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# Initial Fatigue Crack Growth Behavior in a Notched Component (Report I)<sup>†</sup>

– Estimation of Elasto-plastic Stress Distribution in a Notch Field –

Kohsuke HORIKAWA\*, Sang-moung CHO\*\*

## Abstract

*In order to evaluate J-integral by elastic analysis for the short cracks or initial defects existing in the elasto-plastic field of notches, it is needed to obtain elasto-plastic stress distribution in the notch field, when crack is not present, by elastic solutions.*

*In the present study, strain energy density was taken as the intermediary quantity which connect elastic and elasto-plastic states.*

*The estimating formulars on the intermediary quantity were derived by combining the extension of Neuber's rule and elasto-plastic FEM. The distributions of elasto-plastic stresses were calculated from the strain energy density.*

*By taking strain energy density as the intermediary quantity, it was possible to consider stress redistribution by local yielding, and to consider multiaxial stress state in the notch field.*

*As the result, equivalent and principal stresses in the elasto-plastic field of notches could be estimated by only elastic solution and material constants.*

**KEY WORDS:** (Elasto-Plastic Stress Distribution) (Elasto-Plastic Notch Field) (Strain Energy Density) (Initial Fatigue Crack)

## 1. Introduction

Many fatigue cracks have been detected in the vicinity of notches (structural discontinuities) in welded structures subjected to cyclic load<sup>1,2)</sup>. In many cases, fatigue cracks are grown from initial defects by welding or other process. When loading stress is low, Linear Elastic Fracture Mechanics (LEFM) can be applied to estimate fatigue life by considering initial defects<sup>3)</sup>. Also by this procedure, allowable size of initial defects can be assessed<sup>4)</sup>. However, when loading stress is high, the notches in welded structures with residual stress have elasto-plastic behavior. In this case, LEFM can not be applied to assess initial defects.

The object of the present study is the fatigue behavior of the notched component subjected to high stress. And the primary purpose of this study is to develop the evaluating method of initial fatigue crack growth, and then of initial defect in a notch field.

In Fig. 1 (a) on a notched component, when the ratio of crack plastic zone  $r_p$  to crack length  $a$  is small, crack propagation life can be characterized by application of stress intensity factor  $K$  based on LEFM. In this case of a notch field,  $K$  to Mode I or Mode II is calculated by approximate method only, because the analytic solution

has not been obtained. It is said that the accuracy on calculation of  $K$  to Mode I in a notch field may be guaranteed to a certain extent even though by approximating method such as Eq. (1)<sup>5)</sup>.

$$K = 1.1215 \sigma(a) \sqrt{\pi a} \quad (1)$$

Where  $a$  is crack length from a notch tip, and  $\sigma(a)$  is the stress at the corresponding point of crack tip when crack is not present. But if a crack is developed in the plastic zone due to stress concentration of a notch, as shown in Fig. 1 (b), LEFM is not defined any more<sup>6)</sup>. Under condition where the assumption inherent to LEFM are violated, the introduction of Elasto-Plastic Fracture Mechanics (EPFM) may have to be considered. J-integral as the parameter of EPFM is used here on account of the interrelation with LEFM<sup>7,8)</sup>. In order to obtain J-integral for a crack in a notch field, elasto-plastic numerical analysis can be used. But, because that is very complicated, it is desirable to develop the estimating method of J-integral by elastic computation only.

The research which extended the approximating method on  $K$  for a crack in a notch field to J-integral was reported<sup>9)</sup>. Moreover, the research attempted to characterize the initial fatigue crack propagation in a notch field by application of J-integral. However, even at present, the

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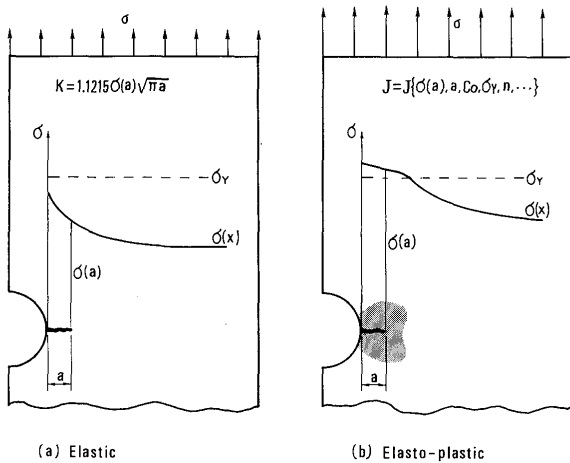


Fig. 1 Applications of fracture mechanics to cracks in notch fields.

calculation method of J-integral and other considerations proposed in the research are under debate<sup>10,11</sup>). By that approach, generally, J-integral for a crack in a notch field can be given as,

$$J = J \{ \sigma(a), a, C_N, \sigma_Y, n, \dots \} \quad (2)$$

where,  $a$  and  $\sigma(a)$  are identical with those of Eq. (1), but  $\sigma(a)$  is the elasto-plastic stress determined by substitution of  $x = a$  in the stress distribution  $\sigma(x)$  of Fig. 1 (b). And  $C_N$  is the correction factor,  $\sigma_Y$  and  $n$  are material constants.

