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Osaka University
Essays on factor mobility, public policy and international trade
(生産要素の移動、公共政策、国際貿易に関する研究)

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A dissertation for the degree of Doctor of Philosophy

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Abstract

As widely documented, recent decades have observed ever-increasing trans-boundary movements of goods, services, labor, capital, information, and knowledge. Changes in the mobility of scarce resources affect the nature of interdependencies between countries and regions, and thus pose new challenges to policy making. What will happen to the global economy, which involves a large number of actors, if effective international coordination schemes are not designed? How should modern society handle such emerging situations? I try to address these questions and hope to organize the findings in meaningful ways to provide the insights into real-world policy practice for the resolution of complicated issues.

This dissertation consists of three distinct essays on public policy in spatial economies. The first two chapters share a focus on the role of strategic interactions among governments facing international capital mobility in shaping economic geography.

The first chapter proposes a framework, based on a reciprocal dumping model, that assesses the effects of tariff competition for mobile firms on the location patterns of the industry as well as welfare implications.
transport costs encourage geographic dispersion in the industry, sufficiently low transport costs result in a core-periphery location where nobody bears tariff burdens. I show that the global economy would be better off under an international coordination scheme, which differs from ones proposed in previous studies.

The second chapter shares the objective of the first but emphasizes the fact that inherent heterogeneity of regions inevitably creates asymmetric industrial linkages. The chapter coauthors, Hikaru Ogawa and Yasuhiro Sato, and I investigate which of the two types of countries—resource-rich or resource-poor—gains from capital market integration and capital tax competition. We develop a framework involving vertical linkages through resource-based inputs as well as international fiscal linkages between resource-rich and resource-poor countries. The analysis shows that capital market integration causes capital flows from the latter to the former and thus improves production efficiency and global welfare. However, such gains accrue only to resource-poor countries, and capital mobility might even negatively affect resource-rich countries. In response to capital flows, the governments of both types of countries have an incentive to tax capital. We thus conclude that such taxation enables resource-rich countries to exploit their efficiency gains through capital market integration and become winners in the tax game.

The third chapter studies the costs and benefits of urban interactions. Urban economics, economic geography, and urban planning have widely
recognized the importance of urban interactions, such as face-to-face communication or a convivial atmosphere, to understand urban phenomena. However, solitary contemplation is indispensable to enhance ability and creativity. I model a situation involving a choice of the frequency of visits to a playing field in a monocentric city by households facing a trade-off between enjoying interactions at the playing field and cultivating their ability through solitary introspection and reflection. Two conflicting magnification forces are generated through urban interactions and human capital spillovers. Positive externalities possibly reinforce a dispersion force. I discuss a first-best policy in this environment.
Acknowledgments

I am able to complete this dissertation only due to the help and support of many teachers, friends, and families.

I am deeply indebted to my advisor, Kazuhiro Yamamoto, who patiently kept on pushing me to finish it. This thesis would never have been possible without his professional guidance and invaluable support. Yasuhiro Sato taught me a lot more than how to think about urban economic issues. I would like to thank Kenzo Abe, Koichi Futagami, Mutsumi Matsumoto, Noriaki Matsushima, Kazuo Mino, Tomoya Mori, Se-il Mun, Tetsuo Ono, Takatoshi Tabuchi and Yoshitsugu Kanemoto for continuous care and insightful feedback. I am immensely grateful to my co-authors Yuki Amemiya, Hiroshi Kitamura, and Hikaru Ogawa who helped my research mature. The shortcomings of the dissertation are my responsibility.

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Finally, deeply heartfelt thanks are due to my parents, Hajime and Yoko, for their understanding and encouragement in my life.
Chapter 1

Tariff policy and transport costs under reciprocal dumping

1.1 Introduction

Market access is a key ingredient in the determination of industrial agglomeration and geographical differentiation. Literature on tariff jumping investment has examined the link between the location of the industry and the tariff policy, which affects market access. However, the effects of market access on international tariff competition and welfare are ambiguous.

Mai, Peng and Tabuchi (2008; MPT hereafter) examined the effects of tariff competition on the spatial distribution of manufacturing activities.

\*This chapter is based on Oshiro (forthcoming).

\textsuperscript{1}For example, Bhagwati (1987), Brander and Spencer (1987) and Motta (1992). Bloegen (2005) provides a survey of the empirical literature.
and explored the welfare implications. The present chapter extends and complements the MPT argument. The purpose of this study is twofold. First, we propose an alternative model for analytically characterizing the equilibrium. This is important because MPT rely on numerical simulations in the case in which transport costs are of intermediate value. Second, we examine the welfare implications, which are rather different from those offered by MPT.

The MPT framework is based on the canonical new trade theory model. Consider a two-country, one-factor (immobile labor), two-goods (numéraire and differentiated varieties) economy. Differentiated varieties are produced in a monopolistic competitive sector with increasing returns. Trading the varieties incurs composite trade costs, encompassing both transport costs and tariff barriers. Although the former are exogenously given, the latter are strategically determined by governments to maximize welfare of its residents who have a nested Cobb-Douglas-CES utility.

On the other hand, our model, described in Section 1.2, introduces strategic interactions among firms that are Cournot competitors, instead of monopolistic competitors, and encounter segmented markets. Consumers have quadratic utility and linear demands.

Thus far, only a few attempts have been made to address the issues regarding tariff competition within a new economic geography framework. A few exceptions are Haufler and Wooton (1999), Ludema (2002), and Takatsuka and Zeng (2012). Behrens (2006) emphasized the importance of the distinction between transport costs and tariff barriers in the new economic geography.
The positive results are as follows. High transport costs induce governments to set low trade barriers that lead to the dispersion of production. Conversely, tariff competition at sufficiently low transport costs lead to a core-periphery structure wherein the core government imposes a sufficiently high tariff and the periphery government eliminates its trade barriers. As a result, nobody bears the tariff burden, i.e., *de facto* free trade arises in equilibrium. MPT also obtained the similar results in terms of positive analysis.

MPT and this chapter do not share their normative properties, and therefore they propose the opposite policy prescriptions. Section 1.3 shows that tariff competition unambiguously leads to inefficient outcomes whenever the transport costs are strictly positive, and hence there are potential gains from policy co-ordination. For high transport costs leading to the dispersed location, the policy coordinations require prohibiting trade, instead of writing a binding free trade agreements proposed in MPT. Furthermore, in contrast to MPT, *de facto* free trade will not be efficient even for sufficiently low transport costs. In such cases, it is necessary to achieve *de jure* free trade agreements to avoid locational distortions induced by tariff competition. We can conclude that the validity of the theory should be scrutinized in order to apply it to policy making in practice.
1.2 The model

Our model combines a reciprocal dumping model à la Brander (1981) with footloose capital. We model the tariff competition between two symmetric countries, labeled \( i \) or \( j \in \{1, 2\} \), for the plants of two identical firms (\( M \)-firms). The \( M \)-firms produce a homogeneous good (\( M \)-good). Without loss of generality, we assume that each country has a continuum of consumers of size one. The inverse demand function for \( M \)-good is:

\[
p_i = a - (n_i q_{ii} + n_j q_{ji}),
\]

where \( a > 0 \) is a constant parameter, \( p_i > 0 \) is the price in country \( i \), \( q_{ij} \geq 0 \) stands for the quantity of an \( M \)-good that firms produce in country \( i \) and sell in country \( j \), and \( n_i \) denotes the number of firms located in country \( i \).

We assume that \( a \) is large enough to ensure that the individual demand for \( M \)-good is positive for any positive access cost.

The firms are freely mobile between these two countries while consumers are internationally immobile. Trade is balanced through a numéraire good (\( A \)-good), which is produced by employing only labor according to constant returns technology. \( A \)-good is traded with zero transaction costs under perfect competition. The equilibrium wages in both the countries are equalized.

Shipping a unit of \( M \)-good from country \( i \) to country \( j \) requires positive

---

3 The assumption about the number of firms can be relaxed. In an \( n \)-firm oligopoly model, the qualitative nature of the analysis remains unchanged.

4 The demand functions are consistent with quasi-linear preferences.
specific tariff $\theta_j \geq 0$ and positive transport costs $\tau > 0$ (in terms of numéraire). Governments impose specific import tariffs to maximize their objective functions and redistribute tariff revenue to consumers. The lump-sum transfer is given by $s_i = \theta_i n_j q_{ji}$.

In the spatially segmented markets, the $M$-firms compete in quantities and their marginal labor requirement is constant and normalized to zero without loss of generality.$^5$ Rents of an identical $M$-firm located in region $i$ are given by:

$$r_i = p_i q_{ii} + (p_j - \tau - \theta_j) q_{ij}, \quad (1.2.2)$$

all of which are equally distributed among all consumers.

The indirect utility function of a consumer in country $i$ can be written as:

$$V_i = \left(\frac{a - p_i}{2}\right)^2 + w_i + \frac{n_1 r_1 + n_2 r_2}{2} + s_i + \omega. \quad (1.2.3)$$

The first term represents country $i$'s consumer surplus in the $M$-good market. The second and third terms represent the wage and profit share, respectively. Note that $w_1 = w_2$. $\omega$ denotes the initial endowment, which is assumed to be large enough to ensure positive demand for the numéraire.

$^5$For simplicity, indeterminate locations have not been considered in this chapter; therefore, negative tariffs have been omitted. MPT assume positive tariffs and emphasize that, in reality, negative tariffs are rare. If allowing negative tariffs (yet positive market access), the equilibrium location configuration remains unchanged under the duopoly.

$^6$When the marginal unit input requirement $a_M$ is strictly positive, what follows continues to hold true if $a - a_M$ is strictly positive.
The game comprises the following three stages. In the first stage, both
the national governments simultaneously and irreversibly select their spe-
cific tariff rates, \( \theta_i \in \mathbb{R}_+ \). In the second stage, firms select the location for
establishing their plant after observing both the tariffs. In the third stage,
firms initiate production in the international market.

The above sequence of moves implies that governments can credibly
make their policy choices before firms make their location choices (the mo-
bility of firms is assumed to be costless here). The order of the adopted
play follows MPT in order to enable a comparison of results.

1.3 Equilibrium

The threshold values of tariffs at which a firm located in country \( j \) is inactive
in country \( i \) are defined as \( \bar{\theta}_i = \bar{\theta}_i(\theta_j) \). If \( \theta_i \geq \bar{\theta}_i(\theta_j) \), then \( q_{ji} = 0 \). The sets
that represent trade patterns are defined as follows:

\[
B = \{ (\theta_1, \theta_2) \in \mathbb{R}_+^2 \mid \theta_1 < \bar{\theta}_1(\theta_2) \text{ and } \theta_2 < \bar{\theta}_2(\theta_1) \},
\]

\[
U^{ij} = \{ (\theta_1, \theta_2) \in \mathbb{R}_+^2 \mid \theta_i \geq \bar{\theta}_i(\theta_j) \text{ and } \theta_j < \bar{\theta}_j(\theta_i) \} \quad \text{for } i \neq j,
\]

\[
A = \{ (\theta_1, \theta_2) \in \mathbb{R}_+^2 \mid \theta_1 \geq \bar{\theta}_1(\theta_2) \text{ and } \theta_2 \geq \bar{\theta}_2(\theta_1) \}.
\]

The sets \( B \), \( U^{ij} \) and \( A \) represent pairs of tariffs that characterize bilateral
trade, unilateral trade (only firms in country \( i \) can export to country \( j \)),
and autarky, respectively.
1.3.1 Third-stage game: Cournot competition

Given the tariff rates and the location of firms, each firm maximizes Equation (1.2.2) in the last stage. The equilibrium price of M-good is given as follows:

\[
p_i = \begin{cases} 
  \frac{a}{n_i + 1} & \text{if } \theta_i \geq \bar{\theta}_i(\theta_j), \\
  \frac{a + n_j(\tau + \theta_i)}{3} & \text{if } \theta_i < \bar{\theta}_i(\theta_j).
\end{cases}
\] (1.3.1)

Using \( n_i + n_j = 2 \) and \( q_{ji} = 0 \), we obtain the no trade threshold:

\[
\bar{\theta}_i = \frac{a}{n_i + 1} - \tau.
\] (1.3.2)

As we will observe below, \( n_i \) is determined as a function of the tariffs.

The equilibrium rents for firms are derived as follows:

- If \((\theta_1, \theta_2) \in A\),
  \[
  r_i = \left( \frac{a}{n_i + 1} \right)^2.
  \] (1.3.3)

- If \((\theta_1, \theta_2) \in U^{ij}\),
  \[
  r_i = \left( \frac{a}{n_i + 1} \right)^2 + \left[ \frac{a - (n_j + 1)(\tau + \theta_j)}{3} \right]^2,
  \] (1.3.4)
  \[
  r_j = \left[ \frac{a + n_i(\tau + \theta_j)}{3} \right]^2.
  \] (1.3.5)

- If \((\theta_1, \theta_2) \in B\),
  \[
  r_i = \left[ \frac{a + n_j(\tau + \theta_i)}{3} \right]^2 + \left[ \frac{a - (n_j + 1)(\tau + \theta_j)}{3} \right]^2.
  \] (1.3.6)

In the above equations, the terms on the right-hand side give the operating profits earned in each country.
### 1.3.2 Second-stage game: equilibrium location of firms

In the second stage, firms can freely move to any location where they can earn a higher profit. Table 1.1 indicates the payoff of this location game. The term \( r_i(n_1) \) denotes the rent of return when a firm operates in country \( i \) as a function of the number of firms located in country 1.

Computing the signs of \( r_1(2) - r_2(1) \) and \( r_1(1) - r_2(0) \), one can determine the equilibrium location configuration for any given pair of positive tariffs. Table 1.2 summarizes the results of the location game. Except in a knife-edge case, the game has a unique equilibrium configuration for any \( (\theta_1, \theta_2) \in \mathbb{R}^2_+ \).

The sets representing location configurations are defined as follows.

\[
C^1 = \{(\theta_1, \theta_2) \in \mathbb{R}^2_+ \mid n_1 = 2\},
\]

\[
I = \{(\theta_1, \theta_2) \in \mathbb{R}^2_+ \mid n_1 = 1\},
\]

\[
C^2 = \{(\theta_1, \theta_2) \in \mathbb{R}^2_+ \mid n_1 = 0\}.
\]

When \( \tau \) and \( \theta_j \) are low enough, a sufficiently high \( \theta_i \) makes firms locate together in country \( i \). Otherwise, equilibrium would involve dispersion.

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<td>Country 1</td>
<td>( r_1(2), r_1(2) )</td>
<td>( r_1(1), r_2(1) )</td>
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<tr>
<td>Country 2</td>
<td>( r_2(1), r_1(1) )</td>
<td>( r_2(0), r_2(0) )</td>
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Table 1.1: Payoff matrix
\[ r_1(2) - r_2(1) \quad r_1(1) - r_2(0) \quad \text{Equilibrium Location} \]

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<td>( n_1 = 2 )</td>
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<tr>
<td>−</td>
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<td>( n_1 = 1 )</td>
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<td>−</td>
<td>−</td>
<td>( n_1 = 0 )</td>
</tr>
<tr>
<td>0</td>
<td>+</td>
<td>( n_1 \geq 1 )</td>
</tr>
<tr>
<td>−</td>
<td>0</td>
<td>( n_1 \leq 1 )</td>
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Table 1.2: Equilibrium location

1.3.3 First-stage game: tariff competition for welfare

The first-stage game considers tariff competition. The welfare maximization problem is divided into the following two components. First, the ‘local’ maxima within each of the subsets are identified from the first-order conditions. Second, the welfare levels are compared to identify whether either of the governments has any incentive to deviate.

Welfare is affected by a tariff increase through the following three channels. First, strategic interactions among firms play an important role in welfare analysis. Protecting the domestic firm in one country increases domestic sales and lowers foreign sales in the market because domestic and foreign outputs are strategic substitutes under Cournot competition. Such a production shift benefits the domestic firm, and consequently enhances domestic welfare by saving on transport costs. At the same time, domestic protection reduces the total supply in the domestic market, thereby in-
creasing the domestic price. As \textit{Brander and Krugman} (1983) emphasized, each government encounters a trade-off between saving transport costs and fostering competition. The level of transport costs and the location configuration of the firms determine which one of the two effects, the production-shifting effect or the anticompetitive effect, is more dominant.

Second, a tariff change generates a rent-shifting effect that creates international externalities. Since the equities of firms are equally owned by domestic and foreign consumers, domestic consumers receive only half of the profits (or losses) from a change in rents.

The first-order conditions within each subset represent the reactions of each of the countries to their rival’s tariff without any changes in the trade patterns and the locations of the firms. Keeping trade flow and firm location unchanged, the optimal trade policy is to reduce the tariff to zero.

Third, trade barriers operate through a tariff-jumping relocation of firms. Raising tariff barriers above a certain level would attract an additional firm into the region because suppliers want to be protected by a high tariff rate and export goods at a low tariff rate. Spatial concentration enhances the competitive pressure on the firms, and therefore a lower price of $M$-good.

\footnote{Firms can only partially transfer the marginal costs to foreign consumers ($\partial p_i/\partial \theta_i \in [0, 2/3] \text{ if } \theta_i < \bar{\theta}_i$). In other words, there exists “reciprocal dumping” which can be regarded as terms-of-trade gains (or losses). We would like to emphasize that dumping will work even if firms co-locate.}

\footnote{All these conditions satisfy the associated second-order conditions. When country $i$ does not import $M$-good, $\theta_i$ is indeterminate.}
benefits consumers who incur no trade costs. Despite this, an increase in the number of firms located in a country will certainly lead to a reduction in that country’s tariff revenue. Note that location configuration influences the impact of both production shifting and anti-competitive effects.

**Benchmark case: international tariff co-ordination**

We assume that there is a world-level benevolent planner who simultaneously establishes the tariff levels in both countries, thereby maximizing the sum of indirect utilities, which is given by \( \max_{\theta_1, \theta_2} (V_1 + V_2) \). The following proposition describes the optimal tariff policy as a function of \( \tau \).

**Proposition 1.1.** Assume that the economy is duopolistic. Coordinated tariff rates are given as follows:

1. For \( \tau \geq a/4 \), \((\theta_1, \theta_2) \in A \cap I\);
2. For \( a/5 < \tau < a/4 \), \((\theta_1, \theta_2) \in B \cap I \) and \( \theta_1 = \theta_2 = 5\tau - a > 0 \);
3. For \( \tau \leq a/5 \), \((\theta_1, \theta_2) = (0, 0) \in B \cap I \).

**Proof.** Calculating the first-order conditions in \((\theta_1, \theta_2) \in B \cap I \), \((\theta_1, \theta_2) \in U^{ij} \cap I \) and \((\theta_1, \theta_2) \in C^i \), one can find the possible optimal level of global welfare in each of the subsets. Then, the global optimum can be found by comparing these values. A full derivation is provided in Appendix 1.A.

