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Characteristics of Restraint Stress-Strain of Slit Weld in a Finite Rectangular Plate and the Significance of Restraint Intensities as a Dynamical Measure[†]

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Abstract

The mechanism of cold weld cracking may be simply stated from the dynamical aspect that the cold crack initiates when the stress and strain induced at a point reach the critical values. In this respect, the stress and strain are the important information to prevent initiation of cold cracking from the dynamical point of view. In connection to this, a series of researches is carried out on this kind of problems. A slit weld in a rectangular plate is chosen as the basic research model because of its two dimensional restraint state in which the constraint of thermal expansion and shrinkage varies along the weld line.

In this paper, experiments are conducted in order to investigate the validity of the already proposed analytical calculation method for the inherent shrinkage which is the direct factor of producing restraint stresses and strains in the perpendicular direction to the weld line and for thus produced restraint stresses and strains. From the result, the characteristics of the restraint stress and strain produced in the weld metal are clarified. At the same'time, the restrain strain which can be estimated by a simple calculation is proposed as a more general dynamical measure for initiation of cold cracking. In addition, the restrain intensities under three loading conditions are investigated of their significance as a dynamical measure in case of estimating the restraint stress and strain.

KEY WORDS: (Slit Weld) (Restraint Stresses) (Restraint Strains) (Restraint Intensity) (Dynamical Measure for Cold Cracking)

1. Introduction

From the dynamical point of view, it is necessary to predict the welding stresses and strains when the countermeasure against initiation of cold weld cracking is considered. On this kind of problem, the slit weld in a rectangular plate (**Fig. 1**) has been used to a series of researches[1-3] as a basic example representing the two dimensional constraint state in which thermal expansion and shrinkage vary along the weld line.

In this paper, the following subject will be investigated in relation with the above: (1) Experimental examination of validity of "Analytical Calculation Method for Restraint Stresses and Strains due to Slit Weld" mentioned in Ref.3), (2) clarification of general characteristics of restraint stresses and strains perpendicular to the weld line in the weld metal by a series of analytical calculations, (3) proposal of the restraint strain which can be simply calculated as a more general measure in place of the restraint intensity which is conventionally defined as the

dynamical measure for cold cracking (at present, three different loading conditions are considered for restraint intensities) and (4) discussion of significance of applying restraint intensities based on the conventional definition to prediction of the severity of mechanical restraint conditions expressed by restraint stresses and strains.
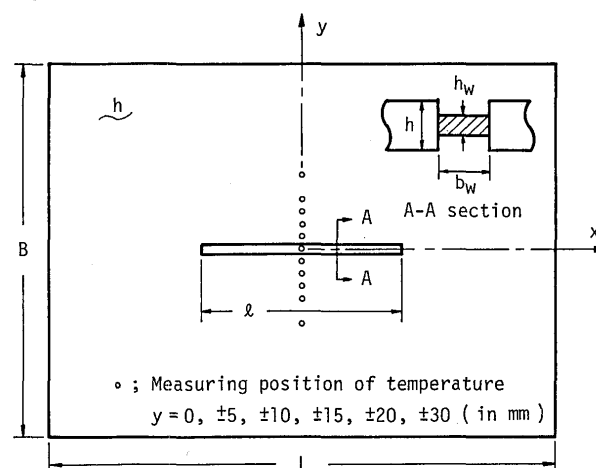

Fig. 1 Slit weld specimen

2. Experimental Examination of Inherent Shrinkage and Restraint Stresses-Strains Estimated by the Analytical Calculation Method

The production mechanism of restraint stresses and strains by the slit weld can be divided into the following two factors[2]. (1) Thermal deformation (inherent shrikage) along the slit occurs at the rigidity recovery temperature of the weld metal at the cooling stage. This deformation tends to restore its original form as the temperature in the base plate approaches to be uniform. (2) The weld metal which has already recovered its rigidity resists this restoration of the deformation. The already proposed analytical calculation method for a finite plate[3] has its base on the analysis[4] of the restraint stresses and strains in an infinite plate caused by the inherent shrinkage occurs. Here, the validity of the calculation results are examined by experiments.

SS41 and SM41A steels were used in this experiment. The specimen sizes, the heat input, the yield stress and the tensile strength of the weld metal are shown in **Table 1**. The CO$_2$ gas arc welder was used. Backing for penetration was applied to specimens No.1 to No.4.

Table 1 Experimental conditions and mechanical properties

No.	B (mm)	L (mm)	ℓ (mm)	h (mm)	h_w (mm)	Groove	Q (J/cm)	σ_Y	σ_u	NOTE
1	150	200	80				2950			
2	300	200	80				3100			SS41
3	150	400	160	3.2		I	3050	46	57	
4	300	400	160				3050			
5	150	200	80	25	5	Y	16500			SM41A

σ_Y : Yield strength, σ_u : Tensile strength of weld metal (kg/mm^2)

2.1 Investigation of temperature distribution and determination of physical constants

The inherent shrinkage which is the direct source of restraint stresses and strains is the thermal deformation along the slit at the rigidity recovery temperature T_m of the weld metal. In order to estimate accurately this inherent shrinkage, accurate temperature distribution at the time when the weld metal is cooled down to T_m is necessary. On the other hand, simple estimation of the temperature distribution is required to derive analytical expression of the above mentioned thermal deformation. As the analytical solution[3] in which the temperature dependency of physical constants is disregarded will be applied, determination of physical constants, therefore, is very critical. Physical constants (specific heat, density) to be used in

calculations are determined by measurement of temperature distribution in experiments to be performed.

