Compressive Strength of Structural Members with Soft Weld Joints†

Yukio UEDA*, Hidekazu MURAKAWA** and Hidetoshi KIMURA***

Abstract

Soft weld joints or under matching weld joints are often employed in the welding of high tensile strength steel to avoid crackings. The compressive strength of structural members, such as columns, plates and pipes, with soft weld joints is analyzed by Finite Element Method and simple mechanical models. Especially, the effects of the width and the location of the soft joint, slenderness ratio, initial deflection and the strain hardening are clarified.

KEY WORDS: (High Tensile Strength Steel) (Soft Weld Joint) (Compressive Strength) (Plastic Constraint) (Buckling) (Column) (Plate) (Pipe)

1. Introduction

With the increasing severity of design conditions, high tensile strength steel has been introduced in various welded structures. In case of conventional high tensile strength steel, soft weld metal which has lower yield stress compared to the base metal is used to avoid crackings. This type of joint is known as soft weld joint[1] and it can be also found in TMCP (Thermo-Mechanical-Control-Process) steel with low carbon equivalent. The mechanical properties of weld joints, such as tensile strength[2)-7)] and fracture toughness[8] have been studied. However, its strength under compressive load[9] is not throughly understood yet because it involves buckling phenomena. To clarify the compressive strength of structural members with soft joints, the authors analyzed the behavior of beams, plates and pipes. Finite Element Methods and simple idealized models are employed and the effects of the soft joint location and geometry, slenderness ratio, initial deflection and strain hardening are clarified.

2. Strength of Soft Weld Joints

The buckling is usually observed in thin structures. The stress state in such structures can be treated as plane stress state. However, if soft weld joints exist, the elastic-plastic stress state near the joints becomes three dimensional stress state and the yield strength and ultimate strength are increased due to so called plastic constraint. The plastic constraint can be separated into in-plane and thickness directions. The former is automatically considered in ordinary Finite Element Method (F.E.M.) based on the plane stress assumption. On the other hand, the latter is neglected. Thus, special care is necessary to count the effect of plastic constraint in the thickness direction when

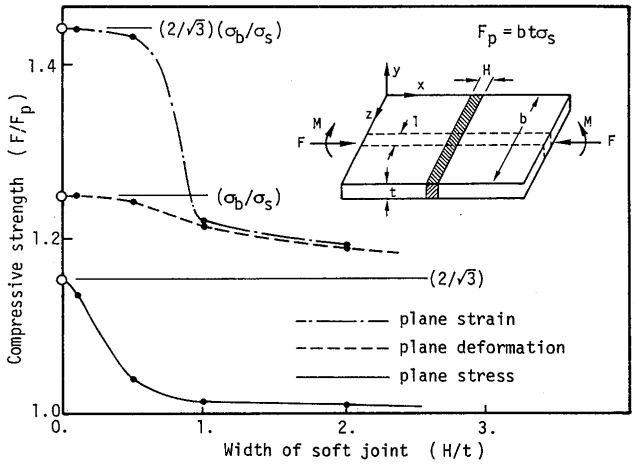

Fig. 1 Effect of soft joint width on its compressive strength.

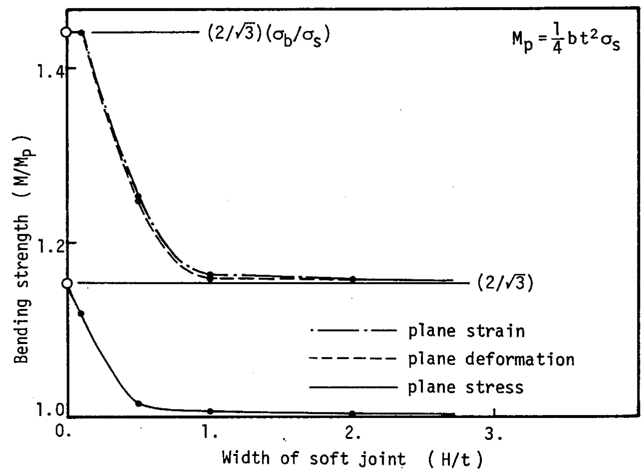

Fig. 2 Effect of soft joint width on its bending strength.

the strength of structures with soft joints is analyzed.

The illustration in **Fig. 1** shows a plate with a welding bead. The force is applied in the direction normal to the bead. The degree of the plastic constraint depends on H/t and H/b, which are relative width of the soft joint com-

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pared to the plate thickness t and width b. If the constraint in the width direction is considered, the deformed state becomes plane deformation in z direction when H/b is small and there is no external constraint in the width direction. When the plate is fixed in the width direction, the deformation state becomes plane strain. On the contrary, if H/b is large, such as thin columns, it can be considered as plane stress state. Thus, the variation of the constraint in the width direction can be represented by three typical states, namely, plane strain, plane deformation and plane stress states.

The effect of the plastic constraint in the thickness direction is examined for the forementioned three states by analyzing a strip with unit thickness as shown in Fig. 1. In this analysis, two dimensional F.E.M. based on the plastic flow theory using Mises yield criterion is employed. The base metal and the weld metal are assumed to be perfect elastic-plastic materials and their Young's moduli and yield stresses are $E = 21000$ kgf/mm^2, $\sigma_b = 80$ kgf/mm^2 and $\sigma_s = 0.8\sigma_b = 64$ kgf/mm^2. In case of the plane deformation state, the plane normal to the weld line is assumed to keep plane and free to move without rotation around x axis.

The computed strength of plate with soft joint is plotted against H/t in Figs. 1 and 2. The strength is normalized by the full plastic compressive force or bending moment of the homogeneous plate with the yield stress which is same as that of the weld metal. The effect of the constraint in the width direction can be seen from the comparison among the three typical states. The plane strain state gives the highest strength and the plane stress state gives the lowest. On the other hand, the constraint in the thickness direction increases as H/t becomes smaller. However, there are upper and lower limits in the strength and they are shown in Figs. 1 and 2 (see Appendix for detail).

Since, the plane deformation state is the closest to the real state of the plate member in most structures, its detail is examined here. When H/t is large, the plastic constraint in the thickness direction is negligible and only constraint in the width direction is effective. Thus, it can be shown from the flow theory of plasticity that the lower limits for both compressive and bending strength are $(2/\sqrt{3})$. However, the upper limits for the small value of H/t are different and they are (σ_b/σ_s) and $(2/\sqrt{3})(\sigma_b/\sigma_s)$ for compression and bending, respectively. As seen from the figures, the strength of the soft joint varies with H/t between these upper and lower limits. Therefore, if $\sigma_b/\sigma_s < 2/\sqrt{3}$, the upper and lower limits for compression coincide and the strength becomes independent on H/t. In case of bending, the strength of soft joint is always smaller than that of base metal unless $\sigma_b < \sigma_s$.