When wasteful transport charges are sufficiently higher than demand, international trade becomes rather expensive. Therefore, it is more effective to impose high tariffs that restrict the import of products and promote
domestic production. However, the planner will lower tariffs once the transport costs become so low that the wastage of resources is less than the loss from the anticompetitive effect.

For any level of transport costs, the benchmark policy requires dispersed locations. In other words, if countries are symmetric, there are agglomeration diseconomies. What matters is the strategic channel. If two firms are located in the same country, no firm can enjoy a cost advantage in the markets. If the firms are located in different countries, in contrast, each firm enjoys an advantage of circumventing the payment of trade costs in its own home market. Consequently, domestic production substitutes for imports in both countries, thereby saving on transport costs. For $\theta_i = \theta_j$ and any $\tau \geq 0$, the sum of equilibrium outputs $\sum_i \sum_j n_i q_{ij}$ is unchanged regardless of whether firms are agglomerated or dispersed. Global welfare will be improved by dispersing the location and equally consuming the outputs due to the diminishing marginal utility of consumption.

Non-cooperative equilibria

We now investigate the case where governments select a tariff rate, $\theta_i$, to maximize the welfare of their citizens, $V_i$.

For $\tau \geq a/4$, the sets $C^1$ and $C^2$ are empty, so that the tariff-jumping channel is eliminated. It results in a unique free-trade equilibrium in this

\footnote{This depends on the assumption of both symmetric countries and duopolistic markets. However, if we relax even one of these assumptions, then the analytical description cannot be pursued over a broad range of parameter values.}
game; that is, $\theta_1 = \theta_2 = 0$.

**Proposition 1.2.** When transport costs are sufficiently high, free trade is a unique subgame perfect Nash equilibrium of the tariff competition for duopoly firms in identical countries. This equilibrium, wherein firms are geographically dispersed, is less efficient than that obtained under tariff coordination.

**Proof.** For $\tau \in [a/4, a/2)$, we have:

$$V_1(0, 0) \in B \cap I - V_1(\theta_1, 0) \in U_{12} \cap I = V_1(0, \theta_2) \in A = (a - 2\tau)^2 > 0.$$

A protected country always profits by opening up its import market. This discussion is completely pertinent for country 2 as well. Proposition 1.1 indicates that free trade is inefficient.

This proposition is different from the benchmark case because the benevolent planner internalizes the rent-shifting effect, which is counterbalanced by summing the welfare of the countries, and hence considers only the strategic effects.

MPT suggested that sufficiently high transport costs result in an equilibrium wherein both governments establish excessive protection, which necessitates a mutually binding agreement for free trade. However, our model implies that the non-cooperative equilibrium is instead characterized by too little protection as countries adopt an inefficient free trade policy. The reason for the difference in the welfare implication is the market structure. Under price competition, there are gains from trade because the anticompetitive effect of trade restriction is stronger than
under quantity competition. Under quantity competition, by contrast, there are losses from trade when transport costs are high. As trade opens up, the increase in consumer surplus must be outweighed by the reduction in the profits of both firms.

Subsequently, the case where $\tau < a/4$ is considered. Figure 1.1 illustrates equilibrium trade as well as location patterns as the functions of the countries’ tariff offers. When transport costs are sufficiently lower than demand, there exists no equilibrium in set $I$.

**Lemma 1.1.** If transport costs are low enough to cause agglomeration ($\tau < a/4$), then a geographically dispersed location is not achievable in equilibrium.
Proof. We can show that there is at least one government which can benefit by unilaterally deviating from any point for \( \tau < a/4 \) such that \((\theta_1, \theta_2) \in B \cap I\), \((\theta_1, \theta_2) \in A\), and \((\theta_1, \theta_2) \in U^{ij} \cap I\). See Appendix 1.B for details.

Propositions 1.1 and 1.2 and Lemma 1.1 indicate that non-cooperative tariff competition is unambiguously harmful for global welfare.

The next proposition characterizes the equilibrium for \( \tau < a/4 \).

**Proposition 1.3.** When transport costs are sufficiently lower than demand \((\tau < (9 - \sqrt{78})a/12 \approx 0.014a)\), there exists a subgame perfect Nash equilibrium such that one country sets its tariff at zero and the other imposes a sufficiently high tariff with industrial agglomeration. An equilibrium in pure strategies does not exist for intermediate transport costs.

Proof. Here we wish to show the existence of the core-periphery equilibrium.

The remaining results are proven in Appendix 1.C.

Lemma 1.1 indicates that all pure-strategy Nash equilibria must belong to either \( C^1 \) or \( C^2 \) for \( \tau < a/4 \) if they exist. For \( \tau < (9 - \sqrt{78})a/12 \approx 0.014a \), a peripheral country has no incentive to deprive the protected core country of a firm by increasing its tariff because:

\[
V_2|_{(\theta_1,0) \in U^{12} \cap C^1} - V_2|_{(\theta_1,a/4-\tau) \in U^{12} \cap I} = \frac{a^2 - 72a\tau + 48\tau^2}{144}.
\]

This equation takes a positive value if \( \tau < (9 - \sqrt{78})a/12 \). In this range, \( V_2|_{(\bar{\theta}_1,0) \in U^{12} \cap C^1} > V_2|_{(\bar{\theta}_1,a/4-\tau) \in U^{12} \cap I} > V_2|_{(\bar{\theta}_1,\bar{\theta}_2) \in A} \) and \( V_1|_{(0,0) \in B \cap I} < V_1|_{(\bar{\theta}_1,0) \in U^{12} \cap C^1} \).

Therefore, \( \theta_1 > a/2 - \tau \) and \( \theta_2 = 0 \) are subgame perfect Nash equilibria. Al-
though another equilibrium could exist, all pure-strategy equilibria resulted in the core-periphery location with “limit tariff” owing to Lemma 1.1.

Even among symmetric countries, we arrived at the asymmetric location of the industry in equilibrium. Tariff competition with low transport costs leads to a core-periphery economy wherein the periphery country imposes a zero tariff for importing goods. When transport costs are sufficiently low, it is rather economical for the periphery country to import goods. The government has a weak incentive to attract firms by increasing the level of tariff protection, which exacerbates the anti-competitive effect and reduces the rents for firms.

In this case, no country collects tariff revenue in equilibrium for low transport costs. This indicates the emergence of de facto free trade. In MPT, de facto free trade with sufficiently low transport costs is optimal; this is contrary to Propositions 1.1 and 1.3. Even without price-distorting tariffs, we find that distortions in the location of firms continue to exist because of the Nash policies. De jure free trade agreements are needed when transport costs are not high but low. The political implication derived from the monopolistic competition setting may therefore be a model-specific result.
1.4 Conclusion

This study has developed a strategic tariff competition model for mobile firms that engage in quantity competition. Proposition 1.2 suggests that tariff competition among symmetric countries generates symmetric access costs and an inefficient equilibrium when transport costs are sufficiently high. Consequently, mutual trade protection may improve welfare levels. On the other hand, tariff competition results in asymmetric access costs and the spatial agglomeration of firms in the process of economic integration. In such a case, nobody bears tariff burdens; however, industrial distributions without free entry are inefficient in such equilibria. Then what is required is a trade agreement that can deter the tariff war (for example, imposing tariff ceilings). These findings are contrary to the those of previous studies, even though both the models have indicated similar relationships between tariff policy and transport costs. Even insofar as a model can replicate certain observations, careless applications of that could easily harm rather than help the economy.

Appendix 1.A Proof of Proposition 1.1

Global welfare (net of constant term $2(1 + \omega)$) in each case is calculated as follows:

$$(V_1 + V_2)|_{(\theta_1, \theta_2) \in B \cap I} = \frac{1}{18} [16a^2 + 22\tau^2 - \theta_1^2 - \theta_2^2 + 10\tau(\theta_1 + \theta_2) - 2a(8\tau + \theta_1 + \theta_2)],$$

$$(V_1 + V_2)|_{(\theta_1, \theta_2) \in U_{ij} \cap I} = \frac{1}{72} [59a^2 + 44\tau^2 + 40\tau\theta_j - 4\theta_j^2 - 8a(4\tau + \theta_j)],$$
\[(V_1 + V_2)|_{(\theta_1, \theta_2) \in C^i} = \frac{2}{9} [4a^2 + 2\tau^2 + \tau\theta_j - \theta_j^2 - a(4\tau + \theta_j)].\]

Solving \(\max_{\theta_1, \theta_2} V_1 + V_2\), one has the following first-order conditions:

- \(\theta_1 = \theta_2 = 5\tau - a\) for \((\theta_1, \theta_2) \in B \cap I\). According to the non-negativity requirement of tariff rates, \(\theta_1 = \theta_2 = 0\) are global welfare maximizers for \(\tau \leq a/5\).

- \(\theta_j = 5\tau - a\) for \((\theta_1, \theta_2) \in U^{ij} \cap I\). For \(5a/24 < \tau < a/4\) and \(\theta_i \geq a/2 - \tau\), \((\theta_i, 5\tau - a) \in U^{ij} \cap I\).

- \(\theta_j = (\tau - a)/2 < 0\) for \((\theta_1, \theta_2) \in C^i\).

All the conditions also satisfy the second-order condition.

Substituting the optimal tariffs from above into global welfare yields the maximum welfare levels within each of the subsets. Figure 1.2 summarizes these calculations.

Appendix 1.B  Proof of Lemma 1.1

We demonstrate that there exists no Nash equilibrium for \(\tau < a/4\) such that \((\theta_1, \theta_2) \in B \cap I\), \((\theta_1, \theta_2) \in A\), and \((\theta_1, \theta_2) \in U^{ij} \cap I\). The following proof relates to country 1 but is completely pertinent to country 2.

For \(\tau < a/4\) and given \(\theta_j = 0\), country \(i\) decides to deprive firms with \(\theta_i \geq \bar{\theta}_i\):

\[V_1|_{(0,0) \in B \cap I} - V_1|_{(\bar{\theta}_i,0) \in U^{ij} \cap C^i} = -\frac{\tau}{18} (4a - 9\tau) < 0.\]

Therefore, free trade is no longer a global Nash equilibrium.
Likewise, for $\tau < a/4$ and given that $\theta_j \geq a/2 - \tau$, country $i$ continues to reduce its tariff until its consumers can import $M$-good:

$$V_1|_{(a/4-\tau,\theta_2)\in U^{1j}\cap I} - V_1|_{(\theta_1,\theta_2)\in A} = \frac{a(3a - 8\tau)}{48} > 0.$$  

In other words, autarky is also unachievable in equilibrium.

In addition, we can show that within the set $U^{ij} \cap I$, exporting country $j$ has an incentive to open up its market:

$$V_1|_{(\theta_1,a/4-\tau)\in U^{1j}\cap I} - V_1|_{(a/4-\tau,a/4-\tau)\in B\cap I} = -\frac{a(3a - 8\tau)}{48} < 0.$$  

Therefore, for $\tau < a/4$, the protected country where one of the firms is not located will reduce its tariff and import $M$-good. There is no Nash equilibrium in $U^{ij} \cap I$ for $\tau < a/4$. 

Figure 1.2: Transport costs and maximized global welfare within each set.
Appendix 1.C  Proof of Proposition 1.3

We show that a periphery country has an incentive to increase its tariff and refuse to import rather than agree to be a periphery for at least $\tau \in (3a/16,a/4)$.

First, we consider $\theta_1 \geq \Phi_2$ for any $\theta_2$.

$$V_2|_{(\theta_1,0)\in U^{12} \cap C^1} - V_2|_{(\theta_1,a/4-\tau)\in U^{12} \cap I} = \frac{a^2 - 72a\tau + 48\tau^2}{144}.$$  

This equation takes a negative value if $\tau > (9 - \sqrt{78})a/12 \approx 0.014a$. Therefore, in the range $\tau \in (3a/16,a/4)$, a periphery country will deprive the protected core country of a firm.

Second, we identify a range wherein a periphery country can become a core by increasing its tariff. When $\theta_1 = \Phi_2$ and $\theta_2 = 0$, then $r_1(2) = r_2(1)$ where $\Phi_2 = \left(1 - \sqrt{1 - 4\tau/a}\right)a/2 - \tau$. $\Phi_2$ is a horizontal intercept of $r_1(2) = r_2(1)$ line in Figure 1.1. Here, $(a/4-\tau) - \Phi_2 > 0$ since $\tau < 3a/16$. In other words, when $\tau > 3a/16$, periphery country 2 never obtains both firms by increasing its tariff for a given $\theta_1 \in [\Phi_2,a/2-\tau)$ such that $(\theta_1,0) \in C^1$.

$$\partial \left( V_2|_{(\theta_1,0)\in C^1} - V_2|_{(\theta_1,\theta_2)\in U^{21} \cap I} \right) / \partial \theta_1 = \frac{a - 5\tau - 5\theta_1}{9} < 0.$$  

The difference decreases in $\theta_1$ for $\tau > 3a/16$ and $\theta_1 \geq \Phi_2$. Therefore, if $V_2|_{C^1} - V_2|_{U^{21} \cap I} < 0$ in the border between $U^{21} \cap I$ and $A$, which is the point at which the difference is smallest, then $V_2|_{C^1} < V_2|_{U^{21} \cap I}$ holds for all
\( \theta_1 \geq \Phi_2 \). The following equation is presented for \( \tau > 3a/16 \):

\[
\lim_{\epsilon \to 0} \left( V_2 \bigg|_{(\Phi_2 + \epsilon, 0) \in C^1} - V_2 \bigg|_{(\Phi_2 + \epsilon, \theta_2) \in U^{21} \cap I} \right) = -\frac{1}{36} \left( 14a\tau - 12\tau^2 - 3a^2 \sqrt{1 - 4\tau/a} \right) < 0.
\]

Propositions 1 and 2 and Lemma 2 indicate that the equilibria are inefficient.
Bibliography


Chapter 2

Capital mobility—a resource curse or blessing? How, when, and for whom?

2.1 Introduction

In the past few decades, we have observed drastic increases in capital flows between regions and countries. Such capital movements have provoked intensive discussions on the direction of capital move and governments’ reaction to capital flows. These issues have been tackled by numerous studies in the literature of tax competition theory, whose long history dates back

*This chapter is based on Ogawa, Sato, and Oshiro (2012).*
at least to Zodrow and Mieszkowski (1986) and Wilson (1986). The literature investigates the role of governments in attracting capital to their jurisdictions by mainly focusing on the effects of capital tax and subsidy policies. A significant strand of the literature emphasizes that regions and countries differ in many aspects and analyzes the case of asymmetric regions and countries. They place due importance on regional disparities in, for instance, population (Bucovetsky 1991; Kanbur and Keen 1993; Ottaviano and van Ypersele 2005; Sato and Thisse 2007; Wilson 1991), capital endowment (DePater and Myers 1994; Peralta and van Ypersele 2005; Itaya et al. 2008), and degree of market competitiveness (Haufler and Mittermaier 2011; Egger and Seidel 2011; Ogawa et al. 2010). In this chapter, we introduce an additional aspect of regional disparities—resource availability—which is undoubtedly key to the production of firms and yet has been overlooked in this literature.\footnote{Wilson (1999), Wilson and Wildasin (2004), and Fuest et al. (2005) provide surveys on the literature of tax competition.}

\footnote{Of course, this does not imply that the tax competition literature neglects other types of policies that might be relevant. For instance, studies such as Bayindir-Upmann (1998), Bucovetsky (2005), Cai and Treisman (2005), Fuest (1995), Matsumoto (1998), Noiset (1995), and Wrede (1997) examined the role of infrastructure and institutions provided by the local governments to benefit production possibilities.}

\footnote{To the best of the authors’ knowledge, Raveh (2011) is the only exception that studies the role of natural resources in tax competition. He incorporated a competitive resource sector into a standard capital tax competition model. However, his focus is on the differences in tax instruments available between countries and not on the resources of a particular country.}
More specifically, we explore the effects of natural resources on the distribution of capital across countries, governments’ reaction to capital flows, and the influence on a regional welfare of capital flows and tax competition. To accomplish this, we develop a tax competition model involving two countries, of which one is endowed with natural resources. There are two sectors in the economy: the numéraire good sector and the resource-based intermediate good sector. The former is characterized by perfect competition, and its production requires capital, labor, and intermediate goods. The latter is characterized by oligopoly à la Cournot, and its production requires capital as a variable input and the numéraire goods as a fixed input. We focus on the circumstances in which the intermediate good can be produced only in places where the natural resources exist, because it is prohibitively costly to transport the resource itself across countries.

Using this framework, we first examine the impact of capital market integration in a laissez-faire economy (without government intervention). We show that once the capital markets are integrated, resource-rich countries can import capital from resource-poor countries. Although such capital movements help improve global production efficiency and increase global welfare, the gains accrue only to resource-poor countries. Resource-rich countries, in contrast, may suffer due to the capital movements. We refer to this as the resource-curse associated with capital market integration. We next investigate the implications of a tax game in our environment. In a tax

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4 Throughout the chapter, the phrase ‘curse’ (or ‘blessing’) is defined as a decrease (or an increase) in welfare in a static sense. Notice that different authors attribute different
game, governments can levy a tax/subsidy on capital. In equilibrium, both
countries levy a tax on capital, the rate being higher in the resource-rich
country than in the resource-poor country. This is consistent with [Slemrod
(2004)], who empirically showed that a country enjoying higher per capita
income from natural resources (oil) is likely to levy higher taxes on corpor-
ate income.\footnote{In addition, this chapter shows that resource-rich countries
gain from tax competition, while resource-poor countries are disadvantaged
by it: there is a \textit{resource-blessing} associated with tax competition. Since the
latter loss dominates the former gain, the tax game reduces global welfare
compared to the laissez-faire economy.}

Besides the tax competition literature, the importance of natural re-
sources is widely recognized in the other fields of economics: beginning with
a seminal article by [Sachs and Warner (1995)], many scholars have widely
discussed the impacts of natural resource wealth on economic growth. This
literature suggests that large natural resource endowments can affect eco-
nomic performance both positively and negatively through the Dutch dis-
ease, institutional quality, armed conflict, volatility of commodity prices,

\footnote{However, a controversy exists over the robustness of this empirical finding. [Dharmap-
ala and Hines Jr. (2009)] concluded that higher corporate tax rates are not observed in
the data of resource-abundant countries.}
financial imperfection, or investment of human capital. However, none of these studies focused on the mechanisms for transferring natural resources to the economy through fiscal externalities arising from factor mobility. Given the increasingly pervasive influence of capital mobility and governments’ concern about it, it is indispensable to understand the features and impacts of possible interactions among the unevenly distributed natural resources, capital mobility and the role of governments.

