Effective Coefficient of Initial Deflection for Simple Prediction Method of Compressive Ultimate Strength of Rectangular Plate†

Yukio UEDA*, Keiji NAKACHO and Shuji MORIYAMA*****

Abstract

In general, a deck plate of a ship hull is subjected to mainly in-plane tensile and/or compressive load due to longitudinal bending moment acting the ship. When the strength of a deck plate is considered, the contribution of the plate elements (panels) subdivided by stiffeners and girders are very important, and the accurate evaluation of their compressive ultimate strength is essential.

In this report, based on the results obtained in this series of research[6)-8)]*, the authors have developed a simple prediction method of the ultimate strength with a limited amount of information on the kind of steel, sizes of deck plate panel, welding condition, etc., which does not require complex calculation such as FEM.*

The main results obtained in this study are as follows.

(1) In order to estimate the influence of actual complex initial deflection, the effective coefficient of initial deflection is rationally determined for the standard initial deflection which is idealized from the actually measured initial deflections. The coefficient indicates that the maximum initial deflection influences the ultimate strength fully for thick plates and less than a half for thin plates.

(2) The ultimate strength of a deck plate panel with initial imperfections (deflection and residual stress) can be evaluated from the ultimate strength formulae proposed in the previous report[7)]*, taking into account of the above effective coefficient of initial deflection. The predicted ones are very accurate in the comparison with those obtained by direct FEM calculation to the actual panels.*

KEY WORDS : (Compressive Ultimate Strength) (Effective Coefficient of Initial Deflection) (Simple Prediction Method) (Stiffened Panel) (Deck Plate of Ship Hull) (Welding Initial Deflection) (Welding Residual Stress)

1. Introduction

In box-girder structures such as hulls and bridges, deck plates play an important role for longitudinal bending strength. There have been a number of reports on the behavior of a plate and a stiffened deck plate subjected to in-plane compressive loads until the compressive ultimate strength. The authors also have studied this subject in series[1)-10)]. As for the compressive ultimate strength of a stiffened rectangular plate, they have analyzed the behavior up to failure under the influence of welding initial imperfections such as initial deflection and residual stress, and clarified its characteristics. Nevertheless, the estimation of compressive ultimate strength of an actual deck plate panel with complex initial imperfections requires either an experiment or a theoretical analysis, such as by the finite element method (FEM), for each panel, with considerable time and labor. Moreover, the results of a limited number of experiments or theoretical calculations do not necessarily provide general information, but specific. Therefore, they proposed two kinds of prediction methods, utilizing the results of their reseach which are summarized in the following.

(1) The entire magnitude of initial deflection of a plate is not always effctive to decrease its compressive ultimate strength.

(2) There is a fundamental difference between thin and thick plates in the behavior up to compressive collapse.

One[6)-8)] of the proposed methods is to derive simple estimating equations for the compressive ultimate strength of rectangular plate with uni-modal deflection and welding residual stress, with the plate sizes and the magnitudes of initial imperfections as parameters, based on the results of numerical analysis, and to estimate the compressive ultimate strength of an actual deck panel on the safety side,

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introducing the idea of effective coefficient of initial deflection. The other[8)-10)] is to estimate the compressive ultimate strength of an equivalent rectangular plate with initial deflection of a single sinuisoidal half-wave, which shows very similar deformation behavior to an actual deck plate panel with complex initial imperfections. By the former method, the compressive ultimate strength can be estimated as simply as by hand calculation on the safety side, though the accuracy is not enough if the effective coefficient of initial deflection presented hitherto is used. The latter method is accurate using an equivalent simple plate, and yet requires measurement of actual initial deflection to determine an equivalent rectangular plate, and elastic-plastic large-deflection analysis for the compressive ultimate strength.

This research aims to propose such a simple prediction method as by hand calculation to obtain accurate estimate of the compressive ultimate strength by utilizing the advantages of the above-mentioned two methods. To begin with, welding initial imperfections (initial deflection and residual stress) should be predicted. These can be obtained accurately by the simple prediction method presented by the authors[11)]. In this paper, first, the above second method is applied to rationally determine accurate and widely applicable effective coefficient of initial deflection, and this coefficient is used in the above first method for effective initial deflection. Consequently, if the sizes of a deck plate panel and its stiffeners, and welding conditions for attachment of the stiffeners to the deck plate are decided at a design stage, the compressive ultimate strength of the panel can be obtained very simply and accurately. Analyses will be performed on deck plate panels of mild steel and high tensile strength steel (HT steel) to demonstrate good applicability of the method.

This paper does not discuss the direct effect of longitudinal stiffeners on the compressive ultimate strength of a deck plate, local buckling of a panel nor overall compressive collapse of a deck.

2. Simple Estimating Equations for Compressive Ultimate Strength of Rectangular Plate

The authors have already presented simple estimating equations for the lowest compressive ultimate strength of a rectangular plate with welding residual stresses and the uni-modal initial deflection[7)-10)]. This set of simple equations may be used to estimate compressive ultimate strength of an actual deck plate panel with complex initial deflection, utilizing effective coefficient of initial deflection, which will be determined in the next chapter. The process of derivation of the equations is outlined in the following.

It is assumed that a rectangular plate (slenderness ratio $\xi=(b/t)\sqrt{\sigma_Y/E}$) is simply supported for its out-of-plane deformation, and its in-plane displacements along the four sides are kept straight. The plate is assumed to be accompanied by the uni-modal welding initial deflection expressed by Eq. (1) and welding residual stresses (the breadth for tensile stress $\sigma_{wmax}=\sigma_Y$: b_t) , as shown in **Fig. 1**.

$$w_0=w_{0s}\cdot\sin(m\pi x/a)\cdot\sin(\pi y/b) \tag{1}$$

where, w_0: initial deflection,

w_{0s}: the maximum magnitude of initial deflection.

For the case where a rectangular plate with such initial imperfections is subjected to compressive load (thrust) in x-direction, its compressive ultimate strength is evaluated for each uni-modal deflection of m in Eq. (1) being 1, 2, 3, …… (w_{os} is constant), so as to obtain the minimum value of ultimate strength for the same magnitude of initial deflection, w_{os}.