It is worthwhile to note that the calculation accuracy of J-integral by Eq. (2) is greatly affected by the trend of the stress distribution  $\sigma(x)$  in a notch field shown in Fig. 1 (b). In the present report, the estimating method of elasto-plastic stress distribution  $\sigma(x)$ , when crack is not present, was treated. It may be the feature of this report that the distribution of equivalent and principal stresses in the notch field under elasto-plastic state can be estimated by elastic solution and material constants only.

### 2. Derivation of the Estimating Method of Elasto-Plastic Stress Distribution in a Notch Field.

Though external load on a overall structure is uniaxial state, stress condition in a notch field is apt to be biaxial (plane stress) or triaxial (plane strain) stress state due to discontinuity of the configuration. Equivalent stress (or effective stress) is, generally, used to calculate J-integral for a crack under multiaxial stress state<sup>8,12</sup>). There are several reports that estimate the stress distribution in a notch field under elasto-plastic condition<sup>9,13</sup>). However, it is difficult to find that the equivalent stress distribution is estimated with mechanical validity. In the present report, the estimating method was derived by combining the extension of Neuber's rule<sup>14</sup>) and elasto-plastic Finite

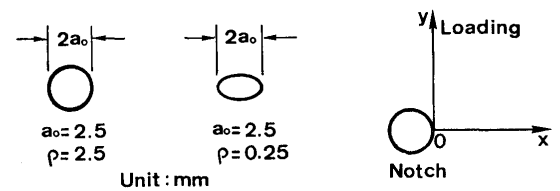
Element Method (FEM). Strain energy density was used as intermediary quantity which connect elastic state to elasto-plastic one.

The major flow of the estimating method is given as.

- (1) Computation of elastic strain energy density distribution in a notch field ..... Elastic FEM
- (2) Using the results of above (1), estimation of elasto-plastic strain energy density distribution ..... Approximating formulas by combining extension of Neuber's rule and elasto-plastic FEM.
- (3) Calculation of elasto-plastic stress distribution from the strain energy density estimated in above (2) ..... Deformation theory

The computer program for numerical analysis, which had been developed by Shiratori et al.<sup>15</sup>), was used for elasto-plastic FEM. And  $J_2$  flow theory under plane stress condition was applied in FEM. In disregard of stress history, the flow theory results to the same as the deformation theory if only loading path under proportional state is considered<sup>16</sup>). For that reason, the deformation theory was used in the estimating method because of its simplicity.

Two kinds of model specimens used in FEM are shown in Fig. 2 (a). The notch root radius  $\rho$  of circular hole was 2.5 mm, elliptical hole was 0.25 mm. The coordinates system was taken as Fig. 2 (b). Material considered in the analysis was aluminum alloy A5083-0 ( $E = 68.6$  GPa,  $\sigma_Y = 172$  MPa,  $n = 0.17$ ).



(a) Shape of notches (b) x-y coordinates

Fig. 2 Shape of notches and x-y coordinates.

### 2.1 Distribution of elastic strain energy density in a notch field

Considering the results of elastic FEM for two kinds of center notched strips ( $2B=36$ mm) under tension load, the procedure for formulation was described in this section.

Elastic stress concentration factor of circular notch was computed as  $K_t = 2.7$ , elliptical notch was  $K_t = 6.0$ .

Taking account of biaxial stresses in the notches (assumed plane stress), elastic strain energy density was computed as follows.

$$W = \int \sigma_{ij} d\epsilon_{ij} \quad (3)$$

The distribution of strain energy density was considered only on the x-axis which is the plane of the maximum

principal stress. That is shown in Fig. 3. The distribution  $W(x)$  is the function of the maximum strain energy density  $W_{\max}$  at the notch tip ( $x = 0$ ), and of the normalized distance  $x/\rho$ . That could be approximated as Eq. (4).

$$W(x) = W_{\max} / (1 + g_{we} \cdot x/\rho) \quad , \quad x/\sqrt{a_0} \rho \leq 1.0 \quad (4)$$

where,  $g_{we}$ , the gradient of strain energy density, was nearly constant up to  $x/\sqrt{a_0} \rho = 1.0$ . The value of  $g_{we}$  was 6.3 in the circular notch, and 6.5 in the elliptical notch. Accordingly in the present study, the notch field was defined as the range of  $x \leq \sqrt{a_0} \rho$ .

The elasto-plastic distribution  $W(x)$  would be estimated by using of the elastic distribution  $W(x)$  of Eq. (4). Namely, the maximum strain energy density  $W_{\max}$  and the gradient  $g_{we}$  of the distribution were estimated separately for elasto-plastic state, and then each estimated result was substituted into Eq. (4). The estimating procedures of  $W_{\max}$  and  $g_{we}$  for elasto-plastic state are described in Sec. 2.2 and Sec. 2.3 respectively.

