Title	Buckling and Ultimate Strength Interactions of Plates and Stiffened Plates under Combined Loads (1st Report) : Inplane Biaxial and Shearing Forces(Mechanics, Strength & Structural Design)
Author(s)	Ueda, Yukio; Rashed, S. M. H.; Paik, J. K.
Citation	Transactions of JWRI. 1989, 18(2), p. 267-280
Version Type	VoR
URL	https://doi.org/10.18910/4168
rights	
Note	

Buckling and Ultimate Strength Interactions of Plates and Stiffened Plates under Combined Loads (1st Report)†

— Inplane Biaxial and Shearing Forces —

Yukio UEDA*, S. M. H. RASHED and J. K. PAIK*****

Abstract

The main portion of a ship structure is usually composed of stiffened plates. Between girders and floors, stiffeners are furnished to plates in one direction, usually the longitudinal direction. Under various loads applied to a ship, such as those due to waves, these stiffened plates are subjected to combined inplane and lateral loads.

In this report, buckling, ultimate and fully plastic strength interaction relationships of plates and unidirectionally stiffened plates subjected to inplane biaxial and shearing forces, are derived and expressed in explicit forms based on the result of theoretical investigation of the nonlinear behavior of plates and stiffened plates.

The accuracy of the interaction relationships is confirmed comparing with the result of analysis by other methods.

With the aid of these interaction relations, bucking load and ultimate strength, or fully plastic strength of this type of stiffened plates subjected to inplane loads may be predicted by hand calculation.

KEY WORDS : (Plates) (Stiffened Plates) (Combined Loads) (Buckling) (Ultimate Strength) (Interaction Relationships)

1. Introduction

The hull of a ship is fundamentally regarded as a thin walled box-girder whose major portion is usually composed of stiffened plates. Stiffeners, furnished to plates, are supported by girders and bulkheads. Under various loads applied to a ship, such as those due to cargo and waves, the hull is subjected to longitudinal shear, bending and torsion. Locally, each portion of the structure is subjected to lateral loads, axial forces, bending moments and shearing forces.

Simple methods to accurately evaluate buckling, plasticity and ultimate strength of components of such complicated structures are very useful to examine their safety.

Plates and stiffened plates, as main components of such structures, are considered. External loads acting on them may be divided into two groups.

(1) Inplane loads, composed of axial forces (compression or tension) in two normal directions, bending moments and shearing forces.

(2) Distributed lateral loads, caused by water pressure or pressure due to weather, liquid or bulk cargo.

In this report, only implane loads are considered. The influence of distributed lateral loads is reported in a following paper.

As rectangular plate and stiffened plate panels are small compared with the overall ship structure, inplane bending moments acting on individual panels are insignificant and may be neglected.

Therefore, rectangular plate and stiffened plate panels are considered to be subjected to uniformly distributed inplane axial forces in two normal directions and inplane shearing forces.

Development of buckling and ultimate strength interaction relationships in the form of equations or graphs for plates and stiffened plates has attracted a lot of international interest for a long time. Works available in the litrature may be divided into two classes. The first is a presentation of results obtained by some numerical techniques taking account of geometric and material nonlinearities. Example of such results may be found in Refs 1,2 and 3. The other class is rather analytical, in which solutions are obtained based on suitable failure criteria. Examples may be found in Refs 4 and 5. A review of available material requires a paper devoted to this purpose and it is not intended to present such a review in this paper. The available information, however, does not cover all the practical range with sufficient accuracy and confidence, in particular with regard to inplane shear effects.

† Received on October 31, 1989

* Professor

** Manager, Advanced Technology Department, Century Research Center Corp., Osaka, Japan

*** Formerly, Graduate Student, Osaka University
Presently, Associate Professor, Pusan National University, Korea

Transactions of JWRI is published by Welding Research Institute of Osaka University, Ibaraki, Osaka 567, Japan

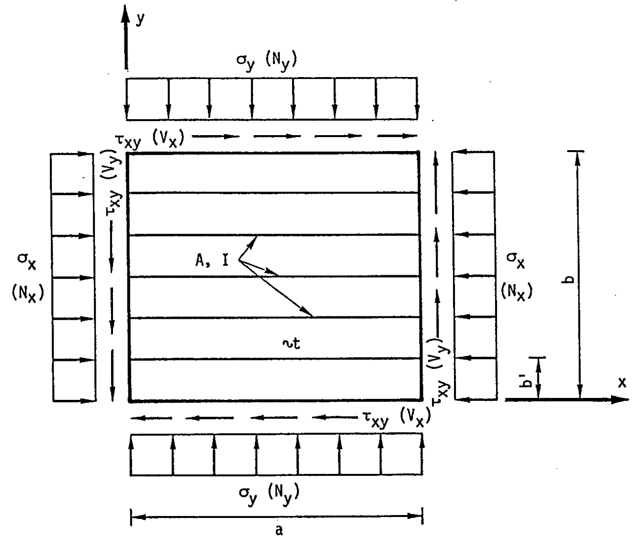

Fig. 1 Stiffened plate and applied loads

In this study, buckling strength, ultimate strength after buckling and full plastic strength under combined inplane biaxial forces and inplane shear are investigated. Strength functions are derived and expressed in terms of applied forces. Comparisons with published results and results of analysis by the finite element method are presented.

2. Nonlinear behavior of stiffened plates

2.1 Object for analysis

A stiffened plate is consiered as a part of a large plate structure such as a deck or a side shell of a ship as shown in **Fig.1.** Length, breadth and thickness of this stiffened plate is $a \times b \times t$, and plate bending stiffness $D = Et^3/12(1-\nu^2)$, where E is the modulus of elasticity and ν is Poisson's ratio.

N similar stiffeners in the x direction are attached to the plate at equal intervals. The sectional area and the moment of inertia of each stiffener is A and I (I includes the effective breadth of the plating associated with the stiffener). It is assumed that stiffeners do not buckle prior to buckling of plates between adjacent stiffeners (stiffeners are usually designed to satisfy this condition). Stiffened plate panels are assumed to be simply supported, uniform inplane compressive or tensile displacements are applied in the two axial directions x and y (i.e. edges remain straight in the plane of the panel) together with uniform shear stress.

In this paper, this loading condition is referred to as a combined load of biaxial forces and inplane shear.

2.2. Nonlinear behavior of stiffened plates

When a stiffened plate is subjected to a combined load as mentioned above, uniform biaxial normal stresses (σ_x and σ_y) and uniform shear stress (τ_{xy}) act on the plate, while only a uniform normal stress in x direction, σ_{xs} acts on the stiffeners. Increasing this combined load, the stiffened plate behaves in different ways according to its dimensions and the combination of the applied load.

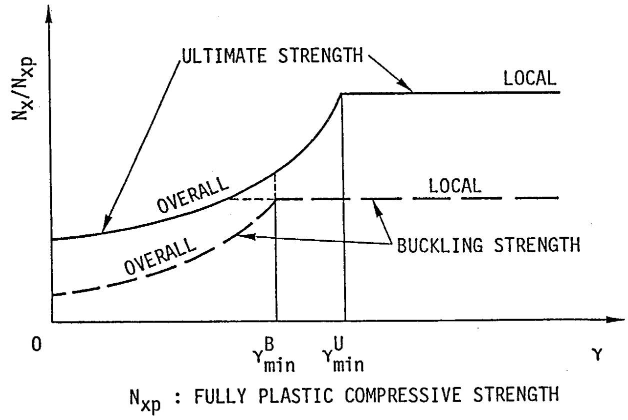

Fig. 2 Relation of buckling strength and ultimate strength of axially compressed stiffened plates to the stiffness ratio of stiffeners

When the stiffened plate has a sufficiently low out-of-plane bending stiffness, it buckles in one of two buckling modes. One is overall buckling, and the other is local buckling. This is controlled by the relative stiffness ratio γ of the stiffeners to the plate ($\gamma = EI/b'D$). When γ is smaller than γ^B_{min} [6], the stiffened plate buckles in an overall mode. The overall buckling strength increases together with γ.

When γ is greater than γ^B_{min} [6], local buckling takes place instead of overall buckling and the buckling strength reaches an upper limit regardless of higher values of γ. This γ^B_{min} [6] is given as the point of intersection between the two buckling curves representing the two buckling modes as shown in **Fig.2.** After buckling, the stiffened plate may support further increment of the load, though with lower inplane stiffness, untill it reaches its ultimate strength after plasticity prevails.

The collapse made at ultimate strength varies according to the value of γ. With compression in the x direction, one of the following modes is produced.

a) When γ is smaller than γ^B_{min}, the stiffened plate buckles in an overall mode, followed by overall collapse produced with the same mode of deformation as that at buckling.

b) When γ is slightly greater than γ^B_{min}, the stiffened plate buckles locally. As its effective stiffness decreases due to buckling, overall collapse may occur either due to spread of plasticity in the stiffeners or due to overall buckling of stiffeners together with the associated effective portions of the buckled plates.

In these two cases, a) and b), ultimate strength increases together with γ.

c) When γ is greater than γ^{B}_{min} [6], stiffeners are strong enough to prevent overall collapse after local buckling. The stiffened plate reaches its ultimate strength by local collapse of plate panels between stiffeners followed by buckling or plastic collapse of stiffeners. In this case ultimate strength does not significantly increase with γ. γ^{u}_{min} is 30 to 50 % greater than γ^{B}_{min}.

