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## THE DIMENSION FORMULA FOR THE RING OF CODE POLYNOMIALS IN GENUS 4

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### 1. Introduction

The purpose of this paper is to study the dimension formula for the invariant ring  $C[f_a]$  for  $a \in F_2^4$ , which may be considered as the ring of code polynomials in genus 4. We also give all characteristic polynomials of elements in  $H_4$ . The main ingredient is the determination of the conjugacy classes of the symplectic group  $Sp(8, 2)$ . Our result will be useful for the investigation of the Siegel modular forms in genus four.

We recall from [10, 11] that the finite group  $H_g$  is (up to  $\pm 1$ ) just the image of the modular group  $\Gamma_g = Sp(2g, \mathbb{Z})$  under the theta representation (of index 1) and that the ring of modular forms of even weight is given by

$$A(\Gamma_g)_{(2)} = \bigoplus_{2|k} [\Gamma_g, k] = (C[f_a]^{H_g} / \{\text{relation}\})^N,$$

where  $N$  denotes the normalization in its field of fractions and “relation” are the theta relations. However, the generators and the dimension formulas for  $A(\Gamma_g)$  are known only for genus  $g \leq 3$ .

On the other hand, the invariant ring  $C[f_a]^{H_g}$  may be considered as the ring of code polynomials in genus  $g$ . In [9], there is the definition of the  $g$ -th weight polynomial for codes (codes mean the binary linear codes) and the connections among codes, lattices, the invariant rings of the finite groups, and the theory of modular forms were studied (cf. [1], [4], [5], [8], [16]). In particular, it was shown that the invariant ring of the group  $\langle H_g, \zeta_8 \rangle$ , which is the subring of  $C[f_a]^{H_g}$ , is generated by the  $g$ -th weight polynomials for self-dual doubly-even codes, where  $\zeta_8$  is the primitive 8-th root of unity. This invariant ring corresponds to the ring of the modular forms of weights divisible by 4.

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## 2. On the group $H_g$

In this section, we study the group  $H_g$ . In addition to Runge, the group  $H_g$  has been studied by several authors, for example, see [4], [7].

Let  $V$  be the  $g$ -dimensional vector space over the field of two elements, i.e.,  $V = F_2^g$ . For  $x, y \in V$ , let  $x \cdot y$  denote the usual dot product. Set

$$T_g := \left( \frac{1+i}{2} \right)^g [(-1)^{a \cdot b}]_{a, b \in V},$$

and for a symmetric  $g \times g$  matrix  $S$ ,

$$D_S := \text{diag}(t^{S[a]}) \text{ with } a \in V,$$

where  $S[a] := aS'a$ . Let

$$H_g := \langle T_g, D_S \mid S \text{ runs over all symmetric matrices in } \text{Mat}_{g \times g}(\mathbb{Z}) \rangle$$

be the subgroup of  $GL(2^g, \mathbb{C})$  generated by the elements  $T_g$  and the  $D_S$ .

We get for the invariant ring  $C[f_a]^{H_g}$  the dimension formula

$$\Phi_{H_g}(t) = \sum_{d \geq 0} (\dim C[f_a]_d^{H_g}) t^d = \frac{1}{|H_g|} \sum_{\sigma \in H_g} \frac{1}{\det(1_{2^g} - t\sigma)},$$

where  $C[f_a]_d^{H_g}$  is the  $d$ -th homogeneous part of  $C[f_a]^{H_g}$ ,  $\Phi_{H_g}(t)$  is called the Molien series of  $H_g$ .

The following lemma gives the simplification we use.

**Lemma 2.1** (Lemma 2.1 [9]). *One has an exact sequence*

$$0 \rightarrow N_g \rightarrow H_g \xrightarrow{\varphi} Sp(2g, 2) \rightarrow 0,$$

where  $N_g := \langle i, D_S^2, T_g^{-1} D_S^2 T_g \rangle$  and the homomorphism  $\varphi$  is given by the conjugation of  $H_g$  on the  $F_2$ -vector space  $N_g / \langle i \rangle$ .  $\square$

Therefore we have the obvious formula

$$(2.1) \quad \Phi_{H_g}(t) = \frac{1}{|H_g|} \sum_i \sum_{n \in N_g} \frac{|C_i|}{\det(1 - tz_i n)},$$

where  $\{C_i\}$  are the conjugacy classes of  $Sp(2g, 2)$  and  $z_i$  is some element of  $H_g$  with  $\varphi(z_i) \in C_i$ . Our computation is done using (2.1).

Finally we list the sizes of groups.

$$\begin{aligned}
|N_g| &= 2^{2+2g}, \\
|Sp(2g, 2)| &= 2^{g^2}(2^{2g} - 1) \cdots (2^2 - 1), \\
|H_g| &= |N_g| \times |Sp(2g, 2)| = 2^{2+2g+g^2}(2^{2g} - 1) \cdots (2^2 - 1).
\end{aligned}$$

In our case ( $g=4$ ),  $|N_4| = 1,024 = 2^{10}$ ,  $|Sp(8, 2)| = 47,377,612,800 = 2^{16}3^55^2 \cdot 7 \cdot 17$ ,  $|H_4| = 48,514,675,507$ ,  $200 = 2^{26}3^55^2 \cdot 7 \cdot 17$ .

REMARK.  $H_1$ ,  $\langle H_1, \zeta_8 \rangle$ ,  $H_2$  are the reflection groups No.8, No.9, No.31 in [14], respectively (cf. Proposition 2.6 [10]).

### 3. On $Sp(8, 2)$ and its conjugacy classes

In this section, we study the symplectic group  $Sp(8, 2)$ . This is identified with the Chevalley group of type  $(C_4)$  over the field of two elements. We give all the characteristic polynomials of elements in  $H_4$ . We remark that we cannot read off representatives of the conjugacy classes of  $Sp(8, 2)$  from Atlas [3] although it is the good reference for the finite simple groups.

As is said,  $Sp(8, 2)$  is one of the Chevalley groups. The properties of such groups are known and we describe what we need.

Let  $\Delta = \{\pm 2\xi_i, \pm \xi_i \pm \xi_j \mid 1 \leq i, j \leq 4\}$  be the root system of type  $(C_4)$ , and choose  $a = \xi_1 - \xi_2$ ,  $b = \xi_2 - \xi_3$ ,  $c = \xi_3 - \xi_4$ ,  $d = 2\xi_4$  for a fundamental system  $\Pi$  of roots. We denote by  $\Delta^+$  the set of positive roots with respect to  $\Pi$ . We write an element  $\alpha a + \beta b + \gamma c + \delta d$  of  $\Delta^+$  as  $\alpha\beta\gamma\delta$ . For example, we write 1023 for  $a + 2c + 3d$ .

$E_{ij}$  is the elementary matrix of size  $8 \times 8$  with 1 in the  $(i, j)$ -entry. For  $1 \leq i, j \leq 4$ ,

$$\begin{aligned}
x_{\xi_i + \xi_j} &:= 1 + E_{i, 4+j} + E_{j, 4+i}, \\
x_{\xi_i - \xi_j} &:= 1 + E_{i, j} + E_{4+j, 4+i} \quad (i < j), \\
x_{2\xi_i} &:= 1 + E_{i, 4+i}, \\
x_{-r} &:= {}^t x_r \quad (r \in \Delta).
\end{aligned}$$

Then  $Sp(8, 2)$  is generated by  $x_r$  ( $r \in \Delta$ ) and is known to be one of the finite simple groups (cf. [3]).

Let  $X_r$  be the group generated by  $x_r$  and put  $B = X_{0100}X_{0010}X_{0110}X_{0001}X_{0011}X_{0111}X_{0021}X_{0121}X_{0221}X_{1000}X_{1100}X_{1110}X_{1111}X_{1121}X_{1221}X_{2221}$ . Then  $B$  is a Sylow 2-subgroup of  $Sp(8, 2)$  and is normal in  $Sp(8, 2)$ .

Put  $n_r = x_r x_{-r} x_r$  and  $N = \langle n_r \mid r \in \Delta \rangle$ .  $N$  is isomorphic to the Weyl group of type  $(C_4)$ . We fix the following correspondence:

$$\begin{aligned}
\xi_1 &\leftrightarrow \underline{1} := [1, 0, 0, 0, 0, 0, 0], \\
\xi_2 &\leftrightarrow \underline{2} := [0, 1, 0, 0, 0, 0, 0],
\end{aligned}$$

$$\begin{aligned}
\xi_3 \leftrightarrow \underline{3} &:= [0, 0, 1, 0, 0, 0, 0], \\
\xi_4 \leftrightarrow \underline{4} &:= [0, 0, 0, 1, 0, 0, 0], \\
-\xi_1 \leftrightarrow \underline{-1} &:= [0, 0, 0, 0, 1, 0, 0], \\
-\xi_2 \leftrightarrow \underline{-2} &:= [0, 0, 0, 0, 0, 1, 0], \\
-\xi_3 \leftrightarrow \underline{-3} &:= [0, 0, 0, 0, 0, 0, 1], \\
-\xi_4 \leftrightarrow \underline{-4} &:= [0, 0, 0, 0, 0, 0, 1].
\end{aligned}$$

An element of  $N$  is uniquely determined by its natural action on  $\underline{1}, \underline{2}, \underline{3}, \underline{4}$ . If  $n \in N$  satisfies  $\underline{1}n = \underline{\alpha}$ ,  $\underline{2}n = \underline{\beta}$ ,  $\underline{3}n = \underline{\gamma}$ ,  $\underline{4}n = \underline{\delta}$ , then we denote  $n$  by  $n(\alpha, \beta, \gamma, \delta)$ . For example, we have

$$n(2, 3, -1, -4) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

In the following, we give the conjugacy classes of  $Sp(8, 2)$  and all characteristic polynomials of elements in  $H_4$ . To determine the conjugacy classes of  $Sp(8, 2)$ , we have to show

- (i) No two elements in the list are conjugate in  $Sp(8, 2)$ ,
- (ii)  $\sum_{0 \leq i \leq 80} |Sp(8, 2)| / |C_{Sp(8, 2)}(c_i)| = |Sp(8, 2)|$ ,

where  $C_{Sp(8, 2)}(c_i)$  denotes the centralizer group of  $c_i$  in  $Sp(8, 2)$ . These statements are proved using GAP [13]. Then we compute  $1024 \times 81$  determinants of size  $16 \times 16$ . Since  $H_4$  is a subgroup of  $SU(2^4, \mathbb{Z}[\frac{1+i}{2}])$  (Proposition 2.6 [10]), the polynomials have the type  $\sum_{0 \leq i \leq 16} a_i t^i$  with  $a_0 = a_{16} = 1$ ,  $\bar{a}_1 = a_{15}$ ,  $\bar{a}_2 = a_{14}$ ,  $\bar{a}_3 = a_{13}$ ,  $\bar{a}_4 = a_{12}$ ,  $\bar{a}_5 = a_{11}$ ,  $\bar{a}_6 = a_{10}$ ,  $\bar{a}_7 = a_9$ , and  $\bar{a}_8 = a_8$  (A bar denotes complex conjugation).

There are 81 boxes below and the  $i$ -th box ( $0 \leq i \leq 80$ ) gives the characteristic polynomials of elements in  $z_i N_4$ . In each box, the first column gives the multiplicity of each polynomial. The next columns give the values of  $a_1, a_2, \dots, a_8$ , respectively. For example, in the first box, the 2 in the first column means that there are 2 occurrences of the polynomial in the form  $1 + (8 - 8i)t - 64it^2 + (-168 - 168i)t^3 - 644t^4 + (-952 + 952i)t^5 + 2240it^6 + (2136 + 2136i)t^7 + 3334t^8 + \dots$ .

