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A Systematic Method of Tuning a Performance Index in Nonlinear Model Predictive Control

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A Systematic Method of Tuning a Performance Index in Nonlinear Model Predictive Control

A dissertation submitted to
THE GRADUATE SCHOOL OF ENGINEERING SCIENCE
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DOCTOR OF PHILOSOPHY IN ENGINEERING

BY

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Abstract

Department of Systems Innovation

DOCTOR OF PHILOSOPHY IN ENGINEERING

A Systematic Method of Tuning a Performance Index in Nonlinear Model Predictive Control

by FATIMA TAHIR

Model predictive control (MPC) is extensively implemented in industry as an efficient way to deal with large multivariable constrained control problems. Its core idea is to choose the control actions by repeatedly solving an optimal control problem in real-time to minimize a given performance criterion over a certain future horizon, possibly subject to constraints on the manipulated inputs and outputs, where the future behavior is computed according to a model of a plant.

Although all real processes are inherently nonlinear, most applications use linear or piecewise linear models because it is easy to obtain a linear model based on the process data and because linear models provide good results when the plant is working in the neighborhood of the operating point. However, in many industrial applications, there is a strong requirement to obtain the best achievable performance in the presence of strict and tight constraints and a linear model is not sufficient to achieve this goal. Therefore, nonlinear MPC is needed for such problems as it permits the utilization of a nonlinear model for prediction. In nonlinear model predictive control (NMPC), more constraints have to be satisfied at the same time and process nonlinearities and constraints are explicitly taken into account in the controller.

Since the performance of NMPC depends on a performance index, systematic and efficient tuning of the performance index is a very important task from a practical viewpoint. In this dissertation, we propose a systematic method for the efficient tuning of the performance index in NMPC. In this tuning approach, first of all, a linear quadratic (LQ) regulator is designed for the linearized model using the inverse linear quadratic (ILQ) regulator design approach, which is based on the optimality conditions for the feedback control law obtained in the inverse regulator problem. After that, the inverse optimality conditions are applied to the designed ILQ regulator to tune the quadratic weights in the performance index of NMPC. After that, the NMPC algorithm is applied to the nonlinear model. Since NMPC is a finite-horizon problem, a terminal cost is added to the performance index of NMPC to make it work similarly to the infinite-horizon problem. Input and state constraints are penalized by adding penalty functions in the integral part of the performance index. The use of the ILQ regulator design method for the tuning of quadratic weights provides some tuning parameters that give a trade-off between the speed of the system's response and the magnitude of the control input. Moreover, this tuning methodology provides a free parameter that can be utilized to adjust the transient responses in the controlled output as well as to obtain a balance between the magnitudes of the control inputs. The effectiveness of this tuning approach is demonstrated by NMPC control of water-levels in a coupled-three-tank system (CTTS).

The tuning algorithm is extended for NMPC of parameter-dependent nonlinear systems and two methods are proposed for the selection of parameter-dependent tuning parameter. The extended tuning algorithm is applied to the speed control of nonlinear mean-value model of spark ignition (SI) engines. The effectiveness of tuning the parameter-dependent tuning parameter is elaborated in simulation results. Load torque is considered as a parameter and the effect of change in its value is suppressed by tuning the parameter-dependent tuning parameter using the proposed method.

Keywords: tuning of performance index, nonlinear model predictive control, inverse linear quadratic regulator design method, coupled three-tank system, parameter-dependent nonlinear systems, speed-control of spark ignition engines

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Abbreviations

MPC	Model predictive control
NMPC	Nonlinear MPC
LQ	Linear quadratic
ILQ	Inverse LQ
MIMO	Multi-input multi-output
MR-ILQ	Model reference ILQ
CTTS	Coupled-three-tank-system
SI	Spark Ignition
RHC	Receding horizon control
GMRES	Genearlized minimum residual
C/GMRES	Continuation/GMRES
PMP	Pontryagin's minimum principle
MVM	Mean-value model
PID	Propotional-integral-derivative

Physical Constants

Constant Name	Symbol	=	Constant Value (with units)
Acceleration due to gravity	g	$=$	$980\ \mathrm{cm/s^2}$
Universal gas constant	R	$=$	$280.058\ \mathrm{J/(Kg.K)}$
Atmospheric pressure	p_a	$=$	1 bar

Symbols

Symbol	Name	Unit (if any)
t	Time	s
n	Number of states	
m	Number of inputs	
$\mathbb{R}^n$	n-dimensional Euclidean space	
$x(t)$	State vector	
$u(t)$	Input vector	
J	Performance index	
$\varphi(x(t+T))$	Infinite horizon cost	
$L(x(t),u(t))$	Integral part in J	
T	Horizon length	
$C(x(t),u(t))$	Vector of Equality constraints	
m_c	Number of equality constraints	
$\mathscr{H}$	Hamiltonian	
λ	Costate vector	
μ	Lagrange multiplier vector	
$U(t)$	Vector of u and μ	
m_{uc}	Dimension of vector $U(t)$	
$\mathscr{F}$	Function of discretized stationary conditions	
A_s	Stable matrix introduced to stabilize $\mathscr{F}=0$	
A	$n \times n$ System matrix	
B	$n \times m$ System matrix	
Q	$n \times n$ Semi positive definite and real matrix	
R	$m \times m$ Positive definite real matrix	
K	$m \times n$ Feedback gain matrix	

P	$n \times n$ positive definite real matrix	
H	$H := BK/2 - A$	
x^*	Desired reference value of state vector	
u^*	Desired reference value of input vector	
M	Similarity transformation	
B_2	Non-singular lower $m \times m$ part of matrix B	
V	$m \times m$ non-singular matrix	
Σ	Main tuning diagonal and positive matrix of order m	
F_1	$m \times (n-m)$ matrix	
I_m	Identity matrix of order m	
σ_i	ith diagonal entry of matrix Σ	
σ	Main tuning parameter	
Γ	$m \times m$ positive diagonal matrix	
γ_i	ith diagonal entry of matrix Γ	
$p_{iec}(x(t), u(t))$	penalty function for inequality constraints	
D	$m \times m$ positive definite matrix	
Y	$n \times n$ positive definite matrix	
Y_1	$(n-m) \times (n-m)$ matrix positive semidefinite matrix	
σ_{lb}	Lower bound of σ	
$P1$	Pump 1 of CTTS	
$P2$	Pump 2 of CTTS	
$P3$	Pump 3 of CTTS	
$V1$	Binary valve 1 of CTTS	
$V2$	Binary valve 2 of CTTS	
$V3$	Binary valve 3 of CTTS	
$V4$	Binary valve 4 of CTTS	
$V5$	Binary valve 5 of CTTS	
$V6$	Binary valve 6 of CTTS	
$V7$	Binary valve 7 of CTTS	
h_1	Water level in Tank-1 of CTTS	cm
h_2	Water level in Tank-2 of CTTS	cm
h_3	Water level in Tank-3 of CTTS	cm
a_{T1}	Cross-section area of Tank-1 of CTTS	cm^2

a_{T2}	Cross-section area of Tank-2 of CTTS	cm^2
a_{T3}	Cross-section area of Tank-3 of CTTS	cm^2
a	Cross-section area of pipes in CTTS	cm^2
v_2	Value of valve V2	
v_3	Value of valve V3	
v_4	Value of valve V4	
v_5	Value of valve V5	
v_6	Value of valve V6	
v_7	Value of valve V7	
k_1	Gain constant of pump P1	
k_2	Gain constant of pump P2	
H_{11}	Height of pipe through valve V3	cm
H_{12}	Height of pipe through valve V4	cm
H_{21}	Height of pipe through valve V5	cm
H_{22}	Height of pipe through valve V6	cm
H_{31}	Height of pipe through valve V7	cm
$v_{ij}, i \neq j$	Off-diagoal entries of matrix V	
p	Parameter	
σ_0	Any $\sigma > \sigma_{lb}$	
$F(\sigma, p)$	Closed-loop system matrix	
$\hat{K}$	Part of K independent of σ	
F_0	Closed-loop system matrix for $\sigma = \sigma_0, p = p_0$	
A_0, B_0	System matrices for $p = p_0$	
M_0	Similarity transformation for $\sigma = \sigma_0$ and $p = p_0$	
$\sigma^*(p)$	Parameter-dependent tuning parameter	
H_∞	H-infinity	
ω_e	Engine speed	rad/s or rpm
τ_l	Load torque	Nm
J_e	Engine crank inertia moment	$Kg.m^3$
p_m	Intake manifold pressure	bar
d_r	Rotational friction constant	
T_m	Intake manifold temperature	K
V_m	Intake manifold volume	m^3

$\dot{m}_{th}$	Air mass flow rate through the throttle	g/s
$\dot{m}_{cyl}$	Air mass flow rate entering the cylinder	g/s
ρ_a	Atmospheric density	$\mathrm{Kg/m^3}$
η	Volumetric efficiency of the engine	
V_d	Engine displaced volume	$\mathrm{m^3}$
T_e	Engine torque	Nm
ω_d	Desired engine speed	rad/s or rpm

This dissertation is dedicated to my beloved parents, sister, brother, husband and son for their selfless love, unparalleled prayers, kind support and endless encouragements.

Chapter 1

Introduction

1.1 Motivation and Literature Survey

Model predictive control (MPC) is extensively implemented in industry as an efficient way to deal with large multivariable constrained control problems. MPC was originated in the end of 1970s [1–3] and it has undergone significant development since then. Its core idea is to choose the control actions by repeatedly solving online an optimal control problem in real-time to minimize a given performance criterion over a certain future horizon, possibly subject to constraints on the manipulated inputs and outputs. The forecasting of process behavior and performance is based on the process model and measurements available at current time. The advantage of repeated online optimization is to incorporate some feedback mechanism.

The success of MPC in industry is due to the fact that it handles multivariable control problems, takes account of actuator limitations and constraints imposed on manipulated and controlled variables, allows operations closer to the constraints, can handle non-minimal phase and unstable systems, and is the most general way of posing the control problem in the time domain. Therefore, MPC is suitable for almost any kind of problem.

Although all real processes are inherently nonlinear, most applications use linear or piecewise linear models because it is easy to obtain a linear model based on the process data and because linear models provide good results when the plant is working in the neighborhood of the operating point [4]. However, in many industrial applications, there is a strong requirement to obtain the best achievable performance in the presence of strict and tight constraints and a linear model is not sufficient to achieve this goal. Therefore, nonlinear MPC is needed for such problems as it permits the utilization of a nonlinear model for prediction. In nonlinear model predictive control (NMPC), more constraints have to be satisfied at the same time and process nonlinearities and constraints are explicitly taken into account in the controller. In NMPC, the development of numerical algorithms for real-time optimization [5–13], the stability of closed-loop systems [14–27] and robustness analysis [28–37] are active areas of research.

Since the performance of NMPC depends on a performance index, systematic and efficient tuning of the performance index is a very important task from a practical viewpoint. Although a lot of work has been carried out for the development of real-time optimization of NMPC, selection of the appropriate performance index is still an open problem and little research has been carried out in this area. This is because the unavailability of a precise solution to a nonlinear optimal control problem makes it difficult to explicitly associate the desired characteristics of the closed-loop response with the free parameters in the performance index of NMPC.

In Ref. [38], the quadratic weights in the cost function of a linear MPC are tuned so that the controller behaves similarly to a given "favorite controller". Tuning is based on controller matching and the solutions of convex optimization problems. However, this approach cannot be used for NMPC as the use of a nonlinear model converts the convex quadratic programming problem to a nonconvex nonlinear program. In another study [39], feedback linearization is combined with NMPC, and the proposed performance index consists of two tuning parameters that give a reasonable trade-off between the transient responses of the controlled output and the control input. This tuning approach is simple and is applicable for nonlinear systems. However, it depends on the input-output linearization of the given nonlinear system which may require large magnitude of control inputs.

1.2 Problem Statement, Main Objective and Design Strategy

We propose a systematic method of tuning the quadratic weights in the performance index of NMPC [40, 41]. In this approach, first of all, we obtain the linear model of a given nonlinear system. Then, we design a linear quadratic (LQ) regulator using the inverse linear quadratic (ILQ) regulator design approach [42], which is based on the optimality conditions for the feedback control law obtained in the inverse regulator problem. After that, we use the inverse optimality conditions to tune the quadratic weights. Then, these weights are used in the performance index of NMPC, and the NMPC algorithm is applied to the nonlinear model. Since NMPC is a finite-horizon problem, a terminal cost is added to the performance index of NMPC to make it work similarly to the infinite-horizon problem as reported in Ref. [43]. Input and state constraints are penalized by adding penalty functions in the integral part of the performance index. The use of the ILQ regulator design method for the tuning of the quadratic weights provides us with some parameters that can be used for a trade-off between the magnitude of the control inputs and the speed of the system's response. It also provides a parameter that can be used to control the transient responses in the controlled output and to adjust the magnitude of the control inputs. After that, the tuning algorithm is extended for the NMPC control of parameter-dependent nonlinear systems and two methods are proposed for the selection of parameter-dependent tuning parameter [44]. The extended tuning algorithm is applied to the speed control of nonlinear mean-value model of spark ignition (SI) engines. The effectiveness of tuning the parameter-dependent tuning parameter is elaborated in simulation results. Load torque is considered as a parameter and the effect of change in its value is suppressed by tuning the parameter-dependent tuning parameter using the proposed method.

1.3 Thesis Overview

The outline of this thesis is as follows: Chapter 2 gives a brief introduction to NMPC and the algorithm used to solve NMPC problem. Chapter 3 describes the significant features of the ILQ regulator design method and the procedure used to design the feedback control law in this approach. In Chapter 4, inverse optimality conditions are employed to find the quadratic weights that guarantee the optimality of the control law designed using the ILQ regulator design method and significance of the tuning algorithm is demonstrated by applying it to a coupled-three-tank-system (CTTS). In Chapter 5, tuning algorithm is extended for NMPC of parameter-dependent nonlinear systems and its effectiveness is elaborated by solving the speed control problem for spark ignition engines so that the effect of constant load disturbances on the closed-loop response is minimized. Chapter 6 concludes this thesis.

In this thesis, the symbols appearing after a semicolon within the arguments of a function represent parameters on which the function depends. For example, $u(t;x(t))$ means the value of the function u at time t with the parameter $x(t)$.

Chapter 2

Nonlinear Model Predictive Control

2.1 Introduction

In this chapter, we briefly introduce the basics of NMPC. In MPC, a model of the process is used to predict the future evolution of the process to optimize the control signal. NMPC uses nonlinear model for prediction. First of all, measurements/estimates of the states of the system are obtained at time t. Then an optimal input signal is computed by minimizing a given cost function over a certain prediction horizon in the future using a nonlinear model of the system. First part of the optimal control input is applied to the system until new measurements/estimates of the states are available and this process is repeated. The advantage of this continuous online optimization is to introduce a sense of feedback mechanism. If there were no disturbances and no modeling errors, and if the optimization problem could be solved for infinite horizons, then the control input computed at time $t = 0$ could be applied to the system for all times $t \geq 0$. However, this is not possible in general due to the fact that presence of disturbances and model-plant mismatch may make the predicted response significantly different from the true closed-loop response. Therefore, in order to introduce feedback into the MPC law, control input is implemented only until the next measurements are available. This provides a degree of robustness to modeling errors and uncertainties.

2.2 Problem Formulation

Consider a nonlinear system governed by the following nonlinear state equations:

$$\dot{x}(t) = f(x(t), u(t)), \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ denotes the state vector, and $u(t) \in \mathbb{R}^m$ denotes the input vector.

The performance index to be minimized is given by

$$J = \varphi(x(t+T)) + \int_{t}^{t+T} L(x(\tau), u(\tau))\, \mathrm{d}\tau, \tag{2.2}$$

where T is the horizon length and time from current time t to time $t+T$ is termed as prediction horizon. Since this prediction horizon recedes into the future, MPC is also known as receding horizon control (RHC).

Equality constraints are given in the following form:

$$C(x(t), u(t)) = 0, \tag{2.3}$$

where C is an m_c-dimensional vector-valued function. If there are inequality constraints on the inputs and/or state variables, they can be incorporated using two different approaches. In the first approach, they are converted into the equality constraints by adding some dummy inputs. After that, penalty functions for the dummy inputs are added to the performance index. In the second approach, appropriate barrier functions are introduced in the performance index to incorporate the inequality constraints.

At each time t, this optimal control problem, specified by the nonlinear model, performance index and constraints, is solved to find the sequence of the optimal control input $u_{optimal}(\tau; x(t))$ for the interval $t \leq \tau \leq t+T$, and then only the initial value of the control input is applied to the system at each time t, i.e.,

$$u(t) = u_{optimal}(t; x(t)). \tag{2.4}$$

Since the actual state $x(t)$ is taken as initial state to minimize the cost function given by Eq. (2.2), MPC is a feedback law of the state $x(t)$.

Therefore, NMPC involves solving an optimal control problem at each time t for a finite horizon using a nonlinear model of the system so that the given performance index is minimized and all the constraints are satisfied, and then applying only the initial value of the optimal control input to the system.

2.3 Continuation/GMRES Algorithm

In this work, a fast numerical algorithm C/GMRES method [9] is used to update control input in real time. In this algorithm, the continuation method [45] is combined with the generalized minimum residual (GMRES) method [46] which is a Krylov subspace method. First of all, horizon is divided into N steps and open-loop optimal control problem is discretized on the τ-axis using forward difference approximation, resulting in a nonlinear algebraic equation for the discretized sequence of control input as follows:

$$x_{i+1}^*(t) = x_i^*(t) + f(x_i^*(t), u_i^*(t))\Delta\tau(t), \tag{2.5}$$

$$x_0^*(t) = x(t), \tag{2.6}$$

$$C(x_i^*(t), u_i^*(t)) = 0, \tag{2.7}$$

$$J = \varphi(x_N^*(t)) + \sum_{i=0}^{N-1} L(x_i^*(t), u_i^*(t))\Delta\tau, \tag{2.8}$$

where

$$\Delta\tau := T/N. \tag{2.9}$$

Taking the current state as the initial state of the discretized problem, the control input sequence $\{u_i^*(t)\}_{i=0}^{N-1}$ is optimized at each time t and only the initial value of this sequence is applied to the system, i.e., the actual control input is given by

$$u(t) = u_0^*(t). \tag{2.10}$$

The control input that minimizes the performance index given by Eq. (2.8) is computed so that Pontryagin's minimum pricinciple (PMP) is satisfied, i.e., Hamiltonian is minimized [77]. Let $\mathscr{H}$ denote the Hamiltonian defined by

$$\mathscr{H}(x, \lambda, u, \mu) := L(x, u) + \lambda^T f(x, u) + \mu^T C(x, u), \tag{2.11}$$

where λ denotes the n-dimensional costate vector, and μ denotes the m_c-dimensional Lagrange multiplier vector associated with the equality constraints. The first-order necessary conditions for the sequence of the control input $\{u_i^*(t)\}_{i=0}^{N-1}$, Lagrange multiplier $\{\mu_i^*(t)\}_{i=0}^{N-1}$ and costate $\{\lambda_i^*(t)\}_{i=0}^{N-1}$ are obtained by the calculus of variation as

$$\frac{\partial \mathscr{H}}{\partial u}(x_i^*(t), \lambda_{i+1}^*(t), u_i^*(t), \mu_i^*(t)) = 0, \tag{2.12}$$

$$\lambda_i^*(t) = \lambda_{i+1}^*(t) + \frac{\partial \mathscr{H}}{\partial u}^T(x_i^*(t), \lambda_{i+1}^*(t), u_i^*(t), \mu_i^*(t))\Delta\tau, \tag{2.13}$$

$$\lambda_N^*(t) = \frac{\partial \varphi}{\partial x}^T(x_N^*(t)). \tag{2.14}$$

The sequences of the optimal control input $\{u_i^*(t)\}_{i=0}^{N-1}$ and the Lagrange multiplier $\{\mu_i^*(t)\}_{i=0}^{N-1}$ must satisfy Eqs. (2.5)–(2.6) and (2.12)–(2.14). Let $U(t)$ be a vector of the control inputs and Lagrange multipliers defined by

$$U(t) := [u_0^{*T}(t), \mu_0^{*T}(t), u_1^{*T}(t), \mu_1^{*T}(t), ..., u_{N-1}^{*}{}^T(t), \mu_{N-1}^{*}{}^T(t)] \in \mathbb{R}^{m_{uc}N}, \tag{2.15}$$

where

$$m_{uc} := m + m_c. \tag{2.16}$$

For a given $U(t)$ and $x(t)$, $\{x_i^*(t)\}_{i=0}^N$ is calculated recursively using Eqs. (2.5) and (2.6), and after that, the sequence of the Lagrange multipliers $\{\lambda_i^*(t)\}_{i=0}^N$ is also calculated recursively from $i = N$ to $i = 0$, using Eqs. (2.13) and (2.14). Since $x_i^*(t)$ and $\lambda_i^*(t)$ are determined by $x(t)$ and $U(t)$ through Eqs. (2.5), (2.6), (2.13) and (2.14), Eqs. (2.7) and (2.12) can be regarded as one equation defined by

$$\mathscr{F}(U(t),x(t),t) := \begin{bmatrix} \frac{\partial \mathscr{H}}{\partial u}^T(x_0^*(t),\lambda_1^*(t),u_0^*(t),\mu_0^*(t)) \\ C(x_0^*(t),u_0^*(t)) \\ \vdots \\ \frac{\partial \mathscr{H}}{\partial u}^T(x_{N-1}^*(t),\lambda_N^*(t),u_{N-1}^*(t),\mu_{N-1}^*(t)) \\ C(x_{N-1}^*(t),u_{N-1}^*(t)) \end{bmatrix} = 0. \tag{2.17}$$

The optimal vector $U(t)$ is obtained by solving Eq. (2.17) in real time. However, solving Eq. (2.17) involves recursive calculation of $\{x_i^*(t)\}_{i=0}^N$ and $\{\lambda_i^*(t)\}_{i=0}^N$ at each sampling time, i.e., nonlinear equations are to be solved at each sampling time which is computationally demanding.

