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ON ALMOST RELATIVE INJECTIVES ON ARTINIAN MODULES

MANABU HARADA

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We have introduced a concept of almost relative projectives (resp. injectives) in [7] (resp. [2]) which is deeply related with lifting modules [9] (resp. extending modules [10]). When we study further those modules, we have understood that it is necessary to generalize [2], Theorem to a case of artinian modules. Namely, we shall give the following theorem (Theorem 2): let $U$ and $\{U_i, I_j\}_{i=1}^{r} \in \mathfrak{A}$ be LE and artinian modules such that $U$ is $I_j$-injective for all $j$ and $U$ is almost $U_i$-injective but not $U_i$-injective for all $i$. Then $U$ is almost $\Sigma_i \oplus U_i \oplus \Sigma_j I_j$-injective if and only if $\Sigma_i \oplus U_i$ is an extending module.

1. Preliminaries

Let $R$ be a ring with identity. Every module $M$ is a unitary right $R$-module. In this paper we mainly study modules with non-zero socles. We shall denote an injective hull and the socle of $M$ by $E(M)$ and $\text{Soc}(M)$, respectively. Let $N$ be a submodule of $M$. If $N \cap N' \neq 0$ for any non-zero submodule $N'$ of $M$, then $N$ is called an essential submodule of $M$. If every proper submodule is essential in $M$, then we call $M$ a uniform module.

We start with definition of almost injective modules following [2]. We take two $R$-modules $U$ and $U_0$. Let $V$ be a submodule of $U$ and $i$ the inclusion. Consider the following diagram and two conditions 1) and 2):

\[
\begin{array}{ccc}
U & \xrightarrow{i} & V \\
\downarrow{\pi} & & \downarrow{h} \\
U' & \xrightarrow{\bar{h}} & U_0
\end{array}
\]

(0)

1) There exists $\bar{h}: U \rightarrow U_0$ such that $\bar{h}i = h$

2) There exist a non-zero direct summand $U'$ of $U$ and $\bar{h}: U_0 \rightarrow U'$ such that $\bar{h}h = \pi i$, where $\pi: U \rightarrow U'$ is the projection of $U$ onto $U'$.

$U_0$ is called almost $U$-injective if the above 1) or 2) holds true for the diagram (0) with any $V$ and any $h$ [2] ($U_0$ is called $U$-injective if we have only 1) [1]).

We frequently use the following property:
Assume that $U$ is indecomposable and $U_0$ is almost $U$-injective. If the $h$ given in (0) is not a monomorphism, the case 2) does not occur, and hence there exists $\tilde{h}: U \to U_0$ with $\tilde{h}i = h$.

We use sometimes this property without any references.

We shall exhibit some properties dual to ones on almost relative projectives, whose proofs are categorical. Hence we shall skip their proofs.

The following is useful in this paper.

**Theorem 1** (dual to [6], Theorem 1). Let $U$ be an indecomposable and non-uniform module and $U_0$ an $R$-module. If $U_0$ is almost $U$-injective, then $U_0$ is $U$-injective.

We always assume every module contains non-zero socle unless otherwise stated. We shall study almost relative injectivity among uniform modules with non-zero socle.

Let $U_1$ and $U_2$ be uniform modules with isomorphic socles $S_1$ and $S_2$, respectively. If for any isomorphism $f: S_1 \to S_2$, $f$ or $f^{-1}$ is extensilbe to an element in $\text{Hom}(U_1, U_2)$ or in $\text{Hom}(U_2, U_1)$, then we say that $U_1 \oplus U_2$ has the extending property of simple modules (briefly EPSM). If $\text{End}(U_i)$ is a local ring for $i=1, 2$ i.e., the $U_i$ are LE modules, then this concept coincides with usual one in [5], §9.6.

**Proposition 1** (dual to [6], Proposition 2). Let $E$ be an indecomposable injective module and $U_1$, $U_2$ submodules of $E$. Assume that either $U_1$ or $U_2$ is artinian. Then $U_1$ is almost $U_2$-injective if and only if i) $\text{J}(T)U_2 \subseteq U_1$ and ii) $U_1 \oplus U_2$ has EPSM, where $T = \text{End}(E)$. In this case if $U_1$ is not $U_2$-injective, then $U_2$ is $U_1$-injective.