Table.

| order = 1, $c_0 = 1$ , $ C_{Sp(8,2)}(c_0)  = 47377612800$ |           |        |               |        |               |          |                 |      |
|-----------------------------------------------------------|-----------|--------|---------------|--------|---------------|----------|-----------------|------|
| 2                                                         | $8 - 8i$  | $-64i$ | $-168 - 168i$ | $-644$ | $-952 + 952i$ | $2240i$  | $2136 + 2136i$  | 3334 |
| 504                                                       | 0         | 0      | 0             | -4     | 0             | 0        | 0               | 6    |
| 2                                                         | $8 + 8i$  | $64i$  | $-168 + 168i$ | $-644$ | $-952 - 952i$ | $-2240i$ | $2136 - 2136i$  | 3334 |
| 256                                                       | 0         | $8i$   | 0             | -28    | 0             | $-56i$   | 0               | 70   |
| 256                                                       | 0         | $-8i$  | 0             | -28    | 0             | $56i$    | 0               | 70   |
| 2                                                         | $-8 - 8i$ | $64i$  | $168 - 168i$  | $-644$ | $952 + 952i$  | $-2240i$ | $-2136 + 2136i$ | 3334 |
| 2                                                         | $-8 + 8i$ | $-64i$ | $168 + 168i$  | $-644$ | $952 - 952i$  | $2240i$  | $-2136 - 2136i$ | 3334 |

| order = 2, $c_1 = x_{0001}$ , $ C_{Sp(8,2)}(c_1)  = 185794560$ |           |        |               |        |               |          |                 |      |
|----------------------------------------------------------------|-----------|--------|---------------|--------|---------------|----------|-----------------|------|
| 2                                                              | $8 - 8i$  | $-64i$ | $-168 - 168i$ | $-644$ | $-952 + 952i$ | $2240i$  | $2136 + 2136i$  | 3334 |
| 504                                                            | 0         | 0      | 0             | -4     | 0             | 0        | 0               | 6    |
| 2                                                              | $8 + 8i$  | $64i$  | $-168 + 168i$ | $-644$ | $-952 - 952i$ | $-2240i$ | $2136 - 2136i$  | 3334 |
| 256                                                            | 0         | $8i$   | 0             | -28    | 0             | $-56i$   | 0               | 70   |
| 256                                                            | 0         | $-8i$  | 0             | -28    | 0             | $56i$    | 0               | 70   |
| 2                                                              | $-8 - 8i$ | $64i$  | $168 - 168i$  | $-644$ | $952 + 952i$  | $-2240i$ | $-2136 + 2136i$ | 3334 |
| 2                                                              | $-8 + 8i$ | $-64i$ | $168 + 168i$  | $-644$ | $952 - 952i$  | $2240i$  | $-2136 - 2136i$ | 3334 |

| order = 2, $c_2 = x_{0100}$ , $ C_{Sp(8,2)}(c_2)  = 8847360$ |       |     |        |     |         |     |        |     |
|--------------------------------------------------------------|-------|-----|--------|-----|---------|-----|--------|-----|
| 4                                                            | 8     | 24  | 24     | -36 | -120    | -88 | 88     | 198 |
| 120                                                          | 0     | -8  | 0      | 28  | 0       | -56 | 0      | 70  |
| 768                                                          | 0     | 0   | 0      | -4  | 0       | 0   | 0      | 6   |
| 120                                                          | 0     | 8   | 0      | 28  | 0       | 56  | 0      | 70  |
| 4                                                            | -8    | 24  | -24    | -36 | 120     | -88 | -88    | 198 |
| 4                                                            | $-8i$ | -24 | $24i$  | -36 | $120i$  | 88  | $88i$  | 198 |
| 4                                                            | $8i$  | -24 | $-24i$ | -36 | $-120i$ | 88  | $-88i$ | 198 |

| order = 2, $c_3 = x_{0100}x_{0121}$ , $ C_{Sp(8,2)}(c_3)  = 2949120$ |       |     |        |     |         |      |         |     |
|----------------------------------------------------------------------|-------|-----|--------|-----|---------|------|---------|-----|
| 4                                                                    | 8     | 32  | 88     | 188 | 328     | 480  | 600     | 646 |
| 240                                                                  | 0     | 0   | 0      | -4  | 0       | 0    | 0       | 6   |
| 128                                                                  | 0     | -8  | 0      | 28  | 0       | -56  | 0       | 70  |
| 128                                                                  | 0     | 8   | 0      | 28  | 0       | 56   | 0       | 70  |
| 4                                                                    | -8    | 32  | -88    | 188 | -328    | 480  | -600    | 646 |
| 512                                                                  | 0     | 0   | 0      | 4   | 0       | 0    | 0       | 6   |
| 4                                                                    | $-8i$ | -32 | $88i$  | 188 | $-328i$ | -480 | $600i$  | 646 |
| 4                                                                    | $8i$  | -32 | $-88i$ | 188 | $328i$  | -480 | $-600i$ | 646 |

| order = 2, $c_4 = x_{0100}x_{0121}x_{1110}$ , $ C_{Sp(8,2)}(c_4)  = 49152$ |       |    |        |    |       |     |        |     |
|----------------------------------------------------------------------------|-------|----|--------|----|-------|-----|--------|-----|
| 16                                                                         | 4     | 8  | 12     | 12 | 4     | -8  | -20    | -26 |
| 32                                                                         | 0     | -8 | 0      | 28 | 0     | -56 | 0      | 70  |
| 384                                                                        | 0     | 0  | 0      | -4 | 0     | 0   | 0      | 6   |
| 16                                                                         | -4    | 8  | -12    | 12 | -4    | -8  | 20     | -26 |
| 512                                                                        | 0     | 0  | 0      | 4  | 0     | 0   | 0      | 6   |
| 32                                                                         | 0     | 8  | 0      | 28 | 0     | 56  | 0      | 70  |
| 16                                                                         | $-4i$ | -8 | $12i$  | 12 | $-4i$ | 8   | $-20i$ | -26 |
| 16                                                                         | $4i$  | -8 | $-12i$ | 12 | $4i$  | 8   | $20i$  | -26 |

| order = 2, $c_5 = x_{0100}x_{1110}$ , $ C_{SP(8,2)}(c_5)  = 737280$ |     |   |      |     |      |     |      |     |
|---------------------------------------------------------------------|-----|---|------|-----|------|-----|------|-----|
| 16                                                                  | 4   | 0 | -20  | -20 | 36   | 64  | -20  | -90 |
| 960                                                                 | 0   | 0 | 0    | -4  | 0    | 0   | 0    | 6   |
| 16                                                                  | -4  | 0 | 20   | -20 | -36  | 64  | 20   | -90 |
| 16                                                                  | -4i | 0 | -20i | -20 | -36i | -64 | -20i | -90 |
| 16                                                                  | 4i  | 0 | 20i  | -20 | 36i  | -64 | 20i  | -90 |

| order = 2, $c_6 = x_{0100}x_{0001}$ , $ C_{SP(8,2)}(c_6)  = 147456$ |         |      |           |     |           |      |           |    |
|---------------------------------------------------------------------|---------|------|-----------|-----|-----------|------|-----------|----|
| 8                                                                   | 4 - 4i  | -16i | -20 - 20i | -36 | -28 + 28i | 48i  | 44 + 44i  | 70 |
| 480                                                                 | 0       | 0    | 0         | -4  | 0         | 0    | 0         | 6  |
| 8                                                                   | 4 + 4i  | 16i  | -20 + 20i | -36 | -28 - 28i | -48i | 44 - 44i  | 70 |
| 64                                                                  | 0       | 8i   | 0         | -28 | 0         | -56i | 0         | 70 |
| 384                                                                 | 0       | 0    | 0         | 4   | 0         | 0    | 0         | 6  |
| 64                                                                  | 0       | -8i  | 0         | -28 | 0         | 56i  | 0         | 70 |
| 8                                                                   | -4 + 4i | -16i | 20 + 20i  | -36 | 28 - 28i  | 48i  | -44 - 44i | 70 |
| 8                                                                   | -4 - 4i | 16i  | 20 - 20i  | -36 | 28 + 28i  | -48i | -44 + 44i | 70 |

| order = 3, $c_7 = n(2, 3, 1, 4)$ , $ C_{SP(8,2)}(c_7)  = 12960$ |     |    |     |    |      |     |      |    |
|-----------------------------------------------------------------|-----|----|-----|----|------|-----|------|----|
| 16                                                              | 4   | 6  | 8   | 17 | 24   | 22  | 28   | 36 |
| 480                                                             | 0   | -2 | 0   | 1  | 0    | -2  | 0    | 4  |
| 480                                                             | 0   | 2  | 0   | 1  | 0    | 2   | 0    | 4  |
| 16                                                              | -4  | 6  | -8  | 17 | -24  | 22  | -28  | 36 |
| 16                                                              | -4i | -6 | 8i  | 17 | -24i | -22 | 28i  | 36 |
| 16                                                              | 4i  | -6 | -8i | 17 | 24i  | -22 | -28i | 36 |

| order = 3, $c_8 = (x_{0011}n(-4, -1, -2, -3))^5$ , $ C_{SP(8,2)}(c_8)  = 77760$ |    |   |     |   |   |     |      |   |
|---------------------------------------------------------------------------------|----|---|-----|---|---|-----|------|---|
| 256                                                                             | -i | 0 | 5i  | 5 | 0 | -10 | 10i  | 0 |
| 256                                                                             | i  | 0 | -5i | 5 | 0 | -10 | -10i | 0 |
| 256                                                                             | 1  | 0 | 5   | 5 | 0 | 10  | 10   | 0 |
| 256                                                                             | -1 | 0 | -5  | 5 | 0 | 10  | -10  | 0 |

| order = 3, $c_9 = x_{0001}n(1, 2, 3, -4)$ , $ C_{SP(8,2)}(c_9)  = 4354560$ |     |     |       |     |       |      |        |      |
|----------------------------------------------------------------------------|-----|-----|-------|-----|-------|------|--------|------|
| 4                                                                          | 8   | 36  | 112   | 266 | 504   | 784  | 1016   | 1107 |
| 504                                                                        | 0   | 4   | 0     | 10  | 0     | 16   | 0      | 19   |
| 4                                                                          | -8i | -36 | 112i  | 266 | -504i | -784 | 1016i  | 1107 |
| 504                                                                        | 0   | -4  | 0     | 10  | 0     | -16  | 0      | 19   |
| 4                                                                          | -8  | 36  | -112  | 266 | -504  | 784  | -1016  | 1107 |
| 4                                                                          | 8i  | -36 | -112i | 266 | 504i  | -784 | -1016i | 1107 |

| order = 3, $c_{10} = x_{0001}n(-2, 3, -1, -4)$ , $ C_{SP(8,2)}(c_{10})  = 3888$ |     |    |     |    |      |    |     |    |
|---------------------------------------------------------------------------------|-----|----|-----|----|------|----|-----|----|
| 64                                                                              | 2i  | -3 | 2i  | -7 | -12i | 2  | -8i | 18 |
| 384                                                                             | 0   | -1 | 0   | 1  | 0    | 2  | 0   | -2 |
| 384                                                                             | 0   | 1  | 0   | 1  | 0    | -2 | 0   | -2 |
| 64                                                                              | 2   | 3  | -2  | -7 | -12  | -2 | 8   | 18 |
| 64                                                                              | -2i | -3 | -2i | -7 | 12i  | 2  | 8i  | 18 |
| 64                                                                              | -2  | 3  | 2   | -7 | 12   | -2 | -8  | 18 |

| order = 4, $c_{11} = x_{0001}x_{1110}$ , $ C_{Sp(8,2)}(c_{11})  = 92160$ |           |        |           |    |             |        |             |    |
|--------------------------------------------------------------------------|-----------|--------|-----------|----|-------------|--------|-------------|----|
| 8                                                                        | $4 - 4i$  | $-12i$ | $-4 - 4i$ | 24 | $36 - 36i$  | $-28i$ | $28 + 28i$  | 78 |
| 240                                                                      | 0         | $4i$   | 0         | -8 | 0           | $-12i$ | 0           | 14 |
| 8                                                                        | $4 + 4i$  | $12i$  | $-4 + 4i$ | 24 | $36 + 36i$  | $28i$  | $28 - 28i$  | 78 |
| 240                                                                      | 0         | $-4i$  | 0         | -8 | 0           | $12i$  | 0           | 14 |
| 8                                                                        | $-4 + 4i$ | $-12i$ | $4 + 4i$  | 24 | $-36 + 36i$ | $-28i$ | $-28 - 28i$ | 78 |
| 8                                                                        | $-4 - 4i$ | $12i$  | $4 - 4i$  | 24 | $-36 - 36i$ | $28i$  | $-28 + 28i$ | 78 |
| 512                                                                      | 0         | 0      | 0         | 4  | 0           | 0      | 0           | 6  |

| order = 4, $c_{12} = x_{0001}x_{1000}x_{1110}x_{1111}$ , $ C_{Sp(8,2)}(c_{12})  = 92160$ |           |        |             |      |               |         |               |     |
|------------------------------------------------------------------------------------------|-----------|--------|-------------|------|---------------|---------|---------------|-----|
| 8                                                                                        | $4 - 4i$  | $-20i$ | $-36 - 36i$ | -104 | $-124 + 124i$ | $252i$  | $220 + 220i$  | 334 |
| 240                                                                                      | 0         | $4i$   | 0           | -8   | 0             | $-12i$  | 0             | 14  |
| 240                                                                                      | 0         | $-4i$  | 0           | -8   | 0             | $12i$   | 0             | 14  |
| 8                                                                                        | $4 + 4i$  | $20i$  | $-36 + 36i$ | -104 | $-124 - 124i$ | $-252i$ | $220 - 220i$  | 334 |
| 512                                                                                      | 0         | 0      | 0           | -4   | 0             | 0       | 0             | 6   |
| 8                                                                                        | $-4 - 4i$ | $20i$  | $36 - 36i$  | -104 | $124 + 124i$  | $-252i$ | $-220 + 220i$ | 334 |
| 8                                                                                        | $-4 + 4i$ | $-20i$ | $36 + 36i$  | -104 | $124 - 124i$  | $252i$  | $-220 - 220i$ | 334 |