Compression in the y direction causes one of the following two modes.

a) When γ is smaller than γ^{B}_{min}, the stiffened plate buckles in an overall mode, then, it collapses in the same mode.

b) When γ is greater than γ^{B}_{min}, the stiffened plate buckles locally, then, it collapses locally. In this case, $\gamma^{B}_{min}=\gamma^{u}_{min}$. Values of γ^{B}_{min} and γ^{u}_{min} depends on the properties of the stiffened plate as well as the ratio of load components.

Therefore, the behavior of a stiffened plate may be classified into 4 calsses depending on the values of γ, γ^{B}_{min} and γ^{u}_{min} as follows.

(1) $\gamma<\gamma^{B}_{min}$ (ultimate strength condition 1)
The stiffened plate buckles and collapses in an overall mode.

(2) $\gamma^{B}_{min}<\gamma<^{u}_{min}$ (ultimate strength condition 2)
Plates between stiffeners buckle locally; Utimate strength is reached by plastification or buckling of stiffeners.

(3) $\gamma>\gamma^{u}_{min}$ (ultimate strength condition 3)
After plates between stiffeners buckle locally, they reach their ultimate strength. Buckling or plasticity of stiffeners follows leading to collapse.

(4) When the stiffened plate has sufficient stiffness such that buckling does not occur untill the fully-plastic strength is reached under the specified loading condition. (Ultimate strength condition 4)

3. Buckling, ultimate strength, and full plastic strength interaction relationships of a rectangular plate subjected to uniform axial stresses in the two principal perpendicular directions and a uniform shearing stress.

The buckling and post-buckling behavior of a rectangular plate between stiffeners or girders is discussed in the following section. A rectangular plate subjected to uniform normal stresses in the two principal perpendicular directions and a uniform shearing stress is considered. Buckling, ultimate and fully plastic strengths are theoretically studied and expressed in explicit interation relationsps.

3.1 Buckling interaction

Buckling interaction relationships of a plate under a combined load of uniform normal stresses σ_x and σ_y and shearing stress τ_{xy} may be expressed by the following equations based on analytical solutions[7].

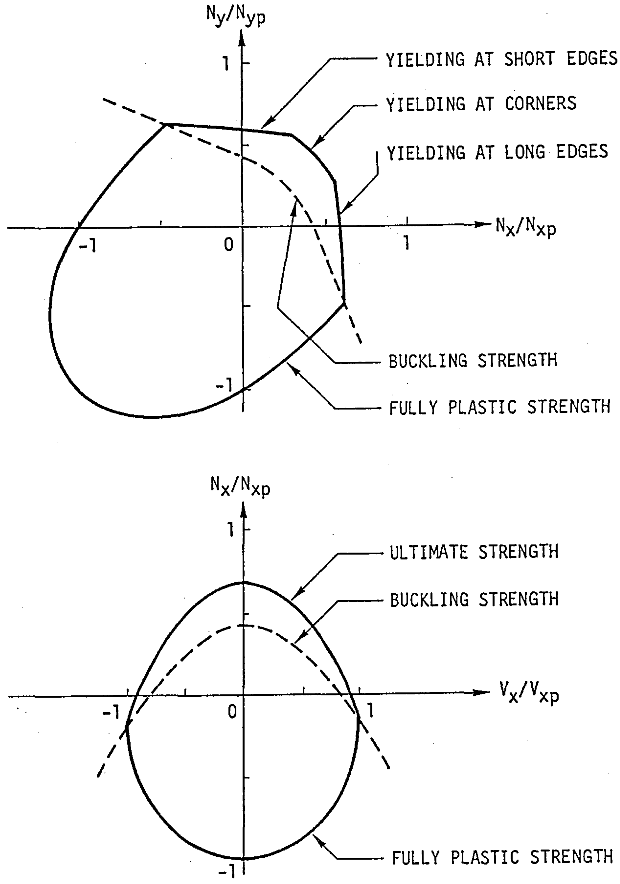

Fig. 3 Buckling strength, ultimate strength and fully plastic strength of a rectangular plate

(1) When σ_x is tensile and σ_y is compressive, $(\sigma_x<0, \sigma_y>0)$,

$$\frac{(m^2+\beta^2)^2}{m^2(1+\beta^2)^2}\frac{\sigma_x}{\sigma_{xcr}}+\frac{\sigma_y}{\sigma_{ycr}}+\left(\frac{\tau_{xy}}{\tau_{xycr}}\right)^2=1 \tag{3.1.a}$$

(2) When σ_x is compressive and σ_y is tensile, $(\sigma_x>0, \sigma_y<0)$,

$$\frac{\sigma_x}{\sigma_{xcr}}+\frac{(1+\beta^2)^2}{(m^2+\beta^2)^2}\frac{\sigma_y}{\sigma_{ycr}}+\left(\frac{\tau_{xy}}{\tau_{xycr}}\right)^2=1 \tag{3.1.b}$$

(3) When σ_x and σ_y are compressive, $(\sigma_x>0, \sigma_y>0)$,

$$\left[\frac{\sigma_x/\sigma_{xcr}}{1-(\tau_{xy}/\tau_{xycr})^2}\right]^{\alpha_1}+\left[\frac{\sigma_y/\sigma_{ycr}}{1-(\tau_{xy}/\tau_{xycr})^2}\right]^{\alpha_2}=1 \tag{3.1.c}$$

Where, for $1/\sqrt{2}\leq\beta\leq\sqrt{2}$, $\alpha_1=\alpha_2=1$,
for $\beta>\sqrt{2}$,

$\alpha_1=0.0293\beta^3-0.0364\beta^2+1.5854\beta-1.0596$
$\alpha_2=0.0049\beta^3-0.1183\beta^2+0.6153\beta+0.8522$

σ_{xcr}, σ_{ycr} and τ_{xycr} are the buckling stresses when each stress acts alone on the plate, $\beta=\alpha(\mathrm{n}1)/\mathrm{b}=\mathrm{a}/\mathrm{b}$: aspect ratio of a plate between stiffeners, m is the number of half waves of buckling when a plate of the aspect ratio β buckles under compression in the x direction only.

Here, when a plate is subjected to compression in both x and y directions, the buckling interation relation may be expressed by several linear interaction equations each for a certain region depending on β and the ratio of σ_x to σ_y. Having many equations is, however, inconvenient. Eq.(3.1.c) is continuous and yields a good approximation for the entire region.

Adopting these equations, a buckling interaction function Γ_B may be expressed in the following equations as shown in **Fig.3.**

(1) When N_x is tensile, and N_y is compressive. ($N_x<0$, $N_y>0$)

$$\Gamma_B=\frac{(m^2+\beta^2)^2}{m^2(1+\beta^2)^2}\frac{N_x}{N_{xcr}}+\frac{N_y}{N_{ycr}}+\left(\frac{V_x}{V_{xcr}}\right)^2-1 \qquad (3.2.a)$$

(2) When N_xx is compressive, and N_y is tensile. ($N_x>0$, $N_y<0$)

$$\Gamma_B=\frac{N_x}{N_{xcr}}+\frac{(1+\beta^2)^2}{(m^2+\beta^2)^2}\frac{N_y}{N_{ycr}}+\left(\frac{V_x}{V_{xcr}}\right)^2-1 \qquad (3.2.b)$$

(3) When N_x and N_y are compressive. ($N_x>0$, $N_y>0$)

$$\Gamma_B=\left[\frac{N_x/N_{xcr}}{1-(V_x/V_{xcr})^2}\right]^{\alpha 1}+\left[\frac{N_y/N_{ycr}}{1-(V_x/V_{xcr})^2}\right]^{\alpha 2}-1 \qquad (3.2.c)$$

Where, N_x, N_{xcr}, N_y, N_{ycr}, V_x, V_{xcr} are obtained by integrating σ_x, σ_{xcr}, σ_y, σ_{ycr}, τ_{xy}, τ_{xycr} over the cross sectional area of a plate between stiffeners (b't or at).

It is to be noted that when the four sides of a rectangular plate are equally subjected to shearing stress τ_{xy}, the shearing forces V_x and V_y in x and y directions are proportional to the lengths of the sides, i.e., $V_x=\mathrm{a}t\tau_{xy}$, $V_y=\mathrm{b}'t\tau_{xy}$ and $V_x/V_y=\mathrm{a}/\mathrm{b}'$.

When Γ_B is smaller than zero, it indicates that the plate has not buckled. Buckling condition is

$$\Gamma_B=0 \qquad (3.3)$$

$\Gamma_B>0$ indicates that the plate has buckled.

