GAME THEORETIC APPROACH TO GLOBAL SUPPLY CHAIN PLANNING WITH UNCERTAIN DEMANDS

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Abstract

With the increase of use of technologies in industries, the globalisation of business structure is driven by the complex environment and various requirements from the customers. Global supply chain planning becomes a key issue for enterprises to win in the global market. Supply chain planning problems are classified according to various features such as types of decisions, operations, competition and coordination. The ultimate objective of global supply chain planning is to achieve global optimisation coordination and cooperation forecasting uncertain markets. In this thesis, the coordination of global supply chain planning between one manufacturer and multiple suppliers under demand uncertainty is investigated. A Stackelberg game theoretic approach is applied to coordinate the manufacturer and the the suppliers under demand uncertainty. It is assumed that the manufacturer is a leader and the suppliers are followers. Therefore, the manufacturer's decision problem is investiaged first. The problem is formulated as a mixed integer nonlinear programming problem including integral terms. Solution approaches are proposed to solve the problem. Next, game theoretic models for global supply chain planning are introduced to coordinate the manufacturer and the suppliers in order to improve profits and enhance competiveness. Another important factor in supply chain coordination is the quality of products, because it changes the price of products. A game theoreic model with the asymmetric quality information is investigated.

This thesis describes the following contributions. First of all, the manufacturer's decision problem with quantity discounts in supply chain planning under demand uncertainty is studied. Quantity discount is a price reduction strategy offered by suppliers to buyers who purchase a large number of products at once, which is considered one of important coordination mechanism. Quantity discounts are applied to reduce the total costs for the manufacturer's decision model. The problem is formulated as a mixed integer nonlinear programming problem (MINLP) with integral terms which is an NP-hard problem. An outer approximation algorithm with a heuristic is proposed to solve the problem.

Secondly, a reformulation of the supply chain planning problem with quantity discounts under demand uncertainty is proposed in order to reduce the computational time. The stochastic model is reformulated by a normalisation technique into an equivalent deterministic form. The stochastic model is treated by the expectation of the objective function. Then, the previously proposed outer approximation approach is used to solve the reformulated problem.

Thirdly, a game theoretic model for supply chain coordination problem is derived. The supply chain coordination problem involves one manufacturer and multi-suppliers under demand uncertainty. The relationship between the manufacturer and the suppliers is modeled by a noncooperative game. The noncooperative game model is analysed by the Stackelberg equilibrium where the manufacturer is regarded as a leader and the suppliers as followers. By deriving suppliers' optimal response functions, the optimal price discounts are obtained.

Finally, a game theoretic model to coordinate single manufacturer and multiple suppliers with asymmetric information under demand uncertainty is addressed. In the

model, it is assumed that the quality information is asymmetric. The quality of components purchased from suppliers is unknown to the manufacturer while it is known to the suppliers. Two scenarios (average case and worst case) are investigated for the manufacturer to estimate the quality of components. Computational experiments are conducted to illustrate the features of the proposed models with different parameters. The results show the validity of the proposed model.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Background

With increased globalisation and offshore sourcing, comprehensive global supply chain management becomes an important issue for enterprises. Supply chain is regarded as one of important topics to be investigated due to business globalisation in the past decades. Many researchers have investigated global supply chain planning from different perspectives, such as inventory planning, transportation network or risk management [32, 64, 81, 90]. Hulsmann et al. [43] gave a theoretical analysis of autonomous cooperation and control (ACC) in order to improve the adaptivity and competitiveness of global supply chains. They investigated five constitutive characteristics of ACC to competence replication, reconfiguration, and detainment to response the complex and changing environment of global supply chains. Jiao et al. proposed an agent-based multi-contract negotiation system for global manufacturing supply chain coordination. A multiobjective mixed integer nonlinear programming (MINLP) model for a global process supply chain optimisation problem was introduced by Liu and Papageorgiou [60], involving the consideration of production, distribution and capacity expansion. Vidal and Goetschalckx [87] presented a model for the optimisation of a global supply chain that maximises the after tax profits of a multinational corporation, and it includes transfer prices and the allocation of transportation costs as explicit decision variables. The model also considers the selection of transportation modes based on approximate expressions for the inventory costs generated by the utilisation of each transportation mode. Goh et al. [31] studied a stochastic model of the multi-stage global supply chain network problem incorporating a set of related risks, namely, supply, demand, exchange, and disruption. They presented a multi-stage stochastic model for supply chain networks by providing a general formulation of the multi-stage supply chain network problem operating under a scenario of a variety of risks.

1.2 Demand Uncertainty

Business globalisation imposes the need to consider elements from a new perspective: new objectives should be considered, different from the ones typically applicable to the individual supply chain echelons. For instance, marketing uncertainty should be managed taking into account the roles of decision makers in global supply chain [12]. Unlike traditional supply chains, global supply chain management requires companies to enhance their competitiveness and capture market share in the global market. In today's ever changing global markets, maintaining an efficient and flexible supply chain is critical for every enterprise, especially when revailing volatilities in the business environment are given [36]. Demand estimation can lead companies to reduce opportunity loss costs in order to improve customer satisfaction because the demand fluctuation can affect production or inventory plans.

Many researchers assume that demand is deterministic to reduce the complexity of supply chain planning. However, it is more realistic to involve the demand uncertainty to the resolve supply chain planning problems. A supply chain design problem for a new market opportunity under demand uncertainty in an agile manufacturing setting was proposed by Pan and Nagi [74]. Afterwards, Hsu and Li [39] presented a series of models to investigate the supply chain design problems for manufacturers in response to economies of scale and demand fluctuations. Their study focuses on the evaluation of reliability and the adjustment of the supply chain network design to respond to different demand fluctuations. Li and Liu [57] studied a multi-period supply chain planning with one supplier and one buyer. The decision policy which is characterised by the unit selling price and the quantity that is coordinated through quantity discounts. Shin and Benton [83] developed a quantity discount model for a single buyer and a single supplier under uncertain demands changing basis of both parties' economic lot sizes. However, the model is based on the assumption of economic order quantity (EOQ) model. Demand uncertainty should be managed taking into account the roles of many players for global supply chain planning. Xiao et al. [92] proposed a game theoretic model of a three-stage supply chain consisting of one retailer, one manufacturer and one subcontractor to study ordering, wholesale pricing and lead-time decisions under demand uncertainty. Hua and Li [40] introduced retailer-dominant noncooperative game models for a newsvendor problem by introducing the sensitivity of the retailer's order quantity to the manufacturer's wholesale price. They found that the manufacturer and the retailer can bargain to cooperate at any level of retail-market demand uncertainty with exogenous retail price. Al-Othman et al. [2] developed a multi-period stochastic planning model for a petroleum organization consisting of all activities related to crude oil production, processing and distribution in uncertain market demands and prices. A stochastic formulation has been developed, which is based on the two-stage problem with finite number of realisations. Afterwards, Awudu and Zhang [4] proposed a stochastic production planning model for a biofuel supply chain under demand and price uncertainty. They formulated the problem as a stochastic linear programming model to maximise the expected profit. Then, they applied Benders decomposition with Monte Carlo simulation technique to solve their proposed model.

1.3 Quantity Discount

Coordination is another important issue that should be addressed when dealing with global supply chain management. Coordinated supply chain performance refers the execution of a precise set of actions. Unfortunately, supply chain members fail to achieve optimal performance due to the conflict interest. For instance, the members are primarily concerned with optimising their own objectives [9]. Thus, discount contracts become an effective mechanism to coordinate the supply chain members. Quantity discount is a price reduction strategy offered by suppliers to buyers who purchase a large number of products at once. The application of quantity discounts contributes to reducing the buyer's total costs and increasing supplier's profit [41, 54, 65]. In this research, quantity discount is used as an important technique to coordinate the manufacturer and the supplier in supply chain planning. Many researchers focus on buyer-vendor coordination by applying various discount contracts in order to optimise supply chain planning [10, 20, 21, 45]. Li and Liu [57] developed an optimisation model for illustrating how to use a quantity discount policy to achieve supply chain coordination. A supplier-buyer system involving selling one type of product with multi-period and probabilistic customer demand is considered. Sarmah et al. [80]

introduced the basic buyer and vendor coordination models, and reviewed the literature dealing with buyer and vendor coordination models that have used the quantity discount under deterministic demand. Li et al. [58] attempted to improve the cooperation of a buyer and seller system in an inventory control system. Afterwards, they discussed how quantity discount works in the system to divide additional profits. The effectiveness of quantity discounts and volume discounts as coordination mechanisms between a vendor and a retailer was analysed by Viswanathan and Wang [88]. Qin et al. [77] considered volume discount and franchise fees as a coordination mechanism in a system consisting of a supplier and a buyer. The problem is analysed by a Stackelberg game. A number of techniques are applied to resolve discount contract problems by researchers in order to obtain the optimal solutions. Tsai [85] solved a nonlinear supply chain model capable of treating various quantity discount functions simultaneously, including linear, single breakpoint, step, and multiple breakpoint functions. Linearisation techniques are utilised to solve a nonlinear model. The piece-wise linear model is approximately solved in order to obtain an approximate global optimum. A mixed integer nonlinear programming model for order allocation considering different capacities, failure probability and quantity discounts was developed by Meena and Sarmah [62]. They showed that the formulated problem is NP-hard in nature and a genetic algorithm was applied to solve it. Lee et al. [53] also applied genetic algorithms to solve the lot-sizing problem with multiple suppliers, multiple periods and quantity discount which is reformulated as a a mixed integer programming model.

1.4 Game Theoretic Model

The game theoretic models have been extensively studied in different manners in the past years. Game theoretic approach, as a well-known method, is widely used in supply chain coordination. Game theory is generally divided into noncooperative games and cooperative games. Nagarajan and Sosic [66] gave an extensive survey on applications of cooperative game theory in supply chain management. In their paper, they emphasised two important aspects of cooperative games: profit allocation and stability. Yang et al. [95] presented an assembly supply chain system consisting of one retailer and two suppliers with forecast updating. Huang et al. [41] introduced a three-level noncooperative game model considering suppliers and components selection, pricing and inventory. The retailers' problem focuses on replenishment cycles and retail prices for products. Furthermore, various contract strategies are widely used in game theoretic models in order to achieve coordination of supply chain planning [50, 72, 105]. Zhao et al. [106] used a cooperative game theoretic approach in a manufacturer-retailer supply chain with option contracts. They developed an option contract model using wholesale price mechanism as a benchmark to coordinate the supply chain and achieve Pareto-improvement. Palsule-Desai [73] introduced revenue sharing contracts as a coordination mechanism to build a two-period supply chain model wherein the actual proportion in which the supply chain revenue is shared among players depends on the quantity of revenue generated. Corbett et al. [17] used shared-savings contracts by considering the double moral hazard framework where the shared-savings contracts combine a fixed service fee with a variable component based on consumption volume. By using this contract, supply chain planning is improved to lead to a more efficient choice by the two parties. Quantity discount is regard as one of important strategies among those contracts which is intensively studied by many researchers in order to reduce total costs and improve profits. Krichen et al. [48] considered an economic order quantity problem involving a single supplier that offers quantity discounts and allows retailers to delay payments. They proposed a solution approach that generates stable coalition structures for the retailers taking into account the delay in payments. A manufacturer-retailer supply chain model was proposed to study the coordination of cooperative advertisement when the manufacturer offers price deductions to customers by Yue et al. [102]. They obtained a necessary and sufficient condition for the price reduction to ensure an increase of manufacturer's profit, and a search procedure for determining such an optimal price reduction. They also considered when price discounts are offered by both the manufacturer and retailer [103]. They showed that the manufacturer always prefers Stackelberg equilibrium, but there was no definitive conclusion for the retailer. In Stackelberg game, the leader' decision is solved considering all possible reactions of its follower in order to maximise its own profits. Meanwhile, the follower`s optimal reaction is determined by considering the leader`s decision as its input parameters.

1.5 Noncooperative Game Theoretic Model

Game theory is generally divided into cooperative game theory and noncooperative game theory. In cooperative games, all players cooperatively make contributions in order to maximise the whole system's profits. In noncooperative games, each play makes decisions independently to maximise its own profits. In this thesis, a global supply chain planning is investigated when the manufacturer appropriately decides the production plan and purchase components from its outsourcing suppliers while retailers properly purchase products from the manufacturer. Each supply chain member makes decisions independently in order to maximise its own profits. Thus, it is more reasonable to apply the noncooperative game in realistic situations. A number of works have done by many researchers to apply noncooperative game to resolve various coordination problems in supply chain planning. Cai et al. [11] studied the optimal pricing and ordering with partial lost sales from a two-stage theoretic perspective. They anlysed solutions of noncooperative games for the buyer and the seller, and provided insights into both the deterministic and stochastic demand models. Erickson [23] treated the strategic interdependence problem involving marketing and operational decisions by a noncooperative differential game. A feedback Nash equilibrium is derived by considering price and advertising costs which are controlled by marketing. Yang et al. [96] developed a model of a general closed-loop supply chain network of raw material suppliers, manufacturers, retailers, consumers and recovery centers which are analysed by the theory of variational inequalities. Guardiola et al. [34] studied the coordination of actions and the allocation of profit in supply chain under decentralised control in which a single supplier supplies to several retailers with good for replenishment of stocks. The decisions of the supplier and the retailers are analysed by a noncooperative game. Meng et al. [63] introduced a novel competitive facility location problem about a firm that intends to entry an existing decentralised supply chain comprised of three tiers of players with competition: manufacturers, retailers and consumers. Variational inequalities are applied for the supply chain network equilibrium with production capacity constraints. The “manufactuer-Stackelberg” game is widely used in supply chain literatures [14, 24, 35, 52, 75]. Yu et al. [101] improved members’ profits of supply chain systems between a manufacturer and its retailers incorporating the inventory policy by studying Stackelberg game problems where advertising, pricing and inventory replenishments are all involved. In the Stackelberg game, players make decisions sequentially where the leader dominates the game. Chen et al. [15] examined manufacturer's pricing strategies in a dual-channel supply chain, in which the manufacturer is a Stackelberg leader and the retailer is a follower. A bi-level programming approach is applied by Sadigh et al. to investigate a multi-product manufacturer-retailer supply chain where demand of each product is jointly influenced by the price and advertising expenditure [78]. They proposed a Stackelberg game framework with two scenarios. Under the competing supply chain, Wu [91] examined the impact of buyback policy on retail price, order quantity and wholesale price in a duopoly of two manufacturer-retailer supply chains. The buyback contract allows that the buyer can return unsold products to the seller after the demand is realised. Two channel policies including vertical integration and manufacturer's Stackelberg are considered. Yu et al. [100] discussed how the vendor can take advantage of inventory and market-related information in a vendor managed inventory system for increasing his profit by using a Stackelberg game.

1.6 Quality

Another issue that must be incorporated into a global supply chain management strategy is quality. The rapid growth of globalisation has increased companies' competitiveness. Many companies in the global business environment are facing fierce competition on price and quality simultaneously. In order to win in the global business environment, competition is shifting from price to quality in many industries in order to achieve high customer satisfaction [28]. Thus, a global supply chain planning concerning price and quality is needed to be optimised simultaneously in order to help multinational enterprises for decision makings in the global business environment.

The quality problem of products associating with defects from suppliers or in the production process is widely investigated [13, 61, 94]. In 1998, Banker et al. [5] studied the impact of various factors on quality in different competitive environments. Xie et al. [94] also introduced a model which involves quality of products in competing supply chains. Both of them assume that demand is influenced by quality levels of products. Franca et al. [27] introduced a method to evaluate the risk of poor quality. Defects are minimised to improve the sigma quality level which a simple statistic that puts a given defect rate on a "six-sigma" scale. Hsieh and Liu [38] established game models in a supply chain consisting of one manufacturer and one supplier, both having imperfect production and inspection processes. They investigated the supplier's and the manufacturer's quality investment and inspection strategies with different degrees of information revealed. Perfect and imperfect quality items were considered in an integrated production-inventory model by Sana [79]. A cost-effective production and distribution system is established to improve response to customers as well as the quality of products.

1.7 Symmetric Quality Information

Quality issues have been studied intensively for supply chain planning. Most of researches assume that quality information is complete. However, it is difficult for enterprises to observe complete quality information. From more practical perspective, it is important to assume asymmetric information due to different business strategies in the global supply chain planning. Bauso et al. [6] dealt with the repeated nonasymmetric congestion games in which the players cannot observe their payoffs at each stage. They provided a consensus protocol that allows the convergence of the players' strategies to the Pareto optimal Nash equilibrium. Kyparisis and Koulamas [51] derived sufficient conditions for the existence and uniqueness of the Stackelberg-Nash–Cournot equilibrium for a supply chain problem with a single manufacturer and multiple asymmetric retailers. They assumed that the manufacturer supplies a homogenous product to all retailers with the retail price determined by a general nonlinear inverse demand function. Lau et al. [52] considered a dominant manufacturer wholesaling a product to a retailer, and the retailer sells it to the consumer. They model the general supply chain problem by a manufacturer-Stackelberg game under a deterministic and asymmetric information framework.

1.8 Motivation

The objective of the thesis is to propose an effective coordination approach to resolve manufacturing, inventory and purchasing in order to achieve global supply chain optimisation. Thus, a game theoretic approach is proposed to coordinate one manufacturer and multiple suppliers in global supply chain management. A Stackelberg game is applied where the manufacturer is assumed to be a leader. The manufacturer's decision problem with quantity discounts is introduced first. The quantity discount problem involving quantity discounts with supplier selection is formulated as a mixed integer nonlinear programming problem which is NP-hard. Therefore, an outer approximation algorithm with a heuristic is proposed in order to solve the problem efficiently. A reformulation of the supply chain planning problem is applied in order to reduce the computational time. Next, quantity discounts in a game theoretic model are addressed to coordinate supply chain members with uncertain demands. Moreover, a game theoretic model with assymetric quality information is proposed.

1.9 Structure of This Thesis

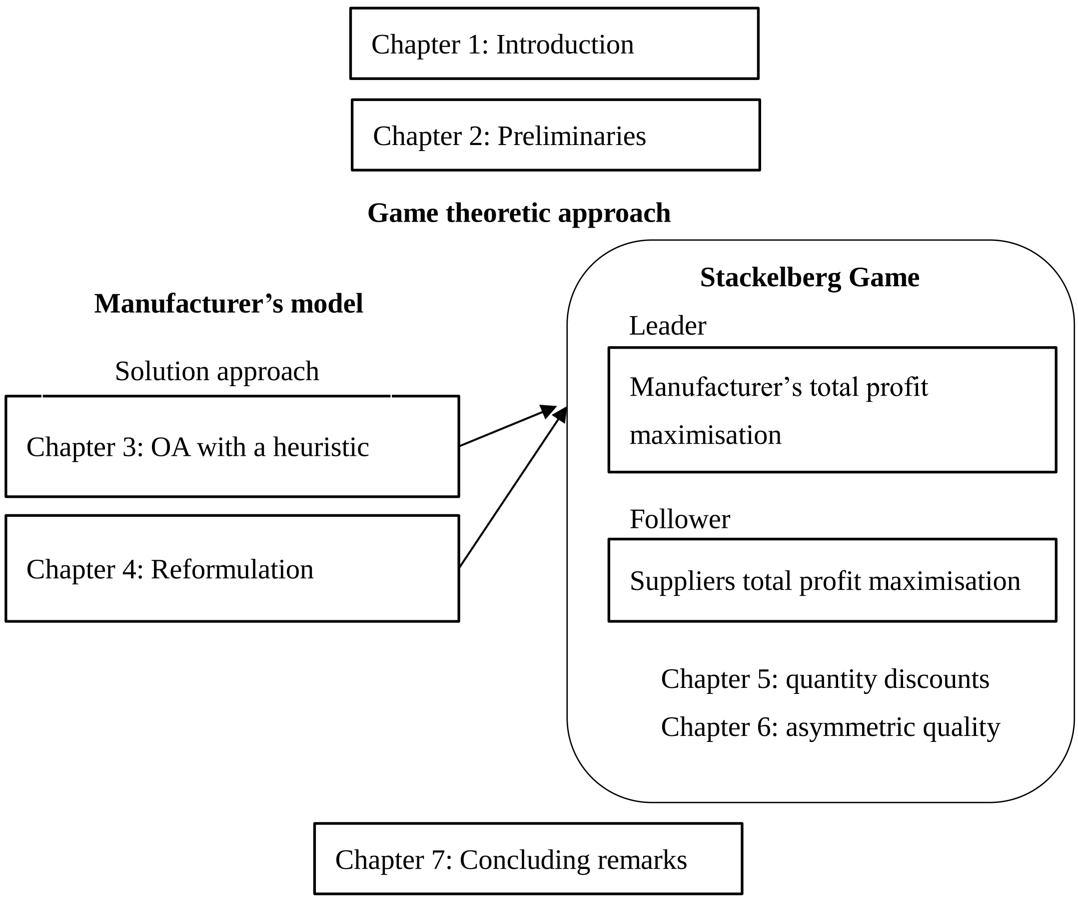

Figure 1.1: The structure of the thesis.

In this thesis, game theoretic approaches are introdocued to coordinate one manufacturer and multiple suppliers in global supply chain planning under demand uncertainty. A Stackelberg game is applied to achieve the coordination between a manufacturer and suppliers. The manufacturer is a leader and suppliers are followers. The chapters of the thesis are related as shown in Fig. 1.1.

In Chapter 1, the background of this thesis is stated. A game theoretic approach to global supply chain planning problem is addressed. Several critical issues should be involved in global supply chain planning.

Chapter 2 presents basic theories which are used in the thesis, such as supply chain management, game theory, pricing game and outer approximization approach for mixted integer nonlinear programming problem.

In the thesis, a Stackelberg game theoretic model is introduced in global supply chain planning where the manufacturer is treated as a leader. Thus, the manufacturer's decision problem with quantity discount contracts under demand uncertainty is addressed first. The quantity discount problem under demand uncertainty is formulated as a mixed integer nonlinear programming (MINLP) problem. Chapter 3 and Chapter 4 focus on solution approaches to this manufacturer's decision model. In Chapter 3, an outer approximation (OA) algorithm with a heuristic is proposed to solve the MINLP problem.

In Chapter 4, both of incremental discount and all units discount models are considered. The outer approximation method is used to solve the problems. In order to reduce the computational complexity, the model is further reformulated into a stochastic model by replacing integral terms with expectation formulation. By replacing the integral terms, the resulting equivalent deterministic optimisation model is a convex programming problem.

In Chapter 5, a noncooperative game theoretic model involving one manufacturer and multiple suppliers is addressed. The noncooperative game model is analysed by a Stackelberg game where the manufacturer is a leader and the suppliers as followers. By deriving suppliers' optimal response functions, optimal price discounts are created.

In Chapter 6, the game theoretic model with the asymmetric quality information is introduced. The quality of components is unknown by the manufacturer while it is known by the suppliers. Two scenarios (average case and worst case) are investigated for the manufacturer to estimate the quality of components. The coordination problem is modelled by a Stackelberg game where the manufacturer is the leader and suppliers are followers.

Finally, concluding remarks of the thesis are given in Chapter 7.

Chapter 2

Preliminaries

In this thesis, game theoretic models in global supply chain management are addressed. Game theoretic approaches are applied to coordinate supply chain members. This chapter introduces fundamentals such as supply chain management, game theory, pricing game and an outer approximation algorithm for mixed integer nonlinear programming problems.

2.1 Supply Chain Management

A supply chain is composed of all parties involved, directly or indirectly, in fulfilling a customer request. The supply chain includes not only the manufacturer and suppliers, but also transporters, warehouses, retailers, and even customers themselves [16]. A supply chain involves the constant flow of information, product and funds between different stages. Supply chain management encompasses the planning and management of all activities involved in sourcing, procurement, conversion, and all logistics management activities. Importantly, it includes also coordination and collaboration with channel partners, which can be supplier, intermediaries, third-party service providers, and customers. Logistics management is a part of supply chain management. Simply, supply chain consists of series of activities and organizations through which materials flow on their way from the initial suppliers to final customers.

