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Influence of Shape of Initial Deflection upon Compressive Ultimate Strength of Long Rectangular Plates†

Yukio UEDA* and Tetsuya YAO**

Abstract

In general, the deck plate of a box girder such as ship hull is subjected to inplane tensile and/or compressive load due to longitudinal bending. The deck plate is a main strength member of a box girder, and is stiffened by stiffeners and girders in the longitudinal and transverse directions to prevent overall buckling under compressive load. When the strength of a deck plate is considered, the plate elements subdivided by stiffeners and girders play an important role, and the accurate evaluation of their compressive strength is necessary, taking into account of the effect of welding deflection and residual stresses.

In this paper, the compressive ultimate strength of a long rectangular plate with initial deflection is theorectically investigated, and the following main conclusions are obtained.

(1) The simple formulae are proposed to calculate the conservative compressive ultimate strength of a long rectangular plate with welding initial deflection and residual stresses.

(2) The mode of initial deflection due to welding is measured on 33 deck panels of a bulk carrier and a pure car carrier, and the coefficients of components of series functions to express initial deflection are calculated.

(3) The characteristic of elastic-plastic large deflection behavior of a long rectangular plate with initial deflection may be described as follows:
In case of thin plate, only one component mode of the initial deflection is amplified and becomes stable as the deflection increases. This component mode is not necessarily the one at buckling and is influential upon ultimate strength. This component is usually pretty smaller than the maximum initial deflection. In case of thick plate, a certain portion of the plate, where the largest curvature of the initial deflection is located, is decisive to progress of plastification which leads to collapse. Then, the maximum curvature of the initial deflection is decisive rather than the maximum deflection. Concerning plate of medium thickness, the mode of the initial deflection may change as the deflection increases. In this process, progress of plastification may be accelerated or decelerated by interaction among the component modes. This phenomena makes lower or higher the ultimate strength.

(4) Based on the characteristic, Deflection method and Curvature method are proposed to predict the compressive ultimate strength of thin plate and thick plate, respectively. It is confirmed that these methods predict the ultimate strengths accurately in comparison with calculated ones.

(5) The in-plane rigidity of the plate decreases as the load increases, since the compressive load is not supported at the central portion of the plate due to the effect of large deflection in the case of thin plate, while it is due to the effect of plastification by bending in the case of thick plate.

KEY WORDS: (Initial deflection) (Compressive ultimate strength) (Rectangular plate) (Stable deflection) (Post-buckling) (Prediction of compressive ultimate strength)

1. Introduction

One of the most common types of structures is a box girder such as a ship hull, or a bridge of certain types. Generally, these box girder type structures are in service under lateral loads, and their deck plates which are the main strength members are subjected to in-plane tensile and/or compressive loads. To increase efficiently the strength of the deck plates under compression they are reinforced by a number of stiffeners, which are provided by fillet welding. Consequently, the deck plate is accompanied by initial imperfections which are residual stresses and distortion due to welding. These imperfections often reduce the strength and rigidity of the plate. When the ultimate strength of the deck plate is considered, the contribution of these subdivided strip plate element is essential.

As to compressive buckling and ultimate strengths of unstiffened and stiffened plates with initial imperfections, many investigations have been performed both theoretically and experimentally. Here, attentions may be restricted on unstiffened plates.

The effects of welding residual stresses on the elastic buckling strength were studied by Y. Yoshiki et al.[1] and N.O. Okarblom[2]. As to these effects on elastic-plastic and plastic buckling strengths, a series of theoretical and experimental investigations were carried out at Lehigh University, and the results were reported by Y. Ueda[3], F. Nishino[4], Y. Ueda and L. Tall[5], F. Nishino et al.[6] and so on. The same problem was also investigated at

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Cambridge University[7),8)].

In the above investigations related to the buckling strength, the analytical methods were employed for theoretical analyses. Contrary to these, it is almost impossible to calculate the ultimate strength in analytical way, since both geometrical and material nonlinearities must be taken into consideration. For such problems, the finite element method is useful. Application of the finite element method to the elastic-plastic large deflection analysis was performed by G.P. Bergan[9)], H. Ohtsubo[10)]. H. Arai[11)] and Y. Ueda et al.[12)]. Applying the finite element method, the compressive ultimate strength of square plates was calculated by H. Ohtsubo[10)], S. Komatsu et al.[13)], the authors[14)−17)] and so on, taking into consideration of the influence of initial imperfection due to welding. Recently, R.S. Dow and C.S. Smith[18)] discussed the effect of localized imperfections on the compressive ultimate strength of rectangular plates.

On the other hand, the compressive ultimate strength of rectangular plates has been investigated based on the concept of an effective width formerly by G. Shnadel[19)], S. Schumann and G. Back[20)], T. Kárnán et al.[21)] and recently by J.B. Dwigit and K.E. Moxham[22)] and H. Becker et al.[23)−25)]. The proposed formulae to calculate the compressive ultimate strength were summarized by D. Faulkner[26)].

The authors have also been investigating this problem[14)−17),27)−30)]. In this paper, considering the effects of initial imperfections due to welding, the compressive ultimate strength of rectangular plates will be discussed based on the results of the elastic-plastic large deflection analysis by the finite element method. First, the compressive ultimate strength is calculated assuming the uni-modal initial deflection on a rectangular plate. Then, the existing modes of initial deflection are measured on *33* panels of the strength deck plates of ship hulls, and the general feature of the modes of initial deflection is shown. Adopting the measured modes of initial deflection, compressive ultimate strength of a rectangular plate with multi-modal initial deflection is calculated, and elastic-plastic large deflection behavior of the plate up to collapse is precisely investigated into. From these accurately calculated results, the characteristic of elastic-plastic large deflection behavior of the plate is studied and the mechanism of influence of initial deflection upon ultimate strength is clarified. Based on this information, prediction methods are proposed to evaluate compressive ultimate strength of a rectangular plate with multi-modal initial deflection. Accuracy of the prediction methods are confirmed comparing their results with the accurately calculated ones.

2. Method of Theoretical Analysis

It is well known that an initially flat plate undergoes a primary buckling under thrust. After the buckling takes place, lateral deflection increases. If the plate is accompanied originally by initial deflection, lateral deflection increases with an increase of load from the beginning of loading, but the increasing rate of the deflection is small until the load reaches near the buckling load. To analyze such behavior of the plate, the geometrical nonlinearity must be taken into account.

For further increase of load, plastification in the plate gradually takes place, and finally the plate reaches to its ultimate strength. For this process, the material nonlinearity must be also taken into account in addition to the geometrical one in the analysis.

Therefore, both material and geometrical nonlinearities should be considered to evaluate theoretically the compressive ultimate strength of the rectangular plate. In this paper, the fundamental equations in the incremental form for the finite element method are used, which were derived by applying the principle of virtual work. The detailed process of derivation of the fundamental equations is represented in Refs. 12) and 15).

A rectangular plate in the analysis is supposed to be a strip plate element subdivided by stiffeners. Therefore, considering the continuity between the strip plate elements, the following boundary conditions are imposed. A plate is assumed to be simply supported along all edges, which implies that there exist no out-of-plane deflection along its four edges. As to in-plane displacements, all straight edges are assumed to move keeping straight.

3. Compressive Ultimate Strength of a Rectangular Plate with Uni-modal Initial Deflection

3.1 Behavior of a rectangular plate under thrust

A series of elastic-plastic large deflection analysis is

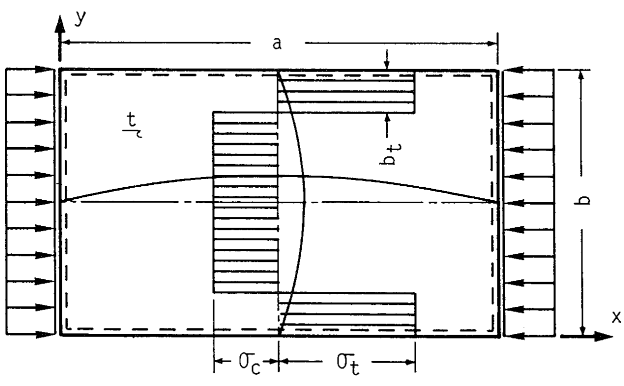


Fig. 1 Rectangular plate with welding residual stresses and uni-modal initial deflection

carried out, using the finite element method, to evaluate the compressive ultimate strength of a rectangular plate illustrated in **Fig. 1**. The plate is supposed to be accompanied by welding residual stresses of a rectangular distribution. These residual stresses are due to the fillet welding of stiffeners along the edges, $y = 0$ and $y = b$, and such distribution may be valid when the continuity condition of the panel is considered. If the residual stresses are assumed to be self-equilibriating in the plate, there exists the following relation between the compressive residual stress, σ_c, tensile residual stress, σ_t, denoting the plate width by b and the width where the tensile residual stresses are acting by b_t.

$$2b_t\sigma_t = (b - 2b_t)\sigma_c \tag{1}$$

Furthermore, due to the fillet welding mentioned above, the plate is accompanied also by initial deflection which is generally of a complex mode. However, in this chapter, first, in order to examine the fundamental behavior of a long rectangular plate accompanied by initial deflection under thrust, it is assumed that initial deflection is simple uni-modal.

In this section, the welding residual stresses are not considered in the analysis. First, taking the plate width, b, the thickness, t, and the yield stress of the material, σ_Y, as *1000 mm*, *12 mm* and *28 kgf/mm²*, respectively, the compressive ultimate strength is calculated. In this analysis, the following two forms of initial deflection are assumed. For the following first one,

$$w_0 = A_{01} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \tag{2}$$

the calculated ultimate strengths are plotted against the aspect ratio, a/b, by ○, △ and □ in **Fig. 2**.

Then, for the second one,

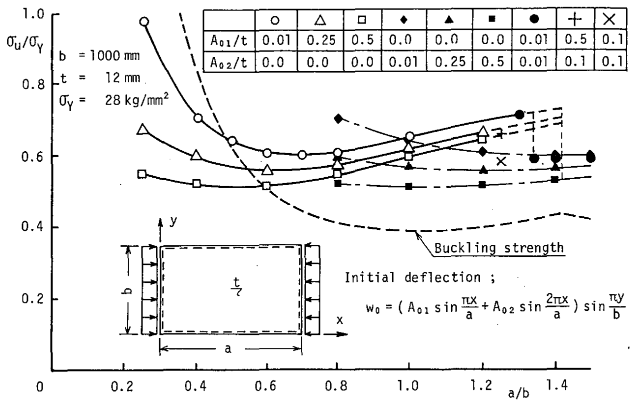


Fig. 2 Compressive ultimate strength of a rectangular plate

$$w_0 = A_{02} \sin \frac{2\pi x}{a} \sin \frac{\pi y}{b} \tag{3}$$

the calculated results are plotted also in Fig. 2 by ◆, ▲ and ■.

