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Development of New Interface Potential for Simulating Mode I & II Fracture Behavior [†]

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Abstract

In order to describe both the opening and shear deformation by one interface element, a new type interface potential was developed, which included an interaction between two modes. This proposed method was applied to the analyses of the fracture strength of co-cured joint between steel and resin, where the stress in the joint was applied by a simple tensile load or a thermal strain. From these computations, the predicted joint strength was found to decrease by increasing the order of stress singularity at the joint interface regardless of the type of applied stress.

1. Introduction

The strength of the bonded joint is influenced by the geometry of the joint, the thickness of the adhesive, its elastic modulus and strength. The roughness of the surfaces to be joined is also influential. To study the influences of various parameters, the level of stress and the order of the singularity in the stress field are commonly employed for the relative evaluation of the strength[1-4]. Although detailed information on the stress field is provided, little information on the criteria of the fracture is obtained from these types of study. This comes from the fact that, the physics of failure itself is not explicitly modeled in these analyses.

On the other hand, crack propagation in both brittle and ductile materials and debonding along the interface in the joint materials are typical examples of fractures. In these types of fracture, the failure is the consequence of surface formation accompanied by the crack extension. Based on this understanding, the authors propose an interface element which explicitly models the formation of new surfaces. This proposed method has been applied to the peeling of thin films from substrates, dynamic crack propagation in elastic materials, Charpy impact tests and welding hot cracking[5-10]. From these analyses, it is found that the interface element may have potential capability, not only to give insight into the criteria of the fracture, but also to make quantitative predictions of strength itself.

So, in this study, as an example of a joint between dissimilar materials which has a stress singularity due to the differences of material properties, the strength of co-cured joints between steel and resin were analyzed by using the finite element method with interface elements. In order to examine the effects of stress singularity on the joint strength, the scarf angle between the steel and the resin were varied. Also, the effects of stress field on the analyses were studied by changing the type of loading, namely a simple tensile load and a thermal load.

2. Interface Element

Essentially, the interface element is the distributed nonlinear spring existing between surfaces forming the interface or the potential crack surfaces as shown by **Fig.1**. The relation between the opening of the interface δ and the bonding stress σ is shown in **Fig.2**. When the opening δ is small, the bonding between two surfaces is maintained. As the opening δ increases, the bonding stress σ increases till it becomes a maximum value σ_{cr} at the opening δ_{cr}. With further increase of δ, the bonding strength is rapidly lost and the surfaces are completely separated. Such interaction between the surfaces can be described by an interface potential. There are rather wide choices for such a potential. The authors employed the

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(a) Before Crack Propagation
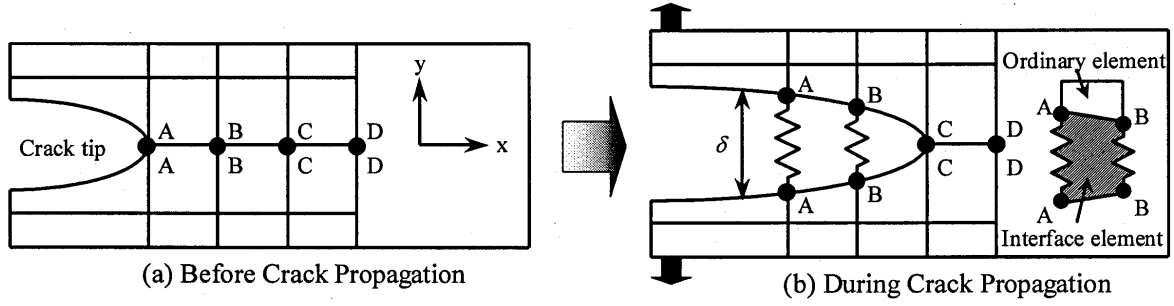
(b) During Crack Propagation

