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The Characteristics of the Source of Welding Residual Stress (Inherent Strain) and Its Application to Measurement and Prediction

Yukio UEDA* and M. G. YUAN**

Abstract

Welding residual stress in a joint is produced as a result of thermal elasto-plastic behavior during welding. The source of residual stress is incompatible strain, called inherent strain in this report which is composed of shrinkage strain of the weld metal, plastic strain in the vicinity of welded zone, etc. Utilizing of the characteristics of inherent strain, measurement and prediction of three dimensional residual stress have become possible with the inherent strain being used as a parameter. Here, the characteristics of inherent strain are described by a simple example and the predicting and measuring methods of welding residual stress are outlined.

KEY WORDS : (Inherent Strain) (Welding Residual Stress) (Welded Joints) (Measurement) (Prediction) (Numerical Analysis)

Introduction

The welding residual stress is inevitably produced in a welded structure. For accurate evaluation of the safety of the structure, accurate prediction of the welding residual stress is indispensable. Prediction methods of welding residual stress may be classified into the following three: (1) thermal elasto-plastic analysis (theoretical analysis), (2) measurement on similar welded joints, and (3) simple formulae based on accumulated experimental data and theoretical information.

From the theoretical viewpoint, thermal elasto-plastic analysis¹⁾ is the most general method to predict welding residual stress taking account of many influential factors. By this method, complex calculation can be performed simulating the entire mechanical behavior during welding and the welding residual stress is obtained at the end of the calculation. There should exist the source to cause the residual stress.

Without tracing the entire process of elasto-plastic behavior, simple predicting methods²⁻⁴⁾ including some simple formulae predict directly welding residual stress, except a few^{2,3)} which are concerned with the source of residual stress. However, no papers have proposed methods to determine the source, except a series of papers⁵⁻⁹⁾ published by one of the authors. In these papers, the characteristics of the source of residual stress was clarified, a method of determining the source was presented and several measuring methods of three-

dimensional residual stress in a body were developed using the source as a parameter. Later, a new aspect of the characteristics was discovered and a predicting method of welding residual stress¹⁰⁻¹³⁾ has been developed. In this method the source is also used as a parameter.

In this paper, the characteristics of the source of welding residual stress will be clarified and the predicting and measuring methods of welding residual stress developed by using the source as a parameter will be introduced.

Characteristics of the Source of Residual Stress

Generally, the stress contained in a self-balanced body is called residual stress. In the body, the source of residual stress should exist, which is incompatible strain at room temperature and the stress produced by the incompatible strain is called inherent stress. In this connection, the source of residual stress is called here inherent strain. In the case of welded joints, the inherent strain may be composed of many components, such as shrinkage strain in the weld metal, plastic strain in the vicinity of welded portion and so on. This is indifferent from the process of generation of residual stress.

The characteristics of inherent strain will be described using a circular plate of mild steel in plane stress state as an example. The plate of R radius is heated up from 0 °C under the following four different conditions, assuming

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that the heating time is very short and no heat transfer occurs:

- (1) The entire plate is heated uniformly to $T^{\circ}\text{C}$;
- (2) A small area inside of r radius is heated to 100°C and cooled down to 0°C ;
- (3) A small area inside of r radius is heated up to 300°C and cooled down to 0°C ;
- (4) Increasing the radius R , a small area inside of r radius is heated up to 300°C and cooled down to 0°C .

In case (1), thermal strain is generated in the entire plate, but no thermal stress is produced since the thermal strain is compatible strain. In case (2), as thermal strain is confined by the surrounding area of unheated zone, thermal stress is produced since thermal strain is incompatible strain. As the behavior is regarded as being elastic in this heating process, no residual stress is generated after the plate is cooled down to 0°C since the incompatible strain disappears at 0°C . In case (3), the behavior is elasto-plastic at the heating stage because large thermal compressive stress is produced to reach the yield stress, and a certain amount of plastic strain is generated above this stress according to the maximum temperature. These thermal strain and plastic strain are incompatible strain. Once the plate is cooled down to 0°C , the thermal strain disappears but the plastic strain remains as incompatible one. Consequently, the plastic strain causes residual stress. For simplicity, plastic strain which may be induced at the cooling stage is neglected. In case (4), as the size of plate is increased, the degree of restraint against thermal strain generated in the central small area is also increased and it approaches rapidly to that for a infinite plate. Consequently, the magnitude of plastic strain generated at the heating stage converges to a certain value (Fig. 1 (a)). However, the residual stress distribution varies according to the size of plate (Fig. 1 (b)).

The pattern of plastic strain distribution is simple and a simple relationship between the magnitude of plastic strain and radius R should exist. For prediction of the residual stress in the circular plate, the thermal elasto-plastic analysis may be applied. On the other hand, it can be calculated by elastic analysis, imposing the inherent strain to a stress-free plate of the same size. The inherent strain should have been obtained separately, either by experiments or theoretical analysis. In the case where the size of a square plate is large, its residual stress due to the heating condition of case (4) may be calculated by imposing this inherent strain. This is the basic idea of the new prediction method of welding residual stress.

