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ON THE DADE CHARACTER CORRESPONDENCE AND ISOTYPIES BETWEEN BLOCKS OF FINITE GROUPS

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Abstract

In [3] Dade generalized the Glauberman character correspondence. In [13] Tasaka showed that the Dade correspondence induces an isotopy between blocks of finite groups under some assumptions. In this paper we obtain a generalization of [13], Theorem 5.5.

1. Introduction

Let p be a prime and $(\mathcal{K}, \mathcal{O}, k)$ be a p -modular system such that \mathcal{K} is a splitting field for all finite groups which we consider in this paper. Let \mathcal{S} denote \mathcal{O} or k . For a finite abelian group F , we denote by \hat{F} the character group of F and by \hat{F}_q the subgroup of \hat{F} of order q for $q \in \pi(F)$ where $\pi(F)$ is the set of all primes dividing the order $|F|$ of F . Let G be a finite group and N a normal subgroup of G . We denote by $\text{Irr}(G)$ the set of ordinary irreducible characters of G and $\text{Irr}^G(N)$ be the set of G -invariant irreducible characters of N . For $\phi \in \text{Irr}(N)$, we denote by $\text{Irr}(G|\phi)$ the set of irreducible characters χ of G such that ϕ is a constituent of the restriction χ_N of χ to N .

HYPOTHESIS 1. G is a finite group which is a normal subgroup of a finite group E such that the factor group $F = E/G$ is a cyclic group of order r . λ is a generator of \hat{F} . $E_0 = \{x \in E \mid \bar{x} \text{ is a generator of } F\}$ where $\bar{x} = xG$. E' is a subgroup of E such that $E'G = E$, $G' = G \cap E'$ and $E'_0 = E' \cap E_0$. Moreover $(E'_0)^\tau \cap E'_0$ is the empty set, for all $\tau \in E - E'$.

Under the above hypothesis, in [3], E.C. Dade constructed a bijection between $\text{Irr}^E(G)$ and $\text{Irr}^{E'}(G')$ which is a generalization of the cyclic case of the Glauberman correspondence in [4].

Theorem 1 ([3], Theorems 6.8 and 6.9). *Assume Hypothesis 1 and $|F| \neq 1$. For each prime $q \in \pi(F)$, we choose some non-trivial character $\lambda_q \in \hat{F}_q$. There is a bijection*

$$\rho(E, G, E', G'): \text{Irr}^E(G) \rightarrow \text{Irr}^{E'}(G') \quad (\phi \mapsto \phi' = \phi_{(G')})$$

which satisfies the following conditions. If r is odd, then there are a unique integer $\epsilon_\phi = \pm 1$ and a unique bijection $\psi \mapsto \psi_{(E')}$ of $\text{Irr}(E|\phi)$ onto $\text{Irr}(E'|\phi')$ such that

$$(1.1) \quad \left(\prod_{q \in \pi(F)} (1 - \lambda_q) \cdot \psi \right)_{E'} = \epsilon_\phi \prod_{q \in \pi(F)} (1 - \lambda_q) \cdot \psi_{(E')},$$

for any $\psi \in \text{Irr}(E|\phi)$. If r is even, and we choose $\epsilon_\phi = \pm 1$ arbitrarily, then there is a unique bijection $\psi \mapsto \psi_{(E')}$ of $\text{Irr}(E|\phi)$ onto $\text{Irr}(E'|\phi')$ such that (1.1) holds for all $\psi \in \text{Irr}(E|\phi)$. In both cases we have

$$(\lambda \psi)_{(E')} = \lambda \psi_{(E')}$$

for any $\lambda \in \hat{F}$ and $\psi \in \text{Irr}(E|\phi)$. Furthermore, the resulting bijection is independent of the choice of the non-trivial character $\lambda_q \in \hat{F}_q$, for any $q \in \pi(F)$.

Assume Hypothesis 1. If $|F| = 1$, then $E = E'$. We call $\rho(E, G, E', G')$ the Dade correspondence, where $\rho(E, G, E', G')$ denote the identity map of $\text{Irr}^E(G)$ when $|F| = 1$. Following [13], for $\phi' \in \text{Irr}^{E'}(G')$, we set $\phi'_{(G)} = \rho(E, G, E', G')^{-1}(\phi')$, and for $\psi \in \text{Irr}(E|\phi)$ and $\psi' \in \text{Irr}(E'|\phi')$, we set $\psi'_{(E)} = \psi$ if $\psi' = \psi_{(E')}$. From (1.1) ψ' is a constituent of $(\lambda \psi)_{E'}$ for some $\lambda \in \hat{F}$, hence $\phi'_{(G)}$ is a constituent of $\phi_{(G')}$. In particular if ϕ is the trivial character of G , then $\phi_{(G')}$ is the trivial character of G' . From the above theorem we have the following also.

Proposition 1. *Assume Hypothesis 1. Let $\phi \in \text{Irr}^E(G)$ and $\phi' \in \text{Irr}^{E'}(G')$. Then $\phi' = \phi_{(G')}$ if and only if there exist $\psi \in \text{Irr}(E|\phi)$, $\psi' \in \text{Irr}(E'|\phi')$ and $\epsilon = \pm 1$ such that*

$$\psi(x) = \epsilon \psi'(x) \quad (\forall x \in E'_0).$$

THE GENERALIZED GLAUBERMAN CASE Let G and A be finite groups such that A is cyclic, A acts on G via automorphism and that $(|C_G(A)|, |A|) = 1$. We set $E = G \rtimes A$, $G' = C_G(A)$ and $E' = G' \times A \leq E$. By [3], Lemma 7.5, E, G, E' and G' satisfy Hypothesis 1. Moreover by [3], Proposition 7.8, in the Glauberman case, that is, if $(|A|, |G|) = 1$, then the Glauberman correspondence coincides with the Dade correspondence.

In the generalized Glauberman case, suppose that $p \nmid |A|$ and $p \nmid |G : C_G(A)|$. Then in [8], H. Horimoto proved that there is an isotopy between $b(G)$ and $b(C_G(A))$ induced by the Dade correspondence where $b(G)$ is the principal block of G . Isotopy is a notion defined in [1].

HYPOTHESIS 2. Assume Hypothesis 1. $(p, r) = 1$. b is an E -invariant block of G covered by r distinct blocks of E .

Assume Hypothesis 2 and that r is a prime power. Moreover let b' be a block of G' containing $\phi_{(G')}$ for some $\phi \in \text{Irr}(b)$. In [13], F. Tasaka proved that if r is odd, or $r = 2$ or $b = b(G)$, and if b' is covered by r blocks of E' , then there is an isotypy between b and b' induced by the Dade correspondence ([13], Theorem 5.5). In this paper we prove that the arguments in [13] can be extended to the general case (see Theorem 6 in §5). Theorem 6 is a generalization of Theorem 5 in [16]. We also show that the Brauer correspondent of b and that of b' are Puig equivalent (see Theorem 8 in §6).

NOTATIONS. We follow the notations in [13], [12] and [15]. Let G be a finite group. We denote by $G_0(\mathcal{K}G)$ the Grothendieck group of the group algebra $\mathcal{K}G$. If L is a $\mathcal{K}G$ -module, then let $[L]$ denote the element in $G_0(\mathcal{K}G)$ determined by the isomorphism class of L . For $\phi \in \text{Irr}(G)$, we denote by $\check{\phi}$, e_ϕ and L_ϕ , the dual character of ϕ , the centrally primitive idempotent of $\mathcal{K}G$ corresponding to ϕ and a $\mathcal{K}G$ -module affording ϕ respectively. We also denote by ω_ϕ the linear character of the center $Z(\mathcal{K}G)$ of $\mathcal{K}G$ corresponding to ϕ . Let H be a subgroup of G . We denote by $(\mathcal{S}G)^H$ the set of H -fixed elements of $\mathcal{S}G$. We denote by Pr_H^G the \mathcal{S} -linear map from $\mathcal{S}G$ to $\mathcal{S}H$ defined by $\text{Pr}_H^G(\sum_{x \in G} a_x x) = \sum_{h \in H} a_h h$ and by Tr_H^G the trace map from $(\mathcal{S}G)^H$ to $Z(\mathcal{S}G)$. For $\alpha \in \mathcal{O}$, we denote by α^* the canonical image of α in k . For $a \in \mathcal{O}G$, we denote by a^* the canonical image of a in kG . For a p -subgroup P of G , we denote by $\text{Br}_P^{\mathcal{S}G}$ the Brauer homomorphism from $(\mathcal{S}G)^P$ onto $kC_G(P)$. Also let $G_{p'}$ denote the set of p -regular elements of G .