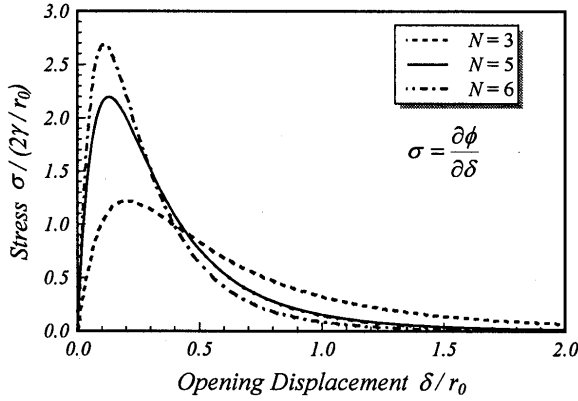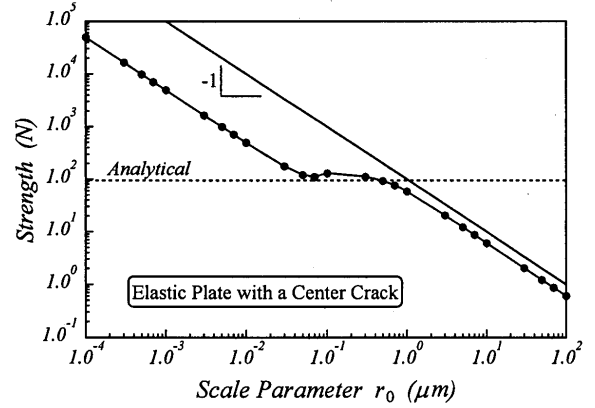**Fig.1** Representation of crack growth using interface element.

Fig.2 Relation between crack opening displacement and bonding stress.

Fig.3 Effect of scale parameter on predicted strength of plate with center crack[9].

Lennard-Jones type potential because it explicitly involves the surface energy γ which is necessary to form new surfaces. Thus, the surface potential per unit surface area ϕ can be defined by the following equation.

$$\phi(\delta) = 2\gamma \cdot \left\{ \left(\frac{r_0}{r_0 + \delta} \right)^{2N} - 2 \cdot \left(\frac{r_0}{r_0 + \delta} \right)^{N} \right\} \quad (1)$$

where, constants γ, r_0, and N are the surface energy per unit area, the scale parameter and the shape parameter of the potential function. From the derivative of ϕ with respect to the opening displacement δ, the maximum bonding stress, σ_{cr}, is obtained as follows when the opening displacement is δ_{cr}.

$$\sigma_{cr} = \frac{4\gamma N}{r_0} \cdot \left\{ \left(\frac{N+1}{2N+1} \right)^{\frac{N+1}{N}} - \left(\frac{N+1}{2N+1} \right)^{\frac{2N+1}{N}} \right\}, \quad (2)$$

$$\delta_{cr} = r_0 \cdot \left\{ \left(\frac{2N+1}{N+1} \right)^{\frac{1}{N}} - 1 \right\} \quad (3)$$

As it is seen from the above equation, the maximum bonding stress σ_{cr} is proportional to the surface energy γ and inversely proportional to the scale parameter r_0.

By arranging such interface elements along the crack propagation path as shown in Fig.1, the growth of the crack under the applied load can be analyzed in a natural manner. In this case, the decision on the crack growth based on the comparison between the driving force and

the resistance as in the conventional methods is not necessary.

Among three parameters involved in the interface energy function, only the surface potential γ has a clear physical meaning, while those of the scale parameter r_0 and the shape parameter N are not very clear. From our previous studies about the peeling of two bonded elastic strips, the scale parameter r_0, the shape parameter N and the mesh division had no influence on the peeling process and the process was mainly governed by the surface energy γ[5,6]. Also, from an analysis of the brittle fracture of an elastic plate with a center crack, the influence of the scale parameter r_0 on the fracture strength was found to be divided into three parts as shown in **Fig.3**[9]. From this analysis, it was found that the failure mode and the stability limit depend on the combination of the deformability of the plate and the mechanical properties of the interface. Moreover, the fracture strength in the failure problems of various structures might be quantitatively predicted by using the interface element if the scale parameter is selected appropriately.