On the slit weld specimen with plate thickness $h = 3.2$mm, temperature was measured along y axis as shown in Fig. 1. When the weld metal at the slit center is cooled to 700°C, temperature was measured. The results are indicated by ⌈●⌋ in **Fig. 2**.

In this paper, the following values of the rigidity recovery temperature and the physical constants are adopted for mild steel.

rigidity recovery temperature : $T_m = 700$°C
initial temperature : $T_i = 15$°C
specific heat : $c = 0.188$cal/g·°C
density : $\rho = 7.66$g/cm^3
heat efficiency : $\xi = 0.75$

With application of these values, the temperature distribution at the time the weld metal is cooled to the rigidity recovery temperature can be accurately predicted from the analytical solution, as indicated by the solid line in Fig. 2.
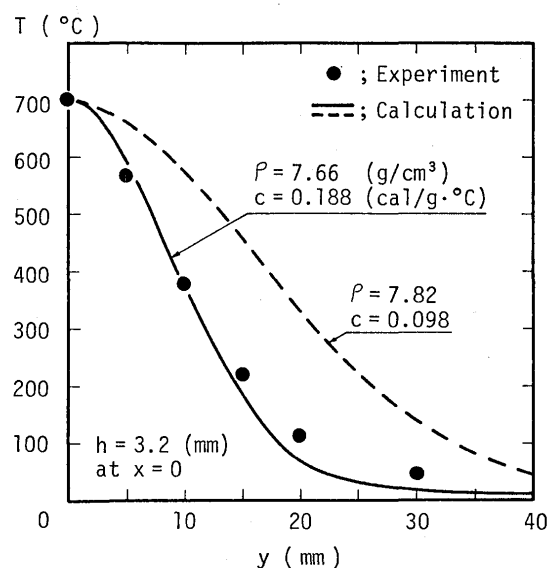

Fig. 2 Temperature distributions at the middle of slit ($x=0$)

2.2 Experimental examination of validity of analytical calculation method

In order to examine the accuracy of analytically calculated inherent shrinkage, and restraint stresses and strains, an experiment is conducted on a slit weld in a finite rectangular plate.

(1) Elastic restraint stresses in the weld metal

When inherent shrinkage S_T is given, the breadth of the weld metal is extremely small in comparison with the breadth of a finite rectangular plate. Therefore, it is only the base plate which contributes to deforma-

tion disregarding[*] the thermal expansion of the weld metal. In this case, S_T and elastic restraint stresses display one to one correspondency. For this reason, restraint stresses (residual stresses) produced in the weld metal are measured instead of S_T to be compared with the result of the analysis.

In this experiment, 2-axes gauges are attached on both surfaces of the weld metals of test specimens No.1 to No.4 so as to measure the residual stresses by the stress relaxation method. The observed residual stresses perpendicular to the weld line are shown by ⌜●⌟ in **Fig. 3**. The solid lines in Fig. 3 show the restraint stresses σ_w calculated by the following equation presented in the preceding report. The results of calculation and experiment indicate fairly good coincidence.
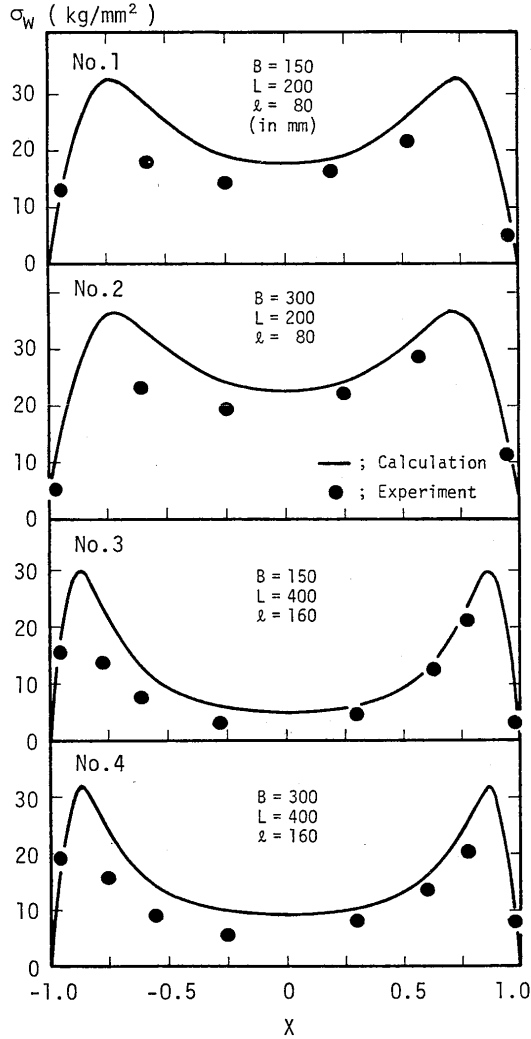

σ_w (kg/mm^2)

Fig. 3 Distributions of residual stresses

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[*] In the preceding report[3] , weld metal was assumed to be per-
fect elastic-plastic. If it is plastic, its extension becomes free but
is restricted in magnitude by the deformation of a base plate.

