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Three Dimensional Numerical Simulation of Various Thermo-mechanical Processes by FEM (Report I)†

— Methods for Improving the convergence of 3-D Analysis of Welding —

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Abstract

The three dimensional thermal-elastic-plastic analysis by FEM has been paid great attention recently with the development of computer techniques,since this method provides the possibility to analyze complex problems such as the welding processes which undergo a high temperature cycle and exhibit temperature dependent material properties. Because of the temperature dependent material nonlinearities and geometrical nonlinearity due to the large deformation during welding, it is a very important problem how to ensure the computing accuracy and get a stable solution, especially at high temperature stages. The investigations show that the stages with drastic change, such as the transition from elastic to plastic, unloading stage and stage in which rapid changes of material properties with temperature occur suffer from the numerical error. Also the mesh division and the temperature increment are found to show significant influences to the accuracy and convergence of the solution.

To improve the accuracy, weight factors from elastic stage to plastic stage considering the temperature dependency of material properties have been introduced. In addition to this, some other methods for improving the convergence are introduced. The effectiveness of the proposed methods is demonstrated through a 3-D analysis of welding as an example in this report.

KEY WORDS :(Three Dimensional Problem) (Welding) (Thermal-Elastic-Plastic Analysis) (Accuracy) (Convergence) (Numerical Simulation) (Finite Element Method)

1. Introduction

It is about 20 years since the Finite Element Method was employed for the thermal-elastic-plastic analysis of welding first in 1970s[1]. However the research works in the past records were almost confined to two dimensional analysis, and those dealing with three dimensional problems[2)-4)] were reported only recently. One reason is that it needs very large memory capacity of computers and long CPU time. Another reason is that it is very difficult to control the accuracy and ensure the convergence of the solution.

With the development of computer techniques, the first reason are solved gradually and a large scale 3-D analysis of welding problems which are close to the real situation is becoming a realistic research target in recent years. Then, the problem of how to improve the computing accuracy and get the stable solution has become a pressing task.

Compared with the usual thermal-elastic-plastic FEM analysis, the thermal and mechanical behaviors in welding have the following characteristics:

(1) local high temperature (over the melting point of materials);
(2) rapid changes of temperature, stress and strain with the time and space due to the quick local heating and cooling;
(3) heating and cooling or loading and unloading in the weldment are observed at the same time;
(4) temperature dependency of material properties, especially at high temperature;
(5) phase transformation and creep phenomenon;
(6) treatments such as grooves filled with the weld metal and repeated thermal cycle due to multi-pass;
(7) large deformation when the structure is thin.

Due to the above characteristics, the ordinary commercial soft wares for 3-D elastic-plastic FEM analysis would not be sufficient for welding analysis. The welding phenomena includes high nonlinearities (plasticity,temperature dependency of material properties, large deformations) and very large gradients both in space and time (local and rapid thermal cycle). Thus,the accuracy of analysis has to be ensured very carefully in all stages of computation processes, otherwise the numerical error accumulates step

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by step until the solution becomes out of control.

The factors which influence the convergence of 3-D thermal -elastic- plastic analysis are investigated. Based on the investigations, methods to improve the convergence are proposed. Further, the effectiveness of the proposed method is demonstrated using 3-D welding problem as an example.

2. Fundamental Relationship and Convergence of Solution

2.1 Equilibrium equation

In the nonlinear analysis, it is necessary to use the incremental forms of equilibrium equation. according to the virtual work theorem, the equilibrium equation at the (n+1)th step can be written as follows,

$$[K]\ \{\Delta u\} = \{\Delta F\} + (F_n - f_n) \qquad (1)$$

where,

$[K]$ = the tangent stiffness matrix;
$\{\Delta F\}$ = the external load increment;
F_n = the external load at step n;
f_n = the internal load at step n.

2.2 Determination of strains and stresses

By solving Eq.(1), the increment of nodal displacement $\{\Delta u\}$ can be obtained, and the relationship between the strain increment and displacement increment is given as follows,

$$\{\Delta \varepsilon\} = [B]\ \{\Delta u\} \qquad (2)$$

where, $[B] = [B_0] + [B_L]$

$[B_0]$ = the matrix corresponding to the linear strain term;
$[B_L]$ = the matrix corresponding to the non-linear strain term due to large deformation.