It is well known that the buckling strength of a simply supported long rectangular plate under uni-axial thrust takes its minimum value for the aspect ratio being a certain integer. However, the ultimate strength is not necessarily to be the minimum value for this aspect ratio. Such aspect ratio as gives the minimum ultimate strength is approximately *0.7* of an integer multiple for this size of plate. This aspect ratio increases up to *1.0* of an integer multiple as the slenderness ratio, b/t, decreases[31].

It is observed from Fig. 2 that the aspect ratio of the one half-wave of the deflection mode which gives the minimum ultimate strength decreases as the magnitude of initial deflection increases. The welding residual stresses shown in Fig. 1 also decrease the aspect ratio giving the minimum ultimate strength[29].

When the aspect ratio is smaller than $\sqrt{2}$, the buckling

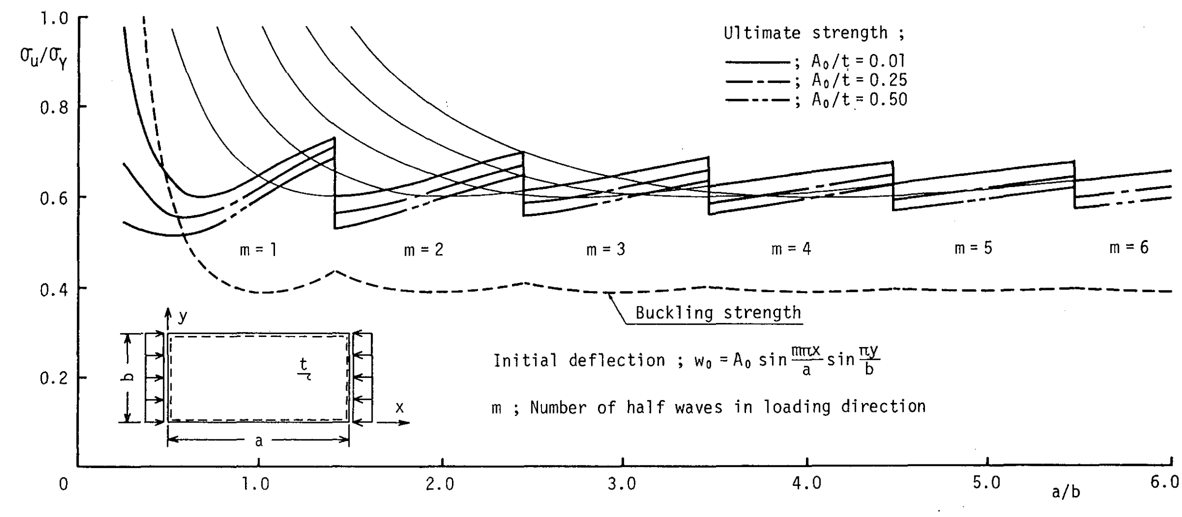


Fig. 3 Compressive ultimate strength of a rectangular plate with the same mode of initial deflection as buckling mode

mode of a perfectly flat plate must be in one half-wave in the loading direction. This deflection mode will be stable until the secondary buckling takes place. For the plate analyzed here, the secondary buckling strength is so high that the plastification followed by the collapse takes place before the secondary buckling occurs.

The ultimate strength of a rectangular plate with uni-modal initial deflection which coincides with the buckling mode may be represented by solid and chain lines in **Fig. 3**, if the buckling mode is stable until collapse. As shown in Fig. 3, there exist abrupt changes in the ultimate strength at the aspect ratios where the buckling modes terminate. For example, the buckling mode terminates from one half-wave to two half-waves at the aspect ratio of $\sqrt{2}$. If the plate of the aspect ratio being $\sqrt{2}$ buckles in one half-wave, the ultimate strength of the plate may be high, but it may be low if it buckles in the two half-waves. Consequently, the ultimate strength curves show saw-tooth appearances with respect to the aspect ratio.

When the lateral deflection of a plate with complex modes of initial deflection becomes large under thrust, the stable deflection mode does not necessarily coincide with the buckling mode near the aspect ratio of $\sqrt{2}$. The stable deflection mode depends upon the magnitudes of the components of the initial deflection and their ratios. This ratio was defined as the critical ratio of the components of initial deflection[27),28)]. For example, the initial deflection is assumed to be expressed by the sum of the two components as follows.

$$w_0 = (A_{01} \sin \frac{\pi x}{a} + A_{02} \sin \frac{2\pi x}{a}) \sin \frac{\pi y}{b} \qquad (4)$$

Then, if the magnitude of initial deflection is taken as $A_{01}/t = A_{02}/t = 0.01$, the finally stable deflection mode is in one half-wave or two half-waves in the loading direction for the plate of aspect ratio being smaller or larger than *1.36*, respectively. The calculated ultimate strength is plotted by ● in Fig. 2. In this case, they are almost the same as those of plates with the same uni-modal initial deflection as the stable one, expressed by Eq.(2) or Eq.(3).

There is another interesting example on the plate of aspect ratio being *1.25*. Two different initial deflections are assumed taking $A_{01}/t = 0.5$, $A_{02}/t = 0.1$ and $A_{01}/t = A_{02}/t = 0.1$, in Eq.(4). The ultimate strength is calculated for these two cases and represented by + and X in Fig. 2. In the former case, the plate collapses in one half-wave in the loading direction, while in the latter case in two half-waves. The difference of the collapse modes is attributed to the difference of the shape and the magnitude of initial deflection[27),28)]. Although the former plate is imposed a larger initial deflection, the ultimate strength is higher than the latter plate. In actual plates, the mode of initial deflection is complicated. In these cases, the ultimate strength may be estimated by adopting the lowest ultimate strength which may be conservative, among those obtained assuming the same maximum magnitude of a uni-modal initial deflection as the actual. Such ultimate strength is illustrated by bold solid lines in **Fig. 4** together with some calculated ultimate strengths assuming various initial deflections. It is observed that the ultimate strength takes almost the constant value as the aspect ratio of the plate increases.

3.2 Minimum ultimate strength of a rectangular plate

A series of elastic-plastic large deflection analysis is performed changing the breadth to thickness ratio, the magnitudes of initial deflection, and welding residual stresses. The magnitude of the tensile residual stress, σ_t, is taken as the yield stress, σ_Y. The calculated minimum ultimate strengths are plotted by ○, △, □ and ◇

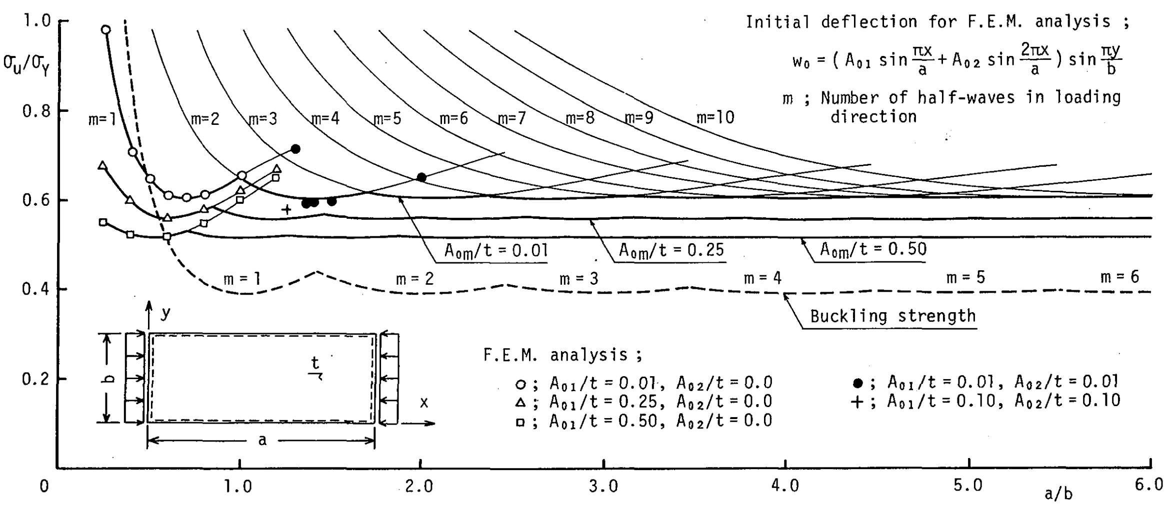


Fig. 4 Compressive ultimate strength of a rectangular plate with initial deflection

