma_{45}}e^{-\frac{1}{2}\ln y\cdot\gamma}e^{r\frac{1+\gamma}{2}\Gamma_{14}}e^{x_3\frac{1+\gamma}{2}\Gamma_{34}}e^{t\frac{1+\gamma}{2}\Gamma_{04}}\xi. \tag{6.7.9}$$

6.8 BPS condition

In this section we try to obtain the condition for preserving some of supersymmetries. When a Dp-brane exists, a part of the original supersymmetry is generally broken. The remaining supersymmetry parameters are spinors of the form (6.7.8) which satisfy the relation [37, 38, 39, 40, 41, 42, 31]

$$\Gamma\epsilon = \epsilon. \tag{6.8.1}$$

This is called "the kappa symmetry projection" where the operator Γ is determined for a Dp-brane as

$$d^{p+1}\xi \cdot \Gamma := \left(-e^{-\Phi}(-\det(G_{\text{ind}} + \mathcal{F}))^{-1/2}e^{\mathcal{F}}\chi\right)\Big|_{(p+1)-\text{form}}, \tag{6.8.2}$$

$$\chi := \sum_n \frac{1}{(2n)!}\hat{E}^{a_s}\cdots\hat{E}^{a_1}\Gamma_{a_1\cdots a_s}K^n(-i), \tag{6.8.3}$$

where ξ^i, $i = 0, \cdots, p$, are worldvolume coordinates, Φ is the dilaton, G_{ind} is the induced metric of the Dp-brane and $\hat{E}^A$ is the pullback of E^A defined as $\hat{E}^A := E^A_M \frac{\partial X^M}{\partial \xi^i}d\xi^i$. We calculated the kappa symmetry projection operators for a D5-brane and for a D1-brane in appendix F and we use the relations obtained in this appendix in the following calculation.

We calculate the kappa symmetry projection operator Γ defined above under our ansatz given in section 6.7. Here we only show the result

$$\Gamma = \frac{1}{W}\Big\{ s\sin\theta\,\mathcal{A}\Gamma_{62}K(-i)\Gamma_{04} + \sin\theta\,\mathcal{B}(-i)\Gamma_{60} - s\,\mathcal{C}(-i)\Gamma_{20} + \mathcal{D}K(-i)\Gamma_{04}\Big\}. \tag{6.8.4}$$

For the detailed calculation, see appendix 6.5. Here we defined a y independent function W as

$$W := y^2\sqrt{-\det(G_{\text{ind}} + \mathcal{F})}. \tag{6.8.5}$$

The induced metric for the D5-brane is

$$ds^2 = -\frac{1}{y^2}dt^2 + s^2 d\psi^2 + \sin^2\theta d\phi^2 + \frac{\beta}{y^2}dy^2 + h_{ij}du^i du^j + \frac{\partial_a\beta}{y}du^a dy, \tag{6.8.6}$$

$$\beta := 1 + s^2 + z^2, \quad h_{ij} := \sum_{\lambda=s,z,\theta}\partial_i\lambda\partial_j\lambda.$$

In the expression (6.8.4), $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$ are the following matrices.

$$\mathcal{A} := -\{s,z\}\Gamma_{13} - \{s,\theta\}\Gamma_{15} - \{z,\theta\}\Gamma_{35} + s^2\{\frac{z}{s},\theta\}\Gamma_{1345}, \tag{6.8.7–i}$$

$$\mathcal{B} := -\{P,\frac{z}{s}\}\Gamma_{13} + \{P,s\}\Gamma_{14} + \{P,z\}\Gamma_{34} - s\{P,\theta\}\Gamma_{15} - z\{P,\theta\}\Gamma_{35} - \{P,\theta\}\Gamma_{45}, \tag{6.8.7–ii}$$

$$\mathcal{C} := -\{Q,\frac{z}{s}\}\Gamma_{13} + \{Q,s\}\Gamma_{14} + \{Q,z\}\Gamma_{34} - s\{Q,\theta\}\Gamma_{15} - z\{Q,\theta\}\Gamma_{35} - \{Q,\theta\}\Gamma_{45}, \tag{6.8.7–iii}$$

$$\mathcal{D} := -\{P,Q\}(1 + s\Gamma_{14} + z\Gamma_{34}), \tag{6.8.7–iv}$$

where $\mathcal{C}$ is obtained from $\mathcal{B}$ by replacing all P's by Q's. We use the notation of "Poisson bracket"

$$\{A,B\} := \epsilon^{ab}\frac{\partial A}{\partial u^a}\frac{\partial B}{\partial u^b} = \frac{\partial A}{\partial u^1}\frac{\partial B}{\partial u^2} - \frac{\partial A}{\partial u^2}\frac{\partial B}{\partial u^1}. \tag{6.8.8}$$

