Title	A Measuring Theory of Three Dimensional Residual Stresses in Long Welded Joints : L_y Method and L_Z Method(Welding Mechanics, Strength & Design)
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Citation	Transactions of JWRI. 1983, 12(1), p. 113-122
Version Type	VoR
URL	https://doi.org/10.18910/4479
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A Measuring Theory of Three Dimensional Residual Stresses in Long Welded Joints†

– L_y Method and L_z Method –

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Abstract

Two of the authors have already proposed the general principles for measurement of residual stresses. In this paper, the authors develop a new measuring theory of three dimensional residual stresses in a long welded joint by using effective inherent strains as parameters, which is based on one of these principles.

First of all, the characteristic of the inherent strain distribution induced in a long welded joint is clarified. This characteristic makes the measuring theory simple. Then, the measurement of three dimensional welding residual stresses in a very thick plate has become practically possible. When residual stress distributions are uniform along the weld line, the theory should assure the exact measured ones.

By this developed theory, the distributions of residual stresses in an electroslag welded joint are measured for the first time. And the estimated residual stresses show a fairly good coincidence with the directly observed stresses on its surfaces.

It can be concluded that the present theory is reliable and practically applicable to measure such complex three dimensional residual stresses.

KEY WORDS: (Residual Stresses) (Inherent Strains) (Three Dimensional Residual Stress) (Measurment of Residual Stress) (L_y Method) (L_z Method)

1. Introduction

It is very important to measure three dimensional residual stresses produced in the interior of a welded joint of a very thick plate by theoretically rational methods because such a welded joint is often applied in the recent large-scaled structures.

In this regard, a series of researches[1-5] have been carried out. In the first place, the general measuring principles of three dimensional residual stresses were proposed. They are (1) one in which inherent strains are dealt as parameters in measurement and (2) the other in which sectioned-forces are dealt as parameters. Based on these two principles, a basic theory was developed by the finite element method and a statistic approach. Validity and applicability of these principles are confirmed by numerical experiments[2].

In this report[5], a measuring theory of three dimensional residual stresses applicable to a long welded joint where residual stresses are uniform along the weld line is proposed based on the measuring principle of residual stresses in which inherent strains are dealt as parameters. Its applicability is proved by actual measurement of three dimensional residual stresses produced in an electroslag welded joint of a very thick plate.

2. Measuring Theory of Three Dimensional Welding Residual Stresses

In general, residual inherent strains are produced in the weld zone and its vicinity of a welded joint as the result of the thermal elastic-plastic phenomenon caused by welding. Some portion of residual inherent strains which contributes to a free expansion-contraction results in the movement of a rigid body and has no relation to residual stresses. The other major portion of inherent strains which directly corresponds to residual stresses is called

† Received on April 30, 1983

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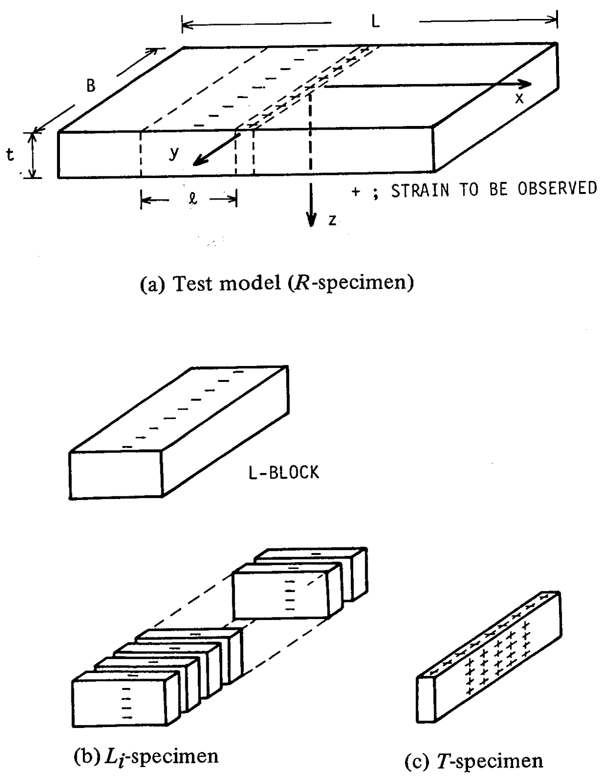
Transactions of JWRI is published by Welding Research Institute of Osaka University, Ibaraki, Osaka 567, Japan

effective inherent stains (hereafter simply called inherent strains). The measuring principle of residual stresses in which inherent strains are dealt as parameters is based on the idea that residual stresses can be obtained from estimated effective inherent strains because they are the source of residual stresses. Validity of this principle has already been basically confirmed by numerical experiments. For practical application of this principle, the measuring methods in which surface strains are observed by such as strain gages are shown here.

As shown in **Fig. 1**, the object of measurement, in this paper, is three dimensional residual stresses produced in the middle of the weld line in a cross section of a butt welded joint of a very thick plate. Taking the characteristic of the inherent strain distribution in the welded joint into consideration, the general measuring theory is simplified as in the following chapters.



(a) Test model (R-specimen)

(b) L_i-specimen

(c) T-specimen

Fig. 1 Test model and procedure of slicing T and L_i-specimens (L_y method)

2.1 Assumption employed in measuring theory

Generally, inherent stains are produced in a welded joint by a unsteady temperature distribution under the surrounding restraint. In a continuously long welded joint, the welding temperature distribution during welding is quasi-static except at the weld ends and in their vicinities, so that the temperature history at a corresponding position in each cross section is approximately identical. Therefore, if the restraint is uniform along the weld line, the residual stress distribution in a long welded joint along the weld line can be regarded as uniform except the portions near the weld ends. In this case, inherent strains are regarded as uniform also along the weld line. As is well known, welding residual stresses are approximately symmetric with respect to the central cross section ($x = 0$), so that inherent strains, γ^*_{xy} and γ^*_{zx}, which cause unsymmetric stresses can be disregarded. With this result, the following assumptions are made for development of measuring theories of three dimensional welding residual stresses.