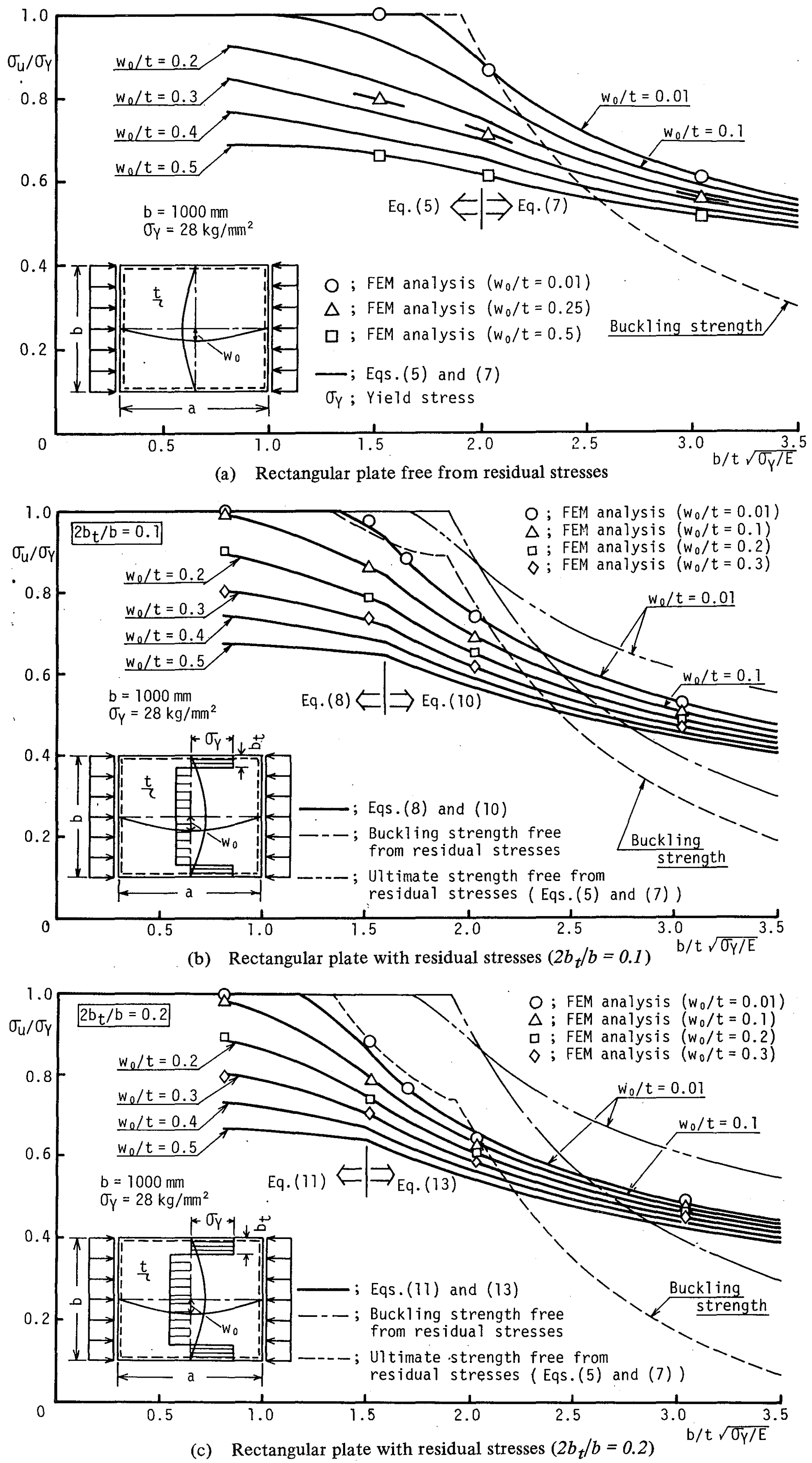


Fig. 5 Minimum ultimate strength of a rectangular plate with initial imperfections under thrust

against the non-dimensional parameters, $b/t \cdot \sqrt{\sigma_Y/E}$, in **Figs. 5** (a), (b) and (c), in which the magnitude of residual stresses varies, such as $2b_t/b$ being *0.0*, *0.1* and *0.2*, respectively.

In these figures, the calculated minimum ultimate strengths are represented by the solid lines and expressed by the following simple formulae, which are derived by applying the method of least squares to the calculated values.

(a) $2b_t/b = 0.0$

i) $0.8 \leqslant \xi \leqslant 2.0$

$$\sigma_u/\sigma_Y = (-2.431\eta^2 + 1.6826\eta - 0.2961)(\xi^2 - 4.0) + (7.2745\eta^2 - 4.7431\eta + 0.6709)(\xi - 2.0) + Z_1 \qquad (5)$$

$$Z_1 = (-0.3597\eta^2 + 0.1748\eta + 0.8598)/(2.2432\eta + 1.3322) + 0.0373\eta + 0.2481 \qquad (6)$$

ii) $2.0 \leqslant \xi \leqslant 3.5$

$$\sigma_u/\sigma_Y = (-0.3597\eta^2 + 0.1748\eta + 0.8598)/(\xi + 2.2432\eta - 0.6678) + 0.0373\eta + 0.2481 \qquad (7)$$

(b) $2b_t/b = 0.1$

i) $0.8 \leqslant \xi \leqslant 1.6$

$$\sigma_u/\sigma_Y = (-0.398\eta^2 + 0.4339\eta - 0.1342)(\xi^2 - 2.56) + (1.0814\eta^2 - 0.7551\eta + 0.1020)(\xi - 1.6) + Z_2 \qquad (8)$$

$$Z_2 = (0.4974\eta^2 + 0.8281\eta + 1.0171)/(2.7942\eta + 1.2908) - 0.1849\eta + 0.1571 \qquad (9)$$

ii) $1.6 \leqslant \xi \leqslant 3.5$

$$\sigma_u/\sigma_Y = (0.4974\eta^2 + 0.8281\eta + 1.0171)/(\xi + 2.7942\eta - 0.3092) - 0.1849\eta + 0.1571 \qquad (10)$$

(c) $2b_t/b = 0.2$

i) $0.8 \leqslant \xi \leqslant 1.5$

$$\sigma_u/\sigma_Y = (-0.3317\eta^2 + 0.6314\eta - 0.2656)(\xi^2 - 2.25) + (0.5369\eta^2 - 0.7798\eta + 0.2854)(\xi - 1.5) + Z_3 \qquad (11)$$

$$Z_3 = (0.292\eta^2 + 1.2936\eta + 0.7471)/(2.897\eta + 1.1189) - 0.2715\eta + 0.2057 \qquad (12)$$

ii) $1.5 \leqslant \xi \leqslant 3.5$

$$\sigma_u/\sigma_Y = (0.292\eta^2 + 1.2936\eta + 0.7471)/(\xi + 2.897\eta - 0.3811) - 0.2715\eta + 0.2057 \qquad (13)$$

where

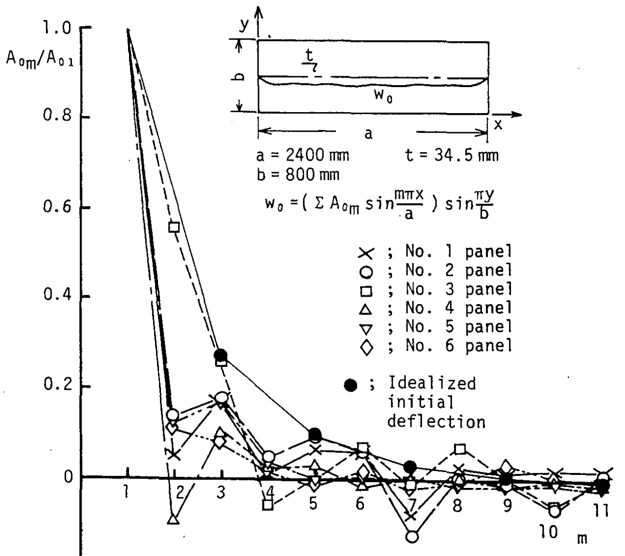
$$\xi = b/t \cdot \sqrt{\sigma_Y/E} \qquad (14)$$

$$\eta = w_0/t \qquad (15)$$

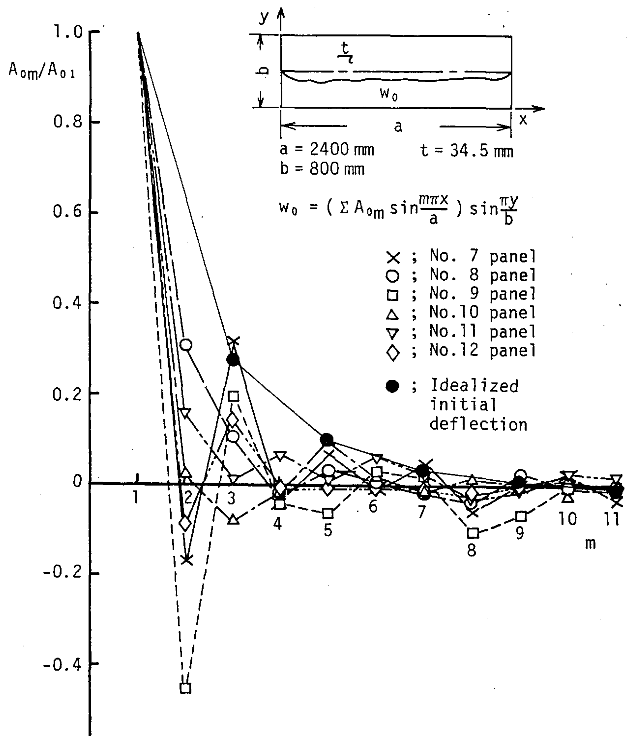
These equations will be used later to predict the conservative ultimate strength of a rectangular plate with multi-modal initial deflection.

4. Initial Deflection due to Welding on Deck Plates of a Ship Hull

In the previous chapter, a uni-modal initial deflection is assumed in the analysis. However, such simple mode of initial deflection does not exist in the actual structures. Here, attentions is restricted on ship structures. As for the maximum magnitude of initial deflection of panels, many

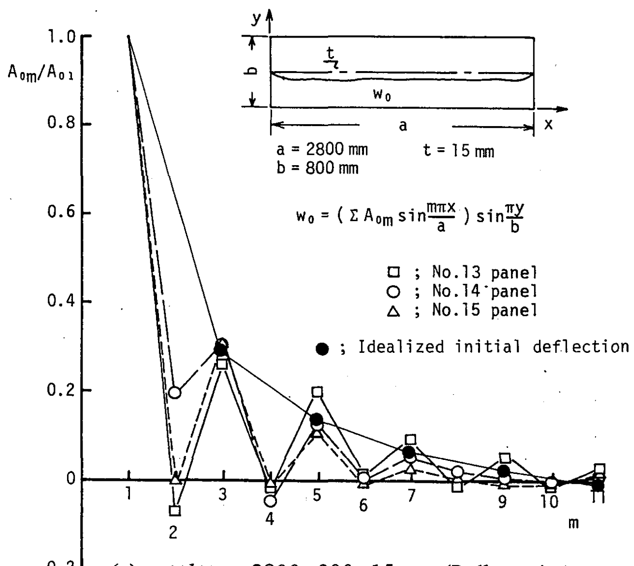


(a) $a \times b \times t = 2400 \times 800 \times 34.5$ *mm* (Bulk carrier)