**Remark 1.** In Proposition 1, if we assume $U_1 \subseteq U_2$, we know that the assumption of "artinian" is superfluous for the first half. If $U_i$ does not contain a simple socle for $i=1, 2$, $U_1 \oplus U_2$ trivially have EPSM. Let $Z$ be the ring of integers. Then $U_1 = U_2 = Z$ trivially satisfy i) and ii) in Proposition 1. However $Z$ is not almost $Z$-injective as $Z$-modules.

**Proposition 2** (dual to [8], Proposition 1). Let $U_0$, $U_1$ and $U_2$ be $R$-modules and $U_1$, $U_2$ indecomposable. Assume that $U_0$ is almost $U_1$-injective, but not $U_1$-injective. Then 1): if $U_0$ is $U_2$-injective, $U_1$ is $U_2$-injective. 2): If $U_0$ is almost $U_2$-injective, but not $U_1$-injective, then we obtain the following fact: i) if $\text{Soc}(U_1) \neq \text{Soc}(U_2)$, $U_1$ is $U_2$-injective and ii) if $0 \neq \text{Soc}(U_1) \neq \text{Soc}(U_2)$, then $U_1$ is almost $U_2$-injective (and $U_2$ is almost $U_1$-injective) if and only if $U_1 \oplus U_2$ has EPSM.

**2. Main theorem**

In this section we shall give the main theorem which is a generalization
of [2], Theorem.

**Lemma 1** (dual to [6], Proposition 5). Let $U_0$, $U_1$, and $U_2$ be $R$-modules with non-zero socle. Assume that i): $U_1$ and $U_2$ are LE modules, ii): $U_0$ is almost $U_1$-injective but not $U_1$-injective and iii): $U_0$ is almost $U_1 \oplus U_2$-injective. Further assume that there exists an isomorphism $f$ of a simple sub-factor module $V_2/V_1$ of $U_2$ onto a simple submodule of $U_1$. Then $f$ is extensible to an element $f': U_2 \rightarrow U_1$ (or $f^{-1}$ is extensible to an element $f': U_1 \rightarrow U_2$, in this case $V_1 = 0$).

**Corollary.** Let $U_0$, $U_1$, and $U_2$ be as in Lemma 1 and satisfy i), ii) and iii) in Lemma 1. Then $U_1$ is almost $U_2$-injective.

Proof. If $U_0$ is $U_2$-injective, $U_1$ is $U_2$-injective by Proposition 2. Hence we may assume that $U_0$ is almost $U_2$-injective, but not $U_2$-injective. Further we may assume from Proposition 2 and Theorem 1 that $\text{Soc}(U_1) \approx \text{Soc}(U_2)$ is simple. It is clear from Lemma 1 that $U_1 \oplus U_2$ has EPSM. Hence $U_1$ is almost $U_2$-injective by Proposition 2.

**Lemma 2.** Let $U_1$ and $U_2$ be artinian and uniform modules with isomorphic socles. Assume that $U_2$ is almost $U_1$-injective. If an isomorphism $f$ of $S_1 = \text{Soc}(U_1)$ onto $S_2 = \text{Soc}(U_2)$ is extensible to an element $F: U_1 \rightarrow U_2$, then $U_2$ is $U_1$-injective.

Proof. Let $g: S_1 \rightarrow S_2$ be any isomorphism. Since $U_2$ is almost $U_1$-injective, $g$ is extensible to $G: U_1 \rightarrow U_2$ or $g^{-1}$ is extensible to $G': U_2 \rightarrow U_1$. We assume the latter case. Then $G'F$ is an endomorphism of $U_1$ and $G'F|S_1 = g^{-1}f|S_1$ is an isomorphism. Hence $G'F$ is a monomorphism, and so an isomorphism, since $U_1$ is artinian. Therefore $G'$ is an isomorphism, and hence $G'^{-1}$ is an extension of $g$. We shall show that $U_2$ is $U_1$-injective. Take any diagram with $V_1$ a submodule of $U_1$:

$$
\begin{array}{c}
U_1 \\
\downarrow h \\
U_2
\end{array}
\begin{array}{c}
V_1 \\
\rightarrow 0
\end{array}
$$

Assume that $h$ is a monomorphism. Then $h|S_1$ is extensible to $H: U_1 \rightarrow U_2$ from the initial part. Since $\ker (h-Hi) \supseteq S_1$, there exists $h': U_1 \rightarrow U_2$ with $hi = h-Hi$, and hence $h = (h + H)i$. If $h$ is not a monomorphism, then we obtain $h': U_1 \rightarrow U_2$ with $hi = h$ by definition.

