Landau-Khalatnikov Damping of Ultrasound in Heavy-Fermion Superconductors

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The Landau-Khalatnikov mechanism associated with the relaxation of the order-parameter amplitude gives a peak in the longitudinal ultrasonic attenuation α near T_c, which quantitatively accounts for the observed peak in UBe_{13} and UPt_3. At lower temperatures this mechanism gives a power-law attenuation.

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Recent experiments of *longitudinal* ultrasonic attenuation in UBe_{13}[1] and UPt_3[2] have shown that there exists a peak just below the transition temperature T_c. It was suggested in Ref. 1 that such a peak might provide evidence of a collective mode associated with internal degrees of freedom of anisotropic pairing as in superfluid ^{3}He. Such a peak has not been observed in ordinary (singlet isotropic) superconductors.[3]

In this paper, we show the following: (i) A peak can be explained by the Landau-Khalatnikov (LK) mechanism associated with the relaxational mode of the order-parameter amplitude.[4,5] This mechanism works *in principle* for *all* types of pairing. (But as explained later for ordinary isotropic pairing, its magnitude is much smaller.) So the attenuation peak itself does not provide evidence for anisotropic pairing. (ii) Also at low temperatures $T \ll T_c$, the LK mechanism gives a nonnegligible contribution[6] $\alpha_r \propto T^n$ to the sound attenuation; the exponent n depends on the nature of the pairing and the mechanism of collision between quasiparticles and impurities.[7]

The essence of the LK mechanism is to take into account the existence of the macroscopic degrees of freedom which are associated with the order parameter η which tends to relax to its local equilibrium η_{le} as determined by the other hydrodynamical variables (in the present case, the dilatation δ of the lattice). Entropy production associated with the relaxation process $\eta \rightarrow \eta_{le}(\delta)$ causes sound attenuation in general. The equation of motion for the dilatation δ is written in terms of the free-energy density $F(\delta,\eta)$ as

$$\rho \frac{\partial^2 \delta}{\partial t^2} = \frac{\partial^2}{\partial z^2} \frac{\partial F(\delta,\eta)}{\partial \delta}, \tag{1}$$

where ρ is the mass density of the lattice and it is assumed that the longitudinal sound is propagating along c ($\parallel \hat{z}$) axis. If $\eta_{le}(\delta)$ were substituted into (1), it would reduce to the usual wave equation. The fluctuating part of the "pressure" $P \equiv \partial F(\delta,\eta)/\partial\delta$ consists of two parts:

$$P' = \left(\frac{\partial P}{\partial \delta}\right)_\eta \delta' + \left(\frac{\partial P}{\partial \eta}\right)_\delta \eta'. \tag{2}$$

Hereafter primed quantities are assumed to vary as $\exp[i(kz-\omega t)]$. By assuming that the dilatation δ' gives the change in the chemical potential μ' of the electronic system, $\mu' \propto \delta'$, and that $k\xi_0 \ll 1$ (ξ_0 being the coherence length) so that motion of η is determined locally, it follows from linear response theory[8,9] that

$$\eta'^k(\omega) = \frac{\chi_{\eta n}(k,\omega)}{\chi_{\eta n}(k,0)} \frac{d\eta}{d\delta} \delta'^k(\omega). \tag{3}$$

Here $\chi_{AB}(k,\omega)$ denotes a linear response function $\langle\langle A_k, B_{-k}\rangle\rangle(\omega)$ and $d\eta/d\delta$ is the isothermal derivative. Then, substituting (3) and (2) into (1), we obtain the velocity $s(\omega)$ and the attenuation $\alpha_r(\omega)$:

$$s(\omega) \simeq s_0 - \frac{s_\infty^2 - s_0^2}{2s_0} \mathrm{Re} I(\omega), \tag{4}$$

$$\alpha_r(\omega) \simeq \frac{s_\infty^2 - s_0^2}{2s_0^3} \mathrm{Im} I(\omega)\omega, \tag{5}$$

where

$$I(\omega) \equiv \lim_{k\rightarrow 0} \frac{\chi_{\eta n}(k,\omega) - \chi_{\eta n}(k,0)}{\chi_{\eta n}(k,0)},$$

$$s_\infty^2 \equiv \frac{(\partial P/\partial\delta)_\eta}{\rho},$$

and

$$s_\infty^2 - s_0^2 \equiv \frac{1}{\rho}\left(\frac{\partial P}{\partial \eta}\right)_\delta \frac{d\eta}{d\delta}. \tag{6}$$

In order to calculate $I(\omega)$, we assume a "polarlike" gap as a simple representative of those with nodes along lines on the Fermi surface, since the state with such a gap seems to explain the existing experiments consistently.[7] We shall mention the results for the "axial-like" gap also.