$$\sigma_w = \frac{E}{\pi} \frac{\alpha(T_m - T_i)h'_{cr}}{l}\left[\frac{1 - e^{-(\pi/4)(1+X)^2(l/h'_{cr})^2}}{1+X}\right.$$
$$\left. + \frac{1 - e^{-(\pi/4)(1-X)^2(l/h'_{cr})^2}}{1-X} - c_1\right]\frac{h}{h_w} \qquad (1)$$

where,

$$c_1 = 1.25/(L/l)^{2.15} \quad [2 \leq L/l < 0.9675(B/l)^{0.8186}]$$
$$c_1 = 1.36/(B/l)^{1.76} \quad [L/l \geq 0.9675(B/l)^{0.8186}] \qquad (2)$$

$\alpha = 1.2 \times 10^{-5}$: coeffi. of linear expansion (1/°C)
$E = 21000$: Young's modulus (kg/mm^2)
h : plate thickness (mm), l : slit length (mm)
h_w : throat thickness (mm), $X = 2x/l$

h'_{cr} in Eq.(1) is introduced to deal both cases, $h < h_{cr}$ and $h \geq h_{cr}$ together and can be expressed as,
(1) In case of $h < h_{cr}$: $h'_{cr} = h_{cr}/h$
(2) In case of $h \geq h_{cr}$: $h'_{cr} = h_{cr}$

$$h_{cr} = 10\sqrt{\frac{0.24\xi Q}{c\rho(T_m - T_i)}}$$: critical plate thickness (mm)

Q : heat input (J/cm)

Here, restraint strains ε_w are composed of only the elastic restraint strains ε_w^e and distribute in the same form as σ_w.

$$\varepsilon_w = \varepsilon_w^e = \sigma_w/E \qquad (3)$$

(2) Fully plastic in the weld metal

According to the result of thermal elastic-plastic analysis in consideration of the effect of moving heat source, when the welded metal at the finishing end of the slit is cooled to the rigidity recovery temperature, deformation along the slit shows its maximum at every point which approximately becomes the inherent shrinkage.
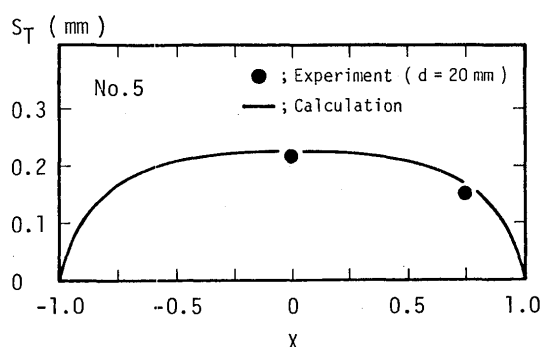
Transient deformation is measured on specimen No.5 by attaching the clip gauge (gauge length $d = $ 20mm) at the slit center and the point 10mm inside from the slit end. The maximum transient deformations are shown by ⌜●⌟ in **Fig. 4**. The solid line in Fig. 4 shows the inherent shrinkage S_T calculated by analytical calculation method, which coincides well with the experimental result.

If weld metal is fully plastic, magnitude and distribution of plastic strains are considered to indicate the severity of mechanical conditions. Plastic restraint strains calculated by an analytical method can be expressed by the following equation and its validity will be confirmed by an experiment.

$$\varepsilon_w^p = (S_T - S_e)/b_w \qquad (4)$$

where,

b_w : slit gap (mm)

Fig. 4 Distribution of inherent shrinkage : S_T

Fig. 5 Elastic deformation of base plate : S_e

S_T in Eq.(4) is the inherent shrinkage and its validity is experimentally confirmed. S_e is the elastic deformation of a base plate. If the weld metal is fully plastic, restraint stresses are yield stresses σ_Y along the entire slit length. In this case, S_e can be simply calculated by the following equation applying the restraint intensity[1] R_p under uniform loading.

$$S_e = \sigma_Y h_w / R_p \qquad (5)$$

where,

$$R_p = (1-\beta_p)\frac{E}{2}\frac{h}{l}\frac{1}{\sqrt{1-X^2}}$$

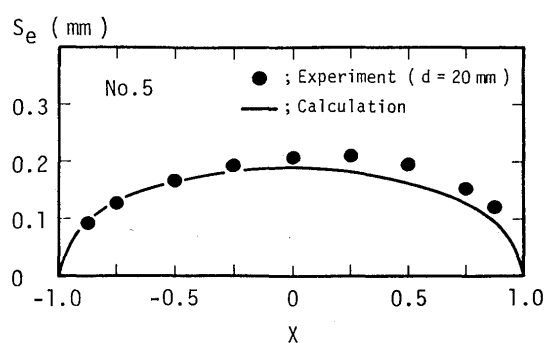: restraint intensity of a finite rectangular plate under uniform loading

$$\beta_p = 0.6/(B/l)^n + 0.75/(B/l) \qquad (6)$$
$$n = 5.8/(B/l)^2 + 2.2 \qquad [B/l \geq 1.8, \ L/l \geq 1.5]$$

In addition, S_e calculated by Eq.(5) is considered to coincide with the deformation with the opposite sign, which is produced by release of restraint stresses in the weld metal along the slit. This will be confirmed by an experiment.

