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Osaka J. Math.
6 (1969), 321-327

A NOTE ON GALOIS COVERING*)

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(Received December 19, 1968)
(Revised April 25, 1969)

In [9], A. Grothendieck introduced the notion of "revetement principal". The purpose of this paper is to give characterizations of the notion and a generalization of the fundamental theorem of Galois theory in this case. In particular we shall know that the notion is a generalization of Galois extension by Chase, Harrison and Rosenberg [4].

In our first section, a notion of Galois covering will be introduced and we shall show that it is related to Galois extension. Moreover we shall know that the notion of Galois covering means "revetement principal". In our second section, a generalization of the fundamental theorem of Galois theory is developed in the case of preschemes by using the notion of the above covering.

We refer to Eléménts de Géométrie Algébrique by Grothendieck by signals such as E.G.A. I (8.7.1) and in this paper we shall use the notions in that book without explanation.

In this paper, all rings are assumed to be commutative and to have an identity.

1. Galois coverings

Let $\Phi=(\varphi, \alpha)$; $X\rightarrow Y$ be a morphism of preschemes. We shall say that the prescheme X is a *quasi-unramified covering* (resp. a *quasi-etale covering*) of Y if

1) Φ is a finite morphism: $X\rightarrow Y$ (resp. Φ is finite and flat)

2) $\mathcal{O}_x/\mathfrak{m}_y\mathcal{O}_x$ is a finite separable extension of $k(y)=\mathcal{O}_y/\mathfrak{m}_y$ for any $x\in X$, $y=\varphi(x)$ where $\mathcal{O}_x$ and $\mathcal{O}_y$ are the fibres of the structural sheeves $\mathcal{O}_X$ and $\mathcal{O}_Y$ of X and Y at the points x and y, respectively and $\mathfrak{m}_y$ is the maximal ideal of $\mathcal{O}_y$.

Now we easily obtain the following lemma by [11, theorem 1].

Lemma 1.1. *Let φ: $X\rightarrow Y$ be a surjective morphism of preschemes. Then the Y-prescheme X is a quasi-unramified covering (resp. a quasi-etale covering)*

1. This paper was written while the auther held a visiting position at Universidad de Buenos Aires. He takes the opportunity to express his thanks to that good situation.

*) Dedicated to Professor K. Asano for the celebration of his 60th birthday.

if and only if, for any affine open covering $\{V_\gamma\}_{\gamma\in I}$ *of* Y, *the following conditions hold; for any* $\gamma\in I$,

1) $U_\gamma=\varphi^{-1}(V_\gamma)$ *is an affine open set of* X
2) *the ring* $\Gamma(U_\gamma, \mathcal{O}_X)$ *is a separable* $\Gamma(V_\gamma, \mathcal{O}_Y)$*-algebra of finite type*
3) *the ring* $\Gamma(U_\gamma, \mathcal{O}_X)$ *is integral over* $\Gamma(V_\gamma, \mathcal{O}_Y)$

(*resp.* 4) $\Gamma(U_\gamma, \mathcal{O}_X)$ *is a finitely generated flat* $\Gamma(V_\gamma, \mathcal{O}_Y)$*-module*).

Let Y be a prescheme, X a Y-prescheme and $\mathfrak{G}$ a finite group of Y-automorphisms of X. We shall denote the sum of n-copies of Y by $\mathfrak{G}_Y$ for $n=(\mathfrak{G}: 1)$. For $\sigma\in\mathfrak{G}$, there exists uniquely a morphism $(1, \sigma)_Y: X\to X\underset{Y}{\times}X$ such that $pr_1\cdot(1, \sigma)_Y=1$ and $pr_2\cdot(1, \sigma)_Y=\sigma$ where 1 is the identity morphism of X and pr_i $(i=1, 2)$ are the projections: $X\underset{Y}{\times}X\to X$. Then the morphisms $(1, \sigma)_Y$ define canonically a morphism of the sum of n-copies of X into $X\underset{Y}{\times}X$ which is denoted by $(1, \mathfrak{G})_Y$.