Under our ansatz the parameter ϵ in eq. (6.7.8) is decomposed by the dependence of y and t:

$$\begin{aligned}
\epsilon &= e^{-\frac{\theta}{2}\gamma\Gamma_{45}}e^{\frac{\phi}{2}\Gamma_{56}}e^{-\frac{1}{2}\ln y\cdot\gamma}e^{r\frac{1+\gamma}{2}\Gamma_{14}}e^{x_3\frac{1+\gamma}{2}\Gamma_{34}}e^{t\frac{1+\gamma}{2}\Gamma_{04}}e^{\frac{\psi}{2}\Gamma_{12}}\epsilon_0 \\
&= e^{-\frac{\theta}{2}\gamma\Gamma_{45}}e^{-\frac{1}{2}\ln y\cdot\gamma}\left(1 + ys\frac{1+\gamma}{2}\Gamma_{14}\right)\left(1 + yz\frac{1+\gamma}{2}\Gamma_{34}\right)\left(1 + t\frac{1+\gamma}{2}\Gamma_{04}\right)\xi \\
&= e^{-\frac{\theta}{2}\gamma\Gamma_{45}}e^{-\frac{1}{2}\ln y\cdot\gamma}\left(\xi + ys\Gamma_{14}\xi_- + yz\Gamma_{34}\xi_- + t\Gamma_{04}\xi_-\right) \\
&= e^{-\frac{\theta}{2}\gamma\Gamma_{45}}\left(\frac{1}{\sqrt{y}}\xi_+ + \sqrt{y}\xi_- + \frac{1}{\sqrt{y}}(ys\Gamma_{14}\xi_- + yz\Gamma_{34}\xi_- + t\Gamma_{04}\xi_-)\right) \\
&=: \sqrt{y}\epsilon_1 + \frac{1}{\sqrt{y}}\epsilon_2 + \frac{t}{\sqrt{y}}\epsilon_3,
\end{aligned} \tag{6.8.9}$$

where we define $\xi := e^{\frac{\phi}{2}\Gamma_{56}} e^{\frac{\psi}{2}\Gamma_{12}} \epsilon_0$ in the second line and $\xi_\pm := \frac{1\pm\gamma}{2}\xi$. The explicit forms of $\epsilon_1, \epsilon_2, \epsilon_3$ are written as

$$\epsilon_1 = e^{-\frac{\theta}{2}\gamma\Gamma_{45}}(1 + s\Gamma_{14} + z\Gamma_{34})\xi_-, \tag{6.8.10–i}$$

$$\epsilon_2 = e^{-\frac{\theta}{2}\gamma\Gamma_{45}}\xi_+, \tag{6.8.10–ii}$$

$$\epsilon_3 = e^{-\frac{\theta}{2}\gamma\Gamma_{45}}\Gamma_{04}\xi_-. \tag{6.8.10–iii}$$

Since the kappa symmetry operator of eq. (6.8.4) is independent of y and t, we can impose the projection condition (6.8.1) for each ϵ_i :

$$\Gamma\epsilon_i = \epsilon_i, \quad i = 1, 2, 3. \tag{6.8.11}$$

The kappa symmetry projections for the D5-brane and the D1-brane give the conditions (6.4.15) and (6.4.6), respectively, which are obtained in appendix (6.4.1 and 6.4.2).

$$\text{D5 condition} \Leftrightarrow (K\Gamma_{3456} + \gamma)\xi = 0, \tag{6.8.12}$$

$$\text{D1 condition} \Leftrightarrow (iK\Gamma_{04} - 1)\xi = 0. \tag{6.8.13}$$

We want to obtain the condition for the functions (6.3.3) such that all spinors restricted by the equations (6.8.12) and (6.8.13) satisfy the projection condition (6.8.11). The condition (6.8.11) is equivalent to

$$e^{\frac{\theta}{2}\gamma\Gamma_{45}}\Big\{s\sin\theta\,\mathcal{A}\Gamma_{62}K(-i)\Gamma_{04} + \sin\theta\,\mathcal{B}(-i)\Gamma_{60} - s\,\mathcal{C}(-i)\Gamma_{20} + \mathcal{D}K(-i)\Gamma_{04} - W\Big\}\epsilon_i = 0, \quad i = 1, 2, 3. \tag{6.8.14}$$