The stress-strain increment relationship is given as,

$$\{\Delta \sigma\} = [D]\ \{\Delta \varepsilon\} - \{C\}\ \Delta T \qquad (3)$$

where, $[D]$ = elastic or elastic-plastic matrix;
$\{C\}$ = vector due to temperature effects.

2.3 Iteration process

The [D] matrix in Eqs.(1) and (3) depends on the stresses at present step σ, and [B] depends on the current displacement u, if the large deformation is considered. Therefore, the iterative procedure has to be included in such nonlinear problems. The sequence of the iteration in the program can be described in the flow chart in Fig.1. $F_{er}(=F_n\text{-}f_n)$ is the non-equilibrium load caused by various nonlinear factors, and it should be reduced less than the given tolerance ε after iterations. If F_{er} is too large, the convergence would not be reached, and it is impossible to get accurate results.

2.4 Error and convergence

In order to evaluate the accuracy of the computed result, the following parameter has been introduced as a measure of error,

$$E_{rr} = \sqrt{\Sigma F_i^2} / \sqrt{\Sigma F_j^2} \qquad (4)$$

where, F_i : the force at the free node i due to the non-equilibrium;
F_j : the reaction force at the fixed node j.

A criterion of the convergence is given using a tolerance ε which is very small (e.g. $\varepsilon = 10^{-4}$). After iterations, if the following condition is satisfied,

$$E_{rr} < \varepsilon \qquad (5)$$

then it can be considered that the calculation of this step is converged, and the computation can be moved to the next step. It may be expected if the error E_{rr} is small at the beginning of the iterations, the convergence and the stability are generally good.

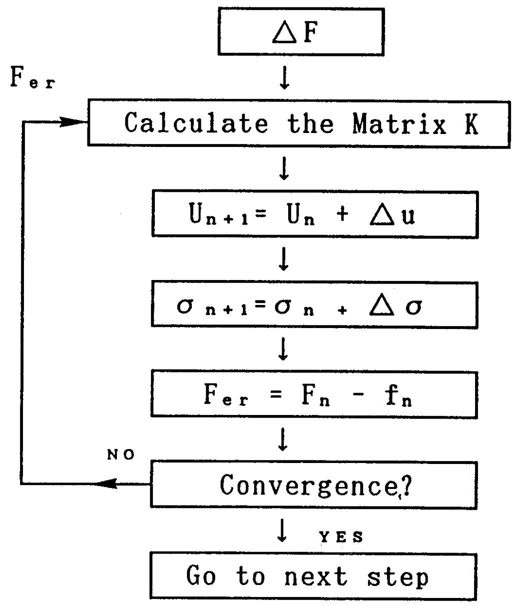


Fig.1 Iteration process.

Obviously, the less of the value of E_{rr}, the better is the stability of the solution.

3. Factors Influencing Accuracy and Convergence of Solution

3.1 Model for analysis

In order to find the factors which may influence the accuracy and convergence, a model which is fixed at two ends as shown in Fig.2 is considered. The thermal cycle with uniform temperature is assumed to be applied. The assumed material properties are:

Thermal expansion coefficient α $=1.1\times10^{-5}$ °C^{-1}

Poison's ratio μ $=0.3$

Young's modulus E_0 $=210$ GPa ; $E_{1100}=20$ GPa

Yield stress σ_{y0} $=290$ MPa ; $\sigma_{y700}=10$ MPa

Thermal cycle 20→1100→20 °C

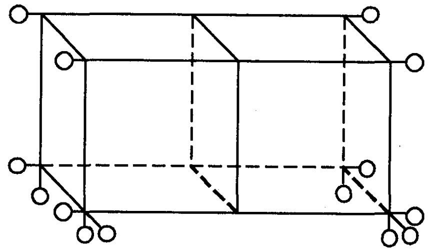


Fig.2 Model for analysis.

3.2 Numerical results

Figure 3-a) shows the temperature dependency of Young's Modulus E and yield stress σ_y. Computed results for transient stresses and error in the whole cycle are shown in Fig.3-b),c) and d), respectively. From these,it can be found that there are some situations in which the error becomes relatively large, namely,

(1) transition from elastic to plastic stage,(A,E)

(2) unloading,(D)

(3) rapid changes of material properties E and σ_y with temperature,(B-G)

(4) dis-match of E and σ_y at high temperature,(C-F).

If the above situations appear at the same time, such as unloading with reyielding, it would cause the further decrease of accuracy, and even fail to reach convergence. Therefor, we should pay attention to these situations and handle them carefully.