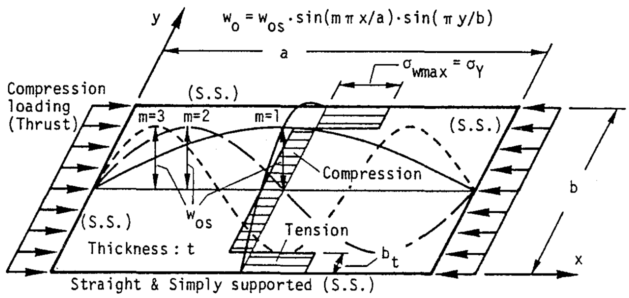

Fig. 1 Rectangular plate with uni-modal initial deflection and residual stresses under compression loading

This minimum value varies periodically to some extent with the change of the aspect ratio a/b of a plate, but the difference in the range of $0.5\leqq a/b$ is so small that the value may be regarded as constant. Accordingly, the minimum value of the compressive ultimate strength of a rectangular plate is expressed approximately as a function of only slenderness ratio ξ for the specified of initial imperfections (the magnitude w_{os} of initial deflection and the breadth b_t for tensile yield residual stress). That is to say, the non-dimensionalized compressive ultimate strength, σ_u / σ_Y, can be expressed as functions of non-dimensional parameters $\eta = w_{os}/t$, $2b_t/b$ and $\xi=(b/t)\sqrt{\sigma_Y/E}$, as follows.

(a) $2b_t/b=0.0$ (in case of no residual stress)

(a-1) $0.8\leqq\xi\leqq2.0$

$$\sigma_u/\sigma_Y=(-2.431\eta^2+1.6826\eta-0.2961)(\xi^2-4.0)+(7.2745\eta^2-4.7431\eta+0.6709)(\xi-2.0)+z_1 \tag{2}$$

$$z_1=(-0.3597\eta^2+0.1748\eta+0.8598)/(2.2432\eta+1.3322)+0.0373\eta+0.2481$$

(a-2) $2.0<\xi\leqq 3.5$

$$\sigma_u/\sigma_Y=(-0.3597\eta^2+0.1748\eta+0.8598)/(\xi+2.2432\eta-0.6678)+0.0373\eta+0.2481 \quad (3)$$

where, $\xi=(b/t)\sqrt{\sigma_Y/E}$, $\eta=w_{0s}/t$

(b) $2b_t/b=0.1$

(b-1) $0.8\leqq\xi\leqq 1.6$

$$\sigma_u/\sigma_Y=(-0.398\eta^2+0.4339\eta-0.1342)(\xi^2-2.56)+(1.0814\eta^2-0.7551\eta+0.102)(\xi-1.6)+z_2 \quad (4)$$

$$z_2=(0.4974\eta^2+0.8281\eta+1.0171)/(2.7942\eta+1.2908)-0.1849\eta+0.1571$$

(b-2) $1.6<\xi\leqq 3.5$

$$\sigma_u/\sigma_Y=(0.4974\eta^2+0.8281\eta+1.0171)/(\xi+2.7942\eta-0.3092)-0.1849\eta+0.1571 \quad (5)$$

(c) $2b_t/b=0.2$

(c-1) $0.8\leqq\xi\leqq 1.5$

$$\sigma_u/\sigma_Y=(-0.3317\eta^2+0.6314\eta-0.2656)(\xi^2-2.25)+(0.5369\eta^2-0.7798\eta+0.2854)(\xi-1.5)+z_3 \quad (6)$$

$$z_3=(0.292\eta^2+1.2936\eta+0.7471)/(2.897\eta+1.1189)-0.2715\eta+0.2057$$

(c-2) $1.5<\xi\leqq 3.5$

$$\sigma_u/\sigma_Y=(0.292\eta^2+1.2936\eta+0.7471)/(\xi+2.897\eta-0.3811)-0.2715\eta+0.2057 \quad (7)$$

Equations (2) to (7), thus non-dimensionalized by yield stress, can be used in common for both mild steel and HT steel panels irrespective of their different yield stresses. However, HT steel panel with $\sigma_{wmax}<\sigma_Y$ (see Fig. 1 and refer to Eq. (16) in Ref. 11) needs some correction, because the non-dimensionalization by yield stress is based on the assumption that the residual stress in the tensile stress region reaches yield stress. For such a case, the breadth for tensile residual stress, b_t, should be corrected as follows, paying attention on the fact that compressive residual stress affects directly the compressive ultimate strength. Keeping self-equilibrium of residual stress, that is, the integrated value of tensile residual stress is equal to that of compressive residual stress, the breadth is to be corrected by Eq. (8) assuming that the magnitude of tensile residual stress is σ_Y.

$$b_t=(\sigma_{wmax}/\sigma_Y)\times b_{t0} \quad (8)$$

where, b_{t0}: breadth for tensile residual stress, calculated by Eq. (15) in Ref. 11

σ_{wmax}: magnitude of tensile residual stress, shown in Eq. (16) in Ref. 11

The minimum value of compressive ultimate strength in Eqs. (2) to (7) is obtained by elastic-plastic large-deflection analysis based on the finite element method. In the analysis, welding imperfections are assumed to be produced in such way that initial deflection w_{0s} is first formed in the panel due to angular distortion of fillet welds and next inherent strains are given to the region of breadth b_t along the longitudinal stiffeners to produce residual stresses, and then the compressive ultimate strength of such panel is analyzed. The increment of deflection due to longitudinal residual stresses is calculated in the first step of elastic-plastic large-deflection analysis. Accordingly, initial deflection by only angular distortion of fillet welds is necessary to use for Eqs. (2) to (7). This initial deflection can be calculated by Eq. (14) in Ref. 11 as w_{0s}. As a matter of fact, the above mentioned increment of deflection due to residual stresses is extremely small, such as smaller than 5% of the one by angular distortion, in almost all deck panels[11]. Therefore, in place of the above deflection, any measured value of initial deflection in an actual panel or in an experimental panel, which is produced not only by angular distortion but also by longitudinal residual stresses, may be applied to Eqs. (2) to (7).

3. Determination of Effective Coefficient of Initial Deflection

Based on theoretical consideration, effective coefficients of initial deflection is determined in the following.

3.1 Definition of effective coefficient of initial deflection

The simple estimating equations mentioned in Chapter 2 give a minimum value of compressive ultimate strength of a rectangular plate with initial deflection of a sinuisoidal wave. In this case, the entire initial deflection is effective on the decrease of compressive ultimate strength. That is, the maximum magnitude of initial deflection is an important factor affecting compressive ultimate strength. While, an actual panel is accompanied by initial deflection of a complex shape, and its maximum magnitude of the deflection is not necessarily effective on the decrease of compressive ultimate strength. The effectiveness of such initial deflection of an actual panel may be represented by the effective coefficient of initial deflection, ζ, defined in the following.

$$\zeta = w_{0s}/w_{0\max} \tag{9}$$

where, w_{0s}: effective maximum magnitude of initial deflection of a sinuisoidal wave, to estimate the compressive ultimate strength of an actual panel by using Eqs. (2) to (7).