## 2.2 Maximum strain energy density $W_{\max}$ for elasto-plastic state.

Under elastic condition, the maximum strain energy density  $W_{\max}$  at the notch tip is equal to  $K_t^2 \cdot W_n$ , where  $W_n$  means net section strain energy density, namely  $W_n = S^2/2E$  ( $S$ : nominal stress on net section). If a notch tip yields,  $W_{\max}$  depends upon not only  $K_t$  and stress  $S$ , but also other factors such as material constants.

By the way,  $W_{\max}$  under elasto-plastic condition was presumed that could be estimated from the following Eq. (5).

$$W_{\max} = K_w \cdot W_n \quad (5)$$

where,  $K_w$  was defined as strain energy density concentration factor. It was regarded that  $W_{\max}$  can be calculated by using the estimated  $K_w$ .

As the relation of stress and total strain, piecewise power hardening rule was used as follows.

$$\sigma = E\epsilon \quad , \quad \sigma \leq \sigma_Y \quad (6-1)$$

$$(\sigma/\sigma_Y) = (\epsilon/\epsilon_Y)^n \quad , \quad \sigma > \sigma_Y \quad (6-2)$$

Thus,  $W_{\max}$  under plastic condition is the sum of linear part up to  $\sigma_Y$  and nonlinear part over  $\sigma_Y$  as following Eq. (7).

$$W_{\max} = \left(\frac{n-1}{n+1}\right) \left(\frac{\sigma_Y^2}{2 \cdot E}\right) + \frac{\sigma_{\max} \cdot \epsilon_{\max}}{n+1} \quad (7)$$

Moreover, supposing that net section stress  $S$  is below  $\sigma_Y$ , Eq. (7) divided by  $W_n (=S^2/2E)$  gives Eq. (8) as follows.

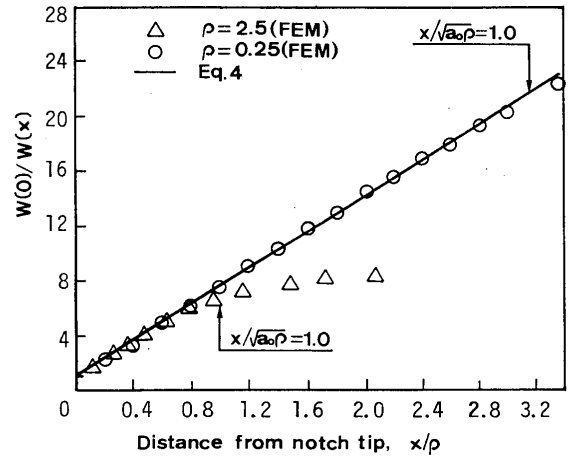


Fig. 3 Distribution of elastic strain energy density  $W(x)$  in notch fields.

$$\frac{W_{\max}}{W_n} = \left(\frac{n-1}{n+1}\right) \left(\frac{\sigma_Y}{S}\right)^2 + \frac{2}{n+1} K_\sigma \cdot K_\epsilon \quad (8)$$

where,  $K_\sigma$  and  $K_\epsilon$  are stress and strain concentration factor respectively based on net section stress.

Neuber's rule was applied to the right side in Eq. (8). And strain energy density concentration factor of the left side in Eq. (8) was denoted by  $K_{WN}$ . Therefore,  $K_{WN}$  could be derived by,

$$K_{WN} = \left(\frac{n-1}{n+1}\right) \left(\frac{\sigma_Y}{S}\right)^2 + \frac{2}{n+1} K_t^2 \quad (9)$$

This  $K_{WN}$  is affected primarily by  $K_t$  and  $S/\sigma_Y$ . Figure 4 shows both results of elasto-plastic FEM and Eq. (9) on strain energy density concentration factor. The results calculated by extension of Neuber's rule have a tendency to be higher than by FEM. That is similar to the general tendency of Neuber's rule<sup>17)</sup>. On that account, the introduction of correction factor  $C_w$  based on  $K_{WN}$  was implemented to estimate  $K_w$  with more accuracy. The correction factor  $C_w$  was defined as follows.

$$C_w = (K_w - K_t^2) / (K_{WN} - K_t^2) \quad (10)$$

It was regarded that, when  $K_t$  is known,  $C_w$  depends on only normalized stress level  $\phi = S/\sigma_Y$ . The variation of the factor  $C_w$  was modeled as sine curve by taking the results of FEM into account, as shown in Fig. 5, and was given as following approximating formula.

$$C_w = 1 + \frac{1}{2} \text{Sin} \left\{ \frac{\pi}{2} K_t \left( \frac{\phi - 1}{K_t - 1} \right) \right\} \quad (11)$$

Therefore, strain energy density concentration factor  $K_w$  could be calculated as following Eq. (12) using the factor  $C_w$  of Eq. (11).

$$K_w = K_t^2 + \left(\frac{1-n}{1+n}\right) \left(K_t^2 - \frac{1}{\phi^2}\right) \cdot C_w \quad (12)$$