(1) Variation in strains caused by cutting is elastic and no inherent strain is produced.
(2) Residual stresses in a thinly sliced plate are in the plane stress state (the plate needs be sliced such thin).
(3) Inherent strains are uniform along the weld line (x-axis) and composed of ϵ^*_x, ϵ^*_y, ϵ^*_z, and γ^*_{yz} which are functions of y and z coordinates.

2.2 Separation of three dimensional inherent strain components

$L_1 \sim L_m$ specimens (Fig. 1 (b)) and T-specimen (Fig. 1 (c)) are cut out from the original R-specimen (Fig. 1 (a)). In this case, inherent strains are still uniform along the weld line because cutting does not produce any new inherent strains. That is, inherent strain distributions which exist in T-specimen and L_i-specimens are the same as one existed in the original R-specimen.

On the other hand, these T and L_i-specimens are so thin that their inherent strain components which are perpendicular to respective plate surfaces do not contribute to their residual stresses. Therefore, residual stresses produced in a sliced T-specimen are composed of only cross-sectional inherent strains (ϵ^*_y, ϵ^*_z, and γ^*_{yz}) and those in a sliced L_i-specimen are composed of only longitudinal inherent strain (ϵ^*_x) (ϵ^*_z in each L_i-specimen is uniform along the weld line having no relation to residual stresses).

As three dimensional inherent strain components are separated into the cross-sectional inherent strains in T-specimen and the longitudinal inherent strain in L_i-specimen, measurement in each specimen can be regarded as a two dimensional problem. This is a significant simplifying method of the measuring theory of three dimensional residual stresses produced in a welded joint when inherent strains are uniform in one direction.

Three dimensional residual stresses $\{\sigma\}$ in R-specimen can be considered to be produced by the sum of inherent strains obtained respectively from T-specimen and L_i-specimens. Consequently, residual stresses $\{\sigma\}$ in R-specimen can be expressed as the sum of stresses $\{\sigma^A\}$ caused by only cross-sectional inherent strains and stresses $\{\sigma^B\}$ caused by only longitudinal inherent strain. That is,

$$\{\sigma\} = \{\sigma^A\} + \{\sigma^B\} \qquad (1)$$

In the next chapters, measuring methods for respective three dimensional stress components $\{\sigma^A\}$ and $\{\sigma^B\}$ are explained.

2.3 Three dimensional stresses $\{\sigma^A\}$ caused by cross-sectional inherent strains

It is possible to estimate $\{\sigma^A\}$ by the three dimensional elastic analysis. In the analysis, cross-sectional inherent strains (ϵ_y^*, ϵ_z^*, and γ_{yz}^*) obtained from the residual stresses of T-specimen are applied to the stress-free R-specimen. On the contrary to this, measurement of this $\{\sigma^A\}$ can be simplified by employing the relation explained in the next.

Inherent strains in a cross section are uniform in the x-direction. When only these inherent strains are given to R-specimen, its cross section at some distance inside from the weld ends still keeps plane even though it is deformed by the inherent strains (hereafter, this type of deformation is called plane deformation). Moreover, stresses produced by giving these inherent strains to the plane strain state ($\epsilon_x = 0$) are balanced in the y-z plane, so that longitudinal stresses produced by this restraint of strains perpendicular to the cross section (plane strain) are also self-equilibriating. That is, the above mentioned plane deformation coincides with the plane strain state[3]. In order to keep stresses in such plane strain state, a cross section needs be distant from the weld ends approximately by the plate thickness.

Residual stresses $\{\sigma^{Ao}\} = \{0, \sigma_y^{Ao}, \sigma_z^{Ao}, \tau_{yz}^{Ao}, 0, 0\}^T$ in the sliced T-specimen are in the plane stress state and can be directly observed. Regarding the relation between the plane stress and the plane strain, the following equations are formulated.

$$\begin{aligned} &\sigma_x^A = \nu\,(\sigma_y^{Ao} + \sigma_z^{Ao})/(1-\nu^2) \\ &\sigma_y^A = \sigma_y^{Ao}/(1-\nu^2),\ \sigma_z^A = \sigma_z^{Ao}/(1-\nu^2) \qquad (2) \\ &\tau_{yz}^A = \tau_{yz}^{Ao}/(1-\nu^2),\ \tau_{xy}^A = \tau_{zx}^A = 0 \end{aligned}$$

where, ν: Poisson's ratio

σ_y^{Ao}, σ_z^{Ao}, τ_{yz}^{Ao}: Residual stresses in T-specimen

With measured $\{\sigma^{Ao}\}$, Eq. (2) makes it possible to directly obtain three dimensional stresses $\{\sigma^A\}$. There is no need to estimate cross-sectional inherent strains because the three dimensional elastic analysis is no more necessary.

Longitudinal elastic strain ϵ_x produced in R-specimen by three dimensional inherent strains (ϵ_x^*, ϵ_y^*, ϵ_z^*, γ_{yz}^*) are divided into ϵ_x^A caused by cross-sectional inherent strains and ϵ_x^B caused by longitudinal inherent strain. The former inherent strain is $\epsilon_x^A = 0$ since it forms the plane strain state in R-specimen. In this consequence, the longitudinal elastic strain produced in the portion of R-specimen approximately at the distance of its plate thickness inside from the weld ends is produced by only longitudinal inherent strain ϵ_x^*.

$$\epsilon_x = \epsilon_x^B \qquad (3)$$

2.4 Three dimensional stresses $\{\sigma^B\}$ produced by longitudinal inherent strain

In this case, the standard method of the measuring theory in which inherent strains are dealt as parameters has to be applied. By the method, inherent strain ϵ_x^* is obtained and three dimensional stresses $\{\sigma^B\}$ are estimated by an elastic analysis using the measured inherent strain ϵ_x^*.

Observation on L_i-specimen aims at estimation of inherent strains ϵ_x^*. Therefore, it is indispensable that ϵ_x^* has an explicit relation with elastic strains remain in sliced L_i-specimens. It is deduced from this that the longitudinal axis of L_i-specimen has to be parallel to the x-axis of R-specimen. On the other hand, the slant angle the surfaces of thin L_i-specimen to the surface of R-specimen may be arbitrary. There are two practical methods of slicing L_i-specimens. One is called L_z-methods by which the normal line to the surfaces of L_i-specimens coincides with the z-direction. The other is L_y-method by which the normal line is set in the y-direction. Details of each methods are described in the next.