Critical regime [$0 < t \equiv 1 - T/T_c \ll 1$].—The response function $\chi_{\eta n}(k,\omega)$ with

$$\eta_k \equiv \sum_p \hat{p}_z (a_{p+k/2} a_{-p+k/2} + \mathrm{H.c.})/\sqrt{2}$$

and $n_k \equiv \sum_p a^\dagger_{p-k/2} a_{p+k/2}$ can be calculated easily with

the random-phase approximation as in Refs. 8 and 9. In the critical regime, the main contribution to the fully renormalized collision rate $1/\tau$ of *quasiparticles* is expected to come, for anisotropic pairing, from nonmagnetic impurity scattering and is approximated by $1/\tau_N$ (τ_N being the lifetime in the normal state). In the hydrodynamical region $\omega\tau \ll 1$, $\chi_{\eta\eta}(k,\omega)$ can be calculated analytically.[8] The result for $I(\omega)$ is

$$I(\omega) = i\omega\tau_r(T)/[1 - i\omega\tau_r(T)], \tag{7}$$

where the relaxation time $\tau_r(T)$ shows the so-called *critical slowing down*[9]:

$$\tau_r(T) \simeq \begin{cases} \frac{1}{16}\pi^2[10/7\zeta(3)]^{1/2}, & \Delta(T)\tau_N \gg 1, \\ \frac{1}{48}\pi(\hbar/k_B T_c)t^{-1}, & \Delta(T)\tau_N \ll 1. \end{cases} \tag{8}$$

Equation (7) has the familiar Debye form but with a lifetime determined by the critical fluctuations. The maximum gap $\Delta(T)$ is defined by $\Delta_p \equiv \Delta\hat{p}_z$. With use of (7), the sound velocity (4) and attenuation (5) are written as

$$s(\omega) \simeq s_0 + \frac{s_\infty^2 - s_0^2}{2s_0}\frac{\omega^2\tau_r^2}{1+\omega^2\tau_r^2}, \tag{9}$$

$$\alpha_r(\omega) \simeq \frac{s_\infty^2 - s_0^2}{2s_0^3}\frac{\omega^2\tau_r}{1+\omega^2\tau_r^2}. \tag{10}$$

One can see from (9) and (10) that $(s_\infty^2 - s_0^2)/2s_0$ is equal to the velocity change Δs across the transition point, and that the attenuation α_r exhibits a peak around $\omega\tau_r(T) = 1$ as a function of T with fixed frequency ω. The peak position is given by $t_p \equiv 1 - T_p/T_c \sim (\omega\tau_N)^2$ for $\Delta(T_p)\tau_N \gg 1$ and $t_p \simeq (\pi\hbar\omega/48k_B T_c)$ for $\Delta(T_p)\tau_N \ll 1$, respectively, which in turn makes it possible to determine τ_N from the peak position. This value can be checked against an independent measurement, say from the resistivity or the attenuation in the normal state. It should be remarked that the peak height $\alpha_p(\omega) \simeq (\Delta s/2s^2)_{T_c}\omega$ is determined completely by the sound velocity and its jump at T_c and is proportional to ω.

If we use the measured values of $\Delta s/s \sim 25\times10^{-6}$ (13×10^{-6}) and $s \sim 8\times10^5$ cm/sec (3.9×10^5 cm/sec) for UBe_{13}[1] (UPt_3),[10] the peak height is predicted as $\alpha_p(\omega) \simeq 9.8\times10^{-2}\omega/2\pi$ cm^{-1} GHz ($1.0\times10^{-1}\omega/2\pi$ cm^{-1} GHz). When we compare these predictions with experiments, we have to take into account carefully the following two factors: (i) These compounds have rather wide transition widths δT_c (~ 40 mK in UBe_{13}[1] and ~ 15 mK in UPt_e[2]). Therefore, if $t_p < \delta T_c/T_c$, the peak is reduced considerably as a result of smearing. (ii) The observed attenuation α should be $\alpha \simeq \alpha_{qp} + \alpha_r$, where α_{qp} is the contribution from quasiparticle collisions[7,11] which decreases rather rapidly when going away from T_c (say around $t \sim 10^{-1}$). This contribution shifts the observed peak position closer to T_c than that given by t_p.[12] With consideration of these two factors, the quantitative agreement between the above predictions for $\alpha_p(\omega)$ and the experiments in UBe_{13}[1] and UPt_3[2] is not bad (see Table I.) Discrepancies between the theory and the experiment in UPt_3 for higher frequencies are due to the factor (ii), while those in UBe_{13} for lower frequencies are due to the factor (i). The values of $T_c - T_p$ used to get the theoretical numbers in Table I are the observed values, ≈ 30 mK at 0.092 GHz in UPt_3 and 50 mK at 2.4 GHz in UBe_{13}. From these values and the relation (8), the lifetime τ_N is estimated as $\tau_N \sim 6\times10^{-10}$ sec for UPt_3[2] and $\tau_N < 5\times10^{-12}$ sec for UBe_{13},[1,13] where $\tau_N\Delta(T_p) \sim 8$ for UPt_3 and $\tau_N\Delta(T_p) < 1$ for UBe_{13}.[14] The position of the peak and its detailed shape depends on the value of $\Delta(T)\tau_N$. But the peak of $I(\omega)$ given in Table I occurs at $\omega\tau_r = 1$. It is calculated from the *observed* change in sound velocity and does not depend on the value of $\Delta(T)\tau_N$.