3.2 Ultimate strength interaction relationships and stress coefficients

3.2.1. Ultimate strength interaction relationships

When a plate buckles under two axial forces in x and y directions, normal stress distribution along a half buckling wave becomes as shown in **Fig.4.** This stress distribution is developed repeatedly along each half buckling wave length of the plate. Under this kind of stress distribution, the ultimate strength of the plate is assumed to be reached when the resultant stress satisfies the yield condtion at the four corners or at the edges and at the middle of each half buckling wave. This ultimate strength may be obtained as follows. First, denoting the uniaxial yield stress by σ_0, Mises' yield condition may be expressed as follows for the case where a shear stress τ_{xy} and normal stresses σ_x and σ_y act simultaneously,

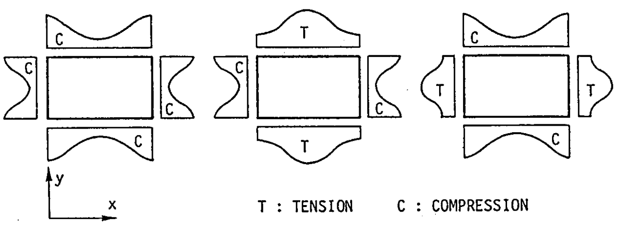

Fig. 4 Stress distribution in a buckled plate

$$\sigma_0^2=\sigma_x^2+\sigma_y^2-\sigma_x\sigma_y+3\tau_{xy}^2 \qquad (3.4)$$

Introducing an effective yield stress, σ_{0v}, $\sigma_{0v}=\sqrt{\sigma_0^2-3\tau_{xy}^2}$ the above yield condition may be rewritten as

$$\sigma_{0v}^2=\sigma_x^2+\sigma_y^2-\sigma_x\sigma_y \qquad (3.5)$$

Here, the normal stresses at each of the above mentioned locations may be expressed in terms of the axial forces, N_x and N_y, and shearing force V_x as follows.

$$\begin{Bmatrix}\sigma_x)_{y=0}\\ \sigma_x)_{y=b'/2}\\ \sigma_y)_{x=0}\\ \sigma_y)_{x=a'/2}\\ \tau_{xy}\end{Bmatrix}=\frac{1}{t}\begin{bmatrix}\frac{1}{b'}(1+\alpha_{\mathrm{xmax}}) & 0 & 0\\ \frac{1}{b'}(1+\alpha_{\mathrm{xmin}}) & 0 & 0\\ 0 & \frac{1}{a}(1+\alpha_{\mathrm{ymax}}) & 0\\ 0 & \frac{1}{a}(1+\alpha_{\mathrm{ymin}}) & 0\\ 0 & 0 & \frac{1}{b'}\end{bmatrix}\begin{Bmatrix}N_x\\ N_y\\ V_x\end{Bmatrix} \qquad (3.6)$$

where

$\sigma_x)_{y=0}$, $\sigma_y)_{x=0}$: the stresses at a corner of half buckling wave,

$\sigma_x)_{y=b'/2}$, $\sigma_y)_{x=a'/2}$: the stresses at the center of a half buckling wave,

σ_{xav}, σ_{yav} : the average values of the stresses perpendicular the sides,

a' : the length of one half buckling wave.

α_x and α_y in the above equation are stress coefficients which express the deviation of the stresses σ_x and σ_y from the mean stresses σ_{xav} and σ_{yav} respectively, due to buckling. Yielding locations vary according to the loading conditions with different values of α_x and α_y as follows,

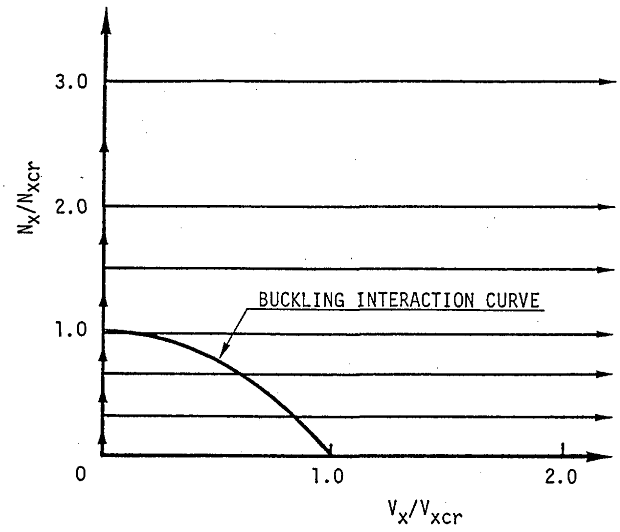

Fig. 5(a) Loading histories

a) In the case of yielding at the corner : α_{xmax}, α_{ymax}

b) In the case of yielding at sides parellel to the x axis (Fig.1) : α_{xmax}, α_{ymin}

c) In the case of yielding at sides parellel to the y axis (Fig.1) : α_{xmin}, α_{ymax}

At the moment when a rectangular plate buckles, the values of α_x and α_y are 0.0. Their values change according to the value of the load and are evaluated in the next section.

Denoting the post-buckling effective width by b_e' and a_e' the relationship between the effective widths and the above mentioned stress coefficients may be expressed as follows.

$$\left.\begin{aligned} b_e' &= b'/(1+\alpha_{x\max}) \\ a_e &= a/(1+\alpha_{y\max}) \end{aligned}\right\} \tag{3.7}$$

dividing Eq.(3.5) by σ_0^2

$$\left(\frac{\sigma_x}{\sigma_0}\right)^2+\left(\frac{\sigma_y}{\sigma_0}\right)^2-\frac{\sigma_x\sigma_y}{\sigma_0^2}=\frac{\sigma_{0v}^2}{\sigma_0^2} \tag{3.8}$$

The ultimate strength interaction relationship is obtained by substituting Eq.(3.6) into Eq.(3.8)

$$\left[\frac{N_x}{N_{xp}}(1+\alpha_x)\right]^2+\left[\frac{N_y}{N_{yp}}(1+\alpha_y)\right]^2 -\frac{N_xN_y}{N_{xp}N_{yp}}(1+\alpha_x)(1+\alpha_y)=1-\left(\frac{V_x}{V_{xp}}\right)^2 \tag{3.9}$$

Where, $N_{xp}=b't\sigma_0$, $N_{yp}=at\sigma_0$, $V_{xp}=at\tau_0$, $\tau_0=\sigma_0/\sqrt{3}$

An ultimate strength function Γ_u of the plate, may be expressed as follows,

$$\Gamma_u=\left[\frac{N_x}{N_{xp}}(1+\alpha_x)\right]^2+\left[\frac{N_y}{N_{yp}}(1+\alpha_y)\right]^2 -\frac{N_xN_y}{N_{xy}N_{yp}}(1+\alpha_x)(1+\alpha_y)+\left(\frac{V_x}{V_{xp}}\right)^2-1 \tag{3.10}$$

and the ultimate strength condition is expressed as,

$$\Gamma_u=0 \tag{3.11}$$

3.2.2 Stress coefficients

The stress coefficients, $\alpha_{x\max}$, $\alpha_{x\min}$, $\alpha_{y\max}$, and $\alpha_{y\min}$ which have been defined in the preceding section are evaluated as follows.

When a rectangular plate has buckled under two axial forces in x and y directions, maximum stresses are developed at the corners of, and the minimum stress at the edges at the middle of a half buckling wave, **(Fig.4).** These maximum and minimum stresses may be analytically evaluated and the stress coefficients may be expressed as follows.

$$\left.\begin{aligned} \alpha^*_{x\max} &= \eta_1+\eta_2\frac{b'}{a}\frac{N_y}{N_x}-\frac{\pi^2m^2D}{a^2}\eta_3\frac{b'}{N_x} \\ \alpha^*_{x\min} &= -\alpha^*_{x\max} \\ \alpha^*_{y\max} &= \eta_2\frac{a}{b'}\frac{N_x}{N_y}+\frac{a^4}{m^4b'^4}\eta_1-\frac{\pi^2D}{b'^2}\eta_3\frac{a}{N_y} \\ \alpha^*_{y\min} &= -\alpha^*_{y\max} \end{aligned}\right\} \tag{3.12}$$

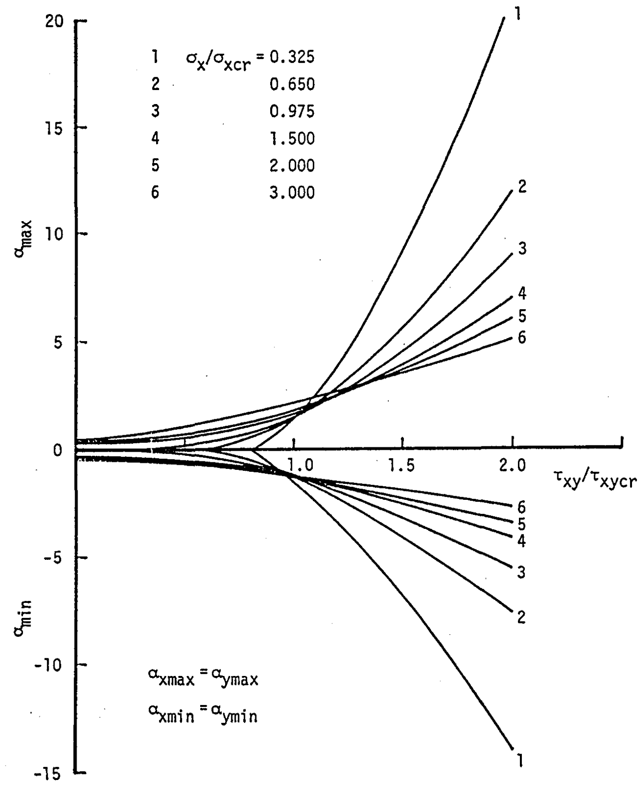

Fig. 5(b) Effect of shear stress on stress coefficients

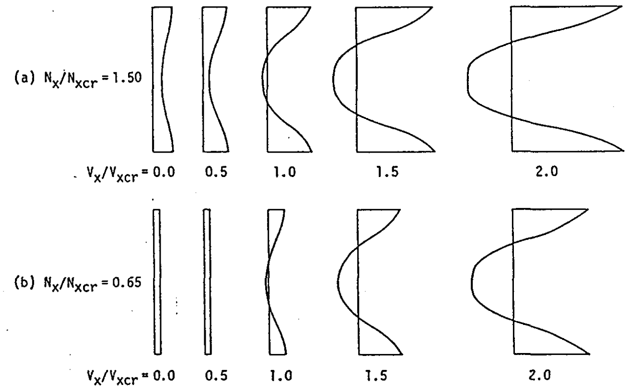

Fig. 6 Axial stress distribution

$$\eta_1=\frac{2m^4b'^4}{a^4+m^4b'^4},\ \eta_2=\frac{2m^2a^2b'^2}{a^4+m^4b'^4},\ \eta_3=\frac{2(a^2+m^2b'^2)^2}{a^4+m^4b'^4}$$

m in the above equations is the number of half buckling waves which depend on the aspect ratio of the plate, $\beta=a/b'$, and the ratio of axial forces, N_y/N_x. m is evaluated as the minimum integer satisfying the following equation.