2.4.1 Measurement by L_z-method[3]

Generally, inherent strains exist in a welded joint for the narrower range than the plate breadth. Accordingly, ϵ_x^* which exists in L_i-specimens sliced by L_z-method produces residual elastic strains there proportionally to its value. That is, it works as the effective inherent strain. For this reason, each sliced L_i-specimen is attached with strain gages and then divided into pieces in order to measure elastic strains. Applying thus attained data to the observation equation $[H^*]$[2,3] formulated for the plane stress state in each L_i-specimen by the finite element method, the most probable value of inherent strain ϵ_x^* in each L_i-specimen can be estimated. The distribution of this inherent strain in each L_i-specimen determines the distribution of ϵ_x^* in R-specimen.

Application of this inherent strain distribution to stress-free R-specimen enables us to obtain three dimensional stresses $\{\sigma^B\}$ by a three dimensional stress analysis using the finite element method. Actual application of this L_z-method to a real welded joint has already been shown in reference 3).

2.4.2 Measurement by L_y-method[4)5)]

Owing to the fact that the inherent strain distribution in a welded joint usually penetrates through the plate thickness (z-axis), the evaluating method of ϵ_x^* by L_y-method differs more or less from that of L_z-method. In this case, the longitudinal effective inherent strain ϵ_x^* can be expressed as the sum of the component ϵ_{xl}^* which linearly changes along z-axis and the other nonlinear component ϵ_{xn}^*.

$$\epsilon_x^*(y, z) = \epsilon_{xl}^*(y, z) + \epsilon_{xn}^*(y, z) \qquad (4)$$

where,

$$\epsilon_{xl}^*(y, z) = a_1(y) + a_2(y) \cdot z \qquad (5)$$

$$\int_0^t \epsilon_{xn}^*(y, z)\, dz = \int_0^t \epsilon_{xn}^*(y, z) \cdot z dz = 0 \qquad (6)$$

a_1, a_2 : functions of only y-ordinate

The first term of the right side of Eq. (4) which is the component linearly distributes at $y = y_i$ along z-axis in L_i-specimen works as the effective inherent strain in R-specimen. However, slicing L_i-specimen out of R-specimen destroys the continuity of the L_i-specimens in y-direction and extinguishes the restraint, so that ϵ_{xl}^* which linearly distributes in L_i-specimen turns to be a noneffective component which does not contribute to residual stresses in the sliced L_i-specimen. Therefore, it is only the nonlinear component, ϵ_{xn}^*, that can be estimated from the residual elastic strain observed in L_i-specimen by the same method as L_z-method. For this reason, in order to determine coefficients, a_1 and a_2, of the linear component ϵ_{xl}^*, L_y-method requires not only observation of residual strains in L_i-specimen but also that of relaxed strains produced by slicing L_i-specimen out of R-specimen.

Generally, the length of a welded joint is sufficiently long in comparison with the plate thickness t, so that the length of L_i-specimen l can exceed $2t$. In this case, it is possible to separate inherent strains into the linear component ϵ_{xl}^* and the nonlinear component ϵ_{xn}^* for the convenience of measurement.

(a) Characteristic of strain distribution in the transverse cross section of R-specimen at $x = x_o$

The strain distribution produced in the transverse cross section of R-specimen in the position $x = x_o$ ($|x_o| < \frac{L}{2} - t$) distant from the weld ends by its plate thickness or more is taken into consideration. Stresses produced by cross sectional inherent strains in this position constitute the plane strain state. Therefore, as shown by Eq. (3), the elastic strain ϵ_x is produced only by the longitudinal inherent strain ϵ_x^*. Deformation produced by this ϵ_x^* in the x-direction of R-specimen corresponds to that which is produced by the x-directional equivalent stress σ_x^{eq} given to the surfaces of the weld ends in the opposite direction. This equivalent stress σ_x^{eq} is produced if the inherent strain ϵ_x^* is imposed on R-specimen keeping the plane strain state in the x-direction. Based on Saint-Venant's principle, the assumption of deformation of a plane plate, "a straingt line perpendicular to a neutral plane of a plane plate sustains its straightness and normality even after the plate is deformed", is considered to be applicable to the portion which is distant from loading points (weld ends) by the plate thickness or more. In this accordance, the whole longitudinal strain e_x in the abovementioned position $x = x_o$ is considered to be straight in the plate thickness direction (generally, they are curved in the plate breadth direction).

At $x = x_o$ in R-specimen,

$$e_x(y, z) = b_1(y) + b_2(y) \cdot z \qquad (7)$$

where, b_1, b_2 : functions of only y-ordinate

On the other hand, like the inherent strain, elastic strain produced in the same portion of R-specimen can be expressed as the sum of the linear component ϵ_{xl} and the nonlinear component ϵ_{xn} which are in the interior of the plate.

At $x = x_o$ in R-specimen,

$$\epsilon_x(y, z) = \epsilon_{xl}(y, z) + \epsilon_{xn}(y, z) \qquad (8)$$

ϵ_{xl} may be expressed as the following equation.

$$\epsilon_{xl} = a_3(y) + a_4(y) \cdot z \qquad (9)$$

where, a_3, a_4 : functions of only y-ordinate

ϵ_{xn} is defined so as to satisfy the condition of Eq. (10).

$$\int_0^t \epsilon_{xn}(y, z)\, dz = \int_0^t \epsilon_{xn}(y, z) \,.\, zdz = 0 \qquad (10)$$

(b) Characteristic of strain distribution in the middle of L_i-specimen

L_i-specimen is cut out from R-specimen including the position where $y = y_i$, setting the x-coordinate of the center of Li-specimen equal to the above-mentioned x_o and making the length l longer than $2t$. The longitudinal center of thus sliced L_i-specimen is distant from its ends by its plate thickness or more. Based on the Saint-Venant's principle, therefore, residual stresses in its longitudinal central portion are under no influence of the boundary. Accordingly, the whole strain e_x which corresponds to the longitudinal deformation forms a straight line in the central portion.