3. Model for Analysis

As an example of a joint between dissimilar materials, co-cured joints between steel and resin were analyzed. **Figures 4 and 5** show models and mesh divisions of the joint where the scarf angle θ was varied

Fig.4 Model and mesh division of co-cured joint between steel and resin ($\theta = 90$ degree)
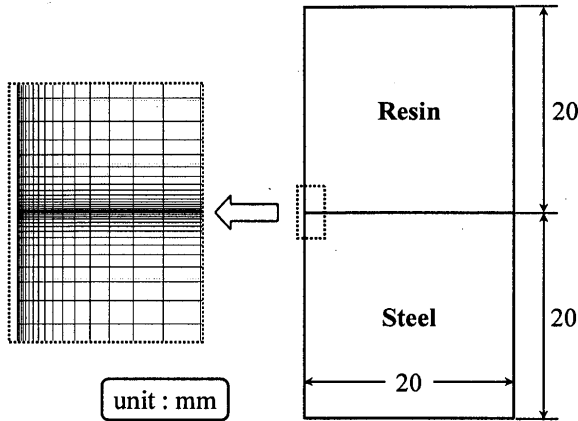

Fig.5 Model and mesh division of co-cured joint between steel and resin ($\theta = 120$ degree)

between 90 and 120 degree. The length, the width and the thickness were 40, 20 and 1 mm, respectively. Young's moduli and Poisson's ratios of steel and resin were assumed to be 210 GPa, 21 GPa, 0.30 and 0.30, respectively. The interface elements were arranged along the interface between steel and resin. In order to fix an initiation of fracture at the left side of the interface, the size of the element was reduced in the geometric series from right side to left side and from top and bottom of the interface as shown in Figs.4 and 5.

In order to examine the effect of scarf angle on the strength of the joint, the mechanical properties of the interface element need to be defined for both the opening and the shear modes since the mode of the failure is a mixed one. In this research, two types of the interface potential were developed. One was an independent model where the two modes were assumed to be independent for simplicity although the interaction between two modes is expected practically. According to this assumption, the interface potential ϕ could be defined as a sum of those for the opening mode ϕ_n and the shear mode ϕ_t as in the following equations.

$$\phi(\delta_n, \delta_t) \equiv \phi_n(\delta_n) + \phi_t(\delta_t) \tag{4}$$

$$\phi_n(\delta_n) = 2\gamma_n \cdot \left\{ \left(\frac{r_{0n}}{r_{0n} + \delta_n} \right)^{2N} - 2 \cdot \left(\frac{r_{0n}}{r_{0n} + \delta_n} \right)^N \right\} \tag{5}$$

$$\phi_t(\delta_t) = 2\gamma_t \cdot \left\{ \left(\frac{r_{0t}}{r_{0t} + |\delta_t|} \right)^{2N} - 2 \cdot \left(\frac{r_{0t}}{r_{0t} + |\delta_t|} \right)^N \right\} \tag{6}$$

Where, δ_n and δ_t were the opening and the shear deformations of the interface. Due to the symmetry of the shear deformation, the interface potential for the shear mode ϕ_t was assumed as a symmetric function of the shear deformation δ_t as shown in **Fig.6**.

On the other hand, the other model was a combined one in which the interface potential ϕ was assumed to be
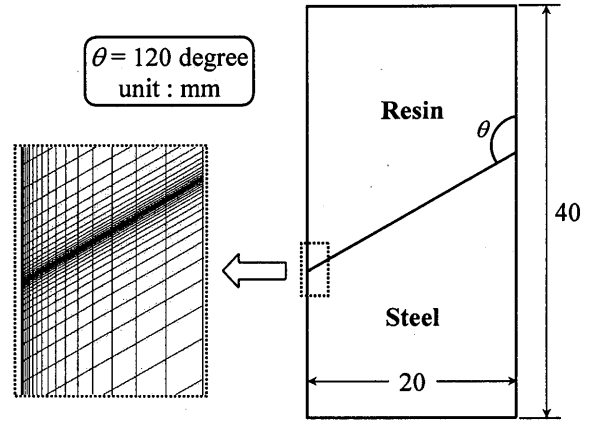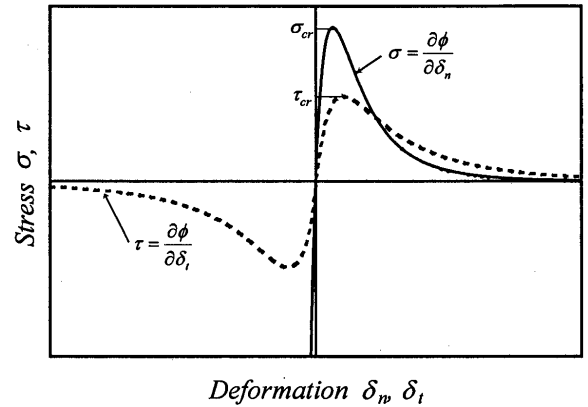