In the regime near T_c, we can see the Ginzburg-Landau form for the free-energy density

$$F(\delta,\eta) = F_0(\delta) + a'(\delta)(T - T_c)\eta^2 + b(\delta)\eta^4 + \ldots.$$

Then, we can show that the quantity $s_\infty^2 - s_0^2$, Eq. (6) is expressed in terms of thermodynamic quantities as

$$s_\infty^2 - s_0^2 = s^2\frac{\Delta C_p}{T_c}B\left(\frac{dT_c}{dP}\right)^2 \quad (T \simeq T_c), \tag{11}$$

where ΔC_p, B, and P denote the specific-heat jump at T_c, the bulk modulus, and the pressure, respectively. We have verified that relation (11), with $\Delta s \simeq (s_\infty^2 - s_0^2)/2s$, holds fairly well both in UPt_3[11] and UBe_{13}.[1]

The reason why the attenuation peak has not been observed in ordinary superconductors is as follows: In the case of isotropic pairing, the lifetime τ_N relevant to

TABLE I. Calculated and experimental magnitudes of the ultrasonic attenuation peak in UPt_3 and UBe_{13}.

f (GHz)	Theory $\alpha_p(\omega)$ (cm^{-1})	Experiment $\alpha_p - \alpha_N$ (cm^{-1})
	UPt_3[a]	
0.92	0.009	0.008
0.15	0.015	0.102
0.27	0.027	0.019
	UBe_{13}[b]	
1.3	0.13	0.075
1.7	0.17	0.014
2.4	0.24	0.24

[a]Reference 2. $T_c = 0.46$ K, $\delta T_c \simeq 15$ mK.
[b]References 1 and 13. $T_c = 0.86$ K, $\delta T_c \simeq 40$ mK.

the relaxation of the order parameter is not given by nonmagnetic impurity scattering (dynamical version of the Anderson theorem) but by the electron-phonon scattering, and is the order[9] of 10^{-8} sec, much larger than the $\sim 10^{-10}$–10^{-12} sec for heavy-fermion systems. Then, in order that the peak position, determined in that case by $t_p \sim (\omega\tau_N)^2$, is located near T_c, the frequency ω should be much less than $2\pi\times 10^{-8}$ $\sec^{-1}$; so that the peak height $\alpha_p(\omega) = (\Delta s/2s^2)\omega$ becomes about 10^{-2} times smaller than those observed in UPt_3 and UBe_{13}, even if $\Delta s/s$ were of the same order of magnitude as in UPt_3 and UBe_{13}.

The relaxational mode of the order-parameter amplitude can also couple with the transverse sound in principle. However, its coupling is proportional to $(d\eta/d\delta)(d\delta/d\delta_t)$, δ_t being the transverse deformation of the lattice [cf. Eq. (3)], so that the coupling is smaller by a factor $|(d\delta/d\delta_t)| \ll 1$ compared with the longitudinal sound. This is consistent with the fact that no attenuation peak near T_c has been observed for the transverse sound.[2]

Since the compression of the lattice in a given direction causes a change in the anisotropy of the interaction and in the density of states of quasiparticles, the longitudinal sound can couple, in principle, with the "orbital" mode associated with the internal degrees of freedom of anisotropic Cooper pairs. This might lead one to think that the attenuation peak might be explained by resonance absorption with the "orbital" modes, whose eigenfrequency is $O(\Delta)$, as in superfluid ^{3}He.[15] In the hydrodynamic region, however, these "orbital" modes become purely damped, so that the attenuation is expected to be given by a formula similar to (5) and (7) with an appropriate relaxation time of the "orbital" mode. If crystal-field anisotropies are included, the mathematical structure is quite similar to that of the orbital dynamics in ^{3}He-A, in which case the origin of the anisotropy was the dipole interaction between nuclear spins of ^{3}He.[8,16] Its relaxation time $\tau_{\rm orb}$ can be estimated in a similar way to that above: The longest τ corresponds to the fluctuations of the "orbital" axis, i.e., $\sum_p \hat{p}_x(a_p a_{-p} + \text{H.c.})$ and $(x \rightarrow y)$, and is given as