In the middle of L_i-specimen,

$$e_x(z) = b_3 + b_4 z \qquad (11)$$

where, b_3, b_4: constants

As stresses in the middle portion of L_i-specimen are in the longitudinal one axial state, the residual elastic strain ϵ_x satisfies the following equation provided that Young's modulus E is constant within L_i-specimen.

$$\int_0^t \epsilon_x dz = \int_0^t \epsilon_x z dz = 0$$

As the result of comparison of this equation with Eq. (10), it is found in the middle of L_i-specimen that,

$$\epsilon_x(z) = \epsilon_{xn}(y_i, z) \qquad (12)$$

The whole strain (Eq. (11)) produced in L_i-specimen is the sum of inherent strain given by Eq. (4) and elastic strain given by Eq. (12), that is,

$$\epsilon_{xn} + \epsilon^*_{xn} = (b_3 - a_1) + (b_4 - a_2)z$$

The above equation can be integrated under consideration of Eq. (6) and Eq. (10) and the result is,

$$b_3 = a_1, \quad b_4 = a_2$$

In the middle of L_i-specimen, the following relation is obtaind.

$$\epsilon_{xn} = -\epsilon^*_{xn} \qquad (13)$$

(c) Measuring method by which inherent strain ϵ^*_x is separated into linear and nonlinear components

The whole strain produced in R-specimen (Eq. (7)) is the sum of inherent strains (Eq. (4)) and elastic strains (Eq. (8)), from which the next equation is derived.

$$b_1 + b_2 z = \epsilon^*_{xl} + \epsilon^*_{xn} + \epsilon_{xl} + \epsilon_{xn} = \epsilon^*_{xl} + \epsilon_{xl} \qquad (14)$$

This equation indicates that the linear component ϵ_{xl} of elastic strains is produced only by the linear component ϵ^*_{xl} of inherent strains. The strain in the x-direction which is relaxed by cutting L_i-specimen out of R-specimen is produced by the discontinuation of the restraint of ϵ^*_{xl} in the y-direction. Therefore, it coincides with the linear component of elastic strains in R-specimen. On the contrary, the nonlinear component ϵ_{xn} is not relaxed at all by this cutting because the deformation produced in the x-direction of L_i-specimen by the inherent strain ϵ^*_{xn} is completely restrained as shown by Eq. (13).

The strain component ϵ_{xl} in R-specimen can be estimated when the linear coefficient in Eq. (9) is determined by calculating relaxed strains produced by cutting L_i-specimens out of R-specimen which is attached with strain gages on the top and bottom surfaces at $x = x_o$. On the other hand, the nonlinear component ϵ_{xn} of elastic strains coincides with elastic strains remain in L_i-specimens and can be directly observed on each small rectangular prism cut parallel to the surfaces from a specimen which is attached with strain gages on the previously sectioned surfaces.

As is stated above, when the inherent strain ϵ^*_x is presumed to be the sum of the linearly changing component in the plate thickness direction, ϵ^*_{xl}, and the nonlinear component ϵ^*_{xn}. The elastic strain relaxed by cutting L_i-specimen out of R-specimen which is linearly changing in the z-direction is produced by the linear component of the inherent strain in R-specimen. While, the elastic strain remains in L_i-specimen is composed of the nonlinear component ϵ_{xn} produced by the nonlinear component ϵ^*_{xn} of the inherent strain in R-specimen. Consequently, the inherent strain ϵ^*_x can be measured being separated into the linear component ϵ^*_{xl} and the nonlinear component ϵ^*_{xn}.

An observation equation of R-specimen $[H^*]$ is formed by the finite element method using the plate bending elements, so that the linear component ϵ^*_{xl} can be estimated from the general inverse matrix of $[H^*]$ using the relaxed strains ϵ_{xl} as observed values. On the other hand, the nonlinear component ϵ^*_{xn} can be directly estimated from Eq. (13) if the elastic strain ϵ_{xn} is observed in the middle of L_i-specimen.

In this way, the longitudinal inherent strain distribution can be completely estimated as the sum of the linear and the nonlinear components. Application of this longitudinal inherent strain distribution to R-specimen enables it to measure three dimensional stresses $\{\sigma^B\}$ by the three dimensional elastic analysis.

Three dimensional residual stresses $\{\sigma\}$ in the original R-specimen can be estimated as the sum of this $\{\sigma^B\}$ and stresses $\{\sigma^A\}$ already mentioned in section 2.3 as to be caused by the cross sectional inherent strains.

As a result, three dimensional residual stresses produced in a certain portion along the weld line can be accurately measured based on the so far stated measuring theory.

3. Measurement of Three Dimensional Residual Stress Distribution in a Welded Joint of a Very Thick Plate

Based on the already developed measuring theory of three dimensional residual stresses in which inherent strains are dealt as parameters, residual stresses in an electroslag welded joint of a very thick plate are actually measured. In this result, applicability and reliability of this measuring theory are demonstrated. L_i-specimens are sliced according to the procedure of L_y-method.

3.1 Specimen and observing points

Initial residual stresses are removed from a thick plate by stress relief annealing prior to the welding. The material of the plate is *SM 50 B*. **Figure 2** depicts the process of producing R-specimen, that is, two very thick plates are joined together by electroslag welding with endtabs attached to the both ends. Then some portions are cut off at the respective weld ends. The welding conditions are shown in **Table 1. Figure 3** shows the size, the system of coordinates of R-specimen and the locations where T-specimen and L_i-specimens are cut out. L_i-specimens are cut out according to the procedure of L_y-method with dimensions of $l=2.5t=250\,mm$. In order to investigate the reliability of residual stresses estimated by this measuring method, directly observed stresses are needed. For this observation, the location of the strain observing points on the top and bottom surfaces of R-specimen are also shown. Observing points of strains