For ϵ_2 (6.8.10–ii),

$$(6.8.14) \Leftrightarrow \Big\{s\sin\theta\,\mathcal{A}\cdot\Gamma_{51} + \sin\theta\,\mathcal{B}\Gamma_{53}e^{\theta\Gamma_{45}} - s\,\mathcal{C}\cdot\Gamma_{31} + \mathcal{D}e^{\theta\Gamma_{45}} - W\Big\}\xi_+ = 0, \tag{6.8.15}$$

where we have used relations obtained from (6.8.12), (6.8.13) and $\gamma\xi_\pm = -i\Gamma_{0123}\xi_\pm = \pm\xi_\pm$,

$$\Gamma_{62}\xi_\pm = \Gamma_{51}\xi_\pm, \tag{6.8.16–i}$$

$$\Gamma_{60}\xi_\pm = \pm i\Gamma_{53}\xi_\pm, \tag{6.8.16–ii}$$

$$\Gamma_{20}\xi_\pm = \pm i\Gamma_{31}\xi_\pm. \tag{6.8.16–iii}$$

The left hand side of (6.8.15) can be written only by using $\Gamma_1, \Gamma_3, \Gamma_4, \Gamma_5$ and $\mathbf{1}$ (identity matrix) and their products. Each coefficient of independent matrices gives the conditions:

$$s\{s,\cos\theta\}-\sin\theta\{P,z\sin\theta\}+s^3\{Q,\frac{z}{s}\}-\cos\theta\{P,Q\}-W=0, \tag{6.8.17–i}$$

$$s\{z,\theta\}-\{P,s\sin\theta\}=0, \tag{6.8.17–ii}$$

$$s\sin^2\theta\{P,\frac{z}{s}\}+\{Q,z\}+\cos\theta\{P,Q\}=0, \tag{6.8.17–iii}$$

$$s^2\sin\theta\cos\theta\{P,\frac{z}{s}\}+sz\{Q,\theta\}-s\sin\theta\{P,Q\}=0, \tag{6.8.17–iv}$$

$$s^3\{\frac{z}{s},\cos\theta\}+\frac{1}{2}\{P,\cos^2\theta\}-s\{Q,s\}+z\cos\theta\{P,Q\}=0, \tag{6.8.17–v}$$

$$\{P,z\cos\theta\}-\{P,Q\}=0, \tag{6.8.17–vi}$$

$$\sin\theta\{P,s\cos\theta\}-s\{Q,\theta\}=0. \tag{6.8.17–vii}$$

In this equations (6.8.17–iv) is not independent and can be lead from (6.8.17–vi) and (6.8.17–vii). For ϵ_3, a similar calculation gives the same conditions. For ϵ_1, the calculation is a bit complicated, but we can do it in the same way.

$$-\frac{s^4}{2}\{\frac{\beta}{s^2},\cos\theta\}+\frac{z^3\sin^2\theta}{2}\{P,\frac{\beta}{z^2}\}-\beta\cos\theta\{P,Q\}-W=0, \tag{6.8.18–i}$$

$$\frac{sz^3}{2}\{\frac{\beta}{z^2},\cos\theta\}+\frac{s^3\sin^2\theta}{2}\{P,\frac{\beta}{s^2}\}+\frac{s}{2}\{Q,\beta\}=0, \tag{6.8.18–ii}$$

$$\frac{1}{2}\{\beta,\cos\theta\}-\frac{z^3}{2}\{Q,\frac{\beta}{z^2}\}-W=0, \tag{6.8.18–iii}$$

$$\frac{1}{2}\{P,\beta\sin^2\theta\}-\frac{s^4}{2}\{Q,\frac{\beta}{s^2}\}+zW=0, \tag{6.8.18–iv}$$

$$\cos\theta\{s^2,z\}+\{P,\beta\cos^2\theta\}=0, \tag{6.8.18–v}$$

$$\frac{1}{4}\{s^2,z^2\}+\frac{z^3\cos\theta}{2}\{P,\frac{\beta}{z^2}\}+\beta\{P,Q\}=0, \tag{6.8.18–vi}$$

$$s\sin\theta\{s,z\}+\frac{s^2\sin\theta\cos\theta}{2}\{P,\frac{\beta}{s^2}\}+\beta\{Q,\theta\}=0. \tag{6.8.18–vii}$$

Consequently, we obtain the 14 equations (6.8.17–i)-(6.8.17–vii) and (6.8.18–i)-(6.8.18–vii). We find independent set of these equations in the next section.

6.9 Equation describing D5-brane and boundary condition

One can check the last seven equations, (6.8.18–i)-(6.8.18–vii), are derived from eqs. (6.8.17–i)-(6.8.17–vii). So we only have to consider eqs. (6.8.17–i)-(6.8.17–vii) which are rewritten as

$$\{s,z\} = -\frac{1}{s\cos\theta}\{P,\beta\cos^2\theta\}, \tag{6.9.1–i}$$

$$\{s,\theta\} = -\frac{1}{s}\{P,z\sin\theta\} + \frac{s^2}{\sin\theta}\{Q,\frac{z}{s}\} - \frac{1}{s}\cot\theta\{P,Q\} - \frac{1}{s\sin\theta}W, \tag{6.9.1–ii}$$