Proposition 1.2. *Let* Y *be a prescheme and* X *a* Y*-prescheme such that the structural morphism* $\varphi: X\to Y$ *is surjective and finite. Let* $\mathfrak{G}$ *be a finite group of* Y*-automorphisms of* X. *If* X *is formally principal homogeneous on* G_Y, *i.e.* $(1. \mathfrak{G})_Y$ *is an isomorphism, then* X *is a quasi-unramified covering of* Y.

Proof. Let $\{V_\gamma\}_{\gamma\in I}$ be any affine open covering of Y. Then it is sufficient to show that a ring $\Gamma(U_\gamma, \mathcal{O}_X)$ is a separable $\Gamma(V_\gamma, \mathcal{O}_Y)$-algebra for every $\gamma\in I$ where $U_\gamma=\varphi^{-1}(V_\gamma)$. Let

$$\delta: U_{\gamma,\sigma_1}\pi U_{\gamma,\sigma_2}\pi \cdots \pi U_{\gamma,\sigma_n}\to U_\gamma\underset{V_\gamma}{\times}U_\gamma$$

be a morphism induced by $(1, \mathfrak{G})_Y$ where $U_{\gamma,\sigma_i}=U_\gamma$ for $\sigma_i\in\mathfrak{G}$. Then δ is an isomorphism. Let $A_\gamma=\Gamma(U_\gamma, \mathcal{O}_X)$ and $B_\gamma=\Gamma(V_\gamma, \mathcal{O}_Y)$. The homomorphism

$$\tilde{\delta}: A_\gamma\underset{B_\gamma}{\otimes}A_\gamma\to A_{\gamma,\sigma_1}\times A_{\gamma,\sigma_2}\times\cdots\times A_{\gamma,\sigma_n}$$

induced by δ is an isomorphism where $A_{\gamma,\sigma_1}\times A_{\gamma,\sigma_2}\times\cdots\times A_{\gamma,\sigma_n}$ means the direct sun of n-copies of A_γ. If σ_1 is the identity of $\mathfrak{G}$, A_{γ,σ_1} is a direct summand of $A_\gamma\underset{B_\gamma}{\otimes}A_\gamma$ as $A_\gamma\underset{B_\gamma}{\otimes}A_\gamma$-module, since $\tilde{\delta}(a\otimes b)=a\cdot\sigma_1(b)\times\cdots\times a\cdot\sigma_n(b)$ for $a\otimes b\in A_\gamma\otimes A_\gamma$. Hence A_γ is a separable B_γ-algebra.
This completes the proof.

Let A be a separable B-algebra finitely generated as B-module for a commutative ring B. Let Ω be an algebraic closure of $B/\mathfrak{m}$ for any maximal ideal $\mathfrak{m}$ of B. If p is a homomorphism; $B\to\Omega$ such that $\mathrm{Ker}\, p=\mathfrak{m}$ (we shall say that such homomorphism p is a *geometric point* of B), then let $V_p^B(A)$ be the set of all homomorphisms $\theta: A\to\Omega$ such that the diagram

$$\begin{tikzcd} A \arrow[dr, "\theta"] & \\ B \arrow[u] \arrow[r, "p"'] & \Omega \end{tikzcd}$$

is commutative where the vertical mapping is the canonical homomorphism. If $\mathfrak{G}$ is a group of B-automorphisms of A, then $V_p^B(A)$ forms a left $\mathfrak{G}$-set defining the operation of $\sigma\in\mathfrak{G}$ by $\sigma\cdot\theta=\theta\sigma^{-1}$ for $\theta\in V_p^B(A)$.

The following proposition is a generalization of theorem of Chase [3].

Proposition 1.3. *Let A be a separable B-algebra which is a finitely generated faithfully projective B-module and let $\mathfrak{G}$ be a finite group of B-automorphisms of A. If there is a bijective correspondence between $V_p^B(A)$ and $\mathfrak{G}$ as left $\mathfrak{G}$-sets for any geometric point p of B, then A is a Galois extension of B with a Galois group $\mathfrak{G}$.*

