The Effect of Welding Tensile Residual Stresses on Fatigue Crack Propagation in Low Propagation Rate Region†

Kohsuke HORIKAWA*, Atsushi SAKAKIBARA and Takeshi MORI*****

Abstract

The effect of welding tensile residual stresses on fatigue crack propagation in low propagation rate(low ΔK) region was studied by experiments. The used material was high tensile strength steel (HT80) of 800 MPa class in tensile strength. Center Cracked Tension (CCT) specimens were employed for experiments, and consisted of three kinds; specimens of base metal only, specimens with longitudinal welded joint (as weld), specimens with longitudinal welded joint (stress relieved). The major results obtained are summarized as follows:

(1) Tensile residual stresses lower the threshold value of ΔK.

(2) When tensile residual stresses were high, the threshold value of ΔK becomes the lower limit and is independent of stress ratio.

(3) The effect of residual stress can be treated as effect of mean stress.

KEY WORDS : (Fatigue) (Fatigue Crack Propagation Rate) (ΔK) (ΔK_{th}) (Welding Residual Stress) (Stress Ratio) (Mean Stress) (Superposition)

1. Introduction

In welded joints of steel structures, frequently there are blowholes, undercuts, cracks and etc..If the welded joints are subjected to cyclic load, fatigue cracks are initiated form such initial defects, propagate and make a component or structure lead to failure. In this case, it is said that the great part of fatigue life is not crack initiation life, but crack propagation life[1]. And in the short crack stage just after fatigue cracks are initiated, fatigue crack propagation rate is low, and this stage forms the great part of crack propagation life. Also in this stage, ΔK is low. Therefore in the present study, attention was paid to fatigue crack propagation behavior in low propagation rate (low ΔK) region, particularly to the effect of welding tensile residual stress on threshold stress intensity factor range ΔK_{th}.

In the medium propagation rate region, there are many studies on the effect of welding tensile residual stress on fatigue crack propagation[2], but in the low propagation rate region there are few studies on it[3]. Especially, the research for the effect of stress ratio in welding tensile residual stress field on ΔK_{th} is not found. Accordingly in the present study, three kinds of base metal only, specimens with longitudinal welded joint (as weld), and specimens with longitudinal welded joint (stress relieved) were taken up as specimens, two to four kinds of stress ratios were given for crack propagation tests in each specimen. And, it is pointed out that the effect of residual stress varies with the level of ΔK, stress

Nomenclature

a	: Crack length, mm
a_i	: Crack length considered at the inauguration of measurement, mm
a_t	: Crack length considered at the termination of measurement, mm
da/dN	: Fatigue crack propagation rate, mm/cycle
K	: Stress intensity factor, $MPa\sqrt{m}$
ΔK	: Stress intensity factor range, $MPa\sqrt{m}$
ΔK_{th}	: Threshold stress intensity factor range, $MPa\sqrt{m}$
ΔK_{eff}	: Effective stress intensity factor range, $MPa\sqrt{m}$
$\Delta K'$	: The value of ΔK when da/dN is 10^{-7} mm/cycle, $MPa\sqrt{m}$
ΔK_i	: The value of K for a_i, $MPa\sqrt{m}$
K_{max}	: The maximum level of K value by cyclic stress, $MPa\sqrt{m}$
K_{min}	: The minimum level of K value by cyclic stress, $MPa\sqrt{m}$
K_{res}	: K value calculated in consideration of residual stress as mean stress, $MPa\sqrt{m}$
R	: Stress ratio
R_{res}	: Stress ratio obtained by considering residual stress as mean stress
U	: Crack opening ratio
BM	: Specimens made of only base metal without weld
WT	: Specimens with longitudinal welded joint
SR	: Specimens with longitudinal welded joint in which welding residual stress was relieved

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ratio and crack length[4]. It was tried, in the present study, to evaluate such complicate effect of residual stress by the principle of linear superposition.

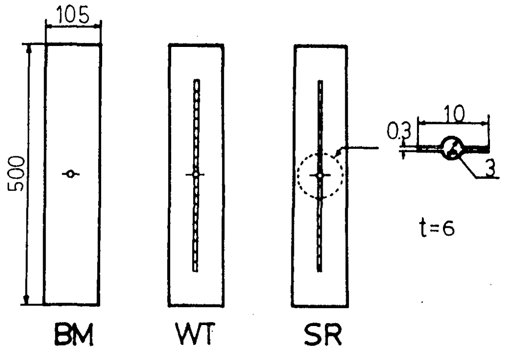

Fig. 1 Specimen configurations.

Table 1 Mechanical properties and chemical composition of material.