$$\{z,\theta\} = \frac{1}{s}\{P,s\sin\theta\}, \tag{6.9.1–iii}$$

$$\{Q,s\} = s^2\{\frac{z}{s},\cos\theta\} + \frac{1}{2s}\{P,\cos^2\theta\} + \frac{1}{2s}\{P,z^2\cos^2\theta\}, \tag{6.9.1–iv}$$

$$\{Q,z\} = -s\sin^2\theta\{P,\frac{z}{s}\} - \cos\theta\{P,z\cos\theta\}, \tag{6.9.1–v}$$

$$\{Q,\theta\} = \frac{\sin\theta}{s}\{P,s\cos\theta\}, \tag{6.9.1–vi}$$

$$\{P,Q\} = \{P,z\cos\theta\}. \tag{6.9.1–vii}$$

By the definition of the Poisson bracket (6.8.8), the bracket can be rewritten in terms of differential forms as

$$\{A,B\}du^1\wedge du^2 = \partial_i A\partial_j B\epsilon^{ij}du^1du^2 = dA\wedge dB = d(A\wedge dB). \tag{6.9.2}$$

Then eqs. (6.9.1–i)-(6.9.1–vii) are expressed in terms of differential forms as follows.

$$d(\sqrt{\beta}(dz-\cos\theta dP)) = 0, \tag{6.9.1–i$'$}$$

$$sds\wedge d(\cos\theta) - \sin\theta dP\wedge d(z\cos\theta) + s^3dQ\wedge d(\frac{z}{s}) - \cos\theta dP\wedge dQ - Wdu^1\wedge du^2 = 0, \tag{6.9.1–ii$'$}$$

$$sdz\wedge d(\cos\theta) + \sin\theta dP\wedge d(s\sin\theta) = 0, \tag{6.9.1–iii$'$}$$

$$d(P+Q)\wedge\frac{dz}{z} - \frac{\sin^2\theta}{s}dP\wedge ds - \sin\theta dP\wedge d(\sin\theta) = 0, \tag{6.9.1–iv$'$}$$

$$sdQ\wedge ds - \frac{1}{2}dP\wedge d((z^2+1)\cos^2\theta) - s^3d(\frac{z}{s})\wedge d(\cos\theta) = 0, \tag{6.9.1–v$'$}$$

$$sdQ\wedge d(\cos\theta) + \sin^2\theta dP\wedge d(s\cos\theta) = 0, \tag{6.9.1–vi$'$}$$

$$d(dP(Q-z\cos\theta)) = 0. \tag{6.9.1–vii$'$}$$

Since eq. (6.9.1–vi$'$) can be written as a total derivative, it is expressed as the derivative of a appropriate function ω according to Poincaré's lemma:

$$d(-(Q+\sin^2\theta P)\frac{d\theta}{\sin\theta\cos\theta} + P\frac{ds}{s}) = 0 \quad\Leftrightarrow\quad -(Q+\sin^2\theta P)\frac{d\theta}{\sin\theta\cos\theta} + P\frac{ds}{s} = d\omega. \tag{6.9.4}$$

Eqs. (6.9.1–iii$'$), (6.9.1–vii$'$) lead to the relation

$$z\cos\theta = P+Q. \tag{6.9.5}$$

Furthermore, eq. (6.9.1–iv′) and eq. (6.9.1–v′) are equivalent to eq. (6.9.1–vi′) and eq. (6.9.1–i′), respectively. We also substitute the explicit form of W into eq. (6.9.1–ii). Then our equations are simplified as follows.

$$d(\sqrt{\beta}(dz - \cos\theta dP)) = 0, \tag{6.9.6–i}$$

$$d\left(-(Q + \sin^2\theta P)\frac{d\theta}{\sin\theta\cos\theta} + P\frac{ds}{s}\right) = 0, \tag{6.9.6–ii}$$

$$z\cos\theta = P + Q, \tag{6.9.6–iii}$$

$$\begin{aligned}\left(\frac{s^2}{\cos^2\theta} + 1\right)\{P, \cos\theta\}^2 + \frac{s^2}{\cos^2\theta}\{Q, \theta\}^2 + \frac{s}{\cos\theta}\{s, \cos\theta\}\Big(\{P, Q\} - z\{P, \cos\theta\}\Big)& \\ + z\{P, \cos\theta\}\{P, Q\} + 2\frac{s^2}{\cos^2\theta}\{P, \cos\theta\}\{Q, \cos\theta\} = 0.&\end{aligned} \tag{6.9.6–iv}$$

This is one of the main results of this thesis.

6.10 Special case

Let us check the consistency of these equations in the well known case [6] where

$$P = 0, \;\; Q = \kappa\cos\theta, \;\; z = \kappa. \tag{6.10.1}$$

We can easily check that this configuration satisfies eqs. (6.9.6–i)-(6.9.6–iv).