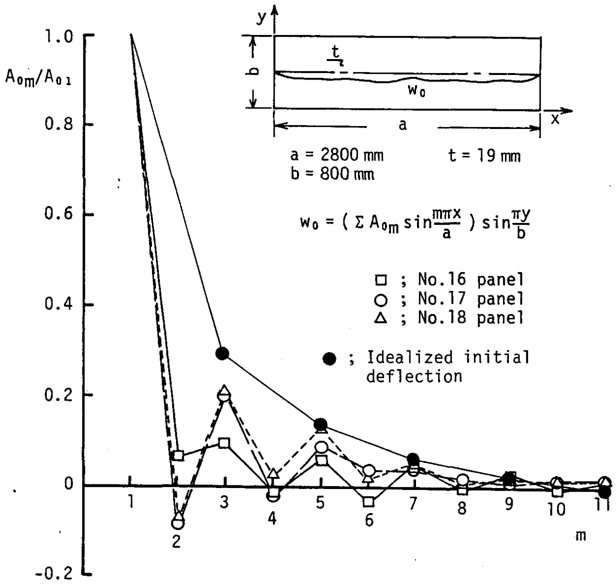


(b) $a \times b \times t = 2400 \times 800 \times 34.5$ *mm* (Bulk carrier)

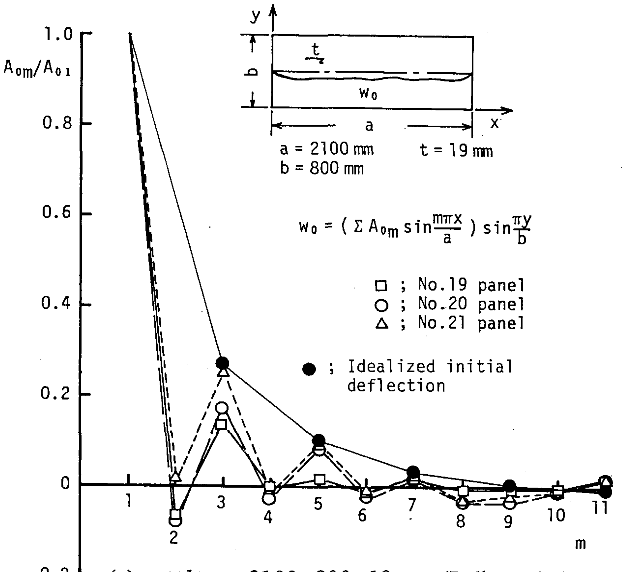
Fig. 6 (a)~(g) Components of measured initial deflection



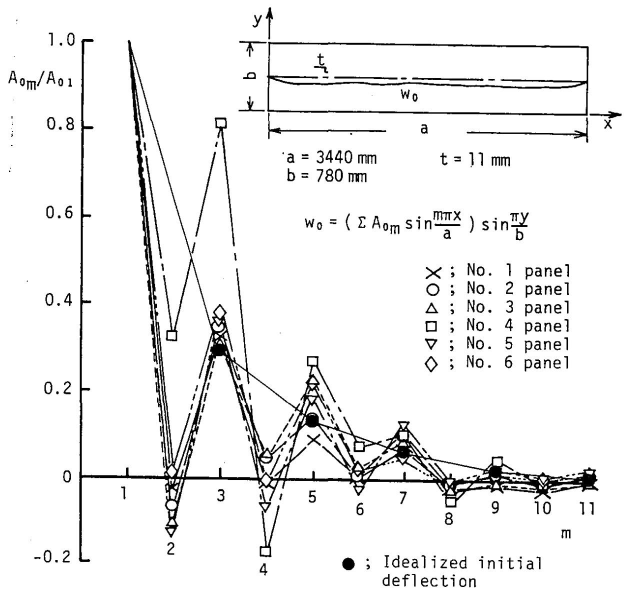
(c) $a \times b \times t = 2800 \times 800 \times 15$ *mm* (Bulk carrier)



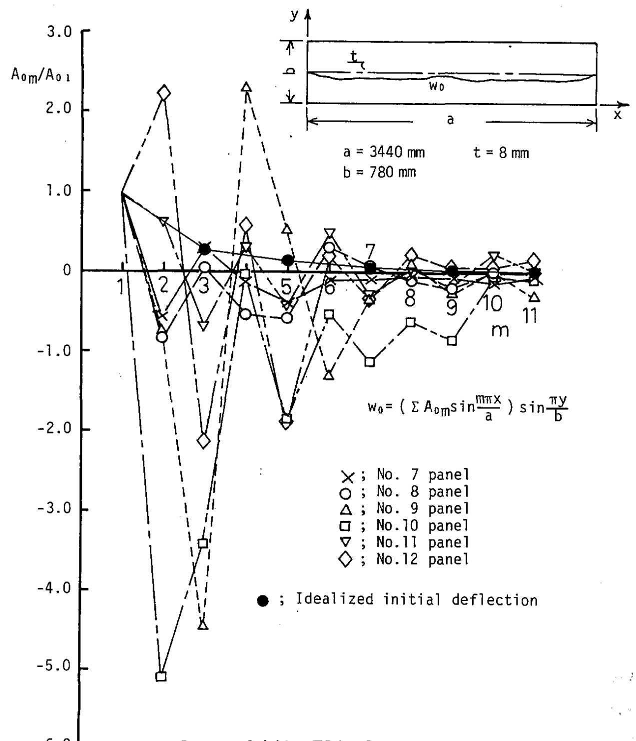
(d) $a \times b \times t = 2800 \times 800 \times 19$ *mm* (Bulk carrier)



(e) $a \times b \times t = 2100 \times 800 \times 19$ *mm* (Bulk carrier)



(f) $a \times b \times t = 3440 \times 780 \times 11$ *mm* (Car carrier)



(g) $a \times b \times t = 3440 \times 780 \times 8$ *mm* (Car carrier)

Fig. 6 (a)~(g) Components of measured initial deflection

measurements have been carried out at various parts of the ship structures[32),33)]. Based on the measured data and actual practice, some quality standards including limitation of the maximum initial deflections were published[33)-36)]. In contrast with this, only few measurements have been carried out as to the mode of initial deflection[37),38)].

In this paper, a series of measurements of the mode of initial deflection is carried out on deck plates of ship hulls. The measurement was carried out on *21* panels of

Table 1 Coefficients of deflection components of measured initial deflection

(Car Carrier)

No.	a × b × t	w_{0max}	A_{01}	A_{02}	A_{03}	A_{04}	A_{05}	A_{06}	A_{07}	A_{08}	A_{09}	A_{010}	A_{011}
1	3440x780x11	-5.290	-5.949	-0.223	-1.936	-0.070	-0.598	-0.148	-0.455	0.031	0.020	0.022	0.008
2	3440x780x11	-5.360	-5.611	0.735	-1.948	-0.075	-0.793	0.075	-0.547	0.116	-0.030	0.013	-0.030
3	3440x780x11	-4.620	-4.643	0.702	-1.456	-0.147	-1.065	-0.073	-0.362	0.146	0.128	0.057	0.083
4	3440x780x11	-5.870	-3.425	-1.500	-2.965	0.525	-0.966	-0.380	-0.332	0.124	-0.125	-0.026	0.005
5	3440x780x11	-5.470	-5.125	0.586	-1.832	0.318	-0.945	0.107	-0.629	-0.013	-0.082	-0.064	-0.057
6	3440x780x11	-5.650	-5.647	0.133	-2.167	0.147	-1.201	0.209	-0.270	0.114	-0.006	0.051	-0.003
7	3440x780x8	-3.970	-2.635	1.455	-0.871	0.328	0.349	0.194	0.154	0.027	0.114	0.074	0.108
8	3440x780x8	1.300	0.605	-0.508	0.037	-0.308	-0.349	0.082	0.058	-0.058	-0.107	0.002	0.014
9	3440x780x8	2.330	0.269	-0.198	-1.195	0.621	0.144	-0.350	-0.085	0.029	-0.066	0.006	-0.080
10	3440x780x8	1.510	-0.142	0.720	0.482	0.012	0.261	0.073	0.158	0.086	0.120	0.006	-0.014
11	3440x780x8	-1.350	-0.497	-0.338	0.339	-0.159	0.200	-0.252	0.154	-0.032	0.031	-0.120	0.017
12	3440x780x8	-2.960	0.460	1.023	-0.981	0.259	-0.862	0.100	-0.156	0.096	0.032	0.023	0.076

(Bulk Carrier) (in mm)

No.	a × b × t	w_{0max}	A_{01}	A_{02}	A_{03}	A_{04}	A_{05}	A_{06}	A_{07}	A_{08}	A_{09}	A_{010}	A_{011}
1	2400x800x34.5	-1.320	-1.346	-0.090	-0.224	-0.036	-0.087	-0.082	0.102	-0.036	-0.014	0.026	-0.028
2	2400x800x34.5	-0.770	-0.746	-0.026	-0.124	0.001	-0.066	-0.023	0.112	0.016	0.016	0.065	-0.008
3	2400x800x34.5	-1.550	-1.235	-0.339	-0.344	0.129	-0.061	-0.015	-0.031	-0.031	-0.033	0.075	-0.018
4	2400x800x34.5	-1.260	-1.247	0.178	-0.090	-0.002	-0.004	0.034	0.023	0.018	0.029	0.009	0.023
5	2400x800x34.5	-1.040	-0.968	-0.176	-0.133	-0.052	0.043	-0.020	0.030	0.007	-0.000	-0.000	0.030
6	2400x800x34.5	-1.090	-1.136	-0.107	-0.102	-0.007	0.012	-0.014	0.015	-0.011	-0.039	-0.004	-0.006
7	2400x800x34.5	-0.590	-0.563	-0.021	-0.179	-0.044	-0.014	-0.039	-0.009	0.011	0.020	-0.043	0.042
8	2400x800x34.5	-1.330	-1.101	-0.481	-0.102	-0.032	-0.020	-0.043	0.032	0.025	0.036	-0.014	0.019
9	2400x800x34.5	-1.680	-1.239	0.483	-0.268	0.050	0.029	-0.029	-0.046	0.110	0.056	0.004	-0.008
10	2400x800x34.5	-1.420	-1.339	-0.094	-0.090	-0.024	-0.176	-0.040	-0.049	-0.027	-0.050	0.009	-0.031
11	2400x800x34.5	-1.210	-1.158	-0.142	-0.091	-0.059	-0.057	-0.062	-0.055	0.038	-0.015	-0.022	-0.035
12	2400x800x34.5	-1.390	-1.481	0.024	-0.283	-0.028	-0.062	-0.013	-0.047	0.003	-0.032	-0.019	-0.022
13	2800x800x15	-2.390	-2.668	0.286	-0.686	0.097	-0.534	-0.008	-0.234	0.058	-0.105	0.044	-0.073
14	2800x800x15	-4.150	-4.535	-0.183	-1.419	0.387	-0.656	0.138	-0.341	0.043	-0.155	0.092	-0.135
15	2800x800x15	-3.740	-3.955	-0.218	-1.204	-0.095	-0.359	-0.025	0.023	-0.080	0.137	0.016	0.015
16	2800x800x19	-3.180	-3.339	-0.188	-0.297	0.059	-0.194	0.128	-0.143	0.030	-0.070	0.014	-0.042
17	2800x800x19	-3.680	-4.187	0.441	-0.867	0.150	-0.388	-0.110	-0.181	-0.045	-0.035	-0.037	-0.074
18	2800x800x19	-3.810	-4.161	0.434	-0.921	-0.013	-0.559	-0.015	-0.243	0.004	-0.083	-0.022	-0.093
19	2100x800x19	-3.010	-3.092	0.336	-0.362	0.056	0.017	0.076	-0.010	0.058	0.060	0.030	-0.013
20	2100x800x19	-3.360	-3.687	-0.049	-0.749	-0.055	-0.382	-0.016	-0.142	0.048	0.041	-0.013	-0.092
21	2100x800x19	-3.110	-3.634	-0.035	-0.937	0.043	-0.336	0.069	-0.078	0.119	0.069	0.027	-0.046