$$\beta\leq[-P+(P^2+4Q)^{1/2}]^{1/2}/2 \tag{3.13}$$

where,

$P=c[m^4-(m+1)^4]/[2(m-2mc-c)+1]$

$Q=m^2(m+1)^2(2m+1)/[2(m-2mc-c)+1]$

$c=(N_y/N_x)(b'/a)$

In the above equations, when N_x or N_y is zero, the corresponding stress coefficient, α_x or α_y, becomes infinite. Actually, when the average stress in one direction is even zero, finite values of stresses in this direction are produced in the plate due to the constraint of the edges. The stress may be evaluated by the following equations.

$$\left.\begin{aligned}
\sigma_x{}^*)_{y=0}&=(1+\eta_1)\frac{N_x}{b't}+\eta_2\frac{N_y}{at}-\frac{\pi^2m^2D}{a^2t}\eta_3\\
\sigma_x{}^*)_{y=b'/2}&=2\frac{N_x}{b't}-\sigma_x{}^*)_{y=0}\\
\sigma_y{}^*)_{x=0}&=\eta_2\frac{N_x}{b't}+\left(1+\frac{a_4}{m^4b'^4}\eta_1\right)\frac{N_y}{at}-\frac{\pi^2D}{b'^2t}\eta_3\\
\sigma_y{}^*)_{x=a/2}&=2\frac{N_y}{at}-\sigma_y{}^*)_{x=0}
\end{aligned}\right\} \tag{3.14}$$

In the actual analysis, stresses are obtained as the product of the stress coefficients and average stresses. In order to avoid numerical troubles when N_x or N_y equals to zero, infinitesimally small values of N_x or N_y may be assumed instead of zero, such that stress coefficients of finite values may be evaluated.

3.2.3. Effect of shearing force on stress coefficients

In the following, the effect of shearing force on stress coefficients is numerically examined. A parametric study is carried out with a square plate. Loading histories of axial compression in one direction and shearing force are plotted in **Fig.5(a).**

At first, axial compression is applied. Then, keeping the compression at a constant value, N*, shearing force is applied. Six different levels of N* are considered and the stress coefficients are calculated using the incremental Galarkin's method[8]. Results are shown in **Fig.5(b).** The effect of shearing force may be classified into the following two types, according to the relative value of axial compression, N_x to the buckling compression, N_{xcr}.

a) When $N_x>N_{xcr}$, (curves 4 to 6 in Fig.5(b))

When a rectangular plate is subjected only to an increasing axial compression N_x, it buckles when $N_x=N_{xcr}$ and stress coefficients change as expressed by Eq.(3.12) ($N_y=0$).

Next, keeping N_x constant and applying an increasing shearing force, the normal stress σ_x increases in the vicinities of edges y=0 and y=b , and decreases in the middle as shown in **Fig.6(a).** The stress coefficients change gradually with the change of shear stress as shown in Fig.5(b).

b) When N_x is smaller than N_{xcr}. (curves 1 to 3 in Fig.5(b))

When a rectangular plate is subjected to an axial compression, N_x smaller than N_{xcr}, it does not buckle. However, keeping the axial compression constant and applying an increasing shearing force, the plate buckles at a certain value of shear stress. As the shear stress continues to increase, normal stress σ_x increases near the edges y=0 and y=b and decreases in the middle as shown in **Fig.6(b).** The stress coefficients change gradually as shown in Fig.5(b).

Changes of the stress coefficients (curves 1 to 6 in Fig.5(b)) may be accurately expressed by the following equations.

$$\left.\begin{aligned}
\alpha_{x\max}&=1.62\frac{N_{xcr}}{N_x}v^{2.4}+\alpha^*_{x\max}(f(V)+1)\\
\alpha_{x\min}&=-1.3\frac{N_{xcr}}{N_x}v^{2.1}+\alpha^*_{x\min}(0.3v+1)
\end{aligned}\right\} \tag{3.15}$$

where,

$\alpha^*_{x\max}\leq0 \qquad f(V)=0.62v$

$\alpha^*_{x\max}>0,\ v\leq1 \qquad f(V)=1.3v^{1.5}$

$\alpha^*_{x\max}>0,\ v>1 \qquad f(V)=1.3v$

$v=|V_x|/V_{xcr}$

The case mentioned above is a rectangular plate subjected to axial force in one direction. When the plate is subjected to compression in two directions, shearing force is assumed to affect stress coefficients in x direction

(α_{xmax} and α_{xmin}) and y direction (α_{ymax} and α_{ymin}) in the same way as under compression in one direction. The stress coefficients may then be expressed by the following equations.

$$\left.\begin{aligned} \alpha_{xmax} &= 1.62\frac{N_{xcr}}{N_x}v^{2.4}+\alpha^*_{xmax}(f(V)+1) \\ \alpha_{min} &= -1.3\frac{N_{xcr}}{N_x}v^{2.1}+\alpha^*_{xmin}(0.3v+1) \\ \alpha_{ymaxx} &= 1.62\frac{N_{ycr}}{N_y}v^{2.4}+\alpha^*_{ymax}(g(V)+1) \\ \alpha_{ymin} &= -1.3\frac{N_{ycr}}{N_y}v^{2.1}+\alpha^*_{ymin}(0.3v+1) \end{aligned}\right\} \quad (3.16)$$

where,

$\alpha^*_{xmax}\leq 0$	$f(V)=0.62v$
$\alpha^*_{xmax}>0,\ v\leq 1$	$f(V)=1.3v^{1.5}$
$\alpha^*_{xmax}>0,\ v>1$	$f(V)=1.3v$
$\alpha^*_{ymax}\leq 0$	$g(V)=0.62v$
$\alpha'^*_{ymax}>0,\ v\leq 1$	$g(V)=1.35v^{1.5}$
$\alpha^*_{ymax}>0,\ v>1$	$g(V)=1.3v$

$v=|Vx|/V_{xcr}$

Substituting these stress coefficients into the ultimate strength interaction function, Eq.(3.11), the ultimate strength interaction relationship of a plate are obtained as plotted in Fig.3.

3.3 Full plastic strength interaction

When a stiffened plate is stiff enough to prevent both local and over-all buckling, it reaches its full plastic strength. Normal stresses, σ_x and σ_y, and shear stress τ_{xy} are regarded to be distributed uniformly in the flat plate panels. Mises' yield condition, Eq.(3.4), may be written as follows.

$$\left(\frac{\sigma_x}{\sigma_0}\right)^2+\left(\frac{\sigma_y}{\sigma_0}\right)^2-\frac{\sigma_x\sigma_y}{\sigma_0{}^2}+\left(\frac{\tau}{\tau_0}\right)^2=1 \quad (3.17)$$

where, $\tau_0=\sigma_0/\sqrt{3}$

Rewriting the above equation in terms of the axial forces and shearing force, the full plastic strength interaction function of a plate Γ_p is obtained in the following form. (Fig.3).

$$\Gamma_p=\left(\frac{N_x}{N_{xp}}\right)^2+\left(\frac{N_y}{N_{yp}}\right)^2-\frac{N_xN_y}{N_{xp}N_{yp}}+\left(\frac{V_x}{V_{xp}}\right)^2-1 \quad (3.18)$$

The full plastic strength condition is expressed as,

$$\Gamma_p=0 \quad (3.19)$$

4. Buckling, ultimate strength and full plastic strength interaction relationship of a stiffened plate subjected to axial forces and shearing force

In this paper, as mentioned in 2.2, the behavior of a stiffened plate is classified into four types according to the value of the relative stiffness ratio γ of the stiffener to the plate. In this section, buckling, ultimate strength, and full plastic strength interaction relationships of a stiffened plate are derived for these four types of behavior.

4.1 Over-all buckling followed by overall collapse. ($\gamma\leq\gamma^B_{min}$)

A stiffened plate in this paper is considered to be stiffened by many stiffeners. When a stiffened of this type plate buckles in an overall mode, its behavior may be approximated as that of an orthotropic plate. Therefore, in this section, a stiffened plate is dealt with as an equivalent orthotropic plate.