$w_{0\max}$: maximum magnitude of initial deflection of an actual panel.

If this effective coefficient of initial deflections, ζ, can be known beforehand, the compressive ultimate strength of an actual panel is easily estimated with the aid of the simple estimating equations, Eqs. (2) to (7).

3.2 Deflection method and curvature method, and effective coefficient of initial deflection

In order to obtain the effective coefficient of initial deflection, an equivalent rectangular plate with initial deflection of a half sinuisoidal wave as shown in **Fig. 2** is considered. This rectangular plate exhibits equivalent compressive behavior up to collapse to that of an actual panel.

An equivalent rectangular plate can be obtained by two methods[8)-10)]; one for thin plates and the other for thick plates, called the deflection method and the curvature method, respectively. First, it is assumed that an equivalent rectangular plate has the same plate breadth b and plate thickness t as an actual panel. Then, the plate length a_o and maximum initial deflection A_o are determined so as to make the behavior of deformation equal to one of an actual panel. Initial deflection of the equivalent rectangular plate is expressed as follows.

$$w_0 = A_0 \cdot \sin(\pi x/a_0) \cdot \sin(\pi y/b) \tag{10}$$

a_o and A_o in Eq. (10) are determined by either of the two methods mentioned above.

(1) Deflection method (for thin plate)

In thin plate, only particular deflection mode becomes predominant and stable above the elastic buckling load. Afterward plastification in the plate develops to collapse of the plate. The behavior above the elastic buckling load is almost the same as that of rectangular plate with the above-mentioned stable component as initial deflection.

Adopted as the length of an equivalent rectangular plate, a_o, and the maximum magnitude of initial deflection, A_o, are the length of one half-wave of a stable mode component among the actual initial deflection modes and the magnitude of the component.

(2) Curvature method (for thick plate)

In thick plate which collapses below the elastic buckling load, plastification initiates on the surface at the position of maximum curvature of the deflection which is very close to that of initial deflection, and develops in the whole section up to collapse with almost the same deflection mode as the initial one.

The primary component of deflection to the maximum curvature is taken out and its one half-wave length is adopted as a_o. The magnitude of initial deflection A_o is so determined that the curvature by only this component equals to the maximum curvature of an actual initial deflection. In this method, alternatively, a_o may be determined as the distance between two zero curvatures in the vicinity of the maximum curvature.

(3) Effective coefficient of initial deflection

The authors have shown that the compressive ultimate strength of thus specified equivalent rectangular plate with only a half sinuisoidal wave is nearly equal to that of an actual panel[8)-10)]. Therefore, the effective coefficient of initial deflection (Eq. (9)) may be rewritten as follows.

$$\zeta = A_0/w_{0\max} \tag{11}$$

3.3 Idealized standard modes of initial deflection and effective coefficient of initial deflection

As is well known, initial deflection of actual panel is

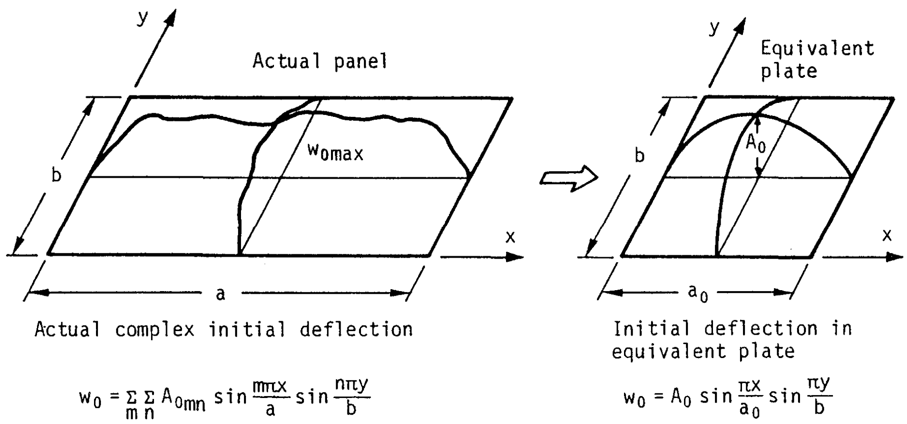

Fig. 2 Actual and equivalent panels

composed of complex deflection modes and is different in all panels, so that it is difficult to estimate the deflection mode and magnitude of initial deflection in each individual panel. In Ref. 11, therefore, idealizing the deformation behavior of a panel due to welding, the procedure to estimate the maximum magnitude (appears at the middle of the panel) of initial deflection due to angular distortion is presented. Similarly, idealizing the modes of initial deflection, based on the results of research by the authors[8)-10)], the deflection mode shown in **Fig. 3** is proposed as a standard one of initial deflection. This deflection mode consists of a half of the one half-wave of the buckling mode near both ends and a segment of cylinder between them, having the maximum initial deflection w_{omax}. The deflection across the transverse section is composed of a half sinuisoidal wave. Thus idealized standard initial deflection mode may be expressed as follows.

$$w_0=\begin{cases} w_{0max}\cdot\left|\sin\dfrac{K\pi x}{a}\right|\cdot\sin\dfrac{\pi y}{b} \\ \qquad\left(0\leqq x\leqq\dfrac{a}{2K},\ \dfrac{2K-1}{2K}a\leqq x\leqq a\right) \\ w_{0max}\cdot\sin\dfrac{\pi y}{b} \qquad \left(\dfrac{a}{2K}\leqq x\leqq\dfrac{2K-1}{2K}a\right)\end{cases} \tag{12}$$

where, a: length of panel, w_{0max}: maximum magnitude of initial deflection,
K: number of half-waves of the buckling mode for specified aspect ratio

In Refs. 9 and 10, the authors compare the correlation between the magnitudes of components of measured initial deflections of deck panels of various sizes and those of this standard initial deflections obtained for the respective deck panels. The result reveals that standard initial deflection modes indicate almost the same general characteristics as the actual ones.