The effect of the plastic constraint in the thickness direction may be taken into account by introducing a fictitious yield stress which is proportional to the strength ratio which is given by Figs. 1 or 2. Considering that compressive and bending deformation occurs simultaneously in compressive buckling of structural members, it is a conservative assumption to use smaller one of the strength ratios determined for the compression and the bending. However, the width of the soft joint is assumed to be large enough and the plastic constraint in the thickness direction is ignored in the analyses discussed in the following chapters.

3. Buckling Strength of Columns

Though columns are simple as structural members, their buckling behavior under compressive load involves fundamental ideas to understand those of plates and shells. Thus, the compressive strength of columns with soft joint is analyzed and the effect of the following factors are discussed.

(a) slenderness ratio
(b) initial deflection
(c) location of soft joint
(d) width of soft joint
(e) strain hardening

The column considered has rectangular crosssection and its length is L as shown in **Fig. 3**. It is assumed that the soft region, the width of which is H, is located l from the end of the column and the plastic constraint in the thickness direction is neglected. The base metal and the soft weld metal are both assumed to be perfect elastic-plastic materials and their yield stresses are $\sigma_b = 80$ kgf/mm^2 and $\sigma_s = 64$ kgf/mm^2, respectively. Further, the sinusoidal form of initial deflection is assumed.

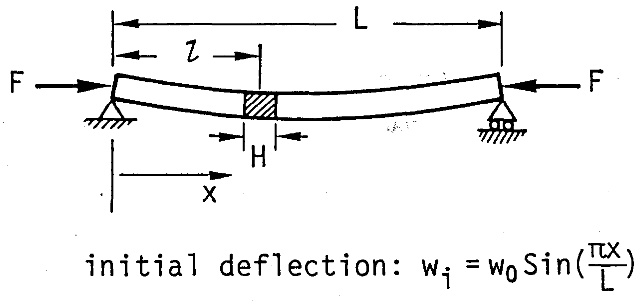


Fig. 3 Column with soft joint under compression.

(a) effect of slenderness ratio

The buckling phenomena can be categorized into elastic and plastic bucklings depending on the slenderness ratio. There are significant differences between elastic and plastic bucklings. Thus, it is important to clarify the effect of the slenderness ratio on the compressive strength of columns with soft joint. The slenderness ratio λ, which is defined by the following equation, is commonly used as a measure to indicate the thinness of beams

$$\lambda^2 = \sigma_Y/\sigma_E = (L/\pi)^2\ (A/I)\ (\sigma_Y/E) \qquad (1)$$

where σ_Y and σ_E are yield stress and elastic buckling stress, and

$$A = tb,\ I = t^3 b/12,\ \sigma_E = \pi^2\ EI/L^2 A$$

From the above definition, it can be seen that elastic buckling occurs if $\lambda > 1$, and plastic buckling occurs if $\lambda < 1$. In this report, however, $\gamma = \lambda^2$ is used as a measure of thinness for convenience. Three types of columns with γ = 2.0, 1.0, 0.5 are considered and these are referred to as elastic buckling model (E. B. M), transition B.M. (T.B.M.) and plastic B. M. (P.B.M.), respectively. Their dimensions are shown in **Table 1**.

Table 1 Dimensions of columns under compression.

	$\gamma = \lambda^2$	t (mm)	b (mm)	L (mm)
E.B.M.	2.0	20.0	20.0	415.6
T.B.M.	1.0	20.0	20.0	293.9
P.B.M.	0.5	20.0	20.0	207.8

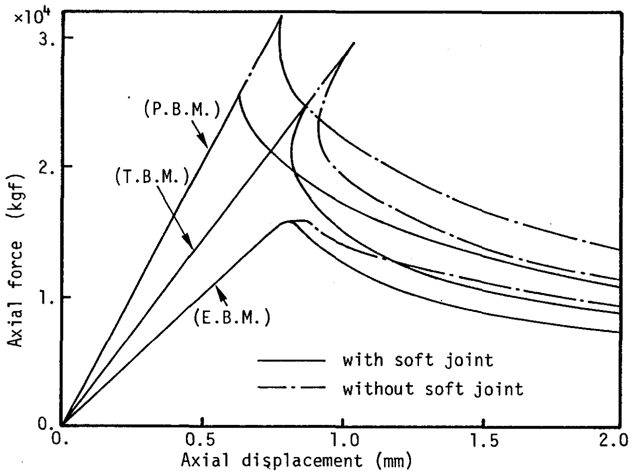


Fig. 4 Load-displacement curves of columns with soft joint at center (H/t = 1, $w_o = t/1000$).

Compressive load-displacement curves computed by F.E.M. are shown in **Fig. 4** for columns with initial deflection, the magnitude of which is $t/1000$. The solid and the chain lines represent columns with soft joint and homogeneous columns without soft joint, respectively. The width of the soft joint is assumed to be equal to the thickness of the column. In case of the elastic buckling model (E.B.M.), the column keeps the strength which is as high as the elastic buckling load immediately after the buckling. Then the load decreases gradually with the plastic deformation due to the bending. On the other hand, sharp decrease of load after reaching the maximum value is observed in the transition (T.B.M.) and the plastic buckling models (P.B.M.). Thus, it can be seen that the transition and the plastic buckling models are sensitive to disturbances. Particularly, the effect of soft joint is significant in these two models. Such differences in the effect of soft joint can be explained qualitatively by the following simple mechanical models. The elastic and the plastic models are considered and the compressive strength is obtained as a intersection of curves representing the two models[9], as shown in **Fig. 5**.

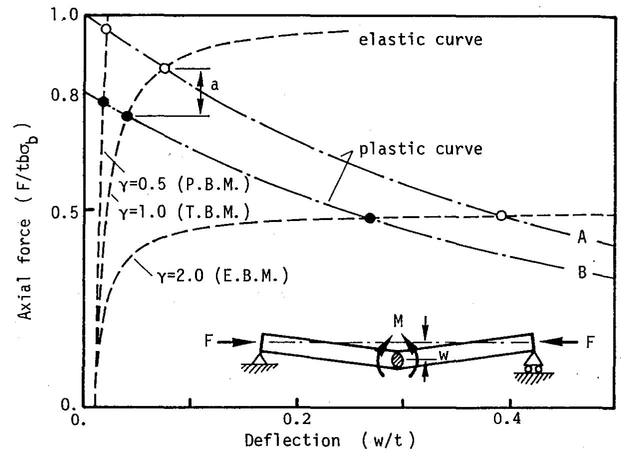


Fig. 5 Prediction of compressive strength of column with soft joint at center ($w_o = t/100$).