In the analysis, the properties of the equivalent orthotropic plate may be taken as follows,

$$\left.\begin{aligned} & E_x=E\left(1+\frac{nA}{bt}\right),\ E_y=E \\ & A_x=bt, \qquad A_y=at \\ & D_x=\frac{nI}{b}+D_{pl},\ D_y=D_{pl},\ 2D_{xy}=\frac{Gt^3}{3}+\nu(D_x+D_y) \\ & D_{pl}=Et^3/[12(1-\nu^2)] \end{aligned}\right\} \quad (4.1)$$

where, E : Young's modulus, D: flexural rigidity, and I: moment of intertia of a stiffener together with the corresponding effective breadth of plating. Directions are indicated by suffix x and y, and the plate by pl.

4.1.1 Buckling interaction relationship

When a stiffened plate is dealt with as an orthotropic plate, buckling interaction relationship may be expressed by the same equation as for an isotropic plate, Eqs.(3.2), that is,

$$\Gamma_B=\Gamma_B(\beta,\ N_{xcr},\ N_{ycr},\ V_{xcr}) \quad (4.2)$$

However, the following expressions are to be substituted for variables of Eq. (3.2).

$$\left.\begin{aligned} & \beta=a/b, \\ & N_{xcr}=\sigma^0{}_{xcr}(bt+nA), \\ & N_{ycr}=\sigma^0{}_{ycr}(at) \\ & V_{xcr}=\tau^0{}_{xycr}(at) \end{aligned}\right\} \quad (4.3)$$

$\sigma^0{}_{xcr}$, $\sigma^0{}_{ycr}$ and $\tau^0{}_{xycr}$ are the buckling stresses of an orthotropic plate when subjected to normal stresses σ_x or σ_y, or shear stress τ_{xy} acting separately.

4.1.2 Ultimate strength interaction relationship

In the case where a stiffened plate reaches its ultimate strength in an over-all collapse mode after over-all buckling, an ultimate strength condition may be described in the same form as for a flat plate, Eq.(3.10).

When Eq.(3.10) is applied, stress coefficients α_{xmax}, α_{xmin}, α_{ymax} and α_{ymin} are obtained by Eq.(3.16). However, b′ in Eq.(3.10) and (3.16) is to be replaced by b. N_{xcr}, N_{ycr} and V_{xcr} are evaluated by Eq.(4.3), and $N_{xp}=\sigma_0(bt+nA)$.

4.2 Local buckling, followed by over-all or local collapse. ($\gamma>\gamma^B{}_{min}$)

A stiffened plate with n stiffeners is regarded as being composed of n stiffeners and (n+1) plate panels whose behavior and strengths are dealt with separately.

The behavior of plate panels between stiffeners has already been studied in section 3. Results of this study are used in the following.

4.2.1 Buckling interaction relationship

When a stiffened plate is subjected to axial forces N_x and N_y, and shearing forces V_x and V_y, and until the stiffened plate buckles locally, uniform stresses σ_x, σ_y and τ_{xy} act on each plate panel. The stiffeners are subjected to uniform stress σ_{xs} in x direction. This stress may be evaluated by the condition that the strain ε_x on the connecting line of a stiffener and the plate is the same in the plate and the stiffener.

$$\sigma_{xs}=\sigma_x-\nu\sigma_y \tag{4.4}$$

Therefore, the relationship between the applied force and the resulting stresses, may be expressed as follows.

$$\left.\begin{aligned} N_x&=\sigma_x(bt+nA)-\nu\sigma_y nA\\ N_y&=\sigma_y at\\ V_x&=\tau_{xy}at\\ V_y&=\tau_{xy}bt \end{aligned}\right\} \tag{4.5}$$

where, A: cross-sectional area of a stiffener.

When a stiffened plate buckles locally due to normal stresses σ_x and σ_y, and a shear stress τ_{xy}, the buckling interaction relationship is expressed by Eq.(3.1). Substituting Eq.(4.5) into Eq.(3.1) the buckling interaction function Γ_B can be represented in terms of N_x, N_y and V_x as follows.

(1) When σ_x is tensile and σ_y is compressive. ($\sigma_x<0$, $\sigma_y>0$)

$$\Gamma_B=\frac{(m^2+\beta^2)^2}{m^2(1+\beta^2)^2}\,\frac{N_x-(\nu nA/at)N\nu}{N_{xcr}}+\frac{N_y}{N_{vcr}}+\left(\frac{V_x}{V_{xcr}}\right)^2-1 \tag{4.6.a}$$

(2) When σ_x is compressive, and σ_y is tensile. ($\sigma_x>0$, $\sigma_y<0$)

$$\Gamma_B=\frac{N_x-(\nu nA/at)Ny}{N_{xcr}}+\frac{(1+\beta^2)^2}{(m^2+\beta^2)^2}\,\frac{N_y}{N_{ycr}}+\left(\frac{V_x}{V_{xcr}}\right)^2-1 \tag{4.6.b}$$

(3) When σ_x and σ_y are compressive. ($\sigma_x>0$, $\sigma_y>0$)

$$\Gamma_B=\left[\frac{[N_x-(\nu nA/at)N_y]/N_{xcr}}{1-(V_x/V_{xcr})^2}\right]^{\alpha_1}+\left[\frac{N_y/N_{ycr}}{1-(V_x/V_{xcr})^2}\right]^{\alpha_2}-1 \tag{4.6.c}$$

The buckling condition is expressed as,

$$\Gamma_B=0 \tag{4.7}$$

where,

$N_{xcr}=\sigma_{xcr}(bt+nA)$

$N_{ycr}=\sigma_{ycr}\cdot at$, $V_{xcr}=\tau_{xycr}\cdot at$

σ_{xcr}, σ_{ycr}, τ_{xycr} : the independent buckling stresses of a plate between two adjacent stiffeners

4.2.2 Ultimate strength interaction relationship

When local buckling occurs in a stiffened plate, one of two ultimate strength modes may take place, depending on the relative stiffness ratio γ of the stiffener to the plate.

(1) $\gamma^B{}_{min}<\gamma<\gamma^u{}_{min}$

As described in section 2, ultimate strength in this case is reached by buckling or yielding of the stiffeners depending on the loading condition. After local plate buckling, the effectiveness of plate panels decreases gradually causing the neutral axis of the stiffened plate to move far from the plate. When the axial force N_x always acts at the neutral axis, a stiffened plate reaches its ultimate strength due to buckling of stiffeners. On the other hand, when the line of action of the axial force is fixed, the stiffened plate becomes subjected to an eccentric axial force causing bending of the stiffeners. In this case, a stiffened plate reaches its ultimate strength due to initial yielding of the stiffeners.

a) Buckling of stiffeners (central loading)

Euler buckling strength of a stiffener is given by the following equation.

$$P_{us}=\frac{\pi^2 EI}{a^2} \tag{4.8}$$

The effective breadth considered here, corresponds to the post buckling stiffness of a plate between two adjacent stiffeners. This is obtained by consideration of $\Delta\varepsilon/\Delta\sigma_x$ and using b_e' of Eq.(3.7).

$$b'_{est}=b'\Big/\left[1+\frac{2m^4b'^4}{a^4+m^4b'^4}(f(V)+1)\right] \tag{4.9}$$

f(V) in the above equation is defined by Eq.(3.16). Now the ultimate strength, N_{xu} may be obtained by adding the ultimate strength of one plate panel to the buckling strength of all stiffeners.

$$N_{xu}=nP_{us}+N'^{\,0}_x,\ N'_x=\frac{\sigma_0 b' t}{2(1+\alpha_x)}\left[\frac{N_y}{N_{yp}}(1+\alpha_y)\right.$$
$$\left.+\sqrt{4-4\left(\frac{V_x}{V_{xp}}\right)^2-3\left[\frac{N_y}{N_{yp}}(1+\alpha_y)\right]}\right] \tag{4.10}$$

The ultimate strength function Γ_u is expressed as,

$$\Gamma_u=\frac{N_x}{N_{xp}}-\frac{nP_{us}+N_x'}{N_{xp}} \tag{4.11}$$

b) Bending of stiffeners (eccentric loading)

In this case a stiffener, together with its effective breadth, is subjected to compression and bending. The stress distribution in a stiffener and associated plate becomes as shown in **Fig.7.** The continuity of strain of a stiffener and the plate on the connection line is satisfied by Eq.(4.12.a). However, the stress σ_{xmax} in the plate is different from that σ^0_{xmax} of a stiffener.

$$\sigma_{xmax}=\sigma^0_{xmax}+\nu\sigma_y \tag{4.12.a}$$

The effective breadth b_e' changes according to the change of σ_{xmax} (stress distribution), and can be obtained from Eqs.(3.7) and (3.16). The effective breadth ratio has the following relationship.