Let b be a block of G . We denote by $\mathcal{R}_{\mathcal{K}}(G, b)$ the additive group of generalized characters belonging to b , by $\text{CF}(G, b; \mathcal{K})$ the subspace with a basis $\text{Irr}(b)$ of the \mathcal{K} -vector space of the \mathcal{K} -valued central functions of $\mathcal{K}G$, and by $\text{CF}_{p'}(G, b; \mathcal{K})$ the subspace containing the elements of $\text{CF}(G, b; \mathcal{K})$ which vanish on p -singular elements of G , where $\text{Irr}(b)$ is the set of ordinary irreducible characters belonging to b . Let (u, b_u) be a b -Brauer element. We denote by $d_G^{(u, b_u)}$ the decomposition map from $\text{CF}(G, b; \mathcal{K})$ onto $\text{CF}_{p'}(C_G(u), b_u; \mathcal{K})$. For $\gamma \in \text{CF}(G, b; \mathcal{K})$ and $c \in C_G(u)_{p'}$, we have $d_G^{(u, b_u)}(\gamma)(c) = \gamma(ucb_u)$. We also denote by ω_b the central character of $Z(\mathcal{O}Gb)$ and by $\text{Bl}(C_G(P), b)$ the set of blocks of $C_G(P)$ associated with b where P is a p -subgroup of G . Let N be a normal subgroup of G . For $\phi \in \text{Irr}(N)$, we denote by $I_G(\phi)$ the inertial group of ϕ in G . For a block \mathbf{b} of N , we denote by $I_G(\mathbf{b})$ the inertial group of \mathbf{b} in G . For a subgroup H and a block \mathbf{c} of H , if \mathbf{c} is associated with a block B of G , then B is denoted by \mathbf{c}^G .

2. Preliminaries

In this section we assume Hypothesis 1. For $x \in E$ (resp. $x \in E'$), we denote by $C(x)$ (resp. $C(x')$) the conjugacy class of E (resp. E') containing x . For $X \subseteq E$, we set $\hat{X} = \sum_{x \in X} x \in SE$.

Lemma 1. *Let $s \in E'_0$ and let Q, R be subgroups of G' centralized by s . Let $a \in G$. If $Q^a = R$, then $a \in C_G(Q)G'$. In particular $N_G(Q) = C_G(Q)N_{G'}(Q)$.*

Proof. By the assumption, $s^a \in C_E(R) \cap E_0$. By [13], Lemmas 3.9 and 2.4, there exists $y \in C_E(R)$ such that $s^{ay} \in C_{E'}(R)$. Since $s^{ay}, s \in E'_0, ay \in E'$. Set $z = ay$. Then $Q^z = R$, hence $a = (zy^{-1}z^{-1})z \in C_E(Q)E'$. Since $C_E(Q) = C_G(Q)\langle s \rangle$ and $E' = \langle s \rangle G'$, $a \in C_G(Q)G'\langle s \rangle$ and hence $a \in C_G(Q)G'$. □

Proposition 2 (see [13], Proposition 3.7). *Let $x \in E'_0, \phi \in \text{Irr}^E(G)$ and $\phi' \in \text{Irr}^{E'}(G')$. Then we have the following.*

- (i) $\text{Pr}_{E'}^E(\widehat{C(x)}e_\phi) = \widehat{C(x')}e_{\phi_{(G')}}.$
- (ii) $\text{Tr}_{E'}^E(\widehat{C(x')}e_{\phi'}) = \widehat{C(x)}e_{\phi'_{(G)}}.$

Proof. Let ψ be an extension of ϕ to E . $\widehat{C(x)}e_\phi$ is a \mathcal{K} -linear combination of the elements in xG . Hence we have

$$\widehat{C(x)}e_\phi = \frac{|C(x)|}{|E|} \sum_{y \in xG} r\psi(x)\psi(y^{-1})y.$$

From Theorem 1, (1.1), $\psi(z) = \epsilon_\phi \psi_{(E')}(z)$ for any $z \in E'_0$. Therefore we have

$$\begin{aligned} \widehat{C(x')}e_{\phi_{(G')}} &= \frac{|C(x')|}{|E'|} \sum_{z \in xG'} r\psi_{(E')}(x)\psi_{(E')}(z^{-1})z \\ &= \frac{|C(x')|}{|E'|} \sum_{z \in xG'} r\psi(x)\psi(z^{-1})z. \end{aligned}$$

From [13], 2.4, we have (i) and (ii). □

3. The Dade correspondence and blocks

Assume Hypothesis 1 and $p \nmid r$. If an element $s \in E'_0$ centralizes a Sylow p -subgroup of G , then the principal block $b(G)$ satisfies Hypothesis 2.

HYPOTHESIS 3. Assume Hypothesis 1. $(p, r) = 1$. b' is an E' -invariant block of G' covered by r distinct blocks of E' .

Assume Hypotheses 2 and 3 and assume that $\phi_{(G')} \in \text{Irr}(b')$ for some $\phi \in \text{Irr}(b)$. In this section we show the Dade correspondence $\rho(E, G, E', G')$ induces a bijection between $\text{Irr}(b)$ and $\text{Irr}(b')$, and the Brauer categories $\mathbf{B}_G(b)$ and $\mathbf{B}_{G'}(b')$ are equivalent.

Theorem 2 (see [13], Proposition 3.5, (1) and (2)). (i) *Assume Hypothesis 2. Then $\{\phi_{(G')} \mid \phi \in \text{Irr}(b)\}$ is contained in a block $b_{(G')}$ of G' .* (ii) *Assume Hypothesis 3. Then $\{\phi'_{(G)} \mid \phi' \in \text{Irr}(b')\}$ is contained in a block $b'_{(G)}$ of G .*

Proof. (i) Let $\phi_1, \phi_2 \in \text{Irr}(b)$ and set $\phi'_i = \phi_{i(G')}$ for $i = 1, 2$. We show ϕ'_1 and ϕ'_2 belong to a same block of G' . We may assume at least one of these characters is of height 0. Let \hat{b} be a block of G covering b and for $i = 1, 2$, let $\hat{\phi}_i$ be a unique extension of ϕ_i to E belonging to \hat{b} recalling Hypothesis 2. Note \hat{b} and b are isomorphic by restriction. Set $(\hat{\phi}_i)' = (\hat{\phi}_i)_{(E')}$ for $i = 1, 2$. By [12], Chapter III, Lemma 6.34, we have the following for a non-trivial linear character λ of F ,

$$(3.1) \quad \sum_{x \in E_{p'}} \hat{\phi}_1(x)\hat{\phi}_2(x^{-1}) \neq 0, \quad \sum_{x \in E_{p'}} \hat{\phi}_1(x)\lambda(x^{-1})\hat{\phi}_2(x^{-1}) = 0.$$

For each $q \in \pi(F)$, let λ_q be a non-trivial linear character in \hat{F}_q . Set $(E_0)_{p'} = E_0 \cap E_{p'}$ and $(E'_0)_{p'} = E'_0 \cap E_{p'}$. We have

$$\begin{aligned} & \sum_{x \in E_{p'}} \hat{\phi}_1(x) \left(\prod_{q \in \pi(F)} (1 - \lambda_q) \cdot \hat{\phi}_2 \right) (x^{-1}) \\ &= \sum_{y \in (E_0)_{p'}} \hat{\phi}_1(y) \left(\prod_{q \in \pi(F)} (1 - \lambda_q) \cdot \hat{\phi}_2 \right) (y^{-1}) \end{aligned}$$

by [13], Lemma 2.4,

$$= \frac{|E|}{|E'|} \sum_{z \in (E'_0)_{p'}} \hat{\phi}_1(z) \left(\prod_{q \in \pi(F)} (1 - \lambda_q) \cdot \hat{\phi}_2 \right) (z^{-1})$$

by Theorem 1,

$$\begin{aligned} &= \epsilon_{\phi_1} \epsilon_{\phi_2} \frac{|E|}{|E'|} \sum_{z \in (E'_0)_{p'}} (\hat{\phi}_1)'(w) \left(\prod_{q \in \pi(F)} (1 - \lambda_q) \cdot (\hat{\phi}_2)'\right) (w^{-1}) \\ &= \epsilon_{\phi_1} \epsilon_{\phi_2} \frac{|E|}{|E'|} \sum_{u \in (E')_{p'}} (\hat{\phi}_1)'(u) \left(\prod_{q \in \pi(F)} (1 - \lambda_q) \cdot (\hat{\phi}_2)'\right) (u^{-1}), \end{aligned}$$

that is,

$$\begin{aligned}
 (3.2) \quad & \sum_{x \in E_{p'}} \hat{\phi}_1(x) \left(\prod_{q \in \pi(F)} (1 - \lambda_q) \cdot \hat{\phi}_2 \right) (x^{-1}) \\
 &= \epsilon_{\phi_1} \epsilon_{\phi_2} \frac{|E|}{|E'|} \sum_{u \in (E')_{p'}} (\hat{\phi}_1)'(u) \left(\prod_{q \in \pi(F)} (1 - \lambda_q) \cdot (\hat{\phi}_2)' \right) (u^{-1}).
 \end{aligned}$$

From (3.1) there exists $\lambda \in \prod_{q \in \pi(F)} \hat{F}_q$ such that

$$\sum_{u \in (E')_{p'}} (\hat{\phi}_1)'(u) (\lambda (\hat{\phi}_2)')(u^{-1}) \neq 0.$$

Then $(\hat{\phi}_1)'$ and $\lambda(\hat{\phi}_2)'$ belong to a same block of E' . Hence ϕ'_1 and ϕ'_2 belong to a same block of G' . (ii) follows from (3.2) and the above arguments. □

Assume Hypothesis 2. We denote by \hat{b}_0 a block of E covering b . For each $\phi \in \text{Irr}(b)$, we denote by $\hat{\phi}$ a unique extension of ϕ which belongs to \hat{b}_0 . For any $i \in \mathbf{Z}$, we denote by \hat{b}_i the block of E which contains $\lambda^i \hat{\phi}$ where $\phi \in \text{Irr}(b)$. For the block b , \hat{b}_i is fixed throughout this paper. Let $\hat{b}_0 = \sum_{x \in E} \alpha_x x$. Then $\hat{b}_i = \sum_{x \in E} \lambda^i (x^{-1}) \alpha_x x$. Moreover we note that for any $t \in E$, $\sum_{x \in G^t} \alpha_x^* x \neq 0$ because $\{(\hat{b}_0)^*, (\hat{b}_1)^*, \dots, (\hat{b}_{r-1})^*\}$ are linearly independent. This fact is used implicitly in the proof of Proposition 5 below.