The supply chain system often appears to be quite complex including many functions within a firm and many organisations along the supply chain. Fig. 2.1 provides a framework of supply chains. The figure shows that there are three major issues which need to be considered when supply chain problems are analysed.

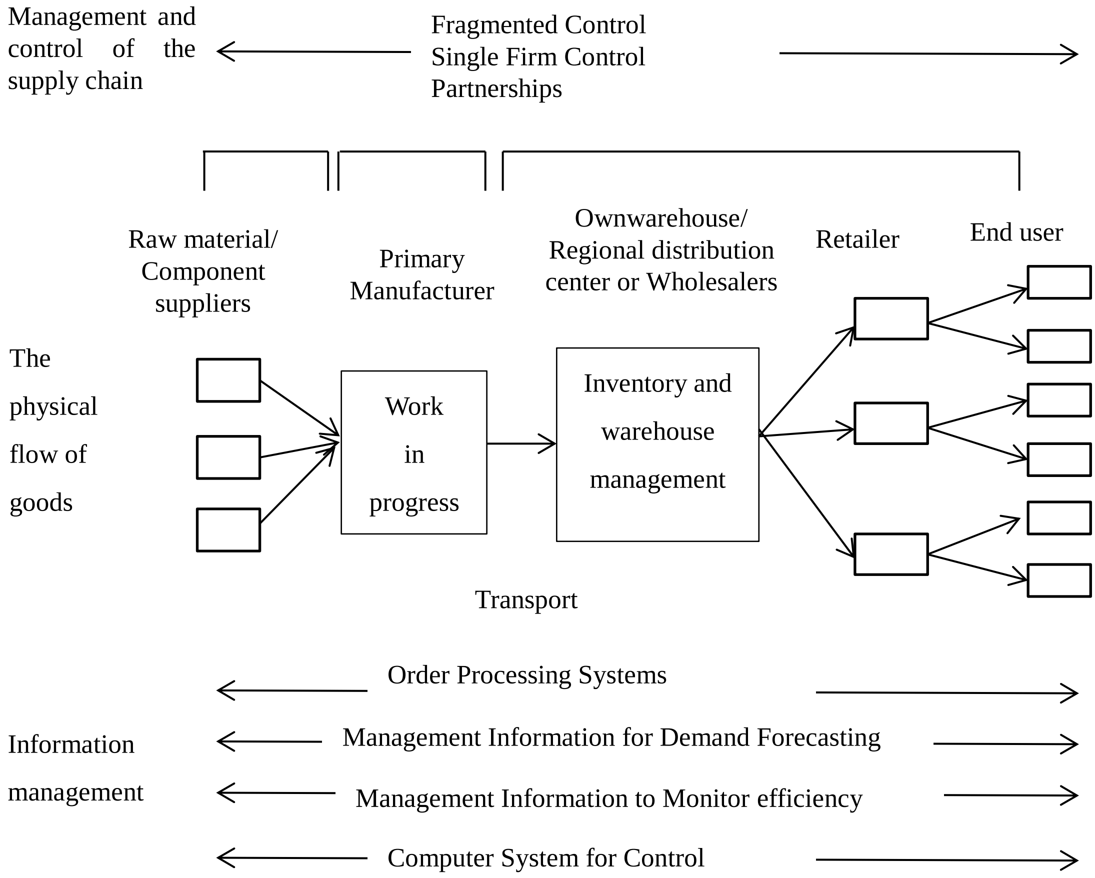

Figure 2.1: The supply chain management model.

- The physical flow of goods
- The information flows and systems which underpin the flow of goods
- The organizational and management structures which control the supply chain

Fig. 2.1 shows that the management and control of the supply chain include fragmented control, single firm control and partnerships. The physical flow of goods is the process that raw materials are delivered from suppliers to manufacturers. The raw materials are used for production, and finished products are stored in the warehouse. Finally, the products are sent through retailers to end users. Along the physical flow of goods, several information management activities are involved, such as order processing, demand forecasting and so on.

2.2 Game Theory

Game theory provides a tool to analyse situations involving conflicts and cooperation between players. A cooperative game is a game where groups of players ("coalitions") may enforce cooperative behavior, hence the game is a competition between coalitions of players rather than between individual players. An example is a coordination game, when players choose the strategies by a consensus decision making process. A noncooperative game is one in which players make decisions independently. The noncooperative game is generally divided into Nash and Stackelberg games for dealing with simultaneous and sequential non-cooperating decision-making by multiple players which are introduced by Kogan and Tapiero [47].

2.2.1 Nash game

A Nash equilibrium is defined as a solution concept of a noncooperative game involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only his own strategy unilaterally.

Consider a game, with the strategies $y_i,\ i=1,...,N$ being feasible actions which N players may undertake. All possible strategies of a player i form a strategy set Y_i of the player. A payoff (objective function) $J_i(y_1, y_2, ..., y_N)$, $i=1,..,N$ is evaluated when each player i selects a feasible strategy $y_i \in Y_i$. Since two-player games can be straightforwardly extended to multiple players and to simplify the presentation, we further assume that there are only two players A and B.

Definition 2.1

A pair of strategies (y_A^*, y_B^*) is said to constitute a Nash equilibrium if the following pair of inequalities is satisfied for all $y_A \in Y_A$ and $y_B \in Y_B$

$$J_A(y_A^*, y_B^*) \geq J_A(y_A, y_B^*) \text{ and } J_A(y_A^*, y_B^*) \geq J_B(y_A^*, y_B). \tag{2.1}$$

The definition implies that the Nash solution is

$$y_A^* = \arg\max_{y_A \in Y_A} [J_A(y_A, y_B^*)] \text{ and } y_B^* = \arg\max_{y_B \in Y_B} [J_B(y_A^*, y_B)], \tag{2.2}$$

and a unilateral deviation from this solution results in a loss. We assume this problem is static, strategy sets are not constrained and the payoff functions are continuously differentiable. The first-order (necessary) optimality condition results in the following system of two equations in two unknowns y_A^* and y_B^*:

$$\left.\frac{\partial J_A(y_A, y_B^*)}{\partial y_A}\right|_{y_A=y_A^*} = 0 \text{ and } \left.\frac{\partial J_B(y_A^*, y_B)}{\partial y_B}\right|_{y_B=y_B^*} = 0. \tag{2.3}$$

In addition, the second order (sufficient) optimality condition which ensures that we maximise the payoffs when

$$\left.\frac{\partial J_A(y_A, y_B^*)}{\partial y_A}\right|_{y_A=y_A^*} < 0 \text{ and } \left.\frac{\partial J_B(y_A^*, y_B)}{\partial y_B}\right|_{y_B=y_B^*} < 0. \tag{2.4}$$

Equivalently, one may determine $y_A^R(y_B) = \arg\max_{y_A \in Y_A} [J_A(y_A, y_B)]$ for each $y_B \in Y_B$ to find the optimal response functions $y_A = y_A^R(y_B)$ of player B and $y_B = y_B^R(y_A)$ of player B, which constitutes a system of two equations in two unknowns.

2.2.2 Stackelberg game

Stackelberg strategy is applied when there is an asymmetry in power or in moves of the players. As a result, the decision-making is sequential rather than simultaneous as in the case of Nash strategy. The player who first announces his strategy is considered to be the Stackelberg leader. The follower then chooses his optimal response to the leader's move. The leader thus has an advantage because he is able to optimise his objective function subject to the follower's optimal response. Formally this implies that if, player A, for example, is the leader, then $y_B = y_B^R(y_A)$ is the same optimal response for player B as determined for the Nash equilibrium. Since the leader is aware of this response, he then optimises his objective function subject to $y_A = y_A^R(y_B) = y_A^R(y_B^R(y_A))$.

Definition 2.2

In a two-person game with player A *as the leader and player* B *as the follower, the strategy* $y_A^* \in Y_A$ *is called a Stackelberg equilibrium for the leader if, for all* y_A,

$$J_A(y_A^*, y_B^R(y_A^*)) \geq J_A(y_A, y_B^R(y_A)), \tag{2.5}$$

where $y_B = y_B^R(y_A)$ is the optimal response function of the follower, when the leader's decision is $y_A \in Y_A$.

It implies that the leader's Stackelberg solution is

$$y_A^R(y_B) = \arg\max_{y_A \in Y_A} [J_A(y_A, y_B)]. \tag{2.6}$$

That is, if the strategy sets are unconstrained and the payoff functions are continuously differential, the necessary optimality condition for the leader is

$$\frac{\partial J_A(y_A, y_B^*)}{\partial y_A}\bigg|_{y_A = y_A^*} = 0. \tag{2.7}$$

To make sure that the leader maximises his profits, we check also the second-order sufficient optimality condition

$$\frac{\partial^2 J_A(y_A, y_B^*)}{\partial y_A^2}\bigg|_{y_A = y_A^*} < 0. \tag{2.8}$$

2.3 Pricing Game

Consider a two-echelon supply chain consisting of a single supplier selling a product type to a single retailer over a period of time. The supplier has ample capacity and the period is longer than the supplier's lead-time which implies that the supplier is able to deliver on time when any quantity q ordered by the retailer. The retailer faces a concave endogenous demand, $q(p)$, which decreases as product price p increase, i.e., $\frac{\partial q}{\partial p} < 0$ and $\frac{\partial^2 q(p)}{\partial p^2} \leq 0$. The supplier incurs unit production cost c and sells at unit wholesale price w, i.e., the supplier's margin is $w - c$.

Let the retailer's price per unit be $p = w + m$, where m is the retailer's margin. Both players, the supplier and the retailer, want to maximise their profits margin times demand which are expressed as $J_s(w) = (w - c)q(w + m)$ and $J_r(p) = mq(w + m)$, respectively as shown in Fig. 2.2.

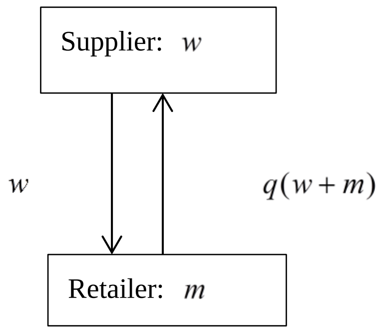

Figure 2.2: Vertical pricing competition.

This leads us to the following problems.

The supplier's problem

$$\max_{w} J_s(w,m) = \max_{w}(w-c)q(w+m) \tag{2.9}$$

$$\text{s.t.} \quad w \geq c\,. \tag{2.10}$$

The retailer's problem

$$\max_{m} J_s(w,m) = \max_{m} mq(w+m) \tag{2.11}$$

$$\text{s.t.} \quad m \geq 0\,, \tag{2.12}$$

$$q(w+m) \geq 0\,. \tag{2.13}$$

Note that from $w \geq c$ and $m \geq 0$, it immediately follows that $p = w + m \geq c$. In contrary to the vertical competition between the two decision-makers, the supply chain may be vertical integrated or centralised. Such a supply chain is characterised by a single decision-maker who is in charge of all managerial aspects of the supply chain. We then have the following single problem as benchmark.

The centralised problem

$$\max_{m,w} J(m,w) = \max_{m,w} [J_r(m,w) + J_s(m,w)] = \max_{m,w} (w+m-c)q(w+m) \tag{2.14}$$

$$\text{s.t.} \quad m \geq 0\,, \tag{2.15}$$

$$q(w+m) \geq 0. \tag{2.16}$$

To distinguish between different optimal strategies, we will use the superscript n for Nash solutions, s for Stackelberg solutions and * for centralised solutions.

System-wide optimal solution

We first study the centralised problem by employing the first-order optimality conditions. The first- order optimality conditions are as follows:

$$\frac{\partial J(m,w)}{\partial m} = q(w+m) + (w+m-c)\frac{\partial q(p)}{\partial p} = 0, \tag{2.17}$$

$$\frac{\partial J(m,w)}{\partial w} = q(w+m) + (w+m-c)\frac{\partial q(p)}{\partial p} = 0. \tag{2.18}$$

Since both equations are identical, only the optimal price matters in the centralised problem, p^*, while the wholesale price $w \geq 0$ and the retailer's margin $m \geq 0$ can be chosen arbitrarily so that $p^* = m + w$. This is because w and m represent internal transfers of the supply chain. Thus, the proper notation for the payoff function is $J(p)$ rather than $J(m,w)$, and only optimality condition is

$$q(p^*) + (p^* - c)\frac{\partial q(p^*)}{\partial p} = 0. \tag{2.19}$$

Let $q(p) = 0$ and $P > c$. Then it is easy to verify that,

$$\frac{\partial^2 J(p)}{\partial p^2} = \frac{\partial q(p)}{\partial p} + \frac{\partial q(p)}{\partial p} + (p-c)\frac{\partial^2 q(p)}{\partial p^2} < 0. \tag{2.20}$$

That is, the centralised objective function is strictly concave in price for $p \in [c, P]$. This implies that the equation has a unique solution.

2.3.1 Nash equilibrium

To determine the Nash equilibrium, which corresponds to simultaneous moves of the supplier and retailer, we next consider the optimality conditions for the supplier's objective function. It is obtained as

$$\frac{\partial J_s(m,w)}{\partial w} = q(w+m) + (w-c)\frac{\partial q(w+m)}{\partial p} = 0 . \qquad (2.21)$$

One can readily verify that the supplier's objective function is strictly concave in w, $\frac{\partial^2 J_s(m,w)}{\partial w^2} < 0$ and, thus, the supplier's optimal response function of Eq. (2.21) is unique as well. As a result, the Nash equilibrium (w^n, m^n) is found by solving the following system of equations:

$$q(w+m) + m\frac{\partial q(w+m)}{\partial p} = 0 , \qquad (2.22)$$

$$q(w+m) + (w-c)\frac{\partial q(w+m)}{\partial p} = 0 . \qquad (2.23)$$

Solving the system composed of Eq. (2.22) and Eq. (2.23), we obtain

$$w - c - m = 0 \quad \text{and} \quad q(c+2m) + m\frac{\partial q(c+2m)}{\partial p} = 0 . \qquad (2.24)$$

Assuming that the solution $w + m = P, q(P) = 0$ cannot be optimal since it leads to zero profit for all supply chain members, we conclude with the following result.

Proposition 2.1

The pair (w^n, m^n), *where* m^n *satisfies the following equation*

$$q(c+2m^n) + m^n \frac{\partial q(c+2m^n)}{\partial p} = 0 , \qquad (2.25)$$

and $w^n = m^n + c$ *constitutes a unique Nash equilibrium of the pricing game with*

$$0 < m^n < \frac{P-c}{2} .$$

2.3.2 Stackelberg equilibrium

Next we assume that the supplier makes the first move by setting the wholesale price. The retailer then decides on what price to set and the quantity to order. To find the Stackelberg equilibrium, we need to maximise the supplier's objective with m subject to the optimal retailer's response $m = m^R(w)$ determined by

$$J_s(m,w) = (w-c)q(w+m^R(w)) \,. \tag{2.26}$$

Differentiating the supplier's objective function we have

$$\frac{\partial J_s(m,w)}{\partial w} = q(w+m^R(w)) + (w-c)\frac{\partial q(w+m)}{\partial p}\frac{\partial m^R(w)}{\partial w} = 0 \,, \tag{2.27}$$

where $\frac{\partial m^R(w)}{\partial w}$ is determined by differentiating Eq. (2.26) with m set equal to

$$m^R(w)\frac{\partial q(w+m)}{\partial p}(1+\frac{\partial m^R(w)}{\partial w}) + \frac{\partial m^R(w)}{\partial w}\frac{\partial q(p)}{\partial p} + m\frac{\partial^2 q(p)}{\partial p^2}(1+\frac{\partial m^R(w)}{\partial w}) = 0 \,. \tag{2.28}$$

Thus,

$$\frac{\partial m^R(w)}{\partial w} = -\frac{\dfrac{\partial q(w+m)}{\partial p} + m\dfrac{\partial^2 q(w+m)}{\partial p^2}}{\dfrac{\partial q(w+m)}{\partial p} + \dfrac{\partial q(w+m)}{\partial p} + m\dfrac{\partial^2 q(w+m)}{\partial p^2}} \,. \tag{2.29}$$

Eq. (2.29) naturally implies the greater the supplier's wholesale price w, the lower the retailer's margin m.

Based on Eq. (2.27) and Eq. (2.29), we conclude that a pair (w^s, m^s) constitutes a Stackelberg equilibrium of the pricing game if there exists a joint solution in w and m of the following equations:

$$q(w+m) + (w-c)\frac{\partial q(w+m)}{\partial p} = 0 \,, \tag{2.30}$$

$$q(w+m) + m\frac{\partial q(w+m)}{\partial p} = 0 \,, \tag{2.31}$$

where we have

$$\frac{\partial m}{\partial w} = -\frac{\dfrac{\partial q(w+m)}{\partial p} + m\dfrac{\partial^2 q(w+m)}{\partial p^2}}{\dfrac{\partial q(w+m)}{\partial p} + \dfrac{\partial q(w+m)}{\partial p} + m\dfrac{\partial^2 q(w+m)}{\partial p^2}}.$$

2.4 Mixed Integer Nonlinear Programming (MINLP)

Mixed Integer Nonlinear Programming (MINLP) refers to the mathematical programming with continuous and discrete variables and nonlinearities in the objective function or constraints. The use of MINLP is a natural approach of formulating problems where it is necessary to simultaneously optimise the system structure (discrete) and variables (continuous).

The form of a general MINLP is

$$\min \quad f(x,y), \tag{2.32}$$

$$\text{s.t.} \quad g(x,y) \le 0, \tag{2.33}$$

$$x \in X, \tag{2.34}$$

$$y \in Y \text{ integer.} \tag{2.35}$$

The function $f(x,y)$ is a nonlinear objective function and $g(x,y)$ is a vector-valued nonlinear constraint function. The variables x, y are decision variables, where x is a real number and y is an integer value. X and Y are bounding-box-type restrictions on the variables.

MINLP problems are difficult to solve precisely, because they inherit all the difficulties of both of mixed integer programs (MIP) and nonlinear programs (NLP): the combinatorial nature of mixed integer programs (MIP) and the difficulty in solving nonconvex (and even convex) nonlinear programs (NLP), because many MIP and NLP become theoretically difficult problems (NP-hard). Thus, it is not surprising that solving

MINLP can be a challenging and daring venture. Fortunately, the component structure of MIP and NLP within MINLPs provide a collection of natural algorithmic approaches, exploiting the structure of each subcomponent.

2.5 Outer Approximation

In order to solve MINLP problems, this section will introduce one of well-known techniques that is Outer-Approximation (OA), which is used to solve MINLP problems widely. Namely, we introduce the OA approach which was developed by Duran and Grossmann [22]. The main characteristics of the MINLP problem are linear with respect to the integer variables and convex with respect to continuous variables. The general MINLP mathematical function can be represented as follow:

$$(P)\ Z = \min c^T y + f(x), \tag{2.36}$$

$$\text{s.t. } g(x) + By \le 0, \tag{2.37}$$

$$x \in X \subset R^n, \tag{2.38}$$

$$y \in U \subset R_+^m, \tag{2.39}$$

where the nonlinear function $f : R^n \to R$ and those in the vector function $g(x)$ are assumed to be differentiable on the compact polyhedral convex set $X = \{ x : x \in R^n, A_1 x \le a_1 \}$; $U = \{y : y \in Y,$ integer, $A_2 y \le a_2\}$ is a finite discrete set. Y corresponds to the unit hypercube $Y = \{0,1\}^m$. B, A_1, A_2 and c, a_1, a_2 are matrices and vectors of comfortable dimensions, respectively. R^n is n-dimensional real vectors. R_+^m is m-dimensional nonnegative real vectors.

The basic ideas of OA are explained as follows. Because of the linearity with respect to discrete variables, the continuous and discrete feasible spaces of program (P) can be treated independently. Furthermore, the continuous space corresponds to the intersection of a finite number of compact convex regions, where each region is determined by a different discrete parameterisation. Hence, linearity with respect to continuous variables can be introduced into the problem (P) if a polyhedral representation is provided for each of those compact convex sets. To achieve this goal, OA of a convex set by intersection of its collection of supporting half-spaces can be used.

Since $f(x)$ in the objective function is convex, problem (P) can be rewritten as the following program with a linear objective function:

$$(P_0)\ Z = \min c^T y + \mu, \tag{2.40}$$

$$\text{s.t.}\quad f(x) - \mu \le 0, \tag{2.41}$$

$$g(x) + By \le 0, \tag{2.42}$$

$$x \in X, y \in U, \mu \in [f_L, f_U]. \tag{2.43}$$

where μ is a scalar variable. f_L and f_U are valid finite bounds given by $f_L = \min\{f(x): x \in X\}$, and $f_U = \max\{f(x): x \in X\}$. It is assumed that the following suitable form of Slater's constraint qualification holds: there exists a point $x \in X$ such that $g(x) + By < 0$ for each $y = U \cap V$, where $V = \{y: g(x) + By \le 0$ for some $x \in X$ }.

Let $F(y)$ be a set defined as follows for each $y \in U \cap V$:

$$F(y) = \{x, \mu : x \in X, \mu \in [f_L, f_U], f(x) - \mu \le 0, g(x) + By \le 0\}, \tag{2.44}$$

We note that $F(y)$ for each $y \in U \cap V$ is a closed convex set.

From the property of convexity, we have

$$f(x) - \mu \ge f(x^i) + \nabla f(x^i)^T (x - x^i) - \mu, \tag{2.45}$$

$$g(x) + By \ge g(x^i) + \nabla g(x^i)^T (x - x^i) + By\ . \tag{2.46}$$

Then, the feasible region of the program (P_0) can be defined by the following infinite set of supporting half-spaces:

$$\begin{cases} f(x^i)+\nabla f(x^i)^T(x-x^i)-\mu \le 0, \forall i \in T, \\ g(x^i)+\nabla g(x^i)^T(x-x^i)+By \le 0, \forall i \in T, \end{cases} \tag{2.47}$$

where $x \in X$, $\mu \in [f_L, f_U]$, $y \in U$. $\nabla f(x^i)$ is the n-gradient vector and $\nabla g(x^i)$ is the $n \times p$ jacobian matrix. The half-spaces (2.47) correspond to the approximation of the convex set defined by $f(x)-\mu \le 0$ and $g(x)+By \le 0$ by the pointwise maximum of the collection of their linear supports.

Examples of outer approximation at a finite number of points are illustrated in Fig. 2.3. In Fig. 2.3, H_{11}, H_{12}, and H_{21}, H_{22}, correspond to supporting half-spaces of g_1 and g_2, respectively, at points x_1 and x_2.

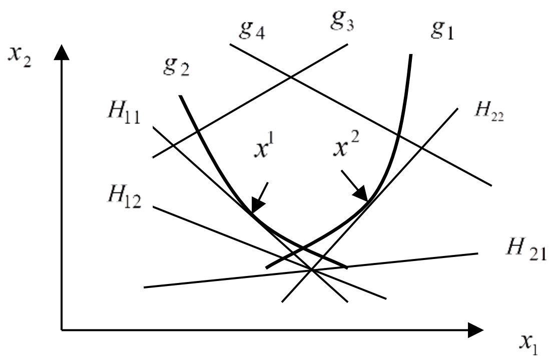

Figure 2.3: Outer-approximation of a convex function in *R*.

Outer approximation of the feasible region of program (P_0) is as shown in Eq. (2.47), renders linearity in the constraints and objective of function of (P). It leads to the following semi-infinite mixed-integer programming formulation:

$$(P_1) \;\; z = \min c^T y + \mu, \tag{2.48}$$

$$\text{s.t. } f(x^i)+\nabla f(x^i)^T(x-x^i)-\mu \le 0, \forall i \in T, \tag{2.49}$$

$$g(x^i) + \nabla g(x^i)^T (x - x^i) + By \le 0, \forall i \in T\ , \tag{2.50}$$

$$x \in X\ , \mu \in [f_L,\ f_U], y \in U\ . \tag{2.51}$$

Lemma 2.1: When the assumptions with respect to functions and sets in problem (P_0) hold, problems (P_1) and (P_0) are equivalent.