Fig.6 Displacement-stress relation at interface for opening and shear modes in Independent Model.

a coupled function of the opening and the shear deformations at the interface as shown in the following equations.

$$\phi(\delta_n, \delta_t) \equiv \phi_a(\delta_n, \delta_t) + \phi_b(\delta_n) \tag{7}$$

$$\phi_a(\delta_n, \delta_t) = 2\gamma \cdot \left\{ \left(\frac{r_0}{r_0 + \delta} \right)^{2N} - 2 \cdot \left(\frac{r_0}{r_0 + \delta} \right)^N \right\}, \tag{8}$$

$$\delta = \sqrt{\delta_n^2 + \delta_t^2} \tag{9}$$

$$\phi_b(\delta_n) = \begin{cases} \dfrac{1}{2} \cdot K \cdot \delta_n^2 & (\delta_n \leq 0) \\ 0 & (\delta_n \geq 0) \end{cases} \tag{10}$$

Where a second term of Eq.(7) was introduced to prevent overlaping in the opening direction and K was a constant having a positive value.

In the serial computations, the surface energies γ_n, γ_t in the independent model and γ in the combined model were set to a constant and the scale parameters r_{0n}, r_{0t} and r_0 were varied, i.e. $\gamma_n = \gamma_t = \gamma = 2$ N/m and $r_{0n} = r_{0t} = r_0 = 1$ nm ~ 100 mm. The scarf angle was assumed to be 60, 90 and 120 degree.
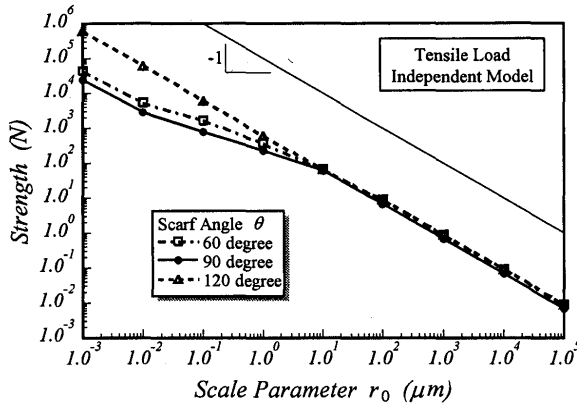
Fig.7 Influence of scale parameter r_0 on predicted bond strength of joint (Independent Model).
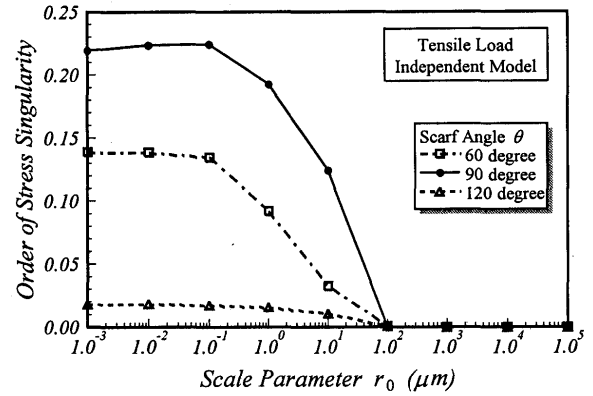

Fig.9 Influence of scale parameter r_0 on order of stress singularity at interface (Independent Model).
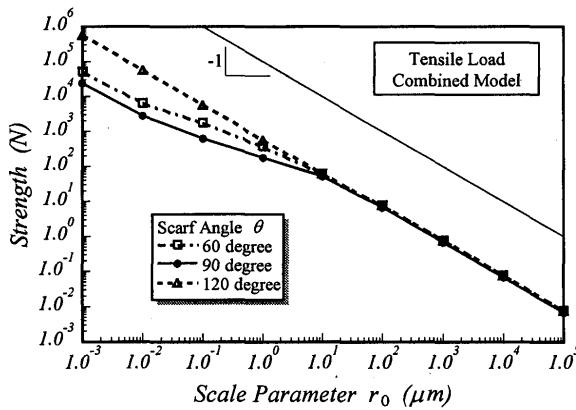