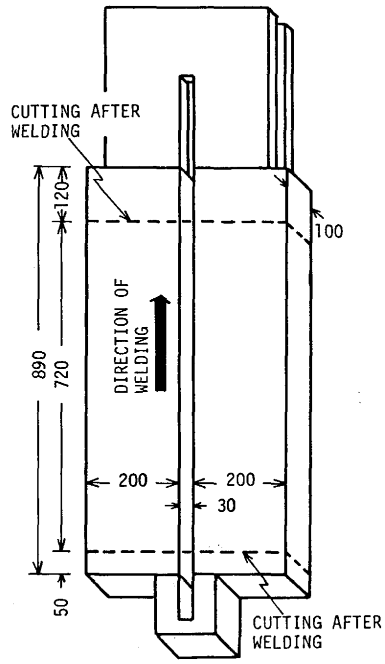


Fig. 2 Test model for electroslag welded joint

Table 1 Condition of electroslag welding

Welding process	Material of welding	Current (A)	Voltage (V)	Velocity (mm/min)	Number of electronodes
Electroslag*	ES-50xMF-38	460	56	7.38	1

* : with weaving

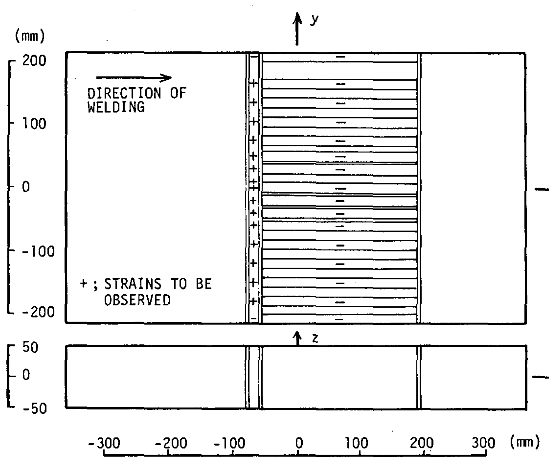


Fig. 3 Locations of observing points on the top and bottom surfaces of R-specimen

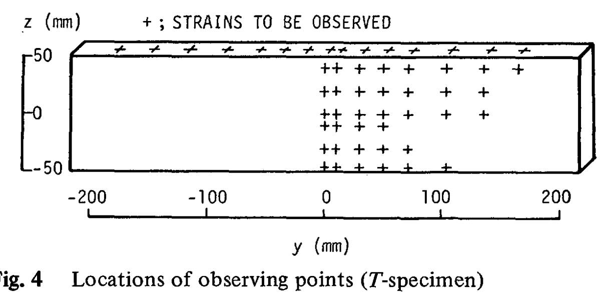


Fig. 4 Locations of observing points (T-specimen)

on the cross section of sliced T_i-specimen are shown in **Fig. 4. Figure 5** shows observing points of strains on the sectional surface of sliced L_i-specimen. Strain gages with gage length *2 mm* are used to observe strains remain in T_i-specimen and L_i-specimens. They are attached to the corresponding points of the respective T-specimen and L_i-specimens as shown in Figs. 4 and 5. The average strains observed on the both surfaces of respective specimens are used as the observed data. The unsteady temperature distribution due to the electroslag welding is approximately symmetric with respect to the three coordinate axes. In this connection, the residual stress distribution can be dealt as symmetric.

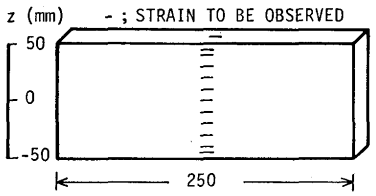


Fig. 5 Locations of observing points (L_i-specimen)

3.2 Measurement of stresses $\{\sigma^A\}$ due to cross sectional inherent strains

Bi-axial strain gages are attached to the sliced T-specimen at the positions shown by Fig. 4. Then the specimen is cut into *10 mm* square pieces centering each bi-axial strain gage in order to measure stresses $\{\sigma^{AO}\}$ in the plane stress state. Substituting the measured stresses into Eq. (2), three dimensional stresses $\{\sigma^A\}$ produced by cross-sectional inherent strains (ϵ_y^*, ϵ_z^*, γ_{yz}^*) can be directly given. The measured examples are shown by **Fig. 6.** As the material properties, Young's modulus $E = 21{,}000\ kg/mm^2$ and Poisson's ratio $\nu = 0.3$ are assumed.

3.3 Measurement of $\{\sigma^B\}$ due to longitudinal inherent strain

As is already stated in the preceding chapter, when L_y-method is applied to measurement of residual stresses, the longitudinal effective inherent strain ϵ_x^* is divided into the component ϵ_{xl}^* which linearly changes in the z-direction of the L_i-specimen and the nonlinear component ϵ_{xn}^*. For these components, the measuring procedure and the measured results are shown in the next sections.

3.3.1 Estimation of linear component ϵ_{xl}^* of longitudinal inherent strain

In this case, the stress distribution is presumed to be symmetric with respect to the middle plane of R-specimen and the relaxed strain ϵ_{xl} observed on the L_i-specimen newly cut out from R-specimen is supposed

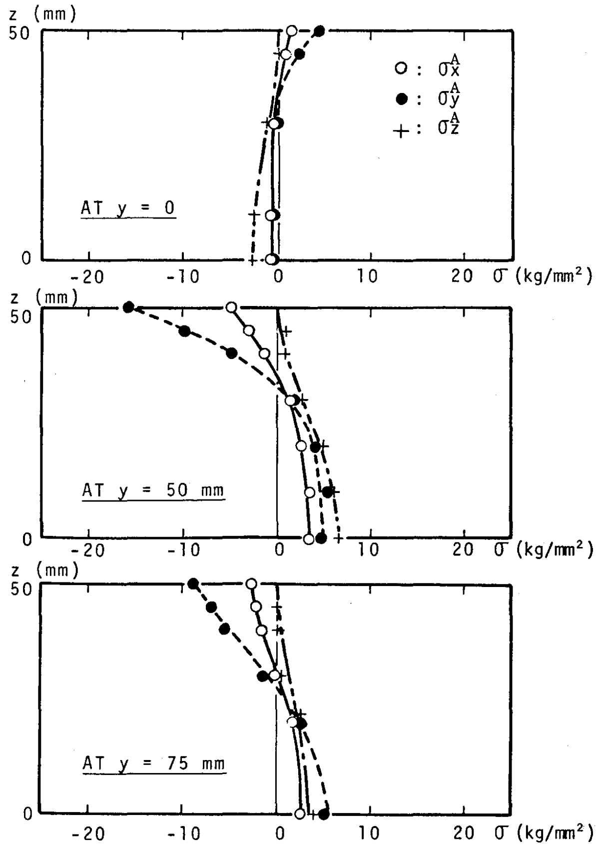


Fig. 6 Measured distributions of stresses $\{\sigma^A\}$ due to inherent strains (ϵ_y^*, ϵ_z^*, γ_{yz}^*) contained in the cross section

