Fracture Analysis of Ductile Materials by Mean of Recent Fracture Theory ($MN - r_p - \sigma_\theta$)†

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Abstract

The current theory is based on the original version of ($MN - r_p \quad \sigma_\theta$) theory for brittle and quasi-brittle materials. It was developed with Linear Elastic Fracture Mechanics LEFM to study the cracking in the elastic stage for non ductile materials and materials with low ductility. In this research a new of the theory version for the fracture of pure ductile materials and fracture in brittle – ductile transition for semi brittle semi ductile materials is proposed. This new proposed theory for ductile fracture is called the DMN-ductile theory. It has a new concept and a new mechanism. It proposed that the fracture direction can be predicted in the elastic stage in LEFM where the critical fracture load will include two parts one from the elastic stage and the other part from the plastic stage. The fracture load would be determined with Elastic – Plastic Fracture Mechanics EPFM. The direction of the fracture will be in LEFM in the beginning of the propagation and will extend in the same direction in the plastic stage for the same fracture increment, but it may change in the nest fracture problems. In addition, it unifies all views of researches on ductile fracture energy approach. It could introduce a new concept for the ductile fracture process for dealing with any crack and for predicting any required fracture aspect.

KEY WORDS: (Ductile material), (Fracture), (Fracture Theory), (Cracking mechanism), (Failure analysi)s.

1. Introduction

Failure analysis for ductile materials was studied using energy theory based on the scalar approach [1-12]. In this paper we study the ductile fracture using the energy method in a new manner. The study includes mode I, mode II, mixed mode under tension and mixed mode under compression. This study introduces a new theory for ductile fracture based on directional concepts by considering the direction of the fracture factor of each cracking mode [16-17]. This theory considers also the direction of the shear stress in ductile fracture. In the case of compression loading the effect of friction between crack surfaces is considered and regarding a negative stress intensity factor at the crack tip of mode I crack under compression stresses. It is a face that failures of pure ductile structures and ductile-brittle structures are different to the failures of pure brittle or quasi-brittle structures because ductile materials exhibit before the failure a large amount of plastic deformation and a continuous evolution of the elastic-plastic boundaries around the crack tip in Large Scale Yielding (LSY). In such cases Linear Elastic Fracture Mechanics LEFM assumptions will be invalid. In brittle materials, the failure could be studied in linear elastic fracture mechanics analysis (LEFM) because the failure in this case usually occurs after Small Scale Yielding (SSY) at the crack tip, while for the ductile structures the material exhibits a large amount of plastic deformation LSY at the crack tip before failure. In this regard the ductile criterion is proposed. The developed criterion is called (DMN) theory. This criterion represents the ductile version of the NN-theory [16-19]. Some researchers proposed other concepts such as a minimum total strain energy criterion [4] and a maximum dilatational strain energy density criterion [3]. These criteria have not been successful. The reason is related to the basic concept of those theories. For example the S- criterion [4] uses the total scalar amount of strain energy density ignoring the fact that the components of the strain energy density have different effects in different types of crack. These effects have a significant role for the fracture aspects, which control the failure of structures. On the other hand, although the T-criterion[3] depends on the components of the strain energy density, which are dilatational and distortional, it assumes that the crack usually extends in the direction of maximum dilatational strain energy density factor for mode I, mode II and mixed mode. This assumption is valid only for mode I but invalid for mode II, according

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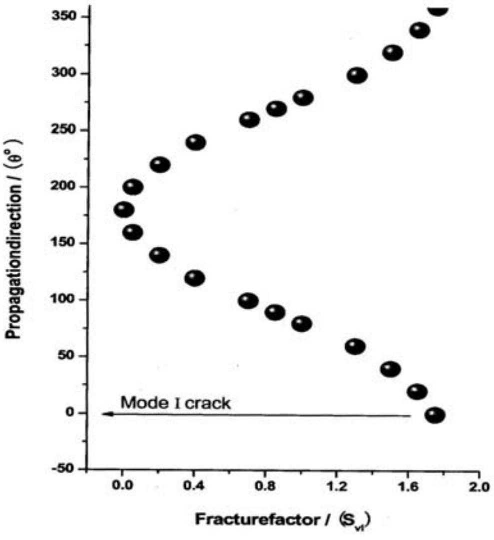

Fig. 1 Representaion of S_{vI}.

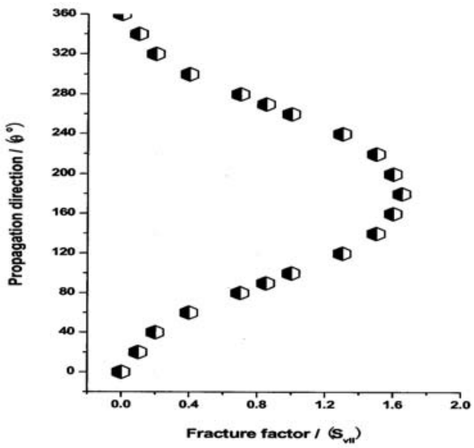

Fig. 2 Representaion of SvII.