This configuration contains no D1-brane and corresponds to the ’t Hooft operator with the trivial Young diagram.

Chapter 7

Summary

In this thesis we aim to test the AdS/CFT correspondence [1] by virtue of non-local operators. According to this conjecture, the system consisting of N parallel D3-branes gives two different theory — IIB superstring in the near horizon limit and $\mathcal{N} = 4$ d=4 super Yang-Mills in low energy limit. Non-local operators have counterparts in string theory in terms of branes or fundamental strings. The interface is introduced by considering a probe D5-brane. This interface divides the whole space into two spaces where two different gauge theory live.

Such a non-local operator generally introduce a parameter in the theory. Thanks to this parameter, it is possible to compare the results from gauge theory and from gravity theory.

In chapter 4 and chapter 5, in addition to the interface we added a chiral primary operator or a test particle. We calculated the expectation value of the operators in the existence of the interface. The gauge calculation is performed using the classical solution. On the other hand, gravity calculation is performed by virtue of the probe D5-brane. We found completely agreement between the gauge and gravity results in the classical level.

In chapter 6 we aimed to relate a brane configuration which consists of parallel N D3-branes, a D5-brane and D1-branes to a representation of a gauge theory object. Our results reveals the procedure to investigate the detailed construction of branes in terms of Young diagrams. Our system preserves a quarter of the supersymmetry. The embedding of the D5-brane in bulk spacetime is determined by the equations (6.9.6). To solve these equation we need the boundary condition of the unknown functions P and Q. The values of these functions are related to the number of the boxes in the Young diagram by the definition of charges: (6.6.1) and (6.6.2).

The existence of solutions is confirmed — at least in the simplest case, solution (6.10.1), corresponding to the trivial Young diagram.

Chapter 8

Discussion and Future problem

About Correlation functions

In this section we investigate the 1/2 BPS interface, in particular the potential between this interface and a test particle. We calculated the potential both in the gauge theory side and the gravity side and found perfect agreement in the leading order. This is strong evidence of the AdS/CFT correspondence including the interface.

In the gravity side we also obtained sub-leading corrections of a power series of λ/k^2. This may be compared to the perturbative corrections in the gauge theory side. It will be an interesting future work to calculate these sub-leading corrections in the gauge theory side and see if they agree with the gravity side.

Here we give a heuristic argument on the perturbative corrections in the gauge theory side, in particular the λ/k^2 behavior of the corrections. In order to calculate the perturbative corrections, we express the field as $\phi_i = \phi_i^{(0)} + \tilde{\phi}_i$ where $\phi_i^{(0)}$, $(i = 4, 5, 6)$ are the classical solution (2.3.7) and $\tilde{\phi}_i$ are the fluctuations of the fields. For simplicity let us perform the following Weyl transformation and go to AdS_4 frame

$$A_\mu \to A_\mu, \quad \psi \to e^{3\Omega/2}\psi, \quad \phi_i \to e^{\Omega}\phi_i, \quad (e^{\Omega} := r/x_3), \tag{8.0.1}$$

where r is a constant. The metric becomes by this Weyl transformation

$$ds^2 = \frac{r^2}{x_3^2}(\eta_{\mu\nu}dx^\mu dx^\nu). \tag{8.0.2}$$

The classical solution (2.3.7) is now simply the constant vacuum expectation value

$$\phi_i^{(0)} = \frac{1}{r}t^i \oplus 0_{(N-k)\times(N-k)}, \quad (i = 4, 5, 6). \tag{8.0.3}$$

This is an analogue as the Higgs mechanism. Actually the gauge field gets mass as the following way. The Lagrangian density includes

$$\mathrm{tr}([A_\mu, \phi_i^{(0)}]^2) = -\frac{1}{2r^2}k^2\mathrm{tr}_{k\times k}(A_\mu A_\mu) + \cdots . \tag{8.0.4}$$

Some of the scalar fields also have mass square term proportional to k^2. These k^2 terms are the leading terms in the Lagrangian density in the large k limit. Therefore the action can be written as

$$S = N\frac{k^2}{\lambda}\int d^4x\sqrt{g}\mathcal{L}', \tag{8.0.5}$$

where $\mathcal{L}'$ is a function of the fields and their derivatives, which satisfies

$$\lim_{k\to\infty}\mathcal{L}' = (\text{finite}). \tag{8.0.6}$$

From this form of the action we expect that the perturbative corrections will be a power series of λ/k^2 in the large k limit.

About Bubbling D5-brane

We already know a solution of our equations satisfying the given boundary conditions — it is the simplest case, as we saw in the previous section 6.10. Then finding other nontrivial solutions is an interesting future work. We do not know these system defines a unique solution yet.