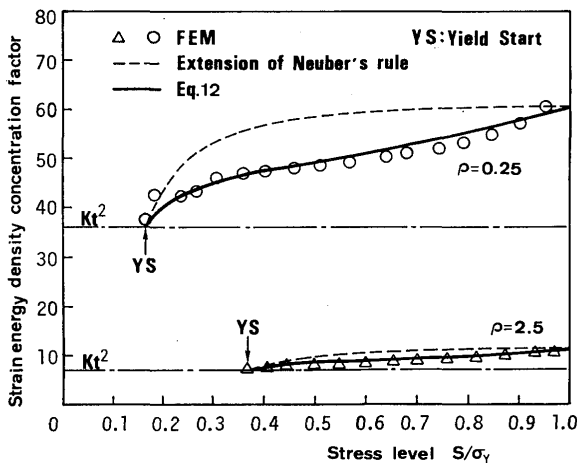


Fig. 4 Relation of strain energy density concentration factor  $K_w$  and stress level.

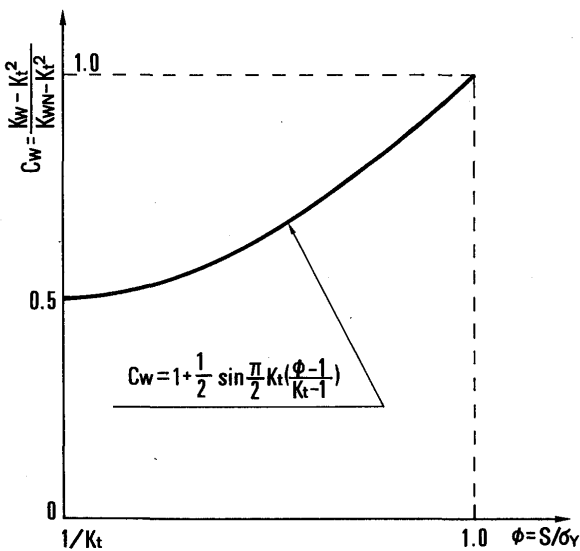


Fig. 5 Correction factor  $C_w$  of strain energy density concentration factor  $K_w$  as a function of stress level.

Solid lines in Fig. 4 indicate the trend of  $K_w$  by Eq. (12). The results by Molski et al., in which  $K_w$  is equal to  $K_t^2$  under elasto-plastic conditions as well as elastic<sup>18)</sup>, are also indicated as dot-and-dashed lines in Fig. 4.

Consequently, substituting  $K_w$  of Eq. (12) into Eq. (5), the maximum strain energy density  $W_{max}$  at the notch tip under elasto-plastic condition could be estimated by using elastic solution and material constants only.

### 2.3 Gradient of strain energy density in a notch field under elasto-plastic condition

It was considered that,  $g_{we}$  in Eq. (4), which is the gradient of strain energy density in a notch field under elastic condition, changes to  $g_{wp}$  under elasto-plastic condition. In order to estimate the gradient  $g_{wp}$ , redistribution factor  $R_g$  of strain energy density defined as follows was applied.

$$R_g = (g_{wp}/g_{we}) - 1 \tag{13}$$

The typical results of FEM are shown in Fig. 6 (a) (b). The redistribution factor  $R_g$  of strain energy density is the function of normalized stress level  $S/\sigma_Y$  or  $W_{max}/W_Y$  ( $W_Y = \sigma_Y^2/2E$ ), notch configuration ( $\rho, a_0$ ), and normalized distance  $x/\rho$  etc. By taking into account the results of FEM, the approximate formula could be obtained as follows.

$$R_g = (n-1) g_0 \{ 1 - (x/r_p)^2 \} \text{EXP} \{ -(x/r_p)^q \} \tag{14}$$

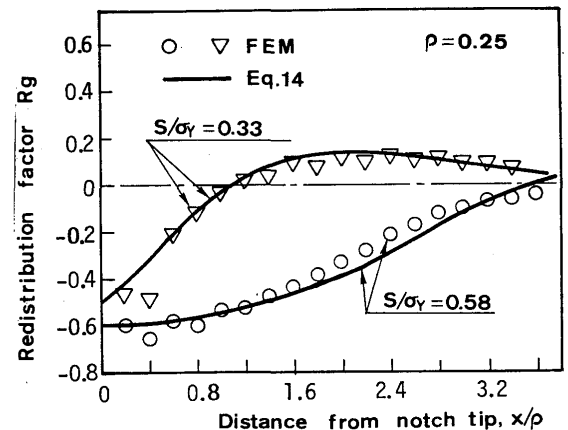
where,

$$g_0 = \tanh \left\{ 0.65 \left( \frac{\rho}{a_0} \right)^{0.1} \left( \frac{W_{max}}{W_Y} - 1 \right)^{0.2} \right\}$$