Proof. Let σ be any element $(\neq 1)$ of $\mathfrak{G}$ and $\mathfrak{M}$ any maximal ideal of A. Then we shall show that there is an element $a=a(\mathfrak{M},\sigma)$ in A such that $a-\sigma(a)\notin\mathfrak{M}$. If $\sigma(\mathfrak{M})\neq\mathfrak{M}$, then it is trivial. Assume $\sigma(\mathfrak{M})=\mathfrak{M}$. Let θ be a geometric point of A such that $\mathfrak{M}=Ker\,\theta$. Since $Ker\,(\sigma\theta)=\mathfrak{M}$, there exists such element $a=a(\mathfrak{M},\sigma)$ in A. Then A is a Galois extension of $A^{\mathfrak{G}}$ with a Galois group $\mathfrak{G}$ and so $A^{\mathfrak{G}}$ is finitely generated as a B-module [4, theorem 1.3]. It is sufficient for proving the proposition to show $B=A^{\mathfrak{G}}$. For any maximal ideal $\mathfrak{m}$ of B, a residue field $B/\mathfrak{m}$ is the fixed ring of $A/\mathfrak{m}A$ under the group of all $B/\mathfrak{m}$-automorphisms of $A/\mathfrak{m}A$. Since any $B/\mathfrak{m}$-automorphism of $A/\mathfrak{m}A$ is induced by an element of $\mathfrak{G}$, we have $A^{\mathfrak{G}}/\mathfrak{m}A^{\mathfrak{G}}=B/\mathfrak{m}$. Then we obtain our result.

Let α be a geometric point of a prescheme Y. Then we can regard α as a pair of a point y of Y and a local homomorphism $\theta_y\colon \mathcal{O}_y\to\Omega$ where Ω is an algebraic closure of $k(y)=\mathcal{O}_y/\mathfrak{m}_y$. If a prescheme X is a quasi-unramified covering of Y, then the set of all geometric points of X with values in Ω over a geometric point α of Y is finite which we shall denote by $E_\alpha^Y(X)$ [c.f. 5]. We can consider $E_\alpha^Y(X)$ as the set of pairs (x,θ) where x is a point of X such that $y=\varphi(x)$ and θ is a local homomorphism: $\mathcal{O}_x\to\Omega$ such that the diagram

$$\begin{tikzcd} \mathcal{O}_x \arrow[dr, "\theta"] & \\ \mathcal{O}_y \arrow[u, "\gamma_x^\#"] \arrow[r, "\theta_y"'] & \Omega \end{tikzcd}$$

is commutative where $(\varphi,\gamma)\colon X\to Y$ is the structural morphism.

Moreover let $\mathfrak{G}$ be a group of Y-automorphisms of X. Then $E_\alpha^Y(X)$ is a left $\mathfrak{G}$-set defining the operation of elements σ of $\mathfrak{G}$ by $\sigma\cdot\beta=(\sigma(x),\theta\sigma^{-1})$ for $\beta=(x,\theta)\in E_\alpha^Y(X)$.

DEFINITION 1.4. Let Y be a prescheme, X a Y-prescheme and $\mathfrak{G}$ a finite group of Y-automorphisms of X. Suppose that the structural morphism; $X\to Y$ is surjective. Then we shall say that X is a *Galois covering* of Y *with a Galois group* $\mathfrak{G}$ if the following conditions hold;

1) X is an etale covering i.e. X is a quasi-etale covering and is finitely presented over Y.

2) There exists a bijective correspondence between $E_\alpha^Y(X)$ and $\mathfrak{G}$ as left $\mathfrak{G}$-sets for any geometric point α of Y.

Lemma 1.5. *Let $\varphi: X\to Y$ be a morphism of preschemes and $\mathfrak{G}$ a finite group of Y-automorphisms of X. We assume that the prescheme X is a quasi-unramified covering of Y. If, for any affine open set V of Y, we set $A=\Gamma(\varphi^{-1}(V), \mathcal{O}_X)$ and $B=\Gamma(V, \mathcal{O}_Y)$, then, for any geometric point p of B, there exist a geometric point α of Y and a bijective correspondence between $E_\alpha^Y(X)$ and $V_p^B(A)$ as left $\mathfrak{G}$-sets.*