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Appendix A

Gamma matrices

$\Gamma_M,\ M = 0, 1, \cdots, 9$ are the 10-dimensional gamma matrices satisfying the algebra

$$\{\Gamma_M, \Gamma_N\} = 2\eta_{MN}. \tag{A.0.1}$$

$\eta_{MN} = \mathrm{diag}(-1, +1, \cdots, +1)$ is the metric of 10-dimensional Minkowski space. We also use the matrices with anti-symmetric indices.

$$\Gamma_{MN} = \frac{1}{2}(\Gamma_M\Gamma_N - \Gamma_N\Gamma_M). \tag{A.0.2}$$

Gamma matrices with more than two indices are defined in the same way.

Appendix B

Elliptic integrals

The definition of elliptic integrals are

$$F(\varphi, h) := \int_0^{\sin\varphi} \frac{du}{\sqrt{(1-u^2)(1-h^2u^2)}} \quad : \text{ the first kind,} \tag{B.0.1}$$

$$E(\varphi, h) := \int_0^{\sin\varphi} du \sqrt{\frac{1-h^2u^2}{1-u^2}} \quad : \text{ the second kind.} \tag{B.0.2}$$

And we give a useful formula

$$F(\varphi, h) - E(\varphi, h) = \int_0^{\sin\varphi} du \frac{h^2u^2}{\sqrt{(1-u^2)(1-h^2u^2)}}. \tag{B.0.3}$$

We just introduced these functions. A reference of this sort is, for example, [43].

Appendix C

Spherical harmonics

C.1 SO(3) $\times$ SO(3) invariant ansatz

Our interface preserves SO(3)×SO(3) symmetry out of SO(6) R-symmetry, rotation in 4,5,6 and 7,8,9 space, see table 2.1. Thus only SO(3)×SO(3) invariant operators can have non-vanishing expectation values. We would like to introduce SO(3)× SO(3) invariant spherical harmonics on S^5.

The S^5 is described as a hypersurface in 6-dimensional Euclidean space whose coordinates are $(x_4, \dots, x_9)$. S^5 is defined by the equation

$$x_4^2 + \cdots + x_9^2 = 1. \tag{C.1.1}$$

We introduce a parameter ψ, $0 \leq \psi \leq \frac{\pi}{2}$ and reexpress this S^5 as the following way.

$$x_4^2 + x_5^2 + x_6^2 = \sin^2\psi, \;\; x_7^2 + x_8^2 + x_9^2 = \cos^2\psi. \tag{C.1.2}$$

Then the metric is written as

$$ds^2 = d\psi^2 + \cos^2\psi d\tilde{\Omega}_2^2 + \sin^2\psi d\Omega_2^2, \tag{C.1.3}$$

where $d\tilde{\Omega}_2^2$ and $d\Omega_2^2$ are line elements of unit S^2.

C.2 Expressed as hypergeometric function

The SO(3)×SO(3) invariant spherical harmonics only depends on the coordinate ψ. Let Y be such a function of ψ; $Y = Y(\psi)$. The Laplacian operating on this Y is written as

$$\Box Y = \frac{1}{\sqrt{g}}\partial_i\sqrt{g}g^{ij}\partial_j Y = \frac{1}{\cos^2\psi\sin^2\psi}\frac{d}{d\psi}\cos^2\psi\sin^2\psi\frac{d}{d\psi}Y(\psi). \tag{C.2.1}$$

After changing the variable $z := \cos^2\psi$, the Laplacian is rewritten as

$$\Box Y = 4z(1-z)\partial_z^2 Y + (6-12z)\partial_z Y. \tag{C.2.2}$$

Then the eigenvalue equation, $\Box Y = -EY$, reads

$$z(1-z)\partial_z^2 Y + \left(\frac{3}{2} - 3z\right)\partial_z Y + \frac{E}{4}Y = 0. \tag{C.2.3}$$

This is a hypergeometric differential equation.

In general a hypergeometric differential equation is given by

$$z(1-z)\partial_z^2 F + (c - (a+b+1)z)\partial_z F - abF = 0, \tag{C.2.4}$$

where a, b, c are real parameters. The solution which is regular at $z = 0$ is the hypergeometric function given by an infinite power series

$$F(a,b,c;z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!}. \tag{C.2.5}$$

Here the Pochhammer symbol $(a)_n = \Gamma(a+n)/\Gamma(a)$ is used.

Since we need the smooth solution on whole S^5, the solution of eq. (C.2.3) must be regular not only at $z = 0$ but $z = 1$. Then the solution must be a hypergeometric function with $a = -\ell, b = \ell + 2, c = 3/2$, $(\ell = 0, 1, 2, 3, \dots)$ and the eigenvalue $E = 2\ell(2\ell + 4)$ is obtained.