Instead of solving $\mathscr{F}(U,x,t) = 0$ itself at each sampling time with an iterative method such as Newton's method, C/GMRES algorithm involves obtaining a differential equation for updating the control sequence through the use of continuation method [45]. That is, the time-derivative of the control input sequence is obtained according to the corresponding time-derivative of the state, and it can be integrated to determine the control input in real time without using iterative optimization methods. We find the time-derivative of $U(t)$ such that $\mathscr{F}(U(t),x(t),t) = 0$ is satisfied. By choosing $U(0)$ so that

$$\mathscr{F}(U(0),x(0),0) = 0, \tag{2.18}$$

the time-derivative of U, i.e., $\dot{U}$ is determined so that

$$\dot{\mathscr{F}}(U,x,t) = A_s \mathscr{F}(U,x,t), \tag{2.19}$$

where A_s is a stable matrix introduced to stabilize $\mathscr{F} = 0$. Using the chain rule of differentiation, Eq. (2.19) yields the following linear equation for $\dot{U}$:

$$\frac{\partial \mathscr{F}}{\partial U}\dot{U} = A_s \mathscr{F} - \frac{\partial \mathscr{F}}{\partial x}\dot{x} - \frac{\partial \mathscr{F}}{\partial t}. \tag{2.20}$$

Then if $\frac{\partial \mathscr{F}}{\partial U}$ is nonsingular, a differential equation is obtained for $U(t)$ as follows:

$$\dot{U} = \left(\frac{\partial \mathscr{F}}{\partial U}\right)^{-1} \left(A_s \mathscr{F} - \frac{\partial \mathscr{F}}{\partial x}\dot{x} - \frac{\partial \mathscr{F}}{\partial t}\right). \tag{2.21}$$

To find the solution $U(t)$ without iterative optimization methods, Eq. (2.21) can be integrated in real time. However, this differential equation still involves high computational cost because of Jacobians $\frac{\partial \mathscr{F}}{\partial U}$, $\frac{\partial \mathscr{F}}{\partial x}$ and $\frac{\partial \mathscr{F}}{\partial t}$ and a large linear equation associated with $(\frac{\partial F}{\partial U})^{-1}$. Moreover, from the definition of F in Eq. (2.17), the Jacobian $\frac{\partial \mathscr{F}}{\partial U}$ is dense because $x_i^*(t)$ and $\lambda_i^*(t)$ are functions of $U(t)$. Therefore, in order to reduce the computational cost in the Jacobians and the linear equation, forward difference approximation is used for the products of Jacobians and vectors, and the GMRES method [46] is used to solve the linear equation. For more details on C/GMRES algorithm, Ref. [9] can be seen.

Chapter 3

Inverse Linear Quadratic Regulator Design Method

3.1 Significant Features and Applications of Inverse Linear Quadratic Regulator Design Method

The ILQ regulator design method is used to design LQ regulators for linear systems from the viewpoint of the inverse regulator problem. Unlike the conventional LQ regulator design techniques, it does not depend on the selection of quadratic weights. Rather, it focuses on the properties of the regulator to be designed, that are more important from a practical viewpoint. Although there is no need to specify a performance index to design an LQ regulator using the ILQ regulator design approach, it gives a feedback control law that is optimal for some quadratic performance index. This method is based on the optimality conditions for a feedback control law as obtained in the inverse problem of LQ regulators, and it has two steps. In the first step, an optimal feedback control law is parameterized using the necessary conditions for optimality. In the second step, the sufficient conditions for optimality are used to determine these parameters so that the optimality of the feedback control law is ensured. It is a simple method from the computational viewpoint as there is no need to solve the Ricatti equation. The most appealing feature of this method is that it provides some tuning parameters in the designed regulator that give the trade-off between the magnitude of the control inputs and the speed of the resulting closed-loop system. Ref. [42] can be seen for more details.

A generalization of the ILQ regulator design method is shown in [47, 48] to extend it for the design of optimal servo systems. In Ref. [49], the use of the ILQ method for the design of optimal servo systems is extended to a class of non-minimum phase multi-input multi-output (MIMO) systems. Design of optimal servo systems is further extended to achieve quadratic stability for linear systems with structured uncertainty in [50, 51]. This approach is applied for robust control of a multivariable magnetic levitation system in [52–54]. In Ref. [55], a new optimality condition is developed for the generalization of the ILQ regulator design method to attain an interesting decoupling property with desired output response. As an extension of the ILQ servo system presented in Ref. [51], design of the model reference ILQ (MR-ILQ) servo system is proposed in [56, 57] to attain tracking robustness by use of a very simple structure. Ref. [58] presents an application of MR-ILQ design method to temperature control of wafer in semiconductor manufacturing systems, as one of its significant application in the industry process. Many other extensions of the ILQ method for design of optimal servo system and its application to important industrial systems can be seen in [59–71].

3.2 Review of Linear Quadratic Regulator and its Inverse Problems

Since the ILQ regulator design method is based on the optimality conditions for feedback control law as obtained in the inverse regulator problem, a brief review of LQ regulator and its inverse problem is presented in this section.

Consider the conventional LQ regulator problem represented by the following state equations and cost function:

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{3.1}$$

$$J = \int_0^\infty (x^T(t)Qx(t) + u^T(t)Ru(t))\,\mathrm{d}t, \tag{3.2}$$

where $x(t) \in \mathbb{R}^n$ denotes the state vector, and $u(t) \in \mathbb{R}^m$ denotes the input vector. A, B, Q and R are real matrices of appropriate dimensions. A and B are the system matrices representing the linear model, $n > m$, and (A,B) is controllable. Q is a symmetric nonnegative definite matrix and R is a symmetric positive definite matrix. In a conventional LQ regulator design problem, the design objective is to find an optimal control law $u(t)$ which minimizes a quadratic cost for given Q and R. As is well known, the optimal control is given by the following linear feedback control law:

$$u(t) = -Kx(t), \tag{3.3}$$

where K is the feedback gain matrix and is expressed as follows:

$$K = R^{-1}B^TP. \tag{3.4}$$

Here P is the unique positive definite solution of the algebraic Ricatti equation

$$A^TP + PA - PBR^{-1}B^TP + Q = 0. \tag{3.5}$$

The feedback gain matrix K is said to be "stable" if all the eigenvalues of closed-loop system matrix have negative real parts, i.e.,

$$\text{Re}\lambda_i(A - BK) < 0,\ i = 1, 2, ..., n. \tag{3.6}$$

The inverse problem of LQ regulators was first addressed by R. E. Kalman [72]. In the inverse problem of LQ regulator, a state feedback gain matrix K is available and we need to find necessary and sufficient conditions on A, B and K such that K is "optimal" in the sense that control input $u(t)$ given by Eq. (3.3) minimizes the cost function J for some quadratic weights $Q \geq 0$ and $R > 0$.

3.2.1 Optimality of Feedback Gain Matrix

Lemma 3.1 The feedback gain matrix K is both optimal and stable if and only if there exist $P > 0$ and $R > 0$ satisfying the following two conditions [42]:

$$PH + H^TP > 0, \tag{3.7}$$

$$B^TP = RK, \tag{3.8}$$

where

$$H := \frac{BK}{2} - A. \tag{3.9}$$

For the proof of Lemma 3.1, [42, 73] can be seen.

Lemma 3.2 For any real $n \times n$ matrix H, the matrix inequality given by Eq. (3.7) has a positive definite diagonal solution P if H satisfies any of the following conditions:

(a) H is copositive, i.e.,

$$H + H^T > 0, \tag{3.10}$$

(b) H is diagonal dominant, i.e.,

$$h_{ii} > \sum_{j=1}^{n} |h_{ij}| - |h_{ii}|,\ 1 \leq i \leq n \tag{3.11}$$

or

$$h_{ii} > \sum_{i=1}^{n} |h_{ij}| - |h_{jj}|,\ 1 \leq j \leq n. \tag{3.12}$$

Here h_{ij} represents the element corresponding to ith row and jth column. Equation (3.11) represents the case when H is row diagonal dominant and Eq. (3.12) represents the case when H is column diagonal dominant. It is easy to see that condition (a) ensures the inequality given by Eq. (3.7) for $P = I$. For the second part of Lemma 3.2, refer to [74].

Lemma 3.3 There exists $P \geq 0$ and $R > 0$ satisfying Eq. (3.8) if and only if the following conditions hold.

(I) KB has m linearly independent real left-eigenvectors.

(II) KB has m real non-negative eigenvalues.

(III) $\text{rank}(KB) = \text{rank}(K)$.

Refer to [75] for more details.

3.3 Design Procedure of the ILQ Method

To design the ILQ regulator for the linear system given by Eq. (3.2), we proceed as follows:

3.3.1 Partitioning of System Matrices

In this step, the system matrices, A and B, of the linear model are partitioned as follows:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \tag{3.13}$$

$$B = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}, \ \det(B_2) \neq 0, \tag{3.14}$$

where A_{11}, A_{12}, A_{21} and A_{22} are $(n-m)\times(n-m)$, $m\times(n-m)$, $(n-m)\times m$ and $m\times m$ matrices, respectively, and B_2 is a nonsingular matrix of dimensions $m\times m$. If A and B cannot be partitioned in this way, we need to apply a similarity transformation to transform them into this form. For example, we can apply a similarity transformation M such that

$$MB = \begin{bmatrix} 0 \\ I_m \end{bmatrix}. \tag{3.15}$$

This similarity transformation M is not unique [76] and there may be many possible candidates for it.

3.3.2 Parameterization of Feedback Gain Matrix

The necessary conditions for optimality [42, 75] are used for the parameterization of the state feedback gain matrix as follows:

$$K = B_2^{-1} V^{-1} \Sigma V [F_1 \quad I_m], \tag{3.16}$$

where V and Σ are $m \times m$ real nonsingular matrices. Matrix Σ is the main tuning matrix. It is a diagonal matrix with positive diagonal entries, i.e.,

$$\Sigma = \text{diag}\{\sigma_{\text{i}}\} > 0. \tag{3.17}$$

Σ may also be expressed as follows

$$\Sigma = \sigma \Gamma, \tag{3.18}$$

where

$$\sigma > 0, \tag{3.19}$$

and

$$\Gamma = \text{diag}\{\gamma_{\text{i}}\} > 0. \tag{3.20}$$

F_1 is an $m \times (n-m)$ matrix that is dependent on the partial pole placement. Although V and F_1 are also free parameters, Σ is the main tuning parameter. If Σ is expressed in the form given by Eq. (3.18), scalar positive number σ is the main tuning parameter.

3.3.3 Determination of the Parameters

This step involves the determination of the matrices V, Σ, F_1 and the lower bounds of $\{\sigma_i\}, i = 1, 2, \ldots, m$ or σ so that the matrix $H = BK/2 - A$ is either copositive or diagonal dominant, which is a sufficient condition for the optimality of the state feedback gain matrix K [42]. The computation of F_1 is dependent on $n-m$ stable poles s_i, which must be distinct from the eigenvalues of the matrix A_{11}. The matrix Σ is the main tuning parameter responsible for the trade-off between the speed of the resulting closed-loop response and the magnitude of the control input. It is computed in such a way that the sufficient condition for the optimality of the state feedback gain matrix K is satisfied. There is a lower bound for σ, and any σ greater than this lower bound will satisfy the sufficient condition for optimality. The matrix V can be chosen arbitrarily. For simplicity, we can choose it to be an identity matrix of order m.

Chapter 4

Tuning of Performance Index in Nonlinear Model Predictive Control

4.1 Tuning of Performance Index using Inverse Optimality Conditions

Although the feedback control law designed using the ILQ method does not depend on the selection of quadratic weights, since it is designed using the necessary and sufficient conditions for optimality, it is optimal for some quadratic cost. This means that we may use these optimality conditions in the reverse direction to determine the quadratic weights for which this feedback control law is optimal. This approach is used to tune the quadratic weights in the performance index of NMPC. This tuning method provides us with tuning parameters σ_i, $i = 1, 2, ..., m$, which can be used to adjust the magnitude of the control inputs and the speed of the response of the resultant system.

First of all, we obtain the linearized model of the given nonlinear model. Then we design an LQ regulator for the linearized model using the ILQ regulator design method. Then inverse optimality conditions are used to tune the quadratic weights in the performance index of NMPC. Finally the NMPC algorithm is applied to the nonlinear model given by Eq. (2.1).

To obtain the linear model, the nonlinear model given by Eq. (2.1) is linearized at an equilibrium point (x^*, u^*). The obtained linearized model can be expressed as follows:

$$\dot{\delta}_x(t) = A\delta_x(t) + B\delta_u(t), \tag{4.1}$$

where

$$f(x^*, u^*) = 0, \tag{4.2}$$

$$\delta_x(t) = x(t) - x^*, \tag{4.3}$$

$$\delta_u(t) = u(t) - u^*, \tag{4.4}$$

$$A = \left(\frac{\partial f}{\partial x}\right)\Big|_{x=x^*, u=u^*}, \tag{4.5}$$

$$B = \left(\frac{\partial f}{\partial u}\right)\Big|_{x=x^*, u=u^*}. \tag{4.6}$$

The performance index in Eq. (2.2) is given by

$$\varphi(x(t+T)) = \frac{1}{2}(x(t+T) - x^*)^T P(\sigma)(x(t+T) - x^*), \tag{4.7}$$

$$\begin{aligned} L(x(t), u(t)) \quad = \quad & \frac{1}{2}(x(t) - x^*)^T Q(\sigma)(x(t) - x^*) + \\ & \frac{1}{2}(u(t) - u^*)^T R(\sigma)(u(t) - u^*) + \\ & p_{iec}(x(t), u(t)), \end{aligned} \tag{4.8}$$

where $P(\sigma)$, $Q(\sigma)$ and R(σ) are the quadratic weights to be tuned by applying the optimality conditions to the designed ILQ regulator. From now on, even if argument (σ) is not used explicitly, it is understood that quadratic weights depend on the main tuning parameter σ. x^* and u^* are the desired references for the state and input variables, respectively, and $p_{iec}(x(t), u(t))$ is the penalty function, which is introduced in the performance index to incorporate the inequality constraints on the states and inputs. The terminal penalty function $\varphi(x(t+T))$ is the infinite-horizon cost for the interval $t+T < \tau \leq \infty$ because of the fact that for the given quadratic weights Q and R, the matrix P is the positive-definite solution to the algebraic Ricatti equation given by Eq. (3.5), and because the following holds for an LQ regulator problem [72, 78],

$$\frac{1}{2}\int_0^\infty (x^T(\tau)Qx(\tau) + u^T(\tau)Ru(\tau))\,\mathrm{d}\tau = \frac{1}{2}x^T(0)Px(0). \tag{4.9}$$

Therefore, the infinite-horizon cost function is given by

$$\frac{1}{2}\int_{t+T}^\infty (x^T(\tau)Qx(\tau) + u^T(\tau)Ru(\tau))\,\mathrm{d}\tau = \frac{1}{2}x^T(t+T)Px(t+T). \tag{4.10}$$

The feedback gain matrix K is optimal if and only if there exist $P > 0$ and $R > 0$ satisfying the two conditions given by Eqs. (3.7) and (3.8) in Lemma 3.1 of Chapter 3. Since the feedback gain matrix K was parameterized using the necessary conditions for optimality and these parameters were determined using the sufficient conditions for optimality, if the feedback gain matrix

K is optimal, we can use Eqs. (3.7) and (3.8) to find matrices P, Q and R that confirm its optimality and satisfy the Ricatti equation given by Eq. (3.5). Thus, using inverse optimality conditions [75], we can tune the quadratic weight matrices P, Q and R as shown in Sections 4.1.1 – 4.1.3. Then, the response of NMPC is similar to that of the LQ regulator designed for the linearized model given by Eq. (4.1) by the ILQ regulator design method, at least in the neighborhood of the equilibrium point (x^*, u^*).

4.1.1 Determination of the Matrix *R*

The matrix R can be determined using the following expression:

$$R = V^T DV, \tag{4.11}$$

where D is a positive definite matrix satisfying

$$\Sigma D = D\Sigma. \tag{4.12}$$

4.1.2 Determination of the Matrix *P*

The matrix P can be determined using the following expression:

$$P = (VK)^T D\Sigma^{-1}(VK) + Y, \tag{4.13}$$

where Y is a positive semidefinite matrix satisfying

$$YB = 0. \tag{4.14}$$

Owing to the particular structure of matrix B, Y can also be written as

$$Y = \text{block-diag}(Y_1, 0),\ Y_1 \geq 0. \tag{4.15}$$

We need to tune matrices D and Y_1 in such a way that matrix $P > 0$ is diagonal. There may be many possible candidates for D and Y_1. The main role of these matrices is to make P diagonal and to adjust the magnitude of the quadratic weights P, Q and R. These matrices have no effect on the closed-loop response because the feedback gain matrix K is independent of these matrices as shown in Eq. (3.16).

4.1.3 Computation of the Matrix Q

After determining the matrix P, we can compute the matrix Q using the following expression:

$$Q = PH + H^T P. \tag{4.16}$$

For a copositive or diagonal dominant matrix H and a positive definite diagonal matrix P, the matrix Q given by Eq. (4.16) is guaranteed to be positive definite. That is, Eq. (3.7) holds.

Although the quadratic weights in the performance index of NMPC have been determined by applying the inverse optimality conditions on the feedback gain matrix K designed using the ILQ regulator design method which uses the linearized model, these weights are then tuned through numerical simulations for nonlinear model in NMPC which takes into account all the constraints on states and inputs. Therefore, the use of linearized model for computing the quadratic weights does not deteriorate the performance of NMPC.

4.1.4 Outline of the Tuning Procedure

The procedure for tuning the quadratic weights can be outlined as follows:

(i) First, linearize the nonlinear model at an equilibrium point.

(ii) Partition the linearized model as shown in Eqs. (3.14) and (3.14).

(iii) Choose $n-m$ stable poles distinct from the eigenvalues of A_{11}.

(iv) Determine the parameter F_1 using the algorithm given in [42]. Choose any arbitrary non-singular matrix V of order m and positive diagonal matrix Γ of order m. Determine the lower bound σ_{lb} of σ using the sufficient conditions for optimality [42]. Choose any σ greater than the lower bound.

(v) Compute the feedback gain matrix using Eq. (3.16).

(vi) Determine the quadratic weights R, P and Q using Eqs. (4.11), (4.13) and (4.16), respectively.

(vii) Use these quadratic weights in the performance index of NMPC given by Eqs. (2.2), (4.7) and (4.8). Penalty functions can be introduced in the performance index to incorporate the inequality constraints on the states and inputs.

(viii) Solve the NMPC problem for the nonlinear model given by Eq. (2.1) and check the plots of the closed-loop response and the control inputs.

(ix) If the response is too slow, increase the value of σ and go back to step (v) and continue.

(x) If the magnitudes of the control inputs are too high, decrease the value of σ and go back to step (v) and proceed.

(xi) If the response satisfies the desired criteria, terminate the algorithm.

Since the matrix $A-BK$ is a Hurwitx matrix, i.e., all the eigenvalues of the matrix $A-BK$ lie in the left half plane, using Lyapunov's indirect method [79], we can conclude that the reference state x^* in the closed-loop nonlinear system is locally asymptotically stable.

4.2 Numerical Example

4.2.1 Coupled Three-Tank System

Consider a coupled three-tank system (CTTS) consisting of one reservoir; three tanks, Tank-1, Tank-2 and Tank-3; three pumps, P1, P2 and P3; seven ON/OFF valves, V1, V2, V3, V4, V5, V6 and V7; seven level sensors and five flow meters [80, 81]. A sketch of the system is shown in Fig. 4.1. The drain valves of Tank-1 and Tank-2 are kept closed whereas all the other valves are kept open. For these simulations, we have kept pump P2 closed to demonstrate the case of $m<n$. The inputs to the system are the voltages applied to pumps P1 and P3, and outputs of the system are the levels of water in each tank.

4.2.1.1 Mathematical Model

Let h_1, h_2 and h_3 denote the level of water in Tank-1, Tank-2 and Tank-3 respectively. u_1 is the voltage applied to the pump P1 and u_2 is the voltage applied to the pump P2. To model the system, it is assumed that there are no thermal losses, thermal delays, pump delays in the system, and hydraulic and friction losses are negligible. Using Bernoulli's law and principle of conservation of fundamental quantities (mass and energy), the nonlinear model of CTTS can be described as follows:

$$a_{T1}\dot{h}_1 = -v_3a\sqrt{2g(H_{11}+h_1)} - v_4a\sqrt{2g(H_{12}+h_1)} + v_2k_1u_1, \tag{4.17}$$

$$a_{T2}\dot{h}_2 = -v_5a\sqrt{2g(H_{21}+h_2)} - v_6a\sqrt{2g(H_{22}+h_2)} + v_4a\sqrt{2g(H_{12}+h_1)} + k_3u_2, \tag{4.18}$$

$$a_{T3}\dot{h}_3 = -v_7a\sqrt{2g(H_{31}+h_3)} + v_6a\sqrt{2g(H_{22}+h_2)} - k_3u_2. \tag{4.19}$$

Table 4.1 shows the description and values of the CTTS parameters appearing in Eqs. (4.17) – (4.19).