The elastic load-deflection curve is given by the following equation which involves γ as a parameter.

$$w\ (x) = w_o\ \mathrm{Sin}\ (\pi x/L)\ /\ (1 - F/F_E) \qquad (2)$$

$$= w_o\ \mathrm{Sin}\ (\pi x/L)\ /\ (1 - \gamma F/F_p)$$

where

$$F_p = tb\sigma_Y,\ F_E = tb\sigma_E$$

The borken lines in the figure show the load deflection curves of a columns with initial deflection, $w_o = t/100$, as an example.

While, the plastic deformation after the plastic hinge is formed is obtained by the rigid-plastic hinge model as shown in Fig. 5. In this model, the moment M, which is acting at the hinge, satisfies the equilibrium condition and the plastic condition, such that

$$M = w\ (l)\ F \qquad (3)$$

$$(F/F_p)^2 + (M/M_p) = 1 \qquad (4)$$

where

$$M_p = (1/4)t^2 b\sigma_Y$$

By eliminating the moment from the above equations, the load-deflection relation is obtained in the following form.

$$w\ (l) = (M_p/F)\ \left\{1 - (F/F_p)^2\right\} \qquad (5)$$

Two chain lines in Fig.5 are the plastic curves give by Eq. (5) assuming that the yield stresses are σ_b and σ_s, respectively. Thus, the intersecting points of the elastic and the plastic curves, which are shown by solid and open circles, represent the compressive strengths of column with soft joint and a uniform column. Then, the reduction of the strength due to the soft joint is given as the difference in heights of these two points. As seen from the figure, reduction of the strength is small for the elastic buckling model. Whereas, it is large for the transition and the plastic buckling models and the reduction ratio is close to the ratio between yield stresses, i.e. $(\sigma_b - \sigma_s)/\sigma_b$. However, the above conclusions are valid only for columns for which one dimensional stress state is assumed. Thus, the behavior is different when the stress state becomes multi dimensional such as in plates or pipes which will be discussed in the following chapters.

(b) effect of initial deflection

The same model can be applied to examine the effect of initial deflection. **Figure 6** shows load deflection curves of transition buckling model with different initial deflections, i.e. $t/1000$, $t/100$, and $t/10$. It can be seen that both compressive strength and its reduction due to the soft joint decrease with the initial deflection. Thus, the initial deflection dose not enhance the effect of the soft joint.

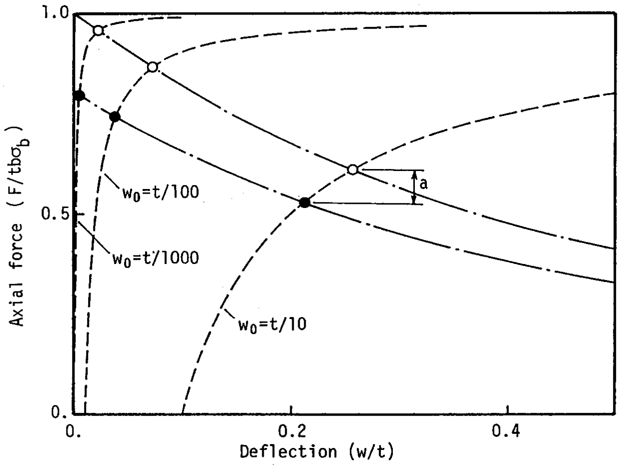


Fig. 6 Effect of initial deflection on strength of column (T. B. M.).

(c) effect of the soft joint location

The effect of the soft joint location on the strength is examined using the transition buckling model with initial deflection of $t/10$ as an example. Three broken lines in **Fig. 7** show relations between the load and the deflection at the location of the soft joint for columns which have a soft joint at $x = L/2$, $L/3$ and $L/8$. These lines are plotted using Eq. 2. The chain lines show the load-deflection curves under platic deformation for cases with plastic hinge formed at base and soft metals, respectively. The solid circles in the figure represent collapse points. The collapse modes are separated into two. Namely, collapse with a plastic hinge at the soft joint and that with a hinge at the center. If, for example, the soft joint locates near the center, the column collapse in the former mode and the reduction of the strength is larger as the soft joint approaches the center. On the other hand, if the soft joint

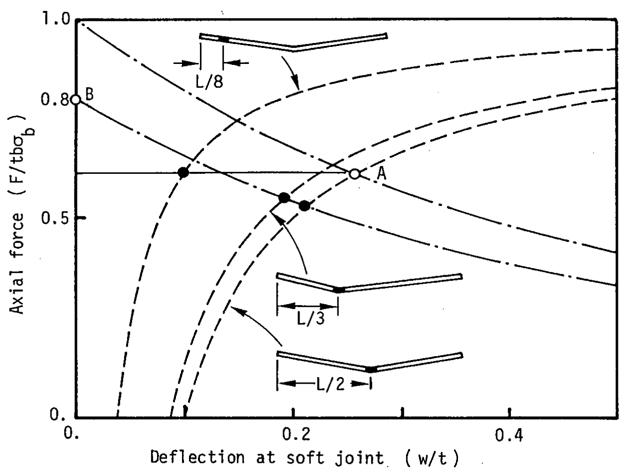


Fig. 7 Effect of soft joint location on strength of column (T. B. M., $w_0 = t/10$).

locates near the end, it collapses in the latter mode. It can be seen, in this case, that the existence of the soft joint shows no effect on the strength. However, this mode of collapse occurs only when the initial deflection is large and the strength of the homogeneous column (point A in Fig. 7) is smaller than the full plastic strength of the soft joint (point B).