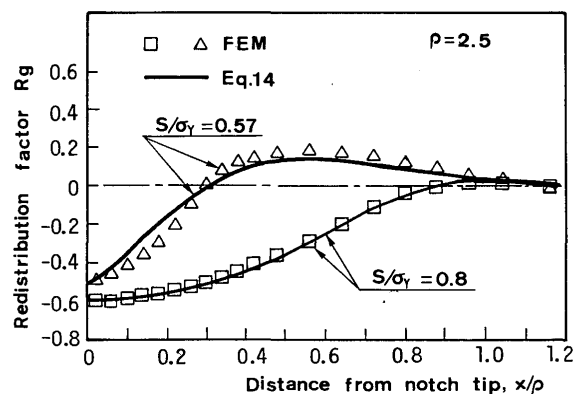
$$r_p = \frac{\rho}{g_{we}} \left( \frac{W_{max}}{W_Y} - 1 \right) \left( \frac{a_0}{\rho} \right)^{0.15}$$

$$q = 6 - 5 \text{sech} \left\{ \frac{1}{8} \left( \frac{\rho}{a_0} \right)^{0.5} \left( \frac{W_{max}}{W_Y} - 1 \right)^{1.5} \right\}$$

Solid lines in Fig. 6 (a) (b) indicate the results calculated by Eq. (14). Also, the factor  $R_g = 0$  means that  $g_{wp}$  is equal to  $g_{we}$ . Therefore, in a notch field under elasto-



(a) Elliptical holes ( $\rho=0.25$ )



(b) Circular holes ( $\rho=2.5$ )

Fig. 6 Variation of redistribution factor of strain energy density due to plastic deformation of notch.

plastic condition, the gradient  $g_{wp}$  of strain energy density could be estimated, from the relation of Eq. (13) using the factor  $R_g$  of Eq. (14), as follows.

$$g_{wp} = g_{we} (R_g + 1) \quad (15)$$

Consequently, under elasto-plastic condition, the maximum strain energy density  $W_{max}$  at the notch tip can be estimated by Eq. (5), and the gradient  $g_{wp}$  of strain energy density in a notch field can be estimated by Eq. (15). As the result, the distribution  $W(x)$  of strain energy density in a notch field under elasto-plastic condition can be obtained by Eq. (4).

While, this obtained  $W(x)$  includes multiaxial stresses in a notch field as given in Eq. (3). And then in order to calculate equivalent stress  $\bar{\sigma}(x)$  and principal stress  $\sigma_y(x)$  in  $y$ -direction (or loading direction) from the strain energy density  $W(x)$ , it is necessary to consider multiaxial stresses separately.

#### 2.4 Evaluation of multiaxial stresses in a notch field

In a notch field of plane stress, multiaxial stress state can be evaluated by considering only biaxial stress coefficient  $B_x$ . Where the coefficient  $B_x$  was defined as  $B_x = \sigma_x/\sigma_y$  on the plane of the maximum principal stress ( $x$ -axis). The results of FEM on the biaxial stress coefficient  $B_x$  is shown in Fig. 7. The coefficient  $B_x$  was almost determined by only the notch configuration and the distance  $x$ . But there was scarcely difference between under elastic and elasto-plastic condition on the coefficient  $B_x$ . Namely, the coefficient  $B_x$  depended little on stress level. Therefore, the coefficient  $B_x$  obtained under elastic condition could be extended to and also used under elasto-plastic state, because the stress state for loading path in a notch field could be regarded as proportional loading state.

The approximating formulas on the coefficient  $B_x$  by the results of FEM were given as following Eq. (16), and indicated in Fig. 7 as solid lines.

$$\rho = 2.5 : B_x = (x^{0.8}) / (3.45 + x^2) \quad (16-1)$$

$$\rho = 0.25 : B_x = (0.67x^{0.5}) / (0.8 + x^2) \quad (16-2)$$

#### 2.5 Calculation of stresses from strain energy density under elasto-plastic condition

Under elastic condition of plane stress, strain energy density  $W$  can be written as follows<sup>20</sup>.

$$W = \frac{(1+\nu)}{E} J_2 + \frac{(1-2\nu)}{6E} I_1 \quad (17)$$

where  $J_2$  is the second invariant of deviatoric stress, and  $I_1$  is the first invariant of stress tensor. Also, each term containing  $J_2$  and  $I_1$  is called as shear strain (or deformation)

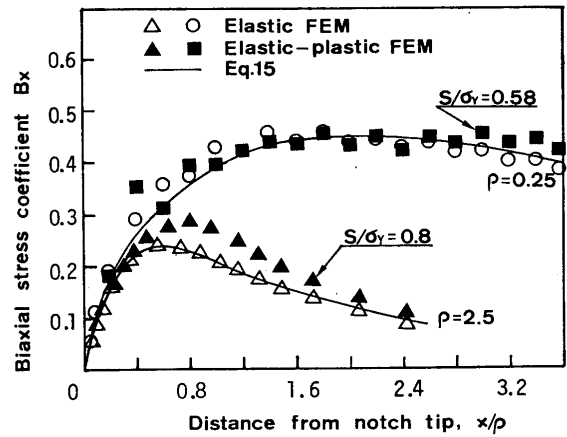


Fig. 7 Variation of biaxial stress coefficient  $B_x$  in notch fields.

energy density  $W_D$  and volume strain (or volume change) energy density  $W_V$  respectively.