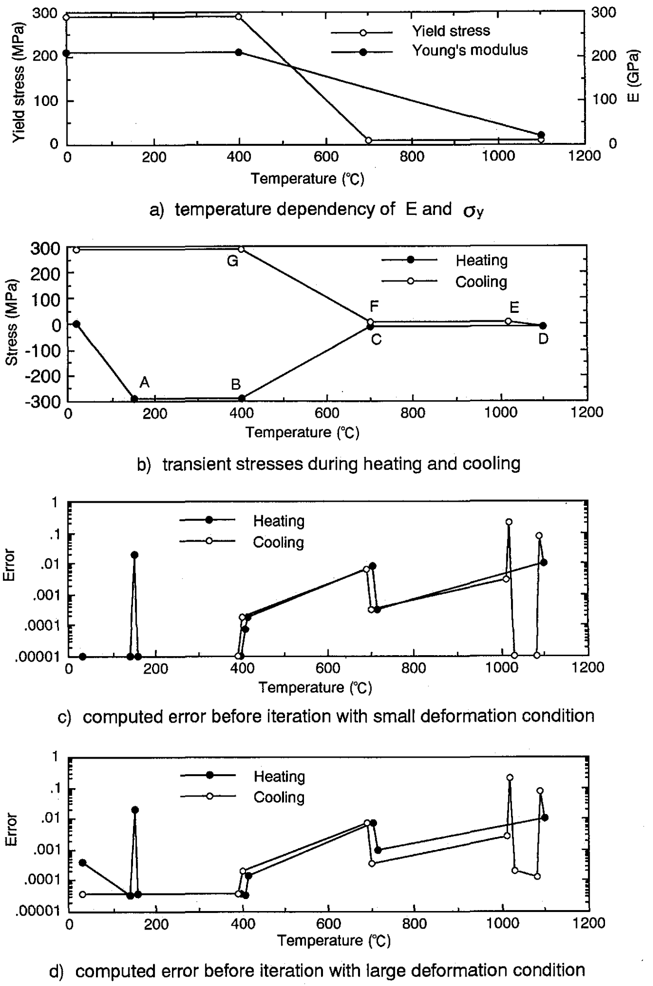


Fig.3 Computed results of transient stresses and error.

3.3 Other influential factors

(1) oscillation in temperature distribution

Welding thermal analysis is the essential prerequisite for solving the thermal-elastic-plastic problems, therefor ensuring the high accuracy of the thermal analysis is the first important step. If there are serious oscillation in temperature distribution due to numerical error, it may obviously influence the following analysis of mechanical behaviour.

(2) size of the temperature step

It may happen that the temperature increment decided based on the thermal analysis by FEM becomes relatively large, especially for the first stage of welding. However, if the size of the temperature increment is too large, the accuracy in the thermal-elastic-plastic analysis would be decreased. Table 1 shows the computed results of the error at the beginning of the iterations and the number of iterations needed to reach the convergence ($E_{rr}<10^{-4}$) in the above model at the stages C, D and E with different size of the temperature increment. It may be seen from Table 1 that the optimum size of the

Table 1 Error and Iteration No.

ΔT (℃)	Err / Iteration No.		
	C	D	E
5	0.0025/2	0.0028/3	0.1198/4
10	0.0078/3	0.0098/4	0.1989/4
25	0.0499/4	0.0444/6	0.5167/6
50	0.2051/5	0.1182/9	1.1081/9

temperature increment can be found according to the situation in the welding process.

(3) mesh division

Because of the locally concentrated heat input, the temperature near the weld bead and heat affected zone changes rapidly with the distance from the center of the heat source. It is necessary to use very fine mesh division in that area. However, the freedoms of the stress analysis is three times of the thermal analysis, and fine mesh requires large computation time. Considering that the variation of stress in space is less than that of temperature, two different mesh divisions, namely fine mesh for thermal analysis and coarse mesh for stress analysis, can be employed to save computer memory and CPU time.

(4) large deformation

Figures 3-c) and d) show that convergence is better when small deformation is assumed compared to the case in which large deformation is considered. Therefor it is better to use small deformation FEM program, unless the large deformation conditions should be considered.