Yield stress	Tensile strength	Elongation
755 MPa	819 MPa	20.6 %

C	Si	Mn	P	S	Cu	Cr	Mo	V	B
0.16	0.06	1.13	0.016	0.006	0.29	0.76	0.21	0.03	0.015

(wt%)

Table 2 Welding condition.

Wire	Y-CS ϕ1.6
Fused flux	NF-16
Welding position	Flat
Welding current	270 A
Arc voltage	30 V
Welding speed	52 cm/min.
Heat input	9.3kJ/cm

Table 3 Testing conditions.

Specimen	R	ΔK_i (MPa$\sqrt{m}$)	a_i (mm)	a_t (mm)
BM-1	-1	17.9	6.09	8.33
BM-2	0	13.8	9.13	18.88
BM-3	0.5	13.9	7.57	18.29
BM-4	0.7	15.1	7.45	24.17
WT-1	-1	13.2	7.23	19.54
WT-2a	0	13.2	7.20	23.81
WT-2b(14)	0	6.6	9.87	14.04
WT-2b(33)	0	6.9	29.54	33.30
WT-3	0.5	14.5	7.35	17.52
SR-1a	-1	17.6	7.77	19.64
SR-1b	-1	19.5	16.56	23.01
SR-2	0	14.9	7.79	17.01

ΔK_i: ΔK at the beginning of measurement,
a_i: Crack length at the beginning of measurement,
a_t: Crack length at the end of measurement.

2. Specimens and Experimental Procedures

The used material was high tensile strength steel(HT80) of 800 MPa class in tensile strength, and specimens in strips were made of plate with 6mm thickness. Quenching and tempering temperature was 930°C and 625°C respectivly. Mechanical properties and chemical composition of material are shown in **Table 1**.

The specimens were processed according to the procedure in Reference (2). **Figure 1** indicates the configuration of center cracked specimens. Specimens were composited of three kinds of base metal (BM) specimens, longitudinal welded joint (WT) specimens and stress relieved longitudinal wleded joint (SR) specimens

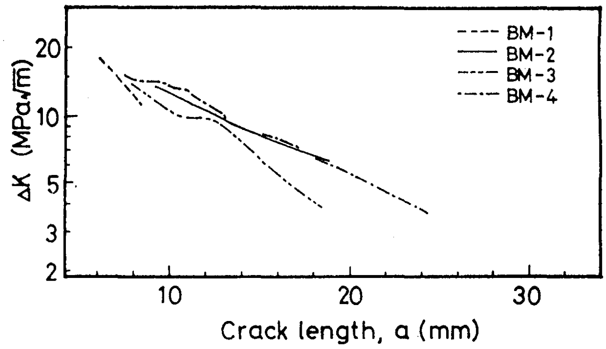

Fig. 2 Relations between Δ K and crack length in base metal specimens (BM).

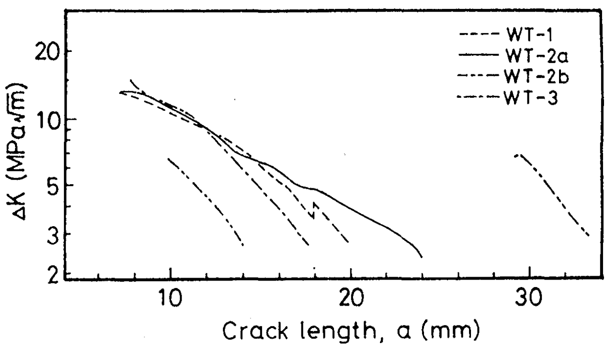

Fig. 3 Relations between Δ K and crack length a in welded joint specimens (WT).

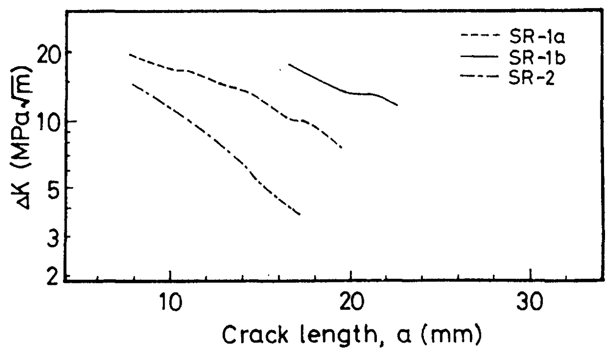

Fig. 4 Relations between Δ K and crack length in stress relieved welded joint specimens (SR).