Proposition 3 (see [13], Proposition 3.5, (3)). *Assume Hypotheses 2 and 3, and assume $b' = b_{(G')}$ using the notation in Theorem 2. Then there exists a block $(\hat{b}_0)_{(E')}$ of E' such that $\text{Irr}((\hat{b}_0)_{(E')}) = \{(\hat{\phi})_{(E')} \mid \phi \in \text{Irr}(b)\}$. If r is odd, then $(\hat{b}_0)_{(E')}$ is uniquely determined, and if r is even, we have exactly two choices for $(\hat{b}_0)_{(E')}$.*

Proof. Let $\phi_1, \phi_2 \in \text{Irr}(b)$ and suppose that ϕ_1 is of height 0. Assume $(\hat{\phi}_1)_{(E')}$ belongs to a block $(\hat{b}_0)_{(E')}$ of E' . Here we note that we have two choices for $(\hat{\phi}_1)_{(E')}$ when r is even by Theorem 1, and hence we have two choices for $(\hat{b}_0)_{(E')}$. By the proof of Theorem 2 and by our assumption, there is a unique linear character $\nu \in \hat{F}$ such that $\nu(\hat{\phi}_2)_{(E')}$ belongs to $(\hat{b}_0)_{(E')}$ and that $\nu = 1$ or ν is a product of some elements of $\{\lambda_q \mid q \in \pi(F)\}$. Hence if r is odd, then $\nu = 1$ because λ_q can be replaced by another non-trivial linear character in \hat{F}_q . If r is even, $\nu = 1$ or $\nu = \lambda_2$, hence $(\hat{\phi}_2)_{(E')}$ belongs to $(\hat{b}_0)_{(E')}$ by replacing ϵ_{ϕ_2} by $-\epsilon_{\phi_2}$ if necessary. This combined with Theorem 1 completes the proof. □

With the notation in the above proposition, we denote by $(\hat{b}_i)_{(E')}$ the block of E' containing $\lambda^i(\hat{\phi})_{(E')}$ ($\phi \in \text{Irr}(b)$) for $i \in \mathbf{Z}$. Moreover, when r is even, we fix one of two $(\hat{b}_0)_{(E')}$, and hence $(\hat{b}_i)_{(E')}$ are fixed.

Lemma 2 (see [13], Lemma 3.3). *Assume Hypothesis 2. We have the following holds.*

- (i) *There exists $s \in E_0$ such that $(\omega_{\hat{b}_i}(\widehat{C}(s)))^* \neq 0$ for all $i \in \mathbf{Z}$.*
- (ii) *For s in (i), $\widehat{C}(s)b \in Z(\mathcal{O}Eb)^\times$, that is, $\widehat{C}(s)b$ is invertible in $Z(\mathcal{O}Eb)$.*

Proof. (i) By the assumption and [12], Chapter III, Theorem 6.24, for any $q \in \pi(F)$, there exists $s(q) \in E$ such that $(\omega_{\hat{b}_i}(\widehat{C}(s(q))))^* \neq 0$ and that $s(q)G$ is a generator of the Sylow q -subgroup of F . Then $(\omega_{\hat{b}_i}(\prod_{q \in \pi(F)} \widehat{C}(s(q))))^* \neq 0$. This implies that there exists $s \in E_0$ such that $(\omega_{\hat{b}_i}(\widehat{C}(s)))^* \neq 0$.

(ii) From (i) $\widehat{C}(s)\hat{b}_i \in Z(\mathcal{O}E\hat{b}_i)^\times$ for any i because $Z(\mathcal{O}E\hat{b}_i)$ is local. Hence $\widehat{C}(s)b \in Z(\mathcal{O}Eb)^\times$. □

Assume Hypothesis 2. By the above lemma and [13], Lemma 2.4, there exists an element $s \in E'_0$ such that $\widehat{C}(s)b \in Z(\mathcal{O}Eb)^\times$. Hence there exists a defect group D of b centralized by s , and hence contained in G' (see [13], Lemma 3.10). Let $P \leq D$. Then by [13], Lemma 3.9, $C_E(P)$, $C_G(P)$, $C_{E'}(P)$ and $C_{G'}(P)$ satisfy Hypothesis 1. Moreover we note $F \cong C_E(P)/C_G(P)$. Let $e \in \text{Bl}(C_G(P), b)$. Then we see that $\text{Br}_P^{\mathcal{O}E}(\widehat{C}(s)b)e^* \in (Z(kC_E(P)e^*))^\times$. This implies that e is covered by r blocks of $C_E(P)$. Similarly assume Hypothesis 3. Let D' be a defect group of b' and $e' \in \text{Bl}(C_{G'}(P'), b')$ for a subgroup P' of D' . Then e' is covered by r blocks of $C_{E'}(P')$.

Theorem 3 (see [13], Proposition 3.11). *Using the same notations as in Theorem 2 we have the following.*

- (i) *Assume Hypothesis 2. Let D be a defect group of b obtained in the above and let $P \leq D$. Let $e \in \text{Bl}(C_G(P), b)$. Then $e_{(C_{G'}(P))} \in \text{Bl}(C_{G'}(P), b_{(G')})$. In particular, $b_{(G')}$ has a defect group containing D .*
- (ii) *Assume Hypothesis 3. Let D' be a defect group of b' and let $P' \leq D'$. Let $e' \in \text{Bl}(C_{G'}(P'), b')$. Then $e'_{(C_G(P'))} \in \text{Bl}(C_G(P'), b'_{(G)})$. In particular, $b'_{(G)}$ has a defect group containing D' .*

Proof. See the proof of [13], Proposition 3.11. □

Assume Hypotheses 2 and 3, and assume $b' = b_{(G')}$ where $b_{(G')}$ is the block determined by Theorem 2. We have

$$\text{Irr}(b') = \{\phi_{(G')} \mid \phi \in \text{Irr}(b)\}$$

by Theorem 2. Let D be a common defect group of b and b' , and let $P \leq D$. Such a defect group exists by the above theorem. Let (D, b_D) be maximal b -Brauer pair and let (P, b_P) be a b -Brauer pair contained in (D, b_D) . By the above theorem, $(D, (b_D)_{(C_{E'}(D))})$

is a maximal b' -Brauer pair and $(P, (b_P)_{(C_{E'}(P))})$ is a b' -Brauer pair. We set

$$(b_P)' = (b_P)_{(C_{E'}(P))}$$

and

$$(b_P^*)' = ((b_P)')^*.$$

For any $u \in C_{E'}(P)$, we denote by $C(u)_{(P)}$ the conjugacy class of $C_E(P)$ containing u , and by $C(u)'_{(P)}$ the conjugacy class of $C_{E'}(P)$ containing u .

Theorem 4 (see [13], Theorem 5.2). *Assume Hypotheses 2 and 3, and assume $b' = b_{(G')}$ where $b_{(G')}$ is the block determined by Theorem 2. Then the Brauer categories $\mathbf{B}_G(b)$ and $\mathbf{B}_{G'}(b')$ are equivalent.*

Proof. Our proof is essentially the same as the proof of [13], Theorem 5.2. Let D be a common defect group of b and b' , and let $P \leq D$. There is an element $t \in C_E(P) \cap E'_0$ such that $\widehat{C(t)_{(P)}}b_P^* \in (Z(kC_E(P))b_P^*)^\times$. By Lemma 2, such an element exists. For any $a \in G'$ we have the following using Proposition 2 and Theorem 2.

$$(3.3) \quad \widehat{C(t^a)'_{(P^a)}}((b_P^*)')^a = \Pr_{C_{E'}(P^a)}^{C_E(P^a)}(\widehat{C(t^a)_{(P^a)}}(b_P^*)^a) \neq 0.$$

In fact we have

$$\begin{aligned} \widehat{C(t^a)'_{(P^a)}}((b_P^*)')^a &= \widehat{C(t)'_{(P)}}(b_P^*)'^a \\ &= (\Pr_{C_{E'}(P)}^{C_E(P)}(\widehat{C(t)_{(P)}}b_P^*))^a = \Pr_{C_{E'}(P^a)}^{C_E(P^a)}(\widehat{C(t^a)_{(P^a)}}(b_P^*)^a) \neq 0. \end{aligned}$$