(d) effect of soft joint width

Since the effect of the soft joint width can not be examined by the simple mechanical model, F.E.M. is employed for this purpose. **Figure 8** shows the load deflection curves of the transition buckling model with various soft joint width. Five solid lines in the figure represent a column which is made of soft metal, columns with soft joint, the widths of which are $2t$, t, $t/10$, and a homogeneous column, respectively from the bottom. The broken and the chain lines represent elastic and plastic curves given by the simple mechanical model. It can be clearly seen that the strength becomes higher as the width of the soft joint is smaller. Since the plastic constraint in the thickness direction is neglected in these analyses, upper limit of the strength coincides with the value which

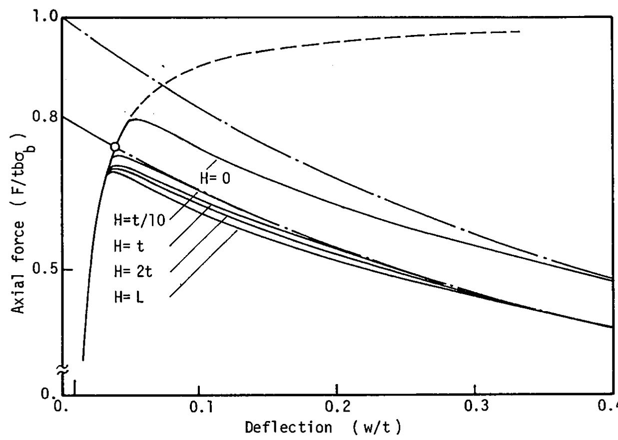


Fig. 8 Changes in behavior of column with soft joint width (T. B. M., $w_0 = t/100$).

is predicted by the simple mechanical model (open circle in Fig. 8). However, when the soft joint width is small, the strength is increased by the plastic constraint in the thickness direction and its magnitude can be estimated from Figs. 1 and 2.

Similar analyses are done for the elastic and the plastic buckling models and the results are put together in **Fig. 9**. It is observed that the effect of the soft joint width is large when the initial deflection is large and it is small for the small initial deflections. Also, it is seen that the strength reduces almost to that of the homogeneous column which is made of soft metal if the soft joint width is larger than the thickness. However, in case of elastic buckling model, the effect of the existence of the soft joint itself is small.

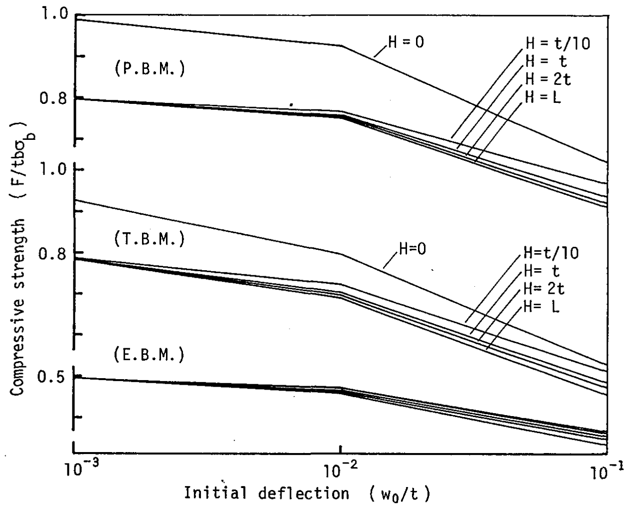


Fig. 9 Effect of soft joint width on compressive strength of column.

(e) effect of strain hardening

Considering that the plastic deformation is concentrated at the soft joint when the column buckles, it is necessary to examine the effect of the strain hardening of the material. The effect is analyzed using F.E.M. The plastic buckling model with a soft joint at its center is considered. Both the base and the soft metals are assumed to be linearly hardening materials and their hardening coefficient h is assumed to be $h = E/10$. The initial deflection is assumed to be $t/100$. The computed compressive strength of columns with various soft joint width are plotted in **Fig. 10**. Solid and open circles in the figure represent strength of columns with and without strain hardening. As seen from the figure, the strength is increased by the strain hardening when the soft joint width is small. Whereas, significant effect is not observed when the soft joint width is large. Qualitative explanation of these phenomena can be given by the simple model which is shown in Fig. 10.

The column with soft joint can be modeled as a composite column made fo three parts with different young's moduli. Then, the buckling load can be obtained as a root of the following nonlinear equation[10].

$$\tan(k_1 l_1)\tan(k_2 l_2) = (E_2/E_1)^{1/2} \quad (6)$$

where,

$$k_1 = (F/IE_1)^{1/2}\ ,\ k_2 = (F/IE_2)^{1/2} \quad (7)$$

$$l_1 = H/2\ ,\ l_2 = (L-H)/2 \quad (8)$$

When it is assumed that the soft metal reaches the yielding and the base metal stays in elastic state, such that

$$A\sigma_s < F < A\sigma_b \quad (9)$$

The stiffnesses E_1 and E_2 become

$$E_1 = Eh/(E+h),\quad E_2 = E \quad (10)$$

Based on the above assumptions, Eq. (6) is solved to determine the buckling strength of the column which has a soft joint with various width. The results are plotted as broken lines for the cases where $h = E/10$ and $h = E/50$. As it is seen from the figure, the strength increases with the decrease of the soft joint width. However, such dependence on soft joint width appears only in the region between A and B. If the width is smaller than A, the buckling load becomes same as the full plastic load of the homogeneous column without soft joint. Whereas, if the width is greater than B, no increase in strength due to the strain hardening can be expected. Though qualitative agreement can be seen between the results obtained by F.E.M. and the composite column model, quantitative agreement is not satisfactory because the effect of the initial deflection and the partial yielding are not considered in the latter.

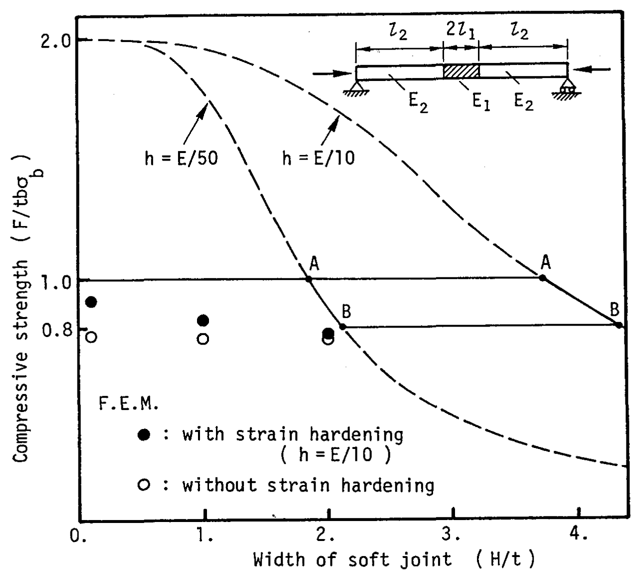


Fig. 10 Effect of strain hardening on compressive strength (P.B.M., $w_0 = t/100$).