In the literature on growth and natural resources, Bretschger and Valente (forthcoming) would be the most closely related to this chapter. Extending the two-country endogenous growth model, they investigate the strategic resource taxation policies of resource-rich and resource-poor economies that are involved in an asymmetric trade structure induced by uneven endowments of natural resources. They showed that a resource-poor country has an incentive to levy taxes on the use of domestic resources at an excessively high rate to reduce resource dependency. In a similar vein, this chapter examines an economy in which the geographical necessity and availability of natural resources induce an asymmetric industrial structure and then inter-industry trade linkages. The main difference is that this chapter

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6 The literature on the so-called “natural resource curse” is comprehensively reviewed by Frankel (2010) and van der Ploeg (2011). For an overview on the recent empirical literature, see Torvik (2009) and Rosser (2006).

7 Wildasin (1993) also constructs a tax competition model with inter-industry trade linkages. In contrast, we characterize the equilibrium arising from tax competition and examine the welfare properties of such equilibrium.
mainly examines the role of a mobile production factor (capital), whereas Bretschger and Valente (forthcoming) does not deal with this issue.

The rest of the chapter is organized as follows. The basic environment is presented in Section 2.2. In Sections 2.3 and 2.4 we study the effects of capital market integration without government intervention and the effects of tax competition, respectively. Section 2.5 discusses the robustness of our main results against possible extensions and Section 2.6 concludes the chapter.

2.2 The basic settings

Consider two countries (1 and 2) in each of which there is a representative individual of measure one possessing two factors of production, labor ($L$) and capital ($K$). Each factor endowment in each country is fixed at unity. We assume that individuals are immobile between countries and inelastically

\[ \bar{x}_i \] is the endowment of $x$ in country $i$, if $\bar{K}_1 = \bar{K}_2 = \bar{L}_1 = \bar{L}_2 \neq 1$, what follows continues to hold true. If $\bar{K}_1 < \bar{K}_2 = \bar{L}_1 = \bar{L}_2$, i.e., if resource-rich country has fewer per capita endowment of capital than resource-poor country as in the real world, then our main results (Propositions 2.2–2.5) remain largely true as long as we restrict attention to interior solutions. Only the statement about country 1’s welfare in Proposition 2.2 has to be modified because as capital income becomes relatively unimportant to wage income, the negative effect that a benefit of natural resources shrinks according to capital market integration is dominated by the positive effect of the integration on production efficiency. See also Peralta and van Ypersele (2005) for tax competition among countries with asymmetric factor endowments.
supply their labor in their own country of residence. In the followings, we consider two scenarios in which capital is either immobile or mobile. In the first case, all factor markets are segmented, and in the second case, individuals can freely choose where to supply their capital, such that both labor markets are segmented but the capital markets are integrated. We first compare these two cases without taxation, and then introduce the tax game to the case in which capital is mobile.

Two goods are produced, a numéraire good (X) and a resource-based intermediate good (M) (e.g., petroleum, steel, and minor metals). X-good is produced using capital, labor, and the intermediate good (M-good) as inputs under perfect competition. The production of M-good requires capital as a variable input and X-good as a fixed input. We assume that the production of M-good does not need labor because such resource-based sectors are considered highly capital intensive and account for only a small part of employment.\footnote{For instance, among all the EU countries, Romania had the highest employment share of the mining and quarrying industry in 2009 (Eurostat, http://epp.eurostat.ec.europa.eu). Still, its employment share of the mining and quarrying industry is only 3.3 percent. The share in most EU countries is less than 2 percent.} Natural resources exist only in country 1, and it is prohibitively costly to transport them to country 2. We call countries 1 and 2 the resource-rich and resource-poor countries, respectively. In country 1, firms start production after paying for the fixed input as entry costs; they exploit the natural resources (e.g., raw crude oil, iron ore, and other mineral ore) and transform them into M-good, using capital. M-good is tradable without
incurring additional costs. The mining industry is an example of the M-good sector. Imagine the production of rare earths. Exploration companies export purified and lighter rare earth elements after separating and refining them near the mine sites. This is because ores mined are so heavy that it would be quite costly to transport them, but purified rare earth elements are light enough to be exported. The concentration of resource-based intermediate production implies that X-good is produced in both countries whereas M-good is produced only in country 1, and both the produced goods are traded freely without costs. Thus, country 2 imports M-good from country 1 while exporting X-good. Figure 2.1 describes the environment of the model.

In the numéraire sector, the profit of the firm is given by

$$\Pi_i = X_i - (r_i + t_i)K_i - w_iL_i - p_M M_i,$$

where $w_i$, $r_i$, and $t_i$ are the labor wage rate, capital price, and capital tax rate in country $i \in \{1, 2\}$, respectively; $p_M$ represents the price of M-good, equalized across countries. The constant returns to scale production

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10 Of course, this is an extreme case. In the other extreme case, the production of M-good is equally possible in country 2 as well. Such a case yields the same allocation as the one observed in the mobile capital case without government interventions in this chapter. The reality lies between the two: one country has some advantage in producing M-good over the other. Our analysis then works to pin down the upper limit of the possible effects of this type of asymmetry.
function for producing X-good in country $i$ is assumed to be quadratic:

$$X_i = \alpha(K_i + M_i) - \frac{\beta}{2L_i}(K_i^2 + M_i^2) - \frac{\gamma}{2L_i}(K_i + M_i)^2,$$

where $\alpha$, $\beta$, and $\gamma$ are constants satisfying $\alpha > 0$, $\beta > 0$ and $\beta + 2\gamma > 0$ to guarantee that the Hessian matrix of $\Pi_i$ is negative definite. $\alpha$ represents the level of productivity, and $\beta$ measures (inversely) the own-price effects on factor demands. $\gamma$ captures the substitutability/complementarity between capital and M-good in production: a positive (resp. negative) $\gamma$ represents

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$^1$\(\alpha\) is assumed to be sufficiently large to ensure that both factor prices and factor employments are positive in equilibrium.
that capital and M-good are Pareto substitutes (resp. Pareto complements),
that is, the marginal product of one input is decreasing (resp. increasing)
in the other input. A quadratic production function is often used in the
literature on tax competition. For example, see Bucovetsky (1991), Elitzur
and Mintz (1996), Peralta and van Ypersele (2006), and Devereux et al.
(2008).12

From a firm’s profit maximization, we obtain the linear factor demand
functions (relative to labor) as follows:

\[ \frac{K_i}{L_i} = \frac{\alpha}{\beta + 2\gamma} - \frac{1}{\beta}(r_i + t_i) + \frac{\gamma}{\beta(\beta + 2\gamma)}(r_i + t_i + p_M), \]  
(2.2.1)

\[ \frac{M_i}{L_i} = \frac{\alpha}{\beta + 2\gamma} - \frac{1}{\beta}p_M + \frac{\gamma}{\beta(\beta + 2\gamma)}(r_i + t_i + p_M). \]  
(2.2.2)

The second terms on the right hand side are decreasing in their own factor
prices. The third terms are either increasing or decreasing in a factor price
index, \((r_i + t_i + p_M)\), depending on the sign of \(\gamma\).

Substituting (2.2.1) and (2.2.2) into the profit function, the profit is
rewritten as

\[ \Pi_i = (\Lambda_i - w_i)L_i, \]
where
\[
\Lambda_i := \frac{2\beta \alpha (\alpha - r_i - t_i - p_M) + \beta [(r_i + t_i)^2 + p_M^2] + \gamma [(r_i + t_i) - p_M]^2}{2\beta (\beta + 2\gamma)}.
\]

In the competitive environment, the labor markets are cleared and the wage rate is determined by the zero profit condition:

\[L_i = 1, \quad (2.2.3)\]
\[w_i = \Lambda_i.
\]

The factor price frontiers are \(\partial w_i/\partial r_i = -K_i/L_i < 0\) and \(\partial w_i/\partial p_M = -M_i/L_i < 0\).

The total demand for M-good is given by \(M := M_1 + M_2\), yielding the inverse demand function for the good:

\[p_M = \frac{2\alpha \beta - \beta (\beta + 2\gamma) M + \gamma \sum_{i=1}^{2} (r_i + t_i)}{2(\beta + \gamma)}. \quad (2.2.4)\]

We assume that the M-good sector is characterized by oligopoly, where \(n\) identical firms (M-firms) producing M-good engage in Cournot competition. Each firm in country 1 determines the quantity of M-good supplied after paying for a fixed requirement, \(F(> 0)\) units of the numéraire good, as the entry cost (e.g., a cost to procure mining concession). Each firm needs one unit of capital to produce one unit of M-good. A firm’s profit is given by

\[\pi = [p_M - (r_1 + t_1)]m - F,\]

where \(m\) gives the firm’s supply of M-good, and \(r_1\) and \(t_1\) are the endogenous capital price and (temporarily exogenous) capital tax rate, respectively. For
given factor prices, the Cournot equilibrium is characterized by the level of output \( m \), the price of M-good \( p_M \), and the number of firms in the M-good sector \( n \). Using \( M = \sum^n m \), the level of outputs in the Cournot equilibrium is

\[
m = \frac{M}{n} = \frac{2\alpha\beta - 2(\beta + \gamma)(r_1 + t_1) + \gamma \sum_{i=1}^{2}(r_i + t_i)}{\beta(\beta + 2\gamma)(n + 1)}.
\]

(2.2.5)

Equations (2.2.4) and (2.2.5) give the equilibrium price of M-goods:

\[
p_M = \frac{\alpha\beta}{(\beta + \gamma)(n + 1)} + \frac{n(r_1 + t_1)}{n + 1} + \frac{\gamma \sum_{i=1}^{2}(r_i + t_i)}{2(\beta + \gamma)(n + 1)}.
\]

(2.2.6)

We assume that firms enter and exit the market freely. Then, the profit of a firm is driven to zero, determining the equilibrium number of firms as follows,

\[
n = \frac{2\alpha\beta - 2(\beta + \gamma)(r_1 + t_1) + \gamma \sum_{i=1}^{2}(r_i + t_i)}{\sqrt{2\beta(\beta + \gamma)(\beta + 2\gamma)F}} - 1.
\]

(2.2.7)

We relax the free entry assumption in a later section.

The capital markets are perfectly competitive. Capital market clearing requires

\[
K_1 + M = 1, \text{ and } K_2 = 1,
\]

(2.2.8)

when the capital is immobile, and

\[
K_1 + M + K_2 = 2
\]

(2.2.9)

\[13\] Amir and Lambson (2000) provide the conditions under which the Cournot equilibrium exists and is symmetric. Our settings satisfy those conditions: The profit is a supermodular function on the relevant domain.

\[14\] We ignore the integer constraint and consider the number of firms as a positive real number.
when the capital is mobile. These market clearing conditions determine the capital prices $r_i$.

## 2.3 Effects of capital mobility

Before considering the tax game, let us examine the effects of capital mobility by comparing the case of immobile capital with that of mobile capital in the absence of policy intervention (i.e., $t_1 = t_2 = 0$). This comparison will form the basis of our analysis of the tax game (Section 2.4).

### 2.3.1 Equilibrium factor prices

The equilibrium is characterized by profit maximization, free entry, and full employment conditions. We start from the case in which there is no capital mobility. Using equations (2.2.1) to (2.2.6) and $t_1 = t_2 = 0$, the market clearing conditions (2.2.8) are rearranged to yield the capital prices as functions of the number of firms $n$:

$$r_1 = \alpha - \gamma - \beta \frac{\beta + 2\gamma + n(\beta + \gamma)}{\beta + 2\gamma + n(3\beta + 4\gamma)}, \quad (2.3.1)$$

$$r_2 = \alpha - \gamma - \beta \frac{\beta + 2\gamma + n(3\beta + 5\gamma)}{\beta + 2\gamma + n(3\beta + 4\gamma)}. \quad (2.3.2)$$

Equations (2.3.1) and (2.3.2) show how the number of firms in the M-good sector affects capital prices: $dr_1/dn > 0$, and $dr_2/dn \leq 0$ if and only if $\gamma \geq 0$. An increase in $n$ would raise the demand for capital in country
1, resulting in an increase in the capital price. Although the increase in the capital price in country 1 raises the marginal cost that M-firms face, a larger number of M-firms would lower the price of M-good by intensifying competition. When capital and M-good are Pareto substitutes (resp. Pareto complements), a lower $p_M$ will decrease (resp. increase) the demand for capital and lower (resp. raise) the capital price in country 2.

Plugging (2.3.1) and (2.3.2) into (2.2.7), we obtain the equilibrium number of M-firms as

$$n^I = \frac{2(\beta + \gamma)}{3\beta + 4\gamma} \left( \sqrt{\frac{\beta \Phi}{F}} - \Phi \right), \quad (2.3.3)$$

where the superscript $I$ indicates that the variable is related to the equilibrium without capital mobility (i.e., the case of immobile capital) and $\Phi$ is defined as

$$\Phi := \frac{\beta + 2\gamma}{2(\beta + \gamma)} > 0.$$

Throughout the chapter, we assume that the entry cost is sufficiently small:

$$F < \frac{\beta}{\Phi}.$$

Thus, the equilibrium number of M-firms is strictly positive.

From (2.3.3), the closed-form expressions of the equilibrium factor prices
are as follows:

\[ r_I^1 = \alpha - \frac{(\beta + 2\gamma)^2 + (2\beta + 3\gamma)\sqrt{\beta F}}{3\beta + 4\gamma}, \]  
\[ (2.3.4) \]
\[ r_I^2 = \alpha - \frac{(\beta + 2\gamma)(3\beta + 2\gamma) - \gamma\sqrt{\beta F}}{3\beta + 4\gamma}, \]  
\[ (2.3.5) \]
\[ p_M^I = \alpha - \frac{\beta^2}{3\beta + 4\gamma} + \frac{\beta + \gamma}{3\beta + 4\gamma} \left( \sqrt{\beta F} - 4\gamma \right), \]  
\[ (2.3.6) \]
\[ w_I^1 = \left( \frac{\beta + 2\gamma}{3\beta + 4\gamma} \right)^2 \left( \beta + 2\gamma + \sqrt{\beta F} \right) + \frac{(\beta + \gamma)(5\beta + 8\gamma)F\Phi}{2(3\beta + 4\gamma)^2}, \]  
\[ (2.3.7) \]
\[ w_I^2 = \left( \frac{\beta + 2\gamma}{3\beta + 4\gamma} \right)^2 \left[ 5\beta^2 + 10\beta\gamma + 4\gamma^2 + \beta F/4 - (\beta + 2\gamma)\sqrt{\beta F} \right]. \]  
\[ (2.3.8) \]

From (2.3.4) and (2.3.5), we find that \( r_I^1 > r_I^2 \). Since the intermediate good sector exists, a resource-rich country can enjoy a higher capital price than that in a resource-poor country. Therefore, we will observe the flow of capital from the resource-poor country to the resource-rich country once the capital markets are integrated.

Next, we introduce capital mobility. If we allow for capital mobility, the capital prices will be equalized between countries:\[ r_1 = r_2 =: r. \]  
\[ (2.3.9) \]

Similar to the case of immobile capital, on the basis of (2.2.1) to (2.2.6), we rearrange the capital market clearing conditions (2.2.9) to yield the capital price as functions of the number of firms \( n \):

\[ r = \alpha - \gamma - t_1 - \frac{[\beta + 2\gamma + n(\beta + \gamma)](2\beta - t_1 + t_2)}{2[\beta + 2\gamma + 2n(\beta + \gamma)]}. \]  
\[ (2.3.10) \]

\[ ^{15} \text{Such equalization of the marginal product of capital across countries is reported in Caselli and Feyrer (2007) and Hsieh and Klenow (2007).} \]
We then derive the equilibrium number of M-firms from (2.2.7) and \( t_1 = t_2 = 0 \). In this case, we obtain the number of firms and factor prices as follows:

\[
\begin{align*}
n^M &= \sqrt{\frac{\beta \Phi}{F}} - \Phi, \\
r^M &= \alpha - \gamma - \frac{1}{2} \left( \beta + \sqrt{\beta \Phi F} \right) \\
p^M_M &= \alpha - \gamma - \frac{1}{2} \left( \beta - \sqrt{\beta \Phi F} \right) \\
w^M_1 &= w^M_2 = \frac{\beta + 2\gamma + \Phi F}{4},
\end{align*}
\] (2.3.11, 2.3.12, 2.3.13, 2.3.14)

where the superscript \( M \) represents the equilibrium with capital mobility.

Since \( F < \beta/\Phi \), the equilibrium number of M-firms is positive (i.e., \( n^M > 0 \)). A simple comparison will show that \( r^I_1 > r^M > r^I_2 \), which is the result of capital export from country 2 to country 1 under an integrated capital market.

### 2.3.2 Welfare

Each individual gains utility from consuming the numéraire good. We take the amount of consumption of a representative individual as the criterion of national welfare. It is equal to the national income \( Y_i \) as follows:

\[
\begin{align*}
Y_1 &= w_1 + r_1 + t_1(K_1 + M), \\
Y_2 &= w_2 + r_2 + t_2K_2.
\end{align*}
\] (2.3.15, 2.3.16)

\textsuperscript{16}We assume that the tax revenues are redistributed equally and in a lump-sum fashion to each individual.

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The national income can also be measured on the production side by using zero profit conditions in each sector:

\[
Y_1 = X_1 - F_n + p_M M_2 + r_1(1 - K_1 - M), \tag{2.3.17}
\]

\[
Y_2 = X_2 - p_M M_2 + r_2(1 - K_2). \tag{2.3.18}
\]

That is, the national income consists of the total market value of final goods (i.e., the output of X-good minus the amount to be used in M-sector as a fixed requirement) plus the net factor income from abroad.

From (2.2.8), the net capital income of both countries is equal to zero when their capital is immobile. In the case of immobile capital, substituting the equilibrium number of M-firms (2.3.3) and the equilibrium factor prices (2.3.4)-(2.3.8) into the welfare functions (2.3.17) and (2.3.18), we obtain the equilibrium national welfare:

\[
Y_{1I} = \alpha - \frac{(\beta + \gamma)[-4(\beta + 2\gamma)^2 + (5\beta + 8\gamma)(\phi F - 2\sqrt{\beta F})]}{2(3\beta + 4\gamma)^2}, \tag{2.3.19}
\]

\[
Y_{2I} = \alpha - \frac{(\beta + \gamma)[8(\beta + \gamma)(\beta + 2\gamma) - \beta \phi F + 2\beta \sqrt{\beta F}]}{2(3\beta + 4\gamma)^2}. \tag{2.3.20}
\]

Welfare is unambiguously higher in country 1 than in country 2 under the assumption that \( F < \beta / \Phi \). This is confirmed by

\[
Y_{1I} - Y_{2I} = \frac{2(\beta + \gamma)(\beta + 2\gamma)}{(3\beta + 4\gamma)^2} \left( \sqrt{\beta} - \sqrt{F \Phi} \right)^2 > 0.
\]

This shows that a resource-rich country benefits from the presence of natural resources, which is intuitively plausible.
Using (2.3.11) to (2.3.14), we see that the welfare level across all countries under capital mobility is the same:

\[ Y_1^M = Y_2^M = \alpha - \frac{(\beta + 2\gamma) - \Phi F + 2\sqrt{\beta F}}{4}. \]  

(2.3.21)

This is a direct result of factor price equalization under free trade.