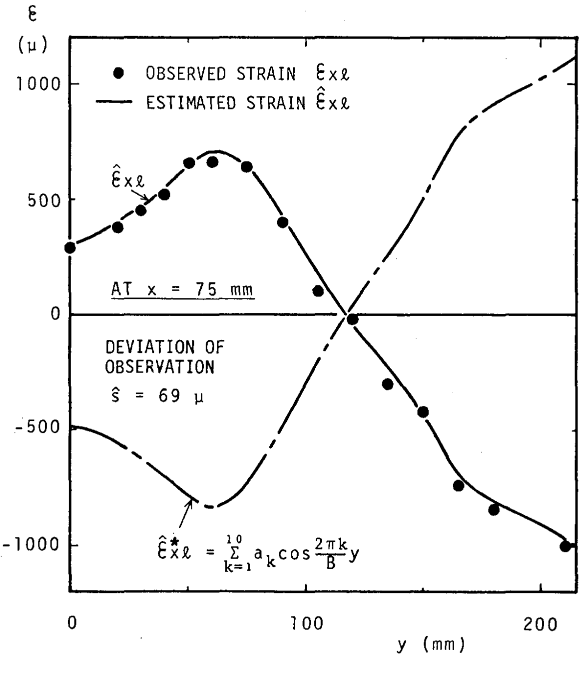


Fig. 7 Most probable values of linear component $\hat{\epsilon}_{xl}^*$ of effective longitudinal inherent strain and relaxed strains $\hat{\epsilon}_{xl}$ due to cutting L_i-specimens out from R-specimen

to be uniform in the z-direction. The average value of ϵ_{xl} on the top and bottom surfaces is shown in **Fig. 7**. Using this data, ϵ^*_{xl} which uniformly (generally linearly) distributes in the plate thickness direction can be estimated. Here, the distribution of effective inherent strain is approximated by the Fourier series.

$$\epsilon^*_{xl} = \sum_{k=1}^{10} a_k \cos(2\pi ky/B) \quad a_k \text{ ; constant}$$

An observation equation is derived with the aid of the finite element method. Using the above mentioned ϵ_{xl} as the observed data, the coefficient a_k in the above equation is determined under the condition that the sum of square of residual becomes the least. Most probable values of the linear component $\hat{\epsilon}^*_{xl}$ of the effective inherent strain are shown also in Fig. 7. The elastic strain distribution $\hat{\epsilon}_{xl}$ reproduced at the measuring points (x=*75mm*) by the finite element method using the inherent strain distribution $\hat{\epsilon}^*_{xl}$ is shown by the solid line in Fig. 7. As is obvious from the figure, the observed strains and the estimated strains show a good coincidence. Therefore, it is recognized that the estimated components of inherent strains are accurate.

3.3.2 Estimation of nonlinear component ϵ^*_{xn} of longitudinal inherent strain

In the stress distribution remaining in L_i-specimens, Eq. (13) applies to the central portion of each specimen. For this reason, the in-plane nonlinear component of longitudinal inherent strain, ϵ^*_{xn}, can be estimated as the opposite signed elastic strain ϵ_{xn}. In stead of ϵ^*_{xn}, some representative examples of elastic strains directly observed in L_i-specimens are shown in **Fig. 8.**

3.3.3 Longitudinal effective inherent strain and three dimensional stresses $\{\sigma^B\}$

The longitudinal inherent strain ϵ^*_x can be given as the sum of the above mentioned in-plane linear component ϵ^*_{xl} and the nonlinear component ϵ^*_{xn} (= $-\epsilon_{xn}$), which are shown in **Fig. 9.** Three dimensional stresses $\{\sigma^B\}$ produced by this inherent strain ϵ^*_x can be analyzed by the finite element method. For the analysis, the number of rectangular solid elements: *420*, the number of nodes: *660*, the number of unknown nodal displacements: *1745*, and the computing time (C.P.U. time): *406* seconds (NEAC System 900 of ACOS Series). **Figure 10** shows the estimated distributions of three dimensional stresses $\{\sigma^B\}$ in the plate thickness direction.