Let $\{U_i\}_{i=1}^n$ be a set of artinian and uniform modules with $\text{Soc}(U_i) = S_i$ as in Lemma 2. Assume that $U_i$ is almost $U_j$-injective for any pair $(i,j)$. If an isomorphism of $S_i$ onto $S_j$ is extensible to $F: U_i \rightarrow U_j$, we denote it by $U_i \approx U_j$. Then if $U_i \leq U_j$ and $U_j \leq U_i$, $U_i \approx U_j$ from the above proof. Hence the relation
M. Harada defines a total order on the isomorphism classes of \( \{U_i\} \), and \( U_i \supseteq U_j \) is equivalent to \( U_i \) being \( U_j \)-injective. We give one more remark. Let \( \{A_i\} \) be a set of uniform modules with isomorphic socles. Then we may assume that the \( A_i \) are submodules of \( E = E(A_i) \). If \( A_i \) is \( A_j \)-injective, \( A_j \subseteq A_i \) (cf. [3], Lemma 9). Hence if we assume that for every pair \((i, j)\) either \( A_i \) is \( A_j \)-injective or \( A_j \) is \( A_i \)-injective, then \( \{A_i\} \) is linearly ordered with respect to inclusion. Further if \( A_i \supseteq A_j, A_i \) is \( A_j \)-injective by assumption.

We remember here the definition of extending modules [10]. Let \( X \) be an \( R \)-module. If for any submodule \( Y \) of \( X \), there exists a direct decomposition of \( X \) such that \( X = X_1 \oplus X_2 \) and \( X_1 \) is an essential extension of \( Y \), then \( X \) is called an extending module [10].

Let \( \{D_i\}_{i=1}^r \) be a set of indecomposable \( R \)-modules and \( U_0 \) an \( R \)-module. Assume that \( U_0 \) is almost \( \Sigma_i \oplus D_i \)-injective. Then \( U_0 \) is almost \( D_i \)-injective for all \( i \) (cf. [2]). We shall divide \( \{D_i\} \) into two disjoint parts \( \{D_i\} = \{U_j\} \cup \{U_k\} \) as follows:

(*) 1) \( U_0 \) is \( I_k \)-injective for all \( k \).
2) \( U_0 \) is almost \( U_j \)-injective but not \( U_j \)-injective for all \( j \).

We note that all \( U_j \) are uniform Theorem 1. The following theorem is a generalization of [2], Theorem.

**Theorem 2.** Let \( U_0 \) be an \( R \)-module, \( \{U_j; I_k\}_{k=1}^r, \{I_j\}_{j=1}^r \) a set of \( R \)-modules satisfying \((*)\). Assume that the \( U_j \) are \( \Sigma \) \( R \)-modules for all \( j \). If \( U_0 \) is almost \( \Sigma_j \oplus U_j \)-injective, then \( \Sigma_j \oplus U_j \) is an extending module. We assume further that the \( U_j \) are artinian. Then the converse is true.

Proof. The first part is clear from Corollary to Lemma 1 and [3], Theorem 4. Conversely, we assume that \( \Sigma_j \oplus U_j \) is an extending module. If \( \mathrm{Soc}(U_0) = 0, \pi = 0 \) for the \( U_j \) are artinian, and Theorem 2 is clear by [1]. Hence we may assume \( \mathrm{Soc}(U_0) \neq 0 \). Now \( U_i \) is almost \( U_j \)-injective by [3], Theorem 4 \((i \neq j)\). Put \( I = \Sigma_j \oplus I_k, U = \Sigma_j \oplus U_j \) and \( W = U \oplus I \). Take any diagram with row exact:

\[
\begin{array}{c}
W = U \oplus I \leftarrow V \leftarrow 0 \\
\downarrow h \\
U_0
\end{array}
\]

We shall show

1. either a) there exists \( \tilde{h} : W \to U_0 \) with \( \tilde{h}i = h \) or b) there exist an non-zero indecomposable direct summand \( U'_1 \) of \( W \) and \( \tilde{h}' : U_0 \to U'_1 \) with \( \pi'_1 i = \tilde{h}' h \), where \( \pi'_1 : W \to U'_1 \) is the projection.