4. Methods to Improve Accuracy and Convergence

4.1 Weight factor for elastic to plastic transition

When the element is in the transition from elastic stage to plastic stage at present step, it is necessary to introduce the concept of weight factor ω ($1 > \omega > 0$), and divide this step into two parts. The first part is elastic stage and the remainder is the plastic stage. The relative proportion of the two stage is ω:(1- ω) . The accurate determination of weight factor ω is important for improving the convergence of the problem.

If only simple elastic relation is used for above transition element, the stress increment is computed using the following equation.

$$\Delta \sigma = D^e \Delta \varepsilon \tag{6}$$

where, D^e : elastic matrix,

$\Delta \sigma$: stress increment,

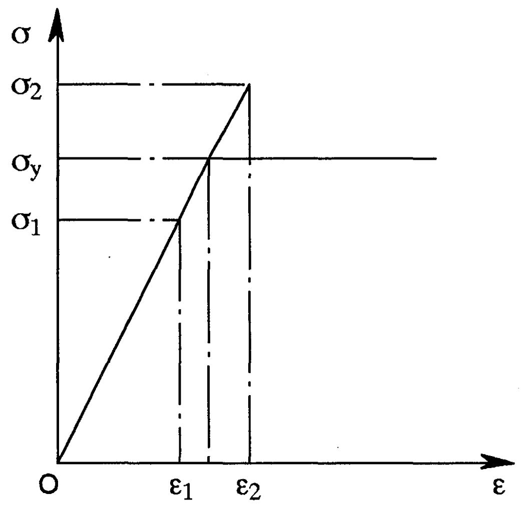


Fig.4 Stress-strain curve for elastic-plastic analysis.

$\Delta \varepsilon$: strain increment.

It can be found that the stress σ_2 has the large error as shown in Fig.4.

One way of defining the weight factor for elastic-plastic transition is given by the following equation as shown in Fig.4.

$$\omega = (\sigma_y - \sigma_1)/(\sigma_2 - \sigma_1) \tag{7}$$

and the stress can be obtained by following formula,

$$\Delta \sigma = D^e(\omega \Delta \varepsilon) + \Sigma[D^{ep}(1- \omega) \Delta \varepsilon /N] \tag{8}$$

Where, σ_1 : equivalent stress at previous step,

σ_2 : equivalent stress at present step,

σ_y : yield stress,

D^{ep} : elastic-plastic matrix,

N : subdivision of plastic increment

In this method, the yield stress σ_y is assumed to be constant, and the correct result can be obtained if the material exhibits no temperature dependence such as simple elastic-plastic problems.

Formula (7) is true for the elastic-plastic problems, however it's not accurate for thermal-elastic-plastic problems if the current yield stress σ_{y2} is used instead of true yield stress σ_y. Because of the temperature dependency of yield stress, the following three cases may happen,

(a) $\sigma_2 > \sigma_{y2} > \sigma_1$: $0 < \omega < 1$

(b) $\sigma_2 > \sigma_1 > \sigma_{y2}$: $\omega < 0$

(c) $\sigma_1 > \sigma_2 > \sigma_{y2}$: $\omega > 1$

If the factor ω becomes less than 0.0 or greater than 1.0 such as in cases (b) and (c), the solution during the iteration may become unstable.

To eliminate such situation, the definition of the weight factor is modified. Figures 5-a) and b) show the relationship between stress and temperature during loading and unloading with reyielding in a time step,respectively. Following Fig.5 the weight factor for transition between elastic and plastic stages can be defined as follows,

for loading,

$$\omega=(\sigma_{y1}-\sigma_1)/(\sigma_{y1}-\sigma_1+\sigma_2-\sigma_{y2}) \tag{9}$$

for unloading with reyielding,

$$\omega=2\sigma_1/(2\sigma_1+\sigma_2-\sigma_{y2}) \tag{10}$$

where, T_1, σ_1, σ_{y1} : temperature, equivalent stress, yield stress at previous step,

T_2, σ_2, σ_{y2} : temperature, equivalent stress, yield stress at present step,

$T_1 \sim T_y$: elastic stage,

$T_y \sim T_2$: plastic stage.

The element reaches yielding at temperature T_y and the yield stress at that moment is given by

$$\sigma_y=\sigma_{y1}(1-\omega)+\sigma_{y2}\omega \tag{11}$$

Eq.(10) is for the unloading with reverse reyielding as shown in Fig.5-b). However, it is possible that the unloading with positive reyielding phenomenon may happen due to the rapid changes of E and σ_y with temperature.In such case, it can be assumed that $\omega=0$.