In particular, if $(P, b_P)^a = (P, b_P)$, then $(P, (b_P)')^a = (P, (b_P)')$. □

Now for $P \leq R \leq D$, we prove $(P, (b_P)') \leq (R, (b_R)')$. We may assume $P \trianglelefteq R$. From (3.3) R fixes $(b_P)'$ because R fixes b_P . Now let $s \in E'_0$ be such that $\widehat{C(s)}b \in Z(\mathcal{O}Eb)^\times$. Then $\widehat{C(s) \cap C_{E'}(P)}(b_P)'$ is fixed by R . Moreover $\widehat{C(s) \cap C_E(P)}b_P^*$ is invertible in $(Z(kC_E(P))b_P^*)^R$. Hence $\text{Br}_{R/P}^{kC_E(P)}(\widehat{C(s) \cap C_E(P)}b_P^*)b_R^*$ is invertible in $Z(kC_E(R))b_R^*$ where $\text{Br}_{R/P}^{kC_E(P)}$ is the restriction to $(kC_E(P))^R$ of the Brauer homomorphism Br_R^{kE} . In particular it does not vanish. Hence we have from Proposition 2

$$\begin{aligned} &\text{Br}_{R/P}^{kC_E(P)}(\widehat{C(s) \cap C_{E'}(P)}(b_P^*)')(b_R^*)' \\ &= \text{Br}_{R/P}^{kC_{E'}(P)}(\Pr_{C_{E'}(P)}^{C_E(P)}(\widehat{C(s) \cap C_E(P)}b_P^*)) (b_R^*)' \\ &= \Pr_{C_{E'}(R)}^{C_E(R)}(\text{Br}_{R/P}^{kC_E(P)}(\widehat{C(s) \cap C_E(P)}b_P^*)) (b_R^*)' \\ &= \Pr_{C_{E'}(R)}^{C_E(R)}(\text{Br}_{R/P}^{kC_E(P)}(\widehat{C(s) \cap C_E(P)}b_P^*)b_R^*) \neq 0. \end{aligned}$$

The last inequality follows from [13], Lemmas 3.9 and 2.4. Therefore

$$\text{Br}_{R/P}^{kC_{E'}(P)}((b_P^*)')(b_R^*)' \neq 0.$$

This implies $(P, (b_P)') \leq (R, (b_R)')$.

For a subgroup T of D and $a \in G$, suppose that $(P, b_P)^a \leq (T, b_T)$. We show that there is an element $e \in C_G(P)$ such that $ea \in G'$ and $(P, (b_P)')^{ea} \leq (T, (b_T)')$. By Lemma 1, we may assume $a \in G'$. Since we have $(P, b_P)^a = (P^a, b_{Pa})$, $(b_P)^a = b_{Pa}$. From (3.3), $((b_P)')^a = (b_{Pa})'$, hence $(P, (b_P)')^a = (P^a, (b_{Pa})') \leq (T, (b_T)')$. Conversely for $c \in G'$, suppose that $(P, (b_P)')^c \leq (T, (b_T)')$. Then we have $((b_P)')^c = (b_{Pc})'$. By (3.3) again, $b_{Pc} = (b_P)^c$, so $(P, b_P)^c = (P^c, b_{Pc}) \leq (T, b_T)$. This implies that the categories $\mathbf{B}_G(b)$ and $\mathbf{B}_G(b')$ are equivalent. This completes the proof.

4. Perfect isometry induced by the Dade correspondence

In Sections 4, 5 and 6, we assume Hypotheses 2 and 3, and $b' = b_{(G')}$ using the notation in Theorem 2. In this section we show b and b' are perfect isometric in the sense of Broué [1]. Moreover we use notations in §3. In particular, we recall that $\text{Irr}(\hat{b}_i)_{(E')} = \{\lambda^i(\hat{\phi})_{(E')} \mid \phi \in \text{Irr}(b)\}$. Now we have $b = \sum_{i=0}^{r-1} \hat{b}_i$, and $b' = \sum_{i=0}^{r-1} (\hat{b}_i)_{(E')}$, and hence we have

$$b'b = \sum_{i=0}^{r-1} \sum_{l=0}^{r-1} (\hat{b}_l)_{(E')} \hat{b}_{l+i}.$$

We put

$$(4.1) \quad b_i = \sum_{l=0}^{r-1} (\hat{b}_l)_{(E')} \hat{b}_{l+i} \quad (\forall i \in \mathbf{Z}).$$

Then $(b_i)^2 = b_i$ and $b_i \in (\mathcal{O}Gbb')^{E'}$ for each i because

$$b_i = \sum_{y \in E'} \sum_{x \in E} \sum_{l=0}^{r-1} \lambda^l(y^{-1}) \lambda^l(x^{-1}) \lambda^i(x^{-1}) \beta_y \alpha_x yx \in \mathcal{O}G$$

by the orthogonality relations where $\hat{b}_0 = \sum_{x \in E} \alpha_x x$ and $(\hat{b}_0)_{(E')} = \sum_{y \in E'} \beta_y y$ ($\alpha_x, \beta_y \in \mathcal{O}$). For each prime $q \in \pi(F)$, let $\lambda_q \in \hat{F}_q$ be a non-trivial character as in Theorem 1. Set $l = |\pi(F)|$. Of course we may assume $l > 0$ for our purpose. Moreover we can write for t ($t \leq l$) distinct primes $q_1, q_2, \dots, q_t \in \pi(F)$

$$\lambda_{q_1} \cdots \lambda_{q_t} = \lambda^{m_{\{q_1, \dots, q_t\}}} \quad (m_{\{q_1, \dots, q_t\}} \in \mathbf{Z})$$

recalling λ is a generator of \hat{F} . Then we have

$$(4.2) \quad \prod_{q \in \pi(F)} (1 - \lambda_q) = 1 + \sum_{t=1}^l (-1)^t \sum_{\{q_1, \dots, q_t\} \subseteq \pi(F)} \lambda^{m_{\{q_1, \dots, q_t\}}}$$

where $\{q_1, \dots, q_t\}$ runs over the set of t -element subsets of $\pi(F)$.

Proposition 4 (see [13], Proposition 4.4). *With the above notations we have*

$$\begin{aligned} & [b_0 \mathcal{K}G] + \sum_{t=1}^l (-1)^t \sum_{\{q_1, \dots, q_t\} \subseteq \pi(F)} [b_{m_{\{q_1, \dots, q_t\}}} \mathcal{K}G] \\ &= \sum_{\phi \in \text{Irr}(b)} \epsilon_\phi [L_{\phi(G')} \otimes_{\mathcal{K}} L_{\check{\phi}}] \end{aligned}$$

in $G_0(\mathcal{K}(G' \times G))$.

Proof. Our proof is essentially the same as the proof of [13], Proposition 4.4. Let $\phi \in \text{Irr}(b)$. In $G_0(\mathcal{K}E')$ we have the following from (4.1), (4.2) and (1.1)

$$\begin{aligned} & [b_0 \mathcal{K}E \otimes_{\mathcal{K}E} L_{\hat{\phi}}] + \sum_{t=1}^l (-1)^t \sum_{\{q_1, \dots, q_t\} \subseteq \pi(F)} [b_{m_{\{q_1, \dots, q_t\}}} \mathcal{K}E \otimes_{\mathcal{K}E} L_{\lambda^{m_{\{q_1, \dots, q_t\}}} \hat{\phi}}] \\ &= [b_0 (L_{\hat{\phi}})_{E'}] + \sum_{t=1}^l (-1)^t \sum_{\{q_1, \dots, q_t\} \subseteq \pi(F)} [b_{m_{\{q_1, \dots, q_t\}}} (L_{\lambda^{m_{\{q_1, \dots, q_t\}}} \hat{\phi}})_{E'}] \\ &\stackrel{(4.1)}{=} [(\hat{b}_0)_{(E')} (L_{\hat{\phi}})_{E'}] + \sum_{t=1}^l (-1)^t \sum_{\{q_1, \dots, q_t\} \subseteq \pi(F)} [(\hat{b}_0)_{(E')} (L_{\lambda^{m_{\{q_1, \dots, q_t\}}} \hat{\phi}})_{E'}] \\ &\stackrel{(4.2), (1.1)}{=} \epsilon_\phi \left([(\hat{b}_0)_{(E')} L_{(\hat{\phi})_{E'}}] + \sum_{t=1}^l (-1)^t \sum_{\{q_1, \dots, q_t\} \subseteq \pi(F)} [(\hat{b}_0)_{(E')} L_{\lambda^{m_{\{q_1, \dots, q_t\}}} (\hat{\phi})_{E'}}] \right) \\ &\stackrel{(4.1)}{=} \epsilon_\phi [L_{(\hat{\phi})_{E'}}]. \end{aligned}$$

This implies that in $G_0(\mathcal{K}G')$

$$[b_0 \mathcal{K}G \otimes_{\mathcal{K}G} L_\phi] + \sum_{t=1}^l (-1)^t \sum_{\{q_1, \dots, q_t\} \subseteq \pi(F)} [b_{m_{\{q_1, \dots, q_t\}}} \mathcal{K}G \otimes_{\mathcal{K}G} L_\phi] = \epsilon_\phi [L_{\phi(G')}].$$

Since $b_i b = b_i$ for any $i \in \mathbf{Z}$, the proof is complete. □

Theorem 5 (see [13], Theorem 4.5). *Assume Hypotheses 2 and 3, and that $b' = b_{(G')}$. Set $\mu = \sum_{\phi \in \text{Irr}(b)} \epsilon_\phi \phi_{(G')} \phi$. Then μ induces a perfect isometry $R_\mu: \mathcal{R}_{\mathcal{K}}(G, b) \rightarrow \mathcal{R}_{\mathcal{K}}(G', b')$ which satisfies $R_\mu(\phi) = \epsilon_\phi \phi_{(G')}$.*

Proof. We note that $b_j \mathcal{O}G$ is projective as a right $\mathcal{O}G$ -module and as a left $\mathcal{O}G'$ -module if $b_j \neq 0$. Hence by [1], Proposition 1.2, μ is perfect. This and the above proposition imply the theorem. \square