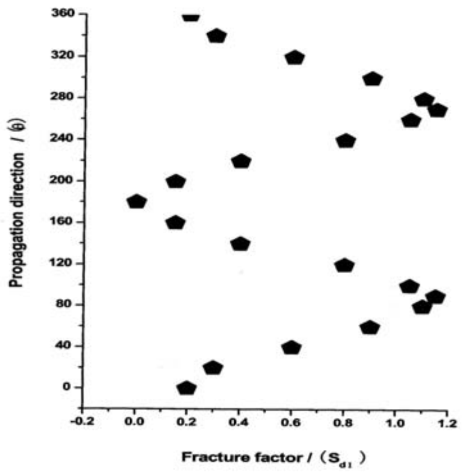

Fig. 3 Representation of S_{dI}.

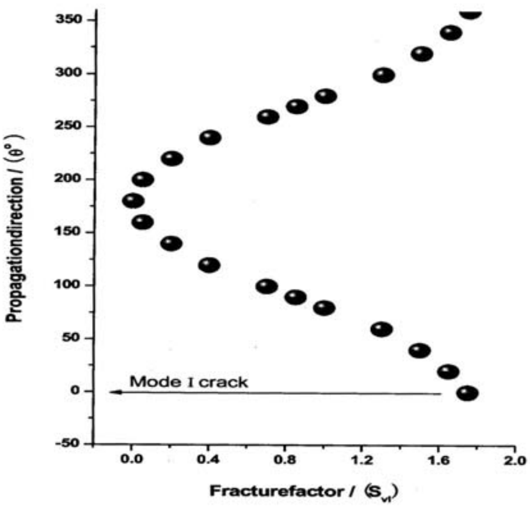

Fig. 4 Representation of SsII.

to the influence of the maximum volumetric strain energy density factor at the crack tip for mode I and mode II as shown in **Figs.1 to 4**. So these theories can not predict mixed mode in addition to mode II cracks.

The concept and fundamental analytical hypotheses of a new theory for ductile failure are introduced. This criterion is called the ductile version of the MN- Criterion (DMN). This criterion depends on the HRR field [13-15] and energy approach [17-19]. By applying this criterion, the failure process, the failure load and cracking properties can be predicted easily. It can be applied for ductile and ductile-brittle materials in the form of Elastic Plastic Fracture Mechanics EPFM. Some researchers have devoted their attention to dealing with the cracks in ductile materials using the energy approach depending on the strain energy density criterion proposed by Sih [4] and the so called HRR field presented by Hutchinson, Rice and Rosengren [13-15]. These previous studies were in scalar form [19] which considers only the quantity of the fracture factors and neglects the effect of fracture direction. In this paper we treated the ductile fracture problem with a new method depending on NN theory [1-17] with a new concept of strain energy components; the dilatational and distortion strain energy density. Theocarais and others [9-11] used the volumetric component as the (T_{max}) criterion. Other researchers used the total strain energy density (S_{min}) criterion [5]. In this research we use the two components of strain energy density; volumetric component (S_v) and distortion component (S_d).

2. Aim and Purpose of Current Research

This work is concentrated on developing a new criterion to deal with the cracks of ductile materials in Elastic-Plastic Fracture Mechanics EPFM. In this paper we treated the ductile fracture problem with a new method depending on NN theory [1-17] with a new concept of strain energy components; the dilatational and

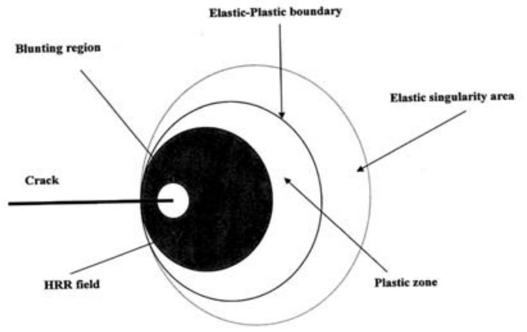

Fig. 5 Representation of HRR field at the vicinity of the crack

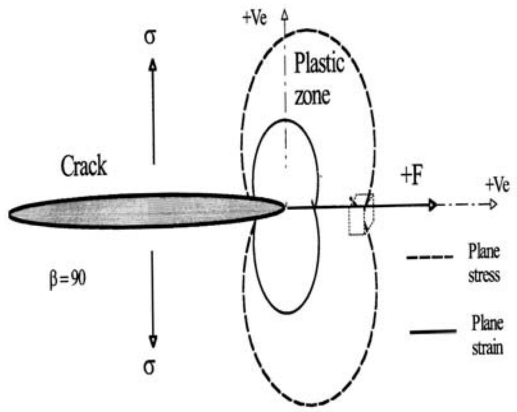

Fig. 6(a) Mode I tension case.

distortion strain energy density. This research is devoted to studying the fracture mechanism, fracture direction, fracture load and threshold fracture factor.