Proof. Let p be any geometric point of B, y the point of Y which is associated with the maximal ideal $\mathfrak{m}=Ker\ p$ in B and Ω an algebraic closure of $k(y)=\mathcal{O}_y/\mathfrak{m}_y$. Then, for the canonical homomorphism $\theta_y: \mathcal{O}_y\to\Omega$, the pair $\alpha=(y, \theta_y)$ is a geometric point of the affine scheme V and so it is a geometric point of X. It is trivial that there is a bijective correspondence between $E_\alpha^Y(X)$ and $E_\alpha^V(U)$ for $U=\varphi^{-1}(V)$. It is sufficient for proving our lemma to show that there exists a bijection between $E_\alpha^V(U)$ and $V_p^B(A)$ as left $\mathfrak{G}$-sets. We shall define a correspondence f: $E_\alpha^V(U)\to V_p^B(A)$ by $f((x, \theta))=\theta\cdot\alpha_x$ for $(x, \theta)\in E_p^V(U)$ where α_x is the canonical homomorphism of A into $\mathcal{O}_x$ $(=A_{\mathfrak{j}_x})$. Since A is integral over B, $\mathfrak{j}_x$ is a maximal ideal of A. We have $f((x, \theta))\neq f((x', \theta'))$ if $(x, \theta)\neq(x', \theta')$ for (x, θ), $(x', \theta')\in E_\alpha^V(U)$, because the kernels of $f((x, \theta))$ and $f((x', \theta'))$ are the maximal ideals $\mathfrak{j}_x$ and $\mathfrak{j}_{x'}$, respectively. Therefore it is trivial that the bijective correspondence between $E_\alpha^V(U)$ and $V_p^B(A)$ as left $\mathfrak{G}$-sets is given by f.

Lemma 1.6. *Let B be a commutative ring, A a commutative B-algebra and $\mathfrak{G}$ a finite group of B-automorhisms of A. Then A is a Galois extension of B with a Galois group $\mathfrak{G}$ if and only if A is a faithful flat B-module and a homomorphism*

$$\delta: A\underset{B}{\otimes}A\to A_{\sigma_1}\times A_{\sigma_2}\times\cdots\times A_{\sigma_n}$$

by $\delta(a\otimes b)=a\sigma_1(b)\times\cdots\times a\sigma_n(b)$ for $a\otimes b\in A\otimes A$, is an isomorphism where $\mathfrak{G}=\{\sigma_1, \sigma_2, \cdots, \sigma_n\}$ and $A_{\sigma_i}=A$ $(i=1, 2, \cdots, n)$.

Proof. Necessity. It is trivial by [E.G.A. O (6.6.1)] that A is a faithful flat B-module. There exist elements x_i, y_i in A $(i=1, 2, \cdots, r)$ such that $\sum_{i=1}^r x_i\sigma_k(y_i)=\delta_{\sigma_1,\sigma_k}$ [4, theorem 1.3]. Hence δ is an epimorphism. On the other hand the ranks of $A \underset{B}{\otimes} A$ and $A_{\sigma_1} \times A_{\sigma_2} \times \cdots \times A_{\sigma_n}$ over B are equal [c.f. 4, lemma 4.1]. Therefore δ is an isomorphism.

Sufficiency. Let $e = \sum_{i=1}^{r} x_i \otimes y_i \in A \underset{B}{\otimes} A$ such that $\delta(e) = 1 \times 0 \times \cdots \times 0$. Then we obtain $\sum_{i=1}^{r} x_i \sigma_k(y_i) = \delta_{\sigma_1, \sigma_k}$. Hence it is sufficient to show that B is the fixed ring of A under G. Set $B_0 = A^{\mathfrak{G}}$. Then A is a Galois extension of B_0 with a Galois group $\mathfrak{G}$, so that $A \underset{B_0}{\otimes} A$ is isomorphic to $A_{\sigma_1} \times \cdots \times A_{\sigma_n}$. By the hypothesis the canonical mapping μ: $A \underset{B}{\otimes} A \to A \underset{B_0}{\otimes} A$ by $\mu(a \underset{B}{\otimes} b) = a \underset{B_0}{\otimes} b$ for $a \underset{B}{\otimes} b \in A \underset{B}{\otimes} A$ is an isomorphism. It implies that if we put $B[B_0]$ a subring of A generated by B_0 over B, $A \underset{B}{\otimes} B[B_0]$ is isomorphic to $A \underset{B}{\otimes} B$ $(\cong A)$. Since A is a faithful flat B-module, we have $B = B[B_0]$ and so $B = B_0$.