Since our proof is very long, we shall divide it into several steps.

**Step 1 Reduction.** Taking a complement of \( V \) in \( W \), we may assume from the proof of Theorem in [2] that
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(#) $V$ is essential in $W$.

**Step 2** Refinement of diagrams. Put $U = U_j = U_1 \oplus \cdots \oplus U_{j-1} \oplus U_{j+1} \oplus \cdots \oplus U_n$ and $V_j = U_j \cap V$. Consider three diagrams:

(2-j)
\[
\begin{array}{c}
U_j \xleftarrow{i} V_j \xleftarrow{0} \\
\downarrow h|V_j \\
U_0
\end{array}
\]

(2-j*)
\[
\begin{array}{c}
U - U_j \xleftarrow{i} (U - U_j) \cap V \xleftarrow{0} \\
\downarrow h|(U - U_j) \cap V \\
U_0
\end{array}
\]

and

(2-j**)
\[
\begin{array}{c}
I \xleftarrow{i} I \cap V \xleftarrow{0} \\
\downarrow h|I \cap V \\
U_0
\end{array}
\]

Since $U_0$ is $I_k$-injective, there exists always $\tilde{h}_j: I \to U_0$ with $\tilde{h}_j |(I \cap V) = h | (I \cap V)$ by [1].

**Step 3** Existence of $h_j: U_j \to U_0$ for all $j$. First we shall show under (#) that

(3) if there exists $\tilde{h}_j: U_j \to U_0$ in (2-j) such that $\tilde{h}_j(i | V_j) = h | V_j$ for each $j$, then there exists $\tilde{h}_0: U \to U_0$ such that $\tilde{h}_0 |(U \cap V) = h | (U \cap V)$. Hence there exists $\tilde{h}: W \to U_0$ with $\tilde{h}_i = h$, i.e. (1)-(a)).

Using [2], Lemma C, we can prove (3) in a similar manner to step 3 in [8] by induction on $n$, the number of direct summands $U_i$.

**Step 4** Existence of $\tilde{h}_j: U_0 \to U_j$ for some $j$. From Step 3 the following case remains: for some $j$ there exist no homomorphisms $\tilde{h}_j': U_j \to U_0$ with $\tilde{h}_j'(i | V_j) = h | V_j$, and hence

there exists $\tilde{h}_j: U_0 \to U_j$ with $\tilde{h}_j(h | V_j) = i | V_j$, i.e.

(4)
\[
\begin{array}{c}
U_0 \xleftarrow{i} V_1 \xleftarrow{0} \\
\downarrow h|V_1 \\
\tilde{h}_j \xleftarrow{h|V_1} U_0
\end{array}
\]

is commutative.

Under the assumption (4) we shall show that we obtain the second half b) in (1). We pick one $U_i'$ in the set $U$ consisting of all the $U_j$ satisfying (4), and take the subset $T = \{U_{i'} | \text{Soc}(U_{i'}) \approx \text{Soc}(U_{i'}) \}$ of $U$. Now we finally choose a largest one in $T$ with respect to the relation $\leq$ given after Lemma 2, say $U_i$. Then $U_i$ is $U_j$-injective for any $U_j (\neq U_i)$ in $T$ by Lemma 2.
Here we fix $U_i$ and $\tilde{h}_i$, i.e.

\[(4') \quad \begin{array}{c}
U_j \leftarrow V_j \leftarrow 0 \\
\downarrow h|V_j \\
\tilde{h}_j \\
U_0 \\
\end{array}
\]

is commutative.

**Step 5-1) $m=0$.** We assume $W=U$. Then we shall show the following (5) by induction on $p$ under the assumption (4) and (4').