| order = 4, $c_{13} = x_{0100}x_{0121}x_{1100}x_{2221}$ , $ C_{Sp(8,2)}(c_{13})  = 36864$ |       |     |        |    |        |      |         |     |
|------------------------------------------------------------------------------------------|-------|-----|--------|----|--------|------|---------|-----|
| 16                                                                                       | 4     | 12  | 28     | 52 | 84     | 116  | 140     | 150 |
| 16                                                                                       | $4i$  | -12 | $-28i$ | 52 | $84i$  | -116 | $-140i$ | 150 |
| 16                                                                                       | $-4i$ | -12 | $28i$  | 52 | $-84i$ | -116 | $140i$  | 150 |
| 96                                                                                       | 0     | 4   | 0      | 4  | 0      | -4   | 0       | -10 |
| 96                                                                                       | 0     | -4  | 0      | 4  | 0      | 4    | 0       | -10 |
| 16                                                                                       | -4    | 12  | -28    | 52 | -84    | 116  | -140    | 150 |
| 768                                                                                      | 0     | 0   | 0      | 4  | 0      | 0    | 0       | 6   |

| order = 4, $c_{14} = x_{0100}x_{0121}x_{1100}$ , $ C_{Sp(8,2)}(c_{14})  = 12288$ |       |    |       |     |        |    |        |     |
|----------------------------------------------------------------------------------|-------|----|-------|-----|--------|----|--------|-----|
| 16                                                                               | 4     | 4  | -4    | -12 | -12    | -4 | 12     | 22  |
| 16                                                                               | -4    | 4  | 4     | -12 | 12     | -4 | -12    | 22  |
| 96                                                                               | 0     | -4 | 0     | 4   | 0      | 4  | 0      | -10 |
| 96                                                                               | 0     | 4  | 0     | 4   | 0      | -4 | 0      | -10 |
| 256                                                                              | 0     | 0  | 0     | 4   | 0      | 0  | 0      | 6   |
| 512                                                                              | 0     | 0  | 0     | -4  | 0      | 0  | 0      | 6   |
| 16                                                                               | $-4i$ | -4 | $-4i$ | -12 | $12i$  | 4  | $12i$  | 22  |
| 16                                                                               | $4i$  | -4 | $4i$  | -12 | $-12i$ | 4  | $-12i$ | 22  |

| order = 4, $c_{15} = x_{0100}x_{0001}x_{1110}$ , $ C_{Sp(8,2)}(c_{15})  = 6144$ |           |   |           |   |           |        |           |     |
|---------------------------------------------------------------------------------|-----------|---|-----------|---|-----------|--------|-----------|-----|
| 32                                                                              | $2 - 2i$  | 0 | $6 + 6i$  | 8 | $-6 + 6i$ | $16i$  | $-2 - 2i$ | -18 |
| 384                                                                             | 0         | 0 | 0         | 0 | 0         | 0      | 0         | -2  |
| 32                                                                              | $2 + 2i$  | 0 | $6 - 6i$  | 8 | $-6 - 6i$ | $-16i$ | $-2 + 2i$ | -18 |
| 32                                                                              | $-2 + 2i$ | 0 | $-6 - 6i$ | 8 | $6 - 6i$  | $16i$  | $2 + 2i$  | -18 |
| 32                                                                              | $-2 - 2i$ | 0 | $-6 + 6i$ | 8 | $6 + 6i$  | $-16i$ | $2 - 2i$  | -18 |
| 512                                                                             | 0         | 0 | 0         | 4 | 0         | 0      | 0         | 6   |

| order = 4, $c_{16} = x_{0100}x_{0001}x_{1110}x_{1111}$ , $ C_{Sp(8,2)}(c_{16})  = 6144$ |           |       |             |     |             |        |             |    |
|-----------------------------------------------------------------------------------------|-----------|-------|-------------|-----|-------------|--------|-------------|----|
| 32                                                                                      | $2 - 2i$  | $-8i$ | $-10 - 10i$ | -24 | $-22 + 22i$ | $40i$  | $30 + 30i$  | 46 |
| 384                                                                                     | 0         | 0     | 0           | 0   | 0           | 0      | 0           | -2 |
| 32                                                                                      | $2 + 2i$  | $8i$  | $-10 + 10i$ | -24 | $-22 - 22i$ | $-40i$ | $30 - 30i$  | 46 |
| 32                                                                                      | $-2 - 2i$ | $8i$  | $10 - 10i$  | -24 | $22 + 22i$  | $-40i$ | $-30 + 30i$ | 46 |
| 32                                                                                      | $-2 + 2i$ | $-8i$ | $10 + 10i$  | -24 | $22 - 22i$  | $40i$  | $-30 - 30i$ | 46 |
| 512                                                                                     | 0         | 0     | 0           | -4  | 0           | 0      | 0           | 6  |



| order = 4, $c_{17} = x_{0100}x_{0121}x_{1000}$ , $ C_{Sp(8,2)}(c_{17})  = 3072$ |     |    |      |    |      |     |      |    |
|---------------------------------------------------------------------------------|-----|----|------|----|------|-----|------|----|
| 16                                                                              | 4   | 8  | 12   | 16 | 20   | 24  | 28   | 30 |
| 128                                                                             | 0   | -4 | 0    | 8  | 0    | -12 | 0    | 14 |
| 192                                                                             | 0   | 0  | 0    | 0  | 0    | 0   | 0    | -2 |
| 128                                                                             | 0   | 4  | 0    | 8  | 0    | 12  | 0    | 14 |
| 16                                                                              | -4  | 8  | -12  | 16 | -20  | 24  | -28  | 30 |
| 256                                                                             | 0   | 0  | 0    | 4  | 0    | 0   | 0    | 6  |
| 256                                                                             | 0   | 0  | 0    | -4 | 0    | 0   | 0    | 6  |
| 16                                                                              | -4i | -8 | 12i  | 16 | -20i | -24 | 28i  | 30 |
| 16                                                                              | 4i  | -8 | -12i | 16 | 20i  | -24 | -28i | 30 |

| order = 4, $c_{18} = x_{0100}x_{0001}x_{1121}$ , $ C_{Sp(8,2)}(c_{18})  = 3072$ |           |     |           |    |           |      |           |     |
|---------------------------------------------------------------------------------|-----------|-----|-----------|----|-----------|------|-----------|-----|
| 32                                                                              | $2 - 2i$  | -4i | $-2 - 2i$ | -4 | $-6 + 6i$ | 12i  | $6 + 6i$  | 6   |
| 32                                                                              | $-2 + 2i$ | -4i | $2 + 2i$  | -4 | $6 - 6i$  | 12i  | $-6 - 6i$ | 6   |
| 32                                                                              | $2 + 2i$  | 4i  | $-2 + 2i$ | -4 | $-6 - 6i$ | -12i | $6 - 6i$  | 6   |
| 32                                                                              | $-2 - 2i$ | 4i  | $2 - 2i$  | -4 | $6 + 6i$  | -12i | $-6 + 6i$ | 6   |
| 64                                                                              | 0         | 4i  | 0         | -4 | 0         | 4i   | 0         | -10 |
| 64                                                                              | 0         | -4i | 0         | -4 | 0         | -4i  | 0         | -10 |
| 384                                                                             | 0         | 0   | 0         | -4 | 0         | 0    | 0         | 6   |
| 384                                                                             | 0         | 0   | 0         | 4  | 0         | 0    | 0         | 6   |

| order = 4, $c_{19} = x_{0100}x_{0010}x_{0011}x_{2221}$ , $ C_{Sp(8,2)}(c_{19})  = 1024$ |           |     |           |    |           |      |           |    |
|-----------------------------------------------------------------------------------------|-----------|-----|-----------|----|-----------|------|-----------|----|
| 32                                                                                      | $2 - 2i$  | -4i | $-2 - 2i$ | 0  | $2 - 2i$  | -4i  | $-2 - 2i$ | -2 |
| 32                                                                                      | $2 + 2i$  | 4i  | $-2 + 2i$ | 0  | $2 + 2i$  | 4i   | $-2 + 2i$ | -2 |
| 256                                                                                     | 0         | 0   | 0         | 0  | 0         | 0    | 0         | -2 |
| 32                                                                                      | $-2 + 2i$ | -4i | $2 + 2i$  | 0  | $-2 + 2i$ | -4i  | $2 + 2i$  | -2 |
| 32                                                                                      | $-2 - 2i$ | 4i  | $2 - 2i$  | 0  | $-2 - 2i$ | 4i   | $2 - 2i$  | -2 |
| 256                                                                                     | 0         | 0   | 0         | 4  | 0         | 0    | 0         | 6  |
| 256                                                                                     | 0         | 0   | 0         | -4 | 0         | 0    | 0         | 6  |
| 64                                                                                      | 0         | 4i  | 0         | -8 | 0         | -12i | 0         | 14 |
| 64                                                                                      | 0         | -4i | 0         | -8 | 0         | 12i  | 0         | 14 |

| order = 4, $c_{20} = x_{0100}x_{0010}x_{1000}$ , $ C_{Sp(8,2)}(c_{20})  = 768$ |     |   |     |    |     |   |     |    |
|--------------------------------------------------------------------------------|-----|---|-----|----|-----|---|-----|----|
| 64                                                                             | 2   | 0 | -2  | -4 | -6  | 0 | 6   | 6  |
| 768                                                                            | 0   | 0 | 0   | 0  | 0   | 0 | 0   | -2 |
| 64                                                                             | -2  | 0 | 2   | -4 | 6   | 0 | -6  | 6  |
| 64                                                                             | -2i | 0 | -2i | -4 | 6i  | 0 | 6i  | 6  |
| 64                                                                             | 2i  | 0 | 2i  | -4 | -6i | 0 | -6i | 6  |

| order = 4, $c_{21} = x_{0100}x_{0010}x_{0011}x_{1110}$ , $ C_{Sp(8,2)}(c_{21})  = 512$ |     |   |     |    |     |   |     |    |
|----------------------------------------------------------------------------------------|-----|---|-----|----|-----|---|-----|----|
| 64                                                                                     | 2   | 0 | -2  | 0  | 2   | 0 | -2  | -2 |
| 64                                                                                     | -2  | 0 | 2   | 0  | -2  | 0 | 2   | -2 |
| 512                                                                                    | 0   | 0 | 0   | 0  | 0   | 0 | 0   | -2 |
| 256                                                                                    | 0   | 0 | 0   | -4 | 0   | 0 | 0   | 6  |
| 64                                                                                     | -2i | 0 | -2i | 0  | -2i | 0 | -2i | -2 |
| 64                                                                                     | 2i  | 0 | 2i  | 0  | 2i  | 0 | 2i  | -2 |

| order = 4, $c_{22} = x_{0100}x_{0010}x_{0011}x_{1110}x_{2221}$ , $ C_{Sp(8,2)}(c_{22})  = 512$ |      |    |     |   |       |     |      |    |
|------------------------------------------------------------------------------------------------|------|----|-----|---|-------|-----|------|----|
| 64                                                                                             | 2    | 4  | 6   | 8 | 10    | 12  | 14   | 14 |
| 64                                                                                             | $2i$ | -4 | -6i | 8 | $10i$ | -12 | -14i | 14 |
| 64                                                                                             | -2i  | -4 | 6i  | 8 | -10i  | -12 | 14i  | 14 |
| 64                                                                                             | -2   | 4  | -6  | 8 | -10   | 12  | -14  | 14 |
| 512                                                                                            | 0    | 0  | 0   | 0 | 0     | 0   | 0    | -2 |
| 256                                                                                            | 0    | 0  | 0   | 4 | 0     | 0   | 0    | 6  |

| order = 5, $c_{23} = x_{0001}x_{0011}n(1, 2, -4, 3)$ , $ C_{Sp(8,2)}(c_{23})  = 3600$ |     |     |      |    |      |     |      |    |
|---------------------------------------------------------------------------------------|-----|-----|------|----|------|-----|------|----|
| 16                                                                                    | -4i | -10 | 20i  | 35 | -52i | -68 | 80i  | 85 |
| 480                                                                                   | 0   | -2  | 0    | 3  | 0    | -4  | 0    | 5  |
| 16                                                                                    | 4   | 10  | 20   | 35 | 52   | 68  | 80   | 85 |
| 480                                                                                   | 0   | 2   | 0    | 3  | 0    | 4   | 0    | 5  |
| 16                                                                                    | 4i  | -10 | -20i | 35 | 52i  | -68 | -80i | 85 |
| 16                                                                                    | -4  | 10  | -20  | 35 | -52  | 68  | -80  | 85 |