**Proposition 2.1.** *If capital is immobile, welfare is higher in the resource-rich country than in the resource-poor country (i.e., \( Y_1^I > Y_2^I \)). If capital is mobile, welfare is the same across both types of countries (i.e., \( Y_1^M = Y_2^M \)).*

### 2.3.3 Welfare implications of capital mobility: a resource curse or blessing?

Here, we examine the impacts of capital market integration on the welfare of each country and global welfare by comparing \( Y_i^M \) with \( Y_i^I \). From (2.3.19) to (2.3.21), we obtain

\[ Y_1^M - Y_1^I = -\frac{\beta(\beta + 2\gamma)}{4(3\beta + 4\gamma)^2} \left( \sqrt{\beta} - \sqrt{\Phi F} \right)^2 < 0. \]

The difference is strictly decreasing in \( F \). Similarly, for country 2,

\[ Y_2^M - Y_2^I = \frac{(7\beta + 8\gamma)(\beta + 2\gamma)}{4(3\beta + 4\gamma)^2} \left( \sqrt{\beta} - \sqrt{\Phi F} \right)^2 > 0. \]

Furthermore, we can also explore the impacts of such changes on global welfare. In our environment, it is natural to consider global income, defined by \( Y_1 + Y_2 \), as the criterion of global welfare. We can readily see that \( Y_1 + Y_2 \) changes as

\[ Y_1^M + Y_2^M - Y_1^I - Y_2^I = \frac{(\beta + 2\gamma)}{2(3\beta + 4\gamma)} \left( \sqrt{\beta} - \sqrt{\Phi F} \right)^2 > 0. \]
Proposition 2.2. Capital market integration negatively affects the resource-rich country (i.e., $Y_1^M < Y_1^I$) but benefits the resource-poor country (i.e., $Y_2^M > Y_2^I$). It further enhances global welfare (i.e., $Y_1^M + Y_2^M > Y_1^I + Y_2^I$).

Proposition 2.2 implies that the national income of the resource-rich country will unambiguously decrease due to capital mobility. In other words, there exists a resource-curse, that is, the resource-rich country does not enjoy the benefits of capital market integration. When capital is immobile, the uneven distribution of natural resources induces a natural resource bonanza: the resource wealth that raises the rate of returns on capital and then increases capital income would make country 1 better off than country 2 (cf. Proposition 2.1). Once the capital markets are integrated, however, country 2 will be able to access the benefits of the natural resources bonanza through capital investment. Corresponding to the capital inflows, country 1 has to pay for the import of capital. Since the negative effects of the shrinkage of the natural resource bonanza always exceed the positive effects of the expansion in both sectors in country 1, capital mobility leads to a resource-curse. In contrast, country 2 always gains from capital movements, because of the increasing capital income and the expanding M-sector.
2.4 Tax game

2.4.1 Non-cooperative tax competition

Given the effects of capital market integration, we next examine governments’ reactions to such integration, and its welfare implications. In the tax game, the government of each country simultaneously chooses its capital tax level in order to maximize national welfare, anticipating market reactions and taking the tax policy of the other country as given. The tax game consists of three stages: first, the governments determine their tax rates; second, firms enter into the markets; and finally, the production of all goods takes place and the market clearing determines all the prices. We solve the model backward to obtain the subgame-perfect Nash equilibrium.

Since the third stage is already described in Section 2.2, we can start from the second stage. Temporarily, we assume that the tax differences are sufficiently small; that is, $2\beta > t_1 - t_2$. This is necessary for M-firms to have the incentive to produce (i.e., the price-cost margin, $p_M - r_1 - t_1$, is positive). As will be shown later, this condition is satisfied in equilibrium.

Just as in the case when capital is mobile and governments are inactive, we use equations (2.2.1) to (2.2.6) and rearrange the market clearing conditions (2.2.9) to obtain the factor prices as functions of the number of firms $n$. We then derive the equilibrium number of M-firms from (2.2.7). In this case,
we obtain the number of M-firms and factor prices as follows:

\[ n^T = \frac{(2\beta - t_1 + t_2)}{2\beta F} \sqrt{\beta \Phi F} - \Phi, \]
\[ r^T = \alpha - \beta - \gamma + \frac{2\beta - 3t_1 - t_2}{4} - \frac{1}{2} \sqrt{\beta \Phi F}, \]
\[ p^T_M = \alpha - \beta - \gamma + \frac{2\beta + t_1 - t_2}{4} + \frac{1}{2} \sqrt{\beta \Phi F}, \]
\[ w^T_1 = \frac{(2\beta - t_1 + t_2 + 4\gamma)^2}{16(\beta + 2\gamma)} + \frac{\Phi F}{4}, \]
\[ w^T_2 = \frac{(t_1 - t_2) [(5\beta + 8\gamma)(t_1 - t_2) + 8(\beta + 2\gamma)\sqrt{\beta \Phi F}]}{16\beta(\beta + 2\gamma)} + \frac{\Phi F + \beta + 2\gamma + t_1 - t_2}{4}, \]

where the superscript \( T \) represents the tax game case. Note that taxation by country 1 has a greater impact on the capital prices than that by country 2: \( \partial r^T / \partial t_1 < \partial r^T / \partial t_2 < 0. \)

In the first stage, each government simultaneously chooses \( t_i \) to maximize \( Y_i \), anticipating the market reactions described in \((2.4.1)-(2.4.5)\) and taking \( t_j \) \((i \neq j)\) as given.

The best response functions are given by\(^\text{17}\)

\[ \frac{\partial Y_1}{\partial t_1} = -(11\beta + 16\gamma)t_1 + (5\beta + 8\gamma)t_2 + \frac{8\beta(\beta + 2\gamma)}{1 \left(1 - \sqrt{\Phi F/\beta}\right) = 0}, \]
\[ \frac{\partial Y_2}{\partial t_2} = \beta t_1 - (7\beta + 8\gamma)t_2 + \frac{8\beta(\beta + 2\gamma)}{8\beta(\beta + 2\gamma) = 0}. \]

Note that we observe a strategic complement in tax decisions. Still, the global concavity of \( Y_i \) with respect to \( t_i \) ensures the existence of the unique

\(^{17}\)The associated second-order conditions are globally satisfied.
non-cooperative Nash equilibrium, in which the tax rates are given by

\[ t_1^T = \frac{\beta(7\beta + 8\gamma)(\beta + 2\gamma)}{2(3\beta + 4\gamma)^2} \left( 1 - \sqrt{\frac{\Phi F}{\beta}} \right) , \]  

\[ t_2^T = \frac{\beta^2(\beta + 2\gamma)}{2(3\beta + 4\gamma)^2} \left( 1 - \sqrt{\frac{\Phi F}{\beta}} \right) . \]

A simple comparison would show that \( t_1^T > t_2^T > 0 \) from \( F < \beta/\Phi \).

**Proposition 2.3.** In a subgame-perfect Nash equilibrium, both countries impose positive capital taxes. In particular, the resource-rich country levies a higher tax rate than the resource-poor country; that is, \( t_1^T > t_2^T > 0 \).\(^{18}\)

This is consistent with the empirical evidence shown in Slemrod (2004).\(^{19}\)

Note that capital taxation in either country reduces the capital price (i.e., \( dr_T^T/dt_1 < 0 \) and \( dr_T^T/dt_2 < 0 \)). Since country 1 is an importer of capital, it has an incentive to raise \( t_1 \) in order to exploit the return to capital and lower capital prices. In contrast, country 2 is an exporter of capital, and hence has a weaker incentive to raise \( t_2 \) to maintain high capital prices. These terms-of-trade effects lead to a higher tax rate in country 1 than in country 2.\(^{19}\)

When country 1 levies a positive tax rate on capital, the amount of capital exported from country 2 declines if country 2 imposes no tax. In

\(^{18}\)If we allow asymmetric factor endowments, the equilibrium tax differential is increasing in the capital endowment of country 1 and decreasing in the capital endowment of country 2. As resource-rich countries own fewer capital, they are more likely to exploit the capital inflows which seek a greater differential in returns to capital in autarky.

\(^{19}\)Corden and Neary (1982) also investigate the terms-of-trade effect between traded and non-traded goods (in their terminology, the resource movement effect). On the other hand, the key mechanism in the current chapter is the terms-of-trade of capital.
such a case, country 2 can regain the rent originated from capital mobility by setting a positive tax rate as long as its tax rate is lower than the tax rate of country 1.

Further, note that capital taxation lowers the price of M-good ($\frac{\partial p_M^T}{\partial t_1} > 0$ and $\frac{\partial p_M^T}{\partial t_2} < 0$), implying that country 1 has an incentive to raise its capital tax rate in order to increase its revenue from the export of M-good; country 2 also has an incentive to raise its capital tax rate to reduce its payment for M-good. However, because $\partial(-p_M M_2)/\partial t_2 = \partial(p_M M_2)/\partial t_1$ holds true in equilibrium, we know that such incentives are counteracted by each other, and do not lead to tax differentials.

Here, the equilibrium tax rates satisfy the condition $2\beta > t_1^T - t_2^T$ as assumed above:

$$2\beta - t_1^T + t_2^T = \frac{\beta(5\beta + 6\gamma) + (\beta + 2\gamma)\sqrt{\beta F}}{2(3\beta + 4\gamma)} > 0.$$  

The next question is, who gains from uncoordinated tax competition? Plugging the equilibrium conditions (2.2.1), (2.2.2), and (2.4.1) to (2.4.7) into (2.3.17) and (2.3.18), we obtain the equilibrium national incomes $Y_1^T$ and $Y_2^T$. We can compare these with $Y_1^M$, i.e., the welfare level under capital mobility in the absence of government interventions (i.e., $t_1 = t_2 = 0$):

$$Y_1^T - Y_1^M = \frac {(15\beta + 16\gamma)(\beta + 2\gamma)}{16(3\beta + 4\gamma)^2} \left( \sqrt{\beta} - \sqrt{\Phi F} \right)^2,$$

$$Y_2^T - Y_2^M = -\frac {3(7\beta + 8\gamma)(\beta + 2\gamma)}{16(3\beta + 4\gamma)^2} \left( \sqrt{\beta} - \sqrt{\Phi F} \right)^2,$$

$$Y_1^T + Y_2^T - Y_1^M - Y_2^M = -\frac {\beta + 2\gamma}{8(3\beta + 4\gamma)} \left( \sqrt{\beta} - \sqrt{\Phi F} \right)^2.$$
Therefore, we have $Y^T_1 - Y^M_1 > 0$, $Y^T_2 - Y^M_2 < 0$, and $Y^T_1 + Y^T_2 - Y^M_1 - Y^M_2 < 0$. These results can be summarized as follows.

**Proposition 2.4.** The resource-rich country gains from tax competition (i.e., $Y^T_1 > Y^M_1$), whereas the resource-poor country loses from it (i.e., $Y^T_2 < Y^M_2$). The latter loss dominates the former gain, and therefore, tax competition hurts global welfare (i.e., $Y^T_1 + Y^T_2 < Y^M_1 + Y^M_2$).

There is a resource-blessing in the sense that the presence of a resource-based sector enables the resource-rich country to gain from fiscal competition. However, the tax differentials created by such competition induce losses in global welfare, resulting in welfare losses in the resource-poor country.

The intuition underlying the resource blessing is as follows. Rearranging the national income (2.3.15), we get

$$Y_1 = (r + t_1 + w_1) + t_1(1 - K_2).$$

The first parenthesis on the right-hand side $(r + t_1 + w_1)$ represents the factor incomes earned by the initial factor endowments in country 1. Substituting (2.4.1)-(2.4.5) into this, we have

$$r + t_1 + w_1 = \frac{(t_1 - t_2)^2}{16(\beta + 2\gamma)} + \alpha - \frac{\beta + 2\gamma - \Phi F}{4} - \frac{1}{2} \sqrt{\beta \Phi F}.$$

This sum of factor incomes earned by the initial endowments increases as the tax differential rises: while the tax differential causes the outflows of capital from country 1 and reduces both the net return to capital $r$ and the
wage, the reallocation of capital across countries encourages more efficient use of capital, which increases the gross return to capital, \( r + t_1 \). At the same time, even though country 1 aggressively levies a higher capital tax than country 2, country 1 is still a net importer of capital (i.e., \( 1 - K_2 > 0 \)). Thus, country 1 can increase its revenue by taxing the capital inflows attracted by the benefits of its natural resource bonanza: that is, \( t_1(1 - K_2) > 0 \). In contrast, country 2 is doubly cursed in the sense that at a subgame-perfect Nash equilibrium, its initial factor endowments lead to the loss of factor incomes, and it loses the opportunity to levy tax on capital.

### 2.4.2 Tax coordination

The inefficiency (losses in global welfare) arising from tax competition makes room for tax coordination to function. Consider a case in which countries coordinate their policies and jointly make a tax offer to maximize global income, \( Y_1 + Y_2 \). The first-order conditions for global welfare maximization are given by

\[
\frac{\partial(Y_1 + Y_2)}{\partial t_1} = \frac{(t_2 - t_1)(3\beta + 4\gamma)}{4\beta(\beta + 2\gamma)} = 0,
\]

\[
\frac{\partial(Y_1 + Y_2)}{\partial t_2} = \frac{(t_1 - t_2)(3\beta + 4\gamma)}{4\beta(\beta + 2\gamma)} = 0.
\]

These conditions require that \( t_1 = t_2 \) as long as the solution is interior.

**Proposition 2.5.** *Global welfare maximization requires that the capital tax rates in the two countries be harmonized to reach the same level.*

\(^{20}\)The second-order conditions are also satisfied.
Note that the level of coordinated tax rates is undetermined\textsuperscript{21}. Tax rate equalization $t_1 = t_2$ leads to factor price equalization, implying that capital distribution goes back to the one observed in the case of mobile capital without government intervention.

Implementation of such tax coordination between countries would require a certain transfer from the resource-poor to the resource-rich country. Otherwise, the resource-rich country has an incentive to deviate from the coordination. One possible way of facilitating a transfer is by aid from the resource-poor country to improve infrastructure for the production of raw materials.

### 2.5 Robustness

In this section, we discuss the extent to which our results are robust against possible extensions. First, we replace our assumption of the free entry of firms in the resource based intermediate good sector (M-sector) to the assumption of entry restriction. Second, we introduce the possibility that M-good can also be produced in the resource-poor country by incurring transport costs. Third, we discuss how our results may change if M-sector firms are publicly rather than privately owned. Finally, we confirm that our results are unaltered if we use a production function different from a quadratic one.

\textsuperscript{21}This indeterminacy is based on the linearity of utility and factor demand functions; for example, see Peralta and van Ypersele (2006) and Itaya et al. (2008).
2.5.1 Restricted entry

Thus far, we have assumed free entry and exit in M-sector. However, we sometimes observe that governments try to reduce and control the number of producers in resource sectors, partially because of political and environmental concern. For instance, Suxun and Chenjunnan (2008) and Conway et al. (2010) reported entry restrictions in the mining industry in China. Here, we show that although the assumption of free entry plays an important role in analytically comparing the welfare outcomes, many of our results are unaltered if the entry of firms in M-sector is restricted.

Leaving aside the assumption of free entry, consider an exogenous number of M-firms. Assume that the excess profits in M-sector are equally redistributed to households in the resource-rich country (i.e., country 1). Then, the national income in country 1 is modified as

\[ Y_1 = w_1 + r_1 + t_1(K_1 + M) + n\pi. \]

Given \( n \), the equilibrium capital prices are given by (2.3.1) and (2.3.2) for the case of immobile capital, and by (2.3.10) for the other cases. Here, we investigate the robustness of the main results: (I) capital market integration induces a resource curse, and (II) tax competition results in a resource blessing.

As for the first point, we obtained the following result: When capital markets are integrated, the resource-rich country will be better off for a

\[ ^{22} \text{In this section, we assume that } F \text{ is sufficiently small so that } n \text{ does not exceed the level under free entry.} \]
sufficiently small \( n \) (in contrast to Proposition 2.2) while the resource-poor country and global welfare will still be better off. After some calculations, we obtain the welfare differentials as follows

\[
\bar{Y}_1^M - \bar{Y}_1^I = -\Psi_1\Psi_4, \\
\bar{Y}_2^M - \bar{Y}_2^I = \Psi_2\Psi_4 > 0, \\
\bar{Y}_1^M + \bar{Y}_2^M - \bar{Y}_1^I - \bar{Y}_2^I = \Psi_3\Psi_4 > 0,
\]

where \( \bar{Y}_i \) are the equilibrium national welfare in country \( i \) when the number of M-firms is fixed in each case, and \( \Psi_1, \Psi_2 > 0, \Psi_3 > 0 \) and \( \Psi_4 > 0 \) are bundles of parameters defined in Appendix A. Superscripts \( I \) and \( M \) again represent that the variables are related to the capital immobile and mobile cases, respectively. Whether capital market integration is beneficial for the resource-rich country depends on the number of M-firms, \( n \):

\[
\text{sgn} [\bar{Y}_1^M - \bar{Y}_1^I] = \text{sgn} [\tilde{n} - n],
\]

where \( \tilde{n} \) is defined as

\[
\tilde{n} := \frac{\Phi}{\beta} \left[ 2(3\beta + 4\gamma) + \sqrt{2(23\beta^2 + 53\beta\gamma + 32\gamma^2)} \right].
\]

When firms can freely enter/exit the market, capital market integration reduces the marginal cost faced by M-firms, which induces the existing firms to expand production and new firms to enter the market. These two effects increase the overall supply of M-good and negatively affects terms of trade: a larger supply of M-good lowers its price, which is the export price.
of country 1, and raises the price of capital, which is the import price of country 1. Such a negative effect dominates the positive effect of increases in the outputs of both X- and M-goods under free entry. When entry is restricted, for a sufficiently small \(n < \tilde{n}\), the resource-rich country can benefit from capital market integration because entry restriction saves the country from the negative change in terms of trade. As a result, the positive effects of output increases dominate the negative effects of change in terms of trade. For a sufficiently large \(n > \tilde{n}\), on the other hand, the protection for the terms of trade by the entry restriction cannot be large enough to overcome the resource curse because a larger number of M-firms leads to a greater natural resource bonanza which will disappear by capital market integration.\(^{23}\) Note also that this result implies that we observe the resource curse under perfect competition in the M-sector (when \(n \to \infty\)). Thus, our result comes from the asymmetry of the production possibility, not from the assumption of Cournot competition.