Table 2 Coefficients of deflection components of idealized initial deflection

a/b	w_{0max}/t	A_{01}/t	A_{03}/t	A_{05}/t	A_{07}/t	A_{09}/t	A_{011}/t	A_{013}/t	A_{015}/t	A_{017}/t	A_{019}/t	A_{021}/t
$a/b < \sqrt{2}$	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\sqrt{2} \leq a/b < \sqrt{6}$	1.000	1.200	0.240	0.034	-0.011	-0.005	0.003	0.002	-0.001	-0.001	0.001	0.000
$\sqrt{6} \leq a/b < \sqrt{12}$	1.000	1.241	0.333	0.124	0.035	0.000	-0.008	-0.005	0.000	0.002	0.002	0.000
$\sqrt{12} \leq a/b < \sqrt{20}$	1.000	1.255	0.371	0.173	0.082	0.032	0.007	-0.004	-0.006	-0.004	-0.001	0.001
$\sqrt{20} \leq a/b < \sqrt{30}$	1.000	1.261	0.390	0.200	0.111	0.060	0.029	0.010	0.000	-0.004	-0.005	-0.004
$\sqrt{30} \leq a/b < \sqrt{42}$	1.000	1.265	0.400	0.216	0.130	0.080	0.047	0.026	0.011	0.003	-0.002	-0.004

the upper deck plate of a bulk carrier of *60,000* td_w, and *12* panels of the upper and the strength deck plates of a pure car carrier with *5,500* cars on board. In these panels, the aspect ratios, a/b, of the panel are between *2.65* and *4.41* and the slenderness ratios, b/t, between *23.19* and *97.50*.

Initial deflection may be expressed in the following most general form,

$$w_0 = \Sigma\Sigma A_{0m} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{16}$$

and an increase of the deflection under thrust may be in the form

$$w = \Sigma\Sigma A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{17}$$

When a plate is subjected to thrust in x-direction, the magnitudes of many components in x-direction change, but only the first component, $\sin\pi y/b$, among the components, $\sin n\pi y/b$, in y-direction will increase with an increase of loading. So, in this paper, the expression of initial deflection is approximated by the following form.

$$w_0 = \Sigma A_{0m} \sin \frac{m\pi x}{a} \sin \frac{\pi y}{b} \tag{18}$$

Based on the measured data along the center line, $y = b/2$, of the panel, the coefficients, A_{0m}, of initial deflection are calculated taking m up to *11*. The results are plotted in **Figs. 6** (a) to (g), and are summarized in **Table 1**. It should be pointed out that the coefficients of odd terms are greater than those of even terms in most

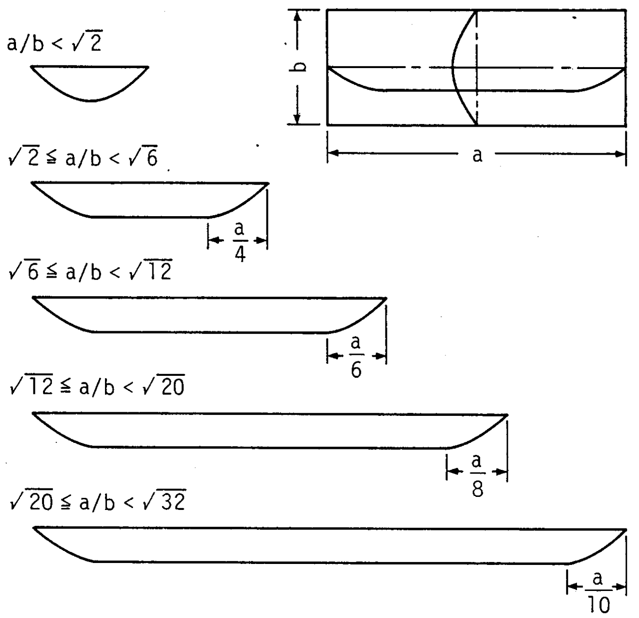


Fig. 7 Idealized modes of initial deflection

cases, which results in the hungry horse mode of initial deflection.

These initial deflections may be idealized as illustrated in **Fig. 7**. For such idealized modes of initial deflection, the coefficients of the deflection components in Eq.(18) are calculated taking m as odd numbers up to *21*. The results are also plotted in Figs. 6 (a) to (g) by ●, and are summarized in **Table 2**. It may be said that these values demonstrate the averaged features of the measured ones except in Fig. 6 (g).

In Fig. 6 (g), the measured results show a different appearance from those in the other figures. This may be attributed to the heat treatment applied to eliminate excessive magnitudes of initial deflection after welding.

5. Compressive Ultimate Strength of a Long Rectangular Plate with Multi-Modal Initial Deflection

5.1 Deformation and plastification of a rectangular plate with multi-modal initial deflection under thrust

When a long rectangular plate with multi-modal initial deflection is subjected to thrust, the process to collapse is somewhat different from that of a short one.

In this section, such behavior will be discussed based on the results of elastic-plastic large deflection analysis by the finite element method. The effect of welding residual stresses is not taken into consideration. When the finite element method is used, the deflection of the plate is determined at the nodes. In this paper, using the calculated nodal displacements along the center line of the plate, $y = b/2$, the deflection is expressed in the following series.

$$w = \Sigma A_m \sin \frac{m\pi x}{a} \sin \frac{\pi y}{b} \tag{19}$$

where, m is taken up to *11* in this study.

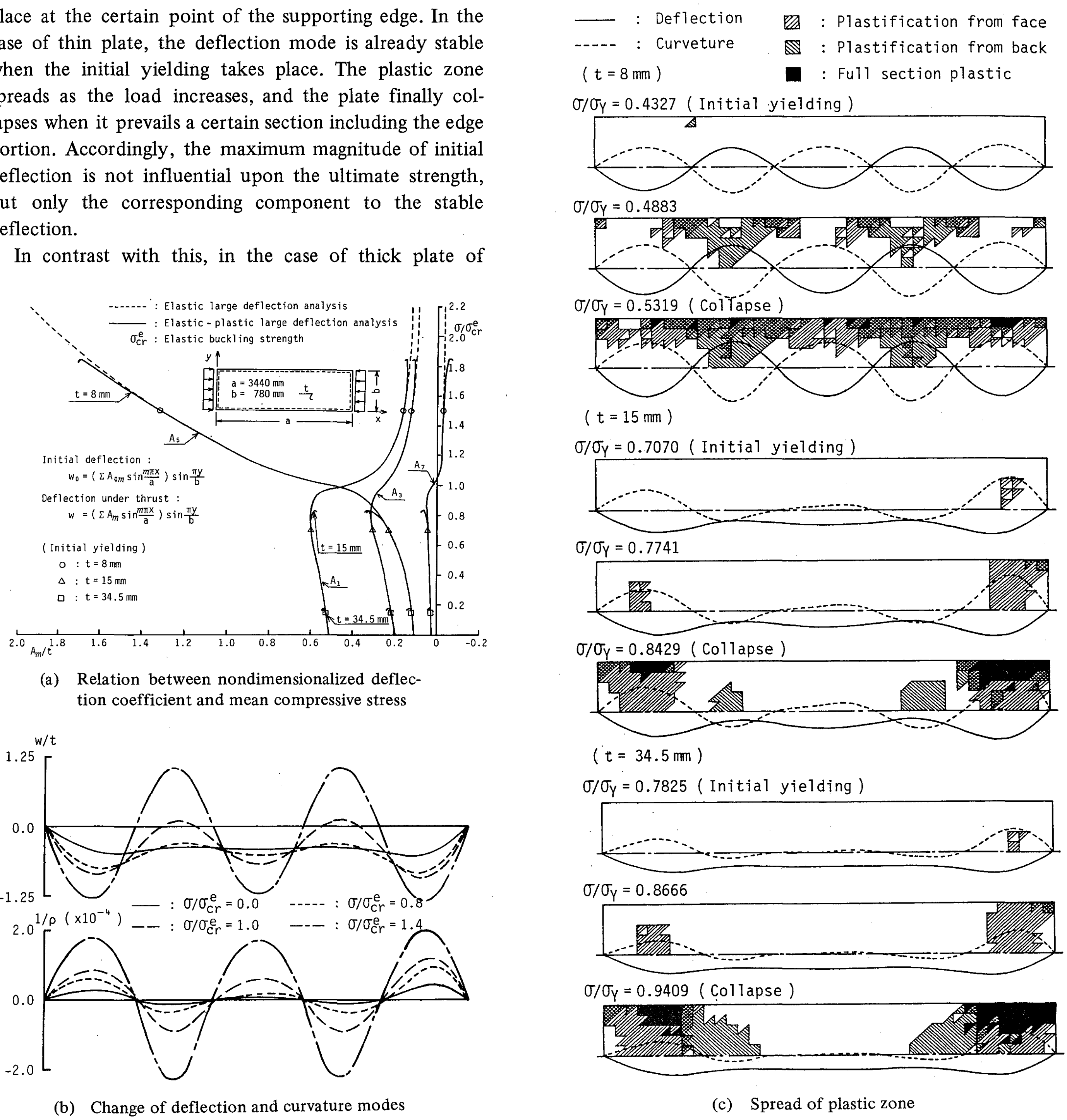
First, the elastic large deflection analysis is performed on the *No. 6* panel of a pure car carrier, which is one of those having a common mode of initial deflection as shown in Table 1. The relations between the non-dimensionalized compressive mean stress by the elastic buckling stress, $\sigma/\sigma_{cr}{}^e$, and the non-dimensionalized coefficients of deflection components by the thickness, A_m/t, are represented by dashed lines in **Fig. 8** (a). In this figure, it should be noted that if the mode of initial deflection, the ratio of the maximum magnitude of initial deflection to the plate thickness and the aspect ratio of the plate are specified, the elastic large deflection behavior of plates is described uniquely in these non-dimensionalized coordinates regardless of the size.