| order = 5, $c_{24} = x_{0010}x_{0111}n(-4, -3, -1, -2)$ , $ C_{Sp(8,2)}(c_{24})  = 300$ |    |   |   |   |     |    |   |   |
|-----------------------------------------------------------------------------------------|----|---|---|---|-----|----|---|---|
| 256                                                                                     | -i | 0 | 0 | 0 | -3i | -3 | 0 | 0 |
| 256                                                                                     | i  | 0 | 0 | 0 | 3i  | -3 | 0 | 0 |
| 256                                                                                     | -1 | 0 | 0 | 0 | -3  | 3  | 0 | 0 |
| 256                                                                                     | 1  | 0 | 0 | 0 | 3   | 3  | 0 | 0 |

| order = 6, $c_{25} = n(-4, -2, -1, 3)$ , $ C_{Sp(8,2)}(c_{25})  = 864$ |           |     |   |    |           |     |           |    |
|------------------------------------------------------------------------|-----------|-----|---|----|-----------|-----|-----------|----|
| 256                                                                    | 0         | -2i | 0 | -1 | 0         | 2i  | 0         | 4  |
| 32                                                                     | $2 + 2i$  | 4i  | 0 | 7  | $8 + 8i$  | 4i  | $6 - 6i$  | 16 |
| 384                                                                    | 0         | 0   | 0 | -1 | 0         | 0   | 0         | 0  |
| 256                                                                    | 0         | 2i  | 0 | -1 | 0         | -2i | 0         | 4  |
| 32                                                                     | $-2 - 2i$ | 4i  | 0 | 7  | $-8 - 8i$ | 4i  | $-6 + 6i$ | 16 |
| 32                                                                     | $-2 + 2i$ | -4i | 0 | 7  | $-8 + 8i$ | -4i | $-6 - 6i$ | 16 |
| 32                                                                     | $2 - 2i$  | -4i | 0 | 7  | $8 - 8i$  | -4i | $6 + 6i$  | 16 |

| order = 6, $c_{26} = n(-4, -1, -3, -2)$ , $ C_{Sp(8,2)}(c_{26})  = 96$ |     |    |   |    |     |    |     |   |
|------------------------------------------------------------------------|-----|----|---|----|-----|----|-----|---|
| 128                                                                    | 0   | 2  | 0 | 1  | 0   | 2  | 0   | 4 |
| 512                                                                    | 0   | 0  | 0 | 1  | 0   | 0  | 0   | 0 |
| 64                                                                     | -2i | -2 | 0 | -3 | 4i  | 2  | 2i  | 4 |
| 64                                                                     | 2i  | -2 | 0 | -3 | -4i | 2  | -2i | 4 |
| 64                                                                     | 2   | 2  | 0 | -3 | -4  | -2 | 2   | 4 |
| 64                                                                     | -2  | 2  | 0 | -3 | 4   | -2 | -2  | 4 |
| 128                                                                    | 0   | -2 | 0 | 1  | 0   | -2 | 0   | 4 |

| order = 6, $c_{27} = n(-4, 2, 1, 3)$ , $ C_{Sp(8,2)}(c_{27})  = 288$ |           |     |           |    |           |      |             |    |
|----------------------------------------------------------------------|-----------|-----|-----------|----|-----------|------|-------------|----|
| 256                                                                  | 0         | 2i  | 0         | -1 | 0         | -2i  | 0           | 4  |
| 32                                                                   | $2 - 2i$  | -4i | $-4 - 4i$ | -9 | $-8 + 8i$ | 12i  | $10 + 10i$  | 16 |
| 384                                                                  | 0         | 0   | 0         | -1 | 0         | 0    | 0           | 0  |
| 32                                                                   | $-2 + 2i$ | -4i | $4 + 4i$  | -9 | $8 - 8i$  | 12i  | $-10 - 10i$ | 16 |
| 256                                                                  | 0         | -2i | 0         | -1 | 0         | 2i   | 0           | 4  |
| 32                                                                   | $-2 - 2i$ | 4i  | $4 - 4i$  | -9 | $8 + 8i$  | -12i | $-10 + 10i$ | 16 |
| 32                                                                   | $2 + 2i$  | 4i  | $-4 + 4i$ | -9 | $-8 - 8i$ | -12i | $10 - 10i$  | 16 |

| order = 6, $c_{28} = x_{0011}x_{0021}n(2, -4, 3, -1)$ , $ C_{Sp(8,2)}(c_{28})  = 288$ |     |    |     |   |     |    |     |   |
|---------------------------------------------------------------------------------------|-----|----|-----|---|-----|----|-----|---|
| 64                                                                                    | -2  | 2  | -4  | 5 | -4  | 6  | -6  | 4 |
| 128                                                                                   | 0   | -2 | 0   | 1 | 0   | -2 | 0   | 4 |
| 64                                                                                    | 2i  | -2 | -4i | 5 | 4i  | -6 | -6i | 4 |
| 128                                                                                   | 0   | 2  | 0   | 1 | 0   | 2  | 0   | 4 |
| 64                                                                                    | -2i | -2 | 4i  | 5 | -4i | -6 | 6i  | 4 |
| 512                                                                                   | 0   | 0  | 0   | 1 | 0   | 0  | 0   | 0 |
| 64                                                                                    | 2   | 2  | 4   | 5 | 4   | 6  | 6   | 4 |

| order = 6, $c_{29} = x_{0001}x_{0011}x_{0021}n(1, -2, -4, 3)$ , $ C_{Sp(8,2)}(c_{29})  = 4608$ |     |    |     |    |     |     |      |    |
|------------------------------------------------------------------------------------------------|-----|----|-----|----|-----|-----|------|----|
| 128                                                                                            | 0   | -4 | 0   | 10 | 0   | -16 | 0    | 19 |
| 512                                                                                            | 0   | 0  | 0   | -2 | 0   | 0   | 0    | 3  |
| 16                                                                                             | -4  | 8  | -8  | 2  | 4   | 0   | -12  | 19 |
| 192                                                                                            | 0   | 0  | 0   | 2  | 0   | 0   | 0    | 3  |
| 16                                                                                             | 4   | 8  | 8   | 2  | -4  | 0   | 12   | 19 |
| 16                                                                                             | 4i  | -8 | -8i | 2  | -4i | 0   | -12i | 19 |
| 128                                                                                            | 0   | 4  | 0   | 10 | 0   | 16  | 0    | 19 |
| 16                                                                                             | -4i | -8 | 8i  | 2  | 4i  | 0   | 12i  | 19 |

| order = 6, $c_{30} = x_{0021}n(1, 4, -3, 2)$ , $ C_{Sp(8,2)}(c_{30})  = 13824$ |     |     |      |    |      |     |      |    |
|--------------------------------------------------------------------------------|-----|-----|------|----|------|-----|------|----|
| 16                                                                             | -4i | -12 | 24i  | 42 | -60i | -80 | 92i  | 99 |
| 96                                                                             | 0   | -4  | 0    | 10 | 0    | -16 | 0    | 19 |
| 768                                                                            | 0   | 0   | 0    | 2  | 0    | 0   | 0    | 3  |
| 16                                                                             | -4  | 12  | -24  | 42 | -60  | 80  | -92  | 99 |
| 96                                                                             | 0   | 4   | 0    | 10 | 0    | 16  | 0    | 19 |
| 16                                                                             | 4i  | -12 | -24i | 42 | 60i  | -80 | -92i | 99 |
| 16                                                                             | 4   | 12  | 24   | 42 | 60   | 80  | 92   | 99 |

| order = 6, $c_{31} = x_{0001}x_{0011}x_{0021}n(-4, -3, -2, 1)$ , $ C_{Sp(8,2)}(c_{31})  = 288$ |     |   |     |    |   |    |     |   |
|------------------------------------------------------------------------------------------------|-----|---|-----|----|---|----|-----|---|
| 768                                                                                            | 0   | 0 | 0   | -1 | 0 | 0  | 0   | 0 |
| 64                                                                                             | 2   | 0 | -4  | -5 | 0 | 4  | 2   | 0 |
| 64                                                                                             | 2i  | 0 | 4i  | -5 | 0 | -4 | -2i | 0 |
| 64                                                                                             | -2  | 0 | 4   | -5 | 0 | 4  | -2  | 0 |
| 64                                                                                             | -2i | 0 | -4i | -5 | 0 | -4 | 2i  | 0 |

| order = 6, $c_{32} = x_{0010}x_{0001}x_{0121}n(1, 4, 3, -2)$ , $ C_{Sp(8,2)}(c_{32})  = 4320$ |     |    |   |     |      |     |      |    |
|-----------------------------------------------------------------------------------------------|-----|----|---|-----|------|-----|------|----|
| 480                                                                                           | 0   | 2  | 0 | 1   | 0    | 2   | 0    | 4  |
| 16                                                                                            | -4  | 6  | 0 | -15 | 24   | -10 | -20  | 36 |
| 480                                                                                           | 0   | -2 | 0 | 1   | 0    | -2  | 0    | 4  |
| 16                                                                                            | -4i | -6 | 0 | -15 | 24i  | 10  | 20i  | 36 |
| 16                                                                                            | 4i  | -6 | 0 | -15 | -24i | 10  | -20i | 36 |
| 16                                                                                            | 4   | 6  | 0 | -15 | -24  | -10 | 20   | 36 |

| order = 6, $c_{33} = x_{0121}n(-1, -3, 2, -4)$ , $ C_{Sp(8,2)}(c_{33})  = 1152$ |         |     |         |     |         |      |         |    |
|---------------------------------------------------------------------------------|---------|-----|---------|-----|---------|------|---------|----|
| 384                                                                             | 0       | 0   | 0       | 2   | 0       | 0    | 0       | 3  |
| 384                                                                             | 0       | 0   | 0       | -2  | 0       | 0    | 0       | 3  |
| 64                                                                              | 0       | -4i | 0       | -10 | 0       | 16i  | 0       | 19 |
| 32                                                                              | 2 + 2i  | 4i  | -4 + 4i | -6  | -2 - 2i | 0    | -2 + 2i | -5 |
| 32                                                                              | -2 - 2i | 4i  | 4 - 4i  | -6  | 2 + 2i  | 0    | 2 - 2i  | -5 |
| 32                                                                              | -2 + 2i | -4i | 4 + 4i  | -6  | 2 - 2i  | 0    | 2 + 2i  | -5 |
| 32                                                                              | 2 - 2i  | -4i | -4 - 4i | -6  | -2 + 2i | 0    | -2 - 2i | -5 |
| 64                                                                              | 0       | 4i  | 0       | -10 | 0       | -16i | 0       | 19 |

| order = 6, $c_{34} = x_{0001}x_{0011}x_{0111}x_{0021}x_{0121}n(1, 2, 3, -4)$ , $ C_{Sp(8,2)}(c_{34})  = 69120$ |           |        |             |       |             |         |               |     |
|----------------------------------------------------------------------------------------------------------------|-----------|--------|-------------|-------|-------------|---------|---------------|-----|
| 480                                                                                                            | 0         | 0      | 0           | 2     | 0           | 0       | 0             | 3   |
| 8                                                                                                              | $4 - 4i$  | $-16i$ | $-24 - 24i$ | $-62$ | $-68 + 68i$ | $128i$  | $108 + 108i$  | 163 |
| 8                                                                                                              | $4 + 4i$  | $16i$  | $-24 + 24i$ | $-62$ | $-68 - 68i$ | $-128i$ | $108 - 108i$  | 163 |
| 8                                                                                                              | $-4 - 4i$ | $16i$  | $24 - 24i$  | $-62$ | $68 + 68i$  | $-128i$ | $-108 + 108i$ | 163 |
| 8                                                                                                              | $-4 + 4i$ | $-16i$ | $24 + 24i$  | $-62$ | $68 - 68i$  | $128i$  | $-108 - 108i$ | 163 |
| 256                                                                                                            | 0         | $4i$   | 0           | $-10$ | 0           | $-16i$  | 0             | 19  |
| 256                                                                                                            | 0         | $-4i$  | 0           | $-10$ | 0           | $16i$   | 0             | 19  |