Applying the deflection method and the curvature

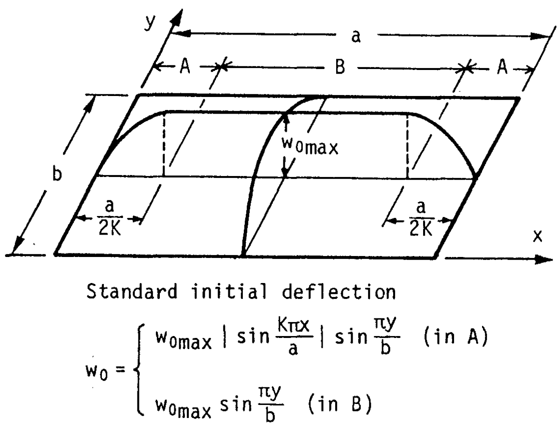

Fig. 3 Idealized standard initial deflection

method to this standard initial deflection, effective coefficients of initial deflections will be obtained.

3.3.1 Determination of effective coefficient of initial deflection by the deflection method (for thin plate)

Equation (12) can be developed into the following sine-series, at y=b/2 (the middle of the breadth).

$$w_0=\sum_m A_m\cdot\sin(m\pi x/a) \tag{13}$$

$$A_m=\begin{cases}\dfrac{4K^2 w_{0max}}{m\pi(K^2-m^2)}\cdot\cos\dfrac{m\pi}{2K} & \text{(in case of m being odd number)}\\ w_{0max}/K & \text{(in case of m being odd number and equal to K)}\\ 0 & \text{(in case of m being even number)}\end{cases}$$

If the number of half-waves of the buckling mode, K, is odd, the stable deflection mode above the elastic buckling load is assumed to coincide with the buckling mode. The magnitude of the stable deflection mode may be derived from Eq. (13) as that of the bucking mode, A_b (when m =K), as

$$A_b=w_{0max}/K \tag{14}$$

Accordingly, the effective coefficient of initial deflection, ζ, expressed by Eq. (11) can be rewritten as

$$\zeta=A_0/w_{0max}=A_b/w_{0max}=1/K \tag{15}$$

In the case where the number of half-waves of the buckling mode, K, is even, the stable deflection mode of an actual panel under thrust does not coincide with the buckling one, but its one higher mode. The effective coefficient of initial deflection can be obtained in the same way as Eq. (15) and approximated as follows.

$$\zeta=1/(K+1) \tag{16}$$

In actual panels, if the ratio of the components of initial deflection exceed the critical ratio[6)], the stable deflection mode may possibly move to the higher mode than the above cases. However, the difference of compressive ultimate strength is considered small enogh since, in case of thin plate, compressive ultimate strength of rectangular plate with uni-model deflection mentioned in Chapter 2 differs only a little for the change of magnitude of initial deflection.

3.3.2 Determination of effective coefficient of initial deflection by the curvature method (for thick plate)

The magnitude and the position of maximum curvature of the standard initial deflection are expressed as follows.

$$(1/\rho)_{max}=-(K\pi/a)^2 w_{0max} \tag{17}$$

(at $x=a/(2K)$ and $x=(2K-1)a/(2K)$)

Curvature of each sinuisoidal wave component of the standard initial deflection at the above position of maximum curvature can be expressed by the following equation.

$$\left(\frac{1}{\rho}\right)_{\max\cdot m}=\begin{cases}-\left(\frac{K\pi}{a}\right)^2\frac{w_{0\max}}{K} & (m=K)\\ -\frac{2w_{0\max}K^2m\pi}{(K^2-m^2)a^2}\sin\frac{m\pi}{K} & (m\neq K)\end{cases} \tag{18}$$

(m takes only odd numbers)

Equation (18) indicates that when the number of half-waves of the buckling mode, K, is odd, the component contributing the most to the maximum curvature (Eq. (17)) of the standard initial deflection is the buckling mode itself. When K is even, it is one higher mode, that is, K+1th mode. First, the case where the buckling mode has odd half-waves is considered. In order that the curvature of the buckling mode expressed by the first equation (for the case of m=K) of Eq. (18) has the same magnitude as that by Eq. (17), the magnitude of the deflection should be increased by K times. If such is the case, the magnitude of the buckling component mode, A_b, is obtained from Eq. (13) as

$$A_b=(w_{0\max}/K)K=w_{0\max} \tag{19}$$

Accordingly, the effective coefficient of initial deflection, expressed by Eq. (11), may be rewritten as follows for thick plate.

$$\zeta=A_0/w_{0\max}=A_b/w_{0\max}=1 \tag{20}$$

Similarly, for the case where the buckling mode has even half-waves, an effective coefficient of initial deflection can be obtained. Although the coefficient in this case is a little less than 1, Eq. (20) may be applied approximately to both cases.

3.3.3 Ranges for application of effective coefficients of initial deflection

Applicable ranges of the effective coefficient of initial deflection, ζ for thin plate and ζ for thick plate, respectively expressed by Eqs. (15), (16) and (20), will be determined for slenderness ratio ξ, using the simple estimating equations, Eqs. (2) to (7), for compressive ultimate strength shown in Chapter 2.

(1) Range for application of effective coefficient of initial deflection for thin plate

As mentioned in 3.2, the behavior of thin plates under thrust may be characterized by the fact that only a single component of initial deflection has become dominant over the others above the elastic buckling load and then development of plastic portion leads to compressive collapse. Accordingly, the compressive ultimate strength, in general, becomes higher than buckling strength. In **Fig. 4**, simple estimating equations for compressive ultimate strength, Eqs. (4) and (5), are illustrated for the case where the breadth for tensile residual stress, b_t, is 0.05b ($2b_t/b=0.1$). In this figure, this characteristic of thin plate is observed in the range of the slenderness ratio, where the solid lines are above the broken one.

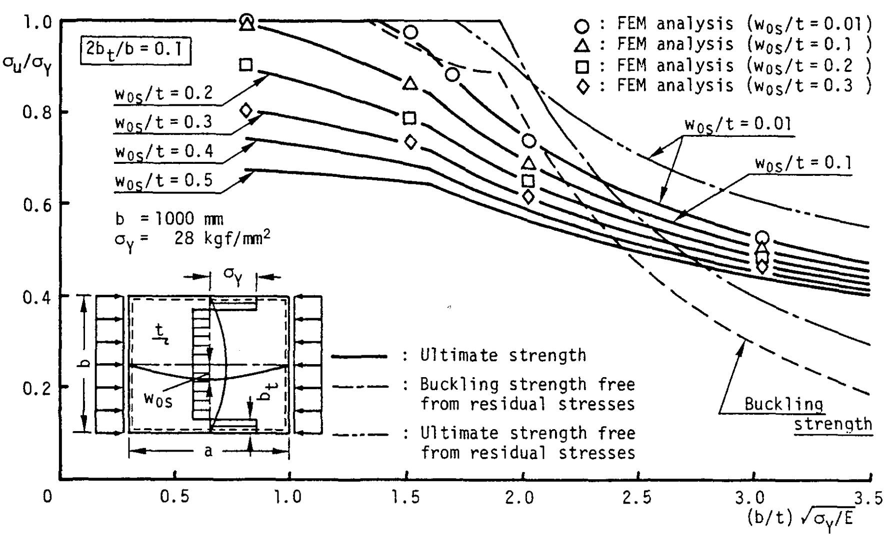

Fig. 4 Minimum compressive ultimate strengths of rectangular plates with uni-modal deflection and residual stresses (In the case of $2b_t/b=0.1$)