Based on the yielding condition of von Mises, on the plane of the maximum principal stress in a notch field,  $J_2 = \bar{\sigma}^2/3$ , and  $I_1 = (\sigma_y + \sigma_x)^2 = \sigma_y^2 (1 + B_x)^2$  were given. Moreover, using the following Eq. (18),

$$\sigma_y = \bar{\sigma} / \sqrt{B_x^2 - B_x + 1} \quad (18)$$

$W$  in Eq. (17) could be given as,

$$W = W_D + W_V \quad (19-1)$$

$$W_D = \frac{(1+\nu)}{E} \frac{\bar{\sigma}^2}{3} \quad (19-2)$$

$$W_V = \frac{(1-2\nu)(1+B_x)^2}{6E(B_x^2 - B_x + 1)} \bar{\sigma}^2 \quad (19-3)$$

Accordingly, equivalent stress  $\bar{\sigma}$  from the relation of Eq. (19) under elastic condition could be obtained as,

$$\bar{\sigma}^2 = (2EW) \left( \frac{B_x^2 - B_x + 1}{B_x^2 - 2\nu B_x + 1} \right) \quad (20)$$

Next, under plastic condition, when the relation of equivalent stress  $\bar{\sigma}$  and equivalent strain  $\bar{\epsilon}$  gives also piecewise power hardening rule as Eq. (6), shear strain energy density  $W_D$  was given as,

$$W_D = \left( \frac{n-1}{n+1} \right) \frac{\sigma_Y^2}{2E} + \frac{\sigma_Y^2}{E(n+1)} \left( \frac{\bar{\sigma}}{\sigma_Y} \right)^{(n+1)/n} \quad (21)$$

On the other hand, volume strain energy density  $W_V$  under plastic condition may be the same relation as under elastic. But, after yielding, the change of  $W_V$  is much smaller than the one of  $W_D$ . Thus, yield stress  $\sigma_Y$  instead of equivalent stress  $\bar{\sigma}$  in Eq. (19-3) was used. The strain energy density  $W$  was given as the sum of  $W_D$  from Eq. (21) and  $W_V$  from Eq. (19-3), and equivalent stress  $\bar{\sigma}$  could be calculated as follows.

$$\frac{\bar{\sigma}}{\sigma_Y} = \left\{ \frac{(n+1)EW}{\sigma_Y^2} + \frac{n-1}{2} + \frac{(2\nu-1)(n+1)(B_x+1)^2}{6(B_x^2-B_x+1)} \right\}^{n/(n+1)} \quad (22)$$

Accordingly, equivalent stress  $\bar{\sigma}$  could be obtained by Eq. (20) and Eq. (22) under elastic and elasto-plastic condition respectively.

Besides, the maximum principal stress  $\sigma_y$  could be calculated by the relation of Eq. (18) using the biaxial stress coefficient  $B_x$ . And equivalent strain  $\bar{\epsilon}$  and the maximum principal strain  $\epsilon_y$  may be also calculated by Eq. (6) and Hencky-Nadai's equation in the  $J_2$  deformation theory respectively.

### 3. Discussion

The stress distributions calculated by the present estimating method and elasto-plastic FEM were compared and discussed below.

There is the report in which the distribution  $K_t(x)$  of elastic stress concentration factor on the plane of the maximum principal stress is used as an intermediary quantity<sup>9)</sup>. That estimating method was also considered, and was called below as "the method by  $K_t(x)$ ". It can be aware that the method by  $K_t(x)$  does not include the effect of multiaxial stress state and only the maximum principal stress  $\sigma_y(x)$  can be estimated by it.

The results on the equivalent stress  $\bar{\sigma}(x)$  by the present estimating method using Eq. (20), Eq. (22) and FEM were depicted in Fig. 8 (a) (b). Also the maximum principal stress  $\sigma_y(x)$  calculated from the method by  $K_t(x)$  was indicated to compare with others in Fig. 8 (a) (b).

While, the distribution on the maximum principal stress  $\sigma_y(x)$  calculated by the present estimating method, FEM and the method by  $K_t(x)$  were compared in Fig. 9 (a) (b). The maximum principal stress  $\sigma_y(x)$  by the present estimating method coincide well with the results by FEM, which indicate a tendency for the stress  $\sigma_y(x)$  to reach the peak at small distance from a notch tip. This good coincidence may be attributed to considering both of the biaxial stress coefficient  $B_x$  and the redistribution of strain energy density due to local yielding in a notch field.

But, as shown in Fig. 8 (a) (b), the stress  $\sigma_y(x)$  by the method by  $K_t(x)$  are higher than by FEM at the notch tip, but lower in the plastic zone. It had been pointed out by Glinka that this tendency is caused to not considering redistribution of stresses due to local yielding and multi-axial stress states<sup>13)</sup>.