Master Program

The set $U \cap V$ is discrete and finite. The concept of projection of program (P) onto the discrete space can be used to identify selected continuous points x^i for outer-approximation in problem (P_1). The projection of program (P) onto y is given by

$$Z = \min_y [\inf_{x \in X} \{ c^T y + f(x) : g(x) + B(y) \le 0\}]. \tag{2.52}$$

For given $y^i \in U \cap V$, the infimum value function of the "inner" problem in Eq. (2.1) is precisely the optimal value of program (P) for fixed y^i. For each $y^i \in U \cap V$, the infimum is attained and corresponds to the optimal value $z(y^i)$ of the nonlinear programming subproblem. We have

$$(S(y)) \quad z(y^i) = c^T y^i + \min f(x), \tag{2.53}$$

$$\text{s.t.} \quad g(x) + By \le 0, x \in X\ . \tag{2.54}$$

It is clear that program (P) is not a convex program in x and y jointly, but fixing y renders it so in x for ($S(y)$). Thus, the first observation is that for y^i to be a candidate for the optimal solution to problem (P), y^i must be such that ($S(y^i)$) is feasible (i.e., $y^i \in U \cap V$), and then the best continuous point x^i associated with y^i is the optimal solution of the corresponding subproblem ($S(y^i)$).

Secondly, according to theorems for characterisation of integer polyhedra and linear programming theory, the mixed-integer solution to problem (P_1) is such that the integer part y^i is an extreme point of the convex hull of feasible integer solutions ($Conv(U \cap V)$), and the continuous part is given by the boundary point ($(x = x^i, f(x^i))$) in the linear support to $f(x)$ which is represented by Eq. (2.45) associated with y. Therefore, as given by projection of program (P) onto y-space, the finite set of continuous points x^i to be considered for outer approximation in problem (P_1) are actually the optimal solutions of the subproblems ($S(y^i)$) defined for the finite number of all integer points $y^i \in U \cap V$. The master program in its final form is then given by the following mixed-integer linear programming program:

$$(M)\ z = \min c^T y + \mu, \tag{2.55}$$

$$\text{s.t. } f(x^i) + \nabla f(x^i)^T (x - x^i) - \mu \le 0, \forall i \in T, \tag{2.56}$$

$$g(x^i) + \nabla g(x^i)^T (x - x^i) + By \le 0, \forall i \in T, \tag{2.57}$$

$$x \in X, \mu \in [f_L, f_U], y \in U, \tag{2.58}$$

where $T = \{ i : x^i \text{ optimal solution to } S(y^i), i = 1, 2, \ldots, k \}$.

Relaxed the master program at iteration k is

$$(M^k)\ z = \min c^T y + \mu, \tag{2.59}$$

$$\text{s.t. } (x, y) \in \Omega^k, \tag{2.60}$$

$$x \in X, \mu \in [f_L, f_U], y \in U, \tag{2.61}$$

where

$$\Omega^k = \{ f(x^i) + \nabla f(x^i)^T (x - x^i) - \mu \le 0,$$
$$g(x^i) + \nabla g(x^i)^T (x - x^i) + By \le 0, i \in T^k \subseteq T \},$$

$$T = \{ i : x^i \text{ optimal solution to } S(y^i), i = 1,2,\ldots,k \}.$$

Then the outline of the solution algorithm is obtained as follows:

(i) at iteration k, solve the solved master program (M^k) that ignores all but some of the constraints in (M) (i.e., ignores $i \in \{T \setminus T^k\}$).

(ii) if the solution to (M^k) and (x, y^{k+1}) does not satisfy certain termination criteria, solve the subproblem to ($S(y^{k+1})$) to determine the continuous point x^{k+1} for OA.

(iii) construct the new relaxed master (M^{k+1}) by intersecting the feasible space at iteration k with the set of closed half-spaces associated with x^{k+1}.

Asuume that (P) has a finite optimal solution for given $x^i \in R^n$ by

$C(x^i) = \{x, y : f(x^i) + \nabla f(x^i)^T (x - x^i) - \mu \le 0, g(x^i) + \nabla g(x^i)^T (x - x^i) + By \le 0, \mu \in R^1\}$.

The concerete algorithm for solving (P) is written as follows:

Step 1: Set $\Omega^0 = R^n \times R^m$, lower bound $Z^0 = -\infty$, upper bound $Z^* = +\infty, i = 1$. Select an integer combination $y \in U$, or $y^i \in U \cap V$ if available.

Step 2: Solve the y^i - paramterised NLP subproblem ($S(y^i)$)

$$Z(y^i) = c^T y^i + \min f(x),$$

$$\text{s.t.} \quad g(x) + By \le 0, x \in X .$$

One of the following cases occurs:

a) Problem ($S(y^i)$) has a finite optimal solution ($(x^i, Z(y^i))$) where $Z(y^i)$ is valid upper bound on the optimal value of MINLP problem (P). In this case, update the current upper bound: $Z^* = \min\{Z^*, Z(y^i)\}$. If $Z^* = Z(y^i)$, set $y^* = y^i, x^* = x^i$. Set $\Omega^i = \Omega^{i-1} \cap C(x^i)$, and go to Step 3.

b) Problem ($S(y^i)$) is infeasible (i.e., y^i not included in V). In this case, go to Step 3.

Step 3: Solve the current relaxed MILP master program (M^i)

$Z^i = \min c^T y + \mu$,

s.t. $(x, y) \in \Omega^i$,

$Z^{i-1} \le c^T y + \mu < Z^*$,

$x \in X,\ y \in U,\ \mu \in R^1$,

$y \in$(set of integer cuts).

One of the following cases occurs:

a) Problem (M^i) does not have a mixed-integer feasible solution. In this case, stop.

b) Problem (M^i) has a finite optimal solution (Z^i, x, y), Z^i is an element in the monotonic sequence of lower bounds on the optimal value of the MINLP program (P); y is a new integer combination to be tested in the algorithm. In this case, set $y^{i+1} = y$ and $i = i + 1$ to indicate a new iteration, and return to Step 2.

The above iterative procedure indicates that algoritm consists of solving an alternating sequence of nonlinear programming subproblems ($S(y)$) and relaxed mixed integer linear master program (M^i). It should be noted that if all the functions in problem (P) are linear, the relaxed master program at the first iteration would be identical to the orginal problem, and hence the above algorithm converge in a finite number of steps to the optimal solution of problem (P). In general, the algorithm consists of solving an alternating sequence of problems:

(1) nonlinear programming subproblem ($S(y)$).

(2) relaxed mixed integer linear master programs (M^i).

Chapter 3

Supply Chain Optimisation with Quantity Discount Policy under Demand Uncertainty

3.1 Introduction

Nowadays, the application of supply chain management is widely used in the field. Supply chain management is defined as an effective and efficient planning strategy including a series of activities such as production, distribution, sales and so on. By information sharing, inventory reduction can be achieved. Meanwhile, delivering goods with the minimum cost by the minimum lead time is one of important strategies in supply chain management. The key to the success of supply chain management is to build rational mathematical models and develop an efficient algorithm [69, 85]. With the development of algorithms, more complicated models of large scale problems including nonlinear objective functions and constraints can be solved [29, 33]. For instance, most of the conventional models assume that the demand is deterministic. However, it is not practical because demand always fluctuates in the real market. Thus, it is common to assume that the demand follows standard a normal distribution [35, 46].

It is necessary to investigate quantity discount models in supply chain planning in order to coordinate supply chain members. For practical manufacturing systems, it is important to consider long term planning such as yearly strategic planning in order to set the schedule for contract numbers, production planning and inventory planning. Based on this planning, the midterm planning is determined. In this chapter, a quantity discount model with the supplier selection for the long term planning is considered. Additionally, production, inventory and supplier selecton are determined simultaneously. Quantity discount is regarded as a pricing strategy between manufacturers and suppliers where prices can be discounted according to purchasing quantities. By quantity discounts, purchasing quantities are increased for suppliers, and the total cost can be reduced. The quantity discount problem is formulated as an MINLP. The objective of this chapter is to propose an efficient algorithm to solve the problem. In general, supplier selection problems are divided into single sourcing models and multiple sourcing models. Quantity discounts are generally divided into incremental quantity discounts and all-units discounts. The unit purchasing prices are constant in incremental discount models if order quantities are in the same quantity interval. However, the discount rate for each unit purchased is based on the order quantity in all-units discounts. In other words, if the order quantity increases, the unit purchasing price decreases in all-units discount models.

The rest of the chapter is organised by the following sections. Section 3.2 describes literature review for related works. In Section 3.3, the quantity discount model proposed by Zhang and Ma [104] is intrdouced. An outer approximation method is applied to solve the problem efficiently in Section 3.4. Computational experiments are demonstrated in Section 3.5. Section 3.6 summarises the chapter and states the future research.

3.2 Literature Review

Supplier selection and quantity discount problems for supply chain optimisation are widely studied by many researchers. Liao et al. [59] proposed supplier selection problem where the demand follows a standard normal distribution which is solved by a genetic algorithm.

There is a number of works which have been done to consider quantity discounts. Burke et al. [9] investigated quantity discounts in the centralised purchasing organization. Benton [8] proposed heuristic approaches to solve quantity discounts considering multiple products and resource capacities. Li and Liu [57] demonstrated the availability of quantity discount models with uncertain demands. Crama et al. [18] presented a nonlinear mixed integer programming model for the optimal procurement with total quantity discounts and its linearisation technique. Shi et al. [82] proposed a quantity discount model to consider the buyer's risk with uncertain demands for supply chain planning. Wang [89] considered quantity discount policies as coordination mechanisms in a decentralised distribution system. Discount policies based on the buyers' individual lot size and the annual volume in a general setting with heterogeneous buyers and the price-sensitive demand are developed. In the chapter, the quantity discount problem under demand uncertainty which is formulated as a mixed integer nonlinear programming problem is addressed.

The quantity discount supply chain optimization problem is mathematically formulated as a mixed integer nonlinear optimization problem (MINLP) [19]. The optimisation approaches to mixed integer nonlinear programming (MINLP) problems have been investigated for many years. A general decomposition approach to mixed integer linear programming problems was introduced by Benders' decomposition [7].

For Benders' decomposition technique, decision variables are partitioned into continuous and binary variables. The dual problem of the inner linear programming problem for fixed feasible integer variables is solved to obtain the upper bound. The lower bound is derived by solving the relaxed master problem with the fixed feasible dual variables. Geoffrion [30] expanded the methodology into general MINLPs. Afterwards, Duran and Grossmann [22] developed an outer approximation algorithm for a class of MINLP problems. Linearity of the integer (or discrete) variables, and convexity of the nonlinear functions involving continuous variables are the main features of MINLPs. The outer approximation scheme for solving a class of MINLPs is defined by a finite sequence of relaxed MILP master programs and NLP subproblems. Theoretically, the convergence of the algorithm can be ensured by the finite steps. However, the size of problems effects the computational time to obtain exact solutions. Thus, it is difficult to solve quantity discount models including supplier selection by the conventional approach due to the large size of the problem. In this chapter, an outer approximation algorithm is proposed to solve the problem. The master problem is formulated by fixing discrete variables. The upper bound is derived by solving the master problem using the linearisation and heuristics. The lower bound is derived by solving the relaxed master problem. The effectiveness of the proposed method can be confirmed by computational results.

3.3 Supply Chain Planning Model

In this section, the supply chain planning problem with quantity discounts under demand uncertainty is introduced. Nomenclatures for the model are used for both of

Chapter 3 and Chapter 4.

3.3.1 Nomenclature

Indices:

i : types of raw materials

j : number of suppliers

k : types of finished products

l : discount intervals

Parameters:

a_k : estimated opportunity loss cost for under stocking of one unit of product k

b_k : estimated inventory holding cost for over stocking of one unit of product k

c_{ijl} : unit prices of raw material i purchased from supplier j at interval l

d_{ijl}^S : lower bound of quantity of raw material i from supplier j at interval l

d_{ijl}^H : upper bound of quantity of raw material i from supplier j at interval l

e_k : unit production cost for product k

$f(z_k)$: probability density function where z_k is a random demand obeying a normal distribution

g_{ik} : number of units of raw material i required to produce one unit of product k

L_j : number of intervals of discounts for supplier j

m_j : management cost associated with supplier j

n_{ij} : amount of internal resource for supplier j required to produce one unit raw material i

Q: resource capacity of the manufacturer

q_j: resource capacity of supplier j

r_k: unit sales revenue for product k

N: maximum number of suppliers

t_k: resource required by the manufacturer to produce one unit of product k

z_k: random demand for product k

$\hat{z}_k$: mean value of random demand for product k

σ_k: deviation of random demand for product k

$OP_k(z_k)$: overproduction cost function for product k

$SH_k(z_k)$: shortfall cost function for product k

V_{jl}^H: lower bound of interval l for the total volume for supplier j

V_{jl}^S: upper bound of interval l for the total volume for supplier j

$UC_k(z_k)$: undesirable cost function due to the shortfall and the overproduction for product k

β_{jl}: all-units discount rate at interval l for supplier j

Decision variables:

u_{ijl}: binary variable which takes 1 if raw material i is purchased from supplier j at interval l, and 0 otherwise.

v_{ij}: binary variable which takes 1 if the manufacturer buys raw material i from supplier j, and 0 otherwise.

w_j: binary variable which takes 1 if supplier j is chosen for any purchased raw material, and 0 otherwise.

x_{ijl} : quantity of raw material i purchased from supplier j at discount interval l

y_k : production quantity of product k during the planning horizon

3.3.2 Problem description

The mathematical model of the supply chain planning problem was formulated by Zhang and Ma [104]. It considers a single-period supply chain planning problem for multiple suppliers and one manufacturer. The supply chain planning problem in Fig. 3.1 describes that supplier $j \, (j = 1,.., J)$ is selected to provide raw materials $i \, (i = 1,.., I)$, then through one manufacturer to produce different kinds of products $k \; (k = 1,.., K)$. The demand for finished products is uncertain but it is assumed that the probability distribution is known. The market demands are independent for each product. Suppliers determine price which depend on the quantity for each order. The problem is to find the optimal production level for the manufacturer, the quantity of raw materials purchased from suppliers, and the price of raw materials which is paid by the manufacturer in order to maximise the expected profit subject to both the manufacturer and supplier's capacities.

3.3.3 Quantity discount model

Quantity discount is described as a mechanism that the purchasing prices depend on the quantity. Generally, the following two types of quantity discounts are commonly applied:

i. Incremental discount: the price is decided by the discount interval of purchasing quantity x_{ij} of raw material i from supplier j given by $[dL_{ij}, dU_{ij}]$. dL_{ij} and

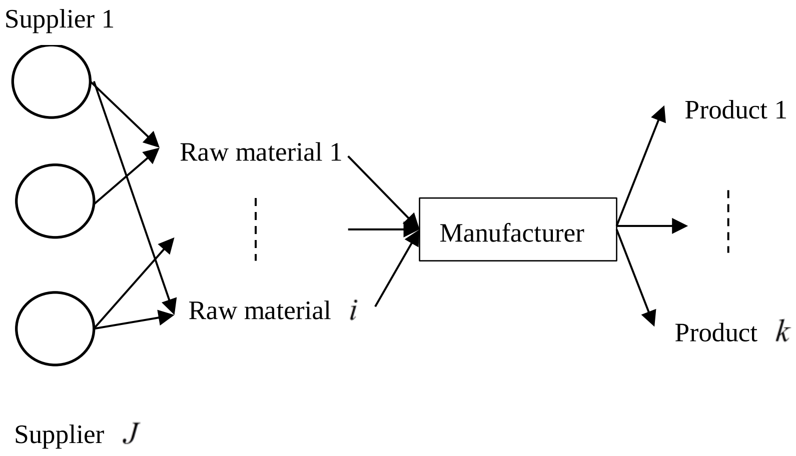

Figure 3.1: Supply chain network model.

dU_{ij} are the minimum and maximum purchasing quantity in the discount interval. With the increase of purchasing quantity, the unit purchasing cost $P(x_{ij})$ is chosen according to the discount interval.

ii. All units discount: if the total purchasing quantity $\sum_i x_{ij}$ of raw material i from supplier j is more than certain amounts, the total purchasing cost $\sum_i P(x_{ij})$ is discounted by the corresponding discount interval.

In this chapter, the incremental discount policy is applied.

3.3.4 Modelling

In this chapter, the total supply chain is considered from the manufacturer's perspective. The decision variables include the purchasing quantity of raw materials, production, supplier selection variables and quantity discount related variables. The constraints consist of the required resources for production, resource capacities for suppliers, quantity discounts and the supplier selection.

The objective function is the maximisation problem which considers the sales revenue, the inventory cost, the opportunity cost, the opportunity cost, the purchasing cost and the production cost. In the model, we consider one manufacturer which purchases different types of raw material $i\ (i=1,..,I)$ within discount interval $l\ (l=1,...,L)$ from the supplier $j\ (j=1,..,J)$ in order to produce product $k\ (k=1,..,K)$ under demand uncertainty.

The quantity discount supply chain planning problem is formulated as the following mixed integer nonlinear programming (MINLP) problem:

$$\begin{aligned} \max \sum_{k=1}^{K} \{ \int_0^{y_k} [r_k z_k - b_k (y_k - z_k)] f(z_k) dz_k + \int_{y_k}^{\infty} [r_k z_k - a_k (z_k - y_k)] f(z_k) dz_k \} \\ - \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{l=1}^{L_j} c_{ijl} x_{ijl} - \sum_{k=1}^{K} e_k y_k - \sum_{j=1}^{J} m_j w_j . \end{aligned} \tag{3.1}$$

Eq. (3.1) represents the expected sales revenue which is minus to the inventory cost and the opportunity loss cost, the purchasing cost, the production cost and the supplier management cost. Actually, the negative value of demand should be considered in the expectation of Eq. (3.1). In real life situations, the demand is always positive. Therefore, the truncated normal distribution with positive demand is considered in the objective function. Thus, it is an approximation of the true maximum expected profit.

The constraints are as follows:

- Required raw materials: the amount of raw materials is required in order to satisfy production,

$$\sum_{k=1}^{K} g_{ik} y_k \le \sum_{j=1}^{J} \sum_{l=1}^{L_j} x_{ijl}, \forall i . \tag{3.2}$$

- Capacity constraints: the production and purchasing quantity should satisfy the resource capacity for factories and suppliers,

$$\sum_{k=1}^{K} t_k y_k \leq Q, \tag{3.3}$$

$$\sum_{i=1}^{I} n_{ij} \sum_{l=1}^{L_j} x_{ijl} \leq q_j w_j, \forall j. \tag{3.4}$$

- Quantity discount related constraints: purchasing quantity cannot exceed the corresponding upper bound and lower bound of the discount interval,

$$x_{ijl} \leq d_{ijl}^{H} u_{ijl}, \forall i, \forall j, \forall l, \tag{3.5}$$

$$x_{ijl} \geq d_{ijl}^{S} u_{ijl}, \forall i, \forall j, \forall l, \tag{3.6}$$

$$\sum_{l=1}^{L_j} u_{ijl} = v_{ij}, \forall i, \forall j. \tag{3.7}$$

- Management cost constraints: management cost occurs if the supplier is contracted by the manufacturer,

$$w_j \geq v_{ij}, \forall i, \forall j. \tag{3.8}$$

- Nonnegative and binary variable constraint:

$$x_{ijl}, y_k \geq 0, u_{ijl}, v_{ij}, w_j \in \{0,1\}, \forall i, \forall j, \forall l. \tag{3.9}$$

- Single source model: only one supplier is allowed to choose if one type of raw materials is purchased,

$$\sum_{j=1}^{J} v_{ij} = 1, \forall i. \tag{3.10}$$

- Multiple sources model: the maximum number N of supplier can be chosen if one type of raw materials is purchased,

$$\sum_{j=1}^{J} v_{ij} \leq N, \forall i\ . \tag{3.11}$$

3.4 Solution Approach

3.4.1 Conventional approach

It is known that there is difficulty in solving the formulated MINLPs. It is impossible to apply the current solver to the MINLP with integral terms. Zhang and Ma [104] proposed an approach by using the external nonlinear programming problem (NLP) solver function combined with the standard branch and bound algorithm. At each node of the search tree, the relaxed problem is an NLP model which is solved by the commercial NLP solver (CONPT, NINOS, SNOPT). However, it is difficult to obtain exact solutions within a reasonable time by using the branch and bound algorithm. Thus, an outer approximation method is proposed to solve the problem.

3.4.2 Outer approximation method

The main characteristics of the following MINLP problem (P_1) include the linearity of integers or discrete variables and the convexity of the nonlinear function involving continuous variables:

$$(P_1) \quad \min\,(c^T y + f(x)), \tag{3.12}$$

$$\text{s.t.} \quad g(x) + By \leq 0, \tag{3.13}$$

$$x \in X = \{x \mid x \in R^n\},\ y \in \{0,1\}^n, \tag{3.14}$$

where x is a continuous variable of n dimensional vector and y is a binary variable of N dimensional vector. X is a nonempty, compact and convex set and the functions f and g are convex in x. f and g are once continuously differentiable.

Fixing y variables to a 0-1 combination, which is denoted as y^h, the problem is formulated as

$$(P_2) \quad \min\ (c^T y + f(x)), \tag{3.15}$$

$$\text{s.t.}\quad g(x) + By^h \le 0, \tag{3.16}$$

$$x \in X. \tag{3.17}$$

The upper bound is obtained from the solution of the primal problem if the problem is feasible. It is obvious that program (P_1) is not a convex program in x and y jointly, but fixing y renders it so in x for program (P_2). Therefore, y^h can be a candidate for the optimal solution to the problem (P_1). Therefore, the problem (P_2) is equivalent to the following minimisation problem (P_3) over x and y variables separately:

$$(P_3) \quad \min_{x} \min_{y^h}\ (c^T y^h + f(x)), \tag{3.18}$$

$$\text{s.t.}\quad g(x) + By^h \le 0, \tag{3.19}$$

$$x \in X,\ y \in \{0,1\}^N. \tag{3.20}$$

The projection of the problem in the space of y variables further enables (P_3) to be written in the following form, which is the relaxed master problem (P_4):

$$(P_4) \quad \min_{x} \min_{y^h}\ (c^T y^h + f(x^h) + \nabla f(x^h)(x - x^h)), \tag{3.21}$$

s.t. (3.19), (3.20),

$$0 \geq g(x^h) + \nabla g(x^h)(x - x^h) + By\,. \tag{3.22}$$

Adding cuts, the problem can be written as follows:

$$(P_5) \quad \min_x \min_{y^h} (c^T y^h + \mu_{OA})\,, \tag{3.23}$$

s.t. (3.19), (3.20), (3.22),

$$\mu_{OA} \geq f(x^h) + \nabla f(x^h)(x - x^h)\,, \tag{3.24}$$

$$\sum_{i \in B^h} y_i^h - \sum_{i \in NB^h} y_i^h \leq |B^h| - 1, \forall h \in B\,, \tag{3.25}$$

where $B^h = \{ i \mid y_i^h = 1 \}, NB^h = \{ i \mid y_i^h = 0 \}$.

The basic idea of the outer approximation is similar to the one in generalised Benders decomposition that an upper bound and a lower bound are derived on the MINLP solution at each iteration. The upper bound is obtained from the solution of the primal problems with fixed y variables ($y = y^h$), while the lower bound is obtained from the solution of the master problem. The master problem is solved with the fixed x^h of the primal solution. The lower bound is derived based on an outer approximation of the nonlinear objective function and constraints around x^h. The solution of the master problem provides information on the next fixed y^{h+1}.