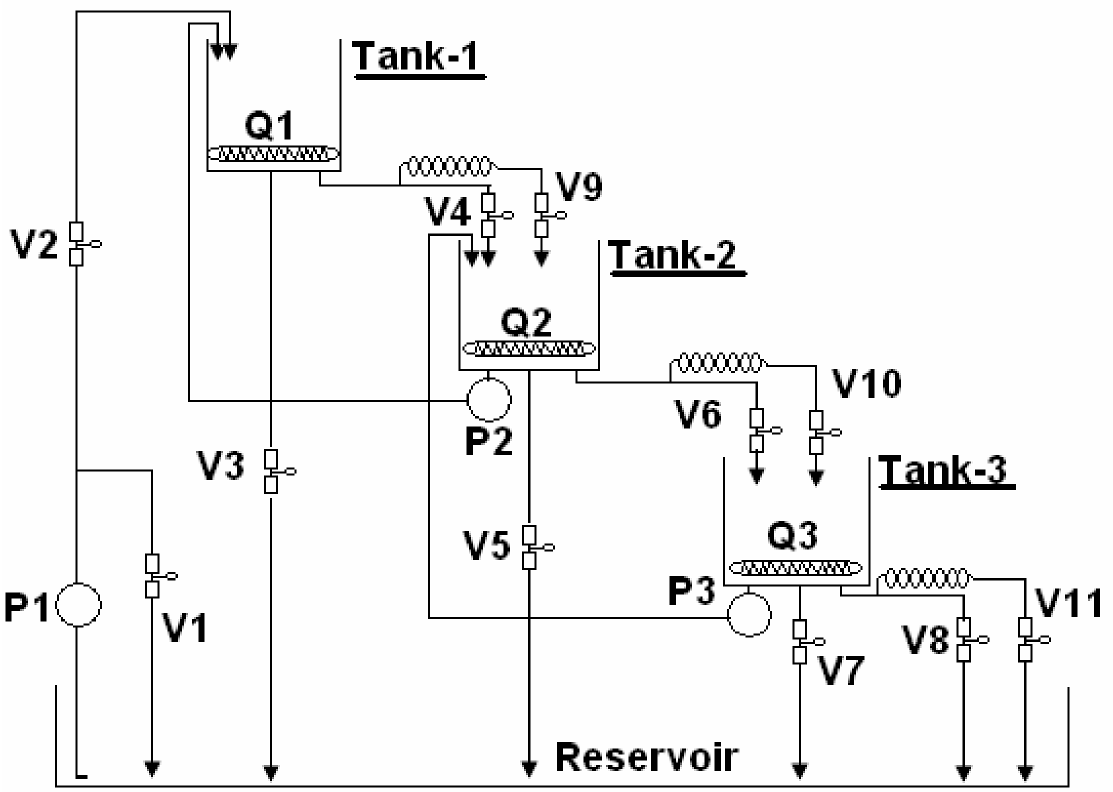

FIGURE 4.1: Sketch of CTTS [81].

Using values given in Table 4.1, simplified state-space representation of the CTTS is:

$$\dot{x}_1 = -0.0845\sqrt{30.48 + x_1} + 0.011u_1, \tag{4.20}$$

$$\dot{x}_2 = 0.0845(\sqrt{30.48 + x_1} - \sqrt{30.48 + x_2}) + 0.011u_2, \tag{4.21}$$

$$\dot{x}_3 = 0.0845(\sqrt{30.48 + x_2} - \sqrt{30.48 + x_3}) - 0.011u_2, \tag{4.22}$$

where x_1, x_2 and x_3 are the state variables representing the water levels in Tank-1, Tank-2 and Tank-3, and u_1 and u_2 are the input variables representing the voltages applied to pumps P1 and P3, respectively.

4.2.1.2 Constraints

The height of each tank is 25 cm; thus, the water level in each tank cannot exceed this value. Also, all the input and state variables must be positive. These are the constraints on the states and inputs and can be represented in the following mathematical form:

$$0 \leq x_i \leq 25, i = 1, 2, 3, \tag{4.23}$$

$$u_j \geq 0, j = 1, 2. \tag{4.24}$$

TABLE 4.1: Values of the CTTS Parameters

Parameters	Desciption	Value with Units
a_{T1}	cross-section area of Tank-1	126.6769cm^2
a_{T2}	cross-section area of Tank-2	126.6769cm^2
a_{T3}	cross-section area of Tank-3	126.6769cm^2
a	cross-section area of pipe	0.2419cm^2
g	acceleration due to gravity	$980\text{cm}/\text{s}^2$
v_2	value of valve V2	1
v_3	value of valve V3	0
v_4	value of valve V4	1
v_5	value of valve V5	0
v_6	value of valve V6	1
v_7	value of valve V7	1
k_1	gain constant of pump P1	1.3887
k_3	gain constant of pump P3	1.3887
H_{11}	height of pipe through valve V3	135cm
H_{12}	height of pipe through valve V4	30.48cm
H_{21}	height of pipe through valve V5	82.55cm
H_{22}	height of pipe through valve V6	30.48cm
H_{31}	height of pipe through valve V7	30.48cm

4.2.1.3 Control Objective

Let x^* and u^* be the desired references for the state and input variables. The control objective is to achieve the required reference levels and inputs such that all the state and input constraints, given by Eqs. (4.23) and (4.24), are satisfied.

4.2.2 Performance Index and its Tuning

The performance index in Eq. (2.2) is given by Eqs. (4.7) and (4.8). The tuning of the quadratic weights P, Q and R is carried out as explained in Section 4.1.

The penalty function $p_{iec}(x(t), u(t))$ is tuned using the barrier function method for the inequality constraints on the states and inputs and is given by

$$\begin{aligned} p_{iec}(x(t),u(t)) \quad = \quad & -p_{x_{1l}}\log_e(x_1(t)) - p_{x_{2l}}\log_e(x_2(t)) \\ & -p_{x_{3l}}\log_e(x_3(t)) - p_{x_{1u}}\log_e(25 - x_1(t)) \\ & -p_{x_{2u}}\log_e(25 - x_2(t)) \\ & -p_{x_{3u}}\log_e(25 - x_3(t)) \\ & -p_{u_1}\log_e u_1(t) - p_{u_2}\log_e u_2(t). \end{aligned} \tag{4.25}$$

For this optimal control problem, suitable values of $p_{x_{1l}}$, $p_{x_{2l}}$, $p_{x_{3l}}$, $p_{x_{1u}}$, $p_{x_{2u}}$, $p_{x_{3u}}$, p_{u_1} and p_{u_2} are found by trial and error to be 0.29, 0.29, 0.29, 0.29, 0.29, 0.29, 0.06 and 0.06, respectively.

Choosing an appropriate value of the horizon length T is also a very important task from a practical viewpoint. A large horizon length ensures the stability of the closed-loop system, whereas a short horizon length results in reduced computational time [82, 83]. Thus, the horizon length should be chosen to obtain a good compromise between the stability of the closed-loop system and the computational cost. For this example, the horizon length is fixed at fifty seconds, which is much smaller than the time constant of the linearized system, thus reducing the computational time. It was observed that if the horizon length is increased beyond this value, it increases the computational time without significantly improving the performance of NMPC.

4.2.3 Simulation Results

The NMPC problem is solved using a fast numerical algorithm called C/GMRES [9]. Simulations are performed using the automatic code generator AutoGenU [84], which generates an executable C-file. The purpose of these simulations is to show the effectiveness of the proposed tuning method and the effect of main tuning parameter σ and other tuning parameters on the closed-loop response and control inputs. Although the quadratic weights in the performance index of NMPC are determined using the ILQ method to design the LQ regulator for a linearized model of the CTTS that does not take into account any constraints on states and inputs, these quadratic weights give satisfactory results even for the nonlinear model given by Eqs. (4.20) - (4.22). The desired references for the state and input variables are as follows:

$$x^* = [\ 6.9541 \quad 20.7813 \quad 6.9536\ ]^T, \tag{4.26}$$

$$u^* = [\ 47 \quad 8\ ]^T. \tag{4.27}$$

For this control problem, the lower bound of σ is 0.005026, which is determined so that the sufficient conditions of optimality are satisfied for the ILQ feedback gain matrix K. For all the simulations, Γ is kept constant at a value chosen to make the lower bound of σ as low as possible. For this control problem, its value is

$$\Gamma = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}. \tag{4.28}$$

For simplicity, V is chosen to be an identity matrix of order 2.

4.2.3.1 Effect of Change in Main Tuning Parameter σ on Closed-loop Response and Control Inputs

First we study the effects of chang in the main tuning parameter σ on the speed of the closed-loop system's response and the magnitude of the control inputs. Figure 4.2 shows plots of the

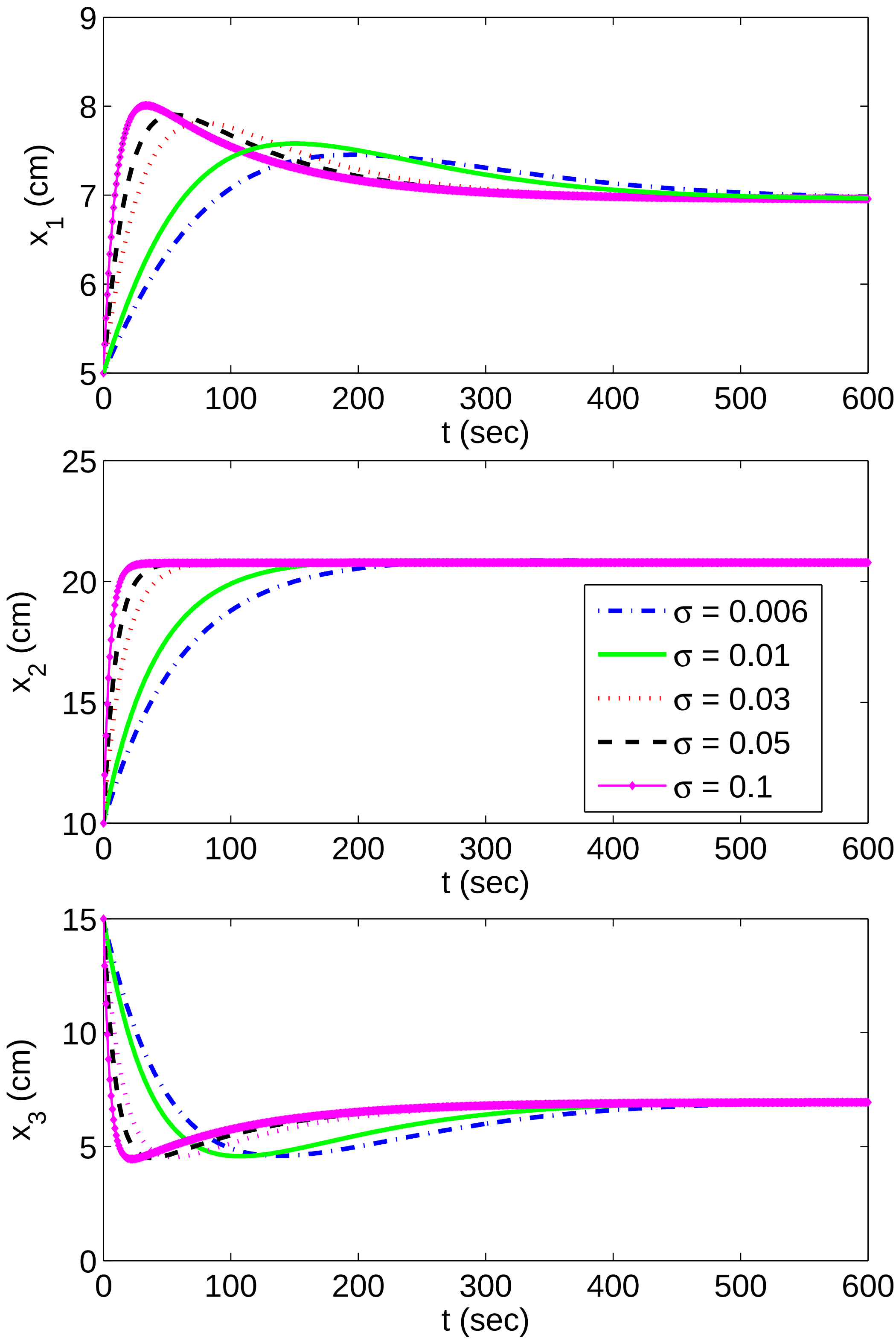

FIGURE 4.2: Effect of change in tuning parameter σ on closed-loop response

state variables, and Fig. 4.3 shows the plots of the control inputs for different values of σ.

The system's response becomes faster with increase in the value of σ as shown in Fig. 4.2.

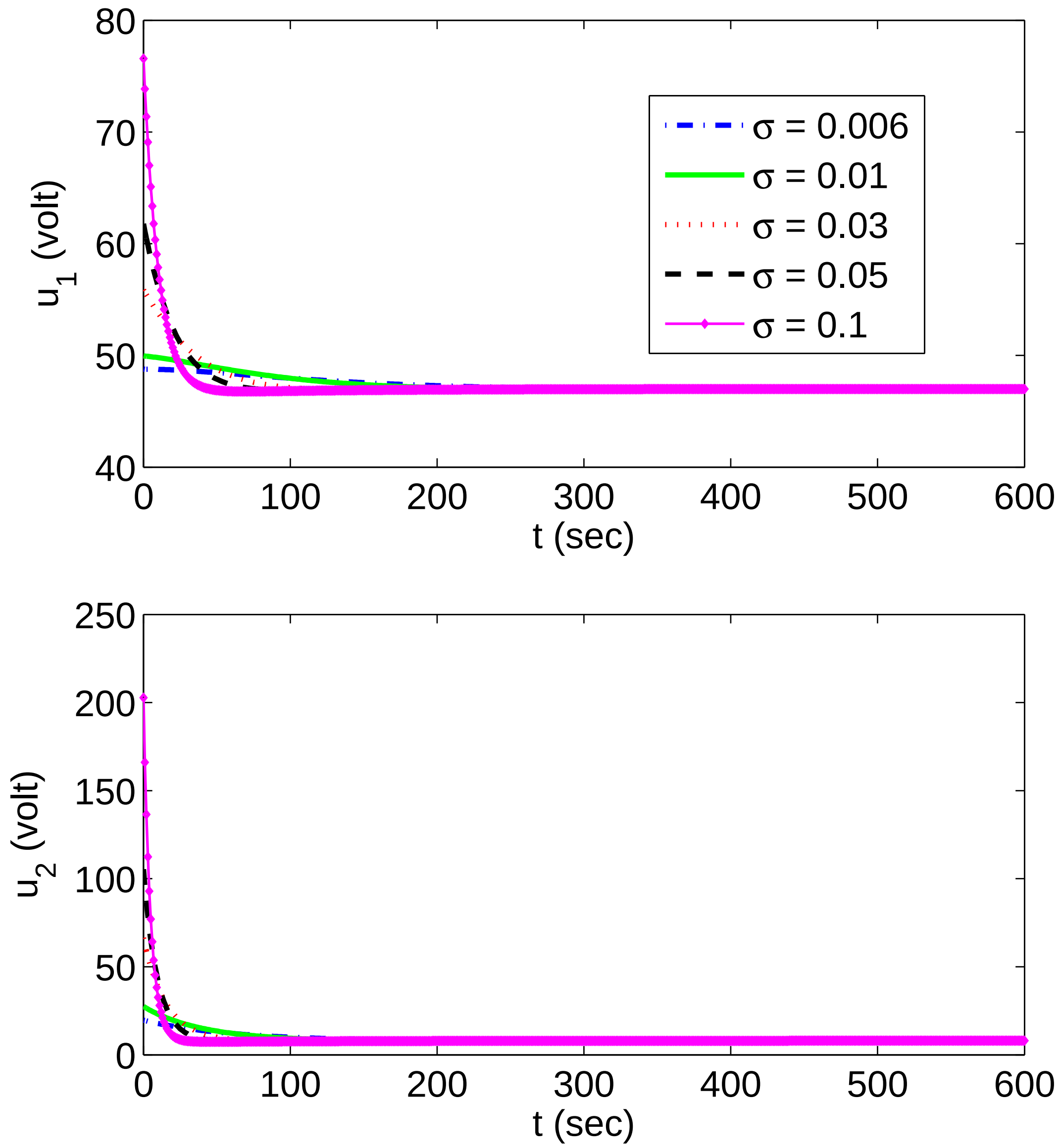

FIGURE 4.3: Effect of chane in tuning parameter σ on control inputs

However, at the same time, the magnitudes of the control inputs increase as shown in Fig. 4.3. Therefore, the tuning parameter σ gives a trade-off between the speed of the resulting closed-loop response and the magnitude of the control inputs. Although a fast response is desirable, too large input magnitudes are not desirable from a practical viewpoint. Therefore, σ should be chosen to achieve a good compromise between the speed of the system's response and the magnitudes of the control inputs.

4.2.3.2 Effect of Change in D

It is observed that, for a fixed value of σ and other parameters, a change in the matrix D does not have a significant effect on the closed-loop response. It can also be seen from Eq. (3.16) that the feedback gain matrix K does not depend on D. Only the quadratic weights are dependent on D as shown in Eqs. (4.11), (4.13) and (4.16). Thus, the main function of the matrix D is to give the quadratic weights the desired form and also to control their magnitudes.

4.2.3.3 Effect of Change in V on Closed-loop Response and Control Inputs

Simulations are also performed to study the effect of change in the parameter V on the closed-loop response and the control inputs. It is observed that this parameter affects the transient response as well as the magnitudes of the control inputs as shown in Fig. 4.4 and Fig. 4.5.

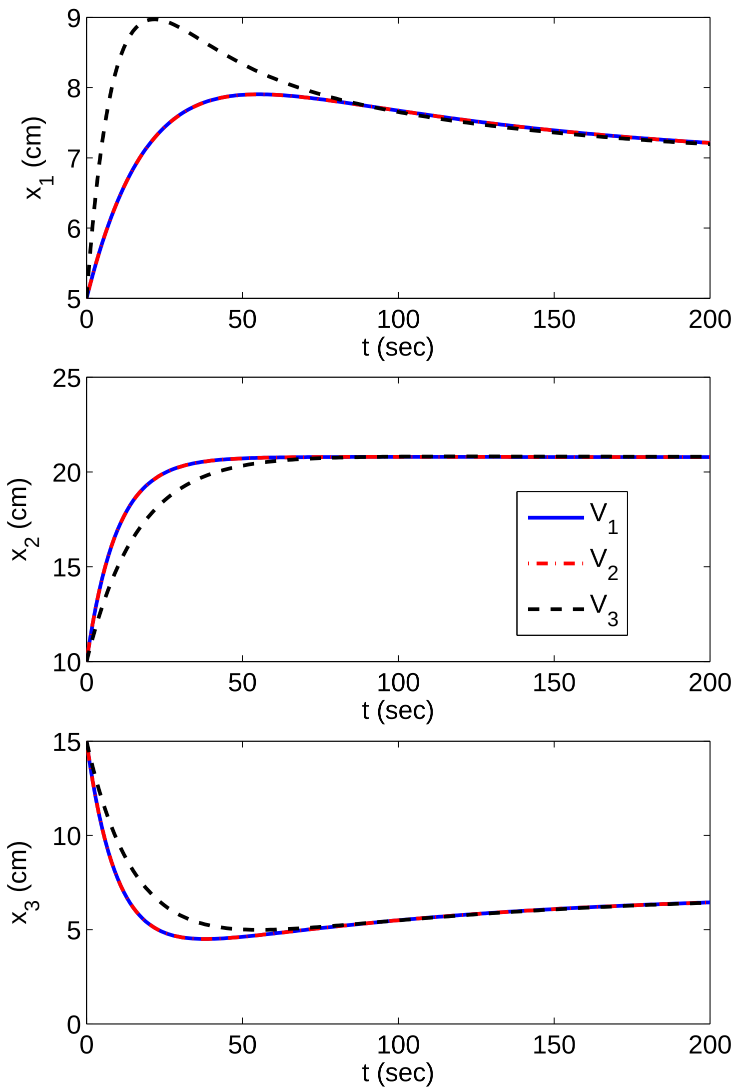

FIGURE 4.4: Effect of change in V on closed-loop response

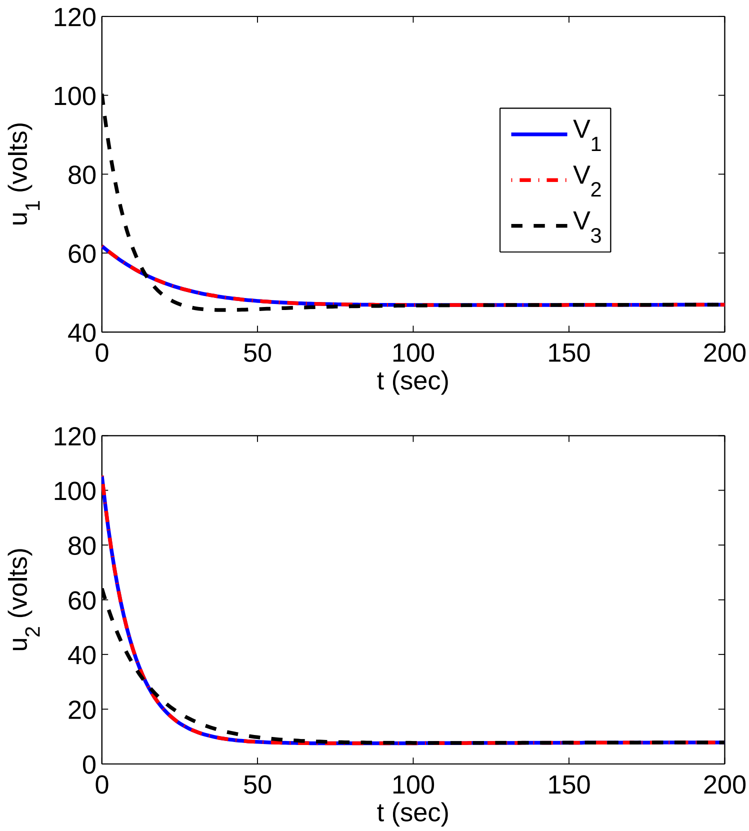

FIGURE 4.5: Effect of change in V on control inputs

Simulations are performed for different matrices V while keeping all other parameters constant. The tuning parameter σ is equal to 0.05 for these simulations. Here, plots are shown for $V = V_i, i = 1,2,3$, where

$$V_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{4.29}$$

$$V_2 = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}, \tag{4.30}$$

and

$$V_3 = \begin{bmatrix} 1 & -3\sqrt{3} \\ 3 & \sqrt{3} \end{bmatrix}. \tag{4.31}$$