Next, let's consider cutting a circular plate containing the residual stress into two parts. By the cutting, redistribution of the residual stress should occur due to

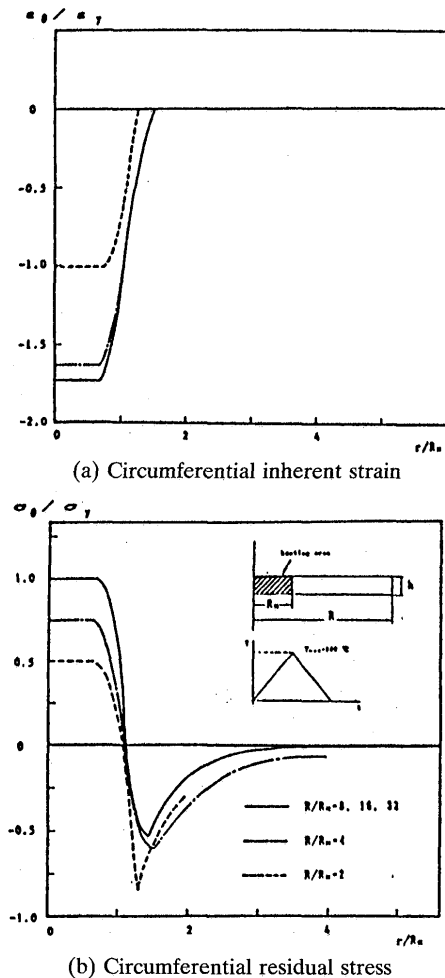


Fig. 1 Distributions of circumferential inherent strain and residual stress with respect to the ratio of disk radius to heating area radius

release of the stress acting on the cutting surfaces. If the cutting procedure is well chosen, the behavior of the plate during the cutting should be elastic and inherent strain does not change. The new residual stress distribution in each part can be computed by applying the released stress on the cutting surfaces in the opposite direction. In contrast with this, if the unchanged inherent strain is imposed in each part of the plate, the entire residual stress distribution can be computed. Inversely, the residual stress in each part of the plate may be measured and the inherent strain may be computed by solving conversely the relationship between inherent strain and residual stress. The inherent strain in the entire plate is composed of the inherent strains in the two separate parts. If the entire strain is imposed to the original stress-free circular plate, the original residual stress is reproduced. In a case of a welded joint, the joint may be separated into such pieces as being thin plates, etc. whose residual stresses can be easily observed. From the observed stresses, the inherent strain distribution in each

piece may be obtained and the entire distribution of welding residual stresses can be computed by applying the entire distribution of inherent strain to the stress-free welded joint. This is the basic procedure of measurement of welding residual stress using inherent strain as a parameter.

In the above, the characteristics of inherent strain were described using a simple example and the new approaches for prediction and measurement of welding residual stress were outlined. Although details of the basic approaches are described in the original papers, those will be demonstrated using actual welded joints in the following sections.

Relationship between Inherent Strain and Residual Stress

The inherent strain $\{\epsilon^*\}$ in a welded joint is produced only in a limited region near the weld. However, welding residual stress $\{\sigma\}$ (or elastic strain $\{\epsilon\}$) due to the inherent strain is produced over the whole weldment.

Their relations are expressed in the following equations.

$$\{\epsilon\} = [H^*]\{\epsilon^*\} \tag{1}$$

$$\{\sigma\} = [D]\{\epsilon\} \tag{2}$$

where,

$[D]$: Stress-strain matrix,

$[H^*]$: Elastic response matrix.

The finite element method is used for practical problems. The number of finite elements in which the inherent strain is produced is assumed to be m and the number of finite elements of the whole joint $n (> m)$.

In order to determine the inherent strains, Eq. (1) is necessary to be solved conversely giving known strains of $\{\epsilon\}$ more than m . When elastic strain $\{\epsilon\}$ is determined by measurement, it may contain various errors. According to the condition which minimize the sum of the square of residual, the most probable value of inherent strain $\{\epsilon^*\}$ is calculated from the following equation.

$$\{\epsilon^*\} = ([H^*]^T [H^*])^{-1} [H^*]^T \{\epsilon\} \tag{3}$$

Substituting $\{\epsilon^*\}$ into Eq. (2), the most probable value of welding residual stress $\{\sigma\}$ at an arbitrary position of the joint can be obtained.

$$\{\sigma\} = [D] [H^*] \{\epsilon^*\} \tag{2'}$$

Predicting Method of Welding Residual Stress¹⁰⁻¹³⁾

In this chapter, the characteristics of the source of welding residual stress generated in a butt joint of plates are studied. It will be shown that the source distributes in a simple form along the weld line and is almost independent of the size of the joint for the specified material and welding condition if it is sufficiently large. Concerning the general theory of predicting welding

residual stress using inherent strain as a parameter, the details is described in the original reference⁵⁻⁹⁾. In the new predicting method, only the elastic analysis is necessary imposing the above mentioned source of residual stress on a stress-free plate, which should have been known either experimentally or theoretically.