This theory represents an extension of the continued efforts to obtain a unified view for all ideas and proposals of the majority of the researchers in the field of development of the fracture theories to predict the mechanism and mechanical aspects of ductile cracking process. In this paper the new developed theory is called (DMN) theory. Depending on the observations of the behavior of different types of cracks during the experimental investigations, the fracture theories have new additional concepts. These concepts are summarized generally in two conditions of fracture for all crack modes. The first condition is responsible of predicting the expectations and probabilities of fracture direction of the crack when the fracture initiation starts. The second condition represents the fracture condition and fractur

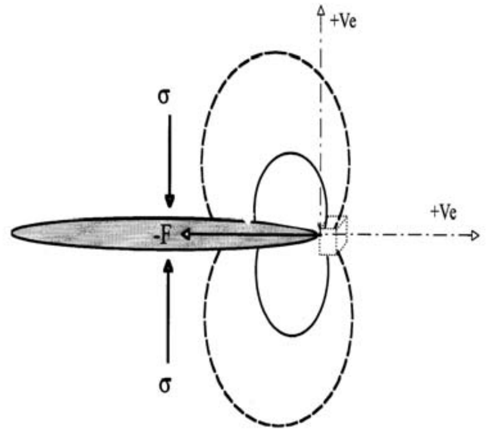

Fig. 6(b) Mode I compression case.

mechanism. It was recognized from experiment that the cracking process is usually accompanied by volume change of the crack under any mode after the plasticity conditions, while the direction of the fracture can be observed before the initiation of fracture during the elastic stage. Using the energy approach under the application of directional fracture approach, the new theory has been proposed. DMN theory proposes that a crack will extend in the direction of extreme plastic value of the energy component responsible for the shape change of the crack. Crack will start extending when the energy component responsible for the volume change of the crack reaches an extreme value. These assumptions can be applied for all cases of fracture modes. In the case of mode I the extreme value of the dilatational component will act in the same direction as the extreme value of the distortional one.

3. Theoretical Analysis Procedures Results

3.1 Singular stress field in a plastic zone around the crack tip under static load

In this research the theory is applied to the HRR field at the crack tip. The applied plastic singularity solution in the HRR field for the crack of a power law hardening material has been already given by Hutchinson, Rice and Rosengren [9-11] for mode I and mode II stress distribution with respect to the crack tip. This solution, HRR singular field; was completed by the solution for mixed mode stress distribution presented by Sih [5]. **Fig.5** represents the HRR singular field.

3.2 Fracture concept of the cracking mechanism

It was recognized from the experimental work [1] on ductile materials that the fracture direction usually can be recognized before the fracture initiation and also before the final formation of the large scale ductile plasticity at the crack tip. This means that the propagation direction should be determined firstly and then the plastic zone formation and then crack propagation will be the final step of the cracking process. This means that the

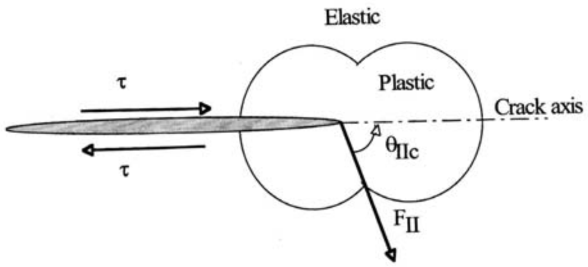

Fig. 7(a) Mode II clockwise case.

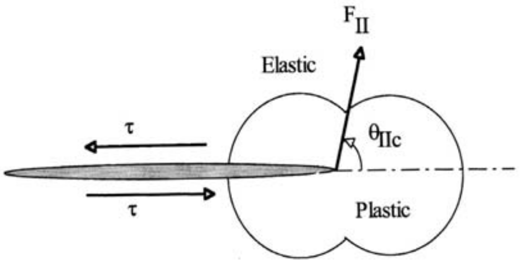

Fig. 7(b) Mode II anti-clockwise case.

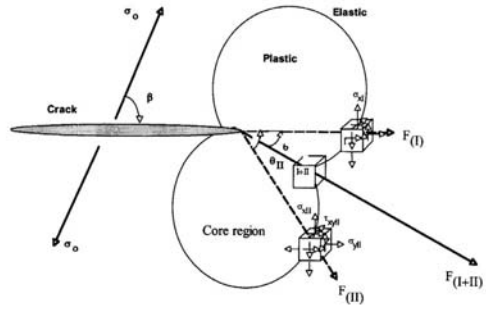

Fig. 8(a) Mixed modes tension case.