As to the second point, although we are unable to completely characterize the welfare properties of tax competition, we show that given \(n\), (i) the resource-rich country levies a higher tax on capital than the resource-poor country, (ii) tax competition is harmful to global welfare, and (iii) tax competition is likely to induce a resource blessing and a resourceless curse. At a unique Nash equilibrium in tax competition, the resource-rich country

\(^{23}\)From (2.3.1) and (2.3.2), we have \(dr_1/dn > 0\) and \(d(r_1 - r_2)/dn > 0\) in the case of immobile capital.
more aggressively levies a tax on mobile capital as in Proposition 2.3:

\[ t_1 - t_2 = \frac{4\beta(\beta + \gamma)(\beta + 2\gamma)n^2}{4(\beta + \gamma)(3\beta + 4\gamma)n^2 + (\beta + 2\gamma)(9\beta + 10\gamma)n + 2(\beta + 2\gamma)^2} > 0. \]

This tax differential is increasing in \( n \).

The global welfare is always worse off due to tax competition as in Proposition 2.4:

\[ \bar{Y}_T^1 + \bar{Y}_T^2 - \bar{Y}_M^1 - \bar{Y}_M^2 = \frac{-\beta\gamma(\beta + 2\gamma)(t_1 - t_2)}{(\beta + 2\gamma + 2n(\beta + \gamma))^2} \Phi\Psi_5(t_1 - t_2)^2 < 0, \]

where \( \Psi_5 > 0 \) is a bundle of parameters defined in Appendix A.

To evaluate the impacts of tax competition on each country’s welfare, it is necessary to compute quintic functions\(^{23}\), and so we shall confirm Proposition 4 by numerical investigations. Figure 2.2 shows sets of parameters \((\beta, \gamma)\) in which tax competition still leads to a resource blessing in the case \( n = 1, 3/2, 2, 3, 10, 20, \) or 100.

The light shaded areas represent a parameter set \((\beta, \gamma)\) such that \( \bar{Y}_T^1 > \bar{Y}_M^1 \) for each \( n \). The dark shaded areas represent a parameter set \((\beta, \gamma)\) such that \( \bar{Y}_T^1 < \bar{Y}_M^1 \) for each \( n \). The white triangles represent the invalid areas in which \( \beta + \gamma \leq 2 \).

The figures indicate that \( \bar{Y}_T^1 > \bar{Y}_M^1 \) may hold true for \( n \geq 2^{25} \) There exists a case of \( \bar{Y}_T^1 \leq \bar{Y}_M^1 \) for a sufficiently small \( n \), however, such \( n \) is

\(^{24}\)If we assume \( \gamma \geq 0 \), then we can analytically show that \( \bar{Y}_T^1 > \bar{Y}_M^1 \) and \( \bar{Y}_T^2 < \bar{Y}_M^2 \) for all \( \beta > 0, \gamma \geq 0 \) and \( n \geq 2 \).

\(^{25}\)Taking the limit as \( n \to \infty \), we obtain \( \bar{Y}_T^1 - \bar{Y}_M^1 = \beta(\beta + 2\gamma)(15\beta + 16\gamma)/[16(3\beta + 4\gamma)^2] > 0 \) and \( \bar{Y}_T^2 - \bar{Y}_M^2 = -3\beta(\beta + 2\gamma)(7\beta + 8\gamma)/[16(3\beta + 4\gamma)^2] < 0 \).
smaller than a plausible domain for oligopolistic markets. Note that when  

\( n \) is sufficiently smaller than 2, the equilibrium tax rate charged by the  

resource-rich country can be negative such that welfare in the resource-rich  
country would deteriorate following the subsidization of larger net inflows  
of capital than in the case of free entry.  

In sum, Propositions 2.2-2.4 are reasonably robust even without free  

entry in M-sector, except that contrary to Proposition 2.2 capital market  
integration will result in Pareto improving outcomes for very small  

\( n \).

2.5.2 Tradable resources

In the baseline model, we have assumed that the production of M-good is  

possible only in the resource-rich country. Of course, this is an extreme  

assumption, and hence, it would be worth examining how the results may  

change if we assume that M-sector can operate even in the resource-poor  
countries if firms pay an additional cost to transport the resources or develop  
new deposits of the resources, or if firms succeed in technological innovation,  
allowing them to produce substitutes to the resource-based intermediate  
goods without particular resource wealth.  

This section relaxes the important assumption that the resource-poor  
countries have no capacity to accommodate M-sector by supposing that M-  
firms can be set up in the resource-poor country by incurring additional  
costs to transport the resource wealth. First note that if there is free entry  
(at least in country 1), no firms operate profitably in country 2 because the
trade cost of natural resources makes the marginal cost in country 2 higher than that in country 1. This scenario results in the same allocations in our benchmark cases, i.e., the trade possibility of M-good does not affect our results.

If entry is restricted, trade possibility may change the prediction of our model. To prove this, we assume that each country has a single M-firm. This case is comparable to that of monopoly described in the previous subsection. Let \( \tau \) be the positive transport cost of natural resources in terms of the numéraire and \( \pi_i \) be the profit of an M-firm in country \( i \):

\[
\pi_1 = [p_M - (r_1 + t_1)]m_1 - F,
\]

\[
\pi_2 = [p_M - (r_2 + t_2) - \tau]m_2 - F,
\]

where \( m_i \) is the sales of each firm, with \( m_1 + m_2 = M \). We assume that \( \tau \) is low enough (in particular, \( \tau < \beta(\beta + 2\gamma)/(3\beta + 4\gamma) \)) for both firms in M-sector to be profitable. National welfare in each country is given by

\[
Y_1 = w_1 + r_1 + t_1(K_1 + m_1) + \pi_1
= X_1 + r_1(1 - K_1 - m_1) + p_M(q_1 - M_1) - F,
\]

\[
Y_2 = w_2 + r_2 + t_2(K_2 + m_2) + \pi_2
= X_2 + r_2(1 - K_2 - m_2) + p_M(q_2 - M_2) - \tau m_2 - F.
\]

When \( \tau = 0 \), the two countries are completely symmetric.

In this economy, there exists a unique equilibrium in each case for all \( \beta > 0, \beta + 2\gamma > 0, \tau < \beta(\beta + 2\gamma)/(3\beta + 4\gamma) \) and sufficiently large \( \alpha > 0 \). Details are shown in Appendix B.
There are two major differences between this extension and the benchmark model. First, when an M-firm operates in country 2, a difference between \( r_1 \) and \( r_2 \) in the case of immobile capital becomes small enough to diminish the natural resource bonanza. Thus if the capital market integrates, welfare in the resource-rich country will always increase, which is in contrast to Proposition 2, because the loss of capital income is fully offset by the improvement in production efficiency through the international reallocation of capital. Welfare in the resource-poor country may increase or decrease with capital mobility. When the production of M-good is costly enough (i.e., \( \tau \) is sufficiently large), the resource-poor country benefits from capital mobility as in Proposition 2. By contrast, when \( \tau \) is small, capital market integration negatively affects the resource-poor country. As the rates of return on capital are equalized, a share of M-good market shifts from the less efficient firm located in country 2 to the more efficient one that has a cost advantage. This shift results in an increase in imports of M-good and thus a decrease in national welfare in country 2, which may dominate the positive effects driven by efficiency gains in X-sector, capital income gains, and transport cost savings.

Second, the direction of inequalities in Proposition 2.4 is reversed: tax competition always negatively affects the resource-rich country but benefits the resource-poor country. In tax competition equilibrium, both countries will subsidize capital at a common rate and the capital price, \( r \), rises at the same rate as the subsidy rate so that the overall capital cost faced by firms,
and hence the capital allocation remains unchanged from the laissez-faire equilibrium. As a result, country 1 that imports capital will merely transfer income to country 2 while global welfare remains unchanged.

In a nutshell, the trade possibility of M-good has no effect on our results under free entry whereas it may change the results in the previous subsection if entry is restricted. In particular, there is a discontinuous change in the welfare implication of capital market integration in country 1 when the resource-based sector operates in both countries. The discontinuity reflects the fact that when M-sector is active but may not necessarily be profitable in country 2, the return to immobile capital jumps so that the resource bonanza becomes small.

### 2.5.3 Publicly owned monopolist

When governments restrict entry of firms in the resource sector, they often put other types of restrictions on firms’ activity, or, place firms under national control. This subsection investigates the impacts of such nationalization. The free entry assumption is implausible in the context of publicly-owned firms. Therefore, we focus on the case of restricted entry. More specifically, we consider that country 1 has a welfare-oriented publicly-owned firm in M-sector. At the third stage of the game, taking the factor prices, \( r_i \), and, \( w_i \), and tax rates, \( t_i \), as given but taking into account the factor

\[ r + t_i. \]

\[ A \text{ non-cooperative game does not implement the allocation under tax coordination, which requires } t_2 - t_1 = 2T 
eq 0. \]
demands of X-sector, the public firm chooses its output $M$ to maximize the following objective function\(^{27}\)

$$\pi_p = \lambda[(pM - r_1 - t_1)M - F] + (1 - \lambda)Y_1.$$ 

The parameter $\lambda \in [0, 1]$ captures (inversely) the importance of welfare considerations in the firm’s objective: when $\lambda$ is lower, national welfare is more important. When $\lambda = 1$, the resulting equilibrium coincides with the one discussed in Section 2.5.1 where $n = 1$. Therefore, when $\lambda$ is high, as expected, our main results are largely unaffected by introducing the public ownership.

A fall in $\lambda$ is likely to leads to an increase in the total output of M-good, $M = M_1 + M_2$, and a decrease in price-cost margins in each case\(^{28}\). In the absence of government interventions, this expansion in production of M-good increases the capital demand and pushes the capital prices up. In the case of immobile capital, it reinforces the natural resource bonanza by widening the difference in the return to capital $r_1 - r_2$. In the case of mobile capital, since the public firm takes the capital price as given, the public firm ignores the terms of trade loss that accrues to a capital-importing country with each additional unit of M-good. Therefore, the likelihood of a resource

---

\(^{27}\)We base our description of public firms on the existing studies in the literature of mixed oligopoly, e.g., [De Fraja and Delbono (1989)], [Pal (1998)], and [Matsushima and Matsumura (2003)].

\(^{28}\)At tax competition equilibrium, the sales of M-good may increase with $\lambda$ when $\lambda$ and $\gamma$ are sufficiently small. Without tax competition, the equilibrium price of M-good must be increasing in $\lambda$. 

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curse due to capital market integration rises as the public firm becomes more welfare conscious (i.e., lower $\lambda$). On the other hand, the lower $\lambda$ is, the higher welfare is in country 2 at interior equilibrium because the public firm lowers the import price of $M$-good, $p_M$, and raises the export price of capital, $r$.  

The impacts of public ownership on tax competition are nonlinear and not obvious. In tax competition, country 1 has an incentive to lower its tax rate and thus reduce the tax differential in order to raise the net return to capital since the public firm that ignores the impacts on $r$ tends to excessively raise $r$. At the same time, since the welfare-oriented firm employs more capital than the profit-maximizing firm, country 1 has an incentive to raise its tax rate to exploit benefits that accrue to the inflows of capital.

Figure 2.3 describes the overall effects of tax competition on welfare with $\alpha = 5$. The horizontal and vertical axes are $\beta$ and $\gamma$, respectively. The light shaded areas represent the domain $(\beta, \gamma)$ such that $Y_{iTP} > Y_{iMP}$. 

---

The derivative of equilibrium welfare in the case of mobile capital with a public firm (superscript $MP$) is

$$\frac{dY_{iMP}^1}{d\lambda} = \frac{4\beta(\beta + \gamma)(\beta + 2\gamma)[\beta - (\beta + 2\gamma)\lambda]}{[5\beta + 6\gamma + (\beta + 2\gamma)\lambda]^4}.$$  
$$\frac{dY_{iMP}^2}{d\lambda} = -\frac{8\beta(\beta + \gamma)^2(\beta + 2\gamma)}{[5\beta + 6\gamma + (\beta + 2\gamma)\lambda]^3} < 0.$$  

Therefore we have $dY_{iMP}^1/d\lambda > 0$ for $\lambda < \beta/(\beta + 2\gamma)$.

---

A Sufficiently large $\alpha$ ensures that all endogenous variables are strictly positive. In addition, we can show that in the tax game the second-order conditions with respect to taxes are satisfied and the equilibrium is uniquely determined for all $\lambda \in [0, 1]$. 

---

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for country $i$ or $Y_1^{TP} + Y_2^{TP} > Y_1^{MP} + Y_2^{MP}$ for the global economy. The dark shaded areas represent $(\beta, \gamma)$ such that $Y_i^{TP} < Y_i^{MP}$ for country $i$ or $Y_1^{TP} + Y_2^{TP} < Y_1^{MP} + Y_2^{MP}$ for the global economy. The white triangles represent the irrelevant area such that $\beta + 2\gamma \leq 0$. The columns in Figure 2.3 provides an overview of how the impacts of tax competition change in relationship to welfare consciousness ($\lambda = 0, 1/3, 2/3, 1$).
Figure 2.2: Numerical examples.

Notes: The horizontal and vertical axes represent $\beta$ and $\gamma$, respectively. The light shaded areas represent a parameter set $(\beta, \gamma)$ such that country 1 gains for each $n$. The dark shaded areas represent $(\beta, \gamma)$ such that country 1 loses for each $n$. The white triangles represent the invalid areas in which $\beta + \gamma \leq 2$. 

(A) $n = 1$.

(B) $n = 3/2$.

(C) $n = 2, 3, 10, 20, \text{ or } 100$. 

$\text{Figure 2.2: Numerical examples.}$

Notes: The horizontal and vertical axes represent $\beta$ and $\gamma$, respectively. The light shaded areas represent a parameter set $(\beta, \gamma)$ such that country 1 gains for each $n$. The dark shaded areas represent $(\beta, \gamma)$ such that country 1 loses for each $n$. The white triangles represent the invalid areas in which $\beta + \gamma \leq 2$. 

$\text{61}$
Figure 2.3: Welfare effects of tax competition with a public firm.

Notes: The horizontal and vertical axes represent $\beta$ and $\gamma$, respectively. The light shaded areas represent a parameter set $(\beta, \gamma)$ such that country $i$ or the global economy gains. The dark shaded areas represent $(\beta, \gamma)$ such that country $i$ or the global economy loses. The white triangles represent the invalid areas in which $\beta + \gamma \leq 2$. 

<table>
<thead>
<tr>
<th></th>
<th>$\lambda = 0$</th>
<th>$\lambda = 1/3$</th>
<th>$\lambda = 2/3$</th>
<th>$\lambda = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>country 1</td>
<td><img src="chart1.png" alt="Image" /></td>
<td><img src="chart2.png" alt="Image" /></td>
<td><img src="chart3.png" alt="Image" /></td>
<td><img src="chart4.png" alt="Image" /></td>
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<tr>
<td>country 2</td>
<td><img src="chart5.png" alt="Image" /></td>
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<td><img src="chart7.png" alt="Image" /></td>
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<tr>
<td>global</td>
<td><img src="chart9.png" alt="Image" /></td>
<td><img src="chart10.png" alt="Image" /></td>
<td><img src="chart11.png" alt="Image" /></td>
<td><img src="chart12.png" alt="Image" /></td>
</tr>
</tbody>
</table>
We find that for sufficiently low $\lambda$ tax competition may Pareto-improve welfare: both countries are better off than in the case without government intervention. In such a case, country 1 levies the capital tax more aggressively than country 2 as in our baseline model. This tax differential causes the international reallocation of capital from country 1 to country 2 and decreases the net return to capital, $r$. The decrease in $r$ weakens an incentive for the public firm to decrease the production of M-good to avoid the loss of net capital income, $r(1 - K_1 - M)$. Furthermore, when capital and M-good are sufficiently complementary, the capital inflows into country 2 stimulates X-sector production and increases the demand for M-good there. It amplifies an incentive for the public firm to increase its output to gain the revenue from exporting the intermediates, $p_M M_2$. These increased production in M-sector would benefit not only the resource-rich country but also the resource-poor country when capital and M-good are sufficiently complementary.

### 2.5.4 Cobb-Douglas production technology

Finally, we briefly discuss the specifications of technology for X-sector. In the baseline model, we have based our arguments on the quadratic production function in X-sector. How valid is this assumption in obtaining our main results? In fact, we can demonstrate that other types of production functions will lead to the same conclusions. As an example, consider a
Cobb-Douglas production function in X-sector:

\[ X_i = AK_i^aL_i^bM_i^{1-a-b}. \]

We maintain all the settings of the baseline model, except for the production function in X-sector. The equilibrium conditions are given in Appendix C. Although it is difficult to characterize the welfare properties analytically, numerical exercises indicate that the Cobb-Douglas production function generates similar results to those shown in the baseline model.

![Graphs showing comparisons among the cases.](image)

Figure 2.4: Comparisons among the cases.

Figures 2.4 and 2.5 depict the levels of the equilibrium welfare in each case with setting \( a = b = 1/3 \) and \( A = 256 \). The domain of \( F \) is chosen

\[ \text{We have checked that other parameter values such as } a = b/4 = 1/6 \text{ lead to similar} \]
to be $n \geq 2$. These numerical results turn out to be thoroughly consistent with the results obtained in the baseline model: capital mobility induces a resource curse, but tax competition creates a resource blessing.

2.6 Concluding remarks

The literature on capital market integration and tax competition has overlooked the role of natural resources. We examined how the availability of natural resources affects capital flow and governments’ reactions to them, results for a sufficiently small entry cost, $F$ (i.e., a sufficiently large number of firms, $n$). These results are available upon request.
who benefits from capital mobility and tax competition, and what are the welfare implications. In so doing, we developed an analytically solvable framework involving vertical linkages through resource-based inputs and international fiscal linkages between resource-rich and resource-poor countries. Our analysis showed that capital market integration yields capital flows from resource-poor to resource-rich countries, improving production efficiency and global welfare. However, such gains accrue only to resource-poor countries, and capital mobility can even make resource-rich countries worse off. Once we introduce the possibility of governments intervening in response to capital flows, both countries can levy a positive tax rate on capital. In particular, resource-rich countries will levy a higher tax rate than resource-poor countries. This tax wedge would make the resource-rich country a winner and the resource-poor country a loser in the tax game. As a result, tax competition negatively affects global welfare. We also discussed the robustness of our results against possible extensions: our results hold true if the resource based sector is sufficiently competitive and trade costs of raw natural resources are sufficiently high.

Our findings shown in Propositions 2.4 and 2.5 imply that while a tax harmonization policy among countries would enhance global welfare, it inevitably will invoke a resource curse if there are no transfers among them. This is because the interests of the two countries are directly in conflict and no Pareto-improvement is possible. It is thus worth investigating a mechanism to implement tax harmonization policies among asymmetric countries,
which will be an important topic for future research.