$$\tau_{\rm orb} \simeq \frac{\pi}{32}\frac{1-g_a}{g_a T_c} N(0) V_0 \begin{cases} \Delta(T)\tau_N, & \Delta(T)\tau_N \gg 1, \\ 1, & \Delta(T)\tau_N \ll 1, \end{cases} \tag{12}$$

where $(1-g_a)V_0$, with $0 < g_a < 1$, is the attractive interaction for the *wrong* components $\hat{p}_x$ and $\hat{p}_y$ if the polarlike state $\Delta_p = \Delta\hat{p}_z$ is assumed. The relaxation time $\tau_{\rm orb}$ shows *critical speeding up* in contrast with τ_r, Eq. (8). The reason is that the fluctuations of the "orbital" axis couple with the external variable, in this case the anisotropy of the thermal distribution of quasiparticles; this gives an "orbital" viscosity of Cross-Anderson type[16] which vanishes more rapidly ($\sim\Delta^3$) than the restoring force ($\sim\Delta^2$) of the "orbital" axis when approaching T_c. Therefore, the resonance $\omega\tau_{\rm orb} = 1$ never occurs as long as $\omega\tau_N \ll 1$, i.e., in the hydrodynamical region. The Carlson-Goldman modes,[17] which is essentially the phase mode of the order parameter, can also couple with the longitudinal sound in principle. However, the coupling is not large enough to give the observed peak: Indeed, the ratio between the peak value α_{CG} due to the phase mode in the pure limit and α_p due to the amplitude mode is estimated as

$$\alpha_{CG}/\alpha_p \simeq [56\zeta(3)/135\pi^3][N(0)V_0]^2 \times (T_c\tau_N)^2\tau_N\omega\,[\Delta(T)/T_c] \ll 1$$

for UPt_3[1] and UBe_{13}.[13]

Low-temperature region [$T < \Delta(T)$].—In the case $\omega\tau \ll 1$ and $T\tau \gg 1$ (τ being the lifetime of quasiparticles in the superconducting state), $I(\omega)$ can be calculated similarly as for the critical regime. Recent analysis[7] of the existing experiments shows that τ should be nearly temperature independent and of the order of τ_N, which is in marked contrast with a naive expectation $\tau(E) \propto 1/N(E)$, $N(E)$ being the density of states of excitations.[18,19] Here we assume according to Ref. 7 that τ is temperature independent. Then, $I(\omega)$ is calculated to be

$$I(\omega) \simeq i\omega\frac{81\pi\zeta(3)}{16}\tau\left(\frac{T}{\Delta}\right)^3 + \ldots . \tag{13}$$

Substituting (13) into (5), we obtain[20]

$$\alpha_r(\omega) \simeq g(T)\frac{\Delta s}{s^2(T_c)}\frac{81\pi\zeta(3)}{16}\tau\left(\frac{T}{\Delta}\right)^3\omega^2, \tag{14}$$

where $g(T)$ is a slowly varying function with $g(T_c) = 1$. With use of the experimental values for $\Delta s/s(T_c)$, α_N (α_N being the attenuation at T_c), and the lifetime τ ($\sim\tau_N$ is estimated from the peak position of attenuation) the ratio α_r/α_N is estimated as $\alpha_r/\alpha_N \sim 26g(T/\Delta)^3$ and $6g(T/\Delta)^3$ for UBe_{13}[1] and UPt_3,[2] respectively. The T^3 dependence of α observed at T down to $\sim T_c/2$ in UPt_3[2] is in agreement with the above prediction, Eq. (14), although it should be remembered that other mechanisms[7] also contribute in this temperature region.

If the gap is "axial-like," e.g., $\Delta_p = \Delta(\hat{p}_x + i\hat{p}_y)$, sound attenuation due to the LK mechanism at low temperatures is given in the form[20]

$$\alpha_r(\omega) \simeq g(T)\frac{\Delta s}{s^2(T_c)}\frac{7\pi^4}{75}\tau\left(\frac{T}{\Delta}\right)^4\omega^2. \tag{15}$$

As far as the peak near T_c is concerned, there is essentially no difference between the predictions for the "axial-like" and the "polarlike" states.

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[20]If we used $\tau = \tau_N N_F/N(E)$ as the lifetime of quasiparticles, we would obtain

$$I(\omega) \simeq i\omega(3\pi^2/8)\tau_N(T/\Delta)^2 + \ldots$$

for (13), and

$$\alpha_r(\omega) \simeq g(T)[\Delta s/s^2(T_c)](3\pi^2/8)\tau_N(T/\Delta)^2\omega^2$$

for (14); in the case of "axial-like" gap,

$$\alpha_r(\omega) \simeq g(T)[\Delta s/s^2(T_c)](\pi^2/15)\tau_N(T/\Delta)^2\omega^2$$

for (15).