On the other hand, Fig. 8 (b) shows changes of the modes of deflection and curvature along the center line, $y = b/2$, of the plate. The aspect ratio of this plate is *4.41*, and the buckling mode is four half-waves in the loading direction. All component modes of the initial deflection increase until the load approaches the buckling one. However, for the mode and magnitude of initial deflection used in the analysis, the deflection mode of five half-waves becomes stable above the buckling load. Thus, the stable deflection mode of a initially deflected plate above the buckling load does not necessarily coincide with the buckling mode depending on the mode and the magnitude of initial deflection. As to such behavior of a plate with initial deflection, some investigations have been carried out[39),40)] and it is known that the stable deflection mode is sometimes the higher one than the buckling mode by one or two degrees[28)].

Next, the elastic-plastic large deflection analysis is performed on the same plate mentioned above, taking the yield stress, σ_Y, as *28* kgf/mm^2. The breadth of the plate is *780 mm* and the thickness is chosen as *8 mm*, *15 mm* and *34.5 mm*. The relation between the non-dimensionalized mean compressive stress and coefficients of deflection components are illustrated by solid lines in Fig. 8 (a), and the spread of plastic zone together with the modes of deflection and curvature in Fig. 8 (c).

In the case of thin plate of *8 mm* thickness, compressive stresses do not increase at the central portion of the plate where deflection becomes large above the buckling load. This is due to the effect of tensile membrane stresses caused by large deflection. Consequently, the compressive load is supported mainly in the portions near the supporting edges, $y = 0$ and $y = b$, where deflection is small. For further load increment, the initial plastification takes place at the certain point of the supporting edge. In the case of thin plate, the deflection mode is already stable when the initial yielding takes place. The plastic zone spreads as the load increases, and the plate finally collapses when it prevails a certain section including the edge portion. Accordingly, the maximum magnitude of initial deflection is not influential upon the ultimate strength, but only the corresponding component to the stable deflection.

In contrast with this, in the case of thick plate of



(a) Relation between nondimensionalized deflection coefficient and mean compressive stress

(b) Change of deflection and curvature modes

(c) Spread of plastic zone

Fig. 8 Deflection and initial yielding of rectangular plate with initial deflection under thrust

34.5 mm thickness, the initial plastification takes place at a certain point of a center line, $y = b/2$. At this load, as the deflection is small, its mode is almost the same as that of initial deflection, and the effect of large deflection does not appear yet. Therefore, the in-plane stress is almost the same all over the plate, though bending stress is different at different points according to the curvature. Consequently, the initial yielding takes place at the point where the curvature takes its maximum value. This point is located along the center line of the plate and just near the point where the curvature of the initial deflection takes the maximum value. For further load increase, the plastic zone spreads from the middle of the width, and the compressive load is not supported in this portion. Consequently, the additional compressive load should be carried in the portions near the supporting edges, and the plate finally collapses when these edge portions become fully plastic. In the case of thick plate, the modes of deflection and the location of the maximum curvature at collapse are almost the same with those of initial deflection. Accordingly, the maximum magnitude of initial deflection is not influential upon the ultimate strength, but the maximum curvature of the initial deflection.

In the case of a plate with medium thickness of *15 mm*, the initial plastification takes place at a certain portion on the center line during the course of the mode change from the initial one to the stable one. Consequently, it sometimes appears that the deflection increases locally, though the fundamental behavior is almost the same as in the case of thick plates.

A series of elastic-plastic large deflection analysis is also performed on the panels represented in Table 1 using the measured initial deflection. From the results of analysis, it is confirmed that the above mentioned behaviors of rectangular plates with multi-modal initial deflection are quite general. Furthermore, a similar series of analyses is carried out on some panels of a bulk carrier, which are accompanied by the initial deflection of the measured mode but different magnitude. The maximum magnitudes of initial deflection are taken as *1/2* and *1/1* times the plate thickness. These calculated ultimate strengths are used to check accuracy of prediction methods which will be proposed in the next section.

5.2 Prediction methods of compressive ultimate strength

5.2.1 Equivalent rectangular plate

It is generally very time-consuming and expensive to perform elastic-plastic large deflection analysis of a long rectangular plate with multi-modal initial deflection. In this section, simple methods will be proposed to predict the compressive ultimate strength. A certain size of rectangular plate with uni-modal initial deflection is considered, which exhibits the equivalent behavior to that of the collapsing portion of the rectangular plate with multi-modal initial deflection.

Denoting the length, width, thickness and the maximum magnitude of initial deflection of this equivalent rectangular plate by a_0, b_0, t_0 and A_0, respectively, the initial deflection is expressed as follows.

$$w_0 = A_0 \sin\frac{\pi x}{a_0} \sin\frac{\pi y}{b_0} \tag{20}$$

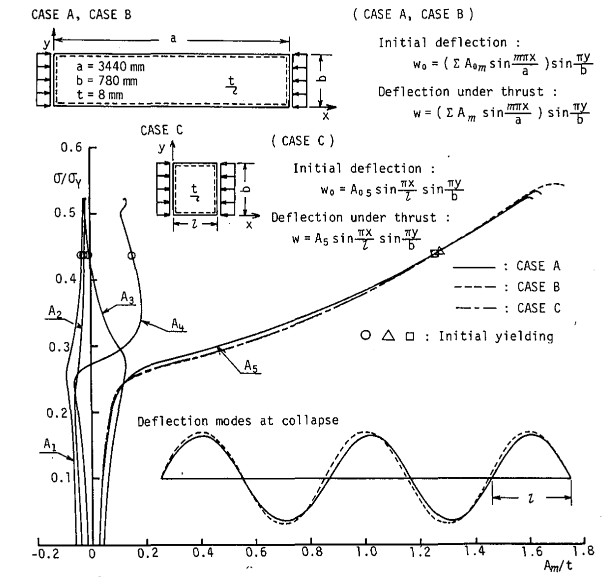
The width, b_0, and the thickness t_0, will be taken equal to the width, b, and the thickness, t, of the original rectangular plate, respectively.

Here, attention is focused on how to determine the length, a_0 and the maximum magnitude of initial deflection, A_0, of the equivalent rectangular plates. In the following, two different methods will be proposed to determine a_0 and A_0. They are, Deflection method for thin plate and Curvature method for thick plate. Once a_0 and A_0 are determined, the ultimate strength of the equivalent rectangular plate will be calculated much easier than that of the original long rectangular plate with multi-modal initial deflection.

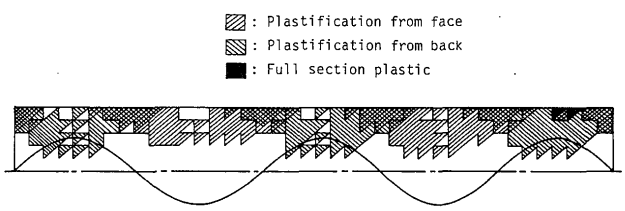
5.2.2 Deflection method (for thin plate)

As described in 5.1, only a certain particular deflection mode becomes predominant above the buckling load, depending on the aspect ratio of the plate, the mode and magnitude of initial deflection. So, it is supposed that the plate may show almost the same behavior as the original if only a stable component of deflection is assumed as an initial deflection neglecting the other components. Here, the calculation is performed on *No. 11* panel of a pure car carrier assuming two types of initial deflection which are Case *A* and Case *B* and the result is represented in **Fig. 9**. Figure 9 (a) shows the relations between the mean compressive stress and the coefficients of deflection components, in which the solid line represents the results of

	A_{01}	A_{02}	A_{03}	A_{04}	A_{05}	A_{06}	A_{07}	A_{08}	A_{09}	A_{010}	A_{011}
CASE A	-0.497	-0.338	0.339	-0.159	0.200	-0.252	0.154	-0.032	-0.031	-0.120	0.017
CASE B	0.0	0.0	0.0	0.0	0.200	0.0	0.0	0.0	0.0	0.0	0.0



(a) Relation between deflection coefficients and mean compressive stress



(b) Spread of plastic zone at collapse (CASE *A*)

Fig. 9 Behavior of thin long plate with initial deflection under thrust (P.C.C. No. *11*) actual panel (Case A) and the dashed line those of the plate with uni-modal initial deflection of the fifth half-wave (Case B).

It may be mentioned that the relation between the mean compressive stress and the coefficient of the fifth deflection component A_5 is almost the same in both cases, but there exists some difference in the ultimate strength. This may be attributed to the influences of other deflection components, which lead to a little difference in the deflection modes of Case A and Case B at collapse. Consequently, the length of one half-wave of the deflection mode is different between these two cases. In Case A, the collapse of the plate takes place at a certain section near the right hand side of the loading edge, where the length of one half-wave is the shortest. In contact with this in Case B, the collapse of the plate takes place at five sections simultaneously which are the centers of each one half-wave.

Here, another equivalent rectangular plate is introduced taking the length, a_0, equal to the length of one half-wave, l, at collapse, from the right hand side edge and the maximum magnitude of initial deflection, A_0, equal to the coefficient of the fifth deflection component, A_{05}, of Case A, respectively. This plate is denoted by Case C, and the results of analysis is plotted by chain line in Fig. 9 (a). The calculated results of Case C have good correlations with those of Case A including the ultimate strength. Thus, for thin plate, the length of the equivalent rectangular plate should be taken as the length of one half-wave of the deflection mode where the collapse is observed. However, in order to know such length of one half-wave, it is required to perform at least an elastic large deflection analysis. On the other hand, the stable deflection mode may be the same as the buckling mode or one or two higher mode than the buckling mode as described in 5.1. So, in this paper, it is proposed to predict the compressive ultimate sgrength in the following manner.