There exists a new direct decomposition of $W_p = U_1 \oplus U_2 \oplus \cdots \oplus U_p = U_1 \oplus (\tilde{U}_2 \oplus \cdots \oplus \tilde{U}_p)$ such that $\tilde{h}_1 h|(W_p \cap V) = \pi_i(p)i|(W_p \cap V)$, i.e.

\[(5) \quad \begin{array}{c}
W_p \leftarrow V \cap W_p \leftarrow 0 \\
\downarrow h|V \cap W_p \\
\pi_i(p) \\
\downarrow \tilde{h}_1 \\
U_0 \\
\end{array}
\]

is commutative,

where $\pi_i(p): W_p \rightarrow U_i$ is the projection with respect to the second decomposition of $W_p$.

Here we assume temporarily that Step 5-1) is completed.

**Step 5-2) $m \neq 0$.** We shall show the following (5') again by induction on $p$ under the assumption (4), (4') and Step 5-1).

\[(5') \quad \text{There exists a new direct decomposition of } W_p = U_1 \oplus \cdots \oplus U_n \oplus I_i, \quad \text{\(\oplus \cdots \oplus I_p = U_1 \oplus (\tilde{U}_2 \oplus \cdots \oplus \tilde{U}_m \oplus I_i \oplus \cdots \oplus I_p)\)} \text{ such that } \tilde{h}_1 h|(W_p \cap V) = \pi_i(p)i|(W_p \cap V).
\]

If we take $W_p = W$ in (5) or (5'), then we obtain b) of (1). Now if $p=1$ in Step 5-1), then (4') is nothing but (5). Further Step 5-1) is similar to Step 5-2), and hence we may show them in the cases $p=n$ and $p=m$. By $X$ we denote $U_n$ if $m=0$ (resp. $I_m$ if $m \neq 0$). Put

\[(6) \quad g = \tilde{h}_1 h: V \rightarrow U_1, \quad W_* = W - X.
\]

**Step 6 New decompositions by induction.** We may show (5) and (5') on $W_p=W$ under the assumption that $W_*$ satisfies (5) and (5'). Namely we obtain a new direct decomposition of $W_*$ by induction hypothesis

\[(7) \quad W_* = U_1 \oplus (U'_2 \oplus \cdots \oplus U'_{m-1}) \quad \text{if } m=0 \quad \text{or} \quad \text{if } m \neq 0,
\]
and a commutative diagram:

\[
\begin{array}{c}
W_\ast \xrightarrow{i} V \cap W_\ast \leftarrow 0 \\
\downarrow h | V \cap W_\ast \\
\pi' \downarrow \rightarrow h \\
U_0 \rightarrow \rightarrow U_1
\end{array}
\]

where \(\pi'_1: W_\ast \rightarrow U_1\) is the projection with respect to the direct decomposition above, i.e.,

\[
g | (V \cap W_\ast) = \pi'_1 i | (V \cap W_\ast)
\]

We fix the direct decomposition of \(W_\ast\) (7). Using (7) we have a decomposition \(W=W_\ast \oplus X\). We consider the diagram:

\[
\begin{array}{c}
X \xleftarrow{i} X \cap V \leftarrow 0 \\
\downarrow h \\
\downarrow g | (X \cap V) \\
U_0 \rightarrow \rightarrow U_1
\end{array}
\]

**Step 7** Existence \(\bar{h}_2: X \rightarrow U_1\). We divide the argument into two cases in (8).

**Step 7-1** \(X=U_0\) \((m=0)\).

i) \(X \in T\). Since \(U_0\) is almost \(X\)-injective, we have the following two cases.

i-1) There exists \(\bar{h}_2': X \rightarrow U_0\) which makes the following diagram commutative:

\[
\begin{array}{c}
X \xleftarrow{i} X \cap V \leftarrow 0 \\
\bar{h}_2' \downarrow \rightarrow h | (X \cap V) \\
U_0 \rightarrow \rightarrow U_1
\end{array}
\]

Putting \(\bar{h}_2 = \bar{h}_2' \bar{h}_1\), we obtain

\(\bar{h}_2: X \rightarrow U_1\) with \(\bar{h}_2 i | (X \cap V) = g | (X \cap V)\).

i-2) There exists \(\bar{h}_2': U_0 \rightarrow X\) satisfying (4). Since \(X \in T\), \(\text{Soc}(X) \cong \text{Soc}(U_i)\) from the definition of \(T\). Hence \(U_i\) is \(X\)-injective by Proposition 2, and so there exists

\(\bar{h}_2: X \rightarrow U_1\) in (8) with \(\bar{h}_2 i | (X \cap V) = g | (X \cap V)\).

ii) \(X \in T\). From the choice of \(U_1\) and Lemma 2 \(U_i\) is \(X\)-injective. Hence we are in the same situation as in i-2).