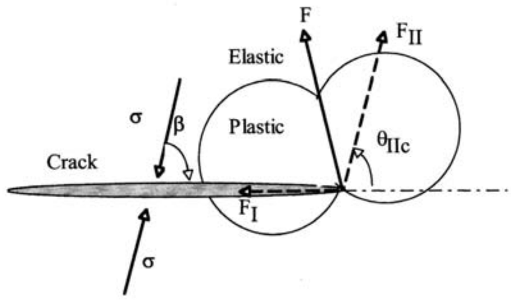

Fig. 8(b) Mixed modes compression case.

propagation direction will be recognized and determined in the elastic stage and the propagation of the crack will happen later, after the elastic and plastic stages. This process can be taken as a general phenomenon for all materials. The propagation direction will be determined in the elastic stage while the fracture will be after the plasticity which will be of small scale for quasi-brittle materials and of large scale for ductile materials. For pure brittle materials the propagation direction and initiation will be in the linear stage with the formation of large scale plasticity and a large amount of plastic energy in addition to the elastic energy. In this regard the new hypotheses of the current theory are presented. For taking into consideration all the possible factors which may affect the cracking process, the new theory called (MN) theory is applied.

3.3 General procedures of current applied cracking approach

The directional and qualitative assessment of the crack behavior is considered. It is very difficult to predict fracture aspects depending on the qualitative analysis only without regard do the direction of these values. The general hypotheses of ductile failure approach $DD\overset{\omega}{F}A$ are;

3.3.1 Single mode cracking conditions

(1) Cracks under pure mode I as shown in **Fig.6 (a)** and **Fig.6 (b)** and mode II cracks as shown in **Fig.7 (a)** and **Fig.7 (b)** will extend in the direction of the optimum elastic value of the applied fracture factor which may be maximum or minimum according to the hypotheses of the applied criterion.

(2) Crack will start to propagate when the elastic and plastic values of the applied fracture factor reach their critical value $\left(DD\overset{\omega}{F}A\right)_{cr}$

For mode I crack:

$$\left(DD\overset{\omega}{F}A\right)_{I}^{el+pl} \geq \left(DD\overset{\omega}{F}A\right)_{I_{cr}}$$

While for mode II crack:

$$\left(DD\overset{\omega}{F}A\right)_{II}^{el+pl} \geq \left(DD\overset{\omega}{F}A\right)_{II_{cr}}$$

3.3.2 Mixed mode cracking conditions

(1) Cracks under mixed mode; **Fig.8 (a) and Fig.8 (b)** will extend in the direction of the vector summation of the elastic values of the applied fracture factors of mode I and mode II for the material under consideration

$$\left(DD\overset{\omega}{F}A\right)^{\beta}=\left(DD\overset{\omega}{F}A\right)_{I}^{\beta} + \left(DD\overset{\omega}{F}A\right)_{II}^{\beta}$$

(2) Cracks will start to extend when the value of the summation of the elastic and plastic values of these factors reach its critical value

$$\left(DD\overset{\omega}{F}A\right)^{\beta} \geq \left(DD\overset{\omega}{F}A\right)_{cr}$$

$$\frac{[\partial s_{dI}]^{e}}{\partial \theta}=0$$

$$\frac{[\partial^{2} s_{dI}]^{e}}{\partial \theta^{2}} \leq 0$$

4. Theoretical Analysis of DMN – Theory and Ductile Failure Hypothesis

The fundamental hypotheses have been developed for all fracture modes (mode I, mode II and mixed modes) as the follows:

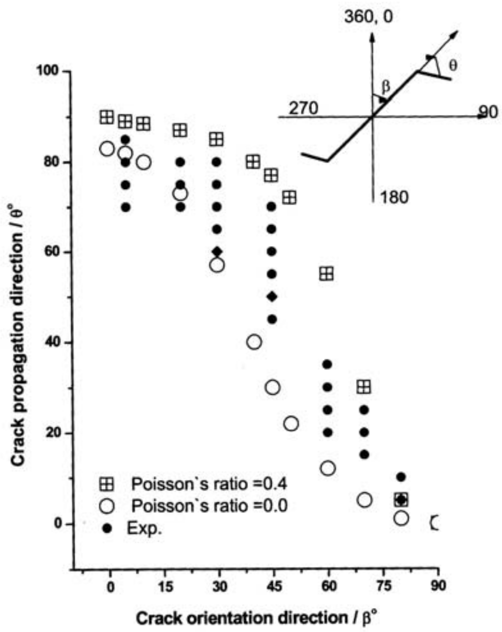

Fig. 9 Directuion of tension cracking.

4.1 Volume change mode crack (mode I)

4.1.1 Conditions of propagation direction

Mode I cracks under tension stress; as shown in Figs.6 (a) and Fig 6 (b); will extend in the direction of elastic minimum value of distortional strain energy density factor $(S_{dI})^{e}$ under the following conditions:

4.1.2 Conditions of propagating load

Cracks will start to extend when the elastic and plastic value of volumetric strain energy density of mode I $(S_{vI})^{e+p}$ reach the critical value

$$[S_{vI}]^{e+p} > S_{vIc}$$

Considering plastic stress intensity factor for mode I shown in **Fig.7 (a)**, where it is:

$$K_{I}^{p} = \left[\frac{kK_{I}^{2}}{\alpha\sigma^{2} I_{n}}\right]^{1/(n+1)}$$

$$I_{1,n} = \int_{-\pi}^{\pi}\left[\left(\frac{n}{n+1}\right)\bar{\sigma}_{e}^{(n+1)}\cos\theta - \left[\sin\theta\left\{\bar{\sigma}_{rr}\left(\bar{u}_{\theta} - \frac{\partial \bar{u}_{r}}{\partial\theta}\right) - \bar{\sigma}_{r\theta}\left(\bar{u}_{r} + \frac{\partial \bar{u}_{\theta}}{\partial\theta}\right)\right\} + \left\{\frac{1}{(n+1)}(\bar{\sigma}_{rr}\bar{u}_{r} + \bar{\sigma}_{r\theta}u_{\theta})\cos\theta\right\}\right]\right]$$

Where
k=1 for plane stress, k=(1-ν^{2}) for plane strain,
(σ)=tension stress,
K_{I}= σ (π a)$^{0.5}$, is elastic stress intensity factor for mode I,
(I_{n}) of **Fig.7 (b)** depends on strain hardening doefficient (n); Hutchinson, Rice and Rosengren [9-11].

In the case of mode I under compression stresses; as shown in Fig.6 (a) and Fig.6 (b); the crack will be closed and the strain energy components of (S_{dI})and (S_{vI}) will have negative directions (-S_{dI}) and (-S_{vI}).

4.2 Shape change mode (mode II crack)

4.2.1 Conditions of propagation direction

Cracks under mode II which may be under clockwise shear stress or under counter clockwise shear stress as shown in Fig.7 (a) and Fig.7 (b); will extend in the

$$\frac{\partial s_{dII}{}^{e}}{\partial \theta}=0$$

$$\frac{\partial^{2} s_{dII}{}^{e}}{\partial \theta^{2}} \leq 0$$

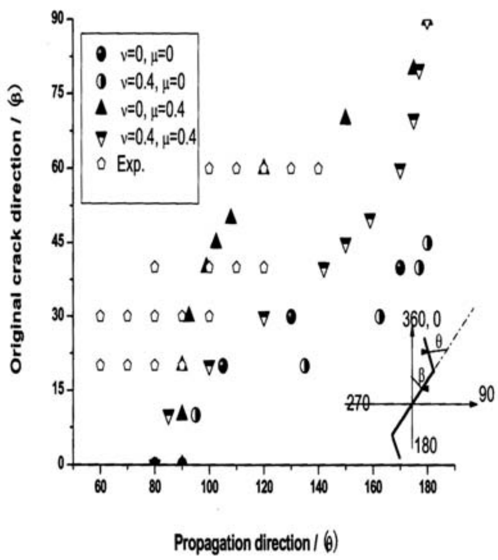

Fig. 10 Direction of compression cracking.

direction of elastic minimum value of distortional strain energy density; $(S_{dII})^e$. They can behave under the following conditions considering that the crack will extend in (+ve) direction under clockwise shear stress and vise versa in the case of crack under pure counter clockwise shear stress ($+\theta_{II}$) and ($-\theta_{II}$) respectively.

4.2.2 Condition of propagating load

Cracks under mode II will start the fracture when the elastic and plastic value of dilatational strain energy components (S_{vII}) reach the critical value under the following condition,

$$(S_{vII})^{e+p} > S_{vIIc}$$

regarding that plastic stress intensity factor for mode II is:

$$K_{\Pi}^{p} = \left[\frac{kK_{\Pi}^{2}}{\alpha \tau^{2} I_{n}} \right]^{1/(n+1)}$$

4.3 Cracks of mixed mode under tension and shear or compression and shear stresses.

4.3.1 Condition of propagation direction

Cracks under mixed mode will extend in the direction (θ_{I-II}) of the summation; (S_{dI-II}); of the vectors of the elastic distortional components of strain energy density factors of both mode I; (S_{dI}); and mode II; (S_{dII}); as in the following equations:

4.3.1.1 Mixed mode cracks under tension stresses

Considering that the crack will extend in a positive direction ($+\theta$) under tension and clockwise shear stresses; as shown in fig.8 (a) and Fig.8 (b), which it will extend in a negative direction; ($-\theta$);

$$\vec{S}_{vI-\Pi}^{tension} = \vec{S}_{vI} + \vec{S}_{v\Pi}$$

$$s_{vI-\Pi}^{tension} = s_{vI} \pm i s_{v\Pi}$$

$$\vec{S}_{vI-\Pi}^{tension} = \left[\left(s_{vI} + s_{v\Pi} \cos\theta_{\Pi} \right)^{2} + \left(s_{v\Pi} \sin\theta_{\Pi} \right)^{2} \right]^{1/2}$$

$$\vec{S}_{dI-\Pi}^{tension} = \vec{S}_{dI} + \vec{S}_{d\Pi}$$

$$s_{dI-\Pi}^{tension} = s_{dI} \pm i s_{d\Pi}$$

$$\vec{S}_{dI-\Pi}^{tension} = \left[\left(\varpi{S}_{dI} + \varpi{S}_{dII} \cos\theta_{II} \right)^{2} + \left(S_{dII} \sin\theta_{II} \right)^{2} \right]^{1/2}$$