In Ref. 11, measured results of initial deflection of actual deck panels of various sizes have been shown. According to the results for the panels of 11mm thickness ($\xi=(b/t)\sqrt{\sigma_Y/E}\fallingdotseq 2.6$), the maximum magnitude of initial deflection is approximately one half of the plate thickness. In case of thin plate, as was shown in 3.3.1, effective initial deflection becomes $1/K$ (K: the number of half-waves of the buckling mode) of the maximum one. Therefore, an application limit of ζ for thin plates may be set at points of intersection of the broken line representing the elastic buckling strength and the solid lines representing the compressive ultimate strength for $w_{os}/t=0.1\sim 0.5$. Depending on the magnitudes of initial deflection (w_{os}/t) and residual stress ($2b_t/b$), the point of intersection varies more or less between slenderness ratio $\xi=2.1\sim 2.6$. Here, $\xi=2.5$ is adopted. Hence ζ for thin plate is applicable for the range where $2.5\leqq\xi=(b/t)\sqrt{\sigma_Y/E}$.

(2) Range for application of effective coefficient of initial deflection for thick plate

In thick plates, plastification extends from the position where the curvature is maximum, and collapse takes place when the average compressive stress increases nearly to the yield stress. Taking this behavior into account, the application limit of ζ for thick plate is set at the slenderness ratio $\xi((b/t)\sqrt{\sigma_Y/E})$, with which the compressive ultimate strength reaches the yield stress, in the following way.

As shown in Ref. 11, the maximum magnitude of initial deflection of a panel with plate thickness being 34.5mm($(b/t)\sqrt{\sigma_Y/E}\fallingdotseq 0.8$) is less than 5% of its plate thickness. Therefore, the application limit is set at the point of intersection of the solid line representing the compressive ultimate strength for $w_{os}/t=0.1$ and the line of $\sigma_u / \sigma_Y = 1.0$, which varies depending on residual stresses more or less. The practical range for application of ζ for thick plate is determined as $\xi=(b/t)\sqrt{\sigma_Y/E}\leqq 1.0$

(3) Effective coefficient of initial deflection (relation with slenderness ratio ξ)

The ranges for application of effective coefficients of initial deflection for thin and thick plates have been determined in terms of the slenderness ratio ξ. For plates of medium thickness whose slenderness ratio is between the above two application limits, the effective coefficient of initial deflection, ζ, is assumed to change linearly with slenderness ratio ξ between ζ for thin plate and ζ for thick plate.

Consequently, the effective coefficients of initial deflection are expressed by the following equations. The equations are also depicted in **Fig. 5**.

$$\zeta=\begin{cases} 1 & (\text{for } \xi\leqq 1.0) \\ 1-(2/3)(1-1/K)(\xi-1) & (\text{for } 1.0\leqq\xi\leqq 2.5) \\ 1/K & (\text{for } 2.5\leqq\xi) \end{cases} \tag{21}$$

where, $\xi=(b/t)\sqrt{\sigma_Y/E}$: slenderness ratio

$$K:\begin{cases} \text{the number of half-waves of the buckling mode (in case the number of buckling mode is odd).} \\ \text{the number of half-waves of the buckling mode + 1 (in case the number of buckling mode is even).} \end{cases}$$

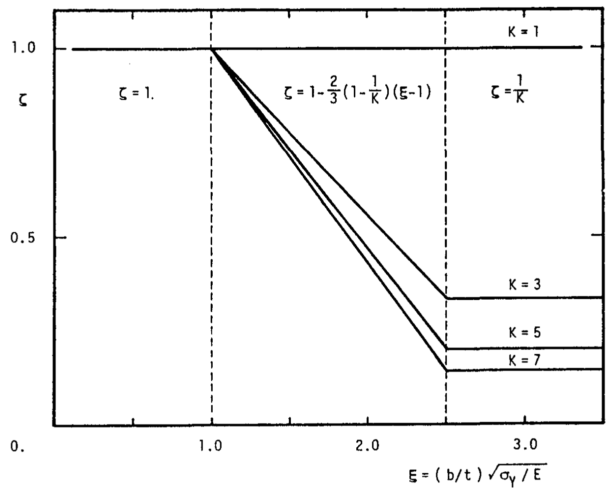

Fig. 5 Effective coefficient of initial deflection

3.4 Procedure of calculation of compressive ultimate strength of a rectangular plate with welding initial imperfections

With the method explained in Ref. 11, it is possible to estimate initial deflection and residual stress produced by welding stiffeners to a rectangular plate. As for initial deflection, effective coefficient of initial deflection discussed in this chapter is used to obtain effective initial deflection. Using this effective initial deflection and residual stress, the compressive ultimate strength of such a rectangular plate as with welding initial imperfections can be calculated by the simple estimating equations of compressive ultimate strength described in Chapter 2. The procedure of these calculations, the simple prediction method, is shown by a flow chart in **Fig. 6**.

4. Applicability of the Simple Prediction Method

In the previous chapters, for deck plate panels in a ship hull, the simple prediction method for the compressive ultimate strength of a rectangular plate with initial imperfections due to welding has been presented with the aid of the effective coefficient of initial deflection. In this chapter, applicability of the method is examined, that is, the compressive ultimate strength of deck plate panels of mild steel and HT steel are estimated by the method,