After welding, contact balls are stricken into the top and bottom surfaces of the specimen spanning the slit such as gauge length being 20mm ($d = 20$mm), then the center line of the weld metal is cut along the slit so that the restraint stresses are released. The elastic deformation produced by this cutting is shown by ⌈●⌋ in **Fig.5**. The solid line in Fig.5 is the calculated one which is obtained by Eq.(5) assuming that the yield stresses and the throat thickness of specimen No.5 are $\sigma_Y = 46$kg/mm^2 and $h_w = 5$mm (measured value), respectively, which shows a good coincidence with the experimental result.

As is stated above, the theoretical values of S_T and S_e correspond well with the experimental results. Naturally, plastic restraint strains calculated by Eq.(4) has a good accuracy. In this case, restraint strains ε_w produced in the weld metal can be obtained as the summation of elastic restraint strains ε_w^e and

plastic restraint strains ε_w^p.

$$\varepsilon_w = \varepsilon_w^e + \varepsilon_w^p = \sigma_Y/E + (S_T - S_e)/b_w \qquad (7)$$

In this analysis, restraint stresses and strains in the vicinity of the starting point of slit weld are more or less overestimated because the idealized instantaneous heat source is applied. However, their estimated values from the slit center to the finishing end are fairly accurate. As the result, the validity of the proposed analytical method is confirmed.

3. General Characteristics of Restraint Stresses and Strains

The general characteristics of restraint stresses and strains due to the slit weld are clarified by a series of calculations applying the analytical method presented in the preceding report[3].

The y-slit weld cracking test specimen with plate breadth $B = 150$mm, length $L = 200$mm, and slit length $l = 80$mm is taken up as the basic model of the analysis. Keeping the same size ratios as the basic model ($B/l = 1.875$, $L/l = 2.5$), other similar specimens with various values of l are also analyzed. Consequently, B and L vary, too. Other sizes including plate thickness, mechanical properties, and welding conditions are specified as follows.

Sizes : $h = 20$mm, $h_w = 5$mm, $b_w = 4$mm, and slit length is variable.

Mechanical properties : yield stress of the weld metal $\sigma_Y = 50$kg/mm^2, coefficient of linear expansion $\alpha = 1.2 \times 10^{-5}(1/^\circ C)$

Welding conditions : heat input $Q = 17000$J/cm ($h_{cr} = 17.5$mm) is uniformly and instantaneously put over the entire slit length.

Applying the above conditions to the analysis, restraint stresses σ_w and plastic restraint strains ε_w^p produced in the weld metal perpendicular to the weld line are calculated. The results are shown in **Fig.6**. It is indicated that if slit length l is long in comparison with h_{cr}, a function of heat input, restraint stresses
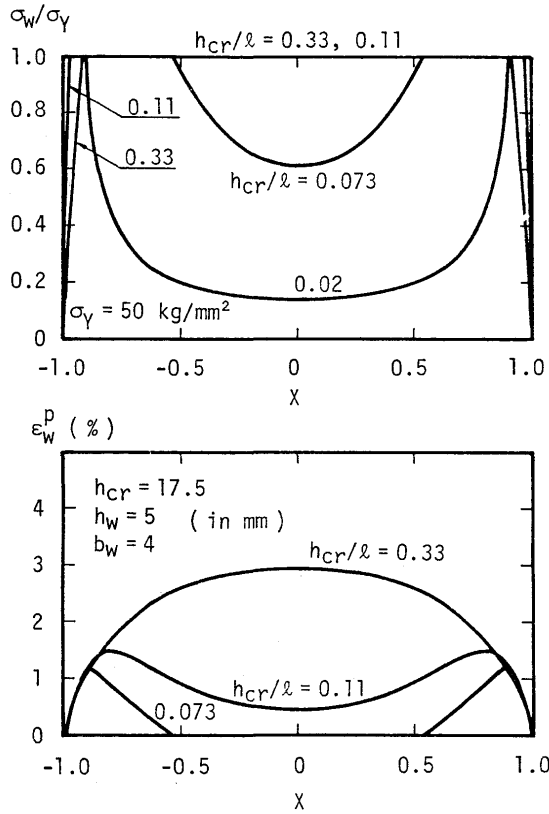
Fig. 6 Distributions of residual stresses and plastic strains

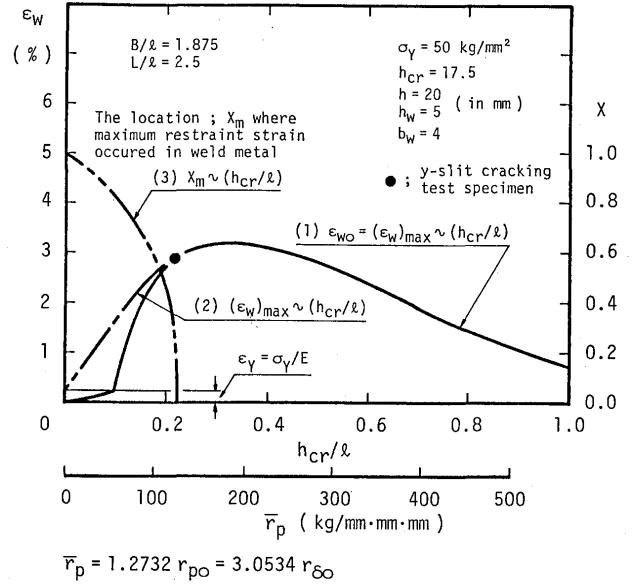$\bar{r}_p = 1.2732\, r_{po} = 3.0534\, r_{\delta o}$