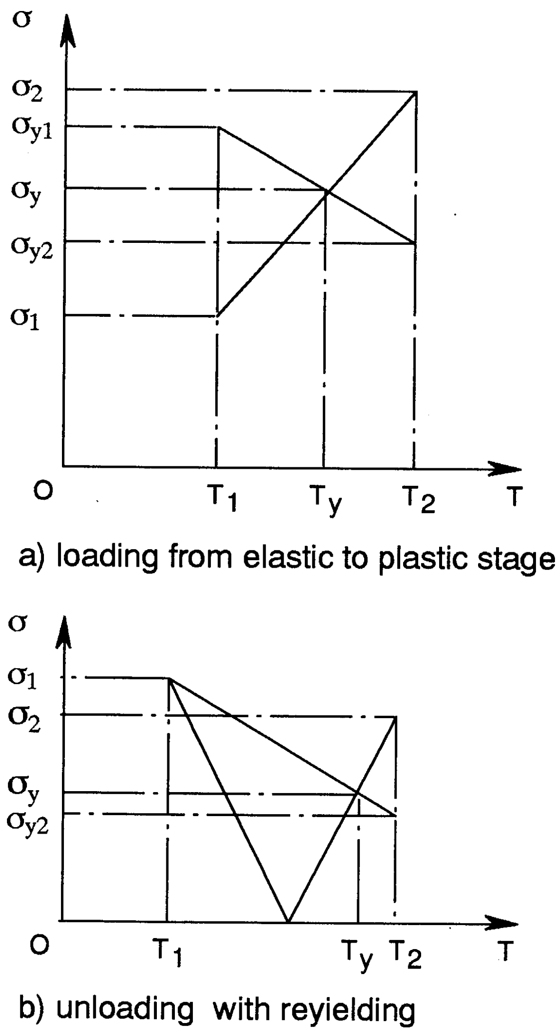


Fig.5 Relationship between stress and temperature in a step.

4.2 Curves for temperature dependent material properties

If the curves representing the temperature dependent material properties are smooth as shown in Fig.6, undesirable effect of temperature dependence upon the accuracy of the computation may be reduced. Besides, usually there is no experiment data of E and σ_y at very high temperature, and reasonably small values are assumed. However, they must satisfy certain conditions. Otherwise, the unreasonably large stress increment may happen during unloading. In order to avoid this phenomenon, one condition may be that the variations of stress due to change of Young's modulus should not be greater than that due to change of thermal expansion. From this criterion, the following condition can be derived.

$$\Delta E\cdot\sigma_y/E < E\alpha\Delta T \tag{12}$$

The condition can be rewritten as,

$$\partial E/\partial T < \alpha E^2/\sigma_y \tag{13}$$

4.3 Step size and mesh division

The size of the temperature step ΔT can be large during the first stage when the phenomena is elastic. Its size must be reduced when the nonlinear effects such as plasticity and big change of material properties due to high temperature appear. Usually, it is desirable to control ΔT less than about 10°C. To determine the size of mesh division, both accuracy of the computation and the capacity of the computer must be considered.

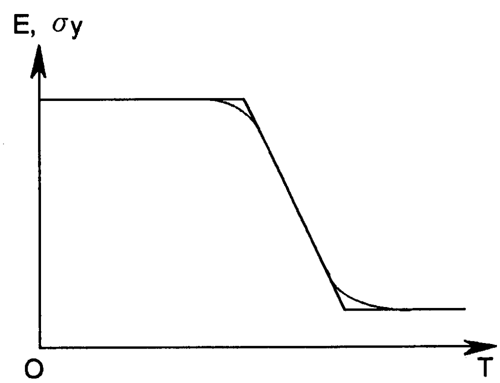


Fig.6 Smoothing of the material properties with temperature.

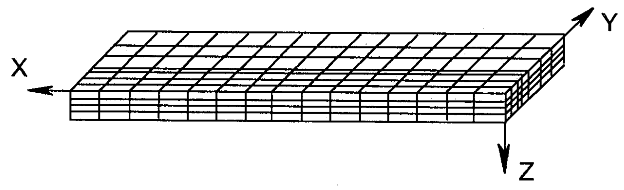


Fig.7 Mesh division.

4.4 Self-correction of non-equilibrium nodal force

The non-equilibrium nodal force caused in each step can be reduced through the iterative correction procedure and the accuracy of the computation is maintained.