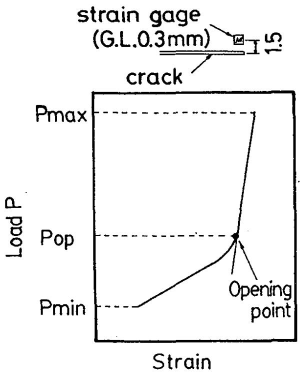

Fig. 5 Definitaion of crack opening point.

as shown in Fig.1. BM specimens were annealed by the condition of 620°C, one hour in order to relieve residual stress. The welded joint specimens (WT) were prepared by processing flatly the welding bead after performing weld on side groov of 5.5mm breadth, 2mm depth processed on the one side of strip. And the weld was performed by microwire-submerged arc welding, the welding conditions are given in **Table 2**. SR specimens were made by performing post-heat treatment for WT specimens under the condition of 620°C, one hour. And the fatigue testing conditions for the above specimens are shown in **Table 3**.

Fatigue crack propagation tests were carried out by servo-hydraulic closed-loop fatigue testing system with 20Ton dynamic capacity, and by axial load control of sine wave. And constant amplitude loading speed was under 30Hz. Fatigue crack length was measured whenever the increment of crack length became about 0.1mm, and by using travelling microscope (X50).

Whenever the increment of crack length became over 0.2mm, load range was decreased by 5% step by step untill fatigue crack propagation rate da/dN was reduced under 10^{-7}mm/cycle. In one step of load range, the measurement of crack length was performed at least twice and da/dN was obtained over two. In principle, when the difference of two da/dN to be continued was smaller than 20%, load range was decreased. But when the difference was over 20%, fatigue crack was made to propagate still more, and load range was decreased after the difference became smaller than 20%. Only two da/dN after one step of load range were validated, and the mean value of the two was plotted. The variation of the relation between ΔK and crack length **a** during fatigue crack propagation test is shown in **Fig.2, 3 and 4**.

ΔK_{th} was determined by extrapolating so that regression curve given by the following formula was superposed well on da/dN- ΔK curve of experimental results.

$$da/dN = C(\Delta K^m - \Delta K_{th}^m) \qquad (1)$$

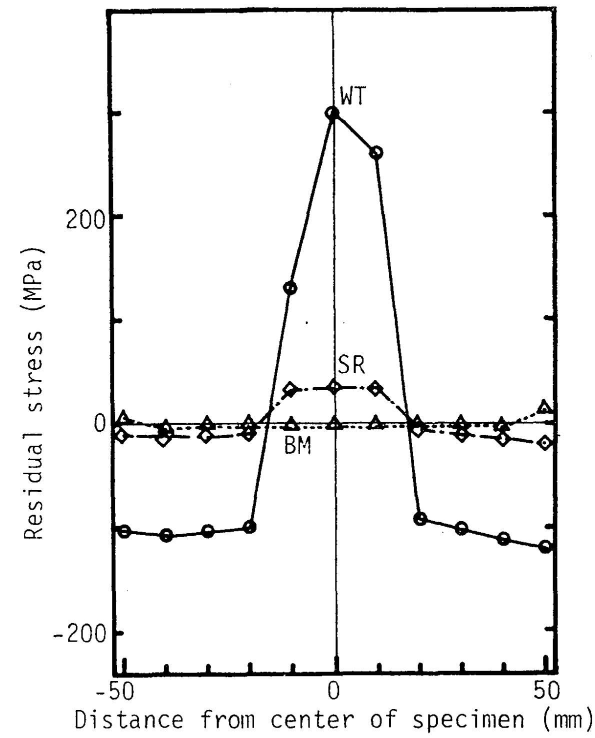

Fig. 6 Initial residual stress distributions.

where, C and m are material constants.

In practice, the constants C and m were obtained by appling the least square method for the linear part of da/dN- ΔK curve of experimental results. In this case, the data in the region of da/dN $\geqq 10^{-5}$mm/cycle of Reference (4) were also included in all present date except BM-4 specimens. Next, ΔK value for da/dN $= 10^{-7}$mm/cycle in da/dN- ΔK curve of expermental results was given as $\Delta K'$. ΔK_{th} was determined by substituting the above C, m and $\Delta K'$ after setting da/dN by 10^{-7}mm/cycle in Eq.(1). But, because Eq.(1) was not fit for regression curve of SR specimens, ΔK_{th} was given by the value of $\Delta K'$. In this case, crack propagation behavior in the region of da/dN $< 10^{-7}$mm/cycle was mainly considered in order to determine ΔK_{th}.

For observation of crack closure behavior, strain gages (0.3mm gage length) were attached at the location to be 1.5mm distance from the crack line supposed that the crack would be propagated, outputs of the strain gages and load were obtained as shown in **Fig.5**. In Fig.5, the crack opening point was determined by the point where the tangent line with the maximum gradient in load-strain curve came off the curve.

3. Distribution of Residual Stress

Figure 6 indicates the distribution of initial residual stress which was obtained by the cutting method. Where, the results on WT and SR specimens were quoted from the data of Reference (4), in which specimen

Table 4 Values of $C, m, \Delta K'$ and ΔK_{th}.