As long as the matrix V is diagonal, irrespective of its diagonal entries, the closed-loop response is the same for different V. The same is observed for the plots of the control inputs. This is because a diagonal V commutes with a diagonal Σ, meaning that it does not affect the feedback gain matrix K because of the product term $V^{-1}\Sigma V$ in Eq. (3.16). However, if V is not diagonal, the effect becomes significant and visible and affects the transient response as shown in Fig. 4.4. The nondiagonal matrix V_3 also causes an increase in the magnitude of the control input u_1 and a decrease in the magnitude of the control input u_2 as shown in Fig. 4.5. This means that the off-diagonal entries in V affect the closed-loop response and the magnitude of the control inputs. To study the effect of each off-diagonal entry, simulations are performed by adding one off-diagonal element to $V_1 = I$, while keeping the other off-diagonal element equal to zero. For this control problem, V is a nonsingular 2×2 real matrix, which we denote as

$$V = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}. \tag{4.32}$$

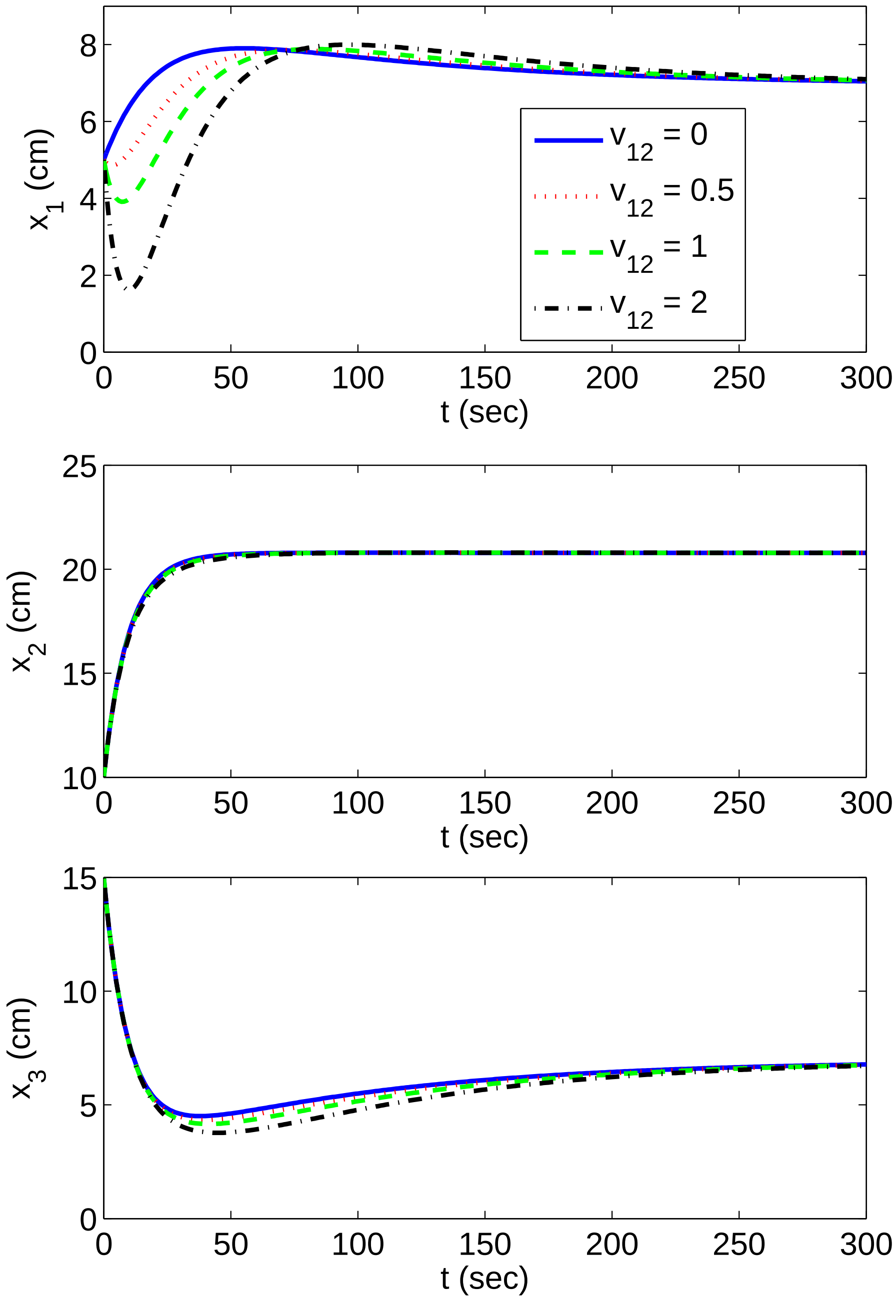

FIGURE 4.6: Effect of change in the off-diagonal entry v_{12} on closed-loop response

For these simulations, v_{11} and v_{22} are kept constant and equal to 1.

Case 1: Effect of Change in v_{12} while keeping $v_{21} = 0$

First of all, the effect of adding a nonzero v_{12} is observed by keeping v_{21} equal to 0. Figure 4.6 shows the effect of v_{12} on the closed-loop response. Increasing the value of v_{12} causes an increase in the undershoot in the plot of state x_1, whereas it does not have a significant effect on states x_2 and x_3. The effect of v_{12} on the control inputs is shown in Fig. 4.7. Increasing the value of v_{12} results in a decrease in the magnitude of u_1 at the start and then a larger overshoot before converging to the desired reference point. If the value of the off-diagonal element v_{12} is too large compared with the diagonal entries, it results in a violation of the constraint on input u_1. Thus, we should either avoid choosing too large values of v_{12} or use stricter criteria to penalize the constraint on input u_1. For this control problem, the off-diagonal entry v_{12} of matrix V can be

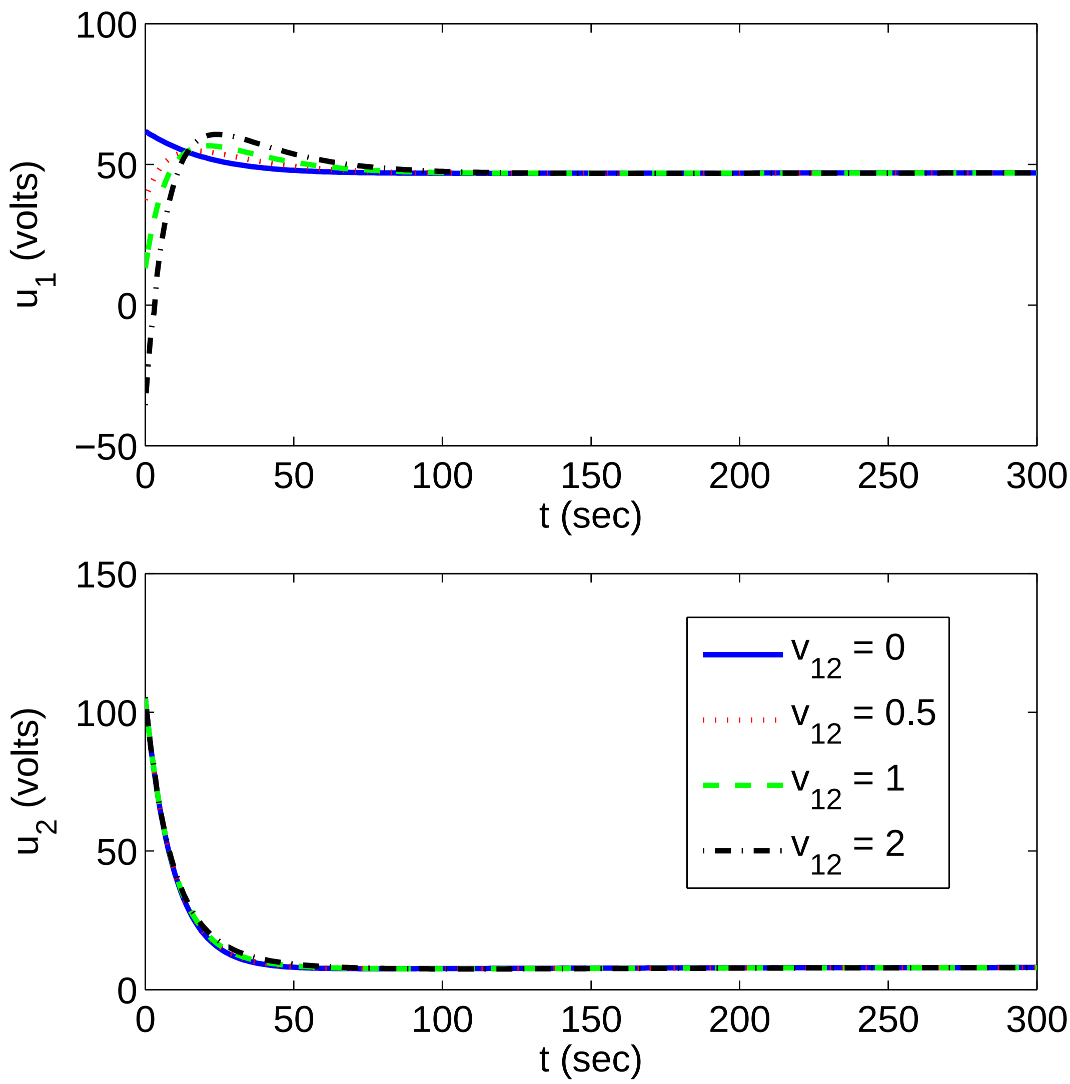

FIGURE 4.7: Effect of change in the off-diagonal entry v_{12} on control inputs

used to change the response of state x_1 and magnitude of control input u_1, as shown in Fig. 4.6 and Fig. 4.7, respectively.

Case 2: Effect of Change in v_{21} while keeping $v_{12} = 0$

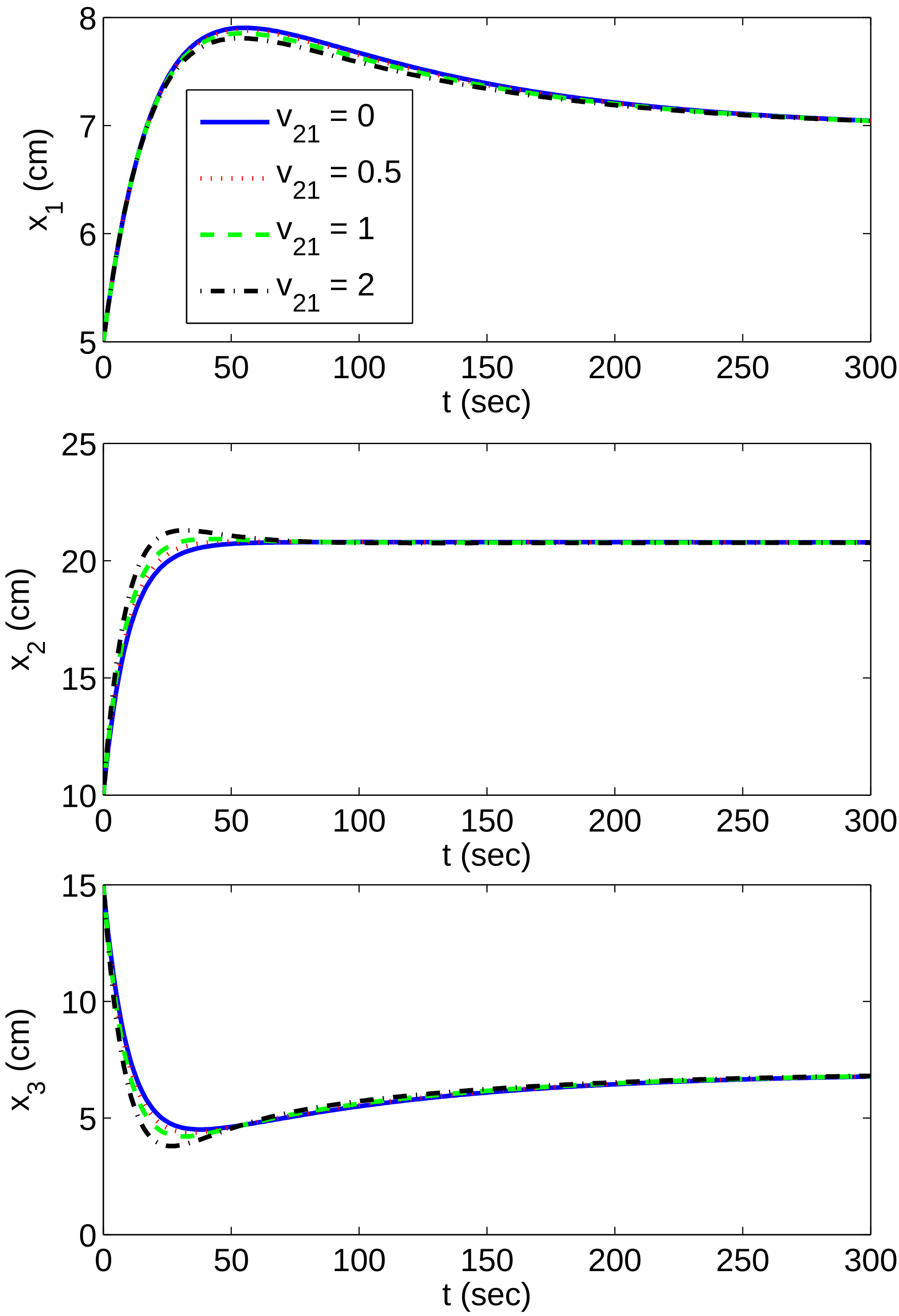

FIGURE 4.8: Effect of change in the off-diagonal entry v_{21} on closed-loop response

The next step is to study the effect of adding a nonzero v_{21} and keeping v_{12} equal to 0. Figure 4.8 shows the effect of v_{21} on the states, whereas Fig. 4.9 shows its effect on the control inputs. As is clear from Fig. 4.8, v_{21} does not have a significant effect on state x_1; however it does affect states x_2 and x_3. Increasing the value of v_{21} causes an increase in the height of the overshoot in the plot of state x_2 and a large undershoot in the plot of state x_3. The parameter v_{21} has no effect on control input u_1; however, it causes an increase in the magnitude of control input u_2 as shown in Fig. 4.9.

The simulation results shown in Fig. 4.6, Fig. 4.7, Fig. 4.8 and Fig. 4.9 demonstrate the effect of change in off-diagonal entries of matrix V on state variables and control inputs. The off-diagonal entries of matrix V can be used to change the response of some particular state and magnitude of some particular control input while not affecting the response of all other states and magnitude of all other control inputs.

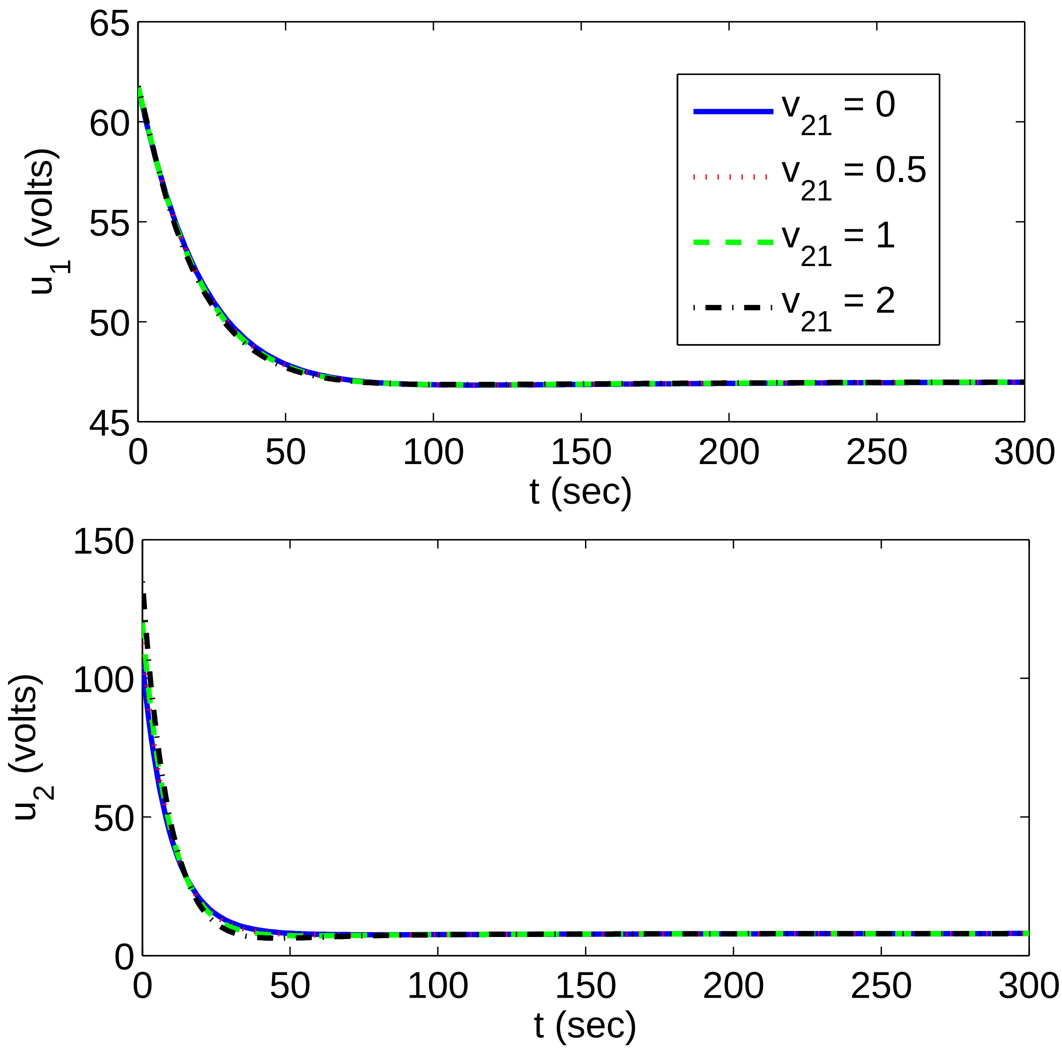

FIGURE 4.9: Effect of change in the off-diagonal entry v_{21} on control inputs

4.2.3.4 Effect of Change in Horizon Length on Closed-loop Response and Control Inputs

For the previous simulation results, horizon length T is kept equal to fifty seconds. Simulations are also performed to study the effect of change in horizon length on the closed-loop response and control inputs. For these simulations, σ is kept equal to 0.01. It is observed that for this system, there is no significant effect of change in horizon length on the state variables as shown in Fig. 4.10. However, change in horizon length does affect the magnitudes of control inputs. For small horizon lengths, there is no significant effect on control inputs. However, increase