Fig. 7 Relation between h_{cr}/l (ratio of heat input and slit length) and plastic strains

Table 2 General characteristics of restraint stresses and strains in weld metal

STRESS STATES	h_{cr}'/l	PATTERN OF RESIDUAL STRESS DISTRIBUTIONS	PATTERN OF PLASTIC STRAIN DISTRIBUTIONS
(1)	$h_{cr}'/l < \sigma_1^*$	elastic ($|X| \leq 0.9$)	no plastic strain
(2)	$\sigma_1^* \leq h_{cr}'/l < \sigma_2^*$	elastic-plastic	△△
(3)	1) $\sigma_2^* \leq h_{cr}'/l < 0.21$	full plastic	◠
(3)	2) $0.21 \leq h_{cr}'/l < 0.35$		approx. ellipse
(3)	3) $0.35 \leq h_{cr}'/l \leq 1$		◠

$$\sigma_1^* = \frac{\zeta}{(10.5 - c_1')}\,\frac{\sigma_Y(h_w/h)}{(E/\pi)\alpha(T_m - T_i)} \quad ; \quad \sigma_2^* = \frac{\zeta}{(2 - c_1')}\,\frac{\sigma_Y(h_w/h)}{(E/\pi)\alpha(T_m - T_i)}$$

are elastic except at the slit ends and their vicinities. On the contrary, as l is shortened, restraint stresses increase and plastic deformation which is limited to slit ends expands to the central portion so as to finally covers the entire slit. In this way, the distributions and magnitudes of restraint stresses and strains produced along the slit weld are determined greatly depending upon the relative proportion of h_{cr}/l.

In **Fig. 7** where the horizontal axis indicates various values of h_{cr}/l, (1) restraint strains ε_{wo} at the slit center is shown by the solid line, (2) the maximum value of restraint strains (ε_w)$_{\max}$ along the slit by the solid with a dot line, and (3) the position along the slit where (ε_w)$_{\max}$ occurs by the solid and two dots line.

With further study, the distributions of restraint stresses and strains in weld metal are classified into three characteristic states by the aid of h_{cr}'/l and σ_1^*, σ_2^* shown in the following equations. They are (1) elastic, (2) elastic-plastic, and (3) fully plastic states (**Table 2**).

$$\begin{aligned} \sigma_1^* &= \bar{\sigma}\,[\zeta/(10.5 - c_1')] \\ \sigma_2^* &= \bar{\sigma}\,[\zeta/(2.0 - c_1')] \end{aligned} \quad (8)$$

where,

$$\bar{\sigma} = \sigma_Y\,(h_w/h)/[(E/\pi)\,\alpha\,(T_m - T_i)]$$

$$h_w = 10\sqrt{\frac{0.24\xi Q}{H\rho\tan\beta}} \quad : \text{ throat thichness (mm)}$$

$H = 1.2 \sim 2.5$: specific deposited heat[5] (Kcal/g), 2β : groove angle

c_1' in the above equation can be calculated by substituting l' for l in Eq.(2). l' can be calculated as the first approximation[3] as follows.

$$l' = \zeta l \quad (10)$$

$$\begin{aligned} \zeta &= 0.0024\,\sigma_Y + 0.765 \quad [30 \leq \sigma_Y < 98] \\ \zeta &= 1.0 \quad [\sigma_Y \geq 98] \end{aligned}$$

(1) Elastic stress state (State (1) in Table 2)

When the slit length is long in comparison with the heat input (or the heat input is small in comparison with the slit length) and $h_{cr}'/l < \sigma_1^*$ is satisfied, restraint stresses σ_w of weld metal are in the elastic

stress state in the area of $|X| < 0.9$ excluding near the slit ends.

(2) Elastic-plastic stress state (State (2) in Table 2)

As the slit length is shortened (or the heat input is increased), plastic state expands from the severely restraint slit ends to the slit center. This stage satisfies $\sigma_1^* \leq h_{cr}'/l < \sigma_2^*$, where plastic restraint strains distribute in the vicinities of the slit ends.

(3) Fully plastic stress state (State (3) in Table 2)

When the slit length is even shorter (or the heat input is further increased) as to satisfy $h_{cr}'/l \geq \sigma_2^*$, the weld metal becomes fully plastic along the slit length. In this case, produced restraint stress σ_w reaches to the yield stress (σ_Y) over the whole slit length. On the other hand, the distribution of plastic restraint strains ε_w^p, can be divided into the following three groups.

1) In the state of $\sigma_2^* \leq h_{cr}'/l < 0.21$, ε_w^p distributes along the slit concavely and the dynamical restraint conditions are severe near the slit ends.

2) In the state of $0.21 \leq h_{cr}'/l < 0.35$, ε_w^p distributes approximately elliptically.

3) In the state of $0.35 \leq h_{cr}'/l \leq 1$, ε_w^p distributes convexly.

In the states of 2) and 3) where $0.21 \leq h_{cr}'/l \leq 1$, dynamical conditions are severer at the slit center than at the slit ends.