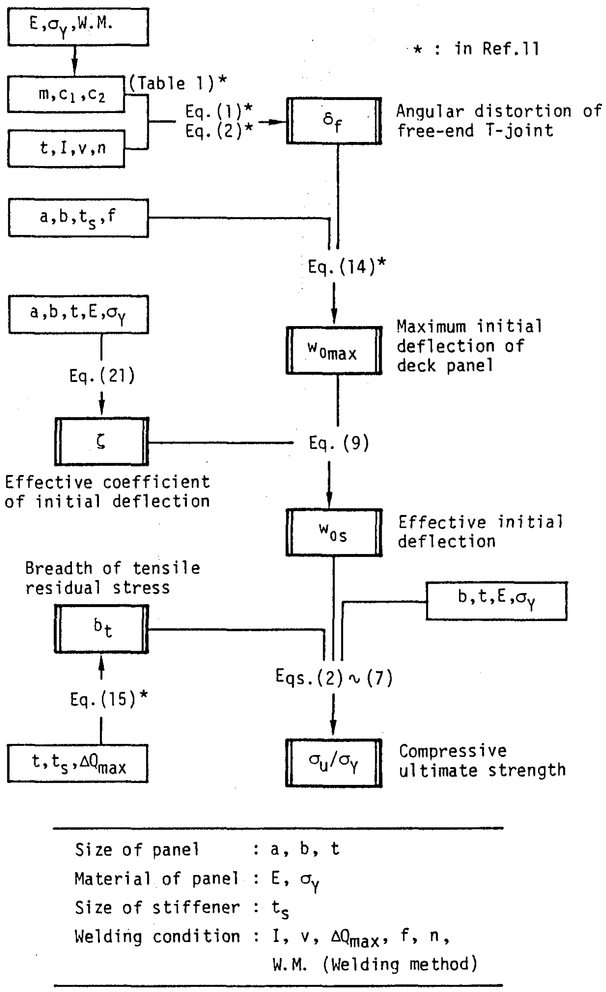

Fig. 6 Flow chart for estimation of initial imperfections and compressive ultimate strength of deck panel

according to the calculating procedure shown in Fig. 6.

4.1 Applicability for deck plate panel of mild steel

First, deck plate panels of mild steel with 28 kgf/mm^2 yield stress will be analyzed.

4.1.1 Analyzed deck panels and their initial imperfections[11)]

The authors, formerly, observed the forms and magnitudes of initial deflections on 21 panels of 4 kinds in a 60,000 ton Bulk Carrier (hereinafter called B.C.) and 12 panels of 2 kinds in a Pure Car Carrier for 5,500 cars load (hereinafter P.C.C.)[8)-10)]. The plate thicknesses were 15, 19 and 34.5mm in B.C. and 8 and 11mm in P.C.C. Among these, the 8 mm thick panels of P.C.C. seemed to have been subjected to straightening, and therefore they are not discussed here. The sizes and welding conditions of the panels treated in this chapter are indicated in **Table 1**.

Using the prediction method shown in Ref. 11, welding initial imperfections of the above mentioned panels of five different sizes can be calculated easily. As a result, the maximum initial deflection w_{0max} and the breadth for tensile residual stress, b_t, (Fig. 1) are obtained for respective five kinds of panels. As for initial deflection, measured values are also available as mentioned above, which scatter though, and their mean values and standard devia-

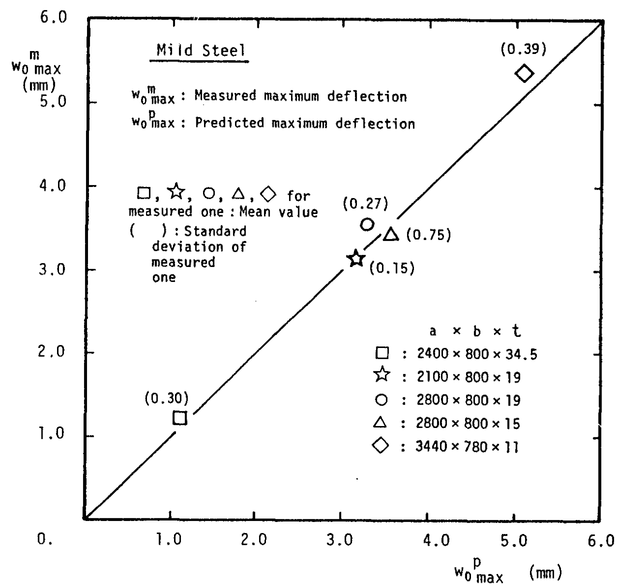

Fig. 7 Comparison between predicted maximum initial deflection of deck panel of mild steel and measured ones

Table 1 Sizes of deck panels and stiffeners, and welding conditions

Kind of Ship	Sizes of Panels and Stiffeners		Number of Deflection-Measured Panels	Conditions of Welding			Leg Length (mm)
	Panel a×b×t (mm)	Long. Stiffener h×ts (mm)		Current (A)	Voltage (V)	Velocity (cm/min)	
Bulk Carrier	2400×800×34.5	400 × 35	12	290 ∼ 300	24 ∼ 46	15	8
	2100×800×19	300 × 28	3				8
	2800×800×19	300 × 29	3				8
	2800×800×15	250 × 19	3				6
Pure Car Carrier	3440×780×11	200×90×9/14 I.A.	6	200 ∿ 250	24 ∿ 46	15	5

tions are calculated. They are compared with predicted ones as shown in **Fig. 7**. It is proved that the prediction method is highly accurate.

4.1.2 Prediction of compressive ultimate strength

The effective coefficients of initial deflection and the simple estimating equations are used to calculate the compressive ultimate strength of the rectangular plate with initial imperfections described in the previous section. Whereas, elastic-plastic large-deflection analysis based on the finite element method (FEM) is conducted to obtain the compressive ultimate strength of the rectangular plate with measured initial deflection of complex deflection modes and estimated residual stress distribution. Adopted in the analysis are three panels for respective sizes (5 different sizes). As for panels of 11mm and 34.5mm in thickness, initial deflection has been measured on six and twelve panels respectively, from which three are chosen at random for each size. Deflection modes of these measured initial deflections and of the standard initial deflections which have the maximum magnitudes predicted by the method presented in Ref. 11 are compared for each size as shown in **Fig. 8**.

For the compressive ultimate strengths of these panels, estimated results by the simple prediction method and analytical results by FEM are compared and shown in **Fig. 9**. As the analytical results, the mean value and standard deviation of three panels are shown for each size. The accuracy of estimation is very high except the case of 34.5mm thick plates. The estimated value for 34.5mm thick plate is $\sigma_u/\sigma_Y=1.0$, whereas the mean value of analytical results is $\overline{\sigma_u/\sigma_Y}=0.924$ showing a little difference. This difference may be attributed to the following reason. In the case of such thick plates, plastification is dominated by bending stress corresponding to curvature rather than deflection. In connection with this, examination of the initial deflection modes shown in Fig. 8 reveals that the maximum curvature of the standard initial deflection mode for 34.5mm thick plate is smaller than the measured ones more or less. This smaller estimation of the maximum curvature leads to predict a higher compressive ultimate strength. This is one of the reason for the difference.