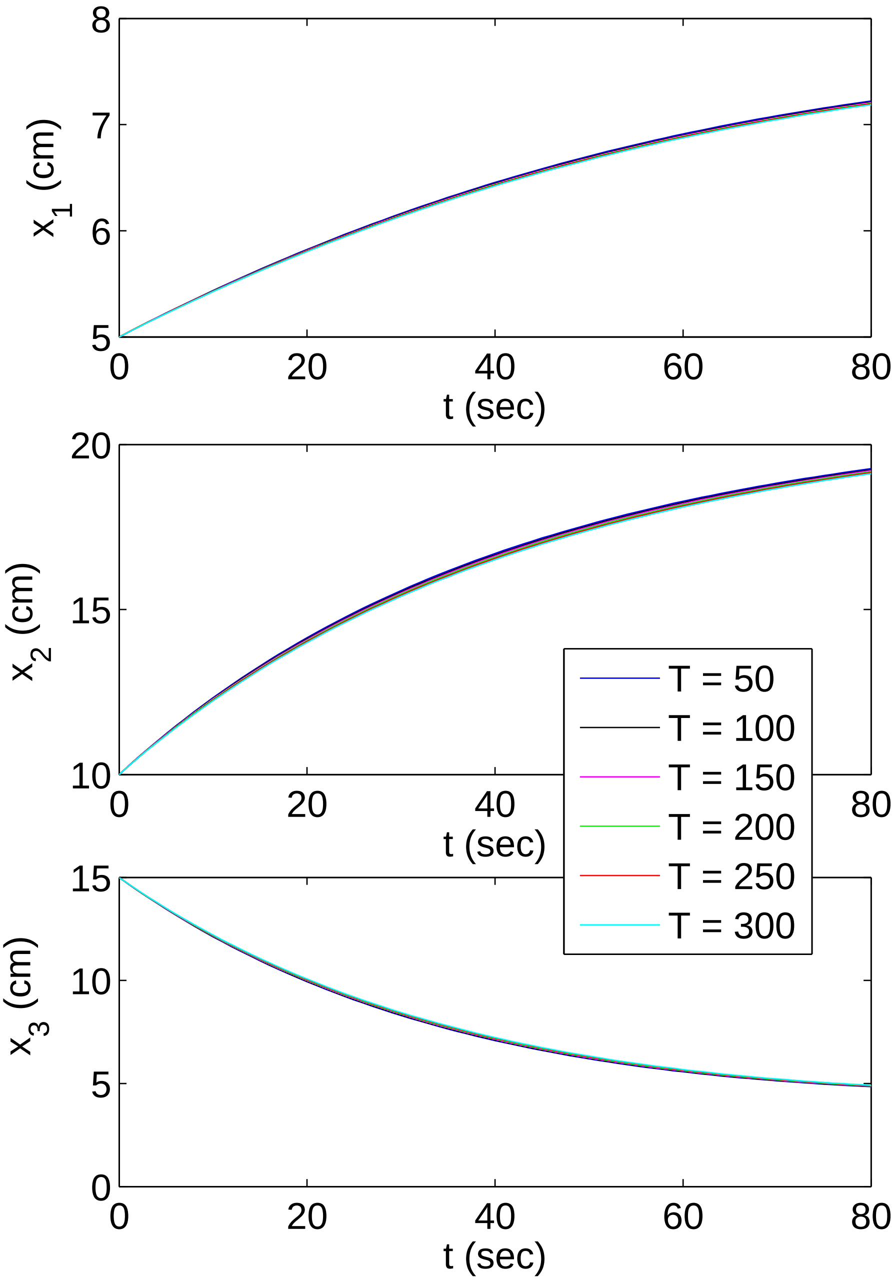

FIGURE 4.10: Effect of change in horizon length on closed-loop response

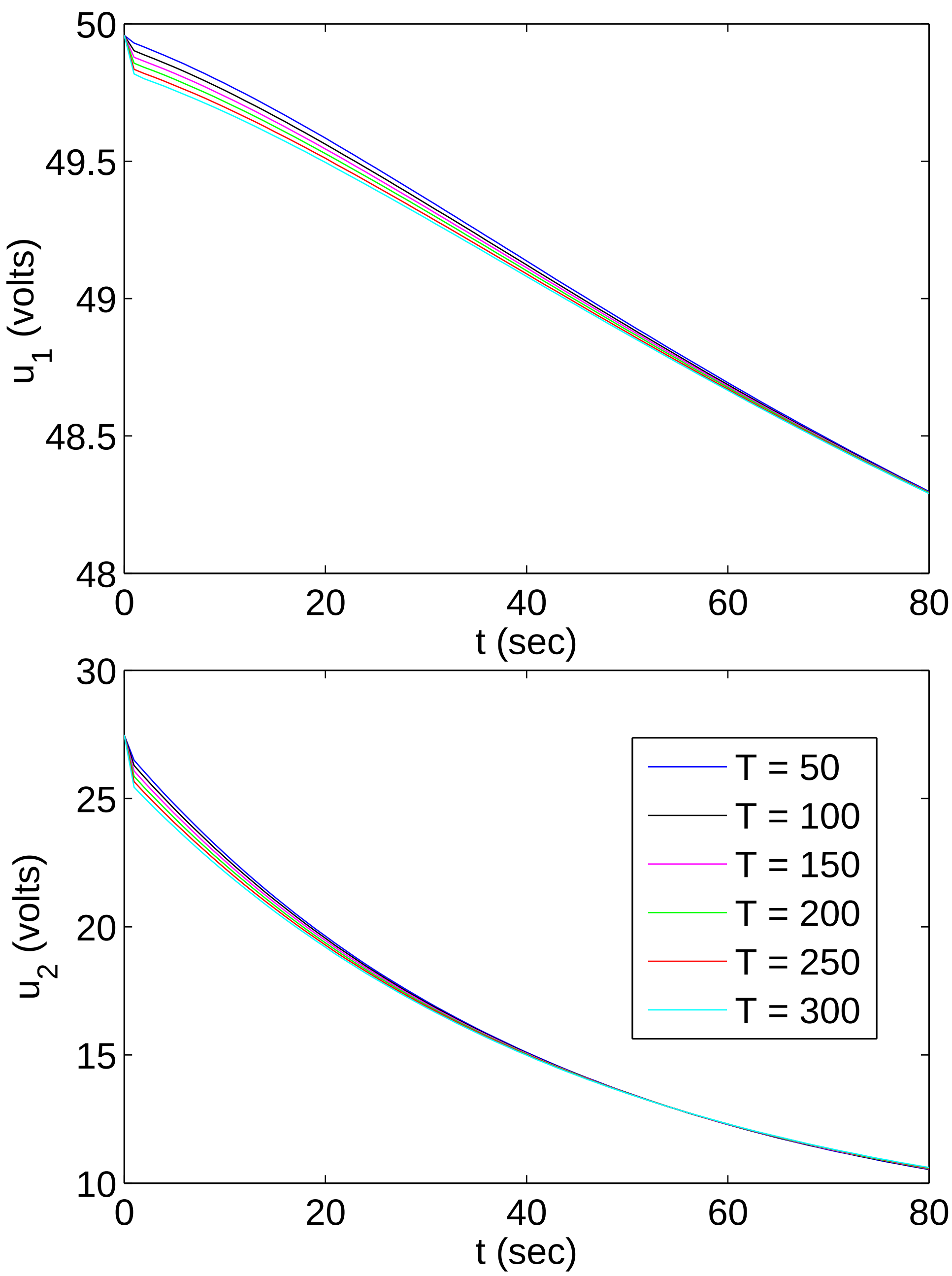

FIGURE 4.11: Effect of change in horizon length on control inpts

in the horizon length after some certain value has a little effect on the initial magnitudes of the control input. For the coupled three tank system, increase in horizon length after some certain value causes a decrease in the initial magnitudes of the control inputs as shown in Fig. 4.11. Since change in horizon length affects only the initial magnitudes of the control inputs and after that magnitudes are almost same for all horizon lengths, Fig. 4.10 and Fig. 4.11 show only the initial part of the trajectories of closed-loop response and control inputs, respectively.

4.2.3.5 Special Case of Desired Reference Level for State Lying at the Boundary of the Constraint

Now we consider a special case in which the state x_2 reaches the maximum allowable value. To demonstrate this case, different equilibrium point has been used. For these simulations, the

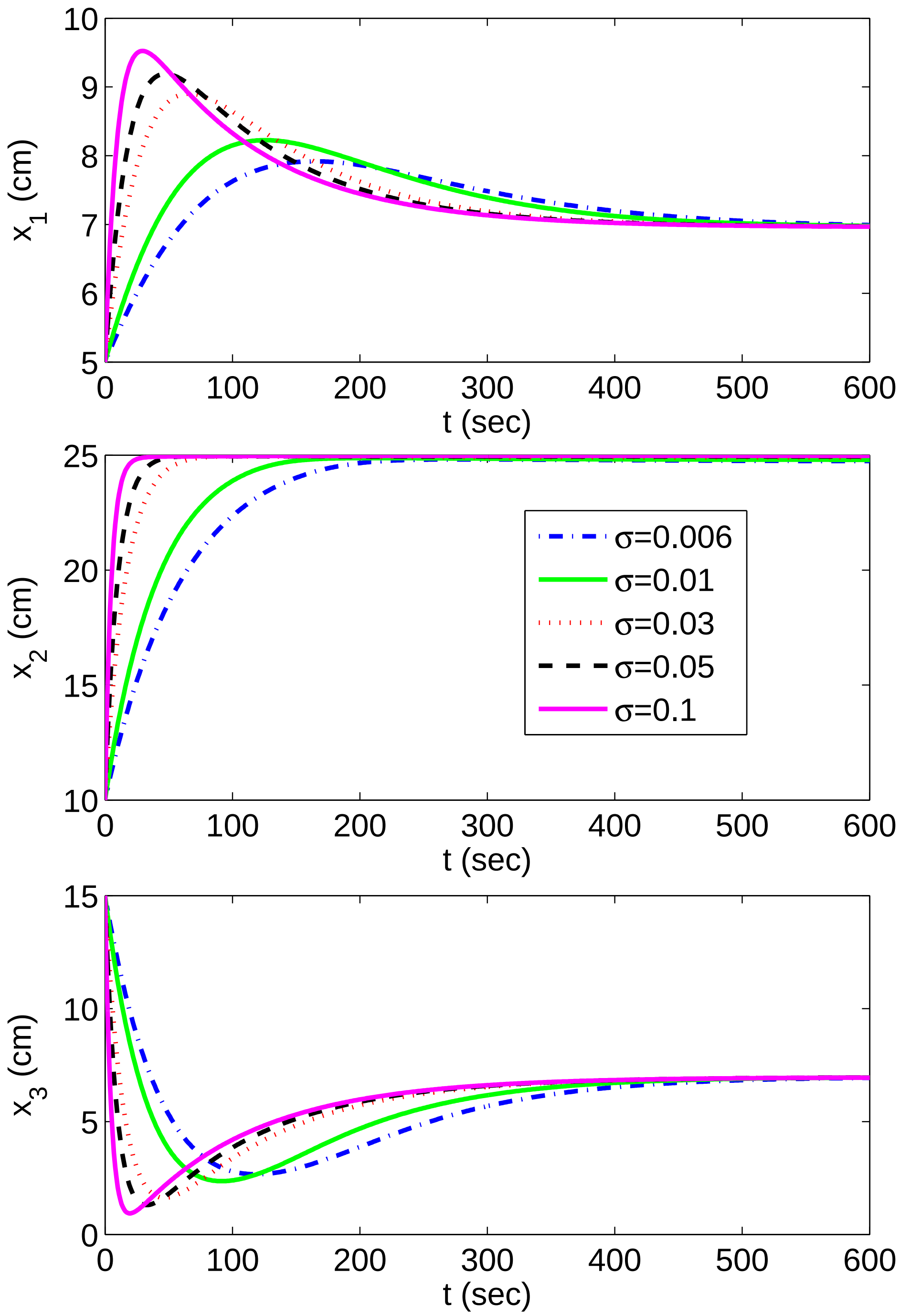

FIGURE 4.12: State x_2 achieving maximum allowable value

desired reference level for state and input variables are as follows:

$$x^* = [\ 6.9541 \quad 25 \quad 6.9541\ ]^T, \tag{4.33}$$

$$u^* = [\ 47 \quad 10.2179\ ]^T. \tag{4.34}$$

All other parameters are same as for the simulation results shown in Fig. 4.2 and Fig. 4.3. Figure 4.12 shows the response of the closed loop system and Fig. 4.13 shows the control inputs. It is clear from Fig. 4.12 that proposed tuning method is efficient to achieve the desired reference for a state even if the constraint for that state variable is active.

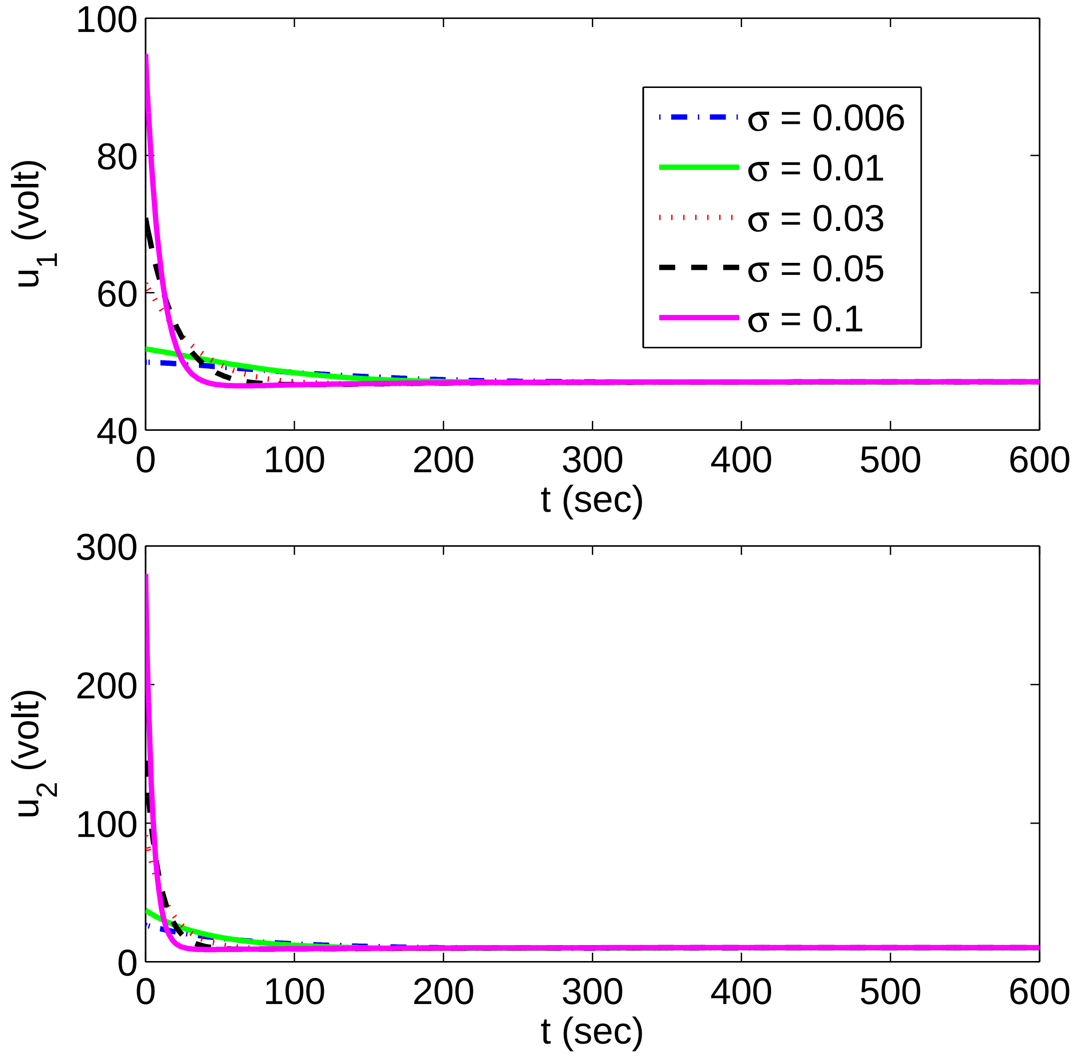

FIGURE 4.13: Control inputs for the case x_2 reaching maximum

The above simulation results show that we have two very important tuning parameters. One is the parameter σ, that gives a trade-off between the speed of the closed-loop response and the magnitude of the control inputs. The other is the matrix V, whose off-diagonal entries can be tuned appropriately to obtain the desired transient response and to adjust the magnitude of the control inputs.

4.3 Summary

In this chapter, we proposed the use of the ILQ regulator design method for tuning the quadratic weights in the performance index of NMPC. This tuning approach is very effective in the sense that the speed of the closed-loop system's response and the magnitude of the control inputs can be adjusted by changing a single tuning parameter σ. This method also provides some other free parameters. Some of these parameters affect the closed-loop response and others are used only to obtain appropriate values for the quadratic weights. Although the quadratic weights in the performance index of NMPC have been computed using the ILQ regulator design method which uses the linearized model, these weights are then used in NMPC algorithm which uses the nonlinear model and takes into account all the constraints on states and inputs. Therefore, the use of the linearized model for computing the quadratic weights does not cause any cost degradation in NMPC, and the error between the linearized model and nonlinear model does not deteriorate the performance of NMPC. Quadratic weights computed using linearized model work well for nonlinear model as shown in simulation results. This is because of the fact that these weights are tuned through numerical simulations for nonlinear model in NMPC. This approach is effective even if reference point of some state is exactly at the boundary of the constraint on that state.

Chapter 5

Extension of Tuning Method for Parameter-Dependent Systems and its Application to Speed Control of Spark Ignition Engine

5.1 Problem Formulation and Design Objective

The problem of systematic tuning of the performance index becomes more challenging if the dynamics of the process under consideration are not fixed; rather they are dependent on some parameters. In this chapter, we extend our tuning approach for NMPC of the parameter-dependent systems. Often, the dynamics of the systems are dependent on some parameters that are known but their values can vary. We describe how to select the main tuning parameter in our tuning approach in Chapter 4 to suppress the effect of parameters on the closed-loop response.

First of all, we redefine NMPC problem for parameter-dependent nonlinear systems. Consider a nonlinear system governed by the following nonlinear state equations:

$$\dot{x}(t) = f(x(t), u(t), p), \tag{5.1}$$

where $x(t)$ and $u(t)$ denote the n-dimensional state and m-dimensional input vectors, respectively, and p denotes the known parameter that is constant over the horizon. However, it is not fixed; i.e., it may change from one constant value to another constant value. The performance index to be minimized is given by

$$J = \varphi(x(t+T), p) + \int_{t}^{t+T} L(x(\tau), u(\tau), p)\, \mathrm{d}\tau, \tag{5.2}$$

where T is the horizon length and time from current time t to time t+T is termed as prediction horizon. The terminal penalty function $\varphi(x(t+T),p)$ is given by

$$\varphi(x(t+T),p) = \frac{1}{2}(x(t+T)-x^*(p))^T P(\sigma)(x(t+T)-x^*(p)), \tag{5.3}$$

and the integral part of the performance index is given by,

$$\begin{aligned} L(x(t),u(t),p) \quad = \quad & \frac{1}{2}(x(t)-x^*(p))^T Q(\sigma)(x(t)-x^*(p)) + \\ & \frac{1}{2}(u(t)-u^*(p))^T R(\sigma)(u(t)-u^*(p)) \\ & + p_{iec}(x(t),u(t),p). \end{aligned} \tag{5.4}$$

$x^*(p)$ and $u^*(p)$ are the desired references for the state and input variables, respectively. Equality constraints are given by Eq. (2.3) and inequality constraints are handled as explained in Section 2.2.

5.2 Computation of Parameter-Dependent Main Tuning Parameter

In the absence of the parameter p, any $\sigma > \sigma_{lb}$ can be picked up to tune the quadratic weights in NMPC to achieve a reasonable trade-off between speed of the closed-loop system's response and magnitude of the control input. Since nonlinear dynamics given by Eq. (5.1) depend on parameter p, system matrices of the linearized model are also dependent on p. This means for a fixed tuning parameter $\sigma > \sigma_{lb}$, if the value of the parameter p changes, closed-loop response will also change, which is not desirable from a practical viewpoint. In order to suppress the effect of change in p on closed-loop response, we need to choose σ as a function of p such that closed-loop response is almost insensitive to variation in p.

To obtain the linear model, the nonlinear model given by Eq. (5.1) is linearized at an equilibrium point $(x^*(p),u^*(p))$. Then, the linearized model can be expressed as follows:

$$\dot{\delta}_x(t) = A(p)\delta_x(t,p) + B(p)\delta_u(t,p), \tag{5.5}$$

where $A(p)$ and/or $B(p)$ depend on the parameter p and,

$$f(x^*(p),u^*(p),p) = 0, \tag{5.6}$$

$$\delta_x(t,p) = x(t) - x^*(p), \tag{5.7}$$

$$\delta_u(t,p) = u(t) - u^*(p), \tag{5.8}$$

$$A(p) = \frac{\partial f}{\partial x}(x^*(p),u^*(p),p), \tag{5.9}$$

$$B(p) = \frac{\partial f}{\partial u}(x^*(p), u^*(p), p). \tag{5.10}$$

Since nonlinear dynamics given by Eq. (5.1) depend on the parameter p, it is obvious that linearized model is also dependent on it. For the sake of simplicity, we often omit explicit mention of it.

Let $F(\sigma, p)$ denote the closed-loop system matrix given by

$$F(\sigma, p) := A(p) - B(p)K(\sigma), \tag{5.11}$$

where K is the feedback gain matrix given by Eq. (3.16). F can also be expressed as

$$F(\sigma, p) = A(p) - \sigma B(p)\hat{K}M(p), \tag{5.12}$$

where

$$\hat{K} := V^{-1}\Gamma V[F_1 \quad I_m], \tag{5.13}$$

and $M(p)$ is a similarity transformation such that

$$M(p)B(p) = \begin{bmatrix} 0 \\ I_m \end{bmatrix}. \tag{5.14}$$

Let A_0 and B_0 denote the system matrices for the case when $p = p_0$ and F_0 denote the closed-loop system matrix for some $\sigma = \sigma_0 > \sigma_{lb}$ and $p = p_0$. Then, F_0 is given by

$$F_0 := F(\sigma_0, p_0) = A_0 - \sigma_0 B_0 \hat{K} M_0, \tag{5.15}$$

where M_0 is a similarity transformation such that

$$M_0 B_0 = \begin{bmatrix} 0 \\ I_m \end{bmatrix}. \tag{5.16}$$

Next we propose two methods to compute the value of σ so that the effect of change in parameter p on closed-loop response is negligible [44].