4. Proposal of a New Dynamical Measure

Up to the present, restraint stresses and strains obtained by the thermal elastic-plastic analysis have been considered to be the most accurate dynamical measure for cold cracking in two dimensional restraint weld joints in which thermal expansion and shrinkage vary along the weld line. Nevertheless, it is economically impossible to execute this analysis for each joint. For this reason, regardless of accuracy, restraint intensities (there are restraint intensities under three different loading conditions and their significance as a dynamical measure is separately stated in the next chapter) are applied as a dynamical measure for cold cracking.

Restraint stresses and strains produced in two dimensional restraint weld joints are under great influence of not only geometrical sizes but also heat input and so on. Therefore, a new measure which should be simply calculated and uniformly expresses the severity of dynamical restraint conditions even under the influence of the above changes is expected.

If produced stresses are elastic, stresses and strains correspond one to one. In case of full plastic

state, stresses are constant, while plastic strains are variable depending on conditions. This implies that plastic strains express the severity of dynamical restraint. Therefore, "restraint strains : ε_w" is proposed here as a new dynamical measure.

Restraint strains ε_w can be given as a summation of elastic restraint strains ε_w^e and plastic restraint strains ε_w^p, as,

$$\varepsilon_w = \varepsilon_w^e + \varepsilon_w^p \tag{11}$$

where,

$$\varepsilon_w^e = \sigma_w/E \qquad [\sigma_w \leq \sigma_Y]$$
$$\varepsilon_w^p = (S_T - S_e)/b_w$$

A concrete analytical calculation method for ε_w in three different characteristic stress states of weld metal has been already reported[3].

5. Significance of Restraint Intensity as a Dynamical Measure for Two Dimensional Restraint State and Its Applicability

5.1 Definition of restraint intensity and its dynamical background

Restraint intensity R applied nowadays is defined as follows, employing load p imposed at the groove and the elastic displacement δ produced by p at the application point of the load.

$$R = p/\delta \tag{12}$$

In case of two dimensional restraint state such as of slit weld joint, welding residual stresses (restraint stresses) are produced in the weld metal. If restraint stresses σ_w perpendicular to the weld line are released by cutting the weld metal along the slit, elastic displacement δ occurs along the slit. If the throat thickness of weld metal is set h_w, $p = \sigma_w h_w$ and restraint intensity can be calculated as defined in Eq.(12).

In this case of two dimensional restraint state, there are basically three methods to calculate restraint intensities. Severity of dynamical restraints expressed by the restraint intensity is investigated in the following three cases.

(1) Restraint intensity (R_a) defined by measured welding residual stresses and elastic displacement occurred at the groove by releasing the weld metal are applied :

In this case, elastic restraint strains ε_w^e can be calculated by dividing measured welding residual stresses σ_w by Young's modulus E.

$$\sigma_w/E = \varepsilon_w^e = \varepsilon_w \tag{13}$$

If restraint strains ε_w express the severity of dynamical restraint state, restraint intensity R_a, therefore, need not be calculated.

(2) Restraint intensity (R_δ) defined by elastic displacement to be released is uniform along the slit (restraint intensity under uniform displacement) :

With elastic displacement $\delta = 1$, R_δ can be calculated by Eq.(12) as defined.

$$R_\delta = p = \sigma_w h_w \qquad (14)$$

As σ_w is the produced residual stresses, Eq.(14) can be rewritten as,

$$(1/E)(1/h_w)R_\delta = \sigma_w/E = \varepsilon_w^e \qquad (15)$$

Hence, R_δ corresponds to elastic restraint strains ε_w^e.

(3) Restraint intensity (R_p) defined by residual stresses are uniform (restraint intensity under uniform loading) :

Assuming uniform stresses $\sigma_w = 1$, R_p can be expresses in accordance with the definition of Eq.(12) as follows.

$$R_p = p/\delta = h_w/\delta \qquad (16)$$

δ in Eq.(16) is the elastic displacement released along the slit in the weld metal. The average elastic restraint strains ε_w^e can be calculated dividing it by slit gap b_w.

$$\varepsilon_w^e = \delta /b_w = 1/[(b_w/h_w)R_p] \qquad (17)$$

In this case, the reciprocal of restraint intensity R_p expresses the distribution of elastic restraint strains ε_w^e which is elliptical along the slit (Eq.(16)).

As the result of investigation of each restraint intensity, it is known that as far as restraint intensities are defined as Eq.(12), they represent elastic restraint strains in a way or another. Therefore, if the basis of measures for the severity of dynamical restraints is set on the newly proposed restraint strains (the sum of elastic and plastic restraint strains), plastic restraint strains cannot be expressed by any restraint intensity.

Actually, there are two cases in the cooling process when welding residual stresses are produced (1) without plastic strains and when (2) with them. As a result, in the case of (1), the severity of dynamical conditions can be expressed by applying an appropriate restraint intensity, while in the case of (2), any restraint intensity is basically inapplicable.

In the state 2) of (3) in Table 2, the distribution pattern of plastic restraint strains ε_w^p is an ellipsoid which is the same as that of shown in Eq.(17). Equation (17) itself expresses elastic restraint strains ε_w^e. In the state where both ε_w^e and ε_w^p distribute elliptically, ε_w^e of Eq.(17) can be multiplied by the coefficient to correspond with the sum of ε_w^e and ε_w^p.