3.4.3 Primal problem

The primal with fixing 0-1 binary variables $y = [\{u_{ijl}\},\{v_{ij}\},\{w_j\}]^T$ to $y^h = [\{u_{ijl}^h\},\{v_{ijl}^h\},\{w_j^h\}]^T$ yields the following optimisation problem:

$$\begin{aligned} \min & -\sum_{k=1}^{K} \{ \int_0^{y_k} [r_k z_k - b_k(y_k - z_k)] f(z_k) dz_k + \int_{y_k}^{\infty} [r_k y_k - a_k(z_k - y_k)] f(z_k) dz_k \} \\ & + \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{l=1}^{L_j} c_{ijl} x_{ijl} + \sum_{k=1}^{K} e_k y_k\,, \end{aligned} \tag{3.26}$$

s.t. (3.2), (3.3),

$$\sum_{i=1}^{I} n_{ij} \sum_{l=1}^{L_j} x_{ijl} \le q_j w_j^h, \forall j, \tag{3.27}$$

$$x_{ijl} \le d_{ijl}^H u_{ijl}^h, \forall i, \forall j, \forall l, \tag{3.28}$$

$$x_{ijl} \ge d_{ijl}^S u_{ijl}^h, \forall i, \forall j, \forall l, \tag{3.29}$$

$$x_{ijl}, y_k \ge 0, \forall i, \forall j, \forall k, \forall l. \tag{3.30}$$

By taking the first partial derivative with respective to y_k, we obtain

$$\frac{\partial L}{\partial y_k} = -(r_k + a_k - e_k) + (r_k + a_k + b_k) F(y_k), \tag{3.31}$$

where L is the Lagrange function of Eq. (3.26). $F(y_k)$ is a cumulative density function. From the condition $\frac{\partial^2 L(y_k)}{\partial y_k} = (r_k + a_k + b_k) f(y_k) \ge 0$, the objective function is convex. The primal problem is a nonlinear optimisation problem with equality and inequality constraints. The problem can be solved exactly by a sequential quadratic programming (SQP) technique.

3.4.4 Master problem

The outer approximation can be obtained by the intersection of a finite set of supporting functions. The following master problem can be written by using solutions obtained in the previous section which is derived by the linearisation technique:

$$\min \left(\mu_{OA} + \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{l=1}^{L_j} c_{ijl} x_{ijl} + \sum_{j=1}^{J} m_j w_j \right), \tag{3.32}$$

s.t. (3.2) - (3.11), (3.28), (3.29),

$$\mu_{OA} \geq -P(y^h) - \frac{\partial P}{\partial y_k}\Big|_{y_k = y_k^h}(y_k - y_k^h), \tag{3.33}$$

$$\sum_{j\in B^h}(w_j + \sum_{i\in B^h} v_{ij} + \sum_{l=B^h} u_{ijl}) - \sum_{j\in NB^h}(w_j + \sum_{j\in NB^h} v_{ij} + \sum_{l\in NB^h} u_{ijl}) \leq |B^h| - 1, \forall h, \tag{3.34}$$

where

$$P(y) = -\sum_{k=1}^{K}\{\int_0^{y_k}[r_k z_k - b_k(y_k - z_k)]f(z_k)dz_k$$
$$+\int_{y_k}^{\infty}[r_k y_k - a_k(z_k - y_k)]f(z_k)dz_k\} + \sum_{k=1}^{K} e_k y_k.$$

The master problem is to determine the binary variable configuration with the outer approximation of the objective function with the linear supporting function, which can be solved by a commercial MILP solver.

3.5 Algorithm

The decomposition algorithm consists of the master subproblem to derive a lower bound, and the primal problem with a fixed y to derive an upper bound. The optimisation algorithm is described as follows:

Step 0 (Initialisation of parameters) Define the maximum number of iterations $H^{\max}$. Initialise the continuous variables x_{ijl}, y_k and binary variables u_{ijl}, v_{ij}, w_j. Define the tolerance parameter ε for evaluating convergence. Set the iteration number $H \leftarrow 1$.

Step 1 (Primal problem) Solve the primal problem for $y_k = y_k^{h-1}$. The optimal solution y_k is obtained by solving the primal problem optimally. If this problem is feasible, store the optimal solution for the primal problem. Update the upper bound.

Step 2 (Relaxed master problem) Formulate the master problem using the current solution of the primal problem derived at Step 1. The linearisation of objective function around y_k^h is executed. The mixed integer linear programming problem is solved. Select the lower bound, and also select corresponding value of y as $y_k^{\min}$. Set $y_k^{H+1} = y_k^{\min}$.

Step 3 (Check for convergence) If $LB > UB - \varepsilon$ or $H \geq H^{\max}$, then the algorithm is terminated. Otherwise $H = H + 1$, and return to Step 1.

3.6 Heuristics

SQP is an iterative method applied to solve the NLP with inequality constraints. If the size of problem increases, the computational time will also increase. In this section, a heuristic approach is applied to production decision. The nonlinear term results in the increased computational time in the model. Thus, the heuristic approach is utilised to determine variables in the nonlinear term in order to reduce computational time. By taking the first partial derivative with respect to y_k, we obtain

$$-(r_k + a_k - e_k) + (r_k + a_k + b_k)F(y_k) = 0. \qquad (3.35)$$

The solution y_k^* of Eq. (3.35) is an infeasible solution of the orginal problem if it cannot satisfy the capacity constraint. If the production decreases, the opportunity loss costs may occur. If the purchasing quantity is increased, the total profit decreases. Thus, the feasible solution should be obtained by minimising the increase of the objective function, while the production is adjusted by increasing the purchasing costs. Let $y_k^{'}$ denote the difference of y_k^*. We can formulate the problem as follows:

$$(P_{H1}) \quad \min \left(\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L_j} c_{ijl}x_{ijl} + \sum_{k=1}^{K} My_k^{'}\right), \qquad (3.36)$$

s.t. (3.27) - (3.30),

$$\sum_{k=1}^{K} g_{ik}(y_k^* - y_k^{'}) \le \sum_{j=1}^{J}\sum_{l=1}^{L_j} x_{ijl}, \forall i \, , \tag{3.37}$$

$$\sum_{k=1}^{K} t_k(y_k^* - y_k^{'}) \le Q \, . \tag{3.38}$$

where M is a large positive number. The problem is formulated as a linear problem. Therefore, it can be solved by the simplex method easily. If feasible solutions exist, we obtain the upper bound by solving the primal problem. However, if they are not optimal solutions by fixing 0-1variables, they cannot be treated as feasible updated solutions by OA. Thus, the following problem is formulated by using the fixed y_k which is obtained from the primal problem:

$$(P_{H2}) \quad \min \left(\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L_j} c_{ijl}x_{ijl} + \sum_{j=1}^{J} m_j w_j\right), \tag{3.39}$$

s.t. (3.4) - (3.9), (3.11),

$$\sum_{k=1}^{K} g_{ik} y_k^h \le \sum_{j=1}^{J}\sum_{l=1}^{L_j} x_{ijl}, \forall i \, , \tag{3.40}$$

where x_{ijl}, u_{ijl}, v_{ij} and w_j are determined by (P_{H2}). If the feasible solution of (P_{H2}) exist, we obtain the upper bound by solving the primal problem. The algorithm for solving the primal problem by heuristics is as follows:

Step 0 The optimal y_k^* is obtained by solving Eq. (3.35).

Step 1 Solve (P_{H1}). If a feasible solution exists, the solution is stored. The upper bound is obtained, and go to Step 2. If a feasible solution does not exist, go to Step 3.

Step 2 Solve (P_{H2}) by fixing y_k. If no feasible solution exists, compare it with the solution obtained at Step 1. Then, choose the better one.

Step 3 Solve the relaxed primal problem.

3.7 Computational Experiments

The model is solved by OA. The Intel Pentium 4 3.4 GHz and 1GB memory computer is used. The primal problem is solved by the fmincon function in Matlab ver7.4.0.287 (R2007a) optimisation toolbox. The relaxed problem is solved by CPLEX MATLAB interface (CPLEXINT) integrating CPLEX 10.1 and MATLAB functions.

3.7.1 Illustrative examples

In order to demonstrate the performance of the proposed method, two types of examples are conducted by using the incremental discount model. We set two examples to demonstrate that the proposed method can solve relatively larger scale problem instances in example 2 efficiently. In example 1, there are 2 products, 5 raw materials, 4 suppliers, and 2 incremental discount intervals. In example 2, there are 5 products, 5 components, 5 suppliers, and 3 incremental discount intervals. The parameters for the experiments are generated randomly in the range shown in Table 3.1 and Table 3.2. The parameter setting is based on the data provided by Zhang and Ma [104]. The update time of the upper bound cannot exceed 10 and the duality gap is less than 1%, and the update time of the lower bound is 1000.

3.7.2 Results and discussion

Five cases of instances are generated and solved by the proposed method. The results are shown in Table 3.3 and Table 3.4, respectively. LBD, UBD, Time and Iter. indicate the lower bound, the upper bound, the computation time [sec.] and the number of

Table 3.1: Parameters for example problems (case 1).

r_k	500-750	e_k	120-135	g_{ik}	0-1
t_k	79.5-80.5	n_{ij}	0.75-1.25	b_k	100-170
μ_k	25.1-25.2	m_j	100-150	Q	3200
σ_k^2	3.7-4.0	q_j	35-45	a_k	10-12
d_{ijl}^S	0.001,10.000	d_{ijl}^H	10,100	c_{ijl}	14-40

Table 3.2: Parameters for example problems (case 2).

r_k	150-250	e_k	23-40	g_{ik}	1-4
t_k	1-3	n_{ij}	1-3	b_k	10-100
μ_k	160-200	m_j	350	Q	2000
σ_k^2	60-80	q_j	450-550	a_k	50-150
d_{ijl}^S	0.001,10.000	d_{ijl}^H	1000,10000	c_{ijl}	5-17

iterations, respectively. DGAP is the duality gap that $DGAP = (UBD\text{-}LBD)/LBD \times 100$ [%]. The results demonstrate that the proposed method can derive near-optimal solutions for problem instances. From the results, we can observe that the computational time increase significantly as the size of the problem increases. This is because the direct integral computation is used for the optimisation of primal problem. The direct integral computation increases the number of iterations. It takes 40 seconds to solve the primal problem and less than 1 second to solve the relaxed problem in Case 1. Thus, we can say that it requires more time to solve primal problems in order to obtain optimal solutions. It takes more than 300 seconds to solve the primal problem in Case 2. The reason is that the size of problem is increased in Case 2.

Table 3.3: Computational results for case 1.

Instance	LBD	UBD	DGAP[%]	Time[s]	Iteration
1	1.95×10^4	1.95×10^4	0.00225	113.0	20
2	2.14×10^4	2.14×10^4	0.00575	90.1	20
3	1.75×10^4	1.75×10^4	0.00608	99.7	20
4	1.99×10^4	1.99×10^4	0.0563	111.0	20
5	2.04×10^4	2.05×10^4	0.0677	94.6	20

Table 3.4: Computational results for case 2.

Instance	LBD	UBD	DGAP[%]	Time[s]	Iteration
1	2.68×10^4	2.70×10^4	1.00	1.08×10^4	630
2	2.88×10^4	2.88×10^4	0.00491	1.09×10^3	40
3	2.88×10^4	2.89×10^4	0.468	9.02×10^2	30
4	2.93×10^4	2.95×10^4	0.781	5.95×10^3	330
5	2.92×10^4	2.93×10^4	0.322	7.78×10^2	30

3.7.3 Performance evaluation

In this section, we compare three methods which are OA-SQP, OA combined with heuristics and Zhang and Ma’s method (optimal) for Case 3. Case 3 consists of 5 raw materials, 4 suppliers, 2 products and 2 discount intervals. The computational environment is the same as the previous section. The result is shown in Table 3.5. From the result, the lower bound and the upper bound are the same for SQP method. And, the computational time is 87.4 seconds. Thus, we can conclude that the OA method is effective to solve the problem. It takes 60 seconds to obtain the exact solutions by Zhang and Ma’s method and the SQP method for small size problems. However, the time will increase significantly by exact algorithms when the size of problems is large.

It only takes less than 2 seconds to obtain solutions by OA combined with a heuristic. However, the duality gap is worse. For the small size problems, the difference between the lower bound and the upper bound does not change even if more computational time is given by OA-Heuristic. Thus, it is more efficient to use the exact algorithm to handle small size problems.

Table 3.5: Computational results for case 3.

Method	LBD	UBD	DGAP[%]	Time[s]	Iteration
Optimal	4.77×10^3	-	-	60	-
OA-SQP	4.77×10^3	4.77×10^3	0	87.4	30
OA-Heuristic	4.34×10^3	5.04×10^3	8.91	1.50	40

3.7.4 Computational results for large scale problems

In this section, computational examples for large scale problems are conducted. In case 4, there are 20 raw materials, 12 suppliers, 10 products and 2 discount intervals. The parameter setting is shown in Table 3.6. We compare OA-Heuristic and OA-SQP for 5 cases. And, the results are shown in Table 3.7 and Table 3.8. We set that the update time of the lower bound is less than 5, the duality gap is less than 1% and the update time of the upper bound is less than 100. From Table 3.7, it takes about 988 seconds to obtain solutions by 19.5% duality gap using OA-Heuristic. The convergence could not be confirmed within 2000 seconds by using OA-SQP. From Table 3.8, it needs 22800 seconds to solve primal problem in case 1. Thus, we conclude that it is effective to handle large scale problems by using OA-Heuristic.

Table 3.6: Parameters for example problems (case 4).

r_k	150-300	e_k	24-55	g_{ik}	0-4
t_k	1-3	n_{ij}	0-3	b_k	10-100
μ_k	120-330	m_j	350	Q	3200
σ_k^2	40-100	q_j	2500-12500	a_k	50-190
d_{ijl}^S	0.001,10	d_{ijl}^H	10,100	c_{ijl}	2-17

Table 3.7: Computational results of OA-Heuristic for case 4.

Instance	LBD	UBD	DGAP[%]	Time[s]	Iteration
1	1.58×10^5	1.83×10^5	13.8	1.24×10^3	76
2	1.84×10^5	2.38×10^5	22.5	1.14×10^3	81
3	1.40×10^5	1.72×10^5	18.4	9.54×10^2	60
4	1.51×10^5	1.83×10^5	17.5	7.33×10^2	51
5	1.18×10^5	1.57×10^5	25.2	8.69×10^2	55

Table 3.8: Computational results of OA-SQP for case 4.

Instance	LBD	UBD	DGAP[%]	Time[s]	Iteration
1	8.99×10^4	2.10×10^5	-	2.28×10^4	1
2	1.31×10^4	2.39×10^5	-	2.26×10^4	1
3	7.95×10^4	2.06×10^5	-	2.30×10^4	1
4	8.26×10^4	2.16×10^5	-	2.29×10^4	1
5	4.91×10^4	1.69×10^5	-	2.30×10^4	1

3.8 Summary

In this chapter, a solution approach to a quantity discount model in supply chain planning with uncertain demand is developed. An outer approximation algorithm with a heuristic is proposed to solve the problem. The master problem is formulated by fixing discrete variables. The upper bound is derived by solving the master problem using the linearisation and heuristics. The lower bound is derived by solving the relaxed master problem. The effectiveness of the proposed method is confirmed by computational results.

Chapter 4

A Reformulation of Supply Chain Planning Problems under Demand Uncertainty

4.1 Introduction

Supply chain optimisation ensures efficient operations that consist of purchasing, manufacturing and all logistics activities function optimally. Quantity discount is a price reduction strategy offered by suppliers to buyers who purchase a large number of products at once. The application of quantity discounts contributes to reducing the buyer's costs and the total profits [54, 57, 65]. Generally, quantity discounts are divided into incremental quantity discounts and all-units discounts. The unit purchasing prices are constant in incremental discount models if order quantities are in the same quantity interval. However, the discount rate for each unit purchased is based on the order quantity for all-units discounts. In other words, if the order quantity increases, the unit purchasing price decreases in all-units discount models.

In this chapter, quantity discount problems under demand uncertainty are addressed. The mathematical model for the incremental discount model introduced in Chapter 3 is used in this chapter as well. The problem is formulated as a mixted integer nonlinear programming problem including integral terms. In Chapter 3, an outer approximization algorithm with a heuristic is proposed to solve the problem. In Chapter 4, an exact algorithm is applied to solve the problem. The model includes integral terms due to the uncertain demand. It requires huge computational efforts to handle the integration of multivariate probability functions. Therefore, the stochastic model is reformulated by a normalisation technique into an equivalent deterministic form to reduce the computational time. Furthermore, the models are extended to consider both of incremental discounts and all-units discounts.

The main contribution of this chapter is to reformulate supply chain planning problems with quantity discounts under demand uncertainty which is formulated as a MINLP with integral terms. The model is reformulated by using a normalisation technique in order to improve efficiency of the algorithm. The supply chain planning problem is formulated as a stochastic model due to demand uncertainty. It is assumed that the uncertain demand obeys a standard normal distribution. Thus, the standardised normal form is used to represent the stochastic model by an equivalent deterministic form.

The rest of the chapter is organised by the following sections. Section 4.2 describes the literature review for related works. In Section 4.3, the incremental discount model is defined, which is formulated as a mixed integer nonlinear programming problem. An outer approximation method is applied to solve the problem efficiently. The normalisation technique is also used to reduce the computational efforts in Section 4.4. The all-units discount model is explained in Section 4.5. Computational experiments are demonstrated in Section 4.6. Section 4.7 summarises the chapter.

4.2 Literature Review

Considerable efforts have been made for supplier selection problems with price discount models. Supplier selection problems can be broadly classified into single sourcing models and multi-sourcing models [1]. While purchasing a range of products, the quantity discount is considered for single item and multiple items models. There are two main streams of the discount structure: quantity discounts and volume discounts. For quantity discount, sales of volume of products do not affect prices and the discount of other products. For volume discount, the discount depends on total amount of sales volume but not the quantity or variety of purchased products. Quantity discounts are generally divided into incremental discounts and all-units discounts. Both of incremental discounts and all-units discounts in the supply chain optimisation model considering multi-sourcing are introduced in this chapter.

Kim et al. [46] presented a mathematical model taking into account the selection of suppliers, decision of ordering products under demand uncertainty with the given capacity limits of suppliers and manufacturers. They developed an iterative algorithm to solve the model using Karush Kuhn Tucker (KKT) condition. Zhang and Ma [104] extended their work to consider the quantity discount and fixed costs. They provided a solution approach using the external nonlinear programming problem (NLP) solver function combined with the standard branch and bound algorithm. At each node of the search tree, the relaxed problem is an NLP model which is solved by the commercial NLP solver. Tsai [85] developed a supply chain model with the quantity discount policy utilizing linearisation techniques. A nonlinear model is approximated to a mixed integer linear programming problem. In Zhang and Ma's model, the all-units discount model is developed to determine the production level, order quantity and supplier selection under demand uncertainty. The model developed by Zhang and Ma is utilised to consider both of all-units discounts and incremental discount [104]. In the all-units discount model, the problem includes a nonlinear term caused by the all-unit discount policy. A technique is applied to linearise the master problem.

The supplier selection problems with the quantity discount models, which are formulated as mixed integer programming problems and nonlinear programming problems, have been studied by using exact algorithms and heuristic algorithms. Crama et al. [18] presented a nonlinear mixed integer programming model for optimal procurement with total quantity discounts and its linearisation technique. Goossens et al. [33] addressed an exact algorithm for procurement problems with the total quantity structure in terms of the fixed charge network flow formulation. Burke et al. [9] analysed a variety of supplier pricing schemes including linear discounts, incremental unit discounts and all unit discounts by a heuristic procedure. Liao and Rittscher [59] considered a multi-objective supplier selection model under stochastic demand conditions. They provided a genetic algorithm for the objective of the multiple cost minimisations for quality minimisation, delivery minimisation and flexibility maximisation. Many researchers considered a variety of supplier selection problems from different perspectives with quantity discounts. The exact algorithms and heuristic algorithms are applied in their works. In this chapter, an exact algorithm is applied to solve quantity discount problems. By using the exact algorithm, a reformulation is required in order to reduce computational time.

In the previous studies, various quantity discount models with different nonlinear quantity discount schemes are widely studied. In this chapter, an exact algorithm is addressed to solve incremental discount and all-units discount problems with uncertain demands which are formulated as mixed integer nonlinear programming problems with integral terms. However, the model includes integral terms due to the uncertain demand. It requires huge computational efforts to handle the integration of multivariate probability functions. Therefore, the key issue is to avoid handling integral terms directly in MINLP problems. The contribution of the chapter is to reformulate the problem in order to reduce the computational time. The stochastic model of MINLP problems is reformulated as equivalent deterministic forms.

4.3 The Incremental Discount Model

Nomenclatures

Indices:

i : types of raw materials

j : number of suppliers

k : types of finished products

l : discount intervals

Parameters:

a_k : estimated opportunity loss cost for under stocking of one unit of product k

b_k : estimated inventory holding cost for over stocking of one unit of product k

e_k : unit production cost for product k

$f(z_k)$: probability density function where z_k is a random demand obeying a normal distribution

L_j : number of discount intervals for supplier j

m_j : management cost associated with supplier j

M : large constant

r_k : unit sales revenue for product k

z_k : random demand for product k

$\hat{z}_k$: mean value of random demand for product k

σ_k : deviation of random demand for product k

$OP_k(z_k)$: overproduction cost function for product k

$SH_k(z_k)$: shortfall cost function for product k

V_{jl}^H : lower bound of interval l for the total volume for supplier j

V_{jl}^S : upper bound of interval l for the total volume for supplier j

$UC_k(z_k)$: undesirable cost function due to the shortfall and the overproduction for product k

β_{jl} : all-units discount rate at interval l for supplier j

Decision variables:

u_{ijl} : binary variable which takes 1 if raw material i is purchased from supplier j at interval l , and 0 otherwise.

v_{ij} : binary variable which takes 1 if the manufacturer buys raw material i from supplier j ,and 0 otherwise.

w_j : binary variable which takes 1 if supplier j is chosen for any purchased raw material, and 0 otherwise.

x_{ijl} : quantity of raw material i purchased from supplier j at discount interval l

y_k : production quantity of product k during the planning horizon

$F(Y_k)$: probability density function subject to $Y_k = \frac{y_k - \hat{z}_k}{\sigma_k}$

$\Phi(Y_k)$: cumulative distribution function subject to $Y_k = \frac{y_k - \hat{z}_k}{\sigma_k}$

In this chapter, the supply chain model which is introduced in Chapter 3 is used. The mathematical model for supply chain planning problem is formulated by Zhang and Ma [104]. Thus, the quantity discount supply chain planning problem is formulated as the following mixed integer nonlinear programming (MINLP) problem:

$$\max \sum_{k=1}^{K}\{\int_0^{y_k}[r_k z_k - b_k(y_k - z_k)]f(z_k)dz_k + \int_{y_k}^{\infty}[r_k z_k - a_k(z_k - y_k)]f(z_k)dz_k\} - \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L_j} c_{ijl}x_{ijl} - \sum_{k=1}^{K} e_k y_k - \sum_{j=1}^{J} m_j w_j, \tag{4.1}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} g_{ik} y_k \le \sum_{j=1}^{J}\sum_{l=1}^{L_j} x_{ijl}, \forall i, \tag{4.2}$$

$$\sum_{k=1}^{K} t_k y_k \le Q, \tag{4.3}$$

$$\sum_{i=1}^{I} n_{ij} \sum_{l=1}^{L_j} x_{ijl} \le q_j w_j, \forall j, \tag{4.4}$$

$$x_{ijl} \le d_{ijl}^{H} u_{ijl}, \forall i, \forall j, \forall l, \tag{4.5}$$

$$x_{ijl} \ge d_{ijl}^{S} u_{ijl}, \forall i, \forall j, \forall l, \tag{4.6}$$

$$\sum_{l=1}^{L_j} u_{ijl} = v_{ij}, \forall i, \forall j, \tag{4.7}$$

$$w_j \ge v_{ij}, \forall i, \forall j, \tag{4.8}$$

$$\sum_{j=1}^{J} v_{ij} \le N, \forall i, \tag{4.9}$$

$$x_{ijl}, y_k \ge 0, \forall i, \forall j, \forall k, \forall l, \tag{4.10}$$

$$u_{ijl}, v_{ij}, w_j \in \{0,1\}, \forall i, \forall j, \forall l \,. \tag{4.11}$$

The objective function of Eq. (4.1) includes integral terms due to demand uncertainty. The objective is to maximise the total profit for the manufacturer. The first term represents the expected revenue minus the inventory holding cost for the manufacturer while the production quantity is more than the actual demand. The second term is the expected revenue minus the penalty cost for shortage while the production quantity is lower than the actual demand. The third, the fourth and the last term are the procurement cost for raw material, the production cost and the management cost, respectively. The management cost for the manufacturer is to maintain supplier j for the management /development activities. Eq. (4.2) is the raw material requirement constraint ensuring that production can be achieved by the purchase. Eq. (4.3) indicates that the production capacity is limited by constraints. Eq. (4.4) describes required resources for producing raw materials, which should not exceed the total quantity of resources reserved by the supplier for the manufacturer. Eq. (4.5) and Eq. (4.6) represent the quantity of raw materials from the supplier that is bounded by the lower and upper bound of a certain quantity discount interval. Eq. (4.7) is the assignment constraint of the quantity discount. Eq. (4.8) is related to the management cost, which is incurred if the supplier is selected to provide any raw material. Eq. (4.9) represents the multiple-sourcing constraint indicating that at most $N(N \geq 2)$ suppliers for the provision of one raw material is allowed. Eq. (4.10) and Eq. (4.11) denote the nonnegative and integer constraints.