5.2.1 Method 1

Choose $\sigma = \sigma^*(p)$ in such a way that trace of the closed-loop system matrix for the linearized system remains constant, i.e.

$$\mathrm{tr}(F(\sigma^*(p), p)) = \mathrm{tr}(F_0), \tag{5.17}$$

where tr denotes trace. Using properties of trace of a matrix [85] and Eq. (5.12),

$$\mathrm{tr}(F(\sigma^*(p),p)) = \mathrm{tr}(A(p)) - \sigma^*(p)\mathrm{tr}(B(p)\hat{K}M(p)). \tag{5.18}$$

Using Eqs. (5.17) and (5.18), we get

$$\sigma^*(p) = \frac{\mathrm{tr}(A(p)) - \mathrm{tr}(F_0)}{\mathrm{tr}(B(p)\hat{K}M(p))}, \tag{5.19}$$

As trace of a square matrix is defined to be the sum of its diagonal entries and it is equal to the eigenvalues of that matrix, keeping the sum of eigenvalues of F constant may preserve some properties of closed-loop response depending on which parts of system matrices are affected by p. If the parameter p does not appear in the diagonal entries of F, this means trace of F will remain constant irrespective of the change in the value of p. In that case, Eq. (5.19) will give the same value of $\sigma^*(p)$ for all values of p.

5.2.2 Method 2

Now we propose a more strict criterion for the selection of σ in order to minimize the effect of change in p on closed-loop response.

Based on the value of p, choose $\sigma = \sigma^*(p)$ such that Frobenius norm of $F - F_0$ is minimized, i.e.,

$$\sigma^*(p) = \mathrm{argmin}_{\sigma} \|F(\sigma,p) - F_0\|_F. \tag{5.20}$$

Let

$$A_1(p) := A(p) - F_0, \tag{5.21}$$

and

$$B_1(p) := B(p)\hat{K}M(p). \tag{5.22}$$

Then,

$$\begin{aligned} \|F(\sigma,p) - F_0\|_F^2 &= \|A_1(p) - \sigma B_1(p)\|_F^2 \\ &= \mathrm{tr}\left((A_1 - \sigma B_1)^T (A_1 - \sigma B_1)\right). \end{aligned} \tag{5.23}$$

Using the fundamental properties of trace of matrices, Eq. (5.23) can be written as follows:

$$\begin{aligned} \|A_1 - \sigma B_1\|_F^2 &= \text{tr}({A_1}^T A_1) - 2\sigma \text{tr}({A_1}^T B_1) \\ &+ \sigma^2 \text{tr}({B_1}^T B_1). \end{aligned} \tag{5.24}$$

Since Eq. (5.24) is quadratic in σ and the coefficient of σ^2 is always positive, its stationary point will be a global minimum. In order to find the value of σ which minimizes $\|F(\sigma, p) - F_0\|_F^2$, we need to solve the following equation:

$$\frac{d(\|F(\sigma, p) - F_0\|_F^2)}{d\sigma} = 0. \tag{5.25}$$

This implies that

$$-2\text{tr}({A_1}^T B_1) + 2\sigma \text{tr}({B_1}^T B_1) = 0. \tag{5.26}$$

Solving this equation for σ, we get;

$$\sigma^*(p) = \frac{\text{tr}(A_1(p)^T B_1(p))}{\text{tr}(B_1(p)^T B_1(p))}, \tag{5.27}$$

where A_1 and B_1 are given by Eqs. (5.21) and (5.22), respectively. The effectiveness of choosing $\sigma^*(p)$ using Eq. (5.27) is shown in simulation results.

5.3 Outline of Tuning Algorithm for Parameter-Dependent Systems

In this section, we briefly describe the procedure of tuning the quadratic weights for NMPC of parameter-dependent systems.

(i) First, linearize the nonlinear model at an equilibrium point as described in Eqs. (5.5) – (5.10).

(ii) Partition the linearized model as shown in Eqs. (3.14) and (3.14).

(iii) Choose $n - m$ stable poles distinct from the eigenvalues of A_{11}.

(iv) Determine the parameter F_1 using the algorithm given in Ref. [42]. Choose any arbitrary nonsingular matrix V of order m and positive diagonal matrix Γ of order m. Determine the lower bound σ_{lb} of σ using the sufficient conditions for optimality [42].

(v) Choose any σ_0 greater than the lower bound σ_{lb}.

(vi) Compute F_0 using Eq. (5.15).

(vii) Compute $\sigma^*(p)$ using Eq. (5.19) or Eq. (5.27).

(viii) Compute the feedback gain matrix using Eq. (3.16).

(ix) Determine the quadratic weights R, P and Q using Eqs. (4.11), (4.13) and (4.16), respectively.

(x) Use these quadratic weights in the performance index of NMPC given by Eqs. (5.2).

(xi) Solve the NMPC problem for the nonlinear model given by Eq. (2.1) and check the plots of the closed-loop response and the control inputs.

(xii) Terminate the algorithm.

5.4 Speed Control of Spark Ignition Engine

In this section, we apply our tuning approach for NMPC of parameter-dependent systems to the speed control of SI engines, which is a very important problem from a practical viewpoint. SI engine is a complicated nonlinear dynamical system and a lot of work has been done for its speed control problem. However, it is still an active area of research due to increasingly stringent regulatory and customer demands. A better speed control strategy can improve fuel economy and comfort. Researchers have investigated engine speed control problem using classical linear control techniques as well as advanced nonlinear control techniques [86, 87]. These techniques include PID control [88], fuzzy logic control [89], adaptive fuzzy logic control, H_∞ control [90, 91] and sliding mode control [92].

5.4.1 Mean-value Model of Spark Ignition Engine

SI engine is a complicated nonlinear dynamical system and its modeling is a difficult task due to its time varying dynamic characteristics. A good model is based on the physical phenomenon and make use of fundamental laws of thermodynamics, fluid mechanics and rotational dynamics. In the last three decades, the mean-value model (MVM) of SI engine has attained significance importance in automotive industry and a lot of research has been done to improve the mean-value modeling [93–95]. An MVM is a continuous control oriented model, i.e., the input-output behavior of the system is modeled with reasonable precision but low computational complexity, and the discrete cycles of the engine are neglected [94].

The sketch of a typical engine system is shown in Fig. 5.1. The fresh air enters the intake manifold through the throttle. Let $\dot{m}_{th}$(g/s) denote the air mass flow rate entering the input manifold through the throttle. Let $\dot{m}_{cyl}$(g/s) denote the air mass flow rate entering the cylinder.

The air mass flow rate $\dot{m}_{th}$ passing through the throttle depends on the throttle valve opening ϕ(deg) and input manifold pressure p_m(bar). The air in the intake manifold is charged into the cylinder during the intake stroke and the fuel is injected into the cylinder directly in the case of direct injection engines. After that, the air-fuel mixture is compressed in the compression stroke and ignited for combustion. The torque is generated during the combustion phase, mainly in the expansion stroke, which acts on the crankshaft rotational motion. The burnt gas is exhausted into the exhaust manifold.

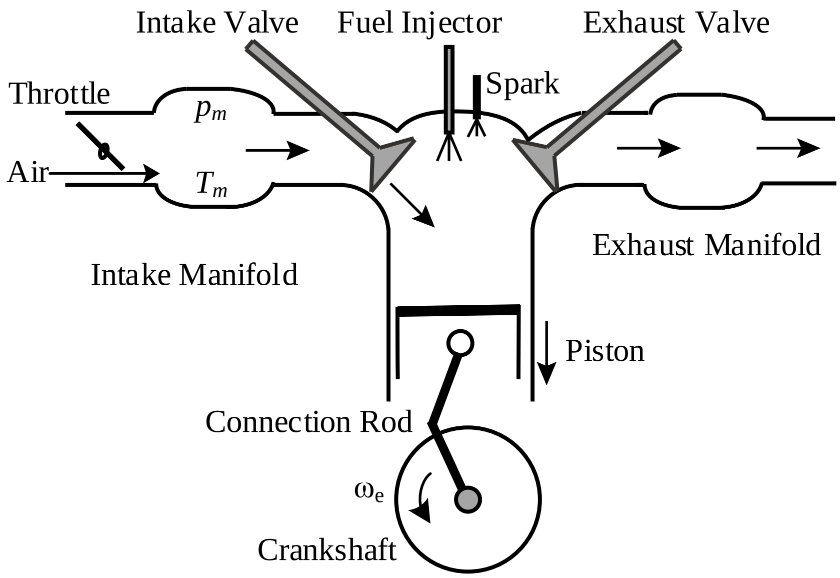

FIGURE 5.1: The sketch of a spark ignition engine system

Engine model for speed control usually includes dynamics of crankshaft rotation and air intake.

5.4.1.1 Crankshaft Rotational Dynamics

Let ω_e(rad/s) and τ_l(Nm) denote the engine speed and load torque, respectively. Using Newton's second law of rotational motion, the dynamics of crankshaft rotational motion can be described as follows [94, 95]

$$J_e\dot{\omega}_e = a_1 p_m - a_2 - d_r\omega_e - \tau_l, \tag{5.28}$$

where where J_e(kg.m^2) is the crank inertia moment, a_1 and a_2 are the constants that are dependent on physical parameters of engine, and d_r is the rotational friction constant.

5.4.1.2 Air Intake Dynamics

The intake manifold can be considered as a constant volume container. Using law of conservation of mass and energy, and ideal gas law, air intake dynamics can be described as follows [94]:

$$\dot{p}_m = \frac{RT_m}{V_m}\left(\dot{m}_{th} - \dot{m}_{cyl}\right), \tag{5.29}$$

where $R(\mathrm{J/(Kg.K)})$ is the universal gas constant, and $T_m(\mathrm{K})$ and $V_m(\mathrm{m}^3)$ denote the input manifold temperature and volume, respectively. Since the air mass flow rate passing through the throttle can be measured, it can be considered as a control input. Let us denote it by u_t, i.e.,

$$u_t = \dot{m}_{th}. \tag{5.30}$$

For a four-stroke engine, the air mass flow rate leaving the manifold is represented as follows [94]

$$\dot{m}_{cyl} = \frac{\rho_a \eta V_d}{4\pi p_a} p_m \omega_e, \tag{5.31}$$

where $\rho_a(\mathrm{kg/m}^3)$, η, $V_d(\mathrm{m}^3)$ and $p_a(bar)$ denote the density of the air at the engine's intake, volumetric efficiency of the engine,engine displaced volume and atmospheric pressure, respectively. Substituting Eqs. (5.30) and (5.31) into Eq. (5.29), we get;

$$\dot{p}_m = a_3 u_t - a_4 p_m \omega_e, \tag{5.32}$$

where

$$a_3 = \frac{RT_m}{V_m}, \tag{5.33}$$

and

$$a_4 = \frac{RT_m \rho_a \eta V_d}{4\pi p_a V_m}. \tag{5.34}$$

Equations (5.28) and (5.32) represent a simple mean-value model of an SI engine describing its key dynamical behavior. For more details on the engine model, [94] can be seen. To perform simulations, data is obtained for a 3.5L SI engine with six cylinders [96]. Maximum value of the manifold pressure p_m can be equal to the atmospheric pressure, i.e., one bar. Minimum value of the manifold pressure p_m is equal to 0.2 bar. Table 5.1 shows the values of the engine parameters for engine speed from 1000 rpm to 2000 rpm. For this speed range, minimum value of u_t is 9.64 grams per second and maximum value of u_t is 96.7 grams per second.

Let

$$T_e = a_1 p_m - a_2 - \tau_l, \tag{5.35}$$

TABLE 5.1: Values of the Engine Parameters

Parameters	Values
J_e	0.5923
d_r	0.1219
a_1	280.8404
a_2	43.2809
a_3	0.1352
a_4	0.2064

where $T_e(\text{Nm})$ is the engine torque. The load torque serves as parameter p in the mean-value model of SI Engine, i.e.,

$$p = \tau_l. \tag{5.36}$$

We assume that load torque τ_l is zero or attains some constant value which can be changed. Therefore, we have

$$\dot{T}_e = a_1 \dot{p}_m. \tag{5.37}$$

Using Eqs. (5.35) and (5.37), mean-value model can be rewritten as follows:

$$\dot{\omega}_e = \frac{T_e}{J_e} - \frac{d_r}{J_e}\omega_e, \tag{5.38}$$

$$\dot{T}_e = a_1 a_3 u_t - a_4 \omega_e T_e - a_4 \omega_e \tau_l - a_2 a_4 \omega_e. \tag{5.39}$$

State-space representation of the above mean-value model is:

$$\dot{x}_1 = -\frac{d_r}{J_e}x_1 + \frac{1}{J_e}x_2, \tag{5.40}$$

$$\dot{x}_2 = -a_4(a_2 + \tau_l)x_1 - a_4 x_1 x_2 + a_1 a_3 u_t, \tag{5.41}$$

where x_1 and x_2 are the state variables representing the engine speed in radians per second and engine torque in Newtons meters, respectively. u_t is the input variable representing the air mass flow rate through throttle.

5.4.2 Constraints and Control Objective

For this control problem, constraints on the state and input variables can be represented in the mathematical form as follows:

$$100\pi/3 \leq x_1(\text{rad/s}) \leq 200\pi/3, \tag{5.42}$$

$$12.8871 \leq x_2(\text{Nm}) \leq 237.5591, \tag{5.43}$$

$$9.64 \leq u_t(\text{g/s}) \leq 96.7. \tag{5.44}$$

The desired references for the state and input variables are as follows:

$$x^* = [\ 157.08 \quad 19.148\ ]^T, \tag{5.45}$$
$$u^* = 53.3. \tag{5.46}$$

The control objective is to achieve the desired engine speed ω_d such that all the state and input constraints, given by Eqs. (5.42) – (5.44) are satisfied.

5.4.3 Cost Function and its Tuning

To tune the quadratic weights in the performance index of NMPC for the speed control of SI Engine, its state-space model given by Eqs. (5.40) and (5.41) is linearized at the desired reference levels for state and input variables. Then LQ regulator is designed using the ILQ regulator design approach. After that, quadratic weights are determined as explained in Chapter 4. Finally these quadratic weights are used in the performance index of NMPC, and NMPC algorithm is applied to the nonlinear state-space model of SI engine. The system matrices of the linearized model are given as follows:

$$A = \begin{bmatrix} -\frac{d_r}{J_e} & \frac{1}{J_e} \\ -a_4(a_2+\tau_l+x_2^*) & -a_4x_1^* \end{bmatrix}, \tag{5.47}$$
$$B = \begin{bmatrix} 0 \\ a_1a_3 \end{bmatrix}, \tag{5.48}$$

where

$$x_1^* = \omega_d, \tag{5.49}$$

and x_2^* is the reference value for engine torque which can be described as

$$x_2^* = d_r\omega_d. \tag{5.50}$$

In the absence of load torque disturbance, state-space model given by Eqs. (5.40) and (5.41) is fixed. Therefore, performance index can be tuned by following the algorithm given in Section 4.1. If load torque is also present, and its value changes from one constant level to another constant level, reference level for control input will increase and closed-loop response will also be affected. In order to suppress the effect of change in parameter $p = \tau_l$, we need to tune the main tuning parameter σ using Eq. (5.19) or Eq. (5.27). Since B does not depend on the parameter τ_l, therefore M-transformation is constant and does not change with change in τ_l. Since trace of a square matrix is equal to sum of elements in its main diagonal, trace of matrix A is fixed for all values of τ_l as can be seen in Eq. (5.47). This means $\sigma^*(p)$ computed using Eq. (5.19) will be the same for all values of τ_l. Simulations results show the comparison of two methods for computing $\sigma^*(p)$.

5.4.4 Simulation Results

The NMPC problem is solved using C/GMRES algorithm [9]. Simulations are performed using the automatic code generator AutoGenU [84], which generates an executable C-file. The quadratic weights in the performance index of NMPC are determined by applying the inverse optimality conditions on the ILQ requlator designed for the linearized model of SI Engine. However, these quadratic weights give satisfactory results even for the nonlinear model given by Eqs. (5.40) and (5.41). For this control problem, the lower bound of σ is $\sigma_{lb} = 0.9621 \times 10^{-5}$, which is determined using the ILQ method.

5.4.4.1 Effect of Change in Main Tuning Parameter σ on Closed-loop Response and Control Inputs

First of all, it is assumed that τ_l is equal to 0 and simulations are performed to study the effect of change in main tuning parameter σ on closed-loop response and control input. Figure 5.2 shows the effects of change in the main tuning parameter σ on speed of the closed-loop system's response and magnitude of the control input [97]. Plot of engine speed is shown in revolutions per minute (rpm), as it is most commonly used unit for engine speed. For these simulations, desired engine speed is 157.08 radians per second, i.e., 1500 revolutions per minute. Dotted line in each plot shows the desired reference level for that state/input variable. System's response becomes faster with increase in the value of σ, that is a good thing from a practical viewpoint. However, at the same time, the magnitude of the control input increases as shown in Fig. 5.2. Therefore, the tuning parameter σ gives a trade-off between the speed of the resulting closed-loop response and the magnitude of the control input. Although a fast response is desirable, too large input magnitudes are not desirable from a practical viewpoint. Therefore, σ should be chosen to achieve a good compromise between the speed of the system's response and the magnitude of the control input.

For these simulations, we need not to introduce any dummy inputs to incorporate the inequality constraints given by Eqs. (5.42) – (5.44) as the plots of all the state and input variables remain within permissible bounds, i.e., constraints on state and input variables are satisfied.

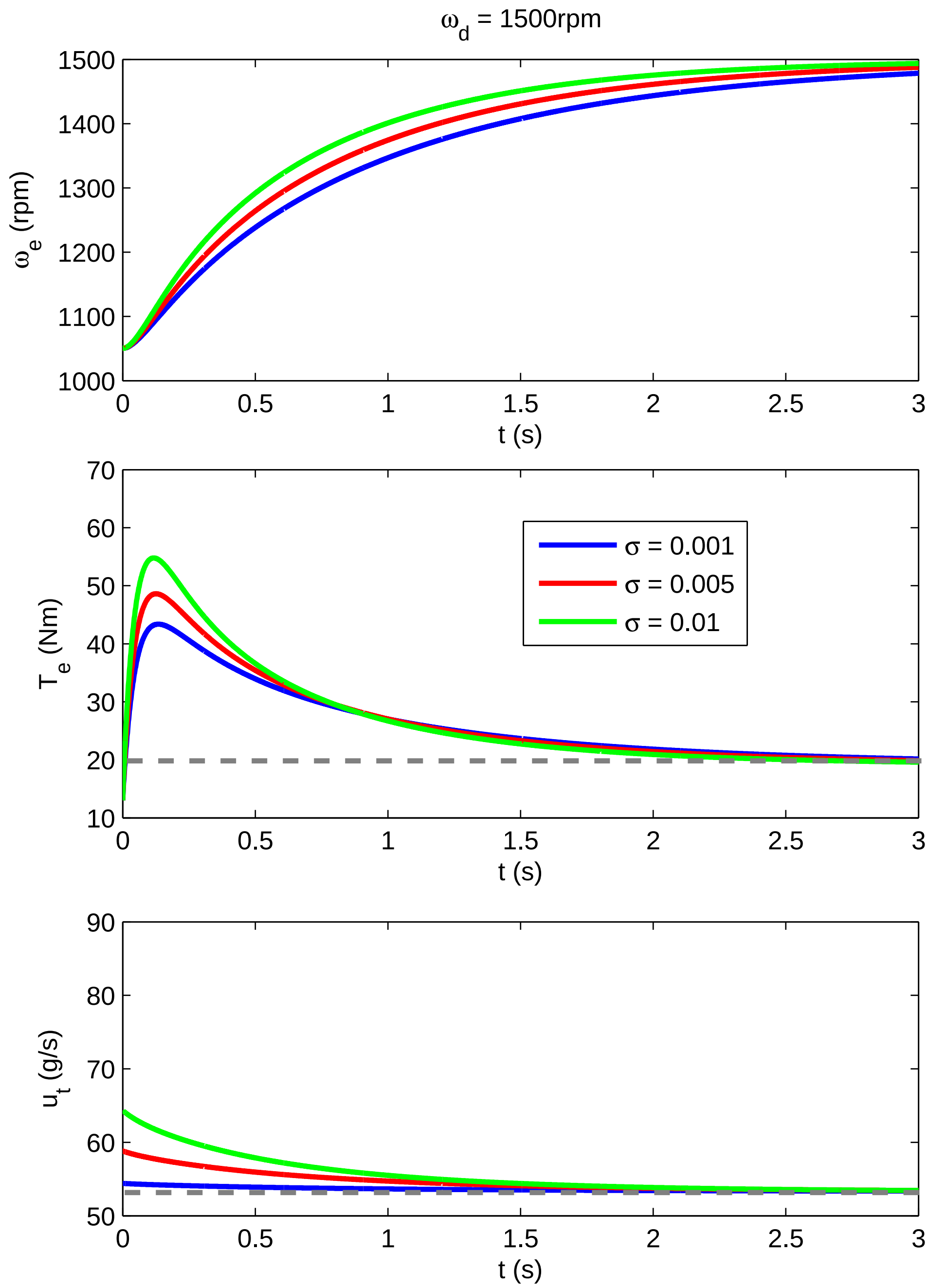

FIGURE 5.2: Effect of tuning parameter σ on closed-loop response and control input

5.4.4.2 Effect of Change in the Parameter $p = \tau_l$ on Closed-loop Response and Control Inputs

Next the effect of change in the value of parameter τ_l on closed-loop response is studied. For all these simulations, the value of σ is kept equal to 0.01. Figure 5.3 shows the effect of change in τ_l on transient response of the state variables. These simulation results correspond to the use of Method 1 for computation of $\sigma^*(p)$. Since trace of matrix $A(p)$ does not depend on p, therefore Method 1 is not so efficient in this case and the effect of change in τ_l on closed-loop response is