Inherent Strain Distribution in Butt Welded Joint

In this study, a butt welded joint of thin plate subjected a moving heat source is adopted as the analysis model (Fig. 2), and the length, width and thickness of the joint are denoted by $2L$, $2B$ and h ($= 6 \text{ mm}$: constant) respectively. The CO_2 gas shielded arc welding is applied, in which electric current $I = 220$ (A), voltage $V = 24$ (V), welding speed $v = 24$ (m/h) and the efficiency of heat-input $\eta = 75\%$ are assumed. This heat-input corresponds to $Q = 594$ (J/mm) for the instantaneous plane heat source.

The thermal elasto-plastic analysis by the finite element method is performed using the temperature dependent physical and mechanical properties of the material shown in reference¹⁰⁻¹³⁾ and assuming that the material is mild steel and the model is in plane stress state. The first analysis is performed on model MO ($2L \times 2B = 1200 \times 1000 \text{ mm}$) in cooperation with thermal conduction analysis. Using the obtained elastic strain for $\{\epsilon\}$ of Eq. (3), the distribution of inherent strain $\{\epsilon^*\} = \{\epsilon_x^*, \epsilon_y^*, \epsilon_{xy}^*\}^T$ is computed and shown by solid lines in Fig. 3. The shearing component of inherent strain, ϵ_{xy}^* , is only produced near the end of the plate.

Neglecting ϵ_{xy}^* , for simplicity, the inherent strain is calculated as $\{\epsilon^*\} = \{\epsilon_x^*, \epsilon_y^*, 0\}^T$. This result is also represented by a dotted line in Fig. 3.

The entire distributions of $\{\epsilon^*\}$ are illustrated in Fig. 4. It is found that ϵ_x^* exists in a narrow width spanning the weld line up to $y = 34 \text{ mm}$. Furthermore, the

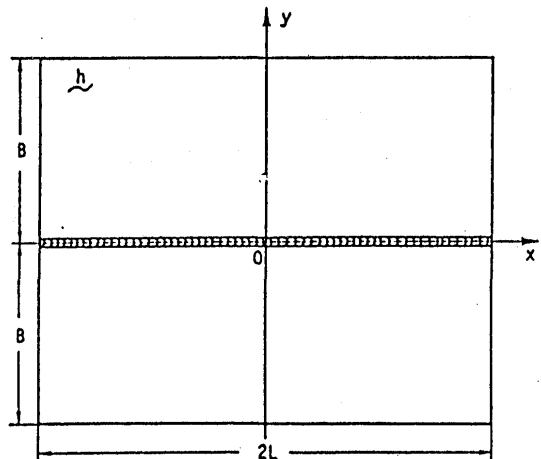


Fig. 2 Model of butt welded joint for analysis

distribution of ϵ_x^* along the direction of welding is the same at each cross-section except near the ends of the plate and the distribution of ϵ_y^* exists only in the vicinities of the both ends and is in a parabola along the weld line.

In order to confirm the accuracy of reproduction of welding residual stress by the inherent strain $\{\epsilon^*\} = \{\epsilon_x^*, \epsilon_y^*, 0\}^T$, welding residual stress is computed by elastic analysis imposing $\{\epsilon^*\}$ to model *MO* in stress-free state. This welding residual stress is almost coincident with that by the thermal elasto-plastic analysis.

Under the welding condition mentioned before, the thermal elasto-plastic analysis is performed on the five models and the inherent strain in each model is computed. The inherent strains are represented in Fig. 5 (a) and (b).

In the case where the width *B* of the plate is sufficiently large ($B \geq 300$ mm) to the specified heat input, the distribution of the inherent strain ϵ_x^* is almost independent of the sizes of the plate. On the other hand, Fig. 5 (b) shows that the magnitude and distribution of the inherent strains (including ϵ_x^* and ϵ_y^*) near the free ends of the five models are nearly the same in spite

of different sizes of the plates.

Figure 6 shows the distribution of residual stress σ_x at the middle cross section of model *MO* analyzed by using the corresponding inherent strain shown in Fig. 4. The distributions of stress σ_x' and σ_x'' are calculated by using ϵ_x^* and ϵ_y^* separately. It should be noted that ϵ_x^* produces fundamental distribution of σ_x and ϵ_y^* influences the inclination of compressive part of σ_x .