Fig.8 Influence of scale parameter r_0 on predicted bond strength of joint (Combined Model).
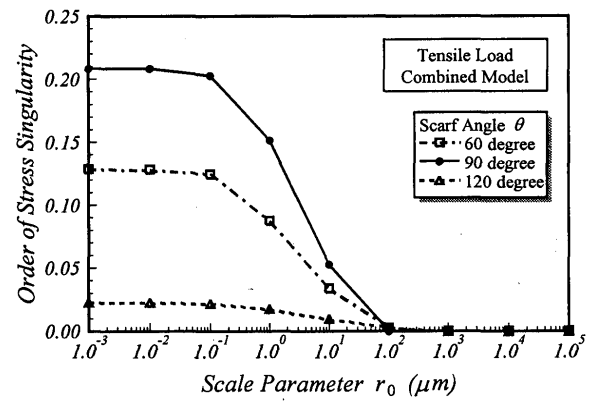

Fig.10 Influence of scale parameter r_0 on order of stress singularity at interface (Combined Model).

4. Tensile Load

By applying forced displacements in the vertical direction at the top and the bottom of joint, strength under a simple tensile load was computed. The influences of scale parameter on the computed tensile strength of the joint are summarized into **Figs.7** and **8** for the independent and the combined models, respectively. In order to examine the effects of stress singularity at the interface between steel and resin, the order of stress singularities in a vertical component of the stress at the interface were calculated from the stress field at the interface before failure and the influences of scale parameter on the stress singularity are also summarized into **Figs.9** and **10**.

In the cases, in which the scarf angle θ was 60 and 90 degree, the effects of scale parameter on the tensile strength can be divided into three parts, the same as in Fig.3, regardless of the model type. When the scale parameter was larger than 10 μm or smaller than 10 nm, the slope of the curves became -1. This could be explained in the following way. According to Eq.(2), the bonding strength and the rigidity of the interface becomes small with an increase of the scale parameter.

So, the joint breaks in the separation mode without significant deformation of steel and resin when the scale parameter is large. On the other hand, the bonding strength becomes larger than the stress induced at the left side of the interface in the FEM model, when the scale parameter is small. In this case, the fracture strength of the joint is governed by the bonding strength σ_{cr}. According to the results of our previous research about the elastic plate with a center crack[9], the scale parameter in the middle part was found to be an appropriate value for studying the effect of the stress singularity on the joint strength quantitatively. Then, from a comparison between the strength and the stress singularity, it was verified that the joint strength decreased by increasing the order of stress singularity.

In the case in which the scarf angle was 120 degree, the effect of the scale parameter on the order of stress singularity was small and the order of stress singularity itself was small. Therefore, the joint strength was considered to be governed by only the bonding strength and the slope of the curve was -1 in all the range of the scale parameter.
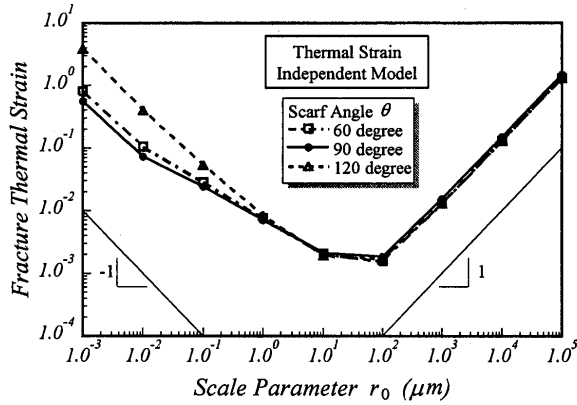
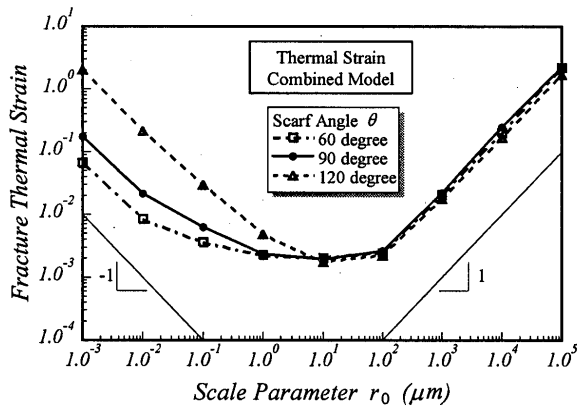**Fig.11** Influence of scale parameter r_0 on predicted fracture strain of bond joint (Independent Model).