Figure 10 shows the maximum principal stresses  $\sigma_y(x)$  by FEM of Okukawa for a wide plate with center notch

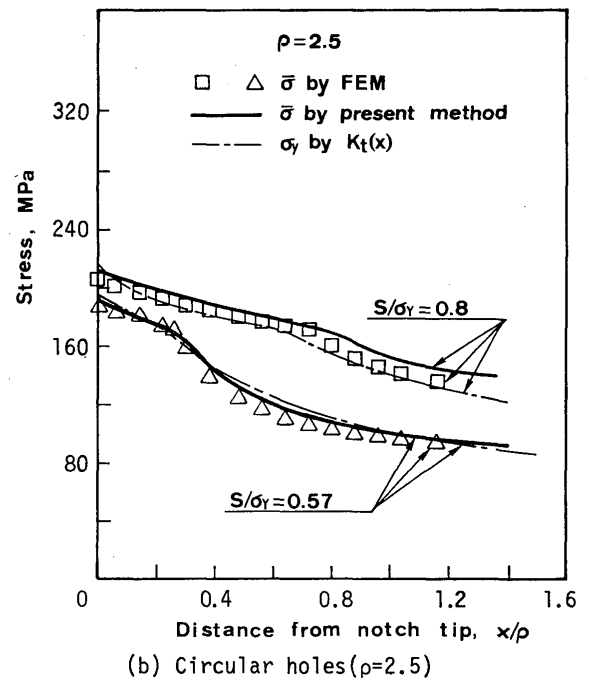
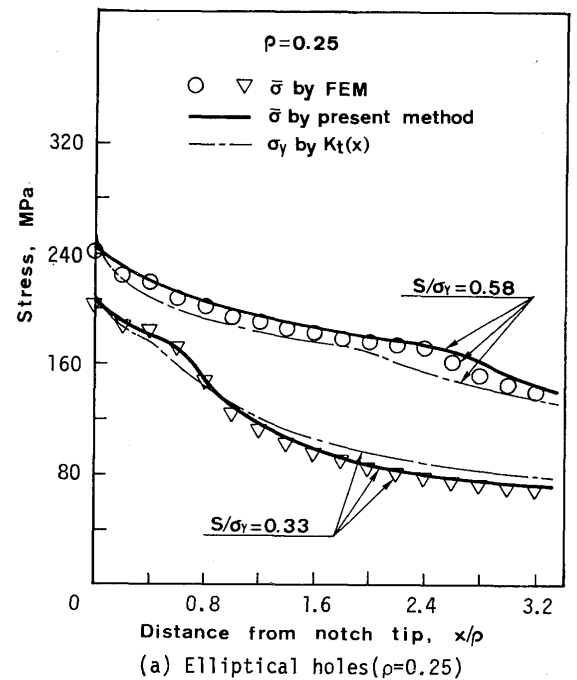
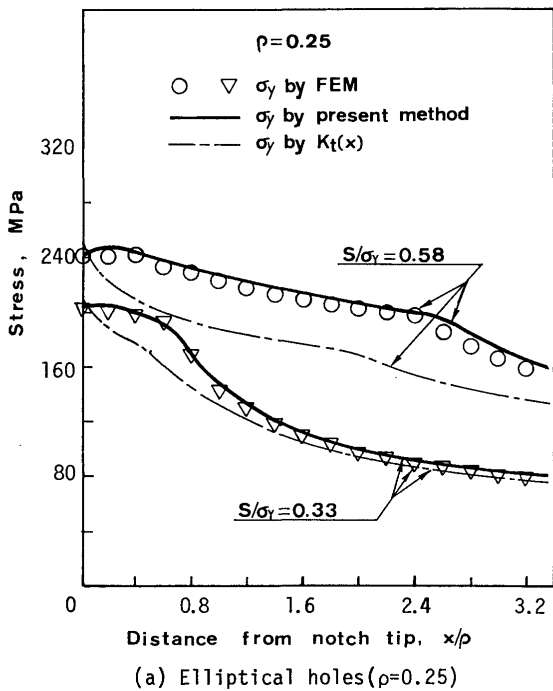


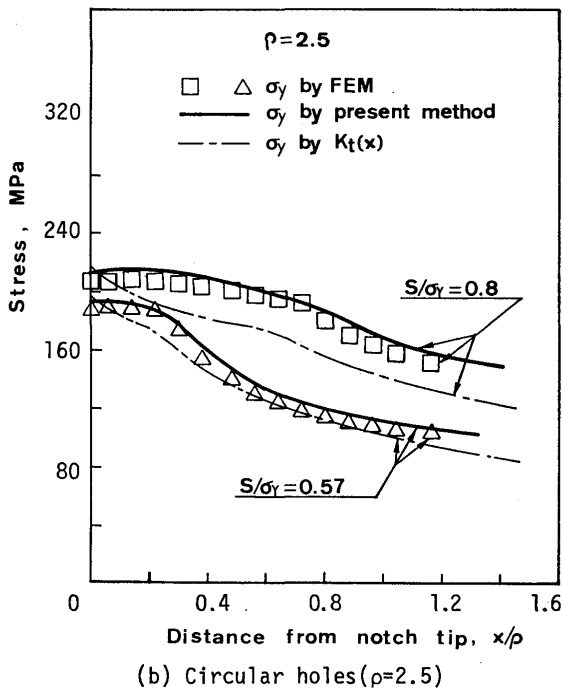
Fig. 8 Comparison of equivalent stress distribution obtained by various methods in notch fields.