5. Isotypy induced by the Dade correspondence

In this section we show that b and b' are isotypic. Here we set

$$\hat{b}'_i = (\hat{b}_i)_{(E')} \quad (i \in \mathbf{Z}).$$

Then D is a defect group of \hat{b}'_i since $p \nmid r$. Let $P \leq D$ and let $(\hat{b}_P)_i$ be a block of $C_E(P)$ such that it covers b_P and it is associated with \hat{b}_i . By our assumption and Lemma 2, $(\hat{b}_P)_i$ is uniquely determined. Similarly there exists a unique block of $C_{E'}(P)$ such that it covers $(b_P)'$ and it is associated with \hat{b}'_i . By applying Proposition 2 for $C_E(P)$, $C_G(P)$ and b_P , let $((\hat{b}_P)_i)_{(C_{E'}(P))}$ be a block of $C_{E'}(P)$ such that $\text{Irr}((\hat{b}_P)_i)_{(C_{E'}(P))} = \{\lambda^i(\hat{\phi}_P)_{(C_{E'}(P))} \mid \phi_P \in \text{Irr}(b_P)\}$, where $\hat{\phi}_P \in \text{Irr}(\hat{b}_P)_0$ is an extension of ϕ_P . Recall that we have two choices for $((\hat{b}_P)_0)_{(C_{E'}(P))}$ when r is even (Proposition 3). Here we set

$$(\hat{b}_P)'_i = ((\hat{b}_P)_i)_{(C_{E'}(P))}$$

and

$$(\hat{b}_P^*)'_i = ((\hat{b}_P)'_i)^* \quad (i \in \mathbf{Z}).$$

Proposition 5 (see [13], Lemma 5.4). *With the above notations, for a subgroup P of D , $(\hat{b}_P)'_i$ is associated with \hat{b}'_i for $i \in \mathbf{Z}$, if we choose appropriately $(\hat{b}_P)'_0$ when r is even.*

Proof. Our proof is essentially the same as the proof of [13], Lemma 5.4. Let $s \in E'_0$. We have

$$\widehat{C}(s)\hat{b}_i = \frac{1}{|C_{E'}(s)|} \sum_{\phi \in \text{Irr}(b)} \left(\sum_{x \in E_0} (\lambda^i \hat{\phi})(s)(\lambda^i \hat{\phi})(x^{-1})x + \sum_{y \in E - E_0} (\lambda^i \hat{\phi})(s)(\lambda^i \hat{\phi})(y^{-1})y \right)$$

since $C_E(s) = C_{E'}(s)$. Similarly we have

$$\begin{aligned} \widehat{C}(s)\hat{b}'_i = \frac{1}{|C_{E'}(s)|} \sum_{\phi \in \text{Irr}(b)} \left(\sum_{x \in E'_0} (\lambda^i(\hat{\phi})_{(E')})(s)(\lambda^i(\hat{\phi})_{(E')})(x^{-1})x \right. \\ \left. + \sum_{y \in E' - E'_0} (\lambda^i(\hat{\phi})_{(E')})(s)(\lambda^i(\hat{\phi})_{(E')})(y^{-1})y \right). \end{aligned}$$

Recall that $\hat{\phi}(x) = \epsilon_\phi(\hat{\phi})_{(E')}(x)$ for $x \in E'_0$. The above equalities, the fact $E'_0 = E' \cap E_0$ and [13], Lemma 2.4 imply the following.

$$(5.1) \quad \Pr_{E'}^E(\widehat{C(s)}\hat{b}_i) - \widehat{C(s)}\hat{b}'_i \in \mathcal{O}[E' - E'_0]^{E'}$$

where $\mathcal{S}[E' - E'_0]^{E'}$ is the \mathcal{S} -submodule of $Z(SE')$ which is spanned by $\{\widehat{C(t)}' \mid t \in E' - E'_0\}$.

In order to prove the proposition, it suffices to show that $(\hat{b}_p)'_0$ is associated with \hat{b}'_0 , if we choose $(\hat{b}_p)'_0$ appropriately when r is even. Suppose that $(\hat{b}_p)'_j$ is associated with \hat{b}'_0 for some j ($0 \leq j \leq r - 1$). We have

$$\begin{aligned} & \Pr_{C_{E'}(P)}^E(\widehat{C(s)}\hat{b}_0)^*(b_p^*)' \\ &= \Pr_{C_{E'}(P)}^E[\Pr_{E'}^E(\widehat{C(s)}\hat{b}_0)]^*(b_p^*)' \end{aligned}$$

from (5.1),

$$\begin{aligned} &= \text{Br}_P^{\mathcal{O}E'}(\widehat{C(s)}\hat{b}'_0 + c)(b_p^*)' \\ &= \text{Br}_P^{\mathcal{O}E'}(\widehat{C(s)}b'\hat{b}'_0 + c)(b_p^*)' \\ &= [\text{Br}_P^{\mathcal{O}E'}(\widehat{C(s)}b') \text{Br}_P^{\mathcal{O}E'}(\hat{b}'_0) + \text{Br}_P^{\mathcal{O}E'}(c)](b_p^*)' \\ &= \text{Br}_P^{\mathcal{O}E'}(\widehat{C(s)}b')(\hat{b}_p^*)'_j + \text{Br}_P^{\mathcal{O}E'}(c)(b_p^*)' \end{aligned}$$

where c is some element of $\mathcal{O}[E' - E'_0]^{E'}$. On the other hand, we can see

$$\begin{aligned} & \Pr_{C_{E'}(P)}^E(\widehat{C(s)}\hat{b}_0)^*(b_p^*)' \\ &= \Pr_{C_{E'}(P)}^{C_E(P)}[\Pr_{C_E(P)}^E(\widehat{C(s)}\hat{b}_0)]^*(b_p^*)' \\ &= \Pr_{C_{E'}(P)}^{C_E(P)}[\Pr_{C_E(P)}^E(\widehat{C(s)})^* \text{Br}_P^{\mathcal{O}E'}(\hat{b}_0)](b_p^*)' \end{aligned}$$

from the argument in the above of Theorem 3 and (5.1) for $C_E(P)$

$$= \Pr_{C_{E'}(P)}^{C_E(P)}[\Pr_{C_E(P)}^E(\widehat{C(s)})^*](\hat{b}_p^*)'_0 + d(b_p^*)'$$

and by Theorem 3

$$\begin{aligned} &= \text{Br}_P^{\mathcal{O}E'}[\Pr_{E'}^E(\widehat{C(s)})] \text{Br}_P^{\mathcal{O}E'}(b')(\hat{b}_p^*)'_0 + d(b_p^*)' \\ &= \text{Br}_P^{\mathcal{O}E'}(\widehat{C(s)}b')(\hat{b}_p^*)'_0 + d(b_p^*)' \end{aligned}$$

where d is some element of $k[C_{E'}(P) - C_{E'_0}(P)]^{C_{E'}(P)}$.

Now we choose an element $s \in C_{E'_0}(P)$ such that

$$\text{Br}_P^{\mathcal{O}E'}(\widehat{C(s)'b'}) \in (kC_{E'}(P) \text{Br}_P^{\mathcal{O}E'}(b'))^\times.$$

Note that $\text{Br}_P^{\mathcal{O}E'}(\widehat{C(s)'b'})$ is a k -linear combination of elements in $sC_{G'}(P)$ because $\widehat{C(s)'b'}$ is an \mathcal{O} -linear combination of elements in sG' . By the above equations

$$\text{Br}_P^{\mathcal{O}E'}(\widehat{C(s)'b'})((\hat{b}_P^*)'_j - (\hat{b}_P^*)'_0) \in k[C_{E'}(P) - C_{E'_0}(P)]^{C_{E'}(P)}.$$

Set $v = (\hat{b}_P^*)'_j - (\hat{b}_P^*)'_0$. The coefficient of any element of $s^{-2}C_{G'}(P)$ in v is zero. Hence $\lambda^j(s^2) = \lambda^{2j}(s) = 1$. Therefore if r is odd, then $j = 0$. If r is even, $j = 0$ or $j = r/2$. Therefore by replacing ϵ_{ϕ_P} by $-\epsilon_{\phi_P}$ for all $\phi_P \in \text{Irr}(b_P)$ if $j = r/2$, we have $(\hat{b}_P^*)'_0$ is associated with \hat{b}'_0 . This completes the proof. \square

Let $P \leq D$. We note again that for any integer i , $(\hat{b}_P)_i$ covers b_P and it is associated with \hat{b}_i . Moreover $(\hat{b}_P)_i$ contains $\lambda^i \hat{\phi}_P$ ($\hat{\phi}_P \in \text{Irr}((\hat{b}_P)_0)$). Let R^P be the perfect isometry between $\mathcal{R}_{\mathcal{K}}(C_G(P), b_P)$ and $\mathcal{R}_{\mathcal{K}}(C_{G'}(P), (b_P)')$ obtained by

$$\rho(C_E(P), C_G(P), C_{E'}(P), C_{G'}(P))$$

(see Theorem 5). Also let $R^P_{p'}$ be the restriction of R^P to $\text{CF}_{p'}(C_G(P), b_P; \mathcal{K})$, where R^P is regarded as a linear isometry from $\text{CF}(C_G(P), b_P; \mathcal{K})$ onto $\text{CF}(C_{G'}(P), (b_P)'; \mathcal{K})$. We set

$$(b_P)_i = \sum_{l=0}^{r-1} (\hat{b}_P)'_l (\hat{b}_P)_{l+i} \in (\mathcal{O}C_G(P)b_P(b_P)')^{C_{E'}(P)}.$$