4.4 Reformulation

The model formulated in the previous section is difficult to handle because the problem includes the nonlinear integral term in MINLPs. In the previous chapter, the outer-approximation algorithm with a heuristic was proposed to solve the MINLP problem. However, it is necessary to reformulate the problem in order to increase the efficiency of the optimisation algorithm.

The primal problem including the integral equation can be simplified from another aspect. Due to demand uncertainty, the model is considered as a stochastic model that product demands are assumed to be multivariate normally distribution random variables. By reformulation, the resulting equivalent deterministic optimisation model is nonlinear programming problems (NLP) with the convex continuous model [76].

While in the deterministic model the planned production quantity is always realised, however, it is not always true in the stochastic case. The stochastic model for quantity discount assumes that product demands fluctuate, which implies that there is a deviation between the realisation of random variables demand z_k and production quantity y_k. Therefore, the sold quantity of product k can be expressed by the minimum between z_k and y_k. The relation can be expressed as follows:

$$\min\ (z_k, y_k).$$

Therefore, the sales revenue from products is

$$RE_k(z_k) = r_k \min\ (z_k, y_k) = \begin{cases} r_k z_k & \text{if} \quad z_k \le y_k, \\ r_k y_k & \text{if} \quad z_k > y_k. \end{cases} \tag{4.12}$$

The undesirable cost from the shortfall and overproduction is

$$UC_k(z_k) = b_k \max\ (0, y_k - z_k) + a_k \max\ (0, z_k - y_k) = \begin{cases} b_k (y_k - z_k) & \text{if} \quad z_k \le y_k, \\ a_k (z_k - y_k) & \text{if} \quad z_k > y_k. \end{cases} \tag{4.13}$$

The stochastic single period quantity discount with supply chain planning model is formulated as follows:

$$\max E[\sum_{k=1}^{K} RE_k(z_k) - UC_k(z_k)] - \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L_j} c_{ijl}x_{ijl} - \sum_{k=1}^{K} e_k y_k - \sum_{j=1}^{J} m_j w_j \ , \tag{4.14}$$

s.t. (4.2) - (4.11),

where we define

$$RE_k(z_k) = r_k \min(z_k, y_k) = \begin{cases} r_k z_k & \text{if } z_k \le y_k, \\ r_k y_k & \text{if } z_k > y_k, \end{cases}$$

$$UC_k(z_k) = b_k \max(0, y_k - z_k) + a_k \max(0, z_k - y_k) = \begin{cases} b_k(y_k - z_k) & \text{if } z_k \le y_k, \\ a_k(z_k - y_k) & \text{if } z_k > y_k. \end{cases}$$

The undesirable cost equation of the shortfall SH_k and the overproduction OP_k is expressed by

$$SH_k(z_k) = a_k \max(0, z_k - y_k) = \begin{cases} 0 & \text{if } z_k \le y_k, \\ a_k(z_k - y_k) & \text{if } z_k > y_k, \end{cases} \tag{4.15}$$

$$OP_k(z_k) = b_k \max(0, y_k - z_k) = b_k[\max(z_k, y_k) - z_k]. \tag{4.16}$$

In order to simplify the calculation of the expectation, the normal distribution can be recast into the standardizing normal form with a mean of 0 and a variable 1. By checking the Z-transformation form with the known mean value μ_k and the standard deviation σ_k^2 of the demand, the expectation can be calculated. Let F be the probability density function, and Φ be the cumulative distribution function.

Let $X_k = \dfrac{z_k - \hat{z}_k}{\sigma_k}$ and $Y_k = \dfrac{y_k - \hat{z}_k}{\sigma_k}$, where $\hat{z}_k$ denotes the mean value of z_k.

Then, we obtain

$$E(RE_k) = \Phi(Y_k)E[r_k z_k | \frac{z_k - \hat{z}_k}{\sigma_k} \le \frac{y_k - \hat{z}_k}{\sigma_k}] + (1 - \Phi(Y_k))E[r_k y_k | \frac{z_k - \hat{z}_k}{\sigma_k} \ge \frac{y_k - \hat{z}_k}{\sigma_k}]$$

$$= r_k \hat{z}_k + r_k \sigma_k \{\Phi(Y_k) E[X_k | X_k \le Y_k] + (1-\Phi(Y_k)) E[Y_k | X_k \ge Y_k]\} . \qquad (4.17)$$

Thus, there are two situations, $X_k \ge Y_k$ and $X_k \le Y_k$. We have

$$E[Y_k | Y_k \le X_k] = Y_k . \qquad (4.18)$$

According to the definition of the expectation of a standard normal distribution excluded $X_k = Y_k$, we have

$$E[X_k | X_k \le Y_k] = \frac{\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Y_k} X_k e^{-\frac{1}{2} X_k^2} dX_k}{\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Y_k} e^{-\frac{1}{2} X_k^2} dX_k} = -\frac{e^{-\frac{1}{2} X_k^2}}{\Phi(Y_k)\sqrt{2\pi}} = -\frac{F(Y_k)}{\Phi(Y_k)} . \qquad (4.19)$$

By substituting Eq. (4.18) and Eq. (4.19) into Eq. (4.17), we obtain

$$E(RE_k) = r_k \hat{z}_k + r_k \sigma_k [-F(Y_k) + (1-\Phi(Y_k)) Y_k] . \qquad (4.20)$$

Therefore, we have

$$E(SH_k) = -a_k \sigma_k [-F(Y_k) + (1-\Phi(Y_k)) Y_k] , \qquad (4.21)$$

$$E(OP_k) = b_k \sigma_k [\Phi(Y_k) Y_k + F(Y_k)] . \qquad (4.22)$$

The resulting primal problem can be formulated as

$$\max \sum_{k=1}^{K} [E(RE_k) - E(SH_k) - E(OP_k)] - \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{l=1}^{L_j} c_{ijl} x_{ijl} - \sum_{k=1}^{K} e_k y_k - \sum_{j=1}^{J} m_j w_j , \qquad (4.23)$$

s.t. (4.2) - (4.11), (4.20) - (4.22).

This primal problem is a nonlinear programming problem with stochastic functions. Even though the reformulation includes the integral term in the normalised function, the integral terms can be computed in advance. Therefore, the problem can be solved efficiently by a SQP method combining external statistical function with statistic tool box in the MATLAB function.

4.5 The All-units Discount Model

The quantity discount models include two types of models, the incremental discount which is explained in the previous section and the all-units discount. In this section, the all-units discount model is introduced. V_{jl}^S denotes the lower bound of interval l for the total volume for supplier j. V_{jl}^H denotes the upper bound. β_{jl} is the all-unit discount rate on interval l Therefore, the procurement cost is formulated as $\sum_{j=1}^{J}(\sum_{i=1}^{I} c_{ij}x_{ij})\sum_{l=1}^{L_j}(1-\beta_{jl})u_{jl}$, where u_{jl} is a binary variable which takes 1 if the discount interval l is applied, 0 otherwise. c_{ij} is the unit price of raw material i purchased from supplier j. x_{ij} is the quantity of raw material i purchased from supplier j.

The supply chain planning problem with the all-units discount policy is formulated as the following nonlinear optimisation problem:

$$\begin{aligned} \max \sum_{k=1}^{K}\{\int_0^{y_k}[r_k z_k - b_k(y_k - z_k)]f(z_k)dz_k + \int_{y_k}^{\infty}[r_k z_k - a_k(z_k - y_k)]f(z_k)dz_k\} \\ - \sum_{k=1}^{K} e_k y_k - \sum_{j=1}^{J} m_j w_j - \sum_{j=1}^{J}(\sum_{i=1}^{I} c_{ij}x_{ij})\sum_{l=1}^{L_j}(1-\beta_{jl})u_{jl}, \end{aligned} \tag{4.24}$$

s.t. (4.3), (4.8), (4.9),

$$\sum_{i=1}^{I} x_{ij} \le \sum_{l=1}^{L_j} V_{jl}^H u_{jl}, \forall j, \tag{4.25}$$

$$\sum_{i=1}^{I} x_{ij} \ge \sum_{l=1}^{L_j} V_{jl}^S u_{jl}, \forall j, . \tag{4.26}$$

$$\sum_{k=1}^{K} g_{ik} y_k \le \sum_{j=1}^{J} x_{ij}, \forall i, \tag{4.27}$$

$$\sum_{i=1}^{I} n_{ij} x_{ij} \le q_j w_j, \forall j, \tag{4.28}$$

$$x_{ij}, y_k \ge 0, \forall i, \forall j, \forall k, , \tag{4.29}$$

$$u_{jl}, v_{ij}, w_j \in \{0,1\}, \forall i, \forall j, \forall l. . \tag{4.30}$$

The all-units discount problem is also formulated as a mixed integer nonlinear programming problem. The model is quite similar to the previous one. Therefore, the previously described reformulation and the outer approximation method which is introduced in Chapter 3 can be applied in this model. The problem includes a nonlinear term caused by uncertain demands. The primal problem with fixing 0-1 binary variables $y = [\{u_{jl}\},\{v_{ij}\},\{w_j\}]^T$ to $y^h = [\{u_{jl}^h\},\{v_{ij}^h\},\{w_j^h\}]^T$ yields the following optimisation problem:

$$\begin{aligned}\max \sum_{k=1}^{K}\{\int_0^{y_k}[r_k z_k - b_k(y_k - z_k)]f(z_k)dz_k + \int_{y_k}^{\infty}[r_k z_k - a_k(z_k - y_k)]f(z_k)dz_k\} \\ -\sum_{k=1}^{K}e_k y_k - \sum_{j=1}^{J}m_j w_j - \sum_{j=1}^{J}(\sum_{i=1}^{I}c_{ij}x_{ij})\sum_{l=1}^{L_j}(1-\beta_{jl})u_{jl},\end{aligned} \tag{4.31}$$

s.t. (4.3), (4.11), (4.25) - (4.29).

The outer approximation can be obtained by the intersection of a finite set of supporting functions. These supporting functions correspond to the linearisation of the objective function. The master problem can be written as

$$\min_{\{x,y,v,w\}}[\mu_{OA} + \sum_{j=1}^{J}\sum_{i=1}^{I}(c_{ij}x_{ij})\sum_{l=1}^{L_j}(1-\beta_{jl})u_{jl} + \sum_{j=1}^{J}m_j w_j], \tag{4.32}$$

s.t. (4.3), (4.8), (4.9), (4.25) - (4.30),

$$\sum_{i\in B^h} y_i^h - \sum_{i\in NB^h} y_i^h \le |B^h| - 1, \forall h, \tag{4.33}$$

$$B^h = \{i \mid y_i^h = 1\}, NB^h = \{i \mid y_i^h = 0\}, \tag{4.34}$$

$$\mu_{OA} \ge -P(y^h) - \frac{\partial P}{\partial y_k}\Big|_{y_k = y_k^h}(y_k - y_k^h), \tag{4.35}$$

$$\begin{aligned}P(y_k) = -\sum_{k=1}^{K}\{\int_0^{y_k}[r_k z_k - b_k(y_k - z_k)]f(z_k)dz_k. \\ + \int_{y_k}^{\infty}[r_k y_k - a_k(z_k - y_k)]f(z_k)dz_k\} + \sum_{k=1}^{K}e_k y_k\end{aligned} \tag{4.36}$$

The linearisation of the master problem can be achieved. The master problem can be reformulated as the following mixed integer linear problem which can be solved by a commercial solver:

$$\min_{\{x,y,v,w\}} [\mu_{OA} + \sum_{j=1}^{J}\sum_{l=1}^{L_j} Q_{jl} + \sum_{j=1}^{J} m_j w_j], \tag{4.37}$$

s.t. (4.3), (4.8), (4.9), (4.25)-(4.30), (4.33) - (4.36),

$$Q_{jl} \geq \sum_{i=1}^{I} c_{ij} x_{ij} (1-\beta_{jl}) - M(1-u_{jl}), \forall j, \forall l, \tag{4.38}$$

where M is a large constant.

4.6 Numerical Examples

4.6.1 Illustrative example

In order to demonstrate the performance of the proposed method, two types of examples are conducted by using the incremental discount model. Two examples are conducted to demonstrate that the proposed method solves relatively larger scale problem instances for Example 2 efficiently. In Example 1, there are 2 products, 2 raw materials, 2 suppliers, and 2 incremental discount intervals. In Example 2, there are 5 products, 5 components, 5 suppliers, and 3 incremental discount intervals. The experiments for comparision are also conducted to examine the efficiency of normalisation technique in the proposed algorithm. The parameters for the experiments are generated from randomly generated numbers in the range shown in Tables 4.1 and 4.2.

Table 4.1: Parameters for example 1.

r_k	500-750	e_k	120-135	g_{ik}	0-1
t_k	79.5-80.5	n_{ij}	0.75-1.25	b_k	100-170
μ_k	25.1-25.2	m_j	100-150	Q	3200
σ_k^2	3.7-4.0	q_j	35-45	a_k	10-12
d_{ijl}^S	0.001,10.000	d_{ijl}^H	10,100	c_{ijl}	14-40

4.6.2 Results and discussion

Five cases of instances are generated and solved by the proposed method. In computational experiments, CPLEX MATLAB interface (CPLEXINT) integrating CPLEX and MATLAB functions is used. The proposed algorithm is encoded in MATLAB language. The primal problem is solved by fmincon function in MATLAB optimisation toolbox. The statistical toolbox is also used for the expectation reformulation by normalisation technique. The MILP master problem is solved by

Table 4.2: Parameters for example 2.

r_k	150-250	e_k	23-40	g_{ik}	1-4
t_k	1-3	n_{ij}	1-3	b_k	10-100
μ_k	160-200	m_j	350	Q	2000
σ_k^2	60-80	q_j	450-550	a_k	50-150
d_{ijl}^S	0.001,10.000	d_{ijl}^H	1000,10000	c_{ijl}	5-17

CPLEXINT function. The maximum number of iterations is set to 100. The convergence condition is that the lower bound is not updated 100 times or duality gap is less than 1%. To reduce computational efforts, the primal problem was solved one time per ten times of replications. An Intel Core2Duo 3.0 GHz processor (E8400) with 3GB memory was used. Computational results for direct integration are shown in Table 4.3.

Table 4.3 shows the results of the integral formulation. Table 4.4 shows the computational results of the reformulated of the model. The results demonstrate that the proposed method can derive near-optimal solutions for problem instances. However, in Table 4.3 and Table 4.4, the computation time becomes much longer as the problem size increases. This is because the direct integral computation is used for the optimisation

Table 4.3: Computational results for direct integral formulation.

Example	LBD	UBD	DGAP[%]	Time[s]	Iter.
1-1	1.962×10^4	1.969×10^4	0.00240	25.8	2
1-2	2.152×10^4	2.172×10^4	0.00575	21.4	2
1-3	1.749×10^4	1.749×10^4	0.00607	39.9	2
1-4	2.008×10^4	2.009×10^4	0.0563	25.9	2
1-5	2.044×10^4	2.045×10^4	0.0677	25.2	2
Ave.	1.974×10^4	1.974×10^4	0.0276	27.6	2
2-1	2.140×10^4	2.761×10^4	22.489	9619	100
2-2	2.863×10^4	2.891×10^4	0.9960	3571	100
2-3	2.871×10^4	2.887×10^4	0.5610	759	11
2-4	2.943×10^4	3.057×10^4	3.7155	5293	100
2-5	2.915×10^4	3.032×10^4	3.8700	11702	100
Ave.	2.746×10^4	2.926×10^4	6.3265	6189	82

Table 4.4: Computational results for reformulation by normalisation technique.

Example	LBD	UBD	DGAP[s]	Time[s]	Iter.
1-1	1.953×10^4	1.958×10^4	0.00236	25.8	2
1-2	2.141×10^4	2.158×10^4	0.81003	21.4	2
1-3	1.749×10^4	1.749×10^4	0.00607	25.6	2
1-4	1.997×10^4	1.997×10^4	0.03560	25.9	2
1-5	2.035×10^4	2.049×10^4	0.07253	25.2	2
Ave.	1.975×10^4	1.982×10^4	0.18532	30.3	2
2-1	2.733×10^4	2.775×10^4	1.5452	1003	100
2-2	2.966×10^4	3.094×10^4	4.1175	702	100
2-3	2.875×10^4	2.889×10^4	0.4681	902	30
2-4	2.928×10^4	3.341×10^4	12.356	1033	100
2-5	2.934×10^4	3.339×10^4	12.113	1029	100
Ave.	2.887×10^4	3.088×10^4	6.1200	934	100

of primal problem. The direct integral computation increases the number of iterations. There are cases (case 1-1 and case 1-4) that upper bounds in Table 4.4 are smaller than lower bounds in Table 4.3 caused by the numerical error between the computation of the expected reformulation and direct integration.

Practically, the size of supply chain planning problem that the manufacturer faces is quite large. The manufacturer should make decision for various types of products from different suppliers. Thus, the development of an efficient algorithm becomes extremely important. It is expected to solve larger scale problems by combining the outer-approximation method and a normalisation technique. The conventional approach proposed by Zhang and Ma [104] can solve only small size problems with almost the same time as the proposed method. However, the proposed algorithm can solve larger size problems more efficiently, because Zhang and Ma's approach is based on branch and bound algorithm.

4.7 Summary

In this chapter, a solution procedure for supply chain planning problems with quantity discounts is addressed. The supply chain problem is formulated as a mixed integer nonlinear programming problem with integral terms. A novel outer approximation algorithm has been applied to solve the MINLP problem. The reformulation of integral function by a normalisation technique for demand uncertainty has been proposed to reduce the computational effort for the primal problem in the decomposition algorithm. Computational experiments have shown that the proposed method can derive a near-optimal solution with a small duality gap with less computational effort. From computational results, it indicates that that the manufacturer's profit can be optimised by quantity discounts with less computational effort.

In this chapter, one single period of supply chain planning is investigated. However, the multi-period supply chain planning could be further studied in order to be applied in a wider industrial area. The detailed consideration of supplier selection such as quality, reliability and so on is not included in this chapter. This is also an interesting area of future work. More efficient algorithms should be developed for large scale problems with the improvement of the computation of bounds in a reasonable computational time.

Chapter 5

Optimal Quantity Discount Contract for Supply Chain Optimisation with One Manufacturer and Multiple Suppliers under Demand Uncertainty

5.1 Introduction

An economic structure is transformed into a new landscape composing of a trilogy of interactive forces that include globalisation, trade liberalisation, and the information technology and communications revolution. Enterprises are facing challenges of the integration of business, technology, process along supply chains in order to succeed in the new economic environment. The integration of the supply chain becomes crucial in the global business environment. Therefore, many companies are trying to achieve the integration of the supply chain by implementing cooperation and collaboration across the value chain. They are expanding, merging, contracting or redesigning their supply chains [43]. In this chapter, an optimal contract strategy is derived from a Stackelberg equilibrium in a two-echelon supply chain system by considering production, inventory and pricing with uncertain demands.

The chapter is motivated by a problem of improving agility, in the context of short life cycle products and increasing product variety. Manufacturers in many industries with high customisation and short life cycle products are willing to improve contract decisions to reduce costs and inventories. They are facing challenges to handle uncertain demands, frequent ordering and the selection of suppliers. For instance, in electronic component industries, both random yield and uncertain demands are common occurrences. For such industries, manufacturers who produce short life cycle products have to adjust manufacturing planning after one period of business planning in order to satisfy customers' diverse requirements. In order to consider the integration of business decisions and manufacturing planning, the planning is executed within one single period as long-term decisions. The life cycle of electronic components is quite short. Thus, the demand of products fluctuates. Efficient contracting decisions for a short life cycle product supply chain considering demand uncertainty become important. In this chapter, the manufacturing planning under demand uncertainty incorporating optimal contract decisions is considered. The optimal contract decisions are developed to reduce negation time and costs so that they can build the long-term partnership with contracted suppliers.

This chapter considers one manufacturer and its multiple suppliers who are involved in purchasing different types of components, assembling and selling multiple finished products within single period. The manufacturer makes contracts with suppliers to improve efficiency of supply chain planning in order to reduce costs and maximise individual profits. The contract decision is made through a negotiation between the manufacturer and multiple suppliers based on prices of components and quantities of purchased components. The relationship between the manufacturer and suppliers is modelled by a noncooperative game. A Stackelberg game is applied where the manufacturer is the leader and the suppliers are followers. In order to obtain an equilibrium, suppliers' optimal response functions are derived analytically. The response functions can be represented by incremental quantity discount policy. Thus, a quantity discount is incorporated into the manufacturer`s model with the selection of suppliers. It is assumed that the manufacturer as a leader and suppliers are followers. By deriving suppliers' optimal response functions, optimal discount schedules are created. Eventually, the resulting function is formulated as a mixed integer nonlinear programming problem with integral terms. In order to reduce the computational complexity, the problem is reformulated by using a normalisation technique which is introduced in chapter 4.

The objective of this chapter is to propose a Stackelberg game theoretic model between the manufacturer and multiple suppliers by considering production, prices and the selection of suppliers simultaneously under demand uncertainty. An optimal quantity discount schedule can be obtained from optimal response functions. There are some contracts such as buyback contracts and revenue-sharing contracts that increase overall profits by making the supplier share some of the buyer's demand uncertainty. In this chapter, suppliers encounter setup and inventory holding costs due to the lot sizing policy. Quantity discounts are effective to coordinate supply chain if the supplier has large fixed costs per lot. Such contracts can encourage the manufacturer to buy in larger lot sizes that can reduce costs for the supplier. An optimal quantity discount is derived from a Stackelberg equilibrium to resolve the manufacturer` decisions with the selection of suppliers with uncertain demands.

This chapter is organised as follows. Relevant literatures are briefly reviewed in Section 5.2. Section 5.3 defines the problem and notations. The mathematical models of a noncooperative game are formulated, and the proposed algorithm is introduced in

Section 5.4 and 5.5. Section 5.6 gives some computational results and analysis. Finally, the summary is given in Section 5.7.

5.2 Literature Review

The game theoretic models have been extensively studied in different manners in the past years. Yu et al. [101] improved the members' profits of supply chain systems between a manufacturer and its retailers incorporating the inventory policy by studying Stackelberg game problems where advertising, pricing and inventory replenishments are all involved. Esmaeili et al. [25] proposed the seller-buyer coordination model by noncooperative and cooperative game theory where the unit marketing expenditure and the unit price charged by the buyer influence the demand of products. The coordination of a supply chain consisting of a manufacturer and a retailer with return policy was proposed by Xiao et al. [93]. Yang et al. [95] presented an assembly supply chain system consisting of one retailer and two suppliers with forecast updating. Those researches mainly focus on inventory management coordinating buyers and vendors by game theory. The partnership between manufacturers and suppliers is not solved. This chapter focuses on the negotiation between one manufacturer and multiple suppliers.