**Step 7-2** \(X=I_m\). Then \(U_1\) is \(X\)-injective by Proposition 2. Thus in any cases we obtain
Step 8  **Final decomposition**  We shall apply \([2]\), the proof of Lemma \(C\) to \((7')\) and \((9)\) (use the assumption \((\#)\) and note the definition of \(f'_i\) in \([2]\), p. 689). Taking \(\pi_1 \otimes \tilde{h}_x: W = W_* \oplus X \to U_1\), we can get a homomorphism \(\bar{h}_x: X \to U_1\) such that

\[
\text{g} = \pi_1 \otimes (\tilde{h}_x + \bar{h}_x') | V
\]

from \([2]\), the proof of Lemma \(C\) (cf. \(f\) in p. 690). Put \(X(-\tilde{h}_x - \bar{h}_x') = \{ -\tilde{h}_x(y) - \bar{h}_x'(y) + y | y \in X\} = W = U_1(\bar{U}_2 \oplus \cdots) \oplus X(-\tilde{h}_x - \bar{h}_x')\). Let \(\pi_1\) be the projection of \(W\) onto \(U_1\) with respect to the above decomposition. We shall show

\[
\pi_1 | V = g = f'_i h.
\]

**Corollary 1.**  Let \(U_0, \{U_j, I_j\}\) be modules satisfying \((*)\) as in Theorem 2. We assume further that \(\text{Soc}(U_j) \neq 0\) for all \(j\) and that for every pair \((i, j)\) either \(U_i\) is \(U_j\)-injective or \(U_j\) is \(U_i\)-injective (e.g., \(\text{Soc}(U_i) \cong \text{Soc}(U_j)\)). Then \(U_0\) is almost \(\Sigma I_j \oplus U_j \oplus \Sigma I_i \oplus I_j�\)-injective if and only if \(\Sigma I_j \oplus U_j\) is an extending module.

Proof.  We note that in the above proof we used only once the assumption, artinian, in Step 4. However it is available to use the same argument in Step 4 from the remark before Theorem 2. Hence the proof is clear from the proof of Theorem 2.

**Corollary 2** ([2], Theorem).  Let \(\{U_0, U_j\}_{i=1}^n\) be \(\text{LE, artinian and uniform modules}\). Then \(U_0\) is almost \(\Sigma I_j \oplus U_j \oplus \Sigma I_i \oplus I_i�\)-injective if and only if

1) \(U_0\) is almost \(U_i�\)-injective for all \(i\).

2) For any pair \(U_i, U_j\) \((i \neq j)\) either \(U_0\) is simultaneously \(U_i�\) and \(U_j�\)-injective or \(U_i \oplus U_j\) has \(\text{EPSM}\).

Proof.  From Proposition 2 and the assumptions 1) and 2) we know that if \(U_0\) is not \(U_k�\)-injective for \(k = i, j\), then \(U_i\) and \(U_j\) are almost relative injective each other, i.e., \(U_i \oplus U_j\) is an extending module. Hence \(U_0\) is almost \(\Sigma I_j \oplus U_j�\)-injective by Theorem 2 and \([3]\), Theorem 4. Conversely if \(U_0\) is almost \(\Sigma I_j \oplus U_j�\)-injective, we have trivially 1). If \(U_0\) is not \(U_i�\)-injective, then \(U_i\) is almost \(U_j�\)-injective by Corollary to Lemma 1, Hence \(U_i \oplus U_j\) has \(\text{EPSM}\).
Remark. The second part of the proof of Theorem 2 is categorical. Hence it is available to get a dual version for almost relative projectives.

References


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