For $u \in D$ we set

$$b_u = b_{(u)}, \quad (b_u)' = (b_{(u)})', \quad (\hat{b}_u)'_0 = (\hat{b}_{(u)})'_0, \quad (b_u)_i = (b_{(u)})_i.$$

Theorem 6 (see [13], Theorem 5.5). *Assume Hypotheses 2 and 3, and assume $b' = b_{(G')}$. With the above notations, b and b' are isotypic with the local system $(R^P)_{\{P(\text{cyclic}) \leq D\}}$.*

Proof. Our proof is essentially the same as the proof of [13], Theorem 5.5. Let $\gamma \in \text{CF}(G, b; \mathcal{K})$, $u \in D$ and let $c' \in C_{G'}(u)_{p'}$. Let $S(u)$ be the p -section of G containing u . We remark that if $v \in S(u)$, then $\widehat{C(v)b}$ is an \mathcal{O} -linear combination of elements of

$S(u)$ by [12], Chapter V, Theorem 4.5. We can see from Proposition 4

$$\begin{aligned} & [(d_{G'}^{(u, (b_u)')} \circ R^{(1)})(\gamma)](c') \\ &= \frac{1}{|G|} \sum_{g \in G} \left[\sum_{\hat{\phi} \in \text{Irr}(b)} \left(\phi(uc'(b_u)'b_0) + \sum_{t=1}^l (-1)^t \sum_{\{q_1, \dots, q_t\} \subseteq \pi(F)} \phi(uc'(b_u)'b_{m_{\{q_1, \dots, q_t\}}}) \right) \phi(g^{-1}) \right] \gamma(g) \\ &= \frac{1}{|G|} \sum_{g \in G} \left[\sum_{\hat{\phi} \in \text{Irr}(b)} \left(\hat{\phi}(uc'(b_u)'b_0) + \sum_{t=1}^l (-1)^t \sum_{\{q_1, \dots, q_t\} \subseteq \pi(F)} \hat{\phi}(uc'(b_u)'b_{m_{\{q_1, \dots, q_t\}}}) \right) \hat{\phi}(g^{-1}) \right] \gamma(g) \end{aligned}$$

from (4.1) and the fact $\hat{\phi} \in \text{Irr}(\hat{b}_0)$

$$\begin{aligned} &= \frac{1}{|G|} \sum_{g \in G} \left[\sum_{\hat{\phi} \in \text{Irr}(\hat{b}_0)} \left(\hat{\phi}(uc'(b_u)'\hat{b}'_0) + \sum_{t=1}^l (-1)^t \sum_{\{q_1, \dots, q_t\} \subseteq \pi(F)} \hat{\phi}(uc'(b_u)'\hat{b}'_{-m_{\{q_1, \dots, q_t\}}}) \right) \hat{\phi}(g^{-1}) \right] \gamma(g) \\ &= \frac{1}{|G|} \sum_{g \in G} \left[\sum_{\hat{\phi} \in \text{Irr}(\hat{b}_0)} \left(\hat{\phi} \left(1 + \sum_{t=1}^l (-1)^t \sum_{\{q_1, \dots, q_t\} \subseteq \pi(F)} \lambda^{m_{\{q_1, \dots, q_t\}}} \right) \right) (uc'(b_u)'\hat{b}'_0) \hat{\phi}(g^{-1}) \right] \gamma(g) \end{aligned}$$

from (4.2)

$$= \frac{1}{|G|} \sum_{g \in G} \left[\sum_{\hat{\phi} \in \text{Irr}(\hat{b}_0)} \left(\prod_{q \in \pi(F)} (1 - \lambda_q) \cdot \hat{\phi} \right) (uc'(b_u)'\hat{b}'_0) \hat{\phi}(g^{-1}) \right] \gamma(g)$$

by applying [12], Chapter V, Theorem 4.5 for E and \hat{b}_0

$$\begin{aligned} &= \frac{1}{|G|} \sum_{x \in S(u)} \left[\sum_{\hat{\phi} \in \text{Irr}(\hat{b}_0)} \left(\prod_{q \in \pi(F)} (1 - \lambda_q) \cdot \hat{\phi} \right) (uc'(b_u)'\hat{b}'_0) \hat{\phi}(x^{-1}) \right] \gamma(x) \\ &= \frac{1}{|C_G(u)|} \sum_{y \in C_G(u)_{p'}} \left[\sum_{\hat{\phi} \in \text{Irr}(\hat{b}_0)} \left(\prod_{q \in \pi(F)} (1 - \lambda_q) \cdot \hat{\phi} \right) (uc'(b_u)'\hat{b}'_0) \hat{\phi}(y^{-1}u^{-1}) \right] \gamma(uy) \end{aligned}$$

by using (1.1) twice, and by Brauer’s second main theorem on blocks ([12], Chapter V, Theorem 4.1) and Proposition 5

$$\begin{aligned} &= \frac{1}{|C_G(u)|} \sum_{y \in C_G(u)_{p'}} \left[\sum_{\hat{\phi} \in \text{Irr}(\hat{b}_0)} \left(\prod_{q \in \pi(F)} (1 - \lambda_q) \cdot (\hat{\phi})_{(E')} \right) (uc'(b_u)'\hat{b}'_0) \hat{\phi}(y^{-1}u^{-1}) \right] \gamma(uy) \\ &= \frac{1}{|C_G(u)|} \sum_{y \in C_G(u)_{p'}} \left[\sum_{\hat{\phi} \in \text{Irr}(\hat{b}_0)} \left(\prod_{q \in \pi(F)} (1 - \lambda_q) \cdot (\hat{\phi})_{(E')} \right) (uc'(\hat{b}_u)'_0) \hat{\phi}(y^{-1}u^{-1}) \right] \gamma(uy) \end{aligned}$$

$$= \frac{1}{|C_G(u)|} \sum_{y \in C_G(u)_{p'}} \left[\sum_{\hat{\phi} \in \text{Irr}(\hat{b}_0)} \left(\prod_{q \in \pi(F)} (1 - \lambda_q) \cdot \hat{\phi} \right) (uc'(\hat{b}_u)'_0 \hat{\phi}(y^{-1}u^{-1})) \right] \gamma(uy)$$

from [12], Chapter V, Theorem 4.11

$$= \frac{1}{|C_G(u)|} \sum_{y \in C_G(u)_{p'}} \left[\sum_{e \in \text{BI}(C_E(u), \hat{b}_0)} \sum_{\rho \in \text{Irr}(e)} \left(\prod_{q \in \pi(F)} (1 - \lambda_q) \cdot \rho \right) (c'(\hat{b}_u)'_0 \rho(y^{-1})) \right] \gamma(uy)$$

from (1.1) for $C_E(u)$

$$= \frac{1}{|C_G(u)|} \sum_{y \in C_G(u)_{p'}} \left[\sum_{e \in \text{BI}(C_E(u), \hat{b}_0)} \sum_{\rho \in \text{Irr}(e)} \left(\prod_{q \in \pi(F)} (1 - \lambda_q) \cdot \rho_{(C_{E'}(u))} \right) (c'(\hat{b}_u)'_0 \rho(y^{-1})) \right] \gamma(uy)$$

recalling $(\hat{b}_u)'_0 = ((\hat{b}_{(u)})_0)_{(C_{E'}(u))}$

$$= \frac{1}{|C_G(u)|} \sum_{y \in C_G(u)_{p'}} \left[\sum_{\hat{\xi} \in \text{Irr}(\hat{b}_u)} \left(\prod_{q \in \pi(F)} (1 - \lambda_q) \cdot \hat{\xi} \right) (c'(\hat{b}_u)'_0 \hat{\xi}(y^{-1})) \right] \gamma(uy)$$

from (4.2)

$$= \frac{1}{|C_G(u)|} \sum_y \left[\sum_{\hat{\xi} \in \text{Irr}(\hat{b}_u)} \left(\hat{\xi}(c'(\hat{b}_u)'_0) + \sum_{t=1}^l (-1)^t \sum_{\{q_1, \dots, q_t\}} \hat{\xi}(c'(\hat{b}_u)'_{-m_{\{q_1, \dots, q_t\}}}) \right) \hat{\xi}(y^{-1}) \right] \gamma(uy)$$

from (4.1)

$$= \frac{1}{|C_G(u)|} \sum_y \left[\sum_{\xi \in \text{Irr}(b_u)} \left(\xi(c'(b_u)_0) + \sum_{t=1}^l (-1)^t \sum_{\{q_1, \dots, q_t\}} \xi(c'(b_u)_{m_{\{q_1, \dots, q_t\}}}) \right) \xi(y^{-1}) \right] \gamma(uy)$$

and from [12], Chapter V, Theorem 4.7

$$= \frac{1}{|C_G(u)|} \sum_y \left[\sum_{\xi \in \text{Irr}(b_u)} \left(\xi(c'(b_u)_0) + \sum_{t=1}^l (-1)^t \sum_{\{q_1, \dots, q_t\}} \xi(c'(b_u)_{m_{\{q_1, \dots, q_t\}}}) \right) \xi(y^{-1}) \right] \\ \times (d_G^{(u, b_u)}(\gamma))(y) \\ = [(R_{p'}^{(u)} \circ d_G^{(u, b_u)})(\gamma)](c')$$

recalling the definition of the perfect isometry $R^{(u)}$, where y runs over $C_G(u)_{p'}$ and $\{q_1, \dots, q_t\}$ runs over the set of t -element subsets of $\pi(F)$. This and Theorem 4 complete the proof. \square