$b_e'/b'=\sigma_{xav}/\sigma_{xmax}$

A modified effective breadth b'_{ec} is defined related to the stiffener stress σ^0_{xmax}. Using Eq.(4.12.a) and considering the same axial force is acting on each plate panel, b_{ec}' may be expessed as follows.

$$b_{ec}'=b'/(1-\nu\sigma_y/\sigma_{xmax}) \tag{4.13}$$

Ultimate strength is obtained assuming the stiffener with its effective breadth as a beam-column. First, the neutral axis is quite colse to middle plane of the plate, so that the stress σ_{xmax} in the plate, may be expressed as,

$$\sigma_{xmax}=P/A_T+\nu\sigma_y \tag{4.12.b}$$

where, P: axial force acting on a stiffener with its effective breadth

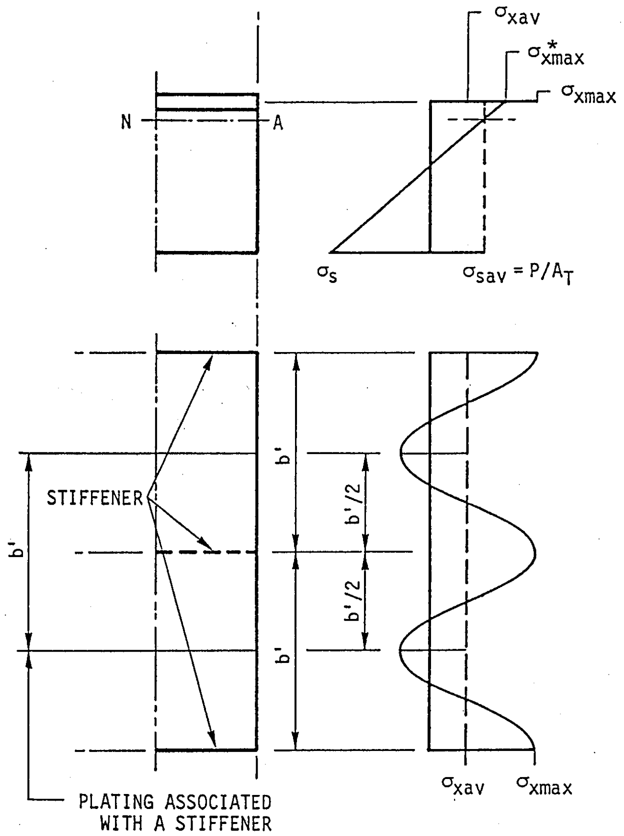

Fig. 7 Stress distribution in a stiffener and associated plating (Eccentric loading)

A_T : total cross-sectional area of each stiffener with modified effective breadth $b'_{ec}Ab'_{ec}t$

The ultimate compressive load P_{us} of the beam-column, is determined by the condition that either i) plasticity of the outmost fibre of the stiffener or ii) compressive collapse of the plate has occurred.

The ultimate strength conditions corresponding to these two criteria may be expressed as follows.

$$\left.\begin{aligned}\sigma_S&=\frac{P}{A_T}-\frac{P_e}{Z_S}\sec\left(\sqrt{P/EI}\cdot\frac{a}{2}\right)\\ \sigma_{pl}&=\frac{P}{A_T}+\frac{P_e}{Z_{pl}}\sec\left(\sqrt{P/EI}\cdot\frac{a}{2}\right)\end{aligned}\right\} \tag{4.14.a}$$

When $\sigma_S=\sigma_o$ or σ^0_{xmaxu},

$$P=P_{us} \tag{4.14b}$$

where, e: the magnitude of eccentricity of loading

Z_S : section modulus of the beam-column corresponding to the outmost fibre of the stiffener

Z_{pl} : section modulus of the beam-column corresponding to the middle plane of the plate

σ_{xmaxu}, σ^0_{xmaxu}: axial stresses acting on the plates with effective breadth b_e' and modified effective breadth b_{ec}', when plate panels between stiffeners collapse

$(\sigma^{0}_{xmaxu}=\sigma_{xmaxu}-\nu\sigma_y)$

The ultimate strength P_{us} may be evaluated as the smaller of those given by Eq.(4.14).

Substituting P_{us} into Eq.(4.11), the ultimate strength function Γ_u may be derived.

Here, however, Eq.(4.14) contains A_T, Z_s, Z_p, and I which are functions of the modified effective breadth b_{ec}', which is expressed in terms of mean stresses. Therefore, when the axial compression P reaches the ultimate compressive load P_{us}, b_{ec}' should be evaluated such that it corresponds to the ultimate load. (Usually, it can be obtained by iteration.)

(2) $\gamma>\gamma^{u}_{min}$

In this case, the plate between stiffeners buckles and collapses locally, while stiffeners do not buckle and may reach their fully plastic state. The ultimate strength of a stiffened plate can be obtained by the sum of the ultimate strengths of the plate panels and the stiffeners. Here, the stiffeners are subjected only to axial force and their ultimate strength is represented by σ_0nA, while the ultimate strength of plate panels is as shown in section 3.

The ultimate strength function Γ_u may now be obtained as follows. When, $N_x>\sigma_0nA+\overline{N}_x$,

$$\Gamma_u=\left[\frac{N_x-\sigma_0nA}{N_{xp}-\sigma_0nA}(1+\alpha_x)\right]^2-\left[\frac{N_y}{N_{yp}}(1+\alpha_y)\right]^2-\frac{(N_x-\sigma_0nA)N_y}{(N_{xp}-\sigma_0nA)N_{yp}}(1+\alpha_x)(1+\alpha_y)-\left(\frac{V_x}{V_{xp}}\right)^2-1 \quad (4.15)$$

α_x and α_y in the above equation may be evaluated by Eq.(3.16).

When, $N_x<\sigma_0nA+\overline{N}_x$,

$$\Gamma_u=N_y/N_{xp}-\overline{N}_y/N_{yp} \quad (4.16)$$

where, $\overline{N}_x$ and $\overline{N}_y$ are the coordinates of the intersection point of Eqs.(3.11) and (3.19).

When eccentricity of the loading occurs after buckling, stiffeners are subjected to bending and axial compression according to increment of the over-all collapse mode changes into the local mode.

The authors[6] have defined the stiffness ratio γ at this transition point as γ^{u}_{min}. A stiffened plate collapses by plasticity caused by bending of stiffeners (with their effective breadths). In this case the ultimate strength is evaluated by Eqs.(4.14) and (4.11).

Finally, the condition of the ultimate strength is expressed by the following equation.

$$\Gamma_u=0 \quad (4.17)$$

4.3 Fully plastic strength interaction curve

When a stiffened plate does not buckle in neither local or overall mode, it reaches its fully plastic strength. In this case, each plate is subjected to uniform stresses σ_x, σ_y and τ_{xy} such that $\sigma^2_{ov}=\sigma^2_0-(3\tau_{xy})^2=\sigma^2_x+\sigma^2_y-\sigma_x\sigma_y$ and each stiffener is subjected to uniform stress equals to σ_0.

The fully plastic strength interaction relationship under biaxial forces N_x and N_y and shearing stress τ_{xy} is expressed in the following form,

$$\left(\frac{N_x\pm\sigma_0nA}{bt}\right)^2+\left(\frac{N_y}{at}\right)^2-\frac{(N_x\pm\sigma_0nA)N_y}{abt^2}=\sigma_{ov}^2 \quad (4.18)$$

Representing the above equation by N_x, N_y and V_x, the fully plastic strength interaction function is derived as follows.

1) (a) $N_y>0$, $N_x<\overline{N}_x-\sigma_0nA$

or

$N_y<0$, $N_x<-\overline{N}x-\sigma_0nA$

$$\Gamma_p=\left(\frac{N_{xp}-\sigma_0nA}{N_x+\sigma_0nA}\right)^2+\left(\frac{N_y}{N_{yp}}\right)^2-\frac{(N_x+\sigma_0nA)N_y}{(N_{xp}-\sigma_0nA)N_{yp}}+\left(\frac{V_x}{V_{xp}}\right)^2-1 \quad (4.19.a)$$

(b) $N_y>0$, $N_x>\overline{N}_x+\sigma_0nA$

or

$N_y<0$, $N_x>-\overline{N}_x+\sigma_0nA$

$$\Gamma_p=\left(\frac{N_{xp}-\sigma_0nA}{N_x-\sigma_0nA}\right)^2+\left(\frac{N_y}{N_{yp}}\right)^2-\frac{(N_y-\sigma_0nA)N_y}{(N_{xy}-\sigma_0nA)N_{yp}}+\left(\frac{V_x}{V_{xp}}\right)^2-1 \quad (4.19.a)'$$

2) $N_y>0$, $\overline{N}-\sigma_0nA\leq N_x\leq\overline{N}_x+\sigma_0nA$

$$\Gamma_p=N_y/N_{yp}-2\sqrt{1-(V_x/V_{xp})^2}/\sqrt{3} \quad (4.19.b)$$

3) $N_y<0$, $-\overline{N}_x-\sigma_0nA\leq\overline{N}_x+\sigma_0nA$

$$\Gamma_p=-N_y/N_{yp}-2\sqrt{1-(V_x/V_{xp})^2}/\sqrt{3} \quad (4.19.c)$$

where, $\overline{N}_x=N_{xp}\sqrt{1-(V_x/V_{xp})^2}/\sqrt{3}$

The fully plastic condition is

$$\Gamma_p=0 \quad (4.20)$$

Buckling strength, ultimate strength and fully plastic strength interaction relationships are schematically illustrated in **Fig.8.**

5. Procedure of analysis and accuracy of interaction relationships

In the preceding sections, buckling strength, ultimate strength and fully plastic strength interaction relationships are derived for a plate and a stiffened plate subjected to inplane biaxial forces and shearing force.