In actual plated structures, such as ships, the length of plate is long enough in comparison with the width so that the important component of residual stress is σ_x caused by ϵ_x^* which distributes nearly in a trapezoidal form in transverse cross section (Fig. 7). Such distribution was determined theoretically^{11,12}, assuming that heat input is

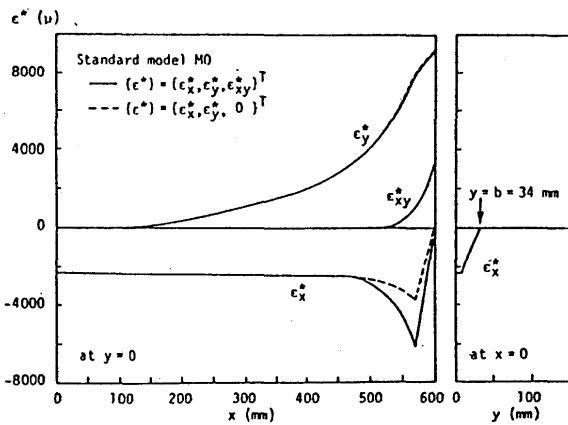


Fig. 3 Inherent strain distribution of ϵ_x^* , ϵ_y^* and ϵ_{xy}^*

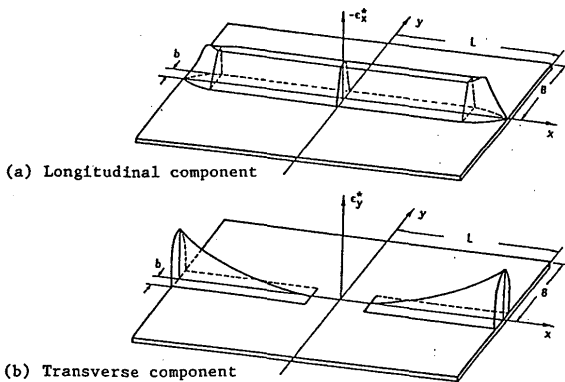
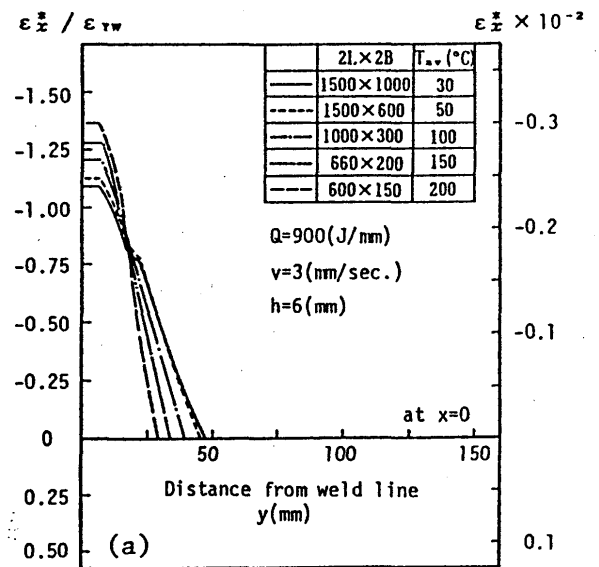
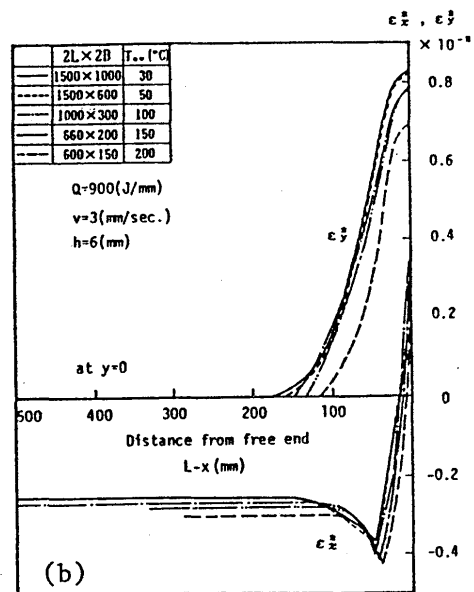


Fig. 4 Schematic representations of inherent strain distributions



(a) ϵ_x^*



(b) ϵ_x^* and ϵ_y^*

Fig. 5 Inherent strain distributions

provided to the weld line instantaneously so that the displacement along the cross section is uniform ($du_x / dy = 0$) except near the free ends.

The width b of inherent strain zone and its magnitude $\hat{\epsilon}_x^*$ in the mechanical heat-affected zone ($0 \leq y \leq y_H$) are expressed as,