Corollary 1 ([8]). *Let G and A be finite groups such that A is cyclic, A acts on G via automorphism and that $(|C_G(A)|, |A|) = 1$. If $p \nmid |A|$ and $p \nmid |G : C_G(A)|$, then the Dade correspondence induces an isotypy between $b(G)$ and $b(C_G(A))$.*

Proof. Let s be a generator of A . Let $E = G \rtimes A$, $G' = C_G(A)$ and $E' = G'A$. Then E, G, E' and G' satisfy Hypothesis 1 by [3], Lemma 7.5. By the assumption $\widehat{C(s)}b(E)$ is invertible in $Z(\mathcal{O}Eb(E))$. Also $sb(E')$ is invertible in $Z(\mathcal{O}E'b(E'))$. Hence the corollary follows from Theorem 6. \square

EXAMPLE. Suppose $p = 5$, and let $G = Sz(2^{2n+1})$, the Suzuki group, $A = \langle \sigma \rangle$ where σ is the Frobenius automorphism of G with respect to $\text{GF}(2^{2n+1})/\text{GF}(2)$. Set $G' = Sz(2) = C_G(A)$, $E = G \rtimes A$, $E' = G' \rtimes A$. Suppose that $5 \nmid 2n + 1$. Then $(2n + 1, |G'|) = (2n + 1, 20) = 1$. Moreover a Sylow 5-subgroup of G has order 5. By the above corollary, the Dade correspondence gives an isotypy between $b(G)$ and $b(G')$.

6. Normal defect group case

In the Glauberman correspondence case if the defect group D is normal in G , there is a Puig equivalence (splendidly Morita equivalence) between b and b' which affords the Glauberman correspondence on the character level ([6], [14]). In the Dade correspondence case we show that b and b' are Puig equivalent if D is normal in G . By our assumption, there exist a defect group D of b and b' , and an element $s \in E'_0$ such that $s \in C_E(D)$ and $\widehat{C(s)}b \in Z(\mathcal{O}Eb)^\times$. Let $\phi \in \text{Irr}(b)$ be of height 0. From [13], Lemma 2.4 and (1.1) in Theorem 1, we have

$$0 \neq (\omega_{\hat{\phi}}(\widehat{C(s)}))^\ast = \left(\epsilon_{\phi} \frac{|E|\phi_{(G')}(1)}{|E'|\phi(1)} \omega_{(\hat{\phi})_{(E')}}(\widehat{C(s)'}) \right)^\ast.$$

Since b and b' have the same defect,

$$(\omega_{(\hat{\phi})_{(E')}}(\widehat{C(s)'})^\ast \neq 0.$$

Hence $\widehat{C(s)' }b' \in Z(\mathcal{O}Eb')^\times$. The element s is used in the next lemma.

Lemma 3. *Let E_1 be a subgroup of $N_E(D)$ containing $C_E(D)$ and set $G_1 = G \cap E_1$, $E'_1 = E' \cap E_1$, and $G'_1 = G' \cap E_1$. Then E_1, G_1, E'_1 and G'_1 satisfy Hypothesis 1. Moreover $(b_D)^{G_1}$ satisfies Hypothesis 2, $((b_D)')^{G_1}$ satisfies Hypothesis 3 and*

$$(6.1) \quad ((b_D)^{G_1})_{(G'_1)} = ((b_D)')^{G_1}.$$

Proof. By our assumption $E = G\langle s \rangle$, hence we have $E_1 = G_1\langle s \rangle = E'_1G_1$, $G'_1 = G_1 \cap E'_1$. Also $E_1/G_1 \cong E'_1/G'_1 \cong F$. Hence the former is clear. On the other hand,

since $\text{Br}_D^{OE}(\widehat{C(s)b})b_D^* \in Z(kE_1(b_D)^*)^\times = Z(kE_1((b_D)^{G_1})^*)^\times$ and $\text{Br}_D^{OE'}(\widehat{C(s)b'}) (b'_D)^* \in Z(kE'_1((b_D)')^*)^\times = Z(kE'_1(((b_D)')^{G_1})^*)^\times$, $(b_D)^{G_1}$ satisfies Hypothesis 2, and $((b_D)')^{G_1}$ satisfies Hypothesis 3. By applying Theorem 3, (i) for E_1 , G_1 and $(b_D)^{G_1}$, we have (6.1). \square

In the above lemma, we set $E_1 = N_E(D)$. Then $(b_D)^{G_1} = (b_D)^{N_G(D)}$ is a Brauer correspondent of b , and $((b_D)')^{N_{G'}(D)}$ is a Brauer correspondent of b' . From now we assume D is normal in G . Then D is normal in E .

Lemma 4. *With the notations in Lemma 3, suppose that E_1 is normal in E . Let $\xi \in \text{Irr}((b_D)^{G_1})$ and $x' \in E'$. We have $(\xi^{x'})_{(G'_1)} = (\xi_{(G_1)})^{x'}$ and $((b_D)^{G_1})^{x'}_{(G'_1)} = (((b_D)')^{G_1})^{x'}$. In particular $I_E(\xi) \cap E' = I_{E'}(\xi_{(G_1)})$ and $I_E((b_D)^{G_1}) \cap E' = I_{E'}(((b_D)')^{G_1})$.*

Proof. Note that $(b_D)^{G_1}$ and $((b_D)^{G_1})^{x'}$ respectively satisfy Hypothesis 2. Let $\hat{\xi} \in \text{Irr}(E_1|\xi)$ and $\xi' = \xi_{(G'_1)}$. By Theorem 1 and (1.1),

$$\left(\prod_{q \in \pi(F)} (1 - \lambda_q) \cdot \hat{\xi} \right)_{E'_1} = \epsilon_\xi \prod_{q \in \pi(F)} (1 - \lambda_q) \cdot (\hat{\xi})_{(E'_1)}$$

where $\epsilon_\xi = \pm 1$. Hence we have,

$$\left(\prod_{q \in \pi(F)} (1 - \lambda_q) \cdot (\hat{\xi})^{x'} \right)_{E'_1} = \epsilon_\xi \prod_{q \in \pi(F)} (1 - \lambda_q) \cdot ((\hat{\xi})_{(E'_1)})^{x'}.$$

Therefore by Theorem 1 we have $(\xi^{x'})_{(G'_1)} = \xi'^{x'}$ because $((\hat{\xi})^{x'})_{G_1} = \xi^{x'}$ and $((\hat{\xi})_{(E'_1)})^{x'}_{G'_1} = \xi'^{x'}$. This implies the lemma because the Dade correspondence $\rho(E_1, G_1, E'_1, G'_1)$ induces the bijection between $\text{Irr}((b_D)^{G_1})$ and $\text{Irr}(((b_D)')^{G_1})$ by Lemma 3. \square

By Lemma 4 we have $I_E(b_D) \cap E' = I_{E'}((b_D)')$. By Lemma 3 $I_E(b_D)$, $I_G(b_D)$, $I_{E'}((b_D)')$ and $I_{G'}((b_D)')$ satisfy Hypothesis 1. Moreover $(b_D)^{I_G(b_D)}$ satisfies Hypothesis 2, and $((b_D)')^{I_{G'}((b_D)')}$ satisfies Hypothesis 3. Also we have

$$(6.2) \quad ((b_D)^{I_G(b_D)})_{(I_{G'}((b_D)'))} = ((b_D)')^{I_{G'}((b_D)')}.$$

By Lemma 3, $DC_E(D)$, $DC_G(D)$, $DC_{E'}(D)$ and $DC_{G'}(D)$ also satisfy Hypothesis 1. Set $K = DC_G(D)$ and $K' = DC_{G'}(D)$. Then $(b_D)^K$ satisfies Hypothesis 2, and $((b_D)')^{K'}$ satisfies Hypothesis 3. Moreover we have

$$((b_D)^K)_{(K')} = ((b_D)')^{K'}.$$

Now suppose that b_D is G -invariant for a while. Then $(b_D)^K$ is G -invariant. Note that as elements of $\mathcal{O}G$, $b = b_D = (b_D)^K$. By Lemma 4, $((b_D)')^{K'}$ is G' -invariant. Since

b is covered by r blocks of E and since $(b_D)^K$ is covered by r blocks of $DC_E(D)$, any block of $DC_E(D)$ covering $(b_D)^K$ is E -invariant. Let $\widehat{(b_D)^K}$ be a block of $DC_E(D)$ covering $(b_D)^K$. In fact the block idempotent of a block of E covering b belongs to $\mathcal{O}DC_E(D)$. If $\xi \in \text{Irr}^G((b_D)^K)$ and $\hat{\xi}$ is an extension of ξ to $DC_E(D)$ belonging to $\widehat{(b_D)^K}$, then G fixes $\hat{\xi}$ and hence E fixes $\hat{\xi}$ because $(b_D)^K$ and $\widehat{(b_D)^K}$ are isomorphic by restriction. Similarly if $\xi' \in \text{Irr}^{G'}(((b_D)')^{K'})$ and $\hat{\xi}'$ is an extension of ξ' to $DC_{E'}(D)$, $\hat{\xi}'$ is E' -invariant. We note that if $\xi \in \text{Irr}^G((b_D)^K)$ then $\xi_{(K')} \in \text{Irr}^{G'}(((b_D)')^{K'})$ by Lemma 4. The following is proved by the analogous way to that of the proof of [10], Lemma 3.2.