In this section, the procedure of analysis is described and the accuracy of the proposed equations are examined through comparisons with results obtained by other methods.

5.1 Procedure of analysis

In this section, precedures are described to calculate the loads which cuases buckling and ultimate strength of a rectangular plate and of a stiffened plate whose sizes and material properties are known.

Load is assumed to be either proportional loading, or that only one of the load components is changing while keeping the others constant. In this way, the load is represented by only one parameter, ρ.

5.1.1 Rectangular plates

Load parameter $\rho=\rho_B$ at buckling strength may be obtained by solving the appropriate equation of Eqs.(3.2) under the condition $\Gamma_B=0$ of Eq.(3.3). Limits of validity are shown with each equation.

Next, the fully plastic strength is obtained by solving Eq.(3.18) under the condition of $\Gamma_p=0$ of Eq.(3.19). After calculating the fully plastic strength load parameter ρ_p, the possibility of buckling is examined by comparing ρ_p with the buckling load parameter ρ_B. If the buckling load parameter ρ_B is smaller than ρ_p, the plate buckles before reaching its full plastic strength. In this case ultimate strength after buckling is to be calculated.

Ultimate strength after buckling is obtained by solving Eq.(3.10) under the condition $\Gamma_u=0$ of Eq.(3.11). Ultimate strength interaction function Γ_u contains the stress coefficients α_x and α_y of Eq.(3.16). These coefficients are substituted into Eq.(3.10) depending on the location of yielding after buckling. There are three possible locations where yielding, of membrane stresses, may occur, (1) at the four corners of the plate, (α_{xmax} and α_{ymax}) (2) at the middles of half buckling waves along sides parallel to the x axis (α_{xmax}, α_{ymin}), and (3) at the middle of the side parallel to the y axis (α_{xmin}, α_{ymax}). Ultimate loads corresponding to these yielding cases are calculated and the lowest one is taken as the ultimate strength.

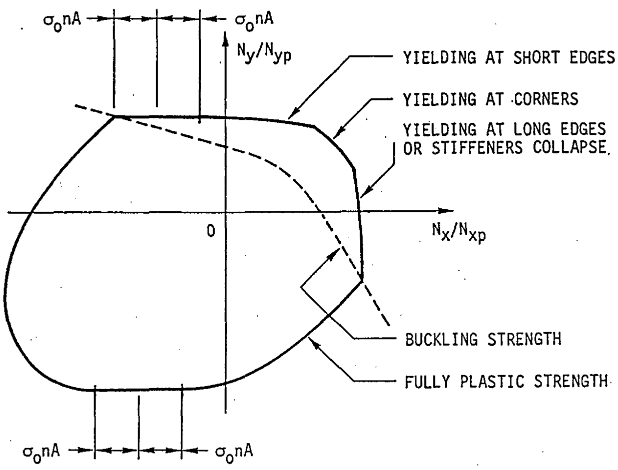

Fig. 8 Buckling strength, ultimate strength and fully plasic strengths of a stiffened plate

In the actual process of calculation, the external load is increased gradually. The stress coefficients and the ultimate strength interaction function Γ_u at the three yielding cases are evaluated at each loading increment. The relationship between the load and the ultimate strength interaction function Γ_u at each case is plotted as illustrated in **Fig.9 (a).** External load satisfies the condition $\Gamma_u=0$ at the intersections of these relationships with the ordinate axis. The smallest load of these intersections is the ultimate strength.

5.1.2 Stiffened plates

At first, the buckling mode and buckling load are to be obtained.

The local buckling load parameter ρ_{Bl} which causes buckling of the plate panels between stiffeners may be obtained by the procedure described in Section 5.1.1. The over-all buckling load parameter ρ_{Bo} may be obtained by imposing the condition $\Gamma_B=0$ on Eq.(4.2). Buckling occurs at the smaller of these two buckling loads, in the corresponding mode.

Next, the fully plastic load parameter ρ_P is evaluated by solving Eq.(4.19) under the fully plastic condition of Eq.(4.20). This load is compared with the buckling load to classify the behavior of a stiffened plate.

(1) $\rho_{BO}, \rho_{Bl} > \rho_P$

Buckling does not occur and the stiffened plate reaches its fully plastic strength.

(2) $\rho_{BO} < \rho_{Bl}, \rho_P$

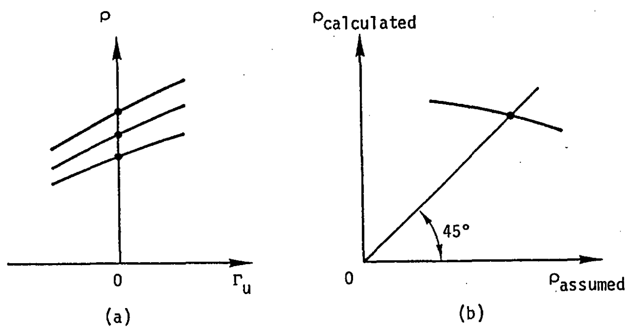

Fig. 9 Iterative procedure to calculate ultimate strength

Over-all buckling occurs and ultimate strength is evaluated using the equations of Section 4.1.2. Procedure of analysis is as shown in Section 5.1.1.

(3) $\rho_{Bl} < \rho_{BO}$, ρ_P

Local buckling occurs and the stiffened plate collapses under concentric or eccentric compressive loading depending upon the type of loading.

a) Concentric loading

There are two cases, one is the case where stiffened plate collapses in overall mode and reaches the corresponding ultimate strength in terms of ρ_{uo}, and the other is where plate panels collapse locally and the stiffened plate reaches the corresponding ultimate strength load at ρ_{ul}. ρ_{uo} can be obtained by Eq.(4.10) as follows. Increasing the load ρ gradually, the effective breadth may be obtained by Eq.(4.9). Using this effective breadth, P_{us} and ρ_{uo} may be obtained by Eq.(4.8) and Eq,(4.10), respectively. The used effective breadth, however, depends on ρ. It corresponds to ρ_{uo} only when $\rho=\rho_{uo}$. To find this load, ρ_{uo} is plotted against ρ as in **Fig.9 (b).**

The point where the coordinates of the two axeses become equal represents the ultimate strength.

On the other hand, ρ_{ul} can be obtained by Eq.(4.15) or Eq,(4.16) under the condition $\Gamma_u=0$.

The procedure of analysis is as shown in 5.1.1. The smaller of ρ_{uo} and ρ_{ul} is the ultimate strength.

b) Eccentric loading

Local buckling reduces the effectiveness of plate panels causing the neutral axis to move away from the plate. When the load is applied at a fixed point, an eccentricity is produced. Ultimate strength in this case may be obtained following conventional incremental load method.

Assuming a small value of the mean stress σ_{sav} acting on stiffeners, the plate stress σ_{xmax} and the effective breadth b'_e are obtained by Eqs.(4.12.b), (3.7) and (3.16). Next b'_{ec} is obtained by modifying $b'e$ by Eq.(4.13).

Calculating properties of the section of a stiffener using this b'_{ec}, the axial load of a stiffener P is obtained by the equation $P=A_T\sigma_{sav}$.

σ_s and σ_{pl} are calculated by Eq,(4.14.a) using this axial load P. These values are examined whether to satisfy either of Eq.(4.14.b).

If they do not satisfy both, the analysis is to be continued increasing the mean stress σ_{sav}. P which satisfies either of the condition $\sigma_S=-\sigma_o$ or $\sigma_{pl}=\sigma^o{}_{xmaxu}$, is the ultimate strength P_{us}.

Now, the ultimate strength function Γ_u, Eq,(4.11) may be evaluated and the ultimate strength load is obtained as the value satisfying the condition $\Gamma_u=0$.

In this analysis, the stress coefficients and effective breadth change according to the load. The ultimate load has to be obtained using the stress coefficients and effective breadth corresponding to this ultimate load. Accordingly the analysis has to be iterative.

5.2 Accuracy of the present interaction relationships

The accuracy of the present ultimate strength equations is examined comparing the result with those reported in the litrature.

(1) Square and rectangular plates subjected to compression in one direction.

Ultimate strength of square plates calculated by three different methods, the finite element method[1)], the combined elastic large deflection and plastic analysis[4)], and the present method is plotted in **Fig.10.**

Results obtained by the finite element method and the present method applied to a rectangular plate are plotted in **Fig.11.**

In the analysis by the finite element method, an initial deflection is assumed, which reduces the ultimate strength, and the aspect ratio corresponding to the minimum ultimate strength changes.

Therefore, both curves in Fig.11 may be regarded to be in good agreement.

Considering both Figs.10 and 11, it may be seen that the present method predicts the ultimate compressive strengths of square and rectangular plates with good accuracy.