$$b = \xi b_0 \tag{4}$$

$$\hat{\epsilon}_x^* = \zeta \sigma_{YW} / E \tag{5}$$

$$b_0 = 0.242 \frac{\alpha EQ}{c \rho h \sigma_{YB}} \tag{6}$$

where ξ is the normalized width of inherent strain zone

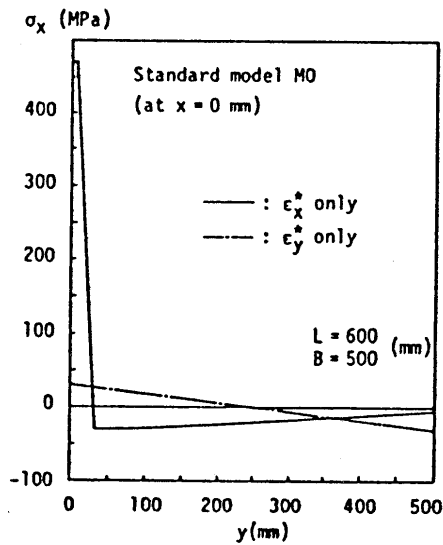


Fig. 6 Stresses produced by ϵ_x^* and ϵ_y^*

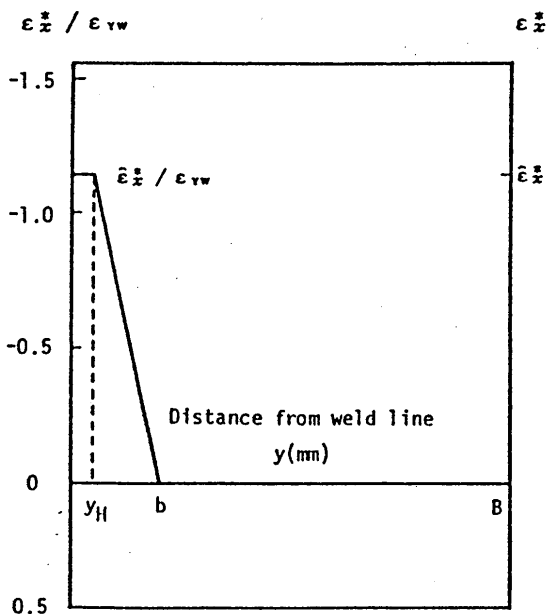


Fig. 7 Idealized distribution of inherent strain ϵ_x^*

with respect to b_0 which is the width for an infinitive width of a plate. ζ is the normalized magnitude of inherent strain. σ_{YB} and σ_{YW} are the yield stresses of the base plate and the weld metal, E , α , c and ρ are Young's modulus, linear thermal expansion coefficient, specific heat and density respectively.

In the above expressions, ξ and ζ are given by complex equations. However, the following simple formulae were derived.

$$\left. \begin{aligned} \xi &\doteq 1 - \frac{0.27 \alpha T_{av}}{\sigma_{YB}} \\ \zeta &\doteq -1 - \frac{0.27 \alpha T_{av}}{\sigma_{YB}} \end{aligned} \right\} \tag{7}$$

$$\text{where, } T_{av} = \frac{Q}{2c \rho h B} \tag{8}$$

These can be applied to the yeild stress σ_{YB} which varies from 230 MP_a to 430 MP_a.

In Fig. 8, using the same material properties, the widths b and magnitudes $\hat{\epsilon}_x^*$ of the inherent strains for different value of T_{av} calculated by Eq. (7) are shown by lines, and those obtained for a moving heat source by the thermal elasto-plastic analysis are plotted by solid circles. It is recognized that the two results have good agreement with each other. This indicates that the formulae (7) are applicable not only to instantaneous heat source, but also to moving heat source, provided that the effective heat input is chosen to be the same in both cases.

The estimated residual stress distributions in the middle cross-sections of eight butt welded plates with different widths and lengths by the proposed method are shown in Fig. 9 using several different marks. The residual stresses in the respective plates by thermal elasto-plastic analysis are also plotted in Fig. 9 using lines. It is observed that the residual stresses in the butt welded plates with different aspect ratio L/B can be predicted accurately by the proposed method, provided that an inherent strain distribution in some butt welded plate had been known correctly from experiment or literature to calibrate the absolute values of b_0 and $\hat{\epsilon}_x^*$ in the mechanical heat-affected zone.

New Measuring Method of Three-dimensional Residual Stresses⁵⁻⁹⁾

In the new measuring methods, the effective inherent strains are estimated and residual stresses are calculated by imposing these obtained effective inherent strains to the object in stress-free state. That is, residual stresses are measured using these inherent strains as parameters.

The object of measurement is uniformly distributed three-dimensional residual stresses produced in a long welded joint as shown in Fig. 10 (a). The source of residual stress (inherent strain) is also uniform along the