The analytical results on actual panels by FEM indicate that inspite of the considerable variation among initial deflections shown in Fig. 8, the compressive ultimate strengths differ little in each size. As having been mentioned in this report, this may be attributed to that the behavior of plate to collapse is almost same according to the slenderness ratio. This is remarkably recognized in the fact that two kinds of panels of plate thickness being t= 19mm have almost the same compressive ultimate strengths, whose slenderness ratios are the same though

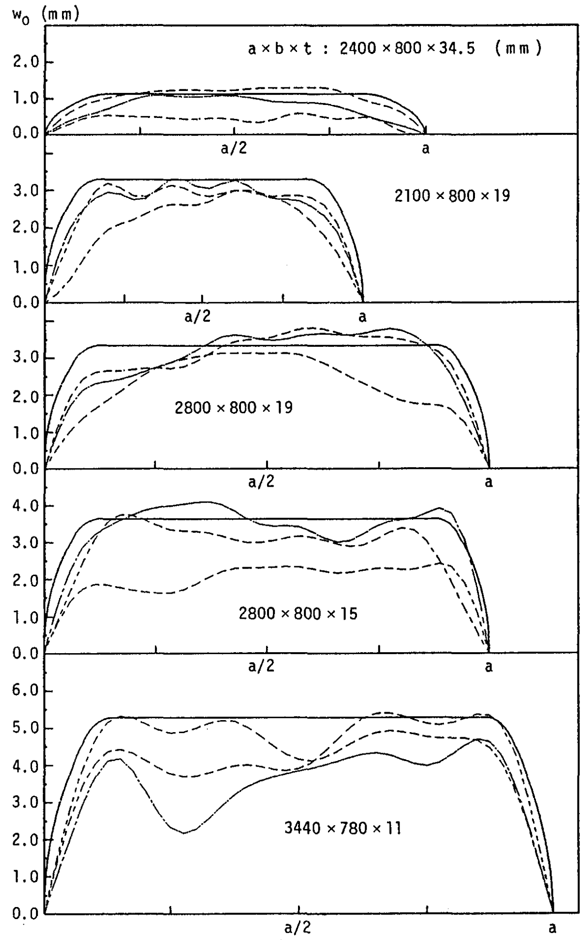

Fig. 8 Comparison between initial deflection of standard one and measured ones along y=b/2

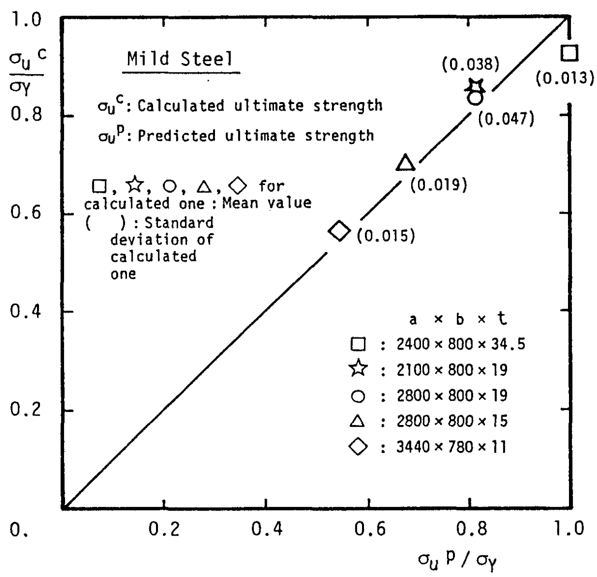

Fig. 9 Comparison between predicted compressive ultimate strength of deck panel of mild steel and calculated ones

the panel lengths are different.

4.2 Applicability for deck plate panel of HT steel

On the assumption that the sizes of deck plate panels and the welding conditions of stiffeners are the same as the mild steel panels shown in Table 1 in the previous section, HT steel (SM53) with 41 kgf/mm^2 yield stress is chosen as the material for analysis.

4.2.1 Initial imperfections

For five kinds of the panels of SM53 steel, maximum initial deflections w_{omax} are estimated by the method presented in Ref. 11 and they are shown in **Table 2**. Only the estimated values are shown since there are no available measured results for deck plates of SM53 steel. The estimated welding initial deflections of deck plates of SM53 steel are smaller than those of mild steel plates. The deflections are approximately 90% of thin plates and 60% of thick plates of mild steel in case of same sizes and welding conditions.

Table 2 Predicted maximum initial deflection of deck panel of HT (SM53) steel

Predicted maximum deflection $w_{0\,max}$ (SM53) (mm)

a × b × t	2400 × 800 × 34.5	2100 × 800 × 19	2800 × 800 × 19	2800 × 800 × 15	3440 × 780 × 11
$w_{0\,max}$	0.69	2.44	2.54	2.80	4.59

The welding residual stresses (the breadth for tensile residual stress, b_t) can be also predicted by the method in Ref. 11. The correction of b_t described in Chapter 2 is not necessary for SM53 steel, since the tensile residual stress reaches the yield stress[11].

4.2.2 Prediction of compressive ultimate strength

Compressive ultimate strengths of the panels of HT steel (SM53) with the above initial imperfections are estimated by the simple method. On the other hand, elastic-plastic large-deflection analysis based on FEM is also conducted as follows: As to the shape of initial deflection, the same ones for the deck panels of mild steel shown in Fig. 8 are used, being decreased in magnitude by the ratio of maximum initial deflections predicted for HT steel panel and mild steel panel. The distribution of residual stresses is, similar to the case of mild steel, assumed as rectangular distribution (see Fig. 1) using the estimated value b_t as the breadth for tensile residual stress. The compressive ultimate strength of the panels of HT steel with such initial imperfections as these is analyzed theoretically based on FEM. Like the case of mild steel, the analysis is conducted on three panels for each size, to obtain the mean values of the compressive ultimate strengths and their standard deviations. The predicted result is compared with this analyzed result and shown in

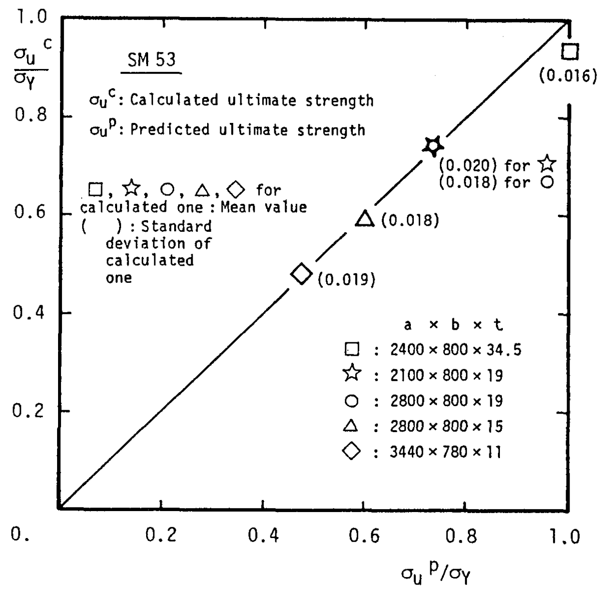

Fig. 10 Comparison between predicted compressive ultimate strength of deck panel of HT (SM53) steel and calculated ones

Fig. 10. Both results correspond very well, though they differ a little in thick plates of 34.5mm in thickness like the case of mild steel.