Therefore the solution of the equation (C.2.3) is expressed in terms of hypergeometric function.

$$Y_\ell(\psi) = C_\ell F(-\ell, 2+\ell, 3/2; \cos^2\psi), \tag{C.2.6}$$

where the normalization factor C_ℓ is determined by

$$\int_{S^5} \sqrt{g}|Y_\ell|^2 = \frac{\pi^3}{2^{2\ell-1}(2\ell+1)(2\ell+2)}. \tag{C.2.7}$$

For the detailed calculation, see an appendix of [25]. The conformal dimension Δ of the corresponding chiral primary operator is $\Delta = 2\ell$.

Appendix D

Representation of Wilson/'t Hooft operators

D.1 Young diagram

In this paper we consider the 't Hooft operator in 4-dimensional $\mathcal{N} = 4$ super Yang-Mills theory with gauge group $SU(N)$. We need to determine an irreducible representation of the gauge group $SU(N)$ to define this operator. This representation is classified in terms of Young diagrams. Let us review briefly properties of these operators.

A Young diagram is expressed by boxes arranged in left-justified rows. An example is shown in figure D.1. There are k_k boxes in the k-th row and l_l boxes in the l-th column. We denote this Young diagram as $R = (k_1, k_2, \cdots, k_K)$, $k_1 \geq k_2 \geq \cdots \geq k_K$. We are considering the representation of the group $SU(N)$. Then the vertical length of the columns is less than N while the horizontal length can take an arbitrary value.

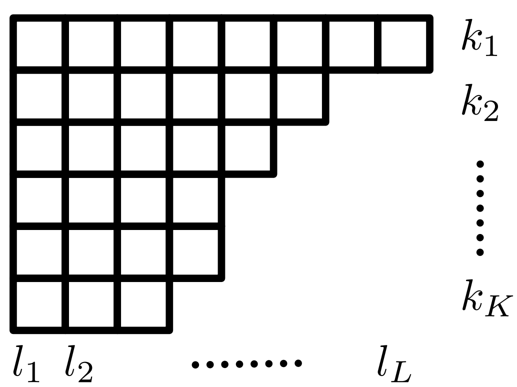

Figure D.1: Young diagram: For $SU(N)$, $l_l < N$

D.2 Representation as branes

The Wilson loop we treat in this paper is just a trivial case, □. Let us survey some cases which correspond to non-trivial Young diagram. These cases have found in [21]. We show here some typical cases of the Young diagram. We denote Dp-brane with k units of charge by Dp_k. In this notation, the horizontal and vertical Young diagrams correspond to the following branes, respectively.

$$D5_k \leftrightarrow Z = e^{iS_{\mathcal{N}=4}} \cdot W_{(1,1,\cdots,1,0,\cdots,0)}, \tag{D.2.1}$$

$$D3_k \leftrightarrow Z = e^{iS_{\mathcal{N}=4}} \cdot W_{(k,0,\cdots,0)}. \tag{D.2.2}$$

Appendix E

Representation of superconformal algebra in $\mathcal{N} = 4$ SYM

The conformal symmetry of 4-dimensional Minkowski space consists of $\mathfrak{psu}(2, \mathbb{C})$ Lorentz group $L, \bar{L}$, translation P, dilatation D, and the conformal boost K. In addition to these superconformal algebra contains $\mathfrak{su}(4)$ rotations R, the supertranslations $Q, \bar{Q}$ and the superconformal boosts $S, \bar{S}$ [44]. They obey the conformal algebra,

$$[M_{\mu\nu}, P_\rho] = -i(\eta_{\mu\rho}P_\nu - \eta_{\nu\rho}P_\mu),\ \ [M_{\mu\nu}, K_\rho] = -i(\eta_{\mu\rho}K_\nu - \eta_{\nu\rho}K_\mu),$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = -i(\eta_{\mu\rho}M_{\nu\sigma} + \text{permutations}),$$

$$[M_{\mu\nu}, D] = 0,\ \ [D, K] = iK_\mu,\ \ [D, P_\mu] = -iP_\mu,\ \ [P_\mu, K_\nu] = 2iM_{\mu\nu} - 2i\eta_{\mu\nu}D,$$

and the following supersymmetry commutation relations,

$$\begin{aligned}
&[D, Q] = -\frac{i}{2}Q,\ \ [D, S] = \frac{i}{2}S,\ \ [K, Q] \simeq S,\ \ [P, S] \simeq Q, \\
&\{Q, Q\} \simeq P,\ \ \{S, S\} \simeq K,\ \ \{Q, S\} \simeq M + D + R.
\end{aligned} \tag{E.0.1}$$