Appendix 2.A Definitions of parameter bundles

\[ \Psi_1 := 2\beta(\beta + \gamma)n^2 - 4(\beta + 2\gamma)(3\beta + 4\gamma)n - 5(\beta + 2\gamma)^2, \]
\[ \Psi_2 := 2(\beta + \gamma)(7\beta + 8\gamma)n^2 + 8(\beta + \gamma)(\beta + 2\gamma)n + (\beta + 2\gamma)^2, \]
\[ \Psi_3 := \Psi_2 - \Psi_1 = 2(\beta + \gamma)(3\beta + 4\gamma)n^2 + 2(\beta + 2\gamma)(5\beta + 6\gamma)n + 3(\beta + 2\gamma)^2, \]
\[ \Psi_4 := \frac{\beta(\beta + \gamma)(\beta + 2\gamma)n^2}{2[\beta + 2\gamma + n(3\beta + 4\gamma)]^2[\beta + 2\gamma + 2n(\beta + \gamma)]^2}, \]
\[ \Psi_5 := \frac{2(\beta + \gamma)(3\beta + 4\gamma)n^2 + 4(\beta + \gamma)(\beta + 2\gamma)n + (\beta + 2\gamma)^2}{[\beta + 2\gamma + 2n(\beta + \gamma)]^2}. \]

Appendix 2.B Equilibrium conditions with tradable resources

We denote the equilibrium value of variable \( x \) by \( \hat{x} \).

- in an autarky equilibrium:
  \[
  \hat{r}_1^f = \alpha - \frac{9(\beta + \gamma)(\beta + 2\gamma) - (2\beta + 3\gamma)\tau}{3(5\beta + 6\gamma)},
  \]
  \[
  \hat{r}_2^f = \hat{r}_1^f - \frac{2}{3}\tau,
  \]
  \[
  \hat{p}_M^f = \alpha - \frac{(\beta + 2\gamma)(2\beta + 3\gamma) - (\beta + \gamma)\tau}{5\beta + 6\gamma}.
  \]
• in a laissez-faire equilibrium:

\[ \hat{r}^M = \alpha - \frac{(\beta + \gamma)(3\beta + 6\gamma + \tau)}{5\beta + 6\gamma}, \]
\[ \hat{p}^M = \hat{p}^I_M. \]

• in a tax game:

\[ \hat{r}^T = \alpha - \frac{(\beta + 2\gamma)(7\beta + 9\gamma) + (4\beta + 5\gamma)\tau}{3(5\beta + 6\gamma)}, \]
\[ \hat{t}^T_1 = \hat{t}^T_2 = -\frac{(\beta + 2\gamma)(2\beta - \tau)}{3(5\beta + 6\gamma)} < 0, \]
\[ \hat{p}^T = \hat{p}^I_M. \]

The restriction of \( \tau < \beta(\beta + 2\gamma)/(3\beta + 4\gamma) \) is required to guarantee that

\[ p_M - r_2 - t_2 - \tau > 0. \]

This restriction also implies \( \tau < 2\beta \), in which \( K_i \) and \( M_i \) are strictly positive. In addition, we assume that \( \alpha > [(\beta + 2\gamma)(7\beta + 9\gamma) + (4\beta + 5\gamma)\tau]/[3(5\beta + 6\gamma)] \) such that \( r_i \) and \( w_i \) are strictly positive.

The welfare differentials are given by

\[ \hat{Y}^I_1 - \hat{Y}^I_2 = \frac{4(\beta + \gamma)(2\beta - \tau)\tau}{3\beta(5\beta + 6\gamma)} > 0, \]
\[ \hat{Y}^M_1 - \hat{Y}^M_2 = \frac{2(\beta + \gamma)(2\beta - \tau)\tau}{3\beta(5\beta + 6\gamma)} > 0, \]
\[ \hat{Y}^T_1 - \hat{Y}^T_2 = \frac{4(\beta + \gamma)(2\beta - \tau)\tau}{3\beta(5\beta + 6\gamma)} > 0, \]
\[ \hat{Y}^T_1 - \hat{Y}^I_1 = \frac{(\beta + \gamma)[12\beta(\beta + 2\gamma) + (29\beta + 30\gamma)\tau]\tau}{18\beta(\beta + 2\gamma)(5\beta + 6\gamma)} > 0, \]
\[
\hat{Y}_2^M - \hat{Y}_2^I = -\frac{(\beta + \gamma)[12\beta(\beta + 2\gamma) - (41\beta + 54\gamma)\tau]}{18\beta(\beta + 2\gamma)(5\beta + 6\gamma)},
\]
\[
\hat{Y}_1^M + \hat{Y}_2^M - \hat{Y}_1^I - \hat{Y}_2^I = \frac{7(\beta + \gamma)\tau^2}{9\beta(\beta + 2\gamma)} > 0,
\]
\[
\hat{Y}_1^T - \hat{Y}_1^M = -\frac{(\beta + \gamma)(2\beta - \tau)\tau}{3\beta(5\beta + 6\gamma)} < 0,
\]
\[
\hat{Y}_2^T - \hat{Y}_2^M = \frac{(\beta + \gamma)(2\beta - \tau)\tau}{3\beta(5\beta + 6\gamma)} > 0,
\]
\[
\hat{Y}_1^T + \hat{Y}_2^T - \hat{Y}_1^M - \hat{Y}_2^M = 0,
\]
\[
\hat{Y}_1^T - \hat{Y}_1^I = \hat{Y}_2^T - \hat{Y}_2^I = \frac{7(\beta + \gamma)\tau^2}{18\beta(\beta + 2\gamma)} > 0.
\]

We can easily see all the signs of welfare differentials for all \(\beta > 0, \beta + 2\gamma > 0\) and \(\tau < \beta(\beta + 2\gamma) / (3\beta + 4\gamma) < 2\beta\) except for \(\hat{Y}_2^M - \hat{Y}_2^I\). One has
\[
\text{sgn} \left[ \hat{Y}_2^M - \hat{Y}_2^I \right] = \text{sgn} \left[ \tau - \frac{12\beta(\beta + 2\gamma)}{41\beta + 54\gamma} \right].
\]

**Appendix 2.C** Equilibrium conditions under a Cobb-Douglas production function

From the profit maximization in X-sector, the inverse demand function for M-good is
\[
p_M = (1 - a - b)A\Psi_6 M^{-a-b},
\]
where
\[
\Psi_6 := \left[ (K_1^a L_1^b)^{\frac{1}{a+b}} + (K_2^a L_2^b)^{\frac{1}{a+b}} \right]^{a+b}.
\]
In the symmetric Cournot equilibrium, the sales of M-good in country \( i \) are

\[
M_i = \left[ \frac{(1 - a - b)(n - a - b)AK_i^aL_i^b}{(r_1 + t_1)n} \right]^{\frac{1}{a+b}},
\]

and the price of M-good is

\[
p_M = \frac{(r_1 + t_1)n}{n - a - b}.
\]

The number of M-firms is determined by the zero profit condition

\[
nF = (p_M - r_1 - t_1)M.
\]

The profit maximization in X-sector and the labor market clearing \((L_i = 1)\) yield the wage rate

\[
w_i = b\Psi_7K_i^{\frac{a}{a+b}}(r_1 + t_1)^{-\frac{1-a-b}{a+b}},
\]

where

\[
\Psi_7 := A\frac{1}{a+b} \left[ \frac{(1 - a - b)(n - a - b)}{n} \right]^{\frac{1-a-b}{a+b}} > 0.
\]

The capital demand in X-sector in country \( i \) is

\[
r_1 + t_1 = (a\Psi_7)^{a+b}K_1^{-b},
\]

\[
r_2 + t_2 = a\Psi_7K_2^{-\frac{b}{a+b}}(r_1 + t_1)^{-\frac{1-a-b}{a+b}}.
\]

The capital demand in M-sector is \( M = M_1 + M_2 \). The capital market equilibrium requires eq. (2.2.8) for the case of immobile capital, and eq. (2.2.9) and \( r_1 = r_2 \) for the other cases.

The tax competition equilibrium requires \( \partial Y_1/\partial t_1 = \partial Y_2/\partial t_2 = 0 \) in addition to the profit maximization, the free entry conditions, and the factor market clearing conditions.
Bibliography


Chapter 3

Solitary city

Solitude, in the sense of being often alone, is essential to any depth of meditation or of character; and solitude in the presence of natural beauty and grandeur, is the cradle of thoughts and aspirations which are not only good for the individual, but which society could ill do without.

John Stuart Mill, “Principles of Political Economy” (1848, ch.6)

3.1 Introduction

Periods of solitude are indispensable for skill acquisition, expertise development and creative thinking. From the psychological literature on expertise, it is known that it takes long time to become an expert even for the most

⋆This chapter is based on Oshiro (2012).
gifted players, and that performance decreases without continued training. In a groundbreaking series of studies, Ericsson et al. (1993) find that experts’ superior performance results from solitary practice for long years. In addition, solitude often helps organize our thoughts and offers a fertile ground for the imagination. Spending much time alone could harm physical and mental wellbeing, but we do not consider such an extreme case of isolation. Rather, based on the fact that expert performance can be accounted for by the amount of time accumulated in solitary practice and contemplation, we focus on the indispensability of solitude in enhancing the ability of economic agents.

On the other hand, many scholars in various disciplines emphasize the role of urban interaction in the development of our understanding and the creation of innovative ideas, which cannot be realized by people remaining isolated. Some business firms require frequent face-to-face contacts to exchange intangible ideas and knowledge. Other people delight in conviviality and togetherness at concert halls. Urban economists have recognized the salient benefits of urban interactions in dense areas as a driving force.

1 See Ericsson (2006) for a review of the evidence.
2 Geniuses are characterized by solitary traits (see Middleton, 1935; Storr, 1989; Cain, 2012). There is some anecdotal evidence: Alan Turing loved to work alone (Copeland, 2012); John Stuart Mill noted that one needs to contemplate alone for in-depth thinking; Albert Einstein said, “I live in that solitude which is painful in youth, but delicious in the years of maturity.”
3 In a psychology context, Long and Averill (2003) and Knafo (2012) discuss the potential benefits of solitude that may outweigh its costs.
of creativity and prosperity, providing insights into the spatial structure of economic activities. However, what is often neglected but economically important is the cost of interactions—time for solitude.

We develop a monocentric city model to draw out the policy implications of a trade-off between solitude and urban interaction. Households decide how many times they will visit a playing field in the city. Here, visitors obtain the benefits of urban interaction, which depend on how much crowded the playing field is. At the same time, such visits will crowd out solitary times necessary to develop human capital. Households balance their private gain and their private loss in choosing their frequency of visits. However, both visiting frequently and not doing so will generate non-pecuniary externalities: social capital and human capital spillovers. We study interdependencies of externality-generating activities and urban spatial structure.

We first characterize the equilibrium properties within this framework. The decentralized economy may generally exhibit multiple equilibria involving an unstable equilibrium. We present examples in which we are able to perform comparative statics regardless of the multiplicity. We show that the two positive externalities generate two reverberating forces that counteract each other. Intuitively, if the playing field becomes more crowded, each household visits there more frequently not only because the initial increase in crowdedness increases a marginal benefit from visits but also because the other households are more likely to visit the playing field. In addition, such behavioral changes of households induce a decrease in the opportunity cost
of visits through the human capital spillovers, resulting in spurring more urban interaction.

After inspecting the decentralized equilibria, we demonstrate a first-best policy of a planner. Since the two externalities are intimately related and inseparable, the optimal policy matches individual behavior with their net cost-benefit effects, which are ambiguous, and could be negligible: the visiting behavior of households, without any interventions, could implement outcomes similar to those of the first-best policy, even in the presence of the externalities. In particular, the extended model presented in Section 3.5 shows that positive externalities possibly work as a dispersion force.

This chapter builds a theoretical framework of urban interactions to assess recent urban policies which take the concept of compact city into consideration and facilitate urban interactions. For instance, Japanese local authorities, particularly peripheral jurisdictions, provide administrative support for urban interactions under the Act Concerning City Center Revitalization. Toyama City is a leading example. The Toyama government invests in the public light rail transit and open space within the central business district. One of their explicit policy targets is to increase pedestrian traffic in the city center on Sundays (Toyama City, 2012). Following Toyama, 104 out of 107 elected cities also set a numerical target for pedestrian traffic under the Act during 2007-2012. Although pedestrian traffic is measurable, its

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4 See also Song and Knaap (2003) for the United States and Scott and Liew (2012) for New Zealand.

5 Other ubiquitous targets are as follows: population in the central business district
economic interpretations are less obvious. Using the model in which urban crowdedness arises from households’ optimization, we show that households are inefficiently isolated at decentralized equilibria whenever a larger city offers higher wages in equilibrium.

3.1.1 Related literature

Most of the existing research on urban non-market interactions focuses on the relationships between knowledge transmission and distribution of agents over space. Fujita (1988), Kanemoto (1990), Helsley and Strange (2007), Berliant and Fujita (2012), and Zenou (forthcoming) present more complicated models of the behavioral dimension of agents, which consider the intensity of urban interaction as endogenously determined. The present work contributes to this strand by highlighting the trade-off inherent in the time constraint.

This work is an extension of Helsley and Strange’s (2007) framework, which embodies Jane Jacobs’s ideas of diversity in cities. Helsley and Strange (2007) allow identical agents to choose how intensely they will inter-

(68 cities), traffic in public or tourist facilities (48 cities), retailers’ sales (36 cities), and the vacancy rate or the number of establishments (26 cities). Source: Cabinet Office, Government of Japan. http://www.kantei.go.jp/jp/singi/chukatu/nintei.html (in Japanese)

To cite a few references, see Beckmann (1976), Fujita and Ogawa (1982), Imai (1982), Tabuchi (1986), Fujita and Smith (1990), Lucas and Rossi-Hansberg (2002), Berliant et al. (2002), Roy and Thill (2004), and Mossay and Picard (2011).
act with others by incurring costs of commuting to a playing field. Because the price mechanism is absent in urban interactions, agents make too few visits, and population densities are too low in equilibrium. They argue that a transport subsidy can improve resource allocation, in principle, by making agents internalize the private contribution to the aggregate benefits. A distinguishing feature of the present chapter is that it regards the loss of time for the refinement of human capital as the opportunity cost of visits, which generate a positive externality.

The chapter is related to Becker's (1965) time allocation model and a Lucas's (1988) endogenous growth model. Within the context of urban policy, we consider a static environment in which households choose their time use for nonwork activities, generating positive externalities and shaping the spatial structure of the economy.

There is extensive literature on the overlapping notions of “social capital,” “social network,” “entrepreneurship capital,” “synergy,” “trust,” and “neighborhood effect.” While it is beyond the scope of this chapter to provide a comprehensive review of these concepts, we draw on important reviews (e.g., Glaeser et al. 2002; Sobel 2002; Durlauf 2004; Durlauf and Fafchamps 2005; Blume et al. 2011; Guiso et al. 2011; Ioannides 2012; Jackson and Zenou 2012). This chapter treats social capital, or crowdedness, as a public good that households voluntarily contribute to and that has an influence on individuals’ non monetary payoff from urban interaction as well as on city-level productivity.
3.2 The model

The model represents a standard application of the monocentric open city framework with urban interactions. We employ a continuum of households and locations over one-dimensional space with unit width. There is a linear symmetric city accommodating a continuum of identical households with measure $N \in \mathbb{R}^+$. Households choose lot size of land $s \in \mathbb{R}^+$, a portion of time dedicated to visiting $v \in \mathbb{R}^+$, residential location $r \in \mathbb{R}^+$, and consumption of the numéraire good $x \in \mathbb{R}^+$. Each household is represented by an additively separable and differentiable quasi-linear utility function

$$U = x - \frac{\sigma}{2s} + \nu(v, k),$$

where $k \in \mathbb{R}^+$ indicates the crowdedness of the playing field that households visit to interact with each other, $\nu : \mathbb{R}_+^2 \rightarrow \mathbb{R}$ is a benefit of urban

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7Brueckner (1987), Fujita (1989) and Anas et al. (1998) provide key reviews in the framework of monocentric city.

8In spite of the analytical tractability of the continuum approach, such an approach confronts mathematical flaws in a class of location models. See Berliant (1985) and subsequent studies.

9Because this formula abstracts from the income effect in the demand for the lot consumption, the model has a partial equilibrium flavor. However, it does not remove the interaction between labor and land markets, thus allowing us to develop a general equilibrium model of urban land use.

10We do not consider the location and number of playing fields. We also ignore the presence of agents managing the playing fields. The playing fields are available free of charge for households.
interaction, and $\sigma \in \mathbb{R}_+$ is a constant parameter representing land use preference.

**Assumption 3.1** (Helsley and Strange 2007). $\nu$ is a $C^2$ function with strictly increasing differences in $(v, k)$ on $\mathbb{R}_+^2$:

$$\frac{\partial \nu(v, k)}{\partial v} > 0, \quad \frac{\partial \nu(v, k)}{\partial k} > 0, \quad \text{and} \quad \frac{\partial^2 \nu(v, k)}{\partial v \partial k} > 0.$$  \hspace{1cm} (3.2.2)

The assumption means that households value not only visiting activities but also brisk traffic in the playing field and that the marginal value of a visit to the playing field increases with the crowdedness there. The variable $k$ can be interpreted as a measure of the social capital stock available in the city (or pure public goods that are acquired with a time cost). \textsuperscript{11}

Each household has initial wealth $\bar{\omega} \in \mathbb{R}_+$, commutes to the central business district (CBD) at location $r = 0$, and supplies inelastically one unit of labor to the numéraire sector. An adequate initial wealth ensures an interior solution in the household’s decision. Following the conventional assumption in the literature on endogenous growth (e.g., Mankiw et al., 1992), we regard the number of labor multiplied by the individual human capital as the effective labor force. Our central assumption is that the individual human capital increases with solitary time at home, i.e., time spent with no interaction at the playing field.

**Assumption 3.2.** A household’s human capital is given by the $C^2$ function $h(v)$ with $h'(v) < 0$. 

\textsuperscript{11}Evidence suggests that social capital in its different dimensions is conductive to subjective well-being (e.g., Bjørnskov et al., 2008; Leung et al., 2011).
It means that we treat non visiting hours as investments in human capital. Another possible interpretation is that the time not visiting is allocated to educate and care for children at home.\footnote{12}

Rental income from urban land accrues to absentee landlords, who live outside the economy.

Let $r_f \in \mathbb{R}_+$ be an urban boundary; then, a budget constraint for a household living at distance $r \in [0, r_f]$ is written in the form

$$x + P(r)s + t(r) = wh(v) + \bar{w},$$

(3.2.3)

where $P(r)$ is the land rent profile on $\mathbb{R}_+$, $w \in \mathbb{R}_+$ is the wage per effective labor unit, and $t(r)$ is the commuting cost function of a class $C^2$ on $\mathbb{R}_+$ with $t(0) \geq 0$ and $t'(r) > 0$. The commuting cost is measured in terms of a numéraire.\footnote{13} As usual, $t'(r) > 0$ implies that the bid-rent function is downward sloping, indicating a rent premium for CBD accessibility. The equilibrium location of households and the equilibrium land rent within the city follow a bid-rent function (see Fujita, 1989).