5.2 Applicability of restraint intensity as a dynamical measure

Based on restraint strains, applicability of three kinds of restraint intensities mentioned above as a dynamical measure is investigated.

(1) Restraint intensity calculated by measured welding residual stresses and elastic displacement : R_a

Since welding residual stresses in weld metal are variable depending on welding conditions, this kind of restraint intensity cannot be calculated only as a geometrical function like other kinds. However, this restraint intensity expresses the severity of dynamical restraint conditions in the elastic region with the most accuracy because the elastic restraint strains (welding residual stresses) applied here are directly measured.

(2) Restraint intensity under uniform displacement : R_δ

In either (1) elastic state or (2) elastic-plastic state in Table 2, restraint stresses and strains produced in the elastic region can be accurately obtained by the aid of R_δ as follows.

$$\sigma_w = m_\delta R_\delta$$
$$\varepsilon_w^e = \sigma_w/E = (1/E)m_\delta R_\delta \qquad (18)$$

where,

$$m_\delta = (6.0 \sim 10.0) \times 10^{-2}$$
$$R_\delta = (1 - \beta_\delta)\frac{E}{\pi}\frac{h}{l'}\frac{1}{1 - X_1^2}$$

    : Restraint intensity of a finite rectangular plate subjected to uniform displacement.

$\beta_\delta = (1 - X_1^2)\beta_{\delta 0}$, $\beta_{\delta 0} = 2\beta_p$, $X_1 = 2x/l'$

β_p : l' is substituted for l in Eq.(6).

l' can be obtained from Eq.(10) as a first approximation.

Consequently, by the aid of restraint intensity R_δ under uniform displacement which can be calculated only as a geometrical function, the magnitude and the distribution of restraint stresses and strains produced in the regions (1) and (2) shown in Table 2 ($h_{cr}'/l < \sigma_2^*$) can be accurately predicted.

(3) Restraint intensity under uniform loading : R_p

In the fully plastic state shown by (3) in Table 2, stresses produced in the weld metal are uniform, while plastic restraint strains ε_w^p can be divided into three characteristic distributions. Among these, ε_w^p in the region of 2) $0.21 \leq h_{cr}'/l < 0.35$ distribute approximately elliptically, where the dynamical conditions are the severest at the slit center. As mentioned above, the magnitude and the distribution of ε_w^p in this region can be expressed by the aid of R_p (Eq.(6)).

$$\varepsilon_w^p = E/(m_p R_p) \qquad (19)$$

where,

$$m_p = \{(1/\pi)\,[\alpha\,(T_m - T_i)\,h'_{cr}/l]\,(h/h_w)^{\psi} - (\sigma_Y/E)\}^{-1}(b_w/h_w)$$

$$\psi = 2 - c_1$$

Nextly, plastic restraint strains ε^p_w in the state of 3) $0.35 \leq h'_{cr}/l \leq 1$ distribute convexly, where dynamical conditions are the severest at the slit center in the same way as in the region 2). For this reason, in spite of the accuracy problem of the magnitude and the distribution of ε^p_w and disregarding the detailed discussions, dynamical conditions can be predicted by Eq.(19). On the other hand, ε^p_w produced in the region 1) $\sigma^*_2 \leq h'_{cr}/l < 0.21$ differ from the above mentioned ε^p_w, where restraint is severer rather at the slit ends than at the slit center. Therefore, in this state, it is hard to apply R_p to predict even the portion where dynamical conditions are the severest.

Consequently, application of restraint intensity under uniform loading, R_p, which can be calculated from the geometrical form, makes it possible to predict by Eq.(19) the magnitude and the distribution of plastic restraint strains ε^p_w in the fully plastic weld metal ($h'_{cr}/l \geq \sigma^*_2$) when $0.21 \leq h'_{cr}/l \leq 1$ is satisfied (including $0.35 \leq h'_{cr}/l \leq 1$ where accuracy is lacking).

(4) Relative expression of severity of dynamical conditions

As is already stated, there is a limit to application of three kinds of restraint intensities as the measure representing the magnitude and the distribution of restraint strains. In this section, it is examined if restraint intensities can relatively represent the severity of dynamical conditions beyond the limit of their application and disregarding detailed discussions. Only the restraint intensity under uniform displacement, R_δ, which can be calculated as a geometrical function and that under uniform loading, R_p, are studied here.

Since there is no need of detailed discussions, it is adequate to examine relative dynamical conditions in comparison applying the average restraint intensities for example. However, the average of restraint intensities under uniform displacement cannot be defined[1]. Therefore, restraint intensities at the slit center, $R_{\delta o}$ and R_{po}, the average restraint intensity under uniform loading, $\bar{R}_p$, are applied in order to precede discussion.

For the same specimen, there is a definite relation[1] between R_δ and R_p. As far as restraint intensities at the slit center, $R_{\delta o}$ and R_{po}, are concerned, there is a definite relation with the average restraint intensity $\bar{R}_p$, too. That is,

$$\bar{r}_p = 1.2732 r_{po} = 3.0534 r_{\delta o} \tag{20}$$

where,

$\bar{r}_p (= \bar{R}_p/h)$: average restraint coefficient (kg/mm $\cdot$ mm $\cdot$ mm)

$r_{po} (= R_{po}/h)$: restraint coefficient at the slit center under uniform loading

$r_{\delta o} (= R_{\delta o}/h)$: restraint coefficient at the slit center under uniform displacement

Application of the correlation in Eq.(20) facilitates the conversion of one kind of restraint intensity into the other. This implies that if the geometry of the specimen is the same and one kind of restraint intensity is applicable as the dynamical measure, other two kinds have similar relations on the average.