5. Numerical Example

5.1 Example Model

A welding arc moving on the surface of a plate ($10 \times 100 \times 150$ mm) along x-direction is analyzed as an example. One half of the plate is analyzed using the mesh division shown in Fig.7 due to the symmetry. The assumed welding condition are:

welding current	:	$I = 170$ A
welding voltage	:	$U = 25$ V
welding speed	:	$V = 2.5$ mm/s
thermal efficiency	:	$E_{ff} = 0.75$
effective radius of arc	:	$R_c = 7$ mm

The heat flux distribution is assumed as Gaussian distribution. The material properties are as same as the simple model discussed in Chapter 7.

5.2 3-D thermal analysis by FEM

The 3-D transient temperature distribution is computed using FEM. Figure 8 shows the temperature fields of the plate at t = 15, 30, 45, 60 seconds after the start of welding.

5.3 Residual stresses and deformations

Using the program for 3-D thermal-elastic-plastic analysis in which all the possible measures to improve the accuracy are implemented, the transient and residual stresses and deformations of the weldment have been computed. Figure 9 shows the distributions of residual stresses σ_x, σ_y, σ_z and displacement in the welded plate. Figure 10 shows the distributions of residual stress σ_x along y direction and σ_y along x direction on the middle of the top surface, respectively. Using the proposed 3-D Finite Element Method, the mechanical behaviour of 3-D welding problems can be analyzed in detail, and further investigation will be extended to more complex and practical welded structures.

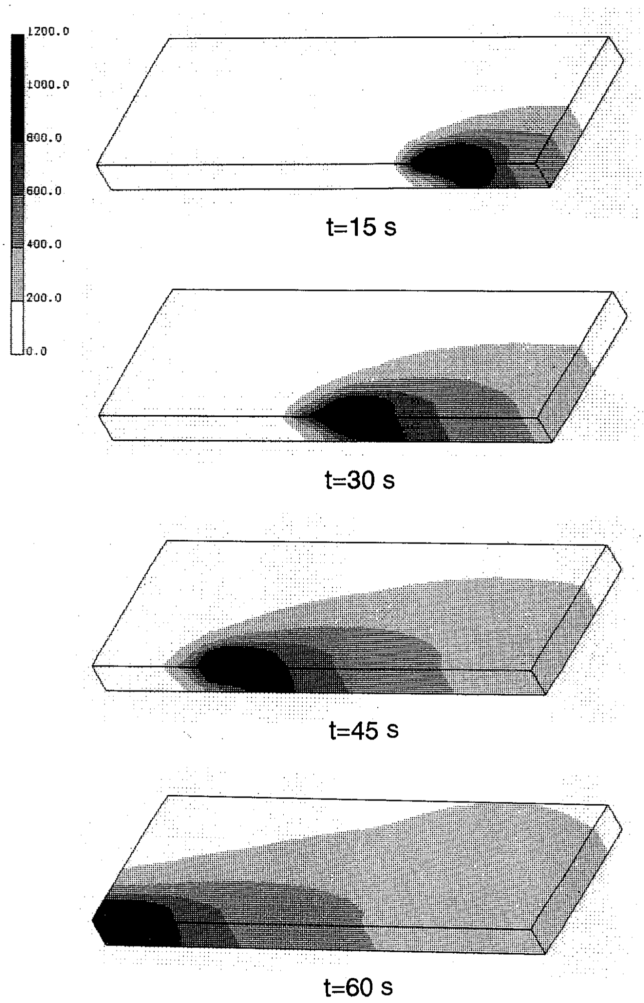


Fig.8 Welding temperature field of the plate.

6 . Conclusions

The following methods are effective for improving the accuracy and convergence of 3-D thermal-elastic-plastic analysis by FEM during welding:

(1) the accurate analysis of welding heat transfer by FEM as a first step;
(2) use of the new weight factors for the transition from elastic to plastic stage in which the temperature dependency of the yield stress is considered;
(3) special consideration on the unloading with reyielding;
(4) smoothing the material property curves and satisfying the condition on E and σ_y at high temperature;
(5) proper selection of size of temperature step and mesh division;