Specimen	C	m	ΔK' (MPa√m)	ΔK_{th} (MPa√m)
BM-1	1.03×10^{-8}	2.32	11.8	11.6
BM-2	4.23×10^{-9}	2.97	6.2	6.0
BM-3	1.32×10^{-8}	2.76	4.0	3.9
BM-4	2.81×10^{-8}	2.39	3.9	3.7
WT-1	1.99×10^{-8}	2.64	2.8	2.4
WT-2a	1.99×10^{-8}	2.64	2.8	2.4
WT-2b(14)	——	—	2.8	2.4
WT-2b(33)	——	—	3.1	2.7
WT-3	1.99×10^{-8}	2.64	2.8	2.4
SR-1a	——	—	7.4	7.4*
SR-1b	——	—	11.9	11.9*
SR-2	——	—	3.9	3.9*

C, m: Crack propagation constants in Eq.(1) when using MPa√m for ΔK and mm/cycle for da/dN.
ΔK' : ΔK at da/dN=10^{-7} mm/cycle.
* : The values are not ones extrapolated but ones when da/dN=10^{-7} mm/cycle.

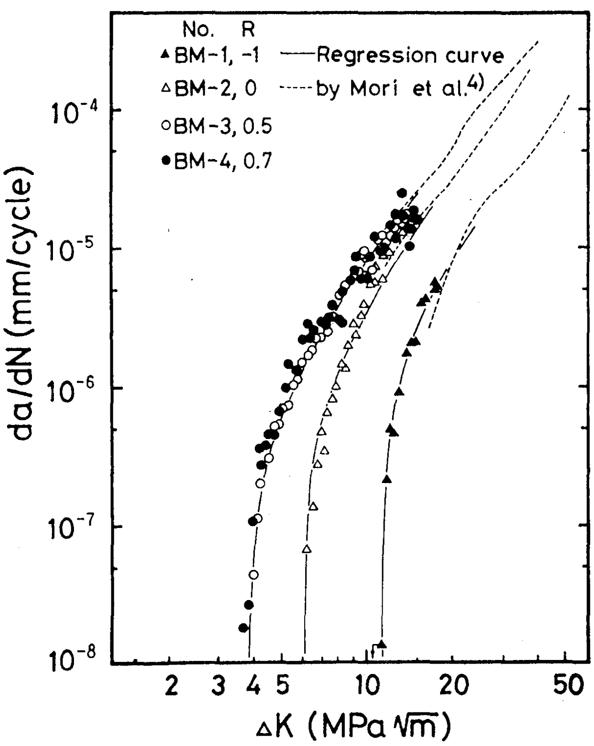

Fig. 7 Relations between fatigue crack propagation rate da/dN and ΔK in base metal specimens (BM).

configuration was the same as it of the present experiment. But, the results on BM specimens were obtained by direct measurement.

In order to investigate the change of residual stress in WT specimens subjected to cyclic load, residual stress was measured after cyclic load. Loading stress range was three kinds of − 54.5 ~ + 54.5MPa, 0 ~ 93.4MPa and 53.9 ~ 112.8MPa. The cyclic load was given by 105cycle, but it was confirmed that the residual stress was scarecely

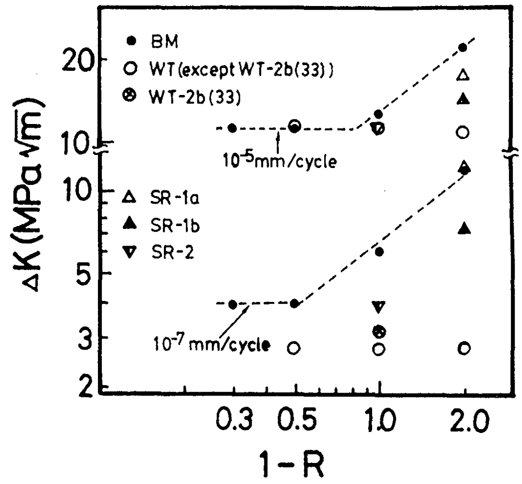

Fig. 8 Relations between Δ K and (1-R) for constant fatifue crack propagation rates da/dN.