There are some works to investigate the partnership between the manufacturer and the supplier in game theoretic models. Huang et al. [42] introduced a three-level dynamic noncooperative game theoretic model considering suppliers and components selection, pricing and inventory. Each supplier faces the problem to make decisions based on prices for components. The manufacturer has to determine the setup time interval for production, wholesale prices, and to make supplier's and component's selection decisions. The retailers' problem focuses on replenishment cycles and retail prices for products. A multiple-suppliers and single manufacturer assembly supply chain was investigated by Leng and Parlar [56]. The suppliers produce components which are assembled by the manufacturer. They discussed the decentralised assembly supply chain to find Nash and Stackelberg equilibrium, and coordinate it by cost-sharing contracts.

As the above examples illustrate, a number of game-theoretic models for optimizing the supply chain have been proposed. Yet, an important issue which is rarely mentioned or omited in those literatures is the consideration of demand uncertainty involved in game theoretic models. Most of the studies assume that demand is price-sensitive or depending on some variables. Hennet and Arda [37] studied different types of contracts to coordinate channel partners which are facing a random demand and a random lead-time along supply chain. They still assumed that the demand is unitary according to a Poisson process. However, there was a work which considers demand uncertainty in a three stage supply chain introduced by Xiao et al. [93]. They resolved coordination of one retailer, one manufacturer and one subcontractor. They developed a newsvendor model to investigate the order quantity, wholesale pricing and lead time decisions. However, contract decisions are provided to integrate manufacturing planning and supplier's decision in supply chain management.

The above literatures employ the game theoretic models to coordinate business partners along the supply chain. In this chapter, quantity discounts are introduced in a game theoretic model to coordinate the manufacturer and its suppliers. Quantity discount is considered as an important pricing strategy. Sarmah et al. [80] introduced basic buyer and vendor coordination models, and reviewed the literature dealing with buyer and vendor coordination models that have used the quantity discount under deterministic demand. Li et al. [58] attempted to improve buyer and seller system cooperation in an inventory control system. Afterwards, they discussed how quantity discount works in the system to divide additional profits. The effectiveness of quantity discounts and volume discounts as coordination mechanisms between a single-vendor and a retailer was analysed by Viswanathan and Wang [88]. Qin et al. [77] considered volume discount and franchise fees as a coordination mechanism in a system consisting of a supplier and a buyer. The problem is analysed by a Stackelberg game.

The main contribution of this chapter is that an optimal discount contract is introduced, which is derived from suppliers' optimal response functions between the manufacturer and multiple suppliers with uncertain demands. The problem is analysed by a Stackelberg game where the manufacturer is a leader and suppliers are followers. In order to obtain an equilibrium of the game, supplier's optimal response functions are derived. The optimal response function can be represented by a quantity discount policy. Thus, quantity discount contracts are incorporated into the manufacturer's model considering the supplier's selection with uncertain demands.

5.3 Problem Description and Notation

Electronic component manufacturing industries are considered as an example in this chapter. In the model, one manufacturer which purchases different types of component $i\,(i=1,..,I)$ from the supplier $j\,(j=1,..,J)$ is considered in order to assemble finished product $k\,(k=1,...,K)$ under demand uncertainty. The manufacturer assembles a variety of finished products under the consideration of the production capacity and the selection of proper suppliers. The manufacturer faces random demands. Each supplier determines its selling prices of components by considering the economic lot size policy. The supply chain model in this chapter is shown in Fig. 5.1. For the game sequence, the manufacturer determines the order quantity of components and selection of suppliers first. Then, the contracted suppliers decide prices of components based on the manufacturer's decisions.

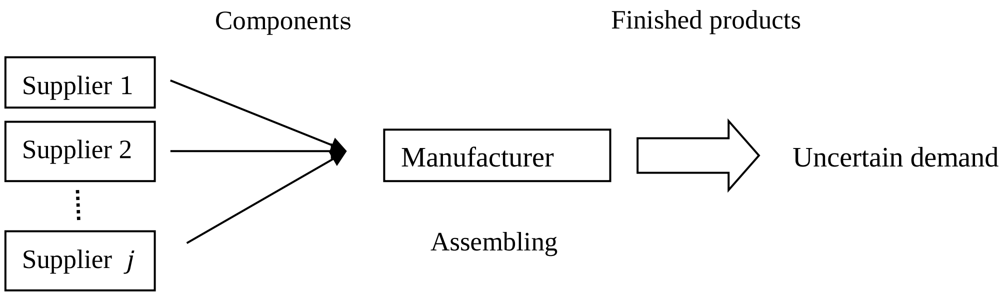

Figure 5.1: Supply chain model.

The manufacturer's model is the maximisation of the total profit which includes expected revenue, understocking, overproduction, procurement, production costs and transaction costs. The model was studied by Kim et al. [46] and Zhang and Ma [104]. The concept of economic lot sizing is applied in the supplier's model. The objective is to optimise the total profit definied by the gross revenue minus the sum of purchasing costs, setup and inventory holding costs. Suppliers make its pricing decisions based on the economic lot sizing policy. For suppliers, large orders usually increase the gross revenue, but it results in the increases of other costs, such as labor costs. Therefore, the suppliers should determine the optimal selling price p_{ij} based on the economic lot size Q_{ij}. The supplier's selling price is determined in a way to recover its fixed and variable costs, such as $p_{ij} = \frac{S_{ij}}{Q_{ij}} + L_{ij}$, which is proposed by Kuzdrall and Britney [49]. S_{ij} is defined as the supplier's fixed cost including the setup cost and any fixed per-order profit. The supplier's variable cost such as the labor cost is represented by L_{ij}.

The objective of this chapter is to propose a Stackelberg game theoretic model with uncertain demands. In the noncooperative game model, the manufacturer as a leader first determines production, order quantities of components and the selection of suppliers. The suppliers are assumed as followers to decide prices and the economic lot sizes.

Indices:

i : types of components

j : number of suppliers

k : types of finished products

Decision variables:

Manufacturer

d_{ij} : order quantity of component i from the supplier j

v_{ij} : binary variable which takes 1 if the manufacturer buys component i from supplier j, and 0 otherwise.

w_j : binary variable which takes 1 if supplier j is chosen for any purchased component, and 0 otherwise.

y_k : production quantity of product k during the planning horizon

Supplier

p_{ij} : unit selling price of component i for supplier j

Q_{ij} : economic lot size of component i for supplier j

Parameters:

Manufacturer

a_k : estimated opportunity cost for understocking of one unit of product k

b_k : estimated inventory holding cost for overstocking of one unit of product k

c_j : resource capacity of the supplier j

e_k : unit production cost for product k

$f(z_k)$: probability density function obeyed by the demand of the product k

g_{ik} : number of units of component i required to produce one unit of product k

m_j : transaction cost associated with supplier j indicating that the manufacturer should be charged in order to make contracts with suppliers

n_{ij} : amount of the internal resource for supplier j required to produce component i

r_k : unit sale revenue cost for product k

U : production capacity of the manufacturer

z_k : random variable of demand for product k obeying by a normal distribution $z_k \sim N(\hat{z}_k, \sigma_k^2)$, where $\hat{z}_k$ is the mean value of z_k and σ_k^2 is the standard deviation of z_k

Supplier

A_{ij} : inventory holding cost of component i for supplier j

L_{ij} : supplier j variable cost such as labor cost of component i

M : sufficiently large positive number

h_{ij} : unit purchasing cost of component i charged to supplier j

S_{ij} : setup cost for component i for supplier j

5.4 The Noncooperative Model

This section introduces a noncooperative model where the manufacturer determines production, order quantities of components and the selection of suppliers. The suppliers decide selling prices of components by considering the economic lot sizing policy.

5.4.1 The Stackelberg game theoretic model

Due to the demand uncertainty, the manufacturer's objective function is formulated as a stochastic model. The manufacturer's objective function is the maximisation of the total profit consisting of sales revenue, overstocking / understocking penalty costs, production costs, purchasing costs and transaction costs. Then the manufacturer's problem becomes

$$\begin{aligned} \max E\{\sum_{k=1}^{K}[r_k \min(y_k, z_k) - b_k \max(0, y_k - z_k) - a_k \max(0, z_k - y_k)]\} \\ - \sum_{k=1}^{K} e_k y_k - \sum_{i=1}^{I}\sum_{j=1}^{J} q_{ij} d_{ij} - \sum_{j=1}^{J} m_j w_j, \end{aligned} \tag{5.1}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} g_{ik} y_k \le \sum_{j=1}^{J} d_{ij} v_{ij}, \forall i, \tag{5.2}$$

$$\sum_{k=1}^{K} t_k y_k \le U, \tag{5.3}$$

$$\sum_{i=1}^{I} n_{ij} d_{ij} \le c_j w_j, \forall j, \tag{5.4}$$

$$w_j \ge v_{ij}, \forall i, \forall j, \tag{5.5}$$

$$d_{ij} \ge 0, y_k \ge 0, w_j \in \{0,1\}, v_{ij} \in \{0,1\}, \forall i, \forall j, \forall k. \tag{5.6}$$

The supplier j's objective function is formulated by the economic lot size policy. The objective function consists of the revenue, purchasing costs, fixed and variable costs, and inventory holding costs. Then the supplier j's problem becomes

$$\max \sum_{i=1}^{I} [p_{ij}d_{ij} - h_{ij}d_{ij} - \frac{(S_{ij} + L_{ij})}{Q_{ij}} - \frac{A_{ij}Q_{ij}}{2}], \tag{5.7}$$

$$\text{s.t.} \quad p_{ij} \geq h_{ij}, \forall i, \forall j, \tag{5.8}$$

$$p_{ij} \geq 0, \forall i, \forall j, \tag{5.9}$$

where we define

$$p_{ij} = \frac{S_{ij}}{Q_{ij}} + L_{ij}.$$

Note that Eq. (5.2) represents that ordered components should not be less than the required production quantity. Eq. (5.3) represents the production capacity. Eq. (5.4) ensures that the internal resource required for the procurement is less than the supplier's capacity. Eq. (5.5) is related to the transaction cost, which is incurred if the supplier is selected to provide any component. Eq. (5.6) is the nonnegative constraint.

For the supplier's model, Eq. (5.7) is the supplier j's total profit objective function consisting of the gross revenue, purchasing costs, setup costs, labour costs and inventory holding costs. Eq. (5.8) gives the price constraint. Eq. (5.9) is the nonnegative constraint.

The manufacturer and suppliers are analysed by a Stackelberg game where the manufacturer is a leader and suppliers are followers. Thus, the supplier's model becomes a constraint for the manufacturer's model when the Stackelberg game theoretic model is solved. The model becomes a bilevel programming problem.

5.4.2 Analysis of the model

The equilibrium of a Stackelberg game is usually solved by a backward induction procedure. It generally works as follows. The follower's (supplier) problem must be first solved to get reaction functions of leader's (manufacturer) decision results. The manufacturer's decision problem is solved considering all possible reactions of its followers for maximizing the profits. For every possible leader's action, every follower's optimal reaction can be determined by considering the manufacturer's decisions as its input parameters. In this chapter, the problem is analysed such that the manufacturer first design production by determining production, purchasing quantities of components and the selection of suppliers in order to satisfy uncertain demands. The negotiation process is illustrated in Fig. 5.2. Therefore, the supplier's optimal response functions should be derived firstly. Then, the manufacturer's decision is solved by substituting supplier's response functions as constraints.

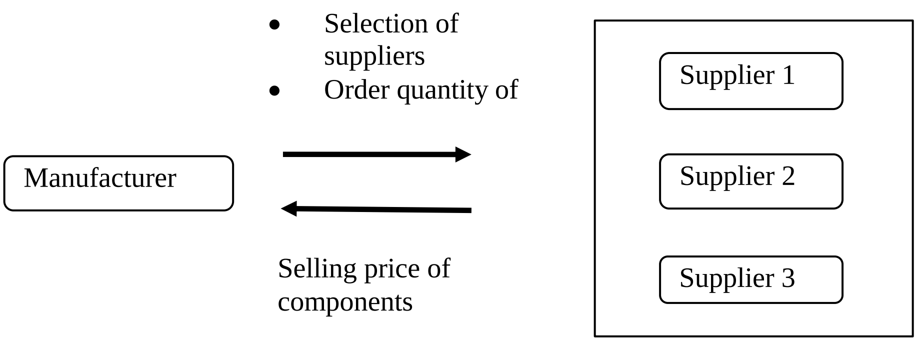

Figure 5.2: Negotiation process.

The backward induction procedure for finding the equilibrium point of a Stackelberg game is complex, because multiple followers are involved. Therefore, the supplier j's optimal response function should be derived first.

First, the objective function can be rearranged as

$$\Pi_j = \sum_{i=1}^{I} [p_{ij} d_{ij} - h_{ij} d_{ij} - \frac{(S_{ij} + L_{ij})(p_{ij} - L_{ij}) d_{ij}}{S_{ij}} - \frac{A_{ij} S_{ij}}{2(p_{ij} - L_{ij})}]. \tag{5.10}$$

In order to obtain supplier j ' s optimal response function, the first partial derivative of supplier's profit function (5.10) with respect to p_{ij} is derived, we have

$$\frac{\partial \Pi_j}{\partial p_{ij}} = \sum_{i=1}^{I} [d_{ij} - \frac{(S_{ij} + L_{ij})d_{ij}}{S_{ij}} + \frac{A_{ij}S_{ij}}{2(p_{ij} - L_{ij})^2}]. \tag{5.11}$$

Because the second partial derivative of Eq. (5.10) with respect to p_{ij} is

$$\frac{\partial^2 \Pi_j}{\partial p_{ij}^2} = -\frac{A_{ij}S_{ij}}{(p_{ij} - L_{ij})^3} \leq 0. \tag{5.12}$$

By solving the equation $\frac{\partial \Pi_j}{\partial p_{ij}} = 0$, we obtain the critical point of the equation with an increasing and concave function.

For each component i, we obtain

$$p_{ij} = L_{ij} + \sqrt{\frac{A_{ij}S_{ij}^2}{2L_{ij}d_{ij}}}. \tag{5.13}$$

Eq. (5.13) shows a nonlinear relationship between the order quantity d_{ij} and the price p_{ij}. We observe that p_{ij} decreases if d_{ij} is increased. Therefore, this relationship can be represented by a quantity discount schedule.

From Eq. (5.13), we notice that the situation that $d_{ij} = 0$ is neglected. When $d_{ij} = 0$, it indicates that supplier j is not selected so that $p_{ij} = 0$. Thus, the following constraint should be added in the manufacturer's model:

$$p_{ij} - Mv_{ij} \leq 0, \tag{5.14}$$

where M is sufficiently large. Eq. (5.14) indicates that $p_{ij} = 0$ if the manufacturer does not purchase component i from supplier j. In other words, when $d_{ij} = 0$, the manufacturer does not purchase components so that $p_{ij} = 0$.

5.5 Solution Procedure

In this model, the supplier's optimal pricing response decisions are solved firstly. Analytically, the relationship of prices and the order quantities of components is found by the supplier' model. Eq. (5.13) implies that if the relationship is satisfied, the supplier's optimal decision can be assured by solving the manufacturer's function. This relation function is the constraint in the manufacturing decision problem. Discount strategies are widely used to encourage buyers to purchase more products. There are two main streams of discount structure: quantity discounts and volume discounts. For quantity discount, sales of the volume of products do not affect prices and discount of other products. The price can be discounted while the quantity of purchased products within certain range of quantities. For the volume discount, discounts depend on the total amount of sales volume but not on the quantity or variety of the purchased products.

By embedding Eq. (5.13) into the manufacturer's objective function, the manufacturing decision problem with the incremental discount policy can be formulated as follows:

$$\begin{aligned} \max\ & E\{\sum_{k=1}^{K}[r_k \min(y_k, z_k) - b_k \max(0, y_k - z_k) - a_k \max(0, z_k - y_k)]\} \\ & - \sum_{k=1}^{K} e_k y_k - \sum_{i=1}^{I}\sum_{j=1}^{J} q_{ij} d_{ij} - \sum_{j=1}^{J} m_j w_j, \end{aligned} \tag{5.15}$$

s.t. (5.2) - (5.6), (5.8), (5.9), (5.13).

The model is a mixed integer nonlinear programming problem (MINLP) with integral terms. It is difficult to solve MINLP problems due to the computational complexity. In order to derive optimal solutions efficiently, it is necessary to reformulate the problem. Therefore, the reformulation technique in Chapter 4 is applied. Thus, the resulting objective function can be formulated as

$$\max \sum_{k=1}^{K}[E(RE_k)-E(SH_k)-E(OP_k)]-\sum_{i=1}^{I}\sum_{j=1}^{J}q_{ij}d_{ij}-\sum_{k=1}^{K}e_k y_k-\sum_{j=1}^{J}m_j w_j\,, \tag{5.16}$$

where we define

$$E(RE_k)=r_k\hat{z}_k+r_k\sigma_k[-F(Y_k)+(1-\Phi(Y_k))Y_k],$$

$$E(SH_k)=-a_k\sigma_k[-F(Y_k)+(1-\Phi(Y_k))Y_k)],$$

$$E(OP_k)=b_k\sigma_k[\Phi(Y_k)Y_k+F(Y_k)].$$

The function is also expressed by the error function. Then, this MINLP problem can be easily implemented in General Algebraic Modelling System (GAMS). There are some special functions which can be implemented in GAMS, such as the error function. The following error function errorf (.) in GAMS implements a variant on this:

$$errorf(x)=\frac{1}{\sqrt{2}}\int_{-\infty}^{x}e^{-\frac{1}{2}t^2}dt\,, \tag{5.17}$$

which is the cumulative distribution function of the standard normal distribution. Thus, $errorf(\frac{(y_k-\hat{z}_k)}{\sigma_k})$ expresses the cumulative distribution function.

The final manufacturing decision problem can be represented as follows:

$$\begin{aligned}\max \sum_{k=1}^{K}\{&r_k\hat{z}_k+r_k\sigma_k[-F(Y_k)+(1-\Phi(Y_k))Y_k]+a_k\sigma_k[-F(Y_k)+(1-\Phi(Y_k))Y_k]\\ &-b_k\sigma_k[\Phi(Y_k)Y_k+F(Y_k)]\}-\sum_{i=1}^{I}\sum_{j=1}^{J}d_{ij}p_{ij}-\sum_{k=1}^{K}e_k y_k-\sum_{j=1}^{J}m_j w_j,\end{aligned} \tag{5.18}$$

s.t. (5.2) - (5.6), (5.8), (5.9), (5.13),

$$\Phi(Y_k)=errorf(\frac{y_k-\hat{z}_k}{\sigma_k})\,, \tag{5.19}$$

$$F(Y_k)=\frac{1}{\sigma_k\sqrt{2\pi}}e^{-\frac{(y_k-\hat{z}_k)^2}{2\sigma_k^2}}\,, \tag{5.20}$$

$$Y_k = \frac{y_k - \hat{z}_k}{\sigma_k}. \tag{5.21}$$

Solution Algorithm

The proposed algorithm consists of the following steps:

Step 0: Initialisation: Set the parameters of the manufacturer and supplier's model.

Step 1: Derive the supplier's optimal response function of Eq. (5.13).

Step 2: Derive an incremental quantity discount schedule.

Step 3: Solve the manufacturer's production planning problem with Eq. (5.18) and the incremental quantity discount schedule derived at Step 1.

Step 4: Obtain the optimal manufacturer's profit and suppliers' profit.

5.6 Analysis with Experiments

This section presents a case study which is aimed at illustrating the features of the proposed model and demonstrating the performance of the algorithm.

5.6.1 Case study

A case study about personal computer peripherals manufacturing is provided. Consider a supply chain consisting of four suppliers $(J = 4)$ and one manufacturer. The manufacturer produces two types of finished products $(K = 2)$ such as a speaker and a monitor from five types of electronic part $(I = 5)$ seen in Fig. 5.3. The demand for the products is assumed to obey a normal distribution with mean value $\hat{z}_k = 15$ for each product. The set of standard deviation σ_k^2 is 12 for each product. A PC with Intel(R) CoreTM i7-3770 3.4 GHz processor and 8GB memory is used for the computation. The program is coded by GAMS23.7.

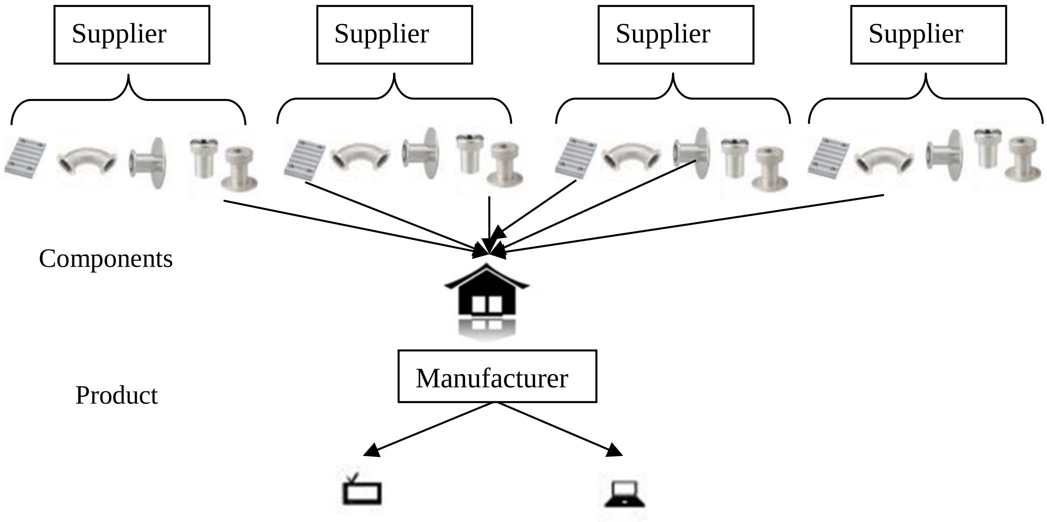

Figure 5.3: Supply chain configuration.

5.6.2 Analysis of results

We set initial parameters and solve the MINLP problem by the solver so-called BONIMN in GAMS. The optimal solutions for the manufacturer and suppliers are shown in Table 5.1 and Table 5.2, respectively.

Table 5.1: Optimal solution for the manufacturer.

Total profit	6029.123
Revenue	8714.552
Production cost	1770
Material cost	783.429
Transaction cost	132

The total computation time to derive the supply chain optimisation problem is 0.843 sec. The relative gap is almost zero. A near-optimal solution is derived by the proposed method. The solution for the manufacturer is shown in Table 5.2. The optimal solution

Table 5.2 Optimal solution for suppliers.

suppl.1	comp.1	comp.2	comp.3	comp.4	comp.5
p_{i1}	0	0	0	0	0
d_{i1}	0	0	0	0	0
$prof_{i1}$	-				
suppl.2					
p_{i2}	0	0	0	0	0
d_{i2}	0	0	0	0	0
$prof_{i2}$	-				
suppl.3					
p_{i3}	2.495	2.404	2.433	2.393	2.393
d_{i3}	49	49	64	81	81
Q_{i3}	8.080	9.900	9.238	12.723	12.723
$prof_{i3}$	201.326				
suppl.4					
p_{i4}	0	0	0	0	0
d_{i4}	0	0	0	0	0
$prof_{i4}$	-				

for suppliers is shown in Table 5.2. In Table 5.2, $prof_{ij}$ is a function defined by

$prof_{ij} = p_{ij}d_{ij} - h_{ij}d_{ij} - \frac{(S_{ij} + L_{ij})d_{ij}}{Q_{ij}} - \frac{A_{ij}Q_{ij}}{2}$. The computational results in Table 5.2 indicate that supplier 3 is selected by the manufacturer. We set transaction costs for each supplier are 135, 130, 132 and 124.