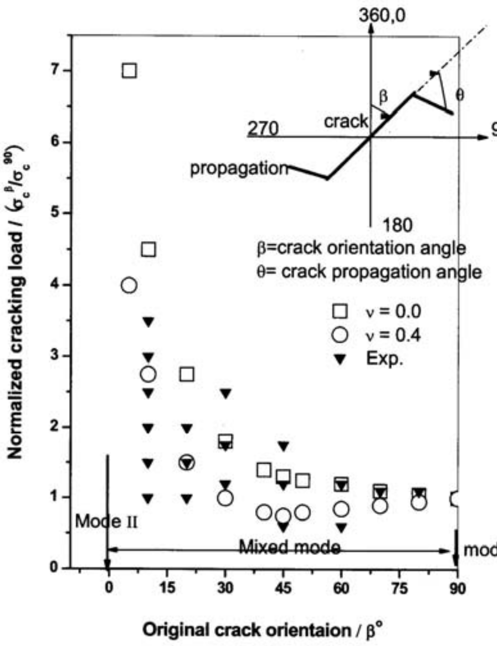

Fig. 11 Load of tension cracking.

under tension and counter clockwise shear stresses.

4.3.1.2 Mixed mode cracks under compression stresses

Considering that the crack will extend in vegative direction; (θ); under tension and clockwise shear stresses; as shown in Fig.8 (a) and Fig.8 (b); and vise versa.

$$s_{vI-\Pi}^{compression} = -s_{vI} \pm i s_{v\Pi}$$

$$\vec{S}_{dI-\Pi}^{compression} = \vec{S}_{dI} + \vec{S}_{d\Pi}$$

$$\vec{S}_{vI-\Pi}^{compression} = \vec{S}_{vI} + \vec{S}_{v\Pi}$$

$$s_{dI-\Pi}^{compression} = -s_{dI} \pm i s_{d\Pi}$$

$$\vec{S}_{dI-\Pi}^{compression} = \left[\left(-s_{dI} + s_{d\Pi} \cos\theta_{\Pi} \right)^{2} + \left(s_{d\Pi} \sin\theta_{\Pi} \right)^{2} \right]^{1/2}$$

$$\vec{S}_{vI-\Pi}^{compression} = \left[\left(-s_{vI} + s_{v\Pi} \cos\theta_{\Pi} \right)^{2} + \left(s_{v\Pi} \sin\theta_{\Pi} \right)^{2} \right]^{1/2}$$

4.3.2 Calculation condition for propagating load

Cracks will initiate the extension when the summation of the vectors of the elastic and plastic value of dilatational components of strain energy density factors; $(S_{vI-II})^d$; of both (S_{vI}) and (S_{vII})reach the critical value; $(S_{vI-II})c$; regarding the direction of the shear stresses as the following equations

(1) Mixed mode crack under tension stresses

$$(S_{VI-II})^{d\,(tension)} > (S_{VI-II})_c^{\,tension}$$

where

$$S_{vI-\Pi}^{tension} = S_{vI} + S_{v\Pi}$$

$$S_{vI-\Pi}^{tension} = \left[\left(s_{vI} + s_{v\Pi}\cos\theta_{\Pi}\right)^2 + \left(s_{v\Pi}\sin\theta_{\Pi}\right)^2\right]^{1/2}$$

(2) Mixed mode crack under compression stresses

$$(S_{VI-II})^{d\,(compression)} > (S_{VI-II})_c^{\,compression}$$

where

$$S_{vI-\Pi}^{compression} = S_{vI} + S_{v\Pi}$$

$$S_{vI-\Pi}^{compression} = \left[\left(-s_{vI} + s_{v\Pi}\cos\theta_{\Pi}\right)^2 + \left(s_{v\Pi}\sin\theta_{\Pi}\right)^2\right]^{1/2}$$

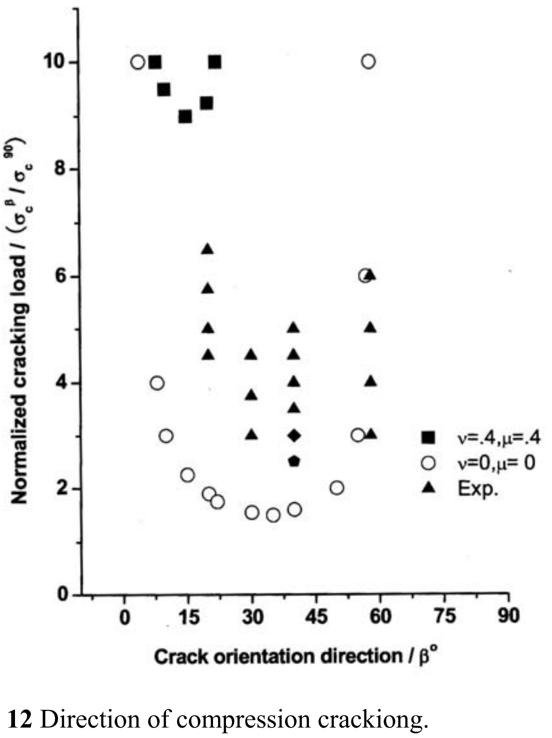

Fig. 12 Direction of compression crackiong.