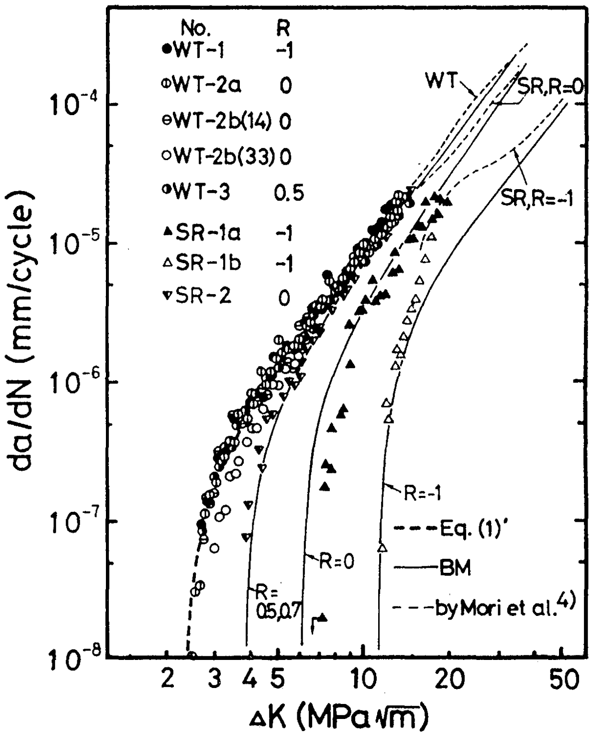

Fig. 9 Relations between fatigue crack propagation rate da/dN and ΔK in every specimen.

changed. Moreover, it can be infered from the above experimental result that residual stress in SR specimens was also scarecely changed by cyclic load, because there was lower residual stress in this specimens than WT specimens.

4. Experimental Results and Discussion

4.1 Fatigue crack propagation rate

Table 4 gives the values of C, m, Δ K', and ΔK_{th}

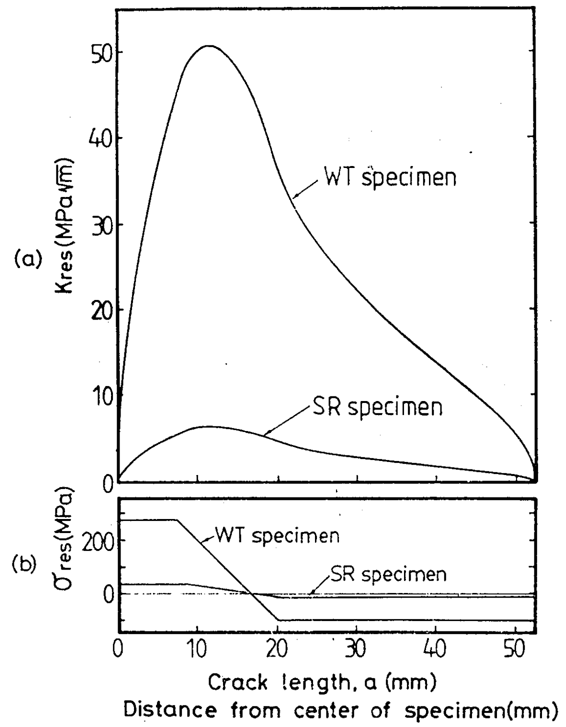

Fig. 10 (a) Relations between K-value due to residual stresses K_{res} and crack length , and (b) intial distributions of residual stresses σ_{res}.

obtained for each specimen. The constant C and m of WT specimens were determined by including the data on WT-1, WT-2a, and WT-3.

Figure 7 shows the relation between da/dN and Δ K obtained for BM specimens. The broken line in the region of da/dN > 10^{-5}mm/cycle means the data quoted from Reference (4). In the BM specimens, da/dN became higher and ΔK_{th} was lowered as stress ratio R was increased in the renge of $R=-1\sim0.5$. But, in the range of $R=0.5\sim0.7$, da/dN and ΔK_{th} were little affected by R. In order to clarify the effect of R, as Ohta et al. tried, the relation between Δ K corresponding to the specified constant crack propagation rate (10^{-5}, 10^{-7}mm/cycle) and (1-R) are indicated in **Fig.8**. The broken lines in Fig.8 shows the relations for BM specimens, the horizontal prats of the lines mean that the effect of R is saturated.

Figure 9 shows the relation between da/dN and ΔK in WT and SR specimens. The regression curves for the BM specimens are also shown to compare with them. The da/dN-ΔK curve for WT specimens is above all of others. This also means that da/dN was accelerated and ΔK_{th} was lowered because of welding tensile residual stress. The da/dN- Δ K relations for WT specimens are nearly in accord with each other except for WT-2b(33). Therefore it can be said that the effect of stress ratio R on the relations for WT specimens does not appeared.

The ΔK_{th}'s for WT-2a, WT-2b(14), and WT-2b(33) at $R=0$ were obtained under the condition that the crack length a_t's at the termination of measurement were about 23, 14, and 33mm respectively. Only in the case of $a_t \doteqdot$ 33mm, the ΔK_{th} was higher by about 13% than those of the other WT specimens. The effect of residual stress on ΔK_{th} at $R=0$ may be nearly saturated in the range of crack length $a_t=14\sim33$mm.