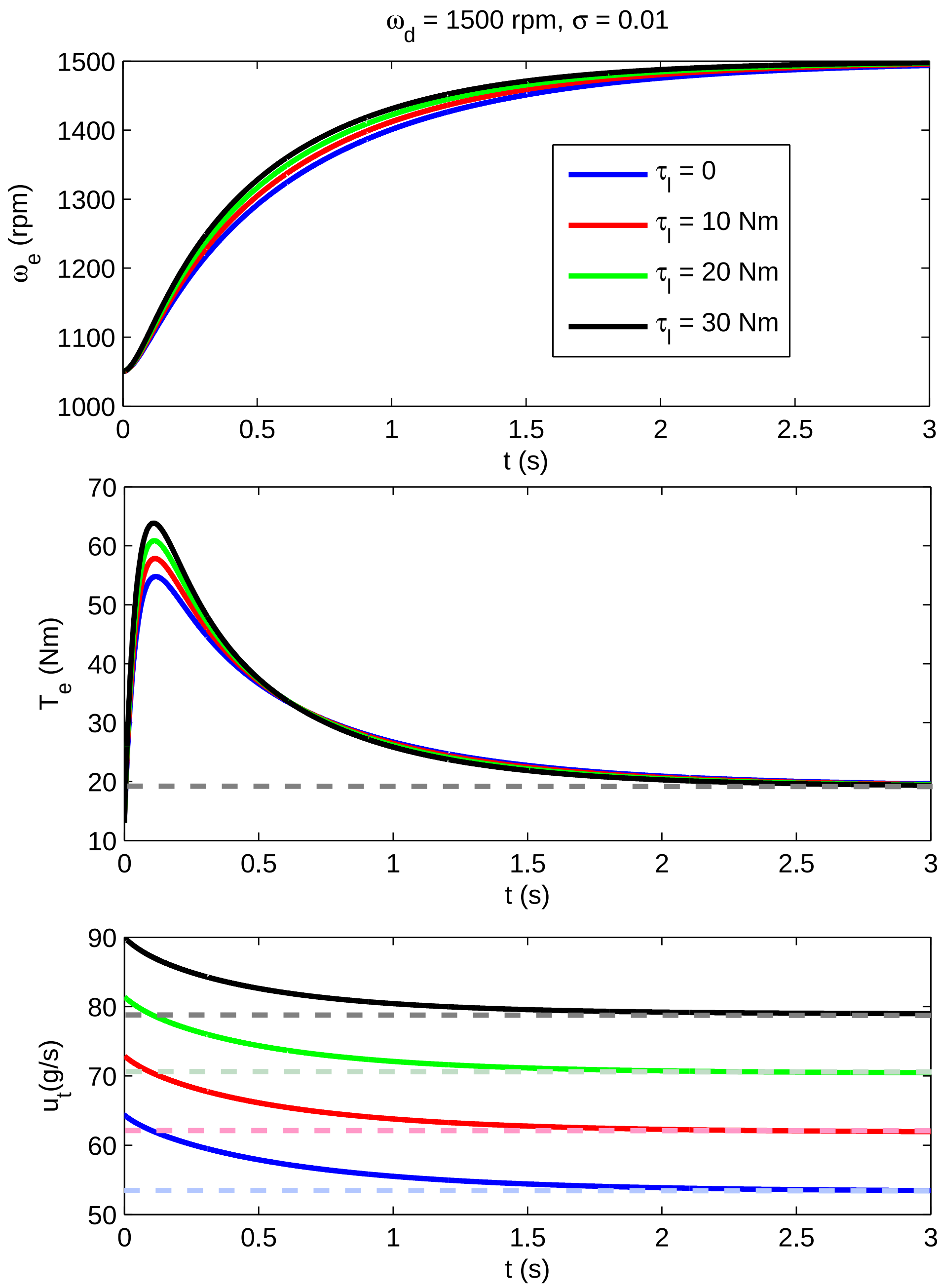

FIGURE 5.3: Effect of load-torque on closed-loop response and control input

not negligible. Transient response of the state variables is affected by change in the value of τ_l and the required reference level for control input also increases with increase in τ_l.

Method 1 for computation of $\sigma^*(p)$ is based on keeping the trace of the closed-loop system matrix fixed. Since trace of the closed-loop system matrix is equal to the sum of its eigenvalues, keeping the trace fixed may preserve some properties of the closed-loop response and may help to suppress the effect of change in the parameter p on the closed-loop response. Since trace of a matrix is equal to the sum of its diagonal entries, Method 1 can be used effectively only for the systems in which parameter p appears in the diagonal entries of the closed-loop system matrix.

5.4.4.3 Suppression of Effect of Change in the Parameter $p = \tau_l$ on Closed-loop Response and Control Inputs

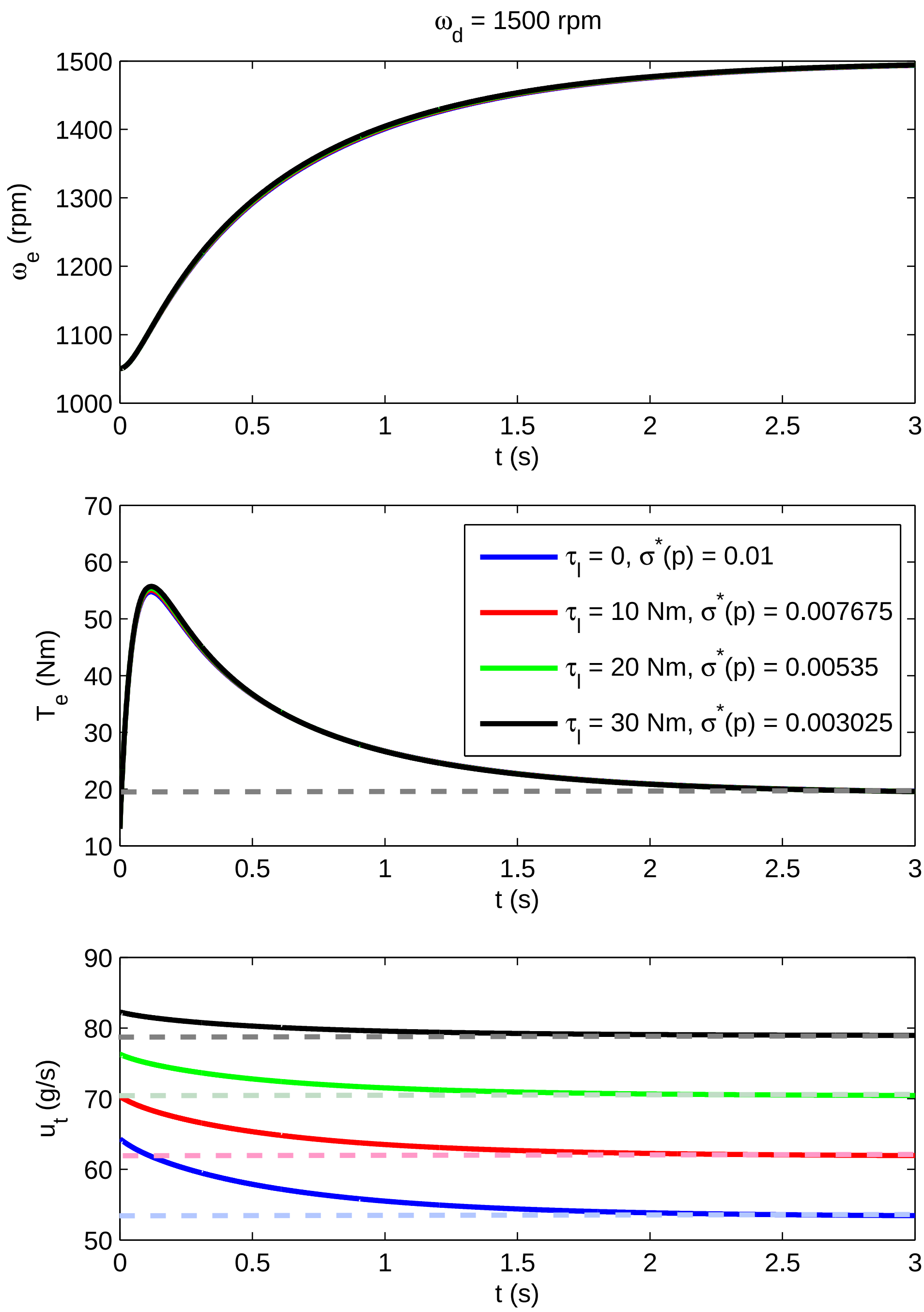

FIGURE 5.4: $\sigma^*(p)$ suppressing the effect of load-torque

Finally, simulations are performed to show the effectiveness of selecting $\sigma^*(p)$ using Eq. (5.27) and results are shown in Fig. 5.4. For these simulations,

$$\sigma_0 := 0.01 >> \sigma_{lb} = 0.9621 \times 10^{-5}. \tag{5.51}$$

For the case of parameter-dependent σ, σ_0 can be denoted as follows:

$$\sigma_0 = \sigma^*(p_0), p_0 = 0. \tag{5.52}$$

For this problem, $p_0 = 0$. As is clear from Fig. 5.4, now the effect of change in τ_l on closed-loop response is negligible. This shows that the proposed method for tuning σ for parameter-dependent systems works well. Moreover, initial values of control input have also reduced.

Since change in the value of the load torque τ_l, that is considered as the parameter p, causes change in the dynamics of the system, therefore $\sigma_{lb}(p)$, the lower bound of σ for each p will be different. Although the computation of $\sigma_{lb}(p)$ is done offline and it does not require so much time, we have selected σ_{lb} the same for all values of p, i.e.,

$$\sigma_{lb} = \sigma_{lb}(p_0) = 0.9621 \times 10^{-5}. \tag{5.53}$$

This is due to the fact that for this problem, increase in the value of the parameter p causes a decrease in the value of $\sigma_{lb}(p)$ as shown in Fig. 5.5(a), i.e.,

$$\sigma_{lb}(p_0) > \sigma_{lb}(p), \tag{5.54}$$

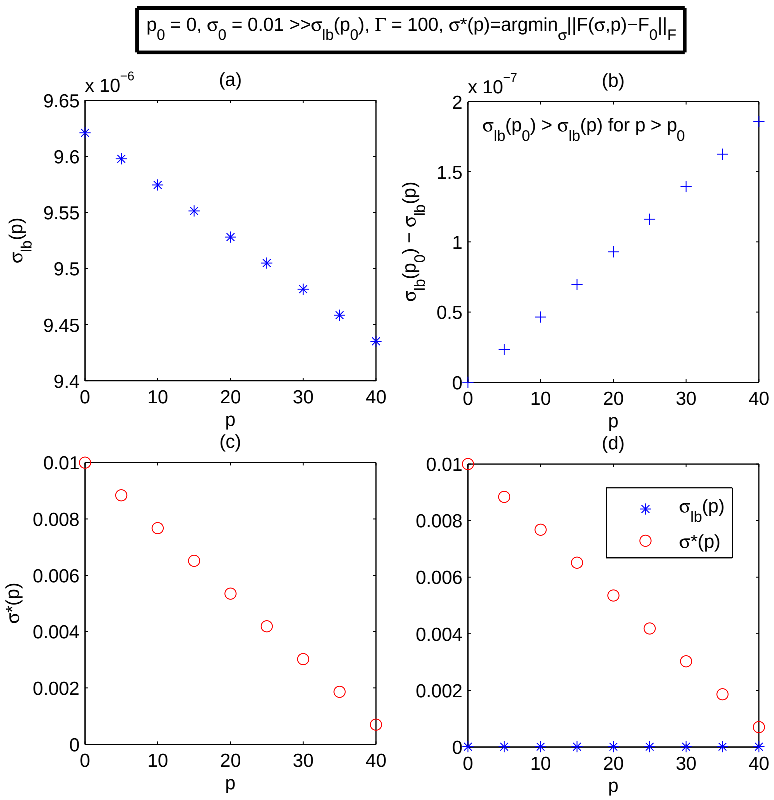

FIGURE 5.5: (a)Lower bound of σ vs p, (b)$\sigma_{lb}(p_0) - \sigma_{lb}(p)$ vs p, (c)Parameter-dependent tuning parameter $\sigma^*(p)$ vs p, (d)$\sigma^*(p) > \sigma_{lb}(p)$

for all $p > p_0$ as shown in Fig. 5.5(b). Therefore, for this control problem, $\sigma_{lb}(p_0)$ can be considered as the lower bound of σ for all the values of p. Figure 5.5(c) shows the values of $\sigma^*(p)$ for different values of p and Fig. 5.5(d) shows that $\sigma^*(p) > \sigma_{lb}(p)$. $\sigma_{lb}(p)$ can be made smaller by increasing the value of Γ.

5.4.4.4 Effect of Change in Horizon Length on Closed-loop Response and Control Input

For the above simulation results, horizon length T is kept equal to 0.01 second. Simulations are also performed to study the effect of change in horizon length on the closed-loop response

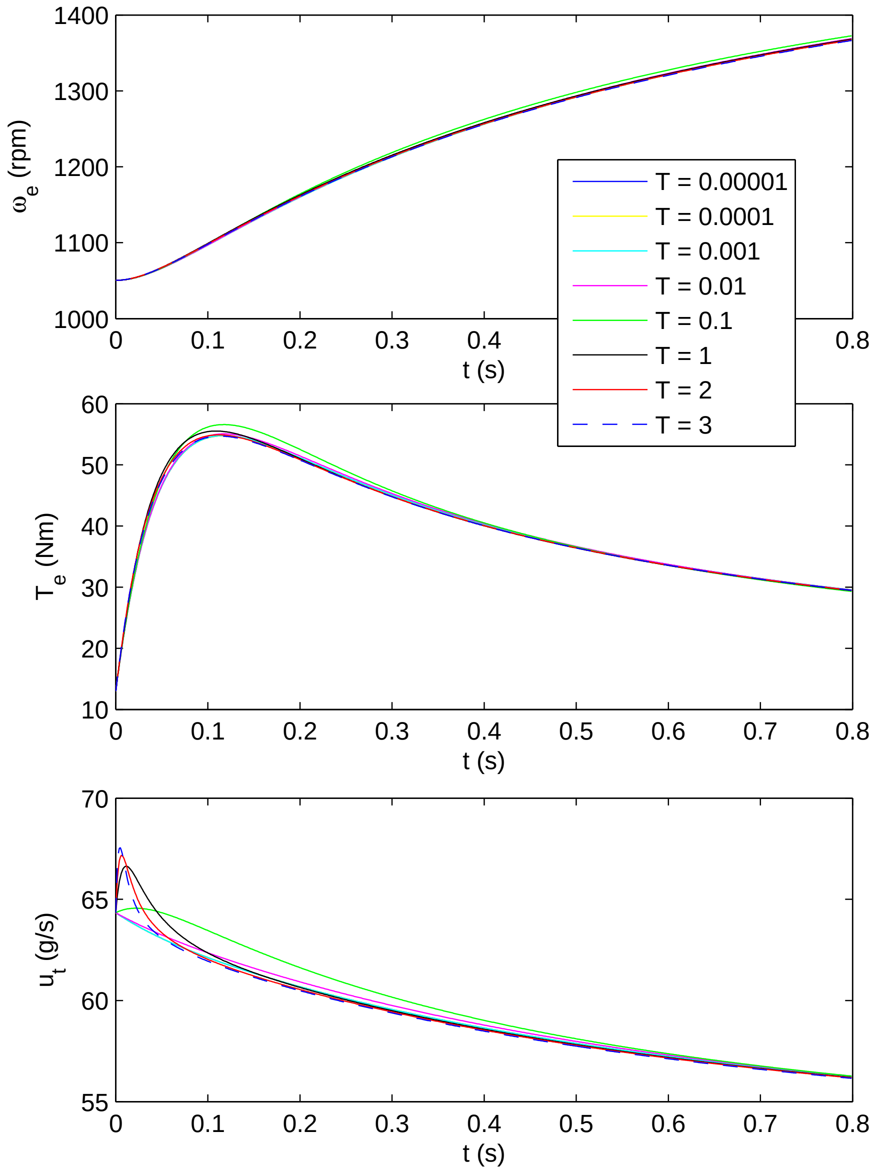

FIGURE 5.6: Effect of change in horizon length on closed-loop response and control input

and control inputs. For these simulations, σ is kept equal to 0.01. It is observed that for this system, there is no significant effect of change in horizon length on the state variables as shown in Fig. 5.6. However, change in horizon length does affect the magnitudes of control inputs. For small horizon lengths, there is no significant effect on control inputs. However, increase in the horizon length after some certain value has a little effect on the initial magnitudes of the control input. For the speed control of nonlinear mean-value model of SI engine, increase in horizon length after some certain value causes an increase in the initial magnitudes of the control input as shown in Fig. 5.6. More the horizon length, more quickly control input rises to a large magnitude and then it decreases. Since change in horizon length affects only the initial magnitudes of the control inputs and after that magnitudes are almost same for all horizon lengths, Fig. 5.6 shows only the initial part of the trajectories of closed-loop response and control inputs.

5.5 Summary

In this chapter, the quadratic performance index of NMPC is tuned by using the ILQ method. This approach is very effective and the speed of closed-loop system response and magnitude of the control input can be adjusted using a single tuning parameter σ. The tuning algorithm is extended to NMPC control of parameter-dependent systems and two methods are proposed to tune parameter-dependent main tuning parameter $\sigma = \sigma^*(p)$. Method 1 is based on keeping the trace of closed-loop system matrix fixed and Method 2 computes $\sigma^*(p)$ so that Frobenius norm of $F - F_0$ is minimized. This tuning approach is applied to control the speed of SI engine which is one of very important practical problems in automotive industry. A nonlinear MVM is used for this purpose. Simulation results shows that Method 2 is more effective and if $\sigma^*(p)$ is computed using Method 2, the effect of change in load torque on the transient response becomes negligible.

Chapter 6

Conclusions

In this thesis, we proposed the use of the ILQ regulator design method for tuning the quadratic weights in the performance index of NMPC. This approach is very effective and the speed of the closed-loop system response and the magnitude of the control inputs can be adjusted using a single tuning parameter σ. This method also provides some other free parameters. Some of these parameters affect the closed-loop response and others are used only to obtain appropriate values for the quadratic weights.

Although the quadratic weights in the performance index of NMPC have been computed using the ILQ regulator design method which uses the linearized model, these weights are then used in NMPC algorithm which uses the nolinear model and takes into account all the constraints on states and inputs. Therefore, the use of the linearized model for computing the quadratic weights does not cause any cost degradation in NMPC, and the error between the linearized model and nonlinear model does not deteriorate the performance of NMPC. Quadratic weights computed using linarzied model work well for nonlinear model as shown in simulation results. This is because of the fact that these weights are tuned through numerical simulations for nonlinear model in NMPC. This approach is effective even if reference point of some state is exactly at the boundary of the constraint on that state. Effectiveness of the tuning algorithm is elaborated by applying it to NMPC control of water levels in a nonlinear CTTS.

After that, tuning algorithm is extended for NMPC of parameter-dependent systems. Two methods are proposed to tune $\sigma = \sigma^*(p)$ for the case of parameter-dependent system. Method 1 is based on keeping the trace of closed-loop system matrix fixed and Method 2 computes $\sigma^*(p)$ so that Frobenius norm of $F - F_0$ is minimized. The extended tuning algorithm is applied to the speed control of nonlinear mean-value model of spark ignition (SI) engines which is a very important practical problem. The effectiveness of tuning the parameter-dependent tuning parameter is elaborated in simulation results. Load torque is considered as a parameter and the effect of change in its value is suppressed by tuning the parameter-dependent tuning parameter using the proposed method. Simulation results shows that Method 2 is more effective and if $\sigma^*(p)$ is computed using Method 2, the effect of change in load torque on the transient response becomes negligible.

The tuning algorithm proposed in this thesis provides a systematic way of tuning the quadratic weights in NMPC. The speed of the closed-loop response and magnitude of the control inputs can be adjusted by changing a single tuning parameter σ which is a scalar positive number. The tuning algorithm is extended for NMPC of parameter dependent systems. The parameter is considered to be known and constant over the horizon. However, it is not fixed, i.e., it may change from one constant value to another constant value.

There are some mild assumptions for the systems for which this tuning algorithm can be used. One assumption is that $n > m$, where n is the number of state variables and m is the number of input variables. Another assumption is that (A, B) of the linearized model is controllable. In future, the tuning algorithm can be further extended to suppress the effect of time-dependent parameters. Algorithm may also be extended for NMPC control of models with structured uncertainties.