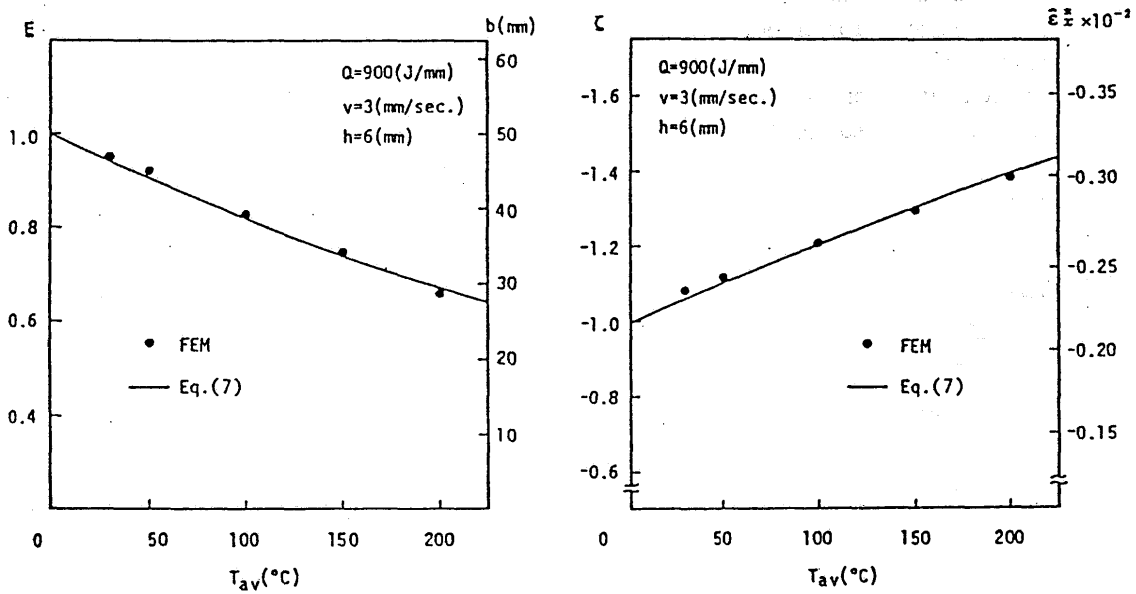


Fig. 8 Width and magnitude of idealized distribution of inherent strain ϵ_x^*

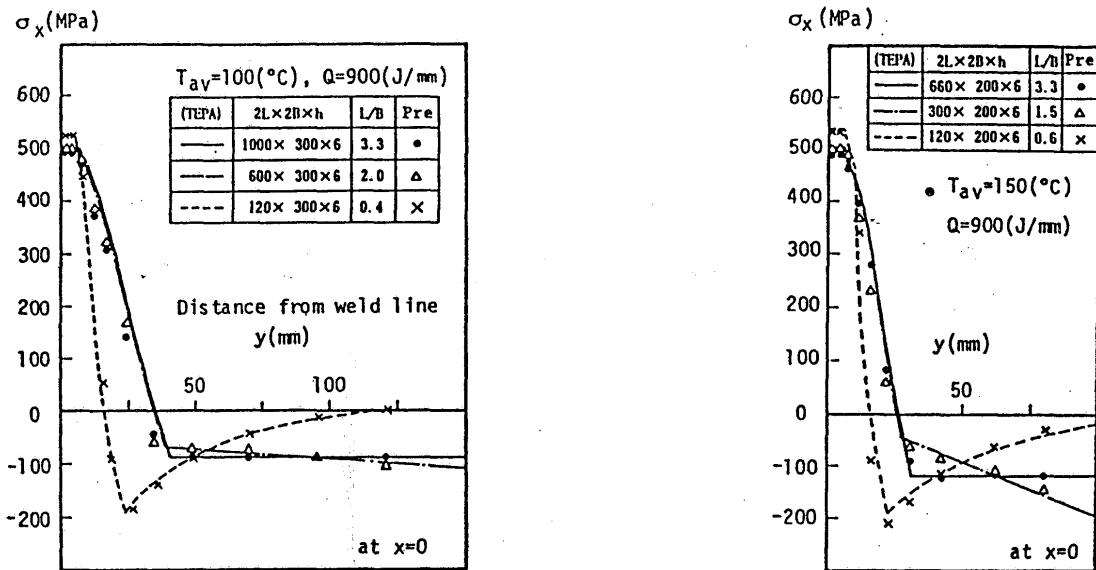


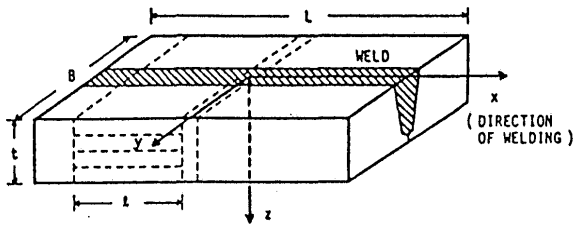
Fig. 9 Welding residual stresses by thermal elasto-plastic analysis and the presented method

longitudinal axis except near the ends and symmetric with respect to the middle cross-section ($x = 0$). Then, the whole effective three-dimensional inherent strain $\{\epsilon^*\}$ is the function of the cross-sectional coordinates (x, y). Taking full advantage of this distribution, $\{\epsilon^*\}$ can be separated into two groups: the longitudinal inherent strain $\{\epsilon_1^*\} = \{\epsilon_x^*\}$, and the cross-sectional one $\{\epsilon_c^*\} = \{\epsilon_y^*, \epsilon_z^*, \gamma_{yz}^*\}$. This enables us to simplify the measurement of residual stresses.

In order to estimate these inherent strains separately, thin plates: one Specimen T and several Specimens L are cut out from the original Specimen R according to L_z method^{6,9)} (Fig. 10). Specimen T is perpendicular and

Specimens L is parallel to x -axis. For this process, the following assumptions are made.

- (1) Cutting is accompanied by only elastic change of strain and does not produce any new plastic strain, that is, the inherent strain is not influenced by cutting.
- (2) In a thinly sliced plate, inherent strain component being perpendicular to the section does not produce any stress. That is, in a thin plate, stress in the plane stress state are produced by only inherent strain component in that plane.