Lemma 5. *Suppose that b_D is G -invariant. Let $\xi \in \text{Irr}^G((b_D)^K)$. Then the factor set α of G/K defined by ξ and the factor set α' of G'/K' defined by $\xi_{(K')}$ are cohomologous when G/K and G'/K' are identified.*

Proof. At first we note again that $G = KG'$ by Lemma 1, $E = DC_E(D)E'$, $E = DC_E(D)G$ and $E' = DC_{E'}(D)G'$. Moreover we have

$$G/K \cong E/DC_E(D) \cong E'/DC_{E'}(D) \cong G'/K'.$$

We may assume $G \neq K$. Let t be a prime dividing $|G : K|$ and let E_t be a subgroup of E containing $DC_E(D)$ such that $E_t/DC_E(D)$ is a Sylow t -subgroup of $E/DC_E(D)$. Set $G_t = G \cap E_t$, $E'_t = E' \cap E_t$ and $G'_t = G' \cap E_t$. By Lemma 3, E_t, G_t, E'_t and G'_t satisfy Hypothesis 1. Moreover $(b_D)^{G_t}$ satisfies Hypothesis 2, $((b_D)')^{G'_t}$ satisfies Hypothesis 3 and that $((b_D)^{G_t})_{(G'_t)} = ((b_D)')^{G'_t}$. Now by a theorem of Gaschütz (see [5], Theorem 15.8.5), we may assume $E = E_t$.

Let $\hat{\xi} \in \text{Irr}(DC_E(D)|\xi)$. From Theorem 1 and (1.1),

$$\left(\left(\prod_{q \in \pi(F)} (1 - \lambda_q) \cdot \hat{\xi} \right)_{DC_E(D)}, (\hat{\xi})_{(DC_{E'}(D))} \right) = \pm 1,$$

where the left hand side is the inner product. Hence there exists an extension $\tilde{\xi}$ of ξ to $DC_E(D)$ such that $(\tilde{\xi}_{DC_E(D)}, (\hat{\xi})_{(DC_{E'}(D))})$ is relatively prime to t . As we stated in the above $\tilde{\xi}$ is E -invariant, and $(\hat{\xi})_{(DC_{E'}(D))}$ is E' -invariant because $\xi_{(K')}$ is G' -invariant. By [2], Theorem 4.4, the factor set of $E/DC_E(D)$ defined by $\tilde{\xi}$ and the factor set of $E'/DC_{E'}(D)$ defined by $(\hat{\xi})_{(DC_{E'}(D))}$ are cohomologous when $E/DC_E(D)$ and $E'/DC_{E'}(D)$ are identified. Similarly by [2], Theorem 4.4, since $\tilde{\xi}$ is an extension of ξ , α and the factor set of $E/DC_E(D)$ defined by $\tilde{\xi}$ are cohomologous when G/K and $E/DC_E(D)$ are identified. Further α' and the factor set of $E'/DC_{E'}(D)$ defined by $(\hat{\xi})_{(DC_{E'}(D))}$ are cohomologous when G'/K' and $E'/DC_{E'}(D)$ are identified, because $(\hat{\xi})_{(DC_{E'}(D))}$ is an extension of $\xi_{(K')}$. Hence α and α' are cohomologous. \square

In the above lemma we can take as ξ the canonical character of b belonging to $(b_D)^K$. Then $\xi_{(K')}$ is the canonical character of (b') because $\xi_{(K')}$ is a constituent of $\xi_{K'}$,

and hence D is contained in the kernel of $\xi_{(K')}$. Moreover $\alpha, \alpha' \in Z^2(G/K, \mathcal{O}^\times)$ since ξ and $\xi_{(K')}$ are respectively characters of a G -invariant $\mathcal{O}K$ -lattice and a G' -invariant $\mathcal{O}K'$ -lattice. By Lemma 5, we see α and α' are cohomologous.

Generally let G be a finite group, b be a block of G with a normal defect group D , and let \mathbf{b} be a G -invariant block of $C_G(D)$ covered by b . Set $K = DC_G(D)$ and let i be a primitive idempotent of $\mathcal{O}C_G(D)\mathbf{b}$. Then we see that i is primitive in $(\mathcal{O}G)^D$ because D is normal in G and i^* is primitive in $kC_G(D)$, and hence $i\mathcal{O}Gi$ is a source algebra of b . Set $B = i(\mathcal{O}G)i$. Let H be a complement of $DC_G(D)/C_G(D)$ in $G/C_G(D)$. Then H is isomorphic to a subgroup of $\text{Aut } D$. For each $h \in H$, we choose $x_h \in G$ such that $h = C_G(D)x_h$. We set $d^h = d^{x_h}$ for any $d \in D$. Moreover let α be a factor set of H defined by the canonical character χ of b , where H and G/K are identified.

Theorem 7. *With the above notations, B is isomorphic to a twisted group algebra $\mathcal{O}^{\alpha^{-1}}(D \rtimes H)$ of the semi direct product $D \rtimes H$ over \mathcal{O} with the factor set α^{-1} (considered as a factor set of $D \rtimes H$), as interior $\mathcal{O}D$ -algebras.*

Proof. For any $h \in H$ we can choose $u_h \in (\mathcal{O}C_G(D)\mathbf{b})^\times$ such that $i^{x_h^{-1}} = i^{u_h}$. Put $v_h = u_h x_h i$. For any $d \in D$, we have

$$(6.3) \quad v_h^{-1}(id)v_h = id^h$$

where v_h^{-1} is the inverse of v_h in B . Then we have

$$B = \bigoplus_{h \in H} i\mathcal{O}Kx_h i = \bigoplus_{h \in H} i\mathcal{O}Kiv_h = \bigoplus_{h \in H} (i\mathcal{O}Di)v_h.$$

Thus B is a crossed product of H over $i\mathcal{O}Di$. As is well known $i\mathcal{O}Di \cong \mathcal{O}D$. Since H is a p' -group, from (6.3) and the proof of Lemma M in [11], B is a twisted group algebra of $D \rtimes H$ over \mathcal{O} with a factor set $\gamma \in Z^2(D \rtimes H, \mathcal{O}^\times)$ which is the inflation of a factor set of H . In fact γ satisfies that

$$v_h v_{h'} = \gamma(h, h')v_{hh'} \quad (\forall h, h' \in H)$$

by replacing v_h by $v_h \delta_h$ for some $\delta_h \in i + iJ(Z(\mathcal{O}D))i$ if necessary. Here $J(Z(\mathcal{O}D))$ is the radical of the center of $\mathcal{O}D$.

For any $a \in \mathcal{O}G$, we denote by \bar{a} the image of a by the natural homomorphism from $\mathcal{O}G$ onto $\mathcal{O}(G/D)$. We set $\bar{G} = G/D$ and $\bar{K} = K/D \leq \bar{G}$. We have

$$\bar{i}\mathcal{O}\bar{G}\bar{i} = \bigoplus_{h \in H} (\mathcal{O}\bar{K}\bar{x}_h \cap (\bar{i}\mathcal{O}\bar{G}\bar{i})) = \bigoplus_{h \in H} \mathcal{O}\bar{v}_h.$$

Also we have

$$\bar{v}_h \bar{v}_{h'} = \gamma(h, h')\bar{v}_{hh'}.$$

Since \bar{i} is a primitive idempotent of $\mathcal{O}\bar{G}$ corresponding to χ , $\bar{i}\mathcal{O}\bar{G}\bar{i}$ is a twisted group algebra of \bar{G} over \mathcal{O} with factor set α^{-1} . This implies that γ and α^{-1} are cohomologous. This completes the proof. \square

Theorem 8. *Assume Hypotheses 2 and 3, and $b' = b_{(G')}$. Further assume the defect group D of b and b' is normal in G . Then b and b' are Puig equivalent.*

Proof. As is well known b and $(b_D)^{I_G(b_D)}$ are Puig equivalent. Hence by Lemma 4 and (6.2), we may assume that b_D is G -invariant. Then from Lemma 5 and Theorem 7, b and b' are Puig equivalent. This completes the proof. \square

By the above theorem, the Brauer correspondent of b and that of b' are Puig equivalent assuming Hypotheses 2 and 3, and $b' = b_{(G')}$.

Corollary 2. *In the above theorem, let $b = b(G)$. Then $a \in \mathcal{O}G'b(G') \mapsto ab(G) \in \mathcal{O}Gb(G)$ is an algebra isomorphism.*

Proof. Since $\mathcal{O}Gb(G)$ is a source algebra of $b(G)$, $\mathcal{O}G'b(G')$ are $\mathcal{O}Gb(G)$ are isomorphic. Therefore $\dim \mathcal{K}Gb(G) = \dim \mathcal{K}G'b(G')$, and hence the Dade correspondence from $\text{Irr}(b(G))$ onto $\text{Irr}(b(G'))$ coincides with restriction, that is, $b(G)$ and $b(G')$ are isomorphic. Hence by [9], Theorem 1 or [7], Theorem 4.1 completes the proof. \square

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