| order = 6, $c_{35} = x_{0001}x_{0111}x_{0021}x_{0121}n(3, -4, -2, 1)$ , $ C_{Sp(8,2)}(c_{35})  = 864$ |       |   |   |      |   |      |       |   |
|-------------------------------------------------------------------------------------------------------|-------|---|---|------|---|------|-------|---|
| 768                                                                                                   | 0     | 0 | 0 | $-1$ | 0 | 0    | 0     | 0 |
| 64                                                                                                    | $-2$  | 0 | 0 | 3    | 0 | $-4$ | 2     | 0 |
| 64                                                                                                    | 2     | 0 | 0 | 3    | 0 | $-4$ | $-2$  | 0 |
| 64                                                                                                    | $2i$  | 0 | 0 | 3    | 0 | 4    | $2i$  | 0 |
| 64                                                                                                    | $-2i$ | 0 | 0 | 3    | 0 | 4    | $-2i$ | 0 |

| order = 6, $c_{36} = x_{0001}x_{0011}x_{0111}x_{0021}x_{0121}n(4, -2, -1, -3)$ , $ C_{Sp(8,2)}(c_{36})  = 144$ |          |      |          |      |           |       |           |      |
|----------------------------------------------------------------------------------------------------------------|----------|------|----------|------|-----------|-------|-----------|------|
| 128                                                                                                            | $-1 - i$ | $i$  | $-1 + i$ | 3    | $-2 - 2i$ | 0     | $-2 + 2i$ | 4    |
| 256                                                                                                            | 0        | $i$  | 0        | $-1$ | 0         | $2i$  | 0         | $-2$ |
| 128                                                                                                            | $1 - i$  | $-i$ | $1 + i$  | 3    | $2 - 2i$  | 0     | $2 + 2i$  | 4    |
| 256                                                                                                            | 0        | $-i$ | 0        | $-1$ | 0         | $-2i$ | 0         | $-2$ |
| 128                                                                                                            | $1 + i$  | $i$  | $1 - i$  | 3    | $2 + 2i$  | 0     | $2 - 2i$  | 4    |
| 128                                                                                                            | $-1 + i$ | $-i$ | $-1 - i$ | 3    | $-2 + 2i$ | 0     | $-2 - 2i$ | 4    |

| order = 6, $c_{37} = (x_{0001}n(-4, -2, -1, -3))^3$ , $ C_{Sp(8,2)}(c_{37})  = 1296$ |          |      |           |      |           |       |           |      |
|--------------------------------------------------------------------------------------|----------|------|-----------|------|-----------|-------|-----------|------|
| 256                                                                                  | 0        | $i$  | 0         | $-1$ | 0         | $2i$  | 0         | $-2$ |
| 256                                                                                  | 0        | $-i$ | 0         | $-1$ | 0         | $-2i$ | 0         | $-2$ |
| 128                                                                                  | $-1 - i$ | $i$  | $3 - 3i$  | $-5$ | $2 + 2i$  | $-8i$ | $-6 + 6i$ | 4    |
| 128                                                                                  | $-1 + i$ | $-i$ | $3 + 3i$  | $-5$ | $2 - 2i$  | $8i$  | $-6 - 6i$ | 4    |
| 128                                                                                  | $1 + i$  | $i$  | $-3 + 3i$ | $-5$ | $-2 - 2i$ | $-8i$ | $6 - 6i$  | 4    |
| 128                                                                                  | $1 - i$  | $-i$ | $-3 - 3i$ | $-5$ | $-2 + 2i$ | $8i$  | $6 + 6i$  | 4    |

| order = 6, $c_{38} = (x_{0001}x_{0011}x_{0111}x_{0121}n(1, -4, 2, 3))^2$ , $ C_{Sp(8,2)}(c_{38})  = 432$ |       |      |       |   |        |       |        |      |
|----------------------------------------------------------------------------------------------------------|-------|------|-------|---|--------|-------|--------|------|
| 64                                                                                                       | 2     | 3    | 6     | 9 | 12     | 14    | 16     | 18   |
| 384                                                                                                      | 0     | 1    | 0     | 1 | 0      | $-2$  | 0      | $-2$ |
| 64                                                                                                       | $-2i$ | $-3$ | $6i$  | 9 | $-12i$ | $-14$ | $16i$  | 18   |
| 384                                                                                                      | 0     | $-1$ | 0     | 1 | 0      | 2     | 0      | $-2$ |
| 64                                                                                                       | $-2$  | 3    | $-6$  | 9 | $-12$  | 14    | $-16$  | 18   |
| 64                                                                                                       | $2i$  | $-3$ | $-6i$ | 9 | $12i$  | $-14$ | $-16i$ | 18   |

| order = 6, $c_{39} = (x_{0001}n(4, 1, -2, 3))^2$ , $ C_{Sp(8,2)}(c_{39})  = 288$ |      |   |      |   |   |      |       |   |
|----------------------------------------------------------------------------------|------|---|------|---|---|------|-------|---|
| 256                                                                              | $-1$ | 0 | $-1$ | 1 | 0 | $-2$ | 2     | 0 |
| 256                                                                              | $i$  | 0 | $-i$ | 1 | 0 | 2    | $2i$  | 0 |
| 256                                                                              | 1    | 0 | 1    | 1 | 0 | $-2$ | $-2$  | 0 |
| 256                                                                              | $-i$ | 0 | $i$  | 1 | 0 | 2    | $-2i$ | 0 |

| order = 6, $c_{40} = (x_{0010}x_{0011}x_{0111}x_{0021}x_{0121}n(-2, -4, 1, 3))^2$ , $ C_{Sp(8,2)}(c_{40})  = 1728$ |      |   |       |      |   |      |       |   |
|--------------------------------------------------------------------------------------------------------------------|------|---|-------|------|---|------|-------|---|
| 256                                                                                                                | 1    | 0 | $-3$  | $-3$ | 0 | 2    | 2     | 0 |
| 256                                                                                                                | $-i$ | 0 | $-3i$ | $-3$ | 0 | $-2$ | $2i$  | 0 |
| 256                                                                                                                | $i$  | 0 | $3i$  | $-3$ | 0 | $-2$ | $-2i$ | 0 |
| 256                                                                                                                | $-1$ | 0 | 3     | $-3$ | 0 | 2    | $-2$  | 0 |

| order = 7, $c_{41} = x_{0011}n(-4, 2, 1, -3)$ , $ C_{SP(8,2)}(c_{41})  = 42$ |     |    |   |   |   |   |     |   |
|------------------------------------------------------------------------------|-----|----|---|---|---|---|-----|---|
| 64                                                                           | -2  | 1  | 0 | 0 | 0 | 0 | -2  | 4 |
| 384                                                                          | 0   | -1 | 0 | 0 | 0 | 0 | 0   | 0 |
| 64                                                                           | 2   | 1  | 0 | 0 | 0 | 0 | 2   | 4 |
| 384                                                                          | 0   | 1  | 0 | 0 | 0 | 0 | 0   | 0 |
| 64                                                                           | -2i | -1 | 0 | 0 | 0 | 0 | 2i  | 4 |
| 64                                                                           | 2i  | -1 | 0 | 0 | 0 | 0 | -2i | 4 |

| order = 8, $c_{42} = x_{0100}x_{0010}x_{0001}$ , $ C_{SP(8,2)}(c_{42})  = 384$ |           |       |           |   |           |       |           |    |
|--------------------------------------------------------------------------------|-----------|-------|-----------|---|-----------|-------|-----------|----|
| 32                                                                             | $2 - 2i$  | $-2i$ | $2 + 2i$  | 8 | $6 - 6i$  | $-2i$ | $6 + 6i$  | 14 |
| 192                                                                            | 0         | $2i$  | 0         | 0 | 0         | $2i$  | 0         | -2 |
| 32                                                                             | $2 + 2i$  | $2i$  | $2 - 2i$  | 8 | $6 + 6i$  | $2i$  | $6 - 6i$  | 14 |
| 192                                                                            | 0         | $-2i$ | 0         | 0 | 0         | $-2i$ | 0         | -2 |
| 32                                                                             | $-2 + 2i$ | $-2i$ | $-2 - 2i$ | 8 | $-6 + 6i$ | $-2i$ | $-6 - 6i$ | 14 |
| 32                                                                             | $-2 - 2i$ | $2i$  | $-2 + 2i$ | 8 | $-6 - 6i$ | $2i$  | $-6 + 6i$ | 14 |
| 512                                                                            | 0         | 0     | 0         | 0 | 0         | 0     | 0         | 2  |

| order = 8, $c_{43} = x_{0100}x_{0010}x_{0001}x_{1111}$ , $ C_{SP(8,2)}(c_{43})  = 128$ |       |    |       |   |       |    |       |    |
|----------------------------------------------------------------------------------------|-------|----|-------|---|-------|----|-------|----|
| 64                                                                                     | 2     | 2  | 2     | 4 | 6     | 6  | 6     | 6  |
| 64                                                                                     | $-2i$ | -2 | $2i$  | 4 | $-6i$ | -6 | $6i$  | 6  |
| 64                                                                                     | $2i$  | -2 | $-2i$ | 4 | $6i$  | -6 | $-6i$ | 6  |
| 64                                                                                     | -2    | 2  | -2    | 4 | -6    | 6  | -6    | 6  |
| 512                                                                                    | 0     | 0  | 0     | 0 | 0     | 0  | 0     | 2  |
| 128                                                                                    | 0     | -2 | 0     | 0 | 0     | 2  | 0     | -2 |
| 128                                                                                    | 0     | 2  | 0     | 0 | 0     | -2 | 0     | -2 |

| order = 8, $c_{44} = x_{0100}x_{0010}x_{0001}x_{1000}$ , $ C_{SP(8,2)}(c_{44})  = 32$ |          |   |          |   |          |   |          |    |
|---------------------------------------------------------------------------------------|----------|---|----------|---|----------|---|----------|----|
| 128                                                                                   | $1 - i$  | 0 | $1 + i$  | 2 | $1 - i$  | 0 | $1 + i$  | 2  |
| 128                                                                                   | $1 + i$  | 0 | $1 - i$  | 2 | $1 + i$  | 0 | $1 - i$  | 2  |
| 128                                                                                   | $-1 + i$ | 0 | $-1 - i$ | 2 | $-1 + i$ | 0 | $-1 - i$ | 2  |
| 128                                                                                   | $-1 - i$ | 0 | $-1 + i$ | 2 | $-1 - i$ | 0 | $-1 + i$ | 2  |
| 512                                                                                   | 0        | 0 | 0        | 0 | 0        | 0 | 0        | -2 |

| order = 8, $c_{45} = x_{0100}x_{0010}x_{0001}x_{0011}$ , $ C_{SP(8,2)}(c_{45})  = 384$ |           |       |           |     |             |        |             |    |
|----------------------------------------------------------------------------------------|-----------|-------|-----------|-----|-------------|--------|-------------|----|
| 32                                                                                     | $2 - 2i$  | $-6i$ | $-6 - 6i$ | -12 | $-10 + 10i$ | $18i$  | $14 + 14i$  | 22 |
| 192                                                                                    | 0         | $-2i$ | 0         | -4  | 0           | $6i$   | 0           | 6  |
| 32                                                                                     | $2 + 2i$  | $6i$  | $-6 + 6i$ | -12 | $-10 - 10i$ | $-18i$ | $14 - 14i$  | 22 |
| 192                                                                                    | 0         | $2i$  | 0         | -4  | 0           | $-6i$  | 0           | 6  |
| 32                                                                                     | $-2 - 2i$ | $6i$  | $6 - 6i$  | -12 | $10 + 10i$  | $-18i$ | $-14 + 14i$ | 22 |
| 32                                                                                     | $-2 + 2i$ | $-6i$ | $6 + 6i$  | -12 | $10 - 10i$  | $18i$  | $-14 - 14i$ | 22 |
| 512                                                                                    | 0         | 0     | 0         | 0   | 0           | 0      | 0           | 2  |

| order = 8, $c_{46} = x_{0100}x_{0010}x_{0001}x_{0011}x_{1000}$ , $ C_{SP(8,2)}(c_{46})  = 32$ |          |       |          |    |          |       |          |   |
|-----------------------------------------------------------------------------------------------|----------|-------|----------|----|----------|-------|----------|---|
| 128                                                                                           | $1 - i$  | $-2i$ | $-1 - i$ | -2 | $-1 + i$ | $2i$  | $1 + i$  | 2 |
| 128                                                                                           | $1 + i$  | $2i$  | $-1 + i$ | -2 | $-1 - i$ | $-2i$ | $1 - i$  | 2 |
| 128                                                                                           | $-1 - i$ | $2i$  | $1 - i$  | -2 | $1 + i$  | $-2i$ | $-1 + i$ | 2 |
| 128                                                                                           | $-1 + i$ | $-2i$ | $1 + i$  | -2 | $1 - i$  | $2i$  | $-1 - i$ | 2 |
| 512                                                                                           | 0        | 0     | 0        | 0  | 0        | 0     | 0        | 2 |