(1) Take the three different lengths of an equivalent rectangular plate, which are the lengths of one half-wave of the buckling mode, and one and two higher deflection modes than the buckling one.
(2) Take the coefficient of the deflection mode corresponding to (1) among the initial deflection components as the maximum magnitude of initial deflection of an equivalent rectangular plate.
(3) Calculate the compressive ultimate strength for these three equivalent rectangular plates performing the elastic-plastic large deflection analysis.
(4) Take the lowest compressive ultimate strength as that of the rectangular plate considered.

In this method, neither elastic nor elastic-plastic large deflection analysis on whole plate is necessary, but the ultimate strength of a shove rectangular plate with uni-modal initial deflection should be calculated in the previous chapter.

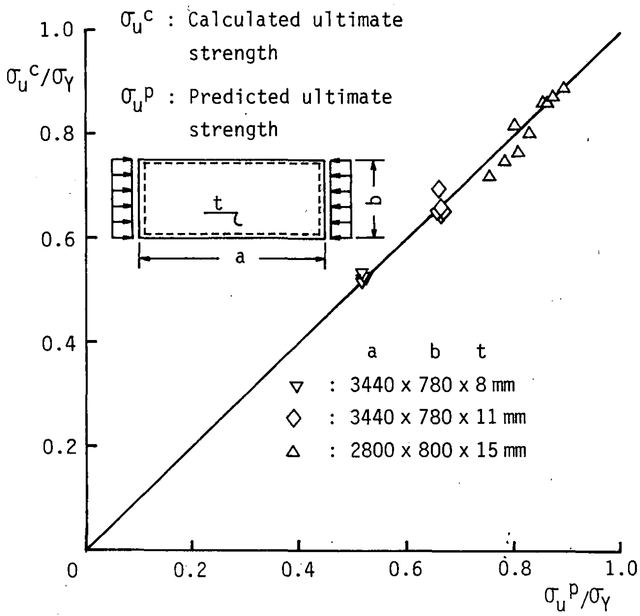


Fig. 10 Comparison of predicted and calculated ultimate strength of thin plate (Deflection method)

Applying this method, the compressive ultimate strengths of the twelve panels of the pure car carrier and the three panels, *Nos.13, 14* and *15* of the bulk carrier are calculated. The results are plotted in **Fig. 10** comparing with the exactly calculated ultimate strengths. It may be said that the predicted values show good correlations with those exactly calculated. This method is developed paying attention to the stable deflection mode above the buckling load, and will be called as "Deflection method" hereafter.

5.2.3 Curvature method (for thick plate)

In the case of thick plate, the initial plastification takes place just near a certain point where the curvature of the initial deflection takes the absolute maximum values. Then, the plastic zone spreads with the further load increase, and the plate finally collapses with almost the same deflection mode as initial deflection. Considering such behavior, the length, a_0, and the maximum magnitude of initial deflection, A_0, will be determined paying attention to the maximum magnitude of the curvature of initial deflection. Therefore, this method will be called as "Curvature method". In this paper, two methods are proposed.

(1) Curvature method-I

When initial deflection is expressed by Eq.(18), the curvature in the x-direction along the center line becomes as follows.

$$1/\rho_0 = -\frac{\pi^2}{a^2}\ \Sigma A_0 m^2 \sin\frac{m\pi x}{a} \tag{21}$$

If the absolute maximum value of the curvature is obtained at $x = x_0$, it is expressed in the following form.

$$(1/\rho_0)_{max} = -\Sigma\ (1/\rho_{0m})_{max} \tag{22}$$

where

$$(1/\rho_{0m})_{max} = \frac{\pi^2 m^2}{a^2}\ A_{0m} \sin\frac{m\pi x_0}{a} \tag{23}$$

Then, such deflection component is necessary as indicates the maximum absolute value of the curvature and takes the same sign as it. If this component is the k-th, the length, a_0, of the equivalent rectangular plate should be a/k. The maximum magnitude of initial deflection, A_0, is determined so that the maximum curvature of the initial deflection of the equivalent rectangular plate is equal to that of the plate considered, and it should be $(a/k\pi)^2(1/\rho_0)_{max}$.

(2) Curvature method-II

There exist two consecutive points where the curvature is zero and the point of the maximum curvature is located between these. In this method, the distance between these two points is taken as the length of an equivalent rectangular plate. These two points are represented by G and F in **Fig. 11**. The maximum magnitude of initial deflection A_0 is taken as $(l/\pi)^2(1/\rho_0)_{max}$ so that the maximum magnitude of initial curvature is the same as that of the original plate considered.

To confirm the validity of Curvature methods I and II,

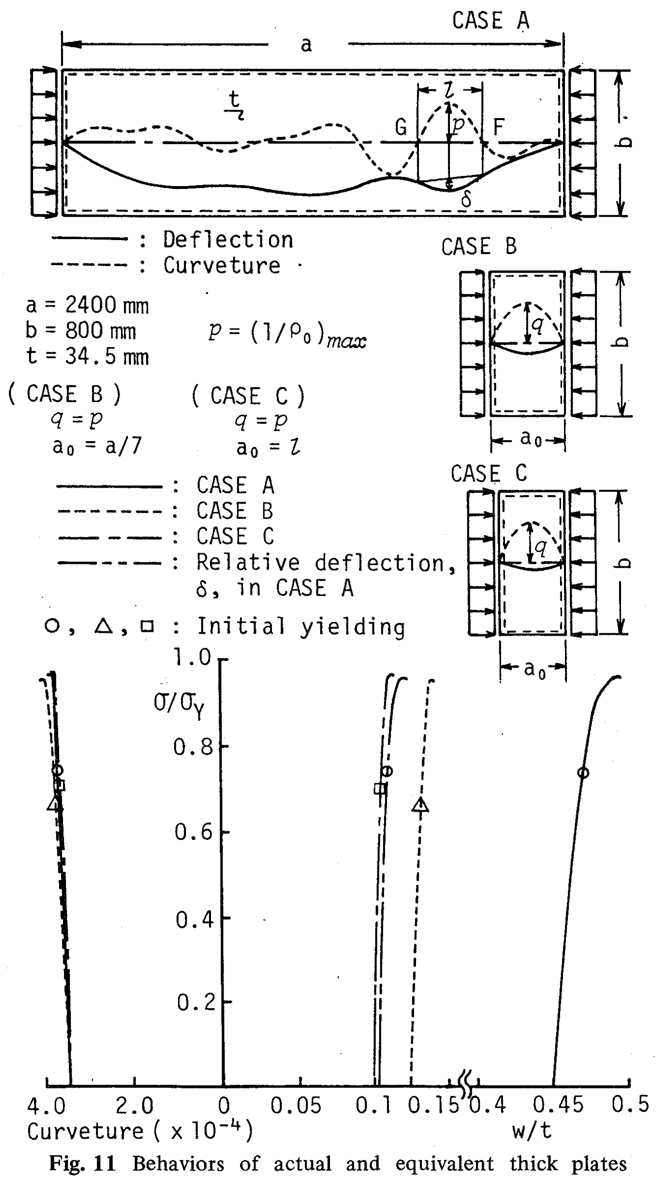
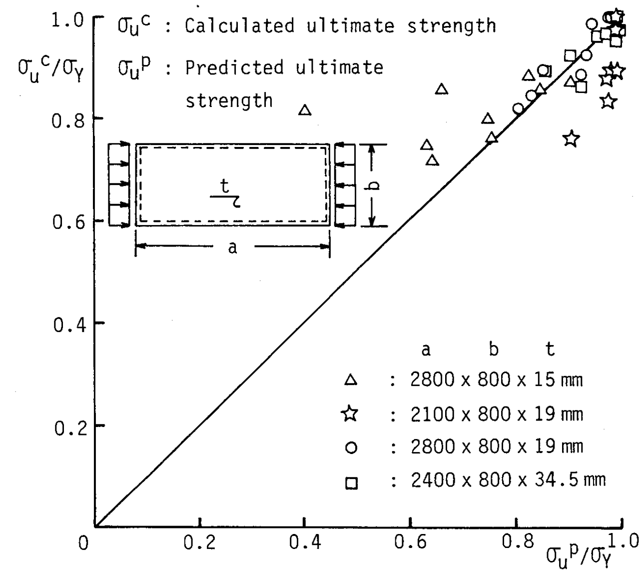
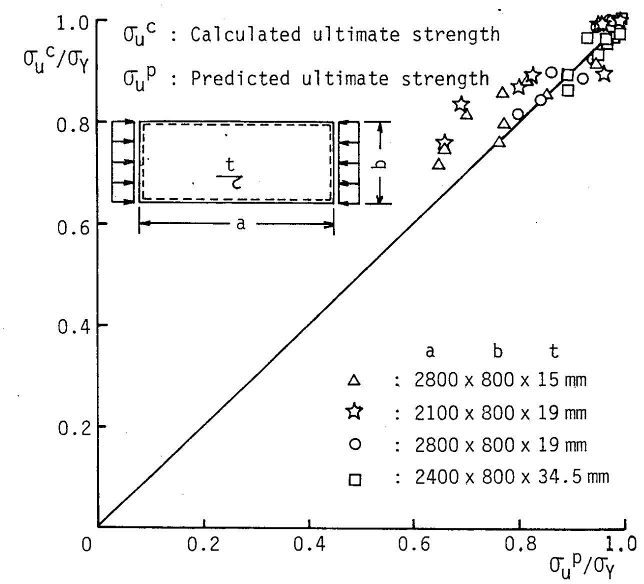


Fig. 11 Behaviors of actual and equivalent thick plates with initial deflection under thrust (B.C. No. *1*)



(a) Curvature method-I



(b) Curvature method-II

Fig. 12 Comparison between predicted and calculated ultimate strengths of thick plate several analyses are performed on *No.1* panel of the bulk carrier assuming the maximum magnitude of initial deflection equal to a half of the plate thickness. In Fig. 11 Case *A* corresponds to the actual panel, and Case *B* and Case *C* to the equivalent rectangular plates by Curvature method-I and Curvature method-II, respectively. For this panel, the most predominant curvature component is the 7-th, and the length of the equivalent rectangular plate is taken as $a/7$ in Curvature method-I.