Their explicit representations can be described as harmonic oscillators [45];

$$[\mathbf{a}^\alpha, \bar{\mathbf{a}}_\gamma] = i\delta^\alpha_\gamma,\ \ [\mathbf{b}^{\dot\alpha}, \bar{\mathbf{b}}_{\dot\gamma}] = i\delta^{\dot\alpha}_{\dot\gamma},\ \ \{\mathbf{c}^a, \bar{\mathbf{c}}_c\} = \delta^a_c, \tag{E.0.2}$$

and other commutators are zero. Here we have used spinor notation, e.g., for a scalar field $\Phi(x)$ at $x = 0$

$$\partial_{\beta\dot\alpha}\partial_{\delta\dot\gamma} \cdots \Phi \simeq \bar{\mathbf{a}}_\beta \bar{\mathbf{a}}_\delta \cdots \bar{\mathbf{b}}_{\dot\alpha} \bar{\mathbf{b}}_{\dot\gamma} \cdots |0\rangle\,. \tag{E.0.3}$$

Using this oscillator representation, superconformal operators are expressed as follows:

$$
\begin{aligned}
&L^{\alpha}{}_{\gamma} \simeq \bar{\mathbf{a}}_\gamma \mathbf{a}^\alpha - \frac{1}{2}\delta^\alpha_\gamma \bar{\mathbf{a}}_\beta \mathbf{a}^\beta, \quad \bar{L}^{\dot{\alpha}}{}_{\dot{\gamma}} \simeq \mathbf{b}^{\dot{\alpha}} \bar{\mathbf{b}}_{\dot{\gamma}} - \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\gamma}} \mathbf{b}^{\dot{\beta}} \bar{\mathbf{b}}_{\dot{\beta}}, \\
&R^a_c \simeq \bar{\mathbf{c}}_c \mathbf{c}^a - \frac{1}{4}\delta^\alpha_\gamma \bar{\mathbf{c}}_b \mathbf{c}^b, \\
&D \simeq \frac{1}{2}\bar{\mathbf{a}}_\alpha \mathbf{a}^\alpha + \frac{1}{2}\mathbf{b}^{\dot{\alpha}} \bar{\mathbf{b}}_{\dot{\alpha}}, \\
&P_{\gamma\dot{\alpha}} \simeq \bar{\mathbf{a}}_\gamma \bar{\mathbf{b}}_{\dot{\alpha}}, \quad K^{\gamma\dot{\alpha}} \simeq \mathbf{b}^{\dot{\alpha}} \mathbf{a}^\gamma, \\
&Q^a_\gamma \simeq \bar{\mathbf{a}}_\gamma \mathbf{c}^a, \quad \bar{Q}_{\dot{\gamma} a} \simeq \bar{\mathbf{c}}_a \bar{\mathbf{b}}_{\dot{\gamma}}, \\
&S^\gamma_a \simeq \bar{\mathbf{c}}_a \mathbf{a}^\gamma, \quad \bar{S}^{\dot{\gamma} a} \simeq \mathbf{b}^{\dot{\gamma}} \mathbf{c}^a.
\end{aligned} \tag{E.0.4}
$$

For the reality condition of these operators, creation and annihilation operators should satisfy

$$
(\bar{\mathbf{a}}_\alpha)^\dagger = \bar{\mathbf{b}}_{\dot{\alpha}}, \ \ (\mathbf{a}^\alpha)^\dagger = \bar{\mathbf{b}}^{\dot{\alpha}}, \ \ (\bar{\mathbf{c}}_a)^\dagger = \mathbf{c}^a. \tag{E.0.5}
$$

Appendix F

Kappa symmetry

This symmetry is need in order for the bosonic and fermionic fields to have equal numbers of degrees of freedom [37, 38, 39, 40, 41, 42, 31]. The kappa-symmetry transformation eliminates half of the sermonic degrees of freedom.

This symmetry projection plays a crucial role in our research. The supersymmetry with the parameters which satisfy

$$\Gamma\epsilon = \epsilon \tag{F.0.1}$$

survives in the presence of a D-brane. Here the projection operator Γ is defined for a Dp-brane in type IIB string theory as

$$d^{p+1}\xi \cdot \Gamma := \left(-e^{-\Phi}(-\det(G_{\text{ind}} + \mathcal{F}))^{-1/2} e^{\mathcal{F}} \chi\right)\Big|_{(p+1)-\text{form}}, \tag{F.0.2}$$

$$\chi := \sum_n \frac{1}{(2n)!} \hat{E}^{a_{2n}} \cdots \hat{E}^{a_1} \Gamma_{a_1 \cdots a_s} K^n(-i), \tag{F.0.3}$$

where ξ^i,$i = 0, \cdots, p$, are worldvolume coordinates, Φ is the dilaton which is zero now, G_{ind} is the induced metric of the Dp-brane and $\hat{E}^A$ is the pull back of E^A defined as $\hat{E}^A := E^A_M \frac{\partial X^M}{\partial \xi^i} d\xi^i$.

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