5. Analytical Results

The analysis of fracture direction and fracture load of both mixed modes under tension and compression stresses can be summarized as follows:
Fig. 12 Direction of compression cracking.

5.1 Failure direction

According to the mentioned conditions of predicting of the fracture direction, the failure directions of mixed mode cracks can be obtained as follows:

$$[\pm\theta]_{cr}^{tension} = \pm\tan^{-1}\left[s_{d\Pi}\sin\theta_{\Pi cr}/\left(s_{dI} + s_{d\Pi}\cos\theta_{\Pi cr}\right)\right]$$

5.1.1 Mixed mode cracks under both normal static tension and in plane static shear stresses can be obtained from the plotted chart shown in Fig.9 based on the relation of Eq.32.

5.1.2 Mixed mode cracks under both normal static compression and in plane static shear stresses can be obtained from the plotted chart shown in Fig.10 based on the relation of Eq.33.

$$[\pm\theta]_{cr}^{compression} = \pm\left\{\tan^{-1}\left[\left(s_{dI} - s_{d\Pi}\cos\theta_{\Pi cr}\right)/s_{d\Pi}\sin\theta_{\Pi cr}\right] + 90\right\}$$

5.2 Failure load

The fracture load of mixed modes can be predicted in normalized relation to the fracture load of mode I. They can be determined as functions of the critical fracture factors of both mixed mode and mode I by developing the following:

5.2.1 Mixed mode cracks under both normal static tension and in plane static shear stresses can be obtained from the plotted chart shown in Fig.11 based on the relation of Eq.34.

$$\left(\sigma_{cr}^{\beta}/\sigma_{cr}^{90}\right)^{tension} = \phi\left(S_{vI-\Pi}^{tension}/S_{vIcr}^{tension}\right)$$

Where

σ^{β}_{cr}= critical load for crack at any orientation to applied load.

σ^{90}_{cr}= critical load for crack at orientation to applied load equal to 90°

$\sigma^{\beta}_{cr}/\sigma^{90}_{cr}$= normalized critical load of the crack at any orientation to the critical load of mode I crack.

5.2.2 Mixed mode cracks under both normal static compression and in plane static shear stresses can be obtained from the plotted chart shown in Fig.12 based on the relation of Eq.35.

$$\left(\sigma_{cr}^{\beta} / \sigma_{cr}^{90}\right)^{compression} = \phi\left(S_{vI-\Pi}^{compression} / - S_{vIcr}^{tension}\right)$$

Where

σ^{β}_{cr}= critical load for crack at any orientation to applied load.

σ^{90}_{cr}= critical load for crack at orientation to applied load equal to 90°

$\sigma^{\beta}_{cr}/\sigma^{90}_{cr}$= normalized critical load of the crack at any orientation to the critical load of mode I crack.

(-) negative sign of (S_{vIcr}) represent the direction of the vector of the fracture factor (S_{vI})

6. Discussions

From the results, the new developed theory; DMN-ductile theory regards all fracture factors with a logical concept and philosophy. It used all energy factors in only one point of view considering that each factor has a certain effect on the full fracture mechanism of the crack. It assumes that the shape change component of strain energy density; distortional strain energy density factor; is the main factor which responsible for the fracture direction at any step of the fracture process and the fracture direction can be determined in the elastic stage before the plasticity. It also assumed that the volume change component of strain energy density factor is responsible for the fracture initiation and crack propagation during the fracture process and fracture load can be determined considering the plasticity in addition to the elasticity. These assumptions have a unified view in applying all the fracture theories in the field of the fracture mechanics science.

The following discussions were derived from the results:

(1) The volume strain energy for both cracks under tension stress (S_{vI}) and cracks under shear stress (S_{vI}) are shown in Fig.1 and Fig.2 respectively. From Fig.1, it is shown that (S_{vI}) has critical values at each of the angles 0° , 180° , and 360° . Mode I crack usually start fracture in the direction of 0° , which is the same direction of 360° while mode I crack would be closed in the direction of 180° . Then, mode I opening mode crack will start propagation at the maximum value of S_{vI} while it will be closed at the minimum value of S_{vI}. From Fig.2 of the volume strain energy for mode II shape change crack mode, it wqs recognized that the critical values were at the directions of 0° , 180° , and 360° while mode II cracks usually propagate at a certain angle between 0° and 90° .

Then S_{vII} could not participate in predicting the direction of the mode II crack propagation.

(2) Figures 3 and 4 indicate the shape strain energy for mode I (S_{dI}) and mode II (S_{dII}). It was recognized that the minimum critical value of (S_{dII}) would determine the direction of fracture of shape change mode II crack.