| order = 8, $c_{47} = x_{0100}x_{0010}x_{0001}x_{0011}x_{1110}x_{1111}x_{2221}$ , $ C_{Sp(8,2)}(c_{47})  = 128$ |     |    |     |   |     |    |     |    |
|----------------------------------------------------------------------------------------------------------------|-----|----|-----|---|-----|----|-----|----|
| 64                                                                                                             | 2   | 2  | 2   | 0 | -2  | -2 | -2  | -2 |
| 64                                                                                                             | -2i | -2 | 2i  | 0 | 2i  | 2  | -2i | -2 |
| 64                                                                                                             | 2i  | -2 | -2i | 0 | -2i | 2  | 2i  | -2 |
| 64                                                                                                             | -2  | 2  | -2  | 0 | 2   | -2 | 2   | -2 |
| 512                                                                                                            | 0   | 0  | 0   | 0 | 0   | 0  | 0   | 2  |
| 128                                                                                                            | 0   | -2 | 0   | 4 | 0   | -6 | 0   | 6  |
| 128                                                                                                            | 0   | 2  | 0   | 4 | 0   | 6  | 0   | 6  |

| order = 9, $c_{48} = x_{0001}n(-4, -1, 3, -2)$ , $ C_{Sp(8,2)}(c_{48})  = 54$ |     |    |     |   |     |    |     |   |
|-------------------------------------------------------------------------------|-----|----|-----|---|-----|----|-----|---|
| 64                                                                            | 2i  | -3 | -4i | 5 | 6i  | -7 | -8i | 9 |
| 64                                                                            | 2   | 3  | 4   | 5 | 6   | 7  | 8   | 9 |
| 64                                                                            | -2i | -3 | 4i  | 5 | -6i | -7 | 8i  | 9 |
| 384                                                                           | 0   | -1 | 0   | 1 | 0   | -1 | 0   | 1 |
| 384                                                                           | 0   | 1  | 0   | 1 | 0   | 1  | 0   | 1 |
| 64                                                                            | -2  | 3  | -4  | 5 | -6  | 7  | -8  | 9 |

| order = 9, $c_{49} = x_{1111}x_{1121}n(-4, -1, -3, -2)$ , $ C_{Sp(8,2)}(c_{49})  = 27$ |    |   |    |    |   |    |    |   |
|----------------------------------------------------------------------------------------|----|---|----|----|---|----|----|---|
| 256                                                                                    | -i | 0 | -i | -1 | 0 | -1 | i  | 0 |
| 256                                                                                    | i  | 0 | i  | -1 | 0 | -1 | -i | 0 |
| 256                                                                                    | 1  | 0 | -1 | -1 | 0 | 1  | 1  | 0 |
| 256                                                                                    | -1 | 0 | 1  | -1 | 0 | 1  | -1 | 0 |

| order = 10, $c_{50} = x_{0011}n(-4, -2, -1, -3)$ , $ C_{Sp(8,2)}(c_{50})  = 80$ |     |    |     |    |     |    |   |   |
|---------------------------------------------------------------------------------|-----|----|-----|----|-----|----|---|---|
| 512                                                                             | 0   | 0  | 0   | -1 | 0   | 0  | 0 | 1 |
| 64                                                                              | -2i | -2 | 2i  | 3  | -2i | 0  | 0 | 1 |
| 128                                                                             | 0   | 2  | 0   | 3  | 0   | 4  | 0 | 5 |
| 64                                                                              | 2i  | -2 | -2i | 3  | 2i  | 0  | 0 | 1 |
| 64                                                                              | 2   | 2  | 2   | 3  | 2   | 0  | 0 | 1 |
| 128                                                                             | 0   | -2 | 0   | 3  | 0   | -4 | 0 | 5 |
| 64                                                                              | -2  | 2  | -2  | 3  | -2  | 0  | 0 | 1 |

| order = 10, $c_{51} = x_{0011}n(-4, -1, -3, -2)$ , $ C_{Sp(8,2)}(c_{51})  = 20$ |    |   |   |   |    |    |   |   |
|---------------------------------------------------------------------------------|----|---|---|---|----|----|---|---|
| 256                                                                             | i  | 0 | 0 | 0 | -i | 1  | 0 | 0 |
| 256                                                                             | -i | 0 | 0 | 0 | i  | 1  | 0 | 0 |
| 256                                                                             | 1  | 0 | 0 | 0 | -1 | -1 | 0 | 0 |
| 256                                                                             | -1 | 0 | 0 | 0 | 1  | -1 | 0 | 0 |

| order = 10, $c_{52} = x_{0011}n(-4, 2, -1, -3)$ , $ C_{Sp(8,2)}(c_{52})  = 240$ |           |       |           |    |           |       |           |   |
|---------------------------------------------------------------------------------|-----------|-------|-----------|----|-----------|-------|-----------|---|
| 32                                                                              | $2 + 2i$  | $4i$  | $-2 + 2i$ | 1  | $2 + 2i$  | 0     | $4 - 4i$  | 9 |
| 384                                                                             | 0         | 0     | 0         | 1  | 0         | 0     | 0         | 1 |
| 256                                                                             | 0         | -2i   | 0         | -3 | 0         | $4i$  | 0         | 5 |
| 32                                                                              | $-2 - 2i$ | $4i$  | $2 - 2i$  | 1  | $-2 - 2i$ | 0     | $-4 + 4i$ | 9 |
| 256                                                                             | 0         | $2i$  | 0         | -3 | 0         | $-4i$ | 0         | 5 |
| 32                                                                              | $-2 + 2i$ | $-4i$ | $2 + 2i$  | 1  | $-2 + 2i$ | 0     | $-4 - 4i$ | 9 |
| 32                                                                              | $2 - 2i$  | $-4i$ | $-2 - 2i$ | 1  | $2 - 2i$  | 0     | $4 + 4i$  | 9 |

| order = 10, $c_{53} = x_{0001}x_{0011}n(2, 1, 4, -3)$ , $ C_{Sp(8,2)}(c_{53})  = 240$ |       |    |       |   |        |     |        |    |
|---------------------------------------------------------------------------------------|-------|----|-------|---|--------|-----|--------|----|
| 64                                                                                    | -2    | 4  | -6    | 9 | -10    | 12  | -12    | 13 |
| 768                                                                                   | 0     | 0  | 0     | 1 | 0      | 0   | 0      | 1  |
| 64                                                                                    | $-2i$ | -4 | $6i$  | 9 | $-10i$ | -12 | $12i$  | 13 |
| 64                                                                                    | $2i$  | -4 | $-6i$ | 9 | $10i$  | -12 | $-12i$ | 13 |
| 64                                                                                    | 2     | 4  | 6     | 9 | 10     | 12  | 12     | 13 |

| order = 12, $c_{54} = x_{0001}n(-4, -3, 2, 1)$ , $ C_{Sp(8,2)}(c_{54})  = 48$ |        |   |   |    |   |       |        |   |
|-------------------------------------------------------------------------------|--------|---|---|----|---|-------|--------|---|
| 512                                                                           | 0      | 0 | 0 | 1  | 0 | 0     | 0      | 0 |
| 128                                                                           | $-1-i$ | 0 | 0 | -1 | 0 | $2i$  | $1-i$  | 0 |
| 128                                                                           | $-1+i$ | 0 | 0 | -1 | 0 | $-2i$ | $1+i$  | 0 |
| 128                                                                           | $1+i$  | 0 | 0 | -1 | 0 | $2i$  | $-1+i$ | 0 |
| 128                                                                           | $1-i$  | 0 | 0 | -1 | 0 | $-2i$ | $-1-i$ | 0 |

| order = 12, $c_{55} = x_{0001}n(-2, 1, -3, -4)$ , $ C_{Sp(8,2)}(c_{55})  = 96$ |       |    |   |    |       |   |       |    |
|--------------------------------------------------------------------------------|-------|----|---|----|-------|---|-------|----|
| 256                                                                            | 0     | 0  | 0 | 2  | 0     | 0 | 0     | 3  |
| 128                                                                            | 0     | -2 | 0 | 2  | 0     | 0 | 0     | -1 |
| 256                                                                            | 0     | 0  | 0 | -2 | 0     | 0 | 0     | 3  |
| 64                                                                             | $2i$  | -2 | 0 | -2 | $-2i$ | 0 | $-2i$ | 3  |
| 128                                                                            | 0     | 2  | 0 | 2  | 0     | 0 | 0     | -1 |
| 64                                                                             | 2     | 2  | 0 | -2 | -2    | 0 | 2     | 3  |
| 64                                                                             | $-2i$ | -2 | 0 | -2 | $2i$  | 0 | $2i$  | 3  |
| 64                                                                             | -2    | 2  | 0 | -2 | 2     | 0 | -2    | 3  |

| order = 12, $c_{56} = x_{0001}n(4, 1, -2, 3)$ , $ C_{Sp(8,2)}(c_{56})  = 24$ |      |   |      |    |   |   |   |   |
|------------------------------------------------------------------------------|------|---|------|----|---|---|---|---|
| 256                                                                          | $-i$ | 0 | $-i$ | -1 | 0 | 0 | 0 | 0 |
| 256                                                                          | -1   | 0 | 1    | -1 | 0 | 0 | 0 | 0 |
| 256                                                                          | $i$  | 0 | $i$  | -1 | 0 | 0 | 0 | 0 |
| 256                                                                          | 1    | 0 | -1   | -1 | 0 | 0 | 0 | 0 |

| order = 12, $c_{57} = x_{0011}x_{0021}n(-4, -1, 3, -2)$ , $ C_{Sp(8,2)}(c_{57})  = 48$ |        |       |         |    |         |       |         |   |
|----------------------------------------------------------------------------------------|--------|-------|---------|----|---------|-------|---------|---|
| 128                                                                                    | $1-i$  | $-2i$ | $-2-2i$ | -3 | $-2+2i$ | $4i$  | $3+3i$  | 4 |
| 128                                                                                    | $1+i$  | $2i$  | $-2+2i$ | -3 | $-2-2i$ | $-4i$ | $3-3i$  | 4 |
| 128                                                                                    | $-1+i$ | $-2i$ | $2+2i$  | -3 | $2-2i$  | $4i$  | $-3-3i$ | 4 |
| 128                                                                                    | $-1-i$ | $2i$  | $2-2i$  | -3 | $2+2i$  | $-4i$ | $-3+3i$ | 4 |
| 512                                                                                    | 0      | 0     | 0       | -1 | 0       | 0     | 0       | 0 |

| order = 12, $c_{58} = x_{0111}x_{0021}n(2, 3, 1, -4)$ , $ C_{Sp(8,2)}(c_{58})  = 144$ |        |       |   |    |         |   |        |   |
|---------------------------------------------------------------------------------------|--------|-------|---|----|---------|---|--------|---|
| 128                                                                                   | $-1-i$ | $2i$  | 0 | 1  | $-2-2i$ | 0 | $-1+i$ | 4 |
| 128                                                                                   | $1+i$  | $2i$  | 0 | 1  | $2+2i$  | 0 | $1-i$  | 4 |
| 128                                                                                   | $1-i$  | $-2i$ | 0 | 1  | $2-2i$  | 0 | $1+i$  | 4 |
| 512                                                                                   | 0      | 0     | 0 | -1 | 0       | 0 | 0      | 0 |
| 128                                                                                   | $-1+i$ | $-2i$ | 0 | 1  | $-2+2i$ | 0 | $-1-i$ | 4 |

| order = 12, $c_{59} = x_{0001}x_{0011}x_{0111}x_{0121}n(1, -4, 2, 3)$ , $ C_{Sp(8,2)}(c_{59})  = 72$ |       |    |       |   |   |    |       |   |
|------------------------------------------------------------------------------------------------------|-------|----|-------|---|---|----|-------|---|
| 64                                                                                                   | $-2i$ | -3 | $2i$  | 1 | 0 | -2 | $4i$  | 6 |
| 384                                                                                                  | 0     | -1 | 0     | 1 | 0 | -2 | 0     | 2 |
| 64                                                                                                   | 2     | 3  | 2     | 1 | 0 | 2  | 4     | 6 |
| 384                                                                                                  | 0     | 1  | 0     | 1 | 0 | 2  | 0     | 2 |
| 64                                                                                                   | $2i$  | -3 | $-2i$ | 1 | 0 | -2 | $-4i$ | 6 |
| 64                                                                                                   | -2    | 3  | -2    | 1 | 0 | 2  | -4    | 6 |