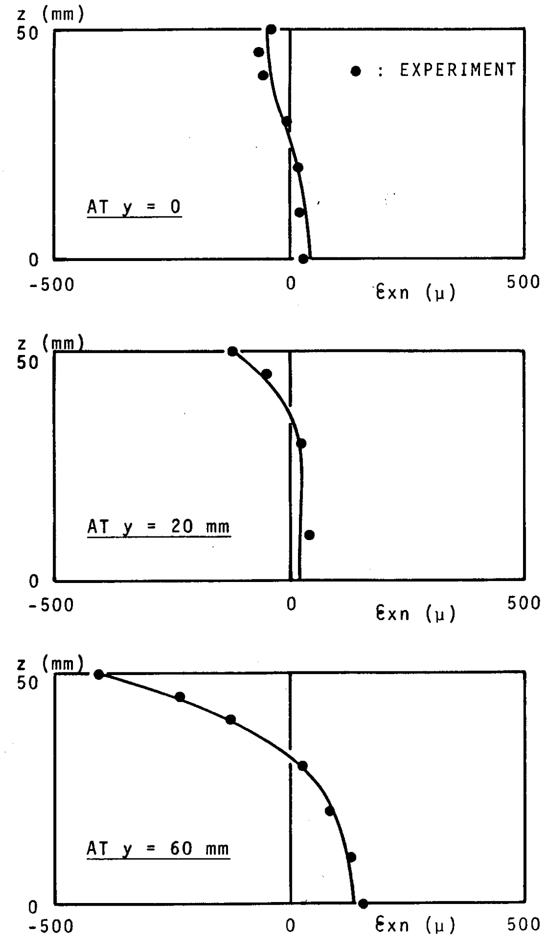


Fig. 8 Remained elastic strains in L_i-specimens

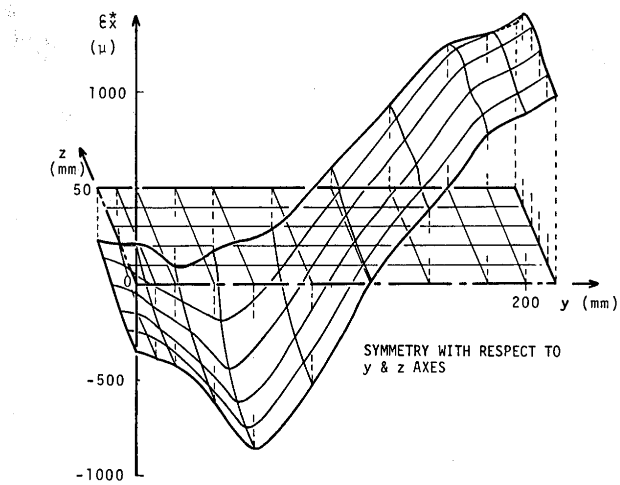


Fig. 9 Distribution of effective longitudinal inherent strain on the cross section of the weld line

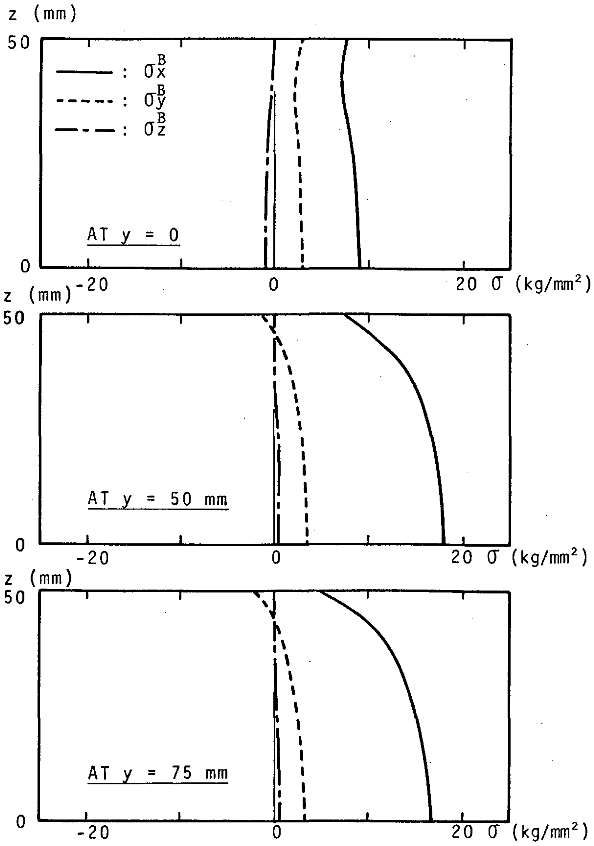


Fig. 10 Estimated distributions of stresses $\{\sigma^B\}$ due to longitudinal inherent strain (ϵ_x^*)

3.4 Distribution of three dimensional welded residual stresses

Three dimensional residual stresses $\{\sigma\}$ in a very thick electroslag welded joint can be obtained as the sum of three dimensional stress components $\{\sigma^A\}$ and $\{\sigma^B\}$ which were measured in 3.2 and 3.3.

Estimated residual stresses on the surface of the central portion of the weld line is shown in **Fig. 11**. It is indicated that the estimated values correspond well to the values directly observed for examination of accuracy. None of these directly measured values is intentionally used in the measurement of three dimensional welded residual stresses by the concerned method. For this reason, it is considered that the assumed measuring theory is valid and the three dimensional residual stress distribution estimated by this theory is reliable. The residual stresses show a peculiar distribution in the region of $y = 50\ mm$. The region, however, corresponds to the heat affection zone (HAZ) appears in the macrophotograph of a cross-section.

The three dimensional residual stress distribution in the inside of the object cannot be directly measured. According to the measuring theory, however, there is no