In Fig.9, comparing the crack propagation characteristics of SR specimens with BM specimens in case of the same R, the crack propagation rate of SR specimens is higher than that of BM specimens for any value of R except for the vicinity of ΔK_{th} in SR-1b specimens. Namely, in SR specimens, the effect of R as well as welding residual stress were appeared to differ from WT specimens. And ΔK_{th}'s of SR-1a and SR-1b at $R=-1$ were obtained under the condition of crack length $a_t=20$, 23mm. In this case, the effect of welding residual stress depended greatly upon crack length, ΔK_{th} for SR-1a specimen was 60% it for BM specimen.

4.2 Treatment of welding residual stress as eman stress

It is impossible to evaluate the effect of welding residual stress by only da/dN-ΔK curve obtained on the base of limited crack length. And the effect of stress ratio R and level of ΔK must be also considered. In order to examine the effect of welding residual stress, to treat it as mean stress was tried by appling the principle of linear superposition. For this treatment, it may be practical to apply the stress ratio R_{res} in which the residual stress is considered as the following Eq.(2).

$$R_{res}=(K_{mim}+K_{res})/(K_{max}+K_{res}) \qquad (2)$$

The above K_{res} was calculated by using the principle of linear superposition, and was the value of K computed by appling the opposite stresses of same level to the residual stress on the crack surfaces. The relation between K_{res} and crack length **a** are shown in **Fig.10(a)**, the distribution of initial residual stress in Fig.6 was simplified as **Fig.10(b)**. It was confirmed in Chap.3 that the initial residual stress is not changed by the stress applied in the present experiment. For the calculation of K_{res}, the formula of G.G.Chell[6], by which K in in the case subject to arbitrary stress on crack surfaces can be computed and in which the effect of width in a specimen is also included, was applied. The variance of K_{res} to depend upon crack length a, as shown in Fog.10(a), corresponds well with the redistribution tendency of residual stress by sawing in Reference (4).

Figure 11 shows the relation between ΔK for specified crack propagation rate (10^{-5}, 10^{-7}mm/cycle) and ($1-R_{res}$). In Fig.11, ΔK-($1-R_{res}$) relations for 10^{-7}mm/cycle in WT and SR specimens except SR-1a and SR-1b specimens almost correspon with ΔK-($1-R_{res}$) relation in

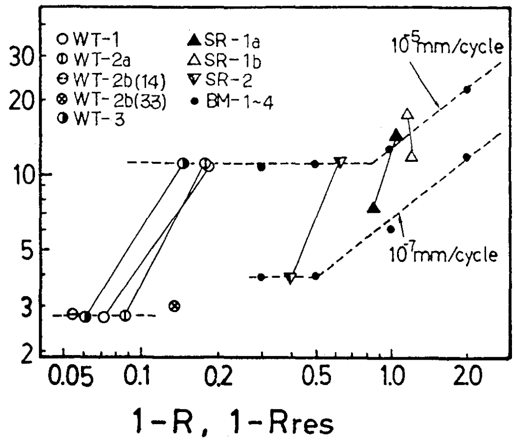

Fig. 11 Relations between ΔK and (1-R_{res}) for constant fatigue crack propagation rates da/dN.

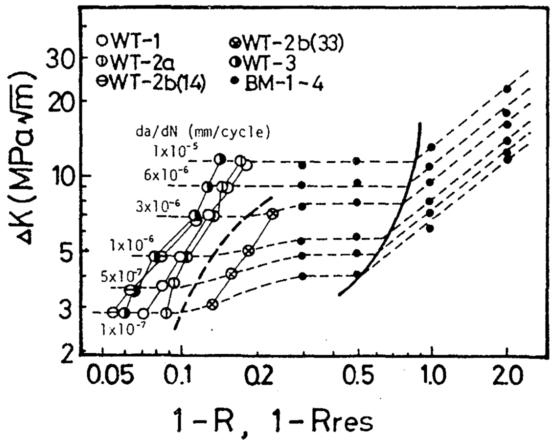

Fig. 12 Relations between ΔK and (1-R_{res}) for constant fatigue crack propagation rates da/dN.