Now we may give characterizations of the notion of Galois covering.

Theorem 1.7. *Let φ: $X \to Y$ be a morphism of preschemes and $\mathfrak{G}$ a finite group of Y-automorphisms of X. Then the following statements are equivalent;*

1) *X is a Galois covering of Y with a Galois group $\mathfrak{G}$.*

2) *There exists an affine open covering $\{V_\gamma\}_{\gamma \in I}$ of Y such that the ring $\Gamma(\varphi^{-1}(V_\gamma), \mathcal{O}_X)$ is a Galois extension of the ring $\Gamma(V_\gamma, \mathcal{O}_Y)$ with a Galois group $\mathfrak{G}$ for every $\gamma \in I$ where φ: $X \to Y$ is the structural morphism.*

3) *φ is an affine and surjective morphism, X is φ-flat and X is formally principal homogeneous on $\mathfrak{G}_Y$.*

Proof. 1) $\Rightarrow$ 2). For any affine open set V of Y, set $U = \varphi^{-1}(V)$. If we put $A = \Gamma(U, \mathcal{O}_X)$ and $B = \Gamma(V, \mathcal{O}_Y)$, then, for any geometric point p of B, there exists a bijection between $V_{\mathfrak{p}}^{B}(A)$ and $\mathfrak{G}$ as left $\mathfrak{G}$-sets (1.5). Hence A is a Galois extension of B with a Galois group $\mathfrak{G}$.

2) $\Rightarrow$ 1). It is prove similarly as the proof of [3, proposition 3.3].

2) $\Leftrightarrow$ 3). It follows trivially from Lemma 1.6.

2. Galois theory

Let $\Phi = (\varphi, \theta)$: $X \to Y$ be a morphism of preschemes which satisfies the following properties;

1) φ is surjective

2) For any affine open set V of Y, the homomorphism θ_V: $\Gamma(V, \mathcal{O}_Y) \to \Gamma(\varphi^{-1}(V), \mathcal{O}_X)$ is injective.

Then a prescheme Z will be called to be an intermediate prescheme of X and Y if there exist morphisms Φ_1: $X \to Z$ and Φ_2: $Z \to Y$ such that Φ_i ($i = 1, 2$) satisfy the same properties as 1), 2) and furthermore $\Phi = \Phi_2 \cdot \Phi_1$.

Proposition 2.1. *Let Y be a prescheme and X a Galois covering of Y with* *a Galois group $\mathfrak{G}$. Then, for any subgroup $\mathfrak{H}$ of $\mathfrak{G}$, there exists the quotient prescheme*[2] *$X/\mathfrak{H}$ of X by $\mathfrak{H}$ and $X/\mathfrak{H}$ is an intermediate prescheme of X and Y. Moreover the quotient prescheme $X/\mathfrak{H}$ is an etale covering of Y and X is a Galois covering of $X/\mathfrak{H}$ with a Galois group $\mathfrak{H}$.*

Proof. It follows from [9] that there exists the quotient prescheme $X/\mathfrak{H}$. Let $\{V_\gamma\}_{\gamma\in I}$ be a covering of Y consisting of affine open sets $V_\gamma=(\mathrm{Spec}\,(B_\gamma), \tilde{B}_\gamma)$ and let φ be the structural morphism: $X\to Y$. If we put $\varphi^{-1}(V_\gamma)=(\mathrm{Spec}\,(A_\gamma), \tilde{A}_\gamma)$ for all $\gamma\in I$, then we have that A_γ is a Galois extension of B_γ with a Galois group $\mathfrak{G}$, so that A_γ is a Galois extension of $A_\gamma^{\mathfrak{H}}$ with a Galois group $\mathfrak{H}$. Moreover $A_\gamma^{\mathfrak{H}}$ is a separable B_γ-algebra of finite type and is integral over B_γ. Since the affine scheme $\{(\mathrm{Spec}\,(A_\gamma^{\mathfrak{H}}), (A_\gamma^{\mathfrak{H}})^\sim)\}_{\gamma\in I}$ give an open covering of $X/\mathfrak{H}$, we obtain our proposition.