(3) It should be recognized that all of the factors of volume strains energy (S_{vI}) (S_{vII}) and shape strain energy (S_{dI}) (S_{dII}) were in the elastic stage. The elastic strain energy factors are responsible for the determination of the fracture direction.

(4) Determination of propagation directions of cracks of mixed modes of tension and shear stresses or compression and shear stresses would depend on both [(S_{vI}) + (S_{dII})] or [(-S_{vI}) + (S_{dII})] respectively.

(5) The fracture direction will be in the same direction as minimum radius ($r_{p\,I\,min}$) or ($r_{p\,II\,min}$) of the plastic zone of mode I or mode II respectively or mixed modes as shown is Figs.5 to 8.

(6) The fracture direction will be in the same direction as maximum circumferential tensile stress ($\sigma_{\theta\,I\,max}$) or ($\sigma_{\theta\,II\,max}$) of the plastic zone of mode I or mode II respectively or mixed modes. The fracture directions for each of mode I, mode II and mixed modes for tension and in plane shear stress or mixed mode of compression and in plane shear stress are calculated and presented in simple relations with the parameters of crack orientation to the direction of loading as shown in Fig.9 and Fig.10.

(7) The cracks will propagate when all of strain energy factors, plastic zone radius and circumferential tensile stress reach the critical values in the same time at once.

(8) The critical loads at which the cracks would propagate were calculated and presented in simple relations with the parameters of crack orientations to the directions of loading as shown in Fig.11 and Fig.12 for all of mode I, mode II and mixed modes.

7. Conclusions

The proposed theory for ductile fracture is called DMS-ductile theory. This theory uses and advanced treatment of elastic-plastic fracture problems. In addition, it unifies all views of researches on ductile fracture energy approach. It could introduce a new concept for the ductile fracture process for dealing with any cracks for predicting any required fracture aspect.

The developed theory is a new version of (MN-r_p- σ_θ). It was developed to deal with quasi ductile and ductile materials in Elastic Plastic Fracture Mechanics EPFM. It includes the fracture analysis in LEFM before yielding and PFM after Large Scale Yielding LSY at crack tip. It can predict fracture direction at the end of the elastic stage and beginning of plastic stage in. It can predict the fracture load which will include both the elastic and plastic stages. The energy at the fracture will include the elastic and plastic values of energy. The analysis was developed at HRR field. The analysis is presented in a closed form solution. The results are plotted in charts to make it easy for use by researchers and engineers.

References

1) Daakhah M. S. and Kobayashi A. S., Eng. Frac. Mech., Vol. 34, No.1, pp. 253-262, (1989).

2) Theocaris P. S. and Philipidis T.P., Int. Journal of

fracture, Vol. 35, pp. 21-37, (1987).
3) Theocaris P. S., Eng. Frac. Mech., Vol. 30, No. 6, pp. 767-777, (1988).
4) Sih G.C. and Madenci, Eng. Frac. Mech., Vol. 6, pp.1159.1171, (1983).
5) Anderianopoulos N. P. and Theocaris P. S., Eng. Frac. Mech., Vol. 30, No. 1, pp. 5-12, (1988).
6) Zheng M., Luo, Z. J. and Zheng X., Eng. Frac. Mech., Vol. 41, No. 1, pp. 103-110, (1992).
7) Chow C. L. and Jilin Xu, Eng. Frac. Mech., Vol. 21, No. 3, pp. 537-545, (1985).
8) Shih C. F., Fracture Analysis, ASTM STP 560, pp. 187-210, (1974).
9) Theocaris P. S. and Anderianopoulos N.P., Int. J. of fracture, Vol. 20, pp. 125-135, (1982).
10) Theocaris P. S., Eng. Frac. Mech., Vol. 19, pp. 975-991, (1984).
11) Theocaris P. S., J. of Engineering Fracture Mechanics, Vol. 24, (1986).
12) Shih C. F., German M.D., Int. J. of Fracture, Vol. 17, pp. 27-43, (1981).
13) Hutchinson J. W., Journal of the Mechanics and Physics of Solids, Vol. 16, pp. 13-31, (1968).
14) Hutchinson J. W., Journal of the Mechanics and Physics of Solids, Vol. 16, pp. 337-347, (1968).
15) Rice J. R. and Rosengren G. F., J. of Mechanics & Physics of Solids, Vol. 16, pp. 1-12, (1968).
16) El-Sheikhy R. and Nishimura N., Proc. of NCFE&IMA, Korea, pp. 98-103, (1999).
17) El-Sheikhy R. and Nishimura N., Proceeding of Int. Conf. of Space Steel Structures., pp., Singapore, (1999).
18) El-Sheikhy R. and Naka M., Proceeding of Int. Conf. ISEC-03, Shunan, Japan, (2005).
19) El-Sheikhy R. and Naka M., Proceeding of Int. Conf. ISEC-03, Shunan, Japan, (2005).