4. Compressive strength of square plates

Ships are the typical plate structures and the ratio of the high tensile strength steel used is 70% in case of the most advanced ship. Ships are welded structures composed of plate members reinforced by stiffeners of various size. The fundamental patterns of welding lines in plate members are shown in **Fig. 11**. The compressive strength of square plate is analyzed for these patterns and the effects of joint location, slenderness ratio and boundary condition on strength are investigated. In the analysis, the soft joint width is assumed to be $2t$ for the joint locating in the center of the plate and t for that at the edge. Assuming that the width of the soft joint is large enough, the plastic constraint in the thickness direction is ignored in the F.E.M. analysis. The initial deflection is assumed to be $t/1000$, which is considered to be extremely small. To clarify the effect of the slenderness ratio, three models, namely elastic, transition and plastic buckling models are analyzed and their dimensions are shown in **Table2.**

The results computed by F.E.M. are shown in **Fig. 12** for a plate with a soft joint which is normal to the applied load as examples. Three solid lines represent load-displacement curves for plastic, transition and elastic buckling models. Also, those of homogeneous plates without soft joint are shown by chain lines for comparison. The behavior greatly changes with the slenderness ratio and very sharp drop in load is observed after the buckling in the transition buckling model.

Same analyses are done for other two patterns shown in Fig. 11 and different boundary conditions. The computed compressive strengths are normalized by that of homogeneous plates and summarized in **Table 3.** The figures in the parentheses are those evaluated by a simple model which is discussed later. The normalized strength of the homogeneous plate made of soft metal is also shown for comparison. It is seen that the strengths of plates with soft joints are much higher than these values.

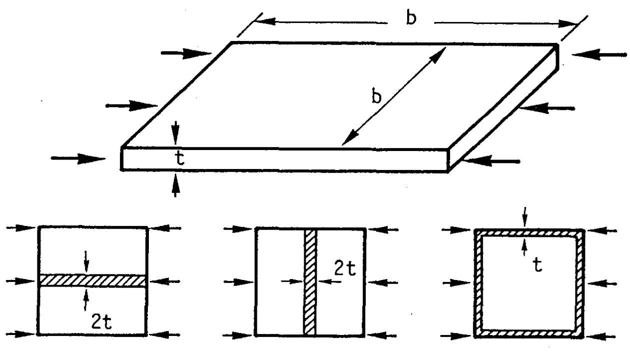


Fig. 11 Fundamental patterns of soft joint location.

Table 2 Dimensions of square plates under compression.

	γ	b (mm)	t (mm) simply s.	t (mm) clamped
E.B.M.	2.0	2000.	45.91	28.93
T.B.M.	1.0	2000.	64.92	40.92
P.B.M.	0.5	2000.	91.81	57.87

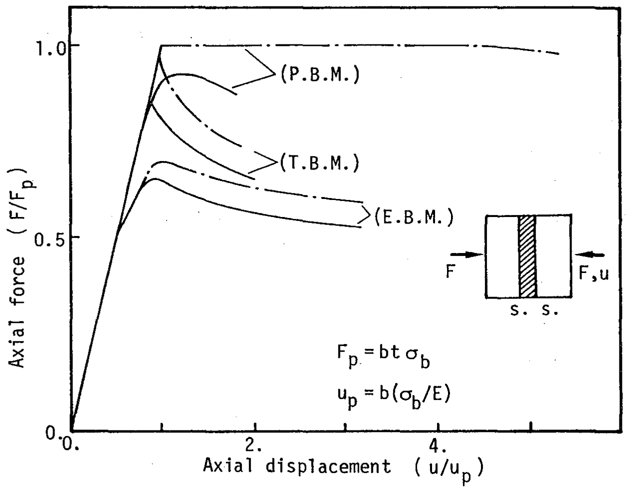


Fig. 12 Load-displacement curves of plates under compression ($w_o = t/1000$).

Table 3 Relative strength of plate with soft joint compared to homogeneous plate computed by F.E.M. and idealized model ($w_o = t/1000$).

	s. s.	s. s.	s. s.	clamped	s. s.
E.B.M.	0.997 (1.000)	0.939 (0.949)	0.980 (0.941)	0.976	0.861
T.B.M.	0.973 (0.987)	0.862 (0.924)	0.913 (0.916)	0.915	0.815
P.B.M.	0.982 (0.982)	0.925 (0.924)	0.923 (0.912)	0.924	0.800

The effect of the soft joint is quite different if the joint location is different. The reduction of the strength due to the soft joint parallel to the load is essentially proportional to the ratio of the crosssectional area and it is as small as 0.3 ~ 3.0%. Whereas it is large when the soft joint is normal to the load. Especially in case of the transition model, the strength is reduced by 14%.

Through the comparison among the elastic, plastic and transition buckling models, it is seen that great reduction in strength occurs in transition buckling model which is most sensitive to disturbances in general. Further, the effect of the supporting condition is examined. As seen from Table 3, the reduction ratios of the strength for the clamped plate are very close to those of the simply supported plate. Thus, the supporting condition has little effect on the reduction ratio of the strength.

So far, the effect of soft joint on the compressive strength has been analyzed for some typical cases. Based on these knowledge, the effect of the soft joint is analyzed in more general form considering the collapse mecha-

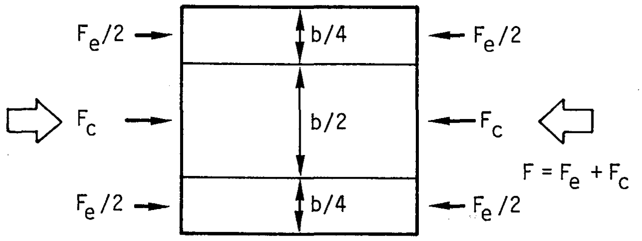


Fig. 13 Idealized model of plate under compression.

nism and the plastic constraint.

To this end, the plate under compressive load is divided into the edge and the center parts as shown in **Fig. 13** and the loads carried by these at the maximum strength are F_e and F_c, respectively. The center part is assumed to carry only the buckling load after elastic buckling and load beyond the buckling is carried by the edge parts. According to this assumption, the effective breadth determined based on the stiffness after buckling[10),11)] can be taken as the width of the edge part and it is $b/4$ for a square plate. Using this model, the strength of homogeneous plate can be estimated in the following manner.

Since the deflection of the edge part is constrained at its edge and the loading condition is close to pure compression, the edge part can carry the load which is same as the full plastic load without buckling. Thus,

$$F_e = \frac{1}{2} bt\sigma_b \qquad (11)$$

Whereas, the strengths of the center parts are different for elastic and plastic bucklings. It is determined by the elastic buckling stress σ_E in the former case and is determined by the yield stress σ_b in the latter case, such that

$$\text{elastic buckling} \quad F_c = bt\sigma_E/2$$

$$\text{plastic buckling} \quad F_c = \mathrm{bt}\sigma_b/2 \qquad (12)$$

Then, the total strength can be given as the sum of F_e and F_c.