Let Y be a prescheme. We consider the following conditions; C) *There exists an affine open covering $\{V_\gamma\}_{\gamma\in I}$ of Y such that, for any pair $(\alpha, \beta)\in I\times I$, there is a sequence; $V_\alpha=V_{\gamma_0}, V_{\gamma_1}, \cdots V_{\gamma_\lambda}=V_\beta$ satisfying $V_{\gamma_i}\cap V_{\gamma_{i+1}}\neq\phi$ for $\gamma_i\in I$.*

Theorem 2.2. *Let Y be a prescheme satisfying the conditions C) and X a Galois covering with a Galois group $\mathfrak{G}$. Let Z be an intermediate prescheme of X and Y. If Z is a quasi-unramified covering of Y, then there exists uniquely a subgroup $\mathfrak{H}$ of $\mathfrak{G}$ such that Z is the quotient prescheme $X/\mathfrak{H}$ of X by $\mathfrak{H}$.*

Proof. Let $\Phi=(\varphi, \theta)\colon X\to Y$ be the structural morphism. Then there exist morphisms $\Phi_1=(\varphi_1, \theta_1)\colon X\to Z$ and $\Phi_2=(\varphi_2, \theta_2)\colon Z\to Y$ which satisfy the conditions that Z is an intermediate prescheme of X and Y. Let $\{V_\gamma\}_{\gamma\in I}$ be an affine open covering of Y as above and set $U_\gamma=\varphi^{-1}(V_\gamma)$, $W_\gamma=\varphi_2^{-1}(V_\gamma)$ for all $\gamma\in I$. Then $\{U_\gamma\}_{\gamma\in I}$ and $\{W_\gamma\}_{\gamma\in I}$ are affine open coverings of X and Z, respectively. Moreover we have $U_\gamma=\varphi_1^{-1}(W_\gamma)$ for all $\gamma\in I$. If we set $U_\gamma=\mathrm{Spec}\,(A_\gamma)$, $W_\gamma=\mathrm{Spec}\,(B_\gamma)$ and $V_\gamma=\mathrm{Spec}\,(C_\gamma)$, we can consider that A_γ is a Galois extension of C_γ with a Galois group $\mathfrak{G}$ and B_γ is a separable C_γ-algebra. Then, by the Galois theory of commutative rings [4, theorem 2.3], there exists uniquely a subgroup $\mathfrak{H}_\gamma$ of $\mathfrak{G}$ such that $B_\gamma=A_\gamma^{\mathfrak{H}_\gamma}$ for every $\gamma\in I$. For any $y\in V_\gamma\cap V_{\gamma'}$, $\varphi_*(\mathcal{O}_X)_y$ $(=A_\gamma\underset{C_\gamma}{\otimes}C_{\gamma\,\mathfrak{j}_y})$ is a Galois extension of $\mathcal{O}_y$ $(=C_{\gamma\,\mathfrak{j}_y})$ with a Galois group $\mathfrak{G}$ and $\varphi_{2*}(\mathcal{O}_Z)_y$ $(=B_\gamma\underset{C_\gamma}{\otimes}C_{\gamma\,\mathfrak{j}_y})$ is a separable $\mathcal{O}_y$-subalgebra of $\varphi_*(\mathcal{O}_X)_y$, so that there exists a subgroup $\mathfrak{H}_y$ of $\mathfrak{G}$ such that $\varphi_{2*}(\mathcal{O}_Z)_y=(\varphi_*(\mathcal{O}_Z)_y)^{\mathfrak{H}_y}$. Then we have $\mathfrak{H}_\gamma=\mathfrak{H}_y$ and so $\mathfrak{H}_\gamma=\mathfrak{H}_{\gamma'}$ for all $\gamma, \gamma'\in I$ which is denoted by $\mathfrak{H}$. Then it is clear that Z is the quotient prescheme $X/\mathfrak{H}$ of X by $\mathfrak{H}$. The uniqueness of such group is trivial.

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2. C.f. [9] for the definition.

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