Eliminating $x$, the objective function for household at distance $r \in [0, r_f]$ is given by $U = wh(v) + \nu(v, k) - P(r)s - \sigma/(2s) - t(r) + \bar{w}$, which has the first-order conditions

$$\frac{\sigma}{2s^2} = P(r),$$

(3.2.4)

\footnote{12} Bernal (2008), Bernal and Keane (2010), and Chyi and Ozturk (2013) find that maternal childcare has positive effects on child cognitive development. Lee et al. (2012) present results of natural experiments for the allocation of nonmarket time in Japan and Korea.

\footnote{13} The assumption that the commuting cost is independent of $v$ will be relaxed later.
\[ wh'(v) + \frac{\partial \nu(v, k)}{\partial v} = 0. \]  

(3.2.5) implies that households equate the marginal benefit of a visit with the marginal loss of wage income from a decrease in periods of solitary practice. We assume that the second-order condition

\[ wh''(v) + \frac{\partial^2 \nu(v, k)}{\partial v^2} < 0, \]  

(3.2.6)
is satisfied for all \( k \in \mathbb{R}_+ \). Let \( \hat{s} = \hat{s}(r) \) and \( \hat{v} = \hat{v}(w, k) \) be the solutions of (3.2.4) and (3.2.5), respectively. Note that \( \hat{v} \) is determined independently of the location of households.

From (3.2.5) and (3.2.6), we obtain the key comparative statics results:

\[ \frac{\partial \hat{v}(w, k)}{\partial w} = - \left( wh''(v) + \frac{\partial^2 \nu}{\partial v^2} \right)^{-1} h'(v) < 0, \]  

(3.2.7)

\[ \frac{\partial \hat{v}(w, k)}{\partial k} = - \left( wh''(v) + \frac{\partial^2 \nu}{\partial v^2} \right)^{-1} \frac{\partial^2 \nu}{\partial v \partial k} > 0. \]  

(3.2.8)

In other words, as \( w \) goes up or \( k \) decreases, households prefer solitude to urban interaction in order to increase their human capital.

The indirect utility at distance \( r \) is

\[ U(r) = wh(\hat{v}) + \nu(\hat{v}, k) - \frac{\sigma}{\hat{s}(r)} + \bar{\omega} - t(r). \]  

(3.2.9)

Free mobility within the city requires that the indirect utility must be equalized: \( U(r) = V \) for some \( V \in \mathbb{R} \) and for all \( r \in [0, r_f] \). This condition implies

\[ \frac{dU(r)}{dr} = 0, \]  

(3.2.10)
for all $r \in [0, r_f]$. In addition, the equilibrium land rent at the urban boundary $r = r_f$ is equal to the reservation value of land, denoted by $\tilde{R} \in \mathbb{R}_+$. Thus,

$$\frac{\sigma}{2\tilde{s}(r_f)^2} = \tilde{R}. \quad (3.2.11)$$

Combining the differential equation in (3.2.10) and the boundary condition (3.2.11), we have

$$\hat{s}(r) = \frac{\sigma}{t(r_f) - t(r) + \sqrt{2\sigma \tilde{R}}}. \quad (3.2.12)$$

By duality, the utility-maximizing lot size $\hat{s}(r)$ must be a bid-maximizing lot size, and hence the equilibrium land rent will be $P(r) = \sigma/[2\hat{s}(r)^2]$ for $r \leq r_f$ and $P(r) = \tilde{R}$ for $r \geq r_f$ when the land market is cleared everywhere.

Substituting (3.2.12) into the indirect utility (3.2.9),

$$V = wh(\hat{v}) + \nu(\hat{v}, k) + \tilde{\omega} - \sqrt{2\sigma \tilde{R}} - t(r_f). \quad (3.2.13)$$

This formula, the indirect utility of households living within the city, depends on the population in the city $N$, which will affect $k$, $r_f$, $w$, and hence $\hat{v}$.

Since $N$ households reside in the linear symmetric city, the population constraint is

$$N = \int_0^{r_f} \frac{2}{\hat{s}(r)} dr = \frac{2r_f \sqrt{2\sigma \tilde{R}}}{\sigma} + \frac{2}{\sigma} \int_0^{r_f} [t(r_f) - t(r)] dr. \quad (3.2.14)$$

\textsuperscript{14}$U(r)$ is left continuous in $r$ at $r = r_f.$
\[ (3.2.15) \] determines the strictly monotonic relationships between \( r_f \) and \( N \).

Differentiating both sides of \((3.2.15)\), we have

\[
\frac{dr_f}{dN} = \frac{\sigma}{2[r_f t'(r_f) + \sqrt{2\sigma R}]} > 0. \tag{3.2.16}
\]

In the familiar “open” city setting, costless intercity migration ensures that the utility level in the city is equal to the exogenous reservation utility, denoted by \( \bar{U} \in \mathbb{R} \). We denote the indirect utility satisfying \((3.2.13)\) and \((3.2.15)\) by \( V(w, k, N) \). Then one of the equilibrium conditions is given by

\[
\bar{U} = V(w, k, N), \tag{3.2.17}
\]

for \( N > 0 \). If \( \max_{N \in \mathbb{R}_+} V(w, k, N) < \bar{U} \), then \( N = 0 \).

Competitive firms employ effective labor to produce the numéraire good under constant returns to scale. Let \( A \in \mathbb{R}_+ \) be the sector-level productivity, which is written in a reduced form, \( A = A(w, k, N) \). Competitive wages are

\[
w = A(w, k, N). \tag{3.2.18}
\]

The arguments of \( A \) reflect human capital spillovers within the city. We refer to human capital spillovers as an externality, considering that individuals’ human capital investment unintentionally affects sector-level productivity and, thus, the city wage level. In contrast, the interaction benefit \( \nu(v, k) \) represents a residual benefit that does not affect productivity. Since an increase in \( w \) induces an increase in the opportunity cost of visits, we naturally assume that \( \partial A/\partial w \geq 0 \). The sign of \( \partial A/\partial k \) depends on the
mechanisms through which urban interactions work. If a crowded environment in the central place catalyzes information acquisition and knowledge transmission among households because of imitation, better matching or serendipity, we will expect that the sign of $\partial A/\partial k$ is positive. On the other hand, it is also possible to think of a scenario in which visits lead to collusive activities between lobbyists and authorities and, in turn, to lower productivity.\footnote{Several papers reveal the detrimental effects of social networks. For example, Fracassi and Tate (2012) investigate that pre-existing network connections between management groups may undermine the effectiveness of internal governance in organizations and therefore reduce firm value. Bentolila et al. (2010) show that social network, while it provides information on jobs, can increase occupational mismatch.} Because human capital encourages “absorptive capacity” (Cohen and Levinthal, 1990), an increase in $k$ may further lower productivity by discouraging solitary practice and the development of human capital in households. The sign of $\partial A/\partial N$ represents a pure thick market effect and may be nonnegative when the aggregate stock of useful human capital in the city matters.

Suppose that the crowdedness $k$ is derived by aggregating the visiting behavior of households in the city:

$$k = K(w, k, N).$$  \hspace{1cm} (3.2.19)

where $K: \mathbb{R}_+^3 \rightarrow \mathbb{R}_+$ is the aggregator of visits. The first two components of $K$ represent an externality in visits. That is, crowdedness depends on the intensity of visits by individuals, $\hat{v}(w, k)$. This would lead to $\partial K/\partial w \leq 0$
and $\partial K/\partial k \geq 0$. Conditional on $w$ and $k$, the total population in the city will contribute to the crowdedness, i.e., $\partial K/\partial N \geq 0$.

**Definition 3.1.** A spatial equilibrium, or equilibrium, is a triplet $(w, k, N) = (w^*, k^*, N^*) \in \mathbb{R}_+^3$ satisfying the zero-profit condition of competitive firms with constant-returns technology (3.2.18), the definition of the crowdedness aggregator (3.2.19), and intercity migration (3.2.17).

**Definition 3.2.** An equilibrium is stable if $dV/dN < 0$ around $(w^*, k^*, N^*)$. An equilibrium is unstable if it is not stable.  

### 3.3 Spatial equilibrium

The existence, uniqueness, and comparative statics results of spatial equilibria crucially depend on the specifications of externalities, $A$ and $K$. To clarify the role of externalities, we first take $w$ and $k$ as given, and then endogenize them.

---

16Intercity migration of farsighted households or its dynamic aspects are beyond the scope of this chapter. Literature on the system of cities as well as new economic geography has intensively explored these topics. Recent contributions, for example, are Oyama (2009), Fujishima (forthcoming) and Mossay (forthcoming).
3.3.1 Without externalities

Suppose that both $w$ and $k$ are exogenously given. Then the indirect utility within the city $V = V(w, k, N)$ decreases with $N$:

$$
\frac{dV}{dN} = -t'(r_f) \frac{dr_f}{dN} < 0.
$$

(3.3.1)

Proposition 3.1. There exists a unique and stable equilibrium for a sufficiently large $w$, $k$, or $\bar{\omega}$, or for a sufficiently small $\bar{U}$, $\bar{R}$, or $\bar{\sigma}$. The equilibrium population in the city is increasing in $w$, $k$, and $\bar{\omega}$, and decreasing in $\bar{U}$, $\bar{R}$, and $\bar{\sigma}$.

Proof. Proposition 3.1 can be easily seen in Figure 3.1 without a formal proof. Note that $\frac{dN^*}{d\sigma} = -\left[\sqrt{R/2\bar{\sigma}} + t'(r_f)\partial r_f/\partial \sigma\right]/[t'(r_f)dr_f/dN] < 0$ where $\partial r_f/\partial \sigma = [r_f\sqrt{R/2\bar{\sigma}} + (1/\sigma) \int_0^{r_f} t(r_f) - t(r)dr]/[r_f t'(r_f) + \sqrt{2\sigma R}] > 0$ from (3.2.15). Similarly, $\frac{dN^*}{d\bar{R}} = -2/t'(r_f) < 0$.

Note that the envelope theorem gives

$$
\frac{dN^*}{dw} = \frac{h(\hat{v})}{t'(r_f)dr_f/dN} > 0.
$$

(3.3.2)

A higher population is associated with a higher wage because a larger population raises urban cost, defined by commuting cost plus land rent, so that households must receive adequate wage compensation to stay in the city. Similarly, we have $\frac{dN^*}{dk} = [\partial \nu(v, k)/\partial k][t'(r_f)dr_f/dN] > 0$.

(3.3.2) and (3.2.7) suggest that a rising wage increases both urban population and periods of solitude. Naively interpreted, economic growth leads to
urban agglomerations and causes people to avoid social connections. However, agglomeration of economic activities may create more attractive spaces so that people would be willing to spend more time for social involvement in the city. The next section incorporates the interdependence of population, wage, and crowdedness.

### 3.3.2 Human and social capital externalities

Let us now consider the case in which $w$ and $k$ are endogenously determined.

Observe, first, that if $\partial A/\partial N = \partial K/\partial N = 0$, there exists a parameter set to guarantee the uniqueness and stability of the equilibrium population for any $w$ and $k$ as in Proposition 3.1. This is because $dV/dN = -t'(r_f)dr_f/dN < 0$. In other words, when pure thick market effects are
absent, the equilibrium city size is automatically determined. However, it does not ensure that (3.2.18) and (3.2.19) have fixed points.

These fixed points can a priori have many configurations, producing uninformative comparative statics of equilibria without imposing specific assumptions on $A$ and $K$. In what follows, we focus on a “nice” economy in which equilibrium wages and equilibrium crowdedness are uniquely determined.

**Assumption 3.3.** There exists a unique fixed point of $w = A(w, k, N)$ and $k = K(w, k, N)$ for any $N \in \mathbb{R}_+$. 

We denote these fixed points by $w = w(N)$ and $k = k(N)$. These functions are assumed to be of class $C^2$ on $\mathbb{R}_+$.

It is useful to decompose the impact of population $N$ on the indirect utility within the city to explicitly parallel the concepts of urban economics:

$$
\frac{dV}{dN} = \frac{\partial V}{\partial w} w'(N) + \frac{\partial V}{\partial k} k'(N) - t'(r_f) \frac{dr_f}{dN}.
$$

(3.3.3)

The three terms on the right-hand side of (3.3.3) represent *agglomeration forces* and *dispersion forces*. Suppose, tentatively, that $w'(N) \geq 0$ and $k'(N) \geq 0$. Then, the first two terms, both of which are positive, capture

\[17\]

From the implicit function theorem, one has

$$
\frac{dw(N)}{dN} = \left[ \left( 1 - \frac{\partial A}{\partial w} \right) \left( 1 - \frac{\partial K}{\partial k} \right) \right]^{-1} \left[ \left( 1 - \frac{\partial K}{\partial k} \right) \frac{\partial A}{\partial N} + \frac{\partial A \partial K}{\partial k \partial w} \right],
$$

$$
\frac{dk(N)}{dN} = \left[ \left( 1 - \frac{\partial A}{\partial w} \right) \left( 1 - \frac{\partial K}{\partial k} \right) \right]^{-1} \left[ \frac{\partial K}{\partial w} \frac{\partial A}{\partial N} + \left( 1 - \frac{\partial A}{\partial w} \right) \frac{\partial K}{\partial N} \right],
$$

when $(1 - \partial A/\partial w) (1 - \partial K/\partial k) - \partial A/\partial k \partial K/\partial w \neq 0$. 

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the agglomeration forces, whereas the last term represents the dispersion force. Suppose, alternatively, that $w'(N) \leq 0$ because a larger population increases crowdedness and decreases both individual human capital and sector-level productivity. Then, the first term acts as a dispersion force.

In the framework of a system of cities (e.g., [Henderson 1974]), the indirect utility takes an inverted U-shaped pattern in $N$. For a small $N$, the agglomeration forces dominate the dispersion forces, and vice versa. To identify which effect dominates in the present model, we have to make additional assumptions.

**Example 3.1.** This example specifies functional forms that are tractable as far as possible. We restrict the domain of $(v, w, k)$ to an open unit cube $(0, 1)^3$.

We make two functional specifications that ensure the uniqueness of the equilibrium. First, productivity is equal to the average level of human capital in the city, as in [Lucas 1988]:

$$A = h(v) = \sqrt{1 - v^2} \text{ for } v < 1.$$  \hfill (3.3.4)

$h(v)$ is strictly decreasing and concave. Second, the sub-utility from interaction has strictly increasing differences in $(v, k)$ on $[0, 1]^2$:

$$\nu(v, k) = v\sqrt{k}.$$  \hfill (3.3.5)

Under these assumptions, the utility-maximizing visits and human cap-
ital are, respectively.\footnote{\label{ft18}The second order condition is satisfied because $\partial^2(wh+\nu)/\partial v^2 = -w(1-v^2)^{-3/2} < 0$ for $v \in (0, 1)$.}

\[ \hat{v}(w, k) = \sqrt{\frac{k}{w^2 + k}} \in (0, 1), \quad (3.3.6) \]

\[ h(\hat{v}) = \frac{w}{\sqrt{w^2 + k}} \in (0, 1). \quad (3.3.7) \]

From $w = A = h$, the equilibrium wage is $w = \sqrt{1 - k} \in (0, 1)$. The indirect utility within the city is independent of $k$ regardless of a specification of $K$ as long as $k \in (0, 1)$:

\[ V = 1 + \bar{\omega} - \sqrt{2\sigma \bar{R} - t(r_f)}. \quad (3.3.8) \]

This decreases with $N$. We therefore obtain a unique and stable equilibrium size for the city as shown in Figure \ref{fig:3.1}.\footnote{\label{ft19}Degenerate equilibria (i.e., $N = 0$, $v = 0$ or $w = h = 0$) can be ignored by restricting attention to appropriate parameter ranges.} The equilibrium population $N^*$ is increasing in $\bar{\omega}$, and decreasing in $\bar{U}$, $\bar{R}$, and $\sigma$.

Note that $wh(\hat{v}) + \nu(\hat{v}, k) = 1$ for any $k \in (0, 1)$. In this example, the crowdedness $k$ has no effect on the indirect utility and therefore on the urban structure. An increase in $k$ produces an incentive for a visit to the playing field. This increases the benefit from interactions, while it reduces human capital and wages. These positive and negative effects completely cancel out each other regardless of the level of $k$. With behavioral changes evaporating the agglomeration forces, the urban cost, which is a dispersion force, governs the urban spatial structure.
Example 3.2. The next example reinforces the agglomeration forces in Example 3.1.

Productivity is assumed to be facilitated by urban interactions:

\[ A = k^\rho h(v) = k^\rho \sqrt{1 - v^2} \text{ for } v < 1, \quad (3.3.9) \]

where \( \rho \in \mathbb{R^+} \) is a constant parameter representing the importance of urban interactions to human capital spillovers. Visits reduce individual human capital but increase crowdedness and, thereby, productivity.\(^{20}\)

The sub-utility from interaction is also redefined as follows:

\[ \nu(v, k) = v k^{\frac{1}{2} + \rho}, \quad (3.3.10) \]

If \( \rho = 0 \), the model is coincident with Example 3.1.

In these specifications, the indirect utility depends on \( k \):

\[ V = k^\rho + \bar{\omega} - \sqrt{2\sigma \bar{R} - t(r_f)}. \quad (3.3.11) \]

Furthermore, we assume that the crowdedness aggregator is given by

\[ K(w, k, N) = v(w, k)N^{\frac{2}{3}}, \quad (3.3.12) \]

where \( \delta \in \mathbb{R^+} \) is constant. That is, there is a thick market effect in urban interaction. Substituting the equilibrium visit and wage\(^{21}\) we then have

\[^{20}\text{It is possible to also consider the case in which } \rho \text{ takes a negative value (e.g., social networks make unproductive collusion sustainable). Then the external effect through productivity strengthens the dispersion force, and the equilibrium is unique and stable as in Figure 3.1.}\]

\[^{21}\text{v} = \sqrt{k} \text{ and } w = k^\rho \sqrt{1 - k}.\]
$k = N^\delta$. Therefore, the indirect utility is rewritten as follows:

$$V = N^{\rho \delta} + \bar{\omega} - \sqrt{2\sigma R} - t(r_f). \quad (3.3.13)$$

We can use (3.3.13) to characterize various equilibrium configurations. When $\rho \delta \geq 1$ and $t(r_f)$ is concave in $N$, the indirect utility is convex. Where the agglomeration forces are strong relative to the dispersion force, there are at most two equilibria on a relevant range of parameters. Figure 3.2 shows an example of multiple equilibria, in which the small equilibrium is stable but the large equilibrium is not. Conversely, if $\rho \delta \in [0, 1)$ and $t(r_f)$ is convex in $N$, the indirect utility (3.3.13) is concave. In addition, if $t'(0)$ is finite, then $\lim_{N \to 0} \frac{dV}{dN} = +\infty$. Accordingly, the indirect utility follows an inverted U-shaped pattern similar to [Henderson (1974)]. In contrast to the

![Diagram of equilibria](image-url)

Figure 3.2: An example of equilibria under strong agglomeration forces. $N_1$ is stable but $N_2$ is unstable.
case where $\rho \delta \geq 1$, the large equilibrium is stable, but the small equilibrium is not.