The same idea is applied to a plate with soft joint and its strength is given in the following form.

$$F = F_e^* + F_c^* \qquad (13)$$

where F_e^* and F_c^* are forces carried by the edge and the center parts of a plate with soft joint. Further, Eq. (13) can be rewritten in terms of F_e, F_c and factors α, β, such that

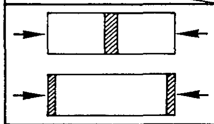
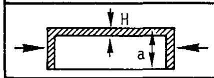
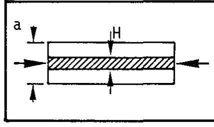
$$F = \alpha F_e + \beta F_c \qquad (14)$$

where

$$\alpha = F_e^* / F_e\,, \quad \beta = F_c^* / F_c$$

The factors α and β vary with the soft joint location and the slenderness ratio. The detail of determining these factors is discussed in the following.

Table 4 Parameters α and β for fundamental patterns.

	α	β	
	$(2/\sqrt{3})(\sigma_s/\sigma_b)$	(E.B.M.)	1
		(T.B.M.) (P.B.M.)	$(2/\sqrt{3})(\sigma_s/\sigma_b)$
	$\{\frac{2(a-H)}{\sqrt{3}\,a} + \frac{H}{a}\}\frac{\sigma_s}{\sigma_b}$		
		(E.B.M.)	1
		(T.B.M.) (P.B.M.)	$\frac{a-H}{a} + \frac{H\sigma_s}{a\sigma_b}$

When the width of the soft joint is greater than the plate thickness, the increase of strength due to the plastic constraint in the thickness direction is small and only the constraint in the width direction should be considered. To determine the factors α and β, the patterns of the soft joints in the edge and the center parts are classified as shown in **Table 4**. When the soft joint is normal to the load, the constraint in the width direction is effective and the strength becomes $2/\sqrt{3}$ times as high as that of soft metal itself. Therefore, the factor α for the edge part, in which compressive deformation is dominant and its strength is determined by the full plastic load, can be estimated as,

$$\alpha = (2/\sqrt{3})\,(\sigma_s/\sigma_b) \qquad (15)$$

Similarly, the strength of the center part in the plastic buckling model is determined by yielding. Thus, for this case, it is given as

$$\beta = (2/\sqrt{3})\,(\sigma_s/\sigma_b) \qquad (16)$$

However, in case of the elastic buckling model, the strength of the center part is determined by elastic buckling and it is assumed that

$$\beta = 1 \qquad (17)$$

On the other hand, when the soft joint is parallel to the load, plastic constraint is not effective. Then, the strength is determined only by the ratio of crosssectional area of the soft joint. Further, if a plate has both normal and parallel soft joints, their effect must be considered together. The factors α and β determined following the above procedure are shown in Table 4.

The compressive strength of a square plate can be estimated using Table 4 and Eq. (14). Estimated values are normalized by the strength of the homogeneous plate and shown in the parentheses in Table 3. Farely good correlation between these values and those by F.E.M. is observed. Especially, the agreement in plastic buckling model is excellent. However, the strength is overestimated for the transition buckling model with a soft joint normal to the load. The reason to this is that the strength of the center part is estimated as the full plastic load. But, in fact, buckling occurred before reaching the full plastic load.

5. Compressive Strength of Pipes

Since high tensile strength steel is widely used as tubular members in offshore structures, the strength of pipes with soft joints is also analyzed. Three simply supported pipes with different radius D to thickness t ratios, namely D/t = 25, 40, 100, are considered. Their dimensions are chosen so that they become transition buckling models when they are treated as simple columns. The dimensions are shown in **Table 5**. The sinusoidal form of the initial deflection is assumed and its magnitude is taken to be $t/10$. Concerning the soft joint location, soft joint at the center normal to the load, which shows the greatest effect in the case of plates, is assumed and its width is assumed to be $2t$. The base and the soft materials are assumed to be linearly hardening materials with same strain hardening coefficient h. The compressive strength is analyzed by F.E.M. using shell elements for two different strain hardening coefficients, $h = E/100$ and $h = E/10$. The plastic constraint in the thickness direction is ignored in these analyses.

The computed strengths are normalized by the full plastic load of the homogeneous pipe and they are shown in **Table 6**. As seen from the table, the reduction of the strength due to the soft joint is small. This is contrast to the case of columns, in which the strength is directly governed by the yield stress of the soft metal and its reduction rate is large. There are two reasons for this difference. The first reason is the plastic constraint in the width or the circumferential direction. This effect is taken into account in pipes. But, it is neglected in columns. Thus, the strength of the pipe is increased up to $2/\sqrt{3}$ times of the full plastic strength of the soft joint itself even for materials with no strain hardening. The second reason is the strain hardening. As shown for columns, the increase of strength depends on H/t. Similarly, it depends on H/D in case of pipes. In general, H/D of pipes is much smaller than H/t of columns. Thus, the strength of a pipe is significantly increased by the strain hardening.

Further, to compare the deformation behavior after collapse, load-displacement curves for the cases with $h = E/100$ are shown in **Fig. 14**. The difference between the pipe with soft joint and the homogeneous pipe is very small. Thus, it can be seen that the effect of the soft joint on the post buckling behavior is not so significant. If the post buckling behaviors are compared among three pipes with different D/t, sharp drops of the load are observed twice for the case in which D/t = 100. The first drop corresponds to the column buckling and the second one is due to the local buckling of the pipe as seen from the deformed crosssection shwon in Fig. 14.

Table 5 Dimensions of pipes under compression.

D/t	25.0	40.0	100.0
D	1000.0	1000.0	1000.0
t	40.0	25.0	10.0
L	17291.	17552.	17817.

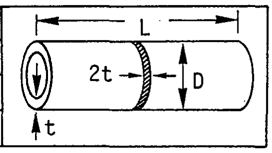


Table 6 Strength of pipes with and without soft joint normalized by full plastic load.