BM specimens. The relation for 10^{-7}mm/cycle in WT specimens is lower than the line extrapolated from the relation for R = 0.5 ~ 0.7 in BM specimens. This trend may result from that R_{res} in WT specimens was higher than 0.87. When the level of R also in BM specimens is as high as R_{res} in WT specimens, it is considered that the relation may approach to it for WT specimens. In the relation between ΔK and $(1-R_{res})$ for da/dN = 10^{-5}mm/cycle, even the relation for WT-3 specimens of R_{res} = 0.86 is on the line extended from it for BM specimens. This phenomenon is considered to be limited to only the vicinity of ΔK_{th}. This may be clarified by subdividing the interval of specified crack propagation rate (**Fig.12**). But, SR specimens are excluded from Fig.12. The relations for SR-1a and SR-1b specimens in Fig.11 are above them for BM specimens because K_{res} evaluated from initial residual stress was lower than the actual value. In the same way, also K_{res} for other specimens might be underestimated though the tendency on the graph is not certain. Particularly, the tendency for SR-1b specimens is remarkable.

In any case, the effect of residual stress may not be underestimated if the relations between ΔK and $(1-R)$ obtained till above R = 0.9 in the vicinity of ΔK_{th} are applied. That is to say, when the philosophy of R_{res} is applied, residual stress can be estimated quantitatively and conservatively. Futhermore, the relations of da/dN- ΔK for WT specimens except for WT-2b(33) specimens did not depend upon R by external load and crack length. This phenomenon may result from that the effect of R_{res} was saturated because R_{res} for those specimens was about 0.9 regardless of R and crack length. The region where the effect of R_{res} is saturated, can be defined as the left part of a bold solid line shown in Fig.12. Because R_{res} in the vicinity of ΔK_{th} is very high as stated above, the relation between ΔK and $(1-R_{res})$ for WT specimens are below the straight lines extrapolated from the cases of R =0.5~0.7 in BM specimens. Therefore, the region where the effect of R_{res} in the vicinity of ΔK_{th} is fully saturated, may be regarded as the left part of a bold broken line in Fig.12. In this case, the effect of R_{res} on ΔK_{th} value is saturated when R_{res} is higher than 0.9. The saturation of R_{res} effect means that the effect of residual stress is saturated.

In the case that the effect of stress relieving treatment is considered, it is not reasonable to discuss by Fig.9 only. But, at first, R_{res} for each condition of the treatment must be evluated, and then the effect of R_{res} must be examined for each case.

4.3 Crack closure behavior

Figure 13 shows the relations between crack opening ratio U and ΔK. These relations correspond relatively with the relations between da/dN and ΔK shown in Fig.7 and Fig.9. **Figure 14** shows the relation between da/dN and ΔK_{eff} determined by appling Fig.13. However, because the crack opening ratio U was scattered considerably, the tendency on U indicated by doted lines in Fig.13 was used for calculation of K_{eff} curves are in narrower band than da/dN- ΔK curves, considerable scattering remains as even. This tendency may result from that the ratio U measured on the surface of specimens in the present study was lower than it in the inside of specimens. Generaly, it is said that fatigue crack propagation is governed by the ratio U in the inside rather than surface of a specimen as indicated in experimental results by Kikukawa et al.[7]. But, when the ratio U measured on the surface is equal to 1.0, it in the inside must be also 1.0.

The following discussion was performed under the two presumptions. The first is to rely the value of U if it is equal to 1.0. And the second is to regard U as lower than the actual value if it is below 1.0.

In the case that the ratio U is always equal to 1.0, the

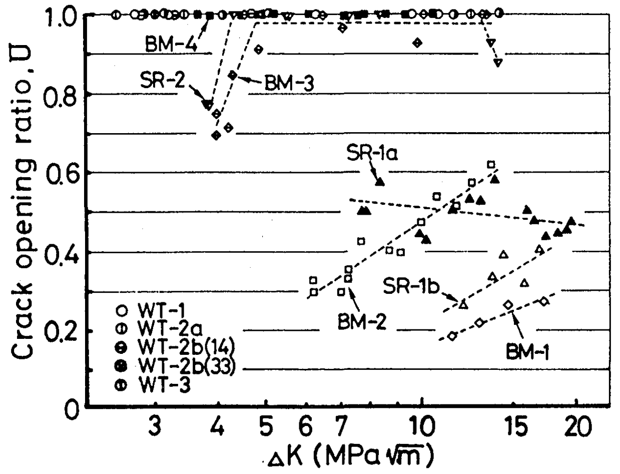

Fig. 13 Relations between crack opening ratio U and ΔK.