Other configurations with particular specifications are possible. Nevertheless, we can perform the comparative statics regardless of multiplicity.

**Proposition 3.2.** Assume that $A$, $\nu$ and $K$ are given by (3.3.9), (3.3.10) and (3.3.12), respectively. Suppose that $\rho$ is positive. Then, as the equilibrium population is larger, households allocate a larger portion of those time to urban interaction, and population density (i.e., $1/s$) is higher at every location within the city regardless of the number and stability of equilibria. If a crowded environment tends to facilitate knowledge transmission (i.e., $k < 2\rho/(1 + 2\rho)$), a large population is associated with a high wage. Otherwise, a large city offers low wages.

**Proposition 3.3.** A population is increasing (resp. decreasing) in $\rho$, $\delta$, and $\bar{\omega}$ at a stable (resp. unstable) equilibrium. A population is decreasing (resp. increasing) in $\bar{U}$, $\bar{R}$, and $\sigma$ at a stable (resp. unstable) equilibrium.

The proof is straightforward and omitted here.

In sum, lessons from these examples are twofold. First, there are two magnification effects that may be in conflict. An inflow of population has a multiplier effect upon crowdedness. The crowded environment attracts further inflows of population through the social capital externality. However, the crowded environment at the same time induces a circular causation leading to low wages. As a large number of households actively participate in
urban interactions, households will earn low wages through the human cap-
ital externality. Low wages decrease time cost of visits, so that households engage in urban interactions more frequently. Example 3.1 shows that the wage-lowering cycle, which is a dispersion force, can cancel out the social multiplier effect, which is an agglomeration force. In Example 3.1 as shown in Section 3.4 households are inefficiently stuck in the wage-lowering cycle with excessive urban interaction.

Second, the multiplicity and instability of equilibrium city sizes emerge when the net agglomeration force is sufficiently strong relative to the urban cost. In Example 3.2 the close interaction, while it lowers the sector-level productivity through the wage-lowering cycle, helps directly improve the productivity by catalyzing information acquisition and knowledge transmission. When this additional agglomeration force driven by urban interactions is important, there is an unstable equilibrium in which once the population changes accidentally a sudden urban grow may take place. The next section shows that if the additional benefit is sufficiently large, the equilibrium visits are insufficient.

3.4 Efficient allocation

We now turn to the normative analysis.\textsuperscript{22}

\textsuperscript{22}For a detailed formulation of the Herbert-Stevens model which we employ here, see Fujita (1985, 1989).
A planner of the city has an objective function:

\[
W := 2 \int_0^{r_f} \left[ \frac{Y(r) - \sigma/(2s(r))}{s(r)} - \bar{R} \right] dr,
\]

(3.4.1)

where

\[Y(r) := wh(v) + \nu(v,k) - t(r) + \bar{\omega} - \bar{U}.
\]

\(W\) represents the surplus of the city, which is equivalent to the differential land rent in the city. Consider a first-best allocation in which taking \(\bar{U}\) as given, the planner can control the visit choices of households directly:

\[
\max_{v,s(r),r_f,N} W, \text{ subject to } N = \int_0^{r_f} \frac{2}{s(r)} dr.
\]

(3.4.2)

The solution is a decentralized equilibrium if both \(w\) and \(k\) are exogenous and hence there are no externalities.

Suppose that the fixed points of (3.2.18) and (3.2.19), denoted by \(w = w(v,N)\) and \(k = k(v,N)\), are well defined and differentiable.

**Proposition 3.4.** With external effects, the first-best interior solution \((v,s(r),r_f,N) \in \mathbb{R}^4_+\) for all \(r \in [0, r_f]\) is characterized by

\[
wh'(v) + \frac{\partial \nu(v,k)}{\partial v} + \left[ h(v) \frac{\partial w}{\partial v} + \frac{\partial \nu(v,k)}{\partial k} \frac{\partial k}{\partial v} \right] = 0,
\]

(3.4.3)

\[
N \frac{\partial Y(r)}{\partial N} = -Y(r_f) + \sqrt{2\sigma R},
\]

(3.4.4)

\[
s(r) = \frac{\sigma}{t(r_f) - t(r) + \sqrt{2\sigma R}},
\]

(3.4.5)

\[
N = \frac{2r_f \sqrt{2\sigma R}}{\sigma} + \frac{2}{\sigma} \int_0^{r_f} [t(r_f) - t(r)] dr,
\]

(3.4.6)

where \(\partial Y(r)/\partial N = [h(v)\partial w/\partial N + (\partial \nu(v,k)/\partial k)(\partial k/\partial N)]\).
Proof. We introduce the following Lagrangian

\[ W - \lambda \int_0^{r_f} \frac{2}{s(r)} \text{d}r + \lambda N, \]

where \( \lambda \) is the Lagrange multipliers. The first-order conditions are \((3.4.3)\),

\[ \lambda = Y(r) - \frac{\sigma}{s(r)} \text{ for all } r \leq r_f, \] \((3.4.7)\)

\[ \frac{1}{s(r_f)} \left[ Y(r_f) - \frac{\sigma}{2s(r_f)} - \lambda \right] = \bar{R}, \] \((3.4.8)\)

\[ \int_0^{r_f} \frac{\partial Y(r)}{\partial N} \frac{2}{s(r)} \text{d}r + \lambda = 0. \] \((3.4.9)\)

and the population constraint \( N = \int 2/s(r) \text{d}r \). From \((3.4.7)\) and \((3.4.8)\), we have \( \sigma/[2s(r_f)^2] = \bar{R} \), which is correspond to \((3.2.11)\). Combining this boundary condition and a differential equation derived by differentiating \((3.4.7)\) with respect to \( r \), we have \((3.4.5)\). \((3.4.6)\) follows \((3.4.5)\) and the population constraint. Observe that \( \partial Y(r)/\partial N \) is independent of \( r \). Substituting \((3.4.7)\), \((3.4.5)\) and the population constraint into \((3.4.8)\), we have \((3.4.4)\).

\((3.4.3)\) and \((3.4.4)\) determine \( v \) and \( N \). Given \( N \), \((3.4.5)\) and \((3.4.6)\) determine \( s(r) \) and \( r_f \) for all \( r \in [0, r_f] \).

\((3.4.5)\) and \((3.4.6)\) are identical to \((3.2.12)\) and \((3.2.15)\), respectively.

In other words, conditional on the total population and individual visits, both the equilibrium lot size and the equilibrium urban boundary achieve the social optimum if productivity and crowdedness are independent of
the internal structure of the city (i.e., $\frac{\partial A}{\partial r_f} = \frac{\partial K}{\partial r_f} = \frac{\partial A}{\partial s(r)} = \frac{\partial K}{\partial s(r)} = 0$).}

In (3.4.4), the optimal population size balances the marginal effects of thick market externalities and the shadow price of accommodating additional settlements, provided that the second-order condition is satisfied. The shadow price consists of a change in the net utility per unit land caused by a change in the population density. In the decentralized economy, $Y(r_f) = \sqrt{2\sigma R}$ from (3.2.17). It implies that the thick market externalities, $\frac{\partial Y(r)}{\partial N}$, distort migration decisions and the equilibrium population.

(3.4.3) prescribes the optimal visit intensity $v$, which incorporates the externalities caused by social interactions and human capital spillovers represented by the brackets in (3.4.3). Under the specifications of (3.3.9), (3.3.10) and (3.3.12) as in Example 3.2, the bracketed term evaluated at a decentralized equilibrium is $hw_v + \nu_k k_v = (2\rho - k)k^\rho / (2v)$. If crowded interaction can serve as important catalysts for knowledge transmission and the corresponding loss of human capital does not inflict significant decline on sector-level productivity, i.e., $k < 2\rho$, then the equilibrium number of visits will be too low. $k < 2\rho$ always holds if $\rho \geq 1/2$ since $k \in (0, 1)$. On the other hand, if a visit does not lead to a greater benefit from urban

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23If not, the urban internal structure will be also inefficient. Glaeser (2000) suggests that there may be a non-monotonic relationship between social capital and density. In contrast, Brueckner and Largey (2008) find that high-density living reduces, rather than spur, social interaction. See also Ahlfeldt et al. (2012) for the effects of population density on worker productivity.
interactions than the cost of decreasing amounts of solitary practice, i.e., $k > 2\rho$, then the decentralized economy gives rise to underinvestment in human capital. This is always the case when $\rho = 0$ as in Example 3.1.

Notice too that since $k < \frac{2\rho}{1 + 2\rho} < 2\rho$, households always visit the playing field with insufficiently low frequency whenever agglomeration of population is associated with high productivity and high wages (see Proposition 3.2). On the other hand, when a large city offers low wages (i.e., $k > \frac{2\rho}{1+2\rho}$), the equilibrium visits can be either excessive or insufficient.

The inefficiency comes from the fact that households do not experience the marginal social impacts of visits. A natural policy to correct this distortion would be imposition of labor income tax/subsidies, rather than transport subsidies proposed by Helsley and Strange (2007).

### 3.5 Extension

#### 3.5.1 Visits incurring commuting cost

We have assumed that the commuting cost is independent of $v$ or $w$. As in Helsley and Strange (2007), the commuting cost is a nonnegligible price of visiting activities. This section relaxes the assumption about the commuting cost function, and shows that such extension strengthens the dispersion forces.

\cite{24} Aguiar and Hurst (2007), and Gelber and Mitchell (2012) provide evidence regarding the effects of taxes on detailed time allocation.
Assumption 3.4. The commuting cost, denoted by $t(r,v)$, is increasing in on $(v,r) \in \mathbb{R}_+^2$:

$$t_r(r,v) := \frac{\partial t(r,v)}{\partial r} > 0 \text{ and } t_v(r,v) := \frac{\partial t(r,v)}{\partial v} > 0.$$  

(3.5.1)

Assumption 3.4 implies that visiting the playing field frequently requires much commuting time. Such visits may represent, for example, while making a day trip to a neighboring town rather than having a good time in cafés or bars near the office with business collaborators.

Under this extension, households consider the marginal impact of interactions on commuting costs. Utility-maximizing visits satisfy the following first-order condition:25

$$wh'(v) + \frac{\partial \nu(v,k)}{\partial v} - \frac{\partial t(r,v)}{\partial v} = 0.$$  

Let $\hat{v} = \hat{v}(w,k,r)$ be a visit satisfying (3.5.2). We obtain a comparative statics result:

$$\text{sign} \left[ \frac{\partial \hat{v}(w,k,r)}{\partial r} \right] = -\text{sign} \left[ \frac{\partial^2 t(r,v)}{\partial r \partial v} \right].$$  

(3.5.3)

If $t_{rv} := \partial^2 t(r,v)/\partial r \partial v < 0$, for example, fixed costs are required to visit the playing field (e.g., spending on an automobile), then a household located away from the workplace will choose to interact more frequently. On the

\footnote{The second-order condition is assumed to be satisfied:

$$wh''(v) + \frac{\partial^2 \nu(v,k)}{\partial v^2} - \frac{\partial^2 t(r,v)}{\partial v^2} < 0.$$}

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other hand, if $t_{rv} > 0$, for example, an additional visit is a burden on households located at distant areas with relatively poor transport infrastructure, then a household near the city center will prefer interactions. In both cases, the time allocation decision of a household depends on its location.

The utility-maximizing lot size $\bar{s} = \bar{s}(r)$ is the same as in the benchmark: $\sigma/(2\bar{s}^2) = P(r)$. The indirect utility at location $r$ is given by

$$U(r) = wh(\hat{v}) + \nu(\hat{v}, k) + \bar{\omega} - \frac{\sigma}{\bar{s}(r)} - t(r, \hat{v}).$$

(3.5.4)

From the locational arbitrage (i.e., $\frac{dU}{dr} = 0$) and the boundary condition (i.e., $P(r_f) = \bar{R}$),

$$\bar{s}(r) = \sigma \frac{\sigma}{\int_r^{r_f} t_r(x, \hat{v}(w, k, x))dx + \sqrt{2\sigma \bar{R}}}.$$  

(3.5.5)

The population constraint is

$$N = \frac{2r_f \sqrt{2\sigma \bar{R}}}{\sigma} + \frac{2}{\sigma} \int_0^{r_f} \int_r^{r_f} t_r(x, \hat{v}(w, k, x))dxdr.$$  

(3.5.6)

Differentiating both sides, we obtain

$$\frac{dr_f}{dN} = \frac{\sigma}{2[r_f t_{rf}(r_f, \hat{v}(w, k, r_f)) + \sqrt{2\sigma \bar{R}}]} > 0.$$  

(3.5.7)

The indirect utility in the city is rewritten as follows:

$$V = wh(\hat{v}) + \nu(\hat{v}, k) + \bar{\omega} - t(r, \hat{v}) - \int_r^{r_f} t_r(x, \hat{v}(w, k, x))dx - \sqrt{2\sigma \bar{R}}.$$  

(3.5.8)

By construction, $\frac{dV}{dr} = 0$ for all $r \in [0, r_f]$.  

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3.5.2 Spatial equilibrium

Consider, first, that both productivity and crowdedness are exogenous leading to the same results as in the benchmark case. Second, we use the specifications provided in Example 3.2 to examine the equilibrium properties in the presence of externalities. Finally, we briefly discuss the implications of this extension.

We begin by assuming that \( w \) and \( k \) are exogenous. Differentiating the indirect utility with respect to \( N \),

\[
\frac{dV}{dN} = -t_r(r_f, \tilde{v}(w, k, r_f)) \frac{dr_f}{dN} < 0. \tag{3.5.9}
\]

Therefore, we can restate Proposition 3.1 in the extended model when externalities are absent.

To endogenize either \( w \) or \( k \), we shall specify the functional forms. In particular, to preclude the analytical difficulty arising from location-dependent visiting, we assume as follows:

Assumption 3.5. \textit{The cross-derivative of the commuting cost is zero:}

\[
\frac{\partial^2 t(r, v)}{\partial r \partial v} = 0. \tag{3.5.10}
\]

For example, if urban interactions take place on the Internet, the marginal commuting cost of interaction will be almost independent of physical distance.

In addition to Assumption 3.5, we assume that \( A = k^\rho \sqrt{1 - v^2} \), \( \nu = v k^{1/2+\rho} \) and \( K = v N^{\delta/2} \), as in Example 3.2. Because \( v \) and \( k \) are determined
by the fixed point of \( v = [k^{1/2+\rho} - t_v] k^{-\rho} \) and \( k = K \), we have
\[
\frac{dv}{dN} = \Delta \left( \frac{1}{2\sqrt{k}} + \frac{\rho t_v}{k^{\rho+1}} \right) = \frac{\Delta}{2k} \left[ \sqrt{k} + 2\rho(\sqrt{k} - v) \right], \quad \frac{dk}{dN} = \Delta \left( 1 + \frac{t_{vv}}{k^\rho} \right),
\]
and \( dw/dN = \rho k Adk/dN - (vk^{2\rho}/A)(dv/dN) \), where
\[
\Delta := \frac{\delta kN}{2} \left[ 1 - \frac{\sqrt{k}}{2v} + \frac{t_{vv}}{k^\rho} - \frac{\rho t_v}{vk^\rho} \right]^{-1},
\]
assuming that the fixed point in the interior of \((v, w, k) \in (0, 1)^3\) is uniquely determined. Differentiating the indirect utility,
\[
\frac{dV}{dN} = \dot{v} k^\rho \Upsilon \Delta - t_v(r_f, \dot{v}(w, k, r_f)) \frac{dr_f}{dN}, \tag{3.5.11}
\]
where
\[
\Upsilon := \rho k \frac{1 - \dot{v}^2}{\dot{v}} \left( 1 + \frac{t_{vv}}{k^\rho} \right) + \frac{t_{vv} 1 + 2\rho}{k^\rho} \frac{1}{2\sqrt{k}} + \frac{\rho \dot{v}}{k}.
\]
As long as the second-order condition for visiting choice is satisfied for all \( v \in (0, 1) \), \( \Upsilon \) takes a positive value. Therefore, if \( \Delta \) is positive, then \( dv/dN > 0, dk/dN > 0 \), and the net agglomeration force, represented in the first term on the right-hand side of (3.5.11), is also positive.

In the benchmark case presented in the previous section, \( \Delta \) is always positive. However, this section allows \( t_v \neq 0 \) so that the sign of \( \Delta \) can be negative: the positive externalities could work as a dispersion force. The reasoning is simple: a visit is costly in terms of time for development of human capital lost as well as commuting costs incurred. Therefore, as \( N \) increases and \( k \) therefore increases, households will regard solitude more...
important than urban interactions considering the increasing costs of visits. If such behavioral changes consequently reduce crowdedness in the city, no cumulative processes that trigger urban agglomeration can result.

When Assumption 3.5 does not hold, households choose different time allocations according to the residential location. In this case, how to aggregate both human and social capital will be an important issue to characterize an equilibrium. Furthermore, an optimal tax/subsidy schedule, if it can be defined, is required to vary along the location of households.

3.6 Conclusion

In the context of urban policy, we have focused on the importance of a trade-off that households face in allocating time for urban interactions and human capital development. While both activities involve positive externalities distorting households' time allocation, it is impossible to separately modify each external effect. Net visit-induced external effects are ambiguous and could be negligible (for example, when \( k \simeq 2\rho \) in Example 3.2). It is possible that a decentralized economy leads to underinvestment in human capital when the positive impacts of urban interaction on productivity are small.

Policy making is associated with many difficulties. Most important is the fact that the two externalities and their net effects are neither empiric-

\[27\] For example, Bénabou (1996) argues that inequality in the distribution of human capital drags down aggregate productivity.
ally measurable nor distinguishable with existing surveys. These issues are active research topics. In particular, we need to know the non-monetary return that urban interaction generates in order to evaluate the urban policy targeting toward urban interaction. The investigation in this chapter should be viewed less as a practical guide to policy and more as an illustration of the abstract point that externalities themselves do not imply any necessary departure from efficiency in decentralized decision making.

The present model imposes some implicit assumptions that it would be interesting to relax. First, we consider the case of homogeneous households. Retirees or homemakers may not face the trade-off that we address. Indigenous residents and migrants may value urban interactions differently. The benefits of interaction depend on the degree of introversion or extroversion. To design better interventions targeting household attitudes to urban interactions, we need to introduce differences in preferences, endowments and socioeconomic status without relying on specific aggregators. Second, we focus only on steady-state equilibria. Accumulation of human and social capital is a dynamic process. It is important to reveal the costs and benefits

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28Helsley and Strange (2000) study interactions between segregation among heterogeneous agents and community policies. Relationships between skill distribution and agglomeration economies are documented empirically by a number of authors including Wheeler (2001) and Bacolod et al. (2009). Murata (2003) views social interaction as a source of heterogeneity of migration costs. See also Brueckner (2006), and Cabralles et al. (2011) for a network formation game with heterogeneous agents and endogenous strength of social ties.
of urban interactions in dynamic models.
Bibliography