	D/t=25	D/t=40	D/t=100
a) without soft joint h=E/100	0.874	0.884	0.842
b) with soft joint h=E/100	0.818	0.851	0.835
b) / a)	0.935	0.962	0.991
c) without soft joint h=E/10	0.875	0.885	0.844
d) with soft joint h=E/10	0.853	0.878	0.843
d) / c)	0.975	0.992	0.999

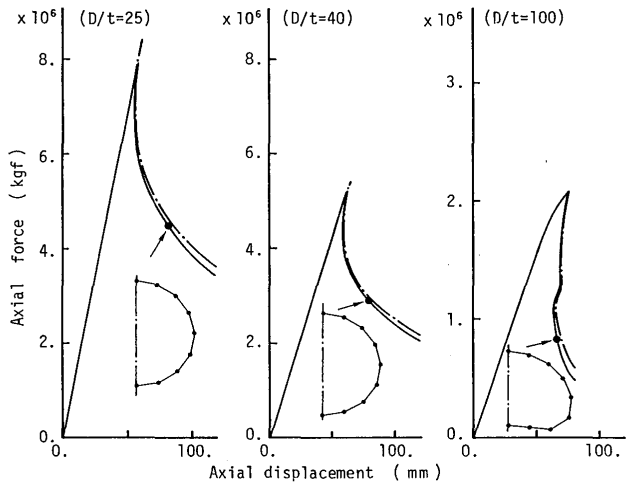


Fig. 14 Load-displacement curves of pipes under compression (T.B.M., $w_0 = t/10$).

6. Conclusion

The compressive strength of structural members with soft weld joints is investigated using simple mechanical models and F.E.M. To this end, the strength of the soft weld joint itself is analyzed and the relation between the strength and the plastic constraint is clarified. Then, the compressive strength of columns, plates and pipes is studied. Especially, the effects of the slenderness ratio, initial deflection, location and width of the soft joint and strain hardening are clarified for columns. Since the plastic constraint in the width direction is small in the case of columns, their strengths are directly governed by the yield stress of the soft material and the reduction of strength is large. However, most of the compressive members of real structures are plates or pipes and solid column is rare. In case of plates and pipes, the plastic constraint in the width or the circumferential direction is effective to raise the strength significantly. In addition to this, if the strain hardening is taken into account, further increase of strength is expected. Especially, in the case of pipes with soft joint, its strength is found to be almost same as that of homogeneous pipe when D/t is large.

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References

1) K. Satoh, Y. Mukai and M. Toyoda, "Welding Engineering", Rikogakusha, pp. 212–215, 1979 (in Japanese).
2) K. Satoh, T. Doi and M. Toyoda, "Size Effects on Static Tensile Properties of Welded Joints Including Soft Interlayer", J. Japan Welding Society, Vol. 37 No. 11 (1968), pp. 1214–1226 (in Japanese).
3) K. Satoh and M. Toyoda, "Effect of Mechanical Heterogeniety on the Static Tensile Strength of Welded Joints", J. Japan Welding Society, Vol. 40 No. 9 (1971), pp. 885–900 (in Japanese).
4) K. Satoh, M. Toyoda and E. Fujii, "Tensile Behaviors and Strength of Soft Welded Joints", J. Society of Naval Architects of Japan, Vol. 132 (1972), pp. 381–393 (in Japanese).
5) Y. Ohsawa, T. Nakano, Y. Ono and I. Suzuki, "Evaluation of Elasto-plastic Behavior of Soft Welded Joints for High Strength Steel Pipes", Nippon Kokan Technical Report, No. 103 (1984), pp. 82–91 (in Japanese).
6) H. Nitoh, D. Sakai, H. Yajima, Y. Inoue, Y. Sogo, K. Satoh and M. Toyoda, "Tensile Strength of Welded Joints for TMCP Type 50 kgf/mm²-Class High Tensile Steel Plates", J. Society of Naval Architects of Japan, Vol. 157 (1985), pp. 304–311 (in Japanese).
7) T. Matsumoto, H. Satoh, S. Kirihara and F. Hataya, "Investigations on Static Strength of Undermatching Welded Joint of HT60 Steel Plates", Quarterly J. Japan Welding Society, Vol. 4 No. 2 (1986), pp. 429–435 (in Japanese).
8) K. Satoh, M. Toyoda and F. Minami, "Fracture Initiation Toughness of Materials with Locally Embrittled Region – with Reference to Electron Beam Welds of HT80 Steel", J. Society of Naval Architects of Japan, Vol. 153 (1983), pp. 223–229 (in Japanese).
9) H. Kawano, Y. Inoue, H. Yajima, H. Nitoh, D. Sakai, Y. Sogo, K. Satoh and M. Toyoda, "Bending and Compressive Strength of TMCP Type HT50 Steel Welded Joints", Transactions of the West Japan Society of Naval Architects, Vol. 70 (1985), pp. 187–198 (in Japanese).
10) S. P. Timoshenko and J. M. Gere, "Theory of Elastic Stability", McGraw-Hill, 1961.
11) S. Levy, "Bending of rectangular plates with large deflections", NACA Report No. 737 (1942), pp. 1–19.

Appendix

Upper and Lower Limits of Strength of Soft Joint

As it is discussed in the text, the plastic constraint at the soft joint is separated into the width and the thickness directions of the plate. The former depends on the relative width of the soft joint compared to the plate width H/b. Typical states of constraints are the plane stress, plane deformation and plane strain states. While, the latter depends on the relative width of the soft joint compared to the plate thickness H/t. The constraint becomes stronger as H/t becomes smaller and the strength increases with the constraint. As shown in Figs. 1 and 2, however, the upper and lower limits of the strength exist.

Roughly speaking, the increase ratio of the strength depends on dimension of the stress state which is determined by the degree of the plastic constraint. For example, if the stress state is one dimensional in which only σ_1 is acting, the stress σ_1 at the yielding is same as the yield stress σ_Y. If the stress state is two dimensional in which σ_1 and σ_2 are acting, the maximum possible value of σ_1 during plastic deformation is $2/\sqrt{3}\,\sigma_Y$, as shown in **Fig. A** describing the Mises yield surface in the principal stress space. Further, it can be seen that there is no bound for σ_1 under plastic deformation if the stress state is three dimensional. Keeping the above relation between the yield condition and the dimension of the stress state in mind, the limit values of the strength of the soft joint are discussed in the following.

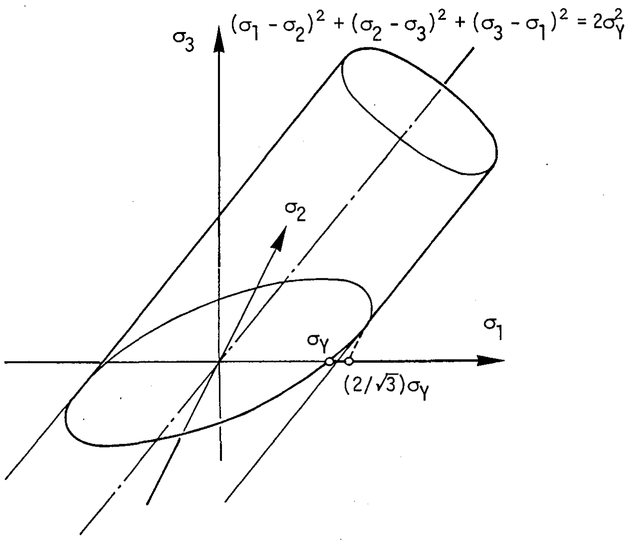

Fig. A Mises yield surface in principal stress space.