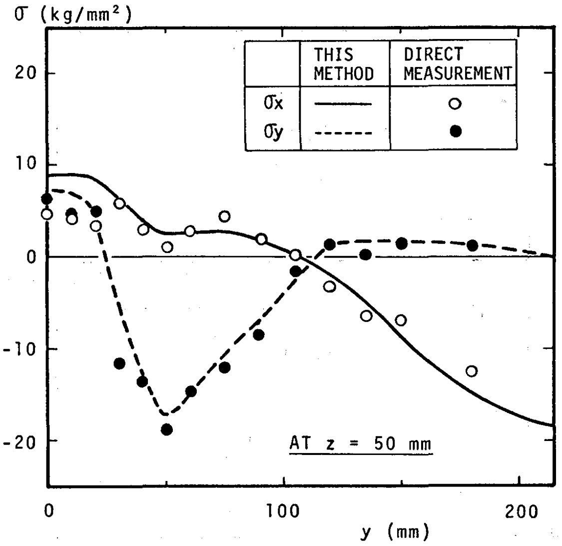


Fig. 11 Welding residual stresses on the surface of R-specimen

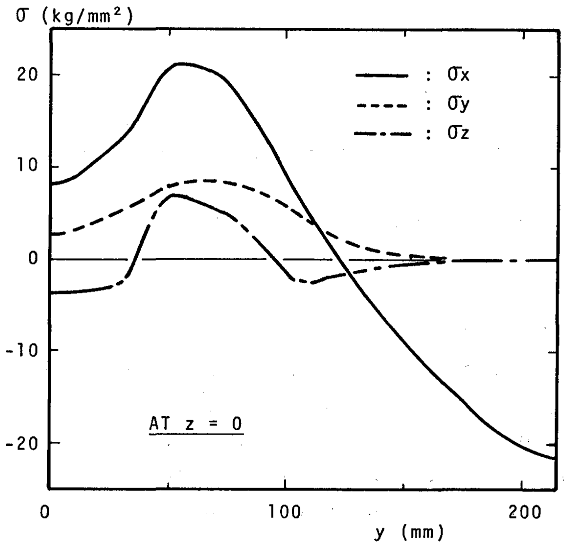


Fig. 12 Estimated residual stresses along y-axis

difference in measurement between stresses on the surface of the joint and those in the interior of it. Therefore, the estimated values of stresses in the inside are considered to be as accurate as those on the surface. **Figure 12** shows the residual stress distribution along the y-axis in the middle of the plate thickness $z = 0$. It indicates that high stresses are produced in HAZ ($y = 60\ mm$). **Figure 13** shows the residual stress distribution in the plate thickness direction. It indicates that in the region of HAZ, the stresses vary greatly between the central plate thickness and the plate surface.

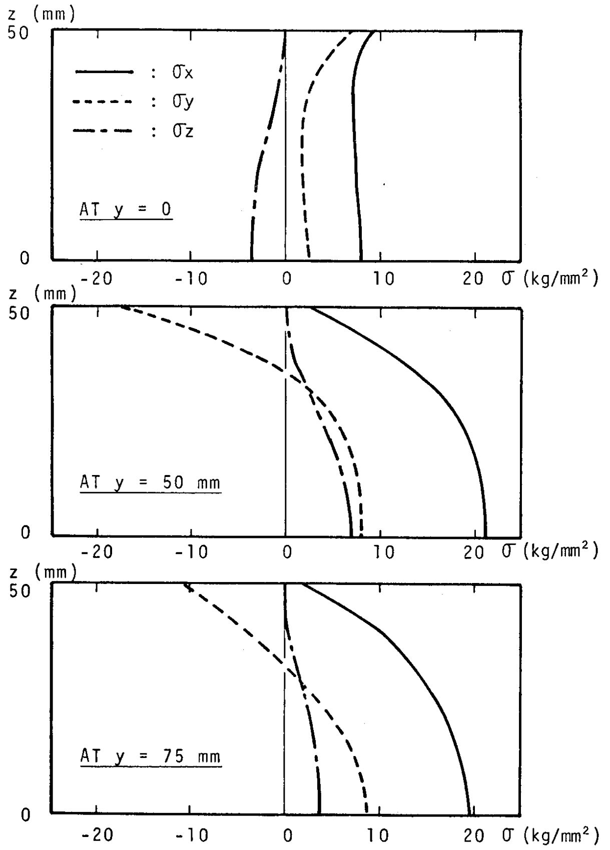


Fig. 13 Estimated residual stresses on the cross section

4. Conclusion

The authors have already proposed the measuring principle of three dimensional residual stresses. In this paper, the measuring theory in which the inherent strains are dealt as parameters is developed utilizing the characteristic of the inherent strain distribution produced in a long welded joint. In order to show the validity and applicability of the measuring theory, three dimensional residual stresses produced in an electroslag welded joint of a very thick plate are measured. The main results are as follows.

(1) In a long welded joint, the inherent strain distribution can be regarded as uniform along the weld line except at the weld ends. Therefore, three dimensional inherent strains are divided into the cross-sectional and the longitudinal components in order to measure three dimensional residual stresses.

(2) Stresses produced by the cross-sectional inherent strains are in the state of plane strain except those in portions within the distance of the plate thickness from the weld ends. Therefore, they can be directly determined by Eq. (2) using residual stresses in the plane stress state which are measured in *T*-specimen.

(3) Stresses produced by the longitudinal inherent strains are estimated by the longitudinal inherent strain which can be obtained from longitudinal elastic strains measured in L_i-specimens sliced along the weld line. It is shown that there are two practical methods of slicing L_i-specimens. In case of L_z-method, the normal line of the sectioned surface of the specimen is in the plate thickness direction and in case of L_y-method, it is in the plate breadth direction.

(4) Three dimensional residual stress distribution produced in the central cross section of a very thick electroslag welded joint is clarified for the first time by the theoretically rational measuring method. L_y-method is applied to slicing L_i-specimens. Measured values of three dimensional residual stresses based on this measuring theory correspond well to the directly observed values on the surfaces of *R*-specimen. In this consequence, it is clarified that the estimated three dimensional residual stress distribution is wholly reliable and this measuring theory is practically applicable.

Acknowledgement

The authors wish to express their thanks to Mr. R. Koki, Research Assistant, Welding Research Institute of Osaka University, for his help in performing experiments.

References

1) Y. Ueda, K. Fukuda and S. Endo: A Study of the Accuracy of Estimated Residual Stresses by the Existing Measuring Methods, Trans. JWRI (Welding Research Institute of Osaka Univ., Japan) 4-2 (1975) pp. 117-121.

2) Y. Ueda, K. Fukuda, K. Nakacho and S. Endo: A New Measuring Method of Residual Stresses with the Aid of Finite Element Method and Reliability of Estimated Values, J. of SNAJ (the Society of Naval Architects of Japan), 138 (1975) pp. 499-507 and Trans. JWRI 4-2 (1975) pp. 123-131.

3) Y. Ueda, K. Fukuda and M. Tanigawa: New Measuring Method of 3-Dimensional Redidual Stresses Based on Theory of Inherent Strain, J. of SNAJ 145 (1979) pp. 203-211 and Trans. JWRI 8-2 (1979) pp. 249-256.

4) K. Fukuda: General Measuring Theory of Residual Stresses and Development of Measuring Method for 3-Dimensional Residual Stresses, Doctoral Dissertation, Osaka Univ., Jan. (1980).

5) Y. Ueda, K. Fukuda and M. Fukuda: A Measuring Theory of Three Dimensional Residual Stresses in Long Welded Joints, J. of JWS (The Japan Welding Society) 49-12 (1980) pp. 845-853.