In the HT steel panel, too, little difference is found in the analytical results by FEM for each size. Comparison of the compressive ultimate strength of the panel of HT steel with that of mild steel shows that the absolute value is larger in HT steel panel for each size, while non-dimensionalyzed compressive ultimate strength by yield stress, σ_u / σ_Y, is almost the same for the panel with t = 34.5mm in plate thickness and becomes remarkably smaller in the panel of HT steel as the plate thickness decreases. This may be attributed to, although the size and welding condition are the same for both steels, the differences in (i) the slenderness ratio, (ii) the magnitude of initial deflection and (iii) decreasing tendency of compressive ultimate strength due to initial deflection, which depends on slenderness ratio. Above all, (i) is considered most influential. That is, slenderness ratio is larger in the panel of HT steel if the size is the same. As is evident in Fig. 4, σ_u / σ_Y does not vary or decreases according as slenderness ratio increases, that is, σ_u / σ_Y is generally inclined to decrease in the panel of HT steel. This fact is very important on determination of HT coefficient which is used to determine the thickness of plates of HT steel from the rules for plates of mild steel.

5. Conclusions

Presented in this study is an accurate method to predict the compressive ultimate strength of deck plate panel of a ship's hull as simply as by hand calculation, using welding initial imperfections estimated by the simple method in Ref. 11.

The main results are as follows:

(1) The effective coefficient of initial deflection was determined rationally as mentioned in the following items, which is used in the calculation of the compressive ultimate strength of a rectangular plate with initial imperfections by the simple estimating equations (see Refs. 7, 9 or 10).

(i) A standard initial deflection mode was proposed, of which longitudinal form is flat in the middle portion and a half of the half-wave of the buckling mode in the vicinities of the both ends.

(ii) Using this standard initial deflection wave, the deflection method was applied for thin plates and the curvature method for thick plates in order to obtain effective coefficients of initial deflection, respectively.

(iii) The ranges for application of respective effective coefficients of initial deflection were determined. For plates of medium thickness, the coefficient was linarly varied between the above coefficients. As a result, effective coefficients of initial deflection can be simply expressed with the number of half-waves of the buckling mode and the slenderness ratio of a panel, as shown by Eq. (21).

(2) Consequently, if sizes of a deck plate panel and stiffeners and welding conditions are specified at the design stage, it is very easy to estimate the compressive ultimate strength of a panel as well as welding initial imperfections. The procedure of prediction is shown in Fig. 6.

(3) For actual deck plate panels of mild steel and HT steel, the compressive ultimate strengths were obtained by the above-mentioned method. The predicted strengths were compared with the result by elastic-plastic large-deflection analysis based on FEM, and the accuracy of the method was proved very high. Furthermore, having examined the compressive ultimate strength obtained for the panels of mild steel and HT steel, it was noted that attention should be paid to determination of HT coefficient.

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References

1) Y. Ueda et al.: Ultimate Strength of Square Plates Subjected to Compression (1st Report) —Effects of Initial Deflection and Welding Residual Stresses—, J. of SNAJ (The Society of Naval Architects of Japan), Vol. 137, 1975, pp. 210-221 (in Japanese).

2) Y. Ueda et al.: Ultimate Strength of Square Plates Subjected to Compression (2nd Report) —Comprehensive Study on Effects of Initial Imperfection—, J. of SNAJ, Vol. 140, 1976, pp. 205-209 (in Japanese).

3) Y. Ueda et al.: Minimum Stiffness Ratio of a Stiffener against Ultimate Strength of a Plate, J. of SNAJ, Vol. 140, 1976, pp. 199-204 (in Japanese).

4) Y. Ueda et al.: Minimum Stiffness Ratio of a Stiffener against Ultimate Strength of a Plate (2nd Report), J. of SNAJ, Vol. 143, 1978, pp. 308-315 (in Japanese).

5) Y. Ueda et al.: Minimum Stiffness Ratio of a Stiffener against Ultimate Strength of a Plate (3rd Report), J. of SNAJ, Vol. 145, 1979, pp. 176-185 (in Japanese).

6) Y. Ueda et al.: Compressive Ultimate Strength of Rectangular Plates with Initial Imperfections due to Welding (1st Report) —Effects of the Shape and Magnitude of Initial Deflection—, J. of SNAJ, Vol. 148, 1980, pp. 222-231 (in Japanese).

7) Y. Ueda et al.: Compressive Ultimate Strength of Rectangular Plates with Initial Imperfections due to Welding (2nd Report) —Effects of Initial Deflection and Welding Residual Stresses—, J. of SNAJ, Vol. 149, 1981, pp. 306-313 (in Japanese).

8) Y. Ueda et al.: Compressive Ultimate Strength of Rectangular Plates with Initial Imperfections due to Welding (3rd Report) — Prediction Method of Compressive Ultimate Strength—, J. of SNAJ, Vol. 154, 1983, pp. 361-371 (in Japanese).

9) Y. Ueda and T. Yao: Influence of Shape of Initial Deflection upon Compressive Ultimate Strength of Long Rectangular Plates, Trans. of JWRI, Vol. 13, No. 1, 1984, pp. 87-103.

10) Y. Ueda and T. Yao: The Influence of Complex Initial Deflection Modes on the Behavior and Ultimate Strength of Rectangular Plates in Compression, J. of Constructional Steel Research, Vol. 5, No. 4, 1985, pp. 265-302.

11) Y. Ueda, K. Nakacho and S. Moriyama: Simple Prediction Methods for Welding Deflection and Residual Stress of Stiffened Panels, Trans. of JWRI, Vol. 15, No. 2, 1987, pp. 197-204, and J. of SNAJ, Vol. 159, 1986, pp. 282-294 (in Japanese).