| order = 12, $c_{60} = x_{0001}x_{0011}x_{0121}n(-1, -2, 4, -3)$ , $ C_{Sp(8,2)}(c_{60})  = 24$ |          |      |          |    |   |       |   |   |
|------------------------------------------------------------------------------------------------|----------|------|----------|----|---|-------|---|---|
| 256                                                                                            | 0        | $i$  | 0        | -1 | 0 | $-2i$ | 0 | 2 |
| 128                                                                                            | $-1 - i$ | $i$  | $1 - i$  | -1 | 0 | 0     | 0 | 0 |
| 256                                                                                            | 0        | $-i$ | 0        | -1 | 0 | $2i$  | 0 | 2 |
| 128                                                                                            | $1 - i$  | $-i$ | $-1 - i$ | -1 | 0 | 0     | 0 | 0 |
| 128                                                                                            | $1 + i$  | $i$  | $-1 + i$ | -1 | 0 | 0     | 0 | 0 |
| 128                                                                                            | $-1 + i$ | $-i$ | $1 + i$  | -1 | 0 | 0     | 0 | 0 |

| order = 12, $c_{61} = x_{0011}x_{0111}x_{0021}x_{0121}n(1, 3, -2, -4)$ , $ C_{Sp(8,2)}(c_{61})  = 576$ |           |       |           |     |             |        |             |    |
|--------------------------------------------------------------------------------------------------------|-----------|-------|-----------|-----|-------------|--------|-------------|----|
| 512                                                                                                    | 0         | 0     | 0         | -2  | 0           | 0      | 0           | 3  |
| 32                                                                                                     | $-2 + 2i$ | $-6i$ | $8 + 8i$  | -18 | $18 - 18i$  | $32i$  | $-26 - 26i$ | 39 |
| 192                                                                                                    | 0         | $-2i$ | 0         | -2  | 0           | 0      | 0           | -1 |
| 32                                                                                                     | $-2 - 2i$ | $6i$  | $8 - 8i$  | -18 | $18 + 18i$  | $-32i$ | $-26 + 26i$ | 39 |
| 192                                                                                                    | 0         | $2i$  | 0         | -2  | 0           | 0      | 0           | -1 |
| 32                                                                                                     | $2 + 2i$  | $6i$  | $-8 + 8i$ | -18 | $-18 - 18i$ | $-32i$ | $26 - 26i$  | 39 |
| 32                                                                                                     | $2 - 2i$  | $-6i$ | $-8 - 8i$ | -18 | $-18 + 18i$ | $32i$  | $26 + 26i$  | 39 |

| order = 12, $c_{62} = x_{0111}x_{0121}n(1, 4, 3, -2)$ , $ C_{Sp(8,2)}(c_{62})  = 576$ |           |       |   |    |           |   |           |    |
|---------------------------------------------------------------------------------------|-----------|-------|---|----|-----------|---|-----------|----|
| 32                                                                                    | $-2 - 2i$ | $2i$  | 0 | -2 | $2 + 2i$  | 0 | $-2 + 2i$ | 7  |
| 192                                                                                   | 0         | $-2i$ | 0 | -2 | 0         | 0 | 0         | -1 |
| 32                                                                                    | $2 + 2i$  | $2i$  | 0 | -2 | $-2 - 2i$ | 0 | $2 - 2i$  | 7  |
| 512                                                                                   | 0         | 0     | 0 | 2  | 0         | 0 | 0         | 3  |
| 192                                                                                   | 0         | $2i$  | 0 | -2 | 0         | 0 | 0         | -1 |
| 32                                                                                    | $2 - 2i$  | $-2i$ | 0 | -2 | $-2 + 2i$ | 0 | $2 + 2i$  | 7  |
| 32                                                                                    | $-2 + 2i$ | $-2i$ | 0 | -2 | $2 - 2i$  | 0 | $-2 - 2i$ | 7  |

| order = 12, $c_{63} = x_{0001}x_{0011}x_{0111}x_{0121}n(4, -3, 1, -2)$ , $ C_{Sp(8,2)}(c_{63})  = 144$ |          |   |           |   |   |       |          |   |
|--------------------------------------------------------------------------------------------------------|----------|---|-----------|---|---|-------|----------|---|
| 128                                                                                                    | $-1 + i$ | 0 | $-2 - 2i$ | 3 | 0 | $2i$  | $-1 - i$ | 0 |
| 128                                                                                                    | $1 + i$  | 0 | $2 - 2i$  | 3 | 0 | $-2i$ | $1 - i$  | 0 |
| 512                                                                                                    | 0        | 0 | 0         | 1 | 0 | 0     | 0        | 0 |
| 128                                                                                                    | $1 - i$  | 0 | $2 + 2i$  | 3 | 0 | $2i$  | $1 + i$  | 0 |
| 128                                                                                                    | $-1 - i$ | 0 | $-2 + 2i$ | 3 | 0 | $-2i$ | $-1 + i$ | 0 |

| order = 12, $c_{64} = (x_{0001}x_{0011}x_{0021}n(-4, -1, -2, -3))^2$ , $ C_{Sp(8,2)}(c_{64})  = 152$ |       |   |       |    |       |    |       |    |
|------------------------------------------------------------------------------------------------------|-------|---|-------|----|-------|----|-------|----|
| 64                                                                                                   | $2i$  | 0 | $4i$  | -2 | $6i$  | -8 | $2i$  | -9 |
| 768                                                                                                  | 0     | 0 | 0     | -2 | 0     | 0  | 0     | 3  |
| 64                                                                                                   | -2    | 0 | 4     | -2 | -6    | 8  | 2     | -9 |
| 64                                                                                                   | 2     | 0 | -4    | -2 | 6     | 8  | -2    | -9 |
| 64                                                                                                   | $-2i$ | 0 | $-4i$ | -2 | $-6i$ | -8 | $-2i$ | -9 |

| order = 12, $c_{65} = (x_{0111}x_{0021}n(-4, -2, -3, -1))^2$ , $ C_{Sp(8,2)}(c_{65})  = 384$ |       |    |       |    |       |    |       |   |
|----------------------------------------------------------------------------------------------|-------|----|-------|----|-------|----|-------|---|
| 64                                                                                           | $2i$  | -4 | $-4i$ | 6  | $6i$  | -8 | $-6i$ | 7 |
| 512                                                                                          | 0     | 0  | 0     | 2  | 0     | 0  | 0     | 3 |
| 64                                                                                           | -2    | 4  | -4    | 6  | -6    | 8  | -6    | 7 |
| 64                                                                                           | $-2i$ | -4 | $4i$  | 6  | $-6i$ | -8 | $6i$  | 7 |
| 64                                                                                           | 2     | 4  | 4     | 6  | 6     | 8  | 6     | 7 |
| 256                                                                                          | 0     | 0  | 0     | -2 | 0     | 0  | 0     | 3 |



| order = 12, $c_{66} = x_{0010}x_{0011}x_{0111}x_{0021}x_{0121}n(-2, -4, 1, 3)$ , $ C_{Sp(8,2)}(c_{66})  = 144$ |    |   |    |   |   |    |     |   |
|----------------------------------------------------------------------------------------------------------------|----|---|----|---|---|----|-----|---|
| 256                                                                                                            | -1 | 0 | -1 | 1 | 0 | 2  | -2  | 0 |
| 256                                                                                                            | -i | 0 | i  | 1 | 0 | -2 | 2i  | 0 |
| 256                                                                                                            | i  | 0 | -i | 1 | 0 | -2 | -2i | 0 |
| 256                                                                                                            | 1  | 0 | 1  | 1 | 0 | 2  | 2   | 0 |

| order = 14, $c_{67} = x_{0011}n(-4, -2, 1, -3)$ , $ C_{Sp(8,2)}(c_{67})  = 14$ |      |    |   |   |   |   |      |   |
|--------------------------------------------------------------------------------|------|----|---|---|---|---|------|---|
| 256                                                                            | 0    | -i | 0 | 0 | 0 | 0 | 0    | 0 |
| 128                                                                            | 1+i  | i  | 0 | 0 | 0 | 0 | 1-i  | 2 |
| 128                                                                            | -1-i | i  | 0 | 0 | 0 | 0 | -1+i | 2 |
| 128                                                                            | -1+i | -i | 0 | 0 | 0 | 0 | -1-i | 2 |
| 256                                                                            | 0    | i  | 0 | 0 | 0 | 0 | 0    | 0 |
| 128                                                                            | 1-i  | -i | 0 | 0 | 0 | 0 | 1+i  | 2 |

| order = 15, $c_{68} = x_{0011}n(-4, 2, -3, 1)$ , $ C_{Sp(8,2)}(c_{68})  = 90$ |     |    |     |    |     |    |     |    |
|-------------------------------------------------------------------------------|-----|----|-----|----|-----|----|-----|----|
| 64                                                                            | 2   | 1  | -2  | -4 | -4  | -1 | 4   | 7  |
| 64                                                                            | -2  | 1  | 2   | -4 | 4   | -1 | -4  | 7  |
| 384                                                                           | 0   | -1 | 0   | 0  | 0   | 1  | 0   | -1 |
| 384                                                                           | 0   | 1  | 0   | 0  | 0   | -1 | 0   | -1 |
| 64                                                                            | -2i | -1 | -2i | -4 | 4i  | 1  | 4i  | 7  |
| 64                                                                            | 2i  | -1 | 2i  | -4 | -4i | 1  | -4i | 7  |

| order = 15, $c_{69} = (x_{0011}x_{0021}n(2, -4, -1, -3))^2$ , $ C_{Sp(8,2)}(c_{69})  = 90$ |    |    |     |   |    |    |     |   |
|--------------------------------------------------------------------------------------------|----|----|-----|---|----|----|-----|---|
| 256                                                                                        | -1 | 1  | -2  | 2 | -1 | 2  | -2  | 1 |
| 256                                                                                        | 1  | 1  | 2   | 2 | 1  | 2  | 2   | 1 |
| 256                                                                                        | i  | -1 | -2i | 2 | i  | -2 | -2i | 1 |
| 256                                                                                        | -i | -1 | 2i  | 2 | -i | -2 | 2i  | 1 |

| order = 15, $c_{70} = x_{0011}n(-4, -1, -2, -3)$ , $ C_{Sp(8,2)}(c_{70})  = 15$ |    |   |   |   |   |   |   |   |
|---------------------------------------------------------------------------------|----|---|---|---|---|---|---|---|
| 256                                                                             | -i | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 256                                                                             | i  | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 256                                                                             | 1  | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 256                                                                             | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| order = 17, $c_{71} = x_{0001}x_{0011}n(-4, -1, -2, 3)$ , $ C_{Sp(8,2)}(c_{71})  = 17$ |    |    |    |   |    |    |    |   |
|----------------------------------------------------------------------------------------|----|----|----|---|----|----|----|---|
| 256                                                                                    | 1  | 1  | 1  | 1 | 1  | 1  | 1  | 1 |
| 256                                                                                    | i  | -1 | -i | 1 | i  | -1 | -i | 1 |
| 256                                                                                    | -1 | 1  | -1 | 1 | -1 | 1  | -1 | 1 |
| 256                                                                                    | -i | -1 | i  | 1 | -i | -1 | i  | 1 |

| order = 17, $c_{72} = (x_{0001}x_{0011}n(-4, -1, -2, 3))^3$ , $ C_{Sp(8,2)}(c_{72})  = 17$ |    |    |    |   |    |    |    |   |
|--------------------------------------------------------------------------------------------|----|----|----|---|----|----|----|---|
| 256                                                                                        | 1  | 1  | 1  | 1 | 1  | 1  | 1  | 1 |
| 256                                                                                        | i  | -1 | -i | 1 | i  | -1 | -i | 1 |
| 256                                                                                        | -1 | 1  | -1 | 1 | -1 | 1  | -1 | 1 |
| 256                                                                                        | -i | -1 | i  | 1 | -i | -1 | i  | 1 |

| order = 18, $c_{73} = x_{0001}n(-4, -2, -1, -3)$ , $ C_{Sp(8,2)}(c_{73})  = 18$ |      |    |   |    |      |    |   |   |
|---------------------------------------------------------------------------------|------|----|---|----|------|----|---|---|
| 256                                                                             | 0    | -i | 0 | -1 | 0    | i  | 0 | 1 |
| 256                                                                             | 0    | i  | 0 | -1 | 0    | -i | 0 | 1 |
| 128                                                                             | 1-i  | -i | 0 | 1  | 1-i  | -i | 0 | 1 |
| 128                                                                             | -1-i | i  | 0 | 1  | -1-i | i  | 0 | 1 |
| 128                                                                             | -1+i | -i | 0 | 1  | -1+i | -i | 0 | 1 |
| 128                                                                             | 1+i  | i  | 0 | 1  | 1+i  | i  | 0 | 1 |