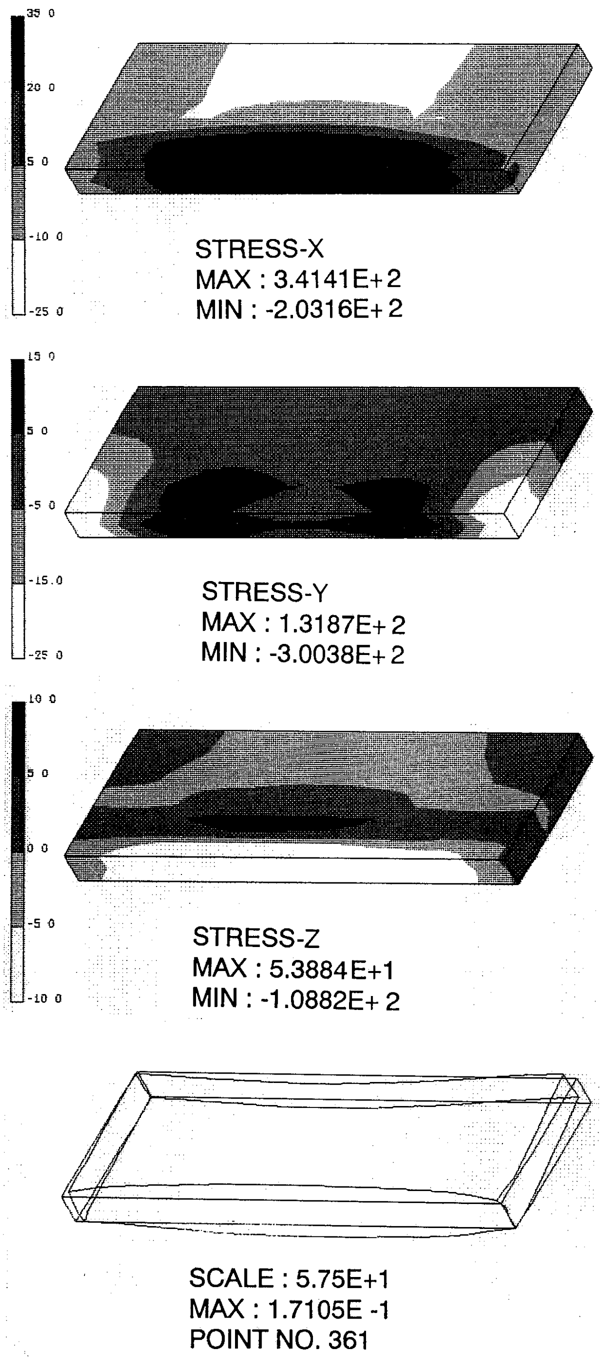
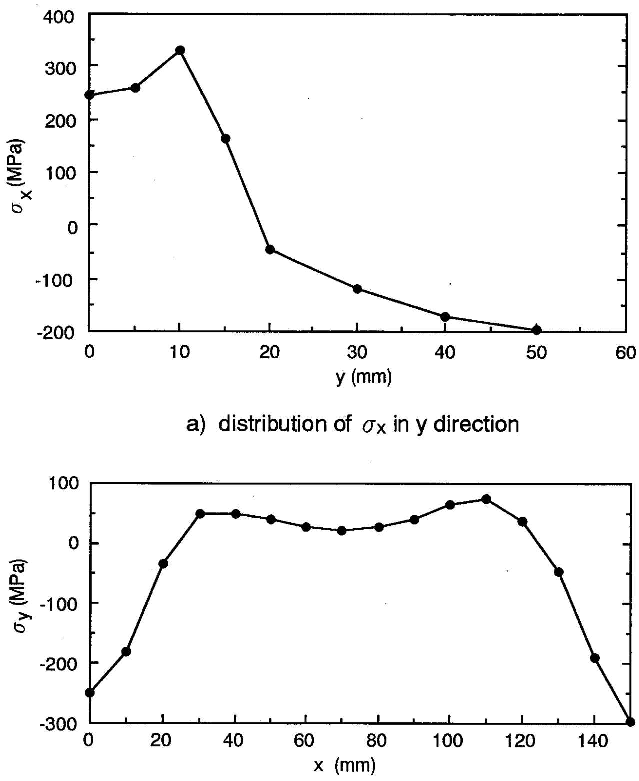


Fig.9 Residual stresses and deformations.

(6) use of the correction of non-equilibrium nodal force through iteration.

A simple example of 3-D welding model has been solved successfully by the above improved Finite Element Method.



a) distribution of σ_x in y direction

b) distribution of σ_y in x direction

Fig.10 Distribution of residual stresses on the middle of top surface.

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