The quantity discount schedule is represented by Eq. (5.13). The parameter setting for quantity discount is as follows: the inventory holding costs are $A_{11} = A_{12} = A_{13} = A_{14} = 3$, $A_{21} = A_{22} = A_{23} = A_{24} = 2$, $A_{31} = A_{32} = A_{33} = A_{34} = 3$, $A_{41} = A_{42} = A_{43} = A_{44} = 2$, $A_{51} = A_{52} = A_{53}$ $= A_{54} = 3$ and the setup costs are $S_{11} = S_{12} = S_{13} = S_{14} = 4$, $S_{21} = S_{22} = S_{23} = S_{24} = 4$,

$S_{31} = S_{32} = S_{33} = S_{34} = 4,\ S_{41} = S_{42} = S_{43} = S_{44} = 5$, $S_{51} = S_{52} = S_{53} = S_{54} = 2$. The labor costs for all suppliers are 3. From the parameter setting for the quantity discount schedule, we know that the inventory holding and the setup cost are the same for four suppliers. However, the labour cost for supplier 3 is cheaper than other suppliers. By substituting those parameters into Eq. (5.13), the selling price of components from supplier 3 is the cheapest for the same amount quantity of components among four suppliers. The computational results also show that the supplier 3 is selected. From the numerical experiments, the effectiveness of the proposed approach can be confirmed by results with a very small duality gap. It also demonstrates that a Stackelberg game equilibrium can be computed effectively by considering the supplier selection, production and inventory under demand uncertainty.

5.6.3 The impact of parameters

In this section, the computational experiments are conducted to illustrate the characteristics of the proposed model. The model is analysed as a Stackelberg game that the manufacturer is considered as a leader. Thus, we focus on decisions to observe that how the decisions influence manufacturing planning in the game theoretic model. One manufacturer with two types of finished products and two types of purchased components is considered in the experiment. In order to analyse the impact of parameters, we conducted the comparative experiments to illustrate that how each cost influences production and profits in the game theoretic model. The parameters are listed in Table 5.3.

Five tests are conducted to analyse the impact of unit sale revenue r_k, opportunity loss cost a_k, inventory cost b_k, deviation of random demand σ_k and mean of random

Table 5.3: Parameters for case study.

Parameter	product 1	product 2		
production cost	50	60		
required resource	100	100		
	supplier 1	supplier2	supplier3	supplier4
management cost	135	130	132	124
resource capacity	500	500	500	500

demand $\hat{z}_k$, respectively. Five parameters including unit revenue, opportunity loss cost, inventory cost, deviation of random demand, and the mean value of random demand are analysed separately for each test. For each test, only one parameter is changed while other parameters are fixed. Four cases for each parameter are conducted. For example, in test 1, revenue r_k is changed once for each case. Then, we observe how the production of two types of product and profit change. The results are presented in Table 5.4.

From Table 5.4, we observe that the profit increases once the sale revenue is increased. However, the production of products stays the same in test 1. Test 2 and test 3 are conducted to illustrate the impact of uncertain demand on manufacturing planning decisions while the manufacturer may encounter shortfall or overproduction. From test 2, it implises that if the manufacturer increases the opportunity loss cost of one kind of products, and the profit is not always increased. The increase of inventory cost could not ensure that the profit always increases in test 3 as well. The influence of uncertainty is analysed in test 4 and test 5. Test 4 shows that with the decrease of deviation of demand, the profit increases. However, we observe that the profits do not always decline if the

Table 5.4: Comparative experiments.

test 1	effects of unit revenue							
	case 1		case 2		case 3		case 4	
	pro.1	pro.2	pro.1	pro.2	pro.1	pro.2	pro.1	pro.2
r_k	300	300	300	320	300	380	400	400
production	15	17	15	17	15	17	15	17
profit	5382.628		5694.639		6468.715		8385.117	
test 2	effects of opportunity loss cost							
	case 1		case 2		case 3		case 4	
	pro.1	pro.2	pro.1	pro.2	pro.1	pro.2	pro.1	pro.2
a_k	135	163	135	180	135	190	160	163
production	15	17	15	17	15	17	15	17
profit	5382.628		5393.588		5262.093		5097.739	
test 3	effects of inventory cost							
	case 1		case 2		case 3		case 4	
	pro.1	pro.2	pro.1	pro.2	pro.1	pro.2	pro.1	pro.2
b_k	20	25	20	35	20	50	25	25
production	15	17	15	17	15	17	15	17
profit	5382.628		5366.134		5476.392		5199.183	
test 4	effects of deviation of demand							
	case 1		case 2		case 3		case 4	
	pro.1	pro.2	pro.1	pro.2	pro.1	pro.2	pro.1	pro.2
σ_k	13	13	12	12	13	10	10	10
production	15	17	15	17	15	17	15	17
profit	5382.628		5848.756		6165.796		6299.796	
test 5	effects of mean of demand							
	case 1		case 2		case 3		case 4	
mean value	pro.1	pro.2	pro.1	pro.2	pro.1	pro.2	pro.1	pro.2
$\hat{z}_k$	15	15	15	13	13	13	10	10
production	15	17	15	17	13	19	10	22
profit	5382.628		5376.519		5348.992		5922.500	

mean value of demand is decreased in test 5.

In this chapter, the impact of key parameters has been analysed. From the results, it is concluded that the manufacturer should design manufacturing planning in order to increase profits according to different strategies of contract decisions.

5.7 Summary

A game theoretic model with one manufacturer and multiple suppliers is proposed. The Stackelberg game theoretic model between one manufacturer and suppliers under demand uncertainty has been introduced. An optimal quantity discount schedule can be derived from a Stackelberg equilibrium. The supplier's optimal response functions can be represented by the incremental discount policy in this chapter. Therefore, quantity discounts have been embedded into the manufacturer's function with the consideration of the supplier's selection. The numerical example has demonstrated the efficiency of the proposed method.

The proposed model is considered as single period manufacturing planning that aims at high customisation and short life cycle product industries. However, it is also interesting to extend this work to consider multi-period planning in order to implement in other industries. Practically, due to the limitation of information sharing, it is difficult to implement full cooperation with channel partners. Thus, the competition among partners can be considered in the model.

Chapter 6

A Game Theoretic Approach to Two-echelon Supply Chains with Asymmetric Quality Information

6.1 Introduction

The growth of globalisation has accelerated the competition on price and quality for multinational enterprises. In order to reduce production costs, global sourcing to countries such as China or India becomes crucial business strategies. In order to win in the global business, the competition is shifting from price to quality in many industries to achieve high customer satisfaction [27]. Thus, supply chain planning problems about price and quality are needed to be optimised in order to help the decision making for multinational enterprises. This chapter is motivated by improving global supply chain planning where quality and price are considered simultaneously. The manufacturer takes a leading role in the global supply chain to determine the optimal outsourcing suppliers in order to reduce costs and enhance the competition.

The coordination and cooperation of supply chain planning are key issues for the global business. As an example of supply chain coordination, Motorola decided to spend $60 million in Singapore to centralise and streamline global supply chain operations with its suppliers and customers [56]. A game theoretic approach is an effective method to improve the coordination/cooperation of supply chain planning. Traditionally, the overall channel performance including vertical or horizontal integration is optimised for all channel members. From a practical point of view, a noncooperative game theoretic approach is effective to coordinate supply chain planning in the global business environment due to different business strategies for each channel member. In this chapter, a global supply chain planning problem about price and quality is introduced by applying noncooperative game to coordinate a manufacturer and suppliers.

This chapter adopts a Stackelberg game where the manufacturer is the leader and suppliers are the followers. The “manufactuer-Stackelberg” game is widely used in supply chain literatures. In the Stackelberg game, players make decisions sequentially where the leader dominates the game. The manufacturer determines an annual production level and the selection of outsourcing suppliers as the leader. The selected suppliers as followers take the manufacturer’s optimal decisions as the input parameters to make decisions.

In the past, researchers focus on noncooperative games considering price and quality. In the games, it is assumed that the complete information can be observed for supply chain planning problems. However, there is a difficulty for each player in observing the complete information in the global supply chain. Thus, it is more practical to assume that the information is asymmetric due to different business strategies in global supply chain planning. Therefore, a two-echelon supply chain with uncertain demands is investigated where the quality information between the manufacturer and suppliers is asymmetric.

The objective of this chapter is to provide a game theoretic model including one manufacturer and its suppliers with uncertain demands where the quality information is asymmetric. The problem is modelled as a Stackelberg game where the manufacturer is a leader. Due to the asymmetric information, the quality is unknown by the manufacturer. Therefore, two scenarios (average case and worst case) are investigated for the manufacturer to estimate the uncertain quality information.

The rest of this chapter is organised as follows. Related literatures are reviewed in Section 6.2. The problem description and modelling are described in Section 6.3. The solution approach is provided in Section 6.4. Numerical examples are shown in Section 6.5. Finally, the chapter is summarised in Section 6.6.

6.2 Literature Review

Supply chain management is regarded as an important strategy to be competitive in global business environment. Thus, it has received much attention with regard to broad activities such as production planning, scheduling and so on [70, 71, 84].

The coordination methods of supply chain planning have been extensively studied in different manners in the past years. Game theory, as a well-known approach, is widely used to achieve the coordination. Yu et al. [100] improved members' profits of supply chain systems between a manufacturer and its retailers incorporating the inventory policy by studying Stackelberg game problems where advertising, pricing and inventory replenishments are all involved. Esmaeili et al. [25] proposed the seller-buyer coordination model by noncooperative and cooperative game theory where the unit marketing expenditure and the unit price charged by the buyer influence the demand of products. The coordination of a supply chain consisting of a manufacturer and a retailer with the return policy was proposed by Xiao et al. [93]. Yang et al. [95] presented an assembly supply chain system consisting of one retailer and two suppliers with forecast updating. However, quality issues are not involved in those works.

Quality issues have been studied intensively in supply chain planning. Product quality in multi-layer supply chain was investigated to consider the impact on production systems by Sana [79]. Due to varying quality levels and prices of products from supplier, the manufacturing faces financial risks resulting from unexpected fluctuation of demand. Thus, the risk consideration is involved in the formulation [67 - 68]. They introduced supernetworks in which supply-side and demand-side risk are included. A game theoretic approach is provided to optimise global supply chain planning concerning quality issues with uncertain demands [97]. However, the incomplete quality information is not studied in their studies. From more practical perspective, it is important to assume the asymmetry in the information due to different business strategies for each entity in the global supply chain planning.

Esmaeili et al. [25] introduced a seller-buyer supply chain model with an asymmetric information structure. They assumed that only buyer knows demand function and is aware of seller’s setup cost and purchasing cost. Lei et al. [55] investigated the impact of asymmetric information on disruption management when disruptions of demand and costs are private information. Most of works related to the asymmetric information in game theoretic models assume that demand information is asymmetric. There are rarely researches considering the asymmetry in the quality information in game theoretic models.

Tse and Tan [86] studied the unclear information of quality risk and visibility in a multi-tier supply chain. They considered the situation of the asymmetric information between manufacturer and supplier. However, they focused on the manufacturer's decision making to manage risk and visibility for supply chain planning. The coordination between manufacturers and suppliers is not investigated.

In this chapter, a game theoretic model including the asymmetric quality information is introduced to coordinate the manufacturer and its suppliers in order to improve quality in global supply chain planning. Quantity discounts are applied to resolve decision making on order quantity, price and production simultaneously for supply chain planning [98]. Afterwards, a game theoretic approach is studied for supply chain planning under demand uncertainty where the quality information is asymmetric which is modelled as a nonlinear programming problem [99]. The main contribution of this chapter is that a game theoretic model under demand uncertainty is propsed where the asymmetry in the quality information is considered. An equilibrium of the proposed game theoretic model is obtained by analysing two scenarios (average case and worst case) in order to estimate the uncertain quality information.

6.3 Problem Description

6.3.1 Supply chain model

One manufacturer which produces finished products $k\ (k = 1,..,K)$ are considered. Components $i\ (i = 1,..,I)$ is provided by suppliers $j\ (j = 1,..,J)$. The supply chain model in this chapter is shown in Fig. 6.1. In order to satisfy uncertain demands, the manufacturer purchases components from suppliers to assemble finished products. The manufacturer determines an optimal annual production level and the estimated defective components under demand uncertainty. Suppliers offer components to the manufacturer

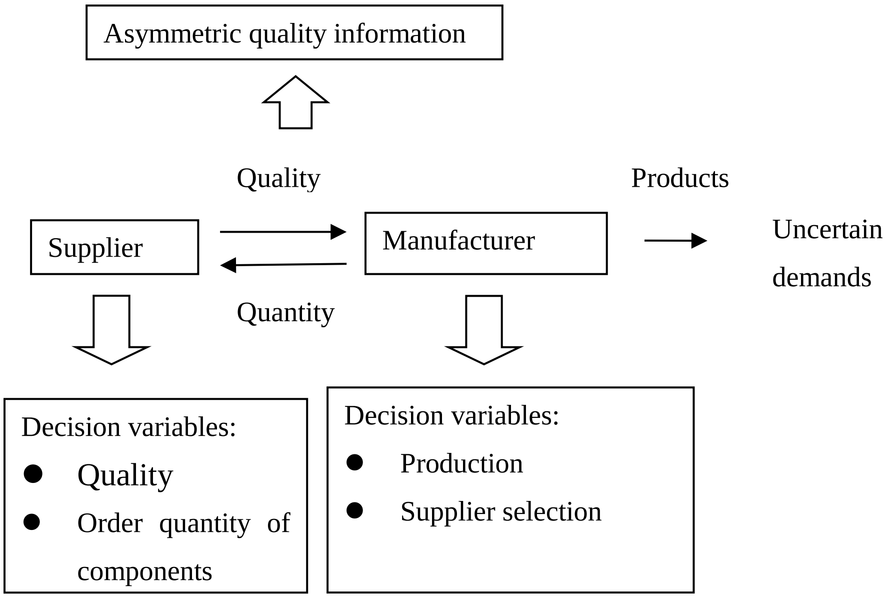

Figure 6.1: Supply chain model.

The asymmetric quality information is considered between one manufacturer and suppliers. Quality of components and quantity of components are determined by suppliers.

There are following assumptions in the model:

1. The quality of components is determined by suppliers. While suppliers offer components, the manufacturer receives defective components due to poor quality. Moreover, the manufacturer cannot observe the full information. However, the information of the average amount and variance of the number of defective components is known by the manufacturer.

2. If the manufacturer receives defective components, the compensation cost will be paid by suppliers.

3. For the manufacturer, the production capacity should be considered. The demand of

finished products obeys a normal distribution where the density function is given.

The problem is analysed by a Stackelberg game where the incomplete quality information is incorporated. In a Stackelberg game, players have the inequal power to make decisions in order to represent a realistic leader-follower relationship. The decisions are made sequentially in order to obtain equilibrium. In this chapter, the manufacturer acts as a leader whose decisions are solved by considering all possible reactions of followers (suppliers)' decisions. The game sequence is as follows:

1. The manufacturer determines production planning and estimation of defective components.
2. The supplier decides quantity of components and quality.

6.3.2 Quality modelling

In this chapter, the quality of components is evaluated by reliability x_{ij} of component. The supplier j pays higher cost for producing one component i if reliability x_{ij} of component i is increased. Let define that poor quality components have low reliability. Thus, it is assumed that production cost h_{ij} for one unit component i for the supplier depends on reliability x_{ij} and quantity d_{ij} of component i for the supplier. The number of defective components is affected by reliability x_{ij}. If reliability x_{ij} is increased, the number of defective components decreases. The production cost is expressed by cost function h_{ij}, such as $h_{ij} = A_{ij} + B_{ij}d_{ij} + C_{ij}x_{ij}$. A_{ij} is the fixed production cost for one unit component i paid by supplier j. B_{ij} is the production cost responsiveness associated with the quantity of component i for supplier j. It indicates that if order quantity d_{ij} is increased, production cost per one unit component increases. The increase of order quantity of components for the supplier causes increase of production. Thus, the supplier must pay more production cost such as machinery wearout costs or labor costs. C_{ij} is the production cost responsiveness of reliability x_{ij} of component i for supplier j. It indicates that higher reliability drives the increase of production cost. The number of defective component i is normally distributed where μ_{ij} is the mean value of the number of defective components which is constant known by both of the manufacturer and suppliers. However, the standard deviation of the number of defective components δ_{ij} is the decision variable for suppliers which is unknown for the manufacturer. With the increase of the standard deviation δ_{ij}, the reliability of components will decrease. Therefore, there is the assumption that reliability $x_{ij} = e^{-\frac{\delta_{ij}^2}{2}}$. For instance, if the supplier decides that the deviation of defective components is equal to 2, the reliability of components is approximately 0.136. With the increase of the deviation δ_{ij}, the reliability of components becomes better as shown in Fig. 6.2.

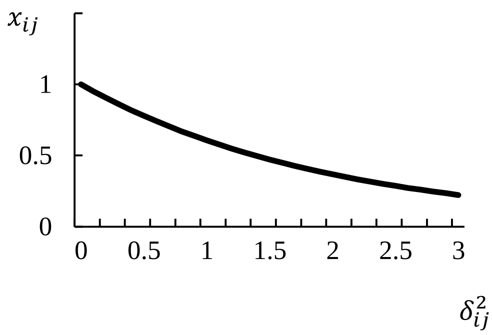

Figure 6.2: Reliability and deviation.

6.3.3 Formulation

Indices

i : component for assembling finished products

j : supplier

k : product

Decision variables

Manufacturer:

y_k : production quantity of finished products k during the planning horizon

P_{ij} : estimated quantity of defective component i for supplier j

v_{ij} : binary variable which takes 1 if the manufacturer buys component i from supplier j

w_j : binary variable to determine the selection of suppliers j for actual purchase which takes 1 if supplier j is contracted, and 0 otherwise.

Supplier:

d_{ij} : order quantity of component i for supplier j

δ_{ij} : standard deviation of the realised quantity of defective component i for supplier j

Parameters

Manufacturer:

U_{ij} : upper bound of estimation of the worst case for component i for supplier j

r_k : wholesale price of finished product k

e_k : unit production cost for product k offering to retailer

a_k : opportunity loss cost for understocking of one unit of product k

b_k : inventory holding cost for overstocking of one unit of product k

c_j : capacity of supplier j

$f(z_k)$: probability density function obeyed by the demand of the product k

n_{ij} : internal resource of component i for supplier j

τ_{ij} : penalty cost for extra defective component i for supplier j

β_{ij} : cost of extra required component i purchased from supplier j

g_{ik} : number of units of component i required to produce one unit of product k

t_k : resource required by the manufacturer to produce one unit of product k

Q : production capacity of the manufacturer

R_{ij} : random variable of the realised quantity of defective component i for supplier j obeys a normal distribution where is μ_{ij} is the mean value and δ_{ij} is the standard deviation of the number of defective component i for supplier j

z_k : random variable of demand for product k from the customers obeying by a normal distribution

Supplier:

h_{ij} : cost of production for one unit component i for supplier j

q_{ij} : selling price of component i for supplier j

γ_{ij} : compensation cost of defective component i paid to the manufacturer by supplier j

x_{ij} : reliability of component i for supplier j

A_{ij} : fixed production cost for one unit component i for supplier j

B_{ij} : production cost responsiveness to order quantity of component i for supplier j

C_{ij} : production cost responsiveness to risk degree of component i for supplier j

Due to the asymmetric quality information, the manufacturer cannot observe full information of defective components during the negotiation process. Therefore, two scenarios (average case and worst case) are investigated for the manufacturer to estimate the number of defective components. For the average case, two situations are considered by the manufacturer. The number of defective components is estimated by minimising the total expected penalty cost. For the worst case, a chance constraint is applied.

6.3.4 The average case model

Let P_{ij} denote the estimated quantity of defective components and R_{ij} be a random variable of realised quantity of defective components. It is assumed that the manufacturer knows partial quality information of components. Therefore, the realised quantity of defective component R_{ij} obeys a normal distribution $(\mu_{ij}, \delta_{ij}^2)$ which is known by the manufacturer.

The manufacturer's objective function of Eq. (6.1) for the average case is the maximisation of the total profit. The first term is the expected sales revenue of finished products where the inventory/opportunity loss cost occurs. The second term is the production cost. The third term is the purchasing cost of components from the selected suppliers. The fifth term is penalty costs of defective components the manufacturer pays. The costs arise when the estimated quantity of defective components is less than the realised quantity, and then the manufacturer has to order extra components from the outsourcing suppliers. The other situation is the manufacturer pays the penalty costs when the estimated quantity is more than the realised quantity. The last term is the compensation cost received from the selected supplier due to the amount of defective components. The formulation is as follows:

$$\max \sum_{k=1}^{K}\{\int_0^{y_k}[r_k z_k - b_k(y_k - z_k)]f(z_k)dz_k + \int_{y_k}^{\infty}[r_k y_k - a_k(z_k - y_k)]f(z_k)dz_k\} - \sum_{i=1}^{I}\sum_{j=1}^{J} q_{ij} d_{ij} v_{ij}$$
$$- \sum_{k=1}^{K} e_k y_k - \sum_{i=1}^{I}\sum_{j=1}^{J}\{\beta_{ij} v_{ij} E[\max(0, R_{ij} - P_{ij})] + \tau_{ij} v_{ij} E[\max(0, P_{ij} - R_{ij})]\} +$$
$$\sum_{i=1}^{I}\sum_{j=1}^{J} \gamma_{ij} v_{ij}\{E[\max(0, R_{ij} - P_{ij})] + E[\max(0, P_{ij} - R_{ij})]\},$$

(6.1)

Subject to:

- Production capacity

$$\sum_{k=1}^{K} t_k y_k \le Q, \tag{6.2}$$

- Required amount of components to assemble products

$$\sum_{k=1}^{K} g_{ik} y_k \le \sum_{j=1}^{J} (d_{ij} - P_{ij}) v_{ij}, \forall i, \tag{6.3}$$

- Supplier's resource

$$\sum_{i=1}^{I} n_{ij} d_{ij} \le c_j w_j, \forall j, \tag{6.4}$$

- Supplier selection

$$w_j \geq v_{ij}, \forall i, \forall j. \tag{6.5}$$

Constraint (6.2) represents the production capacity for finished products. Constraint (6.3) indicates that the amount of non-defective components $d_{ij} - P_{ij}$ is required in order to assemble finished products. Constraint (6.4) is the limited resource constraint for the selected suppliers. It is assumed that there is resource capacity for each supplier to offer components. Constraint (6.5) is the supplier selection constraint. When the manufacturer purchases component i from supplier j such that $v_{ij} = 1$, the supplier j must be selected $w_j = 1$.

The supplier j's problem is formulated including the sales revenue, the cost of components and the penalty cost for defective components as follows:

$$\max \sum_{i=1}^{I} \{d_{ij}(q_{ij} - h_{ij}) - \gamma_{ij}\{E[\max(0, R_{ij} - P_{ij})] + E[\max(0, P_{ij} - R_{ij})]\}\}, \tag{6.6}$$

$$\text{s.t. } h_{ij} = A_{ij} + B_{ij}d_{ij} + C_{ij}x_{ij}, \tag{6.7}$$

$$x_{ij} = e^{-\frac{\delta_{ij}^2}{2}}, \tag{6.8}$$

where Eq. (6.7) represents production cost which depends on the quantity of components and the quality of components. Eq. (6.8) is the reliability function with respect to the standard deviation of amount of defective components.

6.3.5 The worst case model

The worst case is also considered such that the manufacturer makes pessimistic decisions according to its business strategies. Due to different business strategies, the definition of the worst case varies. In this chapter, it is assumed that the worst case is that the actual quantity of defective components R_{ij} is much larger than the estimated number of defective components P_{ij}. In other words, $R_{ij} > P_{ij}$. From the practical perspective, the manufacturers always surffer more loss once they have to purchase extra components. The manufacturer determines an upper bound of this situation in order to design optimal production planning. Therefore, a chance constraint is utilised to estimate uncertainty of defective components. The chance constraint is to give the upper bound when the production cannot be completely achieved. The additional constraint for the manufacturer is given to estimate worst case which is expressed by

$$\Pr[R_{ij} \geq P_{ij}] \leq U_{ij}, \tag{6.9}$$

where U_{ij} is a upper bound on the probability $\Pr$ which is decided by decision makers. The chance-constraint is given to estimate the probability of worse situations that the realised quantity of defective components is greater than estimation, and the probability is bounded by U_{ij}.