Figure 7 depicts the relation between restraint coefficients and maximum restraint strains ($\varepsilon_w)_{max}$, that is curve (2). According to it, as $\bar{r}_p$, r_{po}, and $r_{\delta o}$ increase, ($\varepsilon_w)_{max}$ simply increases until it reaches the critical values of $\bar{r}_p < 200$, $r_{po} < 160$, and $r_{\delta o} < 65$ (kg/mm $\cdot$ mm $\cdot$ mm). On the other hand, the solid line representing the restraint strains at the slit center, ε_{wo}, is bent at the point where the vertical axis is $\varepsilon_Y = \sigma_Y/E$. This means that two stages where the center of the slit is elastic ($h'_{cr}/l < \sigma^*_2$) and plastic ($h'_{cr}/l \geq \sigma^*_2$) are divided at this point. In each stage, ε_{wo} increases according to the increase of $\bar{r}_p$, r_{po}, and $r_{\delta o}$, though there is no uniform increase succeeds from one stage to the other.

Accordingly, within the limits of each stage, restraint coefficients (restraint intensities) can relatively represent the magnitude of ε_{wo}. Whenever $\bar{r}_p$, r_{po}, and $r_{\delta o}$ exceed the formerly mentioned critical values, restraint strains start decreasing and do not express the severity of dynamical conditions any more.

As is known from (1) to (4), it is hard to apply restraint intensities as a dynamical measure to general cases in which geometrical forms and heat inputs vary. Their applicability is limited because they are not so accurate as the newly proposed "restraint strains". However, within the limits, they are applicable as the dynamical measure which simply expresses the severity of dynamical conditions. As the result, it is very doubtful if they can be applied as the dynamical measure over the limit even though the comparison between severities of restrains determined by the geometrical forms may be possible.

6. Conclusion

In this paper, (1) the validity of the analytical calculation method proposed in the preceding report[3] for restraint stresses and strains perpendicular to the weld line is confirmed by an experiment. A series of

calculations are conducted by this method so that (2) the general characteristics of restraint stresses and strains are clarified and (3) the significance of restraint intensities as the dynamical measure under three loading conditions based on the conventional definition is investigated.

Followings are the main results.

1) In the two dimensional restraint state such as of the slit weld, produced restraint stresses and strains vary along the weld line. Their distribution and its magnitude along the slit depend not only on the geometry of the specimen but also greatly on the relative ratio of the function of heat input h'_{cr} to the slit length l (h'_{cr}/l).

2) When the slit length is long in comparison with the heat input (or heat input is small in comparison with the slit length), the dynamical conditions become severe at the slit ends and in their vicinities. On the contrary, according as the slit length is shortened (or heat input increases), the portion where dynamical restraint conditions are severe moves toward the slit center. The same phenomenon can be seen in an infinite plate, too.

3) In place of three kinds of restraint intensities based on the conventional definition, more reasonable "restraint strains" are proposed as the dynamical measure representing the severity of two dimensional restraint state.

4) As far as restraint intensities are based on the conventional definition (Eq.(12)), as the dynamical measure for the two dimensional restraint state, they can represent elastic restraint strains but cannot express any plastic restraint strains.

On the basis of restraint strains calculated by the analytical calculation method, applicability of three kinds of restraint intensities based on the conventional definition as the dynamical measure is investigated. As the result,

5) Restraint intensities provided by measured residual stresses and elastic displacement cannot be calculated from the geometry of the specimen. Nevertheless, they are the most accurate dynamical measure for the elastic state because they are calculated applying the measured elastic restraint strains. However, there is no need of calculating restraint intensities.

6) By the aid of the restraint intensity under uniform displacement, the magnitude and the distribution of restraint stresses and strains produced in the elastic region of the weld metal can be accurately

predicted.

7) Elastic restraint strains which can be calculated by dividing the elastic displacement due to uniformly distributed loading along the slit by the slit gap are proportional to the reciprocal of restraint intensities under uniform loading (Eq.(17)). In the region where the distribution of elastic restraint strains coincides with that of plastic restraint strains produced in the weld metal ($0.21 \leq h'_{cr}/l < 0.35$), the magnitude and the distribution of the plastic restraint strains can be accurately predicted by multiplying the elastic restraint strains (the reciprocal of the restraint intensity under uniform loading) by the coefficient. As for the magnitude and the distribution of plastic restraint strains, even in the region of $0.35 \leq h'_{cr}/l \leq 1$, the severity of the dynamical restraint conditions can be estimated by the aid of the restraint intensity under uniform loading, though more or less accuracy is lacking.

8) If strict discussions on the magnitude and the distribution of restraint strains produced in the weld metal are avoided, the restraint intensities under uniform displacement and uniform loading can be applied to the specimens with different size ratios as the dynamical measure explicitly representing the severity of dynamical restraint conditions. However, careful attention is necessary, because the restraint intensities have to be applied within the limits, otherwise plastic restraint strains decrease though the restraint intensities increase.

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