(a) Experimental model of multi-pass welded joint (R-specimen)

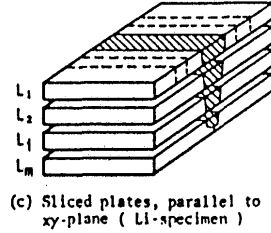
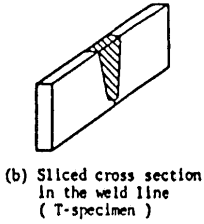


Fig. 10 Experimental model and procedure of slicing Specimens *T* and *L_i*

Separation of Three-dimensional Inherent Strain Components

The inherent strain distributions in Specimens *T* and *L* is the same as that in Specimen *R*, since the cutting is made to satisfy assumption (1). ϵ_x^* being perpendicular to the cross section does not produce stress (assumption (2)) in Specimen *T*. Therefore, the source of residual stress remaining in Specimen *T* is the cross-sectional inherent strain $\{\epsilon_c^*\}$.

Next, ϵ_z^* being perpendicular to the section does not produce stress in Specimens *L* from assumption (2). ϵ_y^* distributes uniformly in the longitudinal direction of Specimens *L*. For this reason, stress by ϵ_y^* is released when Specimens *L* is cut out from Specimen *R*, and ϵ_y^* just expands or shrinks uniformly along the breadth. Accordingly, the source of residual stress remaining in Specimens *L* is only ϵ_x^* . Three-dimensional inherent strain can thus be separated into the cross-sectional inherent strain $\{\epsilon_c^*\}$ in Specimen *T* and the axial one $\{\epsilon_1^*\}$ in Specimens *L*.

Then, they can be estimated by the measurement of two-dimensional stresses remaining in Specimens *T* and *L* respectively. By this procedure, the measurement of three-dimensional residual stress is reduced into that of two-dimensional one.

Therefore, the three-dimensional residual stresses $\{\sigma\}$ produced in Specimen *R* can be obtained as the sum of the stress $\{\sigma^A\}$ produced by the cross-sectional inherent strain $\{\epsilon_c^*\}$ and stress $\{\sigma^B\}$ by the longitudinal inherent

strain $\{\epsilon_1^*\}$.

$$\{\sigma\} = \{\sigma^A\} + \{\sigma^B\} \tag{9}$$

The actual measuring method and the procedure of three-dimensional residual stress components, $\{\sigma^A\}$ and $\{\sigma^B\}$, are described below.

Three-dimensional Residual Stress, $\{\sigma^A\}$, by Cross-sectional Inherent Strain, $\{\epsilon_1^*\}$

As the cross-sectional inherent strain distributes uniformly along the longitudinal axis, the three-dimensional residual stress component $\{\sigma^A\}$ is in the plane strain state in Specimen *R*, since stress in the cross section is self-equilibrating and consequently the resultant stress perpendicular to the section must be zero. In contrast with this, the stress remaining in the thinly sliced Specimen *T* cut out from Specimen *R*, $\{\sigma^{AO}\}$, is in the plane stress state, and can be directly observed by the stress relaxation method. Using the following equations, directly observed $\{\sigma^{AO}\}$ can be converted into the stress $\{\sigma^A\}$ in the plane strain state. As a result, three-dimensional residual stress component $\{\sigma^A\}$ can be obtained without estimating inherent strains.

$$\begin{aligned} \sigma_y^A &= \sigma_y^{AO} / (1 - \nu^2) \\ \sigma_z^A &= \sigma_z^{AO} / (1 - \nu^2) \\ \sigma_x^A &= \nu (\sigma_y^{AO} + \sigma_z^{AO}) / (1 - \nu^2) \\ \tau_{yz}^A &= \tau_{yz}^{AO} / (1 - \nu^2), \tau_{zx}^A = \tau_{xy}^A = 0 \end{aligned} \tag{10}$$

where, ν : Poisson's ratio

Three-dimensional Residual Stress, $\{\sigma^B\}$, by Longitudinal Inherent Strain, $\{\epsilon_1^*\}$

For the measurement of $\{\sigma^B\}$, Specimens *L* are cut out from Specimen *R* as shown in Fig. 10. Elastic strain remaining in each Specimens *L* can be observed when each Specimen *L* is sectioned into pieces in parallel to the longitudinal axis as depicted by the broken lines.

Using the relation between residual stress (strain) and inherent strain, the inherent strain distribution in each Specimen *L* can be determined. The entire distribution is composed of each ones. The residual stress $\{\sigma^B\}$ can be calculated by imposing the entire distribution of $\{\epsilon_1^*\}$ on the stress-free Specimen *R*.

Actual Measurements of Three-dimensional Residual Stress Distributions in Welded Joint

The material of Specimen *R* was mild steel and its initial residual stresses before welding were removed by stress relief annealing. Using submerged arc welding (current: 650 A, voltage: 35 V, welding velocity: 42 cm/min.), Specimen *R* was made by multi-pass butt welding

whose passes were accumulated from the bottom to the top of the specimen. The length, width and thickness of the specimen are $L = 200$ mm, $B = 200$ mm and $t = 50$ mm, respectively. One Specimen T and four Specimens L were cut out from Specimen R . The thickness of these sliced plates was 10 mm. The length of each Specimen L , l , was 70 mm.