In order to facilitate the calculation of chance-constraint, the equation is reformulated into a form introduced by Petkov and Maranas [76]. The chance constraint is equivalently written as

$$\Phi(Y_{ij}) \geq 1 - U_{ij}, \tag{6.10}$$

where the left-hand side $\Phi(Y_{ij})$ is a normal cumulative distribution function obeyed by $(\mu_{ij}, \delta_{ij}^2)$. The cumulative normal distribution is recast into the standardized normal form. $Y_{ij} = \dfrac{P_{ij} - \mu_{ij}}{\delta_{ij}}$ where μ_{ij} is the mean value of R_{ij} and δ_{ij}^2 is the standard deviation of R_{ij}.

Thus, $Y_{ij} \geq \delta_{ij}\Phi^{-1}(1-U_{ij}) - P_{ij} + \mu_{ij}$ which is equivalent to

$$\delta_{ij}\Phi^{-1}(1-U_{ij}) - P_{ij} + \mu_{ij} \leq 0. \tag{6.11}$$

The formulation for the worst case is considered by embedding the chance constraint of Eq. (6.11) into the function of the average case.

6.4 Solution Approach

The manufacturer and suppliers are analysed by a Stackelberg game where the manufacturer is a leader and suppliers are followers. The optimal response functions should be derived firstly. Then, the manufacturer's decision is solved by substituting the optimal response functions as input parameters. Thus, the supplier's model becomes a constraint for the manufacturer's model when we solve the Stackelberg game. The model becomes a bilevel programming problem. The supplier j's objective function is rewritten as:

$$L^1 = \sum_{i=1}^{I}\{d_{ij}(q_{ij} - A_{ij} - B_{ij}d_{ij} - C_{ij}e^{-\frac{\delta_{ij}^2}{2}}) - \gamma_{ij}\{E[\max(0, R_{ij} - P_{ij})] + E[\max(0, P_{ij} - R_{ij})]\}\}. \tag{6.12}$$

Proposition 1 *For any given* $d_{ij} \geq 0$ and $\delta_{ij} \geq 0$, *the supplier* j *'s objective function is given by*

$$L^1 = \sum_{i=1}^{I}\{d_{ij}(q_{ij} - A_{ij} - B_{ij}d_{ij} - C_{ij}e^{-\frac{\delta_{ij}^2}{2}}) - \gamma_{ij}\delta_{ij}[2f(Y_{ij}) - Y_{ij} + 2Y_{ij}F(Y_{ij})]\}, \tag{6.13}$$

with $Y_{ij} = \dfrac{P_{ij} - \mu_{ij}}{\delta_{ij}}$, $F(Y_{ij})$ *is a cumulative distribution function, and* $f(Y_{ij})$ *is a probability density function.*

Proof We utilise a standardised normal distribution form to simplify the stochastic model which is introduced in Chapter 4. Thus, we obtain

$$E[\min(P_{ij}, R_{ij})] = \delta_{ij} + \delta_{ij}[-f(Y_{ij}) + (1 - F(Y_{ij}))Y_{ij}], \tag{6.14}$$

$$E[\max(0, R_{ij} - P_{ij})] = -\delta_{ij}[-f(Y_{ij}) + (1 - F(Y_{ij}))Y_{ij}], \tag{6.15}$$

$$E[\max(0, P_{ij} - R_{ij})] = \delta_{ij}[F(Y_{ij})Y_{ij} + f(Y_{ij})]. \tag{6.16}$$

Thus, the resulting formulation by substituting Eq. (61.4), Eq. (6.15) and Eq. (6.16) into Eq. (6.12) is

$$L^1 = \sum_{i=1}^{I}\{d_{ij}(q_{ij} - A_{ij} - B_{ij}d_{ij} - C_{ij}e^{-\frac{\delta_{ij}^2}{2}}) - \gamma_{ij}\delta_{ij}[2f(Y_{ij}) - Y_{ij} + 2Y_{ij}F(Y_{ij})]\}. \tag{6.17}$$

□

By using Proposition 1, Eq. (6.12) is reformulated as

$$\begin{aligned} L^1 = \sum_{i=1}^{I}\{d_{ij}(q_{ij} - A_{ij} - B_{ij}d_{ij} - C_{ij}e^{-\frac{\delta_{ij}^2}{2}}) - \gamma_{ij}\delta_{ij}[2\delta_{ij}\frac{1}{\sqrt{2\pi}}e^{-Y_{ij}^2} - P_{ij} + \mu_{ij} \\ + 2Y_{ij}(P_{ij} - \mu_{ij})F(Y_{ij})]\}. \end{aligned} \tag{6.18}$$

Proposition 2 *Let* d_{ij}^* *and* δ_{ij}^* *be Stackelberg equilibrium of the game, then supplier* j *'s optimal response functions are given by*

$$\sum_{i=1}^{I}[d_{ij}^* C_{ij} e^{-\frac{\delta_{ij}^2}{2}}\delta_{ij} - 2\gamma_{ij} f(Y_{ij})] = 0, \tag{6.19}$$

$$\sum_{i=1}^{I}(q_{ij} - A_{ij} - 2B_{ij}d_{ij}^* - C_{ij}e^{-\frac{\delta_{ij}^2}{2}}) = 0, \tag{6.20}$$

where $Y_{ij} = \dfrac{P_{ij} - \mu_{ij}}{\delta_{ij}^*}$.

Proof The equilibrium of a Stackelberg game is usually solved by a backward induction procedure. The supplier's optimal response functions should be derived first. Then, the manufacturer's decision is solved by substituting supplier's response functions as the input parameters.

By taking the first partial derivative of Eq. (6.18) with respect to δ_{ij} , we have

$$\frac{\partial L^1}{\partial \delta_{ij}} = \sum_{i=1}^{I}[d_{ij}C_{ij}e^{-\frac{\delta_{ij}^2}{2}}\delta_{ij} - 2\gamma_{ij}\frac{1}{\sqrt{2\pi}}e^{-\frac{Y_{ij}^2}{2}} - 2\gamma_{ij}\delta_{ij}\frac{1}{\sqrt{2\pi}}e^{-\frac{Y_{ij}^2}{2}} \times (-Y_{ij}) \times (-\frac{P_{ij} - \mu_{ij}}{\delta_{ij}^2})$$

$$- 2\gamma_{ij}(P_{ij} - \mu_{ij}) f(Y_{ij}) \times (-\frac{P_{ij} - \mu_{ij}}{\delta_{ij}^2})]$$

$$= \sum_{i=1}^{I}[d_{ij}C_{ij}e^{-\frac{\delta_{ij}^2}{2}}\delta_{ij} - 2\gamma_{ij} f(Y_{ij}) - 2\gamma_{ij} f(Y_{ij})Y_{ij}^2 + 2\gamma_{ij} f(Y_{ij})Y_{ij}^2]$$

$$= \sum_{i=1}^{I}[d_{ij}C_{ij}e^{-\frac{\delta_{ij}^2}{2}}\delta_{ij} - 2\gamma_{ij} f(Y_{ij})]. \tag{6.21}$$

By solving the equation $\dfrac{\partial L^1}{\partial \delta_{ij}} = 0$, we obtain a critical point of the equation as an increasing and concave function subject to δ_{ij}. Because the following condition is satisfied:

$$\frac{\partial^2 L^1}{\partial \delta_{ij}^2} = \sum_{i=1}^{I}[-d_{ij}C_{ij}e^{-\frac{\delta_{ij}^2}{2}}\delta_{ij}^2 - 2\gamma_{ij}\frac{1}{\sqrt{2\pi}}e^{-\frac{Y_{ij}^2}{2}} \times (-Y_{ij}) \times (-\frac{P_{ij} - \mu_{ij}}{\delta_{ij}^2})]$$

$$= \sum_{i=1}^{I}[-d_{ij}C_{ij}e^{-\frac{\delta_{ij}^2}{2}}\delta_{ij}^2 - \frac{2\gamma_{ij}}{\delta_{ij}} f(Y_{ij})Y_{ij}^2] \le 0 . \tag{6.22}$$

Thus, we obtain the optimal solution δ_{ij}^* by solving $\dfrac{\partial L^1}{\partial \delta_{ij}} = 0$.

By taking the partial first derivative of L^1 with respect to d_{ij} and setting the result to zero as follows:

$$\frac{\partial L^1}{\partial d_{ij}} = \sum_{i=1}^{I}(q_{ij} - A_{ij} - 2B_{ij}d_{ij} - C_{ij}e^{-\frac{\delta_{ij}^2}{2}}) = 0\,. \tag{6.23}$$

This equation is obtained from

$$\frac{\partial^2 L^1}{\partial d_{ij}^2} = -\sum_{i=1}^{I} 2B_{ij} \le 0\,. \tag{6.24}$$

We obtain the optimal solution d_{ij}^* by solving Eq. (6.23).

□

By applying Proposition 2, Eq. (6.22) and Eq. (6.23) are the optimal response functions in the Stackelberg game. The manufacturer is treated as a leader, then the Stackelberg game is solved by substituting Eq. (6.22) and Eq. (6.23) in the manufacturer`s model. The Stackelberg game theoreic model is written as follows:

$$\begin{aligned}\max \sum_{k=1}^{K}\{&\int_0^{y_k}[r_k z_k - b_k(y_k - z_k)]f(z_k)dz_k + \int_{y_k}^{\infty}[r_k y_k - a_k(z_k - y_k)]f(z_k)dz_k\} - \sum_{i=1}^{I}\sum_{j=1}^{J} q_{ij}d_{ij}v_{ij} \\ &- \sum_{k=1}^{K} e_k y_k - \sum_{i=1}^{I}\sum_{j=1}^{J}\{\beta_{ij}v_{ij}E[\max(0, R_{ij} - P_{ij})] + \tau_{ij}v_{ij}E[\max(0, P_{ij} - R_{ij})]\} + \\ &\sum_{i=1}^{I}\sum_{j=1}^{J}\gamma_{ij}v_{ij}\{E[\max(0, R_{ij} - P_{ij})] + E[\max(0, P_{ij} - R_{ij})]\},\end{aligned} \tag{6.25}$$

s.t. (6.2) - (6.5), (6.19), (6.20).

The final model is is formulated as a mixed integer nonlinear programming problem which is NP-hard. The normalization technique introduced in Chapter 4 is used to reformulate the problem. The final formulation is expressed as follows:

$$\max \sum_{k=1}^{K}[E(RE_k)-E(SH_k)-E(OP_k)]-\sum_{i=1}^{I}\sum_{j=1}^{J}q_{ij}d_{ij}v_{ij}-\sum_{k=1}^{K}e_k y_k-\sum_{i=1}^{I}\sum_{j=1}^{J}(\beta_{ij}-\gamma_{ij})v_{ij}$$
$$\{-\delta_{ij}[-f(Y_{ij})+(1-F(Y_{ij}))Y_{ij}]\}-\sum_{i=1}^{I}\sum_{j=1}^{J}(\tau_{ij}-\gamma_{ij})v_{ij}\{\delta_{ij}[F(Y_{ij})Y_{ij}+f(Y_{ij})]\} \quad , (6.26)$$

where

$$E(RE_k)=r_k\hat{z}_k+r_k\sigma_k[-F(Y_k)+(1-\Phi(Y_k))Y_k],$$

$$E(SH_k)=-a_k\sigma_k[-F(Y_k)+(1-\Phi(Y_k))Y_k)],$$

$$E(OP_k)=b_k\sigma_k[\Phi(Y_k)Y_k+F(Y_k)].$$

s.t. (6.2) - (6.5), (6.19), (6.20).

By using error functions, the problem can be solved by GAMS (Basic Open-source Nonlinear Mixed Integer programming). The algorithm of solution approach is given as:

Step 0: Initialise parameters of the model.

Step1: Derive the supplier's optimal response functions by Eq. (6.19) and Eq. (6.20).

Step 2: Solve the manufacturer` total profit problem Eq. (6.26) with constraints to obtain optimal decisions.

Step 3: Solve the selected suppliers' problem by substituting the manufacturer's optimal solutions.

6.5 Computational Examples

6.5.1 Case study

In this section, the proposed approach is applied to an illustrative case study. We investigate an electronic device global supply chain depicted in Fig. 6.3. The numerical example is conducted to demonstrate the features of the proposed model. Three outsourcing suppliers $(J=3)$ which offer five electronic parts $(I=5)$ are considered in Fig. 6.3. The manufacturer assembles two types of electronic devices $(K=2)$ to sell to the market. The demand of the finished products from customers is assumed to obey a normal distribution with the mean value of 19 for each product and the standard deviation of 18 for each product, respectively. One type of components is required in order to assemble a finished product. The mean value of the quantity of defective components is 3. The resource capacity for each supplier is 250. The price of components is set from 100 to 150. When the manufacturer purchases the extra components, the unit purchasing cost is 5. The unit compensation cost paid by the supplier is 4. Other parameters are presented in Table 6.1.

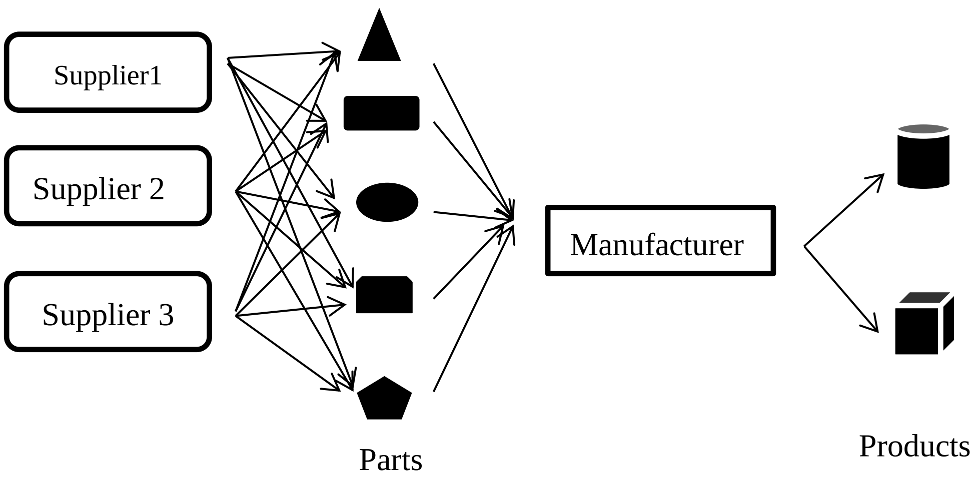

Figure 6.3: Electronic device global supply chain model.

Table 6.1: Parameters setting.

	Product1	Product 2
Unit sales price	2000	2000
Unit product cost	80	100
Capacity	80	80
Opportunity loss cost	150	120
Inventory holding cost	20	30

6.5.2 Analysis of equilibrium

A PC with Intel(R) CoreTM i7-3770 3.4 GHz processor and 8GB memory is used for the computation. The program is encoded by GAMS and solved by BOMININ (Basic Open-source Nonlinear Mixed Integer programming). Since the profit monotonically increases with the decrease of U_{ij}, the discussion of selection of parameter U_{ij} is neglected. The upper bound of the worst case U_{ij} is set to 0.6. A near-optimal solution is derived by the proposed method. The absolute gap and the relative gap are almost zero. The optimal solutions for the manufacturer for the average case and the worst case are shown in Table 6.2.

Table 6.2: Optimal solutions for the manufacturer.

Average case		Product 1	Product 2			
Production		44.5	18.0			
		Component1	Component 2	Component3	Component4	Component5
Estimation	Sup1	0	0	0	0	0
	Sup2	4.000	4.404	4.000	4.404	4.404
	Sup3	3.000	3.000	3.000	3.000	3.000
Quantity of component	Sup1	0	0	0	0	0
	Sup2	75.000	75.041	75.000	75.041	75.041
	Sup3	66.667	66.667	66.667	66.667	66.667
Profit		81256.741				
Worst case		Product 1	Product 2			
Production		44.5	18.0			
		Component1	Component2	Component3	Component4	Component5
Estimation	Sup1	3.342	3.349	3.349	3.342	3.349
	Sup2	4.464	4.464	4.464	4.464	4.464
	Sup3	3.414	3.407	3.407	3.414	3.407
Quantity of components	Sup1	75.053	75.056	75.056	5.053	75.056
	Sup2	75.056	75.056	75.056	75.056	75.056
	Sup3	66.745	66.741	66.741	66.745	66.741

Profit	81220.765

In Table 6.2, we observe that supplier 2 and supplier 3 are selected for the average case. However, all suppliers are selected for the worst case. The total profit increases for the average case compared with the profit for the worst case, although the production of finished products stays the same. In order to avoid the huge loss, the manufacturer estimates that the quantity of defective components for the worst case is more than the quantity for the average case. Thus, the required quantity of components also climbs up for the worst case. From the results, we conclude that the strategy for the manufacturer to avoid the huge loss in the worst case is to increase the estimated quantity of defective components properly.

6.5.3 Sensitivity analysis

In order to illustrate the features of the model, key parameters are analysed. We conduct three cases to analyse the impact of the parameter B_{ij} which is the production cost responsiveness to demand and the parameter C_{ij} which is the production cost responsiveness to quality for two scenarios (average case and worst case). The experiments for comparison are executed to show how each parameter affects the order quantity of components, estimation and the profit, respectively. The results are presented in Table 6.3.

In Table 6.3, it is noticed that the decrease of C_{ij} drives the decrease of profits for both of the average case and the worst case. At the same time, the manufacturer has to order more components from suppliers. Comparing Case 2 with Case 1, we notice that if B_{ij} decreases, the estimatiated quantity of defective components does not change. However, the profits decrease. The suppliers try to cut down the production cost by decreasing the quality of components in order to increase the total profits. However, the manufacturer expects the high quality of components in order to reduce the penalty costs. Thus, meaningful parameters for game theoretic models should be reasonably set. From the computational results, we conclude that it is necessary to optimise demand and quality simultaneously for decision makers in order to obtain the maximum profits.

Table 6.3: Production cost related to parameters.

		Case 1	Case 2	Case3
Average case		A=10	A=10	A=10
		B=0.6	B=0.5	B=0.6
		C=10	C=10	C=8
Profit		80485.908	72056.741	79610.908
Quantity of components	Supplier1	75.000	90.000	76.667
	Supplier2	212.500	255.000	217.500
	Supplier3	66.667	80.000	68.333
Estimation	Supplier1	3.000	3.000	3.000
	Supplier2	9.000	9.000	9.000
	Supplier3	3.000	3.000	3.000
Worst case				
Profit		80453.272	69924.358	79578.960
Quantity of components	Supplier1	75.056	90.053	76.718
	Supplier2	212.668	255.172	217.664
	Supplier3	66.726	80.060	68.391
Estimation	Supplier1	3.342	3.312	3.338
	Supplier2	10.225	9.972	10.044
	Supplier3	3.363	3.331	3.358

In this chapter, the demand of finished products is assumed to be uncertain. In order to demonstrate the impact of demand uncertainty on profits for the average case and the worst case, the experiments for comparison are conducted by analysing the deviation of demand of finished products. The results are given in Table 6.4.

The results show that the profits for both of the average case and the worst case always increase if the deviation is decreased. It indicates that if the demand from the market becomes stable, the manufacturer could obtain more profits. In this chapter, we only conduct very small size problems to illustrate the feature of the model. For further investigations, it is also important to investigate large scale size problems and develop an efficient solution approach in order to solve practical problems for real-world applications.

Table 6.4: Sensitivity analysis of demand deviation.

Case 1	Deviation	
	Product 1= 14	Product 2= 13
Average case (profit)	79610.908	
Worst case (profit)	79578.960	
Case 2	Deviation	
	Product 1= 13	Product 2= 12
Average case (profit)	80870.258	
Worst case (profit)	80838.310	
Case 3	Deviation	
	Product 1= 10	Product 2= 10
Average case (profit)	83425.765	
Worst case (profit)	83393.817	

6.6 Summary

In this chapter, a two echelon supply chain consisting of one manufacturer and its suppliers with uncertain demands is introduced where the quality information is asymmetric. The problem is analysed as a Stackelberg game where the manufacturer is a leader and suppliers are followers. The primary interest of this chapter is to investigate the estimation technique while quality information is asymmetric. Due to asymmetric information, the quality information of components purchased is unknown to the manufacturer. Thus, the manufacturer encounters uncertain amount of defective components after ordering. In order to estimate the number of defective components, two scenarios (average case and worst case) are introduced. This is is a general framework to consider estimation approaches in a game theoretic model. In future, it is interesting to investigate more complex scenarios.

Chapter 7

Conclusion and Remarks

In this thesis, game theoretic approaches are addressed to coordinate a manufacturer and multiple suppliers in global supply chain planning. A Stackelkberg game is applied to analyse the problem where the manufacturer is a leader and the suppliers are followers. The outline of the thesis is summarised as follows.

In Chapter 3, the manufacturer's decision problem with quantity discounts under demand uncertainty is addressed. The supply chain planning problem with uncertain demands is formulated as a mixed integer nonlinear programming problem (MINLP) with integral terms which is NP-hard. An outer approximation algorithm with a heuristic is proposed to solve the problem. By setting the duality gap is less than 1% and the update time of upper bounds cannot exceed 10, the computational results are obtained by the proposed method. The results show that the computational time increases significantly as the size of problems increases. The computational experiments are also conducted to investigate OA-SQP, OA-Heuristic and Zhang and Ma's method. It takes 87.4 seconds to solve the problem by OA-SQP, and 87.4 seconds and 1.5 seconds to solve the problem by OA-SQP and OA-Heuristic, respectively. However, the duality gap of OA-Heuristic is 8.91% which is the worst.

In Chapter 4, a reformulation of the supply chain planning problem is introduced. Both of incremental discount and all units discount models under demand uncertainty are presented. In order to reduce the computational complexity, the models are reformulated by a normalisation technique. The stochastic model is reformulated as an equivalent deterministic form. The computational results are obtained to compare the direct integral formation and the reformulation of the model. For small size instances, the computational time of both of the direct integral formulation and the reformulation of the model cases are no more than 40 seconds. However, the computational time of reformulation cases are significantly reduced compared with of the direct integral formulation cases for large size instances.

In Chapter 5, a game theoretic model with quantity discounts under demand uncertainty is introduced. A Stackelberg game is applied to coordinate a manufacturer and multiple suppliers in supply chain planning. The manufacturer is assumed to be a leader and the suppliers are followers. By deriving suppliers' optimal response functions, the optimal discount schedules are obtained. The total computational time to derive the problem is 0.842 seconds. It demonstrates a near-optimal solution is obtained by the proposed method.

In Chapter 6, the asymmetric quality information in a game theoretic model is addressed. The supply chain system consists of one manufacturer and its suppliers who are involved in purchasing and production with uncertain demands. Due to asymmetric information, the manufacturer cannot observe the complete information of the quality of components. Thus, we investigate two scenarios (average case and worst case) for the manufacturer to estimate the quality of components. A near-optimal solution is obtained by the proposed method. The sensitivity analysis shows as if the demand from the market becomes stable, the manufacturer obtains more profits for both of the average case and the worst case.

The latter half of this thesis focuses on Stackelberg game theoretic approaches to coordinate one manufacturer and suppliers in global supply chain planning. In future, it is also interesting to consider Nash games for different applications of other industries in supply chain planning. A two echelon supply chain is treated as a realistic supply chain model in this thesis. However, it is necessary to extend the problem to consider three echelon supply chains in order to achieve the coordination of global supply chain planning. Moreover, a general estimation approach is proposed to solve the asymmetric quality information problem in a game theoretic model. More sophisticated estimations are required in order to apply to solve practical problems.

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List of Publications

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