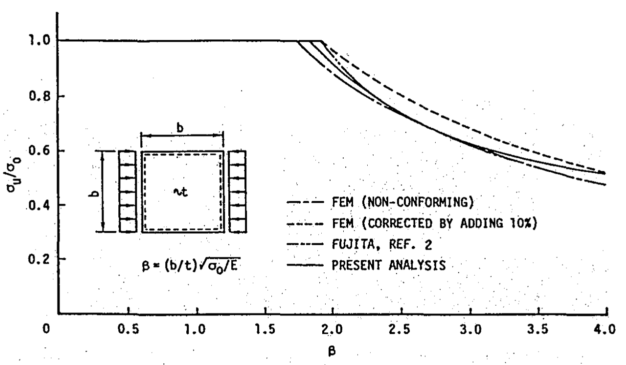

Fig. 10 Ultimate strength of square plates by different methods

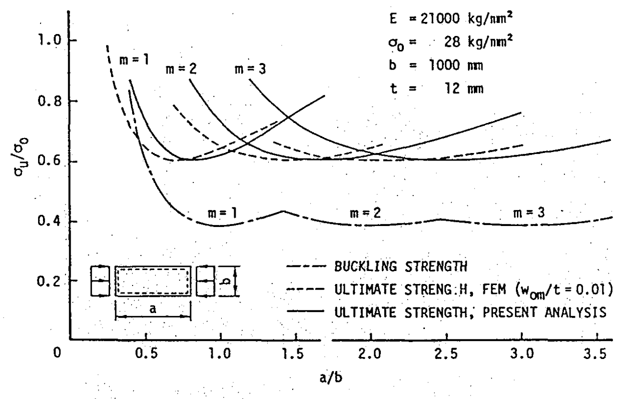

Fig. 11 Ultimate strength of rectangular plates

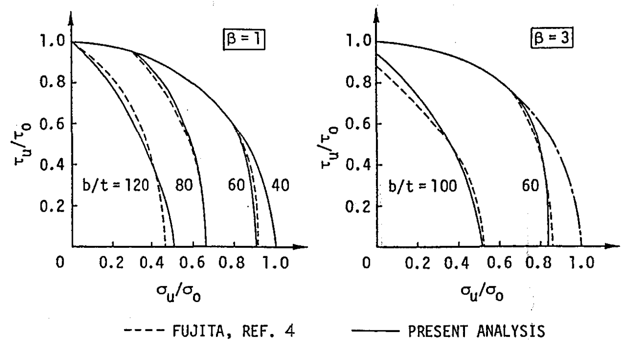

Fig. 12 Ultimate strength of square and rectangular plates subjected to compression and shear

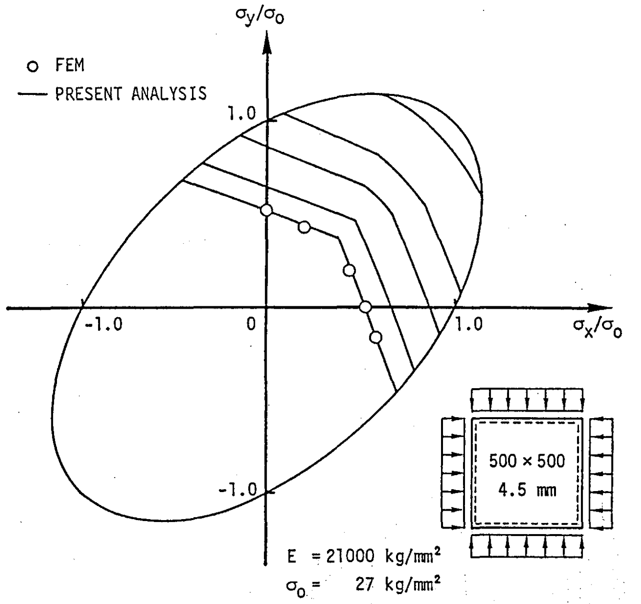

Fig. 13 Ultimate strength of square plates subjected to biaxial loading

(2) Square and rectangular plates subjected to combined loads

Ultimate strength interaction relationship of square plates subjected to compression in one direction and shear is evaluated by the present method and compared to that obtained by the combined elastic large deflection and plastic analysis. Results are plotted in **Fig.12.**

Ultimate strength interaction relationship of square plates subjected to biaxial loads is evaluated by the finite element method and present method. Results are plotted in **Fig.13.**

(3) Stiffened plate subjected to compression in one direction

The relationship between the ultimate strength of a stiffened plate subjected to compression in one direction and the relative stiffness ratio γ of stiffeners is obtained by the finite element method and the present method. Results of analyses by both methods are plotted in **Figs.14** and **15**.

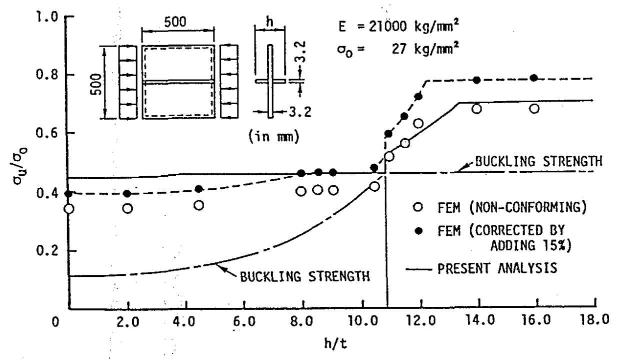

Fig. 14 Ultimate strength of compressed stiffened plates (concentric loading)

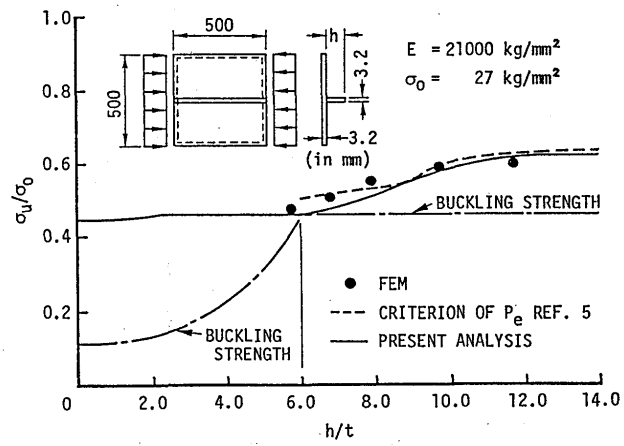

Fig. 15 Ultimate strength of compressed stiffened plates (Eccentric loading)

In the case shown in Fig.14, the compressive load is applied always concentrically on a both-sided stiffened plate, while in the case shown in Fig.15, eccentricity of load is allowed to take place on a one-sided stiffened plate.

In the case shown in Fig.14, non-conforming elements are used in the analysis by the finite element method. The implane displacement of the edges is free. These cause plate strength to decrease.

Actually, the exact solution of buckling strength of these stiffened plates is 15% higher than that predicted by the finite element method. Accordingly, actual ultimate strength is assumed to be higher by the same ratio. Taking this into consideration, the accuracy of the present method is confirmed to be good enough.

6. Conclusions

In this paper, rectangular plates and stiffened plates with stiffeners attached at equal intervals in one direction are studied. The interaction relationships of buckling strength, ultimate strength and fully plastic strength are derived theoretically under inplane biaxial forces and shearing force.

Ultimate strength may be reached in one of three modes. The first is when a stiffened plate collapses in an over-all mode. The second is when plate panels collapse locally and the last is when no buckling occurs. Interaction equations, as functions of biaxial and shearing forces, are presented.

Comparisons of results obtained by these interaction equations with results available in the litrature have confirmed that these equations have sufficient accuracy for practical use.

Acknowledgement

A part of the study presented in this paper is performed by Katsuhiko Omoto, (then, under graduate student, Osaka University) for his graduation thesis. The authors wish to express their appreciation of his effort.

References

1) Y. Ueda and T. Yao, "The influence of complex Initial Deflection Modes on the Behaviour and Ultimate Strength of Rectangular Plates in Compression", Jl of constractional steel Research, Vol. 5, No.4, 1985 (pp. 265-302) "Compressive Ultimate Strength of Rectangular Plates with Initial Imperfections due to Welding", 1st and 2nd reports, SNAJ, Vols. 148, 148 ; 1980,1981 respectively.
2) S. Valsgard, "Numerical Design Prediction of the Capacity of Plates in Biaxial Inplane Compression", Computers & Structures, Vol. 12, 1980 (pp 729-739)
3) J. E. Harding, R. E. Hobbs and B. G. Neal, "Ultimate Load Behaviour of Plates under Combined Direct and Shear Inplane Loading", Steel Plated Structures, Crosby Lockwood Staples, London, 1977.
4) Fujita et al., "Ultimate Strength of Flat Plates Subjected to Combined Load", 1st-3rd reports, SNAJ, Vols. 145, 446, 149; 1979, 1979, 1981 respectively.
5) D. R. Rutledge and A. Ostapenko, "Ultimate Strength of Longitudinally Stiffened Plate Panels", Fritz Eng. Lab. Report No. 248.24, 1968
6) Y. Ueda, T. Yao, "Minimum Stiffness Ratio of Stiffeners Against Ultimate Strength of a Plate", 1st-4th reports, SNAJ, Vols. 140, 143, 145, 148; 1976, 1978, 1979, 1980 respectively.
7) S. Timoshenko and J. Gere, "Theory of Elastic Stability", McGraw-Hill, 1961.
8) Y.Ueda, S. M. H. Rashed and J. K. Paik, "An Incremental Galerkin Method for Plates and Stiffened Plates", Computers & Structures, Vol.27, No.1, 1987 (pp. 145-156)