SR-4 gages with a gage length of 2 mm were used. They were attached in pairs on both faces of the sliced plates which were Specimens T and L and the mean value of each pair of the observed strains was regarded the observed strain at the point.

According to the new measuring methods, three-dimensional residual stress $\{\sigma\}$ was obtained as the sum of residual stress components $\{\sigma^A\}$ and $\{\sigma^B\}$, respectively. The results are shown in Figs. 11 and 12, in which "○" and "●" indicate three-dimensional residual stresses directly observed by the check gages on the surface of Specimen R . The measured values and directly observed values are shown a good coincidence. Figure 12 represents the estimated welding residual stresses on the cross section of the joint.

conclusion

The characteristics of inherent strain produced by welding was described. The new predicting method and measuring method of welding residual stress were introduced, in which inherent strains are dealt as parameters. The new methods were developed without any approximation except the basic assumptions based on the theory of elasticity. Using these methods, the actual prediction and measurement of residual stresses produced in welded joints were performed and the practicability of these new methods and the validity of the theories were shown.

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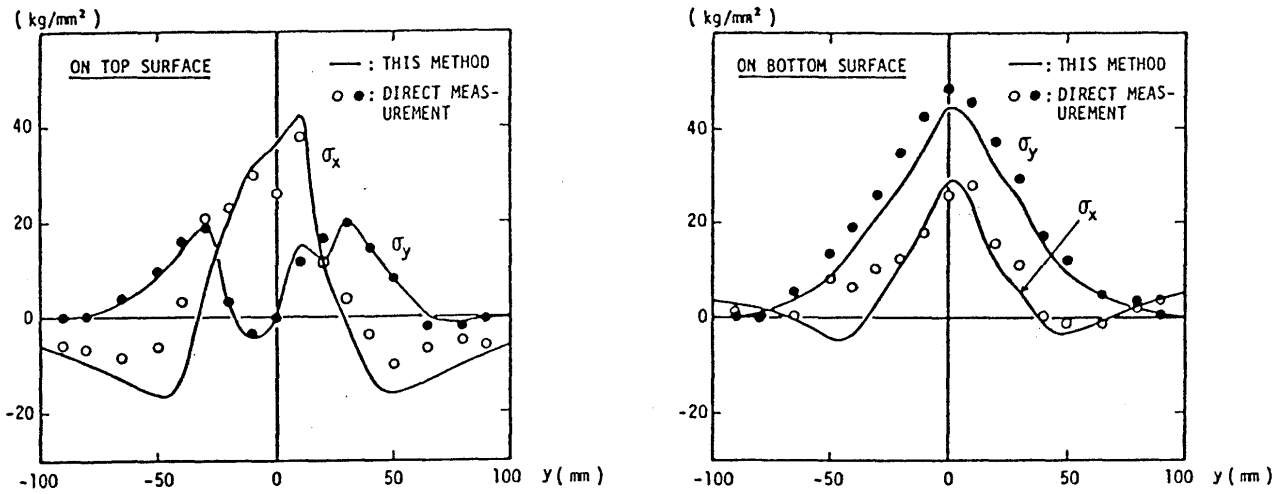


Fig. 11 Welding residual stresses on the top and bottom surfaces in the middle of weld line

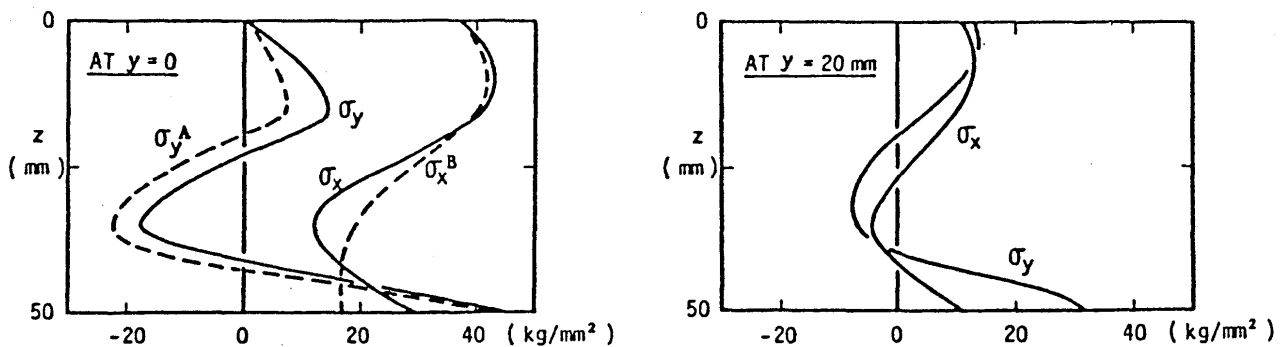


Fig. 12 Estimated welding residual stresses on the cross section ($x = 0$)

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