A. Lower limit under compression

When H/t is large, the strength of the soft joint is limited by that of soft metal and its lower limit is determined in the following manner.

i) plane stress state

When H/t is large enough, the stress state near the center of the soft metal can be considered as one dimensional stress state. Thus, the lower limit of the joint strength F_{min} is given as

$$F_{min} = tb\sigma_s \qquad (a.1)$$

ii) plane strain state

When H/t is large enough, stress in y-direction near the center of the soft metal is negligible, such that

$$\sigma_y = 0 \qquad (a.2)$$

Under this condition, Mises yield condition can be written as

$$\sigma_x^2 + \sigma_z^2 - \sigma_x\sigma_z = \sigma_Y^2 \qquad (a.3)$$

If the material is assumed to be perfect elastic-plastic material, the stress increments $\Delta\sigma_x$, $\Delta\sigma_y$ and $\Delta\sigma_z$ satisfy the following condition.

$$\Delta(\sigma_Y^2) = 2\sigma_x\Delta\sigma_x + 2\sigma_z\Delta\sigma_z - \sigma_x\Delta\sigma_z - \sigma_z\Delta\sigma_x = 0 \qquad (a.4)$$

Further, the stress increment in x direction is zero at the maximum strength, such that

$$\Delta\sigma_x = 0 \qquad (a.5)$$

Substituting Eq. (a.5), following two conditions are derived from Eq. (a.4).

$$\Delta\sigma_z = 0 \qquad (a.6)$$

or

$$2\sigma_z - \sigma_x = 0 \qquad (a.7)$$

If Eq. (a.7) is assumed to hold, Eq. (a.3) can be solved for σ_x and it is shown to be

$$\sigma_x = (2/\sqrt{3})\sigma_s \qquad (a.8)$$

Thus, the lower limit F_{min} is given as

$$F_{min} = (2/\sqrt{3})tb\sigma_s \qquad (a.9)$$

On the other hand, if Eq. (a.6) is assumed to hold, it can be shown from Eqs. (a.2) and (a.5) that all three components of the stress increments and the elastic strain increments are zero, such that

$$\Delta\sigma_x = \Delta\sigma_y = \Delta\sigma_z = 0 \qquad (a.10)$$

$$\Delta\epsilon_x^e = \Delta\epsilon_y^e = \Delta\epsilon_z^e = 0 \qquad (a.11)$$

Considering that the total strain increment in z direction, which is the sum of elastic strain increment $\Delta\epsilon_z^e$ and plastic strain increment $\Delta\epsilon_z^p$, is zero under plane strain condition, it is shown that

$$\Delta\epsilon_z = \Delta\epsilon_z^e + \Delta\epsilon_z^p = \Delta\epsilon_z^p = 0 \qquad (a.12)$$

While the plastic strain increment $\Delta\epsilon_z^p$ is given in the following form from the plastic flow theory.

$$\Delta\epsilon_z^p = \lambda(2\sigma_z - \sigma_x) \qquad (a.13)$$

Finally, it is obtained from Eqs. (a.12) and (a.13) that

$$2\sigma_z - \sigma_x = 0 \qquad (a.14)$$

Since the above equation is identical with Eq. (a.7), the same strength is obtained also from Eq.(a.6).

The above conclusion can be drawn from the dimension of the stress state. The stress state at the center of the soft metal is two dimensional in x and z directions. Thus, it can be shown from Fig. A that the strength of the soft metal is given by Eq. (a.9).

iii) plane deformation state

It is rather difficult to determine the lower limit in a rigorous way. According to the numerical results shown in Fig. 1, the strength for the plane deformation case approaches that of plane strain state as H/t becomes large. Hence, the lower limit for this case is given by

$$F_{min} = (2/\sqrt{3})tb\sigma_s \qquad (a.15)$$

B. Upper limit under compression

The strength of the soft joint increases with the decrease of H/t. The upper limit of the strength is determined by either the strength of the base metal or that of the soft metal depending on the degree of the plastic constraint.

i) plane stress state

In case of the plane stress state, the strength of the base metal F_b is given by

$$F_b = tb\sigma_b \qquad (a.16)$$

On the other hand, the stress state of the soft metal is two dimensional stress state in x and y directions when $H/t \to 0$. Then, the strength of the soft metal F_s becomes

$$F_s = (2/\sqrt{3})tb\sigma_s \qquad (a.17)$$

Therefore, the upper limit of the joint strength F_{max} is given as the smaller on of F_b and F_s, such that

$$F_{max} = F_s = (2/\sqrt{3})tb\sigma_s \qquad (a.18)$$

ii) plane strain state

Since the stress state under the plane strain condition becomes two dimensional in x and z directions, the strength of the base metal is given by

$$F_b = (2/\sqrt{3})tb\sigma_b \qquad (a.19)$$

While, the stress state of the soft metal is three dimensional as $H/t \to 0$ and the strength of the soft metal becomes unlimitedly large. Thus, the upper limit for the plane strain case is given as

$$F_{max} = F_b = (2/\sqrt{3})tb\sigma_b \quad \text{(a.20)}$$

iii) plane deformation

Since the stress component in z direction acting in the base metal is negligible when $H/t \to 0$, the stress state becomes one dimensional. Hence, the strength of the base metal is given as

$$F_b = tb\sigma_b \quad \text{(a.21)}$$

On the other hand, stress in z direction acting in the soft metal is significantly large when $H/t \to 0$. Thus, the stress state in the soft metal becomes three dimensional and the joint strength is determined by that of the base metal, such that

$$F_{max} = F_b = tb\sigma_b \quad \text{(a.22)}$$

C. Limits of strength under bending

The upper and the lower limits of the strength for the bending are same as in compression except for the upper limit under plane deformation condition. The reson for this difference can be explained from the constraint involved in the plane deformation itself. Under the plane deformation condition, it is assumed that the plane normal to z-axis remains plane after deformation and free to move without rotation around x-axis. When the bending moment is applied, the compression side of the column tends to expand in z direction and the tension side tends to contract because of the effect of the Poison's ratio. These deformations in z direction are constrained by the fixed rotation around x axis and the stresses in z direction are created. However, the deformation is skew symmetric about the neutral axis. Thus, the plane normal to z axis does not move in z direction. In other words, strain in z direction is zero. Thus, the plane deformation state becomes identical with the plane strain state under bending.