Bibliography

[1] J. Richalet, A. Rault, J. L. Testud and J. Papon: Algorithmic control of industrial processes; *Proceedings of 4th IFAC Symposium on Identification and System Parameter Estimation*, pp. 1119–1167 (1976)

[2] J. Richalet, A. Rault, J. L. Testud and J. Papon: Model predictive heuristic control: applications to industrial processes; *Automatica*, Vol. 14, No. 2, pp. 413–428 (1978)

[3] C. R. Cutler and B. C. Ramaker: Dynamic matrix control- a computer control algorithm; *Proceedings of Automatic Control Conference* (1980)

[4] E. F. Camacho and C. Bordons: Nonlinear model predictive control: an introductory review; *Assessment and Future Directions of Nonlinear Model Predictive Control* (R. Findeisen et al, Eds.), Lecture Notes in Control and Information Sciences, Vol. 358, pp. 1–16, Springer-Verlag (2007)

[5] T. Ohtsuka and H. A. Fujii: Real-time optimization algorithm for nonlinear receding-horizon control; *Automatica*, Vol. 33, No. 6, pp. 1147–1154 (1997)

[6] T. Ohtsuka: Time-variant receding horizon control of nonlinear systems; *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 1, pp. 174–176 (1998)

[7] T. Ohtsuka: Quasi-Newton-type continuation method for nonlinear receding horizon control; *Journal of Guidance, Control, and Dynamics*, Vol. 25, No. 4, pp. 685–692 (2002)

[8] M. Diehl, H. G. Bock, J. P. J. P. Schlöder, R. Findeisen, Z. Nagy and F. Allgöwer: Real-time optimization and nonlinear model predictive control of processes governed by differential-algebraic equations; *Journal of Process Control*, Vol. 2, No. 4, pp. 577–585 (2002)

[9] T. Ohtsuka: A continuation/GMRES method for fast computation of nonlinear receding horizon control; *Automatica*, Vol. 40, No. 4, pp. 563–574 (2004)

[10] T. Johansen: Approximate explicit receding horizon control of constrained nonlinear systems; *Automatica*, Vol. 40, No. 2, pp. 293–300 (2004)

[11] H. G. Bock, M. Diehl, P. Kühl, E. Kostina, J. P. Schlöder and L. Wirsching: Numerical methods for efficient and fast nonlinear model predictive control; *Assessment and Future Directions of Nonlinear Model Predictive Control* (R. Findeisen et al, Eds.), Lecture Notes in Control and Information Sciences, Vol. 358, pp. 163–179, Springer-Verlag (2007)

[12] M. Diehl, H. J. Ferreau and N. Haverbeke: Efficient numerical methods for nonlinear MPC and moving horizon estimation; *Nonlinear Model Predictive Control: Towards New Challenging Applications* (L. Magni et al, Eds.), Lecture Notes in Control and Information Sciences, Vol. 384, pp. 391–417, Springer-Verlag (2009)

[13] Y. Wang and S. Boyd: Fast model predictie control using online optimization; *IEEE Transactions on Control Systems Technology*, Vol. 18, No. 2, pp. 1630–1635 (2010)

[14] M. Meadows and G. Bornard: On the stability of receding horizon control of nonlinear discrete-time systems; *Systems and Control Letters*, Vol. 23, No. 4, pp. 291–296 (1994)

[15] M. Alamir and G. Bornard: Stability of a truncated infinite constrained receding horizon scheme: the general discrete nonlinear case; *Automatica*, Vol. 31, No. 9, pp. 1353–1356 (1995)

[16] E. S. Meaddows, M. A. Henson, J. W. Eaton and J. B. Rawlings: Receding horizon control and discontinuous state feedback stabilization; *International Journal of Control*, Vol. 62, No. 5, pp. 1217–1229 (1995)

[17] G. De Nicolao, L. Magni and R. Scattolini: Stabilizing nonlinear receding horizon control via a nonquadratic terminal state penalty; *Symposium on Control, Optimization and Supervision, CESA'96 IMACS Multiconference*, pp. 185–187 (1996)

[18] P. B. Sistu and B. W. Bequette: Nonlinear model predictive control: closed-loop stability analysis, *American Institute of Chemical Engineers Journal*, Vol. 42, No. 12, pp. 3388–3402 (1996)

[19] H. Michalska: A new formulation of receding horizon stabilizing control without terminal constraint on the state; *European Journal of Control*, Vol. 3, No. 1, pp. 1–14 (1997)

[20] H. Chen and F. Allgöwer: Nonlinear model predictive control schemes with guarantee stability; *Nonlinear Model Based Process Control* (R. Berber et al, Eds.), pp. 465–494, Kluwer Academic Publishers, Dodrecht (1998)

[21] S. S. Keerthi and E. G. Gilbert: Optimal infinite-horizon feedback laws for a general class of constrained discrete-time systems: stability and moving-horizon approximations; *Journal of Optimal Theory and Applications*, Vol. 57, No. 2, pp. 265–293 (1998)

[22] D. Q. Mayne, J. B. Rawlings, C. V. Rao, P. O. M. Scokaert, Constrained model predictive control: stability and optimality; *Automatica*, Vol.36, pp.789-814 (2000)

[23] F. A. Fontes: A generl framework to design stabilizing nonlinear model predictive controllers; *Systems and Control Letters*, Vol. 42, No. 2, pp. 127–143 (2000)

[24] L. Magni, G. De Nicolao and R. Scattolini: A stabilizing model-based predictive control algorithm for nonlinear systems; *Automatica*, Vol. 37, No. 10, pp. 1351–1362 (2001)

[25] G. De Nicolao, L. Magni and R. Scattolini: Stability and robustness of nonlinear receding horizon control; *Nonlinear Model Predictive Control* (F. Allgöwer et al Eds.), pp. 3–23, Birkhaser (2000)

[26] M. Lazar, W. P. M. H. Heemels, A. Bemporad and S. Weiland: Discrete-time non-smooth nonlinear MPC: stability and robustness; *Assessment and Future Directions of Nonlinear Model Predictive Control* (R. Findeisen et al, Eds.), Lecture Notes in Control and Information Sciences, Vol. 358, pp. 93–103, Springer-Verlag (2007)

[27] V. M. Zavala and L. T. Biegler: The advanced-step NMPC controller: optimality, stability and robustness; *Automatica*, Vol. 45, No. 1, pp. 86–93 (2009)

[28] H. Michalska and D. Q. Mayne: Robust receding horizon control of constrained nonlinear systems; *IEEE Transactions on Automatic Control*, Vol. AC-38, No. 11, pp. 1623–1633 (1993)

[29] Y. Chikkula, J. H. Lee and B. A. Ogunnaike: Robust model predictive control of nonlinear systems using input-output models; *Proceedings of the 1993 American Control Conference*, pp. 2205–2209 (1993)

[30] K. V. Kothare, V. Balakrishnan and M. Morari: Robust constrained model predictive control using linear matrix inequalities; *Automatica*, Vol. 32, No. 10, pp. 1361–1379 (1996)

[31] L. Magni and R. Sepulchre: Stability margins of nonlinear receding-horizon control via inverse optimality; *Systems and Control Letters*, Vol. 32, No. 4, pp. 241–245 (1997)

[32] R. Blauwkamp and T. Basar: A receding-horizon approach to robust output feedback control for nonlinear systems; *Proceedings of the 38th IEEE Conference on Decision and Control*, pp. 4879–4884 (1999)

[33] L. Magni, H. Nijmeijer and A. J. van der Schaft: A receding-horizon approach to the nonlinear H_∞ control problem, *Automatica*; Vol. 37, No. 5, pp. 429–435 (2001)

[34] L. Magni, G De Nicolao, R. Scattolini and F. Allgöwer: Robust receding horizon control for nonlinear discrete-time systems; *Proceedings of the 15th IFAC World Congress* (2002)

[35] L. Magni and R. Scattolini: Robustness and robust design of MPC for nonlinear discrete-time systems; *Assessment and Future Directions of Nonlinear Model Predictive Control* (R. Findeisen et al, Eds.), Lecture Notes in Control and Information Sciences, Vol. 358, pp. 239–254, Springer-Verlag (2007)

[36] M. Lazar, W. P. M. H. Heemels and A. Jokie: Self-optimizing robust nonlinear model predictive control; *Nonlinear Model Predictive Control: Towards New Challenging Applications* (L. Magni et al, Eds.), Lecture Notes in Control and Information Sciences, Vol. 384, pp. 27–40, Springer-Verlag (2009)

[37] V. Adetola and M. Guay: Robust adaptive MPC for constrained uncertain nonlinear systems; *International Journal of Adaptive Control and Signal Processing*, Vol. 25, No. 2, pp. 155–167 (2010)

[38] A. Bemporad: Model predictive control tuning by controller matching; *IEEE Transactions on Automatic Control*, Vol. 55, No. 1, pp. 185–190 (2010)

[39] T. Ohtsuka and K. Ozaki: Practical issues in nonlinear model predictive control: real-time optimization and systematic tuning; *Nonlinear Model Predictive Control: Towards New Challenging Applications* (L. Magni et al, Eds.), Lecture Notes in Control and Information Sciences, Vol. 384, pp. 447–460, Springer-Verlag (2009)

[40] F. Tahir and T. Ohtsuka, Using inverse linear quadratic method for systematic tuning of performance index in nonlinear model predictive control; *Proceedings of the SICE Annual Conference, SICE2012*, pp. 1304–1307 (2012)

[41] F. Tahir and T. Ohtsuka, Tuning of performance index in nonlinear model predictive control by the inverse linear quadratic regulator design method; *SICE Journal of Control, Measurement, and System Integration*, Vol. 6, No. 6, pp. 387–395 (2013)

[42] T. Fujii: A new approach to the LQ design from the viewpoint of the inverse regulator problem; *IEEE Transactions on Automatic Control*, Vol. 32, No. 11, pp. 995–1004 (1987)

[43] H. Chen and F. Allgöwer: A quasi-infinite horizon nonlinear model predictive control scheme with guaranteed stability; *Automatica*, Vol. 34, No. 10, pp. 1205–1217 (1998)

[44] F. Tahir and T. Ohtsuka: Tuning of nonlinear model predictive controller for parameter-dependent systems and its application to the speed control of spark ignition engines;*Transactions of the Institute of Systems, Control and Information Engineers* (2013) (submitted)

[45] E. L. Allgower and K. Georg: *Introduction to Numerical Continuation Methods*, Springer (1990)

[46] C. T. Kelley: *Iterative Methods for Linear and Nonlinear Equations*, Society for Industrial and Applied Mathematics, SIAM (1995)

[47] T. Fujii and T. Shimomura: Generalization of ILQ method for the design of optimal servo systems; *Transactions of the Institute of Systems, Control and Information Engineers*, Vol. 1, No. 6, pp. 194–203 (1988) (in Japanese)

[48] T. Fujii and N. Mizushima: A new approach to LQ design: Application to the design of optimal servo systems; *Transactions of the Society of Instrument and Control Engineers*, Vol. E-1, No. 1, pp. 43–50 (2001)

[49] T. Shimomura and T. Fujii: Extension of ILQ method to non-minimum phase systems or the design of optimal servo systems; *Transactions of the Institute of Systems, Control and Information Engineers*, Vol. 6, No. 11, pp. 498–507 (1993)

[50] T. Fujii and T. Tsujino: An inverse LQ based approach to the design of robust tracking system with quadratic stability; *Robust Control* (L. D. Davisson et al, Eds.), Lecture Notes in Control and Information Sciences, Vol. 183, pp. 106–113, Springer-Verlag (1992)

[51] T. Fujii: Design of tracking systems with LQ optimality and quadratic stability; *IFAC 12th Triennial World Congress*, pp. 435–442 (1993)

[52] M. Sakai, Y. Kuroe, N. Kenji and T. Fujii: Design of a frequency-shaping ILQ controller to suppress vibration in a magnetic levitation system; *Transactions of the Instiute of Systems, Control and Information Engineers*, Vol. 11, No. 5, pp. 267–276 (1998) (in Japanese)

[53] M. Sakai, Y. Kuroe, K. Nakashima and T. Fujii: Application of ILQ-based frequency-shaping control to a magnetic levitation system; *Proceedings of the 1998 IEEE Interntational Conference on Control Applications*, pp. 593–597 (1998)

[54] T. Tsujino, K. Nakashima and T. Fujii: Application of H_∞ control and closed loop identification to a magnetic levitation system; *Asian Journal of Control*, Vol. 1, No. 4, pp. 283–296 (1999)

[55] T. Shimomura, T. Fujii and N. Suda: Generalization of the ILQ regulator design method; *Proceedings of the 1995 American Control Conference*, Vol. 3, pp. 1967–1968 (1995)

[56] S. Kunimatsu and T. Fujii: Design method of ILQ servo system with tracking robustness; *Proceedings of the 4th IFAC Symposium on Robust Control Design* (2003)

[57] S. Kunimatsu, M. Fujii and T. Fujii: Analysis and synthesis of model reference ILQ servo system with tracking robustness; *Transactions of the Institute of Systems, Control and Information Engineers*, Vol. 17, No. 3, pp. 131–138 (2004) (in Japanese)

[58] K. Matsumoto, K. Suzuki, S. Kunimatsu and T. Fujii: Temperature control of wafer in semiconductor manufacturing systems by MR-ILQ design method; *Proceedings of the 2004 IEEE International Conference on Control Applications* (2004)

[59] Y. Kurome, H. Abe and T. Fujii: ILQ design method for optimal servo systems with generalized reference inputs; *Transactions of the Society of Instrument and Control Engineers*, Vol. 32, No. 4, pp. 539–546 (1996) (in Japanese)

[60] Y. Kurome and T. Fujii: Extension of ILQ design theory with respect to reference inputs and control objects; *Transactions of the Society of Instrument and Control Engineers*, Vol. 34, No. 3, pp. 195–202 (1998) (in Japanese)

[61] M. Shiode, Y. Kuroe, T. Fujii and N. Suda: An inverse approach to design of a frequency-shaping optimal servo system with frequency weights; *Transactions of the Institute of Systems, Control and Information Engineers*, Vol. 11, No. 10, pp. 576–584 (1998) (in Japanese)

[62] M. Sakai and T. Fujii: Design method for ILQ robust servo-systems with loop-shaping; *Transactions of the Society of Instrument and Control Engineers*, Vol. 36, No. 4, pp. 340–347 (2000) (in Japanese)

[63] K. Nakamura, M. Sakai, K. Nakashima and T. Fujii: Derivation of stabilizing controller based on minimal-order observer and application to design of frequency-shaping ILQ servo system; *Transactions of the Society of Instrument and Control Engineers*, Vol. 14, No. 2, pp. 62–70 (2001)

[64] S. Kunimatsu, T. Fujii and T. Tsujino: Quadratic stabilization of ILQ servo systems with generalized reference inputs; *Transactions of the Society of Instrument and Control Engineers*, Vol. 17, No. 3, pp. 131–138 (2003) (in Japanese)

[65] M. Sakai and T. Fujii: Extension of ILQ design method by using a free parameter in the feed-forward path; *Transactions of the Society of Instrument and Control Engineers*, Vol. 39, No. 3, pp. 307–309 (2001) (in Japanese)

[66] S. Kunimatsu, T. Fujii and Y. Ito: Discrete time ILQ sesrvo system design method; *Transactions of the Society of Instrument and Control Engineers*, Vol. 17, No. 11, pp. 514–516 (2004) (in Japanese)

[67] S. Kunimatsu and T. Fujii: Analysis and Synthesis of model reference ILQ servo system using robust L_∞ performance measure for a fixed input; *Transactions of the Society of Instrument and Control Engineers*, Vol. 41, No. 1, pp. 46–52 (2005)

[68] S. Magota, S. Kunimatsu, G. Yamamoto, T. Fujii and R. Otsubo: The opening-and-closing speed control of an elevator door by frequency-shaping ILQ design method; *Proceedings of the 2005 CACS Automatic Control Conference* (2005)

[69] S. Magota, S. Kunimatsu, G. Yamamoto, T. Fujii and R. Otsubo: The opening-and-closing speed control of an elevator door by frequency-shaping ILQ design method; *Proceedings of the 2006 American Control Conference*, pp. 1960–1961 (2006)

[70] T. Fujii and O. Kaneko: Synthesis of ILQ control system for disturbance attenuation and its application to thermal control of a plasma CVD reactor; *Proceedings of the 2008 International Symposium on Advanced Control of Industrial Processes*, pp. 370–371 (2008)

[71] T. Fujii, S. Kunimatsu and O. Kaneko: Synthesis of ILQ control system for disturbance attenuation; *Proceedings of the SICE Annual Conference, SICE2008*, pp. 1300–1303 (2008)

[72] R. E. Kalman: When is a linear control system optimal?; *ASME Transactions, Journal of Basic Engineering*, Vol. 86, No. 1, pp. 51–60 (1964)

[73] T. Fujii and M. Narazaki: A complete optimality condition in the inverse problem of optimal control; *SIAM Journal on Control and Optimization'*, Vol. 22, pp. 327–341 (1984)

[74] P. J. Moylan: Matrices with positive principal minors; *Linear Algebra and its Applicatins*, Vol. 17, pp. 53–58 (1977)

[75] A. Jameson and E. Kreindler: Inverse problem of linear optimal control; *SIAM Journal on Control*, Vol. 11, No. 1, pp. 1–19 (1973)

[76] T. Shimomura: *Multiobjective Control Design based on Geometric Structure of the Controller*, Ph.D. dissertation, Osaka University (1998)

[77] A. E. Bryson and Y.-C. Ho: *Applied Optimal Control*, Taylor and Francis (1975)

[78] D. E. Kirk: *Optimal Control Theory, An Introduction*, Dover Publications (2004)

[79] H. K. Khalil: *Nonlinear Systems*, Third Edition, Prentice Hall, Upper Saddle River, NJ 07458 (2002)

[80] F. Tahir: *Stability Analysis of Sampled-Data Control of Nonlinear Systems*, MS Thesis, PIEAS Library, Nilore, Islamabad (2007)

[81] F. Tahir, N. Iqbal and G. Mustafa: Control of a nonlinear coupled three tank system using feedback linearization; *Proceedings of the 3rd International Conference on Electrical Engineering, ICEE2009* (2009)

[82] L. Grune, J. Pannek, M. Seehafer and K. Worthmann: Analysis of unconstrained nonlinear MPC schemes with varying control horizon; *SIAM Journal on Control and Optimization*, Vol. 48, No. 8, pp. 4938–4962 (2010)

[83] K. Worthmann: Estimates on the prediction horizon length in model predictive control, *Proceedings of the 19th International Symposium on Mathematical Theory of Networks and Systems (MTNS2012)* (2012)

[84] T. Ohtsuka: AutoGenU, http://www.symlab.sys.i.kyoto-u.ac.jp/~ohtsuka/code/index.htm

[85] D. S. Bernstein: *Matrix Mathematics: Theory, Facts, and Formulas with Application to Linear Systems Theory*, Princeton University Press (2005)

[86] D. Hrovat and W. J. Johnson: Automotive control systems: past, present, future; *Proceedings of the 1991 International Conference on Industrial Electrlnics, Control and Instrumentation (IECON91)* (1991)

[87] M. Thornhill, S. Thompson and H. Sindano: A comparison of idle speed control schemes; *Control Engineering Practice*, Vol. 8, pp. 519-530 (2000)

[88] A. W. Olbrot and B. K. Powel: Robust design and analysis of third and fourth order time delay systems with application to automotive idle speed control; *Proceedings of the 1989 American Control Conference*, Vol. 2, pp. 1029–1039 (1989)

[89] M. Abate and N. Dosio: Use of fuzzy logic for engine idle speed control; *Society of Automotive Engineers SAE Technical Paper*, Paper No. 900594 (1990)

[90] S. J. Williams , D. Hrovat , C. Davey , D. Maclay , J. W. V. Crevel and L. F. Chen: Idle Speed Control Design Using an H-Infinity Approach; *Proceedings of the 1989 American Control Conference*, pp.1950–1956 (1989)

[91] C. Carnevale and A. Moschetti: Idle speed control with H-Infinity technique; *Society of Automotive Engineers SAE Technical Paper*, Paper No. 930770 (1993)

[92] L. Kjergaard, S. Nielsen, T. Vesterholm and E. Hendricks: Advanced nonlinear engine idle speed control systems; *Society of Automotive Engineers SAE Technical Paper*, Paper No. 940974 (1994)

[93] E. Hendricks and S. Sorenson: Mean value modeling of spark ignition engines; *Society of Automotive Engineers SAE World Congress*, Paper No. 900616 (1990)

[94] L. Guzzella and C. H. Onder: *Introduction to Modeling and Control of Internal Combustion Engine Systems*, Springer-Verlag (2010)

[95] T. Shen and A. Ohata: *Modeling and Control Design for Automotive Engines*, Coronasha Press (2011) (in Japanese)

[96] M. Hong, T. Shen, M. Ouyang, and J. Kako: Torque observers design for spark ignition engines with different intake air measurement sensors; *IEEE Transactions on Control Systems Technology*, Vol. 19, No. 1, pp. 229–237 (2011)

[97] F. Tahir, T. Ohtsuka and T. Shen: Tuning of nonlinear model predictive controller for the speed control of spark ignition engines;*Proceedings of the 2013 CACS International Automatic Control Conference, CACS2013* (2013)

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List of Publications

Refereed Journal Articles

1. F. Tahir and T. Ohtsuka, Tuning of performance index in nonlinear model predictive control by the inverse linear quadratic regulator design method; *SICE Journal of Control, Measurement, and System Integration*, Vol. 6, No. 6, pp. 387–395 (2013)

Refereed International IEEE Conference Papers

1. M. Rehan, F. Tahir, N. Iqbal and G. Mustafa: Modeling, simulation and decentralized control of a nonlinear coupled tank system; *Proceedings of the 2nd International Conference on Electrical Engineering, ICEE2008* (2008)
2. F. Tahir, N. Iqbal and G. Mustafa: Control of a nonlinear coupled three tank system using feedback linearization; *Proceedings of the 3rd International Conference on Electrical Engineering, ICEE2009* (2009)
3. F. Tahir and T. Ohtsuka: Using inverse linear quadratic method for systematic tuning of performance index in nonlinear model predictive control; *Proceedings of the SICE Annual Conference, SICE2012*, pp. 1304–1307 (2012)
4. F. Tahir, T. Ohtsuka and T. Shen: Tuning of nonlinear model predictive controller for the speed control of spark ignition engines;*Proceedings of the 2013 CACS International Automatic Control Conference, CACS2013*, (2013) (Award for Best Paper in Theory)