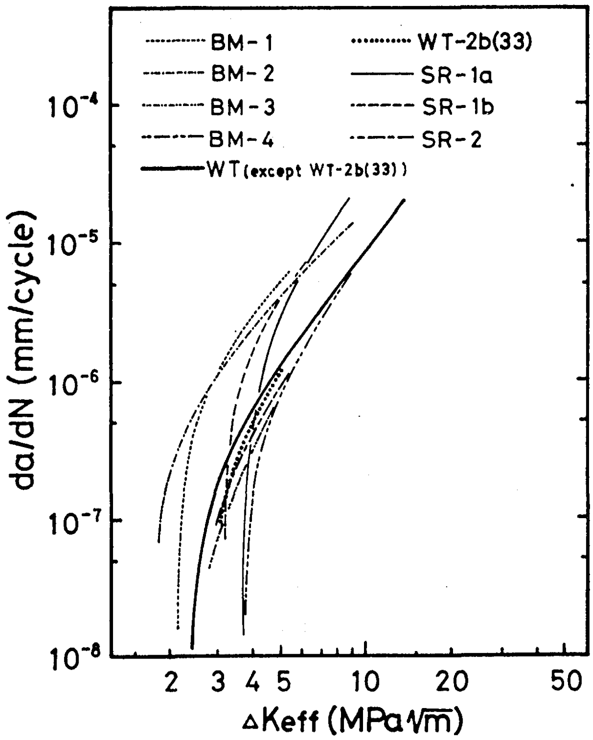

Fig. 14 Relations between fatigue crack propagation rate da/dN and effective stress intensity factor range ΔK_{eff}.

relations between da/dN and Δ N and Δ K can became the upper limit of the relations for given materials. However, although U in both of WT and BM-4 (R=0.7) specimens was always 1.0, there is difference between da/dN- Δ K curves for both specimens in the vicinity of ΔK_{th}. Comparing this phenomenon with the relations between K for specified crack propagation rate and ($1-R_{res}$) in Fig.12, the left part of a bold solid line in Fig.12 corresponds to the state that U is about 1.0. As stated in previous section, in the left part of a bold solid line, Δ K was lowered when R_{res} was increased in the vicinity of ΔK_{th}. Namely, although U is equal to 1.0 in the vicinity of ΔK_{th}, da/dN- Δ K curve is not always the upper limit. When R_{res} is increased, da/dN- Δ K curve is varied and ΔK_{th} is lowered. However, the effect of R_{res} on da/dN- Δ K curve in the left part of a bold broken line in Fig.12, was fully saturated and the curve was not varied. Consequently, it is reasonable that da/dN- Δ K curves for WT specimens except WT-2b(33) in the saturated region can be the upper limit of the curves. Furthemore, ΔK_{th} in this case is the lower limit of it. The regression curve of the da/dN- Δ K relation is shown as a bold broken line in Fig.9, and could be formulated as the follow.

$$da/dN=(\Delta K^m-\Delta K_{th}{}^m) \qquad (1)$$

where, in the case that the units of da/dN and K are mm/cycle and MPa$\sqrt{m}$ respectively, C is 1.99×1.0^{-8}, m is 2.64 and ΔK_{th} is 2.4 MPa$\sqrt{m}$.

5. Conclusions

By using specimens which are ground into Base Metal (BM), Longitudinal Welded Joint (WT) and Stress Relieved Longitudinal Welded Joint (SR) specimens, fatigue crack propagation tests were carried out in the region of low propagation rate (low Δ K region). From this experiment, the effect of stress ratio and welding tensile residual stress on the behavior of fatigue crack propagation was examined. The obtained results can be summerized as the follows.

(1) When stress ratio R becomes high in BM and SR specimens except WT specimens, fatigue crack propagation rate da/dN is accelerated, and threshold stress intensity factor range ΔK_{th} is lowered (Fig.7,9).

(2) In WT and SR specimens, welding tensile residual stress makes da/dN accelerate and ΔK_{th} low (Fig.9).

(3) The effect of welding tensile residual stress can be evaluated in the same way as treating stress ratio R for BM specimens by appling the stress ratio R_{res} in which the residaul stress is considered as mean stress (Fig.11, 12).

(4) When the relation between Δ K and ($1-R_{red}$) are in the left region of a bold broken line in Fig.12, the effect of residual stress is saturated, and da/dN- Δ K curves become the uppper limit of them. When R_{res} is above 0.9, the effect of residual stress on ΔK_{th} is saturated, and ΔK_{th} becomes the lower limit. In this case, da/dN-K relation can be formulated as the follow (Fig.12).

$$da/dN=C(\Delta K^m-\Delta K_{th}{}^m)$$

where, in the case that the units of da/dN and Δ K are mm/cycle and MPa$\sqrt{m}$ respectively, $C=1.99\times10^{-8}$, m=2.64, and ΔK_{th}=2.4MPa$\sqrt{m}$.

(5) When the relation between ΔK and $(1-R_{res})$ are in the left region of a bold broken line in Fig.12, the effect of stress ratio by external load does not appear (Fig.12).
(6) Although cracks are always opened in state of crack opening ratio $U=1.0$, da/dN curves in the vicinity of ΔK_{th} are varied and ΔK_{th} becomes low when R_{res} is increased (Fig.13, 14)

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