It should be noted that the relation between the mean compressive stress and the curvature is almost the same for all cases. Contrary to this, the relation between the mean compressive stress and the deflection is somewhat different with each other. However, the curvature exerts more predominant effect on plastification than the deflection, and then the ultimate strength is almost the same in all cases. Here, in Fig. 11, the chain line with two dots represents the relation between the mean compressive stress and the relative deflection, δ, of the actual plate, which is measured with respect to the line between points G and F. It is interesting that this deflection is very close to that of the equivalent rectangular plate assumed in Curvature method-II.

Applying these two methods, the compressive ultimate strengths of the panels of the bulk carrier are calculated. The results are plotted in **Figs. 12** (a) and (b) comparing with the exactly calculated ones. From these figures, it may be said that both methods predict the ultimate strength fairly well, especially Curvature method-II.

5.3 Discussions

It may be observed from Figs. 10 and 12 that both Deflection method and Curvature method predict fairly well the compressive ultimate strength of a rectangular plate with multi-modal initial deflection. This fact maintains that the background of these estimation methods is quite rational, which is based on the observation of characteristic of the behavior of the rectangular plate under consideration. Here, this background will be discussed in detail from various aspects.

As to Deflection method, the maximum magnitude of initial deflection of the equivalent rectangular plate is taken as the coefficient of the corresponding component of initial deflection, of which value is mostly, much less than *10%* of the plate thickness as indicated in Table 1. In the case of such thin plate, the ultimate strength is not affected so much by small variation in such magnitudes of initial deflection[14),15)]. From these facts, the predicted ultimate strength seems to have a good correlation with the calculated one.

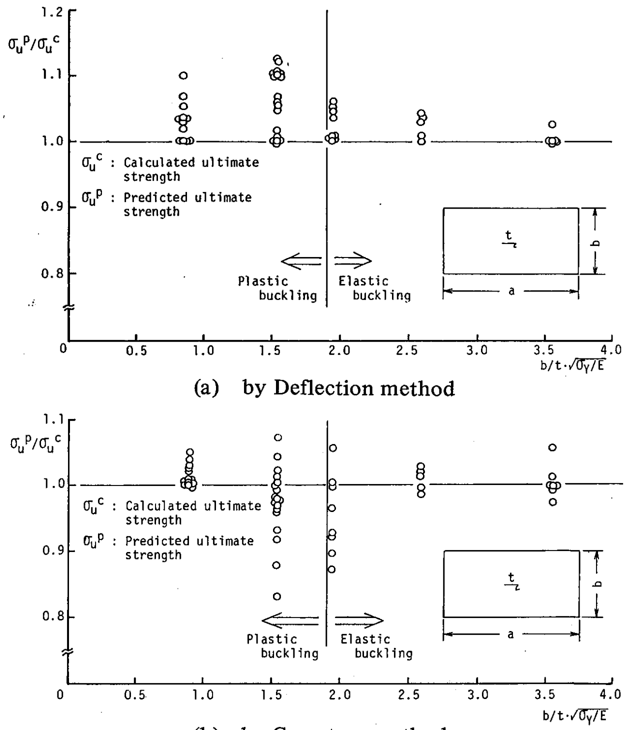
For thick plate, in the case of Curvature method-I, the magnitude and location of the maximum curvature of the initial deflection is obtained and such component as indicates the maximum contribution to the maximum curvature is determined. Here, the coefficients, A_{0m}, of initial deflection becomes smaller with the increase of m, though m^2 increases with the increase of m. Consequently, the component, which composes most of the maximum curvature of initial deflection may change according to the number of the adopted components in an expression of the initial deflection, since the curvature is expressed by the product of A_{0m} and $(m\pi/a)^2$. As to this problem, further investigation is necessary. The most predominant components for the maximum curvature in the case of the panel of a bulk carrier are *3*-rd, 7-th, *8*-th, *9*-th and *11*-th components depending on the individual shape of initial deflection when the components of initial deflection are taken up the the *11*-th.

Contrary to this, the length of an equivalent rectangular plate is determined from the visual viewpoint in the case of the Curvature method-II. However, this method predicts fairly well the ultimate strength of a rectangular plate with multi-modal initial deflection in most cases. This may be due to the following reasons. (1) The change of the deflection mode from the initial state to collapse is very little. (2) The collapse of the portion which is employed as the length of the equivalent rectangular plate is decisive to the collapse of the whole plate.

From Figs. 12 (a) and (b), it may be said that Curvature method-II predicts the ultimate strength better than Curvature method-I, and the former method will be applied hereafter to predict the ultimate strength of a thick plate.

For plate of medium thickness, the mode of the initial deflection may change as the deflection increases. In this process, progress of plastification may be accelerated or decelerated by interaction between the component modes. Therefore, the prediction of the ultimate strength by Curvature method may be lower or higher than the actual one, respectively. In contrast with this, in Deflection method, local plastification is not accounted until the stable deflection mode is attained. Then the prediction should be higher than the actual one.

Here, the application limit of Deflection method and Curvature method will be discussed. Applying both methods to various plates in Table 1, the ratios of the predicted ultimate strength to the calculated one are plotted in **Figs. 13** (a) and (b) against the non-dimensional parameter, $b/t\sqrt{\sigma_Y/E}$, of the plate. It may be said that Deflection method is better than Curvature method for thin plates, while Curvature method is better than Deflection method for thick plates. However, for the plates of medium thickness, both methods are not very good, but may be applicable and their average should be better. So,



(a) by Deflection method

(b) by Curvature method

Fig. 13 Ratio of predicted ultimate strength to calculated one

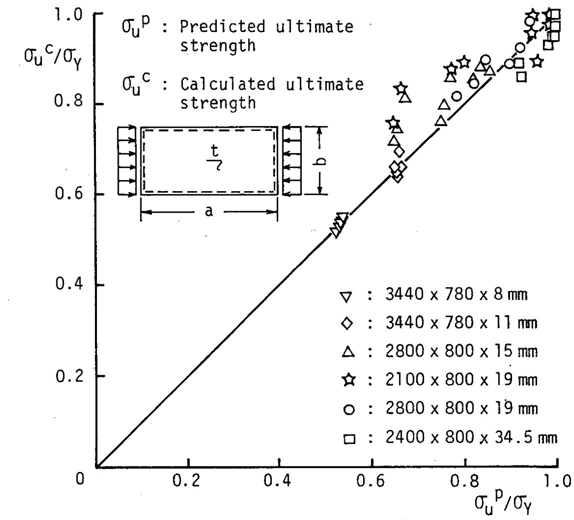


Fig. 14 Comparison between calculated ultimate strength and predicted one by simple formulae

considering the physical background behind both methods, the limit of applications of both methods may be taken as $b/t\sqrt{\sigma_Y/E}=1.9$, of which dimension corresponds to the transition from elastic to plastic buckling of a square plate. In the range of $b/t\sqrt{\sigma_Y/E}\geqslant 1.9$, Deflection method should be applied, while Curvature method should be applied for $b/t\sqrt{\sigma_Y/E}\leqslant 1.9$.

In the above, it is required to perform the elastic-plastic large deflection analysis on an equivalent rectangular plate. However, if such analysis can not be performed, Eqs.(5) to (13) may be used. For this, the plate parameter, ξ, is determined from Eq.(14), if the plate dimensions are given. In the case of thin plate, the magnitude of initial deflection, w_0, in Eq.(15) is taken as the maximum value among the coefficient of the corresponding component in the initial deflection to the buckling mode, or one or two higher mode than buckling. Contrary to this, in the case of thick plate, w_0 is determined according to Curvature method-II. Then, η in Eq.(15) is determined. With these ξ and η, the ultimate strength may be calculated and the results may be conservative for its nature.

For various panels represented in Table 1 disregarding the welding residual stresses, the compressive ultimate strength is calculated using Eqs.(5), (6) and (7). These values are plotted in **Fig. 14** comparing with the exactly calculated ultimate strengths. It may be observed that all the predicted ultimate strengths show good correlation with the calculated ones.

In this chapter, the effects of welding residual stresses are not taken into account. However, the prediction methods of the compressive ultimate strength proposed here may be applicable also to the case where welding residual stresses exist.

6. Conclusions

In this paper, the compressive ultimate strength of a rectangular plate with initial imperfections due to welding is investigated, performing the elastic-plastic large deflection analysis by the finite element method and the following conclusions are derived.

(1) The characteristic of elastic-plastic large deflection behavior of the plate may be summarized as follows. All component modes of the initial deflection increase untill the load approaches the buckling one, irrespective of the size of plate. In case of thick plate, below the buckling load, the portion where the largest curvature of the initial deflection is located is decisive to progress of plastification which leads to collapse. Then, the maximum curvature of the initial deflection plays an important role to the ultimate strength rather than the maximum deflection. When the plate is thin, the deflection increases further before plastification takes place. Above the buckling load, only one component mode of the initial deflection is amplified and becomes stable as the deflection increases. This mode is not necessarily the one at buckling, which is usually one or two higher mode than buckling, and influential upon ultimate strength. Accordingly, the maximum magnitude of initial deflection is not influential upon the ultimate strength, but only the corresponding component to

the stable deflection. This component is usually pretty smaller than the maximum initial deflection.

(2) The aspect ratio of the one half-wave of a deflection mode which gives the minimum ultimate strength does not coincide with that of the minimum buckling strength and such aspect ratio is usually smaller than *1.0*. The ultimate strength of a rectangular plate shows a saw-tooth appearance with respect to the aspect ratio of the plate when the deflection of the buckling mode is kept until the plate collapses. The simple formulae are proposed to calculate the conservative ultimate strength of a rectangular plate with initial deflection and welding residual stresses.

(3) The in-plane rigidity of the plate decreases as the load increases, since the compressive load is not supported at the central portion of the plate due to the effect of large deflection in the case of thin plate, while it is due to the effect of plastification by bending in the case of thick plate.

(4) The mode of initial deflection is measured on *33* panels of the deck plates of two ships as indicated in Table 1, and the coefficients of the component of initial deflection expressed by sine series are calculated taking the components up to *11*-th term.

(5) Two methods are proposed to predict the compressive ultimate strength of a rectangular plate with multi-modal initial deflection. These methods are Deflection method for thin plate and Curvature method for thick plate. The usefulness of these methods is confirmed comparing those predicted ultimate strengths with the exactly calculated ones.

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