| order = 20, $c_{74} = x_{0001}x_{0011}n(-2, 1, -4, 3)$ , $ C_{Sp(8,2)}(c_{74})  = 40$ |      |     |      |    |      |     |       |   |
|---------------------------------------------------------------------------------------|------|-----|------|----|------|-----|-------|---|
| 512                                                                                   | 0    | 0   | 0    | -1 | 0    | 0   | 0     | 1 |
| 128                                                                                   | -1+i | -2i | 1+i  | -1 | 1-i  | 2i  | -2-2i | 3 |
| 128                                                                                   | -1-i | 2i  | 1-i  | -1 | 1+i  | -2i | -2+2i | 3 |
| 128                                                                                   | 1-i  | -2i | -1-i | -1 | -1+i | 2i  | 2+2i  | 3 |
| 128                                                                                   | 1+i  | 2i  | -1+i | -1 | -1-i | -2i | 2-2i  | 3 |

| order = 20, $c_{75} = x_{1111}x_{1121}n(-4, -1, 3, -2)$ , $ C_{Sp(8,2)}(c_{75})  = 40$ |      |   |      |   |      |     |   |    |
|----------------------------------------------------------------------------------------|------|---|------|---|------|-----|---|----|
| 128                                                                                    | 1+i  | 0 | 1-i  | 1 | -1-i | -2i | 0 | -1 |
| 128                                                                                    | -1-i | 0 | -1+i | 1 | 1+i  | -2i | 0 | -1 |
| 512                                                                                    | 0    | 0 | 0    | 1 | 0    | 0   | 0 | 1  |
| 128                                                                                    | -1+i | 0 | -1-i | 1 | 1-i  | 2i  | 0 | -1 |
| 128                                                                                    | 1-i  | 0 | 1+i  | 1 | -1+i | 2i  | 0 | -1 |

| order = 21, $c_{76} = x_{0001}x_{0011}n(-4, -1, -3, -2)$ , $ C_{Sp(8,2)}(c_{76})  = 21$ |    |    |   |   |   |   |    |   |
|-----------------------------------------------------------------------------------------|----|----|---|---|---|---|----|---|
| 256                                                                                     | 1  | 1  | 0 | 0 | 0 | 0 | 1  | 1 |
| 256                                                                                     | -1 | 1  | 0 | 0 | 0 | 0 | -1 | 1 |
| 256                                                                                     | -i | -1 | 0 | 0 | 0 | 0 | i  | 1 |
| 256                                                                                     | i  | -1 | 0 | 0 | 0 | 0 | -i | 1 |

| order = 24, $c_{77} = x_{0001}x_{0011}x_{0021}n(-4, -1, -2, -3)$ , $ C_{Sp(8,2)}(c_{77})  = 48$ |      |   |   |   |      |   |      |    |
|-------------------------------------------------------------------------------------------------|------|---|---|---|------|---|------|----|
| 128                                                                                             | -1-i | 0 | 0 | 0 | -1-i | 0 | -1+i | 1  |
| 512                                                                                             | 0    | 0 | 0 | 0 | 0    | 0 | 0    | -1 |
| 128                                                                                             | -1+i | 0 | 0 | 0 | -1+i | 0 | -1-i | 1  |
| 128                                                                                             | 1-i  | 0 | 0 | 0 | 1-i  | 0 | 1+i  | 1  |
| 128                                                                                             | 1+i  | 0 | 0 | 0 | 1+i  | 0 | 1-i  | 1  |

| order = 24, $c_{78} = x_{0111}x_{0021}n(-4, -2, -3, -1)$ , $ C_{Sp(8,2)}(c_{78})  = 48$ |      |     |       |    |       |     |       |    |
|-----------------------------------------------------------------------------------------|------|-----|-------|----|-------|-----|-------|----|
| 128                                                                                     | 1-i  | -2i | -2-2i | -4 | -3+3i | 4i  | 3+3i  | 5  |
| 512                                                                                     | 0    | 0   | 0     | 0  | 0     | 0   | 0     | -1 |
| 128                                                                                     | -1-i | 2i  | 2-2i  | -4 | 3+3i  | -4i | -3+3i | 5  |
| 128                                                                                     | -1+i | -2i | 2+2i  | -4 | 3-3i  | 4i  | -3-3i | 5  |
| 128                                                                                     | 1+i  | 2i  | -2+2i | -4 | -3-3i | -4i | 3-3i  | 5  |

| order = 30, $c_{79} = x_{0011}x_{0021}n(2, -4, -1, -3)$ , $ C_{Sp(8,2)}(c_{79})  = 30$ |    |    |   |   |    |   |   |   |
|----------------------------------------------------------------------------------------|----|----|---|---|----|---|---|---|
| 256                                                                                    | 1  | 1  | 0 | 0 | -1 | 0 | 0 | 1 |
| 256                                                                                    | i  | -1 | 0 | 0 | -i | 0 | 0 | 1 |
| 256                                                                                    | -i | -1 | 0 | 0 | i  | 0 | 0 | 1 |
| 256                                                                                    | -1 | 1  | 0 | 0 | 1  | 0 | 0 | 1 |

| order = 30, $c_{80} = x_{0011}n(-4, -2, -3, 1)$ , $ C_{Sp(8,2)}(c_{80})  = 30$ |        |      |        |    |         |       |         |    |
|--------------------------------------------------------------------------------|--------|------|--------|----|---------|-------|---------|----|
| 256                                                                            | 0      | $-i$ | 0      | 0  | 0       | $-i$  | 0       | -1 |
| 128                                                                            | $-1-i$ | $i$  | $1-i$  | -2 | $2+2i$  | $-3i$ | $-2+2i$ | 3  |
| 128                                                                            | $1+i$  | $i$  | $-1+i$ | -2 | $-2-2i$ | $-3i$ | $2-2i$  | 3  |
| 128                                                                            | $-1+i$ | $-i$ | $1+i$  | -2 | $2-2i$  | $3i$  | $-2-2i$ | 3  |
| 256                                                                            | 0      | $i$  | 0      | 0  | 0       | $i$   | 0       | -1 |
| 128                                                                            | $1-i$  | $-i$ | $-1-i$ | -2 | $-2+2i$ | $3i$  | $2+2i$  | 3  |

REMARK. The determination of the conjugacy classes of Chevalley groups were studied by several authors. For example, see [2], [6], [15].

#### 4. Main result

We have obtained the surjective homomorphism

$$\varphi: H_4 \rightarrow Sp(8,2)$$

with  $\text{Ker } \varphi = N_4$  and a set  $\{c_i\}_{0 \leq i \leq 80}$  of representatives of conjugacy classes of  $Sp(8,2)$  in the preceding sections. Our main result can be stated as follows:

**Theorem 4.1.** *The Molien series of  $H_4$  is given by*

$$\begin{aligned} \Phi_{H_4}(t) &= N/D \\ &= 1 + t^8 + t^{12} + 2t^{16} + 3t^{20} + 7t^{24} + 7t^{28} + 19t^{32} + 27t^{36} \\ &\quad + 52t^{40} + 87t^{44} + 172t^{48} + 279t^{52} + 550t^{56} + 960t^{60} + 1782t^{64} \\ &\quad + 3183t^{68} + 5485t^{72} + 10288t^{76} + \dots, \end{aligned}$$

where

$$\begin{aligned} D &= (1-t^{12})(1-t^{20})(1-t^{24})^4(1-t^{28})(1-t^{36})(1-t^{48})(1-t^{56})(1-t^{60})(1-t^{68}) \\ &\quad \times (1-t^{72})(1-t^{80})(1-t^{84})(1-t^{120}), \end{aligned}$$

$$N = (1+t^{12})g(t),$$

$$\begin{aligned} g(t) &= 1 + t^8 - t^{12} + 2t^{16} + 3t^{24} + t^{28} + 12t^{32} + 13t^{36} + 34t^{40} + 43t^{44} + 107t^{48} \\ &\quad + 157t^{52} + 335t^{56} + 549t^{60} + 1094t^{64} + 1861t^{68} + 3501t^{72} + 5965t^{76} \\ &\quad + 10728t^{80} + 18041t^{84} + 31051t^{88} + 51025t^{92} + 84427t^{96} + 134865t^{100} \\ &\quad + 215008t^{104} + 333369t^{108} + 513542t^{112} + 773052t^{116} + 1153627t^{120} \\ &\quad + 1688292t^{124} + 2447124t^{128} + 3487706t^{132} + 4922301t^{136} + 6845055t^{140} \\ &\quad + 9427941t^{144} + 12816307t^{148} + 17262549t^{152} + 22980000t^{156} + 30324507t^{160} \\ &\quad + 39594318t^{164} + 51272203t^{168} + 65756890t^{172} + 83679250t^{176} \end{aligned}$$

$$\begin{aligned}
& + 105549085t^{180} + 132161437t^{184} + 164140047t^{188} + 202451163t^{192} \\
& + 247823660t^{196} + 301389903t^{200} + 363960630t^{204} + 436814071t^{208} \\
& + 520802553t^{212} + 617312656t^{216} + 727180701t^{220} + 851846951t^{224} \\
& + 992056493t^{228} + 1149232929t^{232} + 1323941514t^{236} + 1517506330t^{240} \\
& + 1730214602t^{244} + 1963201767t^{248} + 2216376776t^{252} + 2490597453t^{256} \\
& + 2785299743t^{260} + 3100983710t^{264} + 3436532034t^{268} + 3792023955t^{272} \\
& + 4165731123t^{276} + 4557273329t^{280} + 4964284431t^{284} + 5385917115t^{288} \\
& + 5819175192t^{292} + 6262771875t^{296} + 6713128315t^{300} + 7168581264t^{304} \\
& + 7625054128t^{308} + 8080605429t^{312} + 8530781308t^{316} + 8973489231t^{320} \\
& + 9404047193t^{324} + 9820361028t^{328} + 10217690359t^{332} + 10594101406t^{336} \\
& + 10944974102t^{340} + 11268698558t^{344} + 11560953224t^{348} + 11820605627t^{352} \\
& + 12043796179t^{356} + 12230003475t^{360} + 12375970644t^{364} + 12481889419t^{368} \\
& + 12545212616t^{372} + 12566910422t^{376} + 12545212616t^{380} + 12481889419t^{384} \\
& + 12375970644t^{388} + 12230003475t^{392} + 12043796179t^{396} + 11820605627t^{400} \\
& + 11560953224t^{404} + 11268698558t^{408} + 10944974102t^{412} + 10594101406t^{416} \\
& + 10217690359t^{420} + 9820361028t^{424} + 9404047193t^{428} + 8973489231t^{432} \\
& + 8530781308t^{436} + 8080605429t^{440} + 7625054128t^{444} + 7168581264t^{448} \\
& + 6713128315t^{452} + 6262771875t^{456} + 5819175192t^{460} + 5385917115t^{464} \\
& + 4964284431t^{468} + 4557273329t^{472} + 4165731123t^{476} + 3792023955t^{480} \\
& + 3436532034t^{484} + 3100983710t^{488} + 2785299743t^{492} + 2490597453t^{496} \\
& + 2216376776t^{500} + 1963201767t^{504} + 1730214602t^{508} + 1517506330t^{512} \\
& + 1323941514t^{516} + 1149232929t^{520} + 992056493t^{524} + 851846951t^{528} \\
& + 727180701t^{532} + 617312656t^{536} + 520802553t^{540} + 436814071t^{544} \\
& + 363960630t^{548} + 301389903t^{552} + 247823660t^{556} + 202451163t^{560} \\
& + 164140047t^{564} + 132161437t^{568} + 105549085t^{572} + 83679250t^{576} \\
& + 65756890t^{580} + 51272203t^{584} + 39594318t^{588} + 30324507t^{592} + 22980000t^{596} \\
& + 17262549t^{600} + 12816307t^{604} + 9427941t^{608} + 6845055t^{612} + 4922301t^{616} \\
& + 3487706t^{620} + 2447124t^{624} + 1688292t^{628} + 1153627t^{632} + 773052t^{636} \\
& + 513542t^{640} + 333369t^{644} + 215008t^{648} + 134865t^{652} + 84427t^{656} + 51025t^{660}
\end{aligned}$$

$$\begin{aligned}
& + 31051t^{664} + 18041t^{668} + 10728t^{672} + 5965t^{676} + 3501t^{680} + 1861t^{684} \\
& + 1094t^{688} + 549t^{692} + 335t^{696} + 157t^{700} + 107t^{704} + 43t^{708} + 34t^{712} + 13t^{716} \\
& + 12t^{720} + t^{724} + 3t^{728} + 2t^{736} - t^{740} + t^{744} + t^{752}.
\end{aligned}$$

□

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