(HT80)<sup>21)</sup>, and by the present estimating method on the base of elastic solution. Also, equivalent stress  $\bar{\sigma}(x)$  by the present estimating method and the maximum principal stress  $\sigma_y(x)$  by the method by  $K_t(x)$  were plotted to compare. It can be noticed also from Fig. 10 that the present estimating method gives a relatively good coincidence with FEM.

According to the result by FEM and the present estimating method, the stress distributions in elastic region just outside of plastic zone change rapidly as shown in Fig. 8, Fig. 9 and Fig. 10. Also when local yielding is



(a) Elliptical holes( $\rho=0.25$ )



(b) Circular holes( $\rho=2.5$ )

Fig. 9 Comparison of principal stress distribution obtained by various methods in notch fields.

developed, as shown in Fig. 6 (a) (b), the gradient  $g_{wp}$  of strain energy density in plastic zone is smaller than original elastic gradient  $g_{we}$ , but the gradient  $g_{wp}$  in elastic region just outside the plastic zone is larger than the original  $g_{we}$ . That is to say, it can be understood that redistribution of mechanical quantity occurs in elastic region as well as plastic zone.

On the other hand, when Neuber's rule is applied to notch field as well as notch tip, redistribution of mechanical quantity can not be considered, because inter-

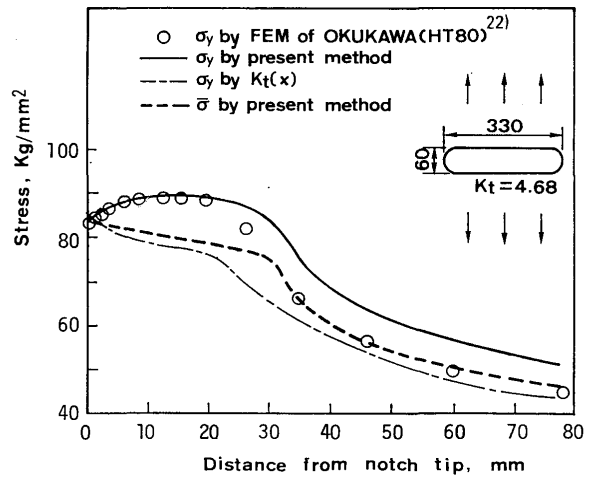


Fig. 10 Comparison of stresses obtained by various methods in notch fields of HT80.

mediary quantity is  $K_t(x)$  to be a kind of configuration factor. Namely, the factor  $K_t(x)$  is determined only by configuration, and not affected by stress level and others. Accordingly, it can not give mechanical validity that the factor  $K_t(x)$  varies with other factors except configuration. But, strain energy density may be governed by configuration, stress level and material constants. Thus, it can be mentioned that considering the redistribution of strain energy density gives mechanical validity.

#### 4. Conclusions

In order to obtain J-integral by elastic analysis for short crack or initial defect in a notch field, it is necessary to estimate equivalent stress distribution under elasto-plastic condition, when crack is not present, on the base of elastic solution and material constants. In the present report, strain energy density was taken as intermediary quantity which connected elastic state to elasto-plastic. And elasto-plastic stress distribution in a notch field was calculated by the estimated strain energy density. The present estimating method was derived by combining the extension of Neuber's rule and elasto-plastic FEM, and based on  $J_2$  deformation theory.

The results obtained are as follows.

- (1) The distribution  $W(x)$  of elasto-plastic strain energy density could be estimated by formulating the distribution of elastic strain energy density as Eq.(4), and by calculating the maximum strain energy density  $W_{max}$  at a notch tip and the gradient  $g_{wp}$  of it for elasto-plastic state respectively. Where, the maximum strain energy density  $W_{max}$  could be obtained by Eq. (5) and Eq. (12), the gradient  $g_{wp}$  of it by Eq. (15).
- (2) It was possible to confirm that biaxial stresses in a notch field under plane stress condition give nearly proportional loading state as shown in Fig. 7.



- (3) On the plane of the maximum principal stress in a notch field under elasto-plastic state, the relation of strain energy density and equivalent stress was derived as Eq. (20) and Eq. (22).
- (4) By taking the strain energy density as intermediary quantity, it was possible to consider multiaxial stress state and redistribution of mechanical quantity due to local yielding in a notch field with mechanical validity.
- (5) The elasto-plastic stress distribution estimated by the present method agreed well with that computed by FEM.

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