Fig.13 Influence of scale parameter r_0 on order of stress singularity at interface (Independent Model).

Fig.12 Influence of scale parameter r_0 on predicted fracture strain of bond joint (Combined Model).

Fig.14 Influence of scale parameter r_0 on order of stress singularity at interface (Combined Model).

5. Thermal Load

Because of the difference of thermal expansion coefficient, there is a stress concentration at the interface of a joint between dissimilar materials in the manufacturing process or in use at high temperature. So, the effect of thermal strain on the joint strength between the steel and the resin was examined as an example. Since the coefficient of linear expansion in the steel is generally smaller than that in the resin, due to the difference of Young's modulus, a thermal strain was applied only to the resin in the analyses. The applied thermal strain at the fracture was defined as the fracture thermal strain. The influences of the scale parameter on the fracture strain are summarized into **Figs.11** and **12** for the independent and the combined models, respectively. Also, **Figs.13** and **14** show the effects of the order of stress singularity in shear stress at the joint interface before the fracture.

As in the simple tensile load, the influences of the scale parameter on the fracture strain also can be divided into three parts at $r_0 = 10$ nm and 10 μm. This also can be described as follows. When the scale parameter is large, the bonding strength becomes small and only the
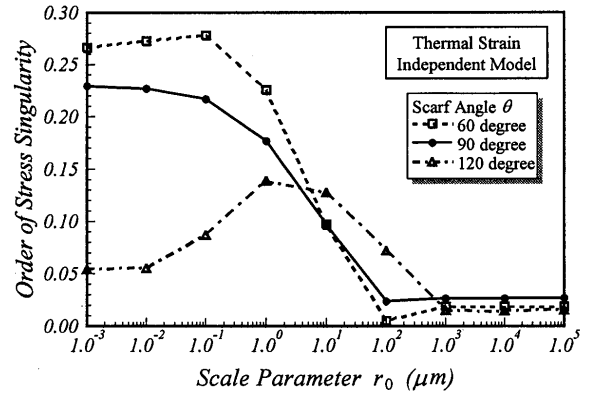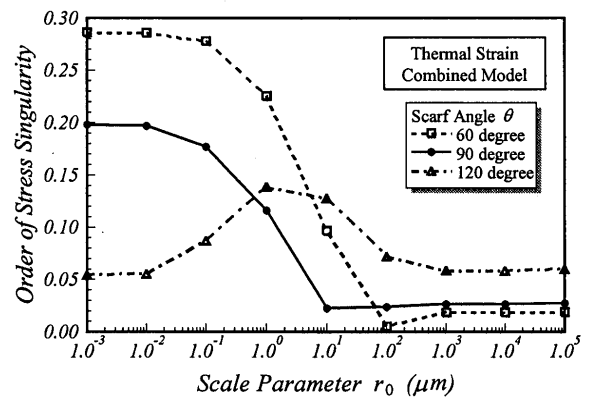
resin is expanded. In this case, the fracture occurs when the shear bonding stress of the interface reaches a maximum value and the shear deformation becomes δ_{cr} which is proportional to the scale parameter r_0 as shown in Eq.(3). So, the slope of the curves was considered to become 1. On the other hand, when scale parameter is small, the fracture is governed by the bonding strength regardless of the stress singularity and the slope of curves becomes -1.

Only in the middle part of the combined model, the fracture thermal strain becomes small with an increase of the order of stress singularity. Therefore, the combined model was considered to have great potential as a tool to study the failure problem, regardless of the type of applied stress.

6. Conclusions

In order to examine the effect of the stress singularity on the strength of joints between dissimilar materials, the fracture strength of co-cured joint between the steel and the resin was analyzed by using finite element method with the interface element. The conclusions can be summarized as follows.

(1) Two types of interface potential were developed to describe both the opening and shear deformation by one interface element. One was an independent model without an interaction between the opening and the shear deformations, and another was a combined model with an interaction.

(2) In the case where the stress was applied by a simple tensile load, the predicted fracture strength was found to decrease by increasing the order of stress singularity at the joint interface regardless of the model type of the interface element.

(3) When the stress was applied by a thermal strain, the computed fracture thermal strain decreased with the increase of the order of stress singularity in only the combined model.

(4) The combined model was considered to have a greater potential as a tool to study the failure problem regardless of the type of applied stress.

References

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